Comparative and Targeted Advertising in Competitive Markets

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Dedication

This dissertation is dedicated to my family.

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[1] This paper is co-written with Nathalie Sonnac.

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Chapter 1

Introduction

"What makes the advertising issue fascinating... is that
it is fundamentally an issue in how to establish truth in economics."
[*Phillip Nelson*]

By its very nature, advertising is a pervasive feature of economic life. It is an increasingly important tool in strategic interactions in competitive markets.

Whether the role of advertising is to enhance the image of the product in the eyes of consumers and change their preferences, or to inform them about the release of a new product in the market, or rather to provide information on prices or qualities of existing products, an important question puzzles the economists: Why do consumers respond to advertising? As economists have struggled with this question, three views have emerged. The first view is that advertising is *persuasive*, that is, it alters consumers' tastes and creates spurious product differentiation and brand loyalty. As a consequence, it has no "real" value to consumers, but rather induces artificial product differentiation. The second view is that advertising is *informative*. According to this approach, many markets are characterized by imperfect consumer information, since search costs may deter a consumer from learning of each product's existence, price and quality. Advertising is the endogenous response that the market offers as a solution: when a firm advertises, consumers receive information. The third view is that advertising is *complementary* to the advertised product. According to this perspective, advertising does not change consumers' preferences, as in the persuasive view; furthermore, it may, but need not, provide information. Instead, it is assumed that consumers possess a stable set of preferences into which advertising enters directly in a fashion that is complementary with the consumption of the advertised product. For example, consumers may value "social prestige," and the consumption of a product may generate greater prestige when the product is (appropriately) advertised. Hence, advertising can influence consumer behavior for different reasons. Accordingly, advertising affects demand, because: (i) it conveys information to consumers, with regard to the existence of sellers, and the price and qualities of products in the marketplace, and (ii) it alters consumers' "wants" or tastes.

Regardless of the role of advertising, economists struggle with another important question: What marketing techniques are more efficient for the advertisers? At this point, it is useful to remark on some recent trends in the marketing literature: comparative versus generic advertising, and targeted versus mass advertising. *Comparative advertising* by one brand against another is such a promotional technique that suggests superiority of one's own brand and stresses the inferiority of the rival's. The European Commission defines it as follows: "comparative advertising is such that explicitly or by implication, identifies a competitor or goods or services offered by a competitor". In other words, "comparative bashing" is identified by one brand comparing itself favorably with a competing brand. This type of advertising exhibits interesting externalities, which are absent in generic advertising. *Targeted advertising* is meant to "target" the individual consumers to whom their respective ads are delivered, that is, it returns from mass-audience advertising towards specific consumer groups.

This dissertation takes a stand in the literature on advertising, wherein economists debate the purpose and the effects of advertising. The first two chapters analyze how the fact that advertising be comparative rather than regular affects price competition and advertising volume itself. In particular, chapter 2 is based on the persuasive view on comparative advertising, and chapter 3 advocates the informative view. Chapter 4 analyzes the trade-off for an advertiser between using targeted advertising and mass-audience advertising.

The first chapter of this dissertation deals with persuasive advertising. A theoretical analysis of advertising wars is performed, where firms engage in deceptive comparative advertising against each other. In a symmetric duopoly set-up with fixed market size it does not matter whether a firm mentions the rival in an advertisement or not, since in both cases the ad reduces the relative valuation of the competing brand. In contrast, if there are three firms in the market, comparative advertising exhibits a directionality, i.e. a firm can choose to target just one of her rivals. Even so, in symmetric scenarios, where none or all firms collude, advertising becomes irrelevant for the price equilibrium irrespectively of whether it is comparative or not. In contrast, if two of the three firms collude, then the fact that advertising is comparative becomes crucial: it is not only true that prices change due to advertising, but also the impact of advertising on prices depends on the derogatory power of advertising. Thus, it is argued in the chapter that there exists a noticeable distinction between comparative advertising and generic advertising due to the difference in effects they bring to the market outcomes. Moreover, we demonstrate how the fact that advertising be comparative rather than regular affects advertising intensity itself: (i) colluding firms suppress their mutual comparative advertising (due to internalizing the negative externality of comparative advertising), (ii) colluding firms decrease the advertising volumes against their rival (from the fear of fierce price competition), (iii) the rival may intensify or trim down the advertising efforts against colluding firms, depending on the derogatory power of comparative advertising. The total amount of advertising in the market decreases. The persuasive advertising is generally believed to be anti-competitive as it tends to make the demand for an advertised product more inelastic. Thus, it is argued that consumers would be better-off in the absence of advertising as they would face lower prices. It is shown in the chapter that the impact of comparative advertising on price competition depends on the vicious power of advertising.

Chapter 3 examines the determinants of the strategic decision of a high quality producer to advertise comparatively against a low quality rival. The intuition suggests that the high quality producer faces a trade-off. First, comparative advertising reveals the existence of rival's brand, which gives the incumbent incentives not to use this promotional technique. This effect is the stronger the more people are unaware of the existence of the competitor in the market. Second, comparative advertising raises product differentiation at the informed segment, which makes it worth it for the incumbent to announce the lower quality of the new brand. This effect is the stronger the larger the difference between the expected quality and true quality of the rival product. Moreover, we investigate the role of price competition in the model. We discover that advertising makes low quality firm decrease her price. The fear of fierce price competition makes the high quality firm refrain from using comparative advertising.

Finally, chapter 4 analyzes the determinants of advertising strategies of an industrial producer on the media market. In particular, we first highlight the pricing strategies of advertising space in magazines not only as a function of the readership size, but also as a function of the quality of the readers. Second, we analyze the relation between the media market and the market of industrial products, that is, the impact of readers' profile on the product prices via the advertising rates. Consequently, the following factors are endogenized: the size of the magazine's demand, and the quality of readers, whose probability of purchasing the industrial product varies with the degree of content specialization of the media. Hence, in the strategic decision to buy advertising space in magazines, the firm will face a trade-off between the large readership size and the interesting profile of a reader. It is shown that a monopolist on the product market is able to segment the market by targeting its ads to certain groups of magazine readers and then practicing price discrimination by internalizing the difference in demand elasticities for the product among the two groups of consumers.

Chapter 2

The Effects of Collusion in a Model of Persuasive Comparative Advertising and Price Competition

2.1 Introduction

Comparative advertising is a promotional technique that suggests superiority of one's own brand and stresses inferiority of the rival's by means of direct comparisons among brands and their characteristics. Since the aim of comparative advertising is to affect both the demand of the advertiser and the rival's, this raises a fundamental question: What difference does it make for the firm to use comparative advertising instead of the regular (non-comparative) one? The intuition suggests that in a duopoly with fixed market size it does not matter whether a firm mentions the rival in an advertisement or not. In both cases, the ad reduces the relative valuation of the competing brand, hence a fraction of rival's consumers switches to the advertised product. Hence the game of regular advertising and the game of comparative advertising are, in this setting, equivalent (let alone the equilibrium of the game).

In contrast, there exist circumstances under which the fact that advertising be comparative rather than regular does make a difference, that is the game played is different. We assume that the distinction between regular and comparative advertising is meaningful if more than two firms co-exist in the market. Under these conditions comparative advertising exhibits a *directionality*, i.e. a firm chooses *which* competitor to target. This directionality is lost in using regular advertising, since advertising oneself will decrease *all* rivals' relative value.

More specifically, the aim of this work is to analyze how the fact that advertising be comparative rather than regular affects price competition and advertising volume itself. We perform a theoretical analysis of advertising wars in an imperfectly competitive market where firms first decide the intensity of their advertising and then compete in prices[1]. The market imperfection comes from the fact that firms are horizontally differentiated as in the Salop's circular city model.

First we analyze the nature of comparative advertising under a non-cooperative scenario. Subsequently we turn to examine the effects of collusion on advertising intensities when *all* firms cooperate and finally when *only two* firms collude[2]. The reader may wonder why we focus on case where firms collude only on advertising. We believe that restricting collusion to advertising is more realistic as collusion on advertising levels, even if prohibited, may be hard to examine and prosecute, hence is more likely to take place, whereas collusion on prices constitutes a price-fixing violation of the antitrust law that can be easily enforced.

Our main results are the following. As anticipated, if firms do not cooperate and firms are symmetric, then both comparative and regular advertising bring similar results: the equilibrium advertising intensities are identical across firms.[3] Hence the consumers' relative valuations for the brands do not change. As a result, the equilibrium prices are

[1] It seems reasonable to think of advertising strategies as longer term than pricing strategies. Whereas prices can often be adjusted rapidly at relatively low cost and the impact on demand will be quickly felt, it may be much more costly to alter advertising strategies, and it can take considerable time until the effect on demand is noticeable. Persuasive advertising by its nature may perhaps be thought of as having long-lasting effects given that, if successful, it affects consumer preferences. In that case it is reasonable to model advertising decisions as long term and pricing decisions as short term.

[2] To motivate the analysis of a partial collusion on advertising, we provide the following example when such a marketing strategy was allegedly used. In 2004 Coca-Cola and Pepsi-Cola decreased their mutual bashing campaigns and they both intensified their advertising efforts to weaken the market position of a new cola-flavoured carbonated beverage, Mecca-Cola, which was marketed as an alternative to Coke and Pepsi to pro-Muslim consumers.

[3] In general, the benefits of advertising (whether comparative or not) for the firm derive from the fact that *ceteris paribus* the firm gains market share thanks to advertising effort. This may occur at the expense of the rivals or through an increase of demand for the entire industry. Since we examine the former case, in the absence of the increase of industry demand it is possible that individual demand shifts neutralize each other.

identical to those in the standard circular city model without advertising. Consequently, such a scenario is reminiscent of the Prisoner's Dilemma. The equilibrium advertising efforts of all firms offset each other. Both comparative and regular advertising prove to be a wasteful activity, yet costly. Therefore, all firms would like to commit to zero advertising. If in fact advertising efforts by close competitors neutralize each other, then there are potential gains from collusive agreements to refrain from comparative advertising. This possibility motivated us to study the sustainability of collusion in our model and its effects on price competition.

If indeed *all* firms collude on advertising intensities, they set advertising levels at zero and each firm gains higher profits than under the non-cooperative scenario. This is an obvious consequence of the cost reductions resulting from suppressing the wasteful and costly advertising[4].

Our more interesting results come from the scenario where only two firms collude and direct their combined comparative advertising against the remaining rival, *the outsider.* Comparative advertising gives rise to a **negative externality**: whenever a colluding firm advertises against her partner, she in fact hurts the partner, helping at the same time the outsider. On the other hand, comparative advertising creates also a **positive externality**, that comes from the "directionality" of advertising: whenever a colluding firm advertises against the outsider, she hurts the outsider, helping at the same time her partner. Colluding firms internalize the negative externality and suppress their mutual bashing. More remarkably, the advertising strategies of colluding firms against the outsider are formed by two opposite effects: (i) the directionality effect which makes each colluding firm internalize the positive externality that helps her partner and consequently increase the comparative bashing against the outsider, (ii) the price effect which makes each colluding firm decrease the advertising against the outsider to prevent her from setting a low price. We show that the price effect dominates the directionality effect. Hence, colluding firms advertise less against the outsider than they did under non-cooperative scenario. The advertising strategies of the outsider against colluding firms are also formed by two opposite effects: (i) the strategic advertising effect, and (ii) the price effect. The former effect comes from the fact that the advertising decisions are strategic substitutes.

[4] Indeed, zero advertising would be socially optimal. This result supports the time honored contention, dating back to Pigou (1924), that advertising efforts by competitors might just neutralize each other and prove wasteful. Netter (1982) reports indirect empirical evidence for the mutual cancelling of advertising efforts. Indeed, some firms "have chosen disarmament after years of ad warfare proved fruitless, such as Unilever's Ragu and Campbell Soup Co.'s Prego" as Neff (1999) reports.

Hence, lower advertising levels from colluding firms make the outsider intensify her advertising effort. The latter effect makes the outsider decrease the advertising volumes from the fear of price reductions by colluding firms. Which effect dominates depends on how vicious comparative advertising is. The outsider will intensify advertising efforts if advertising is not very derogatory (i.e., when it is less comparative), and will decrease them otherwise.

Regarding the effects of the fact that advertising be comparative rather than regular, it is clear that in the symmetric scenarios (where none or all firms collude), there are no effects of advertising on prices. Namely, no matter whether advertising is comparative or not, advertising effects are either zero (full collusion) or they offset each other (no collusion). In contrast, in the partial collusion scenario, as ads become more derogatory, prices are indeed affected. More specifically, prices of colluding firms increase with the vicious power of advertising, whereas the price of the outsider goes down with the value of the parameter.

Before we turn to the policy implications of our results we must describe what an ad is designed to do to consumers. The type of advertising that we analyze in the model attempts to enhance consumer tastes for a certain product, and instead of comparing brands' physical features, presents brands in such a set-up that reduces product substitutability, hence increases product differentiation. We model demand in such a way that consumers are made to wrongly believe that *homogeneous* products are vertically differentiated[5]. Hence, our interpretation of this market is that the nature of comparative advertising is purely deceptive, since its role is to mislead consumers. The decision of which brand to purchase depends on consumers' perception of what the brand is rather than on the actual physical characteristics of the product. This means that this work is not aimed at evaluating the welfare effects of advertising. Using the standard terminology, advertising in our model is persuasive rather than informative. However, we do not analyze the impact of *how* advertising enters consumers' utility on social welfare, i.e., how it changes consumers' preferences and whether it is complementary to the consumption of the advertised product, e.g. creating "social prestige". We analyze advertising from the perspective of the producer and in the absence of repeat purchases. Hence, we simply assume that consumers change their purchase decision, having seen a comparative ad,

[5] In the absence of comparative advertising, all brands are correctly perceived as identical from consumers' point of view.

since it has manipulated them into believing that the advertised product is better than they originally thought. However, we refrain from the discussion on the social welfare implications[6]. Comparative advertising of such type can be subjective or subliminal. The following examples could motivate the concept regarding the nature of comparative advertising employed in the model: an advertisement claiming that the taste of the advertised brand is better than the one of the competing brand, or referring to some non-tangible factors, like the subliminal association of one's product to success, youth, good health, etc. while the rival's to negatively perceived values. One can also consider a case of using "manipulated" representations (pictures and such) of the competing brand, i.e. a case in which the product comparison tends to deceive consumers because of factors left out of the comparison. To illustrate this one can think of an advertisement stating that a chocolate bar contains fewer calories when the difference rests on its being thinner or smaller in size than those of competing chocolate bars - but without mentioning it. Another example could be constituted by unverifiable comparisons (such as "Brand A tastes better than brand B") or could employ unfair factors in the products' presentation, such as famous or popular person speaking in favor of the advertised brand, while someone disliked or a man in the street presenting unfavorably the competing brand. Consequently, comparative advertising becomes bad *per se*. Since it reduces social welfare, the social objective should be to minimize it. Taking into account the welfare implications, the policy recommendations become straightforward: comparative advertising should be banned. However, such an action may prove to be expensive to implement. Moreover, it may be hard to prosecute, not to mention that the product features being compared in an ad may be hard to examine. In such a set-up collusion on advertising turns out to be recommendable, since it brings the total volume of comparative advertising down.

The interest of this work is to analyze the effects of comparative advertising, as opposed to regular advertising, on price competition and on advertising volume itself. The economic literature is still deficient in this field. In current models either the degree of comparativeness of advertising does not play any role in equilibrium outcomes or the advertising is *informative*. Aluf and Shy [WP, 2001] show that comparative advertising increases the degree of product differentiation and therefore the local monopoly power of each firm. However, it is modeled in a duopoly set-up where the fact that advertis-

[6] That is, whether consumers' preferences have really changed and they find the advertised product truly better than before (complementary view), or whether consumers are disappointed, because they have been deceived.

ing be comparative or regular does not change the equilibrium outcomes. Haller and Chakrabarti [WP, 2002] model the effect of comparative advertising on Cournot competition and show that in the symmetric case equilibrium advertising expenses constitute a welfare loss. Anderson and Renault [WP, 2005] investigate the informative comparative advertising in a quality disclosure game. However, the effects of comparative advertising as opposed to its regular counterpart have not been much explored. This work differs from the existing literature on comparative advertising in two respects. First, this paper shows how the fact that advertising be comparative rather than regular affects price competition and advertising volumes. Second, it investigates the effect of collusion on the advertising equilibrium outcome.

The paper is organized as follows. Section 2.2 sets up a model of comparative advertising employed by three firms. In section 2.3, we develop the second stage of the game where firms compete in prices. Section 2.4 presents the first stage of the model in which advertising decisions are taken in a non-cooperative way or through a collusion. Section 2.5 discusses the effects of the derogatory power of comparative advertising. The following section analyzes particular cases of parameters constellations. Finally, section 2.7 concludes. All proves are in Appendix A at the end of the chapter.

2.2 The Model

The product space and consumer preferences

Consider a circular city model originally due to Salop (1979). We represent each product/firm as a point on a circle of unit length. The number of firms is exogenous. For simplicity, let us consider the case of three firms indexed by i (where $i = A, B, C$). Locations of firms are fixed: they are located equidistantly around the circle.[7] All the travel occurs along the circle.

There is a cost of advertising: $a(s_{ij}+s_{ik})^2$, where $a > 0$ is a unit cost of advertising and $s_{ij}, s_{ik} \geq 0$ the advertising intensities by firm i targeted against firms j and k respectively. We assume that at this cost, all consumers will be reached. The advertising cost function is quadratic in the sum of advertising levels. The fact that the cost of advertising be quadratic can be interpreted as including a reduced form of the probability of comparative advertising being prosecuted. The quadratic nature of advertising costs reflects that

[7] Hence the three firms A, B and C are located at respectively: 0, $\frac{1}{3}$ and $\frac{2}{3}$.

the probability of getting caught increases with the total advertising. Technically, the advertising cost function cannot be linear, i.e. the quadratic way in which advertising enters profit functions makes all the optimization problems concave. Linear cost function would cause convexity and result in corner solutions.
All firms have the same production technology. Denoting by c the marginal production cost, and by q_i and $\pi_i(q_i)$ respectively the output and profit levels of the firm producing brand i, the firm's expected profits are given by:

$$\pi_i(q_i) = (p_i - c) \cdot q_i - a(s_{ij} + s_{ik})^2 \tag{2.1}$$

If firm i chooses the advertising level s_{ij} against firm j, then there is a positive gain for firm i in the sense that consumer's valuation of a unit of brand i increases by θs_{ij} whereas consumer's valuation of a unit of brand j decreases by αs_{ij}. Hence the gross consumer's values are modeled as follows:

$$v_i = v + \theta(s_{ij} + s_{ik}) - \alpha(s_{ji} + s_{ki}) \ , \tag{2.2}$$

where v reflects consumers' basic valuation for the product in the absence of advertising. We refer to v as the base willingness to pay. The shift parameter α, $\alpha \geq 0$, can be interpreted as the vicious or derogatory power of comparative advertising. The shift parameter θ, $\theta \geq 0$, represents the regular (non-comparative) advertising effect. Hence $\alpha = 0$ would mean the absence of comparative advertising. For simplicity, we will assume that $\theta \geq \alpha$.[8] Notice that in the absence of comparative advertising, all brands are perceived as identical from consumers' point of view. That is, the product space is completely homogenous to consumers except for the location of each product (no brand is *a priori* better than another).

Consumers are uniformly distributed on the circumference of the unit circle. We assume that consumers are heterogeneous. That is, due to different location, each consumer has a different preference for the brands sold in the market. The demand is totally inelastic: each consumer wishes to buy only one unit of the good, and attaches a gross dollar value of v_i to a unit of good i. In order to ensure full coverage of the market, i.e. to avoid exclusion, we assume that the base willingness to pay v is sufficiently high.

[8] The case of $\theta \leq \alpha$ is discussed briefly in section 2.6.

We will consider quadratic transportation costs. A product which is at a distance x along the circle from a consumer provides a dollar benefit of $v - tx^2$, where we refer to t as the transportation cost per unit of length (this cost may include the value of time spent in travel). Thus, a consumer living at a distance x from the store incurs a cost of tx^2 to go to a nearby store and $t\left(\frac{1}{3} - x\right)^2$ to another nearby store. Each consumer who purchases a product i at distance x away at price p_i derives a surplus from consumption equal to $v_i - tx^2 - p$. Consumers select a brand offering the highest positive perceived value. Figure 2.1 illustrates the position of firm A relative to the positions of firm B and firm C.

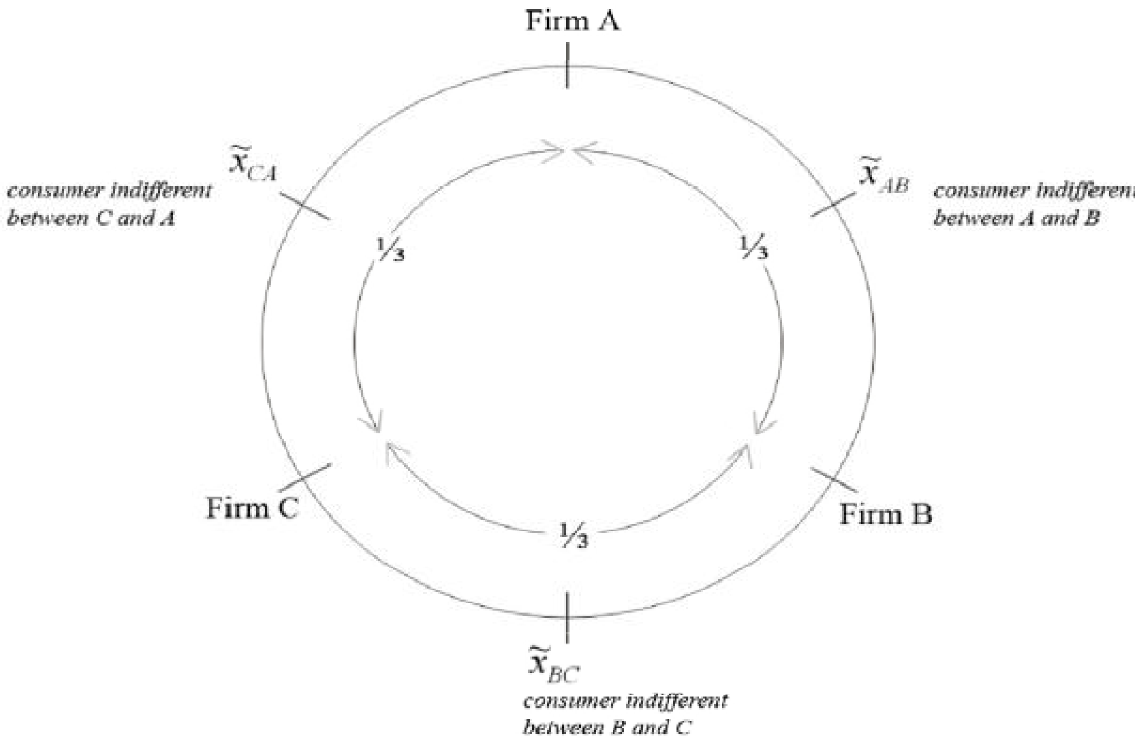

Figure 2.1: The positions of firms on the unit circle.

We now turn to the derivation of the demand curve facing each firm, $q_i\left(p_A, p_B, p_C \mid v_A, v_B, v_C\right)$ for $i = A, B, C$, given the prices and advertising levels of the other firms. A necessary first step is the computation of the location of indifferent consumers.

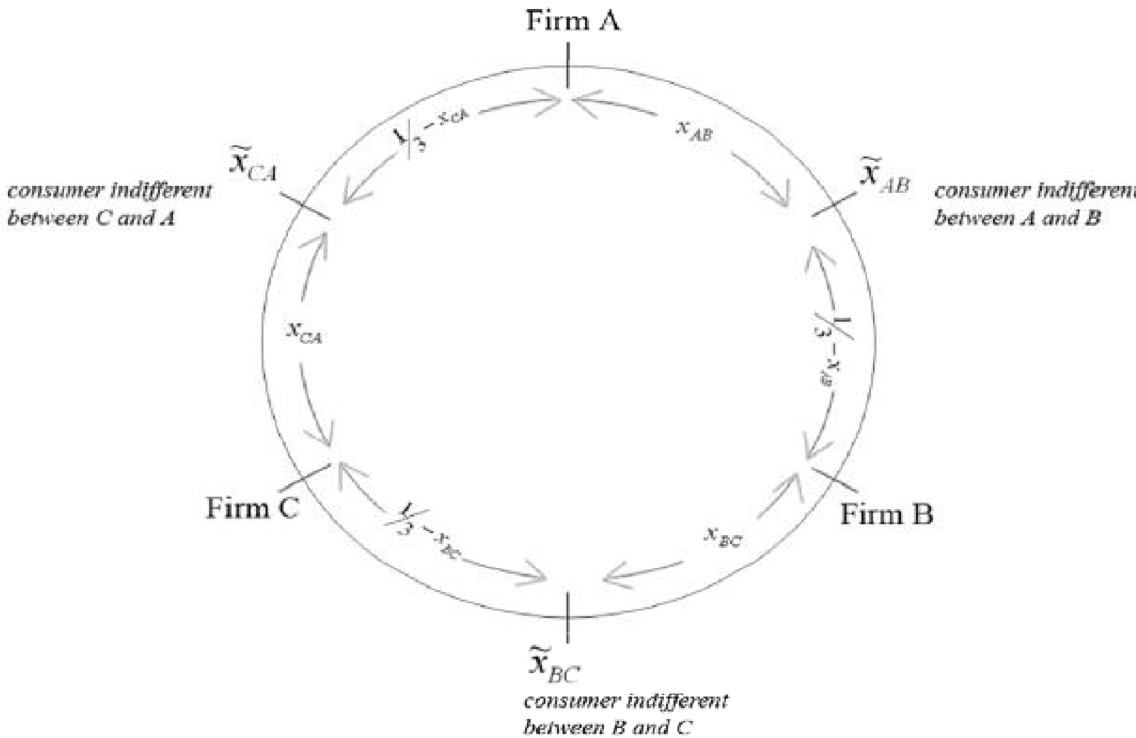

Figure 2.2: Market division among firms

Among the consumers located between firms A and B, there is a consumer who is indifferent between buying at A and B. Let us denote by $\tilde{x}_{AB}$ the location of the indifferent consumer, by x_{AB} the distance between $\tilde{x}_{AB}$ and firm A and by $\frac{1}{3}-x_{AB}$ the distance between $\tilde{x}_{AB}$ and firm B. Now, x_{AB} is the solution of $v_A - tx_{AB}^2 - p_A = v_B - t(\frac{1}{3} - x_{AB})^2 - p_B$. Hence, the distance x_{AB} is depicted in Figure 2.2, and is given algebraically by:

$$x_{AB} = \frac{9(v_A - v_B - p_A + p_B) + t}{6t}. \tag{2.3}$$

All those consumers located between firm A and point $\tilde{x}_{AB}$ would find the product of firm A to be their first choice, whereas consumers between firm B and point $\tilde{x}_{AB}$ would purchase the product from firm B.

