Essais en Macroéconomie Financière

Imen Ghattassi

Université des Sciences Sociales de Toulouse
Midi Pyrénées Sciences Sociales (MPSE)

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Imen Ghattassi

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Patrick Fève

MEMBRES DU JURY

RAPPORTEURS	René Garcia	Professeur, Université de Montréal
	Christian Gouriéroux	Professeur, CREST & University of Toronto
SUFFRAGANTS	Fabrice Collard	Directeur de Recherches, CNRS
	Patrick Fève	Professeur, Université Toulouse I
	Christian Gollier	Professeur, Université Toulouse I
	Nour Meddahi	Professeur, Imperial College of London

Résumé

L'objectif de cette thèse est double. Tout d'abord, la thèse propose un cadre théorique permettant d'évaluer à la fois le rôle de la persistance des habitudes et des nouvelles financières et celui de la dynamique des variables macroéconomiques afin d'expliquer les variations du ratio cours des actions-dividende et de la structure à terme des taux d'intérêt. Les modèles envisagés sont ainsi des extensions du modèle d'évaluation des actifs financiers basé sur la consommation ((C)CAPM). Ensuite, la thèse contribue à l'analyse empirique de ces modèles. L'objectif est ici de tester le pouvoir de prédiction (i) du ratio surplus de consommation et (ii) du ratio cours des actions-dividende. Cette thèse comporte trois chapitres. Le premier chapitre montre que dans le cadre d'un modèle (C)CAPM avec formation des habitudes, la persistance du stock d'habitudes permet de reproduire la persistance du ratio cours des actions-dividende et en partie la prédictabilité des excès de rendements des actifs risqués à long terme. Dans le deuxième chapitre, nous mettons en évidence, théoriquement et empiriquement, la capacité du ratio surplus de consommation à expliquer les variations des rendements d'actifs risqués, à la fois en séries chronologiques et en coupe transversale. Enfin, dans le dernier chapitre de cette thèse, nous considérons un modèle (C)CAPM avec (i) une modélisation générale du niveau de consommation de référence et (ii) une spécification affine des variables exogènes. La représentation affine du taux d'escompte stochastique et des processus exogènes permet de déterminer les solutions analytiques exactes du ratio cours des actions-dividende, du ratio cours du portefeuille de marché-consommation et de la structure à terme des taux d'intérêt.

Mots clés : Modèle d'évaluation des actifs financiers basé sur la consommation, Formation des habitudes, Nouvelles sur le taux de croissance des dividendes, Persistance, Prédictabilié, Solutions analytiques, Processus affines, Ratio cours des actions–dividende, Ratio surplus de consommation, Structure à terme des taux d'intérêt.

Abstract

This thesis analyses different extensions of the Consumption–based Asset Pricing Models (C)CAPM, in which the utility function of the representative agent depends on the past observations. This allows for taking into account the effect of habit formation as well as the impact of the news on dividends. We have two main objectives. The first one is theoretical : to build a "macroeconomic" asset pricing model accounting for some financial stylized facts. The main contribution is to provide a flexible modeling tool to evaluate the role of the preferences and the implications of the joint dynamics of macroeconomic variables in affecting the stock market and the term structure of interest rates. The second empirical objective consists in testing the predictive power of (i) the surplus consumption ratio and (ii) the price-dividend ratio. On this applied part, we propose a Monte Carlo experiment to correct the standard OLS (ordinary least squares) test procedure from its bias in finite sample. In the first chapter, we develop a (C)CAPM model with habit formation when the growth rate of endowments follows a first order Gaussian auto–regression. The habit stock model is found to possess internal propagation mechanisms that increase the persistence of the price–dividend ratio. In the second chapter, we show from a (C)CAPM model with habit formation, that the surplus consumption ratio is a linear predictor of stock returns at long horizons and should explain the cross section of expected returns. This theoretical finding receives support from the U.S data. Finally, in the third chapter, we investigate the asset pricing implications of a consumption–based asset pricing model with a reference level in which exogenous macroeconomic variables follow an first order Compound Autoregressive CaR(1) process (or affine process). The reference level depends on past aggregate consumption and news on dividends. The affine (log) stochastic discount factor and the affine specification for the exogenous variables have the advantage of providing closed form solutions for the price–dividend ratio, the price–consumption ratio and the bond prices.

Key words : Consumption–based asset pricing models, Habit formation, News on dividends, Persistence, Predictability, Analytic solutions, Affine models, Price–dividend ratio, Surplus consumption ratio, Term structure of interest rates.

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Introduction générale

Durant les trente dernières années, une large littérature, empirique et théorique, a été dédiée à l'étude des liens entre les marchés boursiers et les fluctuations économiques. Cette littérature a été motivée par une multitude de faits stylisés. Nous en citons en particulier deux faits mettant en évidence l'intéraction entre la réalité économique et les mouvements des prix des actifs financiers.

Le premier fait traite de la double fonction du taux d'intérêt à court terme. D'une part, suite au modèle initial de la structure à terme de Vasicek [1977], le taux d'intérêt joue le rôle de déterminant fondamental des prix de divers actifs financiers. D'autre part, il joue le rôle d'instrument de politique monétaire que la banque centrale fixe pour contrôler la stabilité de l'économie et assurer la croissance.

Aux Etats-Unis, en baissant le taux d'intérêt peu à peu de plus de 6% en 2001 jusqu'à un minimum de 1% en 2003, la FED (Federal Reserve System) a mené une politique expansionniste, destinée théoriquement à lutter contre les risques de ralentissement de l'économie américaine, avant de repartir en sens inverse pour lutter contre l'inflation depuis juin 2004.

Quant à l'année 2006, elle a été marquée par une hausse des taux d'intérêt par les banques centrales dans l'objectif de juguler l'inflation liée à l'augmentation du prix du pétrole. Cette mesure demeure selon[1] M. Trichet, gouverneur de la banque centrale européenne, un sujet d'inquiétude malgré une récente accalmie des cours.

Le deuxième constat concerne le fait que les fluctuations des marchés boursiers ont tendance à varier avec les cycles réels et à être sensibles à la conjoncture économique. Nous citons un extrait de l'article "La courbe des taux s'aplatit dans la zone euro" paru dans des Echos le 12 Novembre 2006. Cet extrait met en évidence l'intéraction entre les marchés boursiers et les cycles réels :

"La courbe des taux, qui est jugée normale lorsque les rendements des taux longs sont plus élevés que ceux des taux courts s'aplatit sur le vieux continent. Les rendements des obligations à 10 ans s'établissent au même niveau que ceux des obligations à 2 ou 3 ans depuis quelques semaines. Les opérateurs anticipent-ils une ralentissement économique dans la zone euro ? De telles anticipations dans les pays de l'OCDE avaient mené à une inversion de la courbe des taux peu avant la récession de 1991..."

De plus, l'exemple du *lundi noir* met en évidence la tendance des marchés boursiers à fluctuer d'une manière contre–cyclique. Pendant plusieurs jours en Octobre 1987, les marchés boursiers de par le monde ont vu leur valeur diminuer de façon importante, en particulier le lundi 19 Octobre 1987. Toutefois, les principales économies de la planète semblaient prospères en 1987 et une reprise avait suivi la récession de 1981–1982 pendant cinq années consécutives.

Les exemples cités ci–dessus témoignent de l'importance de l'interaction entre les mouvements des marchés financiers et les fluctuations économiques. Cependant, les théories traditionnelles de la finance déterminent les rendements des divers actifs financiers à partir (i) des rendements de portefeuille de marché pour les actifs

[1]Les Echos du 20 Novembre 2006.

risqués tel que le modèle CAPM (Capital Asset Pricing Model) de Sharpe [1964], (*ii*) le taux d'intérêt dans le modèle de structure à terme proposé par Vasicek [1977] ou plus récemment (*iii*) des facteurs observables constitués de taux et de spreads ou des facteurs latents non observables dans les modèles d'absence d'opportunité d'arbitrage (voir Duffie et Kan [1996], Montfort, Gouriéroux et Polimenis [2002], Piazzesi [2003], etc). Certes, ces derniers arrivent à reproduire la dynamique de divers actifs financiers : zeros-coupons, obligations, actions, options sur actions, dérivés de taux, etc. Toutefois, ils sont incapables de répondre aux interrogations suivantes :

1. Quels rôles jouent les variables macroéconomiques dans la détermination et la prédiction des prix des actifs financiers ?

2. Quels risques macro-économiques gouvernent les fluctuations des marchés financiers ?

3. Quelles sont les mécanismes économiques en œuvre ?

Cette thèse s'intéresse aux modèles d'évaluation des actifs financiers basés sur la consommation (C)CAPM. Plus précisément, elle a pour ambition d'étudier le rôle de la formation des habitudes dans la détermination des prix d'actifs financiers.

Dans ce qui suit, nous présenterons d'abord le modèle (C)CAPM standard proposé par Lucas [1978]. Nous exposerons par la suite la notion de formation des habitudes. Le reste de l'introduction générale sera consacré à introduire les trois chapitres de cette thèse.

Nous commençons dans un premier temps par présenter le modèle de Lucas [1978] : ses hypothèses, ses résultats et ses critiques. Le modèle (C)CAPM standard considère une économie d'échange où l'agent représentatif determine son plan de consommation, et par conséquent sa demande d'actifs financiers, en maximisant son utilité inter–temporelle. Lucas [1978] suppose que l'utilité de l'agent représentatif à chaque période ne dépend que de sa consommation courante. De plus, les marchés sont complets et les processus de dotation (consommation et dividende) sont exogènes. Le résultat principal du modèle (C)CAPM standard est le suivant : la prime de risque d'un actif financier est mesurée par le produit de (i) la quantité de risque mesurée par la corrélation conditionnelle de son rendement avec le taux de croissance de la consommation et (ii) le prix unitaire du risque mesuré par le coefficient d'aversion au risque.

L'évaluation quantitative du modèle de Lucas [1978] a révélé une multitude d'énigmes empiriques, en particulier l'énigme de la prime de risque (voir Mehra et Prescott [1985]) et l'énigme du taux sans risque (voir Weil [1989]). Étant donné la faible volatilité du taux de croissance de la consommation, Mehra et Prescott [1985] ont montré que pour reproduire la prime de risque annuelle observée dans les données américaines d'environ 7%, il faut un coefficient d'aversion au risque très élevé. De plus, Weil [1989] a montré que pour répliquer le faible taux d'intérêt sans risque avec une moyenne annuelle de 1.2%, un coefficient d'aversion au risque élevé nécessite un taux de préférence temporelle très élevé et supérieur à 1. Par conséquent, on est face à une contradiction. D'une part, les consommateurs sont très averse au risque et ont tendance à lisser leur consommation d'une manière inter–temporelle. D'autre part, ils ont une préférence au présent très élevée qui les incite à consommer aujourd'hui plutôt que demain.

A la suite des travaux de Mehra et Prescott [1985] et Weil [1989], de nombreux auteurs ont cherché à enrichir le modèle (C)CAPM standard proposé par Lucas [1978] et à en remettre en cause les hypothèses. Sans entrer dans les détails et sans énumérer toutes les modifications apportées au modèle (C)CAPM de base tant la littérature sur le sujet est vaste (voir Kocherlakota [1996], Cochrane [2001, 2005, 2006] et Campbell [2003] pour une revue de cette littérature), nous nous intéressons dans cette thèse aux modifications apportées au comportement de l'agent représentatif et en particulier à l'introduction d'une fonction d'utilité non séparable dans le temps.

Comment modéliser l'hypothèse de la non séparabilité temporelle des préférences ?

Nous citons en particulier deux approches : (i) la fonction d'utilité récursive initialement proposée par Epstein et Zin [1989] et Weil [1989] et (ii) la formation des habitudes, suite aux travaux théoriques de Ryder et Heal [1973], Sundaresan [1989] et Constantidines [1990].

L'approche récursive consiste à supposer que l'utilité du consommateur dépend à la fois de sa consommation courante et de son utilité future espérée, d'où l'appellation de *forward–looking models*. L'utilité récursive permet principalement de différencier deux aspects des préférences : l'aversion pour le risque et le désir de lisser la consommation inter–temporellement. Cette approche a été utilisée pour expliquer les énigmes empiriques liées aux actifs risqués telles que la prime de risque élevée ou la prédictabilité des rendements des actions à long terme (voir Bansal et Yaron [2004], Garcia, Renault et Semenov [2005, 2006] et Garcia, Meddahi et Tedongap [2006]), pour expliquer la dynamique de la courbe des taux (voir Eraker [2006], Garcia et Luger [2006] et Piazzesi et Schneider [2006]) ou pour determiner les prix d'options (voir Garcia et Renault [1998]).

La deuxième approche considère le rôle de la formation des habitudes. Elle consiste à supposer que l'utilité courante du consommateur dépend à la fois de sa consommation courante et de l'historique des consommations passées individuelles et agrégées, d'où l'appellation de *backward–looking models*. L'hypothèse de la formation des habitudes a été utilisée pour expliquer une multitude d'énigmes empiriques dans divers domaines de l'économie. Les travaux théoriques fondateurs de Sundaresan [1989] et Constantidines [1990] ont donné un intérêt croissant à la formation des habitudes en particulier pour l'évaluation des actifs financiers.

La spécification de la formation des habitudes tient compte principalement de trois aspects différents. Tout d'abord, les habitudes de consommation peuvent être internes (Constantidines [1990]) ou externes (Campbell et Cochrane [1999]). Dans le premier cas, le consommateur tient compte de l'historique de sa propre consommation. Dans le second cas, on suppose que l'agent est influencé par les choix des autres. En anglais, on parle de *Catching up with the Joneses*. De plus, la formation des habitudes de consommation peut se limiter à une période (voir Abel [1990]) ou tenir compte de tout l'historique de la consommation (voir Heaton [1995] et Campbell et Cochrane [1999]). Notons que Garcia, Renault et Semenov [2005, 2006] ont proposé un modèle (C)CAPM avec niveau de référence qui englobe (*i*) le modèle (C)CAPM avec formation des habitudes et (*ii*) le modèle (C)CAPM avec utilité récursive.

Dans cette thèse, nous nous focalisons exclusivement sur l'hypothèse de la formation des habitudes. L'objectif est précisément d'étudier la pertinence de l'hypothèse de la persistance des habitudes et sa capacité à expliquer en particulier la prédictabilité des rendements d'actifs financiers à long terme. Dans un premier chapitre, nous étudions le rôle de l'hypothèse de la persistance des habitudes à engendrer la persistance du ratio cours des actions –dividende et la prédictibilité des rendements d'actions à long terme. Dans un second chapitre, nous testons la capacité du ratio surplus de consommation à prédire les rendements d'actions à long terme et à expliquer leur variation en coupe transversale. Enfin, dans un dernier chapitre, nous proposons une solution analytique générale du ratio cours des actions–dividende, du ratio cours du portfeuille de marché–consommation et du la structure à terme des taux d'intérêts dans un context affine.

Le premier chapitre se propose d'étudier le rôle de la persistance des habitudes afin d'expliquer la prédictabilité de l'excès des rendements des actions à long terme. En premier lieu, nous proposons une étude empirique afin d'évaluer le pouvoir de prédiction du ratio cours des actions–dividende. En effet, à la suite des travaux de Fama et French [1988] et Portba and Summers [1988] mettant en évidence l'existence d'une composante prédictible des rendements des actions, de nombreux auteurs ont proposé une variété d'indicateurs financiers et macroéconomiques comme variables explicatives tels que le ratio cours des actions–dividende, le ratio épargne–dividende et le ratio consommation–revenu (voir Fama et French [1988], Campbell et Shiller [1988], Hodrick [1992], Campbell, Lo et Mackinlay [1997] , Cochrane [1997, 2001], Lamont [1998], Lettau et Ludvigson [2001, 2005], Campbell [2003], etc).

Toutefois, concernant le pouvoir de prédiction du ratio cours des actions–dividende, une récente littérature a mis en cause son pouvoir de prédiction vu sa persistance très élevée. Nous citons en particulier les travaux de Stambaugh [1999], Torous, Valkanov et Yan [2004], Ang [2002], Campbell et Yogo [2005] et Ang et Bekaert [2005].

Par conséquent, nous examinons dans la première section du premier chapitre le pouvoir de prédiction du ratio cours des actions–dividende en explorant des données américaines annuelles sur la période 1947–2001. Nous mettons en évidence la capacité de cet indicateur financier à prédire l'excès de rendements des actifs risqués pour la période 1947–1990. Ainsi, son faible pouvoir de prédiction suggéré par des études empiriques récentes sur des périodes qui incluent les dix dernières années s'explique par le boom observé sur les places boursières durant cette période. Ce phénomène est engendré par l'exceptionnelle croissance des valeurs technologiques et ne remet pas en cause le pouvoir de prédiction du ratio en question.

En deuxième lieu, étant donné son incapacité à répliquer le pouvoir de prédiction du ratio cours des actions–dividende, nous proposons une extension du modèle (C)CAPM standard pour apporter des éléments de compréhension de ce fait stylisé. Pour ce faire, nous introduisons la formation des habitudes de l'agent représentatif. Pour situer cette extension par rapport à la littérature théorique, nous notons que des récentes extensions du modèle (C)CAPM ont été proposées pour répliquer le pouvoir de prédiction du prix dividende. La capacité de ces modèles à reproduire ce fait stylisé est engendrée principalement par une aversion au risque variable dans le temps, soit en considérant des agents hétérogènes (voir Chan et Kogan [2001]), soit en considérant la persistance des habitudes (voir Champbell et Cochrane [1999]). Dans le premier chapitre, nous étudions le rôle de la persistance des habitudes, tout en maintenant l'aversion au risque constante.

Les principales caractéristiques du modèle sont les suivantes. Nous considérons une économie d'échange avec agent représentatif où les processus de dotations sont exogènes. Nous supposons que le taux de croissance des dotations (consommation, dividende) suit un processus gaussien auto–régressif d'ordre un. La fonction d'utilité est spécifiée en ratio comme dans Abel [1990,1999] afin de maintenir l'aversion au risque constante. A chaque période, les préférences de l'agent représentatif dépendent du rapport de sa consommation individuelle courante et d'un niveau de référence. Nous proposons plusieurs spécifications de ce dernier. D'abord, nous supposons qu'il ne dépend que la consommation individuelle (habitude interne) ou agrégée (habitude externe, Catching up with the Joneses) de la période précédente. Ensuite, nous suggérons une extension du dernier cas en supposant que le niveau de référence dépend d'une manière dégressive de tout l'historique des niveaux de consommation agrégés, d'où l'appellation "stock d'habitudes". Cette spécification présente l'avantage d'être parcimonieuse en comparaison avec la formulation proposée par Campbell et Cochrane [1999]. De plus, la spécification linéaire du stock des habitudes permet de déterminer la solution exacte du ratio cours des actions – dividende analytiquement ainsi que les conditions garantissant l'existence d'une solution bornée. L'évaluation quantitative de ces extensions du modèle (C)CAPM standard montre que contrairement au modèle standard et au modèle (C)CAPM avec formation des habitudes se limitant à la période précédente, l'introduction de la persistance des habitudes permet de tenir compte en partie de la prédictabilité de l'excès des rendements des actifs risqués.

Dans le deuxième chapitre, nous cherchons à évaluer théoriquement et empiriquement le pouvoir du ratio surplus de consommation à expliquer les variations des exc'e de rendements d'actifs risqués, à la fois en séries chronologiques et en coupe transversale.

Le ratio surplus de consommation est mesuré par (i) le rapport de la différence entre la consommation courante et le niveau de référence et la consommation courante quand la fonction d'utilité est spécifiée en différence ou (ii) le rapport de la consommation et le niveau de référence quand la fonction d'utilité est spécifiée en ratio. Dans chacun des cas, nous montrons théoriquement que cet indicateur macroéconomique est un candidat pour prédire linéairement et négativement les rendements des actifs risqués à tout horizon. Ce résultat théorique généralise la relation inverse proposée par Campbell et Cochrane [1999] se limitant à un horizon d'une période. De plus, il est robuste à la spécification imposée au niveau de référence. La relation linéaire entre le ratio surplus de consommation et l'excès de rendements des actions à long terme a été testée sur des données réelles, en utilisant la méthode des moindres carrés ordinaires. Pour se faire, nous proposons une expérience de Monte Carlo pour évaluer et corriger les biais des coefficients estimés et du coefficient de détermination R^2 ainsi que les distorsions des distributions des tests de nullité, engendrés par (*i*) l'existence d'un effet rétroactif (feedback effect), (*ii*) l'utilisation de données imbriquées et (*iii*) l'évaluation du pouvoir de prédiction linéaire d'une variable persistante en utilisant la méthode des moindres carrées ordinaires. Les résultats empiriques suggèrent que le surplus de consommation est un prédicateur de l'excès des rendements des actions, en particulier à long terme. Notons qu'il explique 35% des variations des rendements des actifs risqués à un horizon de cinq ans. Ce résultat est en adéquation avec la variation temporelle et contre–cyclique de la prime de risque.

De plus, nous montrons que la composante des rendements financiers prédictible par le surplus de consommation n'est pas captée par d'autres prédicateurs tels que le ratio cours des actions – dividende et les approximations du ratio consommation–revenu, *cay* et *cdy*, proposées par Lettau et Ludvigson [2001a, 2001b, 2005].

En s'appuyant sur les mêmes arguments théoriques proposés par Lettau and Ludvigson [2001b] et Jagannathan et al. [2002], le ratio surplus de consommation est utilisé comme variable conditionnelle pour le modèle (C)CAPM, impliquant un modèle en coupe transversale à trois facteurs. Ces derniers sont les suivants : le taux de croissance de la consommation, le surplus de consommation précédent et leur produit. Le modèle à facteurs macro–économiques est estimé par la méthodologie de Fama–MacBeth [1973] en utilisant les rendements trimestriels des 25 portefeuilles de Fama et French sur la période 1952–2001. Nous montrons que le surplus de consommation permet d'expliquer en partie la variation moyenne des rendements des portefeuilles sélectionnés.

Le troisième chapitre propose deux extensions du modèle (C)CAPM présenté dans le premier chapitre. D'une part, nous supposons que les préférences sont non séparables dans le temps. Plus précisément, l'agent représentatif déduit son utilité courante à partir du rapport de sa consommation courante et d'un niveau de référence. Ce dernier dépend (i) du stock d'habitudes, mesuré par une moyenne géométrique des niveaux de consommation passés et aggrégés et (ii) de la déviation courante du processus du taux de croissance des dividendes par rapport à sa moyenne inconditionnelle. En d'autres termes, nous supposons que l'agent représentatif détermine son plan de consommation optimal en tenant compte de l'historique de la consommation agrégée et passée et des nouvelles financières. Tout en maintenant une aversion au risque constante dans le temps, la formation des habitudes engendre de la variation temporelle de la prime de risque et et permet de répliquer la persistance du ratio cours des actions–dividende. L'introduction d'un effet de nouvelles financières dans la détermination du niveau de référence permet de générer un taux sans risque faible en moyenne.

D'autre part, nous supposons que les processus de dotations (consommation et dividende) et le processus d'inflation suivent une classe de processus affines, *Compound Autoregressive Processus (CaR)*, introduite par Darolle, Gouriéroux et Jasiak [2006]. Notons que cette classe de processus *affines* englobent des cas particuliers utilisés dans la littérature tels que (i) le processus $i.i.d$ proposé par Campbell et Cochrane [1999], (ii) le porcessus auto–régressif d'ordre un proposé dans le premier chapitre, (iii) le modèle hétéroscédastique proposé par Bansal et Yaron [2004] ou (iv) le modèle avec composante prédictible du taux de croissance de la consommation et de l'inflation proposé par Schneider et Piazzesi [2006].

La structure *affine* du taux d'escompte stochastique et la spécification *affine* des processus exogènes nous permettent de résoudre le modèle analytiquement. Plus précisément, la principale contribution de ce chapitre consiste à déterminer les solutions analytiques exactes du ratio cours des actions–dividende, du ratio prix du portefeuille de marché–consommation et de la structure à terme des taux d'intérêt. La solution analytique du ratio cours des actions–dividende représente une généralisation des solutions proposées par Abel [1990], Burnside [1998] et Collard, Fève et Ghattassi [2006]. De plus, nous montrons que la structure à terme des taux d'intérêt est *affine*. Notons qu'une large littérature, concernant les modèles basés sur l'absence d'opportunité d'arbitrage, propose des modèles affines pour la structure à terme des taux d'intérêt (voir Duffie, Fillipovic et Singleton [2001] pour une revue de littérature des modèles affines en temps continu et Gouriéroux, Montfort et Polimenis [2002] pour une discrétisation de ces modèles).

Les solutions analytiques proposées dans ce chapitre permettent d'évaluer (i) le rôle du comportement du consommateur et (ii) l'impact de la dynamique des processus exogènes (consommation, dividende et inflation) à expliquer les mouvements des prix d'actifs financiers. Afin de mieux comprendre les mécanismes économiques en œuvre, nous considérons le cas simple d'un environnement de dotation *i.i.d.* De plus, nous supposons que l'inflation suit un processus auto–régressif d'ordre un. Nous proposons par la suite une évaluation quantitative. En supposant ce cas particulier de processus affines, nous montrons que contrairement au modèle (C)CAPM standard, le modèle (C)CAPM avec avec niveau de référence proposé dans le dernier chapitre permet d'expliquer (i) la persistance élevée du ratio cours des actions–dividende, (ii) l'énigme d'excès de volatilité des rendements d'obligations et (iii) la courbe moyenne décroissante des taux d'intérêt réels.

Chapter 1

Predictability and Habit Persistence[1]

This paper highlights the role of persistence in explaining predictability of excess returns. To this end, we develop a CCAPM model with habit formation when the growth rate of endowments follows a first order Gaussian autoregression. We provide a closed form solution of the price–dividend ratio and determine conditions that guarantee the existence of a bounded equilibrium. The habit stock model is found to possess internal propagation mechanisms that increase persistence. It outperforms the time separable and a "catching up with the Joneses" version of the model in terms of predictability therefore highlighting the role of persistence in explaining the puzzle.

Key Words: Asset Pricing, Catching up with the Joneses, Habit Stock, Predictability

JEL Class.: C62, G12.

[1] This chapter reviews the two joint works with Fabrice Collard and Patrick Fève [2006a] and [2006b].

Introduction

Over the last twenty years, the predictability of excess stock returns has attracted a great deal of attention. The initial piece of evidence in favor of predictability was obtained by examining univariate time series properties (See e.g. Poterba and Summers [1988]). The literature has also reported convincing evidence that financial and accounting variables have predictive power for stock returns (See Famma and French [1988], Fama and French [1989], Campbell and Shiller [1988], Hodrick [1992], Campbell, Lo and MacKinlay [1997], Cochrane [1997, 2001], Lamont [1998], Lettau and Ludvigson [2001a, 2005] and Campbell [2003]. The theoretical literature that has investigated the predictability of returns by the price–dividend ratio has established that two phenomena are key to explain it: persistence and time–varying risk aversion (see Campbell and Cochrane [1999], Menzly, Santos and Veronesi [2004] among others). Leaving aside time–varying risk aversion, this paper evaluates the role of persistence in accounting for predictability.

However, recent empirical work has casted doubt on the ability of the price–dividend ratio to predict excess returns (see e.g. Stambaugh [1999], Torous, Valkanov and Yan [2004], Ang [2002], Campbell and Yogo [2005], Ferson, Sarkissian and Simin [2003], Ang and Bekaert [2004] for recent references). In light of these results, we first re–examine the predictive power of the price–dividend ratio using annual data for the period 1948–2001.[2] We find that the ratio has indeed predictive power until 1990. In the latter part of the sample, the ratio keeps on rising while excess returns remain stable, and the ratio looses its predictive power after 1990. Our results are in line with Ang [2002] and Ang and Bekaert [2004], and suggest that the lack of predictability is related to something pertaining to the exceptional boom of the stock market in the late nineties rather than the non–existence of predictability. Furthermore, the predictability of the first part of the sample remains to be

[2] The data are borrowed from Lettau and Ludvigson [2005].

accounted for.

To this end, we develop an extended version of the Consumption based Capital Asset Pricing Model (CCAPM). The model — in its basic time separable version — indeed fails to account for this set of stylized facts, giving rise to a predictability puzzle. This finding is now well established in the literature and essentially stems from the inability of this model to generate enough persistence. Excess returns essentially behave as *iid* stochastic processes, unless strong persistence is added to the shocks initiating fluctuations on the asset market. Therefore, neither do they exhibit serial correlation nor are they strongly related to other variables. But recent theoretical work has shown that the CCAPM can generate predictability of excess returns providing the basic model is amended (see Campbell [2003] for a survey). This work includes models with heterogenous investors (see Chan and Kogan [2001]) or aforementioned models with time varying risk aversion generated by habit formation. This paper will partially pursue this latter route and consider a habit formation model. It should be noted that the literature dealing with habit formation falls into two broad categories. On the one hand, internal habit formation captures the influence of individual's own past consumption on the individual current consumption choice (see Boldrin, Christiano and Fisher [1997]). On the other hand, external habit formation captures the influence of the aggregate past consumption choices on the current individual consumption choices (Abel [1999]). This latter case is denoted "catching up with the Joneses". Two specifications of habit formation are usually considered. The first one (see Campbell and Cochrane [1999]) considers that the agent cares about the difference between his/her current consumption and a consumption standard. The second (see Abel [1990]) assumes that the agent cares about the ratio between these two quantities. One important difference between the two approaches is that the coefficient of risk aversion is time varying in the first case, while it remains constant in the second specification. This has strong consequences for the ability of the model to account for the predictability puzzle, as a time–varying coefficient is thought to be required to solve the puzzle (see Mensly et al. [2004]). This therefore seems to preclude the use of a ratio specification to tackle the predictability of stock returns. One of the main contribution of this paper will be to show that, despite the constant risk aversion coefficient, habit formation in ratio can account for a non–negligible part of the long horizon returns predictability. Note that the model is by no means designed to solve neither the equity premium puzzle nor the risk free rate puzzle, since time varying risk aversion is necessary to match this feature of the data.[3] Our aim is rather to highlight the role of persistence generated by habits in explaining the predictability puzzle, deliberately leaving the equity premium puzzle aside.

We develop a simple CCAPM model à la Lucas [1978]. We however depart from the standard setting in that we allow preferences to be non time separable. The model has the attractive feature of introducing tractable time non separability in a general equilibrium framework. More precisely, we consider that preferences are characterized by a "catching up with the Joneses" phenomenon. In a second step, we allow preferences to depend not only on lagged aggregate consumption but also on the whole history of aggregate consumption, therefore reinforcing both time non–separability and thus persistence. Our specification has the advantage of being simple and more parsimonious than the specification used by Campbell and Cochrane [1999] while maintaining the same economic mechanisms and their implications for persistence. We follow Abel [1990] and specify habit persistence in terms of ratio. This particular feature together with a CRRA utility function implies that preferences are homothetic with regard to consumption. As in Burnside [1998], we assume that endowments grow at an exogenous stochastic rate and we keep with the Gaussian assumption. These features enable us to obtain a closed form solution to the asset pricing problem and give conditions that guarantee that the

[3] Habit formation in ratio is known to fail to account for both puzzles. See Campbell et al. [1997] p. 328–329 and Campbell [2003].

solution is bounded. We then investigate the dynamic properties of the model and its implications in terms of moment matching and predictability over long horizons. We find that, as expected, the time separable model fails to account for most of asset pricing properties. The "catching up with the Joneses" model weakly enhances the properties of the CCAPM to match the stylized facts but its persistence is too low to solve the predictability puzzle. Conversely, the model with habit stock is found to generate much greater persistence than the two previous versions of the model. Finally, the habit stock version of the model outperforms the time separable and the catching up models in terms of predictability of excess returns. Since, risk aversion is held constant in the model, this result illustrates the role of persistence in accounting for predictability.

The remaining of the paper is organized as follows. Section 1 revisits the predictability of excess returns by the price–dividend ratio using annual data for the US economy in the post–WWII period. Section 2 develops a catching up with the Joneses version of the CCAPM model. We derive the analytical form of the equilibrium solution and the conditions that guarantee the existence of bounded solutions, assuming that dividend growth is Gaussian and serially correlated. In section 3, we extend the model to a more general setting where preferences are a function of the whole history of the past aggregate consumptions. We again provide a closed form solution for price–dividend ratio and conditions that guarantee bounded solutions. In section 4, we investigate the quantitative properties of the model and evaluate the role of persistence in accounting for predictability. A last section offers some concluding remarks.

1.1 Empirical Evidence

This section examines the predictability of excess returns using the data of Lettau and Ludvigson [2005].[4]

1.1.1 Preliminary data analysis

The data used in this study are borrowed from Lettau and Ludvigson [2005]. These are annual per capita variables, measured in 1996 dollars, for the period 1948–2001.[5] We use data on excess return, dividend and consumption growth, and the price–dividend ratio. All variables are expressed in real per capita terms. The price deflator is the seasonally adjusted personal consumption expenditure chain–type deflator (1996=100) as reported by the Bureau of Economic Analysis.

Although we will be developing an endowment economy model where consumption and dividend streams should equalize in equilibrium, in the subsequent analysis we acknowledge their low correlation in the data. This led us to first measure endowments as real per capita expenditures on nondurables and services as reported by the US department of commerce. Note that since, for comparability purposes, we used Lettau and Ludvigson data, we also excluded shoes and clothing from the scope of consumption. We then instead measure endowments by dividends as measured by the CRSP value–weighted index. As in Lettau and Ludvigson [2005], dividends are scaled to match the units of consumption. Excess return is measured as the return on the CRSP value–weighted stock market index in excess of the three–month Treasury bill rate.

Table 1.1 presents summary statistics for real per capita consumption growth (Δc_t), dividend growth (Δd_t), the price–dividend ratio (v_t) and the excess return (er_t) for two samples. The first one, hereafter referred as the whole sample, covers

[4] We are thankful to a referee for suggesting this analysis.

[5] More details on the data can be found in the appendix to Lettau and Ludvigson [2005], downloadable from `http://www.econ.nyu.edu/user/ludvigsons/dpappendix.pdf`.

the entire available period and spans 1948–2001. The second sample ends in 1990 and is aimed at controlling for the trend in the price–dividend ratio in the last part of the sample (see Ang [2002] and Ang and Bekaert [2004]).

— Table 1.1 about here —

Several findings stand out of Table 1.1. First of all, as already noted by Lettau and Ludvigson [2005], real dividend growth is much more volatile than consumption growth, 1.14% versus 12.24% over the whole sample. This remains true when we focus on the restricted sub–sample. Note that the volatilities are remarkably stable over the two samples except for the price–dividend ratio. Indeed, in this case, the volatility over the whole sample is about twice as much as the volatility over the restricted sub–sample. This actually reflects the upward trend in the price–dividend ratio during the nineties.

The correlation matrix is also remarkably stable over the two periods. It is worth noting that consumption growth and dividend growth are negatively correlated (-0.13) within each sample. A direct implication of this finding is that we will investigate the robustness of our theoretical results to the type of data we use (consumption growth versus dividend growth). Another implication of this finding is that while the price dividend ratio is positively correlated with consumption growth, it is negatively correlated with dividend growth in each sample. It is interesting to note that if the correlation between dividend growth and the price–dividend ratio remains stable over the two samples, the correlation between the price–dividend ratio and consumption growth dramatically decreases when the 1990s are brought back in the sample (0.18 versus 0.42). The correlation between the excess return and the price–dividend ratio is weak and negative. It is slightly weakened by the introduction of data pertaining to the latest part of the sample.

The autocorrelation function also reveals big differences between consumption and dividend data. Consumption growth is positively serially correlated while dividend growth is negatively serially correlated. The persistence quickly vanishes as the autocorrelation function shrinks to zero after horizon 2. Conversely, the price–dividend ratio is highly persistent. The first order serial correlation is about 0.8 in the short sample, while it amounts to 0.9 in the whole sample. This suggests that a standard CCAPM model will have trouble matching this fact, as such models possess very weak internal transmission mechanisms. This calls for a model magnifying the persistence of the shocks. Finally, excess returns display almost no serial correlation at order 1, and are negatively correlated at order 2.

1.1.2 Predictability

Over the last 20 years the empirical literature on asset prices has reported evidence suggesting that stock returns are indeed predictable. For instance, Campbell and Shiller [1987] or Fama and French [1988], among others, have shown that excess returns can be predicted by financial indicators including the price–dividend ratio. The empirical evidence also shows that the predictive power of these financial indicators is greater when excess returns are measured over long horizons. A common way to investigate predictability is to run regressions of the compounded (log) excess return (er_t^k) on the (log) price–dividend (v_t) evaluated at several lags

$$er_t^k = \alpha_k + \beta_k v_{t-k} + u_t^k \tag{1.1.1}$$

where $er_t^k \equiv \sum_{i=0}^{k-1} r_{t-i} - r_{f,t-i}$ with r and r_f respectively denote the risky and the risk free rate of return.

This procedure is however controversial and there is doubt of whether there actually is any evidence of predictability of excess stock returns with the price-dividend ratio. Indeed, following the seminal article of Fama and French [1988], there has been considerable debate as to whether or not the price–dividend ratio can actually predict excess returns (see e.g. Stambaugh [1999], Torous et al. [2004], Ang [2002], Campbell and Yogo [2005], Ferson et al. [2003], Ang and Bekaert [2004] for recent references). In particular, the recent literature has focused on the existence of some biases in the β_k coefficients, a lack of efficiency in the associated standard errors and upward biased R^2 due to the use of *(i)* persistent predictor variables (in our case v_t) and *(ii)* overlapping observations. Stambaugh [1999], using Monte–Carlo simulations, showed that the empirical size of the Newey–West t–statistic for a univariate regression of excess returns on the dividend yield is about 23% against a nominal size of 5%. This therefore challenges the empirical relevance of predictability of stock returns. In order to investigate this issue, we generate data under the null of no predictability ($\beta_k = 0$ in eq. (1.1.1)):

$$er_t^k = \alpha_k + e_t^k \tag{1.1.2}$$

where α_k is the mean of compounded excess return, and e_t^k is drawn from a Gaussian distribution with zero mean and standard deviation σ_e. We generated data for the price–dividend ratio, assuming that v_t is represented by the following AR(1) process

$$v_t = \theta + \rho v_{t-1} + \nu_t \tag{1.1.3}$$

where ν_t is assumed to be normally distributed with zero mean and standard deviation σ_ν. The values for α_k, θ, ρ, σ_e and σ_ν are estimated from the data over each sample.

We then generated 100,000 samples of T observations[6] under the null (equations (1.1.2) and (1.1.3)) and estimated

$$er_t^k = \alpha_k + \beta_k v_{t_k} + \varepsilon_t^k$$

from the generated data for $k = 1, 2, 3, 5, 7$. We then recover the distribution of the Newey–West t–statistics testing the null $\beta_k = 0$, the distribution of β_k and the distribution of R^2. This procedure allows us to evaluate *(i)* the potential bias in our

[6] We actually generated $T + 200$ observations, the 200 first observations being discarded from the sample.

estimations and therefore correct for it, and *(ii)* the actual size of the test for the null of non–predictability of stock returns.

— Table 1.2 about here —

Simulation results are reported in Table 1.2. Two main results emerge. First of all, the bias decreases with the horizon and is larger in the whole sample. More importantly, the regressions do not suffer from any significant bias in our data. For instance, the bias is -0.077 for k=1 in the whole sample with a large dispersion of about 0.1, which would not lead to reject that the bias is significant at conventional significance level. The bias is much lower in the short sub–sample. Hence, the estimates of predictability do not exhibit any significant bias and do not call for any specific correction. Second, as expected from the previous results, the R^2 of the regression is essentially 0, which confirms that the model is well estimated as v_{t-k} has no predictive power under the null. Hence, the R^2 is not upward biased in our sample.

There still remains one potential problem in our regressions, as the empirical size of the Newey–West t–statistic ought to be distorted. Therefore, in Table 1.3 we report *(i)* the empirical size of the t–statistic should it be used in the conventional way (using 1.96 as the threshold), and *(ii)* the correct thresholds that guarantee a 5% two–sided confidence level in our sample.

— Table 1.3 about here —

Table 1.3 clearly shows that the size of the Newey–West t–statistics are distorted. For example, applying the standard threshold values associated to the two–sided t–statistics at the conventional 5% significance level would actually yield a 10% size in both samples. The empirical size even rises to 16% in the whole sample for the shortest horizon. In other words, this would lead the econometrician to reject the absence of predictability too often. But the problem is actually more pronounced as can be seen from columns 3, 4, 6 and 7 of Table 1.3. Beside the distortion of the size of the test, an additional problem emerges: the distribution are skewed, which implies that the tests are not symmetric. This is also illustrated in Figures 1.3 and 1.4 (see Appendix B) which report the cdf of the Student distribution and the distribution obtained from our Monte-carlo experiments. Both figures show that the distributions are distorted and that this distortion is the largest at short horizons. Therefore, when running regressions on the data, we will take care of these two phenomena.

We ran the predictability regressions on actual data correcting for the aforementioned problems. The results are reported in Table 1.4.

— Table 1.4 about here —

Panels (a) and (b) report the predictability coefficients obtained from the estimation of equation (1.1.1). The second line of each panel reports the t–statistic, t_k, associated to the null of the absence of predictability together with the empirical size of the test. Then the fourth line gives the modified t–statistics, c_k, proposed by Valkanov [2003] which correct for the size of the sample ($c_k = t_k/\sqrt{T}$) and the associated empirical size. The empirical size used for each experiments were obtained from 100,000 Monte Carlo simulations and therefore corrects for the size distorsion problem. Finally, the last line reports information on the overall fit of the regression.

The estimation results suggest that excess returns are negatively related to the price–dividend ratio whatever the horizon and whatever the sample. Moreover, the larger the horizon, the larger the magnitude of this relationship is. For instance, when the lagged price–dividend ratio is used to predict excess returns, the coefficient is -0.362 in the short sample, while the coefficient is multiplied by around 4 and rises to -1.414 when 7 lags are considered. In other words, the price–dividend ratio accounts for greater volatility at longer horizons. A second worth noting fact is that the foreseeability of the price–dividend ratio is increasing with horizon as the R^2 of the regression increases with the lag horizon. For instance, the one year predictability regression indicates that the price–dividend ratio accounts for 22% of the overall volatility of the excess return in the short run. This share rises to 68% at the 7 years horizon. It should however be noticed that the significance of this relationship fundamentally depends on the sample we focus on. Over the short sample, predictability can never be rejected at any conventional significance level, whether we consider the standard t–statistics or the corrected statistics. The empirical size of the test is essentially zero whatever the horizon for both tests. The evidence in favor of predictability is milder when we extend the sample up to 2001. For instance, the empirical size of the null of no predictability is about 17% over the short horizon, and rises to 30% at the 5 years horizon. This lack of significance is witnessed by the measure of fit of the regression which amounts to 29% over the longer run horizon. This finding is related to the fact that while excess return remained stable over the whole sample, the price–dividend ratio started to raise in the latest part of the sample, therefore dampening its predictive power. Taken together, these findings suggest that the potential lack of predictability of the price dividend ratio essentially reflects some sub–sample issues rather than a deep econometric problem. The late nineties were marked by a particular phase of the evolution of stock markets which seems to be related to the upsurge of the information technologies, which may have created a transition phase weakening the predictability of stock returns (see Hobijn and Jovanovic [2001] for an analysis of this issue). This issue is far beyond the scope of this paper. Nevertheless, the data suggest that the price dividend ratio offered a pretty good predictor of stock returns at least in the pre–information technology revolution.

