Elicitation non-paramétrique de la fonction d'utilité et de l'aversion aux pertes sous l'hypothèse "prospect theory"

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ELICITATION NON-PARAMETRIQUE DE LA FONCTION D'UTILITE ET DE L'AVERSION AUX PERTES SOUS L'HYPOTHESE ‹‹PROSPECT THEORY››

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ELICITATION NON-PARAMETRIQUE DE LA FONCTION D'UTILITE ET DE L'AVERSION AUX PERTES SOUS L'HYPOTHESE ‹‹ PROSPECT THEORY››

Résumé de la thèse : Dans ce travail, les fonctions d'utilité des gérants de portefeuilles sont élicitées et leurs degrés d'aversion aux pertes mesurés sous l'hypothèse de la théorie des prospects (1992) et suivant la méthode non-paramétrique d'Abdellaoui et al. (2006). Les résultats obtenus sur le terrain corroborent les résultats obtenus par ces derniers au laboratoire quant à la concavité de la fonction d'utilité pour les gains et la convexité pour les pertes. En ce qui concerne l'aversion aux pertes, nos observations confirment son existence; néanmoins, le gérant de portefeuilles médian est moins averse aux pertes que l'étudiant médian. Les conditions qui caractérisent une expérience réelle du marché mais qui sont difficiles à reproduire dans le contexte artificiel du laboratoire pourraient expliquer les différences de comportement: notamment, la volatilité du marché boursier, les compensations incitatives de Wall Street et le fait que les gérants de portefeuilles acquièrent sur le terrain une gamme de formation et un haut niveau de connaissance qui font qu'ils évaluent les enjeux différenmment des étudiants. La fonction d'utilité doit néanmoins, refléter les préférences de l'individu et l'utilité ne doit pas changer selon la méthode utilisée. En effet, l'étude qualitative des préférences d'étudiants en MBA suivant la méthode non-paramétrique de Baucells et Heukamp (2006) confirme les résultats d'Abdellaoui et al. (2006) pour étudiants. Il est à noter cependant que les étudiants changent de préférence (ne sont plus averses aux pertes mais recherchent le gain) quand l'une des deux loteries offre une plus grande probabilité globale de gain ou une plus grande probabilité de gain maximal combinée avec une perte extrême limitée.

Mots clés : *théorie des prospects, aversion aux pertes, utilité pour les gains et pour les pertes, élicitation de mi-points, dominance stochastique de deuxième ordre, fonction de transformation des probabilités*.

PARAMETER-FREE MEASUREMENT OF THE UTILITY FUNCTION AND LOSS AVERSION UNDER PROSPECT THEORY

Abstract : This work elicits the utility functions of financial practitioners and measures their loss aversion coefficients under prospect theory (1992) using the parameter-free method of Abdellaoui et al. (2006). The measurements in the field corroborate the latter's measurements in the laboratory regarding the concavity of the utility function for gains and convexity for losses. However, although loss aversion exists in the aggregate, the median practitioner is found to be less loss averse than the median student. Conditions that characterize a real market experience but are difficult to realize in the artificial context of the laboratory may account for the behavioral difference. Among them are the schooling in the assessment of prospects, the volatility of the market and the Wall Street's compensation incentives. An important proviso is that the preferences of the students/practitioners analyzed following another method reflect consistent preferences. The qualitative investigation of the preferences of MBA students using the parameter-free method developed by Baucells and Heukamp (2006) supports the results of Abdellaoui et al.'s (2006) for students. A noteworthy result is the strong tendency to shift from loss aversion to gain seeking for the higher overall probability of gain or the higher probability of maximal gain combined with a limited extreme loss.

Keywords : *prospect theory, loss aversion, utility for gains and losses, elicitation of midpoints, second order stochastic dominance, probability weighting function*.

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TABLE DES MATIERES

INTRODUCTION GENERALE

Cette recherche expérimentale a pour objectif principal l'élicitation des fonctions d'utilité de gérants de portefeuille[1] et la mesure de leurs degrés d'aversion aux pertes. Les procédures d'élicitation présupposent que l'individu choisisse entre des alternatives données comme s'il optimisait une fonction fondamentale de préférence. Généralement, cette fonction de préférence résulte du modèle de l'utilité espérée (*EU*). Ce modèle remonte à Cramer (1728) et Bernoulli (1738) qui ont proposé indépendamment que les individus ne maximisent pas des valeurs monétaires espérées mais des utilités espérées, c.-à-d. leur évaluation subjective des valeurs monétaires. Axiomatizé par von Neumann et Morgenstern (1944) pour la prise de décision dans le risque et par Savage (1954) et Anscombe et Aumann (1963) pour la prise de décision dans l'incertain, *EU* est le modèle normatif de choix qui décrit la manière selon laquelle les individus doivent prendre leurs décisions. Cependant pour qu'un modèle soit opérationnel et prescriptivement utile en analyse de décision, le comportement actuel des individus dans des situations de choix simples doit être compatible avec le comportement supposé dans le modèle. Néanmoins, *EU* s'est avérée indéfendable comme modèle descriptif du comportement des décideurs. En effet, les travaux expérimentaux qui ont suivi ces travaux théoriques ont montré que les gens violent systématiquement certains axiomes de *EU* (e.g. Allais, 1953; Ellsberg, 1961; Kahneman et Tversky, 1979).

L'évidence empirique a motivé les chercheurs à développer des théories alternatives de choix qui tiennent compte des violations observées. Ces modèles appelés "utilité non-espérée" ou "généralisations de l'utilité espérée" ont été examinés plus tard dans le *triangle unité* pour trouver si possible le modèle qui se conforme le plus au comportement réel (Harless et Camerer, 1994; Hey et Orme, 1994; Carbone et Hey,

[1] Gérant de portefeuille est pris ici dans le sens général de responsable de la diversifaction d'un portefeuille. Il renvoie aux conseillers financiers et aux gérants de fonds.

1995). Le triangle unité est l'ensemble des distributions des probabilités ou loteries qu'on peut définir sur un ensemble de trois résultats différents. Ce simplexe est généralement représenté par le triangle unité dans un système de deux axes rectangulaires. Si les résultats sont fixes, le triangle contient l'ensemble de toutes les loteries possibles avec le bord gauche caractérisé par une probabilité zéro pour la meilleure conséquence, le bord inférieur par une probabilité zéro pour la moins préférée, et l'hypoténuse par une probabilité zéro pour la conséquence moyenne. Néanmoins, aucune des généralisations de l'utilité espérée ne s'est avérée sensiblement plus conforme que *EU* aux données empiriques dans le triangle entier, c.-à-d. dans toutes les situations risquées possibles. Selon Abdellaoui et Munier (1998), la raison en est que les préférences des décideurs dépendent du risque des prospects auxquels ils font face, c.-à-d. de la région du triangle unité qui représente leur situation. Un des résultats de leur expérience est que les modèles d'utilité non-espérée décrivent les préférences des individus mieux que *EU* en dehors du milieu du triangle. L'enquête de Starmer (2000) qui évalue les données des expériences dans le triangle unité présente le modèle d'espérance d'utilité à dépendance du rang qui ne viole pas la monotonicité comme le "meilleur pari" parmi les alternatives en dehors de l'intérieur de la triangle. Dans ce modèle les probabilités sont remplacées par des pondérations qui reflètent l'impact des événements sur la désirabilité des conséquences. Ces poids de décision qui résultent de la pondération de probabilités cumulées et dont la somme est égale à un sont assignés aux conséquences selon leurs rangs respectifs dans les séries.

Parmi les modèles à dépendance du rang, la théorie des prospects (*CPT*)[2] de Tversky et Kahneman (1992) c.-à-d. la version révisée de leur théorie originale (1979), explique également les régularités du comportement sur le terrain considérées des paradoxes sous *EU*. Camerer (2000) montre que dans plusieurs domaines *CPT* explique les anomalies et les phénomènes de base que *EU* est utilisée pour expliquer grâce aux éléments psychologiques qui y sont intégrés.

[2] *CPT* pour cumulative prospect theory.

LES RÉSULTATS EXPÉRIMENTAUX

Esquissées en bref, les caractéristiques principales de la théorie des prospects sont: 1) la fonction d'utilité définie sur des gains et des pertes relatifs à un point de référence neutre exhibe une sensibilité marginale décroissante aux déviations du point de référence donnant lieu à une fonction d'utilité en forme de *S* c'est-à-dire concave pour les gains ($u'' < 0$) et convexe pour les pertes ($u''> 0$); 2) la fonction d'utilité est plus pentue dans le domaine des pertes que dans le domaine des gains quand la valeur d'une perte est plus faible en valeur absolue que la valeur d'un gain commensuré, une caractéristique marquée par les auteurs comme aversion aux pertes; et 3) les décisions sont basées sur des distributions cumulatives subjectives données par l'équation: $\pi = w(p)$ où *p* renvoie à la distribution cumulative objective, *w* à un traitement subjectif de *p* tels que $w'(.) > 0$, $w(0) = 0$ et $w(1) = 1$. Les éléments psychologiques intégrés dans la théorie sont par conséquent: l'effet de réflexion, l'aversion aux pertes et le traitement subjectif des probabilités. (La théorie des prospects est élaborée au chapitre III, p: 124 du document en anglais).

L'effet de réflexion est un élément psychologique central de la théorie des prospects. D'après Kahneman et Tversky (1979, p: 268), lorsqu'on passe d'un contexte de gains à un contexte de pertes, il y a renversement de préférences. En d'autres termes, si les individus préfèrent des gains sûrs à des gains probables, ils préféreraient des pertes probables aux pertes sûres. L'effet de réflexion explique l'hésitation quant à la réalisation des pertes au marché boursier: Terence Odean (1988) constate que comme Shefrin et Statman (1985) ont prévu, les investisseurs réalisent leurs gains trop tôt et sont peu disposés à réaliser leurs pertes excepté pour le mois de décembre (et ce pour des raisons d'impôts) et que leur comportement les a menés à de moindres bénéfices. Graphiquement, les investisseurs auraient une fonction d'utilité concave pour des gains et convexe pour des pertes. Les élicitations basées sur la méthode du tradeoff qui filtre la pondération des probabilités (Wakker et Deneffe, 1996) généralement corroborent la concavité pour les gains dans l'agrégat et pour la plupart des individus (Abdellaoui, 2000; Abdellaoui, 2002; Bleichrodt et Pinto, 2000). L'évidence pour la convexité pour des pertes est cependant, moins tranchante au niveau de l’individu (Abdellaoui, 2000; Fenema et van Assen, 1999) et d'ailleurs, Levy et Levy (2002) trouvent une utilité concave pour des pertes et une utilité convexe pour des gains pareille à celle de Markowitz (1952).

L’autre élément central des résultats psychologiques de la théorie des prospects est l’aversion aux pertes. Cet élément renvoie à l'assymétrie des gains et des pertes relativement à un point de référence avec les pertes apparaissant plus grandes que les gains commensurés. Kahneman et Tversky (1979, p: 279) définissent l'aversion aux pertes comme suit: un individu est averse aux pertes s'il n’aime pas les paris symétriques et si en plus, l'aversion pour de tels paris augmente avec la valeur absolue des enjeux. Les auteurs montrent aussi que leur définition est équivalente sous l’hypothèse de la théorie des prospects à une fonction d’utilité plus pentue pour les pertes que pour les gains. Le dernier résultat est à la base des diverses définitions de l'aversion aux pertes qui ont suivi: Tversky et Kahneman (1992); Wakker et Tversky (1993); Bowman et al. (1999); Köbberling et Wakker (2005); et Neilson (2002). Ces définitions seront présentées et opérationalisées ultérieurement (p: 34).

L’appui théorique pour l'aversion aux pertes vient de Rabin (2000) qui prouve que des degrés plausibles d'aversion au risque pour des enjeux modestes sous *EU* impliquent des degrés élevés irréalistes d'aversion au risque pour de grands enjeux tandis que l'aversion aux pertes implique une aversion au risque pour des enjeux modestes et une plausible aversion au risque pour de grands enjeux. La suggestion de Rabin favorisant l'utilisation de l'aversion aux pertes pour expliquer l'aversion au risque a été réitérée dans "Anomalies: Aversion au Risque" par Rabin et Thaler (2001).

L’appui empirique pour l'aversion aux pertes vient de Putler (1992) et de Hardie, Johnson, et Fader (1993) pour les élasticités assymétriques des prix. L'aversion aux pertes explique aussi l'effet de dotation (Thaler, 1980; Loewenstein et Adler, 1995) et par conséquent la disparité entre la bonne volonté de payer et la bonne volonté d'accepter (Kahneman, Knetsch et Thaler, 1990; Bateman, Munro, Rhodes, Starmer et Sugden, 1997), le biais du status quo (Samuelson et Zeckhauser, 1988), et l'effet de disposition (Weber et Camerer, 1988, Odean, 1998; Heath, Huddart et Lang, 1999). Selon Benartzi et Thaler (1997) et Barberis, Huang et Santos (2001) l'aversion aux pertes est nécessaire pour expliquer l'*equity premium puzzle*, c'est à dire les primes que les investisseurs exigent pour investir en actions et qui sous l'utilité espérée impliquent un degré d'aversion au risque absurde ainsi qu'a été démontré par Mehra et Prescott (1985).

Le troisième résultat expérimental intégré dans la théorie est la pondération non-linéaire des probabilités. Contrairement à *EU* où l'utilité (de chaque conséquence possible) est pondérée par sa probabilité, dans la théorie des prospects, l'utilité est multipliée par un poids de décision $\pi(p)$ qui est une fonction strictement croissante de p mais qui n'est pas une probabilité (Tversky et Kahneman, 1986). Les poids de décision ont été introduits pour modeler la tendance à surpondérer les faibles probabilités et souspondérer les probabilités modérées et grandes. Preston et Baratta (1948) ont été les premiers à observer cette tendance. Plus récemment, Diamond (1988) et Edwards (1996) ont constaté que des sujets jugeant des risques composés de faible probabilité/grande conséquence ont été plus affectés par la grandeur de la conséquence que par la probabilité tandis que ceux jugeant des risques composés de grande probabilité/faible conséquence ont eu tendance à combiner les deux composantes. Selon MacCrimmon et Larsson (1979) les sujets ont tendance à choisir la loterie avec le gain plus probable pour des probabilités élevées de gain et à choisir la loterie avec le gain le plus élevé pour de faibles probabilités de gain.

D'autres appuis empiriques ont été donnés plus tard par des études paramétriques qui supposent une fonction de probabilité pondérée spécifique: Tversky et Kahneman (1992); Camerer et Ho (1994); Tversky et Fox (1995); Wu et Gonzalez (1996); Abdellaoui (2000) et par des études non-paramétriques: Abdellaoui (2000) et Bleichrodt et Pinto (2000).

MOTIVATION ET OBJECTIFS DE RECHERCHE

La popularité de *CPT* et la possibilité d'expliquer les anomalies sous *EU* par des préférences averses aux pertes ont inspiré des économistes à rechercher des méthodes qui peuvent éliciter des fonctions d'utilité sous *CPT*, tester l'aversion aux pertes et mesurer son degré. Parmi les études qui ont examiné l'aversion aux pertes au niveau individuel on peut distinguer: Bleichrodt et Pinto (2002); Schmidt et Traub (2002) qui ont réalisé plutôt des tests qualitatifs et Abdellaoui et al. (2006) qui ont mesuré quantitativement l'aversion aux pertes.

La disponibilité de la méthode non-paramétrique d'Abdellaoui et al. (2006) et la facilité de son applicabilité (une forme automatisée avec le graphe de l'utilité élicitée simultanément pour des gains et des pertes obtenu immédiatement à la fin de l'expérience) dans un temps relativement court a offert la possibilité d'éliciter les fonctions d'utilité de gérants de portefeuilles pour qui le temps est une denrée rare. Les entrevues étant conduites dans les bureaux des gérants de portefeuilles à leurs institutions financières respectives, les élicitations ont exigé beaucoup de déplacements; néanmoins, les avantages potentiels des élicitations importent aux économistes intéressés par la modélisation du comportement, aux chercheurs des anomalies sur le marché boursier et aux analystes de décision particulièrement s'ils sont corroborés par d'autres investigations sur le terrain.

En effet, l'élicitation de la forme de la fonction d'utilité fournit une perspective descriptive aux modélisateurs d'une règle rationnelle pour la prise de décision, étant donné que les règles normatives doivent tenir compte du comportement actuel de l'individu (Allais, 1953; 1979).

En plus, étant non-paramétrique et donc indépendante de tout choix de fonctionnelle, l'élicitation de la fonction d'utilité serait aussi utile pour les chercheurs intéressés par les anomalies du marché financier. Par exemple, Benartzi et Thaler (1997) ont employé les évaluations de Tversky et Kahneman (1992) pour les coefficients d'aversion aux pertes et les fonctions d'utilité d'investisseurs afin de calculer les valeurs espérées des prospects, actions et obligations visant à expliquer le paradoxe des primes pour les actions. La méthode d'élicitation de Tversky et Kahneman (1992) suppose une forme paramétrique "puissance" pour la fonction d'utilité, l'évaluation de tous les paramètres étant problématique à ce moment-là (ibid, p: 311).

Enfin, un autre avantage est dans l'analyse de décision où les utilités biaisées peuvent avoir comme conséquence des prévisions économiques inexactes. Traditionnellement, l'analyse de décision suppose la normative *EU* pour l'élicitation des fonctions d'utilité. Cependant, pour que le postulat soit valide prescriptivement les préférences du décideur doivent être compatibles avec *EU*. Une utilisation corrective de la théorie des prospects a été suggérée par Kahneman et Tversky (1979); von Winterfeld et Edwards (1986); Fischnoff (1991); et Kahneman et Tversky (2000, p: 157). Bleichrodt, Pinto et Wakker (2001) proposent l'utilisation des utilités corrigées dans les prescriptions des décisions optimales, la correction basée sur des paramètres de préférence trouvés empiriquement.

ANNONCE DU PLAN

Ce travail expérimental élicite donc les fonctions d'utilité des gérants de portefeuilles et mesure leurs degrés d'aversion aux pertes sous *CPT*. Les choix présentés à ces derniers ayant été construits dans le cadre de la théorie de la prise de décision dans le risque, ce travail est divisé en deux parties: La première partie présente le cadre théorique de la prise de décision dans le risque et construit la scène pour le travail expérimental de la deuxième partie.

La première partie se compose d'une introduction et de trois chapitres. L'introduction distingue les étapes principales dans l'évolution de l'utilité espérée jusqu'au travail pionnier du vNM (1944). Le premier chapitre présente l'axiomatization de l'utilité espérée par ces derniers auteurs pour le risque et celle par Savage (1954) pour l'incertain. Le deuxième chapitre explore les violations de certains axiomes de *EU*, présente les théories alternatives de la prise de décision et montre que la théorie des prospects (1992) explique ces violations *et* l'évidence empirique de phénomènes tels que l'aversion aux pertes et l'effet de réflexion par l'intégration d'éléments non-normatifs. Le troisième chapitre explore ces éléments avant de présenter le modèle formel, son axiomatization dans le risque ainsi que la caractérisation de l'aversion au risque sous les différentes théories.

La deuxième partie présente le travail expérimental. D'abord, les fonctions d'utilité des gérants de portefeuilles sont élicitées et leurs coefficients d'aversion aux pertes mesurés suivant la méthode d'Abdellaoui et al. (2006). Les résultats sont ensuite contrastés avec ceux de ces derniers pour étudiants. Ensuite, la méthode de Baucells et Heukamp (2006) est employée pour examiner les préférences d'étudiants en MBA. A chaque expérience, un chapitre est consacré qui commence par une brève introduction montrant le dévelopment progressif de la méthode de celles qui l'ont précédées. L'introduction est suivie de la description de la procédure d'élicitation et de l'application expérimentale. Les résultats sont ensuite contrastés avec ceux de la littérature récente. La conclusion générale est suivie de commentaires finaux et perspectives futures.

PARTIE I: THEORIE DE L'UTILTE ET PRISE DE DECISION FACE AU RISQUE

Cette partie présente d'abord *EU* qui est un point de départ normal puisque les théories alternatives qui suivent sont des généralisations de cette théorie de base. La présentation de ces alternatives est cependant restreinte à deux égards: 1) le risque est distingué de l'incertain et la concentration est sur la décision dans le risque. En effet, l'objectif de la première partie est de présenter le cadre théorique qui convient aux essais empiriques présentés dans la deuxième; or, les choix présentés aux gérants de portefeuilles et aux étudiants en MBA pour inférer leurs préférences ont été conçus pour des situations de risque; 2) l'examen des alternatives se concentre sur des modèles présupposant une fonction simple de préférence, mais défendables en tant que modèles descriptifs du comportement réel. Ce postulat est un principe important de cohérence et il est raisonnable de supposer que les gens souhaitent y obéir même s'il est exigeant. En outre, bien que les violations empiriques d'une fonction simple de préférence présentent un cas pour les modèles non-conventionnels quand ceux-ci ne sont pas jugés en utilisant des critères raisonnables, abandonner la notion de préférences bien définies exige des changements qui augmentent la complexité de la théorie, réduisent son rendement prédictif et la rendent moins compatible avec le reste de la théorie économique (Starmer, 2000). Selon Arrow (1995) "ces modèles sont susceptibles d'être très corrects, c'est juste que leurs prévisions sont beaucoup plus vagues que celles suggérées par la rationalité; la rationalité est unique." Ainsi restreinte, cette partie se compose de trois chapitres: le premier est consacré à *EU* et comprend l'histoire du concept d'utilité et le modèle formel; le deuxième est consacré aux violations de certains axiomes de ce modèle et aux alternatives dévelopées en réponse et le troisième à la théorie de prospects (1992) qui tout en étant cohérente est la plus valide descriptivement grâce aux éléments non-normatifs intégrés dans la théorie; ce dernier chapitre de la première partie comprend le modèle descriptif, le modèle formel ainsi que la caractérisation de l'aversion au risque sous les différentes théories.

CHAPITRE I. LA THEORIE DE L'UTILITE ESPEREE

Ce chapitre commence par situer historiquement le concept d'utilité et établit un langage commun pour le reste du document. Il distingue les étapes principales dans l'évolution de la *EU* jusqu'au travail pionnier du vNM (1944) qui en constitue la version moderne. Ainsi, il montre comment le concept débute avec Bernoulli (1738), évolue avec Bentham (1789) et surtout avec la révolution des marginalistes au dix-neuvième siècle période à laquelle la définition de l'utilité marginale fut établie. Celle-ci est suivie bientôt par la révolution des ordinalistes au vingtième. L'utilité dans le certain et les comparaisons cardinales sont pratiquement abandonnées en faveur d'une vision ordinaliste de l'utilité où le principe de l'utilité marginale décroissante est cependant implicite. Finalement, le concept moderne de vNM (1944) est introduit avant de présenter le modèle formel. (L'histoire du concept de l'utilité est traitée dans la section 1.1 du document en anglais, p: 79).

Plusieurs axiomatisations dans le risque ayant été proposées (e.g. Herstein et Milnor, 1953; Jensen, 1967, Luce et Raiffa, 1957), l'axiomatisation présentée est basée sur Fishburn (1970). D'abord, le cadre général et quelques définitions essentielles sont donnés; les axiomes sont ensuite exposés de manière formelle et le théorème de représentation énoncé. (L'axiomatization est présentée dans la section 1.2, p: 85).

La représentation plus générale de l'utilité espérée subjective (*SEU*) de Savage (1954) suit. Elle peut être considérée comme une combinaison de la théorie de vNM (1944) et de sa duale la théorie de probabilité subjective de Bruno de Finetti (1937). Par rapport à la construction de vNM (1944), il ya donc plus de conditions indiquées. Savage y énonce les conditions qui permettent de montrer l'existence d'une mesure de probabilité subjective sur l'ensemble des états de la nature (un état de la nature étant une description complète d'une situation possible de l'environnement du décideur) et d'une fonction d'utilité similaire à celle de vNM sur l'ensemble des conséquences communes.

L'axiomatisation basée sur Fishburn (1970) est présentée dans la section 1.3. Anscombe et Aumann (1963) ayant suivi une route intermédiaire entre vNM et Savage, une courte description de leur théorie est esquissée.

Les axiomes présentés dans ce chapitre ont une énorme attraction normative et *EU* semble pouvoir être utilisée sans beaucoup de difficultés. Ces travaux théoriques ont été cependant suivis par des travaux expérimentaux qui ont montré que certains des axiomes de cette théorie sont violés systématiquement. Le deuxième chapitre développe en détail ces violations et présente les théories dévelopées comme alternatives.

CHAPITRE II. VIOLATIONS DE L'UTILITE ESPEREE ET THEORIES ALTERNATIVES

Ainsi que le titre l'indique, ce chapitre est divisé en deux parties: la première est concernée par les violations du modèle de l'utilité espérée face au risque et la deuxième par les théories dévelopées en alternatives.

LES VIOLATIONS

C'est la violation de l'axiome de l'indépendance dans le risque qui est la plus discutée en littérature et qui est également responsable de la génération de beaucoup d'alternatives à *EU* et à *SEU* pendant une longue période s'étendant de 1979 jusqu'à ce jour. C'est que comme l'explique Fishburn (1970) le principe d'espérance et la linéarité dans les probabilités ne peuvent pas être retenus sans cet axiome. Ce chapitre présente d'abord les violations de cet axiome face au risque (Allais, 1953) pour expliquer ensuite comment la théorie des prospects (1992) tient compte de ces violations. Une brève description de la violation de cet axiome en contexte dynamique et dans l'incertain suit.

La première section commence par décrire les deux exemples conçus par Allais (1953) qui le premier démontre que la propriété de séparabilité[3] que l'axiome de l'indépendance implique est violée. En effet, ces deux exemples montrent la violation de cette propriété qui est additive *et* multiplicative et sont connus sous le nom d'effet de la conséquence commune et effet de proportionalité respectivement. Ces derniers sont présentés dans la section 2.1.1, Tables 1 et 2 respectivement, p: 97, 99 respectivement). Cette première section montre aussi pourquoi les deux effets violent l'axiome d'indépendance et pourquoi les préférences sont incohérentes sous *EU* alors que la section suivante explique comment la théorie des prospects (1992) tient compte de ces violations (p: 99).

En effet cette dernière incorpore une pondération non-linéaire des probabilités, $p \rightarrow w(p)$ également appelée une fonction de transformation des probabilités. Deux propriétés exigées sur cette fonction réconcilient les préférences d'Allais qui sont contradictoires sous *EU*: La sous-additivité explique la violation de la conséquence commune et la sous-proportionalité explique l'effet de proportionalité (Prelec, 2000). La fonction de transformation des probabilités a une propriété empirique en plus et "peut-être la plus importante" (Prelec, 2001) qui indique que les petites probabilités sont surpondérées et les grandes probabilités sont souspondérées.

La section suivante montre ce que la violation de l'axiome d'indépendance implique pour le choix dynamique. Connue dans ce contexte comme violation de conséquentialisme, elle implique qu'au moins un principe de choix dynamique est violé puisque dans ce contexte quatre conditions impliquent conjointement l'équivalence de l'indépendance (cette section basée sur Wakker (1999) est présentée aux pages 103- 105 avec des figures pour illustrer).

Finalement, les violations des axiomes responsables de la stabilité des préférences sont présentées (p:107) c.-à-d. la transitivité, l'invariance descriptive et l'invariance de procédure. Bien que pour Ramsey (1931) la possibilité que le choix dépende de la forme

[3]Cette propriété est justifiée par l'exclusivité mutuelle des conséquences de la loterie (Machina, 1989; Weber et Camerer, 1987).

spéciale des options offertes soit “absurde”, l’exemple de Kahneman et de Tversky (1979) connu sous le nom de la *maladie d’Asie* prouve que les gens sont influencés par le contexte: les choix sont renversés quand des conséquences initialement présentées en termes de vies sauvées sont présentées en termes de vies perdues. L'inversion de préférence implique que le postulat de l'invariance descriptive est violé. L'effet de réflexion c.-à-d., le renversement des préférences quand on passe d’un contexte de gains à un contexte de pertes est un exemple compatible avec l’effet de contexte.

En raison des violations ci-dessus, beaucoup d'économistes ont conclu que le modèle de l’utilité espérée ne correspond pas du tout aux faits ou ne correspond pas seulement à certains de ces faits, en dépit des arguments qui procèdent avec une logique sans faute des postulats à la conclusion. Pour le développement d'un concurrent sérieux au moins pour quelques objectifs, les théoriciens de décision ont revisité et révisé les axiomes de l'*EU* et ce faisant ont généré un grand nombre de théories allant de pair avec les essais expérimentaux continus de ces théories.

LES THEORIES ALTERNATIVES

Basée sur les revues de ces théories alternatives et des essais expérimentaux les concernant (Camerer, 1989; Schmidt, 2002; et Starmer, 2000) cette partie du chapitre se concentre sur les modèles qui selon ces revues expliquent le mieux les données empiriques actuellement disponibles. Ainsi les théories alternatives présentées dans cette section sont limitées aux modèles à dépendance du rang (Quiggin, 1982; Tversky et Kahneman, 1992) qui ne sont pas linéaires dans la probabilité, particulièrement en raison du degré saisissant de convergence à travers les études concernant la forme de leurs fonctions de transformation des probabilités (Starmer, 2000, p: 359).

Le souci principal étant la réconciliation des prévisions de ces théories avec les faits expérimentaux, la présentation est restreinte à une comparaison dans le triangle unité des formes des courbes d'indifférence entre les ensembles de loteries sous les différentes théories et celles conjecturées des observations au laboratoire et/ou sur le terrain. Les courbes d'indifférence sous *EU* sont discutées en premier lieu (à la page 113 du document) pour être ensuite contrastées avec celles générées des choix qui violent l'axiome d'indépendance. Les courbes d'indifférence sous *RDU* (Quiggin, 1982) et *CPT* (Tversky and Kahneman, 1992) suivent (p: 116, 121 respectivement).

Pour souligner cependant, les inspirations et les idées que les auteurs ont tirées l'un de l'autre et les sauts intuitifs dans la pensée convergente favorisés par la combinaison de la connaissance existente et/ou la tolérance de la dualité, la théorie des prospects (1979), la version originale de la théorie des prospects (1992) est présentée d'abord (p: 113). La théorie anticipée de Quiggin (1982), développée en partie pour inclure certaines caractéristiques de la théorie non-conventionnelle des prospects tout en suivant une stratégie conventionnelle est présentée ensuite. La théorie duale de Yaari (1987), un cas spécial de celle de Quiggin, qui est linéaire dans les conséquences et non-linéaire dans les probabilités et donc la duale de *EU*, est décrite brièvement (p: 120) suivie finalement par la théorie des prospects (1992). En résumé, cette première partie montre que la théorie des prospects réussit à expliquer les anomalies et les phénomènes de base que *EU* est utilisée pour expliquer grâce à une combinaison de réalisme empirique et d'avantages théoriques qui favorisent son utilisation comme approximation de la fonction de préférence que l'individu est supposé optimiser. En raison de la pertinence des éléments psychologiques de cette théorie à la partie expérimentale le troisième chapitre présente le modèle descriptif ainsi que le modèle formel et son axiomatisation pour le risque.

CHAPITRE III . LA THEORIE DES PROSPECTS

La théorie des prospects (1992) en tant qu'une fertilisation croisée de la théorie des prospects (1979) et de celle de Quiggin (1982) présente un mix de validité descriptive et de précision mathématique. Ce chapitre souligne en premier lieu les éléments non-normatifs de la théorie avant de présenter le modèle formel et son axiomatisation dans le risque. Ces éléments psychologiques résultent de ce que l'individu perçoit les conséquences/probabilités relativement à un point de référence. L'intuition de ce point est donnée d'abord et l'importance de sa localisation pour l'ordre des préférences est soulignée suivie d'une section consacrée à l'aversion aux pertes (p: 125, 126 respectivement). Les implications de ces éléments non-normatifs pour la fonction d'utilité sont détaillées ensuite (p: 128).

Bien que ce travail soit concerné principalement par l'élicitation de la fonction d'utilité, la fonction de transformation des probabilités en forme de "*S* inversé" et différenciée pour les gains et les pertes est décrite (p: 129) afin de souligner l'importance de filtrer son impact sur la désirabilité des conséquences.

Le modèle formel pour le risque ainsi que celui pour l'incertain sont présentés suivis de l'axiomatisation de la théorie des prospects (1992) dans le risque basée sur Châteauneuf et Wakker (1999). L'idée centrale de l'axiomatisation de *CPT* dans l'incertain et dans le risque est la *tradeoff consistency* ou cohérence du tradeoff. Cette dernière est également essentielle à la méthode d'élicitation employée dans ce travail expérimental pour encoder les fonctions d'utilité des gérants de portefeuilles. Par conséquent, la définition de l'idée du tradeoff basée sur Wakker (1994) est d'abord présentée (p: 134). L'intuition pour la cohérence du tradeoff est ensuite illustrée par un exemple (p: 135) pris de Wakker et de Tversky (1993, p: 149). L'axiomatisation pour le risque suit (p: 136).

Ce chapitre présente également les caractérisations respectives de l'aversion pour le risque sous *EU*, *RDU*, et *CPT* pour montrer que ce qu'on comprend par aversion au risque a été largement raffiné dans le cadre de *RDU* et d'avantage dans celui de *CPT.*

Les notions de l'aversion de risque définies indépendamment de n'importe quel modèle sont présentées dans une première étape (p: 140) et les liens entre ces notions et les fonctions de préférence sous les différentes théories sont établis dans une deuxième (143-145). Ainsi, des comportements de risque observés de nature différente qui ne sont pas distingués sous *EU* sont séparés sous *RDU* et encore plus sous *CPT* où l'aversion au risque a trois composantes: une fonction d'utilité, une fonction de transformation des probabilités et une aversion aux pertes.

En somme, en plus des avantages théoriques qui caractérisent les théories dépendantes du rang, le réalisme empirique de la *CPT* la prédispose à être une bonne approximation de la fonction fondamentale de préférence que l'investisseur est supposé optimiser. Comme c’est aux mesures des paramètres de la théorie de montrer si l'approximation est bonne, la première expérience élicite la fonction d’utilité des gérants de portefeuilles et mesure leurs coefficients d'aversion aux pertes sous *CPT* testant ainsi la théorie sur le terrain. Les résultats sont ensuite contrastés avec ceux d’Abdellaoui et al. (2006) pour étudiants. Une deuxième expérience examine qualitativement les préférences d’étudiants en MBA également sous *CPT* (Baucells et Heukamp, 2006). Combinées, les deux expériences offrent une comparaison entre gérants de portefeuille *expérimentés* et *en potentiel*.

PARTIE II: ETUDE EXPERIMENTALE DES FONCTIONS D'UTILITE INDIVIDUELLES ET DE L'AVERSION AUX PERTES

La plupart des méthodes d'élicitation employées dans les études empiriques qui s'intéressent à la fonction fondamentale de préférence de l'individu ont supposé des formes paramétriques spécifiques pour la fonction d'utilité rendant de ce fait les inférences au sujet de ces fonctions dépendantes du choix des fonctionelles. Aussi, la méthode choisie pour être utilisée dans cette la première expérience partie est-elle non-paramétrique. En plus, la pondération de la probabilité ne lui pose pas de problème. La pondération de la probabilité est une cause importante des violations de la théorie de l'utilité espérée et des contradictions systématiques parmi les différentes méthodes d'élicitation d'utilité qui devraient donner le même résultat sous *EU* (Hershey et Schoemaker (1985); McCord et de Neufville (1986); Wakker et Deneffe (1996); Bleichrodt et al. (2001); Fischoff (1982); Schkade (1988)). La robustesse contre la pondération de la probabilité est fondamentale pour que la méthode reste valide pour *RDU* et *CPT* et puisse être appliquée dans l'analyse prescriptive de décision.

Aussi, la méthode d'Abdellaoui et al. (2006) est-elle employée pour éliciter entièrement les fonctions d'utilité des gérants de portefeuilles permettant ainsi la mesure de leurs coefficients d'aversion aux pertes. Quand aux préférences des étudiants en MBA, elles seront inférées à partir de loteries construites par Baucells et Heukamp (2006). A chaque expérience un chapitre est consacré qui commence par une brève introduction qui montre que chaque méthode contient tous les éléments essentiels de celles qui l'ont précédée mais non *vice versa.* Autrement dit, c'est le cas où tout ce qui est inférieur est contenu dans ce que est supérieur mais tout ce qui est supérieur n'est pas contenu dans ce qui est inférieur comme Aristote a été le premier à le préciser. A cet égard, pour déterminer ce qui est inférieur dans une séquence, Wilber (1996) suggère de réfléchir à un cas où tout ce qui est supérieur est détruit mais rien de qui est inférieur ne l'est. L'introduction est suivie de l'application expérimentale de la méthode pour chaque chapitre.

CHAPITRE IV. EXPERIENCE I: ELICITATION NON-PARAMETRIQUE DE LA FONCTION D'UTILITE SUR LE TERRAIN

Dans ce chapitre, les fonctions d'utilité des gérants de portefeuilles sont élicitées simultanément pour les gains et pour les pertes et leurs coefficients d'aversion aux pertes mesurés en utilisant la procédure d'élicitation d'Abdellaoui et al. (2006). La première section récapitule la théorie des prospects; la deuxième décrit la procédure d'élicitation (p: 156) ainsi que l'application expérimentale (p: 159); quand à la troisième, elle est concernée par l'analyse des données et comprend les élicitations non-paramétriques, l'ajustement paramétrique et la mesure de l'aversion aux pertes dans l'agrégat aussi bien qu'au niveau individual (p: 165-172). Les sections suivantes présentent les résultats tout en les contrastant avec les résultats d'études précédentes; une section finale discute les résultats de l'expérience (p:184).

LA PROCEDURE D'ELICITATION

La méthode est basée sur l'élicitation de points d'utilité, *mipoints* pour être précis qui est souvent utilisée dans l'axiomatisation des modèles de décision. La procédure consiste en quatre étapes et elle est résumée dans la Table 1 qui suit. La deuxième colonne décrit la quantité élicitée, la troisième l'indifférence qu'on recherche et la quatrième ce que cette indifférence implique sous l'hypothèse *CPT*. La dernière colonne montre les variables qui doivent être spécifiées et les valeurs choisies pour ces variables.

Table 1: Procédure d'Elicitation

	Quantité Élicitée	Indifférence	Sous l'Hypothèse CPT	Variables de Choix
Etape 1	L_1	$(L_1, p'; L^*) \sim (L_0, p'; L)$		$p' = 0.33$
	L_2	$(L_2, p'; L^*) \sim (L_1, p'; L)$	$U(L_0) - U(L_1) = U(L_1) - U(L_2)$	$L^* = -100$
	p_l	$L_1 \sim (L_2, p_l ; L_0)$	$w^-(p_l) = 0.5$	$L=-600$, $L_0 = -1000$
	G_1	$(G_1, p'; G) \sim (G_0, p'; G^*)$		$p' = 0.33$
	G_2	$(G_0, p'; G) \sim (G_1, p'; G^*)$	$U(G_2) - U(G_1) = U(G_1) - U(G_0)$	$G^* = 600$
	p_g	$G_1 \sim (G_2, p_g; G_0)$	$w^+(p_g) = 0.5$	$G = 100$, $G_0 = 1000$
Etape 2	$L_r \in [L_1 , 0]$	$L_r \sim (L_A, p_l ; L_B)$	$U(L_r) = 0.5\, U(L_A) + 0.5\, U(L_B)$	$L_1 = -100000$
Etape 3	l	$L_s \sim (l, 0.5 ; 0)$	$w^-(0.5)U(l) = -s$	$s = 0.25$
	g	$0 \sim (g, 0.5 ; l)$	$w^+(0.5)U(g) = s$	
	G_s	$G_s \sim (g, 0.5 ; 0)$	$U(G_s) = w^+(0.5)U(g) = s$	
Etape 4	$G_r \in [0, G_s]$	$G_r \sim (G_A, p_g ; G_B)$	$U(G_r) =0.5\, U(G_A) + 0.5\, U(G_B)$	

Les Etapes

D'abord, deux conséquences monétaires pour les gains G_1, G_2 et deux pour les pertes L_1, L_2 pour lesquelles la différence d'utilité entre les conséquences successives est constante, sont déterminées en séquence. Ensuite deux probabilités p_g et p_l pour lesquels $w(p_g) = w(p_l) = 0,5$ sont déterminées par la méthode de *l'équivalence des probabilités*. L'étape suivante, *fractile pertes* détermine une séquence d'onze conséquences pour les pertes L_r élicités sur l'intervalle [-100000, 0] pour les utilités suivantes: 0.015, 0.031, 0.062, 0.093, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, où $U(L_r) = -r$ en utilisant la probabilité élicitée pour les pertes.

La troisième étape détermine le gain x_8 qui représente la même variation d'utilité par rapport à la conséquence 0 comme valeur absolue de la variation d'utilité entre la perte $L_{0.25}$ et 0; cette étape lie l'utilité pour les pertes à l'utilité pour les gains à travers l'élicitation de trois conséquences et permet la mesure de l'aversion aux pertes.

La quatrième étape, appelée *fractile gains* élicite une séquence de huit conséquences entre x_8 and 0 pour les utilités suivantes: 0.015, 0.031, 0.062, 0.093, 0.125, 0.156, 0.187, 0.25, où $U(G_r) = r$ en utilisant la probabilité pour les gains déterminée au début.

Le logiciel élicite également des données pour deux tests de cohérence. Pour le premier, la différence d'utilité étant constante entre les éléments successifs dans les séquences élicitées au début de l'expérience ($\{G_0$, G_1 , $G_2\}$ pour les gains et $\{L_0$, L_1 , $L_2\}$ pour les pertes), la cohérence exige que les égalités entre les différences d'utilité soient maintenues pour les pertes et gains obtenus en étapes 2 et 4 de la procédure d'élicitation. Le logiciel permet également un deuxième test de cohérence qui se compose de quatre questions. Pour chacune des loteries (G_r, p_r; L_r), r = 0.031, 0.0625, 0.125, 0.250, la probabilité p_r établie rend le gérant de portefeuilles indifférent entre la loterie d'une part et ne rien recevoir d'une autre part. La même probabilité d'indifférence devrait être obtenue dans les quatre questions.

Les Indifférences

La méthode de bisection est employée pour l'élicitation des indifférences à travers des séries de choix. Le gérant de portefeuilles doit choisir entre deux loteries A = (1000, p; 600) et B = (L_0, p; 100) qui correspondent à (L_1, p'; L^***)** et (L_0, p'; L) présentées comme des “camemberts” sur l'écran de l'ordinateur où les morceaux correspondent aux probabilités. Il doit décider pour chaque itération qui change la valeur de L_0, s'il change de la loterie B à la loterie A. A partir de L_0 = 4000 (L_0 peut changer de 4000 à 8000, l'intervalle pour L_0 étant fixé tel qu'il garantie une préférence forte pour B), le processus consiste à rétrécir l'intervalle contenant L_0 par un certain nombre d'itérations jusqu'à obtenir la valeur de L_0 qui rend le gérant de portefeuilles indifférent entre les deux jeux, c'est à dire (1000, p; 600) $\sim$ (L_0, p; 100). La conséquence L_0 est ainsi déterminée par une série d'itérations pour rendre le joueur indifférent entre les deux loteries. La méthode de bisection employée pour produire les itérations est illustrée dans la Table 2 pour L_1 and $L_{0.0625}$.

Table 2: Illustration de la Méthode de Bisection

Iterations	**Choix Visant l' Elicitation de L_1**	**Choix Visant l' Elicitation de $L_{0.0625}$**
1	**(-1000, 0.33;-600)** vs. (-4000,0.33;-100)	-6080 vs. **(-7800,0.78; 0)**
2	**(-1000, 0.33;-600)** vs. (-2500,0.33;-100)	**-3040** vs. (-100000,0.78; 0)
3	(-1000, 0.33;-600) vs. **(-1750,0.33;-100)**	-4560 vs. **(-100000,0.78; 0)**
4	(-1000, 0.33;-600) vs. **(-2100,0.33;-100)**	**-3800** vs. (-100000,0.78; 0)
5	**(-1000, 0.33;-600)** vs. (-2300,0.33;-100)	**-4180** vs. (-100000,0.78; 0)
Indifférence	-2200	- 4370

La loterie choisie est en caractères typographiques gras et seule la conséquence qui est à éliciter varie, c'est à dire augmente ou diminue en fonction du choix de l'itération. La grandeur du changement est toujours la moitié du changement dans la question précédente. Néanmoins, la conséquence qui en résulte doit être un multiple de 10 et la probabilité résultante un multiple de 0,01, sinon, la valeur est fixée au plus proche multiple de 10 ou de 0,01. La méthode résulte dans un intervalle dans lequel la valeur de l'indifférence doit être choisie. Par exemple, la valeur de l'indifférence pour $L_{0.062}$ doit être entre -4180 et -4560. Par conséquent la valeur est -4370 le mi-point de l'intervalle. Au début des itérations, les valeurs sont généralement choisies de manière que les lotteries aient des valeurs espérées égales. Les exceptions sont L_1, L_2, G_1 et G_2 dont les valeurs au début sont L_0 +3000, L_1 +3000, G_0 +3000 and G_1+3000.

ANALYSE DES DONNEES

Les données ont été analysées en trois étapes (présentées dans la section 4.3). D'abord, la forme de la fonction de l'utilité pour les gains et pour les pertes pour chaque gérant de portefeuilles a été déterminée en suivant l'évolution de la pente de la fonction d'utilité en divers points (p: 166). Trois familles paramétriques ont été ensuite considérées pour l'ajustement paramétrique: La famille puissance, la famille exponentielle et la famille expo-puissance (p:167-168). Pour l'aversion aux pertes et dans l'absence d'une définition généralement admise, le degré de l'aversion aux pertes des gérants de portefeuilles a été mesuré suivant les définitions suivantes (p:169): Kahneman et Tversky (1979); Wakker et Tversky (1993); Neilson (2002); Bowman et al. (1999); et Köbberling et Wakker (2005).

Elicitation Non-Paramétrique de la Fonction d'Utilité

Pour pouvoir suivre l'évolution de la pente de la fonction d'utilité pour les gains et les pertes en divers points, deux pertes élicitées L_r and $L_{r'}$ (G_r et $G_{r'}$ pour gains) sont définies adjacentes si $L_r > L_{r'}$ ($G_r > G_{r'}$) et qu'il n'y a aucune perte (gain) élicitée dans l'intervalle.

$S_L^-(r)$ est défini comme la pente du segment liant (L_r, $U(L_r)$) et ($L_{r'}$, $U(L_{r'})$) où L_r and $L_{r'}$ sont adjacents. De même, $S_L^+(r)$ est défini comme pente du segment liant ($L_{r''}$, $U(L_{r''})$) et (L_r, $U(L_r)$) où $L_{r''}$ et L_r sont adjacents. $S_G^-(r)$ et $S_G^+(r)$ sont définis de façon analogue. $\Delta S_L(r) = S_L^+(r) - S_L^-(r)$ dénote la variation de la pente autour de L_r quand on s'éloigne de zéro. De même, $\Delta S_G(r) = S_G^-(r) - S_G^+(r)$ dénote la variation de la pente autour de G_r quand on s'éloigne de zéro. On peut vérifier facilement que $\Delta S_i(r), i = G, L$ positif, (négatif, zéro) correspond à l'utilité convexe (concave, linéaire). Sept valeurs de $\Delta S_G(r)$ ont été obtenues pour des gains et onze valeurs de $\Delta S_L(r)$ pour des pertes pour chaque gérant de portefeuilles. Pour tenir compte de l'erreur de réponse, les gérants de portefeuilles qui ont au moins quatre/sept $\Delta S_G(r)$ négatifs (positifs) ont été classifiés en tant qu'ayant des utilités concaves (convexes) pour des gains. Ceux qui ont au moins sept/onze $\Delta S_L(r)$ négatifs (positifs) ont été classifiés en tant qu'ayant des utilités concaves (convexes) pour des pertes.

Ajustement Paramétrique

Les données ont été également analysées en supposant trois formes paramétriques: la famille de puissance, la famille exponentielle et la famille expo-puissance. (Le lecteur peut se reférer aussi à la section 4.3.3, p: 167-168 du document en anglais où les équations relatives aux trois familles sont numérotées comme 4.6, 4.7 et 4.8 respectivement).