Repeating this procedure for the remaining part of the market yields:

$$x_{BC} = \frac{9(v_B - v_C - p_B + p_C) + t}{6t}, \tag{2.4}$$

$$x_{CA} = \frac{9(v_C - v_A - p_C + p_A) + t}{6t}. \tag{2.5}$$

Distances x_{BC} and x_{CA} are depicted in Figure 2.2.
Since each firm has customers on her left and her right, the demands for products A, B and C are respectively:

$$\begin{cases} q_A = x_{AB} + \left(\frac{1}{3} - x_{CA}\right) \\ q_B = x_{BC} + \left(\frac{1}{3} - x_{AB}\right) \\ q_C = x_{CA} + \left(\frac{1}{3} - x_{BC}\right) \end{cases}. \tag{2.6}$$

Below we will provide an assumption under which demands are interior. Under this condition, the demand functions faced by firms A, B and C are respectively:

$$\begin{cases} q_A\left(\overrightarrow{p} \mid \overrightarrow{v}(\overrightarrow{s})\right) = \frac{1}{3} + \frac{2v_A - v_B - v_C - 2p_A + p_B + p_C}{2t} \\ q_B\left(\overrightarrow{p} \mid \overrightarrow{v}(\overrightarrow{s})\right) = \frac{1}{3} + \frac{-v_A + 2v_B - v_C + p_A - 2p_B + p_C}{2t} \\ q_C\left(\overrightarrow{p} \mid \overrightarrow{v}(\overrightarrow{s})\right) = \frac{1}{3} + \frac{-v_A - v_B + 2v_C + p_A + p_B - 2p_C}{2t} \end{cases}. \tag{2.7}$$

where $\overrightarrow{p} = \{p_A, p_B, p_C\}$, $\overrightarrow{v} = \{v_A, v_B, v_C\}$ and $\overrightarrow{s} = \{s_{ij}, s_{ik}, s_{ji}, s_{ki}, s_{jk}, s_{kj}\}$.
Not surprisingly, the demands depend on prices and the transportation cost. Each firm can increase her market share, everything else equal, by increasing her level of comparative advertising, since it makes the rival brands less attractive.

Timing of the game

To analyze market behavior in the presence of comparative advertising we develop a set-up in which the three firms interact according to the following two-stage game: in stage one they decide simultaneously on how much to advertise and against whom, and subsequently, in the second stage, given the advertising configuration, firms compete as Bertrand oligopolists. We assume that pricing decisions are taken simultaneously.

We apply the subgame perfect notion to this game. Hence, the game is solved backwards beginning with the price competition stage. We will consider three scenarios, one where in stage 1 all the firms act non-cooperatively, another where all firms collude on advertising intensities, and finally one where only firms A and B collude on advertising intensities. Notice that stage 2 is solved in the same way under all scenarios, since advertising is taken as given in the second stage and we solve the second stage for any level of advertising chosen, it is irrelevant whether firms A and B colluded on advertising levels or not.

The next assumption ensures that the maximization problems for all firms and all scenarios are concave and that the equilibrium of each scenario is interior. Moreover, it allows to rule out casuistics.

Assumption 2.1

$a > \max\left\{\frac{9(\alpha+2\theta)^2}{50t}, \frac{9(2\alpha+\theta)^2}{25t}\right\}$.

Lemma 1 *Under Assumption 2.1, demands are given by (2.7), and profits are non negative.*

We now turn to solving the second stage of the game.

2.3 Stage 2: Price Competition

For purposes of comparative statics it is helpful to define an auxiliary expression $\Gamma_i(\overrightarrow{s}, \alpha, \theta)$ for any $i, j, k = \{A, B, C\}$ and $\overrightarrow{s} = \{s_{ij}, s_{ik}, s_{ji}, s_{ki}, s_{jk}, s_{kj}\}$:

$$\Gamma_i(\overrightarrow{s}, \alpha, \theta) = (\alpha + 2\theta)(s_{ij} + s_{ik}) - (2\alpha + \theta)(s_{ji} + s_{ki}) - (\theta - \alpha)(s_{jk} + s_{kj}) \tag{2.8}$$

All effects of comparative advertising on prices, demands and profits contain the above expression, which will be shown to increase equilibrium price and profits.

Notice that $\Gamma_i(\overrightarrow{s})$ monotonically increases in advertising intensities used by firm i against her competitors, monotonically decreases in advertising levels used by firm i's rivals against firm i, and is also affected by the mutual advertising among firm i's rivals. Now, for $\theta \geq \alpha$, we obtain that $\alpha + 2\theta \geq 2\alpha + \theta \geq \theta - \alpha \geq 0$. In other words, for given advertising intensities, the effect of a firm's own advertising on the value of Γ_i in stronger in absolute terms than the effect of advertising from the firm's rivals against the firm, which in turn is stronger than the effect on Γ_i of mutual bashing among the rivals. In fact, for $\theta > \alpha$, the anti-competitive effect of advertising on prices is the strongest in absolute terms and is exactly equal in strength to the sum of the two pro-competitive effects. Formally, $(\alpha + 2\theta) = (2\alpha + \theta) + (\theta - \alpha)$.

Given the actions of her rivals, each firm maximizes her expected profit in the second stage. Substituting the expression for consumers' valuations (2.2) into demand function (2.7) and subsequently into the profit expression (2.1), each firm i solves $\forall i, j, k = \{A, B, C\}$:

$$\begin{aligned} \underset{\{p_i\}}{Max}\ \pi_i\left(\overrightarrow{p} \mid \overrightarrow{s}\right) &= (p_i - c)q_i - a(s_{ij} + s_{ik})^2 = \\ &= (p_i - c)\frac{[-18p_i + 9(p_j + p_k) + 2t + 9\Gamma_i]}{6t} - a(s_{ij} + s_{ik})^2. \end{aligned} \tag{2.9}$$

The objective function $\pi_i\left(\overrightarrow{p} \mid \overrightarrow{s}\right)$ of firm $i = \{A, B, C\}$ is concave in p_i for all parameters. The first-order conditions yield the price best-response functions $\forall i, j, k = \{A, B, C\}$:

$$p_i(p_j, p_k \mid \overrightarrow{s}) = \frac{9(p_j + p_k) + 2t + 18c + 9\Gamma_i}{36}\ . \tag{2.10}$$

The best-response functions reveal that prices are strategic complements. Also, each best-response function shifts upward with Γ_i which depends on the levels of comparative advertising as explained above. Solving the three best-response functions yields the following proposition.

Proposition 1 *There is a unique second-stage equilibrium in prices, which is characterized by* $p_i^*(\overrightarrow{s}) = \frac{t}{9} + c + \frac{\Gamma_i}{5} \qquad \forall i, j, k = \{A, B, C\}$.

The above equations reveal that the markup (the amount by which the price exceeds unit production cost) depends on the levels of comparative advertising of all firms and on transportation cost, t. More precisely, if no firm advertises or if each firm advertises at the same intensity, consumers perceive all brands as identical homogeneous products. In such cases price competition results in pricing above the unit cost $\left(p_i^* = \frac{t}{9} + c\right)$ and positive profits.

Lemma 2 *Assume $\theta > \alpha$. There are several cases of constellations of advertising intensities which make $\Gamma_i = 0$, hence advertising has no impact on price competition:*
(i) $\Gamma_i(s_{ij} = s_{kl}, \forall i, j, k, l \mid \alpha, \theta) = 0$, *and in particular* $\Gamma_i(s_{ij} = 0, \forall i, j \mid \alpha, \theta) = 0$,
(ii) $\Gamma_i(s_{ij} + s_{ik} = s_{ji} + s_{ki} = s_{jk} + s_{kj}, \forall i, j, k \mid \alpha, \theta) = 0$,
(iii) $\Gamma_i(s_{ij} + s_{ik} = s_{ji} + s_{ki}, \alpha = \theta, \forall i, j, k) = 0$.

The next proposition summarizes the results.

Proposition 2 *Assume that $\theta > \alpha$. A firm's comparative advertising is anti-competitive, whereas the advertising from rivals onto the firm and the one among rivals are pro-competitive. In some cases, identified in Lemma 2, the advertising efforts offset each other and have no impact on equilibrium prices.*

It is also interesting to note the *strategic effect* (commitment effect) which would make firms choose a lower level of advertising intensity in order not to make the rivals aggressive in terms of price cuts: $\frac{\partial p_j^*(\vec{s})}{\partial s_{ij}} < 0$, $\frac{\partial p_i^*(\vec{s})}{\partial s_{ij}} > 0$.

Now we turn to solving the game under three scenarios.

2.4 Stage 1: Advertising decisions

2.4.1 Non-cooperation

First, we analyze the nature of comparative advertising under non-cooperative scenario. Substituting the results of the second stage of the game into the objective function in stage 1, we obtain that each firm seeks to maximize her profits:

$$\underset{\{s_{ij}, s_{ik}\}}{Max}\, \pi_i^{NC}(\vec{s}) = \frac{(5t + 9\Gamma_i)^2}{675t} - a(s_{ij} + s_{ik})^2, \tag{2.11}$$

where the superscript NC stands for the *non-cooperative scenario.*

By inspection of the Γ function, given in (2.8), the profits can be expressed with a single choice variable. Let $s_i = s_{ij}+s_{ik}$ $\forall i,j,k = \{A,B,C\}$. Then the maximization problem can be rewritten as follows

$$\underset{\{s_i\}}{Max}\ \pi_i^{NC}(\overrightarrow{s}) = \frac{(5t+9\Gamma_i)^2}{675t} - a(s_i)^2 \tag{2.12}$$

with $\Gamma_i(\overrightarrow{s},\alpha,\theta) = (\alpha+2\theta)(s_i) - (2\alpha+\theta)(s_{ji}+s_{ki}) - (\theta-\alpha)(s_{jk}+s_{kj})$, slightly abusing of the notation.

Lemma 3 *Under Assumption 2.1, the objective function is concave in s_i for all s_i and any $s_{ji}, s_{ki}, s_{jk}, s_{kj}$.*

The optimal s_i is determined by solving the maximization problem. Advertising decisions are strategic substitutes. Formally,

$$s_i(s_{ji}, s_{ki}, s_{jk}, s_{kj}) = \frac{(\alpha+2\theta)(5t - 9[(2\alpha+\theta)(s_{ji}+s_{ki}) + (\theta-\alpha)(s_{jk}+s_{kj})])}{75at - 9(\alpha+2\theta)^2}$$

For simplicity, we focus on equilibrium where the two components of $s_i, \forall i = \{A,B,C\}$, are identical. We denote the class of equilibria by S, where $S = \{s_{ij}, s_{ik} \mid s_{ij} = s_{ik}, \forall i,j,k = \{A,B,C\}\}$. The equilibrium in the class of S is unique and symmetric.

Under the non-cooperative scenario we face a multiplicity of equilibria[9]. For simplicity, we will analyze the symmetric one, where all firms choose identical (yet strictly positive) advertising intensities, hence the market is equally shared among firms in terms of demand.

Proposition 3 *Under the non-cooperative scenario, if the first-stage the equilibrium advertising is symmetric, it is characterized by $s_{ij}^{*NC} = \frac{\alpha+2\theta}{30a}$ $\quad \forall i,j = \{A,B,C\}$.*

Remark 1 *In equilibrium, prices of all firms are identical to those in the standard circular city model without advertising ($p_i^* = \frac{t}{9} + c$). They are not affected neither by*

[9] See Proof of Proposition 3.

advertising cost nor by advertising intensities which is due to the fact that $\Gamma_i(\overrightarrow{s^*}) = 0$ *in this case.*

When competing firms are symmetric, comparative advertising is harmful for firms, although necessary. It does not provide any increase in market share or profits. Since an increase in the scale of advertising does not influence demand nor prices, it actually results in lower profits. This scenario is reminiscent of the Prisoner's Dilemma. The equilibrium advertising efforts of all firms offset each other and do not change consumer's purchase decision. Hence, comparative advertising proves a wasteful activity, yet costly. Therefore, all firms would like to commit to zero advertising. Indeed, zero advertising would be socially optimal.

The equilibrium advertising levels decrease monotonically with the cost of advertising: $\frac{\partial s_{ij}}{\partial a} < 0$. Moreover, as one would expect, the equilibrium profits increase with a. Hence, more expensive advertising helps firms to commit not to bash each other. The increase in cost of advertising has three effects on firms profits: a direct effect where, ceteris paribus, any given advertising level is more expensive which brings the profits down, and two indirect effects: one which gives firms incentives to advertise less which in turn increases firms' profits and another where higher cost of advertising makes a firm's rival advertise less against the firm, hence the firm responds with a lower advertising intensity as well, since advertising volumes are strategic substitutes. It turns out that the indirect effects dominate the direct one.

Remark 2 *If we depart slightly from the symmetric position of firms, then the above effects should remain, by continuity.*

2.4.2 Collusion of all firms

We now consider the possibility of collusion in the first stage of the game, that is firms decide mutually only upon advertising efforts. Since comparative advertising constitutes Prisoner's Dilemma for the symmetric firms, it is not surprising that if all firms collude, they set the advertising intensities at zero, i.e. $\forall i, j = \{A, B, C\} : \quad s_{ij}^{*FC} = 0$, where the superscript FC stands for the "full collusion scenario". As a result, the impact of advertising on price competition is suppressed due to the fact that $\Gamma_i(\overrightarrow{s^{*FC}} = \overrightarrow{0}) = 0$.

Proposition 4 *If all firms collude on advertising, they set the advertising intensities to zero and price competition is not affected by advertising. The equilibrium price outcome is given by* $\forall i = \{A, B, C\}: \quad p_i^{*FC} = \frac{t}{9} + c.$

Each firm's profits under collusion are higher than in the non-cooperative scenario, which is due to cost reductions resulting from suppressing the wasteful yet costly advertising.

2.4.3 Collusion between firms A and B

Recalling from the introduction the motivation to study partial collusion, we introduce asymmetry to the problem by assuming that firms A and B collude, whereas firm C is the "outsider". This allows us to fully exploit the nature of directionality of comparative advertising. In this case the colluding firms seek to maximize the sum of their profits which is equally shared afterwards. Note that the second stage of this game is identical to its counterpart under non-cooperative scenario, since in both cases prices are competitively chosen.
The joint objective function of colluding firms at stage 1 is given by:

$$\begin{aligned}\underset{\{s_{AB},s_{AC},s_{BA},s_{BC}\}}{Max} \pi_{AB}^{PC}\left(\overrightarrow{s}\right) &= \pi_A^{NC}\left(\overrightarrow{s}\right) + \pi_B^{NC}\left(\overrightarrow{s}\right) \\ &= \frac{(5t+9\Gamma_A)^2}{675t} + \frac{(5t+9\Gamma_B)^2}{675t} - a(s_{AB}+s_{AC})^2 - a(s_{BA}+s_{BC})^2,\end{aligned} \tag{2.13}$$

where the superscript PC stands for *partial collusion*.
Firm C's objective function is the same as under the non-cooperative scenario (see section 2.4.1).
Assumption 2.1 ensures that the collusive profits per firm are higher than the profits under non-cooperative scenario.
By inspection of the best response functions, we observe that the advertising decisions are strategic substitutes. Formally,

$$
\begin{aligned}
s_{AC}(s_{BC}, s_{CA}, s_{CB}) &= \frac{5t(2\alpha+\theta) - 9[2(\alpha+2\theta)(\theta-\alpha)s_{BC} + (\alpha^2+7\alpha\theta+\theta^2)s_{CA} + (\alpha-\theta)^2 s_{CB}])}{75at - 9(2\alpha^2+2\alpha\theta+5\theta^2)} \\
s_{BC}(s_{AC}, s_{CA}, s_{CB}) &= \frac{5t(2\alpha+\theta) - 9[2(\alpha+2\theta)(\theta-\alpha)s_{AC} + (\alpha-\theta)^2 s_{CA} + (\alpha^2+7\alpha\theta+\theta^2)s_{CB}])}{75at - 9(2\alpha^2+2\alpha\theta+5\theta^2)} \\
s_{CA}(s_{AC}, s_{BC}, s_{CB}) &= \frac{5t(\alpha+2\theta) - 9(2\alpha+\theta)(\alpha+2\theta)(s_{AC}+s_{BC}) + (-75at+9(\alpha+2\theta)^2)s_{CB})}{75at - 9(\alpha+2\theta)^2}, \\
s_{CB}(s_{AC}, s_{BC}, s_{CA}) &= \frac{5t(\alpha+2\theta) - 9(2\alpha+\theta)(\alpha+2\theta)(s_{AC}+s_{BC}) + (-75at+9(\alpha+2\theta)^2)s_{CA})}{75at - 9(\alpha+2\theta)^2}.
\end{aligned}
$$

The equilibrium intensities of comparative advertising are given by:

$$
\begin{aligned}
s^{*PC}_{AB} &= s^{*PC}_{BA} = 0, \\
s^{*PC}_{AC} = s^{*PC}_{BC} &= \frac{(2\alpha+\theta)}{30a} \cdot \frac{50at - 9(\alpha+2\theta)^2}{\left(25at - 3(5\alpha^2+8\alpha\theta+5\theta^2)\right)}, \\
s^{*PC}_{CA} = s^{*PC}_{CB} &= \frac{(\alpha+2\theta)}{30a} \cdot \frac{25at - 9(2\alpha+\theta)^2}{\left(25at - 3(5\alpha^2+8\alpha\theta+5\theta^2)\right)}.
\end{aligned}
\tag{2.14}
$$

Remark 3 *Note that comparative advertising gives rise to a* ***negative externality****: whenever firm A advertises against firm B, then firm C's demand is affected as well. Hence, when a colluding firm advertises against her partner, she in fact hurts the partner, helping at the same time the rival. On the other hand, comparative advertising creates also a* ***positive externality****, that comes from the "directionality" of advertising: whenever firm A advertises against firm C, then firm B's demand is affected as well. Hence, when a colluding firm advertises against the outsider, she hurts the outsider, helping at the same time her partner.*

Colluding firms internalize the negative externality and suppress their mutual bashing, hence $s^{*PC}_{AB} = s^{*PC}_{BA} = 0 < s^{*NC}_{AB} = s^{*NC}_{BA}$.

The volumes of advertising attacks by both colluding firms against the outsider are identical, which is due to the fact that one of the components of the joint advertising cost function is $a(s^2_{AC}+s^2_{BC})$.

The outsider's best response to symmetric advertising attacks is to share the advertising efforts equally among the colluding firms.

The next lemma establishes the comparative statics on the differences of advertising intensities *among firms* under the partial collusion.

Lemma 4 *In the partial collusion scenario:*
(i) *the colluding firms suppress their mutual advertising efforts due to the internalization of the negative externality of advertising, formally:* $s_{AB}^{*PC} = s_{BA}^{*PC} = 0$,
(ii) *when* α *is relatively large* [10], *then each colluding firm advertises more against the outsider than the outsider advertises against each colluding firm, formally:* $s_{AC}^{*PC} > s_{CA}^{*PC}$*; in contrast,when* α *is relatively small* [11], *then the outsider advertises more against each colluding firm than each one advertises against the outsider, formally:* $s_{AC}^{*PC} < s_{CA}^{*PC}$.

The following lemma establishes the comparative statics on the differences of advertising intensities of firms *across scenarios.*

Lemma 5 *In comparison to the non-cooperative scenario, in the partial collusion case we have that:*
(i) $s_{AC}^{*PC} < s_{AC}^{*NC}$, *i.e. each colluding firm advertises less against the outsider than in the non-cooperative case,*
(ii) $s_{CA}^{*PC} < s_{CA}^{*NC}$ *when* α *is relatively large (see footnote 11), and* $s_{CA}^{*PC} > s_{CA}^{*NC}$ *when* α *is relatively small (see footnote 12), i.e., the outsider advertises less (resp. more) against each colluding firm than in the non-cooperative case if* α *is relatively large (resp. small).*

Note that s_{AC}^{*PC} is inferior to s_{AC}^{*NC} regardless of whether α is large or small (see footnotes 11 and 12, respectively). This means that under the PC scenario, the colluding firm advertises less against the outsider than she would do under the NC scenario. There are two effects responsible for this result: the **directionality effect** and the **price effect**, that work in opposite directions when determining s_{AC} under the PC scenario. The directionality effect is the pure partial collusion effect. The firm A internalizes the positive externality that helps her partner, and accordingly she has incentives to increase s_{AC}. This effect is amplified with α, which is due to the productivity effect of advertising (the more derogative advertising, the more effective, and the more incentives to use it).

[10] Specifically, it holds when $\alpha \epsilon (\frac{3\sqrt{2}-2}{7}\theta, \theta)$.
[11] Specifically, it holds when $\alpha \epsilon (0, \frac{3\sqrt{2}-2}{7}\theta)$.

To understand the price effect, we calculate the optimal s_{AC} in a case when prices are fixed at the level of NC equilibrium, where prices do not affect the advertising levels. Formally, we obtain:

$$\forall i, j, k = \{A, B, C\}: \qquad s_{AC}^{*PC}(p_i^{*NC}) = \tfrac{2\alpha+\theta}{12a} > s_{AC}^{*NC}(p_i^{*NC}).$$

This means that price competition makes s_{AC}^{PC} go down. The strategic effect on price game works as follows: advertising by firm A would make the rival decrease her price. Formally,

$$\tfrac{\partial p_C(\overrightarrow{s^*})}{\partial s_{AC}} < 0, \quad \text{which is due to:} \quad \tfrac{\partial \Gamma_C(\overrightarrow{s^*})}{\partial s_{AC}} < 0 \ \text{ and } \ \tfrac{\partial p_C(\overrightarrow{s^*})}{\partial \Gamma_C} > 0.$$

It turns out that the strategic effect on price dominates the directionality effect, and altogether firm A decreases s_{AC} under the PC scenario, with respect to the NC scenario. The direction of the total effect is the same when α is large and when α is small. However, the strength of the total effect changes with α. Since the directionality effect is amplified with α, the total effect is stronger when α is small. Accordingly, the decrease in s_{AC} under the PC scenario is larger when α is small.

The same analysis applies to the other colluding firm, i.e. firm B.

Note that s_{CA}^{*PC} is inferior to s_{CA}^{*NC} when α is large (see footnote 11), but is superior to s_{CA}^{*NC} when α is small (see footnote 12). This means that under the PC scenario, the outsider advertises less when α is large and advertises more when α is small. There are two effects responsible for this result: the **strategic advertising effect** and the **price effect**, that work in opposite directions when determining s_{AC} under the PC scenario. The strategic advertising effect comes from the fact that advertising decisions are strategic substitutes. Formally,

$$\tfrac{\partial s_{CA}}{\partial s_{AC}} < 0.$$

We know from the previous paragraph that s_{AC} decreases under the PC scenario, but it decreases more when α is small. Hence, the outsider will increase s_{CA} under the PC scenario, but she will increase it more when α is small. This effect is thus trimmed down with α.

To understand the price effect, we calculate the optimal s_{CA} in a case when prices are fixed at the level of NC equilibrium, where prices do not affect the advertising levels. Formally, we obtain:

$$\forall i,j,k=\{A,B,C\}: \qquad s_{CA}^{*PC}(p_i^{*NC})=\tfrac{\alpha+2\theta}{24a}>s_{CA}^{*NC}(p_i^{*NC}).$$

This means that price competition makes s_{CA}^{PC} go down. The strategic effect on price game works as follows: advertising by firm C would make the rival decrease her price. Formally,

$$\tfrac{\partial p_A(\vec{s^*})}{\partial s_{CA}}<0, \quad \text{which is due to:} \quad \tfrac{\partial \Gamma_A(\vec{s^*})}{\partial s_{CA}}<0 \;\text{ and }\; \tfrac{\partial p_A(\vec{s^*})}{\partial \Gamma_A}>0.$$

It turns out that when α is large, the strategic effect on price dominates, since the strategic advertising effect is relatively weak. Hence, when α is large, the outsider decreases s_{CA} under the PC scenario. In contrast, when α is small, the strategic advertising effect is amplified and dominates the price effect. Hence, when α is small, the outsider increases s_{CA} under the PC scenario.

The direction of the total effect changes with α. When α is small, the strength of the total effect is larger, that is, the lack of the directionality effect and the strength of the strategic advertising effect being sufficiently high change the direction of the total effect. Accordingly, s_{CA} under the PC scenario decreases when α is large and increases when α is small.

The following proposition summarizes the findings.

Proposition 5 *Partial collusion makes each colluding firm advertise less against the outsider. Two opposing factors are responsible for this result:* (i) *the directionality effect, which, due to the internalization of the positive externality of advertising, gives incentives to advertise more against the outsider, and* (ii) *the price effect, which makes colluding firms refrain from bashing the rival due to the fear of fiercer price competition. The latter effect dominates.*

In the PC scenario, the outsider, depending on the derogatory power of advertising, advertises less or more than in the NC scenario. The two opposing factors responsible for the result are: (i) *the strategic advertising effect, which gives incentives to the rival to increase advertising volumes as a response to lower advertising efforts by colluding firms, and* (ii) *the price effect, which trims down the advertising. The latter effect dominates when advertising is very derogatory, otherwise, the former effect is more important.*

It can be observed that in comparison to the non-cooperative scenario the total amount of advertising in the market decreases under partial collusion.

Regarding the division of the market, it is not equally shared among firms due to the asymmetry caused by partial collusion. The equilibrium market shares are given by:

$$\begin{aligned} q_A(\overrightarrow{s^{*PC}}) &= q_B(\overrightarrow{s^{*PC}}) = \frac{50at - 9t(\alpha + 2\theta)^2}{6\left[25at - 3(5\alpha^2 + 8\alpha\theta + 5\theta^2)\right]} \\ q_C(\overrightarrow{s^{*PC}}) &= \frac{50at - 18t(2\alpha + \theta)^2}{6\left[25at - 3(5\alpha^2 + 8\alpha\theta + 5\theta^2)\right]} \end{aligned} \tag{2.15}$$

From the expressions of market shares by inspection we see that the parameter α is crucial for comparative statics of the market shares of different firms under the partial collusion scenario and the comparison of market shares among scenarios. The next proposition summarizes the results.

Proposition 6 *In the partial collusion scenario, when α is relatively large (see footnote 11), then the market share of each colluding firm is superior to the one of the outsider. In contrast, when α is relatively small (see footnote 12), then the outsider steals market shares from both colluding firms.*

Corollary 1 *In the partial collusion scenario, when α is relatively large, then each colluding firm (the outsider) has a higher (lower) market share than in the non-cooperative scenario, where all market shares were equal. The opposite happens when α is relatively small.*

The price equilibrium outcome

The asymmetric advertising intensities make the auxiliary expression $\Gamma_i(\overrightarrow{s^{*PC}})$ different from zero, hence different from the outcome in the symmetric scenarios (when none or when all firms colluded). This means that under partial collusion, price competition is affected by advertising.

The equilibrium prices are given by $p_i^{*PC} = p_i^{*PC}(\overrightarrow{s^{*PC}})$. The explicit formulae are given next.

$$\begin{aligned} p_A^{*PC} &= p_B^{*PC} = \frac{50at(9c+t) - 54c(5\alpha^2 + 8\alpha\theta + 5\theta^2) - 9t(\alpha + 2\theta)^2}{18\left[25at - 3(5\alpha^2 + 8\alpha\theta + 5\theta^2)\right]}, \\ p_C^{*PC} &= \frac{50at(9c+t) - 54c(5\alpha^2 + 8\alpha\theta + 5\theta^2) - 18t(2\alpha + \theta)^2}{18\left[25at - 3(5\alpha^2 + 8\alpha\theta + 5\theta^2)\right]}. \end{aligned} \tag{2.16}$$

Proposition 7 *Under partial collusion, price competition is affected by advertising which is due to asymmetric advertising volumes in equilibrium. Moreover, the degree of comparativeness of advertising, measured by the shift parameter α, has an impact on equilibrium prices.*

Recall that the expressions for stage 2 equilibrium prices are given by: $p_i^{*PC} = \frac{t}{9} + c + \frac{\Gamma_i(\vec{s})}{5}$. For purposes of comparative statics, it is sufficient to analyze the values of the expressions $\Gamma_i(\grave{s})$ to report the impact of partial collusion on the equilibrium prices[12].
The equilibrium values of the auxiliary expressions $\Gamma_i(\overrightarrow{s^{*PC}})$ are given by:

$$\begin{aligned} \Gamma_A(\overrightarrow{s^{*PC}}) &= \Gamma_B(\overrightarrow{s^{*PC}}) = \frac{5t(7\alpha^2 + 4\alpha\theta - 2\theta^2)}{6\left[25at - 3(5\alpha^2 + 8\alpha\theta + 5\theta^2)\right]} \\ \Gamma_C(\overrightarrow{s^{*PC}}) &= \frac{-10t(7\alpha^2 + 4\alpha\theta - 2\theta^2)}{6\left[25at - 3(5\alpha^2 + 8\alpha\theta + 5\theta^2)\right]} \end{aligned} \tag{2.17}$$

It can be observed that $sign\Gamma_A(\overrightarrow{s^{*PC}}) = sign\Gamma_B(\overrightarrow{s^{*PC}}) \neq sign\Gamma_C(\overrightarrow{s^{*PC}})$. Moreover the value of $\Gamma_C(\overrightarrow{s^{*PC}})$ is twice the value of $\Gamma_A(\overrightarrow{s^{*PC}}) = \Gamma_B(\overrightarrow{s^{*PC}})$ This finding is crucial to understand the impact of collusion on equilibrium prices.

