École Polytechnique
Département d'Économie

Thèse

Pour l'obtention du grade de
Docteur de l'École Polytechnique

Discipline : Sciences Économiques

Présentée et soutenue publiquement par

Claudia Saavedra Valenzuela

le 30-IX-2010

Essais d'Économie des Télécommunications
Essays on Telecommunications Economics

Directeurs de thèse :

Rida Laraki

Jean-Pierre Ponssard

Jury:

Marc Bourreau	Professeur, Télécom ParisTech
Marc Lebourges	Directeur Département Europe Économie, France Télécom
Jérôme Pouyet	Professeur associé, Paris School of Economics (rapporteur)
Wilfried Sand-Zantman	Professeur, Univesrité de Toulouse I (rapporteur)

L'Ecole Polytechnique n'entend donner aucune approbation, ni improbation, aux opinions émises dans les thèses. Ces opinions doivent être considérées comme propres à leur auteur.

Acknowledgements

I wish to express my gratitude to my supervisors, Rida Laraki and Jean-Pierre Ponssard. To JPP for introducing me to economics during the masters course and for being a great example of what an economist should be. I especially acknowledge the enthusiastic supervision of Rida, he has made available his support in a number of ways, from providing great technical advice to practical lifestyle tips.

I sincerely thank the members of the jury, Jérôme Pouyet, Marc Bourreau, Marc Lebourges and Wilfried Sand-Zantman for taking the time to read the manuscript and for their kind and constructive comments. It has been an honor to have you as jury.

The research presented in this thesis has been carried out with the financial support of Orange France Telecom Group, which is gratefully acknowledged. I owe my deepest gratitude to Marc Lebourges for many enlightening discussions, he has inspired some of the topics of the thesis, he is an outstanding and accomplished professional. A special thought goes to Antonin, Bertil, Christian, Jean-Samuel, Kevin and Romain, peers at Orange Labs with whom I shared my first years of research, and to Volcy, Julienne and François for their support and friendship at the Regulatory Affairs Department.

I am indebted to my many student colleagues of the Economics Department at the Ecole Polytechnique, Arnaud, Clémence, Delphine, Guy, Idrissa, Jean-Philippe, Julie, Matías, Marc, Nicolas, Pierre, Sabine, Thomas and many others for providing a stimulating and fun environment. Many thanks to Marie-Laure Allain, Claire Chambolle, Jean-François Laslier, Saïd Souam and Sylvain Sorin, for interesting discussions and/or courses. I am also grateful to the assistants in the department, Anh Dao, Eliane, Lyza, Chantal and Christine for helping the department to run smoothly and for assisting me in many different ways.

I am extremely grateful to my parents, Carmen Lucy and Javier, to my little brother JV, and to my family and friends in Bolivia for believing in me and for their support from the distance. I would like to thank the many people who have taught me mathematics back home, a warm thought goes to Hans Müller and don Roberto Zegarra, a first source of inspiration and inclination for the academic path.

Finally, I would like to thank Stéphane. You have lived with me the ups and downs of the thesis with such a graceful patience, giving me the unconditional and constant encouragement needed to make of these years the best of my life. I love you so soo much!

Claudia

General presentation

This thesis contributes to three topics in the regulatory economics of telecommunications. The first chapter explores the debate of network neutrality regulation. It studies the consequences of a regulation when a "must have" content provider is willing to enter in joint investment agreements with Internet service providers. The second chapter investigates investment on next generation access networks in a regulated environment. It considers a framework where returns on investment are uncertain. It illustrates how access contracts with commitment clauses can be more efficient than plain usage-based access charges. The third chapter analyzes investment when technological progress is endogenous. It studies dynamic process innovation in market where n firms can reinvest present profits to reduce future costs. It develops a differential game to capture dynamic effects and describes the role of imperfect competition for technological progress.

Each chapter is summarized as follows:

The network neutrality chapter considers a scenario where Internet access providers are able to negotiate joint investment contracts with a valuable Internet content provider in order to enhance the quality of service and increase industry profits. It builds a model that studies the effect of a net neutrality regulation that would impede joint investments. The analysis shows that an unregulated regime results in higher quality investments, but it also allows access providers to degrade content quality compared to the net neutrality quality level. This might encourage content providers with low bargaining power to enter into exclusive deals as a way of improving their bargaining position instead of choosing a global quality increase. In spite of the welfare loss with exclusivity, consumer surplus is higher under the unregulated regime.

Investment into next generation access networks is characterized by high uncertainties. The second chapter points out that this must be taken into account by regulatory authorities that aim to promote competition without hindering investment incentives. As mandated access to new infrastructures asymmetrically allocates risk on leading investors, mandated access creates a second-mover advantage that can discourage infrastructure roll-out. This chapter builds a model to show that *(i)* richer forms of access contracts encompassing commitment clauses between firms can overcome the investment hold up and *(ii)* they can be more efficient than plain wholesale linear prices in adverse market configurations as they induce a more symmetric allocation of risk.

The third chapter investigates dynamic process innovation in a product differentiated market where n firms can reinvest present profits to reduce future costs. The main focus is set on the market performance when the number competing firms is determined by a social planner. The main contribution consists in developing a differential game to corroborate the central role of imperfect competition on optimal investment highlighted by the industrial organization literature. It is found that increasing the number of firms reduces process innovation, whereas raising the degree of product substitutability increases it. The dynamic approach allows for a clear distinction between the positive effect of increasing the number of firms on static welfare versus the dynamic efficiency loss due to reduced process innovation. It further characterizes the optimal market structure.

Contents

Introduction

The telecommunications industry has been and it will apparently continue to be a heavily regulated industry.

From its inception and for many years, a state-owned or private monopoly provided telecommunications services. The existence of large fixed cost in the industry and the economic impossibility to replicate several infrastructure segments needed to provide telecommunications services justified the monopolistic market structure. The "natural monopoly" was regulated according to rate-of-return schemes where the prices for its services were adjusted so it was permitted to keep all earnings it generated, provided the return on capital was sufficiently close to a specified rate of return target. This scheme proved to sacrifice economic efficiency in two aspects: First, the monopolist had little incentives to reduce its costs. And second, its prices were determined through arbitrary procedures that impeded flexible and innovative pricing structures given that prices for individual services did not need not equal the costs of individual services.

Incentive regulation was then progressively adopted for the industry. One of the most common incentive regulations schemes consists in determining a price cap,[1] that is an average price level, for a basket of services. To the extend that price caps resolve the efficiency shortfalls of rate-of-return regulation, they manifested anticipated limitations. Regulator's imperfect information about the monopoly's actual costs raises concerns in terms of regulator's credibility. High price caps are difficult to sustain from a political point of view and negative revenue balances forces an increase of price caps. Furthermore,

[1]Price cap regulation was designed in the 1980s in the UK to be applied to all of the privatized British network utilities.

the quality of service becomes a concern under price cap regulation. High incentives to reduce costs frequently result on reduction of the quality of services provided by the monopolist.

The constant materialization of technological progress in the telecommunications sector revealed the feasibility of infrastructure competition is some segments of the industry. This encouraged the opening to competition to those segments and to the beginning of the liberalization process across Europe and the US.

Reinforced sector-specific regulation seemed pertinent for accompanying the liberalization process. There still existed infrastructure segments that were considered not replicable,[2] such as the local loop, operated and possessed by the former monopolist to which entrant firms would need to have access to compete in the market. It followed that, with the objective to create a balanced and adequate level playing field for entrants and to do it in a timely manner, policy makers opted for asymmetric ex ante regulation that among other competences would set the charges that give access to the incumbent's bottleneck infrastructure segments. For example, the Directorate-General for Information Society and Media (DG Infso) of the European Commission sets directives to Member States for the access rules to incumbent firms' infrastructures which are periodically revised according to the evolution of the market.[3]

Setting regulated access charges constitutes a difficult task. As mentioned, several factors such as asymmetric information on firms' costs and political discretion imply imperfect regulation. But even if the issue of access to infrastructures could also be treated by competition policy, regulating access charges has been and it is currently at the center of regulatory activity in many countries today. It will continue to be the trend in the future.

Policy makers affirm that competition by infrastructures is the ultimate goal for long term efficiency in the telecommunications market. However, economic analysis

[2] At least in the short term.

[3] The current access Directive (2002/19/EC) went under revision from 2007 to 2009 (the transposition to national law is expected in 2011) and it evolved from rules established in the early 1990s.

highlights that granting access through low charges to incumbent's infrastructures most likely dissuades competitors' incentives to build their own infrastructure. In consequence, regulatory schemes aimed at creating investment incentives in infrastructure have been put in practice. In Europe the theory of the "ladder of investments"[4] aims at using the access charge as a tool to achieve two objectives: to allow entry by competitors an create incentives for competitors to progressively invest in infrastructure.

As technology continues to evolve, new network infrastructures are intended to progressively replace today the old natural monopoly's legacy. The deployment of fiber optic local loop networks would bring greater capacity than the one inherited by the copper access network. The role of incumbent and entrant operators begins to blur. They both have now the possibility to build the new infrastructure from scratch. Yet in Europe policy makers have announced that symmetric regulation and access obligations to whom deploys the network first will apply. In particular, the framework of symmetrical access obligations in urban and rural areas is currently being designed by the French regulatory authority.[5]

This short historical description of different regulatory schemes through time, with focus on access charges, aimed at illustrating our initial statement : The telecommunications industry has been and it will continue to be a heavily regulated industry. Besides, access and interconnection issues account only for a share of regulator's concerns. Issues dealing with universal service and users' rights, such as privacy, relating to electronic communications networks and services appear to be gaining importance in regulators' activities.

Economists have studied the regulatory reforms in telecommunications in order to better understand the impacts of such reforms in the development of the industry.[6] Policymakers have had the opportunity of counting on the analysis provided by economic

[4]See Cave's "Encouraging infrastructure competition via the ladder of investment", published in *Telecommunications Policy* (2006).

[5]See for example ARCEP's decision N. 2009-1106 concerning urban areas.

[6]J. J. Laffont and J. Tirole's *Competition in Telecommunications*, or *The Handbook of Telecommunications Economics* edited by M. Cave, constitute two reference of the early literature.

theory. And inversely, the sector has largely contributed to the development of new and exciting areas in theoretical industrial organization. For instance, concepts like network externalities, switching costs or two-sided markets are inherent to telecommunications services.

Several elements differentiate economic analysis for telecommunications from more "classical" industries. Specifically, the fast rate of change in technology and consequently in the production functions, the convergence of technologies (the boundaries between telephone, internet, television broadcast and mobile phone services are becoming blurred!), and even classical notions such as the "the marginal cost of production" are particular in network industries and they need to be carefully applied in these sectors.

Hence, two reasons motivate a dissertation on regulatory economics of telecommunications. First, to continue to study the impact regulatory reforms have on the development of the industry as technology continues to develop. And second, to contribute to the evolution of the diversification of economic theory as it is interesting in its own right.

In particular, sectors that were not typically regulated such as the Internet and the relationships between content providers, Internet service providers and backbone transit operators have lately entered into the spectrum or regulatory debate and call for guidance of economic theorists. The question whether to regulate the Internet introduces the first chapter of the thesis, the network neutrality debate.

Network Neutrality

The Internet was designed so data carriers would treat data traffic equally, this is without making any difference from the content traffic carried, in a *neutral* way. Throughout the past five years or so the neutrality principle has been challenged at several levels. Let us describe some of the episodes of discrimination.

The first public allegations of the neutrality infringement started in 2004 as some Internet service providers (ISP, this is telecommunications operators providing connectivity access to the Internet) were proved to block and degrade the transmission of third party Internet applications and content.

In the United States, Madison River a telephone company that also provided access to the Internet blocked applications offering voice over Internet Protocol (VoIP).[7] This technology allows to place – generally cheaper – voice calls using the Internet as the base platform. The company was said to have incentives to block VoIP services in order to preserve revenues for its own telephony services.

More recently Comcast, a company that provides Internet access over its cable television network, was investigated and found responsible for interfering with file sharing peer to peer (P2P) applications.[8] The P2P technology allows the exchange of data over the Internet. It is popularly used to exchange music and movies, and it is known to employ great bandwidth capacity. Again, as with Madison River, the motivation of the company to block P2P was said to originate from the cannibalization of revenues of its content division. The company explained that the blocking of P2P was made for efficiency purposes given the congestion caused by the application.

In 2005 the Federal Communications Commission (FCC) reclassified the wireline Internet access services as an information service, removing common carrier restrictions for telephone companies providing access to the Internet. This deregulation in the US opened the possibility for ISP to discriminate or differentiate content transmitted over the Internet.

In that same year, some network operators made public their intention to further charge Internet content providers for network usage. A well known example is the statement by AT&T's former chairman Edward Whitacre:[9]

> Now what they [content providers such as Google, Yahoo, etc] would like to do is use my pipes free, but I ain't going to let them do that because we have spent

[7] Cf. http://hraunfoss.fcc.gov/edocs_public/attachmatch/DA-05-543A2.pdf

[8] Cf. http://fjallfoss.fcc.gov/edocs_public/attachmatch/DOC-284286A1.pdf.

[9] Interview by Businessweek http://www.businessweek.com/magazine/content/05_45/b3958092.htm

> this capital and we have to have a return on it. So there's going to have to be some mechanism for these people who use these pipes to pay for the portion they're using. Why should they be allowed to use my pipes? The Internet can't be free in that sense, because we and the cable companies have made an investment and for a Google or Yahoo! or Vonage or anybody to expect to use these pipes [for] free is nuts!

ISPs were then discussing the possibility of offering online applications and content providers differentiated quality of service in order to increase revenues. Quality of service (QoS) is the ability to provide different priority of transmission to online content, which can guarantee a certain level of performance to a data flow.[10]

For the most part, content providers reject the idea of ISP offering differentiated quality of service. They fear that as gatekeepers of final customers they use their market power to benefit one content over another, distorting competition in the online content market. But they do seem to agree with the practice of differentiating content in the upstream part of the Internet. In effect, the Internet traffic had already been treated with different grades of service. As early as 1998 content providers have been employing Content Delivery Networks (CDN) to improve the performance of their content transmission over the Internet. For example, companies like Akamai Technologies (with an annual revenue of $859.8 million in 2009) or Limelight Networks (revenue of $129.53 million in 2008) specialize in the optimization of the transmission of content on the Internet.

More recently, in 2008 Google was accused of violating the neutrality principle as the company approached major ISP in the US to implement a program, called OpenEdge, to enhance the transmission of its content.[11] Google's proposed arrangement with ISP would place Google servers directly within the network of ISP. This agreement takes the shape of a joint investment that would accelerate Google's service for users increasing the quality of it service and hoping for larger advertising revenues.

---

[10] The term QoS is commonly used in the field of computer networking.

[11] See the story at `http://online.wsj.com/article/SB122929270127905065.html`.

As it has been described, industry players can violate the *neutrality* of the Internet by: blocking online applications and content (Madison River and Comcast), using network management to deal with network congestion (Comcast), using network management to provide higher quality of service for content (CDN like Akamai), differentiating content by investing in capacity (Google's OpenEdge program), or by simply asking a higher compensation for the connectivity service (AT&T former CEO).

In general terms, the debate opposes two groups in the industry: one group that promotes net neutrality (NN) regulation to keep the Internet *neutral* and the other side that opposes any regulation. The first group is composed of online content and application providers, and nonprofit and consumer organizations. The second group is composed of network operators, this is ISP and long distance transit operators, and equipment manufacturers.