1.2 Catching–up with the Joneses

In this section, we develop a consumption based asset pricing model in which preferences exhibit a "Catching up with the Joneses" phenomenon. We provide the closed–form solution for the price–dividend ratio and conditions that guarantee the existence of a stationary bounded equilibrium.

1.2.1 The model

We consider a pure exchange economy à la Lucas [1978]. The economy is populated by a single infinitely–lived representative agent. The agent has preferences over consumption, represented by the following intertemporal expected utility function

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_{t+s} \tag{1.2.4}$$

where $\beta > 0$ is a constant discount factor, and u_t denotes the instantaneous utility function, that will be defined later. Expectations are conditional on information available at the beginning of period t.

The agent enters period t with a number of shares, S_t —measured in terms of consumption goods— carried over the previous period as a means to transfer wealth intertemporally. Each share is valuated at price P_t. At the beginning of the period, she receives dividends, $D_t S_t$ where D_t is the dividend per share. These revenues are then used to purchase consumption goods, c_t, and new shares, S_{t+1}, at price P_t. The budget constraint therefore writes

$$P_t S_{t+1} + C_t \leqslant (P_t + D_t) S_t \tag{1.2.5}$$

Following Abel [1990,1999], we assume that the instantaneous utility function, u_t, takes the form

$$u_t \equiv u(C_t, V_t) = \begin{cases} \frac{(C_t/V_t)^{1-\theta}-1}{1-\theta} & \text{if } \theta \in \mathbb{R}_+ \backslash \{1\} \\ \\ \log(C_t) - \log(V_t) & \text{if } \theta = 1 \end{cases} \tag{1.2.6}$$

where θ measures the degree of relative risk aversion and V_t denotes the habit level.

We assume V_t is a function of lagged aggregate[7] consumption, $\overline{C}_{t-1}$, and is therefore external to the agent. This assumption amounts to assume that preferences are characterized by a "Catching up with the Joneses" phenomenon.[8] More precisely, we assume that[9]

$$V_t = \overline{C}_{t-1}^{\varphi} \tag{1.2.7}$$

where $\varphi \geqslant 0$ rules the sensitivity of household's preferences to past aggregate consumption, $\overline{C}_{t-1}$, and therefore measures the degree of "Catching up with the Joneses". It is worth noting that habit persistence is specified in terms of the ratio of current consumption to a function of lagged consumption. We hereby follow Abel [1990] and depart from a strand of the literature which follows Campbell and Cochrane [1999] and specifies habit persistence in terms of the difference between current and a reference level. This particular feature of the model will enable us to obtain a closed form solution to the asset pricing problem while keeping the main properties of habit persistence. Indeed, as shown by Burnside [1998], one of the keys to a closed form solution is that the marginal rate of substitution between consumption at two dates is an exponential function of the growth rate of consumption between these two dates. This is indeed the case with this particular form of catching up. Another implication of this specification is that, just alike the standard CRRA utility function, the individual risk aversion remains time–invariant and is unambiguously

[7] Appendix A provides a closed form solution to the proposed model under the assumption of internal habit formation. More precisely, we assume that the reference level is function of the individual's own past consumption:

$$V_t = C_{t-1}^{\varphi}$$

[8] Note that had V_t been a function of current aggregate consumption, we would have recovered Galí's [1989] "Keeping up with the Jones". In such a case the model admits that same analytical solution as in Burnside [1998].

[9] Note that this specification of the preference parameter can be understood as a particular case of Abel [1990] specification which is, in our notations, given by $V_t = [C_{t-1}^{D}\overline{C}_{t-1}^{1-D}]^{\gamma}$ with $0 \leqslant D \leqslant 1$ and $\gamma \geqslant 0$.

given by θ.

Another attractive feature of this specification is that it nests several standard specifications. For instance, setting $\theta = 1$ leads to the standard time separable case, as in this case the instantaneous utility function reduces to $\log(C_t) - \varphi \log(\overline{C}_{t-1})$. As aggregate consumption, $\overline{C}_{t-1}$, is not internalized by the agents when taking their consumption decisions, the (maximized) utility function actually reduces to $\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \log(C_{t+s})$. The intertemporal utility function is time separable and the solution for the price–dividend ratio is given by $P_t/D_t = \beta/(1-\beta)$.

Setting $\varphi = 0$, we recover a standard time separable CRRA utility function of the form $\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s (C_{t+s}^{1-\theta} - 1)/(1-\theta)$. In such a case, Burnside [1998] showed that as long as dividend growth is log–normally distributed, the model admits a closed form solution.[10]

Setting $\varphi = 1$ we retrieve Abel's [1990] relative consumption case (case B in Table 1, p.41) when shocks to endowments are *iid*. In this case, the household values increases in her individual consumption vis à vis lagged aggregate consumption. In equilibrium, $C_{t-1} = \overline{C}_{t-1}$ and it turns out that utility is a function of consumption growth.

At this stage, no further restriction will be placed on either β, θ or φ.

The household determines her contingent consumption $\{C_t\}_{t=0}^{\infty}$ and contingent asset holdings $\{S_{t+1}\}_{t=0}^{\infty}$ plans by maximizing (1.2.4) subject to the budget constraint (1.2.5), taking exogenous shocks distribution as given, and (1.2.6) and (1.2.7) given. Agents' consumption decisions are then governed by the following Euler equation

$$P_t C_t^{-\theta} \overline{C}_{t-1}^{\varphi(\theta-1)} = \beta \mathbb{E}_t \left[(P_{t+1} + D_{t+1}) C_{t+1}^{-\theta} \overline{C}_t^{\varphi(\theta-1)} \right] \tag{1.2.8}$$

which may be rewritten as

$$\frac{P_t}{D_t} = \mathbb{E}_t \left[\left(1 + \frac{P_{t+1}}{D_{t+1}} \right) \times \mathbb{W}_{t+1} \times \Phi_{t+1} \right] \times \mathbb{C}_t \tag{1.2.9}$$

[10]Note that this result extends to more general distribution. See for example Bidarkota and McCulloch [2003] and Tsionas [2003].

where $\mathbb{W}_{t+1} \equiv D_{t+1}/D_t$ captures the wealth effect of dividend, $\Phi_{t+1} \equiv \beta[(C_{t+1}/C_t)^{-\theta}]$ is the standard stochastic discount factor arising in the time separable model. This Euler equation has an additional stochastic factor $\mathbb{C}_t \equiv \left(\overline{C}_t/\overline{C}_{t-1}\right)^{\varphi(\theta-1)}$ which measures the effect of "catching up with the Joneses". These two latter effects capture the intertemporal substitution motives in consumption decisions. Note that $\mathbb{C}_t$ is known with certainty in period t as it only depends on current and past aggregate consumption. This new component distorts the standard intertemporal consumption decisions arising in a standard time separable model. Note that our specification of the utility function implies that φ essentially governs the size of the catching up effect, while risk aversion, θ, governs its direction. For instance, when risk aversion is large enough — $\theta > 1$ — catching–up exerts a positive effect on the time separable intertemporal rate of substitution. Hence, in this case, for a given rate of consumption growth, catching up reduces the expected return.

Since we assumed the economy is populated by a single representative agent, we have $S_t = 1$ and $C_t = \overline{C}_t = D_t$ in equilibrium. Hence, both the stochastic discount factor in the time separable model and the "'catching up with the Joneses" term are functions of dividend growth D_{t+1}/D_t

$$\Phi_{t+1} \equiv \beta[(D_{t+1}/D_t)^{-\theta}] \quad \text{and} \quad \mathbb{C}_t \equiv (D_t/D_{t-1})^{\varphi(\theta-1)}$$

Any persistent increase in future dividends, D_{t+1}, leads to two main effects in the standard time separable model. First, a standard wealth effect, stemming from the increase in wealth it triggers ($\mathbb{W}_{t+1}$), leads the household to consume more and purchase more assets. This puts upward pressure on asset prices. Second, there is an effect on the stochastic discount factor (Φ_{t+1}). Larger future dividends lead to greater future consumption and therefore lower future marginal utility of consumption. The household is willing to transfer $t + 1$ consumption toward period t, which can be achieved by selling shares therefore putting downward pressure on prices. When $\theta > 1$, the latter effect dominates and prices are a decreasing function of dividend. In the "catching up" model, a third effect, stemming from habit persistence ($\mathbb{C}_t$), comes into play. Habit standards limit the willingness of the household to transfer consumption intertemporally. Indeed, when the household brings future consumption back to period t, she hereby raises the consumption standards for the next period. This raises future marginal utility of consumption and therefore plays against the stochastic discount factor effect. Henceforth, this limits the decrease in asset prices and can even reverse the effect when φ is large enough.

Defining the price–dividend ratio as $v_t = P_t/D_t$, it is convenient to rewrite the Euler equation evaluated at the equilibrium as

$$v_t = \beta \mathbb{E}_t \left[(1+v_{t+1}) \left(\frac{D_{t+1}}{D_t} \right)^{1-\theta} \left(\frac{D_t}{D_{t-1}} \right)^{\varphi(\theta-1)} \right] \tag{1.2.10}$$

1.2.2 Solution and existence

In this section, we provide a closed form solution for the price–dividend ratio and give conditions that guarantee the existence of a stationary bounded equilibrium.

Note that up to now, no restrictions have been placed on the stochastic process governing dividends. Most of the literature attempting to obtain an analytical solution to the problem assumes that the rate of growth of dividends is an *iid* Gaussian process (see Abel [1990,1999] among others).[11] We depart from the *iid* case and follow Burnside [1998]. We assume that dividends grow at rate $\gamma_t \equiv \log(D_t/D_{t-1})$, and that γ_t follows an AR(1) process of the form

$$\gamma_t = \rho\gamma_{t-1} + (1-\rho)\overline{\gamma} + \varepsilon_t \tag{1.2.11}$$

where $\varepsilon_t \rightsquigarrow \mathbb{N}(0, \sigma^2)$ and $|\rho| < 1$. In the AR(1) case, the Euler equation rewrites

$$v_t = \beta \mathbb{E}_t \left[(1+v_{t+1}) \exp\left((1-\theta)\gamma_{t+1} - \varphi(1-\theta)\gamma_t \right) \right] \tag{1.2.12}$$

[11] There also exist a whole strand of the literature introducing Markov switching processes in CCAPM models. See Cecchetti, Lam and Mark [2000] and Brandt, Zeng and Zhang [2004] among others.

We can then establish the following proposition.

Proposition 1.2.1 *The Solution to Equation (1.2.12) is given by*

$$v_t = \sum_{i=1}^{\infty} \beta^i \exp(a_i + b_i(\gamma_t - \overline{\gamma})) \tag{1.2.13}$$

where

$$\begin{aligned} a_i &= (1-\theta)(1-\varphi)\overline{\gamma} i + \left(\frac{1-\theta}{1-\rho}\right)^2 \frac{\sigma^2}{2}\Bigg((1-\varphi)^2 i - 2\frac{(1-\varphi)(\rho-\varphi)}{1-\rho}(1-\rho^i) \\ &+ \frac{(\rho-\varphi)^2}{1-\rho^2}(1-\rho^{2i})\Bigg) \\ b_i &= \frac{(1-\theta)(\rho-\varphi)}{1-\rho}(1-\rho^i) \end{aligned}$$

First of all it is worth noting that this pricing formula resembles that exhibited in Burnside [1998]. We actually recover Burnside's formulation by setting $\varphi = 0$ — *i.e* imposing time separability in preferences. Second, when the rate of growth of endowments is *iid* over time ($\gamma_t = \overline{\gamma} + \varepsilon_t$), and φ is set to 1, we recover the solution used by Abel [1990] to compute unconditional expected returns:

$$z_t = \beta \exp\left((1-\theta)^2 \frac{\sigma^2}{2} + (1-\theta)(\gamma_t - \overline{\gamma})\right) \tag{1.2.14}$$

In this latter case, the price–dividend ratio is an increasing (resp. decreasing) and convex function of the consumption growth if $\theta > 1$ (resp. $\theta < 1$). Things are more complicated when we consider the general model. Indeed, as shown in proposition 1.2.1 (see coefficient b_i), both the position of the curvature parameter, θ, around 1 and the position of the persistence of dividend growth, ρ, around the parameter of habit persistence, φ, matter.

The behavior of an agent in face a positive shock on dividend growth essentially depends on the persistence of the process of endowments. This is illustrated in Figure 1.1 which reports the behavior of the price–dividend ratio as a function of the rate of growth of dividends for θ below and above 1.

— Figure 1.1 about here —

Let us consider the case $\theta > 1$ (see right panel of Figure 1.1). As we established in the previous section, a shock on dividends exerts three effects: *(i)* a standard wealth effect, *(ii)* a stochastic discount factor effect and *(iii)* a habit persistence effect. The two latter effects play in opposite direction on intertemporal substitution. When $\varphi > \rho$, the stochastic discount factor effect is dominated by the force of habits, as the shock on dividend growth exhibits less persistence than habits. Therefore, the second and the third effects partially offset each other and the wealth effect plays a greater role. The price–dividend ratio increases. Conversely, when $\varphi < \rho$ habit persistence cannot counter the effects of expected stochastic discounting, and intertemporal substitution motives take the upper hand. The price–dividend ratio decreases. Note that in the limiting case where $\rho = \varphi$ (plain dark line in Figure 1.1) the persistence of dividend growth exactly offsets the effects of "catching up" and all three effects cancel out. Therefore, just alike the case of a logarithmic utility function, the price–dividend ratio is constant. The reasoning is reversed when $\theta < 1$ (see left panel of Figure 1.1).

It is worth noting that Proposition 1.2.1 only establishes the existence of a solution, and does not guarantee that this solution is bounded. Indeed, the solution for the price–dividend ratio involves a series which may or may not converge. The next proposition reports conditions that guarantee the existence of a stationary bounded equilibrium.

Proposition 1.2.2 *The series in (1.2.13) converges if and only if*

$$r \equiv \beta \left[(1-\theta)(1-\varphi)\overline{\gamma} + \frac{\sigma^2}{2} \left(\frac{(1-\theta)(1-\varphi)}{1-\rho} \right)^2 \right] < 1$$

As in Burnside [1998], this proposition shows that, given a 4–uplet $(\theta, \varphi, \rho, \sigma)$, $\beta < 1$ is neither necessary nor sufficient to insure finite asset prices. In particular, the solution may converge even for $\beta > 1$ when agents are highly risk adverse. Furthermore, the greater the "catching up", the easier it is for the series to converge. Conversely, β should be lower as ρ approaches unity.

Related to the convergence of the series is the convergence of the moments of the price–dividend ratio. The next proposition establishes a condition for the first two moments of the price–dividend ratio to converge.

Proposition 1.2.3 *The mean and autocovariances of the price–dividend ratio converge to a finite constant if and only if $r < 1$.*

Proposition 1.2.3 extends previous results obtained by Burnside [1998] to the case of "catching up with the Joneses". The literature has shown that this representation of preferences fails to account for the persistence of the price–dividend ratio and the dynamics of asset returns. In the next section we therefore enrich the dynamics of the model.

1.3 Catching–up with the Joneses and Habit Stock

In this section, we extend the previous framework to a more general habit formation process. In particular, we allow habits to react only gradually to changes in aggregate consumption. We provide the closed–form solution for the price–dividend ratio and conditions that guarantee the existence of a stationary bounded equilibrium.

1.3.1 The model

We depart from the previous model in that preferences are affected by the entire history of aggregate consumption per capita rather that the lagged aggregate consumption (see e.g. Sundaresan [1989], Constantidines [1999],Heaton [1995] or Campbell and Cochrane [1999] among others). More precisely, the habit level, V_t, takes the form

$$V_t = X_t^{\varphi}$$

where X_t is the consumption standard. We assume that the effect of aggregate consumption on the consumption standard vanishes over time at the constant rate $\delta \in (0,1)$. More precisely, the consumption standard, X_t, evolves according to

$$X_{t+1} = \overline{C}_t^{\delta} X_t^{1-\delta} \tag{1.3.15}$$

Note that this specification departs from the standard habit formation formula usually encountered in the literature. Nevertheless, in order to provide economic intuition, the evolution of habits (1.3.15) may be rewritten as

$$x_t \equiv \log(X_t) = \delta \sum_{i=0}^{\infty} (1-\delta)^i \log(\overline{C}_{t-i-1}) \tag{1.3.16}$$

The reference consumption index, X_t, can be viewed as a weighted geometric average of past realizations of aggregate consumption. Equation (1.3.16) shows that $(1-\delta)$ governs the rate at which the influence of past consumption vanishes over time, or, otherwise states δ governs the persistence of the state variable X_t. Note that in the special case of $\delta = 1$, we recover the "Catching up with the Joneses" preferences specification studied in the previous section. Conversely, setting $\delta = 0$, we retrieve the standard time separable utility function as habit stock does not respond to changes in consumption anymore.

The representative agent then determines her contingent consumption $\{C_t\}_{t=0}^{\infty}$ and contingent asset holdings $\{S_{t+1}\}_{t=0}^{\infty}$ plans by maximizing her intertemporal expected utility function (1.2.4) subject to the budget constraint (1.2.5) and taking the law of habit formation (1.3.15) as given.

Agent's consumption decisions are governed by the following Euler equation:

$$P_t C_t^{-\theta} X_t^{-\varphi(1-\theta)} = \beta E_t (P_{t+1} + D_{t+1}) C_{t+1}^{-\theta} X_{t+1}^{-\varphi(1-\theta)} \tag{1.3.17}$$

which may actually be rewritten in the form of equation (1.2.9) as

$$\mathbb{E}_t \left[\frac{P_{t+1} + D_{t+1}}{P_t} \times \Phi_{t+1} \right] \times \mathbb{X}_{t+1} = 1 \tag{1.3.18}$$

where Φ_{t+1} is the stochastic discount factor defined in section 1.2.1 and $\mathbb{X}_{t+1} \equiv (X_{t+1}/X_t)^{\varphi(\theta-1)}$ accounts for the effect of the persistent "catching up with the Joneses" phenomenon. Note that as in the previous model, the predetermined variable $\mathbb{X}_{t+1}$ distorts intertemporal consumption decisions in a standard time separable model.

1.3.2 Solution and existence

In equilibrium, we have $S_t = 1$ and $\overline{C}_t = C_t = D_t$, implying that $X_{t+1} = D_t^{\delta} X_t^{1-\delta}$. As in the previous section, we assume that the growth rate of dividends follows an AR(1) process of the form (1.2.11). It is then convenient to rewrite equation (1.3.17) as

$$y_t = \beta E_t \left[\exp((1-\theta)(1-\varphi)\gamma_{t+1} - \varphi(1-\theta)z_{t+1}) + \exp((1-\theta)(1-\varphi)\gamma_{t+1})y_{t+1}\right] \tag{1.3.19}$$

where $z_t = \log(X_t/D_t)$ denotes the (log) habit–dividend ratio and $y_t = v_t \exp(-\varphi(1-\theta)z_t)$.

This forward looking equation admits the closed form solution reported in the next proposition.

Proposition 1.3.4 *The equilibrium price-dividend ratio is given by:*

$$v_t = \sum_{i=1}^{\infty} \beta^i \exp\left(a_i + b_i(\gamma_t - \overline{\gamma}) + c_i z_t\right) \tag{1.3.20}$$

where

$$\begin{aligned}
a_i =& (1-\theta)\overline{\gamma}\left[(1-\varphi)i + \frac{\varphi}{\delta}(1-(1-\delta)^i)\right] + \frac{\mathbb{V}_i}{2}\\
b_i =& (1-\theta)\left[\frac{\rho(1-\varphi)}{1-\rho}(1-\rho^i) + \frac{\varphi\rho}{1-\delta-\rho}((1-\delta)^i - \rho^i)\right]\\
c_i =& \varphi(1-\theta)(1-(1-\delta)^i)
\end{aligned}$$

and

$$\begin{aligned}\mathbb{V}_i =&(1-\theta)^2\sigma^2\Bigg\{\left(\frac{1-\varphi}{1-\rho}\right)^2\left(i-2\frac{\rho}{1-\rho}(1-\rho^i)+\frac{\rho^2}{1-\rho^2}(1-\rho^{2i})\right)\\&+2\frac{\varphi(1-\varphi)}{(1-\rho)(1-\delta-\rho)}\left(\frac{(1-\delta)}{\delta}(1-(1-\delta)^i)-\frac{\rho}{1-\rho}(1-\rho^i)\right.\\&\left.-\frac{\rho(1-\delta)}{1-\rho(1-\delta)}(1-(\rho(1-\delta))^i)+\frac{\rho^2}{1-\rho^2}(1-\rho^{2i})\right)\\&+\frac{\varphi^2}{(1-\delta-\rho)^2}\left(\frac{(1-\delta)^2}{1-(1-\delta)^2}(1-(1-\delta)^{2i})-2\frac{\rho(1-\delta)}{1-\rho(1-\delta)}(1-(\rho(1-\delta))^i)\right.\\&\left.+\frac{\rho^2}{1-\rho^2}(1-\rho^{2i})\right)\Bigg\}\end{aligned}$$

This solution obviously nests the pricing formula obtained in the previous model. Indeed, setting $\delta = 1$, we recover the solution reported in proposition 1.2.1. As shown in Section 1.2.2, the form of the solution essentially depends on the position of the curvature parameter, θ, around 1 and the position of the habit persistence parameter, φ, around the persistence of the shock, ρ. In the generalized model, things are more complicated as the position of the persistence of habits, $1-\delta$, around φ and ρ is also key to determine the form of the solution as reflected in the form of the coefficient b_i. Nevertheless, expression (1.3.20) illustrates two important properties of our model. First, the price–dividend ratio is function of two state variables: the growth rate of dividends γ_t and the habit–dividend ratio z_t. This feature is of particular interest as the law of motion of z_t is given by

$$z_{t+1} = (1-\delta)z_t - \gamma_{t+1} \tag{1.3.21}$$

Therefore, z_t is highly serially correlated for low values of δ, and the price–dividend ratio inherits part of this persistence. A second feature of this solution is that any change in the rate of growth of dividend exerts two effects on the price–dividend ratio. A first direct effect transits through its standard effect on the capital income of the household and is reflected in the term b_i. A second effect transits through its effect on the habit–dividend ratio. This second effect may be either negative or positive depending on the position of θ with regard to 1 and the form of c_i. This implies that there is room for pro– or counter–cyclical variations in the dividend–price ratio. This is critical for the analysis of predictability in stock returns as Section 1.4.3 will make clear. Finally, note that as soon as $\delta < 1$, the price–dividend ratio will be persistent even in the case when the rate of growth of dividends is *iid* ($\rho = 0$) (see the expression for c_i).

As the solution for the price–dividend ratio involves a series, the next proposition determines conditions that guarantee the existence of a stationary bounded equilibrium.

Proposition 1.3.5 *The series in (1.3.20) converges if and only if*

$$r \equiv \beta \exp\left[(1-\theta)(1-\varphi)\overline{\gamma} + \frac{\sigma^2}{2}\left(\frac{(1-\theta)(1-\varphi)}{1-\rho}\right)^2\right] < 1$$

It is worth noting that the result reported in proposition 1.3.5 is the same as in proposition 1.2.2. Hence, the conditions for the existence of a stationary bounded equilibrium are not altered by this more general specification of habit formation. From a technical point of view, this result stems from the geometrical lag structure of habit stock, which implies strict homotheticity of the utility function with respect to habit. From an economic point of view this illustrates that habit formation essentially affects the transition dynamics of the model while leaving unaffected the long run properties of the economy.

Just like in the previous model, it is possible to establish the convergence of the first two moments of the price–dividend ratio.

Proposition 1.3.6 *The mean and the autocovariances of the price–dividend ratio converge to a constant if and only if* $r < 1$.

Propositions 1.3.5 and 1.3.6 provide us with a set of restrictions on the deep and forcing parameters of the economy, which can be used to guarantee the relevance of our quantitative evaluation of the models.

1.4 Quantitative Evaluation

This section investigates the quantitative properties of the model putting emphasis on the predictability of stock returns.

1.4.1 Parametrization

We partition the set of the parameters of the model in two distinct groups. In the first group we gather all deep parameters defining preferences. These parameters are reported in Table 1.5. The calibration takes advantage of propositions 1.2.2 and 1.3.5 to place restrictions on the parameters that guarantee the existence of a forward solution.

— TABLE 1.5 ABOUT HERE —

The two parameters β and θ ruling the properties of the stochastic discount factor (Φ_{t+1}) are set to commonly used values in the literature. More precisely, the household is assumed to discount the future at a 5% annual psychological discount rate, implying $\beta = 0.95$. The parameter of risk aversion, θ, is set to 1.5. We however gauge the sensitivity of our results to alternative values of the degree of risk aversion ($\theta = 0.5$, 5).

The parameters pertaining to habit formation, φ and δ, are first set to reference values, we will then run a sensitivity analysis to changes in these parameters. The parameter ruling the sensitivity of preferences to habit formation, φ, takes on values ranging from 0 to 1. We first study the standard case of time separable utility function, which corresponds to $\varphi = 0$. We then investigate Abel's [1990] case where φ is set to 1. This latter parametrization implies that, in equilibrium, the representative agent values aggregate consumption growth in the case of pure catching up with the Joneses ($\delta = 1$). We also consider intermediate values for φ in our sensitivity analysis. Several values of the habit stock parameter, δ, are considered. We first set δ to 1, therefore focusing on the simple "catching up with the Joneses" model. We then explicitly consider the existence of a habit stock formation mechanism by allowing δ to take on values below unity. Following Campbell and Cochrane [1999], the value for δ is selected such that the model can generate the first order autocorrelation of the price–dividend ratio. This leads us to select a value such that the effect of current consumption on the consumption standard vanishes at a 5% annual rate ($\delta = 0.05$). This result is in line with previous studies which have shown that high persistence in habit stock formation is required to enhance the properties of the asset pricing model along several dimensions (see Heaton [1995], Li [2001], Allais [2004] among others). We however evaluate the sensitivity of our results to higher values of δ and therefore to less persistent habit formation.

The second group consists of parameters describing the evolution of the forcing variables. The latter set of parameters is obtained exploiting US postwar annual data borrowed from Lettau and Ludvigson [2005] (see Section 1.1.1 for more details on the data). The values of these parameters are reported in Table 1.6.

— Table 1.6 about here —

As aforementioned in Section 1.1.1, endowments can be either measured relying on consumption or dividend data. We therefore investigate these two possibilities and estimate the parameters of the forcing variable fitting an AR(1) process on both consumption and dividend growth data. Moreover, as suggested by empirical results in Section 1.1.1, we consider two samples: the first one covers the whole period running from 1948 to 2001, the second one ends in 1990.

Several results emerge from Table 1.6. First of all, the choice of a particular sample does not matter for the properties of the forcing variables. Both the persistence and the volatility of the forcing variable remains stable over the two samples. We therefore use results obtained over the whole sample in our subsequent evaluation. Second consumption and dividend data yield fairly different time series behaviors. For instance, dividend growth data exhibit much more volatility than that found in consumption data. Furthermore, dividend growth is negatively serially correlated while consumption growth displays positive persistence. But it is worth noting that both dividend and consumption data yield weak persistence (respectively -0.25 and 0.34). Then, provided our model possesses strong enough internal propagation mechanism, differences in the persistence of endowment growth ought not to influence much the dynamic properties of the model. This finding will be confirmed in the impulse response analysis (see Section 1.4.2). Therefore, in order to save space and unless necessary, we mainly focus on results obtained relying on dividend data.

1.4.2 Preliminary quantitative investigation

This section assesses the quantitative ability of the model to account for a set of standard unconditional moments characterizing the dynamics of excess returns and the price–dividend ratio.

The model is simulated using the closed–form solution (1.3.20). Since it involves an infinite series, we have to make an approximation and truncate the infinite sum at a long enough horizon (5000 periods). We checked that additional terms do not alter significantly the series. Each experiment is conducted by running 1000 draws of the length of the sample size, T. We actually generated $T + 200$ observations, the 200 first observations being discarded from the sample.

We begin by reporting the impulse response analysis of the model in face a positive shock on endowment growth. Figure 1.2 reports impulse response functions (IRF) of endowments, habit/consumption ratio, excess return and the price–dividend ratio to a standard deviation shock on dividend growth when the endowment growth process is estimated with both dividend and consumption data. IRF are computed taking the non–linearity of the model into account (see Koop, Pesaran and Potter [1996]). Panel (a) of the figure reports IRFs when the forcing variable is measured relying on consumption data, Panel (b) provides results obtained with dividend data. Three cases are under investigation: *(i)* the time separable utility function (TS), *(ii)* the "catching up with the Joneses" (CJ) and *(iii)* habit stock (HS).

— FIGURE 1.2 ABOUT HERE —

Let us first consider the time separable case (TS). In order to provide a better understanding of the internal mechanisms of the model, it is useful to first investigate the *iid* case.[12] A one standard deviation positive shock on dividend growth then translates into an increase in the permanent income of the agent. Therefore, consumption instantaneously jumps to its new steady state value. Since the utility function is time separable, the discount factor Φ_{t+1} is left unaffected by the shock. Asset prices then react one for one to dividends. The price/dividend ratio is left unaffected. Since the excess return is a function of the discount factor, the shock exerts no effect whatsoever on its dynamics. As soon as the *iid* hypothesis is relaxed, stock returns and the price–dividend ratio do react. The choice of data used to calibrate the endowment growth process affects the response of the variables of interest as serial correlation is either positive or negative. Let us first focus on consumption data. In this case (see Panel (a) of Figure 1.2), the autoregressive parameter, ρ, is positive. An increase in endowments leads the agent to expect an increase in her future consumption stream. Hence, the marginal utility of future consumption decreases and so does the discount factor. Therefore, as long as $\theta > 1$, this effect dominates the wealth effect and the price/dividend ratio decreases. Consequently, excess return raises. Note that, given the rather low value of ρ (0.34), the price–dividend ratio

[12] In order to save space we do not report the IRF for the *iid* case. They are however available from the authors upon request.

quickly converges back to its steady state level.

When endowment growth is estimated on dividend data, ρ takes on a negative value (-0.25). Therefore, a current increase in dividend growth is expected to be followed by a relative drop (See Panel (b) of Figure 1.2). Hence, the wealth effect decreases while the marginal utility of future consumption increases and so does the discount factor. Since $\theta > 1$, the latter effect takes the upper hand over the wealth effect and the price–dividend ratio raises. This effect reverses in the next period and so on. Once again, given the low value of ρ, the price–dividend ratio quickly converges back to its steady state level. Excess returns display the opposite oscillations.

Note that the impact effect of a one standard deviation positive shock on the price–dividend ratio is higher when exogenous endowment growth are estimated using dividend data as consumption growth is less volatile. For instance, the effect on the price–dividend ratio does not exceed 0.002% when the endowment growth process is estimated with consumption data compared to 0.02% when the process is estimated with dividend data.

Bringing the "Catching up with the Joneses" phenomenon into the story affects the behavior of price–dividend ratio and excess returns at short run horizon, especially when the endowment process is persistent. Since endowments are exogenously determined, the consumption path is left unaffected by this assumption. The only major difference arises on utility and asset prices, since they are now driven by the force of habit in equilibrium. Consider once again the *iid* case. The main mechanisms at work in the aftermaths of the shock are the same as in the TS version of the model. The only difference arises on impact as the habit term, $\mathbb{C}_t$, shifts and goes back to its steady state level in the next period. As the force of habit plays like the wealth effect, the price–dividend ratio raises. When the shock is not *iid*, the "Catching up with the Joneses" phenomenon can reverse the behavior of the price–dividend ratio when ρ is positive (see Panel (a) of Figure 1.2). Indeed, the force of habit then counters the stochastic discount factor effect. When φ is large enough, habit reverses the impact response of the price–dividend ratio. However, the latter effect is smoothed until the consumption goes back to its steady state level. When $\rho < 0$, the negative serial correlation of the shock shows up in the dynamics of asset prices and excess returns (see Panel (b) of Figure 1.2).

Asset prices and stock returns are largely affected by the introduction of habit stock formation. Both the size and persistence of the effects are magnified. The main mechanisms at work on are the same as in the CJ version of the model: Habits play against the discount factor effect and reverse the behavior of price–dividend ratio. But, in the short run, the magnitude of the effect is lowered by habit stock formation. More importantly, habit stock generates greater persistence than the TS and CJ versions of the model. For example, as shown in Figure 1.2, the initial increase in endowments leads to a very persistent increase in habits even when dividend growth is negatively serially correlated.[13] A direct implication of this is that the effects of habits ($\mathbb{X}_{t+1}$) on the Euler equation is persistent. This long lasting effect shows up in the evolution of the price–dividend ratio that essentially inherits the persistence of habits. A second implication of this finding is that the persistence of the forcing variable does not matter much compared to that of the internal mechanisms generated by habit stock formation. Note that stock returns are less persistent than the price–dividend ratio and that they both respond positively on impact.

The preceding discussion has important consequences for the quantitative properties of the model in terms of unconditional moments. Table 1.7 reports the mean and the standard deviation of the risk premium and the price–dividend ratio for the three versions of the model and the two calibrations of the endowment process.

— Table 1.7 about here —

[13] This persistence originates in the low depreciation rate of habits, $\delta = 0.05$.

The top panel of the table reports the unconditional moments when we calibrate the model with dividend data. It is important to note that since we are focusing on the role of persistence on predictability, we chose to shut down one channel that is usually put forward to account for predictability — time–varying risk aversion — by specifying habit formation in ratio term. By precluding time–varying risk aversion, this modeling generates no risk premium in the model. It should therefore come at no surprise that the model does not generate any risk premium both in the case of time separable utility function and habit formation model. This is confirmed by the examination of Table 1.7. For example, the average equity premium is low in the time separable model, 1.4%, and only rises to 1.49% for the habit stock model and 1.52% for the "Catching up with the Joneses" model when dividend data are used. This is at odds with the 8.35% found in the data (see Table 1.1). The results worsens when consumption data are used as consumption growth exhibits very low volatility. One way to solve the risk premium puzzle is obviously to increase θ. In order to mimic the observed risk premium, the time separable model requires to set θ to 4.5, while lower values for θ can be used in the habit stock model ($\theta = 3.6$) when the endowment process is calibrated with dividend data. Hence, the habit stock model can potentially generate higher risk premia despite risk aversion is not time–varying. It is however worth noting that the model performs well in terms of excess return volatility (17.5 in the data). The habit stock model essentially outperforms the other models in terms of price–dividend ratio, although it cannot totally match its volatility. As suggested in the IRF analysis , the time separable version of the model generates very low volatility (1.2%). The catching up with the Joneses model delivers a slightly higher volatility (around 6%). Only the habit stock model can generate substantially higher volatility of about 14%. It is also worth noting that habit persistence is crucial to induce greater volatility in the price–dividend ratio. This is illustrated in Table 1.8 which reports the volatility of the price–dividend ratio for various values of φ in the CJ model and δ in the HS model.

— TABLE 1.8 ABOUT HERE —

As can be seen from the table, larger values for φ — i.e. greater habit formation — leads to greater volatility in the price–dividend ratio, as it magnifies the force of habit and therefore increases the sensitivity of the price–dividend ratio to shocks. Likewise, the more persistent habit formation — lower δ — the more volatile is the price–dividend ratio.

As can be seen from Table 1.9, introducing habit stock formation enhances the ability of the model to account for the correlation between the price–dividend ratio and, respectively, excess return and endowment growth. For example, the correlation between the ratio and excess return is close to zero in the data. The time separable model generates far too much correlation between the two variables (-0.81 for consumption data and 0.99 for dividend data). Likewise, the catching up model produces a correlation close to unity. Conversely, the habit stock model lowers this correlation to 0.25 with dividend data and 0.15 with consumption data.

— TABLE 1.9 ABOUT HERE —

The same result obtains for the correlation between the price–dividend ratio and endowment growth. Indeed, the habit stock model introduces an additional variable that accounts for past consumption decisions and which therefore disconnects the price–dividend ratio from current endowment growth.

— TABLE 1.10 ABOUT HERE —

The model major improvements are found in the ability of the model to match serial correlation of the price–dividend ratio (see Table 1.10). The first panel of the table reports the serial correlation of the price–dividend ratio for the three versions of the model when the endowment process is calibrated with dividend data. The time separable model totally fails to account for such large and positive persistence, as the autocorrelation is negative at order 1 (-0.25) and is essentially 0 at higher orders. This in fact reflects the persistence of the exogenous forcing variable as it possesses very weak internal propagation mechanism. The "catching up with the Joneses" model fails to correct this failure as it produces exactly the same serial correlations. This can be easily shown from the solution of the model, as adding catching–up essentially re–scales the coefficient in front of the shock without adding any additional source of persistence. The habit stock model obviously performs remarkably well at the first order as the habit stock parameter δ was set in order to match the first order autocorrelation in the data (0.87 in the whole sample). More importantly, higher autocorrelations decay slowly as in the data. For instance, the model remains above 0.67 at the fifth order. Therefore, although simpler and more parsimonious, the model has similar persistence properties as Campbell and Cochrane's [1999]. The second panel of Table 1.10 reports the results from consumption data. The main conclusions remain unchanged. Only the habit stock model is able to generate a very persistent price–dividend ratio. As consumption data are more persistent, the model generates greater autocorrelations coefficients. In this case, setting $\delta = 0.2$, we would also recover the first order autocorrelation. Should endowment growth be *iid*, setting $\delta = 0.12$ would have been sufficient to generate the same persistence. This value is similar to that used by Campbell and Cochrane [1999] to calibrate the consumption surplus process with an *iid* endowment growth process.

As a final check of the model, we now compute the correlation between the price–dividend ratio in the data and in the model when observed endowment growth are used to feed the model. Table 1.11 reports the results.

— Table 1.11 about here —

As can be seen from the table, both the time separable and catching up model fail to account for the data as the correlation between the price–dividend ratio as generated by the model and its historical time series is clearly negative. This obtains no matter the sample period nor the variable used to calibrate the rate of growth of endowments. As soon as habit stock formation is brought into the model the results enhance, although not perfect, as the correlation is now clearly positive. This result is fundamentally related to habit stock formation, and more precisely to persistence. Indeed, when persistence is lowered by increasing δ, the correlation between the model and the data decreases dramatically. For instance, when $\delta = 0.1$ it falls down to zero (0.04) when endowment growth is measured using dividend data.

1.4.3 Long horizon predictability

In this section we gauge the ability of the model to account for the long horizon predictability of excess return. Table 1.12 reports the predictability tests on simulated data. More precisely, we ran regressions of the (log) excess return on the (log) price–dividend ratio evaluated at several lags (up to 7 lags)

$$er_t^k = a_k + b_k v_{t-k} + u_t^k$$

where $er_t^k \equiv \sum_{i=0}^{k-1} r_{t-i} - r_{f,t-i}$. The table reports, for each horizon k, the coefficient b_k and the R^2 of the regression which gives a measure of predictability of excess returns.

— Table 1.12 about here —

— Table 1.13 about here —

The time separable model (TS) fails to account for predictability. Although the regression coefficients, b_k, have the right sign, they are too large compared to those reported in Table 1.4 and remain almost constant as the horizon increases. The predictability measure, R^2, is higher when regression horizon is limited to one period and then falls to essentially 0 whatever the horizon. This should come as no surprise as the impulse response analysis showed that the price–dividend ratio and excess returns both respond very little and monotonically to a shock on dividend growth. A first implication of the little responsiveness of the price–dividend ratio is that the model largely overestimates the coefficient b_k in the regression (around -4). A second implication is its tiny predictive power especially at long horizon, as the R^2 is around 0. This obtains whatever the data we consider. One potential way to enhance the ability of the model to account for predictability may be to manipulate the degree of risk aversion. This experiment is reported in the first panel of Table 1.13. When the degree of risk aversion is below unity, the model totally fails to match the data as all coefficients have the wrong sign and the R^2 is essentially nil whatever the horizon. When θ is raised toward 5, we recover the negative relationship between excess returns and the price–dividend ratio, but the predictability is a decreasing function of the horizon which goes opposite to the data (see Table 1.4).

As shown in the IRF analysis, the “catching up with the Joneses” model possesses slightly stronger propagation mechanisms. This enhances its ability to account for predictability. However, the price–dividend ratio and the excess returns respond only at short run horizon to a positive shock to endowments. In Table 1.12 and 1.13, we report predictability tests for this version of the model for several values of the habit persistence parameter φ. The first striking result is that allowing for “catching up” indeed improves the long horizon predictive power of the model. More precisely, the coefficients of the regression are decreasing with the force of habit. For instance, when $\varphi = 1$, the coefficient b_1 drops to -2.18, to be compared with -3.79 when $\varphi = 0.1$ —low habit persistence — and -4.69 in the time separable model when endowment corresponds to dividend. It should however be noted that, as the horizon increases, b_k remains constant which is at odds with the empirical evidence (see Table 1.4). Moreover, the patterns of R^2 is reversed as it decreases with horizon. Hence, the catching–up model cannot account for predictability for any value of φ although the results improve with larger φ. This comes at not surprise as this version of the model cannot generates greater persistence than the TS model.

In the last series of results, we consider the habit stock version of the model. Results reported in Table 1.12 show that our benchmark version of the habit stock model enhances the ability of the CCAPM to account for predictability compared to the TS and CJ versions of the model. First of all, estimated values of b_k are much closer to those estimated on empirical data (see Table 1.4). For instance, in the short sample where predictability is really significant, b_1 is -0.36 to be compared with -0.44 found in the model. Likewise, at longer horizon, b_7 is -1.01 in the data versus -1.2 in the model. Therefore, the model matches the overall size of the coefficients, and reproduces their evolution with the horizon. It should also be noted that, as found in the data, the R^2 is an increasing function of the horizon. Table 1.13 shows that what really determines the result is persistence. Indeed, lowering the force of habit — setting a lower φ — while maintaining the same persistence (experiment (φ,δ)=(0.5,0.05)) does not deteriorate too much the results. The shape of the coefficients and the R^2 is maintained. Conversely, reducing persistence —higher δ— while maintaining the force of habit (experiment (φ,δ)=(1,0.5)) dramatically affects the results. First of all, the model totally fails to match the scale of the b_k's. Second, predictability diminishes with the horizon.

Concluding Remarks

This paper investigates the role of persistence in accounting for the predictability of excess return. We first develop a standard consumption based asset pricing model à la Lucas [1978] taking "catching up with the Joneses" and habit stock formation into account. Providing we keep with the assumption of first order Gaussian endowment growth and formulate habit formation in terms of ratio, we are able to provide a closed form solution for the price–dividend ratio. We also provide conditions that guarantee the existence of bounded solutions. We then assess the performance of the model in terms of moment matching. In particular, we evaluate the ability of the model to generate persistence and explain the predictability puzzle. We then show that the habit stock version of the model outperforms the time separable and the catching up versions of the model in accounting for predictability of excess returns. Since risk aversion is held constant in the model, this result stems from the greater persistence habit stock generates.