[12] The non-cooperative equilibrium prices are given by: $p_i^{*NC} = \frac{t}{9} + c$, since $\Gamma_i^{NC}(\overrightarrow{s^{*NC}} = \overrightarrow{0}) = 0$ (see section 2.4.2)

From the above expressions by inspection we see that the relative values of α influence the price competition through changes in the expressions $\Gamma_i(\overrightarrow{s^{*PC}})$.
The following proposition recapitulates the new findings and retrieves those from the previous section.

Proposition 8 *When α is relatively large (see footnote 11), then the prices of colluding firms are superior to the one of the outsider. In contrast, when α is relatively small (see footnote 12), then the prices of colluding firms are inferior to the one of the outsider.*

Corollary 2 *In the partial collusion scenario, when α is relatively large, then the colluding firms (resp. the outsider) charge higher (resp. lower) price than in the non-cooperative scenario, where all prices were equal. The opposite happens when α is relatively small.*

The auxiliary expression $\Gamma_i(\overrightarrow{s^*})$ changes with the shift parameter α which stands for the derogatory power of comparative advertising, or the comparativeness of advertising. It implies that the fact that advertising be comparative or regular has an impact on price competition. The effect of α on equilibrium prices is indirect, since α affects directly the equilibrium advertising intensities which in turn shape the equilibrium prices through the $\Gamma_i(\overrightarrow{s^*})$ expressions. The next section analyzes the direct impact of α on equilibrium advertising levels. Here, we analyze how equilibrium prices and market shares change with α, *ceteris paribus.*
More specifically, the more derogatory advertising becomes, the higher price is set by the colluding firms and the lower by the outsider. The equilibrium market shares are affected by the parameter α in the same way. Formally,

$$\begin{aligned} \frac{\partial p_A^{*PC}}{\partial \alpha} &> 0 \\ \frac{\partial p_B^{*PC}}{\partial \alpha} &> 0 \\ \frac{\partial p_C^{*PC}}{\partial \alpha} &< 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial q_A^{*PC}}{\partial \alpha} &> 0 \\ \frac{\partial q_B^{*PC}}{\partial \alpha} &> 0 \\ \frac{\partial q_C^{*PC}}{\partial \alpha} &< 0 \end{aligned}$$

The intuition behind this finding is the following: when advertising becomes more comparative, it changes the equilibrium advertising levels in such a way that the auxiliary expression of each colluding firm $\Gamma_A(\vec{s^*})$ and $\Gamma_B(\vec{s^*})$ go up, which in turn provokes the equilibrium prices and quantities of colluding firms to increase. The effect of an increase in comparativeness of advertising on the outsider is the opposite: when α increases, it changes the equilibrium advertising levels in such a way that the auxiliary expression of the outsider $\Gamma_C(\vec{s^*})$ goes down, which in turn provokes the equilibrium price and market share of the outsider to decrease. The direction of the above derivatives remains unchanged at $\alpha = 0$. This could be interpreted in the following way. When advertising starts to be comparative, thus having a negative impact on the "criticized" firm, then the colluding firms enjoy the presence of the directionality effect.

2.5 Derogatory power of comparative advertising

The comparativeness of advertising, measured by parameter α, affects directly the advertising volumes themselves. The following figures illustrate how advertising volumes change with α under the non-cooperative scenario and under partial collusion.
Figure 2.3 demonstrates the effect of α on the mutual bashing of firms A and B. If those two firms act non-cooperatively, the volume of advertising they use against each other increases with the shift parameter α, i.e. with the derogatory power of advertising. In contrast, if they collude on advertising, they suppress mutual bashing, hence their advertising intensities do not depend on the parameter α.
Figure 2.4 depicts the impact of α on the advertising efforts of colluding firms against the outsider.
Figure 2.5 illustrates the impact of α on the advertising efforts of the outsider against the colluding firms.
Figure 2.6 renders the impact of α on the aggregate advertising efforts in the market.

Note that since we assume in the model that $\theta > \alpha$, then the horizontal limit of each figure is given by $\alpha = \theta$.

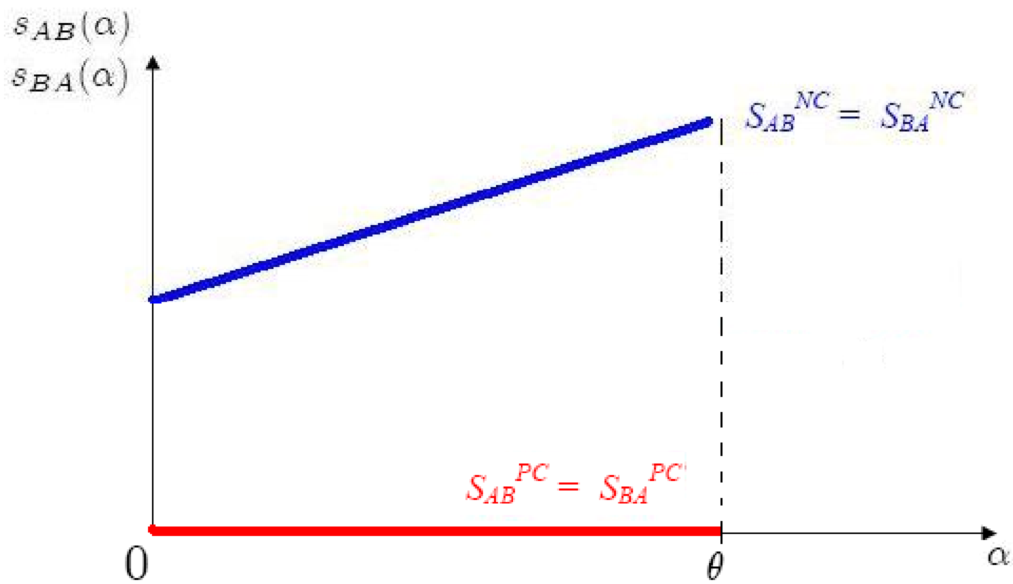

Figure 2.3: The effects of the degree of comparativeness of advertising on advertising volumes among firms A and B.

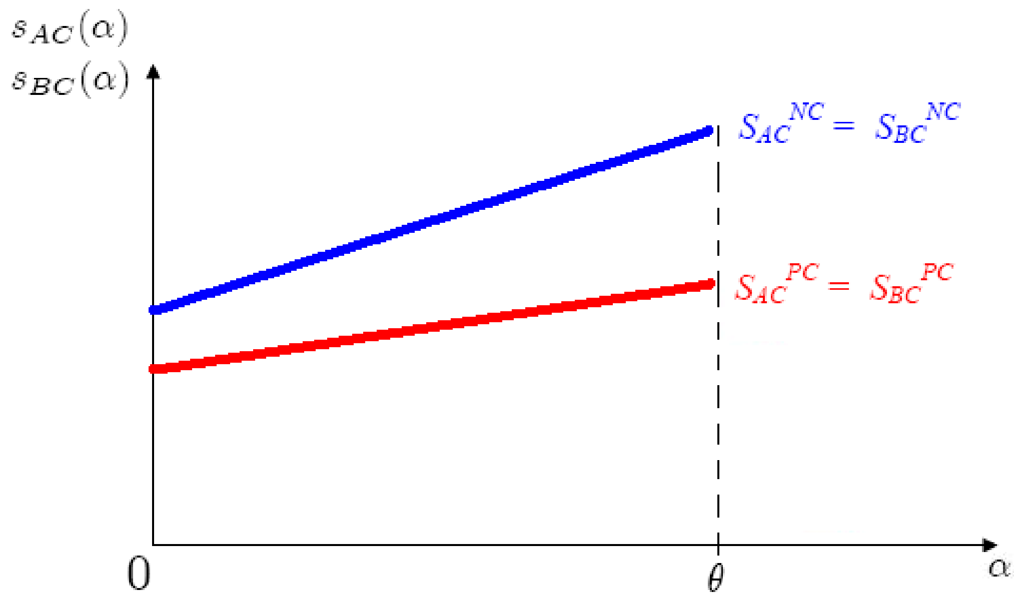

Figure 2.4: The effects of the degree of comparativeness of advertising on advertising efforts of firms A and B against firm C.

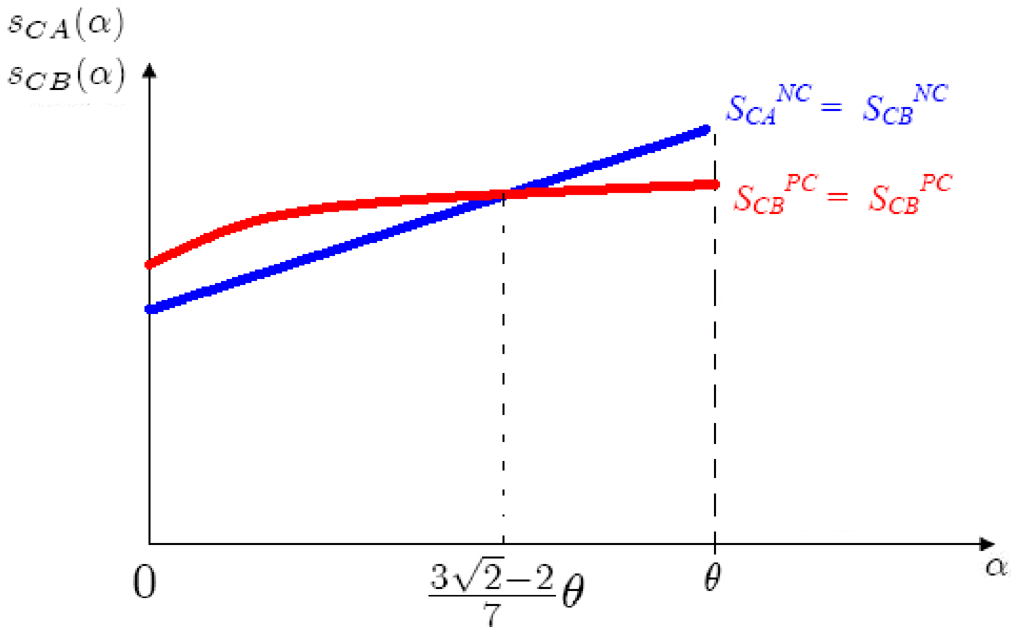

Figure 2.5: The effects of the degree of comparativeness of advertising on advertising efforts of firm C against firms A and B.

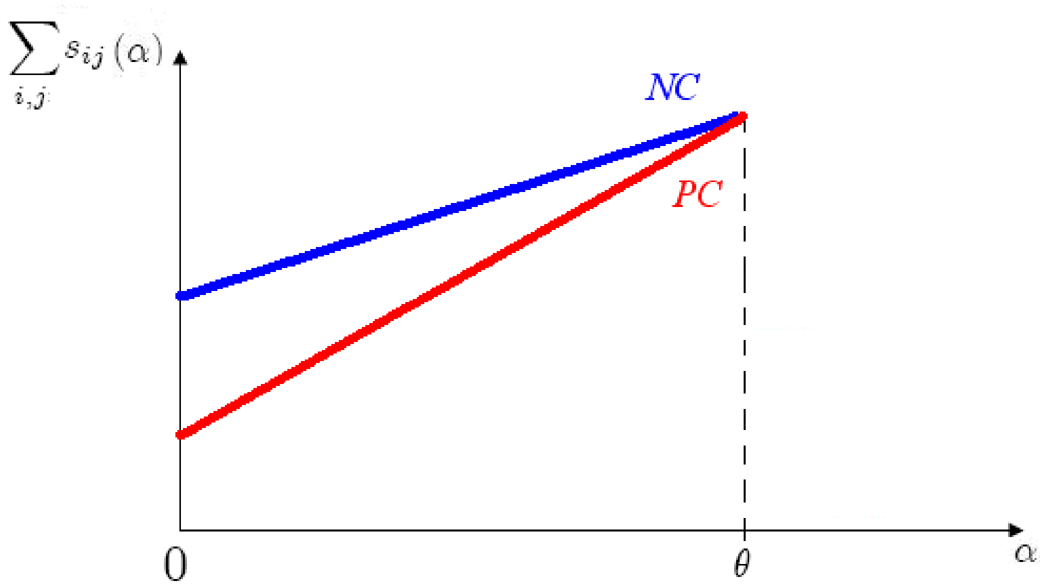

Figure 2.6: The effects of the degree of comparativeness of advertising on the aggregate advertising efforts.

The following table summarizes the impact of the derogatory power of advertising on the equilibrium outcomes of the model. In the table, by "+" we designate a positive sign, and by "-" a negative sign. The columns represent the cases of α large (see footnote 11), and α small (see footnote 12).

	α large	α small
$s_{AC}^{*PC} - s_{AC}^{*NC}$	-	-
$s_{CA}^{*PC} - s_{CA}^{*NC}$	-	+
$s_{AC}^{*PC} - s_{CA}^{*PC}$	+	-
$\sum\limits_{i,j} s_{ij}^{*PC} - \sum\limits_{i,j} s_{ij}^{*NC}$	-	- or +
$p_A^{*PC} - p_C^{*PC}$	+	-
$q_A^{*PC} - q_C^{*PC}$	+	-
$\Gamma_A(\overrightarrow{s^*})$	+	-
$\Gamma_C(\overrightarrow{s^*})$	-	+

Table 2.1: The impact of derogatory power of advertising on the equilibrium outcomes across scenarios.

2.6 Special cases: $\theta = \alpha$ and $\theta < \alpha$

For the specific case of equal shift parameters ($\theta = \alpha$) we obtain that the comparative advertising among rivals does not affect price competition, and the pro- and anti-competitive effects of advertising on prices are equal in strength, *ceteris paribus.* In the non-cooperative case all firms set identical advertising intensities and the auxiliary expression is reduced to zero, regardless of the parameters α, θ: $\Gamma_i(s_{ij} = s_{kl}, \forall i, j, k, l \mid \alpha, \theta) = 0$ $\forall \alpha, \theta$. Similarly, under full collusion $\Gamma_i(s_{ij} = 0, \forall i, j \mid \alpha, \theta) = 0 \ \ \forall \alpha, \theta$. However, the case of $\theta = \alpha$ becomes interesting under partial collusion. In fact, the case previously described as "α large" (see footnote 11) is under the scope of analysis.
For $\theta = \alpha$, we obtain the same results as in the general case of "α large", as it constitutes its limit case[13].

The specific case of shift parameters $\theta < \alpha$ means that the vicious power of comparative advertising is stronger than the regular advertising effect. The current model could

[13] See Table 2.1.

be extended to analyze such a case. However, from the auxiliary expression

$$\Gamma_i(\overrightarrow{s}, \alpha, \theta) = (\alpha + 2\theta)(s_{ij} + s_{ik}) - (2\alpha + \theta)(s_{ji} + s_{ki}) - (\theta - \alpha)(s_{jk} + s_{kj})$$

by inspection one can see that the results would be analogical to those presented in the current model, with a change of sign and direction of the effects. Hence, without loss of generality, we have focused on the case of $\theta \geq \alpha$.

2.7 Conclusions

The basic premise of our analysis is that a firm's advertising against another firm benefits the advertiser and harms the target. We claim that in a symmetric duopoly set-up with fixed market size it does not matter whether a firm mentions the rival in an advertisement or not. In both cases, the ad reduces the relative valuation of competing brand, hence a fraction of rival's consumers switches to the advertised product. For comparative advertising to be meaningful we need more than two firms. Under these conditions comparative advertising exhibits a *directionality*, i.e. a firm can choose which competitor to target.
We construct a model of comparative advertising, in which the fact that advertising be comparative rather than non-comparative does make a difference. In our model two out of three symmetric firms collude and direct their combined comparative advertising against the remaining rival. This is modelled as a two-stage game. In the first stage, firms decide how much to advertise. In the second stage, they engage in price competition.

We use this model to demonstrate how collusion affects the advertising volumes. We find that if firms do not cooperate, advertising levels are positive in equilibrium, but second-stage prices and quantities are the same as with zero advertising. Such a scenario is reminiscent of the Prisoner's Dilemma. The equilibrium advertising efforts of all firms offset each other. Hence, comparative advertising proves a wasteful activity, yet costly. If all firms collude on advertising intensities, they set advertising levels at zero and each firm gains higher profits than under the non-cooperative scenario. This is due to cost reductions resulting from suppressing the wasteful and costly advertising. We then show that if two firms collude: (i) colluding firms reduce their mutual comparative advertising, (ii) colluding firms decrease the advertising against their rival, (iii) the rival may intensify or trim down advertising efforts against colluding firms, and (iv) the total amount of advertising in the market decreases.

As for the effects of the fact that advertising be comparative rather than regular, it is clear that in the symmetric scenarios (none or all firms collude), there are no effects of advertising on equilibrium prices. Namely, no matter whether advertising is comparative or not, advertising effects are either zero (full collusion) or they offset each other (no collusion). In contrast, if we introduce asymmetry by assuming that two of the three firms collude, then the fact that advertising is comparative becomes crucial: prices change with the derogatory power of advertising. More specifically, for relatively small values of the parameter which describes the vicious power of comparative advertising and which shifts consumer's valuation downward, prices of colluding firms decrease with the derogatory power of advertising, whereas the price of the outsider goes up with the value of the parameter. For relatively large values of the parameter we obtain the opposite results.

Partial collusion makes each colluding firm advertise less against the outsider. Two opposing factors are responsible for this result: (i) the directionality effect, which, due to the internalization of the positive externality of advertising, gives incentives to advertise more against the outsider, and (ii) the price effect, which makes colluding firms refrain from bashing the rival due to the fear of fiercer price competition. The latter effect dominates.

In the PC scenario, the outsider, depending on the derogatory power of advertising, advertises less or more than in the NC scenario. The two opposing factors responsible for the result are: (i) the strategic advertising effect, which gives incentives to the rival to increase advertising volumes as a response to lower advertising efforts by colluding firms, and (ii) the price effect, which trims down the advertising. The latter effect dominates when advertising is very derogatory, otherwise, the former effect is more important.

In conclusion, we should point at some of the limiting assumptions we have made, since our results are derived in a framework that in certain respects is fairly restrictive. In particular, it has been assumed throughout that the aggregate demand is exogenously given. Clearly, an important role of advertising is to increase aggregate demand. We have assumed that total demand is not affected by the promotional spending, which only shifts market shares. In practice, promotional competition is also likely to increase the volume of total sales. Furthermore, the endogenous choice of the type of advertising stays out of the scope of the analysis. It would be interesting to reconsider the results of our model using this approach. We hope to report on the results of the extensions.

2.8 Appendix A

Explanation to Assumption 2.1

To make sure that the model is well-defined we need the following conditions to be satisfied for the equilibria under each of the three scenarios analyzed in Section 2.4:
(i) $x_i \in (0, \frac{1}{3})$,
(ii) $q_i \in (0, 1)$,
(iii) $s^*_{ij} > 0$,
(iv) $p^*_i > 0$,
(v) $\pi^*_i > 0$.■

Moreover, consider the difference in profits of firm A in the partial collusion case and in the non-cooperative case:
$\pi_A^{PC} = \frac{(25at-3(2\alpha+\theta)^2)(50at-9(\alpha+2\theta)^2)^2}{2700a\left(25at-3(5\alpha^2+8\alpha\theta+5\theta^2)\right)^2}$; $\pi_A^{NC} = \frac{(25at-3(\alpha+2\theta)^2)}{675a}$.
Hence, $\pi_A^{PC} - \pi_A^{NC} = \frac{(25at-3(\alpha+2\theta)^2)(50at-9(\alpha+2\theta)^2)^2-4(25at-3(\alpha+2\theta)^2)\left(25at-3(5\alpha^2+8\alpha\theta+5\theta^2)\right)^2}{2700a\left(25at-3(5\alpha^2+8\alpha\theta+5\theta^2)\right)^2}$.
To make sure that the collusive profits per firm are higher than the profits under non-cooperative scenario, we need that $\pi_A^{PC} - \pi_A^{NC} > 0$, that is the numerator needs to be positive. Let us rewrite the numerator as $A \cdot B^2 - 4 \cdot C \cdot D^2$, where: $A = (25at - 3(\alpha + 2\theta)^2)$, $B = (50at - 9(\alpha + 2\theta)^2)$, $C = (25at - 3(\alpha + 2\theta)^2)$, $D = \left(25at - 3(5\alpha^2 + 8\alpha\theta + 5\theta^2)\right)$.
Note that from Assumption 2.1 we have that: $A, B, C, D > 0$.
We need that $A \cdot B^2 - 4 \cdot C \cdot D^2 > 0$, which can be written as $\frac{A}{C} > (\frac{2D}{B})^2$. Hence, $\frac{2D}{B} < \sqrt{\frac{A}{C}}$, where $\frac{A}{C} > 1$, since $A > C$.
If $\frac{2D}{B} > 1$ this condition is satisfied for all parameters ranges within Assumption 2.1 (that is: $(7\alpha^2 + 4\alpha\theta - 2\theta^2) > 0$ and $50at - 9(\alpha + 2\theta)^2 > 0$, or both negative).
If $\frac{2D}{B} < 1$,then we need the condition to be satisfied in the following ranges of parameters: $(7\alpha^2 + 4\alpha\theta - 2\theta^2) > 0$ and $50at - 9(\alpha + 2\theta)^2 < 0$, or the inverse, which corresponds

to the following ranges from Assumption 2.1: if $\alpha \in (0, \frac{3\sqrt{2}-2}{7}\theta)$ and $a > \frac{9(\alpha \mid 2\theta)^2}{50t}$, or if $\alpha \in (\frac{3\sqrt{2}-2}{7}\theta, \theta)$ and $a < \frac{9(\alpha \mid 2\theta)^2}{50t}$.In this case the additional requirement is that $\frac{2D}{B} < \sqrt{\frac{A}{C}}$, that is: $\frac{2\left(25at-3(5\alpha^2+8\alpha\theta+5\theta^2)\right)}{(50at-9(\alpha+2\theta)^2)} < \sqrt{\frac{25at-3(2\alpha \mid \theta)^2}{25at-3(\alpha+2\theta)^2}}$.■

Proof of Proposition 1

To make sure that the candidate maximum defines the second stage pricing strategy in equilibrium, we verify the FOC and SOC $\forall i, j, k = \{A, B, C\}$.
FOC: $\frac{\partial \pi_i\left(\overrightarrow{p} \mid \overrightarrow{s}\right)}{\partial p_i} = \frac{[18c-36p_i+9(p_j+p_k)+2t+9\Gamma_i]}{6t} = 0.$
This implies $p_i(p_j, p_k \mid \overrightarrow{s}) = \frac{9(p_j+p_k)+2t \mid 18c \mid 9\Gamma_i}{36}$.
Solving best response functions for all firms establishes the candidate for equilibrium pricing strategy: $p_i^*(\overrightarrow{s}) = \frac{t}{9} + c + \frac{\Gamma_i}{5}$.
SOC: $\frac{\partial \pi_i^2\left(\overrightarrow{p} \mid \overrightarrow{s}\right)}{\partial p_i^2} = -\frac{6}{t} < 0$ since $t > 0$.
Hence the second stage profit function is maximized at $p_i^*(\overrightarrow{s})$ characterized above.■

Proof of Proposition 2

Fix advertising strategies $\overrightarrow{s} = \{s_{ij}, s_{ik}, s_{ji}, s_{ki}, s_{jk}, s_{kj}\}$. Given the auxiliary expression

$$\Gamma_i(\overrightarrow{s}, \alpha, \theta) = (\alpha + 2\theta)(s_{ij} + s_{ik}) - (2\alpha + \theta)(s_{ji} + s_{ki}) - (\theta - \alpha)(s_{jk} + s_{kj})$$

we have that:
$\frac{\partial \Gamma_i(\overrightarrow{s},\alpha,\theta)}{\partial(s_{ij}+s_{ik})} = \alpha + 2\theta > 0,$
$\frac{\partial \Gamma_i(\overrightarrow{s},\alpha,\theta)}{\partial(s_{ji}+s_{ki})} = -(2\alpha + \theta) < 0,$
$\frac{\partial \Gamma_i(\overrightarrow{s},\alpha,\theta)}{\partial(s_{jk} \mid s_{kj})} = -(\theta - \alpha) < 0,$ .given that $\theta > \alpha$.

We also know that prices increase monotonically in Γ_i. It implies that:
$\frac{\partial p_i}{\partial(s_{ij}+s_{ik})} = \alpha + 2\theta > 0,$
$\frac{\partial p_i}{\partial(s_{ji}+s_{ki})} = -(2\alpha + \theta) < 0,$
$\frac{\partial p_i}{\partial(s_{jk} \mid s_{kj})} = -(\theta - \alpha) < 0.$

Hence, a firm's comparative advertising is anti-competitive, whereas the advertising from rivals onto the firm and the one among rivals are pro-competitive.

Assume now that all advertising intensities are equal: $s_{ij} = s_{kl} = s, \forall i, j, k, l$. It follows that:
$\Gamma_i(s_{ij} = s_{kl} = s, \forall i, j, k, l \mid \alpha, \theta) = (\alpha + 2\theta)2s - (2\alpha + \theta)2s - (\theta - \alpha)2s = 0 \; \forall \alpha, \theta$.

It is even more evident if: $s_{ij} = 0, \forall i, j$. It follows that:
$\Gamma_i(s_{ij} = 0, \forall i, j \mid \alpha, \theta) = (\alpha + 2\theta)0 - (2\alpha + \theta)0 - (\theta - \alpha)0 = 0$.

Next consider: $s_{ij} + s_{ik} = s_{ji} + s_{ki} = s_{jk} + s_{kj} = s \;\; \forall i, j, k$. It implies that:
$\Gamma_i(s_{ij} + s_{ik} = s_{ji} + s_{ki} = s_{jk} + s_{kj} = s, \forall i, j, k \mid \alpha, \theta) = (\alpha + 2\theta)s - (2\alpha + \theta)s - (\theta - \alpha)s = 0$.

Finally, assume that $s_{ij} + s_{ik} = s_{ji} + s_{ki} = s$ and $\alpha = \theta$. It follows that:
$\Gamma_i(s_{ij} + s_{ik} = s_{ji} + s_{ki} = s, \alpha = \theta, \forall i, j, k) = 3\theta s - 3\theta s - 0(s_{jk} + s_{kj}) = 0.$■

Proof of Proposition 3

By inspection of the Γ function, given in (2.8), the profits can be expressed with a single choice variable. Let $s_i = s_{ij} + s_{ik} \; \forall i, j, k = \{A, B, C\}$. Then the maximization problem can be rewritten as follows:

$$\underset{\{s_i\}}{Max} \; \pi_i^{NC}(\overrightarrow{s}) = \frac{(5t + 9\Gamma_i)^2}{675t} - a(s_i)^2$$

with $\Gamma_i(\overrightarrow{s}, \alpha, \theta) = (\alpha + 2\theta)(s_i) - (2\alpha + \theta)(s_{ji} + s_{ki}) - (\theta - \alpha)(s_{jk} + s_{kj})$.