The following table resumes arguments that advocates and opponents of a network neutrality regulation.[12]

Advocates in favor of NN concern of	*Opponents of a NN regulation argue*
· blockage, degradation, and prioritization of content and applications	· the Internet is not neutral and never truly has been, and a neutrality rule would effectively set in stone the status quo and preclude further technical innovation · effective network management practices require some data to be blocked altogether · there are efficiencies and consumer benefits from data prioritization · new content and applications also require this kind of network intelligence
· vertical integration by network operators into content applications	· vertical integration by network operators into content and applications and certain bundling practices may produce efficiencies that ultimately benefit consumers

[12]Inspired from a report by the FTC staff after an organized debate on network neutrality in 2007: *Broadband Connectivity Competition Policy*. Available at `www.ftc.gov/reports/broadband/v070000report.pdf`.

· effects on innovation at the edges of the network (innovation by content and applications providers)	· network operators should be allowed to innovate freely and differentiate their networks as a form of competition that will lead to enhanced service offerings for content and applications providers and other end users
· lack of competition in the last-mile broadband services	· there is insufficient evidence of potential harm to justify an entirely new regulatory regime, especially when competition in broadband services is robust and intensifying and the market is generally characterized by rapid, evolutionary technological change.
· legal and regulatory uncertainty in the area of Internet access	· prohibiting network operators from charging different prices for prioritized delivery and other types of quality-of-service assurances will reduce incentives for network investment generally and prevent networks from recouping their investments from a broader base of customers, a price which, in turn, reduce prices for some end users
· diminution of political an other expression on the Internet.	

Table 1: Comparing the arguments for and against regulation.

It is not difficult to notice that the early debate has been carried out in a framework where the debating parties did not actually convey on what exactly such regulation would consist of. The truth is that network neutrality means different things to different people, it depends to who you ask. A modern approach to establish a definition of such rules would be to consult what industry actors have stated on Wikipedia.[13] We can see that there are different "levels" of network neutrality:

> . . . At its simplest, network neutrality is the principle that all Internet traffic should be treated equally. Net neutrality advocates have established different definitions of network neutrality:
>
> **Absolute non-discrimination** Columbia Law School professor Tim Wu: "Network neutrality is best defined as a network design principle. The idea is that a maximally useful public information network aspires to treat all content, sites, and platforms equally."

[13] See http://en.wikipedia.org/wiki/Network_neutrality

Limited discrimination without QoS tiering United States lawmakers have introduced bills that would allow quality of service discrimination as long as no special fee is charged for higher-quality service.

Limited discrimination and tiering This approach allows higher fees for QoS as long as there is no exclusivity in service contracts. According to Tim Berners-Lee: "If I pay to connect to the Net with a given quality of service, and you pay to connect to the net with the same or higher quality of service, then you and I can communicate across the net, with that quality of service." "[We] each pay to connect to the Net, but no one can pay for exclusive access to me."

First come first served According to Imprint Magazine, University of Michigan Law School professor Susan P. Crawford "believes that a neutral Internet must forward packets on a first-come, first served basis, without regard for quality-of-service considerations."

However, what interests us is the economic analysis of network neutrality regulation. The study of such a regulation from an economics point of view is still progressing. We can relate the question of network neutrality regulation to issues already studied by classical economics such as vertical foreclosure, price discrimination, quality differentiation, two-sided markets pricing, or investment incentives and the hold-up problem.

Economists have divided the analyses of network neutrality regulation into two categories. First, a regulation that would prohibit ISP from directly charging content providers for the transmission of their content (like a termination access charge) referred as the *zero-pricing rule*. Second, a regulation that would prohibit ISP from proposing different quality of services and prioritizing traffic over their networks, referred as the *non-discrimination rule*.[14] The introductory section of Chapter 1 details the most relevant articles so far published by economists.

Chapter 1 of the thesis can be classified within the non-discrimination rule. *Bargaining power and the net neutrality debate* proposes to study a framework where, as in the OpenEdge project of Google, a "must-have" content provider would like to enter into joint investment agreement with two competing ISP in order to increase the quality of its

[14] This classification was first proposed by F. Schuett (2010), "Network Neutrality: A Survey of the Economic Literature", *Review of Network Economics*, 9(2).

online content to, at the same time, raise content consumption and hence its advertising revenues. A net neutrality regulation would prohibit such vertical agreements.

One of the issues of paramount importance in such configuration is the exclusivity of content. The Internet has characterized itself so far as an open environment, where all content is provided to consumers but also where all ISP can provide all the available content. What would be the effect of such a regulation? On the one hand, prohibiting vertical agreements would leave the Internet in its actual state where the must-have content is available to all consumers in equal terms. On the other hand, regulation would impede investments that would increase the quality of a valuable content but at the risk of exclusive vertical agreements.

Therefore, the Chapter proposes to examine the effects of the bargaining conditions between parties in a non regulated environment. In particular, it studies the impact of the bargaining power of the content provider together with the degree of competition in the Internet access market over the joint investment agreement or agreements. And it compares this outcome with the regulated environment outcome where each ISP sets the quality of the content when they invest by their own means. Investment can be interpreted as the increase of the capacity of their respective networks.

As it can be noticed in the Table 1 above, investment in networks occupies a central role in the network neutrality debate. Investment in network infrastructure, in particular in the next generation access technologies, introduces the second chapter of the thesis.

Next generation access networks

The second Chapter, *Investment with commitment contracts: the role of uncertainty*, proposes to explore the incentives to invest in a new network infrastructure when firms are subject to ex ante access regulation.

As previously discussed in this introduction, the current local loop telephone network, or access network, is an heritage of the formerly monopoly telecommunications operator. The liberalization process of the telecommunications industry was achieved thanks to regulatory reforms that mandated access, at a regulated price, to the monopoly infrastructure. Today, the increasing usage of the Internet and the evolution of technical progress request the upgrade of the old access network. It is expected that *next generation access networks* (NGA networks), most likely constituted by fiber-based networks, replace the inherited copper access network.

European policymakers have announced that it is a priority for the industry to avoid the establishment of a new monopoly.[15] Consequently, policy measures are being taken to prevent a new infrastructure monopoly. Specific ex ante access obligations will apply to firms with significant market power over a new infrastructure. Which implies that firms are certain when taking investment decisions that they will have to open their network to competitors. As it has been pointed out by economists, ex ante access obligations clearly discourage investment incentives.

The question is here how to conciliate allocative efficiency (through the promotion of service competition) and dynamic efficiency (with the creation of investment incentives) in this new market? As previously discussed, this is not a new question to the sector. In Europe, policymakers have previously tackled this issue with the theory of the ladder of investment.[16] But in that context the industry counted already with one infrastructure. The question was rather pointed to the incentives to invest for entrants.

The NGA presents the problem in a somewhat symmetrical perspective: Both the (now former) entrants and the incumbent have the possibility to invest and deploy a NGA network.

Another ingredient on top of the static-dynamic tradeoff that seem to be relevant in this context is firms' uncertainty about future profits. The retail broadband and previously dial-up access market developed in the past thanks to the content available on the Internet. At first, content and applications like web-surfing and e-mailing were the first drivers of the adoption of dial-up connections. The broadband technology was later

[15] See the Commission's Recommendation on regulated access to Next Generation Access Networks.

[16] Although the theory of ladder of investment has been critically questioned, see for example Bourreau M., Dogan P., and Manant M. (2010) "A critical review of the 'ladder of investment' approach", *Telecommunications Policy*, 34.

adopted because new applications and content requiring greater bandwidth emerged in the content market. For instance P2P applications, streaming video, etc. Today the situation is somehow different. Operators seem to be hesitant about the content drivers for the migration of users from broadband to super fast broadband Internet access. In short, there exists incertitude about the profitability of the market.

The objective of the Chapter is then to study incentives of private investment when firms are subject to ex ante access obligations and when future market profits are uncertain. The Chapter builds a simple game theoretical model of two ex ante symmetric firms in order to study the effects of a regulated access charge on the market outcome. Before making investment decisions, the regulator announces the level of the access charge that the firm without a network will have to pay in order to compete with the one that has built one.

A central element of the analysis resides on the fact that the profitability of the market remains unknown until at least one of the firms has decided to incur the sunk cost of a network. As the rival can always wait for the market profitability to be revealed, it has naturally incentives to behave in an opportunistic way before incurring the sunk cost itself.

The Chapter aims at proving that richer forms of access contracts between firms can attenuate the opportunistic behavior and increase investment incentives. In particular, access contracts with commitment clauses between firms where one firm commits to buy access from the other one independently of the market profitability better allocate the risk of investment between both firms.

As it will be developed throughout the Chapter, the industrial organization literature on the impact of access regulation on broadband investment takes three approaches. Fist, a body of literature focuses on asymmetric frameworks where an entrant who enjoys regulated access to the incumbent's network invest and builds its own network. A second body of literature studies the incentives of an incumbent to upgrade its network when subject to mandated access. And finally, the third body of literature, closer to this Chapter, focuses on a more symmetric situation where both the former incumbent and entrant must build the network from "scratch". In this literature, inspired from the patent race work, the role of the incumbent is endogenous. The object of study is then to determine the investment date when firms know that the sunk cost of the network deployment decreases in time as a result of technological progress.

As in the patent rase literature, cost reduction is the outcome of an technological progress that is exogenous to the model. But actually, and in particular for the telecommunications industry, the progress of technology is rather an endogenous process. Firms reinvest their profits in R&D in order to lower their costs. This is the motivation that leads us to the third Chapter of the dissertation.

Technological progress

The objective of the third Chapter is to revisit the link between competition and innovation in a rather original framework. Two elements motivate a revisit of this classic question.

The first element comes from personal amazement. One can effortlessly observe the constant technological progress in the information technology industry and in particular in the telecommunications sector. A recent study has collected 100 and plus years of data to measure and compare the technological progress in these industries.[17] The study quantifies our observation: the annual progress rate ranges from 20% to 40%!

One can conclude that if studied from a long term point of view, it is pertinent to consider technological progress and innovation as an incremental process rather than a disruptive process as typically addressed by the patent literature.

The second element comes from the particularity of regulated sectors. In many markets and specifically in the mobile telephony market, entry is limited by a social planner. By limiting access to an activity, the social planner has the ability to artificially create

[17] See Koh and Magee (2006), "A functional approach for studying technological progress: Application to information technology", *Technological Forecasting & Social Change*, 73(9).

scarcity which increases market prices. By the same mechanism, a social planner can increase the number of firms in the market and induce a reduction of prices redistributing resources from firms to consumers. We have repeatedly learnt that perfect competition is the optimal market performance. But this is true only in situations where technological advance is unaffected by resource allocation.

It is then important for a social planner to consider which is the optimal level of competition that maximizes welfare in an industry where technological progress is endogenous and incremental.

The question of market structure and investment has been long treated by economists going back to Schumpeter and Arrow. Economics literature on the subject is more polemic than consensual. Empirical economists make the distinction among two different questions: the impact of market structure on innovational *effort* and on innovative *results*.[18] The Chapter and our interest focuses on the former question accepting that greater efforts lead to more innovative results.

Early empirical literature examining the impact of market concentration on research effort has found varying results and little consensus.[19] A share of the literature finds that firms in concentrated industries spend more on research activities whereas exactly another portion find insignificant and even a negative relationship. The variation of results can be attributed to diverse factors. This might be because the empirical tests where realized in different countries, but also there exist deficiencies on the proxies for calculating innovational efforts.[20] A further caveat of empirical studies resides on the fact that standard measures of market concentration quantify only current active firms in the market, hence it might omit potential rivalry and dynamic strategy motives.

[18]It has been observed that smaller firms have more flexible organizational structures which makes them more dynamic and efficient when transforming innovative effort into innovative results.

[19]For a detailed review of the literature see Kamien M. I., and Schwartz N. L. (1975), "Market Structure and Innovation: A Survey", *Journal of Economic Literature*, 13(1).

[20]Usual measures of innovative effort include R&D spending or scientific employees, yet technological improvements are not always made in R&D departments. Further, this measures can be affected by institutional bias. For example, if public policies cut taxes for R&D spending firms might have incentives to blow research spendings out of proportion.

More recently,[21] economists have found that the relationship between competition and innovation is not linear. By plotting a weighted measure of patents against the Lerner index they conclude that an inverted-U best suits the relationship. That is, mildly concentrated industries innovate more. If more innovation implies higher welfare, one can agree that moderately concentrated markets are preferable.

On the theoretical approach the literature has mostly concentrated, as mentioned, on the questions of disruptive innovative process. The originality of this Chapter is then to study this question using game theoretical tools that allow for the explicit account of the incremental nature of technological progress.

The Chapter builds a differential game where n firms invest profits gained today in order to reduce costs incurred tomorrow. The number of firms active in the market is, as in the mobile telephony sector, set by a social planner. As the recent empirical literature, the Chapter highlights the role of imperfect competition in innovation. The differential games approach allows for a clear study of the dynamic effects of competition (investment) versus the static effects (allocation of resources).

Each chapter of the dissertation is self-contained. In order to facilitate its reading, mathematical demonstrations have been arranged in appendices at the end of each chapter.

[21] A well know recent reference is Aghion et al (2005), "Competition and Innovation: An Inverted-U Relationship", *Quarterly Journal of Economics*, 120(2).

Chapter 1

Bargaining power and the net neutrality debate

1.1 Introduction

The Internet was conceived more than 30 years ago to treat all data traffic equally, in a *neutral* way. During these last years this neutrality principle has been challenged at several levels which has led to a debate on whether this original neutrality principle of the Internet should be preserved.

The resulting ongoing debate, referred as *network neutrality*, encompasses the possibility of regulating the Internet in order to maintain the neutrality principle. Different degrees of regulation have been proposed in order to either penalize content blocking, to avoid further content providers taxation by network operators or even to prevent quality of service agreements. The debate clearly opposes two groups in the industry: the side that promotes a regulation regrouping online content and application providers with nonprofit and consumer organizations against network operators and equipment manufacturers who oppose it. In short, the former argue that letting operators have control over online content would distort the Internet content market. The latter oppose to regulation claiming that the Internet has proven to be competitive and that further gains on content quality of service improvement and network investment could be attained without regulatory constraints.

As this paper is written there is formally no regulation that forbids access providers from offering quality enhancement contracts. Competition authorities have stated that blocking online content will not be allowed except for reasonable network management and accordingly all content blocking has been penalized ex post. But all attempts to pass legislation in the US have so far failed.[1] In Europe there is no specific legislation on net neutrality, quality differentiation provision is theoretically allowed as long as this does not lead to anticompetitive effects. The possibility of offering priority services is still open as well as efforts to regulate the market.

The economics literature on network neutrality has focused so far on different and scattered issues. This is probably due to the complexity of the ongoing debate. At this stage, net neutrality means different things to different people. Nonetheless several aspects of the debate find their parallel with more classic economics literature.[2] These include vertical foreclosure, price discrimination, quality differentiation, two-sided market pricing, investments incentives and the hold-up problem.

Hermalin and Katz (2007) apply the theory of product-line restriction to the net neutrality debate. They consider an access provider that brings together consumers and a continuum of content providers. Under an unregulated regime, the platform can sell different levels of quality to content providers, opposed to a net neutrality regime, where the platform chooses a unique quality level for all content. They find that with net neutrality the platform sets a quality service that gives rise to three effects: low-value content providers, who would had otherwise bought low quality from the platform, are excluded from the market; mid-value content buy higher quality services; and high-value content buys lower quality. Net welfare effects of net neutrality are likely to be negative. Moreover, net neutrality would harm low-value or small content providers rather than protect them, leaving consumers with fewer content which opposes the objective of the regulation proponents.

Economides and Tåg (2007) take a two-sided market pricing approach and they

[1]For example Bill numbers H.R. 5252, H.R. 5417, S. 2686, etc.

[2]For a global analysis refer to Kocsis and de Bijl (2007).

suppose, unlike Hermalin and Katz, that all content is homogeneous. They compare welfare in the case where the platform can charge content providers with the regulated net neutrality regime where the platform can only charge consumers. They find, contrary to Hermalin and Katz (2007), that welfare increases with regulation. If content providers are charged, the two-sided pricing mechanism lowers consumers' access prices but reduces the available content providers in the market, which reduces overall welfare.