La famille puissance est définie avec α > 0 pour les gains et β > 0 pour les pertes comme suit:

$$U = U_{\max}(\frac{x}{x_{\max}})^{\alpha} \quad \text{if} \quad x \geq 0 \qquad (1)$$

$$U = -(-\frac{x}{100000})^{\beta} \quad \text{if} \quad x < 0$$

Le cas où α < 0 correspond à une fonction d'utilité décroissante. La fonction puissance pour les gains est concave si α < 1, linéaire si α = 1 et convexe if α > 1; pour les pertes, elle est convexe si β < 1, linéaire si β = 1 et concave si β > 1.

La famille exponentielle est définie avec $\alpha \neq 0$ et for $\beta \neq 0$ comme suit:

$$U = \frac{U_{\max}(\exp(-\alpha(\frac{x}{x_{\max}}))-1)}{(\exp(-\alpha)-1)} \quad \text{if} \quad x \geq 0 \qquad (2)$$

$$U = -\frac{(\exp(-\beta(\frac{x}{100000}))-1)}{(\exp(-\beta)-1)} \quad \text{if} \quad x < 0$$

La fonction est définie comme $U_{\max}(\frac{x}{x_{\max}})$ pour $\alpha = 0$, et comme $-\frac{x}{100000}$ pour $\beta = 0$; la fonction pour les gains est concave si α > 0 et convex si α < 0; pour les pertes, elle est convexe si β > 0 et concave si β < 0.

La famille expo-puissance a été introduite par Abdellaoui, Barrios et Wakker (2002). C'est une variation d'une famille de deux paramètres proposée par Saha (1993). Elle est définie comme suit avec pour $\alpha \neq 0$ et pour $\beta \neq 0$ comme suit:

$$U = \frac{U_{\max}(1-\exp(-(\frac{x}{x_{\max}})^{\alpha}/\alpha))}{1-\exp(-1/\alpha)} \quad \text{if} \quad x \geq 0 \qquad (3)$$

$$U = -\frac{(1-\exp(-(\frac{x}{100000})^{\beta}/\beta))}{1-\exp(-1/\beta)} \quad \text{if} \ x < 0$$

Les cas où $\alpha \leq 0$ et $\beta \leq 0$ n'ont pas été considérés parce que contrairement à l'observation ils mènent à une aversion extrême pour le risque pour les gains et une tendance extrême au risque pour les pertes quand le zéro est parmi les conséquences de la loterie. La famille expo-puissance pour les gains est concave si $\alpha \leq 1$ et convexe si $\alpha \geq 2$; pour les pertes elle est convexe si $\beta \leq 1$ et concave si $\beta \geq 2$.

Mesure de L'Aversion aux Pertes

Plusieurs définitions de l'aversion pour les pertes ont été considérées: Kahneman et Tversky (1979); Wakker et Tversky (1993); Neilson (2002); Bowman et al. (1999); Köbberling et Wakker (2005). Excepté pour cette dernière localisée au point de référence, elles sont toutes globales et certaines sont plus strictes que d'autres. Elles ont été opérationalisées comme suit:

Kahneman and Tversky (1979) ayant suggéré que l'aversion pour les pertes devrait être définie comme $-U(-x) > U(x)$ for $x > 0$, le degré de l'aversion pour les pertes calculé pour chaque gain élicité a été $-\frac{U(-G_r)}{U(G_r)}$ pour tout $G > 0$ où r renvoie aux utilités communes pour les pertes/gains: 0.015, 0.031, 0.06, 0.093, 0.125, 0.15, 0.18, et 0.25.

D'abord les $U(-G_r)$ ont été calculés et ce en utilisant l'interpolation linéaire pour chaque gain obtenu. Le nombre des conséquences obtenues dans le domaine des pertes étant plus grand que le nombre de conséquences obtenues dans le domaine de gains, il a été possible d'obtenir huit valeurs d' $U(-G_r)$ pour chaque gérant de portefeuilles. Pour $G_{0.25}$ excédant 100000, $U(-G_r)$ a été pris en tant que -1.

Le gérant de portefeuilles a été classifié averse aux pertes quand au moins 6 des 8 valeurs des coefficients d'aversion aux pertes calculées avaient excédé 1, neutre pour les pertes quand au moins 6 valeurs avaient été égales à un et chercheur de gain ou "gain seeker" quand au moins 6 valeurs avaient été inférieures à 1. Le moyen/médian de $-\frac{U(-G)}{U(G)}$ a été pris ensuite comme le coefficient d'aversion aux pertes.

Neilson (2002) ayant proposé de définir l'aversion aux pertes par $\frac{U(-x)}{-x} \geq \frac{U(y)}{y}$ pour tout x et y positifs (il fournit aussi une fondation de préférence) un candidat possible est le $\frac{\min(U(L_r)/L_r)}{\max(U(G_r)/G_r)}$. Le portfolio manager a été classifié comme averse aux pertes quand le $\frac{\min(U(L_r)/L_r)}{\max(U(G_r)/G_r)}$ avait excédé un et un chercheur de gain quand le $\frac{\min(U(G_r)/G_r)}{\max(U(L_r)/L_r)}$ avait excédé un.

Wakker et Tversky (1993) ayant défini l'aversion aux pertes comme $U'(-x) \geq U'(x)$ pour tout $x > 0$, le coefficient d'aversion considéré pour les pertes est $\frac{U'(-G_r)}{U'(G_r)}$ où $U'(G_r)$, la pente $\Delta U_r / \Delta G_r$ a été calculée comme suit:

$$U'(G_r) = 1/2[\frac{(U(G_r) - U(G_{r'}))}{(G_r - G_{r'})} + \frac{(U(G_{r''}) - U(G_r))}{(G_{r''} - G_r)}] \qquad (4a)$$

et

$$U'(-G_r) = \frac{(U(L_s) - U(L_{s'}))}{(L_s - L_{s'})} \qquad (4b)$$

si $L_{s'} < -G_r < L_s$ et L_s et $L_{s'}$ sont adjacents, et en définissant,

$$U'(-G_r) = 1/2[\frac{(U(L_s) - U(L_{s'}))}{(L_s - L_{s'})} + \frac{(U(L_{s''}) - U(L_s))}{(L_{s''} - L_s)}] \qquad (4c)$$

si $-G_r = L_s$. (Le lecteur peut se reférer à la page 171 du document où les trois dernières équations sont numérotées comme 4.9*a*, 4.9*b* et 4.9*c* respectivement).

Les pentes pour chaque gain ont été calculées d'abord, puis les pentes pour les pertes commensurées suivant l'équation convenable. Sept valeurs ont été obtenues pour chaque gérant de portefeuilles. Celui-ci a été classifié averse aux pertes quand au minimum six/sept valeurs avaient excédé 1, neutre quand six/sept valeurs avaient été égales à 1 et chercheur de gain quand au minimum six/sept valeurs avaient été inférieures à 1.

Une définition plus forte a été fournie cependant par Bowman et al. (1999): l'aversion aux pertes tient si $U'(-x) \geq U'(x)$ pour tout *x* et *y* positifs. Autrement dit, la fonction d'utilité pour les pertes est partout plus pentue que la fonction d'utilité pour les gains. Par conséquent, le coefficient d'aversion aux pertes a été calculé comme $\frac{\min(U'(L_r)}{\max U'(G_r)}$ excédant un et chercheur de gain comme $\frac{\min(U'(G_r)}{\max U'(L_r)}$ excédant un. $U'(G_r)$ and $U'(L_r)$ ont été calculés comme dans les équations pour les coefficients de Wakker et de Tversky (1993).

Finalement, Köbberling et Wakker (2005) ont défini le coefficient d'aversion aux pertes comme $U'_{\uparrow}(0)/U'_{\downarrow}(0)$ où le numérateur et le dénominateur représentent respectivement les dérivées gauches et droites de l'utilité au point de référence. Par conséquent, le coefficient d'aversion aux pertes qui implique que la fonction d'utilité est plus pentue pour de petites pertes que pour de petits gains a été calculé comme $U(L_{0.015})/L_{0.015}$ sur $U(G_{0.015})/G_{0.015}$ c.-à-d. $G_{0.015}/L_{0.015}$. Le gérant de portefeuilles dont le coefficient avait excédé 1 a été classifié comme averse aux pertes.

LES RESULTATS DE L'EXPERIENCE I.

Les résultats non-paramétriques (tabulés à la page 174, Table 11) montrent que le patron le plus commun, la concavité pour les gains et la convexité pour les pertes exhibé par 58% des gérants de portefeuilles est plus grand que celui de Fenema et van Assen (1999), Abdellaoui (2000) et Etchart-Vincent (2004) dont l'intervalle varie entre (37% et 47%) et proche du 54% de Abdellaoui et al. (2006). Pour l'ajustement paramétrique ‹‹puissance›› (les résultats paramétriques sont tabulés à la page 176-177, Tables 12, 13, 13(*a*), le coefficient médian pour les pertes (0,9) est cependant dans la marge des études mentionnées ci-dessus qui varie entre (0,84) et (0,97). L'aversion aux pertes existe dans l'agrégat avec une pente de 0,450 (R^2 ajusté est 0,906). Quand aux coefficients d'aversion aux pertes obtenus, ils varient avec les définitions utilisées soulignant le besoin d'une définition généralement admise. A cet égard, et excepté pour la définition de Kahneman et Tversky (1979), peu de gérants de portefeuilles ont été caractérisés selon les définitions globales qui mesurent l'aversion aux pertes en divers points et qui semblent excessivement fortes pour des buts empiriques. Les tables 14 et 15 (p: 178-181) présentent les résultats de l'aversion aux pertes pour le gérant de portefeuilles moyen/médian et les comparent aux résultats obtenus par Abdellaoui et al. (2006) pour étudiants.

Contrastés avec les résultats des études précédentes pour étudiants (Schmidt et Traub, 2002; Abdellaoui et al., 2006) pour la définition de Wakker et Tversky (1993) le gérant de portefeuilles médian est moins averse aux pertes (1,08) par rapport à l'étudiant médian (1,43; 1,53) respectivement. Contrastés pour la définition de Kahneman et Tversky (1979) avec Abdellaoui et al. (2006) et Bleichrodt et al. (2001) qui ont estimé un coefficient d'aversion aux pertes selon la même définition les résultats médians sont par contre proches de Abdellaoui et al. (2006) mais moindres que Bleichrodt et al. (2001). (1,69 ; 1,72 ; 2,17 respectivement). Pour la définition locale de Köbberling et Wakker (2005) le gérant de portefeuilles médian est plutôt non averse aux pertes avec un coefficient de 0,74 par rapport à l'étudiant médian de Abdellaoui et al. (2006) dont le coefficient est de 2,52 .

DISCUSSION DES RESULTATS

Les résultats non-paramétriques (discutés aux pages 184-188) indiquent donc que le gérant de portefeuilles est moins averse que l'étudiant et que plus de gérants de portefeuilles que d'étudiants sont agressifs dans des situations où ils doivent décider entre une perte sûre et une perte probable.

Pour ce qui est de l'aversion aux pertes, les gérants de portefeuilles possèdent de par leur activité professionnelle quotidienne une gamme de formation et un haut niveau de connaissance qui font qu'il est plausible qu'ils évaluent les enjeux différemment des étudiants. Certes, il faut aussi souligner que les entrevues ont été entreprises durant la période (2003-2004) qui correspond à une croissance (l'index Standard et Poor est en hausse de 28%, 17% et 10,9 % respectivement par rapport à l'année précédente) et il n'est pas inconcevable que le degré d'aversion aux pertes des gérants de portefeuilles ait diminué avec le mouvement en hauteur du marché boursier. Barberis, Huang and Santos (2001) dont le modèle d'évaluation des biens est basé sur une aversion aux pertes qui change dans un contexte dynamique n'auraient pas pu obtenir la volatilité observée dans le marché boursier sans cette variation.[4]

Cette variation certes consiste en deux volets: des gains antérieurs rendent l'individu moins averse aux pertes alors que des pertes qui succèdent à d'autres rendent l'individu plus averse aux pertes (Thaler et Johnson, 1990; Gertner, 1993); aussi, est-il important de clarifier que le deuxième volet ne contredit pas la convexité de la fonction d'utilité dans le domaine des pertes mais seulement l'hypothèse d'intégration des biens. Comme le remarquent Barberis, Huang et Santos (2001), si les investisseurs intégraients plusieurs années de gains et de pertes, ils seraient en train d'évaluer des niveaux absolus de richesse et non pas des changements dans la richesse. D'ailleurs, Thaler et Johnson (1990) indiquent aussi que dans le cas d'une loterie qui résulte en conséquences limitant la perte maximale mais offrant la chance de devenir quitte les étudiants sont enclins au risque même après une perte.

Gross (1982) qui documente ce phénomène sur le marché boursier y refère comme la "getevenitis disease". Le dictat de la rationalité est douloureux: pour réaliser des pertes, les gérants de portefeuilles doivent auparavant renoncer à l'espoir de s'en sortir quitte.

[4] Le modèle de Barberis, Huang et Santos (2001) a été influencé par Kahneman et Tversky (1979) pour l'aversion aux pertes and par Thaler et Johnson (1990) et Gertner (1993) pour la variation de cette dernière.

Finalement, il faut bien mentionner les compensations auxquelles le gérant de portefeuilles aurait droit en cas de profits. A titre d'exemple, les primes de Wall Street pour l'année 2006 ont varié de $1 million à $3 millions pour le directeur *moyen*, jusqu'à $60 millions pour les maisons d'investissement comme Goldman Sacks, Lehman Brothers et Morgan Stanley (New York Times, Dec 25, 2006).

Les différences de comportement des gérants de portefeuilles par rapport aux étudiants pourraient donc être dues à des facteurs d'occupation. L'expérience II a été enterprise pour analyser d'une manière qualitative les préférences des étudiants en MBA, gérants de portefeuilles en potentiel en utilisant la méthode nouvellement développée de Baucells et Heukamp (2006).

CHAPITRE V. EXPERIENCE II: EXPERIENCE AU LABORATOIRE UTILISANT DES CONDITIONS DE DOMINANCE STOCHASTIQUE

La méthode de Baucells et Heukamp (2006) généralise les conditions de dominance stochastique introduites initialement par Rothschild et Stiglitz (1970) et plus récemment par Levy et Wiener (1998) et Levy et Levy (2002). En effet, les préférences peuvent être inférées des choix entre des loteries construites de façon que l'une domine stochastiquement l'autre. Un individu avec une fonction d'utilité non-décroissante et concave c.-à-d. un individu averse au risque et qui maximize *EU* dis-préférera une loterie qui est dominée par la dominance stochastique de deuxième ordre (*SSD*) et l'inverse est également vrai: si l'individu élimine des alternatives dominées par *SSD*, sa fonction d'utilité est concave. Pour distinguer entre les classes des fonctions d'utilité non-décroissantes qui ne sont pas concaves partout, d'autres conditions sont certes nécessaires. Levy et Levy (2002) développent la dominance stochastique des prospects (*PSD*) et la dominance stochastique de Markowitz (*MSD*) pour différencier entre les fonctions d'utilité qui sont concaves pour les gains et convexes pour des pertes (ayant une forme de *S* comme celle postulée par Kahneman et Tversky (1992)) et ceux qui sont convexes pour des gains et concaves pour des pertes (ont une forme de *S* inversé comme celle postulée par Markowizt (1952)). Ces conditions peuvent être appliquées sous *EU* ou sous n'importe quel modèle dépendant d'un point de référence qui n'incorpore certes pas une fonction de transformation des probabilités.

Aussi, Baucells et Heukamp (2006) étendent-ils ces conditions à *CPT* en incorporant cette dernière ainsi que l'aversion aux pertes et les utilisent comme guides pour concevoir des paires de loteries ou *tasks* qui sont en compétition directe entre les deux théories alternatives: *CPT* (1992) et Markowitz (1952). Ainsi, pour deux loteries *F* et *G* conçues de facon que *F* domine *G* par la dominance stochastique de perspective ($F \succ_{PWSD} G$) et *G* domine *F* par la dominance stochastique de Markowitz ($G \succ_{MWSD} F$) le choix de *F* (*G*) implique que la fonction d'utilité du décideur a la forme *S* (*S* inversé). Les choix des décideurs entre les loteries ainsi conçues indiquent des propriétés de leurs préférences et leurs représentations sous un modèle de choix. Aussi, pour un modèle donné, la condition de dominance stochastique liée à une caractéristique spécifique du modèle permet-elle d'examiner les propriétés qualitatives de ce dernier.

L'expérience II utilise les paires de loteries construites par Baucells et Heukamp (2006) pour examiner de facon qualitative les préférences des étudiants en MBA. Le chapitre V se compose de six sections. La première présente l'intuition pour la dominance stochastique de la théorie des prospects et de Markowitz suivies de la caractérisation des préférences en utilisant ces conditions. La deuxième énonce l'objectif de l'expérience et décrit la source de données. L'analyse des données est présentée dans la troisième section. Les résultats concernant la forme de la fonction d'utilité sont présentés ensuite suivis des résultats concernant l'aversion aux pertes et la probabilité globale des gains/pertes respectivement. Une section finale discute les résultats de l'expérience.

DOMINANCE STOCHASTIQUE SOUS CPT ET MARKOWITZ

L'intuition en bref (élaborée à la page 192) est qu'un individu qui adhère à la théorie des prospects donne plus d'importance aux conséquences près de l'origine qu'aux conséquences extrêmes tandis que le contraire est vrai pour un individu qui adhère à la théorie de Markowitz. Ainsi, au cas où aucune transformation des probabilités n'est postulée, la différence en utilité espérée est donnée par:

$$\Delta = \int_a^b [G(t) - F(t)] U'(t) dt \qquad (5)$$

Où F et G sont les distributions cumulatives des loteries F et G et où l'on suppose pour des raisons de simplicité que F et de G prennent les valeurs 0 et 1 pour certains $a \leq 0$ et $b \geq 0$ respectivement. a et b correspondent aux deux points extrêmes d'inflexion dans la fonction d'utilité de Markowitz et l'on s'attend à ce qu'ils indiquent les niveaux de richesse extrêmes.

Comme l'indique l'équation (5) (c'est l'équation 5.1, p: 192 du document) la différence [$G(t)$ –$F(t)$] est mesurée en proportion avec $U'(t)$; autrement dit, les segments où la pente d' $U(t)$ est large sont plus étirés relativement aux segments où la pente est faible. Les secteurs près de l'origine sont magnifiés pour l'individu qui suit la théorie des prospects puisque la pente est plus grande près de l'origine tandis que les extrémités sont magnifiées pour l'individu qui suit la théorie de Markowizt puisque c'est là où la pente est plus grande.

Cependant, quand la fonction de transformation des probabilités est factorisée dans le processus de décision plus d'importance est donné aux conséquences extrêmes relativement aux conséquences intermédiaires (les détails sont donnés à page 196). Ainsi, quand cette fonction est factorisée dans l'équation (5), la différence d'utilité entre les loteries *F* et *G* est donnée par

$$\Delta = \int_a^0 [w(G(t)) - w(F(t))]U'(t)dt + \int_0^b [w(1-F(t)) - w(1-G(t))]U'(t)dt \qquad (6)$$

En conséquence, en parallèle à l'étirage horizontal, l'axe vertical de probabilité cumulée (0, 1) est étiré par la fonction de transformation des probabilités rendant les prospects près de 0 (possibilité) et près de 1 (certitude) plus souhaitables et magnifiant la différence entre F(t) et G(t) dans ces secteurs. Par conséquent, pour que *CPT* ne soit pas ambigue, il est nécessaire de généraliser les conditions de dominance stochastique pour capturer cet aspect important de *CPT* et également l'autre aspect de la théorie notamment l'aversion aux pertes.

Pour généraliser ces conditions de manière qu'elles caractérisent les préférences d'un individu qui adhère à *CPT* et les préférences d'un individu qui adhère à la théorie de Markowitz, la fonction de transformation des probabilités sous *CPT* en forme de *S* inversé est restreinte à un intervalle $W_c^d = W_c \cap W^d$ où W_c est la classe des fonctions de transformation des probabilités convexes entre *c* et 1 et W^d la classe des fonctions de transformation des probabilités concave entre 0 et *d*. *c* et *d* dénotent en même temps c^+ et c^- (ou d^+ et d^-) qui renvoient à w^+ et w^- les pondérations sous *CPT* pour les gains et les pertes respectivement.

Aussi, U_P est-elle définie comme la classe des fonctions d'utilité en forme de *S* telle que:

$U\in U_P$ si $U'\geq 0$ pour tout $x \neq 0$, $U''\geq 0$ pour $x < 0$ et $U''\leq 0$ pour $x > 0$, et

U_M est définie comme la classe des fonctions d'utilité en forme de S inversé telle que: $U\in U_M$ si $U'\geq 0$ pour tout $x \neq 0$, $U''\geq 0$ pour $x > 0$ et $U''\leq 0$ pour $x < 0$.

Les conditions sont énoncées formellement comme suit:

Proposition 5.3:

$F \succ_{PWSD} G$ si et seulement si $F \succcurlyeq G$ pour tout $U\in U_P$, $w^- \in W_{c^-}^{d^-}$, et $w^+ \in W_{c^+}^{d^+}$. De facon similaire,

$F \succ_{MWSD} G$ si et seulement si $F \succcurlyeq G$ pour tout $U\in U_M$, $w^- \in W_{c^-}^{d^-}$ et $w^+ \in W_{c^+}^{d^+}$;

$F \succ_{SWSD} G$ si et seulement si $F \succcurlyeq G$ pour tout $U \in U_{concave}$, $w^- \in W_{c^-}^{d^-}$, et $w^+ \in W_{c^+}^{d^+}$; et

$F \succ_{S*WSD} G$ si et seulement si $F \succcurlyeq G$ pour tout $U \in U_{convexe}$, $w^- \in W_{c^-}^{d^-}$, et $w^+ \in W_{c^+}^{d^+}$ ■

Ainsi la fonction de transformation des probabilités incorporée est restreinte à c et d dont le choix est le résultat d'un tradeoff. D'un côté, pour un c plus petit ou un d plus grand, la portée des applications de ces conditions augmente mais d'un autre côté, un W_c^d trop restreint ne contiendrait peut-être pas les fonctions désirées. Les choix de Baucells et Heukamp (2006) pour c et d ont été pris dans l'intevalle [0.05 to 0.88] déliéné par une analyse de sensisivité. (Cette partie est traitée aux pages 197-200 où Figure 13 montre la fonction de probabilité W_c^d alors que les figures 12 et 14 montrent la distribution de probabilités cumulative pour les tasks I et VII respectivement).

Pour la condition de dominance stochastique qui tient compte de la fonction de transformation des probabilités et de l'aversion aux pertes, U_L est définie comme la classe des fonctions d'utilité telle que:

$U\in U_L$ si $U'(-x)\geq U'(x)$ pour tout $x \geq 0$, et $U_{PL} = U_P \cap U_L$, et la condition est énoncée formellement comme suit:

Proposition 5.5:

$F \succ_{PWLSD} G$ si et seulement si $F \succcurlyeq G$ pour tout $U \in U_{PL}$, $w^- \in W_{c^-}^{d^-}$ et $w^+ \in W_{c^+}^{d^+}$ tel que $s^- \geq s^+$ ■

L'extension des conditions de dominance stochastique à *CPT* permet de tester: 1) la courbe de la fonction d'utilité et/ou la présence de l'aversion aux pertes postulant une fonction de transformation de probabilité en forme de *S* inversé ; 2) la courbe de cette dernière postulant les caractéristiques empiriques pour la fonction d'utilité de *CPT*; 3) finalement, si l'on postule que toutes les caractéristiques empiriques de *CPT* tiennent, une violation de la condition de domaince stochastique implique une violation de *CPT*.

L'APPLICATION EXPÉRIMENTALE

Les loteries de Baucells et de Heukamp (2006) ont été utilisées pour examiner de façon qualitative les préférences des étudiants en MBA à Arizona State University (ASU). Les étudiants étaient au nombre de 40 et avaient été payés 10$ chacun pour compléter le questionnaire. Plus spécifiquement, ils avaient à choisir pour 20 paires de loteries entre deux investissements *F* et *G* introduits comme suit: "Supposez que vous avez décidé d'investir 10000$ en action *F* ou en action *G*. Laquelle choisirez vous, *F* ou *G* quand il est donné que le dollar gain ou perte dans un mois sera comme suit." (Les paires de loteries ou tasks se trouvent dans l'annexe D).

Comme les loteries *F* et *G* sont conçues de facon qu'elles aient la même valeur mathématique espérée et que $F \succ_{PWSD} G$ et $G \succ_{MWSD} F$ le choix de *F* (*G*) implique le rejet de la fonction d'utilité du décideur en forme de *S* inversé en faveur de celle en forme de *S*. Le format du questionnaire le permettant, deux tasks ont été ajoutés pour étudier l'impact de la probabilité globale de gains/pertes dans les jeux mixtes c'est à dire comportant des gains et des pertes et les étudiants ont été également invités à commenter sur les choix qu'ils ont faits.

ANALYSE DES DONNEES

Les 20 tasks utilisés dans cette expérience ont été initialement construits en réponse à la déclaration de Levy et Levy (2002) que la théorie des prospects importe peu (*is much ado about nothing*). En effet, les sujets de Levy et Levy (2002) qui avaient à choisir *F* ou *G* pour trois tasks (I-III) où $F \succ_{PSD} G$ et $G \succ_{MSD} F$ avaient opté pour *G* dans les proportions suivantes: 71%, 62% et 76% respectivement.

Aussi, les tasks I-III de Baucells and Heukamp (2006) imitent-ils les tasks de Levy and Levy (2002) et discriminent entre l'hypothèse d'une fonction d'utilité en forme de *S* et celle en forme de *S* inversé, le but étant d'obtenir précisément des réponses pareilles c'est à dire $U \notin U_P$. Parce que $F \succ_{PSD} G$, si *w* est linéaire (convexe) partout alors les choix favorisant *G* impliquent que $U \notin U_P$.

Contrastés avec ces tasks (I-III), les tasks qui suivent (IV-VIII) discriminent entre l'hypothèse d'une fonction d'utilité en forme de *S* et celle en forme de *S* inversé factorisant cependant dans la décision la fonction de transformation des probabilités. Ainsi, les loteries *F* et *G* sont des modifications des tasks I-III conçues de facon que $F \succ_{PWSD} G$ et $G \succ_{MWSD} F$ avec $w \in W_{0,1}^{0,4}$, pour $d \leq 0{,}74$ et $c \geq 0{,}1$. Le choix de *F* c'est à dire $U \in U_P$ combiné aves les résultats des tasks I-III implique que la fonction de transformation des probabilités ne peut être linéaire ou convexe partout.

Pour spécifiquement examiner la courbe de cette dernière près de l'origine et postulant que les caractéristiques empiriques de *CPT* pour la fonction d'utilité tiennent, le task IX est construit comme une modification du task I avec un léger changement dans la probabilité attachée aux conséquences communes. Dans *F*, un montant maximal a été ajouté avec une probabilité de 2% et dans *G* un montant minimal a été ajouté avec une probabilité de 2%. Conçu comme $F \succ_{PWSD} G$ and $G \succ_{MWSD} F$ ce task teste conjointement avec le task I si l'hypothèse $w \in W_{0.02}$ peut être rejetée.

Les quatre tasks qui suivent sont construits pour refuter l'argument que la concavité pour les gains et la convexité pour les pertes sont poussées par l'effet de certitude. Aussi, les tasks X-XI sont-ils construits comme des loteries toutes en gains et examinent la convavité pour les gains et les tasks XII-XIII comme des loteries toutes en pertes et examinent la convexité pour les pertes. Dans tous ces tasks aucune conséquence n'est certaine et tous satisfont $F \succ_{PWSD} G$ pour $c \geq 0,1$.

Quant à l'aversion aux pertes, sept tasks sont conçus pour l'examiner. Les tasks XIV-XVI sont conçus comme $F \succ_{W_c^d LSD} G$ et examinent uniquement l'aversion aux pertes sans aucun postulat sur la fonction de l'utilité. Pour ces tasks, le choix de *F* implique donc l'aversion aux pertes. Les tasks XVII-XX par contre sont conçus comme $F \succ_{PW_c^d LSD} G$ et examinent conjointement l'aversion pour les pertes et la concavité/convexité de la fonction d'utilité. Pour les tasks XVII et XX le choix de *F* implique que les étudiants sont averses aux pertes et que leur fonction d'utilité est concave pour les gains alors que le choix de *F* dans les tasks XVIII et XIX implique que les étudiants sont averses auxs pertes et que leur fonction d'utilité est convexe pour les pertes.

Finalement, les deux derniers tasks examinent l'effet de la probabilité globale des gains/pertes. Les étudiants ont été présentés avec les loteries suivantes (Payne, 2005): ($100, 0.2; $50, 0.2; $0, 0.2; $-25, 0.2; $-50, 0.2). Ils devaient dans une première étape ajouter (38$) à 0$ ou à 100$ et dans une deuxième étape ajouter (38$) à 50$ ou à 100$, le choix d'ajouter (38$) à (0$) impliquant une préférence pour une plus grande probabilité globale des gains. (Plus de détails sont donnés aux pages 204-205).

LES RESULTATS

Les tasks ont été donc divisés de facon que plusieurs hypothèses puissent être testées et les résultats obtenus indiquent que:

1) pour la courbe de la fonction d'utilité, la majorité des étudiants a choisi *F*, choix qui implique le rejet de la fonction d'utilité de l'étudiant en forme de *S* inversé en faveur de celle en forme de *S*. En effet, pour les tasks IV-VIII comme prévu les étudiants ont choisi *F* (67.5%, 85%, 87.5%, 85%, et 92.5%) respectivement. Ces proportions sont pareilles à celles de Baucells et Heukamp (2006): 61%, 84%, 66%, 76% et 84% respectivement. Pour les tasks X-XIII, les étudiants ont choisi *F* dans le proportions 85%, 87,5% 72,5 et 85% respectivement montrant que la concavité pour les gains et la convexité pour les pertes ne sont pas poussées par l'effet de certitude. Pour les tasks IV- XIII les valeurs de probabilité sont significatives et rejettent l'hypothèse nulle que % $F = 0.5$. (Ces valeurs se trouvent dans la Table 20).

2) pour la fonction de transformation des probabilités, également comme prévu et pareillement à Levy et Levy (2002) et à Baucells et Heukamp (2006), les étudiants ont opté pour *G* pour les tasks I-III (62,5 %, 60%, 62,5% respectivement). Ainsi, le choix de *F* c'est à dire $U \in U_P$ pour les tasks IV-VIII combiné avec le choix de *G* c'est à dire $U \notin U_P$ dans les tasks I-III implique que la fonction de transformation des probabilités ne peut être linéaire ou convexe partout.

3) pour le comportement de la fonction de transformation des probabilités *près de l'origine*, le revirement des préférences de *G* à *F* pour les tasks I et IX (62,5% à 92,5% respectivement) montre que $w \in W_{0.02}$ est une classe plausible de ces fonctions et souligne le changement brusque de la fonction près de l'origine, suggérant que les décideurs utilisent l'intervalle des conséquences comme critère de décision.

4) pour ce qui est de l'aversion aux pertes, les résultats des tasks XIV-XVI conçus pour déterminer uniquement l'aversion aux pertes montrent que les étudiants sont averses aux

pertes quand les probabilités sont les mêmes ou semblables. En effet, pour le task XIV, 60% des étudiants ont choisi *F* pour éviter une perte. La majorité de ceux qui ont opté pour *G* n'a pas été attirée par la probabilité de 80% de ne rien gagner de *F*.

Pour le task XIV la fréquence de l'aversion aux pertes chez les étudiants à ASU (60%) est plus grande que la fréquence (43%) chez les sujets de Baucells et de Heukamp (2006). Ces derniers sont néanmoins composés d'étudiants et de professionnels qui ont choisi différemment: la proportion des étudiants qui a choisi *F* comparée à celle des professionnels était (48% vs 35%). Ceci supporte l’hypothèse que les professionnels sont moins averses aux pertes que les étudiants. A cet égard, il convient de rappeler qu’une comparaison des résultats de l'expérience I avec ceux d'Abdellaoui et al. (2006) a également montré que l'étudiant médian est plus averse aux pertes que le professionnel médian.

Le task XV où la perte maximale augmente (par rapport au task XIV), montre que l'aversion aux pertes dans des loteries mixtes augmente à mesure que les enjeux augmentent. En effet, la proportion d'étudiants qui ont choisi *F* (67.5%) a augmenté. Certes, la probabilité de ne rien gagner de *F* a diminué de 80% à 60% (pour les tasks XIV et XV respectivement).

Pour le task XVI qui examine l'aversion pour les pertes plus près de l'origine relativement au task XV, moins d'étudiants (55%) ont préféré *F* à *G* préférant la plus grande probabilité pour le gain le plus élevé dans *G* pour des probabilités globales égales de gains/pertes.

5) les tasks XVII-XX sont un essai global pour *CPT*. Excepté pour le task XVII, qui montre une préférence pour la probabilité globale de gain la plus élevée, *F* est clairement préféré à *G* conformément avec les prévisions de *CPT*: 40%, 85%, 70%, 87,5% respectivement. Pour le task XVII, 60% des étudiants ont déclaré avoir choisi *G* pour la pour la grande probabilité globale de gain (70% dans *G* vs 50% en *F*) ou la probabilité

plus élevée du gain maximal et ainsi qu'ils se sont exprimés "500$, ce n'est pas beaucoup à perdre".

6) Finalement, Les deux derniers tasks, où 82%, 67.5% respectivement des étudiants ont choisi *F* confirme que la probabilité globale de gains/pertes est un facteur décision important. (Les tables 17, 18, 19, 20 p: 206-209) résument les résultats de l'expérience II respectivement).

CONCLUSION

Pour généraliser les conditions de dominance stochastique de manière qu'elles caractérisent les préférences d'un individu qui adhère à *CPT*, et les préférences d'un individu qui adhère à la théorie de Markowitz, Baucells et Heukamp (2006) restreignent la fonction de transformation de probabilité en forme de *S* inversé à un intervalle $W_c^d = W_c \cap W^d$. Le choix cependant de *c/d* est le résultat d'un tradeoff pris dans l'intevalle [0.05 to 0.88] déliéné par une analyse de sensisivité et de l'avis même de auteurs un désavantage par rapport à d'autres méthodes (Abdellaoui, 2000).

Leurs conditions tiennent aussi compte de l'aversion aux pertes telle qu'elle est définie par Wakker et Tversky (1993). Ainsi, l'extension des conditions de dominance stochastique à toute la théorie *CPT* permet de les utiliser comme guides pour concevoir des paires de loteries qui sont en compétition directe entre *CPT* (1992) et Markowitz (1952). Ainsi, pour deux loteries *F* et *G* conçues de facon que $F \succ_{PWSD} G$ and $G \succ_{MWSD} F$ le choix de *F* implique le rejet de la fonction d'utilité du l'étudiant en forme de *S* inversé en faveur de celle en forme de *S*. Alors que pour deux loteries *F* et *G* conçues de facon que $F \succ_{W_c^d LSD} G$, le choix de *F* implique que l' hypothèse que la fonction d'utilité est plus pentue pour les pertes que pour les gains n'est pas rejetée.

En bref, les résultats de l'expérience II rejettent l'hypothèse que la fonction d'utilité d'étudiants en MBA est convexe pour les gains et concave pour les pertes et sont compatibles avec une fonction d'utilité en forme de *S*. En outre, ils réaffirment l'importance de l'incorporation de la fonction de transformation des probabilités et montrent que l'aversion aux pertes existe pourvu que les probabilités soient les mêmes ou similaires. Cette condition est importante comme le souligne le résultat du task XVII, le seul qui viole la troisième hypothèse (violation de *CPT*): 60% des étudiants ont déclaré avoir choisi *G* soit pour la probabilité globale élevée de gain ou la probabilité élevée du gain maximal combinée avec une perte extême limitée.

CONCLUSION GÉNÉRALE

La prise en compte des préférences de l'individu dans la prise de décision face à l'incertain remonte au 18ème siècle où Cramer (1728) et Bernoulli (1738) ont proposé indépendamment le modèle de l'utilité espérée. Axiomatizé deux cent ans plus tard par vNM (1944) pour la prise de décision dans le risque et Savage (1954) et Anscombe et Aumann (1963) pour la prise de décision dans l'incertain, ce modèle normatif est aujourd'hui indéfendable comme modèle descriptif du comportement. En effet, les travaux expérimentaux qui ont suivi les travaux théoriques ont montré que les gens violent systématiquement certains axiomes de *EU* (e.g. Allais, 1953; Ellsberg, 1961; Kahneman et Tversky, 1979). L'évidence empirique a motivé les chercheurs à développer des théories alternatives de choix pour tenir compte des violations observées. Parmi ces théories, *CPT* explique la plupart des violations et les phénomènes que *EU* est utilisée pour expliquer grâce à l'intégration dans la théorie d'éléments non-normatifs notamment: l'effet de réflexion, l'aversion aux pertes et le traitement subjectif des probabilités qui découlent de la perception d'un point de référence. Tout en gardant le principe de cohérence, elle raffine en plus ce qu'on comprend par l'attitude au risque en décomposant le risque en trois facteurs: un fonction d'utilité, une fonction de probabilité et une aversion aux pertes.

La popularité de *CPT* et la possibilité d'expliquer les anomalies sous *EU* par des préférences averses aux pertes ont inspiré des économistes à rechercher des méthodes qui peuvent éliciter/examiner (quantitativement/qualitativement) des fonctions d'utilité sous *CPT* et tester l'aversion aux pertes et mesurer son degré. Ainsi, Abdellaoui et al. (2006) élicitent sous *CPT* l'utilité pour les gains et pour les pertes simultanément pour mesurer dans une seconde étape l'aversion aux pertes. Baucells et Heukamp (2006) étendent les conditions de dominance stochastique à *CPT*. Ce travail expérimental utilise la première pour éliciter les fonctions d'utilités des gérants de portefeuilles et la deuxième pour inférer les préférences d'étudiants en MBA.

Les résultats de l'expérience I suivant Abdellaoui et al. (2006) montrent 1) que le patron le plus commun est la concavité pour les gains et la convexité pour les pertes. En d’autres termes plus de gérants de portefeuilles (58%) que d’étudiants (37% à 54%) sont aggressifs en cas de pertes sûres vs des pertes probables. Pour l’ajustement paramétrique “puissance” le coefficient médian pour les pertes est 0,9 (dans la marge des études mentionnées ci-dessus), alors que pour l’exponentielle, le coefficient médian est 0,49; 2) le gérant de portefeuilles médian est moins averse pour les pertes pour les définitions globales alors que pour la définition locale de Köbberling et Wakker (2005) il est plutôt non averse aux pertes: 0,74 vs 2,52 respectivement.

Néanmoins, Les résultats de l'expérience II suivant Baucells and Heukamp (2006) confirment que pour la majorité des étudiants la fonction d’utilité en *S* inversé est rejetée en faveur d’une fonction d’utilité en forme de *S* et que l’hypothèse de l’aversion aux pertes ne l’est pas quand les probabilités sont les mêmes ou similaires. Pour l’aversion aux pertes, la différence entre les fonctions d’utilité de l’étudiant médian et du gérant de portefeuilles médian obtenues selon la méthode Abdellaoui et al. (2006) peut donc être due à des facteurs de profession, notamment: la formation et le niveau de connaissance acquis sur le terrain, la variation du degré d’aversion aux pertes avec le mouvement récent (en hausse) du marché boursier, ainsi qu’aux compensations offertes en fin d’année. Pour ce qui est de la convexité de la fonction d’utilité pour les pertes, il semble qu’il est assez pénible aux gérants de portefeuilles de réaliser leurs pertes et d’admettre avoir eu tort (les arguments sont présentés aux pages 212-216 du document).

REMARQUES ET DIRECTIONS POUR RECHERCHE FUTURE

Baucells and Heukamp (2006) proposent une nouvelle méthode aux expérimentateurs intéréssés par la falsification d'une hypothèse particulière concernant la forme de la fonction d'utilité ou la fonction de transformation des probabilités, sans avoir à l'éliciter.

Pour les expérimentateurs qui s'intéressent à l'exploration entière de la fonction d'utilité, et la quantification du degré d'aversion aux pertes et son occurrence générale, Abdellaoui et al. (2006) offrent une méthode non-paramétrique qui pourrait valider certaines fonctionelles qui sont raisonnables. La méthode est basée sur l'élicitation de mipoints d'utilité qui est souvent utilisée dans l'axiomatisation des modèles de décision et exige moins de mesures en comparaison avec d'autres méthodes (Vind, 2003).

Combinés, les résultats de l'expérience I et ceux de l'expérience II indiquent l'importance de la dépendance d'un point de référence en tant qu'élément de modélisation économique comme préconisé par Rabin (1996) entre autres.

La convexité de la fonction d'utilité est un effet de dépendance d'un point de référence. Comme c'est une tâche assez exigeante (Levy et Wiener (1998) notent que changer de point de référence force l'investisseur à confronter et accepter ses pertes, ce qui est douloureux) une utilisation corrective de la théorie des perspectives en analyse de décision est conseillée ainsi qu' a été suggérée par Bleichrodt, Pinto et Wakker (2001) pour aider les gérants de portefeuilles à prendre de meilleures décisions dans leur interêt aussi bien que celui de leurs clients.

L'aversion aux pertes est un autre effet de la dépendance d'un point de référence. Shalev (2000) décrit l'existence de ce phénomène comme le résultat le plus saisissant des fonctions d'utilité dépendantes d'un point de référence et prolonge l'analyse des jeux pour inclure cette dépendance ainsi que l'aversion aux pertes. La première étape cependant pour examiner ses prévisions est de mesurer l'aversion aux pertes au niveau individuel. Pour une définition généralement admise de cette dernière, il faudrait peut-être la démêler de la fonction de l'utilité. Pour les définitions globales, elle ne peut en être séparée. Cependant, Köbberling et Wakker (2005) qui trouvent la séparation essentielle pour la recherche sur les points de référence variables définissent l'aversion aux pertes localement au point de référence avec une restriction qu'ils comptent néanmoins relaxer dans une recherche future. Leur coefficient d'aversion aux pertes qui permet de classifier tous les gérants de portefeuilles est soutenu par Schmidt and Zank (2002) qui caractérisent l'aversion au risque sous l'hypothèse *CPT* par une condition jointe de la fonction de l'utilité, de la fonction de la transformation des probabilités et de l'aversion aux pertes. Le coefficient de Köbberling et Wakker (2005) est une première approche axiomatique à l'aversion aux pertes en tant que composante logiquement indépendante de l'attitude envers le risque. Néanmoins, il est à noter que si on compare les portfolio-managers across les trois coefficients de Kahneman et Tversky (1979), Wakker et Tversky (1993) d'une part et les coefficients de Köbberling et Wakker (2005) d'une autre part, on trouve une grande différence.

Conçue pour décrire une population générale dans laquelle les différentes composantes de l'attitude au risque (fonction d'utilité, fonction de probabilité et aversion aux pertes) sont assez mixtes, *CPT* décrit un modèle naturel de réflexion et adresse les soucis des gérants de portefeuilles d'une manière que ne peut le faire la normative *EU*. Pourtant les théories qui décrivent le comportement des individus exhibant la prédominance extrême d'une composante tel que l'*EU* et/ou la *DT* sont utiles parce que n'importe quelle position extrême est plus claire et donc plus facilement reconnue et comprise que les positions intermédiaires qui en aucune manière ne contiennent ou ne réconcilient les positions extrêmes (Huxley, 1945). Dans ce sens, une théorie décrivant les individus qui exhibent une prédominance extrême de l'aversion pour les pertes est utile.

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GENERAL INTRODUCTION

This research provides an experimental investigation of the fundamental preferences of financial practitioners. The importance of the individual's preferences in decision making goes back in time to the 18th century when Cramer (1728) and Bernoulli (1738) proposed independently that individuals do not maximize expected monetary values but expected utilities, that is their subjective evaluation of the monetary values.

The expected utility model (*EU*) was not axiomatized however until 1944 by von Neumann and Morgenstern for decision making under risk[5], and a few years later by Savage (1954) and Anscombe and Aumnann (1963) for decision making under uncertainty.

The experimental works that followed the theoretical works showed however that people violated systematically the axioms of *EU* (e.g. Allais, 1953; Ellsberg, 1961; Kahneman and Tversky, 1979). The empirical evidence motivated researchers to develop alternative theories of choice under risk and uncertainty to accommodate the observed violations.

These models termed "non-expected utility" or "generalizations of expected utility" were tested subsequently in a probability triangle to find which if any best approximates actual behavior (Harless and Camerer, 1994; Hey and Orme, 1994; Carbone and Hey, 1995). The probability triangle is the set of probability distributions or lotteries that one can define on a set of three different outcomes. This simplex is generally represented by the unit triangle in a system of two rectangular axes. It is a useful device to display the theories' predictions. If the outcomes are fixed, the triangle contains the set of all possible lotteries with the left edge characterized by a zero probability for the best consequence,

[5] Bernoulli's expected utility, Cramer's and vNM's have the same mathematical form but the first two are assessed for certain outcomes and the third for risky outcomes.

the lower edge by a zero probability for the worst consequence and the hypotenuse by a zero probability for the middle consequence.
None of the generalizations of expected utility however was found to fit the empirical data significantly better than expected utility in the whole triangle, i.e. in all possible risky situations. According to Abdellaoui and Munier (1998), the reason is that decision makers preferences depend on the riskiness of the prospects they are facing, i.e. the region of the probability triangle that represents their situation. A result of their experiment is that non-expected utility models describe individuals' preferences better than *EU* outside the middle of the triangle. Starmer's (2000) survey evaluates the data from the triangle experiments and finds rank dependent utility models which do not violate monotonicity as "probably the best bet" among the alternatives to expected utility outside the interior of the triangle. In these models the probabilities are replaced by decision weights that reflect the impact of events on the desirability of the outcomes. These decision weights which result from weighting cumulative probabilities and sum to one are allocated to the outcomes depending on their respective ranks in the series.

Among the rank-dependent models, prospect theory (Tversky and Kahneman, 1992) based initially on "some pervasive effects people exhibit in their choices", (Kahneman and Tversky, 1979), also explains regularities of behavior in the field that are considered paradoxes under *EU*. Camerer (1988) shows that prospect theory explains the anomalies and the basic phenomena *EU* is used to explain in 10 fields because it integrates psychological insights into economics.

SUPPORT FOR THE EXPERIMENTAL FINDING

Sketched in brief, the main features of prospect theory are: 1) the utility function defined over gains and losses relative to a neutral reference point exhibits diminishing marginal sensitivity to deviations from the reference point giving rise to an *S* shaped utility function that is concave for gains ($u'' < 0$) and convex for losses ($u'' > 0$); 2) the utility function is steeper in the domain of losses than in the domain of gains if the value of a gain is larger in magnitude than the value of a commensurate loss, a characteristic labeled by the authors as loss aversion; and 3) decisions are based on subjective cumulative distributions given by $\pi = w(p)$ where p refers to the objective cumulative distribution, w to a subjective treatment of p such that $w'(.) > 0$, $w'(0) = 0$ and $w'(1) = 1$. The psychological insights integrated into the theory are hence: the reflection effect, loss aversion and the subjective treatment of probabilities.

The reflection effect is a core psychological element of prospect theory. It refers to the finding that the preferences between losses are "mirror images" of the preferences between gains (Kahneman and Tversky, 1979, p: 268) that is, if individuals prefer sure gains to probable gains, they would prefer probable losses to sure losses. The reflection effect explains the reluctance to realize losses in the stock market. Terence Odean (1988) finds that as Shefrin and Statman (1985) have predicted, investors realize their gains too soon and are reluctant to realize their losses except for December for tax-motivated selling and that their behavior has led them to lower returns. Graphically, investors would have a concave utility for gains and a convex utility for losses provided the elicitation method is robust against probability distortion (Abdellaoui et al., 2006). Previous elicitations, using the trade-off (Wakker and Deneffe, 1996) which filters out probability weighting, generally well corroborate the concavity for gains at the aggregate level and for most individuals (Abdellaoui, 2000; Abdellaoui, 2002; Bleichrodt and Pinto, 2000). The evidence for the convexity for losses is however, less clear-cut at the individual level (Abdellaoui, 2000; Fenema and van Assen, 1999) and moreover, Levy and Levy (2002) find a concave utility for losses and a convex utility for gains following Markowitz (1952).

Another core element of prospect theory's psychological findings is loss aversion. It refers to the asymmetry of gains and losses relative to a reference point with losses looming larger than commensurate gains. Kahneman and Tversky's (1979, p: 279) definition of loss aversion is as follows: An individual is loss averse if he dislikes symmetric 50-50 bets and moreover, the aversiveness to such bets increases with the absolute size of the stakes. Kahneman and Tversky show that the above behavioral definition of loss aversion is equivalent under prospect theory to a utility function which is steeper for losses than for gains. The latter result underlies the various definitions of loss aversion that followed: (Tversky and Kahneman, 1992; Wakker and Tversky, 1993; Bowman et al., 1999; Köbberling and Wakker, 2001; and Neilson, 2002).