The first order conditions give rise to the following best response functions:

$$s_i(s_{ji}, s_{ki}, s_{jk}, s_{kj}) = \frac{(\alpha + 2\theta)(5t - 9[(2\alpha + \theta)(s_{ji} + s_{ki}) + (\theta - \alpha)(s_{jk} + s_{kj})])}{75at - 9(\alpha + 2\theta)^2}$$

which, using the expression for $\Gamma_i(\overrightarrow{s}, \alpha, \theta)$, can be written in the following way:

$$s_i = \frac{(\alpha + 2\theta)(5t - 9[\Gamma_i - (\alpha + 2\theta)(s_i)])}{75at - 9(\alpha + 2\theta)^2}$$

Note that the advertising decisions are strategic substitutes.

To make sure that the candidate maximum defines the first stage advertising strategy in equilibrium under non-cooperative scenario, we verify the SOC $\forall i, j, k = \{A, B, C\}$. It is straightforward to see that the objective function is concave in s_i for all s_i and any $s_{ji}, s_{ki}, s_{jk}, s_{kj}$ under Assumption 2.1:
SOC: $\frac{\partial \pi_i^{NC}(\overrightarrow{s})}{\partial s_i} < 0$

Solving best response functions for all firms establishes that there is a multiplicity of equilibria. We analyze the symmetric one where we impose that $s_{ij} = s_{ik}$ for all $i, j, k = \{A, B, C\}$, that is, the two components of s_i are identical.The unique candidate for equilibrium advertising strategy is given by:
$s_{ij}^{*NC} = \frac{\alpha+2\theta}{30a} \quad \forall i, j = \{A, B, C\}$.■

Proof of Proposition 6

First, we will prove that colluding first suppress their mutual bashing.
Let $\pi_{AB}^{PC}(s_{AB}, s_{AC}, s_{BA}, s_{BC} \mid s_{CA}, s_{CB})$ be the joint objective function of colluding firms under partial collusion scenario. Consider that firm A marginally decreases by ε the advertising effort s_{AB} against her partner in collusion, and transfers the effort to increase by ε the advertising volume of s_{AC} against the outsider.
Let $\pi_{AB}^{PC}(s_{AB} - \varepsilon, s_{AC} + \varepsilon, s_{BA}, s_{BC} \mid s_{CA}, s_{CB})$ be the joint objective function of colluding firms with a transfer by firm A of advertising effort from the partner towards the outsider.
Note that such a transfer results in the following changes in consumer valuations of the brands: v_A does not change, v_B goes up by $\varepsilon\alpha$ and v_C goes down by $\varepsilon\alpha$. It changes the second-stage equilibrium prices, given by $p_i(v_i, v_j, v_k) = c + \frac{t}{9} + \frac{18v_i - 9(v_j + v_k)}{45}$, in the following way: p_A does not change, p_B goes up by $\frac{3}{5}\varepsilon\alpha$ and p_C goes down by $\frac{3}{5}\varepsilon\alpha$. Moreover, it does not affect the joint cost function of colluding firms.

Now, consider the difference in profits of colluding firms under both cases (with and without the transfer):
$\pi_{AB}^{PC}(s_{AB} - \varepsilon, s_{AC} + \varepsilon, s_{BA}, s_{BC} \mid s_{CA}, s_{CB}) - \pi_{AB}^{PC}(s_{AB}, s_{AC}, s_{BA}, s_{BC} \mid s_{CA}, s_{CB}) =$
$= \frac{\varepsilon\alpha(5t \mid 9\Gamma_B)}{10t} > 0.$
The above result is positive, since: $\varepsilon > 0$, $\alpha > 0$ and we need $\Gamma_B > -\frac{5t}{9}$ so that the expression of market share of firm B is positive (see Explanation to Assumption 2.1). Hence, the colluding firms maximize the joint profit by choosing the following advertis-

ing strategy: $s^{*PC}_{AB} = s^{*PC}_{BA} = 0$. $\square$

Now, to make sure that the candidate maximum defines the first stage advertising strategy in equilibrium under partial collusion scenario, we verify the FOC and SOC of firms A and B $\forall i, j, k = \{A, B, C\}$.

FOC:

$$\frac{\partial \pi^{PC}_{AB}(\vec{s})}{\partial s_{AC}} = \frac{2[5t(\alpha+2\theta)+9(s_{AC}(2\alpha^2+2\alpha\theta+5\theta^2)+s_{BC}(2\alpha^2+2\alpha\theta-4\theta^2)+s_{CA}(-\alpha^2-7\alpha\theta-\theta^2)+s_{CB}(-\alpha^2+2\alpha\theta-\theta^2))]}{75t} - 2as_{AC} = 0$$

which establishes the s_{AC} best response function.

$$\frac{\partial \pi^{PC}_{AB}(\vec{s})}{\partial s_{BC}} = \frac{2[5t(\alpha+2\theta)+9(s_{AC}(\alpha^2+\alpha\theta-2\theta^2)+s_{BC}(2\alpha^2+2\alpha\theta+5\theta^2)+s_{CA}(-\alpha^2+2\alpha\theta-\theta^2)+s_{CB}(-\alpha^2-7\alpha\theta-\theta^2))]}{75t} - 2as_{BC} = 0$$

which establishes the s_{BC} best response function.

Firm C's objective function is identical to the one under non-cooperative scenario.

Solving best response functions for all firms establishes the candidate for equilibrium advertising strategies:.

$$s^{*PC}_{AC} = s^{*PC}_{BC} = \frac{(2\alpha+\theta)}{30a} \cdot \frac{50at-9(\alpha+2\theta)^2}{\left(25at-3(5\alpha^2+8\alpha\theta+5\theta^2)\right)},$$

$$s^{*PC}_{CA} = s^{*PC}_{CB} = \frac{(\alpha+2\theta)}{30a} \cdot \frac{25at-9(2\alpha+\theta)^2}{\left(25at-3(5\alpha^2+8\alpha\theta+5\theta^2)\right)}$$

SOC: $\frac{\partial \pi^{PC\ 2}_{AB}(\vec{s})}{\partial s^2_{AC}} = \frac{\partial \pi^{PC\ 2}_{AB}(\vec{s})}{\partial s^2_{BC}} = \frac{-50at+6(2\alpha^2+2\alpha\theta+5\theta^2)}{25at}$, $\frac{\partial \pi^{PC\ 2}_{AB}(\vec{s})}{\partial s_{AC}\partial s_{BC}} = \frac{\partial \pi^{PC\ 2}_{AB}(\vec{s})}{\partial s_{BC}\partial s_{AC}} = \frac{12(\alpha^2+\alpha\theta-2\theta^2)}{25t}$.

The determinant of the Hessian matrix in this case is given by: $\frac{4[625a^2t^2-150at(2\alpha^2+2\alpha\theta+5\theta^2)+81\theta^2(2\alpha+\theta)^2]}{625t^2}$. Note that the numerator contains an expression which is quadratic in at and has two roots: $\{\frac{3(4\alpha^2+4\alpha\theta+\theta^2)}{25}, \frac{27\theta^2}{25}\}$. One can prove that $3(4\alpha^2+4\alpha\theta+\theta^2) < 27\theta^2$ by rearranging terms the following way: $3(4\alpha^2+4\alpha\theta+\theta^2)-27\theta^2 = -12(\alpha+2\theta)(\alpha-\theta) < 0$. Hence, $det\ H > 0$ if $.at \in (0, \frac{3(4\alpha^2+4\alpha\theta+\theta^2)}{25}) \cup (\frac{27\theta^2}{25}, +\infty)$. Moreover, from the Proof of Assumption 2.1 we know that at needs to be superior to $\frac{3(\alpha^2+4\alpha\theta+4\theta^2)}{25}$, which is larger than the threshold $\frac{3(4\alpha^2+4\alpha\theta+\theta^2)}{25}$ which was mentioned above for $det\ H > 0$. Hence, the candidate for equilibrium advertising strategies in the partial collusion case maximizes profits if $at > \frac{27\theta^2}{25}$. $\blacksquare$

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Chapter 3

Naming a Rival in Informative Comparative Advertising: A Disclosure Game of Quality (and Existence)

3.1 Introduction

Informative advertising by definition is meant to inform consumers about the existence of the product, its characteristics, its price or price distribution, its location or its vendors. This promotional technique generates awareness of products and provides consumers with information on product quality. *Comparative* informative advertising conveys to consumers the information on products' existence and their qualities, by means of direct comparisons of rival brands.

We observe many examples of industries in which products are heavily advertised and yet there is very little difference in the physical characteristics of the various brands. Producers of close substitutes care for announcing any slight difference in product features (lower quality of the rival) to achieve as much differentiation as possible. This regularity concerns "incumbent" brands, i.e. brands that established their existence on the market (either historically, or through advertising) and only their true quality may be unknown to consumers. Hence, the lower the quality of the rival, the more incentives for the high quality firm to announce it. One example is in the soft drink industry, where the two market leaders, Coca-Cola and Pepsi-Cola, are involved in a long-lasting comparative advertising and marketing war. Other examples include coffee, beer, cigarettes, and detergents. In contrast, the high-quality handbag producer Louis Vuitton does not advertise comparatively against a cheap handbag imitation producer. In our model we offer an explanation of why Louis Vuitton does not advertise the lower quality of its competitor.

The existing literature on non-comparative advertising is unable to answer this question, since advertising a low quality of the rival is ruled out from the start. Grossman and Shapiro (GS 1984) consider a duopoly framework with differentiated product in which *ex ante* consumers are unaware of the existence of either product, but through firms' (non-comparative) advertising consumers are informed of their existence.[1] In further analysis we refer to the effect of revealing the existence of a brand in advertisement as the GS effect. In our model of comparative advertising the GS effect works to the advantage of the rival rather than the advertiser. That is why we refer to it as the *negative* GS effect. In Meurer and Stahl (MS 1994), consumers are informed about the existence of products, but not about the particular characteristics of different brands. (Non-comparative) advertising raises product differentiation among consumers who are informed about the products existence. We further refer to this effect as the MS effect. In this set-up, price competition is relaxed through advertising that informs consumers of product differences.[2]

What we intend to explain in our model is the empirical regularity about the lack of comparative advertising by a producer of a well-known high-quality brand against an unknown entrant producing a much lower quality brand[3]. This empirical observation raises a number of questions for the economic analysis of this type of advertising. When does the incumbent (the high quality firm) have incentives to provide information on the entrant's brand quality? In order for the incumbent to gain some fraction of consumers that thanks to the advertising realize the *true quality* of the entrant's brand (and prefer

[1] In their model the equilibrium level of advertising is positively related to the degree of product differentiation; the gain from informing and attracting an additional customer equals the markup of price over marginal cost, which is higher the more differentiated the two products are (see also Butters, 1877 and Wolinsky, 1984).

[2] The softening effect on pricing strategies and hence the incentive to advertise depends on to what extent products actually differ.

[3] We assume that the quality of the entrant is unknown to consumers either due to its short presence in the market or due to the lack of promotional activities that would inform consumers of the quality.

to consume high rather than low quality), one has to recognize that it happens at the risk of losing some consumers that might escape to the low quality firm once they learn about its *existence*. Then, what are the determinants of this trade-off?

The intuition behind the puzzle suggests that there are two effects responsible for the empirical regularity. First, comparative advertising reveals the *existence of rival brand*, which gives the incumbent incentives not to use this promotional technique. This effect is the stronger the more people are unaware of the existence of the competitor in the market. Second, comparative advertising *raises product differentiation* at the informed segment, which makes it worth it for the incumbent to announce the lower quality of the new brand. This effect is the stronger the larger the difference between the expected quality and the true quality of the rival product. Hence, in line with the example cited before, we propose the following explanation of the empirical observation. In the handbag market many consumers are unaware of the existence of a cheap and low-quality imitation of Louis Vuitton product and, more importantly, the informed consumers have expectations of the entrant quality that are quite correct, which is the reason for the lack of comparative advertising by the high-quality producer against the rival. In contrast, in the soda drinks market, the fraction of consumers who are aware of the existence of the new product is quite large, and/or the informed consumers expectations about the new product are incorrect (sufficiently high), hence it makes it profitable for the incumbent to announce the lower quality of the entrant product, even at the cost of revealing its existence.

More interestingly, we investigate the role of price competition in the model. There is the pro-competitive effect (benefits from improved consumers' information due to disclosure of the information on entrant's quality), which should increase competition and stimulate lower prices. On the other hand, there is the anti-competitive effect of advertising raising the product differentiation at the informed segment. The direction of the relationship between advertising and price competition depends on the level of differentiation between competing firms and on the fraction of consumers who are initially unaware of the existence of the entrant. We obtain that comparative advertising makes the low quality firm set a low price. Thus, the fear of fierce price competition makes the high quality producer refrain from using this promotional technique.

The role of informative advertising and its impact on price competition has been examined in detail in the economic literature, but much less attention has been devoted to understanding the role of *comparative* informative advertising. As mentioned before, Grossman and Shapiro (1984) and Meurer and Stahl (1994) represent two streams of literature on informative (though non-comparative) advertising. The two streams analyze the determinants of informative advertising separately. Butters (1977) considers informative advertising for homogenous products and examines the role of advertising as means of information on product's existence. The same role of informative advertising is further analyzed by Stegeman [1991], Robert and Stahl [1993], McAfee [1994] and others. Bester and Petrakis [1995] offer an interesting extension where all consumers are informed of the existence of firms, but they form expectation as to the price structure. The role of informative advertising as means of information on product's quality is analyzed by Rogerson [1988] who considers a model in which firms advertise prices and also select product qualities. The author finds that consumers infer quality from the advertised price and those that are more willing to pay for quality select firms that advertise higher prices. The literature on informative advertising used to signal product's quality is also well-developed[4]. All the papers cited above analyze only one role of informative advertising: the provision of information on the product's existence or the product's quality. Moreover, they all consider *non-comparative* informative advertising. In contrast, our paper analyses the trade-off which arises when both product's quality and existence is announced. The existing literature on comparative advertising does not consider this trade-off. In fact, the existing models on comparative advertising analyze its persuasive role (as opposed to informative role)[5].

The objective of this study is to understand how comparative advertising differs from non-comparative (regular) advertising in terms of implications for market equilibrium outcomes. A comparative advertisement reveals (apart from the comparison among products) information about the *existence* of the rival. This side effect of comparative advertising may produce the undesirable result of sender's consumers switching to the rival's brand. Obviously, all this is meaningful only if some consumers are initially unaware of the existence of the rival. We design a framework to explain why it is empirically common not to observe comparative advertising for very distinct goods. For this purpose we use the informative comparative advertising setting, in which firms may choose to disclose information on rival's product quality (ergo its existence as well).

The paper is organized as follows. The model and assumptions are presented in section

[4] See Kihlstrom and Riordan (1984), Milgrom and Roberts (1986), Fluet and Garella (2002).
[5] See Haller and Chakabarti (2002), Aluf and Shy (2001).

3.2. The pricing equilibrium under two possible scenarios (advertising or no advertising) is derived in section 3.3, and the advertising equilibrium is derived in section 3.4. The impact of price competition on advertising decision is analyzed in section 3.5. The impact of advertising on profits of the low quality producer is discussed in section 3.6. The analysis of market coverage is briefly developed in section 3.7. Finally, conclusion and extensions are discussed in section 3.8.

3.2 The Model

In our model on informative comparative advertising, we consider a static duopoly set-up, in which firms differ in quality of the products. Firm i produces at zero cost a good of quality q_i and sells it at price p_i, $i = L, H$, where H stands for the high quality producer and L for the low quality producer. Qualities are exogenous in the model and we assume $q_H > q_L$. The size of market is fixed. We also assume there are no repeat sales.

Population of consumers differ in their "taste for quality".[6] Consumers' preferences are described as follows: a consumer, identified by a taste parameter θ, enjoys utility $\theta q_i - p_i$ when consuming a product of quality q_i. We assume that the parameter θ is uniformly distributed between 0 and 1 (the density in the interval [0,1] is 1). Each consumer buys one unit of the good[7] whenever she derives a positive utility of consumption, otherwise a purchase does not take place:

$$U_i = \begin{cases} \theta q_i - p_i & \text{if consumer buys a product with quality } i \text{ where } i = H, L; \\ 0 & \text{if consumer does not buy.} \end{cases}$$

Demand addressed to firm H is defined by the set of consumers who maximize utility when buying product H, rather than product L or refraining from buying. Given (p_H, p_L), we denote by $\overline{\theta}(p_H, p_L)$ the marginal consumer who is indifferent between consuming either of the two products. By definition $\overline{\theta}$ satisfies $\overline{\theta}(p_H, p_L)q_H - p_H = \overline{\theta}(p_H, p_L)q_L - p_L$.

[6] In the analysis of Gabszewicz and Thisse [1979] this difference rests on differences in income. The formulation used here was proposed by Mussa and Rosen [1978].

[7] We assume that each consumer buys only one unit of the good. One may think of college education, computers, or any equipment subject to constant innovation, being the object of comparative advertising, in which case consumer purchases only one unit of the good.

Accordingly, consumers with $\theta >$ (resp. $<$) $\overline{\theta}(p_H, p_L)$ strictly prefer product H (resp. L). Some consumers could also refrain from buying at prevailing prices. In particular, we denote by $\theta_L(p_L)$ the consumer who is indifferent between buying product L and refraining from buying. She is defined as the solution to $\theta_L q_L - p_L = 0$; any consumer of type $\theta < \theta_L(p_L)$ refrains from buying. Similarly, a consumer denoted by $\theta_H(p_H)$ is indifferent between buying product H and refraining from buying. She is defined as the solution to $\theta_H q_H - p_H = 0$; any consumer of type $\theta < \theta_H(p_H)$ refrains from buying. Hence, market is not covered[8].

For simplicity, we assume that both the existence and the true quality of the high quality product are known to *all* consumers. In contrast, only some consumers are informed about the existence of brand L and the true quality of product L is not known to anyone. One can interpret the set-up as a game between an incumbent (high quality producer) and a new rival (low quality producer). However, we do not consider the entry game. Hence, there are two types of buyers. One type (fraction: γ) is initially *unaware* of the existence of the low quality brand, i.e. *only* aware of the existence (and true quality) of the high quality product, whereas the other type (fraction: $1-\gamma$) knows *also* about the existence of the low quality product, although she does not know its true quality. The expected quality of the low quality product[9] is given by q_e. We assume that the qualities are fixed and that consumers anticipate a better quality of the new product than it truly is: $q_H > q_e > q_L$. This makes comparative advertising meaningful. If q_e was inferior to q_L, then firm H would not even engage in comparative advertising.

For simplicity we only allow the high-quality firm to advertise[10] and subsequently they compete in prices. We assume that there is no cost of advertising. Regarding the content of ads, we assume that firm H provides truthful information and a detailed and complete product description in the comparison. Moreover, the amount of advertising is private information. Buyers who do not know about the existence of the new product, they will not know it unless they receive an ad, in which apart from the existence of

[8] For a complete description of possible demands resulting both under full market coverage and/or when market is not fully served, see Choi and Shin [1992], Moorthy [1988], Tirole [1988] and Wauthy [1996], which completes and amends previous results on the subject.

[9] The same logic as the one explained previously, applies when investigating the demand for product L when its quality is unknown (it suffices to replace q_L by q_e).

[10] Note that for sufficiently high value of parameter γ (when almost no-one knows about the existence of the new brand) the low-quality firm would have incentives to advertise and announce its existence at the cost of revealing its low quality as well.

the entrant's product, they also learn its true quality. Hence, we rule out the problem of signaling, since advertising in our model cannot be untruthful, but conveys complete information on products' qualities. The level of advertising is measured in terms of the fraction of buyers who receive an ad (fraction a). The parameter a can be also interpreted as the probability of comparative advertising reaching the consumer. For simplicity, we will only consider cases when $a = 0$ (no advertising) and $a = 1$ (advertising reaches all consumers).

Figure 3.1 depicts the possible actions that may take place in the game.

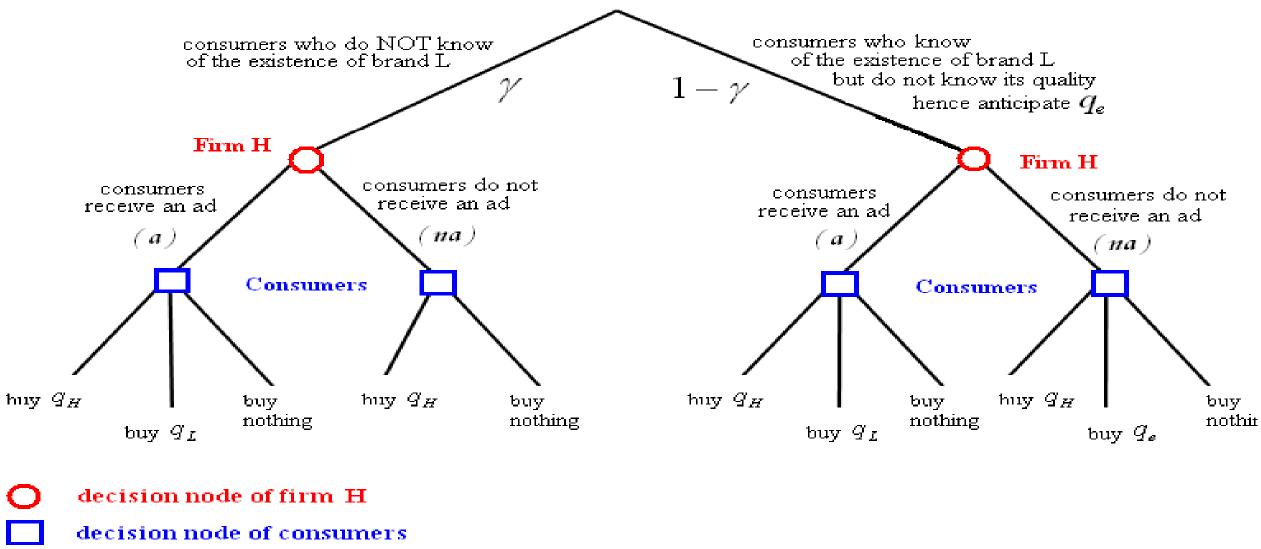


Figure 3.1: Possible actions of the game.

3.3 Price equilibrium

3.3.1 Price equilibrium with no advertising

When $a = 0$, that is when firm H decides not to advertise, then there are many possible demand configurations. We denote by D_i^{γ} the demand addressed to firm i by consumers who are unaware of the existence of firm L and by $D_i^{1-\gamma}$ the demand addressed to firm i by consumers who are aware of the existence of firm L, but who do not know her true quality (and expect the quality to be q_e). Note that the demand addressed to firm H comes from both types of consumers and the demand addressed to firm L comes only from the consumers who know of her existence (prior to any advertising), i.e., D_L^{γ} must be zero. The resulting possible demand configurations are given below, and are denoted as cases: 0 - 4. The configurations are composed of demands from consumers of type γ and consumers of type $(1-\gamma)$, as characterized before. Whenever a demand is strictly positive, we denote it by $+$, otherwise, in case of zero demand, it is denoted by zero[11].

$$\{(D_H^{\gamma}, D_H^{1-\gamma}), (D_L^{\gamma}, D_L^{1-\gamma})\} = \begin{cases} \{(0,0),(0,0)\} & \text{case 0} \\ \{(0,0),(0,+)\} & \text{case 1} \\ \{(+,0),(0,+)\} & \text{case 2} \\ \{(+,+),(0,0)\} & \text{case 3} \\ \{(+,+),(0,+)\} & \text{case 4} \end{cases}$$

where the cases are characterized by:

case 0:	$p_H \geqslant q_H,$	$p_L \geqslant q_e,$
case 1:	$p_H \geqslant q_H,$	$p_L \leqslant q_e,$
case 2:	$(q_H - q_e) + p_L \leqslant p_H \leqslant q_H,$	$p_L \leqslant q_e,$
case 3:	$p_H \in (0, q_H) \cap (0, (q_H - q_e) + p_L),$	$p_L > 0,$
case 4:	$\frac{q_H}{q_e} p_L \leqslant p_H \leqslant (q_H - q_e) + p_L,$	$p_L \leqslant q_e.$

Figure 3.2 illustrates the cases of different demand configurations.

[11] Given that $D_L^{\gamma} = 0$, in total there are eight configurations of demands $\{(D_H^{\gamma}, D_H^{1\ \gamma}), (D_L^{\gamma}, D_L^{1-\gamma})\}$, but only five of them are to be considered. The remaining three, given by $\{(0,+),(0,0)\}$, $\{(0,+),(0,+)\}$ and $\{(+,0),(0,0)\}$, cannot be and are eliminated from the analysis.

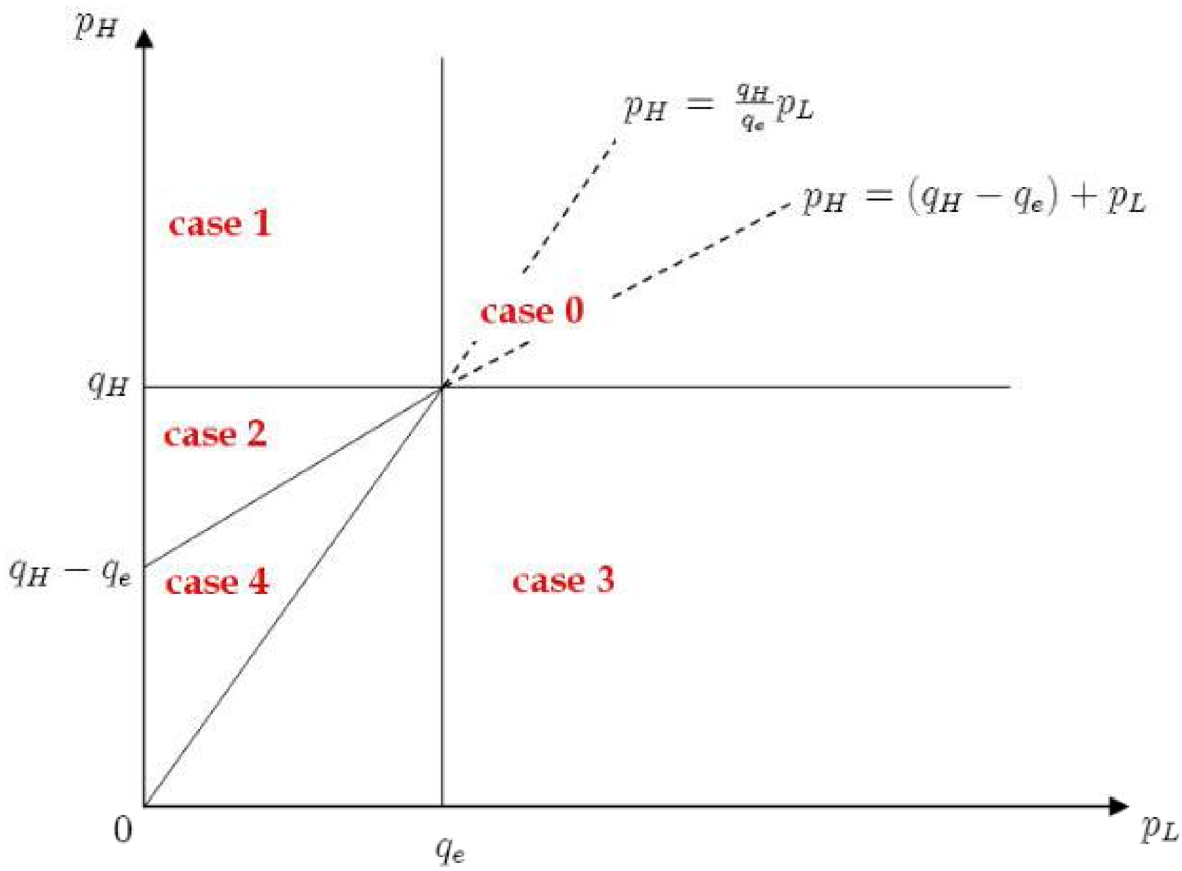


Figure 3.2: Demand configurations under no advertising.