Choi and Kim (2010)[3] analyze the impact of vertical quality of service agreements on investment incentives. They consider an Internet access provider who has the ability to offer priority transmission in a congested network to two competing content providers. In a net neutrality regime both content providers receive the same treatment and hence compete for consumers in equal terms whereas in the unregulated regime the traffic of one content provider is favored over the traffic of the other one. In this framework, they find that content providers face a prisoners' dilemma to pay for priority and they are worse-off without net neutrality. The Internet access provider faces a trade-off, either it invests in expanding its capacity to have a larger revenue from consumers or it maintains scarcity in order to charge content providers for priority. Overall welfare implications are not clearcut, however they show that network investment incentives could be higher under net neutrality, which contradicts the arguments of the opponents of a regulation. On the other hand, investment implications are clear for content providers, the authors affirm that these are undoubtedly reduced under the unregulated regime because the network operator has the power to expropriate content providers' rents.

In general terms, the net neutrality debate has been often perceived as a trade-off between innovation at the edges of the Internet (innovation of content providers) and investment within the Internet (infrastructure investments). But profits, and therefore incentives to innovate and invest, of content and access providers are not necessarily opposing. With a better quality of service online content is improved and generates two sources of revenue, it increments advertising revenues and consumers' willingness to

[3]The forthcoming article of Cheng et al. (2011) study a similar model obtaining similar results.

pay for Internet access. Hence, there might exist a mutual benefit to invest if contracts between content and access providers are agreed before investment, avoiding the hold-up problem encountered by Choi and Kim (2010).

Adding to the existing literature, this paper considers a scenario where a content provider negotiates with two competing Internet access providers for an investment contract to improve the existing quality of service. It complements the existing literature as it focuses on the effects of departing from net neutrality in a context where the bargaining power is no longer concentrated on the Internet access providers' side. This approach is relevant as we observe an increasing concentration in some sectors of the online content provision.[4]

The net neutrality regime is understood as a situation where network operators and the content provider cannot enter in any form of quality agreement. In this regime the network operators invest on the quality of service by their own means. Whereas in the second one, the unregulated regime, the content provider can participate in the investment process by negotiating quality contracts with one, both or none of the network operators. The paper focuses on the effects the content provider's bargaining power has on the quality of service agreements, comparing investments outcomes in each regime.

To this end, two specific features of the Internet are taken into account. First it is supposed that *global connectivity* on the Internet allows consumers to have access to the content even if the content provider does not have an agreement with their network operator. And second, Internet access providers have *control over the last-mile*, or the bottleneck to consumers, which allows them to unilaterally control the consumers perceived quality of the content.

Not surprisingly, allowing contractual relations between content and access providers yields higher investment on quality. Improved quality of content results in more content consumption, bringing higher advertising revenues that are used to cover investment

[4]This is turn has led to heterogenous interconnection agreements between content providers and network operators, favoring big providers, see for example `http://www.wired.com/epicenter/2009/10/youtube-bandwidth/`.

costs. At the same time an unregulated regime might as well lead to adverse effects. If a network operator is excluded from the negotiation process while the other network operator reaches an agreement with the content provider, the excluded operator has incentives to degrade the quality of the content. In effect, the excluded operator does not count with the financial support of the content provider whereas its competitor does which allows him to set a quality superior to the one with independent investment. The quality difference shifts the demand for Internet access towards the operator providing the high-quality content. Hence, facing this disadvantage and anticipating lower retail revenues the excluded operator prefers to degrade the quality of content given that quality is costly.

The ability of access providers to lower content's quality of service has consequential strategic implications for the content provider when deciding whether to enter into joint investment agreements. The content provider can either have an agreement with both network operators and raise its quality for all consumers, or he can enter into an exclusive quality deal with one operator. With exclusivity the content provider faces a trade-off. On the one hand it receives fewer adverting revenues because consumers accessing the content through the excluded operator perceive a degraded quality. On the other hand, the content provider would pay less for the quality given that: first it invests with one operator only and second because the operator with the exclusive deal is willing to accept a smaller financial contribution for the increased quality as exclusivity gives him an advantage on the access market.

A content provider with bargaining power has the ability to negotiate a contract that allows him to keep most of the advertising profit, he then chooses to enter into a simultaneous quality agreement with both access providers. Whereas a small content provider gets his rents extracted during negotiation, he will then use the exclusive agreement as a leverage to increase his position in the negotiation process.

Additionally, the model takes into account the role of competition intensity on the access market. When access competition is not too strong an access provider is less concerned about the quality offered by its competitor. Then, the degradation of content is less severe. With strong competition in the access market the content degradation is aggravated which exacerbates the vulnerability of weak content providers.

Finally, the model finds that consumer welfare is higher when access and content providers are free to negotiate. This result also holds when the content provider decides to enter into exclusive agreements.

The rest of the paper is organized as follows: section 2 presents the framework of the model, section 3 develops the net neutrality benchmark. The non regulated regime is analyzed in section 4, where simultaneous or exclusive quality enhancement contracts between content and access providers can be negotiated. Section 5 discusses competition policy implications, to continue with some extension in section 6 and to finally conclude in section 7. All technical proofs can be found in the appendix.

1.2 The Model

Consider two Internet service providers, denoted by ISP_1 and ISP_2, that compete for consumers who want to have access to a specific Internet content, online service or application offered by a content provider denoted by CP. Consumers pay only for the Internet access service and not for content consumption, content provider's revenues come from online advertising.

Two important and specific features of the Internet are taken in consideration in this model:

Global connectivity The Internet is a global network. A service provider ISP_i can offer the content of CP even if ISP_i has no direct interconnection or contract relation with CP. And vice-versa, the CP receives online advertising revenues without directly remunerating access providers for the connection provided to consumers.

Last-mile control Access providers control what is known as the last mile. Because access providers manage the last segment of the path that connects CP to consumers, they have control over the final transmission quality of the content.

It will be later evident how these two features characterize the hypothesis of the model, demand and costs structures are now described.

Demand structure There is a continuum of consumers of the same type. All consumers use both ISP_1 and ISP_2 to have access to CP.[5] The representative consumer that consumes q_i units of content using access of ISP_i, $i \in \{1, 2\}$, has a quadratic utility $u(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}(q_1^2 + 2\gamma q_1 q_2 + q_2^2)$, where α_i is the quality of service that access providers set in the last mile for the CP. The parameter $\gamma \in [0, 1)$ represents the differentiation between access providers.[6] With γ close to 0, a consumer using access of ISP_i does not reduce his utility from using access of ISP_j to get the content. This would be the case of mobile access *vs.* fixed line access for someone that travels as much as stays at home. On the other hand, with γ close to 1 access providers are perfect substitutes. This could be illustrated by someone having cable and ADSL access at home. As it is frequent in the economic literature, the parameter γ can be reinterpreted as a proxy of competition intensity on the Internet access market.

Given p_i the access price set by the access provider ISP_i, the representative consumer maximizes his net utility $\max_{q_1,q_2} \quad u(q_1, q_2) - p_1 q_1 - p_2 q_2$. This utility function gives rise to a linear demand structure, inverse demands are given by $p_i = \alpha_i - q_i + \gamma q_j$. The linear demand for access provider ISP_i is given by

$$q_i = q\left((\alpha_i, p_i), (\alpha_j, p_j)\right) \equiv \frac{1}{1-\gamma^2}\left(\alpha_i - p_i - \gamma(\alpha_j - p_j)\right).$$

Cost structure In order to set the quality of service α, an access provider must make an investment of $I(\alpha)$. Last-mile quality upgrade investments in telecommunications are known to be very costly.[7] The model supposes that this investment is fixed and it does

[5] Section 6 shows that most results are robust in a context where consumers are horizontally differentiated and choose one access provider only.

[6] We have come to a state where Internet access technologies can take the form of dial-up, landline (over coaxial cable, fiber optic or copper wires), T- lines, Wi-Fi, satellite and cell phones.

[7] See Faulhaber and Hogendorn (2000) or Fijnvandraat and Bouwman (2006).

not depend on variable content consumption. It further supposes that access providers have the basic infrastructure to provide a minimal quality, so the marginal increase for low qualities of service is small compared to a marginal increase for higher qualities. Technically $I(\cdot)$ is set to be an increasing differentiable "very" convex function, with $I'(\cdot)$ a convex function as well,[8] and with $I(0) = I'(0) = 0$. For expositional clarity purposes let us fix the investment cost function to $I(\alpha) = \frac{\alpha^3}{6}$. But no specification of the investment function is necessary to show general results. For the sake of simplicity, other costs and marginal costs are normalized to zero.

Access provider ISP$_i$'s profit from access consumption is

$$\pi_i = p_i q_i - I(\alpha_i)$$

Content provider Revenues for the content provider come from advertising and they are increasing with content consumption. An increase of quality increases content consumption. For simplicity the model supposes that publicity revenues net of operation costs are linear. The content provider has advertising profits

$$\pi_c = q_1 + q_2.$$

The game Two different regulatory regimes will be studied. In the first regime, that will be called the net neutrality regime, the content and access providers are not allowed to enter into any form of agreement. In the second unregulated regime, the content provider can participate in the investment process that determines his quality of service. In general terms and depending on the regime, the timing[9] of the game is as follows: First there is an investment stage where the quality of service $\{\alpha_i, \alpha_j\}$ is fixed according to the regulatory regime; and second the access competition stage where access providers set subscription prices $\{p_i, p_j\}$ for consumers non-cooperatively. The timing is chosen

[8] Other authors that use convex marginal costs for quality in different contexts are Schlee (1996) and Johnson and Myatt (2003)

[9] Details of each stage will be specified further on section 1.4.

to reflect the versatility of price setting after an infrastructure investment, characteristic of the telecommunications sector, which is standard in the economics literature on investment.

1.3 The benchmark: Net neutrality

In this section we set as benchmark the net neutrality regime where the content provider does not participate on the investment process. The timing of the game is as follows

(1) *Investment*, quality of service levels $\{\alpha_i, \alpha_j\}$ are set by access providers non-cooperatively

(2) *Access competition*, access prices $\{p_i, p_j\}$ are fixed by access providers non-cooperatively

In this regime, the content provider plays a passive role in the game. There is no relation between the network operators and the content provider, there are no fees for content access nor fees for network usage, and all decisions are made by access providers only. However the content still generates advertising profits as a consequence of global connectivity.

Access competition The solution concept is sub-game perfect Nash equilibria. Given qualities $\{\alpha_i, \alpha_j\}$ set at stage (1), provider ISP_i sets at stage (2) his price p_i, taking p_j as given, to maximize his profits

$$\max_{p_i} \quad \pi_i = p_i \frac{1}{1-\gamma^2} \left(\alpha_i - p_i - \gamma(\alpha_j - p_j)\right) - I(\alpha_i)$$

The first order condition gives rise to the following reaction function $R_i^p(p_j) = \frac{1}{2}(\alpha_i - \gamma(\alpha_j - p_j))$. Notice that when γ is small ISP_i can price consumers proportionally to the quality he sets, but when γ increases price competition in the access market intensifies and prices adjust to the rival's offer. Solving the system of reaction functions there exists a unique price equilibrium given by:

$$p_i^* = p(\alpha_i, \alpha_j) \equiv \frac{1-\gamma}{2-\gamma}\alpha_i + \frac{\gamma}{4-\gamma^2}(\alpha_i - \alpha_j) \tag{1.1}$$

This prices are the equilibria as long as qualities are sufficiently close such that both operators are active in the market and as long as operators make positive profits.[10] Equilibrium prices at this stage are determined by two factors. The first one accounts for the direct effect that the access provider's quality of service has on consumers and the second one accounts for the quality difference with its rival. Remark that the quality difference has a bigger impact on prices as access by both providers become more substitutes. Further, given that quality costs are fixed and do not depend on consumption levels, equilibrium prices do not depend on the investment cost of quality.

At equilibrium, network operators' profits from access to content depend on their qualities only

$$\begin{aligned}\pi_i = \pi(\alpha_i, \alpha_j) &\equiv p(\alpha_i, \alpha_j) q\left((\alpha_i, p(\alpha_i, \alpha_j)), (\alpha_j, p(\alpha_j, \alpha_i))\right) - I(\alpha_i) \\ &= \frac{1}{1-\gamma^2}\left(\frac{1-\gamma}{2-\gamma}\alpha_i + \frac{\gamma}{4-\gamma^2}(\alpha_i - \alpha_j)\right)^2 - \frac{\alpha_i^3}{6}\end{aligned}$$

and the corresponding profits from content consumption for CP are

$$\begin{aligned}\pi_c(\alpha_i, \alpha_j) &\equiv \sum_{i \neq j = 1,2} q\left((\alpha_i, p(\alpha_i, \alpha_j)), (\alpha_j, p(\alpha_j, \alpha_i))\right) \\ &= \frac{\alpha_i + \alpha_j}{(2-\gamma)(1+\gamma)}\end{aligned}$$

Remark that content profits, taken to be linear and proportional to content consumption, adopt a simple linear form proportional to the quality of service that each operator fixes.

Investment In stage (1), access provider ISP$_i$ sets the quality α_i, taking α_j as given in order to maximize his profits and anticipating the access prices that will be set at equilibrium in stage (2): $\max_{\alpha_i} \pi(\alpha_i, \alpha_j)$. The first order condition of this problem is

$$\underbrace{\frac{2(2-\gamma^2)}{(1+\gamma)(2-\gamma)(4-\gamma^2)}}_{a_\gamma}\alpha_i + \underbrace{\frac{2\gamma(2-\gamma^2)}{(1-\gamma^2)(4-\gamma^2)^2}}_{b_\gamma}(\alpha_i - \alpha_j) - \frac{\alpha_i^2}{2} = 0 \qquad (1.2)$$

[10]Both operators have positive demands if $\frac{\gamma}{2-\gamma^2}\alpha_j \leq \alpha_i \leq \frac{2-\gamma^2}{\gamma}\alpha_j$. They have positive profits if they are not too substitutable, this is if they are able to make some rents to pay for the quality.

Here, $\mathrm{a}_\gamma = \frac{2(2-\gamma^2)}{(1+\gamma)(2-\gamma)(4-\gamma^2)}$ is the marginal revenue gain that an access provider makes from a quality of service increase, independent of what the other access provider sets as quality. It can be easily verified that the parameter a_γ is positive and that it strictly decreases with the substitution parameter. The other parameter $\mathrm{b}_\gamma = \frac{2\gamma(2-\gamma^2)}{(1-\gamma^2)(4-\gamma^2)^2}$ is the marginal revenue gain (or loss) that an access provider makes from offering a higher (or lower) quality of service compared to its rival. With two independent access providers $\mathrm{b}_0 = 0$, but more substitutable operators face more frontal competition as b_γ increases.

The best reply function for access provider ISP_i is

$$R_i^\alpha(\alpha_j) = \begin{cases} \mathrm{a}_\gamma + \mathrm{b}_\gamma + \sqrt{(\mathrm{a}_\gamma + \mathrm{b}_\gamma)^2 - 2\mathrm{b}_\gamma\alpha_j}, & \text{if } \alpha_j \leq \frac{(\mathrm{a}_\gamma+\mathrm{b}_\gamma)^2}{2\mathrm{b}_\gamma} \\ 0, & \text{otherwise} \end{cases}$$

Observe that the reply function is strictly decreasing with the competitors quality. If ISP_j increases his quality, the access provider ISP_i anticipates that in the access pricing stage he will have to reduce his access prices at equilibrium, and having less access revenues ISP_i has no option other than to decrease the quality of service proposed given the high investment costs. With α_j high enough, ISP_j could foreclose ISP_i from the market. Technically this corresponds to the situation where ISP_i has strictly decreasing profits for any set of quality levels.