The theoretical support for loss aversion comes from Rabin (2000) who shows that plausible degrees of risk aversion over modest stakes under expected utility imply unrealistically high degrees of risk aversion over large stakes while loss aversion implies small-scale risk aversion and plausible risk aversion over large stakes. Rabin's suggestion favoring the use of loss aversion to explain risk aversion has been reiterated in "Anomalies: Risk Aversion" by Rabin and Thaler (2001).

The empirical support for loss aversion comes from Camerer et al. (1997) for downward labor supply and from Putler (1992) and Hardie, Johnson, and Fader (1993) for asymmetric price elasticities. Loss aversion explains the endowment effect (Thaler, 1980; Loewenstein and Adler, 1995), and thus the disparity between the willingness to pay and the willingness to accept (Kahneman, Knetsch and Thaler, 1991; Bateman, Munro, Rhodes, Starmer and Sugden, 1997), the status quo bias (Samuelson and Zeckhauser, 1988), and the disposition effect (Weber and Camerer, 1988, Odean, 1998; Heath, Huddart and Lang, 1999). Benartzi and Thaler (1997) and Barberis, Huang and Santos (2001) found loss aversion necessary to explain the large premium investors demand to invest in stocks which was shown to imply an absurd degree of risk aversion under expected utility by Mehra and Prescott (1985).

The possibility of explaining the anomalies under *EU* by loss averse preferences inspired economists to look for methods that can test for loss aversion and measure its degree. Experimental studies in the laboratory of measurement of loss aversion coefficients in the aggregate include Fishburn and Kochenberger (1979); Tversky and Kahneman (1992); Bleichrodt, Pinto and Wakker (2001); and Pennigs and Smidts, (2003). Studies that have tested and measured loss aversion coefficients at the individual level are: Bleichrodt and Pinto (2002); Schmidt and Traub (2002); Abdellaoui et al. (2006).

The third experimental finding integrated into the theory is the non-linear probability weighting. In contrast to expected utility where the utility (of each possible outcome) is weighted by its probability, in prospect theory the utility is multiplied by a decision weight $\pi(p)$ which is a monotonic function of p but is not a probability (Tversky and Kahneman, 1986). The decision weight was introduced to model the tendency to overweight low probabilities and underweight moderate and large probabilities. This tendency was first observed by Preston and Baratta (1948). More recently, Diamond (1988) and Edwards (1996) found that subjects judging low probability/high consequence risks were more affected by the consequence size than by the probability whereas those judging high probability/low consequence risk tended to combine the two. According to MacCrimmon and Larsson (1979) subjects tend to choose the lottery with the more probable gain for high probabilities of gain and tend to choose the lottery with the highest gain for low probabilities of gain.

Further empirical support was later given by parametric studies that is assuming a specific probability weighting function: Tversky and Kahneman (1992); Camerer and Ho (1994); Tversky and Fox (1995); Wu and Gonzalez (1996); and by non-parametric studies: Abdellaoui (2000) and Bleichrodt and Pinto (2000).

RESEARCH'S MOTIVATION

> *"I hope to show that much success has already been had applying prospect theory to field data and to inspire economists and psychologists to spend more time in the wild."*
>
> *Colin Camerer (Kahneman and Tversky, 2000, p: 288)*

Until Abdellaoui et al. (2006) method, however, there was no method to elicit the utilities for gains and losses simultaneously and non-parametrically in a relatively short time. Wakker and Deneffe's (1996) method does not assume any parametric function but elicits the utilities for gains and losses separately, while Tversky and Kahneman's (1992) and

Jullien and Salanie's (2000) studies assume specific forms for utility. The availability of the Abdellaoui et al. (2006) method and the ease of its applicability offered the possibility to elicit non-parametrically the utility for gains and losses simultaneously and consequently to measure the loss aversion/gain seeking coefficient of practitioners in the stock market allowing at the same time a test of prospect theory in the latter field. The elicitations required extensive traveling the interviews being conducted in the practitioners' offices at their respective financial institutions. However, the potential benefits of eliciting the utility of financial practitioners and determining empirically their parameters matter to economists interested in modeling behavior, to researchers of the anomalies in the financial market and to decision analysts especially if they are corroborated by further field investigations.

Eliciting the shape of the utility function provides a descriptive perspective to model builders of a rational rule for decision making. Allais (1953; 1979) argues that rules of how people should behave under uncertainty must take into consideration how individuals actually behave.

Researchers interested in anomalies of the financial market would have available for their studies parameters found independent of any assumption. For instance, regarding the stock market, Benartzi and Thaler (1997) have used the Tversky and Kahneman (1992) estimates of investor utility function and loss aversion coefficients to compute the expected prospect values of stock and bonds in order to explain the equity premium puzzle. Tversky and Kahneman's (1992) method assumes a power form for the utility function despite the drawback of confounding the general test of the theory to a specific form, the estimation of all the parameters being problematic at that time (ibid, p:311).

Another benefit is in decision analysis where biased utilities can result in distorted economic predictions. Traditionally, decision analysis assumes the normative expected utility for calculating optimal decisions and for the elicitation of utilities. For the assumption to be valid prescriptively however, the preferences of the decision maker must be compatible with *EU*. The reluctance to realize losses evident in the practitioners' elicited utilities as will be seen below implies that they find it hard to formulate their decisions to sell a stock independently of its purchase price. The behavior violates *EU* but is compatible with prospect theory. A corrective use of prospect theory has been suggested by Kahneman and Tversky (1979); von Winterfeld and Edwards (1986); Fischnoff (1991); and Kahneman and Tversky (2000, p: 157). Bleichrodt, Pinto and Wakker (2001) propose the use of corrected utilities in prescriptions of optimal decisions, the correction based on parameters preferably found empirically.

RESEARCH'S OBJECTIVES

The empirical investigation of the shape of utility function for gains and losses is the primary objective of this experimental work which is divided in two parts: Experiment I and Experiment II. Each experiment uses a different and newly developed method for inferring preferences. Experiment I uses the Abdellaoui et al. (2006) method to elicit the preferences of financial practitioners and Experiment II uses the Baucells and Heukamp (2006) method to elicit the preferences of MBA students, potential future practitioners.

In Experiment I, financial practitioners' utilities are elicited non-parametrically and simultaneously for gains and losses. Parametric fittings to the power, exponential and the expo-power families are undertaken to find out with which family the non-parametric findings agree. The latter could validate reasonable functional forms and thus the use of certain types of parametric estimation procedures which have the advantage of smoothing response errors.

Abdellaoui et al.'s (2006) method also allows the measurement of loss aversion in the aggregate and at the individual level. There is not however a commonly accepted definition of loss aversion. The method nevertheless is definition-free and allows the measurement of loss aversion at the individual level under both global and local definitions. Abdellaoui et al.'s (2006) method can test thus prospect theory's basic tenets regarding the utility function referred to as value function in prospect theory's framework.

The results in the field for the shape of the utility function and for loss aversion are then contrasted with the results of Abdellaoui et al. (2006) in the laboratory to examine differences in behavior if any between practitioners and students.

Baucells and Heukamp (2006) having also made available a new method[6], Experiment II investigates the preferences of MBA students, potential financial practitioners and tests for loss aversion. More specifically, Experiment II investigates the shape of the utility function according to stochastic dominance criteria Baucells and Heukamp (2006) have newly designed. The latter were motivated in developing them by Levy and Levy (2002) who had found an inverse *S* shaped utility function following Markowitz (1952). Baucells and Heukamp's (2006) experiments using lotteries designed on the above criteria reject the *S* shape and find general evidence for loss aversion. In Experiment II, MBA students at Arizona State University (ASU) were asked in groups of 4-5 to respond to the 20 tasks constructed by Baucells and Heukamp (2006). The latter having hypothesized the impact of the overall probability in mixed gambles, Experiment II has also included tests of this effect and the format of the tasks allowing it, the students were also asked to comment on their choices in order to understand their reasons for any shift in behavior that might occur as in between gain seeking and loss aversion for instance.

[6] The idea of undertaking Experiment II came when Baucells, M. presented the paper he and Heukamp, H. have co-authored at the FUR XI-Paris 2004 in France which I attended. Experiment II was thus implemented based on the paper presented at that time and not on the reviewed paper that was published in 2006.

GENERAL OUTLINE

Both experiments I and II aim at inferring individuals' preferences from the choices presented to them. The choices were constructed within the framework of individual decision making theory under risk. This work is thus divided in two parts: Part I presents the theoretical framework of decision making under risk and Part II the experimental work.

Part I consists of an introduction and four chapters. The introduction situates the major milestones in the history of decision theory up to the early 20th century representation of preferences which was derived under certainty. The distinction between risk and uncertainty is introduced and narrows the theoretical part to the representation of preferences in situations of risk. The prospects presented to the practitioners and to the MBA students to infer their preferences were designed for situations under risk a special case of uncertainty. Consequently, the elicited preferences of the financial practitioners and the MBA students are approximations of their true preferences only under risk. The concentration on risk as opposed to uncertainty will be observed for all four chapters pertaining to the theoretical framework.[7]

Chapter I presents thus expected utility the first individual decision making theory for risk with a brief description of subjective expected utility, the model being a normative theory for both risk and uncertainty. The two models share the same bilinear form and have similar axioms the main difference between the two is the representation of beliefs.

Chapter II explores the violations of the normative axioms of expected utility that dictate how people should behave and presents the alternative models of decision making

[7] Camerer and Weber (1992), Karni and Schmeidler, (1991*a*, part 3) and Fishburn (1988, ch. 7- 9) provide reviews of developments in modeling preferences under uncertainty.

designed to have more descriptive power yet retain “desirable” criteria of rationality like transitivity and monotonicity (Quiggin, 1982).

Chapter III is devoted to the theory that has had “much success in the field”, cumulative prospect theory (1992), a review of the original prospect theory (1979). The chapter explores the model’s psychological elements before presenting the formal model and the axiomatization of the model under risk. The last section is concerned with the characterization of risk aversion under the different theories. The notions of risk aversion defined independently of any model are presented first to establish in a second step the links between these notions and the preference functions under the different theories. This final section shows that the understanding of risk aversion has been refined to a large extent in the alternative theories frameworks’ challenging as a consequence the role of utility in representing at the same time attitude towards consequences and attitude towards beliefs. Thus, observed risk behaviors of different nature which are not distinguished under expected utility are separated under the alternative models into independent components. Moreover, prospect theory’s empirical realism further refines the understanding of risk aversion and argues for its use in applied economics at least in the specific context of portfolio selection the choice domain of financial practitioners and MBA students.

Part II presents the experimental work and consists of two experiments. To each, a chapter is devoted which includes a brief introduction showing the gradual yet portentous development of the elicitation method from its predecessors, the elicitation procedure per se, the data analysis and the results.

In chapter IV the utilities of practitioners are elicited non-parametrically following the Abdellaoui et al.’s (2006) method and their loss aversion coefficients measured. Section 1 presents the parameter-free method. Section 2 is concerned with experimental application. Section 3 presents the data analysis which includes the non-parametric elicitations, the parametric fitting and the measurement of loss aversion in the aggregate as well as the individual level. Section 4 and 5 present the results related to the practitioners' shape of the utility function and their loss aversion respectively. A final section discusses the results and concludes.

In chapter V the shape of utility functions of MBA students are inferred from lotteries based on stochastic dominance criteria developed by Baucells and Heukamp (2006). Section 1 presents the intuition for clarification for stochastic dominance conditions followed by the characterization of preferences using these criteria; section 2, the experiment per se that is the objective, and the source of data; section 3 consists of the analysis of data and section 4 and 5 give the results pertaining to the shape of the utility function and loss aversion of MBA students respectively; section 6 investigates the impact of the overall probability of gain/loss in mixed gambles. A final section discusses the results and concludes.

The general conclusion is followed by some remarks and directions for future research. The bibliography and the Appendix are presented next.

PART I: UTILITY THEORY AND DECISION MAKING UNDER RISK

Part I of this research sets the theoretical scene for the experiments in Part II in which the fundamental preferences of financial practitioners and MBA students in situations of risk are elicited. Elicitation procedures presuppose the individual to decide among given alternatives *as if* he is optimizing some underlying preference function. Generally, the standard preference function in the elicitation procedure results from expected utility the normative model of choice which describes how rational agents ought to choose.

However, for a normative model to be operational and prescriptively useful in applications designed to aid decision makers, the actual behavior of an individual in simple choice settings must be compatible with the behavior assumed in the model. In other words, expected utility must be also defensible as a descriptive model of the behavior of unaided decision makers otherwise, assuming expected utility to elicit the individual's preferences and attitudes towards risk cannot be meaningful.

Expected utility has been found however to be violated systematically in experimental works. The earlier violations found by Allais (1953), Ellsberg (1961) and Kahneman and Tversky (1979), stimulated the research for an alternative model normatively attractive but with more descriptive power to accommodate the violations. The research does not always cohere however because the researchers' purposes are different some focusing on the theories per se, that is on the mathematical properties of their axioms, others on their descriptive validity, others yet on their implications in the field (Camerer and Weber, 1992).

Nevertheless, reviews of the alternative models (Camerer, 1992; Schmidt, 2002; and Starmer, 2000) which organize the data from a large amount of research show a number of stylized facts across the various studies that not only promote what has been called since ancient times a *hedgehog*[8] perspective but also are key ingredients in the selection of the theory that might show, when tested in Experiments I and II, a good approximation of the underlying preference function the individual is assumed to optimize. For if the measurement of the model's postulated parameters or functions in the laboratory and/or the field shows the approximation is good, the model is approximately true and useful despite unrealistic axioms (Camerer and Weber, 1992).

In hindsight, the violations of the expected utility properties which are violations of its axioms can be categorized under two broad headings: those that violate the *form* of the preference function and therefore violate only the independence axiom responsible for restricting it strongly and those that violate the *existence* of a real-valued continuous preference function and therefore challenge the axioms of ordering and continuity.

The alternative models that were developed in response to the violations are categorized accordingly: Those that can be expressed in terms of a single preference function but generalize expected utility by weakening the independence axiom and those that cannot be reduced to a single function. The former models fall under the category of the so-called conventional approach, and the latter fall under the non-conventional approach Starmer (2000). One of these alternative models is however a cross fertilization of the two strategies. Prospect theory (1992) assumes a single preference function, explains the violation of the independence axiom *and* the strong empirical evidence for pervasive phenomena like loss aversion and the reflection effect, which are inconsistent with an evaluation in terms of final wealth as in the conventional approach.

Part I reviews the normative base theory expected utility and the theories designed as alternatives in an attempt to find the model that can as much as possible reconciles rational assumptions with experimental facts in order to be usefully assumed in experimental elicitations. Expected utility is the natural point of departure for

[8] "The fox knows many things, but the hedgehog knows one big thing" (Ignatieff, 1998, the Greek classical poet Archilochus)

understanding the alternatives since they are generalizations of this standard theory. The review is however narrowed in two significant respects:

First, the focus is on modeling choice under risk as opposed to the more general modeling under uncertainty since the objective of Part I is to present the theoretical framework that corresponds appropriately to the empirical tests presented in Part II and the prospects presented to the practitioners and to the MBA students to infer their preferences were designed for situations of risk.

To distinguish between risk and uncertainty is to distinguish between whether the probability, "uncertainty's yardstick", (Fishburn, 1970, p: 101) is known or unknown. The decision maker is in a situation of *risk* if each action leads to one of a set of possible specific outcomes, each outcome occurring with an objective probability, agreed-upon and impersonal. Certainty is a case of risk where the probabilities are 0 or 1.

The decision making is in a situation of uncertainty if each action leads to one of possible specific outcomes, the probability of which is at best subjective (known ambiguously) or at worst indeterminate.[9] Risk is a case of uncertainty where the probability is known unambiguously.

Second, the review of the alternative theories to expected utility under risk concentrates on models presupposing a single preference function, yet defensible as a descriptive models of actual behavior. The rationale for keeping this assumption is that it is an important tenet of coherence and it's not unreasonable to assume that people wish to obey it even if it is a demanding task. Another is that although, the empirical violations of a single preference function do make a case for the non-conventional models when these are not judged using rational criteria, abandoning the notion of well-defined preferences

[9] Subjective probability refers to a personal degree of belief as opposed to an impersonal, agreed-upon degree of belief. Under expected utility it's always known (inferred from bets), however, Ellsberg (1961) showed that because of missing information regarding "the amount, type, reliability and unanimity of information" individuals do not treat subjective probabilities as objective probabilities. In that case, subjective probability is known ambiguously. Camerer and Weber (1992) discuss ambiguity in length.

requires changes that increase the complexity of the theory, reduce its predictive yield and render it less compatible with the rest of economic theory[10] (Starmer, 2000). In Arrow's (1995) words, "these models are apt to be very correct, it's just their predictions are a lot more vague than those implied by rationality; rationality is unique."

Part I, has three chapters and is organized as follows:

Chapter I presents at first the history of the concept of utility which situates the major milestones up to its modern form, the von Neumann and Morgenstern expected utility theory (1944). The axiomatization of this first theory for individual decision making under risk is then presented in terms of simple probability measures following Fishburn (1970). The more general subjective expected utility is presented next showing risk as a case of uncertainty and finally a few words on the directions expected utility theory under risk took in the years that followed its axiomatization.

Chapter II provides 1) an overview of the violations of the properties of *EU* which are violations of its axioms and 2) sets out the alternative theories that can account for the descriptive invalidity of expected utility while retaining the principle of coherence. For the latter, chapter II builds on previous overviews (Camerer, 1992; Schmidt, 2002) and particularly on Starmer's (2000) which evaluates the alternatives against empirical evidence and finds the rank dependent weighting models to be "probably the best bet" (Starmer, 2000, p: 359).

Chapter III explores the different aspects of the most popular among the many alternative models constructed, prospect theory (1992) which in addition to retaining desirable criteria of rationality like transitivity and monotonicity provides a convenient way of modeling the influence of pervasive phenomena like loss aversion and the reflection effect on choice. The last section of the chapter is devoted to the modeling of risk preferences under the different utility theories, the shape of the utility function elicited implying different risk attitudes depending on the model assumed. The equivalence, for

[10] Tversky and Kahneman (1991) and Kahneman, Knetch and Thaler (1991) argue however for abandoning the notion of stable preferences in favor of preferences indexed to a reference level which can be located for particular cases.

instance, of the convexity of the utility function to risk seeking under *EU* does not hold under prospect theory because of the non-linearity of the probability function in the latter. The section begins with the notions of risk aversion defined behaviorally and characterizes subsequently risk aversion under the different theories.

CHAPTER I. EXPECTED UTILITY THEORY.

Chapter I is devoted to the normative theory of decision making. It consists of three sections. Section 1.1 narrates the earlier phases in utility theory up to the middle of the twentieth century. The axiomatizations of expected utility under risk and under uncertainty are presented in section 1.2 and 1.3 respectively. The final section reviews briefly the experimental works which followed the theoretical works.

1.1 *THE HISTORY OF THE CONCEPT OF UTILITY*

> *"To change shape is in the very nature of history, because it is in the nature of history to go on adding to itself".*
>
> *Arnold Toynbee (1972, p: 13)*

History has added 200 years to its length between Bernoulli's (1738) proposal of expected utility maximization and von Neumann and Morgenstern's (1944) modern concept of utility and each portentous addition changed the whole. To establish a common language, this section reviews the major milestones in the history of utility theory during that period. The first is *Daniel Bernoulli*'s (1738) proposal of a theory of expected utility as a basis for a decision making under risk using a logarithmic function of wealth. The embracement of the utility concept by *Jeremy Bentham* (1789) in an attempt to establish it as the basis of social policy constitutes the second.

The principle of diminishing marginal utility and the relationship between demand and utility were not established however until *the marginalist revolution* in the 1870's. This third high point in the history of utility theory is soon followed by the *ordinalist counter-revolution* whereby the utility under certainty and the cardinal comparisons were abandoned in favor of an ordinalist vision of utility in which the principle of diminishing marginal utility is nevertheless implicit.

The *von Neumann and Morgenstern*'s (1944) landmark constitutes the modern concept of expected utility. von Neumann and Morgenstern were not however, the first to incorporate explicitly uncertainty in the preference structure. Earlier, Ramsey (1931) has constructed the first operational model of subjective expected utility in which preferences are represented formally in terms of a utility function and a probability function and Savage's (1954) is a complete axiomatization of this earlier model. The von Newmann and Morgenstern (1944) and the Savage (1954) axiomatizations will be presented in the subsequent sections.

1.1.1 How It Began

The concept of expected utility maximization was introduced by Bernoulli (1738) and Cramer (1728) independently in response to the inadequacy of the expected value model[11] to evaluate a game devised by Bernoulli's cousin the so-called the St-Petersburg puzzle: A fair coin is tossed until heads appears. The player receives 2^n if the first head appears on trial n whose probability of occurrence is $(1/2)^n$. What price is the player expected to pay to enter the game?

The puzzle is that the expected value of the gamble is infinite as can be seen:

Expected Value = $\frac{1}{2}*2+\frac{1}{4}*4+\frac{1}{8}*8+...=1+1+1+\ldots=+\infty$

Yet most people would find it reasonable to pay a relatively small amount to play. Bernoulli suggested that people didn't maximize expected values but expected utilities, that is, they averaged their subjective evaluations of the monetary outcomes rather than the monetary outcomes themselves: "the determination of the value of an item must not be based on its price but rather on the *utility* it yields. The price of an item is dependent

[11]Probability theory and its application to problems of gambling were already highly evolved in the 18^{th} century. The evaluation of a game by the expected value decision rule goes back in time to Pascal (1623-1662) and Fermat (1601-1665).

only on the thing itself and is equal for everyone; the utility however is dependent on the particular circumstances of the person making the estimate." (Bernoulli, 1738, p: 24).

Bernoulli posited utility as a function of wealth that increases with wealth at a decreasing rate: "the utility resulting from an increase in wealth will be inversely proportionate to the quantity of goods previously possessed." The St Petersbourg's expected utility with U a logarithmic function of the monetary outcome is equal to 2.9 a reasonable price. Bernoulli's choice however of a logarithmic function, was "*ad hoc*".[12]

1.1.2 How It Evolved

The concept of utility didn't evolve till 50 years later with Jeremy Bentham (1789) who founded utilitarianism. The principle of diminishing marginal utility was not recognized however till the marginalists' revolution in the 19th century and is implicit in the counter-revolution of the ordinalists that soon followed. The expected utility model intuited by Bernoulli and Cramer 200 years earlier in response to the inadequacy of the expected value model was not axiomatized till the pioneer work of vNM (1944).

Bentham

Utility is referred to fifty years later by Jeremy Bentham (1789) as the pleasure or relief from pain associated with the consumption of a good or a commodity and becomes the foundation of utilitarianism, a theory of social choice in which Bentham advocates aggregating individual utilities into one total utility and maximizing it, to achieve "the greatest good for the greatest number".[13]

The Marginalist Revolution

However, the insight that the principle of diminishing marginal utility rather than total utility is the basis for engaging in commodity exchange was not perceived until the 19th

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[12]Cramer's choice function was the square root function.

[13]Although compatible with Adam Smith's (1776) *invisible hand* (In every human breast...self-regarding interest is predominant over social interest" (Bentham, The Book of Fallacies, 1824)) the utility concept was not retained in 18th century's classical economics.

century with the marginalist revolution associated with Jevons (1871), Menger (1871) and Walras (1874). The marginalists were the first to recognize that the value of a commodity depends on the demand for it,[14] *and* that the demand for a commodity depends on the utility associated with consuming the last unit of the commodity and not on the total utility associated with consuming the commodity. A relationship between demand and utility is thus established (Quiggin, 2004). A rational consumer in an economy where prices are given should make the necessary exchanges (purchases and sales) to move to a new endowment at which the ratio of his marginal utilities for commodities equals the corresponding ratio of their prices. The grand rational optimization of total utility in Bentham's theory becomes a problem of individual rational optimization. For the first time, the macro vision of the economy shifts to a micro vision and the preferences (tastes) of the individual, the consumer, start to play an important role in economic analysis.

The Ordinalist Revolution

However, the marginalists' concept of utility as well as Bernoulli's early statement of expected utility theory and Bentham's utilitarianism assume the existence of a cardinal utility[15] that is numerically measurable. The difficulty of constructing a numerical measure of the unobservable concept of utility and of comparing utility scales between individuals on one hand, and the development of the indifference curves analysis, by Edgeworth (1881) and Pareto (1906) as a tool to measure preferences, on the other hand, led to the rejection of the concept of a cardinal utility as a basis for constructing individual preferences.

The notion of preference as a psychologically primitive concept was emphasized instead. The consumer is assumed to be always able to state which of two alternatives he prefers or else state that he is precisely indifferent between them. If his preferences are also

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[14] As opposed to the classical view of value deriving from production and distribution.

[15] A cardinal utility is an ordinal utility ($x \succcurlyeq y \iff u(x) \geq u(y)$), which is unique up to a positive linear transformation.

transitive it follows from this set of simple axioms, that they can be represented by a real-valued utility function that is preferred choices have higher utility numbers. Since only the ordering property of the utility numbers is meaningful the utility function representing the individual's preferences is said to be ordinal. Preferences represented by indifference curves[16] requiring only the use of ordinal utilities that rank commodity bundles but does not compare the differences between bundles were sufficient for the purposes of demand theory.

A rational consumer then should make the necessary exchanges to move to a new endowment at which the marginal rates of substitution between commodities, which leave total utility unchanged and are observable, replace the ratio of his marginal utilities for commodities.

However, as Quiggin (2004) argues, demand functions can only be well-defined if preferences over commodity bundles are convex that is if a bundle containing an appropriate mixture of two goods is preferred to either of two equally valued bundles each containing only one of the goods. Implicit in this kind of convexity is the principle of diminishing marginal utility.[17] Convexity captures the intuition nevertheless by referring to observable preferences rather than to unobservable utilities.

The vNM Concept of Utility

By the mid 20th century however, the axiomatic method "de rigueur" in mathematics around the turn of the century, was embraced by other disciplines that relied on mathematical methods (Nau, 2004). von Neumann and Morgenstern (1944) moving away from their predecessors' employment of indifference curves for rational optimization used the axiomatic method to deduce formally that an individual strives to maximize his utility. The expected utility model intuited by Bernoulli and Cramer 200 years earlier in

[16]Indifference curves are taken to indicate corresponding trade-offs of goods *A* for *B* or *B* for *A* over the same interval, the movement in either direction presumed to be the same. The reversibility of indifference curves a central assumption of demand theory under certainty implies indifference curves do not cross.

[17] If $x \succcurlyeq z$ and $y \succcurlyeq z$ then $\alpha x+(1-\alpha)y \succ z$ for $\alpha \in]0,1[$. In particular, if x and y are indifferent to each other, then $\alpha x+(1-\alpha)y \succ z$ is strictly preferred to either of them. The statement captures the intuition of diminishing marginal utility namely that ½ x provides "more than half as much utility as x" (Nau, 2004).

response to the inadequacy of the expected value model is in the work of vNM (1944) deduced from a relatively small set of axioms about rational preferences.[18]

For the rigorous description of the endeavor of the rational individual to maximize his utility, vNM (1944) resurrected cardinal utility and axiomatized its maximization over probability distributions under risk. Quiggin's (2004) pertinent remark "no sooner driven out of the front door of economic theory, cardinal utility re-entered it through the back gate of game theory and expected utility theory" illustrates the immediate rekindling of the debate of ordinal vs cardinal utility, the debate ending finally in an agreement: the vNM's utility associated with risk is different from the utility under certainty of the marginalists (Baumol, 1958; Fishburn, 1989).[19] Under vNM's expected utility theory, the individual is assumed to be able to compare different consequences and different combinations of consequences. Thus, if he prefers $z \succ x \succ y$ and $x \succ (y, \frac{1}{2}; z, \frac{1}{2})$ then, it can be inferred that his preference of *x* over *y* exceeds his preference of *z* over *x* and differences in utilities become numerically measurable (vNM, 1944).

The vNM expected utility theory was not however the first to incorporate explicitly uncertainty in the preference structure. Ramsey, a British philosopher and mathematician to whom the theory (that a person's actions are completely determined by his desires and opinions) seemed a useful approximation of the truth, (Ramsey, 1931, p: 75) constructed the first operational model of expected utility in which desires and opinions are quantified and preferences represented formally in terms of a utility function and a

[18] The expected utility model today stands on its own, but originally it was designed as part of vNM's modeling of rational social behavior in the playing of games, the simplest setting in which human rationality is exercised, portraying for the first time decisions as reactions to others actions' rather than reactions to exogenous prices (Herbert Simon's review of vNM's book, p: 559). Rationality is assumed in an attempt to reduce the problem of conflict of interest under uncertainty to a conflict of interest under risk (Luce and Raiffa, 1957, p:14)

[19] The vNM's evaluation of utility can be thought of as a product of two factors: a measure of an increasing or decreasing utilitarian marginal utility under certainty (labeled strength of preference) multiplied by an intrinsic attitude towards risk. The relationship between the two is explored in Winterfeldt and Edwards (1986).

probability function. Savage (1954) is a complete axiomatization of this earlier model of expected utility for uncertainty and vNM (1944) is the complete axiomatization for risk.

This chapter presents first the vNM's axiomatization of expected utility (*EU*) followed by the more general representation of Savage's (1954) subjective expected utility (*SEU*). The axiomatization of expected utility under risk presented in this chapter is based on Fishburn (1970), the literature on individual decision making under risk includes however many axiomatizations (e.g. Herstein and Milnor, 1953; Jensen, 1967, Luce and Raiffa, 1957). Subjective expected utility by Anscombe and Aumann (1963) is sketched in brief.

1.2 *EXPECTED UTILITY UNDER RISK*

The axiomatization of expected utility in von Neumann and Morgenstern's *Theory of Games and Economic Behavior* (1944) is considered a pioneer work. The authors were the first to axiomatize the model when the probabilities are given that is are objective and their work is founded on the formalization of bets. The objective of this section is to introduce the basic concepts and to present the axioms formally. It proceeds as follows: first the general framework and some basic definitions are given, then the axioms and the representation theorem are presented.

1.2.1 vNM Axiomatization of Expected Utility: Axioms and Simple Probability Measures

vNM (1944) proposed a complete set of propositions and axioms necessary and sufficient for the use of expected utility as a rule of choice under risk. In their work, the objects of choice are probability distributions or lotteries defined on a given set of consequences. The decision makers' preferences are then formally represented by a binary relation.

The General Framework

Let $\boldsymbol{X}$ be the set of consequences and A a subset of $\boldsymbol{X}$. A collection of subsets of $\boldsymbol{X}$ denoted a is an *algebra* if $\boldsymbol{X} \in a$; $A \in a \Rightarrow -A \in a$ and $A, B \in a \Rightarrow A \cup B \in a$ (Kreps, 1988, p: 116)

A *probability measure* P is a real-valued function which maps subsets of $\boldsymbol{X}$ into the interval [0, 1] and satisfies the following axioms:

1- $P(A) \geq 0$ for any subset $A \subset \boldsymbol{X}$;

2- $P(\boldsymbol{X}) = 1$;

3- $P(A \cup B) = P(A) + P(B)$ where $A, B \subset \boldsymbol{X}$ and $A \cap B = \phi$;

For a *simple probability measure*, a fourth property must be added:

4- $P(A) = 1$ for some *finite* subset $A \subset \boldsymbol{X}$.

A simple probability measure P such as $P(\{x_1,\ldots,x_n\}) = 1$ for $\{x_1,\ldots,x_n\} \subset \boldsymbol{X}$, can be considered as a lottery that gives x_i with probability p_i : $P = (x_1, p_1; \ldots; x_n, p_n)$ for $i = 1\ldots n$. Each consequence $x_i \in \boldsymbol{X}$ can be represented by a degenerate probability measure that is that gives the consequence with certainty.

If P and Q are simple probability measures on $\boldsymbol{X}$ and $\alpha \in [0, 1]$ then the convex combination $\alpha P + (1-\alpha)Q$ can be interpreted as a combined lottery that gives the lottery P with probability α and the lottery Q with probability (1-α). It can be easily shown that $\alpha P + (1-\alpha)Q$ is a simple probability measure on $\boldsymbol{X}$. The set, $\boldsymbol{P}$, of all simple probability measures on $\boldsymbol{X}$ is closed under convex mixture operations, i.e. $\alpha P + (1-\alpha)Q$ belongs to $\boldsymbol{P}$ is a mixture set.

The Axioms

The preferences of the individual over $\boldsymbol{P}$ the set of simple probability measures defined over the set of the consequences $\boldsymbol{X}$ are represented by a binary relation $\succcurlyeq$ that reads "preferred to or equivalent to". The binary relation $\succcurlyeq$ is used to define two other binary relations: The relation $\sim$ is called indifference and reads "indifferent or equivalent to" i.e., $P \sim Q \Leftrightarrow P \succcurlyeq Q$ and $Q \succcurlyeq P$.

The relation $\succ$ is called a strict preference relation and reads "strictly preferred to" that is, $P \succ Q \Leftrightarrow$ not $Q \succcurlyeq P$.

The binary relation $\succcurlyeq$ is assumed to satisfy the following axioms for all $P, Q, R \in$ ***P***.

Axiom A1: Weak Ordering.

A1 is a fundamental tenet of rationality. It states that $\succcurlyeq$ is a weak order on ***P***, which means that the relation $\succcurlyeq$ is transitive i.e., $P \succcurlyeq Q$ and $Q \succcurlyeq R \Rightarrow P \succcurlyeq R$ and complete i.e., the decision maker can compare lotteries in a consistent way such as $P \succcurlyeq Q$ or $Q \succcurlyeq P$. The relation $\succ$ is asymmetric by definition, and it can easily be established from **A1** that that the relation $\sim$ is transitive.

Axiom A2: The Independence Axiom.

$P \succ Q$, $0 < \alpha < 1$ then $\alpha P + (1-\alpha)R \succ \alpha Q + (1-\alpha)R$

The independence axiom means that if Q is replaced by a preferred lottery P in $\alpha Q + (1-\alpha)R$ then the resulting compound lottery ($\alpha P + (1-\alpha)R$) should be preferred to $\alpha Q + (1-\alpha)R$.

This axiom is also known as the substitutability axiom (Luce and Raiffa, 1957, p: 27). It is strategic to the linearity in the probability property of expected utility and is often considered as a principal normative criterion of the theory along with transitivity (Fishburn, 1970).

Axiom A3*:* An Archimedean axiom.

$P \succ Q$, $Q \succ R$ then $\alpha P + (1-\alpha)R \succ Q$ and also $Q \succ \beta P + (1-\beta)R$ for some $\alpha, \beta \in$ (0, 1).

This axiom rules out lexicographic preferences as well as unbounded utilities for outcomes like heaven or hell. With **A1** this axiom allows to establish the existence of an ordinal utility function on ***X***. It is important to the numerical representation of individual preferences but is not normative as **A2**.

Theorem of Expected Utility under Risk

Theorem 1:

Suppose ***P*** is the set of all simple probability measures on ***X*** and $\succcurlyeq$ a binary relation on ***P***. The following propositions are equivalent:

(i) $\succcurlyeq$ satisfies the axioms **A1**, **A2**, and **A3**

(ii) There exists a real-valued function *U* which represents $\succcurlyeq$ on ***P*** such that:

$$P \succcurlyeq Q \Leftrightarrow \quad U(P) \geq U(Q) \quad \forall P, Q \in \boldsymbol{P} \tag{1.1}$$

$$U(\alpha P + (1-\alpha)Q) = \alpha U(P) + (1-\alpha)U(Q), \forall \alpha \in [0,1], \forall P, Q \in \boldsymbol{P} \blacksquare \tag{1.2}$$

Moreover *U* is unique up to a positive linear transformation: that is if *U* on ***P*** satisfies (1.1) and (1.2) then a real-valued function *V* on ***P*** satisfies (1.1) and (1.2) with *U* replaced by *V* iff there are constants $a > 0$ and b such that $V(x) = a\, U(x) + b \; \forall \; P \in \boldsymbol{P}$.

Theorem 1 implies expected utility maximization on ***P***. Let $\Delta \subset \boldsymbol{P}$ consist of all degenerate probability measures i.e., $P \in \Delta \leftrightarrow \exists \; x \in \boldsymbol{X}$ with $P(x) = 1$ and let elements of Δ be denoted by δ_x, $x \in \boldsymbol{X}$. A function *u* on ***X*** can be defined from *U* on ***P*** the following way. The preference relation is first extended to ***X***:

$$\delta_x \succcurlyeq \delta_y \Leftrightarrow x \succcurlyeq y.$$

Then,

$u(x) = U(\delta_x) \forall \; x \in \boldsymbol{X}$ *u* being the restriction of *U* to ***X***.

Hence, $\forall \; x, y \in \boldsymbol{X}$, $x \succcurlyeq y \Leftrightarrow \quad u(x) \succcurlyeq u(y)$

If $\forall \; P \in \boldsymbol{P}$, $P(\boldsymbol{X}) = 1$, from the linearity property (equation 1.2) the following is obtained: $E(u, P) = \sum_{i=1}^{n} p_i u(x_i)$.

Hence, (1.1) and (1.2) $\Leftrightarrow$ (1.3) the expected utility decision rule:

$$P \succcurlyeq Q \Leftrightarrow E(u, P) \geq E(u, Q) \quad \forall P, Q \in \boldsymbol{P} \tag{1.3}$$

1.3 *SUBJECTIVE EXPECTED UTILITY*

Savage has succeeded in his *Foundation of Statistics* (1954) to treat uncertainty in an entirely subjective manner. In comparison to vNM's construction, there are more conditions stated and some authors like Anscombe and Aumann (1963) have chosen an intermediary road between vNM's and Savage's. This section proceeds as follows: first Savage's axioms and states of the world are presented. Next the phases in the development of Savage's theory are described followed by the formal statement of the theorem of expected utility. A brief description of Anscombe and Aumann's (1963) theory is sketched subsequently.

1.3.1 Savage's Axioms and States of the World

Savage's (1954) subjective expected utility, a model of individual decision making under uncertainty may be viewed as the result of combination of the expected utility of vNM (1944) with its dual subjective probability theory (*SP*)[20] of Bruno de Finetti (1937). Savage (1954) constructed numerical subjective probabilities from comparative subjective probabilities à la de Finetti (1937)[21]and showed that under his axioms, these subjective probabilities obey the laws of probability.

[20] *SP* where utility is linear and probability is subjective is the dual to *SEU*. Nau (2004) shows that the axiomatization of *SP* in terms of acceptable *p*-gambles is equivalent to the axiomatization of *EU* in terms of binary relations i.e. the axioms that the acceptable *p*-gambles must satisfy are the same vNM axioms as reformulated by Jensen (1967, pp: 13).

[21] For Bruno de Finetti, a degree of belief can be expressed *vaguely* as the extent to which an individual is prepared to act on it. The individual's degree of personal belief *P* in event *E*'s occurrence is revealed from the amount he is willing to pay to play the lottery in which the payoff is *W* if *E* occurs. *P* = amount/*W*. Once he bets, either there is the possibility that "he could have a book made against him by a cunning better and would stand to lose in any event" (Ramsey, 1931) or there is no such possibility. In the latter case, the individual's evaluation of the probability is coherent. Otherwise it's incoherent and presents an "intrinsic contradiction." The condition of coherence is the foundation for most of rational choice theory.

The Axioms

In Savage's small world environment, the *set of states of nature* $\boldsymbol{S}$ is given and is an exhaustive set of mutually exclusive states: one and only one state will be the true state of the world but the decision maker is uncertain about which that will be. *Events* are subsets of $\boldsymbol{S}$ and $\boldsymbol{X}$ is the set of outcomes or consequences. The objects of comparison are *acts* which assign an outcome from $\boldsymbol{X}$ to each state of nature from $\boldsymbol{S}$. They are denoted by $f, g, h \in \mathcal{A}$ the set of all *simple acts* or *finite-outcomes* acts.[22] The preference relation on $\mathcal{A}$ is denoted by $\succcurlyeq$ and reads "preferred to or equivalent to". For whatever $f \in \mathcal{A}$ and $B \subset \boldsymbol{S}$, f_B is an act that gives the consequence of f in B. For f and $h \in B$ and 1-B, the object $f_B h$ is the act which gives the consequences of f in B and the consequences of h in 1-B. For $x \in \boldsymbol{X}$ the act $x_B f$ is the act f where all the consequences of the event B are replaced by x.

In addition, an event B is said to be *null* if any pair of acts which differ only on B are indifferent. Savage's theory of decision is deduced from the following axioms.[23]

P1. The relation $\succcurlyeq$ represents a weak ordering on $\mathcal{A}$ i.e., transitive and complete.

This axiom is equivalent to the axiom **A1** in vNM's theory.

P2. Whatever f, g, h and h' and the event B

$f_B h \succcurlyeq g_B h \Leftrightarrow f_B h' \succcurlyeq g_B h'$

This is the sure thing principle, similar to **A2** the independence axiom in vNM's theory.

P3. For all non-null event B, the act f and the consequences x, y,

$x_B f \succcurlyeq y_B f \Leftrightarrow x \succcurlyeq y$

This axiom establishes a relation between preferences for acts and preferences for consequences and is a natural companion to **P2.**

P4. For the consequences x, x', y, y' and if $x \succ y$ and $x' \succ y'$ and $B, C \subset \boldsymbol{S}$, then,

[22] $f(.)$ is said to be a finite-outcome act if its outcome set $f(\boldsymbol{S}) = \{f(s) | s \in \boldsymbol{S}\}$ is finite (Machina and Schmeidler, 1992).

[23] Nau (2004) summarizes the axioms in a table which shows the similarity of the axioms of de Finetti (1937), vNM (1944) and Savage (1954).

$x_B y \succcurlyeq x_C y \Leftrightarrow x'_B y' \succcurlyeq x'_C y'$

Under this axiom, *B* is more probable than *C*. The probabilities are derived from choices and the size of the outcome is irrelevant to the choice if the initial preferences do not change.

P5. There is a least a pair of outcomes such that $x \succ y$.

This axiom ensures that the preference relation is not trivial.

P6. If $f \succ g$ and for all $x \in \boldsymbol{X} \Rightarrow$ there exists a finite partition of $\boldsymbol{S}$ such that if B_i and B_j are any two events of this partition, then

$x_{Bi} f \succ g$ and $f \succ x_{Bi} g$

If *f* is preferred to *g* there exists a partition that renders the act *f* constant for a consequence *x* without varying otherwise, then the initial preference does not change. This axiom establishes a link between subjective probabilities and objective probabilities and gives to subjective probabilities the property of continuity essential to define a utility function among acts.

Theorem of Expected Utility under Subjective Uncertainty

The first phase in the development of Savage's theory is to obtain probabilities from preferences. This is done in two steps. First, a qualitative probability is obtained. Following Fishburn (1970), $\succcurlyeq^p$, a binary relation defined on 2^s the set of all subsets of $\boldsymbol{S}$ that reads "at least as probable as" is defined by:

$B \succcurlyeq^p C \Leftrightarrow x_B y \succcurlyeq x_C y$ verifying $x \succcurlyeq y$; $\forall B, C \subset \boldsymbol{S}$

The second step is to show that the binary relation $\succcurlyeq^p$ possesses under the Savage axioms the attributes of a unique probability measure P^* verifying:

(i) $B \succcurlyeq^p C \Leftrightarrow P^*(B) \geq P^*(C) \quad \forall B, C \subset \boldsymbol{S}$

(ii) $C \subset \boldsymbol{S}$, $0 \le \lambda \le 1 \Rightarrow P^* \mathbf{(}D\mathbf{)} \mathbf{=} \lambda P^* (C)$ for a certain $D \subset C$

Having obtained subjective probability, the second phase is to show that the individual attaches also a subjective utility to the acts' consequences. P^* is used by Savage to construct a set of simple lotteries from the set of simple acts. The construction is based on the idea that a simple act induces a simple probability measure on ***X***. The simple lottery induced by the simple act $f \in \mathcal{A}$ is denoted P_f and it is such that:

$$\forall x \in \boldsymbol{X},\ P_f(x_i) = P^*(\{s \in \boldsymbol{S}:\ f(s) = x_i\}) = P^*(f^{-1}(x_i))$$

Under the axioms **P1**-**P6**, it is shown[24] that

$P_f = P_g \Rightarrow f \sim g$.

f and g are not the same, the implication is that $f \sim x \sim g$, thus $f \sim g$. This condition avoids having $P_f = P_g$ and not $(f \sim g)$.

The preference relation obtained allows to define a binary relation $\succcurlyeq$ on the set of all simple probability measures on $\boldsymbol{X}$ denoted $\boldsymbol{P}$. For P_f and P_g from $\boldsymbol{P}$, and f and g from $\mathcal{A}$, the following binary relation is defined:

$\forall f, g \in \mathcal{A}:\ P_f \succcurlyeq P_g \Leftrightarrow f \succcurlyeq g$.

The binary relation satisfies the vNM axioms[25] and there exists a real-value function u such that for P_f and P_g from $\boldsymbol{P}$,

$$P_f \succcurlyeq P_g \Leftrightarrow \sum_{x \in X} u(x) P_f(x) \ge \sum_{x \in X} u(x) P_g(x)$$

Or equivalently, for f and g from $\mathcal{A}$

$$f \succcurlyeq g \Leftrightarrow \sum_{x \in X} u(x) P_f(x) \ge \sum_{x \in X} u(x) P_g(x)$$

[24] Cf. Fishburn (1970)

[25] Kreps (1988).

Formally, the above results can be stated in the following theorem.

Theorem 2:
Under the axioms **P1- P6**, there exists a subjective probability measure *P** and a utility function *u* defined on ***X*** and unique up to a positive linear transformation such as:

$\forall P_f, P_g \in \boldsymbol{P},\ \ f \succcurlyeq g \Leftrightarrow E\,(P_f, u) \geq E\,(P_g, u)$ ■

Where
$E(P_f, u) = \sum_{i=1}^{n} u(x_i) P^*(f^{-1}(x_i))$, $\{x_i, \ldots, x_n\}$ is the outcome set of the act $f(.)$ and $P_f\,(x_i)$ = $P^*(f^{-1}(x_i))$ = $P^*(\{s \in \boldsymbol{S}:\ f(s) = x_i\})$.

1.3.2 Anscombe and Aumann's Axioms and Horse-Lotteries

Ten years later, Anscombe and Aumann (1963) presented another axiomatization of expected utility under uncertainty. In their model, uncertainty is represented by a horse-lottery rather than a Savage act. A horse-lottery is a mix of a vNM lottery and a Savage act and is represented by lottery "akin in spirit" to the reduced compound lottery with the distinction that one of the lotteries is a simple lottery where the probabilities are known, as in a roulette wheel while the second is a horse lottery where the probabilities are unknown, therefore personal as in a horse race (Anscombe and Aumann, 1963). The order in which the two lotteries are run is immaterial because, Anscombe and Aumann assume as Savage that the utilities of the consequences are the same in all the states. The horse lottery corresponds to a Savage's act, the outcome of the race to the state of the world that obtains and the prize to the consequence; and the objects of comparison are mappings from states to probability distributions over consequences.

In Retrospect

In the years that followed the axiomatization of expected utility, the normative axioms of expected utility were recognized to be inadequate descriptively (Allais, 1953). The research for the most part focused on the violation of the maximization of expected utility and on the violation of the linearity in the probability property (Equation: 1.2). The independence axiom **A2** is implicit in both and its violation prompted theorists to construct alternatives that either weakened the independence axiom or were "robust" against its failures (Machina, 1982, p: 279). However, the two approaches which kept unmodified the two other axioms **A1** and **A3** essential for the existence of a real-valued utility function on ***X*** as in equation (1.1), could not explain some pervasive phenomena most people exhibit most of the time (Kahneman and Tversky, 1979, p: 263). Prospect theory (1992) was developed to accommodate four major phenomena of choice[26] that violate the standard model in addition to the violation of the independence axiom without giving up the principle of coherence. Camerer (1997) shows its success in the field and argues for its use *along side* the expected utility model in economic textbooks and in current research, which confirms the important role the expected utility continues to play despite its descriptive invalidity.

The resilience of the expected utility framework in the face of data challenging its empirical validity is explained by Machina's (1982, p: 277) characterization of the vNM's approach to the theory of individual behavior under risk: "the simplicity and normative appeal of its axioms, the familiarity of the notions it employs (utility functions and mathematical expectation) the elegance of its characterizations of different types of behavior in terms of the properties of the utility function (risk aversion by concavity, the degree of risk aversion by the Arrow-Pratt measure, etc.) and to the large number of results it has produced."