The total demands are given below

$$\{D_H, D_L\} = \begin{cases} \{0, 0\} & \text{case 0} \\ \{0, (1-\gamma)(1-\frac{p_L}{q_e})\} & \text{case 1} \\ \{\gamma(1-\frac{p_H}{q_H}), (1-\gamma)(1-\frac{p_L}{q_e})\} & \text{case 2} \\ \{1-\frac{p_H}{q_H}, 0\} & \text{case 3} \\ \{\gamma(1-\frac{p_H}{q_H}) + (1-\gamma)(1-\frac{p_H-p_L}{q_H-q_e}), (1-\gamma)(\frac{p_H-p_L}{q_H-q_e} - \frac{p_L}{q_e})\} & \text{case 4} \end{cases}$$

Note that the optimization problem has many points of non-differentiability. Hence, our strategy will be to construct a Nash equilibrium by focusing on the equilibrium in case 4[12]. We provide the following procedure of the construction of the equilibrium.

Procedure 1

Step 1

Assume that the equilibrium belongs to case 4. This allows us to use the following formulae for demands under "no advertising" scenario:

$$\begin{aligned} D_H^{na} &= \gamma(1-\frac{p_H}{q_H}) + (1-\gamma)(1-\frac{p_H-p_L}{q_H-q_e}), \\ D_L^{na} &= (1-\gamma)(\frac{p_H-p_L}{q_H-q_e} - \frac{p_L}{q_e}). \end{aligned} \tag{3.1}$$

Step 2

Compute, using the above formulae, the constrained best response functions (constrained since they are only valid inside case 4), as well as the equilibrium candidate $\{p_H^{*na}, p_L^{*na}\}$.

Step 3

Provide conditions on parameters $\{\gamma, q_H, q_e\}$, for which the candidate falls in the interior of the set of price pairs corresponding to case 4[13].

Step 4

Find values for parameters $\{\gamma, q_H, q_e\}$, for which p_L^{*na} is a global and unique best reply to p_H^{*na} and vice versa.

Several remarks are due here. First, and most importantly, Step 3 and Step 4 will allow us to conduct local comparative statics. The idea is that a small departure from the parameters given in Step 4 will not prevent us from using the formulae used to find the equilibrium candidate in Step 2. Second, while the conditions referred to in Step 3 are general, we have not been able to provide general conditions for Step 4. Hence, we content ourselves with providing specific parameters' values for that step.

Formally, using the above procedure, we obtain the following second-stage constrained best response functions:

[12] We focus on case 4, since in this case both firms are active and the candidate best response functions meet within the area of the case for specific values of parameters. Another interesting case to analyze would be case 2, where also both firms are active. However, in this case there are no parameters' values for which the candidate best response functions meet in the area of case 2.

[13] It is not necessary to include q_L in the parametrization, because it is irrelevant under "no advertising".

$$p_H(p_L)^{na} = \frac{(1-\gamma)q_H}{2(q_H-\gamma q_e)}p_L + \frac{q_H(q_H-q_e)}{2(q_H-\gamma q_e)},$$
$$p_L(p_H)^{na} = \frac{q_e}{2q_H}p_H.$$

Hence, the Nash equilibrium candidate prices are characterized by a pair $\{p_H^{*na}, p_L^{*na}\}$ given by:

$$p_H^{*na} = \frac{2(q_H-q_e)q_H}{4q_H-(3\gamma+1)q_e},$$
$$p_L^{*na} = \frac{(q_H-q_e)q_e}{4q_H-(3\gamma+1)q_e}.$$

Remark 4 *Note that for the model to be well-defined we need that* $\frac{q_H-q_e}{4q_H-(3\gamma+1)q_e} > 0$, *which is satisfied since we have assumed that* $\gamma \in (0,1)$ *and* $q_H > q_e$.

The condition, mentioned in Step 3 of the procedure, that ensures the candidate falls in the interior of case 4 is the following: $q_H > \frac{3}{2}\gamma q_e$.

The following figure illustrates by the shaded area the parameters configurations corresponding to the condition characterized in Step 3 of Procedure 1.

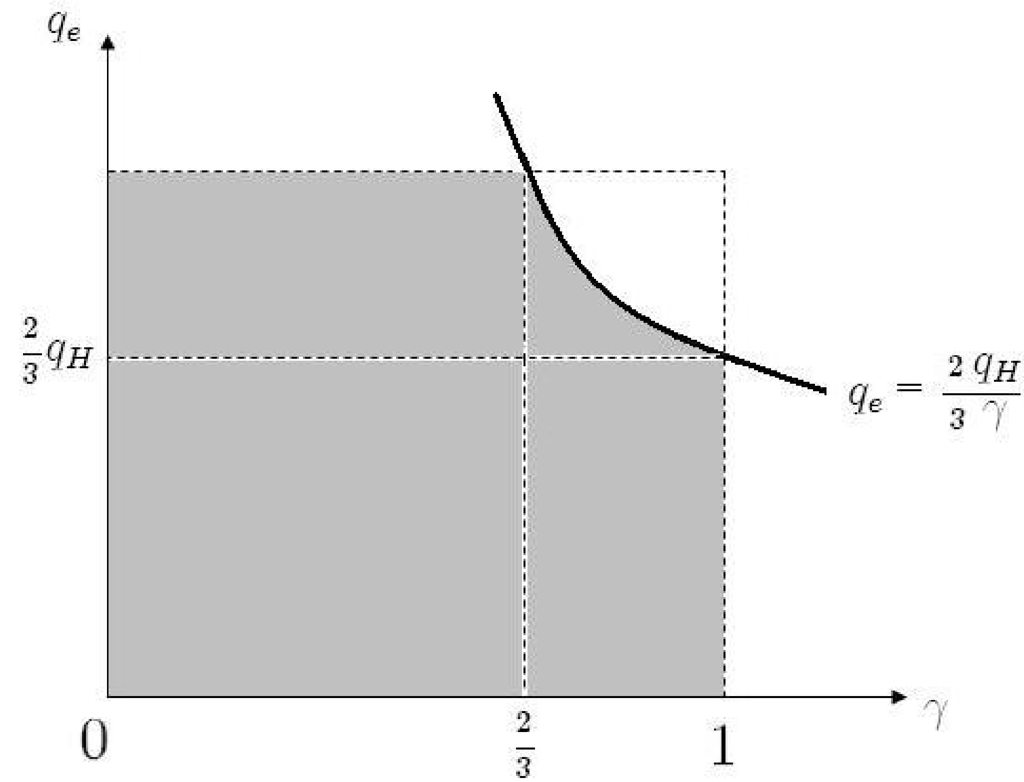


Figure 3.3: Configurations of parameters for which the equilibrium candidate falls in the interior of case 4.

In order to proceed with Step 4, we set the following values for parameters $\{\gamma, q_H, q_e\} = \{\frac{1}{2}, 1, \frac{1}{2}\}$. Note that these values satisfy the condition provided for Step 3.

The following figure shows that the candidate is indeed an equilibrium, i.e. the parameters' values we provided, make p_L^{*na} a global and unique best reply to p_H^{*na} and vice versa.

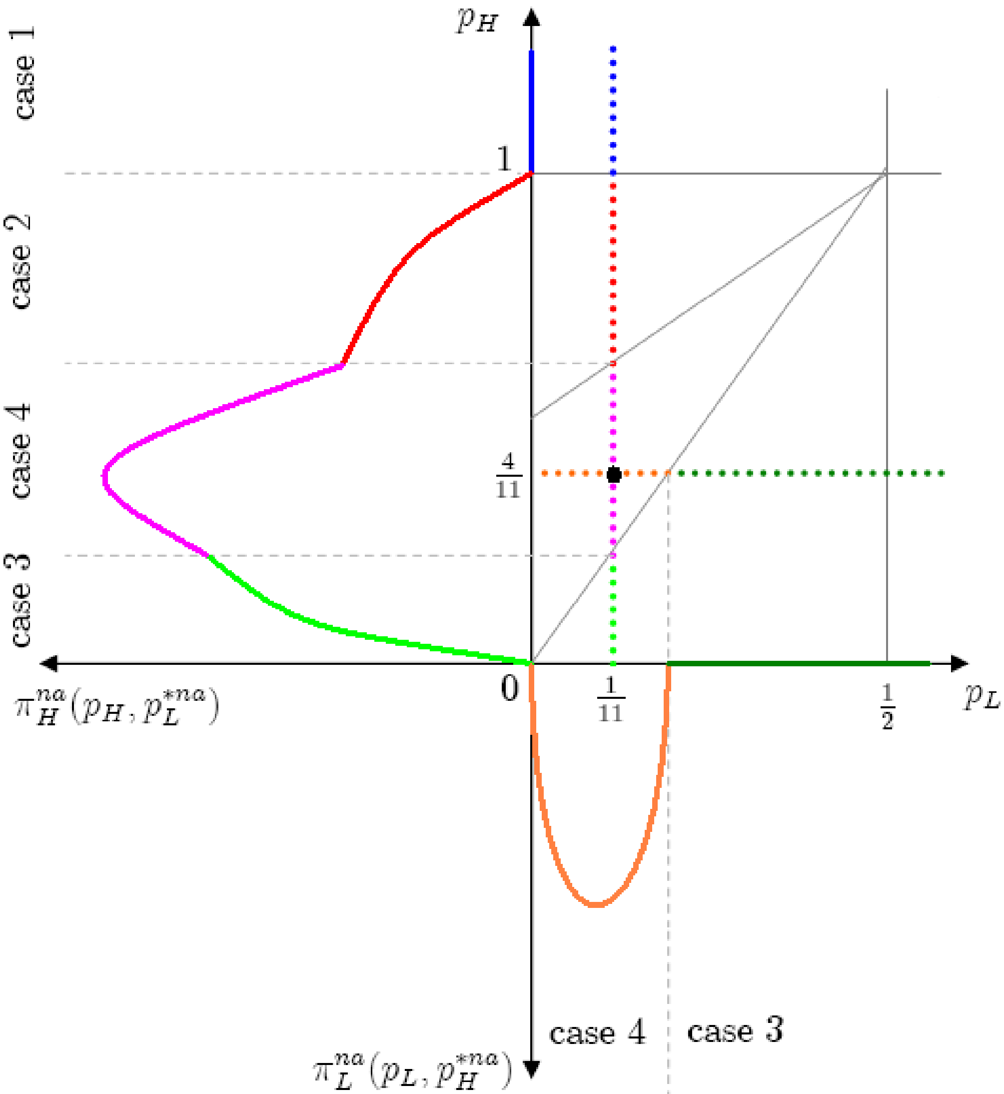


Figure 3.4: Profit functions of each firm at the optimal price of the rival.

We summarize the outcomes of the price game in the following proposition:

Proposition 9 *In the "no advertising" scenario, the constrained Nash equilibrium in prices corresponding to case 4 and the parametrization provided in Step 4 is given by:*

$$\begin{aligned} p_H^{*na} &= \tfrac{2(q_H - q_e)q_H}{4q_H - (3\gamma+1)q_e}, \\ p_L^{*na} &= \tfrac{(q_H - q_e)q_e}{4q_H - (3\gamma+1)q_e}. \end{aligned} \tag{3.2}$$

Note that the Nash equilibrium in prices and the associated market outcomes under the "no advertising" scenario are determined as a function of the degree of product differentiation (q_H, q_L), the expectation of brand L's quality q_e and the proportion of consumers who know of the existence of firm L.

The equilibrium profits are given by:

$$\begin{aligned} \pi_H^{*na} &= \frac{4(q_H - q_e)(q_H - \gamma q_e)q_H}{(4q_H - (3\gamma+1)q_e)^2}, \\ \pi_L^{*na} &= \frac{(q_H - q_e)(1-\gamma)q_e q_H}{(4q_H - (3\gamma+1)q_e)^2}. \end{aligned} \tag{3.3}$$

Remark 5 *The more optimistic consumers are regarding the quality of brand L, i.e. the more they are wrong about its true quality, the less incentives firm H has not to use advertising to announce the true quality of the rival's product:* $\frac{\partial \pi_H^{*na}}{\partial q_e} < 0$.

Remark 6 *Since the market is not fully covered: $D_H^{na} + D_L^{na} < 1$, the proportion of consumers who are not served is positive and denoted by $D_{\varnothing}^{na}$.*

3.3.2 Price equilibrium with advertising

When $a = 1$, that is, when firm H decides to advertise, all consumers get to know about the existence and the quality of brand L.

The demand addressed to firm H comes from the "captive" consumers who remained loyal to firm H after advertising and from the "enlightened" consumers who switched to firm H once they learned the true quality of the competitor, q_L.

The demand addressed to firm L comes from the "captive" consumers who remained loyal to firm L after advertising and from the "enlightened" consumers who switched from firm H to firm L once they learned of her existence.

Note that in case of advertising all consumers get informed about the products' qualities. Hence it corresponds to a not-fully-covered market configuration with perfectly informed consumers. The equilibrium of such a scenario has been provided by Wauthy [1996] who characterizes *inter alia* the demand structure and equilibrium prices of the uncovered market configuration. Henceforth, in this section we use the formulae given by Wauthy. The demands under the "advertising" scenario $\{D_H^a, D_L^a\}$ are characterized by the following functions:

$$\begin{array}{l} D_H^a = (1 - \frac{p_H - p_L}{q_H - q_L}) \\ D_L^a = (\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}) \end{array} \quad \text{if} \quad \frac{q_H}{q_e} p_L \leqslant p_H \leqslant (q_H - q_e) + p_L, \quad p_L \leqslant q_e. \tag{3.4}$$

Nash equilibrium in the price subgame is obtained by computing equilibrium candidates corresponding to the "advertising" market configuration given by the above demand functions. Formally, we obtain the following second-stage best response functions:

$p_H(p_L)^a = \frac{1}{2} p_L + \frac{(q_H - q_L)}{2}$

$p_L(p_H)^a = \frac{q_L}{2q_H} p_H$

Hence, the equilibrium prices are characterized by a pair $\{p_H^{*a}, p_L^{*a}\}$ given by:

$$\begin{aligned} p_H^{*a} &= \frac{2(q_H - q_L) q_H}{4q_H - q_L}, \\ p_L^{*a} &= \frac{(q_H - q_L) q_L}{4q_H - q_L}. \end{aligned} \tag{3.5}$$

Note that the Nash equilibrium in prices and the associated market outcomes under the "advertising" scenario are determined as a function of the degree of product differentiation (q_H, q_L). In this case neither the expectation of brand L's quality q_e nor the proportion of consumers who know of the existence of firm L play any role in the price equilibrium.

The equilibrium profits are given by:

$$\begin{aligned} \pi_H^{*a} &= \frac{4q_H^2 (q_H - q_L)}{(4q_H - q_L)^2}, \\ \pi_L^{*a} &= \frac{q_H q_L (q_H - q_L)}{(4q_H - q_L)^2}. \end{aligned} \tag{3.6}$$

Remark 7 *The higher quality brand L truly has, the lower profits firm H makes when advertising:* $\frac{\partial \pi_H^{*a}}{\partial q_L} < 0$. *Hence, when the difference between true qualities of the brands is relatively small (close substitutes), then firm H faces the risk of losing more consumers when announcing the true quality and the existence of the rival.*

Remark 8 *Since the market is not fully covered:* $D_H^a + D_L^a < 1$, *the proportion of consumers who are not served is positive and denoted by* $D_\varnothing^a$.

3.4 Advertising equilibrium

In this section, we analyze the advertising decision of firm H. We are restricted to local analysis and comparative statics, which is due to the constrained equilibrium of the "no advertising" scenario. Lemma 6 demonstrates the advertising choices of firm H both from the perspective of the levels of consumers' expectation of the entrant's quality q_e, and from the perspective of the proportion γ of consumers who are initially unaware of the existence of brand L.
We introduce symbols: $\overline{\gamma}$, where $\overline{\gamma}(q_e) = \frac{q_H(q_e - q_L)}{q_e(q_H - q_L)}$, and $\overline{q_e}$, where $\overline{q_e}(\gamma) = \frac{q_H q_L}{(1-\gamma)q_H + \gamma q_L}$, to represent the threshold for advertising decision of firm H. The formulae given to $\overline{\gamma}(q_e)$ and to $\overline{q_e}(\gamma)$ come from the same equation which designates the indifference curve of firm H in terms of advertising choices.

We fix $q_L = \frac{1}{3}$ so that the curve of indifference between "advertising" and "no advertising" decision, goes through the constrained equilibrium constructed in section 3.3.1.

Lemma 6 *For any given level* $\gamma_{\mathbf{0}}$ *of* $\gamma \in [0, 1]$ *and for any given level* $q_{e\mathbf{0}}$ *of* $q_e \in [q_L, q_H]$:
(i) if $q_e > \overline{q_e}(\gamma_{\mathbf{0}})$, *i.e. if* $\gamma < \overline{\gamma}(q_{e\mathbf{0}})$, *then* $\pi_H^{*a} > \pi_H^{*na}$,
(ii) if $q_e < \overline{q_e}(\gamma_{\mathbf{0}})$, *i.e. if* $\gamma > \overline{\gamma}(q_{e\mathbf{0}})$, *then* $\pi_H^{*a} < \pi_H^{*na}$.

The following proposition summarizes the main findings of the advertising decisions of firm H with respect to the levels of consumers' expectation of the entrant's quality q_e, and then from the perspective of the proportion γ of consumers who are initially unaware of the existence of brand L.

Proposition 10 *If more people know of the existence of brand L, or if consumers are more optimistic and anticipate a higher quality of the entrant, then the producer of high quality good has more incentives to advertise comparatively, i.e. to reveal the true quality (and existence) of the rival. On the contrary, if fewer people know of the existence of brand L, or if consumers are more pessimistic and anticipate a lower quality of the entrant, then the producer of high quality good has more incentives not to advertise comparatively, i.e. not to reveal the true quality (and existence) of the rival.*

Figure 3.5 serves only for illustration purposes. It depicts the *local* comparative statics across cases: "no advertising" and "advertising".

The curve given by $\gamma = \overline{\gamma}$, that is $q_e = \overline{q_e}$, represents a curve of indifference for firm H, where the firm obtains the same profits in both cases: if she advertises or not. Accordingly, $\pi_H^{*a} = \pi_H^{*na}$.

The region above the curve illustrates the case of $\gamma < \overline{\gamma}$, that is $q_e > \overline{q_e}$, where γ is relatively small and q_e relatively large, that is relatively many consumers know of the existence of brand L and they expect its to quality to be relatively high. In this case firm H has incentives to advertise and let those consumers know how wrong they are about the true quality of brand L. That is, $\pi_H^{*a} > \pi_H^{*na}$.

The region below the curve illustrates the case of $\gamma > \overline{\gamma}$, that is $q_e < \overline{q_e}$, where γ is relatively large and q_e relatively small, that is relatively few consumers know of the existence of brand L and they expect its to quality to be relatively small. In this case firm H does not have incentives to advertise. Hence, $\pi_H^{*a} < \pi_H^{*na}$.

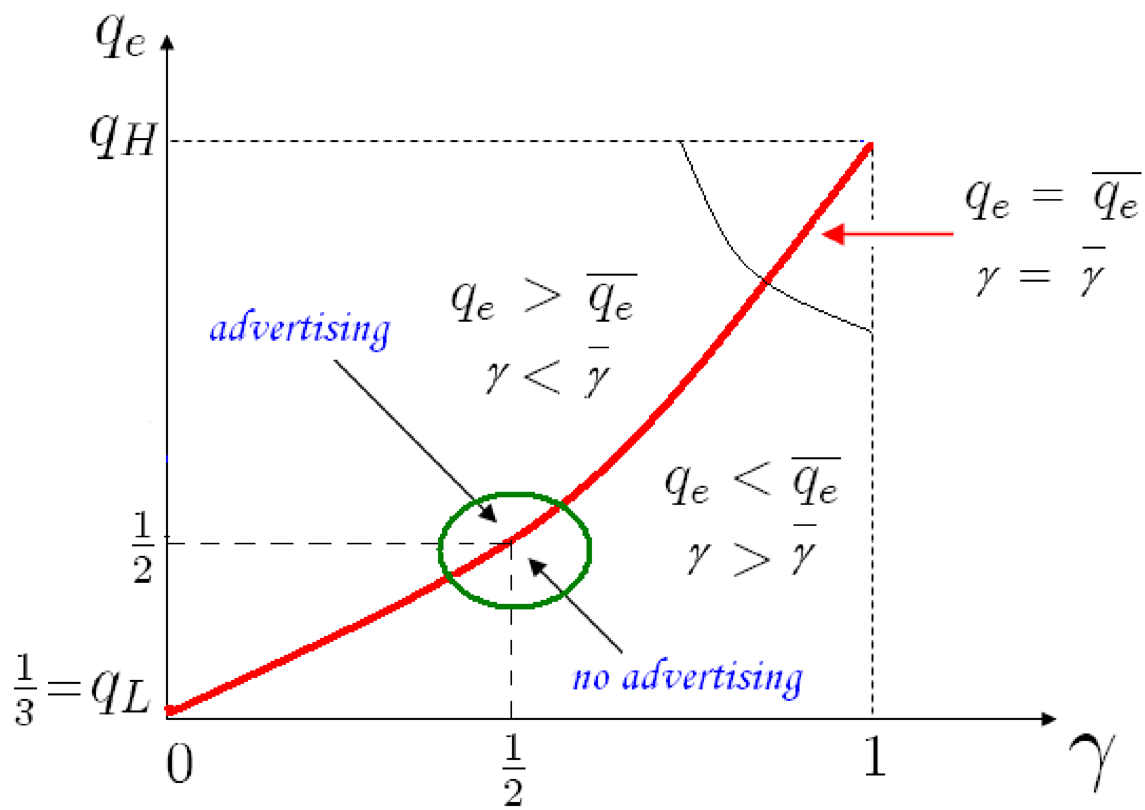


Figure 3.5: Advertising decisions of firm H.

Now we turn to a brief analysis of the impact of the parameters on the advertising decision of firm H.

Comparative advertising raises product differentiation at the informed segment, which makes it worth for firm H announcing the low quality of the new brand. This effect is the stronger the larger the difference between between q_e and q_L. Formally,

$$\frac{\partial \pi_H^{*na}}{\partial q_e} < 0.$$

The smaller the difference between true qualities (the closer substitutes the brands are), the lower profits firm H makes when advertising. Formally,

$$\frac{\partial \pi_H^{*a}}{\partial q_L} < 0.$$

On the other hand, comparative advertising reveals the existence of rival's brand which gives incentives not to use it. This effect is the stronger the larger parameter γ, i.e. the more people are initially unaware of the existence of the competitor in the market. The more people know only about firm H, *ceteris paribus*, the less the firm wants to advertise. Formally,

$$\frac{\partial \pi_H^{*na}}{\partial \gamma} > 0 \;, \frac{\partial \pi_H^{*a}}{\partial \gamma} = 0, \; \frac{\partial (\pi_H^{*a} - \pi_H^{*na})}{\partial \gamma} < 0.$$

3.5 Advertising in the absence of price competition

Under price competition, the low quality producer sets a lower price whenever her true quality is advertised, regardless the parameters of the model:

$$p_L^{*a} < p_L^{*na}. \tag{3.7}$$

On the indifference line, defined previously by $q_e = \overline{q_e}$, we have that $\pi_H^{*a} = \pi_H^{*na}$ and $p_H^{*a} = p_H^{*na}$. The fact that the low quality firm is found out under advertising (so p_L^* decreases) fully compensates the fact that, under advertising, part of the population learns of the existence of the low quality competitor. Hence, demands stay the same. In consequence, on the indifference line we have that $p_H^{*a} = p_H^{*na}$, since we are imposing $\pi_H^{*a} = \pi_H^{*na}$.

The following lemma identifies the impact of advertising choices on equilibrium prices from the perspective of the consumers' perception of the low quality brand and from the perspective of the degree of consumers' unawareness of the existence of firm L.

Lemma 7 *For any given level γ_0 of $\gamma \in [0, 1]$ and for any given level q_{e0} of $q_e \in [q_L, q_H]$:*
(i) if $q_e > \overline{q_e}(\gamma_0)$, that is, if $\gamma < \overline{\gamma}(q_{e0})$, then $p_H^{*a} > p_H^{*na}$,
(ii) if $q_e < \overline{q_e}(\gamma_0)$, that is, if $\gamma > \overline{\gamma}(q_{e0})$, then $p_H^{*a} < p_H^{*na}$.

The following proposition summarizes the main findings of the impact of advertising on price competition with respect to the levels of consumers' expectation of the entrant's quality q_e, and from the perspective of the proportion γ of consumers who are initially unaware of the existence of brand L.

Proposition 11 *Regarding the impact of comparative advertising on price competition, when more consumers know of the existence of brand L, and when consumers are more optimistic and anticipate a higher quality of brand L, then advertising makes the price of the high quality product go up. Otherwise, when fewer consumers know of the existence of brand L, and when consumers are more pessimistic and anticipate a lower quality of brand L, firm H decides not to advertise and it makes her set a higher price.*
The producer of low quality brand sets a lower price whenever her quality is advertised to consumers.

The following figure illustrates the findings identified in proposition 12.

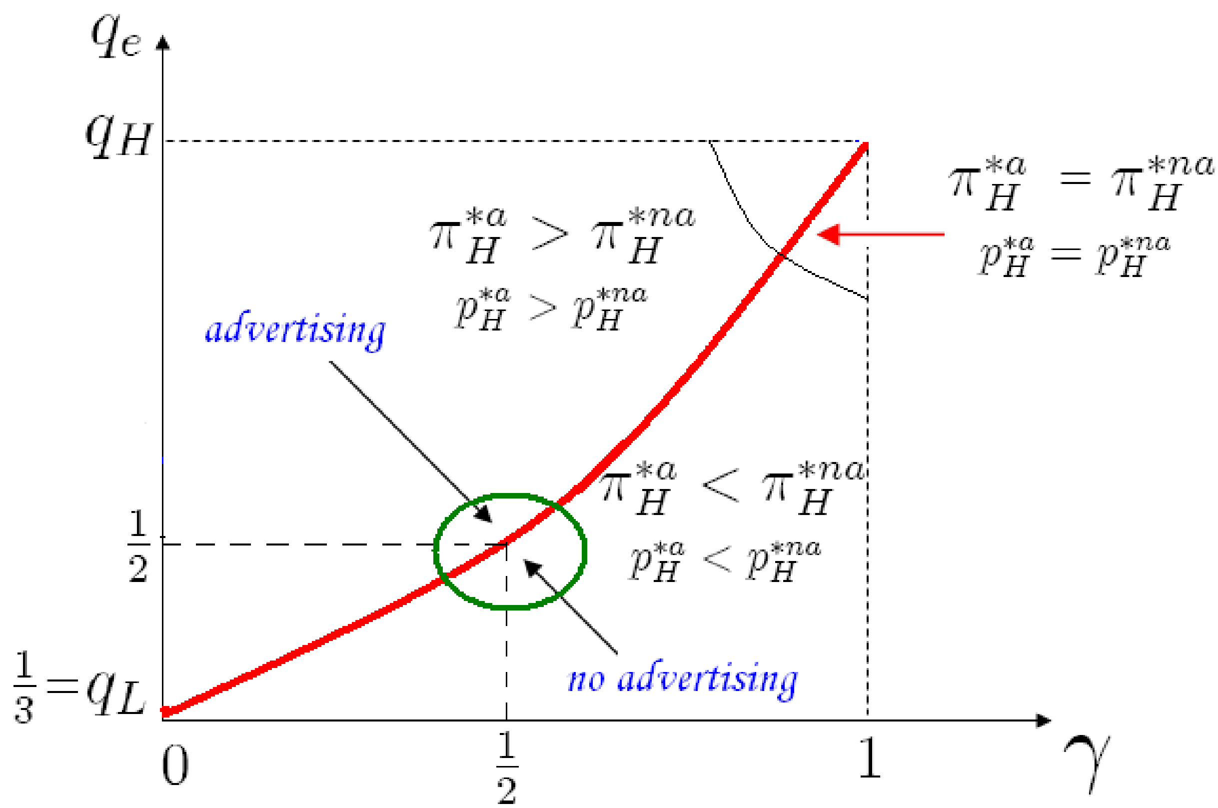


Figure 3.6: The impact of advertising on price competition.