There exists a symmetric solution of the system (1.2) above, or equivalently a fixed point for the best replies system $\alpha_n = R_i^\alpha(\alpha_n)$ given by

$$\alpha_n = 2\mathrm{a}_\gamma$$

This is the equilibria as long as the second order condition holds: $\mathrm{a}_\gamma + \mathrm{b}_\gamma < I''(\alpha_n) = 2\mathrm{a}_\gamma$. Equivalently, this condition holds if marginal revenues from direct consumption exceed marginal revenues from quality competition with the rival: $\mathrm{a}_\gamma > \mathrm{b}_\gamma$ which holds for $\gamma < \hat{\gamma}_n = \sqrt{3} - 1$. If $\gamma > \hat{\gamma}_n$ the profit is not concave, this comes from the fact that quality costs are fixed and do not depend on the quality. However the range of parameters for which equilibria exists can be extended by setting a "more convex" investment cost.

The following proposition resumes the exposed above.

Proposition 1.1. *If access providers are not too substitutable $\gamma < \hat{\gamma}_n$, then there exists a sub-game perfect Nash equilibria which is symmetric where Internet access providers set a quality equal to α_n. The quality decreases as access providers become closer substitutes.*

Proof. All details of technical proofs are found in the appendix. □

There exists as well an asymmetric equilibrium where one operator invests more than the other. However this equilibrium exits for a small range of substitutability between access providers. This equilibrium is not really important in the analysis because total symmetry between access providers suggest that a symmetric equilibrium is more relevant. The interested reader may consult the appendix.

This sets the benchmark for the second regime where CP can participate in the investment process by negotiating a quality enhance with the access providers ISP_i. In what follows, the non regulated regime is analyzed.

1.4 No Regulation

With no regulation, the content provider can participate on the investment process. The investment stage of the game is subdivided in two periods

(1) *Investment.* The content provider CP proposes to negotiate an investment agreement over the terms of a quality level α_i and a fixed monetary transfer T_i to either:

  - Both access providers, having simultaneous contracts
  - One access provider only, where CP enters into a quality exclusive agreement
  - None of them, and access providers invest by their own means, as in the net neutrality regime

(2) *Access competition.* access prices $\{p_i, p_j\}$ for consumers are fixed non-cooperatively by ISP_i and ISP_j.

Given that the transfer fee T negotiated in stage (1.2) is fixed, it does not impact prices set by access providers in stage (2). So the access price equilibrium is the same as in the net neutrality regime, as in equation (1.1). Total profits for the providers are profits from content access or advertising at the agreed quality levels plus or minus the agreed monetary transfer:

$$\Pi_i = \Pi(\alpha_i, \alpha_j; T_i) \equiv \pi(\alpha_i, \alpha_j) + T_i, \qquad \Pi_c(\alpha_i, \alpha_j; T_i, T_j) \equiv \pi_c(\alpha_i, \alpha_j) - T_i - T_j$$

If there is no agreement $T_i = 0$ and the quality α_i is set unilaterally by the access provider.

The outcome of the negotiation process between the content provider and access providers is taken to be the Nash equilibrium of simultaneous generalized Nash bargaining problems.

1.4.1 Simultaneous contracts

Suppose that CP has decided to negotiate with both access providers, and that both access providers decide to enter into negotiation.

Bargaining framework There are four assumptions that determine the solution of the bargaining process. First, as it is usual in the literature of vertical relations for its tractability, the model supposes that negotiations between the pair $\{\text{CP}, \text{ISP}_1\}$ and the pair $\{\text{CP}, \text{ISP}_2\}$ occur simultaneously.[11] The equilibrium concept that is used is somehow close to the *contract equilibrium* first formalized by Crémer and Riordan (1987). This means that at equilibrium, the contract agreed by the pair $\{\text{CP}, \text{ISP}_i\}$ must be immune to unilateral deviations. Second, the solution supposes that contracts negotiated are not contingent on rival pair's disagreement.[12] This means that the bargaining pair

[11] See for example Horn and Wolinsky (1988); O'Brien and Shaffer (1992); Milliou and Petrakis (2007); Allain and Chambolle (2007).

[12] Examples of authors that use this hypothesis are Horn and Wolinsky (1988); O'Brien and Shaffer (1992); McAfee and Schwartz (1994).

$\{$CP , ISP$_i\}$ cannot implement a contract that specifies another outcome if the bargaining process of the other pair $\{$CP , ISP$_j\}$ has failed. Third, as a consequence of the *last-mile control* property, the model supposes that the outside option $\underline{\alpha}_i$ for the access provider ISP$_i$ is a best response to the other pair's agreed quality α_j. This assumption implies that the equilibrium outcome is sub-game perfect. And finally, the model supposes that access providers are completely symmetric, this means that the content provider has the same exogenous bargaining power $\beta \in (0, 1)$ with respect to each one of them.

Having this said, the program that the pair $\{$CP , ISP$_i\}$ solves in this simultaneous setting taking the rival pair's contract $\{\alpha_j^*, T_j^*\}$ as given is

$$\max_{\alpha_i, T_i} \quad \left\{\Pi(\alpha_i, \alpha_j^*; T_i) - \Pi(\underline{\alpha}_i, \alpha_j^*; 0)\right\}^{1-\beta} \left\{\Pi_c(\alpha_i, \alpha_j^*; T_i, T_j^*) - \Pi_c(\underline{\alpha}_i, \alpha_j^*; 0, T_j^*)\right\}^{\beta} \quad (1.3)$$

where outside option quality is

$$\underline{\alpha}_i = \arg\max_{\alpha} \quad \pi(\alpha, \alpha_j^*) \quad (1.4)$$

The monetary transfer that maximizes (1.3) is easily calculated

$$T_i^* = (1-\beta)\left(\pi_c(\alpha_i, \alpha_j^*) - \pi_c(\underline{\alpha}_i, \alpha_j^*)\right) - \beta\left(\pi(\alpha_i, \alpha_j^*) - \pi(\underline{\alpha}_i, \alpha_j^*)\right)$$

Note that the transfer fee T_i^* does not depend on the other pair's monetary transfer. Actually, one can make the parallel of this model with classical vertical relations literature where the producer charges two-part tariffs to distributors. Here the quality level occupies the role of the linear part of the tariff which is usually set to maximize the bargaining pair's joint profits. The fixed part, here the transfer fee, distributes the surplus according to their respective bargaining power. Then, by plugging T_i^* into the program (1.3), we find that it can be easily reduced to

$$\max_{\alpha_i} \left(\pi_c(\alpha_i, \alpha_j^*) - \pi_c(\underline{\alpha}_i, \alpha_j^*)\right) + \left(\pi(\alpha_i, \alpha_j^*) - \pi(\underline{\alpha}_i, \alpha_j^*)\right) \quad (1.5)$$

The first order condition for this problem is

$$\underbrace{\frac{2(2-\gamma^2)}{(1+\gamma)(2-\gamma)(4-\gamma^2)}}_{\mathrm{a}_\gamma}\alpha_i + \underbrace{\frac{2\gamma(2-\gamma^2)}{(1-\gamma^2)(4-\gamma^2)^2}}_{\mathrm{b}_\gamma}(\alpha_i - \alpha_j) + \underbrace{\frac{1}{(2-\gamma)(1+\gamma)}}_{\mathrm{c}_\gamma} - \frac{\alpha_i^2}{2} = 0 \quad (1.6)$$

where a_γ and b_γ are as in equation (1.2), and $c_\gamma = \frac{1}{(2-\gamma)(1+\gamma)}$ is the marginal revenue the content operator gets with an augmentation of the quality α_i.

The solution of the system above has a symmetric equilibrium given by

$$\alpha_s = a_\gamma + \sqrt{{a_\gamma}^2 + 2c_\gamma}$$

Clearly $\alpha_s > \alpha_n$ as long as $c_\gamma > 0$, the quality set with simultaneous contracts is higher than the one set in a net neutral regime if the content provider makes profits with a quality increase. However again, this result holds when the operators are not too substitutable. Equilibria might fail to exist as in the previous regime when $b_\gamma < \sqrt{{a_\gamma}^2 + 2c_\gamma}$, or equivalently for $\gamma < \hat{\gamma}_s \approx 0.89$.

The outside option In a simultaneous setting, the outside option quality $\underline{\alpha}_s$ is the quality that access provider ISP_i would set if no agreement is reached with CP and if the other pair $\{\text{CP}\ ,\ \text{ISP}_j\}$ holds to the equilibrium quality α_s. The access provider ISP_i then solves the program (1.4), it maximizes its revenues $\max_{\alpha_i} \pi_i(\alpha_i, \alpha_s)$. Then $\underline{\alpha}_s$ equals

$$\underline{\alpha}_s = a_\gamma + b_\gamma + \sqrt{{a_\gamma}^2 + {b_\gamma}^2 - 2b_\gamma\sqrt{a_\gamma + 2c_\gamma}}$$

When access providers enjoy a local monopoly positions and $\gamma = 0$, $b_0 = 0$ and the outside option is the same quality as in the net neutral regime. However, if $\gamma > 0$, ISP_i anticipates that the other pair's higher quality forces him to set lower prices, and hence lower revenues. As a result ISP_i sets a quality lower that the quality set under net neutrality. As discussed in section 1.3, R_i^α decreases with α_j, then $\underline{\alpha}_s = R_i^\alpha(\alpha_s) < R_i^\alpha(\alpha_n) = \alpha_n$. In a Nash bargaining setting, the outside option is frequently interpreted as a threat point. Notice that in this case, to set a quality lower than the one in a net neutrality regime is a threat that is credible, given that the access provider maximizes his revenues.

Letting access providers have control over the outside option is a characteristic of this model that distinguishes it from traditional vertical relationships. In a producer-distributor relation, in case of no agreement the producer does not provide the product to the distributor, so the outside option profit for the distributors is zero as he has no product to retail. Here, the access providers as distributors of content do have power over the last segment of the network. As the Internet offers global connectivity the content is available to the Internet access provider always.

The discussion above is summarized in the following proposition:

Proposition 1.2. *Suppose that the content provider CP selects to enter into simultaneous negotiations with both access providers.*

- *If access providers are not too substitutable $\gamma < \hat{\gamma}_s$ there exists a symmetric contract equilibria where the quality is α_s.*
- *The quality set when a simultaneous contract is established is greater than the one set in a net neutrality regime, $\alpha_s > \alpha_n$, however the quality set by an access provider in case of no agreement with the content provider is lower than the net neutrality one $\underline{\alpha}_s \leq \alpha_n$.*

As $\underline{\alpha}_s$ maximizes ISP$_i$'s profit $\pi_i(\alpha_s, \alpha_s) < \pi_i(\underline{\alpha}_s, \alpha_s)$. This implies that the transfer fee is always from the content provider to the access provider, i.e. CP helps to pay for the quality augmentation costs. The transfer fee is

$$T^S = (1-\beta)\Delta_c^S - \beta\Delta^S$$

where $\Delta_c^S = \pi_c(\alpha_s, \alpha_s) - \pi_c(\underline{\alpha}_s, \alpha_s) \geq 0$ is the value for CP of a successful negotiation with one access provider in the simultaneous bargaining setting, and $\Delta^S = \pi(\alpha_s, \alpha_s) - \pi(\underline{\alpha}_s, \alpha_s) \leq 0$ is the minimal compensation an access provider is willing to accept in order to invest α_s. Total profits are

$$\begin{aligned} \Pi_i^s &= \pi(\alpha_s, \alpha_s) + T^S \\ \Pi_c^s &= \pi_c(\alpha_s, \alpha_s) - 2T^S \end{aligned} \tag{1.7}$$

1.4.2 Exclusive contracts

Suppose now that CP has decided to negotiate with only one access provider, lets say ISP$_i$, who accepts to negotiate with him leaving aside the access provider ISP$_j$. As in the simultaneous setting prices set to consumers (and therefore demand for each access provider) do not depend on the agreed transfer fee, they depend on qualities only.

Bargaining framework With exclusive contracts CP negotiates with ISP_i alone, the hypothesis of simultaneous bargaining is no longer needed. However, and for the moment, the game supposes that CP can commit to ISP_i. This implies that if negotiation between {CP , ISP_i} fails CP cannot approach ISP_j to enter in a new negotiation process. Therefore the outside option for the bargaining pair is the net neutrality equilibrium, where access providers invest in quality by their own means. This is a strong hypothesis however largely used for its tractability. Section 1.6.1 relaxes this hypothesis showing that the main strategic insights do not change if CP has no commitment capability and can approach the other access provider if negotiation has failed.

At equilibrium the bargaining pair {CP , ISP_i} agrees on a contract $\{\alpha_E, T^E\}$ that is a best-response to the excluded access provided quality investment α_e, and vice-versa, i.e. a Nash equilibrium. The outside option is the quality that both access providers set in the net neutrality regime, where CP does not participate in the investment process.

In an exclusive contract setting the equilibrium is given by

$$\begin{aligned}
\{\alpha_E, T^E\} = &\quad \arg\max_{\alpha_i, T_i} \quad \{\Pi(\alpha_i, \alpha_e, T_i) - \Pi(\alpha_n, \alpha_n; 0)\}^{1-\beta} \{\Pi_c(\alpha_i, \alpha_e; T_i, 0) - \Pi_c(\alpha_n, \alpha_n; 0, 0)\}^{\beta} \\
\alpha_e = &\quad \arg\max_{\alpha_j} \quad \pi(\alpha_j, \alpha_E)
\end{aligned}$$

Again, the transfer fee between ISP_i and CP distributes the surplus of the bargaining pair according to their respective bargaining power and the quality fee is set to maximize {CP , ISP_i}'s joint surplus as a best response to ISP_j's quality

$$\begin{aligned}
\alpha_E = &\quad \arg\max_{\alpha_i} \quad \pi_i(\alpha_i, \alpha_e) - \pi(\alpha_n, \alpha_n) + \pi_c(\alpha_i, \alpha_e) - \pi_c(\alpha_n, \alpha_n) \\
\alpha_e = &\quad \arg\max_{\alpha_j} \quad \pi(\alpha_j, \alpha_E)
\end{aligned}$$

The first order conditions of the above programs, and the equilibrium qualities are given by the system

$$a_\gamma \alpha_E + b_\gamma(\alpha_E - \alpha_e) + c_\gamma = \frac{\alpha_E^2}{2} \tag{1.8}$$

$$a_\gamma \alpha_e - b_\gamma(\alpha_E - \alpha_e) = \frac{\alpha_e^2}{2} \tag{1.9}$$

The explicit solution of this system is cumbersome, however there are important properties that can be deduced. In an access market where providers are completely independent $\gamma = 0$ equation (1.8) coincides with equation (1.6), the quality set by the access provider with the exclusive deal equals the one in a simultaneous contract setting $\alpha_E = \alpha_s$. Additionally, as equation (1.9) coincides with equation (1.2), the quality set by the excluded access provider is the same as in a net neutral regime $\alpha_e = \alpha_n$. With $\gamma > 0$ the term accompanied by $b_\gamma > 0$ gains importance and the bargaining pair gets more revenues from the quality difference. The content provider can further commit to ISP_i setting an even higher quality $\alpha_E > \alpha_s$ than in the simultaneous contract setting. As access providers become more substitutable it is easier to capture rival's demand by increasing quality. The best response for the excluded operator is to set a quality $\alpha_e < \alpha_n$ lower than in the net neutrality regime.

Foreclosed market The difference in qualities grows up to a point where the excluded access provider lowers his quality until his revenues are so small that he cannot cover quality costs and he is foreclosed from the market. This corresponds to the limit situation where the pair $\{CP , ISP_i\}$ set a quality for which $R_j^\alpha(\alpha_E) = 0$ and this quality is the best for the bargaining pair. Beyond this threshold the pair $\{CP , ISP_i\}$ sets a quality corresponding to a monopoly access, or equivalently to the quality set for $\gamma = 0$, $\alpha_M = a_0 + \sqrt{a_0^2 + 2c_0}$.