[26] These are: the framing effect, the source dependence, the risk seeking and the loss aversion (Tversky and Kahneman, 1992, p: 298)

The purpose of Chapter II the following chapter, is to provide an overview of the violations of the properties of *EU* which are violations of its axioms and to set out the alternative theories that can account for the descriptive invalidity of expected utility while retaining the principle of coherence.

Chapter II: Violations of Certain Axioms of Expected Utility and the Alternative Models

The experimental works which followed the theoretical works of vNM (1944) and Savage (1954) showed that individuals violate systematically the axioms of expected utility under risk and under uncertainty. The early violations were found by Allais (1953) and Kahneman and Tversky (1979) for risk and by Ellsberg (1961) for uncertainty. In these experiments the majority of individuals reversed their preferences. Once the inconsistency was revealed, some decision makers revised their judgments to be consistent with the axioms of the expected utility model but others refused to do so. In the latter case, preference reversal spoke against the reasonableness of the related axiom (Allais, 1953). The empirical evidence motivated researchers to develop alternative theories of choice under risk and uncertainty to accommodate the observed violations. Section 2.1 is concerned with the violations of the independence axiom. Section 2.1.1 presents the case for risk, and sections 2.1.2 and 2.1.3 present the cases for dynamic context and uncertainty respectively. Section 2.2 presents the violations of the axioms responsible for the stability of preferences and section 2.3 is concerned with the alternative models developed.

2.1 *THE VIOLATIONS OF THE AXIOMS OF EXPECTED UTILITY*

The violation of the independence axiom under risk is the most discussed in literature and also the one responsible for the generation of many alternatives to *EU* and *SEU* for a long period of time stretching from 1979 till now. The reason is that without it the expectation principle and the linearity in the probabilities (equation 1.2) cannot be retained. Section 2.1.1 focuses on its violation under risk followed by a brief description for what its violation implies for dynamic contexts in section 2.1.2 and for uncertainty in section 2.1.3. Section 2.2 presents the violations of the axioms responsible for the stability of preferences.

2.1.1 The Violation of the Independence Axiom under Risk

The independence axiom implies a separability property justified by the mutual exclusiveness of the lottery's consequences (Machina, 1989; Weber and Camerer, 1988). The property is twofold: additive and multiplicative. The former implies that replacing a common consequence with the same probability in two lotteries by a different consequence does not influence the preference between the two lotteries. The latter implies that multiplying all the probabilities in two lotteries by the same constant and the remaining probability assigned to a common consequence, does not affect the preference between the two lotteries. The violation of the separability property is known as the common consequence effect and the common ratio effect respectively and in the literature on dynamic choice it is known as the violation of consequentialism. The first evidence of systematic violation of the separatibility property and hence the first evidence of the inconsistency of individuals' choices with equation (1.2) was found by M. Allais (1952).

The Common Consequence Effect

The two sets of choices designed by Allais (1953) to demonstrate the violation of the additive property of the independence axiom are described in Table 1 with M = \$1000000.

Table 1: The Allais Common Consequence Sets

	P = 0.1	**P = 0.89**	**P = 0.01**
Set I			
$\mathbf{S_1}$	1M	1M	1M
$\mathbf{R_1}$	5M	1M	0
Set II			
$\mathbf{S_2}$	1M	0	1M
$\mathbf{R_2}$	5M	0	0

Many people presented with the above choices chose as predicted by Allais S_1 over R_1 attracted by the certainty of receiving 1 M *and* R_2 over S_2 because the consequences were very different for quite similar probabilities. The choices however violate the independence axiom. To see why, consider S_1, R_1, S_2, R_2 as lotteries and A and B as intermediate lotteries such as A = (10/11) 5M + (1/11) 0 and B = 0, then

$S_1 = 0.11\ S_1 + 0.89\ S_1$, and $R_1 = 0.11\ A + 0.89\ S_1$
$S_2 = 0.11\ S_1 + 0.89\ B$, and $R_2 = 0.11\ A + 0.89\ B$

According to the independence axiom, $S_1 \succ R_1$ indicates that $S_1 \succ A$. However, the preference $R_2 \succ S_2$ indicates that $A \succ S_1$ which is a contradiction. Hence, the choices observed by Allais are incompatible with the independence axiom.

Table 1 also shows why these choices are inconsistent with the behavior predicted by expected utility. Going from Set II to Set I, all that is changed is that a 0.89 chance at zero is replaced by a 0.89 chance at 1M, hence an expected utility maximizer would choose S_2 over R_2 provided he has chosen S_1 over R_1 in the first choice otherwise there is an inconsistency as shown by the following:

Assuming expected utility theory and u (5M) = 1 and u (0) = 0, $S_1 \succ R_1$ yields: u (M) > 0.1 + 0.89 u (M) while $R_2 \succ S_2$ yields: 0.1 > 0.11 u (M) which is inconsistent with u (M) > 0.1 + 0.89 u (M).

The violation of expected utility was also observed in the absence of a certain consequence as in Prelec's (1990) example: with M = 10000, S_1 = (2M, 0.02) and R_1 = (3M, 0.01); S_2 = (2M, 0.34) and R_2 = (3M, 0.01; 2M, 0.32). The common consequence added here is (2M, 0.32). People chose S_1 *and* R_2 although the common consequence is rationally irrelevant.

The Common Ratio Effect

Allais (1953) constructed also two other sets of choices to demonstrate the violation of the multiplicative property of the independence axiom. The two sets are described in Table 2.

Table 2: The Allais Common Ratio Sets

	Outcome	**Probability**
Set I		
$\mathbf{S_1}$	3000	1
$\mathbf{R_1}$	4000	0.8
Set II		
$\mathbf{S_2}$	3000	0.25
$\mathbf{R_2}$	4000	0.2

Confronted with Allais' sets, many people chose S_1 over R_1 *and* R_2 over S_2 violating the rule of constant probability ratio $p/pq = \alpha p/\alpha pq$ (in Allais' sets, $p = 1$, $q = 0.8$ and $\alpha = 0.25$) implied by the independence axiom[27] and which justifies the choice of the same lottery when the probability varies while the outcomes remain the same.

The certainty effect plays an important role in the reversal of choices; however even when it is a special case of a more general form, people chose S_1 over R_1 *and* R_2 over S_2, as in the following two sets of choices: Set I: $S_1 = (x, p; 0, 1-p)$ and $R_1 = (y, q; 0, 1-q)$ and Set II: $S_2 = (x, \alpha p; 0, 1-\alpha p)$ and $R_2 = (y, \alpha q; 0, 1-\alpha q)$ where $0 < x < y$, $p > q$ and $0 < \alpha < 1$, The certainty effect applies when $p = 1$.

The Transformation ofs the Probabilities

Prospect theory (1992) designed to explain the Allais paradoxes along with other violations of *EU* incorporates a non-linear probability weighting of probabilities, $p \rightarrow w(p)$ also called a probability weighting function. Two properties required on this

[27] $(x, p) \succ (y, pq)$ implies that $(x, \alpha pr) \succ (y, \alpha pqr)$ where $0 < p, q, r \leq 1$.

function reconcile the Allais preferences found inconsistent under *EU*: *Subadditivity* explains the common consequence violation and *subproportionality* explains the common ratio violation (Prelec, 2000).

Subadditivity

For the interpretation of the common consequence effect through probability weighting it is convenient to describe the Allais (1953) sets in terms of decumulative probability distributions. Therefore the set ***P*** of all simple probability measures on ***X*** defined in section 1.2.1 is transformed into the set ***P**** through a function (.)*: ***P*** $\rightarrow$ ***P**** such that (*P*)* = *P** = $(p_2^*,...,p_n^*)$, where $p_i^* = \sum_{j=i}^{n} p_j, i = 2,...,n.$ In other words, (.)* transforms each lottery into a function *P** assigning to each x_i the probability of receiving x_i or any outcome rank ordered above in ***X*** (Abdellaoui, 2002).

Allais' (1953) sets as alternatives in ***P*** and ***P**** are described in Table 3. For $x_3 > x_2 > x_1$, *P**, *Q**, *R** and *S** are the decumulative distributions corresponding to *P, Q, R* and *S* respectively. The table shows that the Allais preferences *P*S** seem to indicate that people tend to assign a greater weight to the replacement of probability 0.99 by probability 1 than to the replacement of probability 0.1 by probability 0.11.

Table 3: The Allais Paradox (x_1 = 0, x_2 = 1M, x_3 = 5M)

	Alternatives in P	**Alternatives in P***
Set I	P = (0,1,0)	P* = (1, 0)
	Q = (1/100, 89/100, 10/100)	Q* = (99/100, 10/100)
Set II	R = (89/100, 11/100, 0)	R* = (11/100, 0)
	S = (90/100, 0, 10/100)	S* = (10/100, 10/100)

This particular interpretation is fostered by prospect theory (1992) where the Allais preferences imply that *CPT* (*P*) > *CPT* (*Q*) and *CPT* (*S*) > *CPT* (*R*) and thus by definition of *CPT* [28] that $w(1) - w(0.99) > w(0.11) - w(0.1)$ that is the differential weight placed on $w(1) - w(0.99)$ is greater than the differential weight placed on $w(0.11) - w(0.1)$. This property of the weighting function is called subadditivity[29] (Tversky and Wakker, 1995) and it states that the increase in weight produced by adding probability Δ to p is greater when $p + \Delta = 1$ and certainty is reached than when $p + \Delta < 1$.

Subproportionality

Kahneman and Tversky's (1979, p: 282)[30] interpretation of the Allais pattern is that for a fixed ratio, raising the chance of winning from 0.8 to 1 has a greater impact on relative weight than raising the chance of winning from 0.2 to 0.25. Under *CPT*, the Allais preferences imply: $w(0.8)/w(1) < w(0.2)/w(0.25)$. This property of the weighting function is called subproportionality and its preference conditions are a simple generalization of the common ratio pattern observed by Allais (Prelec, 2000):

Subporportionality: for any $0 < \alpha < 1$, $y > x > 0$, and $p > q$, $(x, p) \sim (y, q)$ implies: $(y, \alpha q) \succ (x, \alpha p)$.

The probability weighting function has one more empirical property and "perhaps the first and most important empirical property" (Prelec, 2000) which says that small probabilities are overweighted and large probabilities are underweighted.

---

[28] *CPT's* valuation of $(x, p) = w^{+}(p)u(x)$ for $x > 0$. Chapter III is devoted to *CPT*.

[29] Subadditivity also states that the increase in weight produced by adding Δ to p is greater when $p = 0$ than when $p > 0$. The formal definition is given in section 3.1.3.

[30] The violation of the multiplicative property of the independence axiom conforms to the following rule: if (x, p) is equivalent to (y, pq) then (x, pr) is not preferred to (y, pqr), $0 < p, q, r \leq 1$. By definition of the prospect theory utility function i.e. by equation (3.1):

$\pi(p)\, u(x) = \pi(pq)\, u(y)$ implies $\pi(pr)u(x) \leq \pi(pqr)u(y)$; hence,

$(x, p) = w^{+}(p)u(x)$

$\pi(pq)/\pi(p) \leq \pi(pqr)/\pi(pr)$

Along with the unattractiveness of probabilistic insurance,[31] it has led Tversky and Kahneman (1992) to the four-fold pattern of risk attitudes:

"*Overweighting of small probabilities contributes both to risk seeking for gains and risk aversion for losses when the outcomes are high which explains gambling (optimism) and insurance (pessimism). Underweighting of the probabilities contributes to the prevalence of risk aversion in choices between probable gains and sure things and to risk seeking in choices between probable losses and sure losses*."

The fourfold pattern is empirically supported by later parametric studies: Tversky and Kahneman (1992); Camerer and Ho (1994); Tversky and Fox (1995); Wu and Gonzalez (1996); Abdellaoui (2000) and non-parametrically, Abdellaoui (2000) and Bleichrodt and Pinto (2000).

2.1.2 Implication of the Violation of Independence for Dynamic contexts

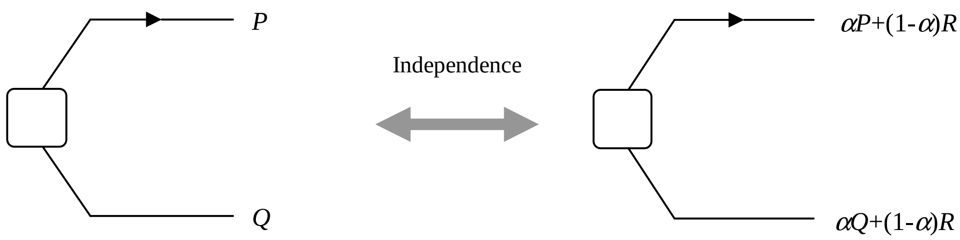
The independence axiom is also assumed in principles of dynamic choice. Wakker (1999) shows that four conditions jointly imply equivalence of independence in dynamic contexts. It follows in view of the evidence for the failure of independence that at least one principle of dynamic choice must be failing too. To recall from section 1.2.1 the independence axiom **A2** states:

$$P \succ Q,\ 0 < \alpha < 1 \quad \text{then} \quad \alpha P + (1-\alpha)R \succ \alpha Q + (1-\alpha)R$$

Figure 1 illustrates the independence axiom. Squares denote decision nodes, circles chance nodes and arrows the preferred path.

[31] That is reducing a probability from p to $\frac{p}{2}$ is less valuable than reducing $\frac{p}{2}$ to zero. Allais' gamble also illustrates that individuals are oversensitive to changes in small probabilities.

Figure 1: Independence



Because the verbal expressions of the conditions are often ambiguous[32], the decision making process in Figure 1 is separated as in Wakker's (1999) into four stages to illustrate each condition.

The first condition is *forgone-event independence* referred to as *consequentialism* by Machina (1989). It is helpful to draw at this point the choice between *P* and *Q* as in Figure 1*b*. Indeed, the latter illustrates the same choice as is 1*a* because the options and their consequences are the same.

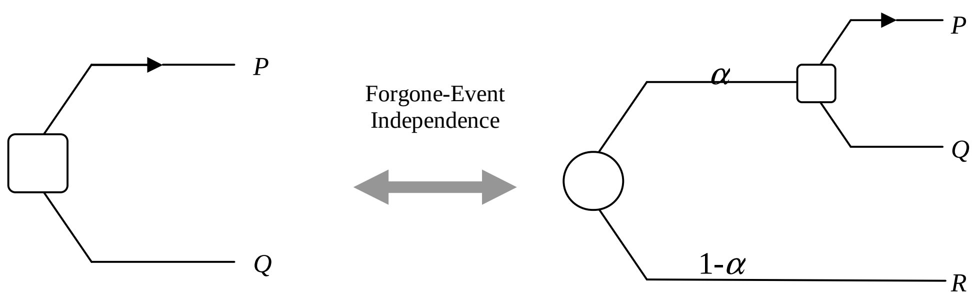


Fig. 1*a* Fig. 1*b*

Forgone-event independence states that if one arrives at the decision node in Figure 1*b* the lower branch is irrelevant to the consequences of choices at that decision node.

[32] The terminologies also vary: Hammond (1986) uses the term consequentialism (currently mostly used for foregone-event independence, Machina, 1989) for equivalences of Figures 1*b*, 1*c* and 1*d* with 1*e*. Burks (1977) calls the equivalence of Figures 1*a* and 1*c* invariance and the equivalence of figures 1*c* and 1*d* with 1*e* normal-form equivalence (Wakker,1999)

The second condition is *dynamic consistency* and is illustrated in Figures 1*b* and 1*c*. It requires that the decision maker commits before the resolution of uncertainty (hence the dashed lines in Figure 1*c*) at the chance node to a decision at a future decision node and does not deviate from his prior commitment once he reaches the decision node.

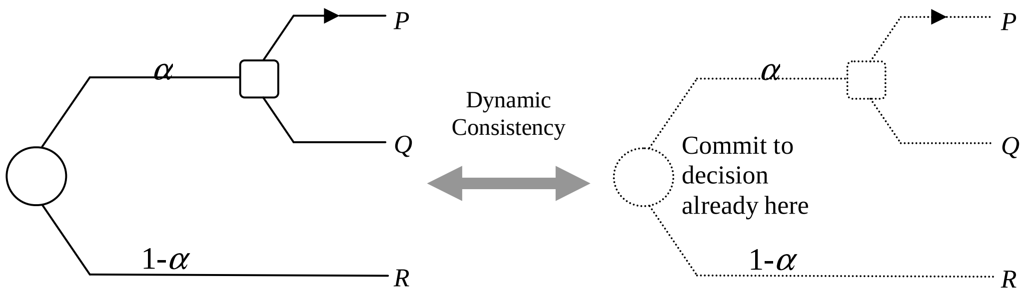


Fig. 1*b* Fig. 1*c*

Context–independence is the third condition and is illustrated in Figures 1*c* and 1*d*. In the latter, the upper branch depicts the prior commitment of going up and the lower branch depicts the prior commitment of going down. The two figures depict the same situation and hence should be treated the same.

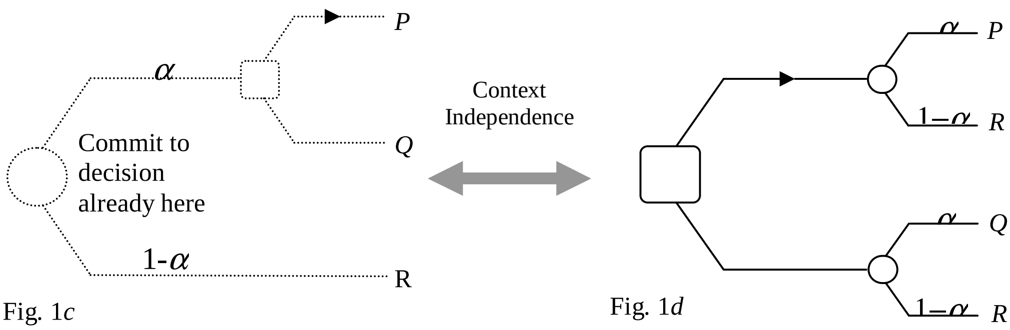


Fig. 1*c* Fig. 1*d*

The last condition is illustrated in Figures 1*d* and 1*e*. Figure 1*e* depicts each lottery at the two chance nodes in Figure 1*d* as probability distributions over consequences. Hence, the two figures should also be treated the same. This condition is called *reduction*.

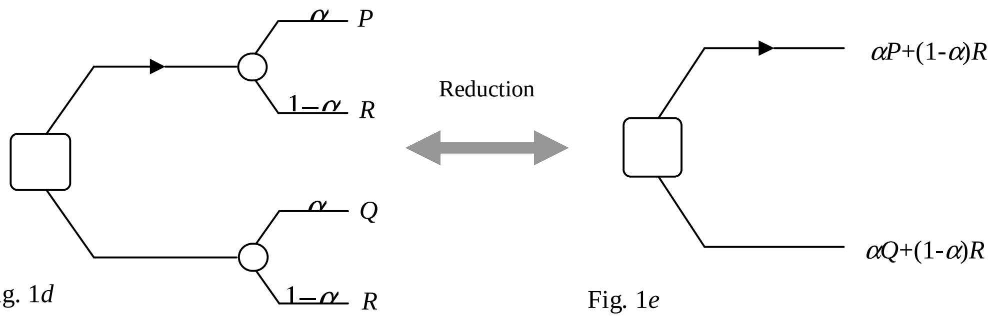


Fig. 1*d* Fig. 1*e*

The four conditions together imply the equivalence of Figures 1*a* and 1*e*, i.e., independence.

Among the studies that have addressed the issue of which condition must be relaxed in order to predict actual behavior are Machina's (1989) and McClennen's (1990) who relaxed consequentialism and Segal's (1990) who relaxed reduction.

2.1.3 Violation under Subjective Uncertainty

The violation of the independence axiom under uncertainty which is a violation of Savage's axioms **P2**, the sure thing principle[33] was first shown by Ellsberg (1961) who designed the following two sets of acts described in Table 4.

Table 4: The Ellsberg Paradox

	States of Nature		
	Red	**Black**	**Yellow**
	30	60	
Set I			
Act I	100	0	0
Act II	0	100	0
Set II			
Act III	100	0	100
Act IV	0	100	100

[33] **P2** with **P3** constitute the independence axiom in Savage's axiomatization of expected utility.

Subjects are presented with an urn known to contain 30 red balls and 60 black and yellow balls in unknown proportions and asked to choose first, an action from Set I, then an action from Set II.

The acts of Set I and the acts of Set II have a common consequence as in Allais' example and differ from the latter only with respect to the information regarding the states of nature of the black and yellow balls. The latter are considered ambiguous because the information about either state of nature is unknown.

The most frequent pattern, Act I (preferring to bet on red) *and* Act IV (preferring to bet against red) violate the sure thing principle. Indeed, Act I implies according to expected utility under uncertainty and after simplification that $\pi(R) > \pi(B)$ and Act IV implies that $\pi(B \cup Y) > \pi(R \cup Y)$. Because π is additive, the implication of Act IV is that $\pi(R) < \pi(B)$ which is a contradiction with the implication of Act I.

Ellsberg's (1961) explanation for the choices and the contradiction is that people's "unease with their best estimates of probabilities" make them prefer objective rather than subjective probabilities which conflicts with **P2**'s implication that once people form their subjective probabilities they use them exactly as objective probabilities. Aversion for ambiguity (absence of information about the states of nature of black and yellow) which refers to the above "unease" is well documented in Heath and Tversky (1991), Camerer and Weber (1992) and Tversky and Fox (1995).

2.2 *VIOLATIONS OF THE AXIOMS OF STABILITY OF PREFERENCES*

The axioms **A1** and **A3** are essential for the existence of a continuous real-valued preference function. The Archimedean axiom is not normative, however relaxing it leads to lexicographic preferences implying extreme attributes and restricted situations. Regarding completeness some authors Nau (2004) favor "dropping" it to relax the requirements on transitivity and independence two normative criteria. Transitivity ensures that the decision maker is not transformed into a money pump,[34] however, it has been found to be violated as well as two other normative criteria implicit in the axiom of ordering: procedure invariance and description invariance.

2.2.1 Transitivity

The earliest evidence on intransitivity goes back to Georgescu-Roegen (1936) and May (1954). Tversky (1969) and Tversky, Slovic, and Kahneman (1990) also show that cyclical choice is robust. Regret Theory (Bell, 1982; Fishburn, 1982; Loomes and Sugden, 1983) is an attempt to remedy to intransitive behavior, but there is no theory yet that is fully consistent with the available data.

2.2.2 Procedure Invariance

Procedure invariance says that preferences over lotteries are independent of the method used that is, it implies that people follow the same procedure for valuing and choosing. Evidence of its violation was provided by Lichtenstein and Slovic (1971), Harold Lindman (1971) and Grether and Plott (1979). Individuals, repeatedly, chose probability bets that offer a larger probability of winning a smaller prize while valuing more dollar bets that offer a small probability of winning a good prize. Tversky, Slovic and Kahneman (1990) show that the preference reversal is due to procedure invariance violation rather than to the violation of either of transitivity or the independence axiom or the violation of the reduction of lottery axiom. The violation of the procedure invariance violation implies the existence of two different processes for valuing and choosing and is supported by Slovic and Lichtenstein (1983) who provide evidence that the latter is fundamentally influenced by the probabilities associated to the outcomes while the former is fundamentally influenced by the outcomes.

---

[34] Fishburn (1970, p: 108-109) and Machina (1989) illustrate the normative appeals of transitivity and the independence axiom. Transitivity also ensures that indifference curves are reversible and do not cross. Knetsch (1990) however demonstrates that owning *A* and being indifferent about trading it for *B* is not the same as owning *B* and being indifferent about trading it for *A* in the presence of loss aversion. In that case, the indifference curves intersect (Kahneman, Knetsch and Thaler, 1991).

2.2.3 Description Invariance

According to Starmer (2000) description invariance is an implicit assumption in any conventional theory and seems so natural to most economists that it is rarely discussed. Descriptive invariance says that preferences over lotteries are purely a function of the probability distributions of consequences implied by the lotteries and do not depend on how these given distributions are described. Nevertheless, the framing effect provides evidence that this assumption fails in practice.

The Framing Effect

Ramsey (1931) referred to the possibility that choice depends on the special form of the offered options as "absurd". However, Kahneman and Tversky's (1979) *Asian disease* shows that people are influenced by the way questions are framed: the choices are reversed when outcomes initially framed in terms of lives saved are framed in terms of lives lost. The preference reversal implies that the assumption of descriptive invariance is violated in response to what has become to be known as a framing effect. Evidence of framing effects is also cited by Slovic (1969), Schoemaker and Kunreuther (1979), Tversky and Kahneman (1981, 1986) and in (Popkin, 1992).

2.2.4 The Reflection Effect

Although the reflection effect is not a violation of description invariance because it involves different options, it is consistent with the framing effect: Kahneman and Tversky (1979) found in one of their experiments that when the sign of outcomes in problems involving positive prospects is reversed, responses change to the exact opposite, that is, behavior towards losses becomes the mirror image of behavior towards gains. Indeed, given the Allais (1953) example described on page 99 with the signs reversed, Kahneman and Tversky's (1979, p: 268) participants reversed their responses. For instance, those who prefered sure gains to probable gains preferred also probable losses to sure losses. The authors labeled this reversal of preferences around 0 the reflection effect. Budescu and Weiss (1987) found 82% of their subjects displayed concavity for gains and convexity for losses. Related evidence include: Fiegenbaum and Thomas (1988); Lowenstein (1988), Terence Odean (1988); Platt and Glimcher (1999);

Smith et al. (2002); Breiter et al., (2001); and Abdellaoui et al. (2006). Yet, the evidence is not indubitable; many empirical studies provide evidence for a linear utility for losses (Edwards, 1955; Hershey and Shoemaker, 1980; Schneider and Lopes, 1986; Cohen, Jaffray, and Said, 1987; Weber and Bottom, 1989; and Lopes and Oden, 1999). Moreover, Levy and Levy (2002) argue for a utility function that is convex for gains and concave for losses.

Sections 2.1 and 2.2 reviewed several empirical effects which seem to invalidate expected utility as a descriptive model. According to Starmer (2000, p: 332) the number of alternative models stimulated by these violations is well into double figures for, to many economists "put bluntly, the standard theory did not fit the facts".

2.3 *THE ALTERNATIVE MODELS*

In view of the above violations, many economists concluded that expected utility theory either does not correspond to the facts at all or correspond to only some of the facts, despite arguments proceeding with faultless logic from the postulates to the conclusion. For the development of a serious contender at least for some purposes, decision theorists revisited and revised the axioms of *EU* and doing so generated a large number of theories going hand-in-hand with ongoing experimental tests of these theories. Section 2.2.1 presents an overview of the recent developments in utility theory and section 2.2.2 presents the alternative models.

2.3.1 Overview of Recent Developments in Utility Theory

This section builds on extent overviews of these alternative theories and their experimental tests (Camerer, 1989; Schmidt, 2002; and Starmer, 2000) and focuses on the models which according to these summaries account best for the currently available empirical data.

Thus, many important alternatives will not be considered. For instance, theories with the betweenness property will not be considered because of the empirical evidence against linear indifference curves implied by the property[35]. Machina's (1982) generalized expected utility will not be considered also because the generalized fanning-out[36] implied by his hypothesis II is ruled out by the presence of fanning-in in numerous studies[37]. The alternative theories presented in this section are thus restricted to the rank dependent weighting models (*RDU*, 1982; *CPT*, 1992) which allow mixed fanning and are not linear in the probability,[38] specially in view of the striking degree of convergence across studies regarding the form of their probability weighting functions (Starmer, 2000, p: 359).

To emphasize however, the inspirations and ideas the authors drew from each other and the intuitive leaps in convergent thinking promoted by combining extent knowledge and /or tolerating dualities[39] prospect theory (1979), the original form of prospect theory (1992) is reviewed first. Quiggin's (1982) anticipated theory, developed in part to build some of the non-conventional prospect theory's features along a conventional strategy is presented next. Yaari's (1987) dual theory (*DT*) a special case of Quiggin's which is linear in consequences and non linear in the probabilities and therefore the dual of *EU*, follows. Prospect theory (1992) a cross fertilization of prospect theory (1979) and anticipated theory (1982) is then contrasted with both. However, prospect theory (1992)'s psychological elements, the formal model and its axiomatization for risk are presented in Chapter III in view of their pertinence to the experimental part of this work. Chapter III's also presents the theories' respective characterizations for risk aversion.

[35] Camerer and Teck-Hua Ho (1994) provide evidence against the linearity of indifference curves.
[36] Fanning out means the indifference curves become steeper (bigger slope) as one moves northwest in the probability triangle, as opposed to fanning in which means the indifference curves become flatter (the slope becomes smaller) in the right hand side of the probability triangle.
[37] Camerer (1989), Chew and William Waller (1986) and Starmer (1992) provide evidence for fanning in.
[38] Table 6 in the Appendix based on Starmer (2000) compares Machina's indifference curves with those implied by theories with the betweeness property and rank dependent theories.
[39] A dual is a connection between two problems which turn out be the very same problem looked at from different angles; for instance Yaari's (1987) theory is the dual of vNM' theory (1944). Duality is borrowed from physical chemistry. Light is dual in the sense that in some experiments its wave properties are most obvious, in others it behaves as a particle.

2.3.2. Theoretical Predictions

The main concern being the reconciling of the theories' predictions with the experimental facts, the presentation is narrowed to a comparison of the forms of the indifference curves between sets of gambles under the different theories and those conjectured from observations in the lab and/or in the field. The indifference curves under expected utility are discussed first to be contrasted with those generated from choices that violate its independence axiom. The indifference curves under the alternative theories developed in response to the violations follow. All indifference curves are presented in the probability triangle in order to compare them visually.

The Probability Triangle

When a lottery $L = (x_1\ p_1;\ x_2\ p_2;\ x_3\ p_3)$ with fixed outcomes such that $x_1 > x_2 > x_3$ is considered in a probability triangle where to recall, the horizontal side represents the probability of the worst consequence x_3 and the vertical side the probability of the best consequence x_1 and where the probability of the third consequence is deduced: $p_2 = 1 - (p_1 + p_3)$, the set of all possible lotteries is contained in the triangle with the vertical side (left edge) characterized by a zero probability for the worst consequence, the horizontal side (lower edge) by a zero probability for the best consequence and the hypotenuse by a zero probability for the middle consequence.

An indifference curve is the set of lotteries with the same utility. Therefore, differentiating totally the utility function with respect to p_1 and p_3, and setting the derivative equal to zero to maintain the utility constant gives the set of indifference curves that characterize the preference function. The ratio $\frac{dp_1}{dp_3}$ is the slope of the tangent line to the indifference curve and is an indicator of its shape at any point in terms of the components of the numerator and the denominator.

Expected Utility

Axioms **A1** and **A3** of expected utility theory imply that preference functions can be represented by well-defined indifference curves in the space of the probability triangle. **A1** implies that lotteries either lie on the same indifference curve or on different indifference curves (completeness) and that these do not cross inside the triangle (transitivity) and **A3** implies that these are not thick and there are no holes in the indifference curves map. **A1** and **A3** do not impose however, any restrictions on the form of the indifference curves.[40]

The utility function of a lottery under expected utility is computed according to:

$$EU(L) = \sum_{j=1}^{n} p_j u(x_j)$$

Accordingly, the utility of lottery (L) is:

$$EU\,(L) = p_1\, u\,(x_1) + p_2\, u\,(x_2) + p_3\, u\,(x_3) \tag{2.1}$$

$$\frac{dp_1}{dp_3} = \frac{u(x_2) - u(x_3)}{u(x_1) - u(x_2)} \tag{2.2}$$

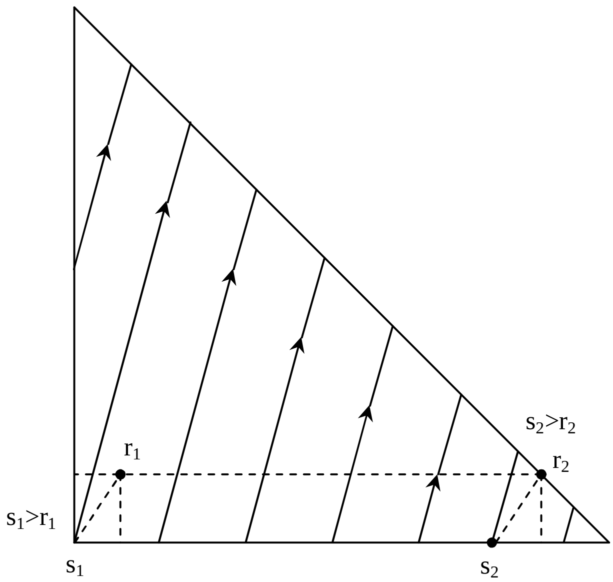
The slope is constant because it's independent of p and implies the indifference curves are linear and parallel with northwest movements along the hypotenuse leading to increasing preferences, i.e. to a higher utility level. The independence axiom **A3** implicit in the linearity property of expected utility theory restricts the indifference curves to linearity, parallelism, and upward sloping leaving however the slope undetermined.

Figure 2 shows the indifference curves under *EU* and also shows the Allais' lotteries [S_1, R_1, S_2, R_2] described in Table 1 above. It can be easily shown that the two lines which join the pairs of lotteries involved in the two choices are parallel. *EU* allows the

[40] Positive affine transformations $v(x) = a\,u(x) + b$, $a > 0$, represent preferences equivalently because the origin and the scale can be arbitrarily defined.

indifference curves to exactly coincide with the lines joining the lotteries [$S_1 \sim R_1$ and S_2, $\sim R_2$], to be less steep [$S_1 \prec R_1$ and $S_2 \prec R_2$] or steeper [$S_1 \succ R_1$ and $S_2 \succ R_2$].

Figure 2: Expected Utility Indifference Curves



However, the indifference curves generated by actual preferences of the Allais' experiments that is, $S_1 \succ R_1$ and $R_2 \succ S_2$ are not parallel. They are flatter in the right hand corner (S_2, R_2) relative to those in the left hand edge of the triangle (S_1, R_1). In other words, the preference in the right hand corner is contrary to the prediction of *EU* given the preference near the left hand border. To conform to actual behavior, indifference curves in any contender for expected utility need to be flatter in the right hand corner like for instance relative to those in the left hand edge of the triangle like.

Original Prospect theory

Original Prospect theory (Kahneman and Tversky, 1979) is a theory designed to account for psychological insights the authors found pervading laboratory and field data (for instance probability transformation) and which are not accounted for in the normative *EU*. The authors' aim is not as much to find out whether or not individuals are rational but to provide a *descriptive* model for actual individual behavior under risk. This review of prospect theory (1979) is based on Camerer (1989)[41] which shows an illustrative figure of prospect theory's indifference curves.

Original Prospect theory (*PT*) differs from expected utility in four points: 1) It applies only to lotteries, referred to as prospects by the authors, with at most two non zero outcomes; 2) the outcomes are "coded" that is perceived as gains or losses relative to a reference point and not final assets with losses looming larger than corresponding gains; 3) lotteries are edited to make them simpler to evaluate for instance using the rule of combination or cancellation for common outcomes; and 4) edited lotteries are evaluated according to one of several expectation like rules that combine the $u(x)$, or the $v(x)$ in the authors terminology,[42] and a decision weight $\pi(p)$ which transforms the probability non-linearly. The decision weight is increasing, subadditive, i.e., a change in probability has less impact as one moves away from the boundaries to the middle, ($\pi(p) + \pi(1-p) < 1$) and discontinuous at the end points 0 and 1. The utility function defined on i.e. deviations from the reference point is generally concave for gains and convex for losses and steeper for losses than for gains.

If $x_3 = 0$ and outcomes x_2 and x_1 are either both gains $x_1 > x_2 > 0$ or both losses $0 > x_2 > x_1$ relative to the reference point and $p_2 + p_1 < 1$ ($p_3 > 0$), the *edited* lottery is evaluated according to:

$$PT(L) = \pi(p_2)\, u(x_2) + \pi(p_1)\, u(x_1) \tag{2.3}$$

Camerer (1989, p: 75) shows that the slope is equal to:

$$\frac{dp_1}{dp_3} = \frac{\pi'(p_2)u(x_2)}{\pi'(p_1)u(x_1) - \pi'(p_2)u(x_2))} \tag{2.4}$$

[41] Camerer (1989, footnote 10) explains how prospect theory's indifference curves were computed.
[42] The utility terminology is used throughout this work; that is there is no reversion to the term "value" to describe utility under prospect theory.

The slope expressed in terms of the decision weight function and of the utility function varies according to the value of p in the numerator and in the denominator causing the indifference curves to be steeper (fan out) in some regions of the triangle (for instance in the lower left hand corner where $p_2 = 1$) and flatter (fan in) in others (for instance near the hypotenuse where p_2 is 0 and fixed).

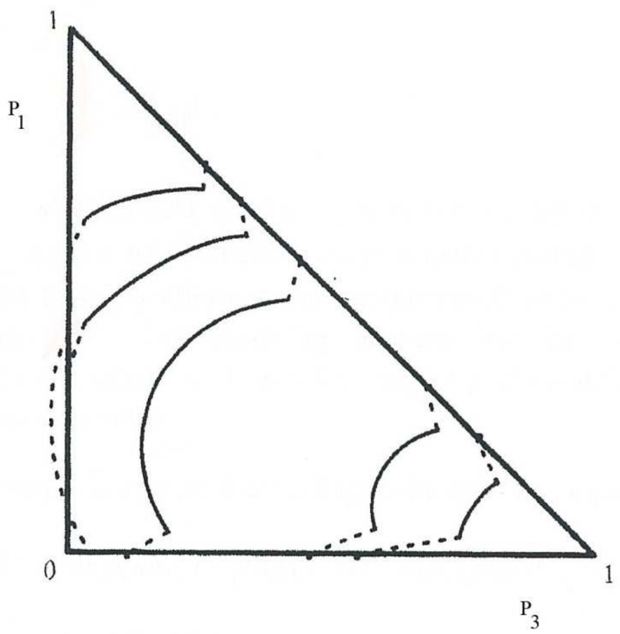
The indifference curves appear on the hypotenuse and near the lower edge very steep and flat respectively. The pattern reflects the preference people have for points inside those edges to points exactly on the edge which in turn reflects people's overweighting of the small probabilities of winning x_2 near the hypotenuse or x_1 near the lower edge.

Thus, indifference curves under prospect theory are not linear (π (p) is assumed to be convex by Kahneman and Tversky except near the end points), do not uniformly fan out and reflect sensitivity to extreme probabilities; however, they allow violation of monotonicity or first order stochastic dominance,[43] the most widely acknowledged criterion for rationality: In the lower left-hand corner, the negatively sloped part of the indifference curve shows that some lotteries stochastically dominate others but all are equally preferred. Kahneman and Tversky assume that people eliminate in the editing phase dominated lotteries if the dominance relation is transparent. The "if" however, leaves open the possibility of intransitive choice among three lotteries. Figure 3 shows the indifference curves of prospect theory (1979).

---

[43] $\forall\ F, G \in D(\boldsymbol{X})$ the set of all cumulative probability distributions functions over $\boldsymbol{X}$, a lottery F is defined to dominate a lottery G by first order stochastic dominance ($F \geq_1 G$) if $F(x) \leq G(x)\ \forall x \in \boldsymbol{X}$ and $F(x) < G(x)$ for at least one $x \in \boldsymbol{X}$.
The definition in terms of cumulative distributions is used for consistency with later sections. There is a one-to-one correspondence between the set $\boldsymbol{P}$ of all probability measures and the set $D(\boldsymbol{X})$ (Herstein and Milnor, 1953). Put differently, $F \geq_1 G$ if the probability that any x is less than x_i under G is greater or equal than the probability that any x is less than x_i under F with at least a strict inequality.

Figure 3: Indifference Curves Assuming Prospect Theory (1979)



Rank Dependent Utility

Quiggin's (1982) anticipated utility theory is the first model that incorporates a probability weighting function and a utility function without violating monotonicity as in (Handa, 1977; Kamarkar, 1979; Kahneman and Tversky, 1979). According to Quiggin, the fundamental problem in these theories is that any two outcomes with the *same* probability need not have the same decision weight; hence, in cases where extremes outcomes are overweighted, at least some intermediate outcomes perhaps with the same objective probability must be underweighted (Quiggin, 1982, p: 326-328). To formalize this insight, the entire cumulative distribution was transformed and each outcome weighted according to its rank relative to other outcomes by a discrete chunk[44] of the transformed cumulative distribution, ensuring monotonicity. The outcomes in each lottery are ranked ordered such as $x_1 \geq x_2 \geq \ldots \geq x_n$ and each outcome is weighted by a

[44] Or by differentials if the cumulative distribution function is continuous.

decision weight depending not only on the probability of the outcome but also on its rank, hence the name of rank dependent utility theory (*RDU*).

The utility of a lottery according to *RDU* is given by:

$$RDU(L) = \sum_{j=1}^{n} \pi_j u(x_j) \qquad (2.5)$$

Where

$$\pi_j = w(\sum_{i=1}^{j} p_i) - w(\sum_{i=1}^{j-1} p_i)$$

For all *j*.

The decision weights π_j sum to 1 and depend on the ranking of the outcomes. The function *w* (.) is the decision weight generated by the probability *p* when associated with the best outcome such that,

For each *j*,

$\pi_j = w(p_1 + \ldots p_j) - w(p_1 + \ldots p_{j-1})$ with $\pi_1 = w^+(p_1)$ for $j = 1$.

One could choose however to use *w** (*p*) the dual of *w* (*p*) that is *w** (*p*) = 1- *w* (1-*p*) for all *p*, which is the decision weight generated by the probability *p* when associated with worst outcome such that,

For each *j*,

$\pi_j = w^*(p_j + \ldots p_n) - w^*(p_{j+1} + \ldots p_n)$ with $\pi_n = w^*(p_n)$ for $j = n$.

This duality follows because the decision weights sum to one for any lottery (*M*, *p*; *m*, *1-p*) with outcomes *M* > *m*. *w* can be called the goodnews weighting function and *w** the badnews weighting function (Diecidue and Wakker, 2001).

The probability weighting function $w(.)$ is a strictly increasing function from [0, 1] to [0, 1] and verifies $w(0) = 0$ and $w(1) = 1$. The weighting function has the shape of an inverted *S* with, $w(0.5) = 0.5$, $w(p) > p$ if $p < 0.5$, $w(p) < p$ if $p > 0.5$, Quiggin (1982) having situated the cross over at 0.5.

The lottery *L* is evaluated according to the following equation:

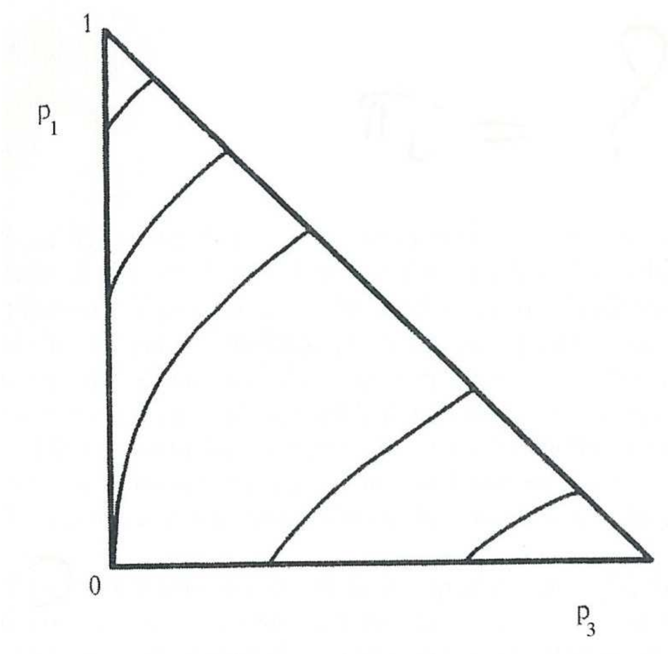
$$RDU\,(L) = w(p_3)u(x_3) + \left[w(p_3 + p_2) - w(p_3)\right]u(x_2) \quad + (1 - w(p_3 + p_2))u(x_1) \qquad (2.6)$$

Camerer (1989, p: 77) shows that the slope is equal to:

$$\frac{dp_1}{dp_3} = \frac{w'(p_3)(u(x_2) - u(x_3))}{w'(1 - p_1)(u(x_1) - u(x_2))} \qquad (2.7)$$

and Figure 4 shows that the indifference curves (the weighting function is assumed convex) are steepest in the left corner (fan out) and get flatter (fan in) as one moves along the lower edge $p_1 = 0$ or the left edge $p_3 = 0$ (vertically upwards) The curves are equal in slope along $p_2 = 0$ this property is called hypotenuse parallelism. Thus, the curves do not uniformly fan out, are not linear, reflect sensitivity to extreme probabilities *and* do not violate monotonicity.

Figure 4: Indifference Curves Assuming *RDU*



Expected utility is the special case of *RDU* where there is no probability weighting, $w(p) = p$ for all $p \in$ (0, 1) and equation (2.7) is reduced to equation (2.2).

Dual theory (Yaari, 1987) is a special case of anticipated utility theory where $u(x) = x$. Yaari's theory has been developed independently of Quiggin's and is the object of the nest section.[45]

[45] Green and Jullien (1988) and Segal (1989) have axiomatized independently a general model of rank dependent utility which contains anticipated utility and dual theory as special cases.

Dual Theory

Dual theory is so called because it's dual to expected utility theory. In contrast to the latter, it's linear in the utility and non-linear in the probability. This duality implies that dual theory is deduced from the same axioms of *EU* with the sole difference that the independence axiom is "laid on its side" (Yaari, 1987, p: 98). Rather than being assumed over convex combinations of probability measures and implying linearity in the probabilities, it is assumed over convex combinations of consequences and implies linearity in the consequences. The following equations compare the functional forms of dual theory, expected utility and rank dependent theory for ranked outcomes such as ($x_1 \geq x_2 \geq \ldots \geq x_n$):

$$DT(L) = \sum_{j=1}^{n} \pi_j x_j \tag{2.8}$$

$$EU(L) = \sum_{j=1}^{n} p_j u(x_j)$$

$$RDU(L) = \sum_{j=1}^{n} \pi_j u(x_j)$$

With $\pi_j = w(\sum_{i=1}^{j} p_i) - w(\sum_{i=1}^{j-1} p_i)$.

As seen from the above equations, dual theory with $u(x_i) = x_i$, has the merit of isolating the implications of the weighting function for risk aversion which is one of the two reasons that prompted Yaari (ibid, p: 95) to look for an alternative to expected utility.[46] Concerning *EU*'s empirical violations, Yaari's second motivation, *DT* accommodates the common consequence effect and the common ratio effect, but as Yaari (ibid, p: 96) acknowledges, behavior inconsistent with the linearity of the utility is often observed. Yaari (ibid, p: 108) also emphasizes that *DT* (as well as *RDU* for that matter) deals with how perceived risk is processed into choice and not how actual risk is processed into perceived risk and thus is not concerned with the violations due to perceptual causes.

[46] The characterization of risk aversion under the different theories is discussed in chapter III.

RDU, as is its special case *DT*, is thus focused on the mathematical connections between the axioms and the numerical representations of preferences and is less descriptive than the non-conventional model *PT* (1979) characterized by a procedural approach and reference dependence, an approach more common to psychology than to economics. The case for both suggested by Starmer (2000) seems to plead as in Camerer and Weber (1992) for a communication between psychologists and decision theorists whereby the former benefit from the latter in mathematical precision and the latter from the former in descriptive validity. Cumulative prospect theory (1992) a cross fertilization of the non-conventional prospect theory (Kahneman and Tversky, 1979) and the conventional anticipated utility theory (Quiggin, 1982) establishes such a common language.

Prospect Theory (1992)

Quiggin's (1982) *RDU* showed that decision weights constructed cumulatively eliminate the possibility of intransitive choice among three lotteries left open by *PT* (1979). By incorporating the idea, Kahneman and Tversky were able to obtain a transitive and monotonic preference function that generalizes to n-outcomes prospects without assuming the editing phase (Tversky and Kahneman, 1992).