Appendix A. Asset Pricing Models with Internal Habit

Appendix A provides a closed–form solution to a standard asset pricing model with *internal* habit formation when the growth rate of endowment follows a first–order Gaussian autoregressive process. More details on the proofs of the main results can be found in Collard, Fève and Ghattassi [2006a].

We consider the problem of an infinitely–lived representative agent who derives utility from consuming a single consumption good. The agent has preferences over both her current and past consumption. She determines her consumption, asset holdings plans so as to maximize the expected sum of discounted future utility

$$\max E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{\mathbb{C}_{t+s}^{1-\theta} - 1}{1-\theta} \right) \tag{1.4.22}$$

subject to her budget constraint the budget constraint

$$P_t S_{t+1} + C_t \leqslant (P_t + D_t) S_t \tag{1.4.23}$$

where $\mathbb{C}_t \equiv C_t / C_{t-1}^{\varphi}$. C_t denotes the agent's consumption of a single perishable good at date t. $E_t(.)$ denotes mathematical conditional expectations. Expectations are conditional on information available at the beginning of period t. $\beta > 0$ is a subjective constant discount factor, $\theta > 0$ denotes the curvature parameter and $\varphi \in [0,1]$ is the habit persistence parameter.

The first order condition that determines the agent's consumption choices is given by

$$\frac{C_t^{-\theta}}{C_{t-1}^{\varphi(1-\theta)}} - \beta\varphi E_t \left[\frac{C_{t+1}^{1-\theta}}{C_t^{\varphi(1-\theta)+1}} \right] = \beta E_t \left[\left(\frac{P_{t+1} + D_{t+1}}{P_t} \right) \left(\frac{C_{t+1}^{-\theta}}{C_t^{\varphi(1-\theta)}} - \beta\varphi \frac{C_{t+2}^{1-\theta}}{C_{t+1}^{\varphi(1-\theta)+1}} \right) \right] \tag{1.4.24}$$

In an equilibrium, $S_t = 1$ for all t so that $C_t = D_t$. Then equation (1.4.24) rewrites:

$$\frac{D_t^{-\theta}}{D_{t-1}^{\varphi(1-\theta)}} - \beta\varphi E_t \left[\frac{D_{t+1}^{1-\theta}}{D_t^{\varphi(1-\theta)+1}} \right] = \beta E_t \left[\left(\frac{P_{t+1} + D_{t+1}}{P_t} \right) \left(\frac{D_{t+1}^{-\theta}}{D_t^{\varphi(1-\theta)}} - \beta\varphi \frac{D_{t+2}^{1-\theta}}{D_{t+1}^{\varphi(1-\theta)+1}} \right) \right] \tag{1.4.25}$$

The growth rate of the endowment $\gamma_t \equiv \log(D_t/D_{t-1})$ is assumed to follow an AR(1) process

$$\gamma_t = \rho\gamma_{t-1} + (1-\rho)\overline{\gamma} + \varepsilon_t$$

where $|\rho| < 1$ and ε_t is an iid process. Furthermore, we depart from the commonly adopted Gaussian distribution assumption and rather consider a truncated normal distribution, such that ε_t is distributed as a truncated gaussian distribution over the support $[-\overline{\varepsilon}, \overline{\varepsilon}]$. The latter restriction implies $E(\varepsilon) = 0$. We further denote $E(\varepsilon^2) = \sigma(\overline{\varepsilon})^2$. As previously noted by Abel [1990], the asset pricing model with habit persistence does not preclude the existence of negative asset prices. The possibility of negative prices comes from *(i)* the log–normal assumption and *(ii)* the marginal utility of consumption which can be negative when the habit persistence and/or relative risk aversion are too large. Using the Markov chain approximation of the law of motion of γ_t, Abel shows that it is possible to determine the upper and lower bounds on the process that guarantee positive prices. Truncating the support of the distribution amounts to impose such a restriction provided $\overline{\varepsilon}$ is not too large. Note that this assumption can be relaxed when $\varphi = 0$ as the price is always positive since the model just reduces to a time separable model.

In order to characterize a solution for equation (1.4.25), it is convenient to rewrite the Euler equation. We first denote $v_t \equiv P_t/D_t$ as the price–dividend ratio. Second, we define $z_t \equiv \exp((1-\theta)\gamma_t - \varphi(1-\theta)\gamma_{t-1})$ and $y_t \equiv v_t[1 - \beta\varphi E_t z_{t+1}]$. It follows that equation (1.4.25) rewrites

$$y_t = \beta E_t \left(1 - \beta\varphi z_{t+2} + y_{t+1}\right) z_{t+1} \tag{1.4.26}$$

Equation (1.4.26) has to be solved for y_t. This forward looking stochastic difference equation admits an exact solution reported in the next proposition.

Proposition 1.4.7 *The equilibrium price–dividend ratio is given by*

$$\frac{P_t}{D_t} = \frac{\beta\varphi\chi\exp(a_0 + b_0(\gamma_t - \overline{\gamma})) + (1-\varphi)\displaystyle\sum_{i=1}^{\infty}(\beta\chi)^i \exp(a_i + b_i(\gamma_t - \overline{\gamma}))}{1 - \beta\varphi\chi\exp(a_0 + b_0(\gamma_t - \overline{\gamma}))} \tag{1.4.27}$$

where

$$a_0 = (1-\theta)(1-\varphi)\overline{\gamma} + (1-\theta)^2\frac{\sigma(\overline{\varepsilon})^2}{2} \qquad\qquad b_0 = (1-\theta)(\rho-\varphi)$$

and

$$\begin{aligned} a_i &= (1-\theta)(1-\varphi)\overline{\gamma}i \\ &\quad + \left(\frac{1-\theta}{1-\rho}\right)^2\frac{\sigma(\overline{\varepsilon})^2}{2}\left[(1-\varphi)^2 i - 2\frac{(1-\varphi)(\rho-\varphi)}{1-\rho}(1-\rho^i) + \frac{(\rho-\varphi)^2}{1-\rho^2}(1-\rho^{2i})\right] \\ b_i &= \frac{(1-\theta)(\rho-\varphi)}{1-\rho}(1-\rho^i) \qquad \text{for} \quad i \geqslant 1 \end{aligned}$$

and

$$\chi = \left(\frac{\Phi\left(\frac{\overline{\varepsilon}-\sigma(\overline{\varepsilon})^2}{\sigma(\overline{\varepsilon})}\right) - \Phi\left(\frac{-\overline{\varepsilon}-\sigma(\overline{\varepsilon})^2}{\sigma(\overline{\varepsilon})}\right)}{\Phi\left(\frac{\overline{\varepsilon}}{\sigma(\overline{\varepsilon})}\right) - \Phi\left(\frac{-\overline{\varepsilon}}{\sigma(\overline{\varepsilon})}\right)}\right) \leqslant 1$$

where $\Phi(\cdot)$ *denotes the cdf of the gaussian distribution.*

Equation (1.4.27) nests many asset pricing formula. For instance, setting $\theta = 1$ (logarithmic utility function), the price–dividend ratio is constant for all states of the nature as $P_t/D_t = \beta\chi/(1-\beta\chi)$. Then habit persistence does not matter for the behavior of the price–dividend ratio, as the parameter φ does not enter in the pricing formula. However, the constant term χ still distorts the price–dividend ratio, as the agents must formulate forecasts on the stream of discounted dividends using a truncated distribution. Nevertheless, as $\overline{\varepsilon} \to +\infty$, χ tends to 1, the price–dividend ratio tends to the usual $P_t/D_t = \beta/(1-\beta)$.

Setting $\varphi = 0$ — *i.e.* imposing time separability in preferences — we recover Burnside's [1998] solution. In this case, the price–dividend ratio rewrites:

$$\frac{P_t}{D_t} = \sum_{i=1}^{\infty}(\beta\chi)^i \exp(a_i + b_i(\gamma_t - \overline{\gamma}))$$

where $a_i = (1-\theta)\overline{\gamma}i + \left(\frac{1-\theta}{1-\rho}\right)^2\frac{\sigma(\overline{\varepsilon})^2}{2}\left[i - 2\frac{\rho}{1-\rho}(1-\rho^i) + \frac{\rho^2}{1-\rho^2}(1-\rho^{2i})\right]$ and $b_i = \frac{(1-\theta)\rho}{1-\rho}(1-\rho^i)$. Once again the truncation assumption affects the solution as long as $\overline{\varepsilon} < +\infty$.

Finally, when the rate of growth of endowments is *iid* over time ($\gamma_t = \overline{\gamma} + \varepsilon_t$) and φ is set to 1, we recover the solution used by Abel [1990] to compute unconditional expected returns (see Table 1, panel C, p 41)

$$\frac{P_t}{D_t} = \frac{z_t}{1 - z_t} \tag{1.4.28}$$

with $z_t = \beta\chi \exp\left((1-\theta)^2 \frac{\sigma(\overline{\varepsilon})^2}{2} + (\theta - 1)(\gamma_t - \overline{\gamma})\right)$. In this latter case, as equation (1.4.28) makes it clear, the price–dividend ratio is an increasing (resp. decreasing) and convex function of consumption growth if $\theta > 1$ (resp. $\theta < 1$). In other words, only the position of the curvature parameter around unity matters.

Note that the solution for the price–dividend ratio involves a series, which convergence properties have not been yet discussed. The following proposition determines a necessary and sufficient condition for the existence of a stationary bounded equilibrium.

Proposition 1.4.8 *The series in (1.4.27) converges if and only if*

$$r \equiv \beta\chi \exp\left[(1-\theta)(1-\varphi)\overline{\gamma} + \frac{\sigma(\overline{\varepsilon})^2}{2}\left(\frac{(1-\theta)(1-\varphi)}{1-\rho}\right)^2\right] < 1 \tag{1.4.29}$$

This proposition makes clear that as in Burnside [1998], $\beta < 1$ is neither necessary nor sufficient to insure finite asset prices. Moreover, it shows that habit persistence help guaranteing a bounded solution. For instance, let us focus on the case $\theta > 1$ and first consider the time separable case (φ=0). If the future path of endowment is uncertain, risk adverse consumers (θ very large) are willing to purchase a large amount of assets today to insure themselves against future bad outcomes — *i.e.* the series goes to infinity. Conversely, when habit persistence is strong enough (large φ), the solution is bounded as the effect of uncertainty is lowered by the smoother consumption path, even for large value of θ. In the limiting case where $\varphi = 1$, the price–dividend ratio takes the form of equation (1.4.28) and therefore the series drops out as the forecasting horizon reduces to one period ahead. Otherwise stated, discounted future risk would be inconsequential. Also note that truncation of the distribution makes conditions for boundedness less demanding as a lower $\bar{\varepsilon}$ reduces the overall volatility of dividends and therefore reduces risk. Households are therefore less willing to purchase assets, which puts downward pressure on asset prices.

Endowed with conditions for boundedness, we are now in a position to discuss the form of the solution. Let us consider the general model, where endowments are serially correlated ($|\rho| \leqslant 1$) and φ is not restricted to either 0 or 1 (see Abel [1990]). In this case, as can be seen from the form of parameter b_i, both the position of the curvature parameter, θ, around 1 and the position of the habit persistence parameter, φ, around ρ matter. This is illustrated in Figure 1.5 that reports the price–dividend ratio as a function of dividend growth for different values for φ.

— FIGURE 1.5 ABOUT HERE —

As can be seen from Figure 1.5, when $\theta > 1$ (resp. $\theta < 1$), the decision rule is increasing (resp. decreasing) with dividend growth when $\varphi > \rho$ (resp. $\varphi < \rho$). The economic intuition underlying this result is clear. Let us consider the case $\theta > 1$ A shock on dividends exerts three effects: *(i)* a standard wealth effect, *(ii)* a stochastic discount factor effect and *(iii)* a habit persistence effect. The two latter effects play in opposite direction on intertemporal substitution. When $\varphi > \rho$, the stochastic discount factor effect is dominated by the force of habits, as the shock on dividend growth exhibits less persistence than habits. Therefore, the second and the third effects partially offset each other and the wealth effect plays a greater role. The price–dividend ratio increases. Conversely, when $\varphi < \rho$ habit persistence cannot counter the effects of expected stochastic discounting, and intertemporal substitution motives take the upper hand. The price–dividend ratio decreases. Note that in the limiting case where $\rho = \varphi$ (plain dark line in Figure 1.5) the persistence of dividend growth exactly offsets the effects of habit persistence and all three effects cancel out. Therefore, just alike the case of a logarithmic utility function, the price–dividend ratio is constant. The reasoning is reversed when $\theta < 1$ (see left panel of Figure 1.5).

A final remark regards the numerical accuracy of the solution. Indeed, although we have a closed–form solution, it involves an infinite series that cannot be exactly computed as it requires truncation. Nevertheless, we can determine the truncation breakpoint that yields an arbitrarily small error. Let us focus on the infinite series and denote the truncated series at horizon K by S_K, such that $S_K = \sum_{i=1}^{K} (\beta\chi)^i \exp(a_i + b_i(\gamma_t - \overline{\gamma}))$. One way to determine a truncation point is to select K such that $\mathbb{P}(\Delta S_K \geqslant \delta) \leqslant \mathbb{E}_t a$ where $\Delta S_K = S_K - S_{K-1}$ and $\delta, \mathbb{E}_t a > 0$. Since $\Delta S_K = \beta^K \exp(a_K + b_K(\gamma_t - \overline{\gamma})) > 0$, Markov's inequality implies that

$$\mathbb{P}(\Delta S_K \geqslant \delta) < \frac{E(\Delta S_K)}{\delta} = \frac{(\beta\chi)^K}{\delta} \exp\left(a_K + \frac{b_K^2}{2} \frac{\sigma(\overline{\varepsilon})^2}{1-\rho^2}\right)$$

It is then easy to select K such that $\frac{(\beta\chi)^K}{\delta} \exp\left(a_K + \frac{b_K^2}{2} \frac{\sigma(\overline{\varepsilon})^2}{1-\rho^2}\right) < \mathbb{E}_t a$, where δ may be given by machine precision and $\mathbb{E}_t a$ a low enough probability.

Appendix B. Distributions

— Figure 1.3 about here —

— Figure 1.4 about here —

Appendix C. Proof of Propositions

Proposition 1.2.1: See proof of Proposition 1.3.4

Proposition 1.2.2: See proof of Proposition 1.3.5

Proposition 1.2.3: See proof of Proposition 1.3.6

Proposition 1.3.4: First of all note that setting $\delta = 1$ in this proof, we obtain a proof for Proposition 1.2.1. Let us denote $v_t = P_t/D_t$ the price–dividend ratio, and $z_t = \log(X_t/D_t)$ the habit to dividend ratio. Finally, letting $y_t = v_t \exp(-\varphi(1-\theta)z_t)$, the agent's Euler equation rewrites

$$y_t = \beta E_t \left[\exp((1-\theta)(1-\varphi)\gamma_{t+1} - \varphi(1-\theta)z_{t+1}) + \exp((1-\theta)(1-\varphi)\gamma_{t+1})y_{t+1}\right]$$

Iterating forward, and imposing the transversality condition, a solution to this forward looking stochastic difference equation is given by

$$y_t = E_t \sum_{i=1}^{\infty} \beta^i \exp\left((1-\theta)(1-\varphi)\sum_{j=1}^{i}\gamma_{t+j} - \varphi(1-\theta)z_{t+i}\right) \tag{1.4.30}$$

Note that, the definition of z_t and the law of motion of habits imply that z_t evolves as

$$z_{t+1} = (1-\delta)z_t - \gamma_{t+1} \tag{1.4.31}$$

which implies that

$$z_{t+i} = (1-\delta)^i z_t - \sum_{j=0}^{i-1}(1-\delta)^j \gamma_{t+i-j} \tag{1.4.32}$$

Plugging the latter result in (1.4.30), we get

$$y_t = E_t \sum_{i=1}^{\infty} \beta^i \exp\left((1-\theta)\sum_{j=1}^{i}\left((1-\varphi) + \varphi(1-\delta)^{i-j}\right)\gamma_{t+j} - \varphi(1-\theta)(1-\delta)^i z_t\right)$$

Let us focus on the particular component of the solution

$$\mathbb{G} \equiv E_t \exp\left((1-\theta)\sum_{j=1}^{i}\left((1-\varphi) + \varphi(1-\delta)^{i-j}\right)\gamma_{t+j}\right)$$

Since we assumed that dividend growth is normally distributed, making use of standard results on log–normal distributions, we have that

$$\mathbb{G} = \exp\left(\mathbb{E} + \frac{\mathbb{V}}{2}\right)$$

where

$$\mathbb{E} = E_t\left((1-\theta)\sum_{j=1}^{i}\left((1-\varphi) + \varphi(1-\delta)^{i-j}\right)\gamma_{t+j}\right)$$

and

$$\mathbb{V} = \mathrm{Var}_t\left((1-\theta)\sum_{j=1}^{i}\left((1-\varphi)+\varphi(1-\delta)^{i-j}\right)\gamma_{t+j}\right)$$

Since γ_t follows an AR(1) process, we have

$$\gamma_{t+j} = \overline{\gamma} + \rho^j(\gamma_t - \overline{\gamma}) + \sum_{k=0}^{j-1}\rho^k\varepsilon_{t+j-k}$$

such that

$$\begin{aligned}
\mathbb{E} =& E_t\left[(1-\theta)\sum_{j=1}^{i}\left((1-\varphi)+\varphi(1-\delta)^{i-j}\right)\left(\overline{\gamma}+\rho^j(\gamma_t-\overline{\gamma})+\sum_{k=0}^{j-1}\rho^k\varepsilon_{t+j-k}\right)\right]\\
=& \left[(1-\theta)\sum_{j=1}^{i}\left((1-\varphi)+\varphi(1-\delta)^{i-j}\right)\left(\overline{\gamma}+\rho^j(\gamma_t-\overline{\gamma})\right)\right]\\
=&(1-\theta)(1-\varphi)\sum_{j=1}^{i}\left(\overline{\gamma}+\rho^j(\gamma_t-\overline{\gamma})\right)+(1-\theta)\varphi\sum_{j=1}^{i}(1-\delta)^{i-j}\left(\overline{\gamma}+\rho^j(\gamma_t-\overline{\gamma})\right)\\
=&(1-\theta)\overline{\gamma}\left[(1-\varphi)i+\frac{\varphi}{\delta}(1-(1-\delta)^i)\right]+(1-\theta)\left[\frac{\rho(1-\varphi)}{1-\rho}(1-\rho^i)+\frac{\varphi\rho}{1-\delta-\rho}((1-\delta)^i-\rho^i)\right](\gamma_t-\overline{\gamma})
\end{aligned}$$

The calculation of the conditional variance is a bit more tedious.

$$\mathbb{V} = \mathrm{Var}_t\left[(1-\theta)\sum_{j=1}^{i}\left(1-\varphi+\varphi(1-\delta)^{i-j}\right)\sum_{k=0}^{j-1}\rho^k\varepsilon_{t+j-k}\right]$$

which after some accounting rewrites as

$$\begin{aligned}
=&\mathrm{Var}_t\left[(1-\theta)\sum_{j=0}^{i-1}\left(\frac{1-\varphi}{1-\rho}(1-\rho^{i-j})+\frac{\varphi}{1-\delta-\rho}((1-\delta)^{i-j}-\rho^{i-j})\right)\varepsilon_{t+j+1}\right]\\
=&(1-\theta)^2\sigma^2\sum_{j=0}^{i-1}\left(\frac{1-\varphi}{1-\rho}(1-\rho^{i-j})+\frac{\varphi}{1-\delta-\rho}((1-\delta)^{i-j}-\rho^{i-j})\right)^2\\
=&(1-\theta)^2\sigma^2\left[\left(\frac{1-\varphi}{1-\rho}\right)^2\sum_{k=1}^{i}(1-\rho^k)^2+2\frac{\varphi(1-\delta)}{(1-\rho)(1-\delta-\rho)}\sum_{k=1}^{i}(1-\rho^k)((1-\delta)^k-\rho^k)\right.\\
&\left.+\left(\frac{\varphi}{1-\delta-\rho}\right)^2\sum_{k=1}^{i}((1-\delta)^k-\rho^k)^2\right]
\end{aligned}$$

Calculating all the infinite series, we end–up with

$$
\begin{aligned}
\mathbb{V} =&(1-\theta)^2\sigma^2\Bigg\{\left(\frac{1-\varphi}{1-\rho}\right)^2\left(i-2\frac{\rho}{1-\rho}(1-\rho^i)+\frac{\rho^2}{1-\rho^2}(1-\rho^{2i})\right)\\
&+2\frac{\varphi(1-\varphi)}{(1-\rho)(1-\delta-\rho)}\Bigg(\frac{(1-\delta)}{\delta}(1-(1-\delta)^i)-\frac{\rho}{1-\rho}(1-\rho^i)-\frac{\rho(1-\delta)}{1-\rho(1-\delta)}(1-(\rho(1-\delta))^i)\\
&+\frac{\rho^2}{1-\rho^2}(1-\rho^{2i})\Bigg)+\frac{\varphi^2}{(1-\delta-\rho)^2}\Bigg(\frac{(1-\delta)^2}{1-(1-\delta)^2}(1-(1-\delta)^{2i})-2\frac{\rho(1-\delta)}{1-\rho(1-\delta)}(1-(\rho(1-\delta))^i)\\
&+\frac{\rho^2}{1-\rho^2}(1-\rho^{2i})\Bigg)\Bigg\}
\end{aligned}
$$

Therefore, the solution is given by

$$y_t=\sum_{i=1}^{\infty}\beta^i\exp\left(a_i+b_i(\gamma_t-\overline{\gamma})+\widetilde{c}_i z_t\right)$$

where

$$
\begin{aligned}
a_i =&(1-\theta)\overline{\gamma}\left[(1-\varphi)i+\frac{\varphi}{\delta}(1-(1-\delta)^i)\right]+\frac{\mathbb{V}}{2}\\
b_i =&(1-\theta)\left[\frac{\rho(1-\varphi)}{1-\rho}(1-\rho^i)+\frac{\varphi\rho}{1-\delta-\rho}((1-\delta)^i-\rho^i)\right]\\
\widetilde{c}_i =&-\varphi(1-\theta)(1-\delta)^i
\end{aligned}
$$

recall that $y_t=v_t\exp(-\varphi(1-\theta)z_t)$, such that the price–dividend ratio is finally given by

$$v_t=\exp(\varphi(1-\theta)z_t)\sum_{i=1}^{\infty}\beta^i\exp\left(a_i+b_i(\gamma_t-\overline{\gamma})+\widetilde{c}_i z_t\right)$$

or

$$v_t=\sum_{i=1}^{\infty}\beta^i\exp\left(a_i+b_i(\gamma_t-\overline{\gamma})+c_i z_t\right)$$

where $c_i=\varphi(1-\theta)(1-(1-\delta)^i)$.

Proposition 1.3.5: First of all note that setting $\delta=1$ in this proof, we obtain a proof for Proposition 1.2.2. Let us define

$$w_i=\beta^i\exp(a_i+b_i(\gamma_t-\overline{\gamma})+c_i z_t)$$

where a_i, b_i and c_i are obtained from the previous proposition. Then, the price–dividend ratio rewrites

$$v_t=\sum_{i=1}^{\infty}w_i$$

It follows that

$$\left|\frac{w_{i+1}}{w_i}\right|=\beta\exp(\Delta a_{i+1}+\Delta b_{i+1}(\gamma_t-\overline{\gamma})+\Delta c_{i+1}z_t)$$

where

$$\begin{aligned}
\Delta a_{i+1} =&(1-\theta)\overline{\gamma}\left[(1-\varphi)+\varphi(1-\delta)^i\right]+(1-\theta)^2\frac{\sigma^2}{2}\Bigg\{\left(\frac{1-\varphi}{1-\rho}\right)^2\left(1-2\rho^{i+1}+\rho^{2(i+1)}\right)\\
&+2\frac{\varphi(1-\varphi)}{(1-\rho)(1-\delta-\rho)}\left((1-\delta)^{i+1}-\rho^{i+1}-(\rho(1-\delta))^{i+1}+\rho^{2(i+1)}\right)\\
&+\frac{\varphi^2}{(1-\delta-\rho)^2}\left((1-\delta)^{2(i+1)}-2(\rho(1-\delta))^{i+1}+\rho^{2(i+1)}\right)\Bigg\}\\
\Delta b_{i+1} =&(1-\theta)\left[(1-\varphi)\rho^{i+1}-\frac{\varphi\rho}{1-\delta-\rho}((1-\rho)\rho^i-\delta(1-\delta)^i)\right]\\
\Delta c_{i+1} =&\varphi(1-\theta)\delta(1-\delta)^i
\end{aligned}$$

Also note that provided $|\rho|<1$ and $\delta\in(0,1)$, we have

$$\begin{aligned}
&\lim_{i\to\infty}\Delta a_{i+1}=(1-\theta)\overline{\gamma}(1-\varphi)+(1-\theta)^2\frac{\sigma^2}{2}\left(\frac{1-\varphi}{1-\rho}\right)^2\\
&\lim_{i\to\infty}\Delta b_{i+1}(\gamma_t-\overline{\gamma})=0\\
&\lim_{i\to\infty}\Delta c_{i+1}z_t=0
\end{aligned}$$

Therefore

$$\lim_{i\to\infty}\left|\frac{w_{i+1}}{w_i}\right|=r\equiv\beta\exp\left((1-\theta)\overline{\gamma}(1-\varphi)+(1-\theta)^2\frac{\sigma^2}{2}\left(\frac{1-\varphi}{1-\rho}\right)^2\right)$$

Using the ratio test, we now face three situations:

i) When $r<1$, then $\lim_{i\to\infty}\left|\frac{w_{i+1}}{w_i}\right|<1$ and the ratio test implies that $\sum_{i=1}^{\infty}w_i$ converges.

ii) When $r>1$, the ratio test implies that $\sum_{i=1}^{\infty}w_i$ diverges.

iii) When $r=1$, the ratio test is inconclusive. But, if $r=1$, we know that

$$\exp\left((1-\theta)(1-\varphi)\overline{\gamma}+\left(\frac{(1-\theta)(1-\varphi)}{1-\rho}\right)^2\frac{\sigma^2}{2}\right)=\frac{1}{\beta}$$

and the parameter a_i rewrites

$$\begin{aligned}
a_i &= \left((1-\theta)(1-\varphi)\overline{\gamma}+\left(\frac{(1-\theta)(1-\varphi)}{1-\rho}\right)^2\frac{\sigma^2}{2}\right)i\\
&\quad+\left(\frac{1-\theta}{1-\rho}\right)^2\frac{\sigma^2}{2}\left[\frac{(\rho-\varphi)^2}{1-\rho^2}(1-\rho^{2i})-2\frac{(1-\varphi)(\rho-\varphi)}{1-\rho}(1-\rho^i)\right]\\
&= -\log(\beta)i+\left(\frac{1-\theta}{1-\rho}\right)^2\frac{\sigma^2}{2}\left[\frac{(\rho-\varphi)^2}{1-\rho^2}(1-\rho^{2i})-2\frac{(1-\varphi)(\rho-\varphi)}{1-\rho}(1-\rho^i)\right]
\end{aligned}$$

After replacement in w_i, we get:

$$w_i=\exp(\tilde{a}_i+b_i(\gamma_t-\overline{\gamma})+c_iz_t)$$

where

$$\widetilde{a}_i = \left(\frac{1-\theta}{1-\rho}\right)^2 \frac{\sigma^2}{2}\left[\frac{(\rho-\varphi)^2}{1-\rho^2}(1-\rho^{2i}) - 2\frac{(1-\varphi)(\rho-\varphi)}{1-\rho}(1-\rho^i)\right]$$

Since $\lim_{i\to\infty}|\widetilde{a}_i| = \left|\left(\frac{1-\theta}{1-\rho}\right)^2 \frac{\sigma^2}{2}\left[\frac{(\rho-\varphi)^2}{1-\rho^2} - 2\frac{(1-\varphi)(\rho-\varphi)}{1-\rho}\right]\right| > 0$, then the series $v_t = \sum_{i=1}^{\infty} w_i$ diverges.

Therefore, $r < 1$ is the only situation where a stationary bounded equilibrium exists.

Proposition 1.3.6: First of all note that setting $\delta = 1$ in this proof, we obtain a proof for Proposition 1.2.3. We first deal with the average of the price/dividend ratio. We want to compute

$$\mathbb{E}(v_t) = \mathbb{E}\left(\sum_{i=1}^{\infty}\beta^i \exp(a_i + b_i(\gamma_t - \overline{\gamma}) + c_i z_t\right) = \sum_{i=1}^{\infty}\beta^i E\left(\exp(a_i + b_i(\gamma_t - \overline{\gamma}) + c_i z_t\right)$$

By the log–normality of γ_t, we know that

$$\mathbb{E}\left(\exp(a_i + b_i(\gamma_t - \overline{\gamma}) + c_i z_t\right) = \exp\left(\mathbb{E}_i + \frac{\mathbb{V}_i}{2}\right)$$

where

$$\mathbb{E}_i = E(a_i + b_i(\gamma_t - \overline{\gamma}) + c_i z_t) = a_i + c_i E(z_t)$$

and

$$\mathbb{V}_i = Var(a_i + b_i(\gamma_t - \overline{\gamma}) + c_i z_t) = b_i^2\frac{\sigma^2}{1-\rho^2} + c_i^2\text{Var}(z_t) + 2c_i b_i \text{Cov}(z_t, \gamma_t)$$

Recall that $z_t = (1-\delta)z_{t-1} - \gamma_t$, therefore

$$E(z_t) = -\frac{\overline{\gamma}}{\delta}$$

and

$$\text{Cov}(z_t, \gamma_t) = \text{Cov}((1-\delta)z_{t-1} - \gamma_t, \gamma_t) = (1-\delta)\rho\text{Cov}(z_{t-1}, \gamma_{t-1}) - \text{Var}(\gamma_t)$$

Hence

$$\text{Cov}(z_t, \gamma_t) = -\frac{\sigma^2}{(1-\rho(1-\delta))(1-\rho^2)}$$

Furthermore, we know that

$$\begin{aligned}\text{Var}(z_t) &= (1-\delta)^2\text{Var}(z_t) + \text{Var}(\gamma_t) - 2(1-\delta)\text{Cov}(z_{t-1}, \gamma_t) \\ &= \frac{\sigma^2}{(1-\rho^2)(1-(1-\delta)^2)}\left[\frac{1+\rho(1-\delta)}{1-\rho(1-\delta)}\right]\end{aligned}$$

Therefore

$$\mathbb{V}_i = b_i^2\frac{\sigma^2}{1-\rho^2} + c_i^2\frac{\sigma^2}{(1-\rho^2)(1-(1-\delta)^2)}\left[\frac{1+\rho(1-\delta)}{1-\rho(1-\delta)}\right] - 2c_i b_i\frac{\sigma^2}{(1-\rho(1-\delta))(1-\rho^2)}$$

Hence we have to study the convergence of the series

$$\sum_{i=1}^{\infty}\beta^i\exp\left(a_i-c_i\frac{\overline{\gamma}}{\delta}+\frac{\sigma^2}{2(1-\rho^2)}\left(b_i^2+\frac{1+\rho(1-\delta)}{(1-\rho(1-\delta))(1-(1-\delta)^2)}c_i^2-\frac{2b_ic_i}{1-\rho(1-\delta)}\right)\right)$$

Defining

$$w_i=\beta^i\exp\left(a_i-c_i\frac{\overline{\gamma}}{\delta}+\frac{\sigma^2}{2(1-\rho^2)}\left(b_i^2+\frac{1+\rho(1-\delta)}{(1-\rho(1-\delta))(1-(1-\delta)^2)}c_i^2-\frac{2b_ic_i}{1-\rho(1-\delta)}\right)\right)$$

the series rewrites $\sum_{i=1}^{\infty}w_i$, whose convergence properties can be studied relying on the ratio test.

$$\left|\frac{w_{i+1}}{w_i}\right|=\beta\exp\left(\Delta a_{i+1}-\Delta c_{i+1}\frac{\overline{\gamma}}{\delta}+\frac{\sigma^2}{2(1-\rho^2)}\left(\Delta b_{i+1}^2+\frac{1+\rho(1-\delta)}{(1-\rho(1-\delta))(1-(1-\delta)^2)}\Delta c_{i+1}^2-\frac{2\Delta(b_{i+1}c_{i+1})}{1-\rho(1-\delta)}\right)\right)$$

Given the previously given definition of a_i, b_i and c_i, we have

$$\begin{aligned}
\Delta a_{i+1}=&(1-\theta)\overline{\gamma}\left[(1-\varphi)+\varphi(1-\delta)^i\right]+(1-\theta)^2\frac{\sigma^2}{2}\left\{\left(\frac{1-\varphi}{1-\rho}\right)^2\left(1-2\rho^{i+1}+\rho^{2(i+1)}\right)\right.\\
&+2\frac{\varphi(1-\varphi)}{(1-\rho)(1-\delta-\rho)}\left((1-\delta)^{i+1}-\rho^{i+1}-(\rho(1-\delta))^{i+1}+\rho^{2(i+1)}\right)\\
&\left.+\frac{\varphi^2}{(1-\delta-\rho)^2}\left((1-\delta)^{2(i+1)}-2(\rho(1-\delta))^{i+1}+\rho^{2(i+1)}\right)\right\}\\
\Delta b_{i+1}^2=&(1-\theta)^2\left[\left(\frac{\rho(1-\varphi)}{1-\rho}\right)^2(2\rho^i(1-\rho)-\rho^{2i}(1-\rho^2))\right.\\
&+\left(\frac{\varphi\rho}{1-\delta-\rho}\right)^2\left(2(\rho(1-\delta))^i(1-\rho(1-\delta))-(1-\delta)^2(1-(1-\delta)^2)-\rho^{2i}(1-\rho^2)\right)\\
&\left.+\frac{2\rho^2\varphi(1-\delta)}{(1-\rho)(1-\delta-\rho)}\left(\rho^i(1-\rho)-\delta(1-\delta)^i+(\rho(1-\delta))^i(1-\rho(1-\delta))-\rho^{2i}(1-\rho^2)\right)\right]\\
\Delta c_{i+1}^2=&(\varphi(1-\theta))^2(2\delta(1-\delta)^i-(1-\delta)^{2i}(1-(1-\delta)^2))\\
\Delta(b_ic_i)=&\varphi(1-\theta)^2\left[(1-\varphi)\rho^{i+1}+\frac{\varphi\rho}{1-\delta-\rho}\left(\rho^i(1-\rho)-\delta(1-\delta)^i\right)\right.\\
&-\frac{\rho(1-\varphi)}{1-\rho}\left(\rho^i(1-\rho)+\delta(1-\delta)^i-(\rho(1-\delta))^i(1-\rho(1-\delta))\right)\\
&\left.-\frac{\varphi\rho}{1-\delta-\rho}\left((\rho(1-\delta))^i(1-\rho(1-\delta))-(1-\delta)^{2i}(1-(1-\delta)^2)\right)\right]
\end{aligned}$$

Then note that

$$\begin{aligned}
&\lim_{i\to\infty}\Delta a_{i+1}=(1-\theta)\overline{\gamma}(1-\varphi)+(1-\theta)^2\frac{\sigma^2}{2}\left(\frac{1-\varphi}{1-\rho}\right)^2\\
&\lim_{i\to\infty}\Delta c_{i+1}=0\\
&\lim_{i\to\infty}\Delta b_{i+1}^2=0\\
&\lim_{i\to\infty}\Delta c_{i+1}^2=0\\
&\lim_{i\to\infty}\Delta(b_{i+1}c_{i+1})=0
\end{aligned}$$

This implies that

$$\lim_{i\to\infty}\left|\frac{w_{i+1}}{w_i}\right| = (1-\theta)\overline{\gamma}(1-\varphi) + (1-\theta)^2\frac{\sigma^2}{2}\left(\frac{1-\delta}{1-\rho}\right)^2 \equiv r$$

Therefore, following proposition 1.3.5, the average of the price–dividend ratio converges to a constant if and only if $r < 1$.

We now examine the autocovariances of the ratio. As just proven, the price–dividend ratio is finite for $r < 1$. Therefore, it is sufficient to show that $\mathbb{E}(v_t v_{t-k})$ is finite for all k. The idea here is to provide an upper bound for this quantity. If the process is stationary it has to be the case that $\mathbb{E}(v_t v_{t-k}) \leqslant \mathbb{E}(v_t^2)$.

We want to compute

$$\begin{aligned}\mathbb{E}(v_t^2) &= \mathbb{E}\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\beta^{i+j}\exp((a_i+a_j)+(b_i+b_j)(\gamma_t-\overline{\gamma})+(c_i+c_j)z_t\right)\\ &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\beta^{i+j}\mathbb{E}\left(\exp((a_i+a_j)+(b_i+b_j)(\gamma_t-\overline{\gamma})+(c_i+c_j)z_t\right)\end{aligned}$$

By the log–normality of γ_t, we know that

$$\mathbb{E}\left(\exp((a_i+a_j)+(b_i+b_j)(\gamma_t-\overline{\gamma})+(c_i+c_j)z_t\right) = \exp\left(\mathbb{E}_{i,j}+\frac{\mathbb{V}_{i,j}}{2}\right)$$

where

$$\mathbb{E}_{i,j} = \mathbb{E}(a_i+a_j+(b_i+b_j)(\gamma_t-\overline{\gamma})+(c_i+c_j)z_t) = a_i+a_j+(c_i+c_j)\mathbb{E}(z_t)$$

and

$$\begin{aligned}\mathbb{V}_i &= \text{Var}(a_i+a_j+(b_i+b_j)(\gamma_t-\overline{\gamma})+(c_i+c_j)z_t)\\ &= (b_i+b_j)\frac{\sigma^2}{1-\rho^2}+(c_i+c_j)\text{Var}(z_t)+2(b_i+b_j)(c_i+c_j)\text{Cov}(z_t,\gamma_t)\end{aligned}$$

From the first part of the proof, we know that

$$\begin{aligned}\mathbb{E}(z_t) &= -\frac{\overline{\gamma}}{\delta}\\ \text{Cov}(z_t,\gamma_t) &= -\frac{\sigma^2}{(1-\rho(1-\delta))(1-\rho^2)}\\ \text{Var}(z_t) &= \frac{\sigma^2}{(1-\rho^2)(1-(1-\delta)^2)}\left[\frac{1+\rho(1-\delta)}{1-\rho(1-\delta)}\right]\end{aligned}$$

Using the definition of a_k, b_k and c_k, $k = i, j$, it is straightforward — although tedious — to show that

$$\mathbb{E}_{i,j} = \left[(1-\theta)(1-\varphi)\overline{\gamma}+\left(\frac{1-\theta}{1-\rho}\right)^2\frac{\sigma^2}{2}(1-\varphi)^2\right](i+j)+(1-\theta)^2\frac{\sigma^2}{2}\Psi_{ij}$$

where

$$\begin{aligned}\Psi_{ij} \equiv& \frac{\rho^2}{1-\rho^2}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)^2\left(2-\rho^{2i}-\rho^{2j}\right)+\frac{\varphi^2(1-\delta)^2}{(1-\delta-\rho)(1-(1-\delta)^2)}(2-(1-\delta)^{2i}-(1-\delta)^{2j})\\ &-\frac{2\rho\varphi(1-\delta)}{(1-\rho(1-\delta))(1-\delta-\rho)}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)(2-(\rho(1-\delta))^i-(\rho(1-\delta))^j)\\ &-2\frac{\rho}{1-\rho}\frac{1-\varphi}{1-\rho}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)(2-\rho^i-\rho^j)+2\frac{1-\delta}{\delta}\frac{\varphi(1-\varphi)}{(1-\rho)(1-\delta-\rho)}(2-(1-\delta)^i-(1-\delta)^j)\end{aligned}$$

and

$$\mathbb{V}_{i,j} =(1-\theta)^2\sigma^2\frac{\rho^2}{1-\rho^2}\mathbb{V}^1_{i,j}+\frac{\varphi^2(1-\theta)^2\sigma^2}{(1-\rho^2)(1-(1-\delta)^2)}\left[\frac{1+\rho(1-\delta)}{1-\rho(1-\delta)}\right]\mathbb{V}^2_{i,j}-\frac{\varphi(1-\theta)^2\sigma^2}{(1-\rho(1-\delta))(1-\rho^2)}\mathbb{V}^3_{i,j}$$

where

$$\begin{aligned}\mathbb{V}^1_{i,j} \equiv& 4\left(\frac{1-\varphi}{1-\rho}\right)^2+\left(\frac{\varphi}{1-\delta-\rho}\right)^2\left((1-\delta)^{2i}+2(1-\delta)^{i+j}+(1-\delta)^{2j}\right)\\ &+\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)^2(\rho^{2i}+2\rho^{i+j}+\rho^{2j})-4\frac{1-\varphi}{1-\rho}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)(\rho^i+\rho^j)\\ &+4\frac{\varphi(1-\varphi)}{(1-\rho)(1-\delta-\rho)}\left((1-\delta)^i+(1-\delta)^j\right)\\ &-2\frac{\varphi}{1-\delta-\rho}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)\left((\rho(1-\delta))^i+(\rho(1-\delta))^j+\rho^i(1-\delta)^j+\rho^j(1-\delta)^i\right)\end{aligned}$$

and

$$\mathbb{V}^2_{i,j} \equiv 4+(1-\delta)^{2i}+(1-\delta)^{2j}-4\left((1-\delta)^i+(1-\delta)^j\right)+2(1-\delta)^{i+j}$$

and

$$\begin{aligned}\mathbb{V}^3_{i,j} \equiv& 4\frac{1-\varphi}{1-\rho}+2\frac{\varphi}{1-\delta-\rho}\left((1-\delta)^i+(1-\delta)^j\right)-2\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)(\rho^i+\rho^j)\\ &-2\frac{1-\varphi}{1-\rho}((1-\delta)^i+(1-\delta)^j)-\frac{\varphi}{1-\delta-\rho}\left((1-\delta)^{2i}+2(1-\delta)^{i+j}+(1-\delta)^j\right)\\ &\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{1-\delta-\rho}\right)\left((\rho(1-\delta))^i+(\rho(1-\delta))^j+\rho^i(1-\delta)^j+\rho^j(1-\delta)^i\right)\end{aligned}$$

Using the triangular inequality, we have $\Psi_{ij} \leqslant \overline{\Psi}$ where

$$\begin{aligned}\overline{\Psi} \equiv& 4\frac{\rho^2}{1-\rho^2}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{|1-\delta-\rho|}\right)^2+4\frac{\varphi^2(1-\delta)^2}{|1-\delta-\rho|(1-(1-\delta)^2)}\\ &+\frac{8\rho\varphi(1-\delta)}{(1-\rho(1-\delta))|1-\delta-\rho|}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{|1-\delta-\rho|}\right)\\ &+8\frac{\rho}{1-\rho}\frac{1-\varphi}{1-\rho}\left(\frac{1-\varphi}{1-\rho}+\frac{\varphi}{|1-\delta-\rho|}\right)+8\frac{1-\delta}{\delta}\frac{\varphi(1-\varphi)}{(1-\rho)|1-\delta-\rho|}\end{aligned}$$

Likewise, the triangular inequality implies that

$$\begin{aligned}\mathbb{V}^1_{i,j} \leqslant & 4\left(\frac{1-\varphi}{1-\rho}\right)^2 + 4\left(\frac{\varphi}{|1-\delta-\rho|}\right)^2 + 4\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right)^2 + 8\frac{1-\varphi}{1-\rho}\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right) \\ & + 8\frac{\varphi(1-\varphi)}{(1-\rho)(|1-\delta-\rho|)} + 8\frac{\varphi}{|1-\delta-\rho|}\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right) \\ \leqslant & 16\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right)^2\end{aligned}$$

similarly

$$\mathbb{V}^2_{i,j} \leqslant 16$$

and

$$\begin{aligned}\mathbb{V}^3_{i,j} \leqslant & 4\frac{1-\varphi}{1-\rho} + 4\frac{\varphi}{|1-\delta-\rho|} + 4\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right) + 4\frac{1-\varphi}{1-\rho} + 4\frac{\varphi}{|1-\delta-\rho|} + 4\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right) \\ \leqslant & 16\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right)\end{aligned}$$

Therefore, we have $\mathbb{V}_{i,j} \leqslant \overline{\mathbb{V}}$, where

$$\begin{aligned}\overline{\mathbb{V}} \equiv & 16(1-\theta)^2\sigma^2\frac{\rho^2}{1-\rho^2}\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right)^2 + 16\frac{\varphi^2(1-\theta)^2\sigma^2}{(1-\rho^2)(1-(1-\delta)^2)}\left[\frac{1+\rho(1-\delta)}{1-\rho(1-\delta)}\right] \\ & + 16\frac{\varphi(1-\theta)^2\sigma^2}{(1-\rho(1-\delta))(1-\rho^2)}\left(\frac{1-\varphi}{1-\rho} + \frac{\varphi}{|1-\delta-\rho|}\right)\end{aligned}$$

Hence, we have that

$$\begin{aligned}\mathbb{E}(v_t^2) \leqslant & \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\beta^{i+j}\exp\left(\left[(1-\theta)(1-\varphi)\overline{\gamma} + \left(\frac{1-\theta}{1-\rho}\right)^2\frac{\sigma^2}{2}(1-\varphi)^2\right](i+j)\right)\exp\left((1-\theta)^2\frac{\sigma^2}{2}\overline{\Psi} + \frac{\overline{\mathbb{V}}}{2}\right) \\ \leqslant & \exp\left((1-\theta)^2\frac{\sigma^2}{2}\overline{\Psi} + \frac{\overline{\mathbb{V}}}{2}\right)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} r^{i+j} \\ \leqslant & \exp\left((1-\theta)^2\frac{\sigma^2}{2}\overline{\Psi} + \frac{\overline{\mathbb{V}}}{2}\right) r\sum_{k=1}^{\infty} nr^n\end{aligned}$$

As long as $r < 1$, the series $\sum_{k=1}^{\infty} nr^n$ converges, such that in this case $\mathbb{E}v_t^2 < \infty$.