Even if foreclosure equilibria might arise, the focus of the paper will be on the shared market equilibria. From now on suppose that access providers are not too close substitutes, or otherwise put, that competition between access providers is not too strong. It can be agreed that this assumption fits well the Internet access market, as it is generally an oligopolistic market. The proposition follows

Proposition 1.3. *Suppose that the content provider CP selects to negotiate only with access provider ISP_i. There exists a threshold $\hat{\gamma}_e$ such that,*

- *If $\gamma < \hat{\gamma}_e$, there exists an equilibrium $\{\alpha_E, \alpha_e\}$ where both access providers are active in the access market. The excluded access provider, ISP_j, offers the content with a lower quality $\alpha_e < \alpha_E$.*

- *The equilibrium qualities satisfy $\alpha_e \leq \underline{\alpha}_s \leq \alpha_n < \alpha_s \leq \alpha_E$*

- *Beyond the threshold the bargaining pair sets quality at the monopoly level excluding ISP_j from the access market.*

With exclusive contracts one part of the population has access to the content with high quality and the other part with low quality. When comparing the average qualitie we have the following corollary.

Corollary 1.1. *The average quality with exclusive contracts is higher than the quality with net neutrality but lower than the quality with simultaneous contracts. This is*

$$\alpha_n < \frac{\alpha_E + \alpha_e}{2} < \alpha_s \tag{1.10}$$

Figure 1.1 bellow illustrates the equilibrium qualities for each regime.

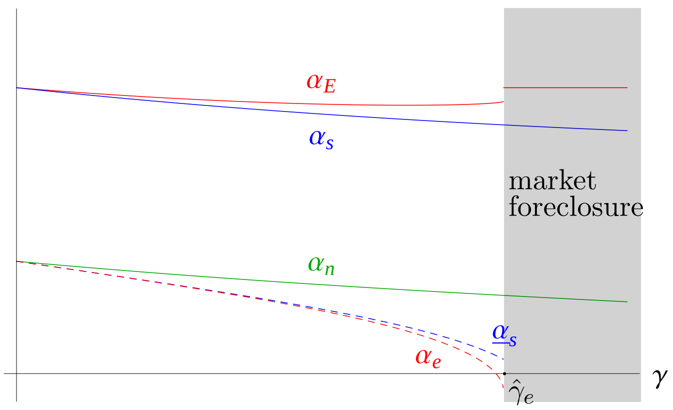

Figure 1.1: Equilibrium qualities for different regimes and different contract choices

The monetary transfer is

$$T^E = (1 - \beta)\Delta_c^E - \beta\Delta^E$$

where $\Delta_c^E = \pi_c(\alpha_E, \alpha_e) - \pi_c(\alpha_n, \alpha_n) > 0$[13] is the value for the content provider of a successful exclusive negotiation with operator ISP_i. Respectively, $\Delta^E = \pi(\alpha_E, \alpha_e) - \pi(\alpha_n, \alpha_n) < 0$ is the minimal compensation ISP_i accepts in order to serve CP with a quality α_E. Total profits for the content provider, the access provider with the exclusivity deal and the excluded access provider are

$$\begin{aligned} \Pi_c^E &= \pi_c(\alpha_E, \alpha_e) - T^E \\ \Pi^E &= \pi(\alpha_E, \alpha_e) + T^E \\ \Pi^e &= \pi(\alpha_e, \alpha_E) \end{aligned} \tag{1.11}$$

respectively.

1.4.3 Equilibrium Analysis

Access providers An important characteristic of the Nash bargaining solution is that a bargaining pair has always incentives to enter into a bilateral negotiation. By construction the final payoff in a successful negotiation is greater than the outside option. In this case however, final payoffs depend as well on the other access provider's actions. Supposing that CP asks both access providers to enter into a negotiation process, they have the option to accept or to refuse the proposal, the table below resumes the outcomes in each case:

$\text{ISP}_i/\text{ISP}_j$	Accept	Refuse
Accept	Π^s, Π^s	Π^E, Π^e
Refuse	Π^e, Π^E	Π^n, Π^n

where $\Pi^n = \pi(\alpha_n, \alpha_n)$, and the rest as described in (1.7) and (1.11).

Actually, an access provider has incentives to negotiate with CP independently of what the other access provider does, and independently of CP's bargaining power.

[13] Corollary 1.1 implies $\Delta_c^E > 0$

Proposition 1.4. *Negotiate with the content is a dominant strategy for access providers. Moreover, when the content is powerful, access providers face a prisoners dilemma.*

Content provider For the content provider the situation is different. As he anticipates that access providers accept to negotiate with him, he has the final choice on the equilibrium outcome.

The following lemma will be of use, after giving some insights of its consequences the following paragraph analyzes content provider's contract choice.

Lemma 1.1. *Suppose that access providers are not too substitutable, then*

(i) The value for CP of a successful negotiation with one network operator is larger with simultaneous contracts than with exclusive contracts, $\Delta_c^E \leq \Delta_c^S$*, with equality at* $\gamma = 0$*.*

(ii) Access providers' profit difference is smaller with simultaneous contracts than with exclusive contracts, $\Delta^S \leq \Delta^E < 0$*, with equality at* $\gamma = 0$*.*

(iii) Joint gains with simultaneous contracts are higher than with an exclusive deal, $\Delta_c^E + \Delta^E \leq \Delta_c^S + \Delta^S$*.*

Recall that profits from content consumption for CP are proportional to the quality offered by each provider $\pi_c(\alpha_i, \alpha_j) = \mathrm{c}_\gamma(\alpha_i + \alpha_j)$. In a simultaneous contract setting, given that contract terms were assumed to be non-contingent, the value of the agreement for CP with one operator is proportional to the *quality loss* it would face if negotiation fails with this operator:

$$\Delta_c^S = \mathrm{c}_\gamma(\alpha_s + \alpha_s) - \mathrm{c}_\gamma(\underline{\alpha}_s + \alpha_s) = \mathrm{c}_\gamma \underbrace{(\alpha_s - \underline{\alpha}_s)}_{\text{quality loss}}$$

For independent access providers this difference equals $\Delta_c^S\big|_{\gamma=0} = \frac{\alpha_s - \alpha_n}{2}$, an increase in the substitutability factor exacerbates the profit difference as $\underline{\alpha}_s < \alpha_n$, making CP gain more from a successful negotiation with an access provider.

With exclusivity it is the opposite. The quality loss from the failed negotiation with the exclusive access provider is compensated by a quality gain coming from the excluded access provider who is no longer subject to lower the quality

$$\Delta_c^E = c_\gamma \Big(\underbrace{(\alpha_E - \alpha_n)}_{\text{quality loss}} - \underbrace{(\alpha_n - \alpha_e)}_{\text{quality gain}} \Big)$$

So this vaguely shows (the formal proof is detailed in the appendix) that the quality compensation effect results in higher value for CP in the simultaneous contract setting $\Delta_c^S \geq \Delta_c^E$. This surplus difference increases with substitutability.

The second point of the lemma is interpreted as follows. First it is clear that either with exclusive or simultaneous contracts the profit difference of an access provider is negative. In either case higher quality costs exceed access revenues. Nevertheless an access provider's profit difference is smaller with an exclusive deal because he obtains larger revenues from consumers given that its rival sets a low quality.

Points *(i)* and *(ii)* of lemma 1.1 have a direct impact on the tariff payed by the content provider. They imply that the monetary transfer payed in symmetric agreements to each operator is higher than the transfer payed with an exclusive contract

$$T^S = (1-\beta)\Delta_c^S - \beta\Delta^S \geq (1-\beta)\Delta_c^E - \beta\Delta^E = T^E$$

The Nash solution suggests that when bargaining, the partner that benefits more from the agreement should "compensate more" the other. And so, the fact that CP pays more to ISP_i in a simultaneous setting is explained by A_i's preferences for an exclusive agreement than a simultaneous one, versus CP's preferences for a simultaneous one.

With simultaneous agreements quality increases globally as both operators set α_s. When access providers are independent, CP's advertising revenues are high enough to cover both operators tariffs. With more substitutable access providers, the threat point $\underline{\alpha}_s$ decreases making T^S increase. At one point high tariffs no longer justify maintaining simultaneous quality contracts. Then, CP chooses to propose an exclusive contract and pay to one access provider only despite the advertising revenues decrease with the excluded quality α_e.

Nevertheless, the tariff payed by CP depends on his bargaining power. A tariff payed by weak content provider depends more on Δ_C than the tariff payed by a more powerful content. Therefore, small content providers are more vulnerable face to operators threats previously described.

This effect is illustrated with the extreme case of $\beta = 0$ and then with $\beta = 1$. Small CP gets his profits completely extracted if he negotiates with one access provider only: $\Pi_c^E\big|_{\beta=0} = \pi_c(\alpha_E, \alpha_e) - \Delta_c^E = \pi_c(\alpha_n, \alpha_n) = \Pi_c^n$, ending up with net neutrality profits. Given the capacity of access providers to set a quality that is lower than the net neutrality one he prefers exclusivity $\Pi_c^s\big|_{\beta=0} = \mathrm{c}_\gamma(2\alpha_s) - 2\mathrm{c}_\gamma(\alpha_s - \underline{\alpha}_s) \leq 2\mathrm{c}_\gamma\alpha_n = \Pi_c^n = \Pi_c^E\big|_{\beta=0}$. On the other hand, a powerful CP covers the remaining investment costs once all A's extra revenues from content have been used to pay for the quality increase, then CP prefers the global quality raise.

In general, a content provider strictly prefers a simultaneous contract if

$$\pi_c(\alpha_s, \alpha_s) - 2\left\{(1-\beta)\Delta_c^S - \beta\Delta^S\right\} > \pi_c(\alpha_E, \alpha_e) - \left\{(1-\beta)\Delta_c^E - \beta\Delta^E\right\}$$

putting β on one side of the inequality, this is equivalent to

$$\beta > \frac{2\mathrm{c}_\gamma(\alpha_n - \underline{\alpha}_s)}{2(\Delta^S + \Delta_c^S) - (\Delta^E + \Delta_c^E)} \equiv \hat{\beta}(\gamma) \tag{1.12}$$

The following proposition formalizes the exposed above

Proposition 1.5. *There exists $\hat{\beta}$ such that:*

- *if $\beta > \hat{\beta}$, the content provider selects a simultaneous contract*
- *if $\beta \leq \hat{\beta}$, the content provider selects an exclusive contract*
- *The bargaining power threshold $\hat{\beta}$ strictly increases with γ.*

Figure 1.2 illustrates proposition 1.5 with comparative statics.

The following corollary is a direct consequence of propositions 1.4 and 1.5,

Corollary 1.2. *The market equilibrium outcome does not change if the timing of the game is reversed at the investment stage, ie. access providers approach the content providers asking for a monetary compensation.*

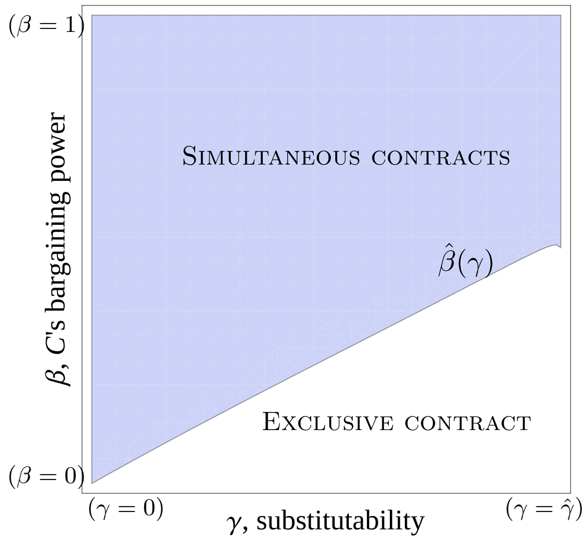

Figure 1.2: Comparative statics, CP's choice of contract

The timing of the game was chosen for simplicity, but also because recently a content provider, partisan of net neutrality, approached major US network operators to propose joint investments with the objective to improve the quality of its content[14]. Given that the debate on net neutrality is carried on the hypothesis that network operators take the initiative to make quality deals, this corollary shows that the equilibrium outcome is robust in this setting[15].

1.5 Competition policy implication

This section discusses the effect of a net neutrality regulation on consumer welfare.

Proposition 1.6. *Consumer welfare is higher under an unregulated regime than under net neutrality even if the content provider selects an exclusive contract. However, simultaneous contracts would be preferred to exclusivity deals.*

[14]See `http://googlepublicpolicy.blogspot.com/2008/12/net-neutrality-and-benefits-of-caching.html`

[15]However this holds as long as there is one content provider, with content competition the analysis is not straightforward.

Consumer welfare is calculated as the net consumer surplus. Supposing that both access providers set they same quality $\alpha_i = \alpha_j = \alpha$, consumer surplus can be rewritten as $CS(\alpha) = 2(\alpha - p)q - (1 + \gamma)q^2$, where $p = \frac{1-\gamma}{2-\gamma}\alpha$ is the access price at stage 2 and $q = \frac{1}{(1+\gamma)(2-\gamma)}\alpha$ is the content consumption levels at price p. Observe that with simultaneous contracts the higher investment on quality positively impacts consumer welfare twofold: consumption of content increases and the surplus associated with the increased consumption of content increases as well. It is straightforward to verify that consumers benefit from the higher quality under the non regulated regime $CS(\alpha_s) > CS(\alpha_n)$.

With exclusive contracts consumer welfare is still higher than with no agreements under net neutrality. Even though the quality set by the excluded provider is lower than the net neutrality one, the quality set by the exclusive provider compensates this loss and demands adjust accordingly. This is a direct consequence of lemma 1.1. Given that average quality with exclusive contracts is higher than the quality with net neutrality total content consumption is higher with exclusive contracts than under net neutrality.

Net neutrality advocates have considered at least three versions of a net neutrality regulation. The first one, referred as *absolute non-discrimination* demands that Internet access providers and network operators respect the end-to-end principle, where no network management is permitted (except for dealing with viruses and similar malicious content), and hence no quality agreements between providers is allowed. The second one, referred as *limited discrimination without quality of service tiering* is a version of net neutrality where operators are allowed to set quality of service for different contents but no fees are charged to content providers. These two regulatory frameworks correspond to the net neutrality regime in the model and hence correspond to a lower quality outcome, reducing consumer welfare.

The third version called *limited discrimination and tiering* allows agreements between content and access providers for better quality levels restricting exclusive deals between them. This flexible regulatory framework is closest to the model except that the exclusivity ban aims to avoid foreclosure issues in the content provider market. In doing so, an Internet access provider would be compelled to open the quality service to all content providers. An analogous regulation for the model presented in this paper could be referred as content neutrality, but a regulator could not demand multiple investments for the content provider. Nevertheless, our framework suggest a natural policy recommendation.

The results of the model imply that competition authorities should be concerned with small content providers. The exclusive quality deals are a consequence of the bargaining power disadvantage they face. If access providers were constrained to setting at least a net neutrality quality level for all content without further degrading it, content providers would not contemplate exclusive deals and a global quality enhancement would be agreed. It suffices to inspect condition (1.12) to remark that if the outside option for the access providers was restricted to the net neutrality level $\underline{\alpha}_s \geq \alpha_n$ then for all $\beta > 0$, the content provider would enter into simultaneous agreements. If one agrees that the quality of today's Internet is the one made with independent investment, then regulation could take it to be the minimal net neutrality quality benchmark and allow only improvements.