Prospect theory (1992) differs from prospect theory (1979) in four points: 1) it applies to prospects with an arbitrary number of outcomes rather than to two non-zero outcomes at most; 2) although the outcomes are still "coded" that is perceived as gains or losses relative to a reference point with losses looming larger than corresponding gains, no editing phase is required; 3) the utility function in the evaluation rule is the same as in *PT* but the decision weight function transforms the entire cumulative distribution and needs not be the same for gains and losses; 4) *CPT* applies to both risk and uncertainty. Hence, the main difference between *PT* and *CPT* is a transformation of cumulative probabilities rather than of individual probabilities, which makes *CPT* a rank dependent theory since the decision weight attached to an outcome depends on the rank of the outcome relative to the other outcomes.

Nevertheless, it is a more descriptive and a more general representation. A comparison with anticipated utility shows that *CPT* differs from the latter in two respects: 1) the utility function is defined on changes in wealth (gains and losses) rather than on final wealth with losses looming larger than corresponding gains; 2) there are two weighting functions, one for gains and one for losses and the transformation is on decumulative probabilities in the gain domain and on cumulative probabilities in the domain of losses. Hence, the weighing function under *CPT* is equal to the sum of two weighting functions under anticipated utility which are computed separately for gains and losses. *RDU* corresponds to the special case where the weighting function for the losses is the dual of the weighing function for gains: $w^{-}(p)=1-w^{+}(p)$. The shape however of the weighting function is the same inverse *S*, under the two models.

CPT's indifference curves shown in Figures 5(*a*), 5(*b*) are constructed by Tversky and Kahneman (1992, p: 314). They are non linear, present mixed fanning and do not violate monotonicity. The curves for positive lotteries resemble those for negative lotteries but are not the same.

Figure 5 (*a*): Indifference Curves Assuming CPT for Positive Lotteries

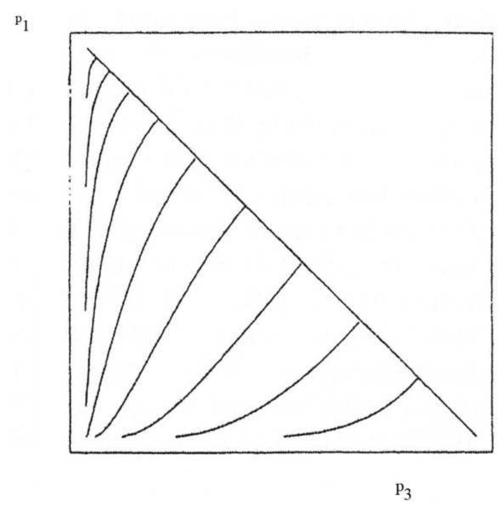
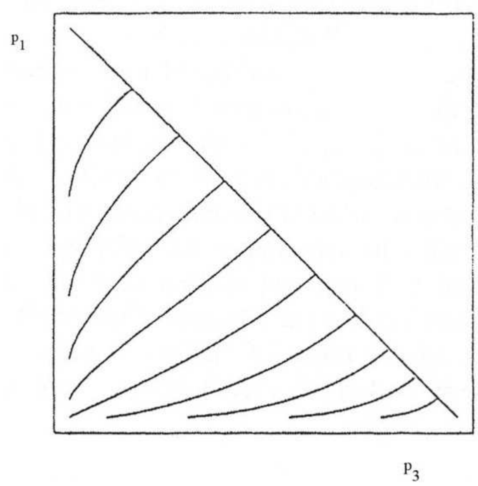


Figure 5 (*b*): Indifference Curves Assuming CPT for Negative Lotteries



Two important features distinguish thus *CPT* from *RDU*: 1) the utility function defined relative to a reference point is characterized by non-normative properties pertaining to perception and judgment as in losses looming larger than gains; 2) *CPT* generalizes rank dependent utility by allowing for different risk attitudes. Chapter III presents first the descriptive model followed by the formal model and its axiomatization for risk. The characterization of risk attitude in the different models is consigned to the last section of chapter III.

CHAPTER III. CUMULATIVE PROSPECT THEORY

> *"The goal that we set for ourselves was to assemble the minimal set of modifications of expected utility theory that would provide a descriptive account of everything we knew about a severely restricted class of decisions: choices between gambles."*
>
> (*Kahneman and Tversky, 2000, preface:* x)

What they knew from observations and a series of experimental and empirical investigations was that choices between gambles presented patterns that could be related to a simple perceptual cause: that of a reference point relative to which outcomes and/or probabilities are considered. They suggested new hypotheses which they were able to verify and provided labels for the patterns to ease their identification in more complex contexts. Two of these phenomena: the reflection effect and loss aversion pertain to the utility function and underlie the empirical realism relative to which prospect theory is considered a better description of individuals preferences than rank dependent theory. Although, this work is concerned primarily with the elicitation of the utility function, the weighting function is described in order to emphasize the importance of filtering its impact on the desirability of the outcomes. Section 3.1 presents the descriptive model followed by the formal model in section 3.2. Finally, the axiomatization for cumulative prospect theory for risk is presented in section 3.3 based on Chateauneuf and Wakker (1999).

3.1 *THE DESCRIPTIF MODEL*

Two principles, diminishing sensitivity to deviations from a reference point and loss aversion are invoked by Tversky and Kahneman (1992) to explain the characteristics of the utility function and the weighting function of cumulative prospect theory. The objective of this section is to present these non-normative elements in the theory. It proceeds as follows: first the intuition for the reference point is given and the importance of its localization for the order of preferences is emphasized. Loss aversion is presented next followed by the implications of these non-normative elements for the utility function and the weighting function in sections 3.1.2 and 3.2.3 respectively.

3.1.1 THE NON-NORMATIVE ELEMENTS IN THE THEORY

The Reference Point

The intuition of the reference point is best understood in relation to psychophysics. Psychophysics is the mapping of physical stimuli into psychological responses (Stevens, 1957; Sinn, H.1983)[47] and is characterized by diminishing sensitivity: discriminability is good in the central range where the most frequent stimuli occur and the mapping is almost linear; however, at the very high (low) stimuli, the sensitivity diminishes and the mapping is asymptotic. In other words, the cost of the nervous system adapting to the middle range is the decreased sensitivity at the ends of the stimuli continuum.

Kahneman and Tversky hypothesized that the same principle applies also to non-physical attributes such as prestige and wealth: past and present experience define a central range, to which people habituate and relative to which stimuli (in this case numbers) are perceived (Kahneman and Tversky, 1979, p: 277). The reference point refers to this adaptation level while diminishing sensitivity reflects the diminishing impact of a number as one moves away from the reference point. In the consequence domain for instance, a difference between a yearly salary of $60,000 and a yearly salary of $ 70,000 has a bigger impact when current salary is $50,000 than when it is $40,000 (Tversky and Kahneman, 1991). Similarly, the impact of a loss of $10,000 is greater when the reference point is $40,000 than when it is $50,000.

[47] Changes in the same stimulus do not yield necessarily changes of the same nature in the sensation or the perception. Put differently, the psychological response is a concave function of the magnitude of the physical change (Kahneman and Tversky, 1979, p: 278)

The Localization of the Reference Point

The reference point is usually taken as the current asset position to which outcomes are evaluated as changes in wealth (Tversky and Kahneman, 1991; Rabin, 2000; and Rabin and Thaler, 2001).[48] It's influenced nevertheless by many factors, among them, recent losses, aspirations, or expectations and therefore, may shift from the status quo across situations. The localization of the reference point is important for the order of preferences (Kahneman and Tversky, 1979, pp: 286-287)[49]. For instance, an investor who has integrated his assets (set the reference point to zero on the scale of wealth) is likely to choose differently from an investor whose reference point is his current asset position.[50] Also an investor who has not adapted to recent losses is likely in the domain of losses to behave more aggressively.[51] The difficulty for financial practitioners to control for losses is well documented in Glick (1957) and Kleinfield (1983).

Loss Aversion

Loss aversion refers to the asymmetrical treatment of gains and losses relative to a reference point: outcomes that are perceived as losses are experienced more keenly than outcomes perceived as gains.

Empirically, Kahneman and Tversky found that most people reject symmetric bets of the form (-$100, 0.5; $100, 0.5): "the aggravation that one experiences in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount…moreover, the aversiveness to symmetric bets increases with the size of the stakes" (Kahneman and Tversky, 1979, p: 279). Their finding was confirmed later by numerous empirical studies showing that loss aversion is a major factor in observed risk aversion (Thaler, 1980, Shefrin and Statman, 1985; Cachon and Camerer, 1996; Gneezy

[48] Markowitz (1952) was the first to argue for considering future outcomes as changes of wealth from a customary level.

[49] Schmidt (2003) and Sugden (2003) give general preference axiomatizations for varying reference points and Bleichrodt, Pinto, and Wakker (2001) use shifts of the reference point to compare preferences at different reference points, assuming the same utility at the same point for the varying reference points.

[50] Let $x<y<z<0,(x,p;z,1-p)\succ(y,1)$ shows risk seeking but when the assets are integrated, $(w+y,1)\succ(w+x,p;w+z,1-p)$ shows risk aversion.

[51] A person who has just lost 2000 and is facing a choice between a sure gain 1000 and 50/50 chance of winning 2000 or nothing, is likely to code the choice as between (-2000,0.5) and -1000 rather than a choice between (2000, 0.5) and 1000 and to prefer the former to the latter (Kahneman and Tversky, 1979, p: 286).

and Potters, 1997; Thaler et al., 1997; Bateman et al., 1997; Benartzi and Thaler, 1997; Payne, Laughhunn and Crum, 1981**;** Schoemaker and Kunreuther 1982; Hershey and Schoemaker 1985; Samuelson and Zeckhauser, 1988; Kahneman, Knetsch and Thaler, 1990; Tversky and Kahneman,1991; and Barberis, Huang and Santos, 2001).

Intuitively, to evaluate the attractiveness of possible prospects, the conscious mind integrates the past and the present subjective imaginings of the future (Shackle, 1955, Kahneman and Tversky, 1979, Damasio, 2003). The constrained feelings that obtain seem to be fundamental components of decision making: Damasio et al. (2000) and Rustichini et al, (in press)[52] demonstrate that people who cannot respond emotionally to the contents of their thoughts show defects in planning and judgment and lack the aversion to ambiguity or to losses. The feeling that arises in conjunction with an action resulting in a loss is aggravation accompanied with the desire to avoid a similar situation enlarging thus the risk which lies at the interface of what one desires and what one wishes to avoid. [53]

The theoretical support for loss aversion came later from Rabin (2000). Observed only in mixed gambles loss aversion reconciles small risk aversion for small stakes[54] which *EU* cannot explain, with realistic degrees of risk aversion for higher stakes. Evidence for risk aversion for small stakes in one shot mixed gambles is given by Samuelson's (1963) which shows that individuals reject favorable mixed gambles of the type: ($11, 0.5; $-10, 0.5) and by Kahneman and Tversky's (1979) which shows that individuals reject symmetric bets of the type ($11, 0.5; $-11, 0.5). Rabin's suggestion favoring the use of

[52]Antonio Damasio (2000) and his group of neuroscientists undertook a study with people with damage in the ventro-medial part of the pre-frontal cortex (VMPFC) which showed gross defects in the people's planning and judgment despite a high level of performance in language and intelligence tests. (A person with VMPFC damage cannot respond emotionally to the content of his thoughts). Also, Rustichini, Dickhaut, Ghirardato, Smith and Pardo (in press) show that people with VMPFC damage show lack of aversion to ambiguity or losses in situations similar to the Ellsberg Paradox.

[53] "The thing I fear most is fear" (Montaigne, 1588, Essais, Book I, 18, 'De la Peur'). "Fear serves as a magnet for fear" (Wollheim, 1999, p: 63-65).

[54] Officer and Halter's (1968) shows that even for small amounts of money farmers have non-linear utilities.

loss aversion to explain risk aversion- has been reiterated in “Anomalies: Risk Aversion” by Rabin and Thaler (2001).

3.1.2 The Utility Function

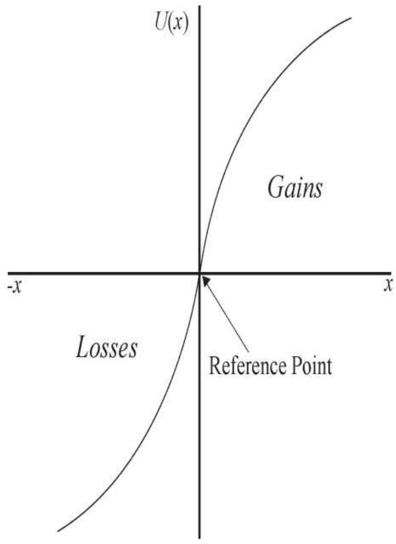
Because of diminishing sensitivity, the utility function is concave for gains, $u'' \leq 0$ and convex for losses, $u'' \geq 0$ and the reversal of preferences around the reference point is labeled the *reflection effect.* The empirical evidence of the phenomenon is listed in section 2.2.

Kahneman and Tversky (1979) also hypothesize that loss aversion is equivalent to a utility function that changes abruptly at the reference point. Graphically, the utility function u is steeper for losses than for gains $u'(x) < u'(-x)$ for $x \geq 0$.

Or for all $x > y \geq 0$, $u(x) - u(y) \leq u(-y) - u(-x)$

The utility function is thus: (1) defined on deviations from the reference point; (2) generally concave for gains and convex for losses it has an *S* shape; (3) steeper for losses than for gains. The proposed utility function is steepest at the reference point in marked contrast to the utility function postulated by Markowitz’(1952). Figure 6 shows the utility function assuming prospect theory.

Figure 6: The Utility Function Assuming Prospect Theory



3.1.3 The Weighting Function

The weighting function has two reference points, certainty and impossibility, which correspond to the end scales 0 and 1 of the probability domain. Diminishing sensitivity implies that a difference from 0.55 to 0.6 in probability has less impact than the difference between 0.05 and 0.1 or between 0.9 and 0.95. The implication is consistent with the overweighting of extreme probabilities and the underweighting of intermediate probabilities and explains the Allais paradoxes.[55]

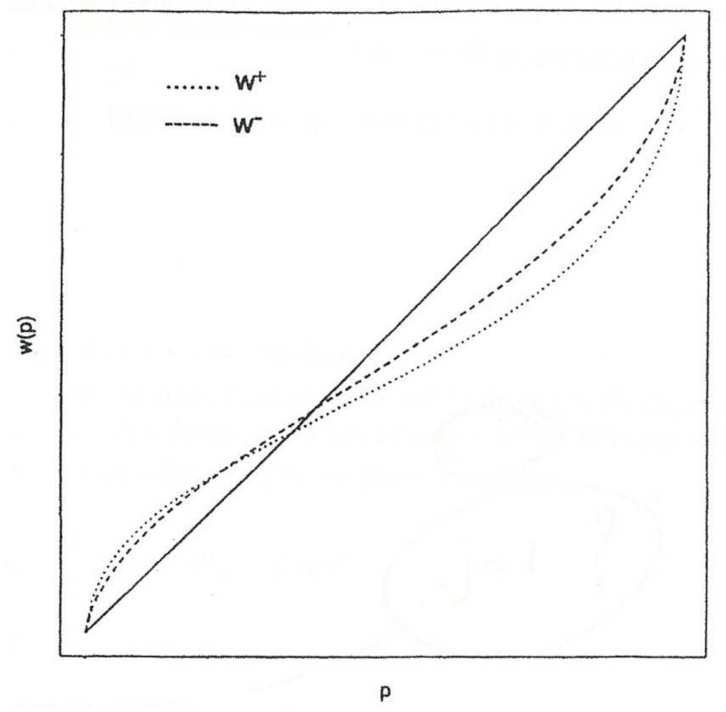
[55]Ranking the outcomes not only eliminated prospect theory's violations of stochastic dominance but also fitted well with the diminishing sensitivity principle hypothesized by Kahneman and Tversky (1979).

The weighting function is regressive: first it's greater than *p* then smaller than *p* (intersects the diagonal from above) asymmetric, inverse *S*-shape (concave first then convex). Quiggin (1982) has situated the cross over at 0.5 but later studies support a cross over value between 0.3 and 0.4 (Camerer and Ho, 1994; Tversky and Fox, 1995; Wu and Gonzalez, 1996, 1998; Prelec, 1998; Abdellaoui, 2000; Bleichrodt and Pinto, 2000). The weighting function is also subadditive.[56]

Another important characteristic is its sign dependency: there are two weighting functions, one for gains and one for losses that are similar in shape but not identical. The inverted *S* shape function is modeled as a single parameter function (Tversky and Kahneman, 1992; Prelec, 1998) or as a two parameter function (Gonzalez and Wu, 1999; Lattimore et al, 1992) to distinguish the elevation of the function (which refers to the degree of overweighting/ underweighting) from its curvature (which refers to how people discriminate intermediate probabilities). Abdellaoui (2000), who estimated the probability weighting function for the two models, showed that: 1) that when the weighting function was assumed to have Tversky and Kahneman (1992) single-parameter form, the median estimates obtained were very close to those obtained by Tversky and Kahneman (1992) for gains and losses which were 0.61 and 0.69 respectively and 2) that when the weighting function was assumed to have the Lattimore et al. (1992) two-parameter function, it exhibited more elevation for losses than for gains. That is the probability weighting function exhibits less pronounced curvature and more elevation for losses than for gains. Figure 7 shows the weighting functions for gains and losses estimated by Tversky and Kahneman (1992).

[56] Lower subadditivity is also known as the possibility effect: $w(q)-w(0)\geq w(p+q)-w(p)$ whenever $p+q\leq 1-\varepsilon$ for constant $\varepsilon\geq 0$ and upper subadditivity is also known as the certainty effect: $w(1)-w(1-q)\geq w(p+q)-w(p)$ whenever $p\geq\varepsilon'$ for constant $\varepsilon'\geq 0$ (Abdellaoui, 2000).

Figure 7: The Weighting Functions for Gains and Losses



3.1 *THE FORMAL MODEL*

Cumulative Prospect theory applies to objective and subjective and uncertainty. This section presents the formal model under both.

3.2.1 Decision under Risk

Let $\boldsymbol{X} = \{x_1,\ldots, x_n\}$ be a rank-ordered finite set of monetary outcomes including a neutral outcome 0. To recall, in decision making under risk, a lottery is described by a finite probability distribution over ***X.*** It is denoted by $(x_1,p_1; \ldots x_n, p_n)$ yielding outcome x_j with probability p_j for $j = 1\ldots, n$; the probabilities are non-negative and sum to one.

Following *CPT*, the utility of a lottery depends on a utility function *u* and a probability weighting function *w*. The function *u* defined on gains and losses is a continuous strictly increasing function satisfying $u(0) = 0$. The function *w* is a strictly increasing function from [0,1] to [0,1] with $w(0) = 0$ and $w(1) = 1$.

Furthermore, the weighting function is differentiated for gains and losses into w^+ and w^- leading to sign dependence with $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$.

For the lottery $L = (x_1,p_1; \ldots x_n, p_n)$ in which $x_1 \geq \ldots x_k \geq 0 \geq x_{k+1} \geq \ldots \geq x_n$ and $0 \leq k \leq n$, all outcomes are gains if $k = n$ and all outcomes are losses if $k = 0$. [57]

The *CPT* functionals are represented by equations (3.1), (3.2), (3.3):

$$CPT(L) = \sum_{j=1}^{n} \pi_j u(x_j) \tag{3.1}$$

Where

$u(x_i) = (x_i)^{\alpha}$ [58] for positive changes in wealth and

$u(x_i) = -\lambda(-x_i)^{\beta}$: for negative changes in wealth[59] where λ is the loss aversion factor and α, β the parameters that define the shape of the (Tversky and Kahneman, 1992) utility function fitted to a power function. The decision weights are defined as follows:

$$\pi_j = w^+(\sum_{i=1}^{j} p_i) - w^+(\sum_{i=1}^{j-1} p_i) \quad \text{for all } j \leq k \tag{3.2}$$

---

[57] The Chateauneuf and Wakker's (1999) notations for the axiomatization are adopted throughout for consistency.

[58] There is no need in prospect theory to adopt the power function for utility. Tversky and Kahneman fitted parametrically their findings to the power because preference homogeneity, that is multiplying the outcomes of prospect by a constant $k > 0$ multiplies its cash equivalent by the same constant, is both necessary and sufficient under their theory to represent utility as a power function.

[59] A decision maker is well advised to assign a greater weight lamda to negative than to positive outcomes, to reflect the asymmetry between the experience of gains and losses (Tversky and Kahneman, 1991). Utility is a ratio scale under *CPT*, i.e. it's unique up to a multiplication by a positive constant.

$$\pi_j = w^-(\sum_{i=j}^{n} p_i) - w^-(\sum_{i=j+1}^{n} p_i) \text{ for all } j > k. \quad (3.3)$$

$\pi_1 = w^+(p_1)$ for $j = 1$ and $\pi_n = w^-(p_n)$ for $j = n$. These decision weights do not necessarily sum to one.

3.2.2 Decision under Uncertainty

Under uncertainty, the utility of a lottery is represented by $(x_1, A_1 ; x_2, A_2 ; \ldots x_n, A_n)$. A_j is a subset of a state space $\boldsymbol{S}$ called an event, $(A_1, A_2, \ldots A_n)$ is a partition of $\boldsymbol{S}$ and $x_i \in \boldsymbol{X}$ is the outcome associated with the states contained in A_j

Under *CPT*, the utility of the lottery $(x_1, A_1 ; x_2, A_2 ; \ldots x_n, A_n)$ in which $x_1 \geq \ldots x_k \geq 0 \geq x_{k+1} \geq \ldots \geq x_n$ is given by the following formula:

$$U(L) = \sum_{j=1}^{k} \pi_i^+ u(x_i) + \sum_{j=k+1}^{n} \pi_i^- u(x_i)$$

Where the decision weights are defined by :

$$\pi_j^+ = W^+(A_1 \cup \ldots \cup A_j) - W^+(A_1 \cup \ldots \cup A_{j-1})$$

$$\pi_j^- = W^-(A_j \cup \ldots \cup A_n) - W^-(A_{j+1} \cup \ldots \cup A_n)$$

With $\pi_1 = W^+(A_1)$ for $j = 1$ and $\pi_n = W^-(A_n)$ for $j = n$.

For gains and losses under risk and uncertainty, decision weights as well as subjective probabilities show subadditivity.[60] Research findings show that experts (option traders) have $w(p) = p \ \ \forall p \in (0,1)$ because of their familiarity with probabilities and calculus (Fox, Rogers and Tversky, 1996) while both lay and experts have subadditive probability judgment (Tversky and Koehler, 1994) i.e. they are not immune to biases and heuristics.

60 Subadditivity under uncertainty is defined as follows: $LSA: W(A) \geq W(A \cup B) - W(B)$ provided $W(A \cup B) \leq W(S-E)$; $USA: 1 - W(S-A) \geq W(A \cup B) - W(B)$ provided that $W(B) \geq W(E')$ where E and E' are boundary events (Abdellaoui, Vossmann and Weber, 2005).

CPT is axiomatized by Chateauneuf and Wakker (1999) for risk and by Wakker and Tversky's (1993) for uncertainty[61].

3.3 *THE AXIOMATIZATION OF CPT FOR RISK*

A central idea to the axiomatization of *CPT* under uncertainty and under risk is tradeoff consistency. The latter is also essential to the elicitation method used in this experimental work to encode the utilities of financial practitioners. Therefore, before presenting it formally, this section provides first the definition of the idea of tradeoff based on Wakker (1994). The intuition for tradeoff consistency is then illustrated through an example taken from Wakker and Tversky's (1993, p: 149) which axiomatizes in one stroke Savage's *SEU*, Schmeidler's (1989) and Gilboa's (1987) *CEU* (cumulative expected utility), *RDU* and *CPT*. The axiomatization for risk by Chateauneuf and Wakker (1999) follows.

3.3.1 The Idea of Tradeoff

This subsection defines the idea of trade-off as Wakker (1994)[62] has adapted it for decision making under risk. The notation with the star superscript below indicates the relation is quaternary; it might be interpreted as a revealed ordering of strength of preferences. The outcomes in the lotteries are rank-ordered.

Definition 3.1:
For four outcomes $\alpha, \beta, \gamma, \delta$, we write $[\alpha;\beta] \succcurlyeq^* [\gamma;\delta]$ or for short $\alpha\beta \succcurlyeq^* \gamma\delta$ if
$(x_1, p_1; \ldots; \alpha, p_j; \ldots; x_n, p_n) \succcurlyeq (y_1, p_1; \ldots; \beta, p_j; \ldots; x_n, p_n)$ and

$(x_1, p_1; \ldots; \gamma, p_j; \ldots; x_n, p_n) \preccurlyeq (y_1, p_1; \ldots; \delta, p_j; \ldots; x_n, p_n)$
For some j, $p_1, \ldots y_n$

Substituting *RDU* for the preferences at constant probabilistic risk and canceling the common positive decision weights shows:

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[61] Other axiomatizations of *CPT* include: Luce (1991); Luce and Fishburn (1991); Tversky and Kahneman (1992); Schmidt (2001); Schmidt and Zank,(2001).
[62] The author has used it previously in several papers for decision making under uncertainty (e.g. Wakker, 1989*a*)

If *RDU* holds, then:

$\alpha\ \beta \succcurlyeq^{*}\ \gamma\delta \quad \Rightarrow\ u(\alpha) - u(\beta) \geq u(\gamma) - u(\delta)$

$\alpha\ \beta \succ^{*}\ \gamma\delta$ holds if any of the weak preferences is strict, then if *RDU* holds, and

$\alpha\ \beta \succ^{*}\ \gamma\delta \quad \Rightarrow\ u(\alpha) - u(\beta) > u(\gamma) - u(\delta)$

The $\succ^{*}$ agrees with the ordering of utility differences. In words, the replacement of the outcomes α, β by γ, δ has caused a reversal of preference. The latter being observed under constant probabilistic risk must be explained by the change in the outcomes; the tradeoff $\alpha\beta$ (receiving α instead of β must be a stronger improvement (less serious loss) than the tradeoff $\gamma\delta$.

3.3.2 The Intuition for Tradeoff Consistency

The main idea is that if inconsistencies in revealed tradeoff comparisons uncover deviations from the model assumed, then tradeoff consistency is necessary (it turns out to be also sufficient for *CPT* given some natural conditions (Chateauneuf and Wakker, 1999, p: 142)). To illustrate, the following example is taken from Wakker and Tversky (1993, p: 149).

Consider the lottery (x_1, x_2) yielding \$$x_1$ if state 1 obtains and \$$x_2$ if state 2 obtains and the following pattern of preferences:

(11, 20) $\succcurlyeq$ (10, 21) and

(31, 20) $\prec$ (30, 21)

Applying *SEU* to both gives:

$P_1 *\ u(11)\ +\ P_2 * u(20) \geq\ P_1 *\ u(10) +\ P_2 * u(21)$ and

$P_1 *\ u(31)\ +\ P_2 * u(20) <\ P_1 *\ u(30) +\ P_2 *\ u(21)$

Where $P_1 * P_2 *$ are the decision maker's probabilities for states 1 and 2. Under subjective expected utility (*SEU*) the following ordering of value differences *u*(11) – *u*(10) > *u*(31)-*u*(30) is obtained which implies that receiving 11 instead of 10 is a stronger improvement than receiving 31 instead of 30 and can be interpreted as a revealed ordering of strengths of preferences: [11; 10] $\succ^*$ [31; 30].

If however the following pattern of preferences is also observed:
(40, 31) $\succcurlyeq$ (41, 30) and
(40, 11) $\preccurlyeq$ (41, 10)

The opposite ordering of utility differences: $u(31)-u(30) \geq u(11)-u(10)$ is observed revealing an inconsistency with *SEU* but not with *RDU* or *CPT* where the outcomes 20, 21 are the best outcomes in the first preference and the worst in the second. A decision maker who pays more attention to the worst outcome will assign more weight to the outcomes 20, 21 when they are the least desirable outcomes. If the weights attached to their states are not the same the inference $u(11)-u(10) > u(31)-u(30)$ is not valid (Wakker and Tversky, 1993, p: 150-151) but the inference $u(31)-u(30) > u(11)-u(10)$ remains valid because the state 2 yields the least desirable outcomes in both preferences.

Hence, when the two lotteries are comonotonic,[63] the decision weights are the same and cancel. In that case, contradictory inequalities of utility differences are avoided. Under *CPT* because sign dependency allows decision weights to differ depending on whether they are associated with gains or losses, restricting comonotonicity to sign comonotonicity (the critical outcomes must be either all gains or all losses) is necessary.

3.3.3 The Axiomatization

This section provides first the basic definitions of decision under risk and cumulative prospect theory. Rank dependence is accommodated by having the outcomes ranked in the lotteries and sign dependence is accommodated by requiring all outcomes to have the same sign. It is shown next that the * relations elicit orderings of utility differences under cumulative prospect theory. Tradeoff consistency is then defined and the theorem stated formally.

[63]Two acts *x* and *y* are comonotonic if $x_i > x_j$ wherever $y_i > y_j$ for any two states *i* and *j*, assuming constant probabilities throughout.

The Basic Concepts

A function U represents $\succcurlyeq$ if, for all prospects, P, Q, $P \succcurlyeq Q$ iff $U(P) \geq U(Q)$; then $\succcurlyeq$ is a weak order, complete, transitive. $\succcurlyeq$ is continuous if the sets:

$\{(x_1,\ldots,x_n) \in X^n : (x_1, p_1;\ldots; x_n, p_n) \succcurlyeq (y_1, p_1;\ldots; y_n, p_n)\}$ and

$\{(x_1,\ldots,x_n) \in X^n : (x_1, p_1;\ldots; x_n, p_n) \succcurlyeq (y_1, p_1;\ldots; y_n, p_n)\}$

are closed sets for every n and fixed n-tuple of probabilities $(p_1,\ldots ; p_n)$ and lottery $(y_1$ $p_1;\ldots; y_n\ p_n)$. Stochastic dominance is satisfied if $\{(x_1\ p_1;\ldots; x_n\ p_n) \succcurlyeq (y_1\ p_1;\ldots; y_n\ p_n)\}$ whenever $x_j \succcurlyeq y_j$ for all j with a strict preference for at least one j with $p_j > 0$.

Tradeoff Consistency

For a lottery $P = (x_1, p_1;\ldots; x_n, p_n)$ and an outcome $\alpha, \alpha_j P$ is defined as resulting from P by replacing x_j by α, i.e.

$$\alpha_j P = (x_1, p_1;\ldots; x_{j-1}, p_{j-1};\ldots; \alpha, p_j; , x_{j+1}, p_{j+1};\ldots; x_n, p_n)$$

Because the weighting function under *CPT* is differentiated for gains and losses into w^+ and w^- leading to sign dependence, it's crucial that all outcomes have the same sign. Hence, a prospect is divided into its gain part and into its loss part. For any lottery P then, the lottery P^+ is obtained if all consequences $x_j < 0$ are replaced by 0 and the lottery P^- is obtained if all consequences $x_j > 0$ are replaced by 0.

The following relations can be used theoretically and empirically to elicit the ordering of utility differences.[64] We write $[\alpha;\beta] \succcurlyeq^* [\gamma;\delta]$ or $\alpha\beta \succcurlyeq^* \gamma\delta$ for short if four outcomes $\alpha, \beta, \gamma, \delta$, are *all gains or all losses* and there exists $P = (x_1\ p_1;\ldots; x_n\ p_n)$ and $Q = (y_1\ p_1;\ldots; y_n\ p_n)$ with the same probability tuple $p_1\ldots p_n$ and an index j with $p_j > 0$ such that

$\alpha_j P \succcurlyeq \beta_j Q$ and

$\gamma_j P \preccurlyeq \delta_j Q$

[64] As shall be seen in Part II, this work uses the * relations to elicit the utilities of financial practitioners.

We write $\alpha\beta \succ^{*} \gamma\delta$ if in the lower preference we have $\prec$ instead of $\preccurlyeq$

$\alpha_j P \succcurlyeq \beta_j Q$ and (3.4)

$\gamma_j P \prec \delta_j Q$ (3.5)

Substituting the *CPT*'s formulas described in (3.1), (3.2) and (3.3) in (3.4) and (3.5) and satisfying and sign-comonotonicity, i.e., as Chateauneuf and Wakker (1999) emphasize, the decision weights for $\alpha_j P$ and $\gamma_j P$ are the same and are written as π_i, $i = 1,\ldots n$, and the decision weights for $\beta_j Q$ and $\delta_j Q$ are also the same and are written as λ_i, $i = 1,\ldots n$. The π_i's may differ from the λ_i's because of different signs of outcomes. The *j*th outcomes in all lotteries have the same sign however, hence $\pi_j = \lambda_j$ is given by equation (3.2) if the outcomes are positive and by equation (3.3) if they are negative. This equality is crucial and is emphasized by: $\mu = \pi_j = \lambda_j$

$$\sum_{i \neq j} \pi_i u(x_i) + \mu u(\alpha) \geq \sum_{i \neq j} \lambda_i u(y_i) + \mu u(\beta) \text{; hence}$$

$$\mu(u(\alpha) - u(\beta)) \geq \sum_{i \neq j} \lambda_i u(y_i) - \sum_{i \neq j} \pi_i u(x_i)$$

And

$$\sum_{i \neq j} \pi_i u(x_i) + \mu u(\gamma) < \sum_{i \neq j} \lambda_i u(y_i) + \mu u(\delta) \text{; hence}$$

$$\mu(u(\alpha) - u(\delta)) < \sum_{i \neq j} \lambda_i u(y_i) - \sum_{i \neq j} \pi_i u(x_i) \text{; hence}$$

$$\mu(u(\alpha) - u(\beta)) > \mu(u(\gamma) - u(\delta))$$

$$\alpha\ \beta \succ^{*} \gamma\delta \Rightarrow u(\alpha) - u(\beta) > u(\gamma) - u(\delta) \quad (3.6)$$

$$\alpha\beta \succcurlyeq^{*} \gamma\delta \Rightarrow u(\alpha) - u(\beta) \geq u(\gamma) - u(\delta) \quad (3.7)$$

Tradeoff consistency holds if there are no outcomes $\alpha, \beta, \gamma, \delta$

such that both $\alpha\beta \succ^{*} \gamma\delta$ and $\gamma\delta \succcurlyeq^{*} \alpha\beta$

Formally, the above results can be stated in the following theorem.
Theorem: The following statements are equivalent:

(i) Cumulative Prospect theory holds with a continuous value function

(ii) $\succcurlyeq$ Satisfies the following conditions:
(a) Weak ordering
(b) Continuity
(c) Stochastic dominance
(d) Tradeoff consistency[65]

To recapitulate, cumulative prospect theory, a descriptive decision making model for risk and uncertainty, hypothesizes a neutral reference point relative to which future outcomes are evaluated as gains or losses rather than increases or decreases in total wealth. This notion, the cornerstone of the theory came to the authors from observations that people are in general risk averse for gains and risk seeking for losses and that they are extremely reluctant to accept mixed prospects.[66] The perception of a reference point implies 1) diminishing sensitivity for the two part cumulative functional of the valuation rule and 2) loss aversion. These two components of psychological nature are invoked by Tversky and Kahneman (1992) to explain the characteristic reflection pattern of attitudes towards risk in terms of the utility function and the probability weighting functions. The following section is devoted to the characterization of risk under *EU*, *RDU*, and *CPT*.

3.4 *THE CHARACTERIZATION OF RISK ATTITUDE*

An important property of expected utility theory is that different notions of risk aversion defined independently of any model are equivalent to the concavity of the vNM utility function that is to the diminishing marginal utility principle.

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[65] Sign-dependence is accommodated by the requirement that all outcomes should have the same sign.
[66] The behavior has been documented in a review by Fishburn and Kochenburger (1979) around the time they wrote their first version of the theory. Fishburn and Kochenberger's is a review of five independent studies by Barnes and Reinmuth (1976), Grayson (1960), Green (1963), Halter and Dean (1971), and Swalm (1966).

Yaari (1987) has however shown that “risk aversion and diminishing marginal utility of wealth are horses with different colors” and an agent can be risk averse without a concave utility. Moreover, Chateauneuf and Cohen (1994) have shown that an agent can be risk averse even with a convex utility provided his probability weighting function synonymous to probabilistic risk aversion is sufficiently convex.

These two components of risk aversion could be further differentiated as suggested by the two “empirically desirable”[67] generalizations of rank dependent utility by Tversky and Kahneman (1992) who brought in loss aversion and allowed for different risk attitudes for gains than for losses to explain the complex patterns of risk behavior displayed in even very simple contexts.

The following section presents the behavioral notions of risk aversion and establishes the links between these notions and the concavity/convexity of the utility function. The characterizations of risk aversion under *EU*, *RDU* and *CPT* are presented next.

3.4.1 Notions of Risk Aversion

There are many ways to define risk aversion. Most notably, Rothschild and Stiglitz' (1970) definition of *strong risk aversion* based on the notion of an increase in riskiness and Pratt's (1964) and Arrow's (1965), notion of *weak risk aversion* which represents more the intensity of risk aversion than risk aversion in the strict sense.

Strong Risk Aversion

The notion of an increase in riskiness is directly linked to the notion of second order stochastic dominance (*SSD*). The formal definition of first order stochastic dominance $F \geq_1 G$ (the cumulative distribution of F is uniformly below the cumulative distribution of

[67]Köbberling and Wakker (2005)

G) was given previously and it can be shown that if $F \succcurlyeq G$ according to *FSD* then necessarily that implies that the expected utility of *F* is greater than the expected utility of *G* for any agent with a monotonic non-decreasing utility function.[68] All of expected utility, rank dependent utility, dual theory and cumulative prospect theory satisfy first order stochastic dominance.

SSD ranks cumulative distributions in terms of their relative riskiness that is in terms of the *spread* of their probability mass when neither distribution is uniformly below the other. If $F \succcurlyeq G$ according to *SSD* (the probability mass under *F* is less spread out than it is under *G*), then necessarily that implies that the expected utility of *F* is greater than the expected utility of *G* for any agent with a non-decreasing *concave* utility function.[69] Thus *FSD* implies *SSD* but not vice versa. Formally, second order stochastic dominance is defined as follows:

Definition 3.2:
$\forall\ F, G \in D(\boldsymbol{X})$, a lottery *F* is defined to dominate a lottery *G* by second order stochastic dominance ($F \geq_2 G$) if $G(x) - F(x) \geq 0\ \forall x \in \boldsymbol{X}$

If, in addition, *F* and *G* have the same mean ($E(F) = E(G)$) it's said that *G* is a mean preserving increase in spread (*MPIS*) constructed by moving probability mass away from the center of the distribution to its tail in such a manner that the mean remains the same. The following definition of strong risk aversion by Rothschild and Stiglitz (1970) delineates a risk averse individual as someone who always dislikes mean–preserving spreads.

Definition 3.3:
An agent has strong aversion for risk if for any two lotteries *F* and *G* having the same mean such that *F* dominates *G* by second order stochastic dominance, he prefers *F* to *G*.

$\forall\ F, G \in D(\boldsymbol{X}), E(F) = E(G), F \geq_2 G \Rightarrow F \succcurlyeq G.$

[68] For proof of the *FSD* rule cf. (Hadar and Russell, 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970)
[69] Ibid for proof of the *SSD* rule.

Weak Risk Aversion

Risk aversion can be also defined in terms of the *certainty equivalent* that is the amount of cash one is willing to accept with certainty in lieu of facing a lottery *P*. An agent is indifferent between the lottery *P* and its *certainty equivalent lottery* which is the sure-thing lottery that yields the same utility as the lottery *P*, i.e. $\delta_{CE(P)} \sim P$; however, the certainty equivalent *CE* (*P*) which is the inverse of the utility of the lottery varies according to the shape of the utility function as can be seen from Figure 8 in Appendix A. When the shape of the utility function is concave, the certainty equivalent is less than the expected value *E* (*P*). The difference denoted by $\pi(P) = E(P) - CE(P)$ is known as the risk premium i.e. the maximum amount one is willing to forego in order to obtain an allocation without risk (Pratt, 1964). *Ad modum,* when the shape of the utility function is convex, $CE\ (P) > E(P)$ and when it is linear, $CE\ (P) = E(P)$ (Eeckhoudt and Gollier, 1992, p:26-28).

An agent is risk averse if $CE\ (P) < E(P)$ or $\pi(P) > 0$

An agent is risk seeking if $CE\ (P) > E(P)$ or $\pi(P) < 0$

An agent is risk neutral if $CE\ (P) = E(P)$ or $\pi(P) = 0$

Definition 3.4:
An agent has weak risk aversion if for any lottery $P \in \boldsymbol{P}$ he prefers to this lottery the certainty of its expectation: $\forall P \in \boldsymbol{P}, E\ (P) \succcurlyeq P$

It's possible to order the notions of risk aversion. Strong risk aversion implies weak risk aversion but not vice-versa. The following section characterizes risk aversion under the different theories.

3.4.2 Characterization of Risk Aversion under the Different Theories

The following section establishes the links between the notions of risk aversion defined independently of any model presented above and the convexity and concavity of the utility function under the different theories. That is it characterizes risk aversion under *EU*, *RDU* and *CPT* respectively.

The Characterization of Risk Aversion under *EU*

Proposition 3.1:
If an agent satisfies the hypotheses of the expected utility model, then the three statements are equivalent:

i) The agent has a strong aversion for risk
ii) The agent has a weak aversion for risk
iii) The agent's utility function is concave ■

Arrow (1964) and Pratt (1965) were able independently to find a measure for the degree of risk aversion in terms of the properties of the utility function by approximating the risk premium of a lottery. Using approximation formulas of a continuous and differentiable function, they showed that under *EU*, the risk premium is equal to $-\frac{1}{2}\sigma^2 \frac{u''(w)}{u'(w)}$ where σ^2 is the variance of the lottery and $\frac{-u''(w)}{u'(w)}$ is the measure of risk aversion.

This local measure $(\frac{-u''(w)}{u'(w)})$ known as the *degree of absolute risk aversion* is fundamentally specific of the individual at a certain level of wealth and enables the comparison of degrees of risk aversion between two individuals who might have the same wealth, might be endowed with the same lottery and yet demand different risk premiums because the curvatures of their utility functions are different. This measure of the intensity of risk aversion is independent of the notions of risk aversion defined above and does not carry over to the other models. The approximation gives different results under different models (Eeckhoudt and Gollier 1992, p: 26-27).

The Characterization of Risk Aversion under *RDU*

The characterization of risk under *EU* excludes thus the possibility of risk seeking behavior for a decision maker exhibiting a concave utility function. However under rank dependent utility theories, the decision maker's behavior is characterized by two functions *u* and *w* which could explain a mixture of risk seeking and of risk aversion (Quiggin, 1991). Yaari (1987) having shown that an agent with a convex probability weighting function can be risk seeking without having a convex *u*, Chateauneuf and Cohen (1994) further demonstrate that an agent can be risk averse even if he has a convex *u* provided a weak definition of risk aversion is adopted. The provision is important for when a strong definition of risk aversion is considered, global risk seeking behavior is inconsistent with the diminishing marginal utility of wealth as shown by Chew, Karni and Safra (1987). The latter defined strong risk aversion for *RDU* as follows:

Proposition 3.2:
An agent satisfying the hypotheses of rank dependent utility has a strong aversion for risk iff his probability weighting function $w(.)$ is convex and his utility function u (.) is concave; w(.) and u (.) are differentiable ■

A convex weighting function can be interpreted as a form of pessimism.[70] The intuition is developed in Diecidue and Wakker (2001)[71]. The decision maker may decide *deliberately and consciously* that the best/worst outcomes should receive more attention, more importance weight than the intermediate outcomes; hence under pessimism improving the ranking of the outcome decreases the decision weight. A weak probabilistic risk aversion has been defined by Quiggin (1982) and Yaari (1987) as follows:

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[70] Assume a lottery yields outcome x with probability p. Let q denote the ranking position of x, i.e. the probability of receiving a lower or equal outcome. The decision weight of x then is $w(p+(1-q)) - w(1-q)$ which is under pessimism decreasing in q iff w is convex (Diecidue and Wakker, 2001, p: 288).

[71] Psychology's contributions to the intuition of rank dependence comprise Birnbaum (1974), Lopez (1987) independently of Quiggin, Weber (1994) and others and are listed in Diecidue and Wakker (2001).

Proposition 3.3:
An agent satisfying the hypotheses of *RDU* has weak risk aversion iff for all $p \in [0, 1]$, $w(p) \leq p$; *u* (.) is differentiable and concave ■

Probabilistic aversion in the weak sense gives a necessary and sufficient condition of risk aversion under the assumption of diminishing marginal utility. Chateauneuf and Cohen (1994) provide a sufficient condition for weak risk aversion under the assumption of increasing marginal utility as follows:

Proposition 3.4:
An agent satisfying the hypotheses of rank dependent utility and having a convex *u* (.) can be risk averse in the weak sense if his function *w*(.) is sufficiently convex, i.e if he is sufficiently pessimistic ■

For strong risk aversion, *u* (.) is concave, and *w* (.) convex as in (Chew, Karni and Safra, 1987), that is a pessimistic agent with a concave *u* (.) is universally risk averse.

The links established thus far between the different notions of risk aversion and the preference functionals under *RDU* can be extended to dual theory a special case of *RDU*. For *CPT*, the results obtained under *RDU* can be extended to prospects restricted to either gains or losses but not to mixed prospects.

The Characterization of Risk Aversion under *CPT*

CPT a reference dependent model has three distinct notions of risk: an intrinsic utility, a probability weighting function and loss aversion. Since loss aversion is only observed in mixed prospects the characterization of risk aversion under *CPT* distinguishes among positive prospects (all gains), negative prospects (all losses) and mixed prospects.

The conditions for strong risk aversion under *CPT* and *RDU* (the special case of *CPT* where the weighting function for losses is the dual of the weighting functions for gains, i.e., $w^{-}(p) = w^{+}(1-p) \forall p \in (0,1)$ coincide if the prospects considered are only positive or only negative.

Proposition 3.5:
For either gains or losses an agent satisfying the hypotheses of *CPT* has a strong aversion for risk iff his probability weighting function is convex and his utility function is concave▪

For mixed prospects, risk aversion under *CPT* is characterized through a joint condition on utility curvature, probability weighting and loss aversion by Schmidt and Zank (2002). Their characterization supports the Köbberling and Wakker (2005) index of loss aversion which is the first axiomatically founded index proposed. Table 6 summarizes the different characterizations of risk aversion under the different theories.

Table 6: Characterization of Risk Aversion under the Different Theories

	EU	**DT**	**RDU**	**CPT**	**CPT**	**CPT**
				(+) Prospects	**(-) Prospects**	**M. Prospects**
	u(.)	*w(.)*	*u(.) , w(.)*	*u(.) , w(.)*	*u(.) , w(.)*	*u(.), w(.), LA*
Strong Aversion	*u (.)* concave	*w (p)* convex	*u (.)* concave, *w(p)* convex	*u(.)* concave, *w (p)* convex	*u (.)* concave, *w(p)* convex	Sufficient Condition
Weak Aversion	*u (.)* concave	$w(p) \leq p$	Sufficient Condition	Sufficient Condition	Sufficient Condition	Sufficient Condition

COMMENTS AD FINEM ON PART I

Rank dependent utility theories explain systematic patterns termed paradoxical under *EU* and refine the understanding of risk aversion by accounting for observed risk behavior that is inconsistent with the characterization of risk under *EU*. The refinement is twofold: 1) under *RDU* where all notions of risk aversion are not confounded with a concave utility function, an individual with an increasing marginal utility can be risk averse in the weak sense of Chateauneuf and Cohen (1994); 2) *RDU* theories distinguish between two behaviors of different nature which are not discernable under *EU*. The different rationales are clearly differentiated in *RDU*'s treatment of the portfolio selection problem where it is the custom theoretically and in practice to assume the presence of a risk free security and to conduct the selection and analysis of the portfolio in two phases: the first is the scale or the allocation between a riskless and risky assets and the second is the mix or the allocation within the category of risky assets.[72]

Yaari (1987) shows that between a riskless asset and a portfolio of risky assets, a risk averse investor (a pessimistic investor) under *DT* does not diversify but stays put until plunging is justified (Yaari, 1987, p: 10), as opposed to a risk averse investor under *EU* who will always diversify (Markowitz, 1952; Tobin, 1958). Yaari (1987) also conjectures that within the portfolio of risky assets, a risk averse investor diversifies according to the maximin criterion where minimizing the worst result is independent of the utility.