We can now consider the autocovariance terms

$$\begin{aligned}\mathbb{E}(v_t v_{t-k}) = & \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\beta^{i+j}\exp\Bigg[a_i + a_j - (c_i+c_j)\frac{\overline{\gamma}}{\delta} + \frac{1}{2}\Bigg(b_i^2\mathrm{Var}(\gamma_t) + b_j^2\mathrm{Var}(\gamma_{t-k}) + c_i^2\mathrm{Var}(z_t) \\ & + c_j^2\mathrm{Var}(z_{t-k}) + 2b_ib_j\mathrm{Cov}(\gamma_t,\gamma_{t-k}) + 2b_ic_i\mathrm{Cov}(\gamma_t,z_t) + 2b_ic_j\mathrm{Cov}(\gamma_t,z_{t-k}) \\ & + 2b_jc_i\mathrm{Cov}(\gamma_{t-k},z_t) + 2b_jc_j\mathrm{Cov}(\gamma_{t-k},z_{t-k}) + 2c_ic_j\mathrm{Cov}(z_t,z_{t-k})\Bigg)\Bigg]\end{aligned}$$

where[14]

$$
\begin{aligned}
\mathrm{Cov}(\gamma_t,\gamma_{t-k}) &= \rho^k\frac{\sigma^2}{1-\rho^2} = \rho^k\mathrm{Var}(\gamma_t)\\
\mathrm{Cov}(z_t,z_{t-k}) &= \frac{\sigma^2}{1-\delta-\rho}\left(\frac{(1-\rho^2)(1-\delta)^{k+1}-(1-(1-\delta)^2)\rho^{k+1}}{(1-\rho(1-\delta))(1-\rho^2)(1-(1-\delta)^2)}\right)\Phi_{zz,k}\mathrm{Var}(z_t)\\
\mathrm{Cov}(z_t,\gamma_{t-k}) &= \frac{\sigma^2}{1-\delta-\rho}\left(\frac{\rho^{k+1}}{1-\rho^2}-\frac{(1-\delta)^{k+1}}{1-\rho(1-\delta)}\right) = \Phi_{z\gamma,k}\mathrm{Cov}(z_t,\gamma_t)\\
\mathrm{Cov}(\gamma_t,z_{t-k}) &= -\frac{\sigma^2}{(1-\rho^2)(1-\delta(1-\delta))}\rho^k = \rho^k\mathrm{Cov}(\gamma_t,z_t)
\end{aligned}
$$

with

$$
\begin{aligned}
\Phi_{zz,k} &= \frac{(1-\delta)(1-\rho^2)(1-\delta)^k-\rho(1-(1-\delta)^2)\rho^k}{(1-\rho^2)(1-\delta)-(1-(1-\delta)^2)\rho}\\
\Phi_{z\gamma,k} &= \frac{(1-\delta)(1-\rho^2)(1-\delta)^k-\rho(1-\rho(1-\delta))\rho^k}{(1-\rho^2)(1-\delta)-\rho(1-\rho(1-\delta))}
\end{aligned}
$$

Note that, by construction, we have $|\Phi_{zz,k}|<1$ and $|\Phi_{z\gamma,k}|<1$.

$\mathbb{E}(v_tv_{t-k})$ can then be rewritten as

$$
\mathbb{E}(v_tv_{t-k}) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\beta^{i+j}\exp\left[a_i+a_j-(c_i+c_j)\frac{\overline{\gamma}}{\delta}+\frac{1}{2}\mathbb{V}_{i,j,k}\right]
$$

where

$$
\begin{aligned}
\mathbb{V}_{i,j,k} \equiv{}& \left(b_i^2+b_j^2+2b_ib_j\rho^k\right)\mathrm{Var}(\gamma_t)+\left(c_i^2+c_j^2+2c_ic_j\Phi_{zz,k}\right)\mathrm{Var}(z_t)\\
&+2\left(b_ic_i+b_jc_j+b_ic_j\rho^k+c_jc_i\Phi_{z\gamma,k}\right)\mathrm{Cov}(\gamma_t,z_t)
\end{aligned}
$$

Since $b_ib_j>0$ and $|\rho|<1$, we have $b_i^2+b_j^2+2b_ib_j\rho^k\leqslant(b_i+b_j)^2$. Likewise $c_ic_j>0$ and $|\Phi_{zz,k}|<1$, so that $\left(c_i^2+c_j^2+2c_ic_j\Phi_{zz,k}\right)\leqslant(c_i+c_j)^2$. Finally, we have $b_nc_\ell>0$, $(n,\ell)\in\{i,j\}\times\{i,j\}$ and both $|\rho|<1$ and $|\Phi_{z\gamma,k}|<1$, such that $\left(b_ic_i+b_jc_j+b_ic_j\rho^k+c_jc_i\Phi_{z\gamma,k}\right)\leqslant(b_i+b_j)(c_i+c_j)$. This implies that

$$
|\mathbb{V}_{i,j,k}|\leqslant(b_i+b_j)^2\mathrm{Var}(\gamma_t)+(c_i+c_j)^2\mathrm{Var}(z_t)+(b_i+b_j)(c_i+c_j)|\mathrm{Cov}(z_t,\gamma_t)|
$$

Hence $\mathbb{E}v_t^2$ is un upper bound for $\mathbb{E}(v_tv_{t-k})$. Therefore, as $\mathbb{E}v_t^2$ is finite for $r<1$, so is $\mathbb{E}(v_tv_{t-k})$.

[14]These quantities can be straightforwardly obtained from the Wold representations of γ_t and z_t:

$$
\gamma_t=\overline{\gamma}+\sum_{i=0}^{\infty}\rho^i\varepsilon_{t-i} \text{ and } z_t=\sum_{i=0}^{\infty}\frac{\rho^{i+1}-(1-\delta)^{i+1}}{1-\delta-\rho}\varepsilon_{t-i}
$$

Figure 1.1: Decision Rules

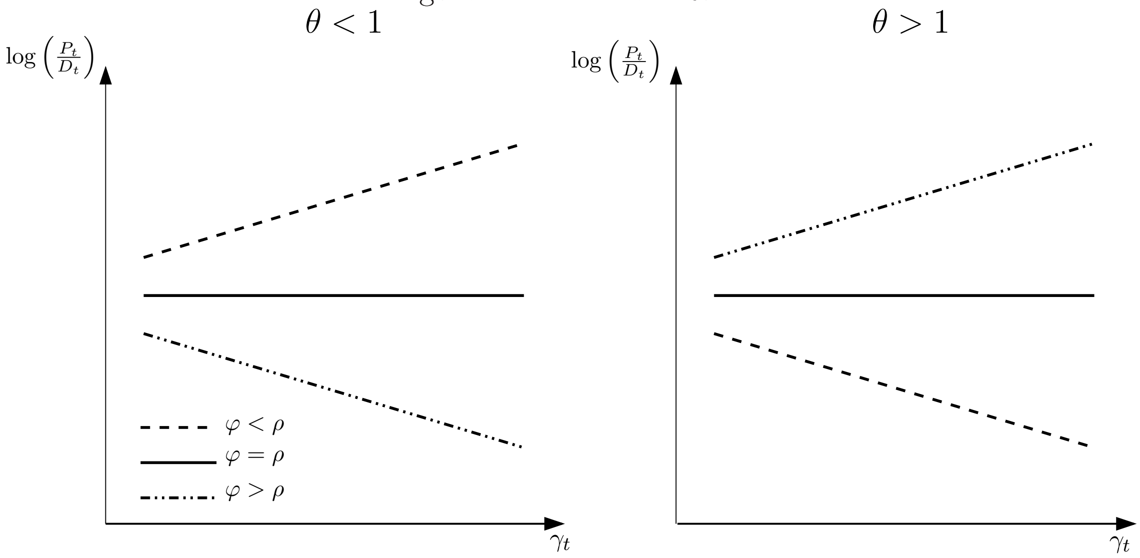
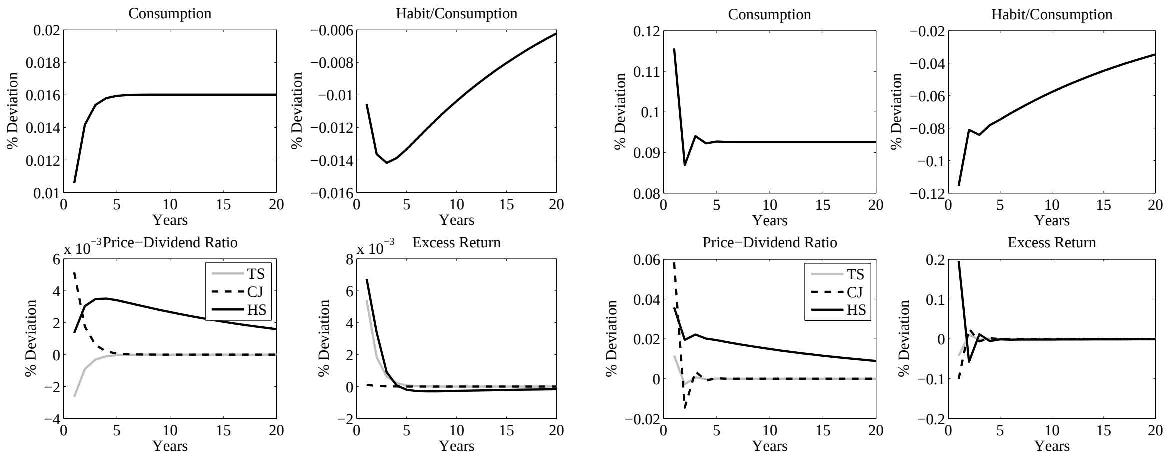


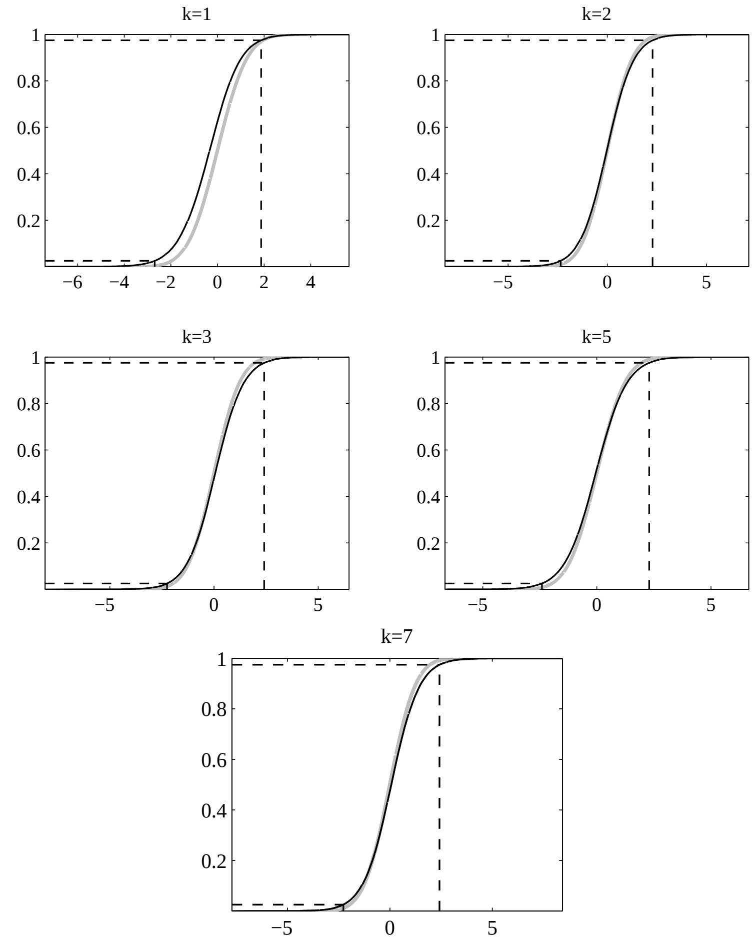
Figure 1.2: Impulse Response Functions

(a) Consumption Data (b) Dividend Data



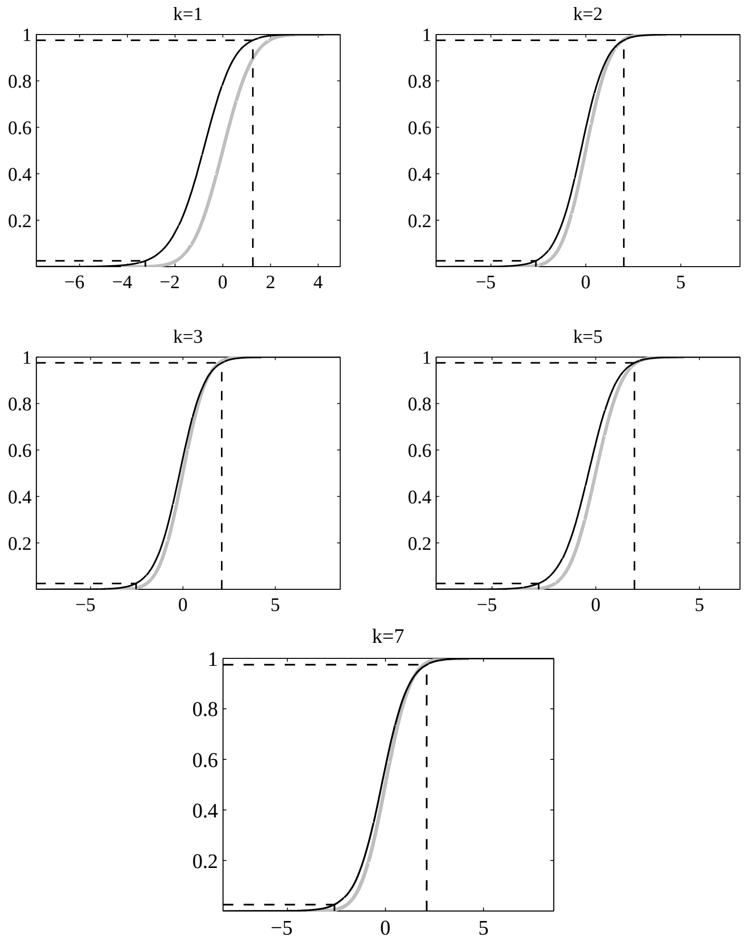
Note: TS: time separable preferences ($\varphi = 0$), CJ: Catching up with the Joneses preferences ($\varphi = 1$ and $\delta = 1$), HS: habit stock specifications ($\varphi = 1$ and $\delta = 0.05$)

Figure 1.3: Distorsion of Distributions (Short Sample)



Note: The dark plain line corresponds to the empirical distribution obtained from 100,000 Monte Carlo simulations. The gray plain line is the student distribution. The two dashed lines report the actual thresholds of a two–sided test for a 5% size.

Figure 1.4: Distorsion of Distributions (Whole Sample)



Note: The dark plain line corresponds to the empirical distribution obtained from 100,000 Monte Carlo simulations. The gray plain line is the student distribution. The two dashed lines report the actual thresholds of a two–sided test for a 5% size.

Figure 1.5: Price–Dividend Ratio P_t/D_t

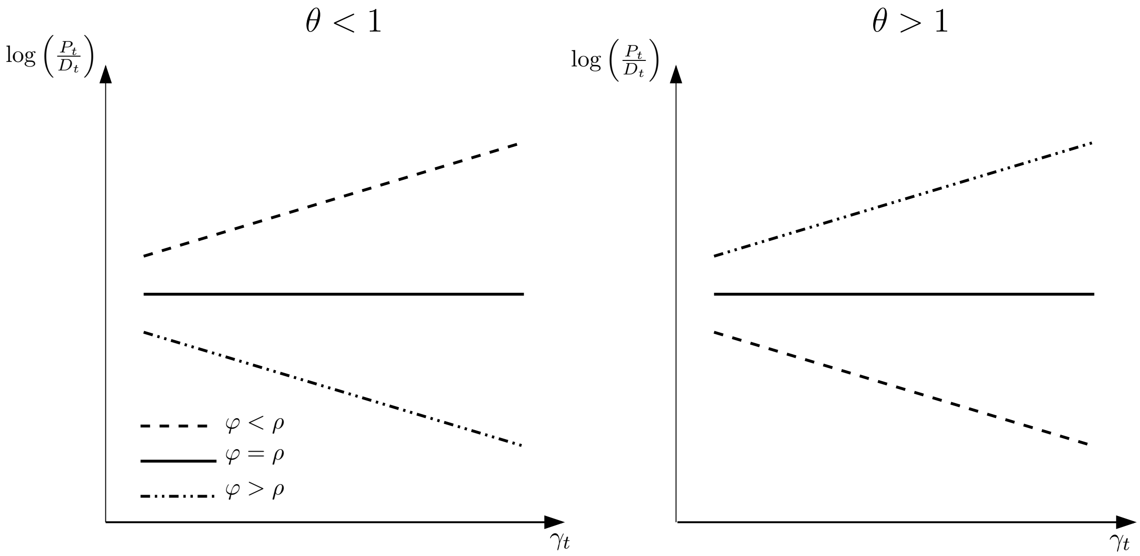


Table 1.1: Summary Statistics

	Sub–sample (1948:1990)				Whole sample (1948:2001)			
	Δc_t	Δd_t	v_t	er_t	Δc_t	Δd_t	v_t	er_t
Mean	2.12	4.85	0.59	7.79	2.01	4.01	0.74	8.35
Std. Dev.	1.17	12.14	0.24	17.82	1.14	12.24	0.39	17.46
				Correlation matrix				
Δc_t	1.00	-0.12	0.42	-0.00	1.00	-0.13	0.18	-0.06
Δd_t		1.00	-0.35	0.65		1.00	-0.33	0.65
v_t			1.00	-0.16			1.00	-0.09
er_t				1.00				1.00
				Autocorrelation function				
$\rho(1)$	0.29	-0.26	0.82	-0.09	0.34	-0.25	0.87	-0.07
$\rho(2)$	-0.03	-0.09	0.66	-0.33	0.05	-0.03	0.72	-0.25
$\rho(3)$	0.01	0.06	0.56	0.17	-0.00	0.02	0.59	0.09
$\rho(4)$	-0.05	0.11	0.47	0.39	-0.04	0.09	0.47	0.32

Table 1.2: Predictability Bias

	Short sample		Whole sample	
k	β_k	R^2	β_k	R^2
1	-0.054	0.001	-0.077	0.009
	(0.142)		(0.101)	
2	-0.005	0.000	-0.033	0.001
	(0.189)		(0.130)	
3	0.013	0.000	-0.023	0.000
	(0.199)		(0.140)	
5	-0.011	0.000	-0.060	0.001
	(0.288)		(0.189)	
7	0.024	0.000	-0.039	0.000
	(0.360)		(0.242)	

Note: β_k and R^2 are average values obtained from 100,000 replications. Standard errors into parenthesis.

Table 1.3: Simulated Distributions

	Short sample			Whole sample		
k	Emp. Size	$\widetilde{t}_{inf}$	$\widetilde{t}_{sup}$	Emp. Size	$\widetilde{t}_{inf}$	$\widetilde{t}_{sup}$
1	0.099	-2.692	1.872	0.160	-3.242	1.259
2	0.088	-2.349	2.280	0.097	-2.626	2.001
3	0.092	-2.253	2.410	0.094	-2.543	2.098
5	0.094	-2.405	2.293	0.102	-2.756	1.865
7	0.094	-2.268	2.422	0.096	-2.595	2.107

Note: These data are obtained from 100,000 replications.

Table 1.4: Predictability Regression

	(a) Short sample: 1948–1990				
k	1	2	3	5	7
β_k	-0.362	-0.567	-0.679	-1.102	-1.414
t_k	-3.847	-3.205	-3.683	-5.494	-6.063
	[0.003]	[0.006]	[0.002]	[0.000]	[0.000]
c_k	-0.594	-0.501	-0.582	-0.891	-1.011
	[0.002]	[0.003]	[0.001]	[0.000]	[0.000]
R^2	0.223	0.314	0.434	0.625	0.680
	(b) Whole sample: 1948–2001				
k	1	2	3	5	7
β_k	-0.145	-0.243	-0.275	-0.532	-0.912
t_k	-1.882	-1.261	-1.133	-0.879	-0.868
	[0.169]	[0.183]	[0.193]	[0.312]	[0.266]
c_k	-0.258	-0.175	-0.159	-0.126	-0.127
	[0.169]	[0.183]	[0.193]	[0.309]	[0.261]
R^2	0.078	0.104	0.101	0.166	0.296

Note: t_k and c_k respectively denote the t–statistics associated to the null of a_k=0, and Vlakanov's corrected t–statistics of the null ($c_k = t_k/\sqrt{T}$ where T is the sample size). Empirical size (from simulated distributions) into brackets.

Table 1.5: Preferences Parameters

Parameter		Value
Stochastic Discount Factor (Φ_{t+1})		
Curvature	θ	1.500
Constant discount factor	β	0.950
Habit Formation (C_t, X_{t+1})		
Habit persistence parameter	φ	[0,1]
Depreciation rate of habits	δ	[0.05,1]

Table 1.6: Forcing Variables

		Dividend Growth		Consumption Growth	
		1948–1990	1948–2001	1948–1990	1948–2001
Mean of dividend growth	$\overline{\gamma}$	4.85%	4.01%	2.12%	2.01%
Persistence parameter	ρ	-0.26	-0.25	0.29	0.34
Std. dev. of innovations	σ	11.30%	11.50%	1.10%	1.00%

Table 1.7: Unconditional Moments

	TS		CJ		HS	
	Mean	St Dev	Mean	St Dev	Mean	St Dev
			Dividend data			
$r - r_f$	1.40	17.81	1.52	30.12	1.49	20.30
$p - d$	0.66	0.01	0.74	0.06	0.74	0.14
			Consumption data			
$r - r_f$	0.01	0.45	0.01	1.53	0.01	0.67
$p - d$	0.69	0.00	0.74	0.01	0.74	0.01

Note: TS: time separable preferences ($\varphi = 0$), CJ: Catching up with the Joneses preferences ($\varphi = 1$ and $\delta = 1$), HS: habit stock specifications ($\varphi = 1$ and $\delta = 0.05$)

Table 1.8: Price to Dividend Ratio Volatility

Catching–up ($\delta = 1$)							
φ	0	0.1	0.2	0.5	0.7	0.9	1
$\sigma(p-d)$	0.01	0.02	0.02	0.04	0.05	0.06	0.06
Habit Stock ($\varphi = 1$)							
δ	0.05	0.10	0.20	0.50	0.70	0.90	1
$\sigma(p-d)$	0.14	0.11	0.08	0.06	0.06	0.06	0.06

Table 1.9: Unconditional Correlations

	Dividend Data			Consumption Data		
	TS	CJ	HS	TS	CJ	HS
corr$(p-d, r-r_f)$	0.99	0.98	0.25	-0.81	0.94	0.15
corr$(r-r_f, \gamma)$	0.99	0.98	0.99	0.81	0.94	0.98
corr$(p-d, \gamma)$	1.00	1.00	0.26	-1.00	1.00	0.11

Note: TS: time separable preferences ($\varphi = 0$), CJ: Catching up with the Joneses preferences ($\varphi = 1$ and $\delta = 1$), HS: habit stock specifications ($\varphi = 1$ and $\delta = 0.05$)

Table 1.10: Serial Correlation in Price–Dividend Ratio

Order	1	2	3	5	7
Dividend Data					
HS	0.88	0.86	0.81	0.74	0.67
CJ	-0.25	0.05	-0.03	-0.01	-0.01
TS	-0.25	0.05	-0.03	-0.01	-0.01
Consumption Data					
HS	0.98	0.93	0.88	0.79	0.70
CJ	0.30	0.07	-0.00	-0.03	-0.04
TS	0.30	0.07	-0.00	-0.03	-0.04

Note: TS: time separable preferences ($\varphi = 0$), CJ: Catching up with the Joneses preferences ($\varphi = 1$ and $\delta = 1$), HS: habit stock specifications ($\varphi = 1$ and $\delta = 0.05$)

Table 1.11: Correlation Between the Model and the Data

	Dividend Growth		Consumption Growth	
	1948–1990	1948–2001	1948–1990	1948–2001
TS	-0.35	-0.35	-0.43	-0.22
CJ	-0.29	-0.32	-0.05	-0.07
HS	0.35	0.37	0.42	0.42

Note: TS: time separable preferences ($\varphi = 0$), CJ: Catching up with the Joneses preferences ($\varphi = 1$ and $\delta = 1$), HS: habit stock specifications ($\varphi = 1$ and $\delta = 0.05$)

Table 1.12: Predictability: Benchmark Experiments

	Dividend Data						Consumption Data					
	TS		CJ		HS		TS		CJ		HS	
k	b_k	R^2	b_k	R^2	b_k	R^2	b_k	R^2	b_k	R^2	b_k	R^2
1	-4.69	0.11	-2.18	0.20	-0.44	0.04	-1.30	0.64	-0.09	0.01	-0.04	0.03
2	-3.74	0.06	-1.70	0.11	-0.56	0.06	-1.68	0.32	-0.17	0.02	-0.16	0.05
3	-4.20	0.06	-1.88	0.11	-0.73	0.08	-1.75	0.20	-0.26	0.02	-0.29	0.08
5	-4.41	0.05	-1.92	0.09	-1.00	0.12	-1.66	0.10	-0.40	0.03	-0.53	0.13
7	-4.77	0.05	-2.03	0.09	-1.20	0.16	-1.50	0.06	-0.57	0.04	-0.75	0.17

Note: TS: time separable preferences ($\varphi = 0$), CJ: Catching up with the Joneses preferences ($\varphi = 1$ and $\delta = 1$), HS: habit stock specifications ($\varphi = 1$ and $\delta = 0.05$)

Table 1.13: Predictability: Sensitivity Analysis

	TS: θ				CJ: φ				HS: (φ, δ)					
	0.5		5		0.1		0.5		(0.5,0.5)		(0.5,0.05)		(1,0.5)	
k	b_k	R^2	b_k	R^2	b_k	R^2	b_k	R^2	b_k	R^2	b_k	R^2	b_k	R^2
1	1.61	0.05	-1.91	0.24	-3.79	0.12	-2.60	0.16	-1.96	0.11	-0.93	0.05	-1.34	0.10
2	1.37	0.03	-1.49	0.14	-3.01	0.07	-2.04	0.09	-1.75	0.07	-1.07	0.06	-1.27	0.08
3	1.61	0.03	-1.64	0.14	-3.37	0.06	-2.26	0.09	-2.02	0.08	-1.36	0.08	-1.47	0.08
5	1.82	0.03	-1.66	0.12	-3.52	0.06	-2.33	0.07	-2.22	0.08	-1.81	0.11	-1.63	0.09
7	2.12	0.03	-1.74	0.11	-3.79	0.05	-2.48	0.07	-2.48	0.08	-2.23	0.15	-1.82	0.09

Note: TS: time separable preferences ($\varphi = 0$), CJ: Catching up with the Joneses preferences ($\varphi = 1$ and $\delta = 1$), HS: habit stock specifications ($\varphi = 1$ and $\delta = 0.05$)

Chapter 2

Surplus Consumption Ratio and Expected Asset Returns[1]

This paper shows, from a Consumption based Asset Pricing Model (C) CAPM with habit formation, that the surplus consumption ratio is a linear predictor of stock returns at long horizons and should explain the cross section of expected returns. This theoretical finding receives support from the U.S data which suggest that the surplus consumption ratio is a strong predictor of excess returns at long horizons. This paper also provides empirical evidence that the surplus consumption ratio captures a component of expected returns, not explained by the proxies of consumption to wealth ratio, *cay* and *cdy*, proposed by Lettau and Ludvigson [2001a, 2005]. Moreover, the (C)CAPM with habit formation performs far better than the standard (C)CAPM to explain for the average returns across the Fama-French (25) portfolios sorted by size and book-to-market characteristics.

Key Words: Habit formation, Surplus consumption ratio, Expected returns, Time series predictability, Cross section returns

JEL Class.: G12, E21

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Introduction

Over the last twenty years, there has been a long debate about the linkage between macroeconomics and financial markets. The central question is concerned with the macroeconomic risks driving the risk premia in financial markets. This challenge is motivated by the fact that expected returns are time varying and are correlated with business cycles. Extensive research has been devoted to answer this question. For instance, Lettau and Ludvigson [2001a, 2001b, 2005] have investigated the linkage between excess returns and the consumption to wealth ratio. They have shown that the consumption to wealth ratio is a good predictor of excess returns at short and intermediate horizons. Besides, used as a conditioning variable, the consumption to wealth ratio improves the performance of the standard Consumption based Asset Pricing Model (C)CAPM in explaining the cross section of expected returns. In their seminal paper, Campbell and Cochrane [1999] have shown, from a (C)CAPM with nonlinear infinite–horizon habit level, that low surplus consumption ratio forecasts high expected returns at the next period. However, the literature has not investigated (i) the long horizon predictive power of the surplus consumption ratio – one exception is Li [2005] – and (ii) its ability to explain the cross section of expected returns.

By linking the surplus consumption ratio to the consumption to wealth ratio, Li[2005] proposed to investigate the ability of the surplus consumption ratio to predict excess returns, providing the predictive power of the consumption to wealth ratio at short and intermediate horizons, as shown by Lettau and Ludvigson [2001]. One of the main contribution of this paper will be to show theoretically, from a (C)CAPM with habit formation, that the surplus consumption ratio is a linear predictor of excess returns at any horizon. Moreover, the economic model predicts negative relationship between the surplus consumption ratio and excess returns at any horizon. This theoretical finding is robust to the specification placed on the habit level.

This paper also presents empirical evidence, which supports the theoretical finding. To investigate empirically the predictive power of the surplus consumption ratio and the dividend to price ratio, Li [2005] has used VAR estimation, proposed by Hodrick [1992], to mitigate the finite sample bias that may rise in studying long horizon returns. However, this econometric methodology does not take into account the high persistence of the explanatory variables. This paper proposes a Monte Carlo experiment accounting for the biased coefficients estimators and the distorted distribution of test statistics due to (*i*) the feedback effect, (*ii*) the highly persistent explanatory variable and (*iii*) the overlapping data. Out-of-sample estimations are also run to evaluate the presence of a look-ahead bias. We find that the surplus consumption ratio is indeed a strong predictor of excess returns at long horizons, as in Li [2005]. For instance, the surplus consumption ratio explains 35% of the variability of excess returns at 5–year horizon. Moreover, empirical findings suggest that the surplus consumption ratio predicts a component of expected excess returns which is not captured by the proxies of the consumption to wealth ratio, *cay* and *cdy*, proposed by Lettau and Ludvigson [2001, 2005]. However, in contrast with Li (2005), the dividend–price ratio fails to predict excess returns at any horizon.

Following the same line of arguments as in Lettau and Ludvigson [2001b], the surplus consumption ratio is used as conditioning variable to the conditional standard (C)CAPM. This implies a three macroeconomic factors' model. The factors are the consumption growth, the lagged surplus consumption ratio and their product. We also show that the unconditional version of the (C)CAPM with habit formation delivers the same conclusion as the conditional standard (C)CAPM. Using data on the Fama and French (25) portfolios, we show empirically that the surplus consumption ratio helps to explain the cross section of average returns on size and book to market sorted portfolios.

The main finding of this paper is that the risk of an asset can not only be measured by its covariance with the contemporaneous consumption growth, as suggested by the standard (C)CAPM, but by its covariance with the contemporaneous consumption as well as its covariance with past consumption growth. The level of habit or, equivalently, the surplus consumption ratio, is the key variable that summarizes the information contained on the past consumption pattern.

The paper is structured as follows. Section 2 presents the general theoretical framework, based on the (C)CAPM with habit formation, that forms the basis of our empirical work. Sections 3 and 4 confront the theoretical findings to the U.S data. Section 3 investigates the long horizon predictability. Section 4 explores the ability of the surplus consumption ratio to explain the cross section variations of expected returns. A final section concludes.

2.1 Theoretical Framework

This section derives, from a (C)CAPM with habit formation, a general framework linking the surplus consumption ratio with expected returns.

We consider an endowment economy with a representative consumer. The preferences of the representative agent are represented by the following intertemporal utility function:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i u_{t+i}$$

where $\beta > 0$ is the subjective discount factor and u_t denotes the instantaneous utility function. Expectations are conditional on information available at the beginning of period t.

The preferences of the agent are assumed to be time nonseparable. The agent derives her utility for period t from her individual current consumption C_t as well as a reference level X_t:

$$u_t = U(C_t, X_t)$$

This class of preferences is now common in the Consumption based Asset Pricing

literature[2]. The reference level, or the level of habit X_t captures the influence of the history of aggregate and individual consumption choices on the current individual choices. For tractability, the habit formation is assumed to be external: the habit level X_t depends only on the history of aggregate consumptions $\{\overline{C}_{t-\tau}, \tau \succeq 0\}$.
The instantaneous utility function is specified in difference, following Campbell and Cochrane [1999]:

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\theta} - 1}{1-\theta}$$

where θ denotes the utility curvature parameter. Note that X_t should be below C_t to ensure positive and finite marginal utility. At this stage, no further specification will be placed on habit level X_t.
The representative agent determines her contingent consumption plan by maximizing her intertemporal utility function subject to the budget constraint. Hence, agents' consumption decisions are governed by the following Euler equation:

$$1 = \beta \mathbb{E}_t \left[R_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \left(\frac{S_{t+1}}{S_t} \right)^{-\theta} \right] \tag{2.1.1}$$

where $S_t = \frac{C_t - X_t}{C_t}$ denotes the surplus consumption ratio and R_{t+1} the gross return on the stock market portfolio. It is useful to consider the log-linear version[3] of Equation (3.2.4):

$$\mathbb{E}_t \Delta s_t = c_{t+1} - \frac{1}{\theta} \mathbb{E}_t \left(r_{t+1} - \delta \right) + \mathbb{E}_t s_{t+1} \tag{2.1.2}$$

where $\delta \equiv (1-\beta)/\beta$. Iterating forward equation (2.1.2), the surplus consumption ratio is given by:

$$s_t = \mathbb{E}_t \sum_{i=1}^{\infty} \left(\triangle c_{t+i} - \frac{1}{\theta} \left(r_{t+i} - \delta \right) \right) + \lim_{i \to \infty} \mathbb{E}_t s_{t+i}. \tag{2.1.3}$$

Equation (2.1.3) suggests that s_t is a good candidate to predict stock returns or consumption growth at long horizons. Furthermore, it indicates that stock returns

[2] See Cochrane [2001, 2006] for a survey.
[3] Throughout, lowercase letters are used for variables in logarithm.

and the surplus consumption ratio are negatively related at any horizon. This finding generalizes the inverse relation between the surplus consumption ratio and expected stock returns at the one period horizon proposed by Campbell and Cochrane [1999] and Li [2001]. Note that this theoretical result does not depend neither on the specification placed on S_t or X_t nor on the the law of motion of the consumption growth. Moreover, similar conclusions are obtained when the utility function is specified in ratio, as in Abel [1990, 1999]:

$$U(C_t, X_t) = \frac{(C_t/X_t)^{1-\theta} - 1}{1-\theta}$$

Indeed, the associated Euler equation is given by:

$$1 = \beta \mathbb{E}_t \left[R_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \left(\frac{X_{t+1}}{X_t} \right)^{\theta-1} \right] \tag{2.1.4}$$

Iterating forward the log–linear version of the Euler equation (2.1.4), one gets:

$$z_t = \mathbb{E}_t \sum_{i=1}^{\infty} \left(\frac{1}{\theta-1} \triangle c_{t+i} - \frac{1}{\theta-1} \left(r_{t+i} - \delta \right) \right) + \lim_{i\to\infty} \mathbb{E}_t z_{t+i}. \tag{2.1.5}$$

where $z_t = \ln(C_t/X_t)$ denotes the consumption to habit ratio. Equation (2.1.5) suggests that the surplus consumption ratio[4] z_t, is a candidate to predict excess stock returns at any horizons.

It is worth comparing equation (2.1.3) with a similar one obtained by Lettau and Ludvigson [2001a, 2005]:

$$c_t - w_t = \mathbb{E}_t \sum_{i=1} \rho_w^i \left(r_{w,t+i} - \triangle c_{t+i} \right) \tag{2.1.6}$$

Following the same line of arguments as in Lettau and Ludvigson [2001b], equation(3) suggests that the surplus consumption ratio should forecast predictable changes in asset prices and therefore explain the cross sectional average returns.

[4] Note that: $\ln(C_t - X_t/C_t) \simeq -X_t/C_t$. That's why consumption to habit ratio is also called surplus consumption ratio.

Indeed, used as a conditioning variable for the standard consumption based asset pricing model, the surplus consumption ratio delivers a linear three factors model.

Consider the standard (C)CAPM, proposed by Lucas [1978], when preferences are time separable. More precisely, the utility function is given by: $u_t = (C_t^{1-\theta} - 1)/1 - \theta$. Therefore, the unconditional version of the stochastic discount factor (SDF) representation of the pricing model[5] is given by:

$$1 = \mathbb{E}[\exp^{\ln \beta + \theta \Delta c_{t+1}} R_{t+1}]$$

where Δc_{t+1} is the single factor of the unconditional standard (C)CAPM. As documented by Jagannathan and al.[2002], the pricing model can be tested using its conditional SDF representation by incorporating conditional information. Indeed, the conditional Euler equation can be written as:

$$\mathbb{E}_t[M_{t+1}R_{t+1}] = \mathbb{E}[M_{t+1}R_{t+1}/\zeta_t] = 1$$

where ζ_t is a vector of economic variables observed at the end of period t. Lettau and Ludvigson [2001b] showed that the factors associated to the conditional SDF representation are Δc_{t+1}, ζ_t and $\zeta_t \Delta c_{t+1}$. As the surplus consumption ratio is a candidate to predict stock returns, it can be used as a conditioning variable to the

[5] Following Jagannathan and al. [2002], we refer to pricing models of the form:
(i)

$$\mathbb{E}[R_{i,t}] = \mathbb{E}[R_{f,t}] + \beta_i \lambda$$
$$R_{i,t} = \alpha_i + \beta_i f_t + u_{i,t} = F_t \beta + u_{i,t}$$

as the beta representation;
(ii)

$$\mathbb{E}[M_{t+1}(F_{t+1})R_{i,t+1}] = 1$$

as the*unconditional* SDF representation;
(iii)

$$\mathbb{E}_t[M_{t+1}(F_{t+1})R_{i,t+1}] = 1$$

as the *conditional* SDF representation.

F_t are the risk factors.

standard (C)CAPM. This implies a linear three factor-model when factors are the consumption growth Δc_t, the lagged surplus consumption growth s_{t-1} and their product $s_{t-1}\Delta c_t$. In Appendix A, we show that the unconditional SDF representation of the Campbell and Cochrane [1999] model and their specification imposed to the surplus consomption ratio (Eq. 2.2.7) deliver the same conclusion.

Therefore, this section suggests that the surplus consumption ratio should forecast predictable changes in asset prices at any horizon, as the wealth to consumption ratio. Moreover, the surplus consumption ratio delivers a linear three factors model that rivals the conditional (C)CAPM proposed by Lettau and and Ludvigson [2001b] and the Fama and French [1993] three factors model in explaining the cross section of expected returns. Both implications will be evaluated empirically in next sections.

2.2 Long Horizon Predictability

This section explores empirically the time series relation between the surplus consumption ratio s_t and excess returns. The data used in this study are borrowed from Lettau and Ludvigson [2005][6]. There are annual variables for the period $1948-2001$. We use data on excess returns, consumption, dividend, asset holdings and price-dividend ratio. All variables are expressed in real per capita terms.