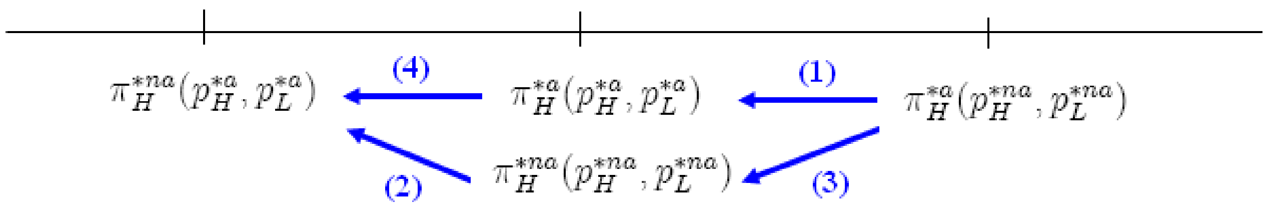
In order to single out the effect of price competition on advertising, we first fix prices at the equilibrium levels of "no advertising" scenario and we analyze whether the decision to advertise or not changes. Subsequently, we repeat the analysis for prices fixed at the equilibrium levels of "advertising" scenario. Both analysis are conducted locally for the parameter values set before.

Recall that at the constrained equilibrium under "no advertising", we have that $\pi^{*a}_H(p^{*a}_H, p^{*a}_L) = \pi^{*na}_H(p^{*na}_H, p^{*na}_L)$, that is for the parameter values chosen before along with $q_L = \frac{1}{3}$ and when prices are endogenous, firm H is indifferent between advertising and not advertising. Moreover, at the constrained equilibrium we have that $p^{*a}_H = p^{*na}_H$.

Now we turn to calculating the profits of firm H under both scenarios, first at the prices fixed at the equilibrium levels of "no advertising" scenario, $\pi_H^{*a}(p_H^{*na}, p_L^{*na})$ and $\pi_H^{*na}(p_H^{*na}, p_L^{*na})$, and then at the prices fixed at the equilibrium levels of "advertising" scenario, $\pi_H^{*a}(p_H^{*a}, p_L^{*a})$ and $\pi_H^{*na}(p_H^{*a}, p_L^{*a})$. By pairwise comparisons, we obtain the following results:

$$\begin{array}{cr}
\pi_H^{*a}(p_H^{*na}, p_L^{*na}) > \pi_H^{*a}(p_H^{*a}, p_L^{*a}) & (1) \\
\pi_H^{*na}(p_H^{*na}, p_L^{*na}) > \pi_H^{*na}(p_H^{*a}, p_L^{*a}) & (2) \\
\pi_H^{*a}(p_H^{*na}, p_L^{*na}) > \pi_H^{*na}(p_H^{*na}, p_L^{*na}) & (3) \\
\pi_H^{*a}(p_H^{*a}, p_L^{*a}) > \pi_H^{*na}(p_H^{*a}, p_L^{*a}) & (4)
\end{array}$$

The resulting order of profits of firm H and the direction of the above inequalities are illustrated in the figure below:



According to inequality (3), when prices are fixed at the equilibrium levels of "no advertising", firm H is better off advertising. Inequality (4) implies that under prices fixed at the equilibrium levels of "advertising", firm H is also better off advertising. Hence, around the equilibrium with endogenous prices, in which firm H is indifferent in terms of advertising decision, the firm prefers to advertise regardless of whether prices are fixed at the "no advertising" or "advertising" equilibrium level.

The fact that in the constrained equilibrium we obtain $p_H^{*a} = p_H^{*na}$ implies that price competition affects advertising decision via changes in p_L^*. Recall that in the constrained equilibrium we have that $p_L^{*a} < p_L^{*na}$. Inequality (1) implies that if firm H advertises, she is better off under the prices fixed at the "no advertising" equilibrium level than at the "advertising" equilibrium level. This means that a low p_L^* affects negatively the profits of firm H. Intuitively, when all consumers thanks to advertising learn the true quality of

the rival, firm L is forced to decrease her price. This makes the high quality firm worse off. Hence, refraining from advertising works as a commitment device.
The intuition behind inequality (2) is straightforward. If firm H does not advertise, she prefers prices to be fixed at the "no advertising" equilibrium level, i.e. she prefers p_L^* to be higher. Otherwise, she would loose consumers who know of the rival's existence and would be attracted by the low price of the competitor.

Proposition 12 *Comparative advertising makes the low quality firm set a low price. Thus, the fear of fierce price competition makes the high quality producer refrain from using this promotional technique.*

Hence, advertising has a positive effect of informing consumers of the true low quality of the rival, but also has two negative effects, one direct and one indirect. The direct negative effect arises since advertising informs consumers of the existence of a competitor. More interestingly, there appears an indirect effect. Namely, advertising makes the rival set a low price. Hence, some consumers may escape from the incumbent not only due to learning the rival's existence, but also due to the low price set by the competitor.

The following figure illustrates the impact of price competition on the advertising decision of firm H.

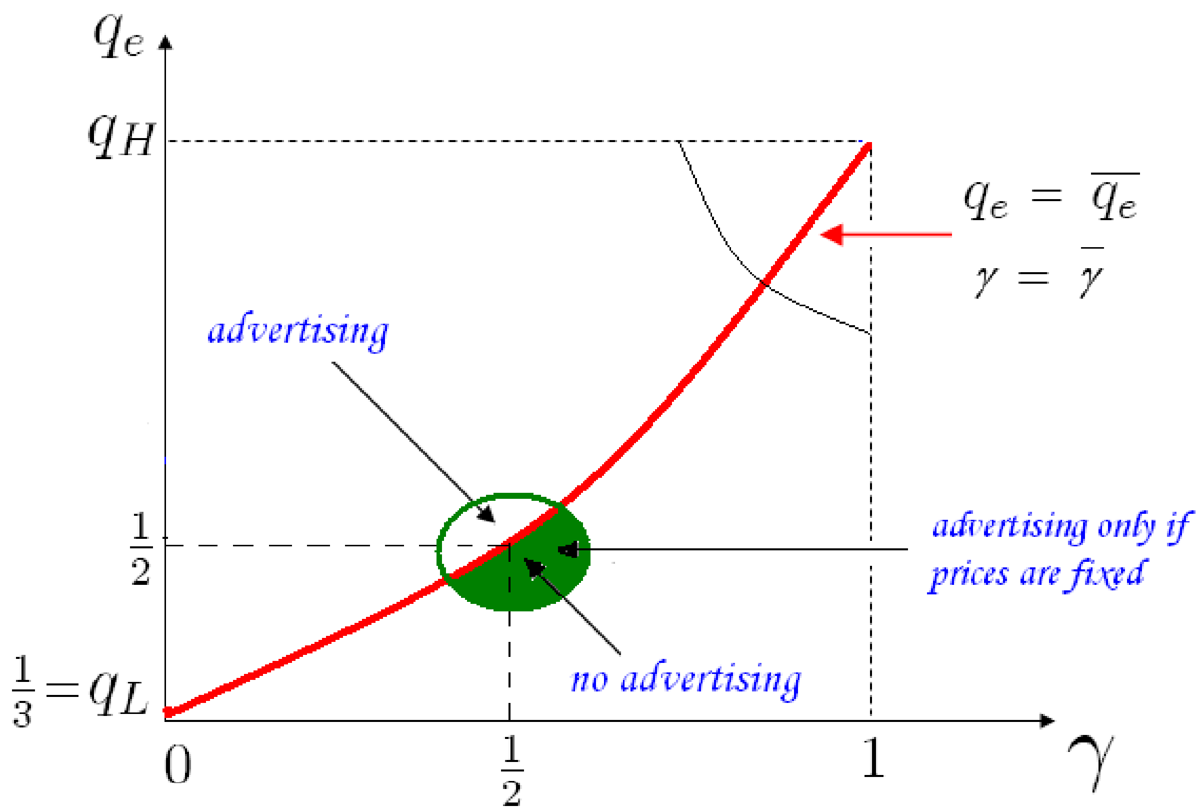


Figure 3.7: The impact of price competition on the advertising decision of firm H.

Note that in the shaded area of Figure 3.7 firm H advertises only if prices are fixed, whereas under endogenous prices she would refrain from advertising. This area can be characterized by relatively low values of q_e, and relatively high values of γ. The following proposition interprets the finding.

Proposition 13 *Price competition makes comparative advertising less attractive when consumers are rather pessimistic, and when relatively few consumers know of the existence of the low quality rival.*

Now we turn to a brief analysis of the impact of parameters on the pricing decisions of both firms.
It is intuitive that the equilibrium price of the high quality producer increases with her quality, regardless whether firm H advertises or not:

$$\frac{\partial p_H^{*na}}{\partial q_H} > 0,$$
$$\frac{\partial p_H^{*a}}{\partial q_H} > 0 \text{ and } \frac{\partial \pi_H^{*a}}{\partial q_H} > 0.$$

The direction of the latter effect can be explained by advertising raising product differentiation at the informed segment which has an anti-competitive effect on the equilibrium price.
By analogy, the equilibrium price of the low quality producer also increases with the quality of the incumbent, regardless whether firm H advertises or not:

$$\frac{\partial p_L^{*na}}{\partial q_H} > 0,$$
$$\frac{\partial p_L^{*a}}{\partial q_H} > 0 \text{ and } \frac{\partial \pi_L^{*a}}{\partial q_H} > 0.$$

The intuition behind the latter finding arises from the observation that for a given q_L, higher q_H means a larger degree of vertical differentiation in case of advertising, which leads the high quality producer to increase the price, and subsequently the producer of low quality good replies accordingly by raising her price likewise.
By analogy, the larger the true quality of brand L, the smaller the degree of vertical differentiation, hence the lower price firm H sets: $\frac{\partial p_H^{*a}}{\partial q_L} < 0$.

The proportion γ of consumers who know of the existence of the low quality producer also shapes the equilibrium prices. This effect is only tractable under the "no advertising" scenario, since in case of advertising everybody gets to know the existence of brand L and the equilibrium outcomes do not depends on this parameter.
When firm H decides not to advertise, parameter γ, that is the fraction of consumers who are unaware of brand L, is anti-competitive: the higher γ, *ceteris paribus*, the larger prices are set by both firms:

$$\frac{\partial p_H^{*na}}{\partial \gamma} > 0, \ \frac{\partial p_L^{*na}}{\partial \gamma} > 0.$$

Similarly, the consumers' expectation towards the quality of brand L determines the outcome of price competition. This effect can be analyzed only in the "no advertising" case. Otherwise, all consumers get to know via advertising the *true* quality of brand L.

In the "no advertising" case, the larger the expected quality of brand L, *ceteris paribus*, the higher price is set by firm H: $\frac{\partial p_H^{*na}}{\partial q_e} < 0$. The direction of this effect can be explained by a decline in the degree of "expected" differentiation, which makes the price of brand H go down.

However, the expected quality of brand L may increase or decrease the equilibrium price of firm L, depending on whether both brands are (truly) close substitutes or not:

$$\frac{\partial p_L^{*na}}{\partial q_e} > 0 \quad \text{if} \quad q_e \in (0, \frac{2}{(2+\sqrt{3(1-\gamma)})} q_H)$$
$$\text{or } \frac{\partial p_H^{*na}}{\partial q_e} < 0 \quad \text{if} \quad q_e \in (\frac{2}{(2+\sqrt{3(1-\gamma)})} q_H, q_H). \ .$$

3.6 The impact of advertising on the low quality producer

In the model we have assumed that qualities are exogenous. If it was not so, that is if firms were to choose quality levels in an additional stage of the game[14], then under the advertising scenario, provided the market is uncovered, an equilibrium would exhibit firm H choosing the best available quality q_H^*, and firm L would choose a fixed proportion of this quality, given by $q_L^* = \frac{4}{7} q_H^*$.

The way the equilibrium profits of firm L are shaped depends on the this threshold of q_L. We find that $\frac{\partial \pi_L^{*a}}{\partial q_L} > 0$ if $q_L \in (0, \frac{4}{7} q_H)$ or $\frac{\partial \pi_L^{*a}}{\partial q_L} < 0$ if $q_L \in (\frac{4}{7} q_H, q_H)$.

In order to represent the thresholds for changes in the equilibrium profits of firm L we introduce symbols: $\overline{\overline{\gamma}}$, where $\overline{\overline{\gamma}}(q_e) = \frac{q_e(16q_H - 7q_L) - 16(q_H - q_L)}{9q_e q_L}$, and $\overline{\overline{q_e}}$, where $\overline{\overline{q_e}}(\gamma) = \frac{16q_H(q_H - q_L)}{16q_H - (7+9\gamma)q_L}$, and we remind the thresholds identified in the previous sections: $\overline{\gamma}(q_e) = \frac{q_H(q_e - q_L)}{q_e(q_H - q_L)}$, and $\overline{q_e}(\gamma) = \frac{q_H q_L}{(1-\gamma)q_H + \gamma q_L}$. The following lemma reports the parameters constellations for which the equilibrium profits of firm L are higher or lower when her true quality is advertised.

Lemma 8 *For any given level γ_0 of $\gamma \in [0,1]$ and when $q_L \in (0, \frac{4}{7} q_H)$, then $\overline{q_e}(\gamma) < \overline{\overline{q_e}}(\gamma)$:*
*(i) if $q_e > \overline{\overline{q_e}}(\gamma_0)$, which means that consumers anticipate **higher** quality of brand L, then firm H advertises and firm L obtains higher profits: $\pi_L^{*a} > \pi_L^{*na}$,*
*(ii) if $\overline{q_e}(\gamma_0) < q_e < \overline{\overline{q_e}}(\gamma_0)$, which means that consumers anticipate an **intermediate***

[14] This result comes from Wauthy (1996).

quality of brand L, then firm H advertises and firm L obtains lower profits: $\pi_L^{*a} < \pi_L^{*na}$,
(iii) *if* $q_e < \overline{q_e}(\gamma_{\mathbf{0}})$, *which means that consumers anticipate **lower** quality of brand L, then firm H does not advertise and firm L obtains lower profits:* $\pi_L^{*a} > \pi_L^{*na}$.

For any given level $\gamma_{\mathbf{0}}$ *of* $\gamma \in [0,1]$ *and when* $q_L \in (\frac{4}{7}q_H, q_H)$, *then* $\overline{\overline{q_e}}(\gamma) < \overline{q_e}(\gamma)$*:*
(i) *if* $q_e > \overline{q_e}(\gamma_{\mathbf{0}})$, *which means that consumers anticipate **higher** quality of brand L, then firm H advertises and firm L obtains higher profits:* $\pi_L^{*a} > \pi_L^{*na}$,
(ii) *if* $\overline{\overline{q_e}}(\gamma_{\mathbf{0}}) < q_e < \overline{q_e}(\gamma_{\mathbf{0}})$, *which means that consumers anticipate an **intermediate** quality of brand L, then firm H does not advertise and firm L obtains higher profits:* $\pi_L^{*a} < \pi_L^{*na}$,
(iii) *if* $q_e < \overline{\overline{q_e}}(\gamma_{\mathbf{0}})$, *which means that consumers anticipate **lower** quality of brand L, then firm H does not advertise and firm L obtains lower profits:* $\pi_L^{*a} > \pi_L^{*na}$.

The following proposition interprets the findings identified in the previous lemma:

Proposition 14 (i) *When consumers are more optimistic and anticipate higher quality of brand L, then firm H advertises. In this case (regardless the difference in true qualities of both brands) the profits of firm L increase which is due to the market stealing effect of announcing by firm H the existence of a rival.*
(ii) *When consumers are more pessimistic and anticipate lower quality of brand L, then firm H does not advertise. In this case (regardless the difference in true qualities of both brands) the profits of firm L decrease with respect to the potential profits under advertising. Since in this case relatively fewer people know of L's existence, even though consumers are more pessimistic about her quality, firm L still would make higher profits under advertising due to revealing her existence to the vast part of the market constituted by unaware consumers.*
(iii) *When consumers anticipate an intermediate quality of brand L, then the difference in true qualities of both brands plays a role. In this case, if the true quality of brand L is low, then firm H advertises and the profits of firm L decrease which is due to the revelation of the low quality of brand L. In contrast, when true quality of brand L is high, then firm H does not advertise and the profits of firm L increase, since consumers are not so much mistaken in their expectation of the brand's quality and we assumed that* $q_e > q_L$.

The following two figures illustrate the findings.

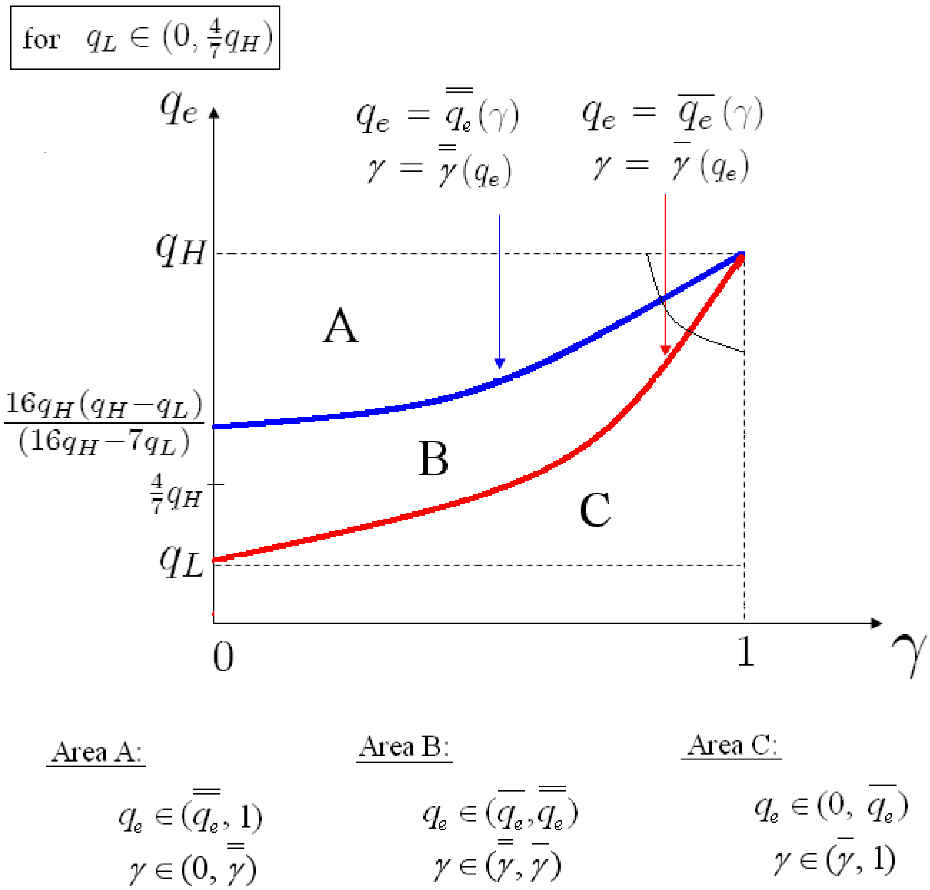


Figure 3.8: Constellations of firm L's profits when her true quality is low.

In figure 3.8 the regions indicated by letters A, B and C, have the following properties:
Area A: advertising, $\pi_L^{*a} > \pi_L^{*na}$
Area B: advertising, $\pi_L^{*a} < \pi_L^{*na}$
Area C: no advertising, $\pi_L^{*a} > \pi_L^{*na}$

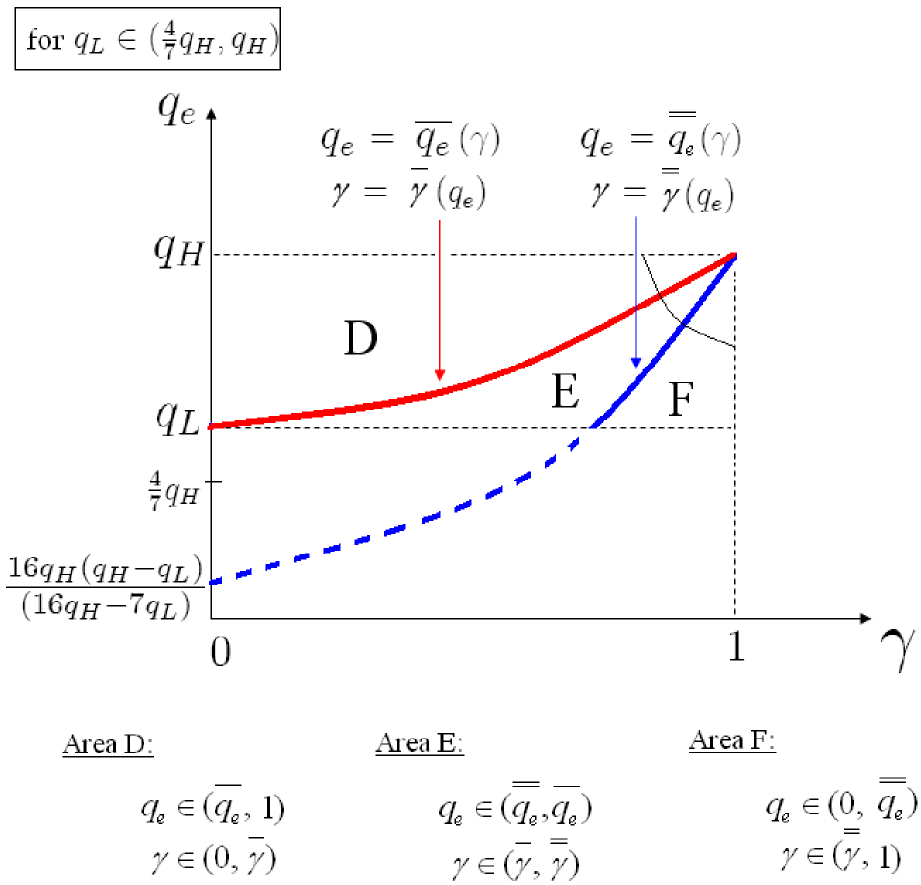


Figure 3.9: Constellations of firm L's profits when her true quality is high.

In figure 3.9 the regions indicated by letters D, E and F, have the following properties:

Area D: advertising, $\pi_L^{*a} > \pi_L^{*na}$

Area E: no advertising, $\pi_L^{*a} < \pi_L^{*na}$

Area F: no advertising, $\pi_L^{*a} > \pi_L^{*na}$

3.7 Market coverage

Since the market is not fully covered: $D_H^i + D_L^i < 1$, where index i represents the advertising scenarios $i = \{a, na\}$, the proportion of consumers who are not served is positive and denoted by $D_\varnothing^i$. It follows that the market exclusion can be analyzed as follows: $D_\varnothing^i = 1 - (D_H^i + D_L^i)$, where $i = a, na$.
In this section we investigate whether advertising entails market exclusion or not.

Lemma 9 *A simple calculation of market exclusion under each scenario implies that:*
(i) $D_\varnothing^a - D_\varnothing^{na} > 0$ *when* $\gamma < \overline{\gamma}(q_e) = \frac{q_H(q_e - q_L)}{q_e(q_H - q_L)}$ *that is when* $q_e > \overline{q_e}(\gamma) = \frac{q_H q_L}{(1-\gamma)q_H + \gamma q_L}$. *In this region of parameters' constellation firm* H *advertises,*
(ii) $D_\varnothing^a - D_\varnothing^{na} < 0$ *when* $\gamma > \overline{\gamma}(q_e) = \frac{q_H(q_e - q_L)}{q_e(q_H - q_L)}$ *that is when* $q_e < \overline{q_e}(\gamma) = \frac{q_H q_L}{(1-\gamma)q_H + \gamma q_L}$. *In this region of parameters' constellation firm* H *does not advertise.*

The following proposition provides intuition behind the finding.

Proposition 15 *When firm* H *advertises, the market exclusion is larger than under "no advertising". When firm* H *decides not to advertise, the market exclusion is also more important than in the other scenario.*

3.8 Conclusion

The primary message of this analysis is that there is a trade-off when using comparative advertising, a trade-off that does not take place in case of using non-comparative advertising. We have demonstrated what determines the trade-off of gaining some fraction of consumers (who thanks to advertising realize the true quality of the entrant's brand) at the risk of losing some consumers (that might escape to the low quality firm once they learn about its existence). As noted earlier, the existing literature on comparative advertising does not analyze this issue. In this sense, this is why the analysis is innovative and contributes to understanding the differences between the impact of regular and comparative advertising on market outcomes.

We have constructed a model of informative comparative advertising with heterogenous goods in which advertising reveals not only information on rival's quality, but her existence as well. The revelation of the rival brand's quality raises product differentiation at the informed segment, which makes it worth it for the incumbent to announce the lower quality of the new brand. The intensity of this effect increases with the difference between the expected quality and true quality of the rival product. On the other hand, the revelation of rival's existence gives the incumbent incentives to refrain from advertising. This effect is amplified when many consumers are unaware of the existence of the competitor in the market.

Hence, our findings are in line with the empirical observation that motivated this study. In a market where many consumers are unaware of the existence of a low quality rival brand and/or informed consumers have expectations of the entrant quality that are quite correct, we observe a lack of comparative advertising by the high-quality producer against the rival. However, in a market where the fraction of consumers who are aware of the existence of the low quality product is quite large, and/or the informed consumers expectations about the new product are incorrect (sufficiently high), it makes it profitable for the incumbent to announce the lower quality of the entrant product, even at the cost of revealing its existence.

Even though we have been able to conduct local comparative statics, we have explained the impact of the parameters used in the model on the decision of the revelation of the rival's quality and existence. If consumers perceive both brands as closer substitutes, the high quality producer has incentives to announce the difference in the products' qualities, thereby increasing product differentiation. On the other hand, if consumers are more pessimistic and anticipate lower quality of the entrant, then the producer of high quality good does not have incentives to advertise comparatively, i.e. to reveal the true quality (and existence) of the rival, since consumers are not very wrong about the expectation.

We have also studied the implications of price competition on advertising decisions of high quality producer. We have found that informative comparative advertising makes the low quality producer decrease her price. Hence, the high quality firm from the fear of fierce price competition refrains from using comparative advertising. Accordingly, advertising has a positive effect of informing consumers of the true low quality of the rival, but also has two negative effects, one direct and one indirect. The direct negative effect arises since advertising informs consumers of the existence of a competitor. More interestingly, there appears an indirect effect. Namely, advertising makes the rival set a low price. Hence, some consumers may escape from the incumbent not only due to learning the rival's existence, but also due to the low price set by the competitor.

It would be interesting to extend the analysis to include endogenous choice of qualities. Moreover, we have assumed that prices do not act as signals of qualities. In practice, it is likely to be the case. Hence, it constitutes an interesting extension of the model. Finally, it would be useful to find a global equilibrium of the "no advertising" scenario of the game, which would not only allow us to perform global comparative statics, but also to report globally on the impact of product substitutability on the decision on the revelation of the rival's quality and existence.

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Chapter 4

New determinants of advertising rates and pricing strategy in the product market: an application to the press industry[1]

4.1 Introduction

The aim of this paper is to understand the determinants of advertising strategies of an industrial producer on the media market. In particular, we seek to highlight the pricing strategies of advertising space in magazines not only as a function of the readership size, but also as a function of the *quality* of the readers[2]. We analyze the relation between the media market and the market of industrial products. Generally, the literature dedicated to media issues, and in particular to the concept of *two sided markets*[3], considers the strategy from the point of view of the media platform. The authors seek to identify the market structure (concentration, diversity, and so on), price structure (media price and

[1] This paper is co-written with Nathalie Sonnac.