Gayant (2004) proposes *deux logiques de décision différentes* for the scale and the mix: between a risky asset and a riskless asset, risk aversion is due to a desire for partial security reflecting the investor's unwillingness to eliminate an opportunity for gain while within a portfolio of risky assets risk aversion is due to a desire for full security through minimizing the worst outcome. His findings that the maximin criterion is "standard" under *DT* and "at the limit" under *EU* the two limit cases of *RDU* which isolate the

[72]The division is called the separation theorem: "breaking down the portfolio selection problem first among and then within asset categories seems to be a permissible and perhaps even indispensable simplification both for the theorist and for the investor himself" Tobin (1958). The division finds its parallel in practice. All fund managers interviewed distinguish between selection among asset categories and selection within a category.

effects of utility and probability transformation, support Yaari (1987) and confirm that "at the minimum utility cannot exclusively represent risk aversion".

The understanding of risk attitude is further refined under *CPT*: risk attitude has three components that are affected jointly by a gamble: a reference dependent utility function, loss aversion and a probability weighting function. The reason is that in addition to retaining the *RDU*'s mathematical precision precious to decision theorists, *CPT* provides a convenient way of modeling the influence of pervasive phenomena like loss aversion and the reflection effect on choice.

Research in behavioral finance (Thaler, 1993; Kahneman and Tversky, 2000) indicates that in financial markets, the control of losses is the major problem for investors (or their planners).[73] The difficulty for controlling losses stems in a large part from the evaluation of possible prospects as gains or losses relative to a reference point rather than positing the decision problem in terms of final wealth as in the normative *EU*. Preferences indexed to a reference point, as in *CPT* explain the tendency for risk seeking in the domain of losses and the difficulty because of loss aversion to perceive the sale price of a stock independently of its purchase price ending in lower earnings. The many studies included in Thaler (1993) also point to the importance on decision making of occupational factors that is psychological factors that seem to unfold in the practitioner's environment. Moreover, Barberis, Huang and Santos (2001) find that both loss aversion and variation of loss aversion with the past movement of the stock market are necessary to explain the equity premium puzzle.

Thus, in addition to the theoretical advantages that characterize rank dependent theories, *CPT*'s empirical realism argues in favor of its use as a good approximation of the underlying preference function the investor is assumed to optimize leaving it up to the measurement of the theory's parameters in the field to show whether the approximation is good.

[73]During the interviews where the financial practitioners were also asked about their major problems the control of losses emerged as the number one preoccupation for all.

This work proposes to use the method Abdellaoui et al. (2006) made available to elicit under prospect theory and non-parametrically i.e. without any assumptions of the form of the functional, the utility functions of practitioners in the field of finance. It also proposes in Experiment II to use a different method to infer the preferences of MBA students.

Part II thus is composed of two experiments. Experiment I investigates the fundamental preferences of financial practitioners by eliciting the utility function completely and measuring the degree of loss aversion at the individual level using Abdealloui et al.'s method. Experiment II investigates qualitatively the preferences of MBA students, potential financial practitioners without eliciting the utility function using Baucells and Heukamp (2006). Part II thus consists of a general introduction and two chapters. Each chapter is devoted to one experiment where the method and the experiment *per se* are described, and the results discussed in light of the recent literature. Part II also addresses the problems encountered such as the need for an agreement on loss aversion's definition, the difficulty of empirically disentangling all three components of risk attitude and the possible influence occupational factors could have on the practitioners' risk attitude in the dynamic context of the stock market. A general conclusion follows leading to some final remarks and direction for future research.

PART II: EXPERIMENTAL INVESTIGATION OF INDIVIDUAL UTILITY FUNCTIONS AND LOSS AVERSION

Part I has shown much evidence that people willingly violate expected utility theory and that cumulative prospect theory explains most of the violations. Indeed, its authors built it to fit the individual-level data they have gathered from their experiments motivated as they were by the observation of a "*remarkable discrepancy*" in the literature on preferences between gambles: "the theoretical analysis implied that the carriers of utility were states of wealth but the outcomes were always described as gains or losses" (Kahneman, 2000, p: ix). The observation eventually led them to a utility function that is concave for gains and convex for losses (*S*-shape) and steeper for losses than for gains. Part II's concern is whether the psychologically plausible *S*-shape is approximately true and useful.

Many empirical studies with the same concern have confirmed the latter shape for the utility function. However, most[74] of the elicitation methods used across these studies have assumed specific parametric forms for the utility function making thus the inferences about these functions dependent on the choice of the functionals.

Although Abdellaoui (2000)[75] was the first paper to elicit cumulative prospect theory model non-parametrically under risk by means of the tradeoff method[76], it's Abdellaoui et al (2006) that have made available a non-parametric method to elicit the utility function for gains and losses *simultaneously* allowing the measurement of loss aversion a gain/loss exchange rate[77]. The availability of the method and the ease of its applicability

[74] Wu and Gonzalez (1996, 1998) elicit the utility function without any parametric assumption by testing preference condition but the approach is demanding.
[75] Abdellaoui et al. (2005) elicits *CPT* under uncertainty without any prior knowledge of probability or utility. Both do not include the measurement of loss aversion.
[76] Initially proposed by Wakker and Deneffe (1996).
[77] So far, loss aversion has not been separated from the utility function either empirically or theoretically. Köbberling and Wakker (2005) disentangle loss aversion from the utility function under a "severe restriction"

in a relatively short time offered the possibility to "spend some time in the wild" specifically in the stock market where time is an expensive commodity.

The method was used hence to elicit non-parametrically and simultaneously the utility functions of financial practitioners for gains and losses and to measure their individual loss aversion allowing hence a test of prospect theory in the field. The results were to be compared subsequently to those obtained in previous studies which have however investigated mainly students' preferences. Another experiment was also undertaken to infer the preferences of MBA students, potential financial practitioners using this time a newly developed method based on stochastic dominance criteria which Baucells and Heukamp (2006) have made available.

Probability weighting is not a problem in either method albeit for different reasons, the two being based on different concepts. The robustness against probability weighting is fundamental if the method is to remain valid for *RDU* and *CPT* and to be applied in prescriptive decision analysis. Indeed it has been shown that probability weighting is a major cause of violations of *EU* and of systematic inconsistencies among different methods of utility elicitation that should yield the same result under *EU* (Fischoff (1982); Hershey and Schoemaker (1985); McCord and de Neufville (1986); Wakker and Deneffe (1996); Schkade (1988); Bleichrodt et al. (2001)).

Part II, in sum, consists of two experiments aiming at examining individuals' fundamental preferences under risk in the specific domain of finance: Experiment I uses Abdellaoui et al.'s (2006) method in the field and Experiment II uses Baucells and Heukamp's (2006) in the lab. To each, a chapter is devoted which begins with a brief introduction showing the gradual yet portentous development of the method from its predecessors.

In chapter IV, the utilities of practitioners are elicited non-parametrically and simultaneously for gains and losses and their loss aversion coefficients are measured. Section 1 presents the elicitation procedure. Section 2 describes the design of the experiment. Section 3 is concerned with the data analysis which includes the non-parametric elicitations, the parametric fitting and the measurement of loss aversion in the aggregate as well as the individual level. Section 3 and 4 present the results related to the practitioners' shape of the utility function and their loss aversion respectively. The results in the field are then contrasted with the results of Abdellaoui et al. (2006) in the laboratory and those of other previous studies. A final section concludes.

In chapter V, the utility functions of MBA students are elicited from lotteries based on stochastic dominance criteria developed by Baucells and Heukamp (2006). Section 1 presents the intuition for the stochastic dominance conditions for clarification followed by the characterization of preferences; section 2, the experimental application that is the objective, and the source of data, section 3 consists of the analysis of data and section 4 and 5 give the results pertaining to the shape of the utility function and loss aversion of MBA students respectively. A final section concludes followed by a general conclusion and some remarks and directions for future research.

CHAPTER IV. EXPERIMENT I.

FIELD INVESTIGATION USING A PARAMETER-FREE METHOD FOR THE ELICITATIONS

In so far as classification is needed before evaluation, Fishburn (1967) provides a list and a classification of 24 methods for estimating utility and Peter Farquhar (1984) describes the state of the art in utility assessment 17 years later reviewing hence many methods that were not examined in Fishburn (1967). Farquhar classifies the methods under 1) preference comparison methods, 2) probability equivalence methods, 3) value equivalence methods and 4) certainty equivalence methods. The methods however turned out in time not to be equivalent under *EU*, the inconsistencies among them shown to be systematic rather than random undermining the validity of the method's use in prescriptive decision analysis.

Hershey and Schoemaker (1984) among others provide evidence of "serious discrepancies" between the two most common methods used for elicitation the certainty equivalent method and the probability equivalent method. To correct for the use of the former as well as for the chaining of responses and the lack of control over ranges and reference points, McCord and Neufville' (1984) propose the lottery equivalent elicitation method. Their procedure although simple, suffers under *EU* from probability weighting problems.

The misperception of the latter being an important violation of *EU*, Wakker and Deneffe (1996) develop the tradeoff method from the saw tooth method, one of 24 methods listed and classified by Fishburn (1967). The method uses the same probability across the various lotteries completely eliminating the distortions of utility measurement which are due to probability weighting. Abdellaoui's (2000) and Abdellaoui et al.'s (2005) show that it can elicit probabilities indirectly assuming *CPT* under risk and under uncertainty respectively and Bleichrodt et al. (2001)[78] suggest its use to correct the certainty equivalent and the probability equivalent for probabilistic weighting. The method is also "well-suited" for the axiomatization of *CPT* under risk (Chateauneuf and Wakker, 1999) and under uncertainty (Wakker and Tversky, 1993) as was shown in section 3.3 for the former.

Abdellaoui et al. (2006) develop a new elicitation method for prospect theory. Their elicitation procedure encodes the utilities for gains and losses simultaneously allowing hence, the measurement of loss aversion non-parametrically. Although the method itself could as well be used joint with matching, the way it was implemented in their experiment was choice-based. Previous studies have found that inferring indifferences from a series of choices leads to fewer inconsistencies than asking subjects directly for their indifference values,[79] and according to Tversky, Sattath and Slovic (1988), when subjects are asked to reconsider inconsistent answers they modified the matching in the direction of the choice.[80]

A drawback of the tradeoff method is that the utility function elicited is not independent of a reference point relative to which outcomes could be perceived as gains or losses. Another, is that it allows error propagation if the assessment of the first utility is not well done, since the measurements are chained and later responses are based on previous ones. Nevertheless, both Bleichrodt and Pinto (2000) and Abdellaoui, Vossman and Weber (2005) who have used similar elicitation methods and have examined in detail the effect of error propagation on chained measurements found it to be negligible.

[78] Bleichrodt et al. (2001) suggest the tradeoff to correct quantitatively the certainty equivalent for probabilistic transformation and the probability equivalent for loss aversion and probabilistic transformation rather than qualitatively as in Hodgkinson et al (1999) or in Payne et al. (1999) or in Arkes (1991).

[79]Luce (2000) provides a review of these studies.

[80]This observation suggests that choice and matching are both biased in opposite directions (the primary dimension may be overweighted in the former and underweighted in the latter, the answers reflecting perhaps a routine compromise rather than the result of a critical reassessment (Tversky, Sattath and Slovic, 1988)

Chapter IV consists of five sections. Section 1 describes the background for the elicitation procedure used based on Abdellaoui et al. (2006). Section 2 describes the procedure itself and its experimental application. Data analysis follows in section 3. The results related to the shape of the utility function and to loss aversion are presented next in sections 4 and 5 respectively. A final section concludes.

4.1 *THE BACKGROUND*

The Abdellaoui et al.'s (2006) elicitation procedure is a *parameter-free* method to *completely* elicit the utility function under prospect theory. It allows thus the measurement of loss aversion at the individual level without making any parametric assumptions. This section reviews in brief the main features of prospect theory to provide a background for the following section, which describes the elicitation procedure and applies it in the field.

In the experiment, the practitioner is asked to choose between two lotteries with at most two distinct outcomes. The discussion is thus restricted to such lotteries; nevertheless, the estimations are valid for both Kahneman and Tversky (1979) and Tversky and Kahneman (1992) which coincide on the two-outcome domain. Outcomes are monetary and are expressed as changes from the reference point, that is, as gains or losses. The reference point is assumed to be zero and all outcomes mixed and non-mixed are assumed to be rank-ordered. The individual evaluates each lottery and chooses the lottery that offers the highest overall utility. The overall utility is expressed in terms of three functions: a probability weighting function w^+ for gains, a probability weighting function w^- for losses and a utility function *U*. The functions w^+, w^- assign a probability weight to each probability. They are strictly increasing and satisfy $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$. The utility function *U* assigns a real number to each outcome, which reflects the desirability of that outcome. The function *U* is increasing and satisfies *U*(0) = 0. *U* is a ratio scale, i.e. the unit of the function is arbitrarily chosen.

The evaluation of the lottery (x, p; y) depends among other things on the sign of the outcomes x and y. For a non-mixed lottery that is involving only gains or only losses, the utility computed by:

$$w^i(p)U(x) + (1 - w^i(p))U(y), \tag{4.1}$$

Where i = + for gains and i = - for losses.

For a mixed lottery it is computed by: (4.2)

$$w^+(p)U(x) + w^-(1 - p))U(y).$$

Kahneman and Tversky (1979) assumed that the probability weighting functions for gains and losses overweight small probabilities and underweight moderate and high probabilities, giving rise to an inverse *S*-shape probability weighting function. The utility function is assumed to be concave for gains and convex for losses and steeper for losses than for gains.

4.2. *THE EXPERIMENT*

The procedure used in the experiment is based on the elicitation of utility midpoints pointed at before in Köbberling and Wakker (2005) and which have been often used in axiomatizations of decision models, and it is noteworthy that it requires few measurements to elicit a given number of utility midpoints.

4.2.1. *THE ELICITATION PROCEDURE*

The procedure consists of four steps and is summarized in Table 7 below. The second column describes the quantity assessed, the third the indifference that is sought and the fourth the implication of this indifference under prospect theory. The final column shows the choices for the variables that have to be specified. The font size in Table 7 is smaller than in the other tables in order not to overcrowd the page.

Table 7: Four-Steps Elicitation Procedure

	Assessed Quantity	Indifference	under Prospect Theory	Choice Variables
Step 1	L_1	$(L_1, p'; L^*) \sim (L_0, p'; L)$		$p' = 0.33$
	L_2	$(L_2, p'; L^*) \sim (L_1, p'; L)$	$U(L_0) - U(L_1) = U(L_1) - U(L_2)$	$L^* = -100$
	p_l	$L_1 \sim (L_2, p_l; L_0)$	$w^-(p_l) = 0.5$	$L = -600$, $L_0 = -1000$
	G_1	$(G_1, p'; G) \sim (G_0, p'; G^*)$		$p' = 0.33$
	G_2	$(G_0, p'; G) \sim (G_1, p'; G^*)$	$U(G_2) - U(G_1) = U(G_1) - U(G_0)$	$G^* = 600$
	p_g	$G_1 \sim (G_2, p_g; G_0)$	$w^+(p_g) = 0.5$	$G = 100$, $G_0 = 1000$
Step 2	$L_r \in [L_1, 0]$	$L_r \sim (L_A, p_l; L_B)$	$U(L_r) = 0.5\, U(L_A) + 0.5\, U(L_B)$	$L_1 = -100000$
Step 3	l	$L_s \sim (l, 0.5; 0)$	$w^-(0.5)U(l) = -s$	$s = 0.25$
	g	$0 \sim (g, 0.5; l)$	$w^+(0.5)U(g) = s$	
	G_s	$G_s \sim (g, 0.5; 0)$	$U(G_s) = w^+(0.5)U(g) = s$	
Step 4	$G_r \in [0, G_s]$	$G_r \sim (G_A, p_g; G_B)$	$U(G_r) = 0.5\, U(G_A) + 0.5\, U(G_B)$	

The first step, a central step in the procedure for decision making under risk, is the elicitation of probabilities that have a decision weight of 0.5.[81] The elicitation of the probability for gains or for losses for which $w^+(p_g) = 1/2$ and $w^-(p_l) = 1/2$ requires three measurements, that is, three indifferences. Once this is known however, just one measurement is needed for the determination of a utility midpoint.

For the elicitation of p_l a sequence of losses L_0, L_1 and L_2 that are equally spaced in terms of utility, i.e. $U(L_0) - U(L_1) = U(L_1) - U(L_2)$. More specifically, a probability p' is chosen and three losses L^*, L, and L_0, with $L^* > L > L_0$. Then losses L_1, L_2 are elicited such that a subject is indifferent between the lotteries (L_1, p'; L^*) and (L_0, p'; L) and between the lotteries (L_2, p'; L^*) and (L_1, p'; L). Because $L^* > L$, $L_2 < L_1 < L_0$ is a must. Under prospect theory, the indifferences $(L_{i+1}, p'; L^*) \sim (L_i, p'; L)$, $i = 0, 1$ imply that

[81] For decision under uncertainty, the elicitation of events that have a decision weight of 0.5 can be interpreted as a generalization of Ramsey's (1931) "ethically neutral events" (events with subjective probability of 0.5) under expected utility to prospect theory (Abdellaoui et al., 2006).

$$U(L_i)-U(L_{i+1})=\frac{1-w^{-}(p)}{w^{-}(p)}(U(L^*)-U(L),i=0,1. \quad (4.3)$$

Because the expression on the right hand side is constant, it follows that: $U(L_0)$ – $U(L_1)$ = $U(L_1)$ – $U(L_2)$. Hence, L_1 is a utility midpoint of L_0 and L_2. This procedure for eliciting utility have been pointed out previously by Abdellaoui (2000) and Köbberling and Wakker (2003).

Having elicited L_1 and L_2 the probability p_l is determined that makes the subject indifferent between L_1 for sure and the prospect (L_2, p_l; L_0). Under prospect theory, $U(L_1)=w^{-}(p_l)U(L_2)+(1-w^{-}(p_l))U(L_0)$. L_1 being the utility midpoint of L_0 and L_2 gives $w^{-}(p_l)=1/2$. The elicitation of p_g is similar except that now three monetary gains G_0 > G^* > G are fixed beforehand.

In the second step of the elicitation determines the utility for losses is determined by eliciting utility midpoints. Once p_l is known the utility midpoint of any two losses L_A and L_B can be measured by eliciting one indifference only as is shown in step 2 in Table 7. $U(L_1)$ is set equal to -1 for some $L_1 < 0$ which is allowed by the uniqueness properties of the utility function in prospect theory. The outcome $L_{0.5}$ is then determined such that the subject is indifferent between $L_{0.5}$ for sure and the lottery (L_1, p_l; 0). Under prospect theory, this indifference implies that $U(L_{0.5}) = -0.5$. The utility is then elicited on the interval [L_1, 0]. For example, by setting $L_A = L_{0.5}$ and $L_B = 0$, the outcome $L_{0.25}$ is elicited for which $U(L_{0.25}) = -0.25$.

The third step is the crucial step in the measurement of loss aversion. In it, the utility of losses is linked to the utility of gains by eliciting three indifferences. In the first indifference one of the outcomes elicited in step 2 L_s is chosen and the loss l is determined such that the subject is indifferent between L_s and (l_1, *1/2*; *0*). It follows that $w^{-}(1/2)U(l)=-s$. The second indifference determines the gain g that makes the subject indifferent between 0 for sure and the lottery (g, *1/2*; l). It follows that $w^{+}(1/2)U(g)=s$.

The gain G_s that the subject finds equivalent to the lottery (*g*, *1/2*; 0) has then the utility *s* and is the mirror image of L_s in terms of utility.

The fourth and final step of the elicitation procedure determines the utility for gains. As for losses, the probability p_g allows to measure the utility midpoint of any two gains G_A G_B by eliciting just one indifference. The utility midpoint of and G_s and 0 is determined first and then the utility for gains on the interval [0, G_s].

4.2.2 *THE EXPERIMENTAL APPLICATION*

The procedure outlined above was used for the complete elicitation under prospect theory of the utility of financial practitioners. It was possible thus, to test in the field the theory's assumption that the shape of the utility is concave for gains and convex for losses and to measure the practitioners' individual degrees of loss aversion, according to the various definitions that have been put forward in the literature.

The General Setup

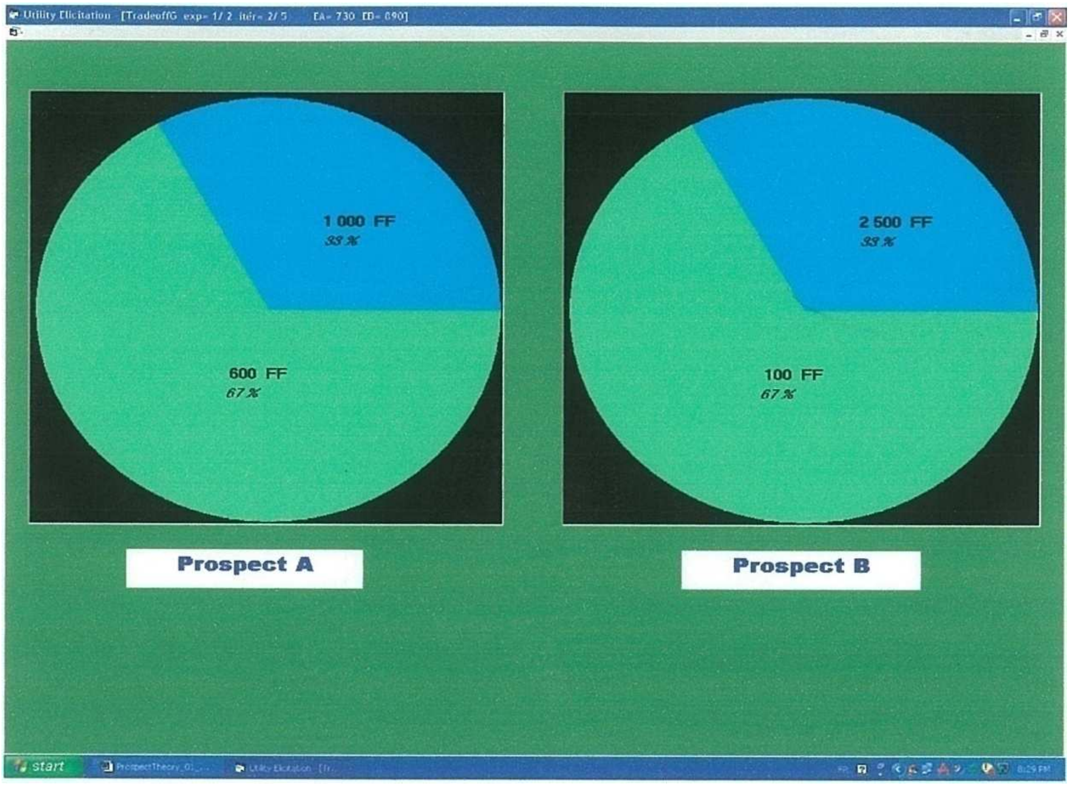
Forty six practitioners participated in the study. Most were financial advisors responsible for managing the portfolios of their respective clients, some however were money managers, that is, portfolio managers in whose funds, the financial advisors invest a fraction of their clients' wealth. They were all affiliated with multi-national financial institutions. The interviews were obtained through personal contact and were conducted individually on a computer in their respective offices: Cleveland-Ohio, Boston-Massassuchets, Manhattan-NewYork, Atlanta-Georgia, Phoenix-Arizona and Beirut-Lebanon. There were no systematic differences among them and their data was pooled. Their median age is 40. During the interviews, the practitioners were encouraged to go on their own pace and were told that there were, no right or wrong answers. The session took an average of 30 mn. Table 8 in the Appendix has the names of the practitioners' institutions and their locations.

Except for the source of the data, and the type of interaction between interviewer and interviewee, the general setup and the details of the experiment were the same as in Abdellaoui et al. (2006). The outcomes were in dollars and they were substantial in order to be able to detect the curvature of the utility function, utility being approximately linear over small intervals (Wakker and Deneffee, 1996). The practitioners were not directly asked for the specific outcome value leading to indifference, previous studies having shown that inferring indifferences from a series of choices leads to fewer inconsistencies than asking subjects directly for their indifference value. Instead, every value was assessed through a series of binary choice questions.

The question was framed as follows: [82] The following hypothetical choices are designed to investigate your attitude towards risk. Please take your time and try to predict your choices as accurately as possible. The responses are anonymous and there is no correct answer, hence, no reason *not* to state your true preference. However, it was pointed out to the practitioner that the money used for the investment was not his own nor the client's which could vary widely from one to another[83] but the company's. In this way, the choices reflected the preferences of the practitioner as a professional.

The following display illustrates the first choice as it was presented to the practitioner on the computer screen. The two lotteries A and B corresponding to (L_1, p'; L^*) and (L_0, p'; L) where L_0, p' and L^* are fixed, are displayed as pie charts with the sizes of the pieces of the chart corresponding to the probabilities.

[82] Analogously to the question asked in Experiment II presented in Appendix D.
[83] Alignment of preferences is important between the practitioner and the client.



The Steps Detailed

Starting from L = 4000 (L can vary from 4000 to 8000; the interval for L is fixed such that it guarantees a strong preference for one lottery over the other) it was explained to the portfolio manager that the process consists of narrowing the interval containing L through a number of iterations until the outcome that made him indifferent between the two lotteries is found, that is $(1000, p'; 600) \sim (L_0, p'; 100)$.[84]

[84] A second screen shot Display II also in Appendix B illustrates the use of the scroll bar installed to help the decision maker understand what is required of him that is, illustrate visually that there should be a value for which preferences between the two lotteries switched. Practitioners were savvy however and did not need it.

The outcome *L* is thus determined through a series of iterations to make the player indifferent between two lotteries. Each binary choice question corresponded to an iteration in a bisection process and the indifference point is taken as the midpoint of the last two bracketing choices as can be seen from Table 9 below which illustrates the process for L_1 and $L_{0.0625}$.

Table 9: An Illustration of the Bisection Method

The Iterations	**Choices in Elicitation of L_1**	**Choices in Elicitation of $L_{0.0625}$**
1	**(-1000, 0.33;-600)** vs. (-4000,0.33;-100)	-6080 vs. **(-7800,0.78; 0)**
2	**(-1000, 0.33;-600)** vs. (-2500,0.33;-100)	**-3040** vs. (-100000,0.78; 0)
3	(-1000, 0.33;-600) vs. **(-1750,0.33;-100)**	-4560 vs. **(-100000,0.78; 0)**
4	(-1000, 0.33;-600) vs. **(-2100,0.33;-100)**	**-3800** vs. (-100000,0.78; 0)
5	**(-1000, 0.33;-600)** vs. (-2300,0.33;-100)	**-4180** vs. (-100000,0.78; 0)
Indifference V.	-2200	- 4370

The chosen lottery is printed in bold. The table shows that only the outcome to be elicited is varied. Depending on the choice of the iteration, this outcome was increased or decreased. The size of the change was always half the size of the change in the previous question, under the restriction that the resulting outcome should be a multiple of 10 and the resulting probability a multiple of 0.01. Otherwise, the value was set equal to the closest multiple of 10 or of 0.01. The method resulted in an interval within which the indifference value should lie and *the midpoint of the interval was taken as the indifference value*. For example, in Table 9, the indifference value for $L_{0.0625}$ should lie between -4180 and -4560. Hence, the indifference value taken was -4370.[85] The starting

[85] After the final iteration, the practitioners were offered a chance to continue with the next choice or to repeat the precedent anew, in order to to minimize the impact of response errors. Also, the number of iterations was not the same for each step of the elicitation procedure. Indifference values were elicited in five iterations in steps 1, 2 and 4 of the method. In step 3, seven iterations were used, the pilot sessions undertaken by Abdellaoui et al. (2006) having shown that these numbers were sufficient to obtain the indifference values with good precision.

values in the iterations were generally chosen so that the lotteries had equal expected values. The exceptions were L_1, L_2, G_1 and G_2 whose starting values were L_0 +3000, L_1 +3000, G_0 +3000 and G_1+3000.

Once L_1 is determined, it replaces L_0 in the lottery and another choice question is used to make him indifferent between the two lotteries: (L_2, p'; L^*) ~ (L_1, p'; L). The probability p_l that makes the subject indifferent between L_1 for sure and the prospect (L_2, p_l; L_0), is then determined relying thus on the *probability equivalence* method. The same sequence is constructed for gains. Thus as stated above and illustrated in Table 7, in step one, two monetary outcomes for gains and two for losses for which the difference in utility between successive outcomes is constant, are determined in sequence as well as the two above mentioned probabilities p_g and p_l for which $w(p_g) = w(p_l) = 0.5$.

The following step, *fractile* [86] losses, determines a sequence of eleven outcomes for losses L_r elicited on the interval [-100000, 0] for the following utilities for losses: 0.015, 0.031, 0.062, 0.093, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, where $U(L_r) = -r$ using the probability elicited for losses. Having determined p_l and $U(L_1)$ set equal to –1 for some $L_1 < 0$ which is allowed by the uniqueness properties of the utility function in prospect theory, the outcome $L_{0.5}$ is then determined such that: (L_1, p_l; 0) ~ $L_{0.5}$

Immediately after, the step called Mirror Image *LG* determines the gain G_{max} that represents the same utility variation with respect to the 0 outcome as the absolute value of the utility variation between the loss $L_{0.25}$ and 0; this step links the utility for losses to the utility for gains through the elicitation of three outcomes and allows the measurement of loss aversion.

The fourth step, the *fractile* gains, elicits a sequence of eight outcomes between G_{max} and 0 for the following utilities for gains: 0.015, 0.031, 0.062, 0.093, 0.125, 0.156, 0.187,

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[86] As was pointed out to me, the elicitation procedure relies on two response modes: the probability equivalence and the fractile. The possible implication for the interpretation of the results will be discussed in the section entitled: Concluding Remarks for Experiment I.

0.25, where $U(G_r) = r$ using the probability for gains determined at the beginning. More data was collected for losses than for gains to improve the operationalization of Kahneman and Tversky's (1979) definition of loss aversion. Also, many points were elicited close to zero to operationalize Köbberling and Wakker's (2005) definition.

The software also elicits data for two consistency tests for the measurement of utility. The first requires that the utilities of the outcomes elicited at the beginning of the experiment $\{L_0, L_1, L_2\}$ exhibit the same difference obtained between the successive elements elicited in step 2 for losses after factoring out probability weighting under *CPT*. *Ad modem*, it requires that the the utilities of the outcomes $\{G_0, G_1, G_2\}$ exhibit the same difference as obtained between the successive elements elicited in step 4 for gains. In other words, consistent measurement of utility requires that the equalities between utility differences be preserved. The utility of L_i and G_i , $i = 0, 1, 2$ were determined through linear interpolation as shown in the next section 4.3.1. The quality of these interpolations was however good because the L_i and G_i were generally concentrated near zero where there were many observations.

The software also allowed for a second consistency test which consists of four questions presented to the practitioner at the end of the elicitations for gains. Each question aimed at determining the indifference point of the paractitioner between a mixed lottery (G_r, p_r; L_r), r = 0.031, 0.0625, 0.125, 0.250, and receiving nothing that is, at determining the probability p_r that that made the negative outcomes mirror images in terms of utility of the commensurate positive outcomes. Equation (4.2) and the results derived before show that in each question, $w^+(p_r) = w^-(1-p_r)$ is obtained and because w^+ and w^- are strictly increasing, the same indifference probability should be found in all four questions.

4. 3 *DATA ANALYSIS*

This section shows how the shape of the utility for gains and of the utility for losses for each subject was determined, that is, it looks at the evolution of the slope of the utility function at various points. It also explains how the various definitions of loss aversion are operationalized for the measurement of loss aversion, at the individual level. This section presents first the analysis for the two consistency tests required for the non-parametric elicitations; the data analysis for the latter is presented next, followed by the analysis for the parametric fittings assuming power, exponential and expo-power functions. The analysis concerning the measurement of the loss aversion coefficients in the aggregate as well as the individual level follows.

4.3.1 Consistency Tests for Non-Parametric Elicitation

To test whether these equalities between utility differences between the successive elements (L_0, L_1, L_2) and (G_0, G_1 and G_2) are preserved in the elicitation of the utility x_r for losses and for gains in steps 2 and 4 of the elicitation procedure respectively, linear interpolation was used to calculate the utility of these L_i and G_i, i = 0,1,2 labeled a_0, a_1, a_2 for gains and b_0, b_1, and b_2 for losses:

For gains $$U(a) = [\frac{U(x_2) - U(x_1)}{(x_2 - x_1)}](a - x_1) + U(x_1) \quad (4.4)$$

For losses $$U(b) = [\frac{U(x_2) - U(x_1)}{(x_2 - x_1)}](b - x_1) + U(x_1) \quad (4.5)$$

The x_1 and x_2 were chosen such as $x_1 < a$; $x_2 > a$; the opposite holds for b. For example, to calculate the utility of an elicited gain a_1= 2200 for a practitioner (number 7) the two outcomes 7060 and 10580 corresponding to the utilities 0.0015 and 0.003 were chosen as x_1 and x_2 respectively.

The trade-off method yields the same difference in utility scale and accordingly, the hypotheses to be tested for the first consistency test are:

For gains: $U(a_1) - U(a_0) = U(a_2) - U(a_1)$

For losses: $U(b_1) - U(b_0) = U(b_2) - U(b_1)$

The second consistency test checks the equality of the four probabilities established for each of the lotteries (G_r, p_r; L_r), r = 0.031, 0.0625, 0.125, 0.250, towards the end of the experiment that made the four negative outcomes mirror images in terms of utility of the commensurate positive outcomes.

4.3.2 Non-Parametric Elicitation of the Utility Function for Gains and Losses

The shape of the utility for gains and of the utility for losses was determined by looking at the evolution of the slope of the utility function at various points. Two elicited losses L_r and $L_{r'}$ (for gains G_r and $G_{r'}$) are *adjacent* if $L_r > L_{r'}$ ($G_r > G_{r'}$) and there is no elicited loss (gain) in between. $S_L^{\uparrow}(r)$ is defined as the slope of the segment linking (L_r, $U(L_r)$) and ($L_{r'}$, $U(L_{r'})$) where L_r and $L_{r'}$ are adjacent. Similarly, $S_L^{\downarrow}(r)$ is defined as the slope of the segment linking ($L_{r''}$, $U(L_{r''})$) and (L_r, $U(L_r)$) where $L_{r''}$ and L_r are adjacent. $S_G^{\uparrow}(r)$ and $S_G^{\downarrow}(r)$ are defined analogously. $\Delta S_L(r) = S_L^{\downarrow}(r) - S_L^{\uparrow}(r)$ denotes the variation of the slope around L_r when moving towards zero. Similarly, $\Delta S_G(r) = S_G^{\downarrow}(r) - S_G^{\uparrow}(r)$ denotes the variation of the slope around G_r when moving away from zero. It is easily verified that $\Delta S_i(r), i = G, L$ positive, (negative, zero) corresponds to convex (concave, linear) utility.

Seven values of $\Delta S_G(r)$ were obtained for gains and eleven values of $\Delta S_L(r)$ for losses. To account for response error, practitioners with at least four/seven negative (positive) $\Delta S_G(r)$ were classified as having concave (convex) utilities for gains. Practitioners with at least seven/eleven negative (positive) $\Delta S_L(r)$ were classified as having concave (convex) utilities for losses. Fenema and van Assen (1999) as well as Abdellaoui (2000) and Etchart-vincent (2004) have used similar criteria.

4.3.3 Parametric Fitting of the Data

The practitioners' data was also analyzed assuming three parametric forms: the power family, the exponential family and the expo-power family.

The power family, characterized by a constant relative risk aversion (*CRRA*),[87] that is, individuals make the same proportional investments in risky assets, allows different degrees of *CRRA*, which in turns allows fine tuning economic models to fit empirical data and is used predominantly when large stakes are relevant.[88] The power function is defined as follows, with $\alpha > 0$ for gains and for losses with $\beta > 0$.

$$U = U_{\max}\left(\frac{x}{x_{\max}}\right)^{\alpha} \quad \text{if} \quad x \geq 0 \tag{4.6}$$

$$U = -\left(-\frac{x}{100000}\right)^{\beta} \quad \text{if} \quad x < 0$$

The case $\alpha < 0$ corresponds to a decreasing utility function. The power function for gains is concave if $\alpha < 1$, linear if $\alpha = 1$ and convex if $\alpha > 1$; for losses, it is convex if $\beta < 1$, linear if $\beta = 1$ and concave if $\beta > 1$.

The exponential family is characterized by a constant absolute risk aversion (*CARA*) under which rich people will not be attracted to risk no matter how richer they become;

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[87]The risk aversion coefficients associated with the power function (CRRA) and the exponential function form (CRRA) are shown in Table 10 in Appendix C.

[88]It is conveniently used under lognormal probability distributions because the resulting risk-neutral probability distributions which are often used to model stock prices are lognormal.

thus it is practical to use over small to moderate stakes. The exponential function is defined as follows for $\alpha \neq 0$ and for $\beta \neq 0$.

$$U = \frac{U_{\max}(\exp(-\alpha(\frac{x}{x_{\max}})) - 1)}{(\exp(-\alpha) - 1)} \quad \text{if } x \geq 0 \tag{4.7}$$

$$U = -\frac{(\exp(-\beta(\frac{x}{100000})) - 1)}{(\exp(-\beta) - 1)} \quad \text{if } x < 0$$

For $\alpha = 0$, it is defined as $U_{\max}(\frac{x}{x_{\max}})$ and for $\beta = 0$ as $-\frac{x}{100000}$. The exponential function for gains is concave if $\alpha > 0$ and convex if $\alpha < 0$; for losses, it's convex if $\beta > 0$ and concave if $\beta < 0$.[89]

The expo-power family was introduced by Abdellaoui, Barrios and Wakker (2002) and is variation of a two-parameter family proposed by Saha (1993). The expo-power is defined as follows for $\alpha \neq 0$ and for $\beta \neq 0$.

$$U = \frac{U_{\max}(1 - \exp(-(\frac{x}{x_{\max}})^{\alpha} / \alpha))}{1 - \exp(-1/\alpha)} \quad \text{if } x \geq 0 \tag{4.8}$$

$$U = -\frac{(1 - \exp(-(\frac{x}{100000})^{\beta} / \beta))}{1 - \exp(-1/\beta)} \quad \text{if } x < 0$$

The cases where $\alpha \leq 0$ and $\beta \leq 0$ were not considered because contrary to observation they lead to extreme risk aversion for gains and extreme risk seeking for losses when zero is among the outcomes of a lottery. The expo-power for gains is concave if $\alpha \leq 1$ and convex if $\alpha \geq 2$; for losses, it is convex if $\beta \leq 1$ and concave if $\beta \geq 2$. obtenues pour le mean: 5.881 E -05 et pour le median: 2.042 E -05 dans la footnote

[89] As was pointed out to me it is $\frac{\alpha}{x\max}$ and not α that should be averaged across individuals. The values obtained for the mean and median are respectively: 5.88 E-05 and 2.042 E-05.

4.3.4 Loss Aversion Measurement

Abdellaoui et al. (2006) was the first paper to measure loss aversion at the individual level non-parametrically. It has done so however in the lab. This work applies their method in the field where there is much evidence that loss aversion can explain a variety of data. This section shows how the data obtained is analyzed in the aggregate and at the individual level.

In the Aggregate

Loss aversion refers to the asymmetry of the value function: a steeper shape for losses than for gains Kahneman and Tversky (1979). Accordingly, the means of the outcomes for gains and losses corresponding to the same utilities i.e. 0.015, 0.031, 0.062, 0.093, 0.125, 0.250 are regressed linearly.

Individual Loss Aversion

There are many definitions of loss aversion. In the absence of a commonly accepted definition, the degree of loss aversion in this work is measured according to the following definitions: Kahneman and Tversky (1979); Wakker and Tversky (1993); Neilson (2002); Bowman et al (1999); and Köbberling and Wakker (2005) respectively. Wakker and Tversky (1993), Neilson (2002) and Bowman et al.'s (1999) imply both the Köbberling and Wakker's (2005) and the Kahneman and Tversky's (1979) definitions.

Kahneman and Tversky (1979) have suggested that loss aversion should be defined as $-U(-x) > U(x)$ for $x > 0$ which suggests that the loss aversion coefficient could be defined as the mean or median of $-\frac{U(-x)}{U(x)}$ over relevant *x*. Kahneman and Tversky (1992) implicitly used $-\frac{U(-\$1)}{U(\$1)}$ as an index of loss aversion. Thus to test for loss aversion in the Kahneman and Tversky (1979) sense, the loss aversion coefficient computed for each gain amount elicited was $-\frac{U(-G_r)}{U(G_r)}$ for all $G > 0$ and where *r* refers to the utilities of the eight amounts of gains G_r elicited: 0.015, 0.031, 0.062, 0.093, 0.125, 0.156, 0.187, and 0.25.

First the $U(-G_r)$ were calculated using linear interpolation for each gain elicited. Since the number of outcomes elicited in the loss domain is greater than the number of outcomes elicited in the gain domain, it was possible to obtain eight values of $U(-G_r)$ for each practitioner. When $G_{0.25}$ exceeded 100000, the $U(-G_r)$ was taken as -1. A practitioner was classified loss averse when at least 6 out of the 8 values of the loss aversion coefficients computed exceeded 1, loss neutral when at least 6 values were equal to one and not loss averse when at least 6 values were less than 1. A coefficient of loss aversion was then computed for each practitioner as the mean/median of $-\frac{U(-G)}{U(G)}$ over relevant G.

Neilson (2002) proposed to define loss aversion by $\frac{U(-x)}{-x} \geq \frac{U(y)}{y}$ for all positive x and y and provided a preference foundation. A possible candidate for the coefficient of loss aversion according to this definition is the ratio of the infinum of $\frac{U(-x)}{-x}$ over the supremum of $\frac{U(y)}{y}$. Hence the loss aversion coefficients were computed as the ratio $\frac{\min(U(L_r)/L_r)}{\max(U(G_r)/G_r)}$ where the gains and losses correspond to the same utilities: 0.015, 0.031, 0.062, 0.093, 0.125, 0.25. A practitioner was classified as loss averse when the ratio $\frac{\min(U(L_r)/L_r)}{\max(U(G_r)/G_r)}$ exceeded one and not loss averse when the ratio $\frac{\min(U(G_r)/G_r)}{\max(U(L_r)/L_r)}$ exceeded one.

Wakker and Tversky (1993) defined loss aversion as the requirement that $U'(-x) \geq U'(x)$ for all $x > 0$, that is the slope of the utility function at each loss is at least as large as the slope of the utility function at the absolutely commensurate gain and provide a preference axiomatization. Their definition could be related to a loss aversion coefficient of the mean or median of $\frac{U'(-x)}{U'(x)}$. Thus the loss aversion coefficient computed were $\frac{U'(-G_r)}{U'(G_r)}$ where r corresponds to the following utilities: 0.015, 0.031, 0.06, 0.093, 0.125, 0.15, 0.18 and where $U'(G_r)$ the slope $\Delta U_r / \Delta G_r$ was computed as:

$$U'(G_r) = 1/2(S_G^{\downarrow}(r) + S_G^{\uparrow}(r)) \qquad (4.9a)$$

i.e.,:

$$U'(G_r) = 1/2[\frac{(U(G_r) - U(G_{r'}))}{(G_r - G_{r'})} + \frac{(U(G_{r''}) - U(G_r))}{(G_{r''} - G_r)}]$$

with $G_{r'}$ and G_r are adjacent and with G_r and $G_{r''}$ adjacent, and by defining

$$U'(-G_r) = S_L^{\uparrow}(r) \qquad (4.9b)$$

i.e.,:

$$U'(-G_r) = \frac{(U(L_s) - U(L_{s'}))}{(L_s - L_{s'})}$$

if $L_{s'} < -G_r < L_s$ and L_s and $L_{s'}$ are adjacent,

$$U'(-G_r) = 1/2(S_L^{\downarrow}(r) + S_L^{\uparrow}(r)) \qquad (4.9c)$$

if $-G_r = L_s$.

The slopes around each gain were computed first. Then the slopes around the commensurate losses were computed according to the appropriate equation. Seven values were obtained for each practitioner. A practitioner was classified as loss averse when at least 5 out of seven values exceed one, loss neutral when six out of seven are equal to one and not loss averse when at least six out of seven are less than one. A loss aversion coefficient was computed for each practitioner as the mean (median) of the seven coefficients.

A stronger definition was provided by Bowman et al (1999): loss aversion holds if $U'(-x) \geq U'(x)$ for all positive x and y. That is the slope of the utility function for losses is everywhere steeper than the slope of the utility function for gains. Hence, the loss aversion coefficient was computed as the $\frac{\min(U'(L_r)}{\max U'(G_r)}$ exceeding one, and not loss averse as the $\frac{\min(U'(G_r)}{\max U'(L_r)}$ exceeding one. The $U'(G_r)$ and $U'(L_r)$ were computed as in equations 4.9*a* and 4.9*c* for the Wakker and Tversky (1993) coefficients.

Köbberling and Wakker (2005) defined the coefficient of loss aversion as $U'_{\uparrow}(0)/U'_{\downarrow}(0)$ where the numerator and the denominator stand respectively for the left and right derivatives of the utility at the reference point[90]. Hence, the loss aversion coefficient which implies that *U* is steeper for small losses than for small gains was computed as the ratio of $U(L_{0.015})/L_{0.015}$ over $U(G_{0.015})/G_{0.015}$ i.e. $G_{0.015}/L_{0.015}$. Subjects whose coefficients exceeded 1 were classified as loss averse. This definition is local and exhaustive in the sense that every practitioner could be classified as opposed to the others which are global and where it is possible that some practitioners are left unclassified.

An implication of Köbberling and Wakker (2005)'s definition of loss aversion is that some modeling problems are encountered when constant relative risk aversion (CRRA) is assumed for small to modest stakes in mixed prospects but not when (CARA) that is the exponential is assumed.

4.4 *RESULTS RELATED TO THE SHAPE OF THE UTILITY FUNCTION*

This section presents the results of Experiment I. The results of the two consistency tests are presented first followed by a test for probability weighting. The results for the shapes of the utility functions for gains and losses elicited non-parametrically are given next followed by the parametric fittings. Whenever convenient, they are summarized in tables. The shape of the utility function of the median practitioner is illustrated in Figure 10.

[90] The ratio was informally suggested by Benartzi and Thaler (1995).

4.4.1 Consistency results

The paired-t tests performed show that the null hypotheses $U\ (a_1) - U\ (a_0) = U\ (a_2) - U(a_1)$ for gains and $U(b_1) - U(b_0) = U(b_2) - U(b_1)$ for losses are not rejected (p = 0.06 for gains and 0.65 for losses).

For the utilities 0.25, 0.125, 0.06, 0.03 the median (mean) probability values are: 0.71(0.68), 0.66(0.66), 0.71(0.66), 0.69(0.60). Friedman test shows that χ^2 = 0.881 when μ = 3 degrees of freedom and the significance is 0.830. Paired t-tests for $p_{0.25}$ and $p_{0.125}$ show p = 0.559; for $p_{0.25}$ and $p_{0.06}$, p = 0.640; for $p_{0.25}$ and $p_{0.03}$, p = 0.140; for $p_{0.125}$ and $p_{0.06}$, p = 0.888; for $p_{0.125}$ and $p_{0.03}$, p = 0.122; for $p_{0.06}$ and $p_{0.03}$, p = 0.041 when 45 is the degree of freedom. The individual elicited probabilities p_l and p_g are shown in Table 16 in Appendix C.

4.4.2 Probability Weighting Tests

Another test was undertaken to test the equality of the probabilities for gains and for losses. Two probabilities were elicited at the beginning of the experiment using $w\ (p)$ = 0.5 for gains and losses: for the mean: p (gains) = 0.58 and p (losses) = 0.49; for the median: p (gains) = 0.64; p (losses) = 0.46. Wilcoxon test results in p = 0.409 and paired t-test results in p = 0.363.
The difference between p (gains) and p (losses) being far from significant, the possibility that they are equal cannot be rejected. Because the probability equivalent method was used to elicit these probabilities the practitioners have been rendered more risk averse and more so for losses than for gains according to Hershey and Schoemaker (1985).

4.4.3 The Utility Function for Gains and Losses

For the shape of the utility function, the non-parametric results are given first followed by those of the parametric fittings.