Before confronting equation (2.1.3) to the U.S data, some assumptions have to be placed on the evolution of the surplus consumption ratio s_t. As benchmark, we consider the (C)CAPM with habit formation proposed by Campbell and Cochrane [1999]. The benchmark model presents two key ingredients. First, the utility function is specified in difference. This implies a time varying coefficient of risk aversion. The second ingredient is that the surplus consumption ratio is nonlinear and moves slowly in response to consumption. The nonlinearity is essential to keep habit always below consumption and therefore to guarantee positive and finite marginal utility.

[6] More details on the data can be found in the appendix to Lettau and Ludvigson [2005], downloadable from http://www.econ.nyu.edu/user/ludvigsons/dappendixe.pdf.

Then, we test the robustness of the empirical findings to alternative specifications of the utility function and the surplus consumption ratio. Moreover, the forecasting power of the surplus consumption ratio will be compared to the predictive power of alternative macroeconomic and financial indicators: the consumption to aggregate wealth ratio [7] $c_t - w_t$ and the dividend–price ratio $d_t - p_t$.

Following Campbell and Cochrane [1999], the surplus consumption ratio s_t is assumed to evolve as:

$$s_t = (1-\phi)\overline{s} + \phi s_{t-1} + \lambda(s_{t-1})(\Delta c_t - g) \tag{2.2.7}$$

where Δc_t is the consumption growth and g denotes the average consumption growth. The sensitivity function $\lambda(s_t)$ is defined as follows:

$$\lambda(s_t) = \begin{cases} \frac{1}{\overline{S}}\sqrt{1-2(s_t-\overline{s})} - 1 & \text{if } s_t \leqslant s_{max} \\ \\ 0 & \text{otherwise} \end{cases}$$

where $\overline{s} = \log \overline{S} = \log\left(\sigma\sqrt{\frac{\theta}{1-\phi}}\right)$ and $\log(S_{max}) = \overline{s} + \frac{1}{2}(1-\overline{S}^2)$. The parameter σ denotes the standard deviation of consumption growth.

Despite s_t is not observable, equation (2.2.7) can be used to generate a time series for s_t. This requires to set ϕ, g, σ and θ. The utility curvature parameter, θ is set to 2, a commonly used value in the literature. The parameters g and σ are estimated using annual consumption data, implying $g = 2.01\%$ and $\sigma = 1.14\%$. The parameter ϕ is set to match the first order serial correlation of the dividend–price ratio, implying $\phi = 0.91$. All these values are close to those used by Campbell and Cochrane [1999]. The forecasting power of the surplus consumption ratio will be compared to the predictive power of alternative financial and macroeconomic indicators: the dividend–price ratio $d_t - p_t$ and the consumption to aggregate wealth ratio $c_t - w_t$. Following Lettau and Ludvigson [2001a, 2005], we use the deviation

[7] See Lettau and Ludvigson [2001a, 2005].

Table 2.1: Summary Statistics

	Autocorrelations				Correlation Matrix			
	ρ_1	ρ_2	ρ_3	ρ_4	$d-p$	cay	cdy	s
$d-p$	0.89	0.75	0.63	0.53	1.0	0.53	-0.11	0.25
cay	0.57	0.14	0.05	0.01		1.0	0.32	-0.14
cdy	0.47	0.20	0.24	0.16			1.0	-0.20
s	0.89	0.71	0.52	0.34				1.0

Note: This table reports summary statistics for the dividend–price ratio $d_t - p_t$, the surplus consumption ratio s_t, the proxies for the consumption to aggregate wealth ratio cay_t and cdy_t. The sample period spans 1948 to 2001.

from the estimated shared trend among consumption, asset holding and labor income, denoted by cay_t, and the deviation from the estimated shared trend among consumption, dividend and labor income, denoted by cdy_t, as proxies for the unobservable consumption to wealth ratio. Table 2.1 presents summary statistics for $d_t - p_t$, cay_t, cdy_t and s_t. The correlations and the autocorrelations coefficients of cay, cdy and $d-p$ are similar to those reported in Lettau and Ludvigson [2005].

Two main results emerge. First, the price–dividend ratio and the surplus consumption ratio are highly persistent. Their first-order autocorrelations are 0.89 and their second-order autocorrelations are 0.75 and 0.71, respectively. As documented by Lettau and Ludvigson [2005], cay_t and cdy_t are less persistent and their autocorrelations die out more quickly. Second, s_t is weakly correlated to other indicators. The correlations between s_t and cay_t or cdy_t are -0.14 and -0.20, respectively.

A common way to investigate excess returns predictability at long horizons is to run regressions for the compounded (log) excess returns $er_{t,t+k} = \sum_{i=1}^{k}(r_{t+i} - r_{f,t+i})$ on the explanatory variable x_t evaluated at several lags:

$$er_{t,t+k} = \alpha_k + \beta_k x_t + u_{t+k,t} \qquad (2.2.8)$$

Several econometric issues arise when assessing the forecasting power of the surplus consumption ratio s_t and the dividend–price ratio $d_t - p_t$. As documented by

Table 2.2: Predictability Bias

	Panel A		Panel B	
	s_t		$d_t - p_t$	
k	β_k	R^2	β_k	R^2
1	0.0016	0.01	0.04	0.01
	(0.05)		(0.057)	
2	0.003	0.01	0.08	0.02
	(0.09)		(0.16)	
3	0.04	0.02	0.11	0.03
	(0.13)		(0.21)	
4	0.005	0.03	0.15	0.05
	(0.17)		(0.25)	
5	0.006	0.03	0.18	0.06
	(0.21)		(0.30)	
6	0.007	0.04	0.22	0.07
	(0.25)		(0.34)	

Note: This Table reports average values of β_k and R^2 obtained from 100.000 simulations. Standard errors are into parentheses.

Valkanov [2003] and Stambaugh [1999)] among others, highly persistent explanatory variables and the existence of a strong correlation between unexpected returns and the innovations of the explanatory variables ought to distort OLS estimators in finite sample. In order to investigate this issue, we follow Valkanov [2003] and run a Monte Carlo experiment under the null of no predictability ($\beta_k = 0$ in equation (2.2.8)) assuming that the explanatory variable s_t or $d_t - p_t$ follow $AR(1)$ processes. Such a procedure enables to recover (i) the distribution of β_k and R^2 under the null of no predictability and (ii) the distributions of the Newey–West t-statistics and the rescaled $t/\sqrt{T}$ proposed by Valkanov [2003]. Simulations results[8] are reported in Table 2.2 and in Figures 2.2 and 2.3.

Several results emerge. First, the average values of the estimated β_k coefficients

[8] Figures are reported in Appendix B.

are upward biased in both cases. However, the bias remains small and not statistically significant at any horizon. Second, when s_t is used as regressor, the average value of R^2 is close to 0 at the 1–year horizon and does not exceed 0.04 at the 6–year horizon. The R^2 is larger at any horizon in the case of $d_t - p_t$. This indicates that s_t appears to be more immune to bias that the conventional $d_t - p_t$. As illustrated in Figures 2.2 and 2.3, which report the cumulative distribution function of the Student distribution and the distributions obtained from the Monte-carlo experiments, the t–statistics and the Newey–West t–statistics exhibit distorsions. Accordingly, the empirical sizes associated to the Newey-West t–statistics and the corrected t–statistics proposed by Valkanov [2003] reported below to evaluate the predictive power of s_t and $d_t - p_t$ are obtained from the Monte Carlo simulations.

Table 2.3 reports the results of univariate long horizon regressions of excess returns using actual data s_t and $d_t - p_t$. When s_t is used as regressor, the estimated coefficients $\hat{\beta}_k$ have negative sign – i.e the model receives empirical support from the data. Furthermore, the surplus consumption ratio is statistically significant at any horizon and the R^2 increases with the horizon. For instance, at 5–year horizon, the surplus consumption ratio accounts for 35% of the variability of excess returns. In contrast to the surplus consumption ratio, the dividend–price ratio is never statistically significant. To further investigate this issue, we run regressions of the asset holding growth and consumption growth on s_t and $d_t - p_t$ at long horizons. As shown in Table 2.4, only s_t predicts asset holding growth at long horizons. The coefficients slopes are negative, statistically significant and increase with horizon. The R^2 rises with the horizon and reaches 47% at 5–year horizon and 52% at 6–year horizon. These results confirm that s_t is a good predictor of excess returns at long horizons. In contrast to s_t, the dividend yield $d_t - p_t$ is never statistically significant and R^2 is almost close to 0. Both s_t and $d_t - p_t$ fail to predict consumption growth at any horizon.

Table 2.3: Univariate Long-horizon Regressions - Excess Stock Returns

	$\sum_{i=1}^{k}(r_{t+i} - r_{f,t+i}) = \alpha_k + \beta_k x_t + \varepsilon_{t,t+k}$					
k	1	2	3	4	5	6
	surplus consumption ratio s_t					
β_k	-0.13	-0.22	-0.31	-0.43	-0.58	-0.67
t_{NW}	-3.49	-3.62	-3.86	-3.84	-4.03	-4.00
	(0.03)	(0.03)	(0.03)	(0.03)	(0.025)	(0.03)
$t/\sqrt{T}$	-0.47	-0.49	-0.52	-0.52	-0.54	-0.54
	(0.009)	(0.018)	(0.006)	(0.003)	(0.002)	(0.01)
R^2	0.07	0.10	0.19	0.31	0.35	0.32
	dividend–price ratio $d_t - p_t$					
β_k	0.14	0.24	0.27	0.34	0.53	0.75
t_{NW}	2.13	1.72	1.35	1.17	1.32	1.58
	(0.4)	(0.5)	(0.7)	(0.8)	(0.8)	(0.8)
$t/\sqrt{T}$	0.29	0.23	0.18	0.16	0.18	0.21
	(0.15)	(0.35)	(0.7)	(0.8)	(0.8)	(0.8)
R^2	0.09	0.12	0.11	0.12	0.18	0.25

Note: t_{NW}: Newey-West t-statistics associated to the null of the absence of predictability, $t/\sqrt{T}$: modified t–statistics proposed by Valkanov [2003]. Empirical size into parentheses. The empirical sizes were obtained from the 100.000 Monte Carlo simulations. The sample is annual and spans the period 1948 to 2001.

Table 2.4: Univariate Long-horizon Regressions: Asset Holding Growth and Consumption Growth

	Asset Holding Growth $a_{t+k} - a_t = \alpha_k + \gamma_k x_t + \varepsilon_{t,t+k}$						Consumption Growth $c_{t+k} - c_t = \alpha_k + \gamma_k x_t + \varepsilon_{t,t+k}$					
k	1	2	3	4	5	6	1	2	3	4	5	6
						s_t						
γ_k	-0.02	-0.05	-0.07	-0.11	-0.15	-0.17	0.00	-0.00	-0.00	-0.01	-0.02	-0.03
t_{NW}	-2.24	-2.61	-3.24	-3.37	-4.81	-4.44	0.13	-0.60	-1.01	-1.33	-1.68	-2.08
$t/\sqrt{T}$	-0.31	-0.35	-0.44	-0.59	-0.65	-0.60	0.01	-0.08	-0.13	-0.18	-0.22	-0.28
R^2	0.05	0.09	0.19	0.35	0.47	0.52	0.00	0.00	0.02	0.05	0.07	0.11
						$d_t - p_t$						
γ_k	0.01	0.00	-0.01	-0.04	-0.03	-0.02	-0.00	-0.00	-0.01	-0.02	-0.02	-0.02
t_{NW}	1.01	0.36	-0.59	-0.86	-0.56	-0.27	-1.54	-1.63	-1.43	-1.23	-1.06	-0.96
$t/\sqrt{T}$	0.09	0.04	-0.09	-0.16	-0.13	-0.07	-0.21	-0.22	-0.19	-0.16	-0.14	-0.13
R^2	0.01	0.00	0.00	0.02	0.01	0.00	0.02	0.01	0.02	0.03	0.03	0.03

Note: t_{NW}: Newey-West t-statistics associated to the null of the absence of predictability, $t/\sqrt{T}$: modified t–statistics proposed by Valkanov [2003]. Empirical size into parentheses. The empirical sizes were obtained from the 100.000 Monte Carlo simulations. The sample is annual and spans the period 1948 to 2001.

The predictive power of s_t is now compared to cay_t and cdy_t, the proxies for the consumption to wealth ratio proposed by Lettau and Ludvigson [2001, 2005]. Table 2.5 reports results of (i) the univariate regressions of long-horizon excess returns using cay_t and cdy_t and (ii) the multivariate regressions of long–horizon excess returns using s_t, cay_t and cdy_t. Consistent with previous results, s_t remains statistically significant when we add cay_t and cdy_t as dependent variables and the sign of the regressions coefficients corresponding to s_t is unchanged. Moreover, the introduction of s_t increases the R^2 specially at long horizons. For instance, at 4–year horizon, the R^2 increases from 33% when we consider only cay_t as predictor variable to 45% when we add s_t. Note that cay_t, cdy_t and s_t together explain about 51% and 63% of the variability of excess returns respectively at 5 and 6–year horizons whereas cay_t and cdy_t capture together only 36% at 5–year horizon and 52% at 6–year horizon. The findings reported in Table 2.5 suggest that there is a component of expected returns — captured by the surplus consumption ratio — that moves independently of cay_t and cdy_t.

As documented by Lettau and Ludvigson [2001, 2005], a look–ahead bias may arise from the fact that the coefficients ϕ, g and σ used to generate the surplus consumption ratio are estimated from the whole sample. To address this issue, Table 2.6 reports results for out–of–sample predictions. The results are consistent with previous experiments regardless the starting date of the out–of–sample regressions. The coefficients are negative and increase with horizons. The R^2 starts low then increases substantially at 4 and 5–year horizons. This result confirms that the surplus consumption ratio is a good predictor of long–horizon excess returns.

Robustness

To check the robustness of empirical results presented above, we evaluate the sensitivity of the predictive power of the surplus consumption ratio to (i) the degree of curvature of the utility function θ and (ii) the specification of the surplus consump-

Table 2.5: Long-horizon Regressions - Excess Stock Returns

	k–period Regression: Excess Returns					
k	1	2	3	4	5	6
cay_t	5.87	10.50	11.93	12.54	16.30	21.65
	(3.74)	(5.61)	(7.64)	(7.03)	(6.46)	(7.91)
	[0.26]	[0.4]	[0.40]	[0.33]	[0.37]	[0.51]
cdy_t	1.50	5.54	6.36	6.89	8.29	11.80
	(1.82)	(6.57)	(5.48)	(4.86)	(4.52)	(4.79)
	[0.03]	[0.20]	[0.24]	[0.23]	[0.23]	[0.37]
cay_t	5.54	9.93	10.55	9.51	11.72	17.43
	(4.23)	(5.36)	(9.17)	(4.55)	(5.73)	(9.12)
s_t	-0.09	-0.15	-0.21	-0.32	-0.40	-0.37
	(-2.85)	(-3.26)	(-6.38)	(-3.31)	(-2.98)	(-3.18)
	[0.27]	[0.48]	[0.47]	[0.45]	[0.49]	[0.57]
cdy_t	1.09	4.93	5.46	5.64	6.76	10.30
	(1.28)	(8.32)	(9.46)	(10.08)	(7.45)	(8.56)
s_t	-0.12	-0.16	-0.25	-0.37	-0.51	-0.57
	(-3.55)	(-2.94)	(-3.57)	(-3.96)	(-4.50)	(-5.46)
	[0.05]	[0.23]	[0.34]	[0.44]	[0.48]	[0.58]
cay_t	5.69	8.95	8.80	6.26	7.28	10.11
	(3.60)	(4.16)	(3.96)	(1.39)	(1.63)	(2.08)
cdy_t	-0.34	1.69	2.18	3.25	3.96	6.41
	(-0.51)	(1.24)	(2.75)	(1.90)	(2.99)	(4.73)
s_t	-0.10	-0.14	-0.20	-0.32	-0.43	-0.43
	(-2.69)	(-3.18)	(-5.51)	(-3.07)	(-3.11)	(-3.76)
	[0.26]	[0.48]	[0.48]	[0.47]	[0.51]	[0.63]

Note: Newey–West (1987) t–statistics into parentheses. Adjusted R^2 statistics into brackets.

Table 2.6: Out-of-Sample Regressions: Excess Returns

First forecast period	1958		1968		1978	
horizon k (year)	β_k	R^2	β_k	R^2	β_k	R^2
1	-0.09	0.07	-0.13	0.12	-0.06	0.03
2	-0.16	0.13	-0.20	0.16	-0.12	0.08
3	-0.23	0.24	-0.25	0.26	-0.18	0.18
4	-0.32	0.39	-0.34	0.39	-0.20	0.18
5	-0.43	0.44	-0.45	0.45	-0.23	0.21
6	-0.49	0.44	-0.50	0.44	-0.25	0.27

tion ratio.

First, we gauge the ability of the model to replicate the long–horizon predictability of the surplus consumption ratio on excess returns, when s_t is defined by equation (2.2.7), for different values of the curvature of utility function θ. This experiment is reported in Table 2.7 for the following values of $\theta = 0.5, 1.5$ and 5. As shown in Table 2.7, we recover the same pattern whatever the value of θ. Indeed, the negative relationship between excess returns and the surplus consumption ratio remains unchanged. Moreover, s_t is statistically significant at any horizon. In addition, the predictability is an increasing function of horizon. The higher the prediction horizon, the higher the measure of fit R^2.

Second, we test the ability of alternative specifications of the level of habit to replicate the long horizon predictability results implied by the Campbell and Cochrane [1999] model. More precisely, we first assume that the utility function takes ratio form:

$$U(C_t, X_t) = \frac{(C_t/X_t)^{1-\theta} - 1}{1-\theta}$$

Compared to the difference form utility function, the ratio specification keeps the marginal utility positive but eliminates changing risk aversion. Then, we specify alternative specifications of the level of habit. Three cases are under investigation. The first case (i) is the "Catching up with the Joneses Joneses" model proposed

Table 2.7: Sensitivity Test

Horizon	$\theta = 0.5$		$\theta = 1.5$		$\theta = 5$	
h	β	R^2	β	R^2	β	R^2
1	-0.06	0.05	-0.12	0.07	-0.19	0.07
	(-3.14)		(-3.32)		(-3.74)	
2	-0.11	0.08	-0.19	0.10	-0.33	0.12
	(-2.94)		(-3.38)		(-4.04)	
3	-0.14	0.13	-0.27	0.18	-0.48	0.23
	(-3.08)		(-4.57)		(-3.55)	
4	-0.19	0.19	-0.37	0.29	-0.67	0.36
	(-2.86)		(-3.51)		(-4.76)	
5	-0.27	0.22	-0.51	0.33	-0.88	40
	(-2.67)		(-3.62)		(-5.06)	
6	-0.32	0.20	-0.59	0.30	-1.01	0.37
	(-2.41)		(-3.58)		(-4.98)	

by Abel [1990]: the habit level is function only of lagged aggregate consumption $X_t = \overline{C}_{t-1}$. Case (*ii*) is the "linear finite habit stock" model proposed by Li [2005] in which the level of habit is function of a finite sequence of past aggregate consumption $X_t = \left(\frac{1-\phi}{1-\phi^T}\right) \sum_{\tau=1}^{T} \phi^{\tau-1}\overline{C}_{t-\tau}$. The parameter ϕ governs the persistence of the habit stock. The parameter T indicates the duration of habit. When T is set to 1, we recover the Abel [1990] model. Finally, case (*iii*) is the "linear infinite habit stock" model proposed by Collard, Fève and Ghattassi [2006] in which the habit level is function of all past aggregate consumptions[9] $X_t = \overline{C}_t^{\delta} X_{t-1}^{1-\delta}$. The parameter δ governs the rate of depreciation of the habit stock.

Following Li [2005] and Collard, Fève and Ghattassi [2006], the parameters ϕ and δ are set to fit the first order serial correlation of the price–dividend ratio. It follows that $\phi = 0.92$, $T = 10$ and $\delta = 0.05$. As shown in Table 2.8, only habit persistence models that take into account the pattern of past consumptions, for instance models with infinite or finite linear habit stocks, manage to replicate the long horizon

[9] $X_t = \overline{C}_t^{\delta} X_{t-1}^{1-\delta}$ implies $x_t = \delta \sum_{i=1}^{\infty} (1-\delta)^{i-1} \overline{c}_{t-i}$.

Table 2.8: Univariate Long-horizon Regressions - Excess Stock Returns

horizon h (in year)					
1	2	3	4	5	6
Catching up with the Joneses (Abel [1990])					
-4.64	-6.15	-4.44	-6.29	-11.71	-14.01
(-2.11)	(-2.19)	(-1.93)	(-2.08)	(-2.83)	(-3.01)
[-0.29]	[-0.30]	[-0.26]	[-0.28]	[-0.39]	[-0.41]
0.09	0.08	0.04	0.07	0.16	0.19
Linear infinite habit stock ($\delta = 0.17$)					
-2.27	-3.70	-5.08	-7.23	-9.41	-10.42
(-3.59)	(-4.12)	(-4.84)	(-5.32)	(-5.39)	(-4.93)
[-0.48]	[-0.56]	[-0.65]	[-0.72]	[-0.73]	[-0.67]
0.09	0.13	0.22	0.38	0.44	0.40
Linear finite habit stock ($T = 10$ and $\phi = 0.92$)					
-2.01	-3.39	-4.74	-6.54	-8.37	-9.06
(-3.41)	(-4.02)	(-5.17)	(-5.85)	(-5.98)	(-5.30)
[-0.51]	[-0.60]	[-0.78]	[-0.88]	[-0.90]	[-0.80]
0.09	0.15	0.29	0.48	0.52	0.49

Note: The table reports OLS estimates of the regressors,t–statistics, Newey-West [1987] corrected t–statistics, the $\frac{t}{\sqrt{T}}$ test suggested in Valkanov [2003] , and adjusted R^2 statistics. The sample is annual and spans the period 1948-2001.

predictability of excess returns.

2.3 Cross Section of Expected Stock Returns

In this section, we explore the ability of the surplus consumption ratio to explain the cross section of expected returns. More precisely, we estimate the linear three factors model when the risk factors are the consumption growth, the lagged surplus consumption ratio ans their product. We compare the performance of the (C)CAPM with habit formation to alternative models: (*i*) the Fama-French three factors model, (*ii*) the unconditional version of the Capital Asset Pricing Model CAPM, (*iii*) the unconditional version of the Consumption Asset Pricing Model (C)CAPM and (*iv*) the conditional (C)CAPM proposed by Lettau and Ludvigson [2001b]. As benchmark, the surplus consumption ratio is generated using the specification (2.2.7) proposed by Campbell and Cochrane [1999]. Then, we evaluate the sensitivity of the empirical results of the benchmark model to the degree of curvature of the utility function[10].

The financial data used in this study are borrowed from the web site of Kenneth French[11]. We use data on (*i*) the value weighted returns of 25 Portfolios on the NYSE, AMEX and NASDAQ sorted by size and book-to-market value, (*ii*) the value weighted returns R_{vw} on the NYSE, AMEX and NASDAQ, (*iii*) the three month treasury bill as proxy for the risk–free rate and (*iv*) the two excess returns capturing the value and the size premia, denoted respectively SMB and HML. We convert the nominal returns to real returns then we convert the monthly real returns to quarterly real data spanning the first quarter of 1952 to the first quarter of 2005, that is, 212 observations for each of the 25 portfolios.

The macroeconomic data are borrowed from the web site of Martin Lettau[12]. We use quarterly data on (*i*) the real per capita consumption data for nondurables

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[10] As mentioned in the previous section, we set $\theta = 2$ to generate a series of surplus consumption ratio.

[11] We refer the reader to the Fama and French articles [1992, 1993, 1996] for more details.

[12] We refer the reader to the Lettau and Ludvigson articles [2001a, 2001b, 2005] for more details.

and services excluding shoes and clothing [13] and (*ii*) the *cay* as a proxy for the unobservable consumption to aggregate wealth ratio. Data are spanning from the first quarter of 1952 to the first quarter of 2005.

[13] Same results are obtained when we use quarterly real per capita consumption data for non-durables and services borrowed from NIPA.

Table 2.9: Cross Sectional Regressions: Fama-MacBeth Regressions Using 25 Fama-French Portfolios

					factors					R^2
Row	cst	R_m^e	SMB	HML	s_{-1}	cay_{-1}	Δc	$s_{-1}\Delta c$	$cay_{-1}\Delta c$	$(\overline{R}^2)$
1	3.85	-0.91								0.06
	(4.17)	(-0.85)								0.02
	(4.15)	(-0.95)								
2	4.03	-1.83	0.47	1.27						0.76
	(3.54)	(-1.45)	(1.26)	(3.29)						0.74
	(3.54)	(-1.38)	(1.26)	(3.28)						
3	1.86						0.30			0.14
	(3.24)						(1.56)			0.10
	(2.70)						(1.31)			
4	4.34					-1.12	0.21		-0.12	0.62
	(4.23)					(-2.78)	(1.00)		(-0.40)	0.56
	(2.94)					(-1.86)	(0.66)		(-0.26)	
5	1.02				-1.13		-0.008	-0.20		0.52
	(1.43)				(-4.55)		(-0.05)	(-1.22)		0.45
	(0.65)				(-2.10)		(-0.02)	(-0.56)		

Note: This table reports the Fama–MacBeth cross sectional coefficients. For each coefficient, two t-statistics are reported in parentheses. The top statistic uses the uncorrected Fama–MacBeth standard errors. The bottom statistic uses the Jagannathan and Wang [1998] correction.

1. Unconditional CAPM: $F_t =[\ cst\ R_{vw}^e]$
2. Fama-French 3 factors Model: $F_t =[\ cst\ R_{vw}^e\ SMB\ HML]$
3. Unconditional (C)CAPM: $F_t =[\ cst\ \Delta c_t]$
4. LL [2001b]: $F_t =[\ cst\ cay_{t-1}\ \Delta c_t\ cay_{t-1}\ \Delta c_t]$
5. CC [99]: $F_t =[\ cst\ s_{t-1}\ \Delta c_t\ s_{t-1}\Delta c_t]$

Note that Δc and cay are expressed in 100 basis points.

We use the beta representation of each model as the basis of the empirical work:

$$\mathbb{E}[R^e_{i,t}] = \mathbb{E}[R_{i,t} - R_{f,t}] = \beta_i \lambda \tag{2.3.9}$$

$$R^e_{i,t} = \beta_i f_t + u_{i,t} = F_t \beta + u_{i,t} \tag{2.3.10}$$

The linear beta representation is estimated by the 2 pass Fama–MacBeth regressions. As mentioned by Lettau and Ludvigson [2001b] and Jagannathan and al. [2002], among others, the Fama–MacBeth procedure is well adapted to a moderate number of quarterly time–series observations and a reasonably large number of asset returns. Table 2.9 reports the estimated coefficients, their uncorrected and Shanken-corrected t-statistics, the R^2 and the adjusted $\overline{R}^2$ for the cross–sectional regressions.

We first examine the unconditional capital asset pricing model CAPM. The single factor f_t in the unconditional CAPM is the market portfolio R_m as a proxy for the total wealth return. It is well commonly assumed that the value weighted returns R_{vw} on the NYSE, AMEX and NASDAQ is a good proxy for the market portfolio return R_m. As shown in the first panel of Table 2.9, the unconditional CAPM fails to explain the cross section of expected returns. The R^2 of the regression is only 6% and the adjusted $\overline{R}^2$ is about 0.02. Moreover, the estimated coefficient $\widehat{\lambda}$ is negative and it is not significantly different of zero.

The second panel of Table 2.9 presents results relative to Fama–French Model [1992, 1993]. The three factors of the Fama–French model are the market portfolio R_m and the two excess returns capturing the value and the size premia SMB and HML. The Fama–French model explains 76% of the cross sectional variability of expected returns. In addition, the t-statistic on the HML factor is highly statistically significant even after correction for sampling errors.

We now turn to the consumption–based asset pricing models. We first examine the single factor unconditional (C)CAPM. As can be seen in Panel 3 of Table 2.9, the unconditional (C)CAPM has little power to explain the cross section of expected returns. Indeed, the estimated coefficient $\widehat{\lambda}$ is not significant and the adjusted $\overline{R}^2$ does not exceed 10%. The Conditional (C)CAPM proposed by Lettau and Ludvigson [2001b] performs better than the unconditional version. As reported in Panel 4, the conditional (C)CAPM explains about 56% of the cross–sectional variations in returns. As documented by Lettau and Ludvigson [2001b], the scaling variable cay_{t-1} is statistically significant. However, the estimated coefficient associated to the lagged consumption to aggregate ratio, cay_{t-1}, has the wrong sign. Moreover, the results are not stable over time. As can be seen in Table 2.10, when we consider the subperiods $1952-2000$ and $1952-1995$, the estimated coefficient associated to cay_{t-1} has opposite signs. Furthermore, it is not significantly different from zero. In contrast with the conclusions of Lettau and Ludvigson [2001b], the cross section of average returns seems to be explained by the product of consumption growth and the lagged consumption to wealth ratio $cay_{t-1}\Delta c_t$ rather than cay_{t-1}.

The last panel of Table 2.9 presents the results of the Campbell and Cochrane [1999] model. The associated factors are the consumption growth, the lagged consumption to wealth ratio and their product. The model performs better than the unconditional (C)CAPM as it explains about 45% of the cross sectional variations of expected returns. Moreover, the estimated coefficient $\widehat{\lambda}$ associated to s_{t-1} has the right sign and it is statistically significant. These conclusions remain unchanged when we estimate the model for the subperiods $1952-2000$ and $1952-1995$.

Finally, we test the robustness of these empirical findings to alternative values of the coefficient of relative risk aversion $\theta = 2$, 5 and 10. Table 2.11 shows that the empirical implications of the (C)CAPM remain unchanged whatever the value of $\theta = 2$, 5 and 10. Indeed, the estimated coefficient associated to the lagged surplus consumption ratio is always negative and statistically significant even after correction for errors.

Table 2.10: Fama–MacBeth Regressions

Panel(A): 1952:2000							
			factors				R^2
Row	cst	s_{-1}	cay_{-1}	Δc	$s_{-1}\Delta c$	$cay_{-1}\Delta c$	$(\overline{R}^2)$
1	2.23			0.22			0.16
	(3.95)			(1.27)			0.13
	(3.57)			(1.15)			
2	2.06		0.14	0.22		0.42	0.58
	(2.84)		(0.45)	(1.23)		(1.98)	0.52
	(2.18)		(0.35)	(0.95)		(1.54)	
3	1.55	-0.75		0.06	-0.11		0.64
	(2.17)	(-3.08)		(0.39)	(-0.66)		0.59
	(1.32)	(-1.90)		(0.24)	(-0.41)		
			factors				R^2
Row	cst	s_{-1}	cay_{-1}	Δc	$s_{-1}\Delta c$	$cay_{-1}\Delta c$	$(\overline{R}^2)$
Panel(B): 1952:1995							
1	1.61			0.003			0.25
	(2.55)			(1.82)			0.22
	(2.08)			(1.42)			
2	2.48		-0.09	0.18		0.50	0.56
	(3.39)		(-0.30)	(0.85)		(2.03)	0.50
	(2.41)		(-0.21)	(0.61)		(1.46)	
3	1.92	-0.61		0.04	0.07		0.63
	(2.66)	(-3.01)		(0.30)	(0.32)		0.57
	(1.43)	(-1.66)		(0.16)	(0.17)		

Note: This Table reports the Fama–MacBeth cross sectional coefficients. For each coefficient, two t-statistics are reported in parentheses. The top statistic uses the uncorrected Fama–Macbeth standard errors. The bottom statistic uses the Jagannathan and Wang [1998] correction. Note that Δc and cay are expressed in 100 basis points.

1. Unconditional (C)CAPM: $F_t =[\ cst\ \Delta c_t]$
2. LL [2001b]: $F_t =[\ cst\ cay_{t-1}\ \Delta c_t\ cay_{t-1}\ \Delta c_t]$
3. CC [99]: $F_t =[\ cst\ s_{t-1}\ \Delta c_t\ s_{t-1}\Delta c_t]$

Table 2.11: Sensitivity Analysis

	factors				R^2
	cst	Δc	s_{-1}	$s_{-1}\Delta c$	$(\overline{R}^2)$
θ =2	1.02	-0.008	-1.13	-0.20	0.52
	(1.43)	(-0.05)	(-4.55)	(-1.22)	0.45
	(-0.65)	(-0.02)	(-2.10)	(-0.56)	
$\theta = 5$	0.77	-0.00	-0.60	-0.06	0.43
	(1.06)	(-0.05)	(-4.50)	(-0.66)	0.35
	(0.48)	(-0.02)	(-2.11)	(-0.31)	
$\theta = 10$	0.72	-0.01	-0.37	-0.03	0.37
	(0.98)	(-0.08)	(-4.32)	(-0.58)	0.28
	(0.47)	(-0.04)	(-2.15)	(-0.29)	

Note:
This table reports the Fama-MacBeth cross sectional coefficients. For each coefficient, two t-statistics are reported in parentheses. The top statistic uses the uncorrected Fama–MacBeth standard errors. The bottom statistic uses the Jagannathan and Wang [1998] correction. The term R^2 denotes the adjusted cross-sectional R^2 statistic and the $\overline{R}^2$ adjusts for the degree of freedom.

Concluding Remarks

This paper investigates the role of the surplus consumption ratio in predicting excess returns. First, we derived, from the (C)CAPM with habit formation, a linear relation linking surplus consumption ratio and long–horizon stock returns. Empirical results showed that the surplus consumption ratio is a good predictor of long–horizon excess returns but has little ability to forecast at short horizons. Compared to alternative indicators – the proxies for the consumption to wealth ratio (cay), (cdy) and the dividend yield ($d - p$) –, we found that the surplus consumption ratio performs better than the dividend yield. Moreover, the expected stock returns component explained by the surplus consumption ratio is not captured by the consumption to aggregate wealth ratios.

Second, we showed that the (C)CAPM with habit formation can be written as a linear three macroeconomic factors' pricing model. The factors are the consumption growth, the lagged surplus consumption and their product. On the empirical side, we found that the (C)CAPM with habit formation performs far better than the standard (C)CAPM by accounting for the cross section expected returns across the Fama–French (25)portfolios.

Appendix A. Cross Section Implications of Campbell and Cochrane [1999]

We consider the stochastic discount factor associated to CC[99] model:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\theta} \left(\frac{S_{t+1}}{S_t}\right)^{-\theta} \tag{2.3.11}$$

Plugging the expression of the surplus consumption ratio (2.2.7) into the stochastic discount factor (2.3.11), we obtain:

$$M_{t+1} = \tau_0(s_t) + \tau_1(s_t)\Delta c_{t+1} \tag{2.3.12}$$

where:

$$\tau_0(s_t)/\beta = 1 - \theta(1-\phi)(\overline{s} - s_t) + \theta g \lambda(s_t)$$

$$\tau_1(s_t)/\beta = -(1 - \lambda(s_t))\theta\Delta c_{t+1}$$

Hence, the stochastic discount factor associated to the Campbell and Cochrane [1999] model can be written as a linear beta model with time varying coefficients τ_0 and τ_1. The source of the variation of these parameters is the surplus consumption ratio s_t.

A linearization of τ_0 and τ_1 allows us to rewrite the linear beta model with time varying coefficients (2.3.12) as a linear beta model with constant coefficients. Assuming $\tau_0 = \eta_0 + \iota_0 s_t$ and $\tau_1 = \eta_1 + \iota_1 s_t$, the stochastic discount factor M_{t+1} can be written as follows:

$$M_{t+1} = b_0 + b_1 s_t + b_2 \Delta c_{t+1} + b_3 s_t \Delta c_{t+1} \tag{2.3.13}$$

Plugging the expression (2.3.11) into the Euler equation (3.2.4), we obtain:

$$1 = \mathbb{E}\left[(b_0 + b_1 s_t + b_2 \Delta c_{t+1} + b_3 s_t \Delta c_{t+1}) R_{i,t+1}\right] \tag{2.3.14}$$

It is straightforward to show that the equation (2.3.14) implies the following unconditional beta representation:

$$\mathbb{E}[R_{i,t}] = E[R_{f,t}] + \beta_{\Delta c}\lambda_{\Delta c} + \beta_{s_{-1}}\lambda_{s_{-1}} + \beta_{s_{-1}\Delta c}\lambda_{s_{-1}\Delta c} \tag{2.3.15}$$

Hence, the unconditional version of the Campbell and Cochrane [1999] model can be written as an unconditional multi-factor model. The factors are the consumption growth, the lagged surplus consumption ratio and their product.

Appendix B. Distributions

– FIGURE 2.2 ABOUT HERE –

– FIGURE 2.3 ABOUT HERE –

Appendix C. Average Pricing Errors

Figure 2.4 displays the pricing errors for each portfolio (ij) for the following models: (i) the unconditional capital asset pricing model CAPM, (ii) the unconditional standard consumption–based asset model (C)CAPM, (iii) the three factors Fama–French model (FFF), (iv) the conditional (C)CAPM proposed by Lettau and Ludvigson [2001b] and the unconditional (C)CAPM with habit formation proposed by Campbell and Cochrane [1999]. The index i denotes the size rank and the index j the book to market value rank. For instance, the portfolio 11 has the smallest size and the lowest book to market value. The pricing errors are generated using Fama–MacBeth methodology.

– FIGURE 2.4 ABOUT HERE –

Figure 2.1: Time–Series Variation of er_t, $d_t - p_t$ and s_t

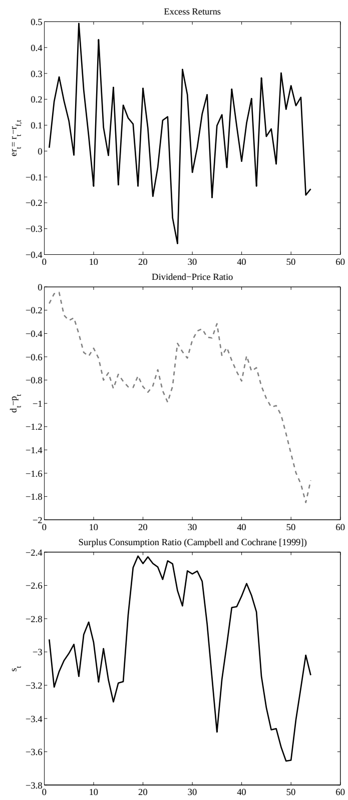
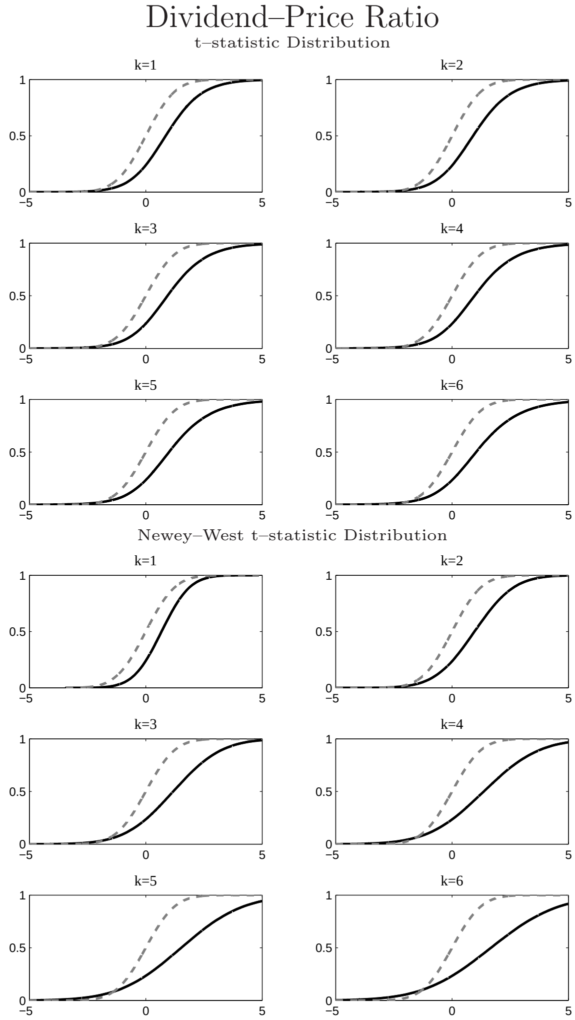
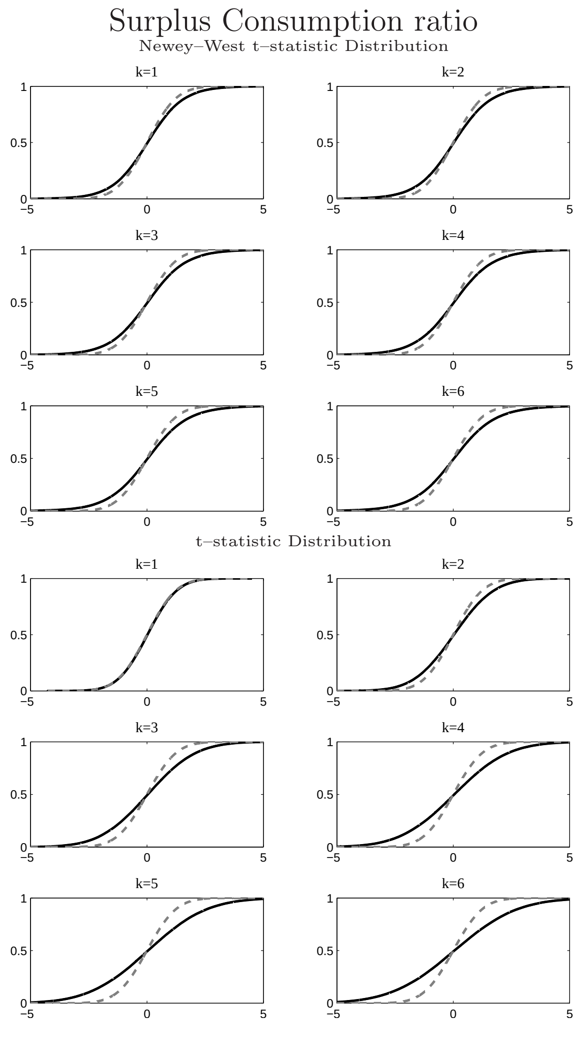


Figure 2.2: Simulations Results (1)



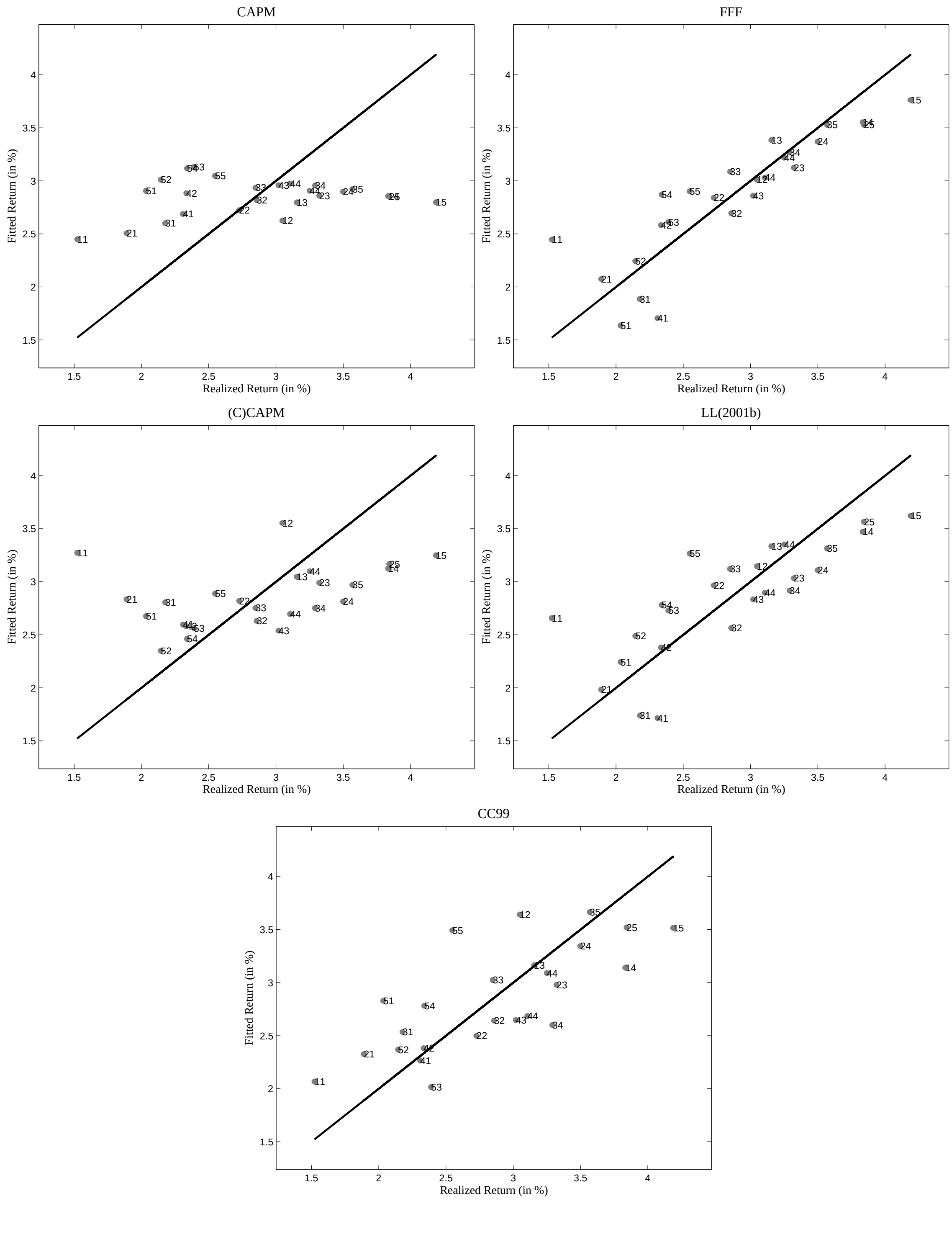
Note The dark line plain line corresponds to the empirical distribution obtained from the Monte Carlo experiment using $d_t - p_t$. The gray line is the student distribution.