[2] By *quality* of the readers we understand the probability of purchasing the industrial product.

[3] In a two-sided market, two groups interact through an intermediary or platform, and intergroup network externalities are present in that members of one group are directly affected by the number from the other group that use the same platform. In the press market, for example, the two groups that interact are readers and advertisers, and the platform is the editor's newspaper. Advertisers benefit when the newspaper has more readers, since those readers represent potential consumers for the advertised products.

advertising rates[4]), the levels of these prices and the ratio advertising over media content. They endogenize the readers' attitude towards advertising[5]. In this paper, our motivation is different. We study the impact of readers' profile on the product prices via the advertising rates. Consequently, we analyze the strategic interaction between the product market and the media market by endogenizing the following variables in the media market: not only the size of the demand, but also the quality of readers, whose probability of purchasing the industrial products varies with the degree of content specialization of the media. In this sense our model fits the literature on targeted advertising[6].

We consider the press market which is comprised of two magazine editors who differ in their degree of content specialization, that is, a general magazine that offers different columns to the readers (news, entertainment, culture, and so on) and a magazine specialized in one specific domain (fishing, sport, music, computer science, and so forth)[7]. The dichotomy between media market and the readership is the same. Hence, the readership

[4]The advertising rates are the prices of a unit of advertising space in a newspaper set to advertisers.

[5]In the literature on two-sided markets, Anderson and Gabszewicz (2004) analyze the performance of the broadcasting market, which delivers two goods, the program to viewers and the audience to the advertisers. The authors provide a comprehensive and recent review of work on advertising and the media. They find that competition for viewers of the demographics most desired by advertisers implies that programming choices will be biased towards the tastes of those with such demographics. Anderson and Coate (forthcoming) provide a theory of commercial broadcasting and advertising. In particular, their model permits a welfare analysis concerning how well the commercial broadcast market fulfills its two-sided role of delivering programming to viewers and enabling advertisers to contact potential consumers. In terms of the literature on two-sided markets, each viewer can use only one platform (i.e., watch only one program) and thus single-homes. By contrast, a firm can use both platforms (i.e., advertise on both programs) and may thus multi-home. As Armstrong (2004) shows, in such a situation a "competitive bottleneck" arises: platform competition is more intense over the party that single-homes. See also Caillaud and Jullien (2003) and Rochet and Tirole (2003).

[6]In the literature on targeted advertising, Roy (2000) considers a model in which firms may target the individual consumers to whom their respective ads are delivered. Working with a duopoly model, he finds that the two firms divide the entire market into mutually exclusive captive segments within which each firm operates as a local monopolist. Adams and Yellen (1977) demonstrate that a non-discriminating monopolist might supply too much advertising to consumers, supply advertising to the wrong types of consumers, or supply too many brands of the product. Galeotti and Moraga-Gonzales (2004) study a simultaneous move game of targeted advertising and pricing in a market with various consumer segments. In this setting they explore the implications of market segmentation on firm competitiveness. Dukes and Gal-Or (2003) show that media have incentives to minimize the extent of differentiation between them, which has an implication of the assumed role of advertising as information and as an ultimate nuisance to the audience. Iyer, Soberman and Villas-Boras (2005) examine advertising strategy when competing firms can target advertising to different groups of consumers within a market. They find that targeted advertising leads to higher profits regardless of whether or not firms can set targeted prices, and that the targeting of advertising can be more valuable for firms in a competitive environment than the ability to target pricing.

[7]We assume that both magazines are printed with the same periodicity.

profiles correspond to magazine profiles.

The advertising strategy of the producer consists in considering both the readership sizes of the magazines (the numbers of copies sold), and also the different types of readers (generalist versus targeted). In marketing terminology, the general readers constitute "the readership", whereas the specialized readers are considered as "the useful readership".

We assume that the general magazine enjoys larger readership size that the one of the specialized magazine. Majority of the economic literature analyzes the first strategic determinant of advertising pricing, namely the readership size. In this paper, we also endogenize the *quality* of the readers, that is the target. The readers of each magazine have distinct probabilities of purchasing the industrial product. Therefore, there are two fundamental elements of the analysis: on the one hand, we assume that the probability of selling the industrial product[8] increases with the readership size, on the other hand, the probability of purchasing the industrial product by a "targeted reader" is superior to the one by a "general reader", which translates into a superior efficiency of advertising in a specialized magazine.

In our model we assume that advertising is informative[9]. The demand for industrial products arises only from the readers of the two magazines (the general and the specialized one). Thus, only the strategy of purchasing the advertising space by the firm (together with pricing strategy of the industrial product) determines the total amount of product sales.

Consequently, in the strategic decision to buy advertising space in magazines, the firm will face a trade-off between the large readership size and the interesting profile of a reader. The editors will anticipate those two determinants of the demand for advertising space and will set the advertising rates and cover prices of magazines accordingly, taking into consideration the fact that setting lower cover prices increases the readership and allows to increase advertising rates. We allow the producer to internalize the difference in demand elasticities for the product among the two groups of consumers (readers of each magazine) and consequently, to discriminate the product price for different types of consumers (readers), taking into account the advertising expenses incurred in order

[8] Note that the probability of *selling* the product by a producer is the same thing as the probability of *purchasing* the product by a consumer.

[9] We do not take a stand in the economic debate that confronts informative and persuasive advertising.

to create demand for the product. Since the producer anticipates that a product sale is more probable to a reader of the specialized magazine than to a reader of the general one, the firm will charge the *full* price to "inelastic" consumers and a reduced price to the "elastic" ones. To illustrate how such a price discrimination could proceed we provide the following example: the producer inserts a reduction coupon into the general magazine to attract the "elastic" consumers who otherwise would not purchase the product. Such a coupon entitles consumers to a price reduction. As mentioned before, each reader buys only one magazine, hence the insertion of a coupon into the general magazine does not increase her readership. We eliminate the case of readers switching magazines in order to get the coupon by assuming that the readers of the specialized magazine do not know of the existence of the coupon in the other magazine. This assumption stays in line with the assumption of readers' not knowing of the existence of the product prior to seeing an advertisement in a magazine.

Regarding the determinants of the advertising rates, we obtain that a larger readership of the general magazine provokes a higher advertising rate, thus the producer will increase her price for the industrial product sold to the "general" readers to make up for higher advertising expenses.

The other determinant of advertising rates, the readers' profile, has a different impact for each magazine. The advertising rate set by the *specialized* magazine increases with the degree of readers devotion to the industrial product. It is due to the fact that: (i) since the advertising rate increases with the degree of readers' devotion, so does the product price, and (ii) the advertising rate is affected by the readership volume which changes with the product devotion via the magazines' prices. The advertising rate set by the *general* magazine also depends on the level of product devotion, yet in a non-monotonic way. When the product devotion is small, then an increase in the price of the general magazine declines her demand, which in turn drags the advertising rate down. However, the higher product price for general readers increases the producer's willingness to pay for advertising space and compensates the effect of reduced readership of the general magazine. Consequently, the advertising rate set by the general magazine goes up with the product devotion. In contrast, when the product devotion is large, then the increase in the price of general magazine is sufficiently large to decline the magazine's demand enough to overtake the effect of a higher willingness to pay for advertising space (ergo, a higher product price for general readers). In other words, large product devotion by the readers increases the producer's willingness to invest in advertising in the specialized magazine, rather than the general one. Consequently, the general magazine is forced to decrease the advertising rate to attract the demand for advertising space.

Regarding the determinants of price discrimination, we obtain that, on the one hand, an increase in the readers' preference for the general magazine makes the producer less willing to use price discrimination, that is, it is less worth it to offer large price reduction to more numerous general readers. On the other hand, an increase in the product devotion amplifies the price discrimination, that is, it increases the product price reduction offered to the readers of the general magazine.

The paper is organized as follows. Section 4.2 sets up the model description. In particular, we provide the timing of the three-period game, and subsequently we describe the three markets (the readers, the media and the industrial product) and we explain the interaction between them. In section 4.3, we analyze the equilibrium of the sequential game. Section 4.4 discusses the results. In particular, we explain the impacts of the readership size and the readers' profile both on advertising rates and on the product prices. Finally, in section 4.5 we conclude.

4.2 The Model

Consider a model with two competing editors $i = 0, 1$, producing differentiated magazines at no cost per copy. The editors operate in two interrelated markets: the readers' market, where magazines are sold to the readers and the advertisers' market, where advertising space is sold to advertisers in order to promote their products. We suppose that magazines differ in the degree of specialization of their content. To represent this diversity among magazines, we assume that the set of their contents is the unit interval [0,1], with 0 corresponding to the most general content (the least specialized) and 1 to the most specialized one. To each possible degree of content specialization there corresponds a specific reader for whom that degree is the ideal one[10]. The farther the magazine's degree of specialization from this ideal point, the lower the utility of the specific reader.

[10] Consumers are uniformly distributed on the unit interval.

For simplicity, we will only consider the case with magazines located at the extremes of the interval[11].

In fact, one can argue that there are numerous possible profiles of specialization of a given magazine. Specialized magazines target readers corresponding to a specific profile, like sport, computer science, beauty, travel, etc. In our model there is one general magazine with zero degree of specialization in any field, and n magazines specialized each in a different field. Consumers have different profiles of interest, hence different reading preferences. Each field of interest is represented by one unit line, on which there are two competing editors and the readers are distributed uniformly. It is important to note that the readers who share the same field of interest (with different intensities) are distributed on *one* of the unit lines corresponding to the profile.

The following figure illustrates the structure of the press market, with one general magazine located centrally on the star and at zero of each unit line.

We make the following assumptions to describe our model.

Assumptions

1. There exist n specialized magazines, one in each of n profiles of specialization.

2. There is a unit line of consumers distributed uniformly in each profile, hence the total size of the population of magazine readers is equal to n.

3. Each reader purchases only one magazine. We assume that the magazines are edited periodically with the same frequency in time. There is no overlapping of readers, that is each reader is identified by a unique point on *one* unit line only.

4. Each specialized magazine competes with the general magazine for the consumers on the unit line corresponding to the specialized magazine.

5. We will consider just one profile, one unit line, and assume that the remaining $n-1$ profiles are clones of the one we analyze. Hence, in each profile of specialization,

[11] The editors' locations on the unit interval are fixed and exogenously given at its extreme points, representing the minimal and the maximal degree of content specialization. In other words, the editors do not choose the location on the line.

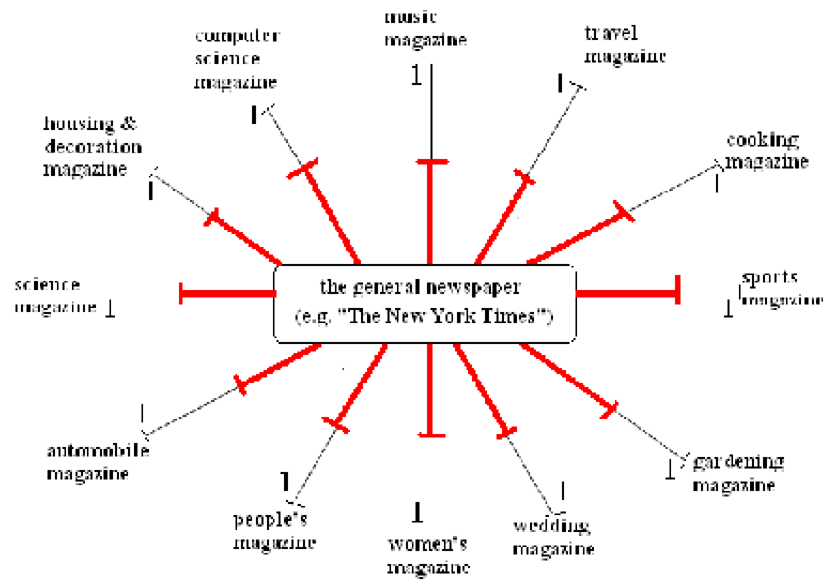

Figure 4-1: An exemplary structure of the press market.

i.e. on each unit line, the optimal ratio of readerships is the same. Accordingly, the readership of the general magazine is n times her readership on one unit line, whereas the readership of a given specialized magazine is just a given fraction on one unit line.

6. There is a monopoly structure in the industrial product market in each specialization, i.e. there is only one advertiser on each profile line. Moreover, each advertiser makes identical decision on the amount of advertising space to be purchased in the two magazines corresponding to her profile line.

7. If a producer of an industrial product, corresponding to one profile, advertises in the general magazine, such an action constitutes a waste of resources from all remaining $n-1$ unit lines of consumers. In other words, we assume that only consumers on the unit line corresponding to profile of the product are potential consumers of the producer[12].

[12]Note that the general newspaper may face majority or minority share on each profile line, but still her readership is superior to the one of each specialized newspaper. However, an advertiser cares only about the structure of readership shares on the specific profile line corresponding to her industry.

The timing

In order to solve the editor's problem of determining magazines' prices at the newsstand, as well as advertising rates, and the advertiser's problem of determining advertising volumes for each magazine as well as product price(s), we consider a three-period sequential game played between editors and the advertiser (i.e. the producer).

In period 1, editors select newsstand prices and advertising rates.

In period 2, the advertising strategies, i.e. the volumes of advertising, are chosen by the advertiser. Entering in this second period the newsstand prices that have been selected in period 1 determines total readership sizes of magazines, which have impact on advertiser's choice of advertising space to be purchased in one or both magazines.

In period 3, once advertisements appeared in the magazines, potential demand for advertiser's product has been created and product price(s) are chosen by the producer. We will assume that the advertiser acts in the product market as a price discriminating monopoly.

The readers' market

We suppose that each reader purchases one magazine. As mentioned before, first we consider only one unit line of specialization to find the ratio of readerships of the general magazine and the specialized one. The reader's utility of consumption of magazine 0 is measured by

$$U_0 = v_0 - tx^2 - p_0$$

where v_0 represents the base utility of reading magazine 0, t is the transportation cost, x represents the distance between magazine 0's location, and the ideal point of the reader, and p_0 denotes the price of magazine 0. The parameter x measures the intensity of readers' preferences for the degree of specialization of magazines. Note that readership is neutral with respect to advertising[13].

[13] Consumers do not enjoy any utility nor suffer any disutility from advertising. Hence, they are neither ad-lovers nor ad-avoiders and they do not loose (nor gain) in base utility when the advertising volume increases. The way advertising volumes may change consumer's total utility is via newspaper prices. For the literature that analyzes the issue of readers' attitude to advertising, see Sonnac (2000), Gabszewicz, Laussel and Sonnac (2002), Gabszewicz, Garella and Sonnac (2007).

By analogy, a reader located at a distance $(1-x)$ from magazine 1 derives a utility of reading the magazine given by

$$U_1 = v_1 - t(1-x)^2 - p_1.$$

Note that the base utilities of reading each magazine are different[14].
Consequently, the reader who is indifferent between buying one or the other magazine is located at a distance $\overline{x}$ from magazine 0 and $1-\overline{x}$ from magazine 1, where $\overline{x}$ is the solution for

$$v_0 - tx^2 - p_0 = v_1 - t(1-x)^2 - p_1$$

and is given by:

$$\begin{aligned} \overline{x} &= \frac{p_1 - p_0 + t + \Delta}{2t}, \\ 1-\overline{x} &= \frac{p_0 - p_1 + t - \Delta}{2t}, \end{aligned} \tag{4.1}$$

with $\Delta = v_0 - v_1$. All readers located at the left of $\overline{x}$ buy magazine 0, and all those at the right buy magazine 1.
Accordingly, the corresponding demand functions in the newsstand sales market of editor 0 and of editor 1 are respectively

$$\begin{aligned} D_0(p_0, p_1) &= \overline{x} = \frac{p_1 - p_0 + t + \Delta}{2t}, \\ D_1(p_0, p_1) &= 1-\overline{x} = \frac{p_0 - p_1 + t - \Delta}{2t}. \end{aligned} \tag{4.2}$$

The demand functions identified above represent the readership volumes of the magazines.

Note that for the readership volumes of magazines to be equal we need not only $\Delta = 0$, but also identical cover prices of the magazines. This means that even if the readers located on the unit line are equally divided between both magazines by their preferences, the market shares of both editors are equal only in case of identical cover

[14] Since we assume that parameter v_i, which can be interpreted as the *pleasure of reading,* is not equal for readers of both magazines, the difference in this base utility ($\Delta = v_0 - v_1$) will affect the equilibrium outcomes. In particular, the asymmetry of market shares will depend on Δ. The larger Δ, the larger readership is enjoyed by magazine 0, and the smaller by magazine 1. A priori, we do not impose any sign on Δ.

prices.

Assuming a zero marginal cost, editor 0 will choose the magazine's cover price p_0 that maximizes the sum of her profits from the readers 'market given by $p_0 \cdot nD_0(p_0, p_1)$ and revenues from the media market, that will be characterized in the next section.

Editor 1 will choose the cover price p_1 that maximizes the sum of her profits given by $p_1 \cdot D_1(p_0, p_1)$, and revenues from the media market characterized in the next section.

The media market

We denote by s_0 and s_1 the unit prices of advertising space sold to the advertiser by editor 0 and editor 1, respectively. These rates are selected by each editor to maximize their revenues from the sales of advertising space. The total revenue of editor 0 from all n "directions" is given by

$$\underset{s_0}{max}\ V_0(s_0, s_1) = ns_0a_0, \tag{4.3}$$

and the revenue of editor 1 is given by

$$\underset{s_1}{max}\ V_1(s_0, s_1) = s_1a_1, \tag{4.4}$$

where a_0 and a_1 are the advertising volumes sold to the advertiser.

Given the advertising rates, the advertiser's demands for advertising space in both magazines, a_0 and a_1, are such that maximize the advertiser's profit from sales in the product market, less advertising expenses. We consider informative advertising only, where messages inform readers of the existence of the particular brand, its location on the line (at point 1), and its price. Unless a consumer sees an ad in a magazine, she remains completely uninformed about the product's existence and refrains from consumption altogether. Hence, we assume that (only) advertising creates potential demand for the product. By potential we mean that a reader of a magazine may or may not see an advertisement placed in the magazine. We assume that a given reader will see at least one advertisement placed in magazine i on behalf of the advertiser (producer) with some probability that depends upon the extent of advertising volume a_i, $i = 0, 1$, chosen by the advertiser. We designate this probability by $\varphi(a_i)$, where $\varphi(a_i) \in [0, 1)$, $\varphi(0) = 0$, $\varphi\prime(a_i) > 0$, $\varphi\prime\prime(a_i) < 0$. Hence, the probability of reaching a consumer is increasing at a decreasing rate with the intensity of advertising, reflecting decreasing marginal returns on advertising. We propose a specific functional form for the probability of advertising reaching consumers. It is given by

$$\varphi(a_i) = \frac{a_i}{1 + a_i}. \tag{4.5}$$

Note that it satisfies the properties established above[15].

Accordingly, the advertising volumes chosen by the advertiser will maximize her revenue from product sales, less advertising expenses. The revenue from product sales will depend *inter alia* on demand for the product coming from two groups of consumers: from readers of magazine 0, and readers of magazine 1. We allow the producer to internalize the difference in demand elasticities for the product among the two groups of consumers (readers of each magazine) and consequently, to discriminate the product price for different types of consumers (readers). Since the producer anticipates that a product sale is more probable to a reader of the specialized magazine than to a reader of the general one, the firm will charge the *full* price to "inelastic" consumers and a reduced price to the "elastic" ones.

We denote by r_0 and r_1 the product prices for readers of magazine 0 and readers of magazine 1, respectively. Note that r_0 is the reduced price and can be written as a fraction of the *full* price r_1. Later in the model we will denote $\frac{r_0^*}{r_1^*} = \gamma$, with $\gamma \in [0, 1]$. That is, $(1-\gamma)$ characterizes the % price reduction for general readers. The total revenue from product sales is given by:

$$R(r_0, r_1) = \frac{a_0}{1 + a_0} r_0 d_0(r_0) + \frac{a_1}{1 + a_1} r_1 d_1(r_1), \tag{4.6}$$

where $\{r_0, r_1\}$ are the product prices, and $\{d_0(r_0), d_1(r_1)\}$ are the demand functions for the product coming from among readers of magazine 0 and 1, respectively, the construction of which is explained in the subsequent section[16].

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[15] The probability function we use could be easily replaced by the standard functional form used in the literature and pioneered by Butters [1977], given by $\varphi(a_i) = 1 - e^{-a_i}$.

[16] Note that the demand for the product from readers of a newspaper i, $d_i(r_i)$ is not necessarily equal to the total readership of newspaper i, $D_i(p_i, p\ _i)$, regardless of the probability of advertising reaching a consumer.

The advertising cost function of the advertiser is given by:

$$C(a_0, a_1) = s_0 a_0 + s_1 a_1 \tag{4.7}$$

Hence, the producer will choose such advertising levels (and subsequently the prices in the product market) that maximize her profits:

$$\max_{a_0, a_1} \pi(a_0, a_1) = \frac{a_0}{1+a_0} r_0 d_0(r_0) + \frac{a_1}{1+a_1} r_1 d_1(r_1) - s_0 a_0 - s_1 a_1 \tag{4.8}$$

The product market

The advertiser, having purchased advertising space in one or both magazines, faces demand for her industrial product. We assume that the advertiser on the product market is a monopoly that discriminates prices. We construct the demand for the advertiser's product in the following way. First, as mentioned before, the demand for the product is a composite of the product demand from readers of magazine 0, $d_0(r_0)$, and the demand from readers of magazine 1, $d_1(r_1)$. Second, we assume that there will be a fraction α among readers of magazine 0 and a fraction β among readers of magazine 1 that will purchase the industrial product, regardless of the probability of consumer seeing an ad. We endogenize the parameters α and β in a way to make them functions of r_0 and r_1 respectively. Hence, the product demands coming from those two group of consumers are related in the following way:

$$\begin{aligned} d_0(r_0) &= \alpha(r_0) D_0(p_0, p_1), \\ d_1(r_1) &= \beta(r_1) D_1(p_0, p_1), \end{aligned} \tag{4.9}$$

where $D_0(p_0, p_1)$ and $D_1(p_0, p_1)$ are the readerships of magazine 0 and magazine 1, respectively. Since the product demands are designated as fractions of magazine readers, we will subsequently refer to them as *readers-buyers*. Subsequently, we endogenize the parameters α and β as follows.

We suppose that there exists a positive correlation between the readers' preferences for magazine's profile and their utility of purchasing the industrial product being in line with the specialized profile of magazine 1.The nature of this relationship is assumed to be a linear bijection with a parameter b, $b > 0$, measuring its strength[17]. More specifically, the population of readers is ranked in the unit interval [0,1] by order of increasing base utility of consumption of the industrial product. Accordingly, a reader of any magazine, identified by location x on the line, enjoys a utility of $bx - r_i$ when purchasing the product at a price r_i, and zero otherwise. Since we allow the producer to discriminate prices, we will consider the choice of whether to buy the product among readers of each magazine separately.

A given reader of magazine 0, that is identified by a location x on the unit line with $x < \overline{x}$, where $\overline{x} = D_0(p_0, p_1) = \frac{p_1 - p_0 + t + \Delta}{2t}$ is the location of the indifferent reader, having seen an ad now chooses whether to buy the product or not. The utilities enjoyed by the reader of magazine 0 in case of purchase and otherwise are given by

$$\begin{aligned} Y^{buy}(x &< \overline{x}) = bx - r_{\mathbf{0}}, \\ Y^{not\_buy}(x &< \overline{x}) = 0. \end{aligned}$$

Hence the reader of magazine 0 who is indifferent between purchasing the product and not, is identified by location $\overline{x_0} = \frac{r_0}{b}$ on the unit line. All readers of magazine 0 located at the right of $\overline{x_{\mathbf{0}}}$ buy the product, and all those located at the left do not buy it.

By analogy, a given reader of magazine 1, that is identified by a location x on the unit line with $x > \overline{x}$, where $\overline{x} = D_0(p_0, p_1) = \frac{p_1 - p_0 + t + \Delta}{2t}$ is the location of the indifferent reader, having seen an ad now chooses whether to buy the product or not. The utilities enjoyed by the reader of magazine 1 in case of purchase and otherwise are given by

$$\begin{aligned} Y^{buy}(x &> \overline{x}) = bx - r_1, \\ Y^{not\_buy}(x &> \overline{x}) = 0. \end{aligned}$$

Hence the reader of magazine 1 who is indifferent between purchasing the product and not, is identified by location $\overline{x_1} = \frac{r_1}{b}$ on the unit line. All readers of magazine 1 located at the right of $\overline{x_1}$ buy the product, and all those located at the left do not buy it.

[17] One can think of the industrial product being a computer game, newspaper 0 being "The New York Times" with no degree of specialization on computer science and newspaper 1 being "Computer World" enjoying the maximum degree of specialization in the field. In such a set-up, our assumption simply says that there is an increasing bijection from the readers' preferences for the purchase of "Computer World" onto the base utility of playing a computer game.

We denote the probability that a reader of magazine 0 (resp. magazine 1) purchases the advertiser's product by α (resp. β). Parameters α and β can also be interpreted as fractions of readership of each magazine that purchase the advertiser's product.

We obtain the following result for α:

$$\alpha = \frac{\overline{x} - \overline{x_0}}{\overline{x}}. \tag{4.10}$$

Hence, substituting $D_0(p_0, p_1)$ for $\overline{x}$, we obtain

$$\begin{aligned} \alpha D_0(p_0, p_1) &= D_0(p_0, p_1) - \overline{x_0}, \\ \text{that is,} \quad \alpha D_0(p_0, p_1) &= D_0(p_0, p_1) - \frac{r_0}{b}. \end{aligned}$$

Taking into account the following constraint: $d_0(r_0) \leq D_0(p_0, p_1)$, that is $\alpha \in [0, 1]$, the corresponding demand for the industrial product coming from readers of magazine 0 is given by:

$$d_0(r_0) = \min\{D_0(p_0, p_1) - \frac{r_0}{b}, D_0(p_0, p_1)\}.$$

Note that for $r_0, b > 0$, we obtain that:

$$\begin{aligned} d_0(r_0) &= D_0(p_0, p_1) - \frac{r_0}{b}, \\ \text{that is,} \quad d_0(r_0) &= \frac{p_1 - p_0 + t + \Delta}{2t} - \frac{r_0}{b}. \end{aligned} \tag{4.11}$$

Hence, α is inferior to 1 in both cases, unless the industrial product is given for free, $r_0 = 0$.

We obtain the following result for β:

$$\beta = \frac{1 - \overline{x_1}}{1 - \overline{x}}. \tag{4.12}$$

Hence, substituting $D_1(p_0, p_1)$ for $(1 - \overline{x})$, we obtain

$$\begin{aligned} \beta D_1(p_0, p_1) &= 1 - \overline{x_1}, \\ \text{that is,} \quad \beta D_1(p_0, p_1) &= 1 - \frac{r_1}{b}. \end{aligned}$$

Taking into account the constraint: $d_1(r_1) \leq D_1(p_0, p_1)$, that is $\beta \in [0, 1]$, the corresponding demand for the industrial product coming from readers of magazine 1 is given by:

$$d_1(r_1) = \min\{1 - \frac{r_1}{b}, D_1(p_0, p_1)\}.$$

Hence, two parallel cases arise.

Case 1 $\quad 1 - \frac{r_1}{b} < D_1(p_0, p_1)$, which implies that:

$$d_1(r_1) = 1 - \frac{r_1}{b}. \tag{4.13}$$

Case 2 $\quad 1 - \frac{r_1}{b} \geq D_1(p_0, p_1)$, which implies that:

$$\begin{aligned} d_1(r_1) &= D_1(p_0, p_1), \\ \text{hence, } r_1 &= b(1 - D_1(p_0, p_1)). \end{aligned}$$

In means that in Case 2, $\beta = 1$. The following figure illustrate the demands for the industrial product under both cases.