Actually, the European Commission has recently incorporated this principle. In November 2009 it amended its Directive on universal service and users' rights relating to electronic communication networks and services in order to include the article 22(3) which stipulates that[16]:

> In order to prevent the degradation of service and the hindering or slowing down of traffic over networks, Member States shall ensure that national regulatory authorities are able to set minimum quality of service requirements on an undertaking or undertakings providing public communications networks.

The debate on the implementation of such requirements is now expected to take place.

[16]See `http://eur-lex.europa.eu/JOHtml.do?uri=OJ:L:2009:337:SOM:EN:HTML`

1.6 Extensions

The section extends the basic model to consider alternative access market structure, and the commitment assumption for the content provider.

1.6.1 Content provider with no commitment

A rather strong hypothesis in the exclusive bargaining framework is that the content provider can commit to one access provider when offering exclusive deals. This meant that if negotiation fails between {CP , ISP$_i$} then access providers invested by their own means, leading to net neutrality quality levels. Here, this hypothesis is relaxed allowing the content provider to turn to the other access provider if negotiation has failed with the first one. However it is still supposed that once negotiation has failed between an access provider and the content provider, they cannot enter into renegotiation if the other pair has not achieve an agreement either.

Suppose that CP chooses to deal with ISP$_i$ first. Supposing that ISP$_i$ refuses or that they do not reach an agreement, CP turns to ISP$_j$. If no agreement is reached between them, then access providers set qualities equal to (α_n, α_n) and we are in the exclusive dealing problem with commitment. The pair {CP , ISP$_j$} sets the quality equal to α_E and the excluded provider ISP$_i$ sets a quality α_e. Providers at this stage have profits as in (1.11). This profits become then the outside option for the bargaining pair {CP , ISP$_i$}. The bargaining pair set a contract that solves

$$\max_{\alpha_i, T_i} \{\pi(\alpha_i, \alpha_j) + T_i - \Pi^e\}^{1-\beta} \left\{\pi_c(\alpha_i, \alpha_j) - T_i - \Pi_c^E\right\}^{\beta}$$

with α_j the quality set by ISP$_j$ that maximizes its profits $\max_{\alpha_j} \pi(\alpha_j, \alpha_i)$. As the resulting equilibrium qualities do not depend on the outside option, they qualities are the same as in the other cases $(\alpha_i = \alpha_E, \alpha_j = \alpha_e)$, and the monetary transfer is

$$T_i = (1-\beta)\left\{\pi_c(\alpha_E, \alpha_e) - \left(\pi_c(\alpha_E, \alpha_e) - T^E\right)\right\} - \beta\left\{\pi(\alpha_E, \alpha_e) - \pi(\alpha_e, \alpha_E)\right\}$$

Final profits for the content provider are

$$\Pi_c^{E'} = \pi_c(\alpha_E, \alpha_e) - T^E + \beta\left\{T^E + \pi(\alpha_E, \alpha_e) - \pi(\alpha_e, \alpha_E)\right\}$$

Observe that the last term of CP's profit is positive: the access provider prefers to negotiate with CP than its outside option $\pi(\alpha_E, \alpha_e) + T^E \geq \pi(\alpha_n, \alpha_n)$, but $\pi(\alpha_n, \alpha_n) > \pi(\alpha_e, \alpha_n) > \pi(\alpha_e, \alpha_E)$ since α_n is a best response to α_n and that A's profit decreases with the rivals quality. Then, for all β a content provider that does not commit to an access provider has higher profits $\Pi_c^{E'} \geq \Pi_c^E$.

Without committing to ISP_i the content provider makes extra rents in exclusivity for two reasons: his outside option (exclusive dealing with ISP_j) is better than net neutrality profits, and ISP_i's outside option (being excluded from negotiation) is worse than net neutrality. However, the main result that weak content providers prefer exclusive contracts and strong content providers prefer simultaneous contracts holds with the difference that exclusivity is preferred by a larger range of values.

Figure 1.3 illustrates the exposed above.

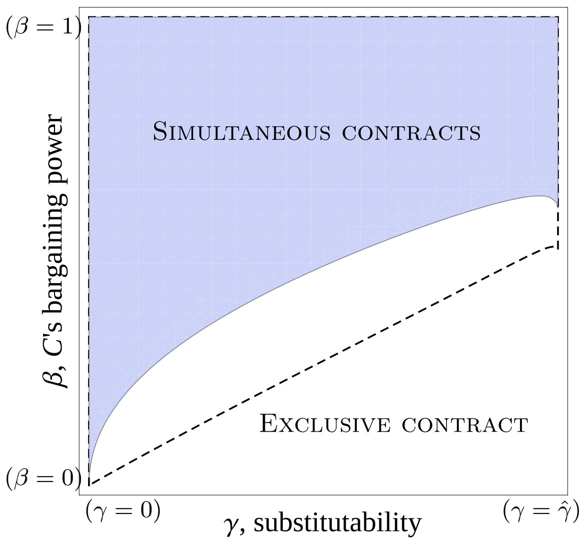

Figure 1.3: Comparative statics for a content with no commitment capability

1.6.2 Access providers horizontally differentiated

Seminal papers on competition in telecommunications have chosen to model demand by a horizontally differentiated population (cf. Laffont et al. (1998); Armstrong (1998); Cambini and Valletti (2004)) rather than with a representative consumer. This choice of modeling is intuitively justified as a consumer has one telephone subscription only. But the Hotelling model presents some limitations on respect to the elasticity of the demand, for this reason the paper was presented using a linear elastic demand. However the results are robust with un underlying Hotelling model.

In order to gain some elasticity assume now that consumers, who are uniformly distributed on the $[0,1]$ segment, have a variable increasing and concave surplus of $v(\alpha_i)$ with α_i the quality set by ISP$_i$. The net utility of consumer located at x and choosing ISP$_i$ is $\bar{v} + v(\alpha_i) - \frac{1}{2\gamma}|x - x_i| - p_i$, with $\bar{v}$ is the fixed utility of having access to the Internet, which it is supposed to be sufficiently hight that it keeps the market covered. The parameter $\gamma \geq 0$ is again the competition intensity between access providers. Then the indifferent consumer is located at

$$\tilde{x} = \frac{1}{2} + \gamma \left\{ v(\alpha_i) - p_i - v(\alpha_j) + p_j \right\}$$

Market shares for operators are $q_1 = \tilde{x}$ and $q_2 = 1 - \tilde{x}$.

The cost structure for the access provider is as in the main model. The content provider has increasing concave advertising revenues $r(\alpha)$ per consumer which reflect the variable content consumption with quality variations.

Content and access provider's profits from content are

$$\pi_i = q_i p_i - I(\alpha_i), \quad \pi_c = r(\alpha_i) q_i + r(\alpha_j) q_j.$$

Access competition For either regime, as long as the quality difference offered by the access providers is not very big $|v(\alpha_i) - v(\alpha_j)| \leq \frac{3}{2\gamma}$ access providers set prices at equilibrium and have corresponding market shares equal to

$$p_i^* = p(\alpha_i, \alpha_j) = \frac{1}{2\gamma} + \frac{v(\alpha_i) - v(\alpha_j)}{3}, \quad q_i^* = q(\alpha_i, \alpha_j) = \frac{1}{2} + \gamma \frac{v(\alpha_i) - v(\alpha_j)}{3}$$

Remark that for independent access providers $\gamma = 0$ access providers have equal market shares $q_i^* = \frac{1}{2}$, otherwise if $\alpha_i > \alpha_j$ then $q_i^* > \frac{1}{2}$ and $q_j < \frac{1}{2}$. It is easier for access providers to steal consumers as they become more substitutable.

Characterizing quality levels Given the convexity of the investment I and the concavity of v and r, the equilibrium qualities for each scenario can be characterized by the following equations

$$\begin{aligned}
I'(\alpha_n) &= \frac{v'(\alpha_n)}{3} \\
I'(\alpha_s) &= \frac{v'(\alpha_s)}{3} + \frac{r'(\alpha_s)}{2} \\
I'(\underline{\alpha}_s) &= 2q(\underline{\alpha}_s, \alpha_s)\frac{v'(\underline{\alpha}_s)}{3} \\
I'(\alpha_E) &= 2q(\alpha_E, \alpha_e)\left(\frac{v'(\alpha_s)}{3} + \frac{r'(\alpha_s)}{2}\right) + \gamma\frac{v'(\alpha_E)}{3}\left(r(\alpha_E) - r(\alpha_e)\right) \\
I'(\alpha_e) &= 2q(\alpha_e, \alpha_E)\frac{v'(\alpha_e)}{3}
\end{aligned}$$

Proposition 1.7. *If competition between access providers is not too intense (γ not too big) there exist a unique equilibrium in qualities with $\alpha_e \leq \underline{\alpha}_s \leq \alpha_n < \alpha_s \leq \alpha_E$.*

The proof is straightforward as I' is increasing and v' and r' decreasing. In all, similar results regarding the equilibrium outcome are obtained with this model.

1.7 Discussion

This paper has presented a model that studies the relations between Internet access providers relations with a possibly powerful Internet content provider. Two regulatory regimes where considered. A regulated one, called net neutrality where providers could not cooperate in quality enhancement agreements. And a second one, where the content provider could bargain with both access and obtain simultaneous quality contracts, or with only one of them entering into quality exclusive deals.

Some of the important insights of the model reveal that small content providers face a rather weak position when dealing with both operators simultaneously. This is given by the capacity operators have to degrade content's quality if no agreement is reached. On the other hand, a very powerful content can extract access providers' surplus leading them to a prisoners dilemma situation where investing in quality leads them to profit losses compared to the net neutrality regime.

It has been showed that an unregulated regime leads to global higher quality increasing consumers welfare. Competition authorities should consequently take measures to protect small content, but to let established popular content increase their quality through vertical agreements.

The paper has focused on a powerful content provider because the Internet content and services market is somehow concentrated and none of the preceding studies have considered it. The author is aware that the problem of entry barriers of new content has not been treated and it will be done in the near future.

1.8 *Appendix:* Proofs of Propositions

Proof of proposition 1.1. It is to verify that $\alpha_n = 2\mathrm{a}_\gamma$ decreases with γ given that a_γ decreases with γ:

$$\frac{d\mathrm{a}_\gamma}{d\gamma} = -\frac{2\left(4 - 2\gamma(1+\gamma)\right) + \gamma^3(3+2\gamma)}{(2-\gamma)^3(1+\gamma)^2(2+\gamma)^2} < 0$$

for $\gamma \in [0,1]$. In general, it can be showed using the implicit function theorem for a general convex cost function that the symmetric equilibrium decreases with competition.

The system (1.2) admits another solution where one access provider invests more than the other:

$$\alpha_i^* = \mathrm{a}_\gamma + 2\mathrm{b}_\gamma - \sqrt{{\mathrm{a}_\gamma}^2 - 4{\mathrm{b}_\gamma}^2}, \quad \alpha_j^* = \mathrm{a}_\gamma + 2\mathrm{b}_\gamma + \sqrt{{\mathrm{a}_\gamma}^2 - 4{\mathrm{b}_\gamma}^2}$$

this solution exists only for $\mathrm{a}_\gamma \geq 2\mathrm{b}_\gamma$. This is an equilibrium as long as second order condition holds for both operators, that is for $\mathrm{a}_\gamma < \sqrt{5}\mathrm{b}_\gamma$. The two conditions set the competition parameter $0.53 < \gamma < 0.56$. □

Proof of proposition 1.2. The other asymmetric equilibria profile is

$$\alpha_i^* = \mathrm{a}_\gamma + 2\mathrm{b}_\gamma - \sqrt{{\mathrm{a}_\gamma}^2 - 4{\mathrm{b}_\gamma}^2 + 2\mathrm{c}_\gamma}, \quad \alpha_j^* = \mathrm{a}_\gamma + 2\mathrm{b}_\gamma + \sqrt{{\mathrm{a}_\gamma}^2 - 4{\mathrm{b}_\gamma}^2 + 2\mathrm{c}_\gamma}$$

with the restriction of $\frac{{\mathrm{a}_\gamma}^2 + 2\mathrm{c}_\gamma}{5} < {\mathrm{b}_\gamma}^2 < \frac{{\mathrm{a}_\gamma}^2 + 2\mathrm{c}_\gamma}{4}$, that corresponds to $0.77 < \gamma < 0.79$.

Existence of equilibrium. It remains only to verify that the joint surplus is positive and that the bargaining solution actually exists. But this follows straightforward, the maximization program defined in (1.3) for $\alpha_i = \underline{\alpha}_s$ assures that at least $\Delta C + \Delta A = 0$, then as $\alpha_s \neq \underline{\alpha}_s$, the joint surplus is then positive at it is a maximizer. $\square$

Proof of proposition 1.3. It only rests to characterize the segment where access providers share the market. The cornered market equilibrium corresponds to a situation where the equations (1.8) and (1.9) hold with the excluded provider indifferent between leaving the market, i.e. with $R_i^\alpha(\alpha_E) = 0$. This is for

$$\hat{\alpha}_e = \mathrm{a}_{\hat{\gamma}_e} + \mathrm{b}_{\hat{\gamma}_e}, \quad \hat{\alpha}_E = \frac{(\mathrm{a}_{\hat{\gamma}_e} + \mathrm{b}_{\hat{\gamma}_e})^2}{2\mathrm{b}_{\hat{\gamma}_e}}$$

that corresponds to $\hat{\gamma}_e$ that solves $8{\mathrm{b}_\gamma}^2\mathrm{c}_\gamma = ({\mathrm{a}_\gamma}^2 - {\mathrm{b}_\gamma}^2)({\mathrm{a}_\gamma}^2 - 5{\mathrm{b}_\gamma}^2)$. This last equation sets the threshold to $\hat{\gamma}_e \approx 0.319$. Using simple numerical methods we can calculate the maximal difference in qualities: $\alpha_E(\hat{\gamma}_e) - \alpha_e(\hat{\gamma}_e) \approx 1.618$

Existence. The verification of the existence of the bargaining Nash equilibrium is straightforward. Note that if the bargaining pair sets $\alpha_i = \alpha_n$ the excluded access provider would then set $\alpha_j = \alpha_n$, so the gain in revenues are at least zero, then $\Delta^E + \Delta_c^E \geq 0$. $\square$

Proof of corollary 1.1. I show that the mean quality with simultaneous contracts is higher than the mean quality with exclusive deals. Suppose the contrary, that $\alpha_s < \frac{\alpha_E + \alpha_e}{2}$. Then, given the convexity of $I'(\alpha) = \frac{\alpha^2}{2}$ and that $I'(0) = 0$ we have that

$$I'(\alpha_s) \leq 0 + \frac{2\alpha_s}{\alpha_E + \alpha_e} I'\left(\frac{\alpha_E + \alpha_e}{2}\right) \leq \frac{2\alpha_s}{\alpha_E + \alpha_e}\left(\frac{I'(\alpha_E)}{2} + \frac{I'(\alpha_e)}{2}\right)$$

Then using equations (1.6), (1.8) and (1.9)

$$\mathrm{a}_\gamma \alpha_s + \mathrm{c}_\gamma = I'(\alpha_s) \leq \frac{2\alpha_s}{\alpha_E + \alpha_e}\left(\frac{I'(\alpha_E) + I'(\alpha_e)}{2}\right) = \frac{2\alpha_s}{\alpha_E + \alpha_e}\left(\frac{\mathrm{a}_\gamma(\alpha_E + \alpha_e) + \mathrm{c}_\gamma}{2}\right)$$

which implies that $\alpha_E < \alpha_E + \alpha_e \leq \alpha_s$ and this is not possible given that $\alpha_E > \alpha_s$.