Figure 2.3: Simulations Results (2)



<u>Note</u> The dark line plain line corresponds to the empirical distribution obtained from the Monte Carlo experiment using s_t. The gray line is the student distribution.

Figure 2.4: Realized vs Fitted returns (b): 25 Fama-French Portfolios



Chapter 3

Affine Equilibrium Asset Pricing Models with a Reference Level[1]

This paper investigates the asset pricing implications of a consumption asset pricing model with a reference level in which exogenous macroeconomic variables follow a first order Compound Autoregressive CaR(1) process (or affine process). The affine (log) stochastic discount factor and the affine specification for the exogenous variables have the advantage of providing closed form solutions for the price–dividend ratio and bond prices. Moreover, the proposed model admits a discrete affine term structure model. This paper provides a flexible modeling tool to evaluate the role of the preferences and the implications of the joint dynamics of macroeconomic variables in affecting the stock market and the term structure of interest rates. On the empirical side, our model replicates under the assumption of *iid* endowment environment: (*i*) the high persistence of the dividends yield, (*ii*) the excess volatility puzzle and (*iii*) the downward sloping real yield curve.

Key Words: Time nonseparable preferences, Reference level, Analytic solutions, Affine processes, Price dividends ratio, Affine term structure model

JEL Class.: C62, G12

[1] I am indebted to Nour Meddahi for his encouragement and supervision of my work. Part of this research was conducted when I was visiting CIREQ.

3.1 Introduction

Models in financial markets are typically based on the existence of some stochastic discount factor (SDF) M_{t+1} that prices the return on any asset i following the moment condition:

$$\mathbb{E}_t \left[M_{t+1}R_{i,t+1}\right] = 1 \tag{3.1.1}$$

These asset pricing models use either no arbitrage or equilibrium arguments to derive the stochastic discount factor. On the one hand, arbitrage–free framework, mostly used in the term structure literature, provide a good tool to describe and forecast movements of financial markets. However, these models stay silent about the main economics mechanisms at work. On the other hand, in equilibrium–based models, the stochastic discount factor is based on preferences and therefore on consumption data. For instance, Lucas (1978) and Breeden (1979) developed the so-called standard Consumption Asset Pricing Model (C)CAPM in which the risk of an asset can be measured by the conditional covariance of its return with per capita consumption growth. In their seminal papers, Mehra and Prescott (1985) and Weil (1989) showed that the quantitative evaluation of the standard (C)CAPM present two anomalies: the equity premium puzzle and the risk free rate puzzle. By relaxing the assumptions of the standard (C)CAPM, a large theoretical asset pricing literature has been devoted to resolve these two puzzles and to provide (C)CAPM models that are consistent with actual data on asset prices. Kocherlakota (1996), Campbell (2003) and Cochrane (2006) provide a thorough survey of this literature. In this paper, we will focus on two modifications of the assumptions of the standard model: the specification of the joint dynamics of macroeconomic variables and the specification of preferences. More precisely, we propose a consumption–based asset pricing model in an economy where the investors derive their utility from current individual consumption and an external reference level, and in which the exogenous macroeconomic variables follow a general class of affine processes.

Following Lucas (1978), several papers have considered the properties of asset prices in fully specified endowment economies. Campbell and Cochrane (1999) and Wachter (2006) solved for equilibrium asset prices in a (C)CAPM with habit formation and when the endowment follows an *iid* process. Piazzesi and Schneider (2006) provided solutions for the equilibrium yield curves under the assumptions of recursive preferences and a state–space specification for the consumption growth rate and the inflation. In our model, the exogenous macroeconomic variables – consumption, dividend and inflation – are assumed to follow a general class of affine processes, or equivalently, first order compound autoregressive processes, $CaR(1)$, proposed by Darolles, Gouriéroux and Jasiak (2006). It is important to note that several asset pricing models have been built with constrained versions of affine processes. An incomplete list includes Campbell and Cochrane (1999), Wachter (2006), Burnside (1998), Bansal and Yaron (2004) and Piazzesi and Schneider (2006). More importantly, imposing affine specification for the state variables allows us to study the implications of non–normal distributions for asset pricing.

The proposed (C)CAPM model assumes time non–separable preferences. Indeed, recent theoretical work has shown that relaxing time separable assumption about individual behavior has led to plausible improvements of the standard model. Kocherlakota (1996) provides thorough survey of this literature. Typically, two classes of preferences have been developed to improve the standard time separable preferences. First, Epstein and Zin (1989 and 1991) propose the recursive preferences as a generalization of the standard time–separable preferences. The main feature of recursive preferences is that the constant coefficient of relative risk aversion is not constrained to be equal to the reciprocal of the elasticity of inter–temporal substitution. A large body of asset pricing models has been built on the recursive utility function (see in particular Bansal and Yaron (2004), Eraker (2006), Garcia and Luger (2006), Piazzesi and Schneider (2006), among others).
Habit formation models propose an alternative approach to improve the standard

(C)CAPM. They capture the influence of the aggregate past consumptions (external habit) or / and the influence of individual's own past consumptions (internal habit) on the individual current consumption choice. For instance, Campbell and Cochrane (1999) introduce habit formation to derive time varying coefficient of relative risk aversion and therefore to explain a wide variety of the movements of stock markets. Wachter(2006) extends the model to the term structure implications. Garcia, Renault and Semenov (2005, 2006) have proposed a (C)CAPM model with a reference level that nests both habit formation models and non–expected recursive utility models. In line with these models, this paper proposes a consumption asset pricing model that is close in spirit to Campbell and Cochrane (1999), Garcia, Renault and Semenov (2005, 2006) and Wachter (2006) models. But, our model presents two main differences. First, the representative agent cares about the ratio of consumption to the reference level $U(C/V)$ rather than (i) the difference between them $U(C-V)$ as in Campbell and Cochrane (1999) or (ii) both the ratio of consomption to the reference level and the reference itself $U(C/V, V)$ as in Garcia, Renault and Semenov (2005, 2006). Second, one of the main contribution of this paper will be to propose a new specification for the reference level V. Indeed, Campbell and Cochrane (1999) have proposed a pure habit formation model by introducing a non linear habit stock that moves slowly in response to consumption. Garcia, Renault and Semenov (2005, 2006) have proposed a reference level that coincides with the optimal aggregate consumption at the same period. Therefore, the expectations of the reference level are assumed to depend on both past consumptions (as in habit formation models) and / or the return on the market portfolio (as in recursive utility models). The reference level proposed in this paper is specified to capture (i) the influence of past aggregate consumptions (*habit formation effect*) and (ii) the influence of cash flows news (*financial news effect*) on the current individual consumption choice. Moreover, the habit stock is assumed to follow an (log) linear specification. This new feature allows us to provide closed form solutions. Despite a constant coefficient of relative risk aversion, habit formation effect is essential for us to capture time variation in the equity premium and high persistence of the price–dividend ratio. The introduction of financial news effect allows us to generate a low risk free rate.

In our proposed model, the affine (log) stochastic discount factor and the affine exogenous variables allows us to derive closed form solutions for asset prices in general affine context. More precisely, we provide the closed form solutions for the price–dividend ratio, the price–consumption ratio and the real and nominal bond prices. Having the exact solutions for the asset pricing model is useful for at least two reasons. It allow us to better understand the role of preferences and the implications of the joint dynamics of the exogenous variables in affecting the financial markets. Moreover, the closed form solutions allow us to compare the exact solutions to numerical solutions used to approximate the non obvious closed form.

This paper enlarges significantly the class of asset pricing models that admit an analytic solution. For instance, Burnside (1998) provides a closed form solution for the price–dividend ratio in a standard asset pricing model when the growth rate of the endowment is a first–order Gaussian autoregressive process. Abel (1990) derives exact solutions for risky asset and one-period interest rate in a (C)CAPM with "Catching up with the Joneses" and when the endowment is an *iid* process. Collard, Fève and Ghattassi (2006) introduce habit persistence and persistent shocks into the story. This paper extends the exact solutions for asset pricing models proposed by Burnside(1998), Abel (1990) and Collard, Fève and Ghattassi (2006) along a number of dimensions. First, closed form solutions are available for a general class of affine exogenous variables. Second, the current consumer behavior is assumed to be influenced by habit persistence effect as well as cash flows news. Finally, exact solutions for the asset pricing model include the term structure of interest rates. More importantly, the proposed discrete model admits an affine term structure model. It is worth noting that a large literature on bond pricing, mostly formulated in an arbitrage–free framework, provide affine models. Duffie, Fillipovic and Singleton (2000) provide a survey of this literature for continuous time. Gouriéroux, Monfort and Polimenis (2002) extended the general approach of affine term structure to discrete time. While in arbitrage –free models, factors are extracted from bond yields, we focus in this paper on the explanatory power of macroeconomic variables. More precisely, all the factors of the proposed affine term structure model are exogenous and therefore no restriction on the dynamics of the state variables is needed to verify the arbitrage–free conditions. Understanding which macroeconomic variables move asset prices is important for at least two major reasons. First, it is well known that the equity premia and the expected returns are time–varying and counter–cyclical. Second, the short–term interest rate is a policy instrument that the central bank controls to achieve its economic stabilization goals.

To access the quantitative implications of our model, we consider a simple setting of affine processes which has appeared frequently in the literature. Indeed, following Campbell and Cochrane (1999) and Wachter (2006), we consider the simple case of *iid* endowment –consumption and dividend– environment. In contrast, inflation follows a first order Gaussian autoregressive process. The quantitative evaluation of the proposed model shows that it outperforms the standard (C)CAPM in predicting substantial equity premium and low risk free rate at moderate value of risk aversion. In addition, the proposed model replicates (i) the high persistence of the dividends yield, (ii) the excess volatility puzzle and (iii) the downward sloping real yield curve.

The rest of the paper is structured as follows. Section 2 presents the general setting. Section 3 provides closed form solutions for stock and bond prices. It also derives an application to the *iid* endowment environment. Section 4 presents the empirical implications of the model considering an *iid* endowment environment and a Gaussian AR(1) inflation. Section 4 concludes. A technical appendix collects the proofs of propositions.

3.2 The model

In the following we develop a (C)CAPM model with a reference level in which the exogenous macroeconomic variables – endowments and inflation – follow affine processes.

3.2.1 Preferences

We consider a representative agent economy when the preferences of the representative consumer are assumed to be time nonseparable. The agent derives her instantaneous utility from her individual current consumption C_t as well as a reference level V_t:

$$u_t = U(C_t, V_t)$$

where V_t summarizes the influence of available information on today's utility. For instance, in habit formation models, V_t depends on past and current individual or/and aggregate consumptions. In our proposed model, we assume that the reference level is external: it does not depend on the individual's own past consumption decisions. At this stage, no further assumption will be placed on V_t. The reference level will be defined later.

Following Abel (1990), the instantaneous utility function is specified in ratio:

$$U(C_t, V_t) = \frac{\left(\frac{C_t}{V_t}\right)^{1-\theta} - 1}{1-\theta} \tag{3.2.2}$$

where the parameter θ denotes the utility curvature parameter and has the restriction of $\theta > 0$. We hereby depart from a strand of the literature that specifies habit formation in term of the difference between current consumption and a reference level, as in Campbell and Cochrane (1999) and Wachter (2006). Compared to the specification in difference $U(C_t - V_t)$, the ratio specification presents two main differences. First, when the utility function is specified in difference, V_t should never fall below C_t to ensure positive and finite marginal utility. The ratio specification keeps the marginal utility[2] positive whatever the positive value of the reference level V_t. Second, when the utility function is specified in difference, the risk aversion is time–variant. Alike the standard CRRA utility function, the ratio specification keeps the risk aversion time–invariant and is given by θ. Note that the assumption of external reference level V_t and the strict positivity of the parameter $\theta > 0$ insure the concavity[3] of the utility function. Note that Garcia, Renault and Semenov (2005, 2006) have proposed a more general utility function by assuming that the utility function depends on the ratio of current consumption and a reference level as well as the reference level itself.

As documented by Heaton (1995) and Cochrane (2005), the backward–looking preferences are used to model either the local substitution of consumption (*duration*) or/and its long run persistence (*complementarity*). In our model, the reference level is assumed to capture only complementarity effect. In other words, the utility of the agent is assumed to depend on her own consumption as well as a standard living which is common to all the others. Equivalently, preferences are assumed to take into account the "Catching up with the Joneses" phenomenon. Formally, the complementarity effect implies that the marginal utility is an increasing function of the reference level. As the marginal utility verifies

$$\frac{\partial U_c(C_t, V_t)}{\partial V_t} = (\theta - 1)C_t^{-\theta} V_t^{\theta - 2}$$

we restrict our analysis to $\theta > 1$ to insure the complementarity effect. The consumer is assumed to be risk averse.

The budget constraint of the representative agent is given by:

$$W_{t+1} = R_{w,t+1} \left(W_t - C_t \right) \tag{3.2.3}$$

where W_t denotes the aggregate wealth and $R_{w,t}$ the return on the (unobservable)

[2]The marginal utility is given by $\frac{\partial U(C_t, V_t)}{\partial C_t} = C_t^{-\theta} V_t^{\theta - 1}$

[3]The second order condition is $\frac{\partial^2 U(C_t, V_t)}{\partial^2 C_t} = -\theta C_t^{-\theta - 1} V_t^{\theta - 1} < 0$

market portfolio, or equivalently on the aggregate wealth, known at the beginning of period t. In our model, the aggregate wealth includes financial and non financial revenues. The financial revenues consist on the financial asset holdings. The agent enters period t with a number of shares S_t – measured in terms of consumption goods – carried over the previous period as a means to transfer wealth inter–temporally. The total number of shares which are traded in this economy is normalized to 1. At the beginning of each period t, each share is valuated at price P_t and pays dividends D_t. Then, the financial income of the consumer at each period t is equal to $(P_t + D_t)\, S_t$. The non financial income includes labor income and taxes transfers, among others. Note that ignoring the non financial income implies the equality of dividend and consumption. Indeed, imposing $W_{t+1} = (P_t + D_t)\, S_t$ and $R_{w,t+1} = (P_{t+1} + D_{t+1})\,/P_t$, the budget constraint (3.2.3) rewrites

$$(P_t + D_t)S_t = P_t S_{t+1} + C_t$$

Since the economy is populated by a single representative agent and the total number of shares is normalized to 1, we have $S_t = 1$ and therefore $C_t = D_t$.
In our general setting, the budget constraint (3.2.3) is expressed in terms of aggregate wealth. Therefore, we make the distinction between consumption and dividend. This generalization of the standard model of Lucas (1978) is nowadays common in the (C)CAPM models, as actual data on consumption and dividends present large differences in term of mean, persistence and others characteristics.

The representative agent maximizes her inter–temporal utility function:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{(C_{t+i}/V_{t+i})^{1-\theta} - 1}{1-\theta}$$

subject to her budget constraint (3.2.3). The parameter $\beta > 0$ denotes the constant subjective discount factor. Expectations are conditional on information available at the beginning of period t.
Because reference level is external, the one–period stochastic discount factor (SDF)

is given by:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \left(\frac{V_{t+1}}{V_t} \right)^{(\theta-1)}$$

The SDF represents the date t real prices of the contingent claims that pay off one unit of consumption in $t+1$. In business cycle models, the SDF is often called the inter–temporal rate of substitution. It governs the willingness of the consumer to transfer consumption inter–temporally.

The first–order condition that determines the agent's consumption choices is given by the following Euler equation:

$$1 = \mathbb{E}_t \left[M_{t+1} R_{i,t+1} \right] \tag{3.2.4}$$

where $R_{i,t+1}$ is the return of any asset i.
For instance, let $R_{t+1} = \frac{D_{t+1}+P_{t+1}}{P_t}$ be the return on risky asset. Then, the Euler equation (3.2.4) rewrites:

$$\frac{P_t}{D_t} = \mathbb{E}_t \left[M_{t+1} \frac{D_{t+1}}{D_t} \left(\frac{P_{t+1}}{D_{t+1}} + 1 \right) \right] \tag{3.2.5}$$

Let $R_{w,t+1} = \frac{C_{t+1}+P_{t+1}}{P_t}$ be the return on the aggregate wealth or, equivalently, on the (unobservable) market portfolio. Then, the price–consumption ratio is determined as follows:

$$\frac{P_t}{C_t} = \mathbb{E}_t \left[M_{t+1} \frac{C_{t+1}}{C_t} \left(\frac{P_{t+1}}{C_{t+1}} + 1 \right) \right] \tag{3.2.6}$$

For real bonds, the Euler equation (3.2.4) implies:

$$P_{n,t} = \mathbb{E}_t \left[M_{t+1} P_{n-1,t+1} \right] \tag{3.2.7}$$

where $P_{n,t}$ denotes the real price of a real bond maturing in n periods, at period t. The payoff of real bonds is fixed in terms of units of the consumption good. At maturity, the real bond pays one unit of the consumption good, implying the boundary condition:

$$P_{0,t} = 1 \tag{3.2.8}$$

For nominal bonds, the Euler equation (3.2.4) rewrites:

$$\frac{P^{\$}_{n,t}}{\Pi_t} = \mathbb{E}_t \left[M_{t+1} \frac{P^{\$}_{n-1,t+1}}{\Pi_{t+1}} \right] \tag{3.2.9}$$

where $P^{\$}_{n,t}$ denotes the nominal price of a nominal bond maturing in n periods, at period t and Π_t the price level at period t. The payoff of nominal bonds is fixed in terms of units of money. At maturity, the nominal bond pays one unit of money, implying the boundary condition:

$$\frac{P^{\$}_{0,t}}{\Pi_t} = \frac{1}{\Pi_t} \tag{3.2.10}$$

Note that the real risk free rate verifies $R_{t+1} = 1/P_{1,t}$ and the nominal risk free rate is given by $R^{\$}_{t+1} = 1/P^{\$}_{1,t}$.
To solve the pricing formulas (3.2.5), (3.2.6), (3.2.7) and (3.2.9), we need to specify (i) the reference level V_t and (ii) the law of motion of the endowments[4] Δc_t and Δd_t and inflation $\Delta \pi_t$.

3.2.2 Specification of the reference level

The reference level V_t is assumed to be defined as follows:

$$\Delta v_t = \log \frac{V_t}{V_{t-1}} = \varphi_1 \Delta x_t + \varphi_2 (\Delta d_t - g_{\Delta d}) \tag{3.2.11}$$

where the variable $x_t = \log X_t$ denotes the consumption index. The parameters $\varphi_1 > 0$ and $\varphi_2 > 0$ rule the sensitivity of preferences to respectively the consumption index and the contemporaneous deviation of the dividends growth rate Δd_t from its mean $g_{\Delta d}$. The consumption index X_t is assumed to evolve according to

$$X_t = \overline{C}^{\delta}_{t-1} X^{1-\delta}_{t-1} \tag{3.2.12}$$

where $\overline{C}_t$ denotes the aggregate consumption at period t. Note that the habit stock X_t is known at period $t-1$.

[4] Throughout, lowercase letters are used for variables in logarithm.

As shown in the definition (3.2.11), the reference level V_t presents two components $\varphi_1 \Delta x_t$ and $\varphi_2(\Delta d_t - g_{\Delta d})$ that summarize the influence of respectively the past aggregate consumptions and the cash flows news on the individual current consumption choice.

The first component $\varphi_1 \Delta x_t$ is a familiar term in habit formation models[5]. To better understand the habit formation phenomenon, it is convenient to rewrite the equation (3.2.12):

$$x_t = \delta \sum_{i=0}^{\infty} (1-\delta)^i \bar{c}_{t-i-1} \tag{3.2.13}$$

The expression (3.2.13) shows that the (log) consumption index x_t can be viewed as a weighted average of all past (log) aggregate consumptions. Consequently, X_t can be viewed as the habit stock level. It follows that Δx_t summarizes the influence of past consumptions levels on today's utility and the parameter $\varphi_1 > 0$ measures the degree of the "Catching up with the Joneses". More importantly, habit over consumption develops slowly, implying a long run persistence of habit level. The parameter $(1-\delta)$ rules the rate of depreciation of past aggregate consumptions.

Our specification of the reference level (3.2.11) departs from the standard habit formation literature – which specifies the reference level only in terms of individual and/or aggregate consumptions – and adds a new term. This additional term, $\varphi_2(\Delta d_t - g_{\Delta d})$ captures the influence of financial news – measured by the deviation of current dividend growth rate from its unconditional mean– on the individual current consumption choice. The introduction of this term is motivated by recent literature which emphasizes the role of cash flows news to explain asset prices. For instance, Bansal, Dittmar and Lundblad (2005) provide empirical evidence pertaining the ability of cash flows to explain cross sectional variation in risk premium. Crose, Martin and Ludvigson (2006) study the impact of cash flows news on the beliefs of agent concerning the exogenous variables. Our proposed model, while

[5] See in particular Campbell (2003) for a survey of this class of models.

similar in motivation to Bansal, Dittmar and Lundblad (2005) and Crose, Martin and Ludvigson (2006), assumes that cash flows news affect the consumer behavior rather than the beliefs of agent concerning the exogenous variables. More precisely, we assume that contemporaneous good financial news incite the agent to increase her current individual consumption.

The specification (3.2.11) nests several specifications of the reference level proposed in the (C)CAPM literature. For instance, setting $\varphi_2 = 0$, we recover the habit stock model proposed by Collard, Fève and Ghattassi (2006). When $\varphi_2 = 0$ and $\delta = 1$, we recover the "Catching up with the Joneses" model proposed by Abel (1990). Setting $\varphi_1 = 0$ and $\varphi_2 = 0$, we recover the time separable utility function of the standard (C)CAPM of the form $u_t = (C_t^{1-\theta} - 1)/(1-\theta)$.

To better understand the economic intuition behind the introduction of the reference level, it is convenient to rewrite the (log) stochastic discount factor:

$$m_{t+1} = \ln\beta - \theta\Delta c_{t+1} + \varphi_1(\theta-1)\Delta x_{t+1} + \varphi_2(\theta-1)(\Delta d_{t+1} - g_{\Delta d}) \qquad (3.2.14)$$

As mentioned above, the stochastic discount factor M_{t+1} can be interpreted as (i) the real price at time t of the contingent claims that pay one unit of consumption goods at time $t+1$ and (ii) the inter–temporal rate of substitution.

The first two terms on the right hand side of (3.2.14) are familiar from the power utility function. It governs the inter–temporal consumption decisions in the standard (C)CAPM. A positive shock of future consumption growth rate leads to greater future consumption and therefore lower future marginal utility of consumption. Therefore, the agent is willing to transfer future consumption toward current period. She borrows from the future to smooth consumption, driving downward asset prices.

The last two terms on the right hand side of (3.2.14) specify how the habit formation and the financial news affect the inter–temporal consumption decisions. The specifications imposed to both reference level and utility function imply that

φ_1 and φ_2 govern the size of respectively the habit formation and financial news effects, while θ governs their directions. For instance, when $\theta > 1$, an increase in the habit level $\Delta x_{t+1} > 0$, known at date t, implies a positive increase in the reference level, driving up the inter–temporal rate of substitution. Likewise, a positive future shock of dividends growth rate $(\Delta d_{t+1} - g_{\Delta d})$ increases the inter–temporal rate of substitution.

We start by analyzing how the habit formation affects the stochastic discount factor. As explained above, a positive future consumption growth rate implies that the consumer is willing to transfer $t + 1$ consumption toward t. However, when the consumer brings future consumption C_{t+1} back to period t, she hereby raises the consumption standard X_{t+1} for the next period. This raises future marginal utility $[\partial U(C_{t+1})/\partial C_{t+1}]$ and therefore plays against the standard stochastic discount factor. Note that habit reacts gradually to consumption, implying a long run persistence of the habit formation effect.

An additional effect stemming from dividend news, comes into play. This is reflected in the term $\varphi_2(\theta - 1)(\Delta d_{t+1} - g_{\Delta d})$. We assume that good financial news provide incentives to increase consumption. Therefore, a positive future shock of dividend growth rate drives up the future marginal utility. This puts upward pressions on asset prices and expected returns decrease.

A final remark regards the form of the stochastic discount factor. Equation (3.2.14) shows that the (log) SDF presents an exponential affine specification. According to Gouriéroux, Montfort and Polimenis (2002), the affine specification of the SDF is obtained in a general equilibrium framework, when the representative agent has time separable power utility function and the endowment process depends in an appropriate way of exogenous factors. As detailed earlier, the proposed model considers time nonseparable preferences in a general equilibrium setting. It thus fulfils the required conditions to justify an affine (log) SDF representation. To complete the description of the model, the exogenous processes are specified in the next section.

3.2.3 The dynamics of exogenous processes

The pricing model is completed by specifying the law of motion of exogenous variables: (i) endowments –i.e consumption and dividend – and (ii) inflation. These macroeconomic variables are assumed to be linear functions of the state variable Y_t of dimension n:

$$\Delta c_t = \gamma_c' Y_t$$
$$\Delta d_t = \gamma_d' Y_t$$
$$\Delta \pi_t = \gamma_\pi' Y_t$$

where γ_c, γ_d and γ_π are $(n \times 1)$ vectors. In the general case, we make the distinction between consumption and dividend. Setting $\gamma_c = \gamma_d$, we recover the equality between them. It is important to note that no restriction will be placed on the number of the state variables n.

The state variable Y_t is assumed to follow a first order Compound Autoregressive Process, $CaR(1)$, introduced by Darolles, Gouriéroux and Jasiak (2006).

Definition 2.3.1: *The vector process Y of dimension n is compound autoregressive of order 1, if and only if the conditional distribution of Y_{t+1}, given Y_t, admits the affine conditional Laplace transform function:*

$$\mathbb{E}_t \left(\exp z^{'} Y_{t+1} \right) = \exp(a'(z) Y_t + b(z)) \tag{3.2.15}$$

where $a(z) \neq 0$ for any multi–variable z with complex components, such that the conditional expectation exists.

As the state variable Y_t is assumed to be exogenous, the functions $a(z)$ and $b(z)$ are known explicitly. Given the exponential–affine form of the conditional Laplace transform functions, CaR(1) processes are also so–called affine processes. The general class of CaR(1) processes nests some endowment environments proposed in the

(C)CAPM literature. Appendix A reports the functions $a(z)$ and $b(z)$ associated with the specification of Piazzesi and Schneider (2006) and Bansal and Yaron (2004).

The state variable Y_t is assumed to contain exogenous macroeconomic factors and does not include asset prices. This specification presents two main advantages. First, no restriction on the dynamics of Y_t is needed to verify the arbitrage–free conditions. Second, several researchers have focused on the linkage between real economy and financial markets. Cochrane (2006) provides thorough survey of this literature. For instance, recent papers evaluate the empirical linkage between macroeconomic factors and financial markets (see in particular Lettau and Ludvigson (2001, 2005), Cochrane and Piazzesi (2005) and Ludvigson and Ng (2006) among others). As in these papers, one of the main contribution of this work is to provide an analytical tool to evaluate how the joint dynamics of macroeconomic variables affect financial markets. It is worth noting that in our general setting, no restriction is placed on the number n of the macroeconomic variables.

3.3 Model Solution

This section calculates the asset prices. Affine state variables and affine (log) stochastic discount factor allow us to compute the closed form solutions. The availability of the closed form solutions for asset prices enhances our understanding of (i) the economic mechanisms driving the stock market and the term structure of interest rates and (ii) the implications of the joint dynamics imposed to exogenous variables. This section focuses explicitly on the analytic solutions for the price–dividend ratio, the price–consumption ratio and the real and nominal bonds yields that will be derived under the assumption of a general class of affine exogenous variables. The analytic solutions are presented in next propositions. Proofs are reported in Appendix B. Then, these analytic formulas will be applied in *iid* context. The main goal of this exercise is to better understand the economic intuition behind financial implications.

3.3.1 General setting

This section provides the closed form solutions for the price–dividend ratio, the price–consumption ratio and the real and nominal band yields assuming that the exogenous variables follow a class of affine processes.

3.3.1.1 The price–dividend ratio and the price–consumption ratio

The forward looking equations (3.2.5) and (3.2.6) of the price–dividend ratio and the price–consumption ratio, derived in the previous section 3.2.1, admit the closed form solutions reported in proposition 3.3.1.

Proposition 3.3.1 *(i) The solution for the price–dividend ratio P_t/D_t is given by:*

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \beta^n \exp\left(\varphi_1(\theta-1)((1-\delta)^n-1)z_t + A_d(n,0)'Y_t + B_d(n,0) + C_n\right)$$

where $z_t = \log(X_t/C_t)$ denotes the (log) habit to consumption ratio and $C_n = -n\varphi_2(\theta-1)g_{\Delta d}$.

$A_d(n,0)$ and $B_d(n,0)$ are derived recursively as follows:

$$A_d(n,n) = B_d(n,n) = 0$$

and for $0 \leq j < n$:

$$A_d(n,n-j-1) = a(\alpha^d_{n,n-j} + A_d(n,n-j))$$
$$B_d(n,n-j-1) = b(\alpha^d_{n,n-j} + A_d(n,n-j)) + B_d(n,n-j)$$

where $\alpha^d_{n,j} = \left(-\theta + \varphi_1(\theta-1)(1-(1-\delta)^{n-j})\right)\gamma_c + (1+\varphi_2(\theta-1))\gamma_d$

(ii) The solution for the price–consumption ratio P_t/C_t is given by:

$$\frac{P_t}{C_t} = \sum_{n=1}^{\infty} \beta^n \exp\left(\varphi_1(\theta-1)((1-\delta)^n-1)z_t + A_c(n,0)'Y_t + B_c(n,0) + C_n\right)$$

where $z_t = \log(X_t/C_t)$ denotes the (log) habit to consumption ratio and $C_n = -n\varphi_2(\theta-1)g_{\Delta d}$.

$A_c(n,0)$ and $B_c(n,0)$ are derived recursively as follows:

$$A_c(n,n) = B_c(n,n) = 0$$

and for $0 \leq j < n$:

$$\begin{aligned} A_c(n,n-j-1) &= a(\alpha^c_{n,n-j} + A_c(n,n-j)) \\ B_c(n,n-j-1) &= b(\alpha^c_{n,n-j} + A_c(n,n-j)) + B_c(n,n-j) \end{aligned}$$

where $\alpha^c_{i,j} = \left(-(\theta-1) + \varphi_1(\theta-1)(1-(1-\delta)^{i-j})\right)\gamma_c + \varphi_2(\theta-1)\gamma_d$

The Proposition 3.3.1 illustrates some properties of our model. First, in our general setting, we make the distinction between (i) consumption as the payoff of aggregate wealth and (ii) dividend as the payoff of equities. Therefore, Proposition 3.3.1 reports the closed form solutions for both price–dividend and price–consumption ratios. Setting $\gamma_c = \gamma_d$, dividend equals consumption and makes the two solutions coincide. Second, the equilibrium–based approach allows us to derive the solutions for price-dividend and price–consumption ratios as function of only exogenous macroeconomic variables: the state variable Y_t and the habit to consumption ratio z_t. Finally, the pricing formulas essentially depend on the preferences of the representative agent (β, θ, φ_1, φ_2 and δ) and the dynamics of the state variable, well defined by functions $a(z)$ and $b(z)$. However, given the recursive form of these formulas, we need to specify explicitly the functions $a(z)$ and $b(z)$ to better understand the role of the preference parameters and the joint dynamics of the macroeconomic variables in determining the price–dividend and the price–consumption ratios. For this purpose, next section 3.3.2 will apply the general formulas reported in Proposition 3.3.1 in *iid* context.

Proposition 3.3.1 only establishes the existence of the solutions for the price–dividend and the price–consumption ratios and does not guarantee that these solutions are bounded. Indeed, the solution involves an infinite series which may or may not converge. This requires conditions of convergence to guarantee the existence of bounded solutions. Given the recursive form of the formulas in the general setting, we need to specify explicitly the functions $a(z)$ and $b(z)$ to derive conditions of convergence.

3.3.1.2 The term structure of interest rates

The term structure model of interest rates describes the evolution of bond prices of various maturities n at any given time t. The following proposition provides the closed form solutions for the real and nominal bond prices.

Proposition 3.3.2 *(i) The solution for the price of a real bond maturing in n periods $P_{n,t}$ is given by:*

$$P_{n,t} = \beta^n \exp\left(C_n + \varphi_1(\theta-1)((1-\delta)^n - 1)z_t + A_b(n,0)Y_t + B_b(n,0)\right)$$

where $C_n = -n\varphi_2(\theta-1)g_{\Delta d}$.
The coefficients $A_b(n,i)$ and $B_b(n,i)$ are defined as follows:

$$A_b(n,n) = B_b(n,n) = 0$$

and for $0 \leq i < n$:

$$\begin{aligned} A_b(n,n-i-1) &= a(\tau_{n,i} + A_b(n,n-i)) \\ B_b(n,n-i-1) &= b(\tau_{n,i} + A_b(n,n-i)) + B_b(n,n-i) \end{aligned}$$

where $\tau_{n,i} = (-\theta + \varphi_1(\theta-1)(1-(1-\delta)^{n-i}))\gamma_c + \varphi_2(\theta-1)\gamma_d$

(ii) The solution for the price of a nominal bond maturing in $n \geq 1$ periods $P^{\$}_{n,t}$ is given by:

$$P^{\$}_{n,t} = \beta^n \exp\left(C^{\$}_n + \varphi_1(\theta-1)((1-\delta)^n - 1)z_t + A^{\$}_b(n,0)Y_t + B^{\$}_b(n,0)\right)$$

where $C^{\$}_n = -n\varphi_2(\theta-1)g_{\Delta d}$.
The coefficients $A^{\$}_b(n,i)$ and $B^{\$}_b(n,i)$ are defined as follows:

$$A^{\$}_b(n,n) = B^{\$}_b(n,n) = 0$$

and for $0 \leq i < n$:

$$\begin{aligned} A^{\$}_b(n,n-i-1) &= a(\tau^{\$}_{n,i} + A^{\$}_b(n,n-i)) \\ B^{\$}_b(n,n-i-1) &= b(\tau^{\$}_{n,i} + A^{\$}_b(n,n-i)) + B^{\$}_b(n,n-i) \end{aligned}$$

where $\tau^{\$}_{n,i} = (-\theta + \varphi_1(\theta-1)(1-(1-\delta)^{n-i}))\gamma_c + \varphi_2(\theta-1)\gamma_d - \gamma_\pi$

Several findings stand out from Proposition 2. First, it is important to note that our general setting admits an affine discrete term structure model. The affine representations of the state variables and the (log) stochastic discount factor are the key features of providing an affine discrete model. In addition, the explanatory factors are exclusively macroeconomic variables, including the state variable Y_t and the habit to consumption ratio z_t. Therefore, no arbitrage–free restriction is imposed to the solutions for bond prices. However, Proposition 3.3.2 only establishes the existence of the solutions for the term structure of interest rates but does not guarantee the positivity of the real yields.

By definition, the real (nominal) interest rate is the yield on a real (nominal) bond maturing next period. therefore, setting $n = 1$, we recover the expressions of the real $r_{f,t+1}$ and nominal $r^{\$}_{f,t+1}$ interest rates:

$$\begin{aligned} r_{f,t+1} = &- \log(\beta) + \lambda_1(\theta - 1)\delta z_t + \lambda_2\varphi(\theta - 1)g_{\Delta d} - a(-\theta\gamma_c + \lambda_2\varphi(\theta - 1)\gamma_d)'Y_t \\ &- b(-\theta\gamma_c + \lambda_2\varphi(\theta - 1)\gamma_d) \end{aligned}$$

and

$$\begin{aligned} r^{\$}_{f,t+1} = &- \log(\beta) + \lambda_1(\theta - 1)\delta z_t + \lambda_2\varphi(\theta - 1)g_{\Delta d} - a(-\theta\gamma_c + \lambda_2\varphi(\theta - 1)\gamma_d - \gamma_\pi)Y_t \\ &- b(-\theta\gamma_c + \lambda_2\varphi(\theta - 1)\gamma_d - \gamma_\pi) \end{aligned}$$

In summary, Propositions 3.3.1 and 3.3.2 provide the analytical pricing formulas for the price–dividend ratio, the price–consumption ratio, the real and nominal interest rates and the real and nominal yields of various maturities n. The pricing formulas obviously depend on the preference parameters and the law of motions of the state variable Y_t and the habit to consumption ratio z_t. However, it is difficult to study the role of the consumer behavior and the dynamics of macroeconomic variables, given the recursive form of the closed form solutions. In next section we therefore apply the general formulas in fully specified environment. We consider the special case of an *iid* endowment economy in which the inflation follows a first order Gaussian autoregressive process.

3.3.2 A special case

In the sequel, we present the exact solutions for the (C)CAPM model with a reference level derived in section 3.2.2 under the assumption of *iid* endowment –i.e. consumption and dividend – economy and a first order Gaussian autoregressive inflation. The simple case of an *iid* endowment was appeared frequently in the asset pricing literature. An incomplete list includes Abel (1990, 1999 and 2005), Campbell and Cochrane (1999) and Wachter (2006). Note that the proposed dynamics imposed to the macroeconomic variables (*iid* consumption and dividend growth rates and a Gaussian $AR(1)$ inflation) verify the affine specification described in section 3.2.3.

More specifically, we assume that the consumption growth rate Δc_t and the dividend growth rate Δd_t are imperfectly correlated and follow *iid* gaussian processes of the form:

$$\Delta c_{t+1} = g_{\Delta c} + \varepsilon_{\Delta c,t+1} \text{ where } \varepsilon_{\Delta c} \rightsquigarrow NID(0, \sigma^2_{\Delta c})$$
$$\Delta d_{t+1} = g_{\Delta d} + \varepsilon_{\Delta d,t+1} \text{ where } \varepsilon_{\Delta d} \rightsquigarrow NID(0, \sigma^2_{\Delta d})$$

where $g_{\Delta c}$ and $g_{\Delta d}$ denote respectively the means of the consumption and dividend growth rates and $\sigma_{\Delta c}$ and $\sigma_{\Delta d}$ denote the standard deviation of the corresponding innovations. Let $\rho_{\Delta c \Delta d}$ represent the covariance of the innovations.

Given the high persistence of inflation observed in actual data, we simply assume that inflation process follows a Gaussian $AR(1)$ of the form:

$$\Delta\pi_{t+1} = (1-\phi)g_{\Delta\pi} + \phi\Delta\pi_t + \varepsilon_{\Delta\pi,t+1}$$

where $\varepsilon_{\Delta\pi,t} \rightsquigarrow NID(0, \sigma^2_{\Delta\pi})$ and $|\phi| < 1$ to guarantee a stationary process.

Let Σ denote the covariance matrix of innovations of consumption growth, dividend growth and inflation $[\varepsilon_{\Delta c,t}\ \varepsilon_{\Delta d,t}\ \varepsilon_{\Delta\pi,t}]'$:

$$\Sigma = \begin{pmatrix} \sigma^2_{\Delta c} & \rho_{\Delta c \Delta d} & \rho_{\Delta c \Delta\pi} \\ . & \sigma^2_{\Delta d} & \rho_{\Delta d \Delta\pi} \\ . & . & \sigma^2_{\Delta\pi} \end{pmatrix}$$

Then, the affine representation of the exogenous processes (3.2.15) is satisfied with:

- $Y_t = [\ \Delta c_t\ \Delta d_t\ \Delta\pi_t]'$ the state variable
- $\gamma_{\Delta c} = [1\ 0\ 0]'$, $\gamma_{\Delta d} = [0\ 1\ 0]'$ and $\gamma_{\Delta\pi} = [0\ 0\ 1]'$
- $a(z) = z \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi \end{bmatrix}$ and $b(z) = z' \begin{bmatrix} g_{\Delta c} \\ g_{\Delta d} \\ g_{\Delta\pi} \end{bmatrix} + \frac{1}{2} z'\Sigma z$

Therefore, the general formulas derived in section 3.3.1 can be used to compute the explicit prices of bonds and stocks in this *iid* endowment environment economy. The main goal of this example is to better understand the economic mechanisms and the role of the consumer behavior.

3.3.2.1 The price–dividend ratio

In the following, we apply the closed form solution for the price–dividend ratio reported in Proposition 3.3.1 in the *iid* context.