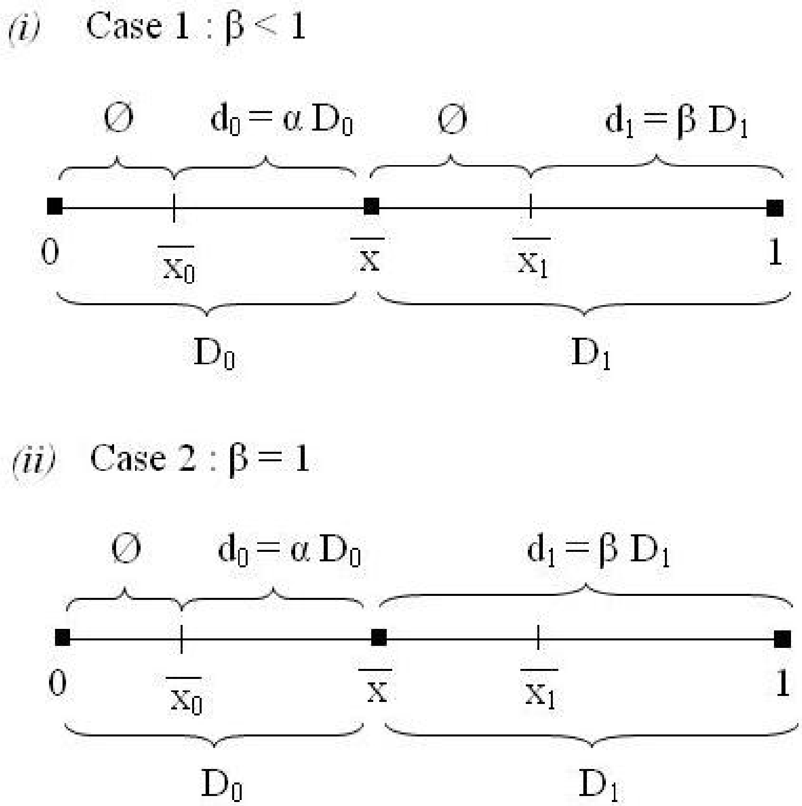

Figure 4.2: The demands for the industrial product as fractions of the magazines' demands.

In this analysis we restrict our attention to Case 1, since it is more interesting to show the optimal policy of the producer when he probability of purchasing the product by a reader of magazine 1 is strictly inferior to 1. In the last section we briefly report on the difference in results among cases.

Now we turn to determining the optimal pricing policy of the product of the advertiser. As mentioned before, the advertiser faces different demand elasticities from the two groups of consumers, hence he has incentives to increase the price on the less elastic segment of the market. We assume there is no cost of production. The producer will choose such product prices that maximize her profits:

$$\max_{r_0,r_1} \pi(r_0,r_1) = \frac{a_0}{1+a_0} r_0 d_0(r_0) + \frac{a_1}{1+a_1} r_1 d_1(r_1) - s_0 a_0 - s_1 a_1. \tag{4.14}$$

4.3 The equilibrium analysis

In this section we provide the optimal strategies for all players in each stage of the game. Subsequently, in the following section, we will perform comparative statics in order to detect the effects responsible for shaping the equilibrium outcomes.

To determine the sequence of optimal actions we use the concept of backwards induction.

Stage 3

Given any possible outcomes of the first two stages, the advertiser chooses in stage 3 the optimal product prices. Solving the advertiser's optimization problem:

$$\max_{r_0,r_1} \pi(r_0,r_1) = \frac{a_0}{1+a_0} r_0 d_0(r_0) + \frac{a_1}{1+a_1} r_1 d_1(r_1) - s_0 a_0 - s_1 a_1,$$

where $d_0(r_0)$ and $d_1(r_1)$ are given by (4.11) and (4.13) respectively, we obtain the equilibrium outcomes in stage 3, $r_0^* = r_0(p_0, p_1 \mid t, b, \Delta)$, $r_1^* = r_1(p_0, p_1 \mid t, b, \Delta)$, characterized by:

$$\begin{aligned} r_0^*(p_0,p_1) &= \tfrac{b(-p_0+p_1+t+\Delta)}{4t}, \\ r_1^*(p_0,p_1) &= \tfrac{b}{2}. \end{aligned} \tag{4.15}$$

Stage 2

Given any possible outcomes of the first stage, the advertiser selects in stage 2 the optimal advertising volumes as a solution to the following problem:

$$\max_{a_0,a_1} \pi(a_0,a_1) = \frac{a_0}{1+a_0} r_0^* d_0(r_0^*) + \frac{a_1}{1+a_1} r_1^* d_1(r_1^*) - s_0 a_0 - s_1 a_1$$

where $d_0(r_0^*) = \alpha(r_0^*) D_0(p_0, p_1)$ and $d_1(r_1^*) = \beta(r_1^*) D_1(p_0, p_1)$. For transparency of the results we will denote $\alpha(r_0^*)$ as α^*, and $\beta(r_1^*)$ as β^*.
The equilibrium outcomes in stage 2, $a_0^* = a_0(p_0, p_1, s_0, s_1 \mid t, b, \Delta)$ and $a_1^* = a_1(p_0, p_1, s_0, s_1 \mid t, b, \Delta)$, are given by

$$\begin{aligned} a_0^*(p_0, p_1, s_0) &= \tfrac{(-p_0+p_1+t+\Delta)}{2t}\sqrt{\tfrac{b}{4s_0}} - 1, \\ a_1^*(p_0, p_1, s_1) &= \sqrt{\tfrac{b}{4s_1}} - 1. \end{aligned} \tag{4.16}$$

The outcomes of both cases can also be written in the following way:

$$\begin{aligned} a_0^*(r_0^*, D_0, s_0, b) &= \sqrt{\tfrac{r_0^*(bD_0 - r_0^*)}{bs_0}} - 1, \\ a_1^*(r_1^*, D_1, s_1, b) &= \sqrt{\tfrac{r_1^*(b - r_1^*)}{bs_1}} - 1. \end{aligned}$$

that is without substituting the stage 3 outcomes for $\{r_0^*, r_1^*, D_0, D_1\}$, which are not functions of the strategic variables at stage 2, $\{a_0^*, a_1^*\}$.

Stage 1

The editors choose simultaneously in stage 1 the optimal advertising rates and magazines' prices solving the following problems:

$$\begin{aligned} &\underset{p_0, s_0}{max}\ p_0 \cdot nD_0(p_0, p_1) + s_0 a_0 \\ &\underset{p_1, s_1}{max}\ p_1 \cdot D_1(p_0, p_1) + s_1 a_1 \end{aligned}$$

where $i = 0, 1$.

We obtain two sets of best response functions. They differ only in the form of $p_0(p_{1,} s_0)$:

$$\begin{aligned} p_0(p_{1,} s_0) &= \tfrac{2p_1 - \sqrt{bs_0} + 2t + 2\Delta}{4} \\ s_0(p_0, p_1) &= \tfrac{b(-p_0 \mid p_1 + t + \Delta)^2}{64t^2} \\ p_1(p_0) &= \tfrac{p_0 \mid t - \Delta}{2} \\ s_1 &= \tfrac{b}{16} \end{aligned} \qquad or \qquad \begin{aligned} p_0(p_{1,} s_0) &= \tfrac{2p_1 + \sqrt{bs_0} + 2t + 2\Delta}{4} \\ s_0(p_0, p_1) &= \tfrac{b(-p_0 + p_1 \mid t + \Delta)^2}{64t^2} \\ p_1(p_0) &= \tfrac{p_0 + t - \Delta}{2} \\ s_1 &= \tfrac{b}{16} \end{aligned}$$

Note that $s_0(p_0, p_1)$ can be written as $s_0(D_0 \mid b) = \frac{bD_0^2}{16}$, or as $s_0(r_0^*, \alpha^* D_0) = \frac{\alpha^* D_0 r_0^*}{4}$, and $s_1(r_1^*, \beta^* D_1) = \frac{\beta^* D_1 r_1^*}{4}$, that is without substituting for equilibrium outcomes the results from stage 3 and 2.
Both sets of best response functions give rise to the same equilibrium outcome identified by

$$\begin{aligned} p_0^* &= \tfrac{(b+16t)(3t+\Delta)}{b \mid 48t} \\ s_0^* &= \tfrac{4b(3t+\Delta)^2}{(b+48t)^2} \\ p_1^* &= \tfrac{2t(b+8(3t-\Delta))}{b \mid 48t} \\ s_1^* &= \tfrac{b}{16} \end{aligned} \tag{4.17}$$

Replacing the stage 1 equilibrium outcomes into the optimal strategies in stage 2 and 3 we obtain a complete equilibrium description.

Lemma 10 *The equilibrium of the three-period sequential game is identified by the following outcomes:*
$\{p_0^*, s_0^*\} = \{\frac{(b+16t)(3t+\Delta)}{b \mid 48t}, \frac{4b(3t+\Delta)^2}{(b \mid 48t)^2}\}$,
$\{p_1^*, s_1^*\} = \{\frac{2t(b+8(3t-\Delta))}{b+48t}, \frac{b}{16}\}$,
$\{a_0^*, a_1^*\} = \{1, 1\}$,
$\{r_0^*, r_1^*\} = \{\frac{4b(3t+\Delta)}{b+48t}, \frac{b}{2}\}$.

Note that for the equilibrium to be well-defined we need that $b + 8(3t - \Delta) > 0$.

Lemma 11 *For the equilibrium outcomes of the game we provide the following pairwise comparisons:*
$p_0^* > p_1^*$,
$s_0^* < s_1^*$,
$r_0^* < r_1^*$.

Lemma 12 *In equilibrium the market shares of editors and fractions of readers-buyers are characterized by:*
$D_0^* = \frac{8(3t+\Delta)}{b+48t}$, $\quad D_1^* = \frac{b+8(3t-\Delta)}{b+48t}$, $\quad$ *hence* $D_0^* < D_1^*$,
$\alpha^* = \frac{1}{2}$, $\quad \beta^* = \frac{b+48t}{2(b+8(3t-\Delta))}$, $\quad$ *hence* $\alpha^* < \beta^*$,
$\alpha^* D_0^* = \frac{4(3t+\Delta)}{b+48t}$, $\quad \beta^* D_1^* = \frac{1}{2}$, $\quad$ *hence* $\alpha^* D_0^* < \beta^* D_1^*$.

We now turn to discussing the results. In particular, we explain the impacts of the readership size and the readers' profile both on advertising rates and on the product prices. Moreover, we analyze the incentives for the producer to use price discrimination.

4.4 Results

4.4.1 The impact of readership size on advertising rates

Since the press industry is in large part financed by industrial firms who buy advertising space to promote their products, a magazine requires a sufficiently sizable readership in order to make the magazine attractive as a media support for the advertisers. The impact of an advertising message increases with the size of the audience.

In our model we have assumed that the insertion of an advertising message in the general magazine by a producer of a specific product, creates demand on the specific unit line of readers, but is a waste of resources on all remaining $1-n$ unit lines of consumers. Hence, the producer is only attracted by the relative readership size of the general magazine, i.e. the readership on the specific unit line corresponding to the product profile. Moreover, what actually impacts the sales is not the *total* readership of the general magazine on the unit line, but the fraction of her readers who are willing to buy the product at a given price, which we previously called the fraction of *readers-buyers*. This effect can be measured directly by αD_0 and indirectly by parameter Δ and cover prices of the magazines.

For all configurations of parameters $\{\Delta, t, b\}$ we obtain that the general magazine sets a lower advertising rate than the specialized one[18]. It is due to the fact that on the specific unit line the fraction of readers of magazine 0 who are willing to buy the product at a given price is smaller than its counterpart at magazine 1. Hence, magazine 1 enjoys the economies of *readers-buyers* scales. Even though we obtain that $D_0^* < D_1^*$, that is the readers market is dominated by the specialized magazine, for sufficiently large n, i.e. $n > (\frac{b+8(3t-\Delta)}{8(3t+\Delta)})^2$, the total readership of the general magazine is larger than its rival's: $nD_0^* > D_1^*$. However, as mentioned above, what influences the advertising rates is the relative readership, i.e. the fraction of readers-buyers, measured by parameters α and β. We find that in equilibrium $\alpha^* < \beta^*$, and more importantly, $\alpha^* D_0^* < \beta^* D_1^*$, which is the reason for higher advertising rates set by editor 1.

[18]Note that for sufficiently large n, i.e. $n > (\frac{b+48t}{8(3t-\Delta)})^2$, we obtain that the general newspaper's *total revenues* from the media market are larger than those of a specialized newspaper: $ns_0^* a_0 > s_1^* a_1$, even though the *unit* advertising rate is smaller: $s_0^* < s_1^*$.

Note that in stage 3 of the equilibrium analysis we obtain that

$$\begin{aligned} r_0^*(p_0, p_1) &= \frac{b}{2} D_0, \\ r_1^*(p_0, p_1) &= \frac{b}{2}(D_0 + D_1). \end{aligned}$$

Hence, for any configuration of magazine prices we obtain that

$$r_0^*(p_0, p_1) < r_1^*(p_0, p_1). \tag{4.18}$$

The intuition behind the result is the following. Since the readers of magazine 0 have larger price elasticity of demand for the industrial product, the producer will set a lower price to attract those consumers. The larger the readership share of magazine 0 on the unit line, expressed by $D_0(p_0, p_1 \mid \Delta)$, the less it's worth it for the producer to use a large price discount for the elastic segment of the readers, because a large price reduction to an important fraction of consumers would decrease total profits from sales. Formally,

$$\begin{aligned} \frac{\partial r_0^*(p_0, p_1)}{\partial D_0} &> 0 \\ \frac{\partial (r_1^*(p_0, p_1) - r_0^*(p_0, p_1))}{\partial D_0} &< 0 \end{aligned}$$

The reason for that to happen is the following. The larger the readership D_0 of the general magazine on one unit line, the larger advertising rate s_0 the magazine sets, which in turn implies a larger price r_0 for the industrial product sold to the "general" consumers. Formally,

$$\frac{\partial s_0}{\partial D_0} > 0, \quad \frac{\partial r_0}{\partial s_0} > 0.$$

Since $D_0 = D_0(p_0, p_1 \mid \Delta)$, we can analyze the indirect impact of p_0, p_1 and Δ on the equilibrium prices of the industrial product. Note that

$$r_1^*(p_0, p_1) - r_0^*(p_0, p_1) = \frac{p_0 - p_1 + t - \Delta}{4t}.$$

We obtain the following results on the impact of Δ on equilibrium prices of the product:

$$\frac{\partial r_0^*}{\partial \Delta} > 0 \quad \text{and} \quad \frac{\partial (r_1^* - r_0^*)}{\partial \Delta} < 0.$$

The result is due to the fact that s_0 and r_0 are strategic complements, as well as the fact that larger readership of the general magazine (amplified by Δ) raises the advertising rates, thus increases the product price of the advertiser. Formally,

$$\frac{\partial D_0}{\partial \Delta} > 0, \ \frac{\partial s_0}{\partial D_0} > 0, \ \frac{\partial r_0}{\partial s_0} > 0.$$

Proposition 16 *The more preference the readers located on a unit line exhibit towards the general magazine, the larger the readership of the magazine provoking higher advertising rate, thus the higher price the producer will set for the industrial product sold to the "general" readers.*

The magazines prices $\{p_0, p_1\}$ affect the prices of the product $\{r_0^*, r_1^*\}$ in the following way:

$$\frac{\partial r_0^*(p_0, p_1)}{\partial p_0} < 0 \quad \text{and} \quad \frac{\partial (r_1^*(p_0, p_1) - r_0^*(p_0, p_1))}{\partial \Delta} < 0.$$

Corollary 3 *The result is due to the fact that higher price of the general magazine p_0 leads to reduced readership of the magazine, which decreases the advertising rate, thus entailing lower price for the industrial product sold to the "general readers".*

Formally,

$$\frac{\partial D_0(p_0, p_1)}{\partial p_0} < 0, \ \frac{\partial s_0^*}{\partial D_0} > 0, \ \frac{\partial r_0^*}{\partial s_0} > 0.$$

4.4.2 The impact of specialization (readers' profile) on advertising rates

The parameter b measures the readers' devotion to the industrial product. This parameter determines the rates for advertising space in each magazine.

It is intuitive that the advertising rate set by a specialized magazine increases with the parameter b, since the magazine anticipates the impact of the high quality of her readers onto the advertiser's profits from product sales. Formally,

$$\frac{\partial s_1^*}{\partial b} > 0$$

There are two effects responsible for the fact that advertising rate in the specialized magazine increases with the parameter b. First, the advertising rate s_1 set by magazine 1, and the product price r_1 for her readers, are strategic complements. Since the product price increases with the degree of readers' devotion, so does the advertising rate. Second, the advertising rate, s_1, is affected by the readership volume D_1, which changes with the parameter b via the magazines' prices, $\{p_\mathbf{0}, p_1\}$. Formally,

$$\begin{array}{ll} \text{Effect 1:} & \frac{\partial r_1^*}{\partial b} > 0, \\ \text{Effect 2:} & \frac{\partial D_1(p_0,p_1)}{\partial b} > 0. \end{array}$$

The direction of Effect 2 is due to the following. The higher the degree of product devotion, the higher prices the magazines set for the readers. However, the general magazine will set a higher price. Formally,

$\frac{\partial p_1^*}{\partial b} > 0, \frac{\partial p_0^*}{\partial b} > 0, \frac{\partial (p_0^* - p_1^*)}{\partial b} > 0.$

Since an increase in b provokes a higher increase in $p_\mathbf{0}^*$ than in p_1^*, and since the readerships are correlated (the size of the readers market is fixed), the readership of magazine 0 will decrease and the one of magazine 1 will increase. Formally,

$\frac{\partial D_0(p_0,p_1)}{\partial (p_0^* - p_1^*)} < 0, \quad \frac{\partial D_1(p_0,p_1)}{\partial (p_0^* - p_1^*)} > 0.$

Note that both effects work in *the same direction*. Moreover, the advertising rate s_1 is affected *positively* by both the product price r_1 and the readership volume D_1. Consequently, the advertising rate set by the specialized magazine increases with the readers' devotion to the industrial product. The following proposition discusses the result.

Proposition 17 *The advertising rate set by a specialized magazine increases with the degree of readers devotion to the industrial product. It is due to the fact that:* (i) *since the product price increases with the degree of readers' devotion, so does the advertising rate, and* (ii) *the advertising rate is affected by the readership volume which changes with the parameter* b *via the magazines' prices.*

Regarding the advertising rate set by the general magazine, it may increase or decrease with the parameter b. Formally, s_0^* decreases with b if $b > 48t$, and increases with b if $b < 48t$. The following figure illustrates how the advertising rates change with b.

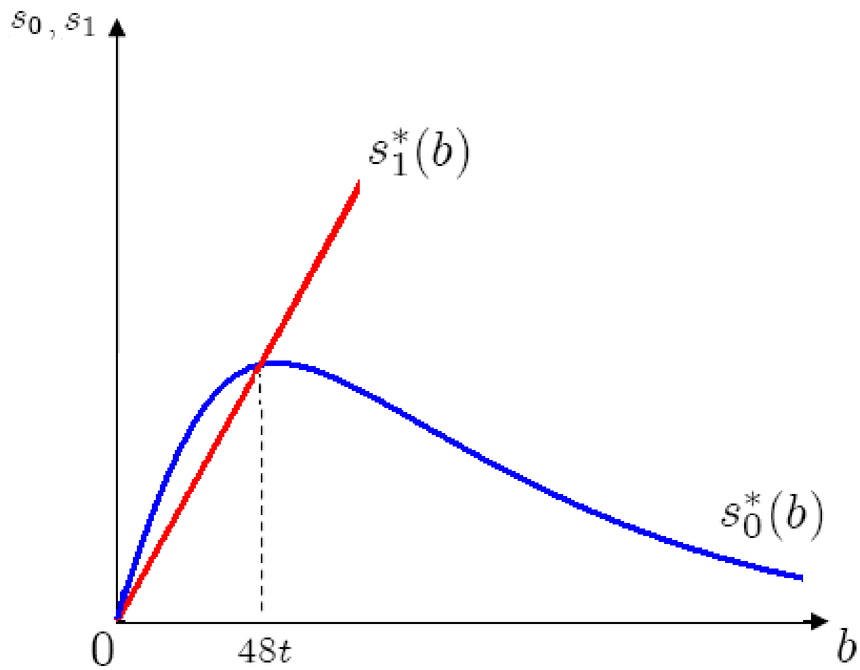

Figure 4.3: The impact of readers' devotion to the product on advertising rates.

There are also two effects responsible for how the advertising rate set by the general magazine changes with the parameter b. First, the advertising rate by magazine 0, s_0, and the product price for her readers, r_0, are strategic complements. Since the product price increases with the degree of readers' devotion, so does the advertising rate. Second, the advertising rate, s_0, is affected by the readership volume, D_0, which changes with the parameter b via the magazines' prices, $\{p_0, p_1\}$. Formally,

$$\begin{array}{ll} \text{Effect 1':} & \frac{\partial r_0^*}{\partial b} > 0, \\ \text{Effect 2':} & \frac{\partial D_0(p_0,p_1)}{\partial b} < 0. \end{array}$$

The direction of Effect 2' is due to the same logic as before. The higher the degree of product devotion, the higher prices the magazines set for the readers. However, the general magazine will set a higher price. Formally,

$\frac{\partial p_1^*}{\partial b} > 0, \frac{\partial p_0^*}{\partial b} > 0, \frac{\partial (p_0^* - p_1^*)}{\partial b} > 0.$

Since an increase in b provokes a higher increase in p_0^* than in p_1^*, and since the readerships are correlated (the size of the readers market is fixed), the readership of magazine 0 will decrease and the one of magazine 1 will increase. Formally,

$\frac{\partial D_0(p_0,p_1)}{\partial (p_0^* - p_1^*)} < 0, \quad \frac{\partial D_1(p_0,p_1)}{\partial (p_0^* - p_1^*)} > 0.$

Note that the two effects work in *opposite directions*. Consequently, depending on which effect dominates the advertising rate set by the general magazine may increase or decrease with the readers' devotion to the industrial product. Recall that the advertising rate, s_0, is affected *positively* by both the product price, r_0, and the readership volume, D_0. Therefore, if Effect 1' dominates then the advertising rate goes up with b. In contrast, if Effect 2' dominates then a higher readers' devotion to the industrial product will make the magazine charge a lower advertising rate for the producer. In other words, an increase in the product devotion increases the product price for general readers and increases the price of magazine 0 which reduces the demand for the magazine. However, when b is sufficiently small, the decrease in the magazine demand is compensated by the increase in the product price, whereas when b is sufficiently large, then the large increase in the magazine price declines the magazine's demand a lot and makes the Effect 2' sufficiently strong to dominate the other effect.

Proposition 18 *When the product devotion is small, then the product price for general readers compensates the effect of smaller readership of the general magazine and decides about the advertising rate. In contrast, when the product devotion is large, then the increase in the price of general magazine is sufficiently large to decline the magazine's demand and overtake the effect of a higher product price for general readers.*

Moreover, the advertising rates are determined by fractions α and β of readers who will purchase the product at a given price. Obviously, the larger the fraction of each readership that is willing to purchase the good, the higher advertising rates the magazines set. Formally,
$\frac{\partial s_0(\alpha)}{\partial \alpha} > 0, \frac{\partial s_1(\beta)}{\partial \beta} > 0.$
Note that in equilibrium, we obtain that α is constant. Hence, when considering the composite of αD_0, only D_0 decides on the direction of the total effect of b on s_0. The equilibrium formula for β does depend on b. However, it is lost in the composite of βD_1 which is constant in equilibrium.

4.4.3 Determinants of price discrimination

Whenever there exists a demand for the specialized magazine, the producer has incentives to discriminate product prices among both types of readers who differ in their demand elasticities for the product. The firm is even willing to pay more per unit of advertising space in the specialized magazine. Accordingly, the producer sets a higher price to 'inelastic" consumers and a reduced price to the "elastic" ones. Formally, $r_0^* < r_1^*$.
In equilibrium we observe that the optimal product price set to the readers of magazine 0 increases with Δ. This means that the higher market share of readers of general magazine (thus the lower the one of the specialized magazine), the less incentives for the producer to use a large discount for the general readers. First, a large price reduction to an important fraction of consumers would decrease total profits from sales. Second, due to the reduced profits from smaller fraction of specialized readers ("inelastic" consumers), the firm increases the price for the elastic segment of demand. Formally,
$\frac{\partial r_0^*}{\partial \Delta} > 0, \frac{\partial (r_1^* - r_0^*)}{\partial \Delta} < 0.$

The reduced price r_0 can be expressed as a fraction of the *full* price r_1. We denote the ratio of the product prices as $\gamma = \frac{r_0^*}{r_1^*}$. Consequently, $1-\gamma$ denotes the % price reduction for general readers. In equilibrium we obtain the following value for parameter γ :

$$\begin{aligned} \gamma &= \frac{8(3t+\Delta)}{b+48t}, \qquad (4.19) \\ \text{that is, } (1-\gamma) &= \frac{b+24t-8\Delta}{b+48t}. \end{aligned}$$

We can observe that if parameter Δ goes up, then the price reduction goes down. This means that lower market share of readers of specialized magazine decreases the total revenue they bring to the producer. Hence, the firm increases the price (i.e. decreases the price reduction) for general readers if they become more numerous.

Proposition 19 *An increase in the readers' preference for the general magazine makes the producer less willing to use price discrimination, that is, it is less worth it to offer large price reduction to more numerous general readers.*

Moreover, if parameter b goes up then the price reduction goes up. This means that when more people are devoted to the product, the firm increases the price r_1^* more than r_0^*. Hence, the % price reduction for general people is larger. Formally,
$\frac{\partial r_0^*}{\partial b} > 0$, $\frac{\partial r_1^*}{\partial b} > 0$, $\frac{\partial r_1^*}{\partial b} > \frac{\partial r_0^*}{\partial b}$,
$\frac{\partial \gamma}{\partial b} < 0$, $\frac{\partial (1-\gamma)}{\partial b} > 0$.

Proposition 20 *An increase in the product devotion amplifies the price discrimination, that is, it increases the product price reduction offered to the readers of the general magazine.*

4.5 Discussion

The analysis has been aimed at understanding the determinants of pricing strategies of advertising space in magazines which differ in the degree of content specialization, ergo in the profile of their readers. Moreover, we have sought to identify the link between the pricing strategy of the industrial product of an advertiser and the advertising rates. We have shown that the producer faces a trade-off between the large readership size and the interesting profile of a reader. In addition, we have demonstrated how the trade-off for the producer depends on the readers characteristics' and their devotion to the consumption of industrial product.

More specifically, we have provided two determinants of the advertising rates: (i) the readership size and (ii) the readers' profile. Regarding the former effect, we have obtained that the more preference the readers located on a unit line exhibit towards the general magazine, the larger the readership of the magazine provoking a higher advertising rate, thus the producer will increase the price for the industrial product sold to the "general" readers. Regarding the latter determinant of the advertising rates, its impact somewhat differs for each magazine. The advertising rate set by the *specialized* magazine increases with the degree of readers devotion to the industrial product. The advertising rate set by the *general* magazine also depends on the product devotion, yet in a non-monotonic way.

Regarding the determinants of price discrimination, we have analyzed the impact of both determinants of product prices onto the incentives to price-discriminate. We have obtained that, on the one hand, an increase in the readers' preference for the general magazine makes the producer less willing to use price discrimination, that is, it is less worth it to offer large price reduction to more numerous general readers. On the other hand, an increase in the product devotion amplifies the price discrimination, that is, it increases the product price reduction offered to the readers of the general magazine.

4.6 References

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