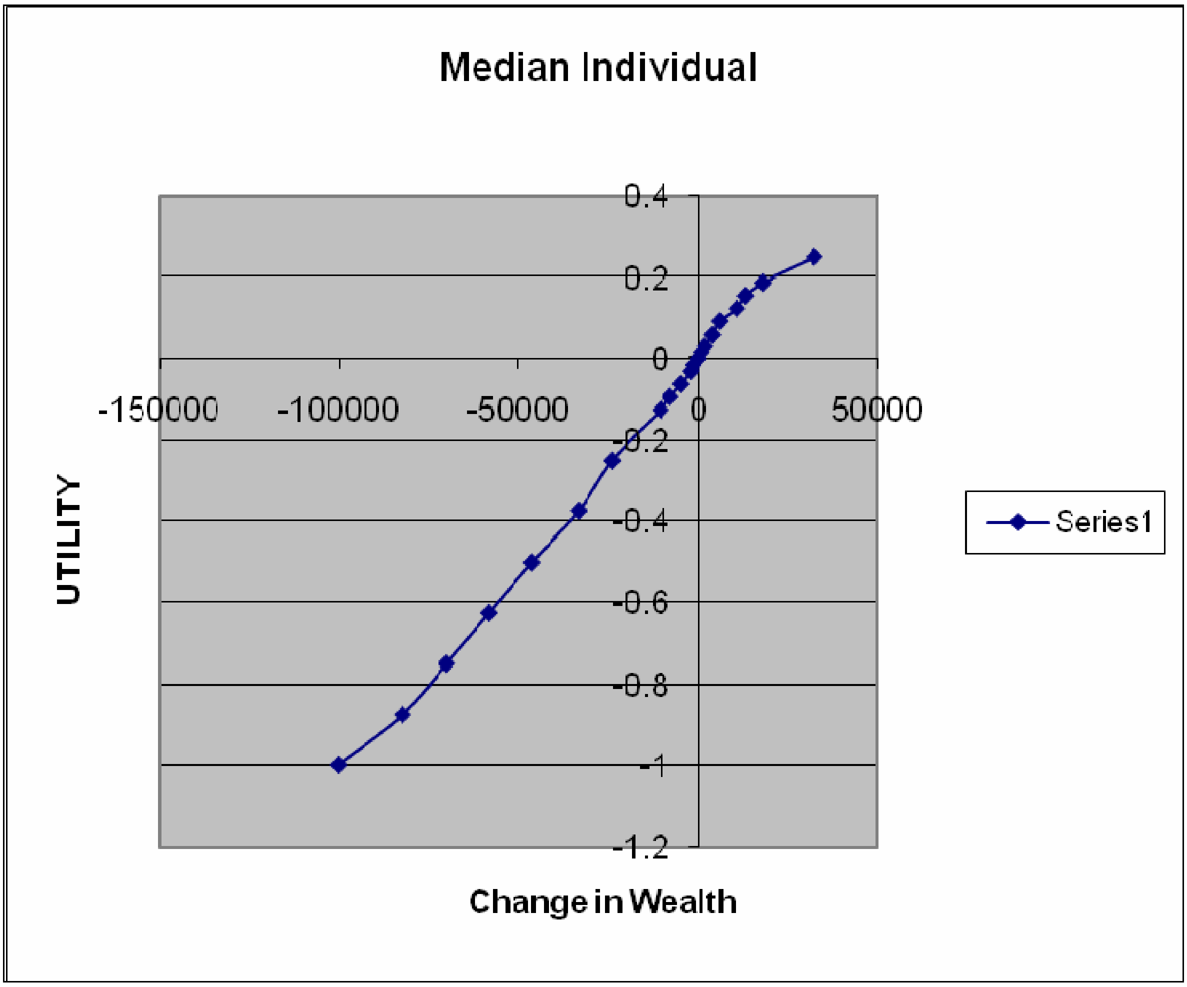
Non- Parametric Results

The most common pattern at the individual level was a concave utility for gains and a convex utility for losses. Previous studies have corroborated the concavity for gains but the evidence on the convexity of the utility function for losses has been less clear-cut. Thus, it is important to compare the results to studies that have estimated the utility of losses at the individual level under prospect theory following the trade-off method (Wakker and Deneffe, 1996) and avoiding the bias due to probability weighing. The proportion (58%) is higher than Fenema and van Assen's (1999), Abdellaoui's (2000) and Etchart-Vincent's (2004) which range between (37% and 47%) and is close to Abdellaoui et al.'s (2006). Table 11 summarizes the results and Figure 10 shows the shape of the utility function for the median practitioner: concave for gains and convex for losses with a slight kink at the reference point.

Table 11: Non-Parametric Classification of the Practitioners

	Losses					
		Concave	**Convex**	**Mixed**	**Linear**	**Total**
Gains	**Concave**	4	27	11	0	42
	Convex	0	1	3	0	4
	Mixed	0	0	0	0	0
	Linear	0	0	0	0	0
	Total	2	28	14	0	46

Figure 10: The Shape of the Utility of the Median Practitioner



Parametric Fittings Results

The parametric estimations confirm also the concavity for gains and convexity for losses. Table 12 summarizes the parametric fittings (α, β) for the three models: Power, Exponential and Expo-Power. The individual parameters for the three models are found in Table 16 (*b*) in the Appendix.

Table 12: Parametric Fittings for Utility for Gains and Losses

	Power		**Exponential**		**Expo-Power**	
Parameter	α	β	α	β	α	β
Mean	0.85549	1.11999	1.28799	0.64304	1.11247	1.37079
Median	0.75734	0.90049	1.01799	0.49964	1.01059	1.16821

Compared to the above mentioned studies, the median power coefficient for losses (0.9) is within their range which varies between (0.84) and (0.97) while the median exponential (1.01) is close to the (0.98) of Abdellaoui et al. (2006). Table 13 classifies the practitioners according to the shape of their utilities while Table 13 (*a*) shows the parametric crossing of gains by losses for the power function, the exponential and the exponential-power respectively.

Table 13: Parametric Classification in Number and Proportion

		Losses		**Gains**	
Power	**Convex**	27	0.58	12	0.26
	Concave	18	0.39	31	0.67
	NA	1	0.02	3	0.06
Expo	**Convex**	28	0.61	12	0.26
	Concave	17	0.37	34	0.74
	NA	1	0.02	0	
Expo-Power	**Convex**	18	0.39	2	0.043
	Concave	7	0.15	21	0.45
	NA	0	0	3	0.06

Table 13 (*a*): Parametric Crossing of Gains by Losses

Gains	**Losses**								
	Power	concave	convex	**Exponential**	concave	convex	**Expo-Power**	concave	convex
	concave	12	16	concave	15	18	concave	7	1
	convex	0	6	convex	2	10	convex		2

4.5 *RESULTS RELATED TO THE LOSS AVERSION OF PRACTITIONERS*

This section presents the results for loss aversion first at the aggregate level then at the individual level according to the definition used.

4.5.1 Loss Aversion in the Aggregate

The means for the outcomes for gains as the independent variable and losses as the dependent for each of the utilities (0.015625, 0.03125, 0.06, 0.09375, 0.125, 0.25) were regressed linearly through the origin. The adjusted R^2 is 0.906 and the slope b is 0.450. The lower and upper bounds of the confidence interval 0.317 and 0.583 respectively, confirm the alternative hypothesis of $b \neq 1$ i.e. reject the null hypothesis of $b = 1$.

$b = 1$ refers to a symmetrical treatment of gains and losses relative to zero. For loss aversion to be observed the outcomes for gains need to be greater than the outcomes for losses for the same utility in absolute value that is $b < 1$. In that case losses are perceived more keenly than gains to paraphrase Kahneman and Tversky (1979). $b > 1$ indicates that the satisfaction one experiences from gaining is greater than the pain associated with losing the same amount.

4.5.2 Individual Loss Aversion

Table 14 displays the results of individual loss aversion under the various definitions and shows as in Abdellaoui et al. (2005) that which definition is adopted matters.

Table 14: Individual Loss Aversion Results under the Various Definitions

Coefficient	Loss Averse	Not Loss Averse	Neutral	Mean	Median	Un-classified
$-U(-G_r)/U(G_r)$	18	13	0	2.19	1.17	15
$minU(L_r)/L_r/maxU(G_r)/G_r$	11	6	0	1.02	0.56	29
$U'(-G_r)/U'(G_r)$	10	12	0	1.5	1.08	24
$minU'(L)/maxU'(G)$	3	2	0	0.28	0.21	41
$G_{0.015}/L_{0.015}$	17	22	3	5.95	0.74	4

According to the definition of Kahneman and Tversky (1979) 18 practitioners were found to have at least 6 coefficients > 1 and therefore were classified as loss averse; 13 were found to have coefficients < 1 and were classified as not loss averse. The mean (median) was 2.19 (1.17). The percentage of loss averse practitioners (30%) is lower than Abdellaoui et al.'s (2006) result (81%) but comparable to Bleichrodt and Pinto's (2002) who have used the same definition and found a range from 2.5% to 30% using the same definition. The proportion of practitioners motivated by gains (28%) is higher than either the latter's proportion which varies between 0% and 2.5% or Abdellaoui et al.'s (2006) 16%.

For Neilson's (2002) coefficient, the mean (median) was: 1.023 (0.56). Eleven practitioners have been found to have ratios that exceed 1 and were classified as loss averse while 6 with ratios of *minG/maxL* exceeding 1 were classified as not loss averse. According to the definition of both Neilson (2002) and Kahneman and Tversky (1979), the loss averse practitioners outnumber the not loss averse. Nevertheless and as can be seen from the table fewer practitioners were classified according to Neilson's (2002) rather strict definition.

For Wakker and Tversky's (1993) definition of loss aversion, 10 were found to be loss averse and 12 not loss averse. The mean (median) was 1.5 (1.08) respectively. According to this definition, 28% of the practitioners were found loss averse vs 37% practitioners who were more focused on gains, as opposed to the 33% loss averse vs 24% not loss averse of Schmidt and Traub's (2002) who have used the same definition and the 64% loss averse vs 12% not loss averse of Abdellaoui et al.'s (2006).

Bowman et al.'s (1999) definition of loss aversion coefficient which implies that *U* is everywhere steeper for losses than for gains is clearly strict. It resulted in 3 having *min*(*U*'(*L*)/*max U*'(*G*) > 1 and 2 having *min*(*U*'(*G*)/*max U*'(*L*) >1. The mean (median) is 0.28 (0.21).

For Köbberling and Wakker's (2005) definition which is exhaustive in the sense that every practitioner could be classified in one of the categories, they almost tie: 17 have a loss aversion coefficient > 1 while 22 have a loss aversion < 1. Four were unclassified because they had zero for the loss outcome immediately around the reference point.

Comparing the mean (median) coefficient of loss aversion found in this experiment with Abdellaoui et al.'s (2006) and shows that the median practitioner is less loss averse than the median student as is illustrated below in Table 15.

Table 15: Comparison of the Mean (Median) of the Practitioner and the Student

Coefficient	Practitioners Mean	Practitioners Median	Students Mean	Students Median
$-U(-G_r)/U(G_r)$	2.19	1.17	2.15	1.72
$minU(L_r)/L_r/maxU(G_r)/G_r$	1.02	0.56	0.83	0.51
$U'(-G_r)/U'(G_r)$	1.5	1.08	2.02	1.53
minU'(L)/maxU'(G)	0.28	0.21	0.62	0.5
$G_{0.015}/L_{0.015}$	5.95	0.74	4.99	2.52

Under the definition of Kahneman and Tversky (1979), both the mean and median values for the ratio *-U(-Gr)/U(Gr)* do not show consistent decrease with the size of the gains and losses involved unlike Abdellaoui et al. (2006) findings and Bleichrodt and Pinto's (2002) who observed that, in the health domain, the degree of loss aversion decreased with the size of the outcomes. Also under the definition of Wakker and Tversky (1993) and this time similarly to Abdellaoui et al.'s (2006) this effect was not observed. Table 16(*a*) immediately below show the practitioners' individual loss aversion Parameters.

Table 16 (*a*) : Practitioners' Individual Loss Aversion Parameters.

Practitioner	Loss Aversion Coefficients				
Number	K & T	Neilson	W &T	Bowman	K&W
2	2.252397	0.965014		0.074148	8.2
3	2.643696	0.011753		0.017925	0.006369
4	18.38589	0.868356		0.518233	1
5	1.063342	0.720599			1
6	5.745162	5.43307	3.12E+00	-0.00065	78.44444
7	3.651059	2.41908	3.28E+00	1	3.861842
8	0.973608	0.43299	4.16E-01	0.135577	1.913978
9	0.114753	0.146225	9.21E-02	0.03109	0.13
10	0.592003	0.36184	6.05E-01	0.271553	0.215
11	0.207838	0.105	1.43E-01	-0.00073	0.117241
12	2.143848	0.436503	1.15E+00		
13	1.043437	0.314074	7.00E-01	0.211715	2.307692
14	1.923994	1.511111	1.40E+00	0.289171	5.05
15	0.955635	0.116911	1.18E+00	0.167907	0.098765
16	1.246142	0.660161	1.81E+00	0.624368	0.52505
17	4.311981	3.793549	3.05E+00	1.115939	14
18	0.646741	0.209524			0.588235
19	1.43059	0.586139	1.08E+00	0.161058	0.686792
20	0.86868	0.63938	8.27E-01	0.251432	0.512397
21	0.651103	0.289249	8.12E-01	0.268143	0.221198
22	1.330008	0.106701	2.25E+00	-0.57779	1
23	4.745734	5.44762			
24	2.002662	0.222395	6.98E+00	0.270407	0.244
25	3.290824	0.320893		0.009312	0.329615
26	1.411629	0.688714	1.73E+00	0.644695	0.788599
27	0.75738	0.242836	8.23E-01	0.247444	0.236559
28	2.616173	0.246233	1.20E+00		
29	0.701446	0.071101	5.67E-01	0.090685	0.088415
30	2.102039	1.258002	1.73E+00	-0.18234	2.837438
31	1.089896	0.205941	5.47E-01	0.047378	1.5
32	2.161788	1.421814	2.49E+00	0.748583	4.575
33	1.943728	1.009852	9.82E-01	0.264105	7.148936
34	2.263393	0.711339	7.35E-01		
35	0.200373	0.123941	2.68E-01	0.089968	0.123563
36	5.509023	2.087996		0.039388	1.836042
37	0.888638	0.673549	7.96E-01	0.598969	1.01
38	0.539273	0.047074		0.04634	0.25
39	1.54285	0.612441	2.09E+00	0.61691	0.665323
40	0.357804	0.171257		0.150345	0.376623
41	3.711391	3.04878	2.19E+00	0.215276	50.5
42	5.733892	6.016739	3.73E+00	1.518924	50.75
43	0.802212	0.395661		0.039654	0.299517
44	0.308329	0.096127	3.41E-01	0.111308	0.131579

45	1.325308	0.551901	1.46E+00	0.423769	2.962963
46	1.743035	0.693555	1.01E+00	0.284807	3.095238
47	0.788702	0.579079	9.67E-01	0.388622	0.6

As the table shows there are some blanks under the definitions of Wakker and Tversky (1993), those of Bowman et al.'s (1999) and those of Köbberling and Wakker (2005). For Bowman et al.'s (1999) definition which is restrictive, calculating $\frac{\min(U'(L_r)}{\max U'(G_r)}$ involves initially calculating a ratio of differences equal to $\frac{u(x_r)-u(x_{r-1})}{x_r - x_{r-1}}$ where r is the reference utility and varies accordingly. The blanks correspond to $x_r - x_{r-1}$ equal to zero, that is equal outcomes for successive utilities. They belong to practitioners who have the numbers: 5, 12, 18, 23, 28, 34. The first one # 5 has equal outcomes for r = 5, 6, # 12 has also equal outcomes for r = 5 and 6 while those numbered 12, 23, 28, 34 have equal outcomes for r = 1 and 2.

For Wakker and Tversky (1993) definition, the blanks correspond to the practitioners whose elicited outcomes were identical for two consecutive utilities. The practitioners' numbers for these are: 2, 5, 18, 23, 38, 40, 43 mostly for r whose utility is 0.125, 0.156, while those numbered 3, 4, 25, 36 have huge coefficients.

For Köbberling and Wakker's (2005) definition, those numbered 12, 23, 28, 34 have zero for the first loss outcome.

Because the findings for the shape of the utility function show different curvatures for the utility of gains and the utility of losses, one may expect that the global measures of the utility function, which measure loss aversion at different points, do not indicate loss aversion unambiguously. A comparison of the practitioners loss aversion coefficients across definitions in Table 16(*a*) shows that there are big differences between the global measures of Wakker and Tversky's (1993) and Kahneman and Tversky's (1979) coefficients on one hand and the local coefficient of Köbberling and Wakker's (2005) on the other hand. Bowman et al.'s (1999) and Neilson's (2002) are global measures but have turned out to be too restrictive for empirical purposes. The results hence, argue in favor of a separation of loss aversion from the curvature of the utility function.

CONCLUDING REMARKS FOR EXPERIMENT I

Part I has shown extensively, that prospect theory can explain a variety of field and experimental data that are paradoxes under expected utility. Abdellaoui et al.'s (2006) method allows a test of the theory. Applied in the financial field, the method proved to be unproblematic. The availability and the ease of the application of a method to elicit prospect theory's utility on the whole domain is important for the theory's application and test.

An advantage of Abdellaoui et al.'s (2006) procedure which is based on the elicitation of utility midpoints is the control over the endpoints (in Table 9, L_2 is specified). Vind's (2003) is a similar method which elicits utility midpoints for given endpoints but requires more measurements, three to one per utility midpoint. It also assumes instead of prospect theory a general additive representation which is not directly applicable to prospect theory.

The elicitation procedure, nevertheless, uses a mix of response modes. First, in step one the probability equivalence method is used to elicit a probability which is then, used to elicit outcomes through the fractile method. According to Hershey and Shoemaker (1985) the two methods of utility assessment are not equivalent and the order in which they are presented, matters. For both gains and losses, the authors predict that for the above mentioned order the practitioners would relatively be less risk averse in the second mode (p: 1222). Put differently, if *p*' is elicited such that $G_1 \sim (G_2, p'; G_0)$ from a subject, then by asking the same subject to state G such that $G \sim (G_2, p'; G_0)$, the G obtained is $< G_1$. Thus, it is possible that the probability of gain has been adjusted upwards in the first mode showing an increase in risk aversion and the sure amount adjusted downward in the second mode (p: 1216). Also, to reduce order effects, counterbalancing the elicitation steps for gains and losses across practitioners might reduce the possible exaggeration or the dampening of the curvature of the utility function by certain features of the elicitation procedure. Most of the practitioners have indeed shown some aspects of fatigue by the time they finished the fractile losses. It did look that losses were tiring to consider and the fractile for losses long.

It remains to be said regarding the applicability of the method that the utility graph immediately obtained at the end of the computerized session motivated the financial practitioners to take time to discuss the psychological elements of *CPT* and most of them asked for a summary of the experiment's results to be sent to them. The reason is that they perceived *CPT* as reflecting the major aspects of their decision making process which could be summarized in an expression often used by them and quoted by Thaler (1993, p: 513): "small profits and large losses."

Regarding the results *per se*, the elicitation of the utility function in Experiment I shows that the most common pattern for the utility function for financial practitioners is concavity for gains and convexity for losses. The proportion (58%) is higher than Fenema and van Assen's (1999), Abdellaoui's (2000) and Etchart-Vincent's (2004) which range between (37% and 47%) and is close to the (54%) of Abdellaoui et al.'s (2006). For parametric fittings, the median power coefficient for losses (0.9) is within their range which varies between (0.84) and (0.97).

Loss aversion exists in the aggregate (b = 0.450, adjusted R^2 = 0.906). At the individual level the coefficients of loss aversion vary however with the definitions used emphasizing the need for a commonly accepted definition. Few practitioners could be characterized according to the global definitions of Neilson (2002) and Bowman et al.'s (1999) which seem overly strong for empirical purposes. Contrasted with the results of previous studies that have estimated a loss aversion coefficient according to the same definition, the findings show that for *Wakker and Tversky*'s (1993) definition, the practitioner's median is found lower than both Schmidt and Traub's (2002) and Abdellaoui et al.'s (2006) i.e. 1.08 vs 1.43 vs 1.53 respectively. *For Kahneman and Tversky*'s (1979) definition it is close to Abdellaoui et al.'s (2006), lower than Bleichrodt et al.'s (2001) i.e. 1.69 vs 1.72 vs 2.17 respectively. Finally, for the local definition of Köbberling et Wakker (2005) the median portfolio manager is rather *not loss averse* relative to the median student of

Abdellaoui et al. (2006): 0.74 vs 2.52 respectively. In brief, as Table 15 which compares the practitioners' and students' means (medians) shows the former is less loss averse than the latter.

Professionals who are exposed to a range of training and high level of knowledge differ in their assessment of the stakes from the students. The difference in the degree of loss aversion may be due to the practitioners' range of training and high level of knowledge relative to the students.

Also, the interviews were conducted during the period (2003-2004) which corresponds to a growth (the Standard and Poor index was up by 28.17% and 10.9% respectively relative to the preceding year). It is not inconceivable thus that the degree of loss aversion of the practitioners diminished during that period with the upward movement of the stock market. Barberis, Huang and Santos' (2001) asset pricing model is based on changing risk aversion over time generated by introducing loss aversion over financial wealth fluctuations and allowing the degree of loss aversion to be affected by prior investment performance.[91] Without the variation of loss aversion with past movements in the stock market, the authors couldn't account for the high volatility of stock returns observed.

In addition to original prospect theory (1979) their model is influenced by the psychological findings in Thaler and Johnson (1990) and Gertner (1993). The latter two studies examine the effect of a sequence of gains and losses for small stakes and high stakes respectively and show evidence of house money effect[92] i.e., prior gains cushion following losses rendering the individual less loss averse.

The two studies also show that the controverse is true, i.e., the individual is more loss averse after having incurred losses. The finding implies that the convexity of the utility

[91] Barberis, Huang et Santos (2001) show that time-varying loss aversion degree is necessary alongside loss aversion to account for the equity premium puzzle.

[92] The terminology "playing with the house money" refers to the gamblers increased willingness to bet when ahead.

function can not be due to the integration of sequential gambles[93] but does not refute the risk seeking in the domain of losses. Indeed, another result of Thaler and Johnson (1990) is that students are risk seeking when the lottery results in consequences which offer the chance of a breakeven and limit the maximal loss.

Gross (1982) well-documents the *get even* phenomenon in the stock market which he refers to as the “geteveniitis disease”. The remedy is painful: to realize losses and perceive sunk costs as irrelevant to the decision making, as dictated by rationality, practitioners need to give up the hope they might get even before they get out. [94]

Last but not least, an important incentive for the practitioner to seek gain is the compensation package he is eligible to in case of profits. For instance, for the record profits of the year 2006, the “Wall St Bonus” according to the New York Times (Dec. 25, 2006)[95] varied from $1mil to $3 mil for an *average* managing director to $60 mil for the investment houses like Goldman Sacks, Lehman Brothers and Morgan Stanley and more for a select group of hedge fund managers and private equity executives.

The variation of loss aversion with the volatility of the market (5% only of the practitioners do beat the market, the majority gains in a bull market and loses in a bear market) and the luring compensation package practitioners could get in case of profits are factors to be considered. Nevertheless, the utility function must reflect the preferences of the individual and the utility must not change with the method being used. Hence, the preferences of the students/practitioners analyzed following a different method must reflect consistent preferences. Experiment II presented next investigates non-parametrically albeit qualitatively the preferences of MBA students, potential financial

[93] If investors did integrate many years of stock market gains and losses, they would essentially be valuing absolute levels of wealth and not the *changes* in wealth that are so important to prospect theory (Barberis, Huang and Santos, 2001).

[94] Daniel Kahneman and Jonathan Renshon recently argued in Foreign Policy magazine that the American administration’s unwillingness to face reality in Irak reflects a basic human aversion to cutting one’s losses, the same instinct that makes the gambles stay at the table hoping to break even (Krugman, P., The New York Times, Jan, 8, 2007)

[95] For instance, the bonus awarded to Blankfein, L.C. CEO of Goldman Sacks is $54.3 mil (New York Times, Dec 25, 2006).

practitioners and tests for loss aversion using stochastic dominance criteria newly developed by Baucells and Heukamp (2006).

CHAPTER V. EXPERIMENT II.

LABORATORY INVESTIGATION USING STOCHASTIC DOMINANCE CONDITIONS

Just as Abdellaoui et al.'s (2006) elicitation method contains all the essentials of Wakker and Deneffe (1996) but *not vice versa*, Baucells and Heukamp's (2006) newly developed conditions contain all the essentials of Levy and Levy's (2002) but *not vice versa*. That is their conditions include Levy and Levy (20002) and other properties. In that sense, the newly developed conditions also represent a case of all of the lower is in the higher but not all of the higher is in the lower as was pointed out first by Aristotle[96]. To recapitulate in brief, preferences can be inferred from choices among lotteries constructed such as one stochastically dominates the other. An individual with a concave non-decreasing utility function i.e. a risk averse expected utility maximizer, will dis-prefer a lottery that is dominated by *SSD* and the converse is also true: if the individual eliminates *SSD* dominated alternatives, he has a concave utility function.

To discriminate between classes of non-decreasing utility functions which are not concave throughout, other conditions are needed. *Ad initium*, Moshe Levy and Haim Levy (2002) develop Prospect Stochastic Dominance (*PSD*) and Markowitz Stochastic Dominance (*MSD*) to differentiate between utility functions which are concave for gains and convex for losses (have an *S* shape as postulated by Kahneman and Tversky (1992)) and those which are convex for gains and concave for losses (have an inverse *S* as postulated by Markowizt (1952)).[97] Their experimental investigation (Levy and Levy, 2002) which uses these conditions rejects the *S* shape and supports the inverse *S*.

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[96] Aristotle first pointed out that the impulse of evolution is this *not vice versa* which invariably establishes a hierarchy, an increasing order of wholeness. For instance, a molecule includes atoms yet has properties that are not merely the sum of its atoms. To spot the higher from the lower in any sequence, Wilber (1996, p: 28) suggests a thought experiment where all of the higher is destroyed and none of the lower.

[97] The utility function as defined by Kahneman and Tversky (1979, p: 279) is the exact opposite of Markowitz' (1952) postulated utility function as shown by Figure 11 illustrated further below.

It turns out however, as Baucells and Heukamp's (2004) and Wakker's (2003) show[98] that the reason the *S* shape is rejected is that Levy and Levy (2002) have assumed that the probability weighting function is unlikely to play a role for probabilities $\geq$ 0.25. Had they not, the results would have been compatible with both theories.[99]

To be able to discriminate empirically between the two theories, Baucells and Heukamp develop stochastic dominance conditions which incorporate probability weighting and loss aversion. Their experimental investigation using these conditions (Baucells and Heukamp, 2006) "rules out" the possibility that the utility function is concave throughout, or has an inverse *S* shape and confirm that the *S* shape holds assuming the curvature of the utility function is the same on each side of the real line. However, loss aversion is only evident when the probability for gains/losses is similar or the same in the two lotteries. In brief, for mixed gambles and for the purpose of prediction: the overall probability matters and loss aversion competes with the convexity of the utility function for losses (Baucells and Heukamp, 2006).

Experiment II, investigates the risk preferences of MBA students at ASU using the Baucells and Heukamp (2006) lottery pairs. More specifically, the students were asked to answer a questionnaire of 20 tasks and to choose for each between two lotteries constructed as head to head competition between the *CPT* and Markowitz' (1952) theories.

Baucells and Heukamp (2006) having hypothesized the impact of the overall probability in mixed gambles, Experiment II has also included tests of this effect. The questionnaire format allowing it, the students were asked to comment on the choices they have made. The statements either confirmed or disturbed the motivation inferred for a particular

[98]Wakker (2003, p: 981) and Baucells and Heukamp (2004) independently and using different methods show that the results are compatible with *CPT*'s predictions contrary to Levy and Levy 's (2002) claims.

[99]For lottery *F* in Levy and Levy's (2002) experiment 2, all 4 outcomes have p = 0.25 and $w(0.25)$ = 0.29 close to 0.25; but the correct decision weights for the four outcomes which depend on cumulative probabilities, are respectively: 0.29, 0.16,0.13 and 0.29, showing the decision weights for the extreme outcomes to be twice as much as those for the intermediate outcomes (Wakker, 2003).

choice particularly when a switching from one theory to the other occurs. The method however does not quantify loss aversion.

Chapter V consists of six sections. Section 1 gives the intuition for prospect and Markowitz stochastic dominance followed by the characterization of preferences using these conditions. Section 2 states the objective of the experiment and describes the source of the data. The analysis of the data is presented in section 3. The results related to the shape of the value function are presented next, followed by the results related to loss aversion and the results related to the overall probability of gain /loss respectively. A final section concludes.

5.1 *PROSPECT AND MARKOWITZ STOCHASTIC DOMINANCE*

The insight in brief behind Levy and Levy's (2002) prospect dominance condition (*PSD*) and Markowitz dominance condition (*MSD*) is that a prospect theory follower gives more importance to outcomes near the origin than to extreme outcomes while the opposite is true for a Markowitz follower assuming no probability weighting. However, when probability weighting is factored in the decision making process more importance is given to extreme outcomes relative to intermediate outcomes. Hence, for *CPT* not to remain ambiguous, it's necessary to generalize the *PSD*, *MSD* conditions to capture this important aspect of *CPT* and also the other remaining aspect of the theory namely loss aversion. This is the motivation for the Baucells and Heukamp's (2006) newly developed *SD* conditions. Section 5.1.1 presents first Levy and Levy's (2002) conditions to be followed by Baucells and Heukamp's (2006) in section 5.1.2.

5.1.1 Levy and Levy's (2002) Stochastic Dominance Criteria

This section presents for clarification the insight that led Levy and Levy (2002) to claim that prospect theory is "much ado about nothing". It will be followed with the preference characterization.

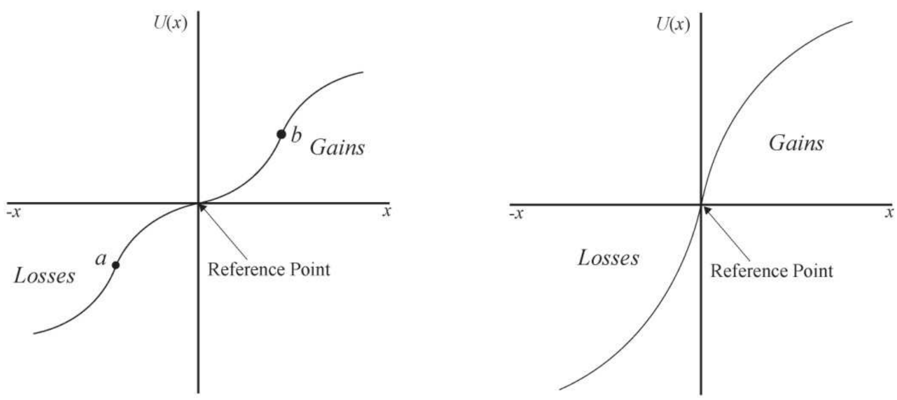
The Intuition

If prospect stochastic dominance (Markowitz stochastic dominance), that is *PSD* (*MSD*), is to be used analogously to *SSD* then, a lottery *F* preferred or equivalent to *G* ($F \succcurlyeq G$) according to *PSD* (*MSD*) ought to necessarily imply that the expected utility of *F* is greater than the expected utility of *G* for any agent with an *S* shape (inverse *S* shape) utility function. Assuming no probability weighting the difference in expected utility is given by[100]:

$$\Delta = \int_a^b [G(t) - F(t)]U'(t)dt \qquad (5.1)$$

Where *F* and *G* are the cumulative distributions of lotteries *F* and *G* and where it is assumed for simplicity that both *F* and *G* take the values 0 and 1 for some $a \leq 0$ and $b \geq 0$ respectively. *a* and *b* correspond to the two extreme inflection points in the Markowitz utility function and are expected to be at extreme wealth levels. Figure 11 contrasts the latter utility function with its exact opposite prospect utility function pointing at the same time to a commonality between the two: in both, decision makers base their decisions on changes in wealth relative to some reference point.

Figure 11: Markowitz and Prospect Theory Utility Function



[100] Equation (5.1) is obtained by integrating by parts the difference in expected utility of the two lotteries *F* and *G* (Rothshild and Stiglitz, 1970).

According to equation (5.1) the difference [$G(t)$ –$F(t)$] is scaled in proportion to $U'(t)$, which is to say that segments where the slope of $U(t)$ is high are stretched more relative to segment where it is small. The areas near the origin are magnified for a prospect theory follower since the slope is higher near the origin while the extremes are magnified for a Markowizt' follower since this is where the slope is higher.

The prospect stochastic dominance and Markowitz stochastic dominance conditions developed by Levy and Levy (2002) capture this insight and characterize the preferences of a decision maker who maximizes the expectation of the utility function assuming no probability weighting. Depending on whether the decisions are based on total wealth or changes in wealth[101], *PSD* and *MSD* cover *EU* or any reference model of which however $w(p)$ is not a part.

Levy and Levy's (2002) Preference Characterization

The Levy and Levy (2002) conditions characterize both the *S* shape and the inverse *S* shape utility functions. Individuals however, may have different types of preferences as can be seen from the results of Experiment I: alongside the *S* shape utility function characterizing the majority of practitioners, a concave utility function for gains and losses for instance is also representative of the practitioners' preferences. The *S* shape and the inverse *S* shape utility functions are considered then as two classes of preferences which are subsets of a general utility function which describes individuals who prefer more to less. This function is monotonic and non-decreasing characterized by a first derivative which is never negative. Formally:

$U \in U_1$ if $U' \geq 0$.

The preference class of non-decreasing concave preference functions $U_{concave}$ is characterized in addition by a non-increasing second derivative. Formally:

$U \in U_{concave}$ if $U' \geq 0$ and $U'' \leq 0$.

While the class of non-decreasing convex preference functions U_{convex} is characterized by a non-decreasing second derivative:

[101] Levy and Levy (2002, p: 1338) show that *FSD*, *SSD*, *PSD MSD* can be stated in terms of total wealth or changes in wealth. Levy (1992) is a review article of stochastic dominance rules.

$U \in U_{convex}$ if $U' \geq 0$ and $U'' \geq 0$.

These classes are subsets of U_1 as are also the two utility functions *S* shape (inverse *S* shape) illustrated in Figure 11 since they are all non-decreasing functions ($U' \geq 0$). Formally, with U_P, the set of *S* shape utility functions,

$U \in U_P$ if $U' \geq 0$ for all $x \neq 0$, $U'' \geq 0$ for $x < 0$ and $U'' \leq 0$ for $x > 0$,

and with U_M, the set of inverse *S*- shape utility functions,

$U \in U_M$ if $U' \geq 0$ for all $x \neq 0$, $U'' \geq 0$ for $x > 0$ and $U'' \leq 0$ for $x < 0$.

Proposition 5.1:

Define *F* and *G* as above. Then, $F \succ_{PSD} G$ for all *S*-shape utility functions, $U \in U_P$, if and only if

$\int_y^0 [G(t) - F(t)]dt \geq 0$ for all $y \leq 0$ and

$\int_0^x [G(t) - F(t)]dt \geq 0$ for all $x \geq 0$ hold

where there is a strict inequality for some pair (y_0, x_0) and for some $U_0 \in U_P$ ■

Proposition 5.2:

Define *F* and *G* as above. Then, $F \succ_{MSD} G$ for all inverse *S*-shape utility functions, $U \in U_M$ if and only if

$\int_a^y [G(t) - F(t)]dt \geq 0$ for all $y \leq 0$ and

$\int_x^b [G(t) - F(t)]dt \geq 0$ for all $x \geq 0$ hold

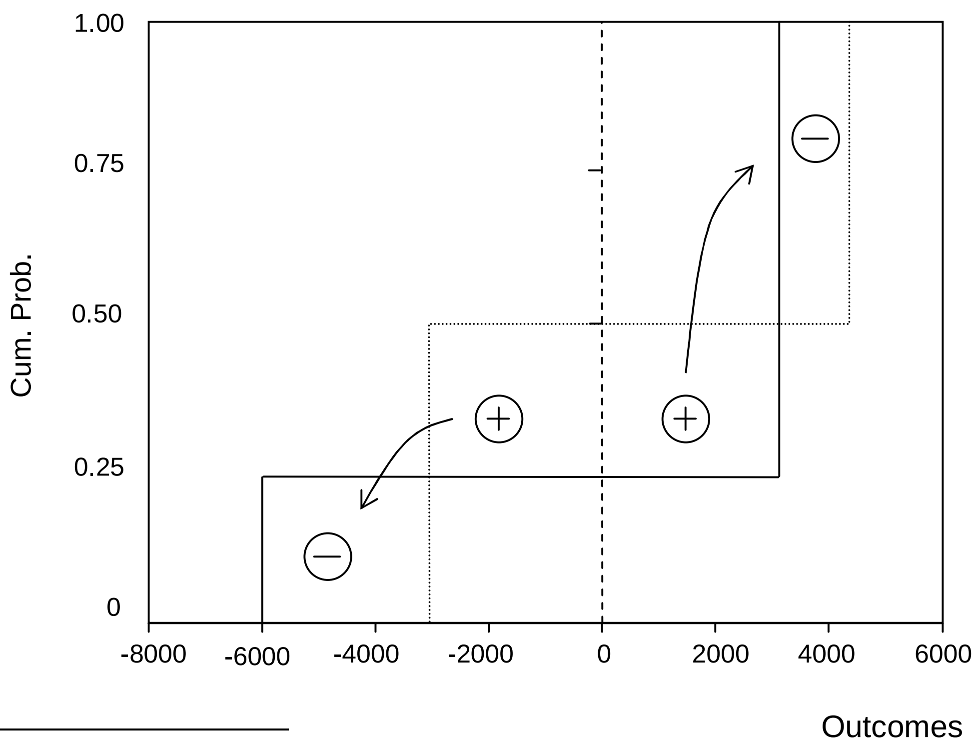
(with at least one strict inequality) ■

Markowitz stochastic dominance rule is not necessarily the opposite of prospect stochastic dominance rule. However, they are opposites if the two distributions have the same mean. Thus, for *F* and *G* with equal means, $F \succ_{PSD} G$ iff $G \succ_{MSD} F$.

These conditions were used by Levy and Levy (2002) as a guide in designing pairs of lotteries or tasks that are head to head competitions between the two alternative theories. The choices of the decision maker between the so-designed lotteries reveal properties of his preferences and their representations under a choice model. Thus, for a given model, the prospect stochastic dominance (Markowitz stochastic dominance) condition related to a specific feature of the model allows for testing the qualitative properties of the latter.

For two lotteries *F* and *G* designed such as $F \succ_{PSD} G$ and $G \succ_{MSD} F$ the choice of *F* (*G)* implies that the utility function of the decision maker has an *S* shape (inverse *S* shape). Figure 12 illustrates the cumulative distributions for the two lotteries *F* and *G* of Task I[102] designed such as $F \succ_{PSD} G$ and $G \succ_{MSD} F$. The solid lines represent *F* and the dashed lines *G*. The signs correspond to the signs of [*G*(*t*) –*F*(*t*)]. Because *F*(*t*) < *G*(*t*) in the areas which are closer to the origin, these will be magnified for a prospect theory follower.

Figure 12: The Cumulative Distributions for Task I: $F \succ_{PSD} G$ and $G \succ_{MSD} F$.



[102] All tasks are described in the questionnaire in Appendix D.

However, as argued above, Levy and Levy (2002) have assumed that the probability weighting function is unlikely to play a role for probabilities $\geq$ 0.25 motivating Baucells and Heukamp (2006) to develop stochastic dominance criteria that account for the probability weighting function. Their preference conditions are presented next preceded by the insight behind them for clarification.

5.1.2 Baucells and Heukamp's (2006) Stochastic Dominance Criteria

To be able to discriminate between prospect theory (1992) and Markowitz' utility theory, Baucells and Heukamp (2006) extend the stochastic dominance conditions to *CPT* by incorporating the probability weighting function and loss aversion. The intuition behind the extension is presented first followed by the preference characterization.

The Intuition

The difference in expected utility was given in equation (5.1) assuming no probability weighting. However, when probability weighting of extreme events is factored into equation (5.1), the difference in utility between lotteries *F* and *G* is given by:

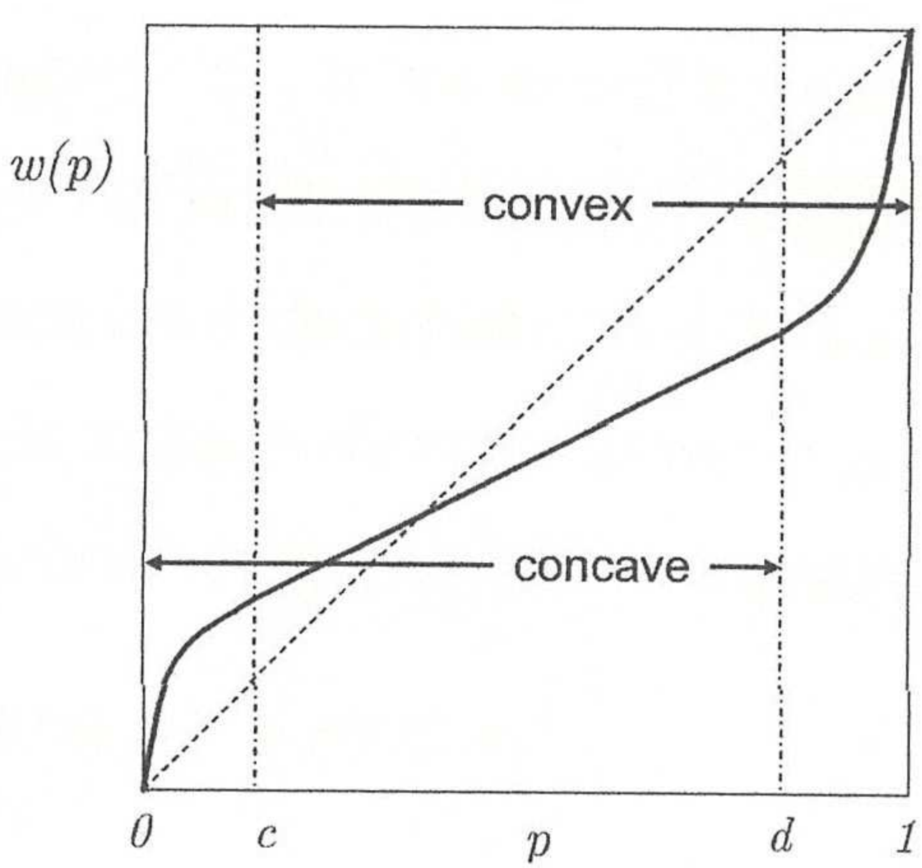
$$\Delta = \int_a^0 [w(G(t)) - w(F(t))]U'(t)dt + \int_0^b [w(1-F(t)) - w(1-G(t))]U'(t)dt \qquad (5.2)$$

Accordingly, in parallel of the horizontal stretching, the vertical cumulative probability axis (0, 1) is stretched by the probability weighting function making the prospects near 0 (possibility) and near 1 (certainty) more desirable and magnifying the difference between *F*(*t*) and *G*(*t*) in these areas. Thus, while $F \succ_{PSD} G$ for a prospect theory follower equation (5.2) can still yield a preference for *G*. To resolve the ambiguity, stochastic dominance conditions that incorporate the probability weighting function need to be used in constructing the lotteries.

Accounting for the Probability Weighting Function

The qualitative features that a descriptively relevant probability weighting function (*pwf*) exhibits are concavity for low values of *p* (*p* close to 0) and convexity for high values of *p* (*p* close to 1). To capture these features, five classes of *pwfs* are defined: W_0 is the class of convex probability weighting functions and W^1 the class of concave *pwfs*, W_c is the class of *pwfs* convex between *c* and 1, W^d the class of *pwfs* concave between 0 and *d* and $W_c^d = W_c \cap W^d$ is their linear intersection ($w(p) = p$) which contains segments that are convex between *c* and 1 and segments that are concave between 0 and *d*. The *c* and *d* denote both c^+ and c^- (or both d^+ and d^-) which apply to w^+ and w^- respectively. $W_c^d = W_c \cap W^d$. If $0 < c \leq d < 1$, then W_c^d is necessarily linear and agrees with the Kahneman and Tversky (1992) inverse *S* shape *pwf*: "shallow in the middle interval and changes abruptly towards the ends of the probability interval (0,1). Figure 13 illustrates the W_c^d class:

Figure 13: the W_c^d class of probability weighting function

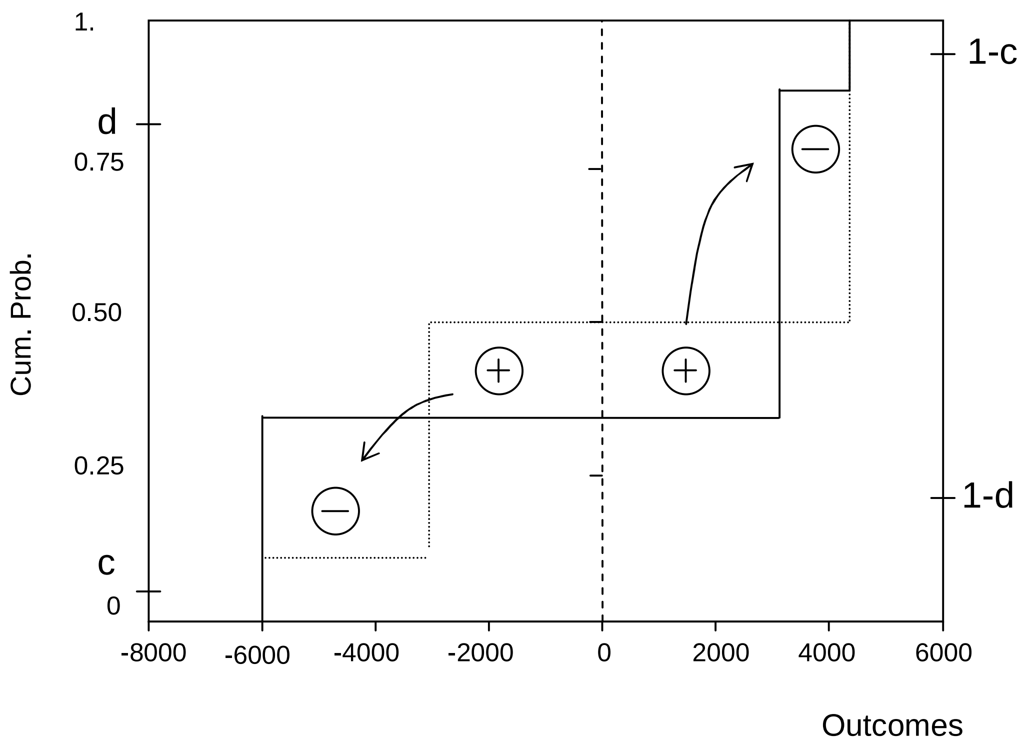


The classes of probability weighting function thus defined, the areas where the vertical stretching and the horizontal stretching are compounding each other that is, the intervals of the payoff line in which the curvatures of *u* and *w* are conjugate[103] are delineated restricting the *pwf* to W_c^d. First order stochastic dominance conditions are imposed then in the intervals where the vertical stretching runs against the horizontal stretching that is in the intervals where the utility function and the *pwf* do not have conjugate curvatures in order to extend the *SD* conditions to *CPT*. Figure 14 shows for $c \leq 1/6$, the cumulative

[103] In the loss domain, a convex *u* and a convex w^- are conjugate as are a concave *u* and a concave w^-. In the gain domain, a concave *u* and a convex w^+ are conjugate as are a convex *u* and a concave w^+.

distributions for the two lotteries F and G of Task VII designed such as $F \succ_{PWSD} G$ and $G \succ_{MWSD} F$.

Figure 14: The Cumulative Distributions for Task VII: $F \succ_{PWSD} G$ and $G \succ_{MWSD} F$.



Formally, Baucells and Heukamp's (2006) stochastic dominance conditions which account for the probability weighting are stated as follows:

Proposition 5.3:

$F \succ_{PWSD} G$ iff $F \succcurlyeq G$ for all $U \in U_P$, $w^- \in W_{c^-}^{d^-}$, and $w^+ \in W_{c^+}^{d^+}$. Similarly,
$F \succ_{MWSD} G$ iff $F \succcurlyeq G$ for all $U \in U_M$, $w^- \in W_{c^-}^{d^-}$ and $w^+ \in W_{c^+}^{d^+}$;

$F \succ_{SWSD} G$ iff $F \succcurlyeq G$ for all $U \in U_{concave}$, $w^- \in W_{c^-}^{d^-}$, and $w^+ \in W_{c^+}^{d^+}$; and
$F \succ_{S*WSD} G$ iff $F \succcurlyeq G$ for all $U \in U_{convexe}$, $w^- \in W_{c^-}^{d^-}$, and $w^+ \in W_{c^+}^{d^+}$ ■

Accordingly, to predict preferences between two lotteries when neither stochastically dominates the other, the probability weighting function must be restricted. The choice of c and d is ultimately the result of a tradeoff. When $c = 1$ and/or $d = 0$, the four conditions are reduced to first order stochastic dominance. Thus, by decreasing c or increasing d, first order stochastic dominance is imposed on a smaller range which increases the scope of application of the different stochastic dominance conditions. However, the W_c^d might become too narrow and might not contain the desired functions. Their choice of c and d was selected from a range of [0.05 to 0.88] according to the sensitivity analysis they undertook.