The price–dividend ratio is given by:

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \beta^n \exp\left(\Lambda_n z_t + \Gamma_n\right) \tag{3.3.16}$$

where

$$\Lambda_n = \varphi_1(\theta - 1)\left((1-\delta)^n - 1\right)$$

and

$$\begin{aligned}\Gamma_n = {} & n g_{\Delta d} + g_{\Delta c}\left[-n\theta + n\varphi_1(\theta-1) - \varphi_1(\theta-1)\frac{1-(1-\delta)^n}{\delta}\right] + \frac{1}{2} n\sigma_{\Delta d}^2 \left(1 + \varphi_2(\theta-1)\right)^2 \\ & + \frac{1}{2}\sigma_{\Delta c}^2 \left[n\theta^2 + \varphi_1^2(\theta-1)^2\left(n - 2\frac{1-(1-\delta)^n}{\delta} + \frac{1-(1-\delta)^{2n}}{1-(1-\delta)^2}\right) - 2\varphi_1\theta(\theta-1)\left(n - \frac{1-(1-\delta)^n}{\delta}\right)\right] \\ & + \left(1 + \varphi_2(\theta-1)\right)\rho_{\Delta c\Delta d}\left[-n\theta + n\varphi_1(\theta-1) - \varphi_1(\theta-1)\frac{1-(1-\delta)^n}{\delta}\right]\end{aligned}$$

First of all, setting $\varphi_1 = 0$, $\varphi_2 = 0$, we recover the standard (C)CAPM in which preferences are time separable. Setting $\varphi_1 = 1$, $\varphi_2 = 0$ and $\delta = 1$ – *i.e* imposing that the reference level depends only on the lagged aggregate consumption $V_t = \overline{C}_{t-1}-$, we recover the "Catching up with the Jonese" case proposed by Abel (1990). When $\varphi_1 = 1$, $\varphi_2 = 0$ – *i.e* imposing that the reference level depends only on habit stock level $V_t = X_t-$, we recover the utility function proposed by Collard, Fève and Ghattassi (2006).

Let us first consider the case of time separable preferences. The price–dividend ratio rewrites

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \beta^n \exp\left(n\left(g_{\Delta d} - \theta g_{\Delta c}\right) + \frac{1}{2}n\left(\theta^2\sigma^2_{\Delta c} + \sigma^2_{\Delta d} - 2\theta\rho_{\Delta c \Delta d}\right)\right) \tag{3.3.17}$$

Several findings emerge from equation (3.3.17). First, the parameter β measures the time preferences. When β is high, the agent prefers consuming next periods. She saves more, leading to higher prices. Second, an increase in future dividend growth ($g_{\Delta d}$) implies that investors expect dividends to rise in the future. Risky assets are more attractive, leading to higher current prices. Conversely, an expected increase in future consumption growth rate leads investors to borrow from future to smooth consumption. They save more, leading to lower prices. Finally, the term captures the precautionary savings. Indeed, the volatility of consumption and dividend – $\sigma^2_{\Delta c}$ and $\sigma^2_{\Delta d}$ – measures the economic uncertainty. When the volatility of consumption and/or dividend increase, the investors save more, driving up the price–dividend ratio. Note that the positive correlation between consumption and dividend implies that in recession periods, the cash flows of the risky asset falls and therefore the risky asset does not insure the consumer against bad states. Risky assets are less attractive, leading to lower prices.

To analyse how cash flows affect the solution for the price–dividend ratio, we relax the restriction $\varphi_2 = 0$ and we keep $\varphi_1 = 0$. It implies that

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \beta^n \exp\left[n\left(g_{\Delta d} - \theta g_{\Delta c}\right) + \frac{1}{2}n\left(\theta^2\sigma^2_{\Delta c} + \sigma^2_{\Delta d}\left(1 + \varphi_2(\theta-1)\right)^2 - 2\theta\left(1 + \varphi_2(\theta-1)\right)\rho_{\Delta c \Delta d}\right)\right] \tag{3.3.18}$$

Equation (3.3.18) illustrates the financial news effect. Compared to the standard time separable preferences, the introduction of the financial news effect increases the precautionary savings on the one hand and amplifies the effect of positive correlation between consumption and dividend on the other hand.

To better understand the habit formation affect, we first consider the "Catching up with the Joneses" case. Setting $\varphi_2 = 0$ and $\delta = 1$, we assume that the reference level depends only on lagged aggregate consumption $V_t = \overline{C}_{t-1}^{\varphi_1}$. The price–dividend ratio rewrites

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \beta^n \exp\left(c_n - \varphi_1(\theta - 1)\Delta c_t\right) \tag{3.3.19}$$

where

$$c_n = n\left(g_{\Delta d} + \left(\varphi_1(\theta-1) - \theta\right) g_{\Delta c}\right) + \frac{1}{2} n \left[\sigma^2_{\Delta c}\left(\varphi_1(\theta-1) - \theta\right)^2 + \sigma^2_{\Delta d} - 2\rho_{\Delta c \Delta d}\left(\varphi_1(\theta-1) - \theta\right)\right]$$

As can be seen in equation (3.3.19), the "Catching up with the Joneses" phenomenon reduces the standard inter–temporal substitution effect of the time separable preferences. This is reflected in the term $[\varphi_1(\theta-1) - \theta]\, g_{\Delta c}$. Indeed, as explained above, a positive future consumption growth implies that the consumer is willing to transfer consumption from future to today, implying an increase in the reference level for the next period. This raises future marginal utility and therefore plays against the inter–temporal transfer of consumption from $t+1$ to t. This puts upward pressions on asset prices and the price–dividend ratio therefore increases.

In the general setting, things are more complicated as can be seen in equation (3.3.16). Nevertheless, the general solution illustrates two important properties of our model. First, the price–dividend ratio is function of the habit to consumption ratio z_t. This relation is of particular interest as the law of motion of z_t is given by:

$$z_{t+1} = (1-\delta) z_t - \Delta c_{t+1}$$

Therefore, z_t is highly serially correlated for low values of δ and the price–dividend ratio inherits part of this persistence. Second, when investors are risk averse ($\theta > 1$), the price–dividend ratio increases with the consumption to habit ratio, the inverse of z_t. As the price–dividend is often taken to be a measure of the business cycle (see in particular Campbell and Cochrane (1999) and Wachter (2006)), this confirms the intuition that the habit to consumption ratio is a counter–cyclical variable. In recession period, consumption falls, driving upward the habit to consumption ratio.

Since the solution for the price–dividend ratio (3.3.16) involves an infinite series, some conditions of convergence should be derived. The next proposition reports the condition of convergence that guarantee the existence of a bounded equilibrium.

Proposition 3.3.3 *The series (3.3.16) converge if and only if:*

$$r_{cv} = \beta \exp[g_{\Delta c}\left(-\theta + \varphi_1(\theta-1)\right) + g_{\Delta d} + \frac{1}{2}\sigma^2_{\Delta d}\left(1+\varphi_2(\theta-1)\right)^2 + \frac{1}{2}\sigma^2_{\Delta c}\left(-\theta+\varphi_1(\theta-1)\right)^2 \tag{3.3.20}$$

$$+\rho_{\Delta c \Delta d}\left(1+\varphi_2(\theta-1)\right)\left(-\theta+\varphi_1(\theta-1)\right)] < 1$$

The computation of the rate of convergence r_{cv} is reported in Appendix C. It should be noted that the condition of convergence may be satisfied for $\beta > 1$. When β is high, investors prefer consuming tomorrow as opposed to today and hence they save more. Prices may explore and real interest rates may be negative. Hence, many researchers (for example Mehra and Prescott (1985), Bansal and Yaron (2004) and Eraker (2006)) typically exclude the case of $\beta > 1$. As early documented by Kocherlakota (1990), it is possible for equilibria to exist in endowment economy with a representative agent even though $\beta > 1$. Burnside (1998) proposes a standard (C)CAPM model in which endowment follows a first order Gaussian autoregressive process. He shows that asset prices are finite even when $\beta > 1$. Likewise, condition of convergence reported in Proposition 3.3.3 does not exclude the case of $\beta > 1$. Figure 1 reports the the region of convergence. Four cases are under investigation. First, we assume that consumption equals dividend and we consider (i) the time separable preferences case (TS), (ii) the pure habit formation model (HS) and (iii) the general setting (GS, $C \neq D$). Finally, the case (vi) considers the general setting (GS) assuming the distinction between consumption and dividend.

– FIGURE 1 ABOUT HERE –

As can be seen in the expression (3.3.20), the condition of convergence may be satisfied for $\beta > 1$ for (i) positive growth rate of consumption $g_{\Delta c}$, (ii) sufficiently higher risk aversion θ and (iii) lower consumption volatility $\sigma_{\Delta c}$. The economic intuition is when consumption is expected to grow and investors are very risk averse, the future marginal utility diminishes enough to offset the effect of time preferences. When the volatility of consumption is high, the precautionary savings increase and amplify the effect of time preferences. Likewise, the habit persistence and the financial news effects reduce the willing of the consumer to smooth her consumption inter–temporally and hence amplify the effect of time preferences. A final remark regards the effect of the distinction between consumption and dividend. As dividend is much more volatile than consumption, the precautionary savings increase and therefore the high volatility of dividend reduces the set of admissible values of $\beta > 1$.

3.3.2.2 Interest rates

In the section, the explicit formulas for the real and nominal interest rates are derived. From the proposition (3.3.2), it follows that the real (log) risk free rate $r_{f,t+1}$, known at period t, equals:

$$r_{f,t+1} = -\log(\beta) + \theta g_{\Delta c} + \varphi_1(\theta - 1)\delta z_t - \frac{\theta^2}{2}\sigma_c^2 - \frac{\varphi_2^2}{2}(\theta - 1)^2\sigma_d^2 + \varphi_2\varphi_1\theta(\theta - 1)\rho_{\Delta c \Delta d} \tag{3.3.21}$$

Several results emerge. Indeed, the formula (3.3.21) has some familiar terms from the time separable case (the power utility case) and others that are due to the introduction of the reference level.

First, setting $\varphi_1 = \varphi_2 = 0$, we recover the solution for the real short rate when the utility function is time separable. It follows that the real short rate is equal to $-\log(\beta) + \theta g_{\Delta c} - \frac{\theta^2}{2}\sigma_c^2$. The first term represents the sensitivity of the real interest rate to the constant discount factor β. The real interest rate is high when β is low. Indeed, when the agent is impatient, she prefers to consume now and does not want to save, driving up interest rates. The second term $\theta g_{\Delta c}$ represents the inter–temporal smoothing effect. A positive expected consumption growth leads investors to borrow from the future to smooth consumption. Note that the parameter θ controls the sensitivity of the the real interest rate to the consumption growth. Indeed, in the time separable utility function, the coefficient of risk aversion θ controls the risk aversion as well as the inter–temporal substitution. When θ is high or, put differently, the inter–temporal elasticity of substitution is low, the risk free rate is high. The term $-\frac{\theta^2}{2}\sigma_c^2$ captures the precautionary savings. When consumption is more volatile, the agent want to save more to provide a hedge against times of low consumption growth. It implies a decreasing in interest rates.

Second, the time nonseparable property of the utility function, generated by the introduction of a reference level V_t, interest rate is function of the additional terms. The term $\varphi_1(\theta - 1)\delta z_t$ captures the influence of habit formation. When $\theta > 1$, the interest rate is an increasing function of habit. Intuitively, we assume that the consumer develops habits for higher or lower past consumptions and therefore, the stock of habit X_t captures her standard of living or the trend of consumption. The persistence of habit implies that the standard of living, depending on past consumptions, has an impact on how you fell about more consumption today. Formally, when the habit stock is higher or, equivalently the habit to consumption is higher, the agent increases her current consumption to maintain her standard living. She saves less and the interest rate increases. Moreover, as the real interest rate is function of the state variable z_t, the model captures the time variation of the risk free rate despite the constant coefficient of risk aversion. Moreover, imposing $\theta > 1$, the solution takes into account the counter–cyclical[6] variation of the risk free rate observed in data. In recession periods, current consumption falls relative to habit stock X_t and therefore risk averse investors borrow more against future periods to adjust consumption and r_f increases.

Finally, as the reference level takes into account the innovations of the dividend growth, the agent faces a second source of uncertainty: innovations of dividend growth. This feature is reflected in the additional precautionary savings $-\frac{\varphi_2^2}{2}(\theta-1)^2\sigma_d^2$. Moreover, a positive correlation between consumption and dividend growth rates $\rho_{\Delta c \Delta d} > 0$ implies that a positive future shock on dividend provides incentives to consume more next period, leading to an increase in future consumption. Therefore, investors borrow from the future to smooth consumption, driving up the risk free rate. This is reflected in $\varphi_2\varphi_1\theta(\theta-1)\rho_{\Delta c \Delta d}$.

From the proposition (3.3.2), it follows that the nominal (log) risk free rate $r^{\$}_{f,t+1}$, known at period t, equals:

$$r^{\$}_{f,t+1} = r_{f,t+1} + (1-\phi)g_{\Delta\pi} + \phi\Delta\pi_t - \frac{1}{2}\sigma^2_{\Delta\pi} - (\theta - \varphi_1(\theta-1))\rho_{\Delta c\Delta\pi} + \varphi_2(\theta-1)\rho_{\Delta d\Delta\pi} \tag{3.3.22}$$

The equation (3.3.22) shows that the inflation process is a leading variable that affects nominal yields. Indeed, compared to the expression of the real interest rate, all the additional terms depend on inflation. To understand the dynamics of nominal interest rate, the inflation premium is derived. Following Wachter (2006), the inflation premium on the nominal risk-free asset is defined as the spread between the expected real return on the one-period nominal bond $r^{\$}_{f,t+1} - \mathbb{E}_t(\Delta\pi_{t+1})$ and the real risk free rate. It follows that:

$$r^{\$}_{f,t+1} - \mathbb{E}_t(\Delta\pi_{t+1}) - r_{f,t+1} = -\frac{1}{2}\sigma^2_{\Delta\pi} - (\theta - \varphi_1(\theta-1))\rho_{\Delta c\Delta\pi} + \varphi_2(\theta-1)\rho_{\Delta d\Delta\pi} \tag{3.3.23}$$

As shown by the formula (3.3.23), the inflation premium depends on (i) the variance of inflation, (ii) the correlation between inflation and consumption growth $\rho_{\Delta c\Delta\pi}$ and

[6] As mentioned above, the habit to consumption ratio z_t varies counter–cyclical.

(iii) the correlation between inflation and dividend growth $\rho_{\Delta d \Delta \pi}$.

The term proportional to the variance of inflation is due to Jensen's inequality. The other terms reflect the inflation risk. The nominal interest rate can be considered as risky asset. Indeed, higher inflation lowers the real return of nominal risk free rate. Therefore, a negative correlation between consumption and inflation implies that nominal interest rate is an unattractive asset. It follows a positive inflation premium. This is reflected in the term $-(\theta - \varphi_1(\theta - 1))\rho_{\Delta c \Delta \pi}$.
Moreover, assume that $\rho_{\Delta \pi \Delta d} < 0$ and $\rho_{\Delta c \Delta d} > 0$. Therefore, a positive future shock on dividend implies an increase in future consumption. It follows that risk averse investors increase their current consumption to smooth consumption and sell assets. Furthermore, the dividend growth rate is negatively correlated with inflation, driving up the real return on the one–period nominal bond. Nominal bonds become more attractive, driving down the inflation premium. This is reflected in term $\varphi_2(\theta - 1)\rho_{\Delta d \Delta \pi}$.

3.3.2.3 The yield curve

Let $y_{n,t} = -\frac{1}{n}\log(P_{n,t})$ denote the real yield of a real bond. It follows, from Proposition 3 that:

$$y_{n,t} = A_n z_t + C_n \tag{3.3.24}$$

where

$$A_n = \frac{\varphi_1(\theta - 1)}{n}(1 - (1 - \delta)^n)$$

$$C_n = -\log(\beta) + g_{\Delta c}\left(\theta - \varphi_1(\theta - 1)\left(1 - \frac{1 - (1 - \delta)^n}{n\delta}\right)\right) + \frac{1}{2}V_n$$

and

$$\begin{aligned} V_n = & -\sigma^2_{\Delta c}[\theta^2 + \varphi_1^2(1 - \theta)^2\left(1 - 2\frac{1 - (1 - \delta)^n}{n\delta} + \frac{1 - (1 - \delta)^{2n}}{n(1 - (1 - \delta)^2)}\right) \\ & - 2\varphi_1\theta(\theta - 1)\left(1 - \frac{1 - (1 - \delta)^n}{n\delta}\right)] \\ & - \sigma^2_{\Delta d}\left[\varphi_2^2(\theta - 1)^2\right)] + 2\rho_{\Delta c \Delta \pi}\left[\varphi_2\theta(\theta - 1) - \varphi_1\varphi_2(\theta - 1)^2\left(1 - \frac{1 - (1 - \delta)^n}{n\delta}\right)\right] \end{aligned}$$

Let $y^{\$}_{n,t} = -\frac{1}{n}\log(P^{\$}_{n,t})$ denote the nominal yield of a nominal bond. It follows from Proposition 4 that:

$$y^{\$}_{n,t} = A^{\$}_n z_t + B^{\$}_n(\Delta\pi_t - g_{\Delta\pi}) + C^{\$}_n \tag{3.3.25}$$

where

$$A^{\$}_n = A_n$$
$$B^{\$}_n = \frac{\phi}{n}\frac{1-\phi^n}{1-\phi}$$

and

$$\begin{aligned}
C^{\$}_n = C_n - g_{\Delta\pi} - \frac{\sigma^2_{\Delta\pi}}{2(1-\phi)^2}\left(1 - 2\frac{\phi}{n}\frac{1-\phi^n}{1-\phi} + \frac{\phi^2}{n}\frac{1-\phi^{2n}}{1-\phi^2}\right) \\
+ \rho_{\Delta d\Delta\pi}\left(\frac{\varphi_2(\theta-1)}{1-\phi}\left(1 - \frac{\phi}{n}\frac{1-\phi^n}{1-\phi}\right)\right) \\
- \rho_{\Delta c\Delta\pi}\left(\frac{\theta - \varphi_1(\theta-1)}{1-\phi}\left(1 - \frac{\phi}{n}\frac{1-\phi^n}{1-\phi}\right)\right. \\
\left. + \frac{\varphi_1(\theta-1)}{1-\phi}\left(1 - \frac{\phi}{n}\frac{1-\phi^n}{1-\phi} - \frac{1}{\delta}(1-(1-\delta)^n) + \frac{\phi}{n}\frac{1-\phi^n(1-\delta)^n}{1-\phi(1-\delta)^n}\right)\right)
\end{aligned}$$

As mentioned earlier, the term structure of the interest rates is an affine discrete model. Equation (3.3.24) shows that the single factor driving the real bond prices is the habit to consumption ratio z_t while the equation (3.3.25) indicates that nominal bond prices are function of the state variable z_t as well as the inflation $\Delta\pi_t$.

The long term premium can be computed analytically. Subtracting equation (3.3.24) from equation (3.3.21), we have

$$y_{n,t} - y_{1,t} = \varphi_1(\theta-1)\left(\frac{1-(1-\delta)^n}{n} - 1\right)z_t + \Delta_n \tag{3.3.26}$$

where

$$\begin{aligned}
\Delta_n = -g_{\Delta c}\left(\frac{1-(1-\delta)^n}{n\delta}\right) \\
- \frac{1}{2}\sigma^2_{\Delta c}\left[\varphi_1^2(1-\theta)^2\left(-2\frac{1-(1-\delta)^n}{n\delta} + \frac{-(1-\delta)^{2n}}{n(1-(1-\delta)^2)}\right) + 2\varphi_1\theta(\theta-1)\left(\frac{1-(1-\delta)^n}{n\delta}\right)\right] \\
+ \rho_{\Delta c\Delta\pi}\varphi_1\varphi_2(\theta-1)^2\left(\frac{1-(1-\delta)^n}{n\delta}\right)
\end{aligned}$$

The analytic solutions (3.3.25), (3.3.24) and (3.3.26) are function of the preference parameters and the joint dynamics of consumption, dividend and inflation. To better understand the internal mechanisms of the proposed model, we first consider the time separable preferences case – by setting $\varphi_1 = 0$ and $\varphi_2 = 0$. Equation (3.3.24) shows that real bond prices are constant over time and maturity. Moreover, the single factor of the nominal bond prices is inflation. Hence, inflation is the leading variable that generates both time–series and cross section variations of nominal yields. This is reflected by the term $\frac{\phi}{n}\frac{1-\phi^n}{1-\phi}(\Delta\pi_t - g_{\Delta\pi})$, which is exactly the mean of the expected changes in future inflation:

$$\mathbb{E}_t\left[\frac{1}{n}\sum_{i=1}^{n}(\Delta\pi_{t+i} - g_{\Delta\pi})\right]$$

The introduction of habit persistence generates both time–series and cross section variations of real bond prices. As a result, the real yields on real bonds increase with the habit to consumption ratio z_t, implying counter–cyclical variation. Likewise, the nominal yields are time–varying and vary counter–cyclically. In addition, long real yields are less sensitive to the business cycle variable z_t that short real yields, implying pro–cyclical long term premium. As shown in equation (3.3.26), in recession periods, the spread between the yield long term bond and the yield of one–period bond decreases.

As the dividend growth rate is assumed to be *iid* process, the cash flows news effect does not affect the time variation of real and nominal bond prices. However, despite the fact that term structure model is derived analytically, it is difficult to assess the implications of both habit persistence and cash flows news in affecting the shape of the real and nominal average yield curves. Therefore, we evaluate quantitatively the implications of the proposed model in the next section.

3.4 Empirical Investigation

This section investigates the quantitative properties of the model when the endowment – i.e consumption and dividend – environment is *iid* and when inflation follows a first order Gaussian autoregressive process. For this purpose, we first review some empirical facts observed on data. Then, we calibrate the model described in section 3.3.2 and we investigate its empirical implications.

3.4.1 Data and facts

The data used in this section are borrowed[7] from Piazzesi and Schneider (2006) and Garcia, Meddahi and Totongap (2006). There are quarterly postwar US data for the period 1952:2–2002:4. The macroeconomic data are the aggregate real consumption and dividend growth rates and inflation. Table 3.1 reports the corresponding summary statistics.

– TABLE 3.1 ABOUT HERE –

Several empirical facts stand out from Table 3.1. First of all, as already noted by Garcia, Meddahi and Tedongap (2006), the dividend growth rate is much more volatile than consumption growth rate, 0.48% versus 5.32%. The high volatility of the dividend growth is due to seasonality in dividend payments. Second, the inflation process is highly persistent. The first order serial correlation is about 0.85 while the fourth–order serial correlation is about 0.72. Consumption and dividend growth rates are dramatically less persistent and their serial correlations die out more quickly. Indeed, the first order serial correlation of the consumption growth rate does not exceed 0.35 and persistence vanishes as the autocorrelation function shrinks to 0.03 at fourth order. Likewise, the first–order and the second–order serial–correlation coefficients of the dividend growth rate are respectively -0.42%

[7] More details on data can be found in Garcia, Meddahi and Tedongap (2006) and in Piazzesi and Schneider (2006).

and −0.01. Finally, the correlation matrix shows that consumption growth and dividend growth rates have a weak but positive correlation of 0.12. Moreover, the inflation process is negatively correlated with both consumption and dividend growth rates. The correlation coefficients are respectively −0.36 and −0.05, that confirms the economic mechanisms derived in section 3.3.2.

The financial data[8] are the real interest rate, the real return on stock, the price–dividend ratio and the nominal yields of nominal bonds. The first panel of Table 3.2 reports some empirical facts about the market stock.

– TABLE 3.2 ABOUT HERE –

As well documented by Campbell (2003), the average real equity premium is high (1.5% per quarter) and the average real interest rate is low (0.35% per quarter). Moreover, the return on stock is more volatile than the real risk free rate, 7.95% versus 0.58%. The first panel of Table 3.2 also documents the high persistence of the price–dividend ratio. The first–order serial–correlation function is about 0.93 and remains substantially high at longer horizons. The autocorrelation function of the real risk free rate is fairly high but substantially lower than for the price–dividend ratio. For instance, the first–order and the fifth–order serial correlation coefficients of the real interest rate are respectively about 0.66 and 0.45.

The second panel of Table 3.2 reports the summary statistics of nominal bonds. As well documented in the term structure literature, the nominal bonds are characterized by an upward sloping and concave average yield curve in maturity. The annualized average yield on the three-month bond is 1.33% to reach an average of 1.57% at 5–year bond. Panel 2 of Table 3.2 also demonstrates that the volatility of nominal bonds is a decreasing function in maturity. For instance, the standard deviation of 1–quarter and 5–year nominal yield are respectively 0.71% and 0.68%.

[8] More details on stock market data can be found in Garcia, Meddahi and Tedongap (2006) and more details on the nominal bond prices can be found in Piazzesi and Schneider (2006).

Moreover, the serial correlation functions of nominal bond yields show that nominal yields are persistent and long yields are more persistent than short yields. The first–order autocorrelations of the yields of bond maturing in 1–quarter and 5–year are respectively 0.93 and 0.96. When we turn to the 5^{th}–order autocorrelations, they achieve respectively 0.75 and 0.85.

3.4.2 Calibration

We partition the set of the parameters of the model in two distinct groups: (i) the deep parameters defining preferences and (ii) the forcing parameters defining exogenous variables.

The parameters describing the evolution of the forcing variables Δc_t, Δd_t and $\Delta \pi_t$ are obtained exploiting the US postwar quarterly data. The values of these parameters are reported in Table 3.3.

– TABLE 3.3 ABOUT HERE –

The deep parameters are the constant discount factor β, the curvature of the utility function θ, the rate of persistence of the habit stock δ and the parameters, φ_1 and φ_2, ruling the sensitivity of preferences to the reference level. Often, the (C)CAPM models are calibrated to match the target financial stylized facts. For instance, Campbell and Cochrane (1999) calibrate their model to match the risk free rate, the first order serial correlation of the price–dividend ratio and the mean of the Sharpe ratio (the ratio of the unconditional mean to unconditional standard of excess returns. Wachter (2006) adds a free preference parameter to the nonlinear structure of the surplus consumption ratio to match the low volatility of the risk free rate and to generate positively sloping yield curves. Piazzesi and Schneider (2006) propose to study the nominal yield curves. Therefore, they select values for the preference parameters to match the average short and long end of the nominal yield curve. Note that their resulting values of the preference parameters depend on the maturity of the short rate, 1–quarter or 1–year. In other words, the choice of the deep parameters depend on the empirical facts that the model proposes to replicate. As the purpose of this paper is to study how the consumer behavior and the dynamics of the macroeconomic variables affect financial variables, the choice of the deep parameters (β, θ, φ_1, φ_1, δ) will depend on which economic effect we want to emphasize.

The preference parameters are set as follows. First, as the first–order serial correlation[9] of the price–dividend ratio depends only on the parameter δ:

$$corr\left(\frac{P_t}{D_t}, \frac{P_{t-1}}{D_{t-1}}\right) \simeq 1 - \delta$$

it follows that δ is set to 0.05. Indeed, we need a low degree of persistence of the habit stock $\delta = 0.05$ and a strict positive parameter ruling the sensitivity of preferences to habit persistence $\varphi_1 > 0$ to insure a high persistent price–dividend ratio.

Second, the average risk free rate is given by:

$$\mathbb{E}(r_{f,t}) = -\log(\beta) + \theta g_{\Delta c} - \varphi_1(\theta-1)g_{\Delta c} - \frac{\theta^2}{2}\sigma_c^2 - \frac{\varphi_2^2}{2}(\theta-1)^2\sigma_d^2 + \varphi_2\theta\rho_{\Delta c\Delta d}\varphi(\theta-1) \tag{3.4.27}$$

An shown in the expression (3.4.27), the choice of the parameter δ does not affect the mean of the real interest rate. The mean of the risk free rate depends only on the deep parameters β, θ, φ_1 and φ_2 and the forcing parameters reported in Table 3.3.

We consider as a benchmark the general setting with $\varphi_1 = 1$ and $\varphi_2 = 1$. Then, the parameter β is set to 0.98, which is a common value used in macro literature when data are expressed in quarterly frequency. At the end, the parameter θ is set to 5 match the mean of the real interest rate.

Keeping $\beta = 0.98$ and $\theta = 5$ fixed, we also evaluate the sensitivity of the empirical results to two special cases: (i) the pure habit stock model ($\varphi_1 = 1$ and $\varphi_2 = 0$) and (ii) the standard (C)CAPM with time separable utility function ($\varphi_1 = 0$ and

[9] see Appendix D for more details.

$\varphi_2 = 0$). Then, we gauge the sensitivity of the proposed model to alternative values of θ and β.

3.4.3 Results

This section assesses the quantitative ability of the model to account for a set of standard unconditional moments characterizing the dynamics of the price–dividend ratio, the price–consumption ratio, the real interest rate, excess returns on equities and on market portfolio and the real and nominal yield curves. The model is simulated [10] using the closed form solutions presented in section 3.3.2. We ran 10.000 draws of the length of the sample size, T. We actually generated $T + 100$ observations, the first observations being discarded from the sample.

As a benchmark, we consider the general setting when the preference parameters[11] $\beta = 0.9832$, $\theta = 5$, $\delta = 0.05$, $\varphi_1 = 1$ and $\varphi_2 = 1$. We also run a sensitivity analysis to the changes in the parameters φ_1 and φ_2. In particular, three cases are under investigation: (i) the standard (C)CAPM with time separable preferences model TS ($\varphi_1 = 0$ and $\varphi_2 = 0$), (ii) the habit stock model HS ($\varphi_1 = 1$ and $\varphi_2 = 0$) and (iii) the general setting GS ($\varphi_1 = 1$ and $\varphi_2 = 1$).

We begin by reporting the quantitative evaluation of the stock prices, under the assumption that endowment – consumption and dividend – growth rates follow *iid* processes. Table 3.4 reports the averages, the standard deviations and the autocorrelation functions of the real risk–free rate, the excess returns on equities and on the portfolio market, the price–dividend ratio and the price–consumption ratio.

– TABLE 3.4 ABOUT HERE –

[10] While the unconditional means, the standard deviations and the autocorrelation function of the price–dividend ratio P_t/D_t and the real and nominal yields are derived analytically, as shown in Appendix D, it isn't the case of $\log(P_t/D_t)$, $\log(P_t/C_t)$ and excess returns. Therefore, we use Monte Carlo simulations to compute the corresponding unconditional moments.

[11] Tables 3.5 and 3.7 report the sensitivity of the quantitative evaluation of the proposed model to alternative values of β, θ, φ_1 and φ_2.

Let us first consider the time separable case ($\varphi_1 = \varphi_2 = 0$). As well documented by the (C)CAPM literature, the time separable preferences with moderate value of coefficient of relative risk aversion ($\theta = 5$) imply (i) a high real risk–free rate (5.87% per quarter versus 0.35% observed in data) and (ii) a very low excess returns on equities and on market portfolio (respectively 0.05% and 0.01% versus 1.33% observed in data). Furthermore, the closed form solutions (3.3.16) and (3.3.21) reported in section 3.3.2 indicate that the assumptions of time separable preferences and iid endowment economy imply constant price–dividend ratio and real risk–free rate.

To better understand the internal mechanisms of our model, it is useful to investigate the habit stock model ($\varphi_1 = 1$ and $\varphi_2 = 0$). Table 3.4 reports the corresponding results. Several findings emerge. First of all, the introduction of persistent habit stock enhances essentially the ability of the (C)CAPM model into account the high persistence of asset prices. When δ is set to 0.05, the habit stock model implies a high persistent price–dividend ratio. For instance, the implied first and fourth order serial correlations are respectively 0.93 and 0.74. Likewise, the price–consumption ratio presents the same autocorrelation function. Indeed, the habit to consumption ratio z_t is the single factor of the model and hence it is the leading variable that generates persistence. Note that the first order serial correlation of the price–dividend ratio matches exactly the data statistic because we chose δ to fit this quantity of interest. Furthermore, excess returns display a series of small negative serial correlations that generate univariate mean reversion. The same result is obtained for the actual data on excess returns (see the first panel of Table 3.2).

By imposing a constant coefficient of relative risk aversion, it should come at no surprise that the habit stock model fails to account for the high equity premium observed in data, about 1.33% per quarter. Nevertheless, the habit persistence model outperforms the time separable model in replicating (i) a higher excess returns on equities, 0.61% versus 0.01% per quarter and (ii) a lower real risk free rate, 2.58%

versus 5.87% per quarter. The excess return on market portfolio remains very low, about 0.01% per quarter. A final remark regards the volatility of stock prices. As can be seen in Table 3.4, setting $\theta = 5$ and $\beta = 0.9832$, the habit stock model fails totally to match the high volatility of the price–dividend ratio (5% versus 34% in the data). Likewise, the model generates a very low volatility for excess returns on equities and on the market portfolio (respectively 1.86% and 1.97% versus 7.95% in the data).

The introduction of the financial news effects – by setting $\varphi_2 = 1$ – essentially outperforms the habit stock model in terms of low risk free rate and high excess returns on the market portfolio. Indeed, compared to the habit stock model, the general setting generates a lower risk free rate (0.35% versus 2.58%) and a higher excess returns on market portfolio (0.50% versus 0.01%). Note that the general setting produces the exact unconditional mean of the real risk–free return because the preference parameters $\beta = 0.9832$ and $\theta = 5$ are set to replicate this statistic.

As shown in the exact solutions reported in section 3.3.2, the habit to consumption ratio remains the single factor of the model. therefore, (i) the autocorrelation functions of the price–dividend ratio, the price–consumption ratio, the real risk free rate and the excess returns and (ii) the volatility of the risk free rate remain unchanged.

Despite the small increase in the volatility of price–dividend ratio and excess returns (respectively 5.44% and 2.48%), the general setting fails to account for the high volatilities observed in data (respectively 34.0% and 7.95%). In addition, the general setting generates substantial unconditional means of the (log) price–dividend and (log) price–consumption ratios, respectively 9.77 and 7.69, versus 3.4 observed in actual data. Indeed, given the high volatility of dividend, the introduction of the cask flows news effect implies an increase in the precautionary savings and therefore leads to higher prices. Setting $\varphi_2 = 1$, the rate of convergence verifies $r_{cv} = 1.00$ and therefore asset prices explose.

Table 3.6 reports the empirical evaluations of the real and nominal bond prices under the assumption that endowment growth rates follow *iid* processes and inflation follows a first order Gaussian autoregressive process. Three cases are under investigation: (*i*) time separable preferences model TS, (*ii*) the habit stock HS model and (*iii*) the general setting GS.

– TABLE 3.6 ABOUT HERE –

The first panel of the table 3.6 displays the time separable preferences setting. First of all, as shown in the solutions for the term structure model, reported in section 3.3.2, the time separable preferences imply constant real bond prices over time. Moreover, the real prices of real bonds do not depend on maturity. The average real yields are about 23.49% whatever the maturity of the real bond. It also follows that the single factor affecting nominal bonds is inflation. Therefore, the autocorrelation functions of nominal bonds, maturing from 1 quarter to 4 year, are exactly the autocorrelation function of the inflation process. As nominal bonds react gradually to the current inflation changes, both unconditional mean and standard deviation are decreasing functions of maturity. For example, the average 1–year and 5–year nominal yields are respectively 27.22% and 27.18% per year and their standard deviations are about 1.57% and 0.69%.

The second panel of Table 3.6 reports the quantitative evaluation of the habit stock model – by imposing $\varphi_1 = 1$ and $\varphi_2 = 0$. Several findings emerge. First of all, setting $\beta = 0.9832$ and $\theta = 5$, the habit stock model generales a high real risk free rate, implying high real and nominal bond yields. For example, the annualized average yield on the 1–year real bond is 10.32% and the annualized average yield on the 1–year nominal bond is 14.06%. Second, the habit stock model generates upward–sloping real and nominal average yield curves. It should come at no surprise that we obtain the same dynamics as in Wachter (2006). Indeed, Wachter (2006) proposes an extended version of the habit formation model of Campbell and

Cochrane (1999), where the utility function is specified in difference:

$$U_t = \frac{(C_t - X_t)^{1-\theta} - 1}{1-\theta}$$

The reference level X_t is specified as follows

$$\begin{aligned} s_{t+1} &= \log\left(\frac{C_{t+1} - X_{t+1}}{C_{t+1}}\right) \\ &= (1-\Phi)\overline{s} + \Phi s_t + \lambda(s_t)\left(\Delta c_{t+1} - \mathbb{E}(\Delta c_{t+1})\right) \end{aligned}$$

The sensitivity function $\lambda(s_t)$ verifies

$$\begin{aligned} \lambda(s_t) &= (1-\overline{S})\sqrt{1 - 2(s_t - \overline{s})} - 1 \ \textit{when}\ s_t < s_{max} \\ &= 0 \quad \textit{otherwise} \end{aligned}$$

where $\overline{S} = \sigma_{\Delta c}\sqrt{\frac{\theta}{1-\Phi-b/\theta}}$ and $s_{max} = \overline{s} + \frac{1}{2}(1-\overline{S}^2)$. The parameter Φ is set 0.97 to match the first–order serial correlation of the price–dividend ratio. Wachter (2006) shows that the surplus consumption ratio is the leading variable that affects the dynamics of the real and nominal bond prices. Moreover, the surplus consumption ratio s_t is approximately equal to $\sum_{j=1}^{40}\Phi^j\Delta c_{t-j}$. Our habit stock model presents similar characteristics. The habit to stock ratio can be viewed as a weighting average of past consumption growth. Indeed, the habit to consumption ratio rewrites

$$z_t = \delta\sum_{j=0}^{\infty}(1-\delta)^j\Delta c_{t-j}$$

Moreover, the parameter $(1-\delta)$ is chosen to match the first–order serial correlation of the price–dividend ratio. Finally, z_t is the single factor of the real term structure model while z_t and $\Delta\pi_t$ are the factors of the nominal term structure. It implies that the dynamics of our habit stock model and the model proposed by Wachter (2006) presents some similarities. In particular, both models generate decreasing yield curves. Moreover, the standard deviations of the real and nominal yields are decreasing in maturity.

The third panel of Table 3.6 displays the quantitative evaluation of the general setting by imposing $\varphi_1 = 1$ and $\varphi_2 = 1$. The parameters β and θ are chosen to fit the mean of the real interest rate. Therefore, the general setting generates reasonable average real and nominal yields. For instance, the implied annualized average 1–year and 5–year nominal yields are respectively 1.43% and 5.11%.

Compared to the habit stock model, the habit to consumption z_t remains the single factor driving the real yields. Likewise, inflation and habit to consumption ratio are the factors that drive the nominal yields. Therefore, standard deviations and autocorrelation functions are unchanged.

More importantly, the introduction of financial news effect reverses the real yield curve. Indeed, compared to the habit stock model, the general setting implies a downward sloping real yield curve. Evidence of a negatively sloping real yield curve was discussed by Piazzesi and Schneider (2006) and Ang and Bekaert (2005). In addition, as shown in Table 3.7, the negative slope of the real yield curve is accentuated with higher values of risk aversion.

The general setting also implies a downward sloping nominal yield curve, conversely to what we observe in actual data (as can be seen in the second panel of Table 3.2). This finding comes at no surprise. Indeed, Eraker (2006) has shown that inflation neutrality (*i.e* an increase in current inflation does not affect the future endowment growth rates) implies a negatively sloping nominal real yield curve. Assuming a negative correlation between expected inflation and consumption growth rate turns the downward real yield curve into a upward nominal yield curve. According to Piazzesi and Schneider (2006), "if inflation is a bad news for consumption growth, the nominal yield curve slopes up".

A final remark regards the fact that the nominal yields always lie above the real yields, whatever the imposed values of φ_1 and φ_2. This feature of the model is implied by the fact that the expected inflation is positive.

The preliminary evaluation of the proposed model, reported in this section, suggests in particular two axis of further research. On the one hand, using the utility function introduced in this paper, we will explore the implications of other settings for the parameters φ_1 and φ_2 between zero and 1. On the other hand, we will test alternative joint dynamics of the exogenous variables. More precisely, two cases will be investigated: (i) the introduction of predicted component to the consumption process as in Piazzesi and Schneider (2006) and (ii) the introduction of macroeconomic uncertainty, measured by time varying consumption volatility as in Bansal and Yaron (2004). Finally, as shown in Table 3.5, the proposed model can generate negative values of the real risk–free rate and therefore negative real yields. Hence, some restrictions on the preference parameters and the joint dynamics of the exogenous variables should be derived to insure the positivity of the real yield curve.

Concluding Remarks

This paper delivers a general framework for analyzing stock and bond prices. We first develop a (C)CAPM model with reference level. The reference level summarizes the influence of habit formation and financial news on the current individual consumption choice. Closed form solutions for asset prices are provided under the assumption of affine class of exogenous endowments. Explicit formulas include the price–dividend ratio, the price–consumption ratio and the real and nominal bond yields. Then, the solutions derived in general affine context are applied in *iid* endowment environment, as in Campbell and Cochrane (1999) and Wachter (2006) to emphasize the economic mechanisms behind the financial implications. On the empirical side, our proposed model produces low real risk free rate and substantial equity premium at moderate value of risk aversion. Moreover, our model takes into account the high persistence of the price–dividend ratio and the persistence of the bond prices. However, the introduction of financial news effect reduces the region of convergence of the price–dividend ratio and may induce negative yields on bonds.

The preliminary quantitative evaluation of the model suggests several extensions. First, the imposed specification of the macroeconomic variables –an *iid* endowment economy and a first–order Gaussian autoregressive process – ignore some important components of the joint dynamics of consumption, dividend and inflation. For instance, it would be interesting to account for (i) the predicted component of the consumption growth and the bad effect of inflation news on future consumption growth as in Piazzesi and Schneider (2006) and (ii) the macroeconomic uncertainty measured by time varying consumption volatility as in Bansal and Yaron (2004). As shown in Appendix A, the fully specified processes proposed by Piazzesi and Schneider (2006) and Bansal and Yaron (2004) verify the affine representation. Second, a complementary line of research would be to study the influence of persistent financial news on current individual consumption choice. Finally, an interesting extension would be to consider non-normal distributions.