In the same, when competition is not too intense and the market is not foreclosed for the excluded access provider the average quality with exclusive deals is higher than with net neutrality. Remember that for $\gamma = 0$: $\alpha_e = \alpha_n < \alpha_s = \alpha_E$, so $\alpha_n < \frac{\alpha_E + \alpha_e}{2}$, and as $\gamma > 0$, the quality lost with α_e is somehow recovered with α_E. It is enough to check if the inequality holds for $\hat{\gamma}$, the competition level where the excluded access provider sets its quality at lowest before leaving the market. At this point we have that $\alpha_e = \mathrm{a}_{\hat{\gamma}} + \mathrm{b}_{\hat{\gamma}}$ and $\alpha_E = \frac{(\mathrm{a}_{\hat{\gamma}} + \mathrm{b}_{\hat{\gamma}})^2}{2\mathrm{b}_{\hat{\gamma}}}$, but then the condition $\alpha_n = 2\mathrm{a}_{\hat{\gamma}} < \frac{(\mathrm{a}_{\hat{\gamma}} + \mathrm{b}_{\hat{\gamma}})(\mathrm{a}_{\hat{\gamma}} + 3\mathrm{b}_{\hat{\gamma}})}{4\mathrm{b}_{\hat{\gamma}}} = \frac{\alpha_E + \alpha_e}{2}$ holds if $0 < (\mathrm{a}_{\hat{\gamma}} - 3\mathrm{b}_{\hat{\gamma}})(\mathrm{a}_{\hat{\gamma}} - \mathrm{b}_{\hat{\gamma}})$ which is verified given that $\mathrm{a}_{\hat{\gamma}} \approx 0.44 > 0.27 \approx 3\mathrm{b}_{\hat{\gamma}}$. □

Proof of proposition 1.4. Suppose first that the rival ISP$_j$ refuses CP's proposal, then if ISP$_i$ accepts the negotiation call of CP he would have exclusive contract profits Π_A^E. If it refuses to enter into negotiation its profits correspond to those in a net neutral regime. Given that net neutrality profits correspond to the outside option of access provider ISP$_i$, by construction the following inequality follows

$$\Pi^E = \pi(\alpha_E, \alpha_e) + T^E \geq \pi(\alpha_n, \alpha_n) = \Pi^n$$

Suppose now that the rival ISP$_j$ is willing to negotiate with CP as well. Having a simultaneous contract with CP is a best option for ISP$_i$ than being excluded.

$$\Pi^s = \pi(\alpha_s, \alpha_s) + T^S \geq \pi(\underline{\alpha}_s, \alpha_s) \geq \pi(\alpha_e, \alpha_E) = \Pi^e$$

The first inequality holds given that by definition a successful negotiation brings profits higher than the outside option. The second one is verified in two steps: first $\pi(\underline{\alpha}_s, \alpha_s) \geq \pi(\alpha_e, \alpha_s)$ given that $\underline{\alpha}_s$ is a best response to α_s, and second $\pi(\alpha_e, \alpha_s) \geq \pi(\alpha_e, \alpha_E)$ because $\alpha_s \leq \alpha_E$ and the profits of ISP$_i$ decrease with the rivals quality.

Finally, suppose that the content provider is very powerful with β close to 1. First observe that with a reasoning analog to the preceding one $\pi(\alpha_n, \alpha_n) \geq \pi(\underline{\alpha}_s, \alpha_n) > \pi(\underline{\alpha}_s, \alpha_s)$. Then observe that a powerful content provider pays only $T^S\big|_{\beta=1} = -\Delta^S$, hence access providers have higher profits with net neutrality than with the simultaneous contracts

$$\Pi^n = \pi(\alpha_n, \alpha_n) > \pi(\underline{\alpha}_s, \alpha_s) = \pi(\alpha_s, \alpha_s) - \Delta^S = \Pi^s$$

□

Proof of lemma 1.1. The proof starts with a result that will be of use:

Intermediary result It will be of use to first show that $\alpha_E - \alpha_s < \alpha_n - \alpha_e$, i.e. that the threat point quality decreases faster than what the exclusive quality increases. This is a general demonstration given the convexity of $I'(\alpha)$.

First remark that given that $\alpha_e < \alpha_n < \alpha_E$ we can write $\alpha_n = (1-t)\alpha_e + t\alpha_E$ where $t = \frac{\alpha_n - \alpha_e}{\alpha_E - \alpha_e}$. Then using first-order conditions (1.2), (1.8) ,(1.9) and the convexity of I' we have that

$$\mathrm{a}_\gamma \alpha_n = I'(\alpha_n) \leq (1-t)I'(\alpha_e) + tI'(\alpha_E) = \mathrm{a}_\gamma((1-t)\alpha_e + t\alpha_E) - \mathrm{b}_\gamma(\alpha_E - \alpha_e) + t(\mathrm{c}_\gamma + 2\mathrm{b}_\gamma(\alpha_E - \alpha_e))$$

which implies that

$$\alpha_e + \frac{\mathrm{b}_\gamma(\alpha_E - \alpha_e)}{\mathrm{c}_\gamma + 2\mathrm{b}_\gamma(\alpha_E - \alpha_e)}(\alpha_E - \alpha_e) < \alpha_n$$

With the same reasoning taking $\alpha_e < \alpha_s < \alpha_E$ the following inequality holds

$$\alpha_e + \frac{\mathrm{c}_\gamma + \mathrm{b}_\gamma(\alpha_E - \alpha_e)}{\mathrm{c}_\gamma + 2\mathrm{b}_\gamma(\alpha_E - \alpha_e)}(\alpha_E - \alpha_e) < \alpha_s$$

Finally adding this two inequalities we have that $\alpha_E + \alpha_e < \alpha_s + \alpha_n$.

Proof of (i) With the intermediate result it is straightforward to show that the surplus made by CP is greater with simultaneous contracts.

$$\Delta_c^s = \mathrm{c}_\gamma(\alpha_s - \underline{\alpha}_s) \geq \mathrm{c}_\gamma(\alpha_s - \alpha_n) > \mathrm{c}_\gamma(\alpha_E + \alpha_e - 2\alpha_n) = \Delta_c^E$$

with equality at $\Delta_c^S\big|_{\gamma=0} = \frac{\alpha_s - \alpha_n}{2} = \Delta_c^E\big|_{\gamma=0}$.

Proof of (ii) Writing $\mathrm{d}_\gamma = \frac{\gamma^2}{(1-\gamma^2)(4-\gamma^2)^2}$, profits from content can be rewritten as

$$\pi(\alpha_i, \alpha_j) = \frac{\mathrm{a}_\gamma + \mathrm{b}_\gamma}{2}\alpha_i^2 - \mathrm{b}_\gamma \alpha_i \alpha_j + \mathrm{d}_\gamma \alpha_j^2 - \frac{\alpha_i^3}{6}$$

First, notice that

$$\Delta^S\big|_{\gamma=0} = \left(\frac{\alpha_s^2}{4} - \frac{\alpha_s^3}{6}\right) - \left(\frac{\alpha_n^2}{4} - \frac{\alpha_n^3}{6}\right) = \Delta^E\big|_{\gamma=0}$$

I can conveniently use the fact that $I(\alpha) = \frac{\alpha^3}{6} = \frac{\alpha}{2}I'(\alpha)$[17] and using first order conditions (1.2), (1.6), (1.8) and (1.9) profits can be rewritten and

$$\begin{aligned}
\Delta^E &= \frac{\mathrm{a}_\gamma + \mathrm{b}_\gamma}{6}(\alpha_E^2 - \alpha_n^2) - \frac{2\mathrm{b}_\gamma}{3}(\alpha_E\alpha_e - \alpha_n^2) - \frac{\mathrm{c}_\gamma}{3}\alpha_E - \mathrm{d}_\gamma(\alpha_n^2 - \alpha_e^2) \\
\Delta^S &= \frac{\mathrm{a}_\gamma + \mathrm{b}_\gamma}{6}(\alpha_s^2 - \underline{\alpha}_s^2) - \frac{2\mathrm{b}_\gamma}{3}\alpha_s(\alpha_s - \underline{\alpha}_s) - \frac{\mathrm{c}_\gamma}{3}\alpha_s
\end{aligned}$$

Comparing this two expressions is straightforward.

□

Proof of proposition 1.6. I show that consumer surplus without regulation but with exclusive contracts is higher than consumer surplus with no investments, but smaller than with simultaneous quality enhance contracts:

$$CS(\alpha_n, \alpha_n) < CS(\alpha_E, \alpha_e) < CS(\alpha_s, \alpha_s)$$

Recall that consumer surplus is

$$CS = u(q_1, q_2) - p_1 q_1 - p_2 q_2$$

using equilibrium prices given by (1.1), we can simplify the above expression to

$$CS = \frac{1}{(2-\gamma)^2(1+\gamma)}\left(\frac{4-3\gamma^2}{2(2+\gamma)^2(1-\gamma)}(\alpha_i - \alpha_j)^2 + \alpha_i\alpha_j\right)$$

It is then sufficient to show that:

$$\alpha_n^2 \leq \frac{4-3\gamma^2}{2(2+\gamma)^2(1-\gamma)}(\alpha_E - \alpha_e)^2 + \alpha_E\alpha_e \leq \alpha_s^2.$$

By corollary 4, $\alpha_n < \frac{\alpha_E + \alpha_e}{2}$, which implies that

$$\alpha_n^2 < \frac{(\alpha_E + \alpha_e)^2}{4} = \frac{1}{4}(\alpha_E - \alpha_e)^2 + \alpha_E\alpha_e < \frac{4-3\gamma^2}{2(2+\gamma)^2(1-\gamma)}(\alpha_E - \alpha_e)^2 + \alpha_E\alpha_e$$

given that $\frac{4-3\gamma^2}{2(2+\gamma)^2(1-\gamma)} \geq \frac{1}{2}$ for all $\gamma \in [0, 1]$. Then the left side of the inequality holds.

For the right side, first observe that for $\gamma = 0$ the inequality holds: $\frac{1}{2}(\alpha_s - \alpha_n)^2 + \alpha_s\alpha_n < \alpha_s^2$. Then observe that for $0 < \gamma < \hat{\gamma}_e$, using Corollary 1.1

$$\alpha_s^2 > (\alpha_E + \alpha_e - \alpha_n)^2 > \frac{4-3\gamma^2}{2(2+\gamma)^2(1-\gamma)}(\alpha_E - \alpha_e)^2 + \alpha_E\alpha_e$$

[17]but with general convex costs the same results hold

given that $\frac{4-3\gamma^2}{2(2+\gamma)^2(1-\gamma)} < \frac{2}{3}$ for $\gamma < \hat{\gamma}_e$.

□

Bibliography

Allain, M. L., and Chambolle, C. (2007). Anti-competitive effects of resale-bellow-cost laws. *Cahier de Recherche de l'Ecole Polytechnique* no. 2009-09. Available at http://ideas.repec.org/p/hal/wpaper/hal-00367492_v1.html.

Armstrong, M. (1998). Network Interconnection in Telecommunications. *The Economic Journal*, 108(448), pp. 545-564.

Binmore, K., Rubinstein, A., and Wolinsky, A. (1986). The Nash Bargaining Solution in Economic Modelling. *The RAND Journal of Economics*, 17(2), pp. 176-188.

Cambini, C., and Valletti, T. M. (2004). Access charges and quality choice in competing networks. *Information Economics and Policy*, 16(3), pp. 391-409.

Cheng, H. K., Bandyopadhyay, S., and Guo, H. (2011). The Debate on Net Neutrality: A Policy Perspective. Forthcoming in *Information Systems Research*, 22(1). Available at http://ssrn.com/abstract=959944.

Choi, J. P., and Kim, B. C. (2010). Net Neutrality and Investment Incentives. *The RAND Journal of Economics*, 41(3), pp 472-597.

Crémer, J., and Riordan, M. H. (1987). On governing multilateral transactions with bilateral contracts. *The RAND Journal of Economics*, 18(3), pp. 436-451.

Economides, N., and Tåg, J. (2007). Net neutrality on the Internet: A two-sided market analysis. *NET Institute Working Paper* No 07-45. Available at http://ssrn.com/abstract=101912.

Faulhaber, G. R., and Hogendorn, C. (2000). The market structure of broadband telecommunications. *The Journal of Industrial Economics*, 48(3), pp. 305-329.

Fijnvandraat, M., and Bouwuman, H. (2006). Flexibility and broadband evolution. *Telecommunications Policy*, 30, pp. 424-444.

Hermalin, B. E., and Katz, M. L. (2007). The economics of product-line restrictions with applications to the network neutrality debate. *Information Economics and Policy*, 19(2), pp. 215-248.

Horn, H., and Wolinsky, A. (1988). Bilateral monopolies and incentives for merger. *The RAND Journal of Economics*, 19(3), pp. 409-419.

Kocsis, V., and de Bijl, P. (2007). Network neutrality and the nature of competition between operators. *International Economics and Economic Policy*, 4(2), pp. 159-184.

Laffont, J. J., Rey, P., and Tirole, J. (1998). Network competition: I. overview and nondiscriminatory pricing. *The RAND Journal of Economics*, 29(1), pp. 1-37.

McAfee, P., and Schwartz, M. (1994). Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity. *The American Economic Review*, 84(1), pp. 210-230.

Milliou, C., and Petrakis, E. (2007). Upstream horizontal mergers, vertical contracts. *International Journal of Industrial Organization*, 25(5), pp. 963-987.

O'Brien, D. P., and Shaffer, G. (1992). Vertical control with bilateral contracts. *The RAND Journal of Economics*, 23(3), pp. 299-308.

Chapter 2

Investment with commitment contracts: the role of uncertainty

2.1 Introduction

Infrastructure industries have experienced an essential change in regulation. More than thirty years ago, firms in these industries were considered natural monopolies and were therefore protected from entry but subject to rate of return regulation. Nowadays competition is actively promoted in many countries and infrastructure-based firms face several forms of incentive regulation.

The passage from a natural monopoly to market competition was made by mandating access to the incumbent firm's infrastructure. This infrastructure was in most countries a heritage of the formerly State-owned monopoly. Technological progress and the increasing utilization of network will require the upgrade and eventually the renovation of this inherited infrastructure. In telecommunications, the roll-out of Next Generation Access networks ("NGA networks") based on fibre optic cable is aspired to replace cooper-based broadband services.

Anticipating the replication of a monopoly infrastructure, regulatory authorities will likely apply ex ante regulation for whoever builds a network. For instance, the European Commissions guidelines draft for regulation of next generation access networks encompass mandated access for operators that roll out fibre optic networks.[1]

Fostering retail market competition without hindering investment can be a challenging objective. The economics literature on R&D rivalry and adoption of new technology has pointed out that if a firm suffers a profit reduction when its rival innovates then a firm has incentives to be the first one to adopt the new technology and win the investment race. However when imitation is possible and the profit reduction effect is not significant, there is a second-mover advantage as firms wait for others to incur R&D fixed costs and then imitate; as a consequence investment is delayed, see Katz and Shapiro (1987).[2] Mandating access to firms in network industries that drastically reduces investment profits can induce the same late adoption effect. In particular, when industries are characterized by the irreversibility of much of the infrastructure cost and by risk due to uncertainty on future profits, firms are not willing to invest if access policy asymmetrically allocates the risk to the firm that invests first.

The industrial organization literature on the impact of access regulation on broadband investment takes three approaches.[3] Fist, a body of literature focuses on asymmetric frameworks where an entrant who enjoys regulated access to the incumbent's network invest and builds its own network (see Bourreau and Doğan (2005); Cave (2006); Schutz and Trégouët (2008), etc.). A second body of literature studies the incentives of an incumbent to upgrade its network when subject to mandated access (see Kotakorpi (2006); Camacho and Menezes (2009); Nitsche and Wiethaus (2010)) The third body of litera-

[1]See the European Commission draft recommendation on regulated access to Next Generation Access Networks available at: `http://ec.europa.eu/information_society/policy/ecomm/library/public_consult/nga` and `http://ec.europa.eu/information_society/policy/ecomm/library/public_consult/nga_2`.