Accounting for Loss Aversion

To incorporate loss aversion, Wakker and Tversky's (1993) definition is used to define the class of utility functions possessing loss aversion. Since the latter guarantees that the stretching of the horizontal axis at $-x$ is at least as large as the stretching of the horizontal axis at $x > 0$. This allows to use positive first order stochastic dominance segments in the losses domain where $F(-x) < G(-x)$ are used to counteract negative first order stochastic dominance segments in the gains domain where $G(x) < F(x)$. Loss aversion entails comparisons between the positive and the negative domain hence the sign-dependent *pwfs* need to be constrained. Assuming $c^+ < d^+$ and $c^- < d^-$ the slope of the linear segment of w^+ is defined as s^+ and the slope of the linear segment of w^- is defined as s^- and the condition $s^- \geq s^+$ is imposed. The slope of w^- being larger than the slope of w^+ ensures that areas of positive *FSD* in the negative domain can counteract areas of negative *FSD* in the positive domain. Defining U_L as the class of utility functions such that:

$U \in U_L$ if $U'(-x) \geq U'(x)$ for all $x \geq 0$,

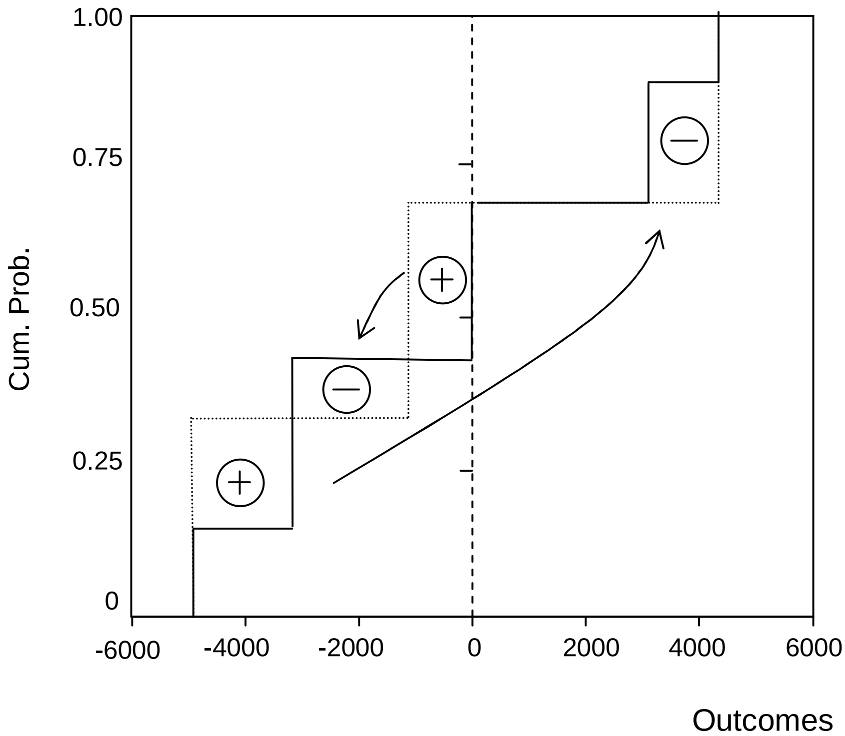
Baucells and Heukamp's (2006) stochastic dominance condition which accounts for loss aversion is stated formally as follows:

Proposition 5.4:

$F \succ_{PLSD} G$ iff $F \succcurlyeq G$ for all $U \in U_L$, $w^- \in W_{c^-}^{d^-}$ and $w^+ \in W_{c^+}^{d^+}$ such that $s^- \geq s^+$ ■

The probability weighting functions for positive outcomes and those for negative outcomes are both restricted since under *CPT* the decision weights for the former are calculated independently from the decision weights for the latter. The condition $s^- \geq s^+$ is consistent with the empirical finding that w^- exhibits less deformation than w^+ (Tversky and Kahneman, 1992; Abdellaoui, 2000). Furthermore, it reinforces loss aversion since it implies that the stretching of the vertical axis for negative outcomes is larger than the vertical stretching for the positive outcomes. Figure 15 illustrates the cumulative distributions of Task XIX designed such as to test for loss aversion such as $F \succ_{PW_c^d LSD} G$ for $W_{0.15}^{0.65}$.

Figure 15: The cumulative distributions for Task XIX



Finally, with $U_{PL} = U_P \cap U_L$, the full *CPT SD* condition which accounts for probability weighting and for loss aversion is stated formally as follows:

Proposition 5.5:

$F \succ_{PWLSD} G$ iff $F \succcurlyeq G$ for all $U \in U_{PL}$, $w^- \in W_{c^-}^{d^-}$ and $w^+ \in W_{c^+}^{d^+}$ such that $s^- \geq s^+$ ■

The extension of the *SD* conditions to *CPT* allows for testing hypotheses about the curvature of the utility function and/or loss aversion assuming the *pwf* is inverse *S* shaped. It also allows to test the curvature of the latter assuming the empirical specifications for *CPT* for the utility function hold. Finally, if one assumes all the empirical specifications for *CPT* holds, then a violation of the *SD* condition implies a violation of the *CPT* model.

5.2 *EXPERIMENTAL APPLICATION*

The tasks constructed by Baucells and Heukamp (2006) using the newly developed stochastic dominance conditions were as in Levy and Levy (2002) head to head competition between two prospects *F* and *G* having the same mean: $F \succ G$ for the *S*-shape functions and $G \succ F$ for the inverse *S*. Therefore, the preferences of the subjects can be inferred from their choices of *F* or *G*. For two lotteries *F* and *G* designed such as $F \succ_{PWSD} G$ and $G \succ_{MWSD} F$, as in Task VII for instance with c = 1/6 and d = 2/3, the choice of *F* implies that the hypothesis of $U \in U_P$ is not rejected i.e. is consistent with an *S* shape utility function and that the hypothesis of $U \in U_M$ is rejected.

Lotteries with at least three outcomes are required because the design of the tasks using the stochastic dominance conditions implies always the addition on common extreme outcomes. Experiment II uses these tasks to investigate the risk preferences of MBA students. Section 5.2.1 states the objectives of Experiment II, section 5.2.2 describes the source of the data.

5.2.1 Objective of the Experiment

Experiment II has three objectives: the first is to investigate what type of utility function MBA students have, i.e., is it *S*-shape as in *CPT* or inverse *S* as in Markowitz; the second is to find out whether they are loss averse or not and the third is to shed light on the effect of the overall probability of gain.

5.2.2 Source of Data

The 40 subjects were first and second year MBA students at Arizona State University, (Phoenix-Arizona, U.S.). They were contacted through the student services coordinator associate following the recommendation of William Boyes, Professor of economics at ASU. The interviews were conducted in groups of 4-5 in the suites, private rooms, the MBA students at the W.P. Carey School of Business at ASU have access to for team work. The students were working and/or had worked previously. (One requirement for MBA at ASU is a minimum of three years of experience). They were paid $10 for the 30 mn on average needed to complete the questionnaire.

The questionnaire consisted of 22 tasks. For 20 of these, the students had to choose between two investments *F* and *G* introduced as in Baucells and Heukamp (2006) and Levy and Levy (2002) that is as follows: “Suppose that you decided to invest $10000 either in stock *F* or Stock *G*. Which stock would you choose, *F* or *G* when it’s given that the dollar gain or loss one month from now will be as follows.”

The students were also asked to state the reason for their choice. For the last 2 tasks as in (Payne, 2005) students were asked whether they preferred to increase the overall probability for gains when given a chance to. The questionnaire handed out to the students is in Appendix D[104].

[104] The questionnaire is based on the paper Baucells, M. presented at the FUR (2004) at GRID, Cachan, France which he has co-authored with Heukamp H. and which I attended.

5.3 *ANALYSIS OF DATA*

The 20 tasks used in this experiment were designed initially in response to Levy and Levy's (2002) claim that prospect theory is *much ado about nothing*. The basis of the authors' claim is that subjects who were given a choice in three tasks I-III, where $F \succ_{PSD} G$ and $G \succ_{MSD} F$ opted for G the Markowitz dominating lottery in the following proportions: 71%, 62% and 76% respectively. To refute Levy and Levy's (2002) claim Baucells and Heukamp (2006) use the tasks I-III as they are initially designed (assuming no probability weighting) to ensure similar responses from subjects then, construct modifications of these tasks which incorporate the probability weighing function.

Thus, tasks I-III mimic Levy and Levy's (2002) tasks to ensure that $U \notin U_P$ where U_P is the set of *S*-shape utility functions. Because $F \succ_{PSD} G$ if w is linear (convex) throughout, then the choices favoring G imply that $U \notin U_P$.

In contrast to tasks I-III where no probability weighting is assumed, tasks IV through VIII, discriminate between the hypotheses of an *S* shape utility function or an inverse *S* shape, assuming the empirical specification of the probability weighting function holds. These tasks are modifications of tasks I-III and exhibit $F \succ_{PWSD} G$ and $G \succ_{MWSD} F$ with $w \in W_{0.1}^{0.74}$, for $d \leq 0.74$ and $c \geq 0.1$. F is the expected answer if $U \in U_P$ and G is the expected answer if $U \in U_M$. Thus, the choice of F in these tasks i.e. $U \in U_P$, given the results of Tasks I-III, implies that the probability weighting function can be neither linear nor convex throughout.

To specifically examine the curvature of the latter function (*pwf*) and its change near the origin, task IX was designed as a modification of task I with a slight change in the probability attached to common outcomes. It exhibits $F \succ_{PWSD} G$ and $G \succ_{MWSD} F$ and tests conjointly with task I whether $w \in W_{0.02}$ and whether $w \in W_0$ can be rejected respectively.

Before however, testing for loss aversion, four tasks are designed to refute the argument that the concavity for gains and the convexity for losses are driven by the certainty effect. Thus, tasks X-XI are constructed as all gains gambles, and test the concavity for gains and tasks XII- XIII are all losses gambles and test the convexity for losses. In all tasks, none of the outcomes is certain and all satisfy the $F \succ_{PWSD} G$ for $c \geq 0.1$.

An important feature of *CPT* is loss aversion and seven tasks are designed to test for it. The tasks XIV-XX which test for loss aversion are divided into two categories. Tasks XIV-XVI have $F \succ_{W_c^d LSD} G$ and test *solely* loss aversion with no assumption on the utility function. For these tasks the choices favoring *F* imply then loss aversion. The remaining tasks XVII- XX test a *joint* hypothesis of loss aversion and concavity for gains/convexity for losses and exhibit all $F \succ_{PW_c^d LSD} G$ and can be interpreted as a global test for *CPT*. For tasks XVII and XX, the choices favoring *F* imply that the subjects are loss averse and that the shape of their utility function is concave for gains, while for tasks XVIII and XIX, choices favoring *F* imply that the subjects are loss averse and that the shape of their utility function is convex for losses.

Finally, Baucells and Heukamp (2006) having hypothesized the impact of the overall probability in mixed gambles, the last two tasks were included to investigate the effect of the overall probability of winning or losing. The students were presented with the following mixed lottery designed by Payne (2005): ($100, 0.2; $50, 0.2; $0, 0.2; $-25, 0.2; $-50, 0.2) and they were asked in a first step to add ($38) to either $0 or $100 and in a second step to add ($38) to either $50 or $100. The choice of adding the ($38) to the ($0) would imply a preference to increase the overall probability of gain.

5.4 RESULTS RELATED TO THE SHAPE OF THE UTILITY FUNCTION

The results related to the shape of the utility function are summarized in Table 17 below which also compares them to those obtained by Baucells and Heukamp (2006).

Table 17: Results of Experiment II Regarding the Shape of the Utility Function in %

Task	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**	**XII**	**XIII**
B.& H.	36	42	33	61	84	66	76	84	81	79	74	75	64
ASU	37.5	40	37.5	67.5	85	87.5	85	92.5	92.5	85	87.5	72.5	85

For tasks I-III which mimic the Levy and Levy (2002) lotteries, the proportions of students who chose *F* over *G* are 37.5 %, 40 %, and 37.5% respectively that is, results comparable to Levy and Levy's (2002) are obtained .

For tasks IV-XIII designed to investigate the utility function in mixed lotteries, the high majority of the students chose *F*. Regarding the relatively low proportion (67%) for task IV, which is complex and presents identical probabilities, the reasons the students gave were mostly centered on the middle class outcomes, which agrees with the editing of extreme outcomes that have the same probabilities. However, in task VI, which also presents common outcomes with common probabilities, the higher overall probability of gain invoked by most students increased the proportion to 87%.

For task IX which is a slight modification of task I (in *F* a maximum amount has been added with a probability of 2% and in *G* a minimum amount has been added also with a probability of 2%) the major reversal of preference from *G* for task I to *F* for task IX (37.5% to 92.5%) shows that $w \in W_{0.02}$ is a plausible class of *pwfs* and emphasizes the abrupt change of the *pwf* near the origin ($w \notin W_0$). It also suggests that decision makers use the range of outcomes for as a decision criterion.

For tasks X-XIII, the responses are overwhelmingly in favor of a higher *p* of gain and minimizing extreme loss. The *p*-values are significant and reject the null hypothesis that % *F* = 0.5 for tasks IV- XIII. The *p*-values for binomial for all tasks are in Table 20.

5.5 RESULTS RELATED TO LOSS AVERSION

The results pertaining to tasks XIV-XVI designed to test solely for loss aversion are summarized in Table 18 below which also compares them to the results obtained by Baucells and Heukamp (2006).

Table 18: Results for Tasks XIV-XVI in %

Task	**XIV**	**XV**	**XVI**
B.&H.	43	61	64
ASU	60	67.5	55

For task XIV, 60% of the students chose *F* to avoid a loss. The majority of the 40% who picked *G* said they were not attracted to the 80% probability of no gain in *F*. For this task the students at ASU show more loss aversion than Baucells and Heukamp's (2006) subjects (60% vs 43%) respectively. However, the latter's subjects were composed of students *and* professionals who for the same task have chosen differently: the students' percentage of *F* compared to the professionals' was (48% vs 35%) for task XIV. That is students show more loss aversion than professionals for losses in Baucells and Heukamp's (2006). To recall, a comparison of the results of Experiment I with those of Abdellaoui et al. (2006) has also shown that the median student is more loss averse than the median practitioner.

For task XV where the stakes are higher (maximum loss has increased) than in XIV the proportion of students who chose *F* increased (67.5%) pointing to an increase in loss aversion in mixed lotteries as the size of outcomes increases. Students also invoked the decrease in the probability of no gain to 60% going from XIV to XV. In Baucells and Heukamp (2006), both the proportions of students and professionals who chose *F* increased (65% vs 54% respectively).

For task XVI which examines loss aversion closer to the origin relative to task XV, less students (55 %) preferred *F* to *G* preferring the higher *p* for the higher gain in *G* for the same overall probabilities of gain/loss, the overall probability of loss having increased from XIV to XV to XVI.

Tasks XVII-XX present a global test for *CPT*. Except for task XVII, which shows a preference for the higher overall probability of gain, *F* is clearly preferred to *G* in line with the predictions of *CPT*. They are summarized in Table 19 below:

Table 19: Results for Tasks XVII-XX in %

Task	**XVII**	**XVIII**	**XIX**	**XX**
B. & H.	46	70	74	77
ASU	40	85	70	87.5

For task XVII as in Baucells and Heukamp, 60% of the students chose *G* for the higher probability of gain (70% in *G* vs 50% in *F*), or the higher probability for the higher gain *and* in their own words “$500 is not too much to lose”. For task XVIII and XIX 85%, 70% respectively of the students chose *F* which necessitates a convex utility function for losses. For task XX, 87.5% chose *F* which necessitates a concave utility function.

5.6 RESULTS RELATED TO THE OVERALL PROBABILITY OF GAIN/LOSS

For the last two tasks, 82% vs 67.5% respectively of the students confirmed that the overall probability of gain/loss is an important factor in deciding between two investments. Table 20 summarizes all the results for Experiment II and shows their p-values.

Table 20: Results for all Tasks and their P-Values for Binomial

Task	B & H %	ASU %	p-values
I	36	37.5	0.113846
II	42	40	0.205903
III	33	37.5	0.113846
IV	61	67.5	0.026857
V	84	85	9.55E-06
VI	66	87.5	2.10E-06
VII	76	85	9.55E-06
VIII	84	92.5	7.62E-08
IX	81	92.5	7.62E-08
X	79	85	9.55E-06
XI	74	87.5	2.10E-06
XII	75	72.5	0.004427
XIII	64	85	9.55E-06
XIV	43	60	0.205903
XV	61	67.5	0.026857
XVI	64	55	0.527089
XVII	46	40	0.205903
XVIII	70	85	9.55E-06
XIX	74	70	0.011412
XX	77	87.5	2.10E-06
XXI		85	9.55E-06
XXII		67.5	0.026857

CONCLUSION

It has been recognized since their introduction by Rothschild and Stiglitz (1970) that second order stochastic dominance criteria offer a framework to test different features of the theory under which they apply. To recall from section 3.4.1, if $F \succcurlyeq G$ according to second order stochastic dominance then necessarily that implies that the expected utility of *F* is greater than the expected utility of *G* for any agent with a non-decreasing *concave* utility function. Recently, Levy and Levy (2002) have designed and used stochastic dominance criteria that apply within *EU* or any reference dependent model that does not however assume probability weighting an important source of departure from expected utility maximization.

Baucells and Heukmap (2006) have thus designed stochastic dominance conditions that apply within *CPT*, the theory which, being developed as a descriptive alternative to *EU* has been quite successful at explaining its violations. When these conditions are used as a guide to design pairs of lotteries, the choices of the decision maker which reveal his preferences and their representation under *CPT* allow a non-parametrical test of the qualitative properties of *CPT*.

The result is that joint hypothesis on the curvature of the utility function and the probability weighting functions can be tested in three ways: 1) If one assumes the *CPT*'s empirical specifications for the probability weighting function hold, then the corresponding stochastic dominance conditions can be used to test hypotheses about the curvature of the utility function and/or about loss aversion; 2) if one assumes the *CPT*'s empirical specifications for the utility function hold, then a test of the hypothesis on the shape of the probability weighting function can be undertaken and 3) if one assumes all the *CPT*'s empirical specifications hold, then a violation of the corresponding stochastic dominance condition implies a violation of the *CPT* model.

Experiment II undertaken using the Baucells and Heukmap's lotteries tests these joint hypotheses on 40 MBA students at ASU. The results show the following:

1) assuming the *CPT*'s empirical specifications for the probability weighting function hold, the results of tasks XI, XIII and VII show that the hypothesis of $U \in U_P$ was not rejected i.e., is consistent with an *S* shape *U* while the hypotheses of $U \in U_M$ and $U \in U_{convex}$, $U \in U_M$ and $U \in U_{concave}$, and $U \in U_M$ were rejected respectively. Also loss aversion's results tested in tasks XIV, XIV and XVI show that the hypothesis of $U \in U_L$ was not rejected.

2) assuming the *CPT*'s empirical specifications for the utility function hold, the curvature of the probability weighting function was tested in Tasks I and IX, the result of which, the hypothesis that $w \in W_0$ was rejected but not the hypothesis that $w \in W_{0.02}$.

3) assuming all the *CPT*'s empirical specifications hold, consistency with *CPT* was tested in tasks XIV and XVII-XX. Except for task XVII where the hypothesis $U \in U_{PL}$ is rejected the results show consistency with *CPT*. The results of tasks XXI and XXII provide evidence however, that the overall probability of gain matter and task XVII is a case where students have reported that their choice for *G* (and hence the shift from loss aversion to gain seeking) was motivated by the higher overall probability of gain (70%) and/or the high probability of the maximal gain while the loss in not extreme.

In brief, the results of Experiment II as their (2006)'s reject the hypothesis that the utility function is convex for gains and concave for losses and are consistent with an *S*-shape utility function. In addition, they show consistency with loss aversion provided the probabilities are similar or the same and last but not least they confirm the importance of accounting for the probability weighing function.

GENERAL CONCLUSION

Baucells and Heukamp (2006) have generalized and extended second order stochastic dominance conditions for expected utility to *prospect theory*. A new method has been added thus for the experimentalists who are interested in falsifying a particular hypothesis about the shape of the utility function or about the probability weighing function without having to elicit these functions.

For those experimentalists interested however in exploring the entire utility function under *prospect theory* Abdellaoui et al. (2006) offer a complete parameter-free elicitation procedure.

The availability of the two methods motivated this experimental work which aims at inferring individuals' preferences from the choices presented to them and which consists of two experiments. Experiment I applies Abdellaoui et al's (2006) method in the field to elicit completely and non-parametrically the utility functions of financial practitioners and to measure their individual loss aversion degrees without however committing to any particular definition of loss aversion. Experiment II infers the preferences of MBA students using Baucells and Heukamp's method (2006).

Generally, the standard preference function presupposed in elicitation procedures results from expected utility the normative model of choice which describes how rational agents ought to choose. Nevertheless, for a normative model to be operational and prescriptively useful the actual behavior of an individual in simple choice settings must be compatible with the behavior assumed in the model. Expected utility has been found to be violated systematically in experimental works to the extent it is not defensible as a descriptive model of actual behavior as was shown in chapter II. For the selection of a theory which might show, when tested experimentally a good approximation of the underlying preference function the individual is assumed to optimize, Part I has reviewed the models presupposing a single preference function, an important tenet of coherence, yet defensible as a descriptive models of actual behavior. It was shown that prospect theory explains the anomalies and the basic phenomena *EU* is used to explain by integrating psychological insights into economics. These include the reflection effect, loss aversion and the subjective treatment of probabilities (the section "support from experimental findings" in the general introduction relates the evidence at a glance). In brief, prospect theory has been successful at organizing empirical departures from *EU* maximization due to a combination of empirical realism and theoretical advantages.

The availability of a method to elicit preferences under prospect theory is thus important for practical interests. Prospect theory has refined the understanding of risk aversion to a large extent. Under *CPT* risk attitude three components that are affected jointly by a gamble: a reference dependent utility function, loss aversion and a probability weighting function challenging as a consequence the role of utility in representing solely risk attitude. The notions of risk aversion defined behaviorally i.e. defined independently of any model are not equivalent anymore to the concavity of the utility function and the elicited shape of the latter implies different risk attitudes depending on which model assumed. The equivalence, for instance, of the convexity of the utility function to risk seeking under *EU* does not hold under prospect theory because of the non-linearity of the probability function in the latter[105].

Moreover, a *parameter-free* method that does not depend on the appropriateness of the selected function has several advantages: 1) it could validate reasonable functional forms and thus the use of certain types of parametric estimation procedures. These have the advantage of smoothing response errors while relatively good estimates can be obtained with a smaller set of lotteries; 2) non-parametric measurements give insights into the psychological reasoning that underlies the data because elicited utilities can be directly traced back to observed choices; and 3) they give empirical meaning to the concepts

[105] For example, if a subject indicates that he is indifferent between a sure loss of $40 and the two-outcome prospect $(-\$100, \frac{1}{2}; \$0)$, then equation (4.1) reveals that this risk seeking preference is consistent with a concave utility for money if $w^{-}(\frac{1}{2})<0.4$.

underlying the decision theory and hence are particularly useful to prescriptive decision analysis.

Experiment I which elicits the utility function of financial practitioners under prospect theory following Abdellaoui et al.'s (2006) corroborate their findings that the most common pattern is concavity for gains and convexity for losses. A comparison however of the individual loss aversion degrees with Abdellaoui et al.'s (2006) results for students shows that the median practitioner is less loss averse than the median student.

The results of Experiment II which infers the preferences of MBA students using Baucells and Heukamp (2006) turn out to be consistent with an *S*-shape utility function and with loss aversion provided the probabilities are similar or the same. Hence, the preferences of the students are consistent across methods. A noteworthy result of Experiment II is that the majority of the students (60%) who have shifted from being loss aversion to being not loss averse in task XVII invoked either the higher overall probability of gain or higher probability of maximal gain combined with a limited extreme loss. The result hints to more than one mechanism of risk attitude being affected *jointly* by a gamble and that the behavior loss averse/not loss averse depends on which mechanism[106](s) is or are triggered.

Professionals who are exposed to a range of training and high level of knowledge and are offered a powerful incentive son a yearly basis, differ in their assessment of the stakes from the students. Also, the interviews were conducted during the period (2003-2004) which corresponds to a growth (the Standard and Poor index was up by 28.17% and 10.9% respectively relative to the preceding year). It is not inconceivable thus that the degree of loss aversion of the practitioners diminished during that period with the upward movement of the stock market (Barberis, Huang and Santos, 2001).

[106] Mechanisms are frequently occurring and easily recognizable causal patterns that are triggered under generally unknown conditions or with indeterminate consequence. Type (B) obtains when two causal chains are triggered leaving the net effect indeterminate, type (A) obtains when one cannot predict which of the causal chains will be triggered (Elster, J., 1999)

Reflecting on what could induce risk seeking in the domain of losses, some aspects of the practitioners' work discussed during the interviews come to mind. In order to realize losses as dictated by rationality, practitioners need to give up the hope they might "get even" before they get out. This is an occupational aspect that MBA curriculums may find it advantageous to emphasize and to offer some *hard rules* against. The explicit formulation of decision problems in terms of final assets to eliminate risk seeking is precisely the *one* advice Daniel Kahneman chose to give to financial practitioners on CNBC the American channel for financial news upon winning the Nobel Prize.[107]

Finally, what emerged from the discussions with practitioners is that they find it difficult to admit having been wrong to their peers and clients. Being evaluated according to their performance in a very competitive environment which depends on astuteness in judgment, it seems to be a "hard pill to swallow" (Kleinfield, 1983). According to them, the soundness of a judgment looms very large in the real that is uncertain environment they work in, because it is the starting point of what they call the investment chain process. Indeed, a major concern of money managers is the prediction of the direction of macro events (trade deficits, inflation/deflation…)[108] with which begins the investment process, i.e. their judgments of the probability of occurrence of uncertain events. Ultimately, it seems this is what distinguishes the performance of one practitioner from another independently of the market's ups and downs. Among the portfolio managers interviewed some were able to predict correctly the probability of the 2001 market crash and "went short" sustaining an above average performance during that period.[109] Accordingly, *CPT* seems to describe a natural thought-pattern in a general population in which the different components of risk attitude are quite mixed and addresses the

[107] Amos Tversky who has died was ineligible for the prize because the Nobel is not awarded posthumously.

[108] The investment process as a chain was described as follows by one hedge fund manager: Macro→ Sector→ Company→ Sustainability →Alignment. The analysts are responsible for the sector and the company while the manager covers the macro, the sustainability of the fund and the alignment of its objectives with the clients'.

[109] Under risk with probability weighting filtered, these few have in common in addition to their sound judgment, a convex utility for gains and an almost linear utility for losses.

practitioner's concerns in a way the normative *EU* does not. Yet theories that describe the behavior of individuals exhibiting extreme predominance of one component such as *EU* and/or *DT* are useful because any extreme position is more uncompromisingly clear and therefore more easily recognized and understood than the intermediate positions which do not in any way contain or reconcile the extreme positions (Huxley, 1945). In that sense, a theory describing individuals who exhibit extreme predominance of loss aversion may be helpful.

REMARKS AND DIRECTIONS FOR FUTURE RESEARCH

Combined, the results of Experiment I and those of Experiment II point to the importance of *reference dependence* as part of economic modeling as advocated by Rabin (1996) among others.

The convexity of the utility function in the domain of losses is an effect of a reference-dependence. More precisely, it is a psychological framing effect that results from shifting reference points. What is required then of the practitioner who refuses to take losses perceived as such relative to his reference point is to bring about a stronger and opposite affect *a la Spinoza*[110] by explicitly formulating his decision problem in terms of final wealth. The required task is demanding and points, pending the results of further research on reference-indexed preferences, to the benefits of using corrective models[111] in decision analysis to help the practitioners make better decisions for their own interest as well as the interest of their clients.

Loss aversion is another effect of reference-dependence. Shalev (2000) describes the existence of this phenomenon as the most striking result of reference-dependent utility functions and extends the analysis of games which models interactive choice such as the behavior in markets to include both reference dependence and loss aversion. The first step however to test his prediction that different degrees of loss aversion would lead to different equilibrium strategies in game theory is to measure loss aversion at the individual level.

[110] The 16th century philosopher has, according to Damasio (2003) intuited in *Ethics* the new findings in neuro-science hundreds of years ago: Feelings are foundational components of the mind and a negative affect cannot be neutralized except by a positive and stronger affect brought about by reasoning and intellectual effort.

[111] As argued by Bleichrodt, Pinto and Wakker's (2001).

The task so far has been problematic. Kahneman et al.'s (1991) estimates of the observed disparities between the willingness to pay (WTP) and the willingness to accept (WTA), show the median and mean WTA values to be between 1.4 and 16.5 times as large as the corresponding WTP values. The comparison however may have been affected by factors such as substitution and income effects. The problem encountered in Experiment II is the absence of an agreed-upon definition of loss aversion which emphasizes the need of a precise meaning for the latter.

Perhaps, to be *more uncompromisingly clear and therefore more easily recognized*, loss aversion which is formalized in prospect theory needs to be disentangled from the two other components of risk aversion in the theory, namely the curvature of utility for gains and losses and probability weighting. The separation also essential for research on varying reference points[112] is however difficult theoretically and empirically.

Empirically, the interpretation of utility as independent of other factors and prior to risk is so far controversial (a summary of the debates is given in Abdellaoui, Barrios and Wakker, 2003) and there is no independent empirical implication yet for probability weighting although attempts have been made at establishing a psychological rationale for the probability weighting function (Weber, 1994; Gonzalez and Wu, 1999; Wakker, 2003).

Theoretically, Schmidt and Zank (2005) argue that the equivalence of the behavior to a utility steeper for losses than for gains under the original prospect theory, does not hold under *CPT* unless the two weighting functions for gains and losses are convex and

[112]Bleichrodt, Pinto and Wakker (2001) observe the loss aversion index by comparing the kink of the utility function at a point when it is the reference point with the kink at the same point when another point is the reference point. They do so however assuming basic utility is the same for different reference points although in a reference dependent model, the utility is determined not only by the outcome but also by the relationship of the outcome to the reference point.

coincide. [113] Their characterization (2001) of risk aversion in *CPT* through a *joint* condition on utility curvature, probability weighting and loss aversion supports however the Köbberling and Wakker's (2005) index which arises naturally in their framework.

Köbberling and Wakker (2005) consider loss aversion as a *logically independent* component of risk attitude. Nevertheless, their work is based on the assumption that the utility function is smooth at the zero point allowing the different processing of gains and losses to be captured by the kink.

The axiomatic foundation of Köbberling and Wakker's (2005) degree of loss aversion without these restrictions awaits perhaps the observation of certain features that have yet to reveal themselves. In response to what a friend of Einstein has told him once in a jest:[114]

"*The mathematician can do a lot of things, but never what you want him to do just at the moment*."

Einstein explained that the theorist's work falls in two tasks of entirely different nature. He must first discover certain features and then use his skills to draw the conclusions which follow from them.[115] Amos Tversky and Daniel Kahneman have showed that these features which are not allowed to destroy each other may dwell in more than one

[113] Schmidt and Zank (2001) argue that for the definition of loss aversion expressed in terms of the properties underlying utility function to be useful in decision analysis *ad instar* the Rothschild and Stiglitz' (1970) definition of risk aversion, it must have also the same behavioral implications in different theories.

[114] Einstein quoted his friend in his Inaugural Address to the Russian Academy of Sciences in 1914 (Barnes and Noble Books, 1934).

[115] A case in point is that before Quiggin (1982, p: 328) observed that "while individuals may distort the probability of an extreme outcome in some way, they need not treat intermediate outcomes with the same probability in the same fashion" there was nothing he could do to formalize the observation.

discipline[116] and Daniel Kahneman has won for that insight the Nobel prize an award that honors the most prize-worthy discovery in a year's nominations.

[116]Camerer, Lowenstein and Prelec (2003) point in the direction of neuroscience as potential candidate, if not directly then indirectly, through its impact on psychology.

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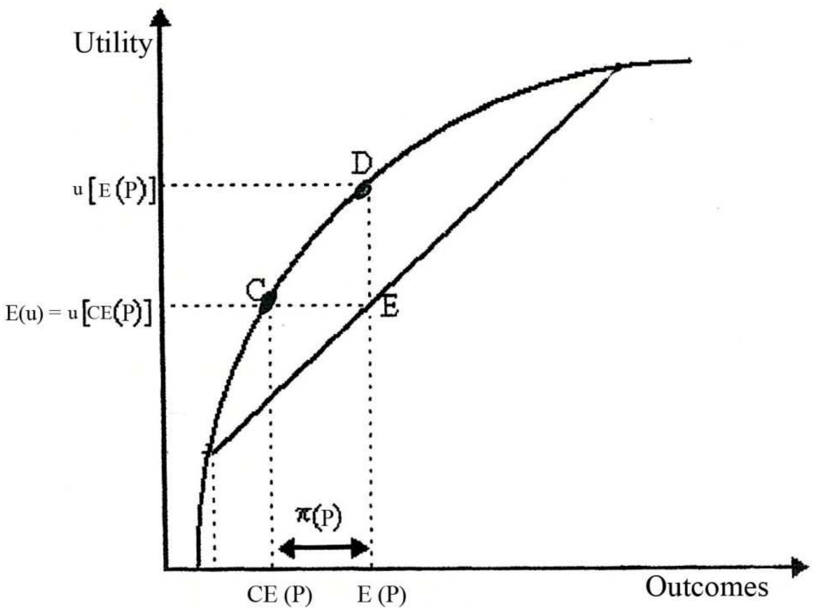
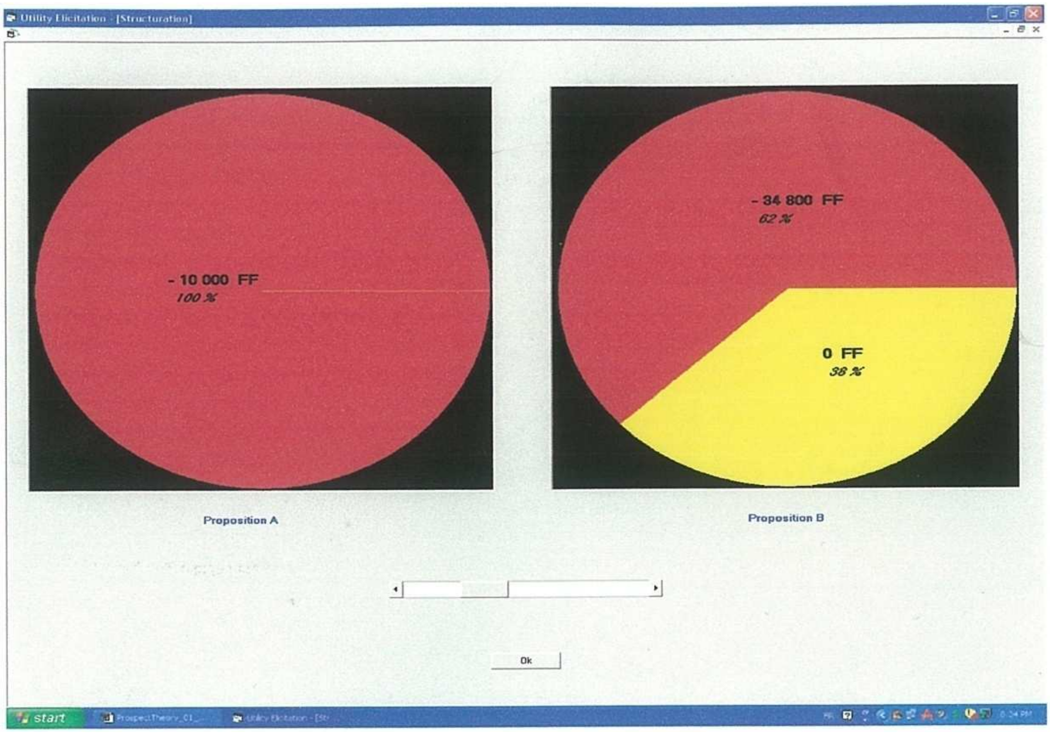
APPENDIX A: DATA FIGURES

Figure 8 : The Certainty Equivalent for a Concave Utility Function



APPENDIX B: DISPLAY

Display II: Illustration of the Use of the Scrollbar



APPENDIX C: DATA TABLES

Table 5: Classification of Alternative Theories

Models	**Indifference Curves**	**Probabilities**
vNM (1944)	Parallel, and Linear	Objective
Machina (1982)	Smooth, not necessarily Linear, and Fan-out	Objective
Chew and MacCrimmon (1979)	Linear, not Parallel, and Fan-out	Objective
Gul-Neilson (1992)	Linear, Fan-in, and Fan-out	Objective
Chew,Segal and Epstein (1991)	Not Linear, Mixed Fanning, May switch from concave to convex	Objective
Quiggin (1982)	Concave, Fan-out, and Parallel at Hypotenuse	Decision Weights
Kahneman and Tversky (1992)	Not-Linear and Mixed Fanning	Decision Weights

Table 8: The Practitioners' Institutions and their Locations

Financial Institution	Location
Martingale Asset Manag.	Boston- US
GMT Capital Corp.	Atlanta-US
GMT Capital Corp.	Atlanta-US
Boyd Watterson	Cleveland-US
Boyd Watterson	Cleveland-US
Lorain National Bank	Cleveland-US
Lorain National Bank	Cleveland-US
Smith Barney	Cleveland-US
Smith Barney	Cleveland-US
Smith Barney	Cleveland-US
Smith Barney	Cleveland-US
Smith Barney	Cleveland -US
UBS PaineWebber	Cleveland-US
Butler Wick &Co., Inc	Cleveland-US
Smith Barney New York	NewYork-US
Smith Barney New York	NewYork-US
Smith Barney New York	NewYork-US
Smith Barney New York	NewYork-US
Corey Capital Inc.	Phoenix-US
Corey Capital Inc.	Phoenix- US
Corey Capital Inc.	Phoenix- US

Wells Fargo	Phoenix-US
Wells Fargo	Phoenix- US
Coble Pension &Wealth Ma.	Phoenix-US
First National Bank	Beirut-Lebanon
Saradar Bank	Beirut-Lebanon
Allied Bank	Beirut-Lebanon
Byblos Bank	Beirut-Lebanon
Comgest	Beirut-Lebanon
Financial Funds Advisors	Beirut-Lebanon
Audi Bank	Beirut-Lebanon
Capital Trust	Beirut-Lebanon
Middle East Capital Group	Beirut-Lebanon
Audi Bank	Beirut-Lebanon
Societe Generale - Fidus	Beirut-Lebanon
Capital Trust	Beirut-Lebanon
Intercontinental Bank	Beirut-Lebanon
Meryll Lynch	Beirut-Lebanon
Credit Agricole Indo- Suez	Beirut-Lebanon
Meryll Lynch	Beirut-Lebanon
Credit Agricole Indo- Suez	Beirut-Lebanon
Audi Bank	Beirut-Lebanon
Saoudi Lebanese Bank	Beirut-Lebanon
El-Rashed Investment	Beirut-Lebanon
Lebanese Canadian Bank	Beirut-Lebanon
Banque du Liban et D'outre mer	Beirut-Lebanon

Table 10: Exponential and Power Risk Aversion Coefficients

	Utility	**Risk Aversion**	**Risk Tolerance**	**Relative Risk Aversion**
Function	u(x)	-u''(x)/u'(x)	-u'(x)/u''(x)	-x u''(x)/u'(x)
Exponential	$u(x) = -e^{-rx}$	r (CARA)	1/r	r x
Power	$u(x) = 1/c\,(x)^c$ $c < 1, c \neq 0$	(1-c)/(x)	(x)/(1-c)	1-c (CRRA)

Table 16: Main Parameters for Practitioners

Practitioner	Probabilities		Power		Exponential		Expo-Power	
Number	P1	P2	Alpha	Beta	Alpha	Beta	Alpha	Beta
2	61	41	2.380998	0.441945	-2.51716	3.554083	2.7409045	0.663343
3	23	87	0.432224	1.882478	2.265403	-1.8518	0.6711281	2.152872
4	71	51	1.215347	0.882535	-0.68419	0.753242	1.4867638	1.298799
5	63	61	1.105859	0.559464	-0.27301	2.885824	1.3764401	0.764835
6	93	35	1.486456	0.624727	-1.25701	1.412348	1.795828	0.896381
7	88	71	1.011	0.722583	0.048198	1.351406	1.2730101	0.949645
8	58	51	1.471043	0.930471	-1.19596	0.242682	1.7700915	1.214302
9	83	37	0.874757	0.773768	0.451493	1.026298	1.1323114	1.000048
10	54	57	0.809379	1.084596	0.583785	-0.27368	1.0706879	1.331851
11	28	57		0.670522	8.715305	1.610599		0.901659
12	24	6	0.550002	0.382677	2.343562	3.95954	0.7857909	0.617033
13	87	41	0.696171	0.940786	1.175854	0.150988	0.9441502	1.236092
14	56	55	1.189312	0.58929	-0.52336	1.818705	1.4732133	0.827597
15	24	70	0.640664	1.447426	0.928459	-1.1093	0.9214748	1.742205
16	43	52	0.730674	1.709349	1.065489	-1.65546	0.9792663	1.99568
17	71	39	1.054633	0.682597	-0.14255	1.348408	1.3241301	0.9244
18	75	14	0.539749	0.705178	2.592722	1.213546	0.7724195	0.961374
19	86	8	1.032571	1.086253	-5.52004	-0.16608	1.2385047	1.363351
20	37	55	0.757344	0.680393	0.970483	1.432612	1.0105909	0.913767
21	29	35	0.504055	0.943881	2.805818	0.223523	0.7352558	1.204265
22	36	77	0.531561	1.827176	1.925866	-1.89741	0.7851091	2.149494
23	66	34	0.58038	0.317038	2.100867	5.43035	0.8193522	0.541587
24	22	84	0.43282	1.717576	3.691506	-1.47282	0.6565923	2.02411
25	25	78	0.368992	6.061721	4.357306	-6.39013	0.5849558	6.451708
26	65	85	0.762166	1.675184	0.894451	-1.54494	1.0180301	1.959153
27	32	34	0.531043	1.114002	2.315585	-0.33631	0.7664623	1.401521
28	26	87	0.841566	0.37596	0.421047	3.663079	1.1246377	0.622335
29	21	48		1.48548	9.150792	-1.16162		1.757882
30	69	63	1.63816	1.082252	-1.49885	-0.20868	1.9244205	1.362707
31	19	34	0.553635	0.529052	2.310261	2.158697	0.7885674	0.77949
32	31	56	0.61065	0.80902	1.867491	0.702876	0.8464555	1.073669
33	71	45	1.876947	0.759428	-1.8342	0.967491	2.1957023	1.01301
34	40	30	0.843514	0.294514	0.509515	5.44487	1.109064	0.517718
35	80	70	0.497438	1.677583	3.03907	-1.62348	0.72059	1.948343
36	89	96	0.790695	2.118735	0.899013	-2.13339	1.0341132	2.443526
37	51	42	0.912529	0.903647	0.309114	0.347053	1.1774802	1.164783
38	98	20		0.425302	5.383797	4.393571		0.639634
39	68	31	0.57544	1.556574	1.99136	-1.27005	0.8175628	1.865881
40	79	32	0.58038	1.725289	2.100867	-1.54789	0.8193522	2.023833
41	78	34	0.996992		-0.086	5.657763	1.254488	0.562262
42	56	44	1.042085	0.578304	-0.21605	1.902523	1.316069	0.820593

43	79	66	0.948722	0.7453	0.090909	1.231125	1.229641	0.968369
44	80	35	0.608574	2.119463	1.697247	-2.12598	0.8566023	2.44121
45	86	57	0.579813	0.848784	2.07892	0.499637	0.814938	1.110919
46	73	28	0.577194	0.900487	2.145681	0.323158	0.8145957	1.171631
47	77	42	0.622703	1.010525	1.768889		0.8596306	1.281592

APPENDIX D: The Questionnaire

The following hypothetical choices are designed to investigate your attitude towards risk. Try to be as accurate as possible in predicting your choices. The responses are anonymous and there is no correct answer, hence, no reason *not* to state your true preference. After each choice you make, please state why you chose it in the blank box below.

A- Suppose that you decided to invest $ 10000 either in stock F or in stock G. Which stock would you choose, F, or G when it's given that the dollar gain or loss one month from now will be as follows.

Task I:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-6000	1/4	-3000	1/2
3000	3/4	4500	1/2

Please write F or G :
Please state the reason for your choice:

Task II:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-1600	1/4	-1000	1/4
-200	1/4	-800	1/4
1200	1/4	800	1/4
1600	1/4	2000	1/4

Please write F or G :
Please state the reason for your choice:

Task III:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-3000	1/4	-1500	1/2
3000	3/4	4500	1/2

Please write F or G :
Please state the reason for your choice:

Task IV:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-5000	1/6	-5000	1/6
-3000	1/6	-2000	1/6
-500	1/6	-1500	1/6
2000	1/6	1000	1/6
3000	1/6	4000	1/6
5000	1/6	5000	1/6

Please write F or G :
Please state the reason for your choice:

Task V:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-3000	30%	-3000	10%
3000	60%	-1500	40%
4500	10%	4500	50%

Please write F or G :
Please state the reason for your choice:

Task VI:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-6000	10%	-6000	10%
-3000	20%	-1500	40%
3000	60%	4500	40%
6000	10%	6000	10%

Please write F or G :
Please state the reason for your choice:

Task VII:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-6000	1/3	-6000	1/6
3000	1/2	-3000	1/3
4500	1/6	4500	1/2

Please write F or G :
Please state the reason for your choice:

Task VIII:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-6000	30%	-6000	10%
3000	60%	-3000	40%
4500	10%	4500	50%

Please write F or G :
Please state the reason for your choice:

Task IX:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-6000	26%	-6000	2%
3000	72%	-3000	48%
4500	2%	4500	50%

Please write F or G :
Please state the reason for your choice:

Task X:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
1000	1/2	0	1/2
2000	1/2	3000	1/2

Please write F or G :
Please state the reason for your choice:

Task XI:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
0	10%	0	50%
1000	40%		
2000	40%		
3000	10%	3000	50%

Please write F or G :
Please state the reason for your choice:

Task XII:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-3000	1/2	-2000	1/2
0	1/2	-1000	1/2

Please write F or G :
Please state the reason for your choice:

Task XIII:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-3000	50%	-3000	10%
		-2000	40%
		-1000	40%
0	50%	0	10%

Please write F or G :
Please state the reason for your choice:

Task XIV:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-1000	10%	-1000	50%
0	80%		
1000	10%	1000	50%

Please write F or G :
Please state the reason for your choice:

Task XV:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-3000	20%	-3000	50%
0	60%		
3000	20%	3000	50%

Please write F or G :
Please state the reason for your choice:

Task XVI:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-3000	20%	-3000	50%
-1000	30%		
1000	30%		
3000	20%	3000	50%

Please write F or G :
Please state the reason for your choice:

Task XVII:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-500	10%	-500	30%
0	40%	500	20%
1500	40%	1000	20%
2000	10%	2000	30%

Please write F or G :
Please state the reason for your choice:

Task XVIII:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-2000	30%	-2000	10%
		-1000	60%
0	60%		
1000	10%	1000	30%

Please write F or G :
Please state the reason for your choice:

Task XIX:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-5000	15%	-5000	35%
-3000	30%	-1000	30%
0	20%		
3000	20%		
5000	15%	5000	35%

Please write F or G :
Please state the reason for your choice:

Task XX:

F		G	
Gain/Loss	Probability	Gain/Loss	Probability
-1500	20%	-1500	50%
1500	60%		
4500	20%	4500	50%

Please write F or G :
Please state the reason for your choice:

B-Consider now an investment whose possible outcomes and their probabilities are the following:

Gain/Loss	Probability
100	0.2
50	0.2
0	0.2
-25	0.2
-50	0.2

1) If you could add a sum of money ($38) to either the outcome that paid $100 or the outcome that paid $ 0, which outcome would you choose?

Please write your answer:
Please state the reason for your choice:

2) If you could add a sum of money ($38) to either the outcome that paid $100 or the outcome that paid $ 50, which outcome would you choose?
Please write your answer:
Please state the reason for your choice:

LIST OF FIGURES

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