Appendix A. CaR(1) Processes

In this appendix, we show that some dynamics imposed to exogenous variables proposed in the Consumption Asset Pricing literature follow $CaR(1)$ process. In each case, we specify (i) the state variable Y_t, (ii) the coefficients $\gamma_{\Delta c}$, $\gamma_{\Delta d}$ and $\gamma_{\Delta\pi}$ linking the macroeconomic variables Δc_t, Δd_t and $\Delta\pi$ with the state variable Y_t and (iii) the functions $a(z)$ and $b(z)$ that define the conditional Laplace transform function of Y_t.

$$\mathbb{E}_t(\exp z'Y_{t+1}) = exp(a(z)Y_t + b(z))$$

- **Example 1**: Piazzesi and Schneider (2006)
  Piazzesi and Schneider (2006) assume that the vector of consumption growth and inflation z_t =[Δc_t $\Delta\pi_t$]' has the following state space representation

$$\begin{aligned} z_{t+1} &= \mu_z + x_t + \varepsilon_{t+1} \\ x_{t+1} &= \Phi_x x_t + \Phi_x K \varepsilon_{t+1} \end{aligned}$$

  where $\varepsilon_t \rightsquigarrow N(0,\Sigma)$. The state variable x_t is 2-dimensional and contains expected consumption and inflation. The matrix Σ, Φ_x and K are 4-dimensional. Then the affine representation is satisfied with:

  - the state variable $Y_t = \begin{bmatrix} z_t' & x_t' \end{bmatrix}$'
  - $\gamma_{\Delta c}$= [1 0 0 0]' and $\gamma_{\Delta\pi}$=[0 1 0 0]'
  - $a(z) = z' \begin{bmatrix} O_{2\times2} & \mathbb{I}_{2\times2} \\ O_{2\times2} & \Phi_x \end{bmatrix}$ and $b(z) = z' \begin{bmatrix} \mu_z \\ O_{2\times1} \end{bmatrix} + \frac{1}{2} z' \begin{bmatrix} \mathbb{I}_{2\times2} \\ Q_x K \end{bmatrix} \Sigma \begin{bmatrix} \mathbb{I}_{2\times2} \\ Q_x K \end{bmatrix}' z$

  Note that the joint dynamics of macroeconomic variables proposed by Wachter (2006) is a special case of this example. Indeed, Wahter (2006) assumes that the consumption growth is *iid* and the inflation process follows a Gaussian $ARMA(1,1)$

$$\begin{aligned} \Delta c_{t+1} &= g_{\Delta c} + \varepsilon_{\Delta c,t+1} \\ \Delta\pi_{t+1} &= (1 - g_{\Delta\pi})\Phi_{\Delta\pi}\pi_t + \Psi\varepsilon_{\Delta\pi,t} + \varepsilon_{\Delta\pi,t+1} \end{aligned}$$

  where $\begin{bmatrix} \varepsilon_{\Delta c,t+1} & \varepsilon_{\Delta\pi,t+1} \end{bmatrix}' \rightsquigarrow N(0,\Omega)$.

  It is easy to show that this specification satisfies the state space representation of Example 1 with:

  $z_t = \begin{bmatrix} \Delta c_t \\ \Delta\pi \end{bmatrix}$ and $x_t = \begin{bmatrix} 0 \\ \Phi_{\Delta\pi}(\pi_t - g_{\Delta\pi}) + \Psi\varepsilon_{\Delta\pi,t} \end{bmatrix}$

  $\mu_z = \begin{bmatrix} g_{\Delta c} & g_{\Delta\pi} \end{bmatrix}$

  $\Phi_x = \begin{bmatrix} 0 & 0 \\ 0 & \Phi_{\Delta\pi} \end{bmatrix}$, $\Phi_x K = \begin{bmatrix} 0 & 0 \\ 0 & \Phi_{\Delta\pi} \end{bmatrix}$ and $\Sigma = \Omega$

- **Example 2**: Bansal and Yaron (2004)
  Bansal and Yaron (2004) propose the following system:

$$\begin{aligned}
\Delta c_{t+1} &= g_{\Delta c} + x_t + \sigma_t \eta_{t+1} \\
\Delta d_{t+1} &= g_{\Delta d} + \phi x_t + \varphi_d \sigma_t u_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\
\sigma^2_{t+1} &= \sigma^2 + \nu_1(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1}
\end{aligned}$$

  where e_t, u_t, η and $w_t \rightsquigarrow N.i.i.i(0,1)$.

  x_t presents the persistent predictable component and σ_t the time–varying economic uncertainty incorporated in consumption growth rate.

  Then, the affine representation is satisfied with:

  - the state variable $Y_t = \begin{bmatrix} \Delta c_t & \Delta d_t & x_t & \sigma^2_t \end{bmatrix}$'
  - $\gamma_{\Delta c}$= [1 0 0 0]' and $\gamma_{\Delta \pi}$=[0 1 0 0]'
  - $a(z) = z' \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & \phi & 0 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & \nu_1 \end{bmatrix} + \frac{1}{2} z' \text{ diag}\left(1, \varphi^2_d, \varphi^2_e, 0\right) z \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}'$
  - $b(z) = z' \begin{bmatrix} g_{\Delta c} \\ g_{\Delta d} \\ 0 \\ (1-\nu_1)\sigma^2 \end{bmatrix} + \frac{1}{2} z' \text{ diag}(0,0,0,\sigma_w) z$

Appendix B. Proof of Propositions 1 and 2

Proposition 3.3.1: **Proof.**
(i) The return of a traded risky asset R_{t+1} verifies the following Euler equation:

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \left(\frac{V_{t+1}}{V_t} \right)^{(\theta-1)} R_{t+1} \right] \tag{3.4.28}$$

As $R_t = (P_t + D_t)/P_{t-1}$, Equation (3.4.28) rewrites:

$$\begin{aligned} \frac{P_t}{D_t} &= \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \left(\frac{V_{t+1}}{V_t} \right)^{(\theta-1)} \left(\frac{D_{t+1}}{D_t} \right) \left(\frac{\frac{P_{t+1}}{D_{t+1}} + 1}{\frac{P_t}{D_t}} \right) \right] \\ &= \mathbb{E}_t \left[\beta \exp\left(-\theta \Delta c_{t+1} + (\theta - 1)\Delta v_{t+1} + \Delta d_{t+1} \right) \left(\frac{P_{t+1}}{D_{t+1}} + 1 \right) \right] \end{aligned}$$

where

$$\Delta v_{t+1} = \varphi_1 \Delta x_{t+1} + \varphi_2 (\Delta d_{t+1} - g_{\Delta d})$$

Let's $z_t = \log(X_t/C_t)$ note the habit to consumption ratio. We obtain:

$$\Delta x_{t+1} = \log \left(\frac{X_{t+1}}{X_t} \right) = z_{t+1} + \Delta c_{t+1} - z_t$$

Let's note $y_t = \left(\frac{P_{t+1}}{D_{t+1}} \right) \exp(\lambda_1 \varphi_1 (\theta - 1)) z_{t+1}$. Hence, Euler equation rewrites:

$$\begin{aligned} y_t = \mathbb{E}_t \beta \exp \Bigg(\Bigg(&- g_{\Delta d} \varphi_2 (\theta - 1) + (-\theta + \varphi_1 (\theta - 1)) \Delta c_{t+1} + (1 + \varphi_2 (\theta - 1)) \Delta d_{t+1} \Bigg) \\ &\Bigg(v_{t+1} + \exp(\varphi_1 (\theta - 1) z_{t+1}) \Bigg) \Bigg) \end{aligned}$$

Iterating forward and imposing the transversality condition, a solution to this forward looking equation is given by :

$$y_t = \sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \exp C_n + (-\theta + \varphi_1(\theta - 1)) \sum_{j=1}^{n} \Delta c_{t+j} + (1 + \varphi_2(\theta - 1)) \sum_{j=1}^{i} \Delta d_{t+j} + \varphi_1(\theta - 1) z_{t+i}$$

where $C_n = -n\varphi_2(\theta - 1) g_{\Delta d}$.

Note that, from the definition of z_t, we have:

$$z_{t+i} = (1 - \delta)^i z_t - \sum_{j=1}^{i} (1 - \delta)^{i-j} \Delta c_{t+j}$$

which implies that:

$$\begin{aligned} y_t = \sum_{n=1}^{\infty} \beta^n \exp &\left(C_n + \varphi_1(\theta - 1)(1 - \delta)^n z_t \right) \\ \mathbb{E}_t \exp &\left(\sum_{j=1}^{n} \left(-\theta + \varphi_1(\theta - 1)(1 - (1 - \delta)^{n-j}) \right) \Delta c_{t+j} + (1 + \lambda_2 \varphi(\theta - 1)) \sum_{j=1}^{n} \Delta d_{t+j} \right) \end{aligned}$$

Imposing $g_{\Delta c,t} = \gamma_c' Y_t$ and $g_{\Delta d,t} = \gamma_{cd}' Y_t$, we obtain:

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \beta^n \exp \varphi_1(\theta-1)(1-(1-\delta)^n) z_t \mathbb{E}_t \exp\left(\sum_{j=1}^{n} (\alpha_{n,j}^d)' Y_{t+j}\right)$$

where

$$\alpha_{i,j}^d = \left(-\theta + \varphi_1(\theta-1)(1-(1-\delta)^{n-j})\right)\gamma_c + (1+\varphi_2(\theta-1))\,\gamma_d$$

Furthermore, we can make use of the properties of affine processes defined by (3.2.15) and the law of the iterated expectations to compute the price–dividend ratio.

$$\begin{aligned}
&\mathbb{E}_t \exp\left(\sum_{j=1}^{n} (\alpha_{n,j}^d)' Y_{t+j}\right) \\
&= \mathbb{E}_t ... \mathbb{E}_{t+n-2} \mathbb{E}_{t+n-1}\left(\exp\left(\sum_{j=1}^{n-1} (\alpha_{n,j}^d)' Y_{t+j} + (\alpha_{n,n}^d)' Y_{t+n}\right)\right) \\
&= \mathbb{E}_t ... \mathbb{E}_{t+n-2}\left(\left(\exp \sum_{j=1}^{n-1} (\alpha_{n,j}^d)' Y_{t+j}\right)\left(\mathbb{E}_{t+n-1} \exp(\alpha_{n,n}^d)' Y_{t+n}\right)\right) \\
&= \mathbb{E}_t ... \mathbb{E}_{t+n-2}\left(\left(\exp \sum_{j=1}^{n-1} (\alpha_{n,j}^d)' Y_{t+j}\right)\left(\exp\left(a(\alpha_{n,n}^d)' Y_{t+n-1} + b(\alpha_{n,n}^d)\right)\right)\right) \\
&= \mathbb{E}_t ... \mathbb{E}_{t+n-2} \exp\left(\sum_{j=1}^{n-2} (\alpha_{n,j}^d)' Y_{t+j} + \left(\left(a(\alpha_{n,n}^d)' + (\alpha_{n,n}^d)'\right) Y_{t+n-1} + b(\alpha_{n,n}^d)\right)\right)
\end{aligned}$$

Therefore, $A_d(n,i)$ and $B_d(n,i)$ are defined as follows

$$\mathbb{E}_t \exp\left(\sum_{j=1}^{n} (\alpha_{n,j}^d)' Y_{t+j}\right) = \mathbb{E}_t ... \mathbb{E}_{t+n-i} \exp\left(\sum_{j=1}^{n-i} (\alpha_{n,j}^d)' Y_{t+j} + A_d'(n,n-j) Y_{t+n-i} + B_d(n,n-i)\right)$$

It implies

$$\mathbb{E}_t \exp\left(\sum_{j=1}^{n} (\alpha_{n,j}^d)' Y_{t+j}\right) = A_d'(n,0) Y_t + B_d(n,0)$$

where $A_d(n,0)$ and $B_d(n,0)$ are obtained as follows:

$$\begin{aligned}
A_d(n,n) &= 0 \\
B_d(n,n) &= 0
\end{aligned}$$

and for $0 \leq j < n$:

$$A_d(n, n-j-1) = a(\alpha^d_{n,n-j} + A_d(n, n-j))$$
$$B_d(n, n-j-1) = b(\alpha^d_{n,n-j} + A_d(n, n-j)) + B_d(n, n-j)$$

(ii) Likewise, the price–consumption ratio verifies

$$\frac{P_t}{C_t} = \sum_{n=1}^{\infty} \beta^n \exp \varphi_1(\theta - 1)(1 - (1-\delta)^n) z_t \mathbb{E}_t \exp\left(\sum_{j=1}^{n} (\alpha^c_{n,j})' Y_{t+j}\right)$$

where

$$\alpha^c_{i,j} = \left((1-\theta) + \varphi_1(\theta-1)(1-(1-\delta)^{n-j})\right)\gamma_c + \varphi_2(\theta-1)\gamma_d$$

Proposition 3.3.2: **Proof.**

(i) Let $P_{n,t}$ denote the real price of a real bond maturing in n periods. Hence, $P_{n,t}$ is determined by the following Euler equation:

$$P_{n,t} = \mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\theta}\left(\frac{V_{t+1}}{V_t}\right)^{\varphi(\theta-1)} P_{n-1,t+1}\right]$$

Then:

$$\exp^{\varphi_1(\theta-1)z_t} P_{n,t} = \mathbb{E}_t\left[\beta \exp^{(-\theta+\varphi_1(\theta-1))\Delta c_{t+1} + \varphi_2(\theta-1)(\Delta d_{t+1} - g_{\Delta d})} \exp^{\varphi_1(\theta-1)z_{t+1}} P_{n-1,t+1}\right]$$

Iterating forward and imposing $P_{0,t} = 1$, we obtain:

$$P_{n,t} = \mathbb{E}_t \beta^n \exp\Bigg(\sum_{i=1}^{n} -g_{\Delta d}\varphi_2(\theta-1) + (-\theta + \varphi_1(\theta-1))\Delta c_{t+i} + \varphi_2(\theta-1)\Delta d_{t+i}$$
$$+ \varphi_1(\theta-1)(z_{t+n} - z_t)\Bigg)$$

The definition of z_t and the law of motion of habits imply that

$$z_{t+n} = (1-\delta)^n z_t - \sum_{j=0}^{i-1}(1-\delta)^j \Delta c_{t+i-j}$$

Therefore, the solution rewrites:

$$P_{n,t} = \beta^n \exp(C_n + \varphi_1(\theta-1)((1-\delta)^n - 1)z_t)\mathbb{E}_t\Bigg(\exp\sum_{i=1}^{n}(-\theta + \varphi_1(1-\theta)(1-(1-\delta)^{n-i}))\Delta c_{t+i}$$
$$+ \sum_{i=1}^{n}\varphi_2(\theta-1)\Delta d_{t+i}\Bigg)$$

Let's note

$$\tau_{n,i} = (-\theta + \varphi_1(\theta - 1)(1 - (1 - \delta)^{n-i}))\gamma_c + \varphi_2(\theta - 1)\gamma_d$$

Then

$$P_{n,t} = \beta^n \exp\left[C_n + \varphi_1(\theta - 1)((1 - \delta)^n - 1)z_t + A_b(n, 0)' Y_t + B_b(n, 0)\right]$$

where $A_b(n, i)$ and $B_b(n, i)$ are defined as follows:

$$A_b(n, n) = 0$$
$$B_b(n, n) = 0$$

and for $0 \leq i < n$:

$$A_b(n, n - i - 1) = a(\tau_{n,i} + A_b(n, n - i))$$
$$B_b(n, n - i - 1) = b(\tau_{n,i} + A_b(n, n - i)) + B_b(n, n - i)$$

(ii) Likewise, the price of a nominal yield verifies

$$P^{\$}_{n,t} = \beta^n \mathbb{E}_t \exp\left[C_n + \varphi_1(\theta - 1)((1 - \delta)^n - 1)z_t + \sum_{i=1}^{n} (\tau^{\$}_{n,i})' Y_{t+i}\right]$$

where

$$\tau^{\$}_{n,i} = (-\theta + \varphi_1(\theta - 1)(1 - (1 - \delta)^{n-i}))\gamma_c + \varphi_2(\theta - 1)\gamma_d - \gamma_\pi$$

Appendix C. Conditions of Convergence

The price–dividend ratio ratio (3.3.16) writes

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} w_n$$

where $w_n = \beta^n \exp \delta_n z_t + c_n$. It follows that

$$\left| \frac{w_{n+1}}{w_n} \right| = \beta \exp \left(\Delta\delta_{n+1} + \Delta c_{n+1} \right)$$

where
$\Delta\delta_{n+1} = \delta_{n+1} - \delta_n = \varphi_1(1-\theta)\delta(1-\delta^n)$

and

$\Delta c_{n+1} = c_{n+1} - c_n = g_{\Delta d} + g_{\Delta c}\left[-\theta + \varphi_1(\theta-1) - \varphi_1(\theta-1)(1-\delta)^n\right] + \frac{1}{2}\sigma^2_{\Delta d}\left(1 + +\varphi_2(\theta-1)\right)^2$
$+\frac{1}{2}\sigma^2_{\Delta c}\left[\theta^2 + \varphi_1^2(\theta-1)^2\left(1 - 2(1-\delta)^n + (1-\delta)^{2n}\right) - 2\varphi_1\theta(\theta-1)\left(1-(1-\delta)^n\right)\right]$
$+(1+\varphi_2(\theta-1)\rho_{\Delta c\Delta d}\left[-\theta + \varphi_1(\theta-1) - \varphi_1(\theta-1)(1-\delta)^n\right]$

As $|\delta| < 1$, we have

$\lim_{n\to\infty} \Delta\delta_{n+1} = 0$

and

$\lim_{n\to\infty} \Delta c_{n+1} = g_{\Delta c}\left(-\theta\varphi_1(\theta-1)\right) + g_{\Delta d} + \frac{1}{2}\sigma^2_{\Delta d}\left(1+\varphi_2(\theta-1)\right)^2 + \frac{1}{2}\sigma^2_{\Delta c}\left(-\theta\varphi_1(\theta-1)\right)^2$
$+\rho_{\Delta c\Delta d}\left(1+\varphi_2(\theta-1)\right)\left(-\theta\varphi_1(\theta-1)\right)$

Let us define

$r_{cv} = \beta \exp[g_{\Delta c}\left(-\theta\varphi_1(\theta-1)\right) + g_{\Delta d} + \frac{1}{2}\sigma^2_{\Delta d}\left(1+\varphi_2(\theta-1)\right)^2 + \frac{1}{2}\sigma^2_{\Delta c}\left(-\theta\varphi_1(\theta-1)\right)^2$
$+\rho_{\Delta c\Delta d}\left(1+\varphi_2(\theta-1)\right)\left(-\theta\varphi_1(\theta-1)\right)]$

It follow that

$$\left| \frac{w_{n+1}}{w_n} \right| \to r_{cv} \text{where } n \to \infty$$

Using the ratio test, we face three situations

i) When $r_{cv} > 1$, the series $\sum_{n=1}^{\infty} w_n$ diverges

ii) When $r_{cv} < 1$, the series $\sum_{n=1}^{\infty} w_n$ converges

iii) When $r_{cv} = 1$, the ratio test is inconclusive. But if $r_{cv} = 1$, we have

$\ln 1/\beta = g_{\Delta c}\left(-\theta\varphi_1(\theta-1)\right) + g_{\Delta d} + \frac{1}{2}\sigma^2_{\Delta d}\left(1+\varphi_2(\theta-1)\right)^2 + \frac{1}{2}\sigma^2_{\Delta c}\left(-\theta\varphi_1(\theta-1)\right)^2$
$+\rho_{\Delta c\Delta d}\left(1+\varphi_2(\theta-1)\right)\left(-\theta\varphi_1(\theta-1)\right)$

Therefore

$$\begin{aligned} c_n &= -n\ln\beta + g_{\Delta c}\left[-\varphi_1(\theta-1)\frac{1-(1-\delta)^n}{\delta}\right] \\ &+ \frac{1}{2}\sigma^2_{\Delta c}\left[\varphi_1^2(\theta-1)^2\left(-2\frac{1-(1-\delta)^n}{\delta} + \frac{1-(1-\delta)^{2n}}{1-(1-\delta)^2}\right) + 2\varphi_1\theta(\theta-1)\left(\frac{1-(1-\delta)^n}{\delta}\right)\right] \\ &- (1+\varphi_2(\theta-1)\rho_{\Delta c\Delta d}\left[\varphi_1(\theta-1)\frac{1-(1-\delta)^n}{\delta}\right] \end{aligned}$$

Therefore, the elements of the series simplify to

$w_n = \exp\left(\delta_n z_t + \hat{c}_n\right)$

where

$$\hat{c}_n = g_{\Delta c}\left[-\varphi_1(\theta-1)\frac{1-(1-\delta)^n}{\delta}\right]$$
$$+\frac{1}{2}\sigma_{\Delta c}^2\left[\varphi_1^2(\theta-1)^2\left(-2\frac{1-(1-\delta)^n}{\delta}+\frac{1-(1-\delta)^{2n}}{1-(1-\delta)^2}\right)+2\varphi_1\theta(\theta-1)\left(\frac{1-(1-\delta)^n}{\delta}\right)\right]$$

At each time t, it can be shown that there exists some $\zeta_t > 0$ such that $w_j \geq \zeta_t$ for all j. It implies that $\sum_{j=1}^{n} w_j \geq n\zeta_t$ so that $\sum_{n=1}^{\infty} w_n$ does not exist.

Appendix D. Unconditional Moments: Analytic Formulas

The habit to consumption ratio z_t verifies

$$z_t = (1-\delta)z_{t-1} - \Delta c_t$$

It implies

$$z_t = -\sum_{i=0}^{\infty}(1-\delta)^i \Delta c_{t-i}$$

Given the specification of consumption growth $\Delta c_t \to N(g_{\Delta c}, \sigma^2_{\Delta c})$, the habit to consumption ratio verifies

$$\mathbb{E}(z_t) = -\frac{g_{\Delta c}}{\delta}$$

$$\mathrm{var}(z_t) = \frac{\sigma^2_{\Delta c}}{1-(1-\delta)^2}$$

$$\mathrm{corr}(z_t, z_{t-1}) = (1-\delta)$$

The price–dividend ratio ratio verifies the following expression

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \beta^n \exp \Lambda_n z_t + \Gamma_n$$

It implies the following pricing formulas:

- (i) the unconditional mean of the price–dividend ratio

$$\mathbb{E}\left(P/D\right) = \sum_{i=1}^{\infty} \beta^i \exp\left(\Lambda_i\left(-\frac{g_{\Delta c}}{\delta}\right) + \Gamma_i + \frac{1}{2}\Lambda_i^2 \frac{\sigma_{\Delta c}}{1-(1-\delta)^2}\right)$$

- the serial–covariance function of the price–dividend ratio

$$\begin{aligned}
\mathrm{cov}\left(\frac{P_t}{D_t}, \frac{P_{t-j}}{D_{t-j}}\right) &= \mathbb{E}\left(\left(\frac{P_t}{D_t}\right)\left(\frac{P_{t-j}}{D_{t-j}}\right)\right) - \mathbb{E}\left(\frac{P_t}{D_t}\right)\mathbb{E}\left(\frac{P_{t-j}}{D_{t-j}}\right) \\
&= \sum_{i=1}^{\infty}\sum_{n=1}^{\infty} \beta^{n+i} \exp\{(\Lambda_n + \Lambda_i)\left(-\frac{g_{\Delta c}}{\delta}\right) + (\Gamma_n + \Gamma_i) \\
&+ \frac{1}{2}\left(\Lambda_n^2 + \Lambda_i^2 + 2\Lambda_n\Lambda_i(1-\delta)^j\right)\frac{\sigma^2_{\Delta c}}{1-(1-\delta)^2}\} \\
&- \sum_{i=1}^{\infty}\sum_{n=1}^{\infty} \exp\{(\Lambda_n + \Lambda_i)\left(-\frac{g_{\Delta c}}{\delta}\right) + (\Gamma_n + \Gamma_i) + \frac{1}{2}\left(\Lambda_n^2 + \Lambda_i^2\right)\frac{\sigma^2_{\Delta c}}{1-(1-\delta)^2}\}
\end{aligned}$$

Given the formula (3.3.24) of the real yields, it follows

- the unconditional mean

$$\mathbb{E}\left(y_{n,t}\right) = A_n\left(-\frac{g_{\Delta c}}{\delta}\right) + C_n$$

- the serial–covariance function

$$\text{cov}\left(y_{n,t}, y_{n,t-i}\right) = A_n^2 (1-\delta)^i \frac{\sigma_{\Delta c}^2}{1-(1-\delta)^2}$$

- the cross–section covariance function

$$\text{cov}\left(y_{n,t}, y_{n',t}\right) = A_n A_{n'} \frac{\sigma_{\Delta c}^2}{1-(1-\delta)^2}$$

Given the formula (3.3.25) of the nominal yields, it implies

- the unconditional mean

$$\mathbb{E}\left(y_{n,t}^{\$}\right) = A_n^{\$}\left(-\frac{g_{\Delta c}}{\delta}\right) + C_n^{\$}$$

- the unconditional variance

$$\text{var}\left(y_{n,t}^{\$}\right) = \left(A_n^{\$}\right)^2 \frac{\sigma_{\Delta c}^2}{1-(1-\delta)^2} + \left(B_n^{\$}\right)^2 \frac{\sigma_{\Delta \pi}^2}{1-\phi^2} + 2A_n^{\$}B_n^{\$}\rho_{\Delta c \Delta \pi}\frac{\sigma_{\Delta c}\sigma_{\Delta \pi}}{\sqrt{(1-(1-\delta)^2)(1-\phi^2)}}$$

- the serial–covariance function ($i \geq 1$):

$$\begin{aligned}\text{cov}\left(y_{n,t}^{\$}, y_{n,t-i}^{\$}\right) = {} & \left(A_n^{\$}\right)^2 (1-\delta)^i \frac{\sigma_{\Delta c}^2}{1-(1-\delta)^2} + \left(B_n^{\$}\right)^2 \phi^i \frac{\sigma_{\Delta \pi}^2}{1-\phi^2} \\ & + A_n^{\$}B_n^{\$}\rho_{\Delta c \Delta \pi}\frac{\sigma_{\Delta c}\sigma_{\Delta \pi}}{\sqrt{(1-(1-\delta)^2)(1-\phi^2)}}\end{aligned}$$

Figure 3.1: Convergence Regions

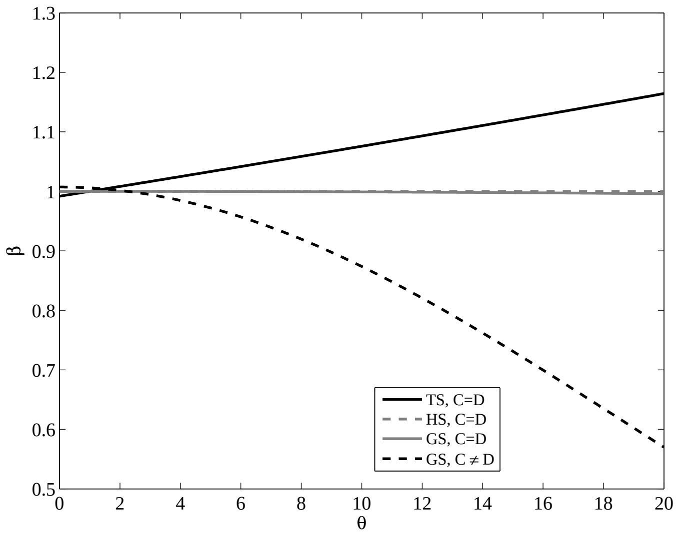


Table 3.1: Macroeconomic Variables: Descriptive Statistics

			Autocorrelation function					Correlation Matrix		
	mean %	std %	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	Δc	Δd	$\Delta \pi$
Δc	0.82	0.48	0.35	0.18	0.21	0.03	-0.14	1.00	0.12	-0.36
Δd	0.10	5.32	-0.42	-0.01	0.04	0.045	0.09		1.00	-0.05
$\Delta \pi$	0.93	0.64	0.85	0.80	0.78	0.72	0.65			1.00

Note: The macroeconomic variables – i.e real consumption growth rate Δc, real dividend growth rate Δd and inflation π – are expressed in quarterly frenquency. The sample period is the second quarter of 1952 to the fourth quarter of 2002.

Table 3.2: Financial Variables: Descriptive Statistics

Panel A: Quarterly Real Prices of Stocks and Risk Free Rate

			Autocorrelation function				
	mean	std	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$
$p - d$	3.382	0.340	0.939	0.921	0.891	0.906	0.845
r_f	0.35%	0.58%	0.664	0.474	0.598	0.589	0.455
r	1.66%	8.23%	0.05	-0.08	-0.03	0.01	-0.00
$r - r_f$	1.31%	7.95%	0.05	-0.08	-0.03	0.00	-0.01

Panel B: Annualized Quarterly Nominal Bonds

			Autocorrelation function				
	mean	std	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$
$1quarter$	1.33%	0.71%	0.93	0.88	0.87	0.81	0.75
$1year$	1.43%	0.72%	0.94	0.90	0.87	0.82	0.77
$2year$	1.48%	0.71%	0.95	0.91	0.88	0.85	0.80
$3year$	1.52%	0.69%	0.95	0.92	0.90	0.86	0.82
$4year$	1.55%	0.69%	0.96	0.93	0.90	0.87	0.83
$5year$	1.57%	0.68%	0.96	0.94	0.91	0.88	0.85

Note: Panel A and B report the summary statistics of respectively (i) quarterly real interest rate r_f, real return on equities r and price–dividend ratio $p - d$, and (ii) annualized quarterly bonds yields. The sample period is the second quarter of 1952 to the fourth quarter of 2002.

Table 3.3: Forcing Variables

					Correlation Matrix of innovations		
	Δc	Δd	$\Delta \pi$				
Mean (%)	0.82	0.10	0.93	Δc	1.00	0.12	-0.19
SD of innovations (%)	0.48	5.32	0.33	Δd		1.00	-0.07
Persistence			0.85	$\Delta \pi$			1.00

Note: The data used to estimated the endowment processes Δc and Δd and inflation $\Delta \pi$, as defined in section 3.3.2, are expressed in quarterly frequency.

Table 3.4: Real Stock Returns and Interest Rates (Per Quarter)

φ_1	φ_2	θ	β	mean	std	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$
				Risk free rate r_f						
0	0	5	0.98	5.87(%)	0.00	–	–	–	–	–
1	0	5	0.98	2.58(%)	0.27(%)	0.92	0.86	0.80	0.74	0.68
1	1	5	0.98	0.35(%)	0.27(%)	0.92	0.86	0.80	0.74	0.68
				Excess return on aggregate wealth $r_w - r_f$						
0	0	5	0.98	0.01(%)	0.00	1.00	–	–	–	–
1	0	5	0.98	0.03(%)	1.97(%)	-0.01	-0.006	-0.006	-0.007	-0.005
1	1	5	0.98	0.51(%)	2.46(%)	-0.01	-0.006	-0.006	-0.007	-0.005
				Excess return on equities $r - r_f$						
0	0	5	0.98	0.05(%)	0.00	1.00	–	–	–	–
1	0	5	0.98	0.61(%)	1.86(%)	-0.01	-0.006	-0.006	-0.007	-0.005
1	1	5	0.98	0.47(%)	2.48(%)	-0.01	-0.006	-0.006	-0.007	-0.005
				Price–consumption ratio $p - c$						
0	0	5	0.98	2.95	0.00	1.00	–	–	–	–
1	0	5	0.98	4.01	4.08(%)	0.92	0.86	0.80	0.74	0.68
1	1	5	0.98	7.69	5.40(%)	0.92	0.86	0.80	0.74	0.68
				Price–dividend ratio $p - d$						
0	0	5	0.98	2.84	0.00	1.00	–	–	–	–
1	0	5	0.98	2.84	4.28(%)	0.92	0.86	0.80	0.74	0.68
1	1	5	0.98	9.77	5.44(%)	0.92	0.86	0.80	0.74	0.68

Table 3.5: Sensitivity Analysis

Real Stock Returns and Interest Rates

					r_f		$r_w - r_f$		$r - r_f$		$p - c$		$p - d$	
φ_1	φ_2	β	θ	r_{cv}	mean(%)	std(%)	mean(%)	std(%)	mean(%)	std(%)	mean	std(%)	mean	std(%)
1	1	0.9823	2	0.98	2.47	0.07	0.000	0.86	0.30	0.84	4.07	1.04	3.93	1.01
1	1	0.9823	5	1.00	0.36	0.27	0.51	2.46	0.47	2.48	7.69	5.40	9.77	5.44
1	1	0.9823	10	1.12	-8.78	0.61	0.60	4.98	0.60	4.98	51.16	12.26	60.78	12.26
1	1	1.008	5	1.03	-2.22	0.27	3.05	2.48	3.05	1.48	18.88	5.45	21.45	5.45
1	1	0.97	5	0.99	1.26	0.25	-0.000	2.22	-0.000	2.36	4.84	4.74	5.71	5.12
1	0	0.9823	2	0.97	2.60	0.07	0.00	0.85	0.5	0.82	4.01	0.01	3.73	0.01
1	0	0.9823	10	0.97	2.49	0.61	0.11	3.85	0.69	3.59	4.02	0.09	3.74	0.08
1	0	0.999	5	0.99	0.89	0.27	0.18	2.38	0.63	2.24	5.97	5.19	4.95	4.80
1	0.5	0.9933	5	0.98	0.36	0.25	0.15	2.39	0.16	5.21	5.96	5.21	5.92	5.20
1	0.5	0.98	5	0.98	2.27	0.27	0.00	2.05	0.02	2.05	4.22	4.29	4.21	4.28

Table 3.6: Annualized Real and Nominal Bond Yields

TS: $\theta = 5$, $\beta = 0.98$, $\varphi_1 = 0$ and $\varphi_2 = 0$

	Real Yields						Nominal Yields					
	mean (%)	std(%)	$\rho(1)$	$\rho(2)$	$\rho(3)$	TB	mean (%)	std(%)	$\rho(1)$	$\rho(2)$	$\rho(3)$	IP
$1q$	23.49	0.00	1.00	–	–	0.00	27.23	2.10	0.83	0.69	0.57	3.74
$1y$	23.49	–	–	–	–	–	27.22	1.57	0.83	0.69	0.57	3.73
$2y$	23.49	–	–	–	–	–	27.21	1.14	0.83	0.69	0.57	3.72
$3y$	23.49	–	–	–	–	–	27.20	0.87	0.83	0.69	0.57	3.71
$4y$	23.49	–	–	–	–	–	27.19	0.69	0.83	0.69	0.57	3.70
$5y$	23.49	–	–	–	–	–	27.18	0.69	0.83	0.69	0.57	3.69

HS: $\theta = 5$, $\beta = 0.98$, $\varphi_1 = 1$ and $\varphi_2 = 0$

	mean (%)	std(%)	$\rho(1)$	$\rho(2)$	$\rho(3)$	BP	mean (%)	std(%)	$\rho(1)$	$\rho(2)$	$\rho(3)$	IP
$1q$	10.31	1.08	0.92	0.86	0.80	–	14.05	2.52	0.85	0.73	0.62	3.74
$1y$	10.32	0.98	0.92	0.86	0.80	0.01	14.06	1.98	0.86	0.74	0.64	3.73
$2y$	10.34	0.87	0.92	0.86	0.80	0.03	14.06	1.55	0.86	0.75	0.65	3.72
$3y$	10.35	0.77	0.92	0.86	0.80	0.04	14.06	1.26	0.87	0.76	0.67	3.70
$4y$	10.36	0.69	0.92	0.86	0.80	0.05	14.06	1.06	0.88	0.77	0.68	3.69

GS: $\theta = 5$, $\beta = 0.98$, $\varphi_1 = 1$ and $\varphi_2 = 1$

	Real Yields						Nominal Yields					
	mean (%)	std(%)	$\rho(1)$	$\rho(2)$	$\rho(3)$	BP	mean (%)	std(%)	$\rho(1)$	$\rho(2)$	$\rho(3)$	IP
$1q$	1.43	1.08	0.92	0.86	0.80	–	5.15	2.52	0.85	0.73	0.62	3.72
$1y$	1.43	0.98	0.92	0.86	0.80	-0.00	5.11	1.98	0.86	0.74	0.64	3.68
$2y$	1.42	0.87	0.92	0.86	0.80	-0.01	5.07	1.55	0.86	0.75	0.65	3.64
$3y$	1.42	0.77	0.92	0.86	0.80	-0.01	5.03	1.26	0.87	0.76	0.67	3.61
$4y$	1.41	0.69	0.92	0.86	0.80	-0.02	5.01	1.06	0.88	0.77	0.68	3.59

Note: BP and IP denote respectively the term bond premia and the inflation premia.

Table 3.7: Sensitivity Analysis

Annualized Real and Nominal Bond Yields

$\varphi_1 = 1$, $\varphi_2 = 1$, $\beta = 1.004$ and $\theta = 1.5$

	Real yields			Nominal yields		
	mean(%)	std(%)	TP	mean(%)	std(%)	IP
$1q$	1.43	0.13	–	5.16	2.13	3.73
$1y$	1.43	0.12	0.000	5.15	1.59	3.72
$2y$	1.43	0.10	0.000	5.13	1.16	3.70
$3y$	1.43	0.09	0.000	5.11	0.89	3.68
$4y$	1.43	0.08	0.000	5.10	0.71	3.67
$5y$	1.43	0.07	0.000	5.09	0.58	3.66

$\varphi_1 = 1$, $\varphi_2 = 1$, $\beta = 0.995$ and $\theta = 4$

	Real yields			Nominal yields		
	mean(%)	std(%)	TP	mean(%)	std(%)	IP
$1q$	1.43	0.81	–	5.16	2.37	3.72
$1y$	1.43	0.73	0	5.13	1.84	3.69
$2y$	1.43	0.65	-0.001	5.09	1.40	3.65
$3y$	1.43	0.58	-0.003	5.07	1.12	3.63
$4y$	1.43	0.52	-0.005	5.05	0.93	3.61
$5y$	1.43	0.47	-0.007	5.03	0.79	3.60

$\varphi_1 = 1$, $\varphi_2 = 1$, $\beta = 0.896$ and $\theta = 10$

	Real yields			Nominal yields		
	mean(%)	std(%)	TP	mean(%)	std(%)	IP
$1q$	1.38	2.45	–	5.08	3.48	3.70
$1y$	1.36	2.21	-0.02	4.99	2.92	3.63
$2y$	1.33	1.96	-0.05	4.89	2.43	3.56
$3y$	1.29	1.75	-0.08	4.81	2.08	3.51
$4y$	1.26	1.57	-0.12	4.74	1.81	3.47
$5y$	1.23	1.41	-0.15	4.68	1.60	3.45

Note: BP and IP denote respectively the term bond premia and the inflation premia.

Conclusion générale

Le lien entre les marchés financiers et les fluctuations économiques est un élément clé dans la détermination des prix d'actifs financiers et l'interprétation de leurs dynamiques. Ce lien entre les variables macroéconomiques et les valeurs financières se justifie par une multitude de faits stylisés. Nous citons en exemple le fait que les anticipations futures de la prime de risque varient avec les cycles réels. Ainsi, une large littérature, empirique et théorique, s'est focalisée sur cette problématique.

Cette thèse a permis de proposer un cadre théorique permettant d'évaluer à la fois le rôle de la persistance des habitudes et des nouvelles financières et celui de la dynamique des variables macroéconomiques afin d'expliquer les variations du ratio cours des actions-dividende et de la structure à terme des taux d'intérêt. Les modèles envisagés sont ainsi des extensions du modèle d'évaluation des actifs financiers basé sur la consommation ((C)CAPM). De plus, cette thèse a permis d'apporter une contribution à l'analyse empirique de ces modèles, en particulier l'étude du pouvoir de prédiction du ratio cours des actions–dividende et du ratio surplus de consommation.

Dans un premier chapitre, nous avons proposé une extension du modèle (C)CAPM avec formation des habitudes en introduisant un niveau de consommation de référence linéaire et tenant compte de tout l'historique de la consommation agrégée. De plus, le taux de croissance des dotations a été supposé suivre un processus auto–

régressif gaussien d'ordre un. Nous avons déterminé la solution analytique exacte du ratio cours des actions–dividende ainsi que les conditions de convergence garantissant l'existence d'une solution stationnaire bornée. Nous avons montré que le modèle (C)CAPM avec persistance des habitudes permet de reproduire la persistance du ratio cours des actions-dividende et en partie la prédictabilité des excès de rendements des actifs risqués à long terme.

Dans le deuxième chapitre, nous avons montré, théoriquement et empiriquement, la capacité du ratio surplus de consommation à expliquer les variations des rendements d'actifs risqués, à la fois en séries chronologiques et en coupe transversale. De plus, nous avons proposé dans ce chapitre une expérience de Monte Carlo permettant de tenir compte des biais d'estimation de la méthode des moindres carrés ordinaires utilisés pour tester le pouvoir de prédiction linéaire du ratio surplus de consommation.

Dans le dernier chapitre de cette thèse, nous avons considéré un modèle (C)CAPM avec (i) une modélisation générale du niveau de consommation de référence et (ii) une spécification affine des processus exogènes. D'une part, nous avons supposé que le niveau de référénce tient compte de tout l'historique de la consommation agrégée ainsi que des nouvelles sur le taux de croissance des dividendes. D'autre part, nous avons supposé que les variables macroéconomiques (le taux de croissance de la consommation, le taux de croissance des dividendes et l'inflation) sont exogènes et suivent un processus affine (*Compound Autoregressive process of order one* CaR(1)).

La représentation affine du taux d'escompte stochastique et des processus exogènes nous a permis de déterminer les solutions analytiques exactes du ratio cours des actions–dividende, du ratio cours du portefeuille de marché–consommation et de la structure à terme des taux d'intérêt. De plus, nous avons montré que la structure à terme est affine. Notons qu'une large littérature, concernant les modèles basés sur l'absence d'opportunité d'arbitrage, propose des modèles affines pour la structure à terme des taux d'intérêt (voir Duffie, Fillipovic et Singleton [2001] pour une revue de littérature des modèles affines en temps continu et Gouriéroux, Montfort et Polimenis [2002] pour une discrétisation de ces modèles).

Afin de mieux comprendre les mécanismes économiques en œuvre, nous avons considéré le cas simple d'un environnement de dotation *i.i.d* et nous avons supposé que l'inflation suit un processus auto–régressif d'ordre un. L'évaluation empirique de ce cas particulier du modèle proposé dans le dernier chapitre de cette thèse a montré que contrairement au modèle (C)CAPM standard, le modèle (C)CAPM avec niveau de référence permet d'expliquer (*i*) la persistance élevée du ratio cours des actions – dividende, (*ii*) l'énigme d'excès de volatilité des rendements d'obligations et (*iii*) la courbe moyenne décroissante des taux d'intérêt réels.

Une des perspectives qui nous semble particulièrement intéressante serait d'explorer d'autres dynamiques du taux de croissance de la consommation, du taux de croissance des dividendes et de l'inflation. En particulier, nous proposons de considérer des dynamiques tenant compte (*i*) d'une composante prédictable du taux de croissance de la consommation (voir Piazzesi and Schneider [2006]) ou/et (*ii*) de l'incertitude économique mesurée par une volatilité du taux de croissance de la consommation variable dans le temps (voir Bansal et Yaron [2004]). D'autre part, étant donné qu'aucune restriction n'a été imposée au nombre de variables d'état, il serait également intéressant d'enrichir le modèle en tenant compte d'autres variables macro–économiques et financières.

De plus, une extension possible de ce travail consiste aussi à tenir compte de la persistance du taux de croissance des dividendes dans la formulation du niveau de référence. Par ailleurs, dans cette thèse, l'évaluation quantitative des modèles proposés se base sur la technique de calibration. Il pourrait être intéressant d'estimer ces modèles.

Enfin, comme dans Garcia, Renault et Semenov [2006], il serait pertinent d'enrichir le modèle proposé dans le dernier chapitre de cette thèse en adaptant la formulation proposée afin d'englober les modèles (C)CAPM avec utilité récursive.

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