[2]This is the case if most of the investment has to be done only once and not continuously. When innovation is sequential and consecutive it has been recently demonstrated that imitation has a positive effect on innovative outcomes, see Bessen and Maskin (2009). One can agree that the roll-out of NGA networks corresponds mainly to a fixed investment.

[3]See Cambini and Jiang (2009) for a literature review on broadband investment and regulation.

ture, which is the closest to our purpose, focuses on a more symmetric situation where both the former incumbent and entrant must build the network from "scratch". In this literature, inspired from the patent race work, the incumbent role is endogenous to the model (see Vareda and Hoernig (2009); Hori and Mizuno (2006, 2009); Gans (2001)). The object of study is then to determine if investment is timely when firms know that access to their network will be mandated. In general, it finds that with high access charges first investment is made earlier than with low access charges. The regulatory authority must then compromise static efficiency in order to induce early investment.

This paper studies the private incentives to invest on a new network when it cannot be financed by direct subsidies and revenues from the market are the only source of funding. Concisely, it finds that for adverse market configurations, more sophisticated access contracts including commitment clauses between firms can be more efficient than simple linear wholesale access tariffs.

Section 2.2 describes the model of two ex ante symmetric firms that must make an irreversible investment decision while facing future uncertain profits together with possible access obligations. Section 2.3 builds the benchmark of perfect information on future profits to illustrate the effect of the access charge level on the equilibrium outcome. As highlighted by the literature, for example by Bourreau and Doğan (2005), low access charges induce service-based competition, whereas high access charges encourage facilities-based competition as long as profits are high enough to cover fixed costs for both firms.

Section 2.4 introduces uncertainty on future market profitability. With unknown future profits, firms delay investment when they know rivals are willing to invest. In effect, mandated access policies, as Guthrie (2006) points out, gives "investment flexibility" to firms: they invest in realized good states and seek access in bad states. Given that industry profits are revealed only when one of the firms has invested, it is profitable to delay investment waiting for the other firm to incur the sunk cost. As a result, uncertain profits combined with mandated access policies do not induce simultaneous “timely” investment.

If the roll-out of NGA is desired, the wholesale price must be sufficiently high to encourage one of the firms to invest first.

Section 2.5 characterizes the market configuration induced by a linear wholesale price set by the regulator prior to investment decisions. Again, with a low access charge the firm that has delayed investment will seek access instead of building its own network in good and bad states. Whereas with a higher access charge, the firm that has delayed the deployment of the network will continue to seek access if the industry is revealed not very profitable, but it will bypass the firm that has invested first in good states by rolling out its network. This opportunistic behaviour reduces the expected wholesale profit for the firm that has invested first. Given anticipated rent losses in good states, a firm will roll out the network only if it is further compensated for bearing all the risk. Considering this, the literature and authorities have introduced the notion of “risk premium” access charges: a wholesale mark-up aimed at compensating the investing firm, see Pindyck (2007).

However, when the sunk cost is rather high and industry profits are expected to be low, a firm anticipating the wholesale rent reduction in good states will no longer be willing to invest even if monopoly rents are allowed in bad states. Then, only a firm in monopoly would incur the sunk cost.

Long-term contracts or contracts with commitment clauses have been used in other industries for a better risk management. For example, long-term contracts in electricity generation markets are used to encourage entry by creating reliance on long-term relationships c.f. Onofri (2003). They have also been used in the telecommunications industry for quite a while. They are usually agreements to best cope with heavy investment like transoceanic fiber optic deployments, they are known as Indefeasible rights of use (IRU) aggrements. This idea is then applied to NGA investment. If the rival firm contractually commits to seeking access in either state, then the firm that would had otherwise not invested secures wholesale profits that cover the sunk cost, solving the hold-up problem. Similarly, an access contract with commitment clauses allows a reduction of the risk premium wholesale price. The network-providing firm renounces to higher wholesale revenues in bad states by assuring wholesale revenues in good states. The network-seeking firm gives up facilities-based rents in good states for a lower access charge in bad ones. The paper shows that ex ante commitment clauses between firms can be beneficial even if firms privately agree on the wholesale price access level.

2.2 The model

Consider two risk-neutral identical firms that plan to enter into an infrastructure-based industry. The firms need a network facility to be able to compete in the retail market.

A firm builds its facility at a fixed sunk cost $I > 0$, investment costs on infrastructure are considered irreversible. However, it is not necessary that both firms deploy a network in order to compete in the retail market. The firm without a network may use its rival's network to provide the service in exchange of a usage-based access charge $w \geq c$, where $c \geq 0$ is the infrastructure usage costs. For simplicity, other retail costs are normalized to zero.

Industry profit flows are uncertain for both firms as well as for the regulator until investment is made. This could be, for example, due to uncertainty on future demand or due to common exogenous industry shocks. This simple model supposes that the variation of profit flows is reduced to two possible states of the world: a good or optimal state, represented by the parameter θ_H, which occurs with probability $\mu \in [0, 1]$; and a bad or adverse state, represented by the parameter θ_L such that $0 < \theta_L < \theta_H \leq 2$.[4] The state of the world is revealed *only* when at least one of the firms has rolled out the infrastructure. If none of the firms invests, the service is not provided and profits from the service provision remain of course uncertain. The model assumes that the opportunity cost of providing the service is normalized to zero.

[4]We suppose that $\theta_H \leq 2$ in order to have welfare decrease with the access charge. See section 2.3.1 for details.

Throughout the paper, firms compete in the retail market by setting downstream capacities, i.e. with Cournot competition. The model uses a simple linear inverse demand function $p = 1 - Q$, with $Q = q_1 + q_2$ the total downstream capacity.[5]

If only one firm has rolled out the infrastructure profits are asymmetric. The profit for the firm that has invested, who will be called from now on the *network provider*, is

$$\Pi^P = \theta \left\{ q_1(p-c) + q_2(w-c) \right\} - I$$

with q_1 its downstream capacity, and q_2 its rival's capacity. The profit for the other firm, called the *network seeker*, is then

$$\Pi^S = \theta \left\{ q_2(p-w) \right\}$$

For both firms the exogenous industry shock $\theta = \theta_H, \theta_L$ is set according to nature.[6]

If both firms build their facilities, profits are symmetric and equal to

$$\Pi^C = \theta \left\{ q_i(p-c) \right\} - I$$

Firms need two periods to build a network from scratch. The first period is used to undertake the necessary civil engineering work and the second to lay out cables and other necessary equipment. If one firm has a built civil infrastructure, as regulation contemplates, must give access to ducts and conduits for other firms to lay out the optical equipment.[7]

The timing of the game is as follows:

0. Nature privately sets the state θ_H or θ_L

[5]The model can be adapted to price competition in the downstream market. Once firms set their quantities at equilibrium, the profit flows that depend on the access charge have equivalent properties. The Cournot competition can be interpreted as capacity on selling the final service: advertising, points of sale, etc.

[6]The basic structure of profits in the model is inspired from Hori and Mizuno (2006, 2009).

[7]Supposing that firms need two periods to build a network greatly simplifies the structure and the resolution of the game. See the appendix for a graphical representation of the game tree.

1. The regulator sets the mandatory access conditions.

2. Firms simultaneously decide to invest or to delay investment. If at least one firm has invested, the state of the world is revealed.

3. If only one firm invested, the firm that delayed investment observes the announced state of the world and chooses:

   - To invest and deploy its own network or *bypass* the firm.

   - To not dot invest and *seek access* to the competitor's network.

   If both firms have invested in stage 2., they continue with the roll out of the network.
   If both of them have decided to delay investment, they can decide to invest now, but none of them would make profits in stage 4.

4. Firms compete in the retail market.

2.2.1 Retail market competition

In the last stage, firms set their downstream capacity, which determines the margins over variable costs corresponding to the non-stochastic part of the firms profits. Cournot equilibrium is standard.

Service-based competition (SBC) If only one firm has rolled out a network, profits at equilibrium are

$$\begin{aligned} \Pi^P(w,\theta) &= \theta\pi_p(w) - I \qquad (2.1) \\ \Pi^S(w,\theta) &= \theta\pi_s(w) \end{aligned}$$

Where $\pi_p(w) = \frac{(1-2c+w)^2}{9} + \frac{(w-c)(1-2w+c)}{3}$, and $\pi_s(w) = \frac{(1-2w+c)^2}{9}$ are the non-stochastic part of profits.

The market is shared by both firms as long as the access charge is not higher than $w_m = \frac{1+c}{2}$ which is the access charge that the network provider would have set to maximize its profits and corner the other firm. For future reference, denote by $\pi_m = \frac{(1-c)^2}{4}$ profit flows of the vertical integrated firm in monopoly. The profit in monopoly is $\Pi^M(\theta) = \theta\pi_m - I$.

Facilities-based competition (FBC) If both firms have rolled out a network, equilibrium margins over variable costs are symmetric and equal to $\pi_c = \frac{(1-c)^2}{9}$, total profits are

$$\Pi^C(\theta) = \theta\pi_c - I \tag{2.2}$$

These profit flow functions at retail equilibrium have some natural intuitive properties. First, profits for the network provider increase with the access charge, whereas they decrease for the network seeker. Second, profit flows for the access provider are superior to profit flows for the access seeker as long as the access charge is above marginal costs. Third, access charges at marginal cost set zero wholesale margins for the network provider and firms have the same profit flow as in facilities-based competition. Finally, the difference in margins set by an access charge $w > c$ between firms is represented by the following relation:

$$\frac{\pi_p(w) - \pi_c}{\pi_c - \pi_s(w)} = \frac{5}{4} \tag{2.3}$$

Note as well the relation between profit flows in competition and in monopoly

$$\pi_m = \frac{9}{4}\pi_c.$$

2.3 Benchmark: Perfect information

2.3.1 Welfare

Social welfare is defined by the sum of profits and consumer surplus minus costs. For a given θ, welfare in service-based competition is

$$W_{SBC}(w, \theta) = CS(w) + \theta\left(\pi_p(w) + \pi_s(w)\right) - I$$

where consumer surplus is $CS(w) = \frac{(2-w-c)^2}{18}$. One can readily verify that welfare decreases with the access charge as long as $\theta_H \leq 2$.

With facilities-based competition welfare is

$$W_{FBC}(\theta) = CS_c + 2\theta\pi_c - 2I + \phi$$

where $CS_c = \frac{2(1-c)^2}{9}$ is the consumer surplus and $\phi \geq 0$ encompasses the expected dynamic benefits of FBC that our static framework cannot explicitly describe. In effect, FBC is expected to remove heavy regulation requirements in the industry and it is widely accepted as a necessary condition for long-term efficiency.

Other than the dynamic efficiency gains, observe that consumers enjoy a higher surplus with FBC than in SBC if the access charge is above network costs $w > c$. However they enjoy the same surplus if the access charge equals marginal costs $CS_c = CS(c)$. Similarly if $w = c$ profit flows for the access provider and access seeker equal FBC profit flows $\pi_p(c) = \pi_s(c) = \pi_c$. Then, if investment is possible with $w = c$, a social planner opts for SBC if $\phi < I$. If the access charge is $w > c$, then is socially preferable to have FBC if infrastructure costs are low and SBC otherwise.

Figure 2.1 can be understood in terms of geographical density. In urban areas deployment costs are rather low compared to rural areas.

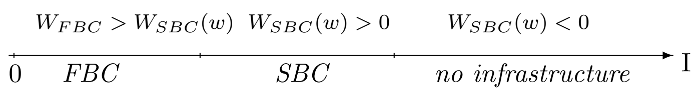

Figure 2.1: Preferred type of competition when no subventions are allowed.

As a general rule, if the regulator promotes SBC, it will do so by setting the lowest access charge that induces investment.

We are interested in the particular range of values where FBC competition is viable only in a good state of the world:

$$\Pi^C(\theta_H) \geq 0 \geq \Pi^C(\theta_L) \tag{2.4}$$

This condition bounds throughout the paper the range of investment costs of interest

$$\underline{I} \equiv \theta_L\pi_c \leq I \leq \theta_H\pi_c \equiv \bar{I}.$$

2.3.2 Equilibrium with perfect information

This section develops the reference point for the incomplete information game. With perfect information both the regulator and the firms know the state of the world, ie. future industry profits. Yet the regulator has the availability to determine the market outcome according to the access charge he sets in the first stage of the game.

Suppose first that the state of the world is θ_L. If one firms has invested at stage 2, the other firm chooses to seek access at stage 3 given that bypassing leads to negative profits. Then, the investment game at stage 2 can be represented by the matrix:

	Invest	Delay investment
Invest	$\Pi^C(\theta_L)$, $\Pi^C(\theta_L)$	$\mathbf{\Pi^P(w, \theta_L)}$, $\mathbf{\Pi^S(w, \theta_L)}$
Delay investment	$\mathbf{\Pi^S(w, \theta_L)}$, $\mathbf{\Pi^P(w, \theta_L)}$	0 , 0

for all access charge $c \leq w \leq w_m$ set by the social planner.

In a bad state of the world the only possible equilibrium is SBC as long as the firm that invests makes no deficit $\Pi^P(w, \theta_L) \geq 0$. We suppose that if firms are indifferent between investing and not investing, they invest. In this way, the regulator can induce the roll-out of at least a network by setting $w \geq w_L$, where w_L saturates the budget constraint $\Pi^P(w_L, \theta_L) = 0$. This access charge can be explicitly calculated:

$$w_L = w_m - 3\sqrt{\frac{\theta_L \pi_m - I}{5\theta_L}}$$

The access charge that encourages investment reduces monopoly profits for the network provider as it lowers the tariff he would have set otherwise in an unregulated environment: w_m the monopoly foreclosing access charge. Remark that regulated service-based competition is viable in a bad state as long as investment is feasible for a firm in monopoly in a bad state, that is, the radical of the above expression is real.

In a good state θ_H, the regulator can decide which market configuration he promotes.

Denote by $\tilde{w}$ the access charge that makes a firm indifferent between bypassing and seeking access (provided the other one has invested): $\Pi^C(\theta_H) = \Pi^S(\tilde{w}, \theta_H)$. Explicitly,

$$\tilde{w} = w_m - 3\sqrt{\frac{\theta_H \pi_c - I}{4\theta_H}} \tag{2.5}$$

By setting a "high" access price $w \geq \tilde{w}$ we have that $\Pi^C(\theta_H) \geq \Pi^S(w, \theta_H)$. Then given that one firm has invested in stage 2, its rival faced to the choice of bypassing or seeking access will bypass. In this way, FBC is the equilibrium induced by the regulator.[8]

Whereas with a "low" access charge $c \leq w < \tilde{w}$, a firm seeks to have access to rival's network - provided that there is one - if given the option of bypassing or seeking access. By replacing θ_L for θ_H, the matrix represented above describes the game at stage 2 in a good state of the world. It is straightforward to verify that in a good state a firm invests at marginal cost $\Pi^P(c, \theta_H) = \theta_H \pi_p(c) - I = \theta_H \pi_c - I \geq 0$. Then SBC is the equilibrium of the game.

Bourreau and Doğan (2005) were among the first to formalize the idea that if the regulator wishes to encourage facilities-based competition, access charges should not be too low. The exposed above is resumed in the following lemma.

Conclusion 2.1. *Suppose that firms and the regulator have knowledge of the state of the world.*

- *In θ_L, the equilibrium is SBC if the access charge is w_L.*
- *In θ_H, if the access charge equals c then the equilibrium is SBC. If on the other hand the access charge is set to $\tilde{w}$, the equilibrium is FBC.*

2.4 Uncertainty

This section describes in more detail the investment game under uncertainty and it shows, as the previous section, how the access charge set by the regulator can create investment incentives and, consequently, determines the market equilibrium outcome.

[8]Note that this outcome is obtained either by both firms investing at stage 2, or by a sequential investment in stages 2 and 3.

Suppose that at stage 2 one firm invests and the other one delays investment. As developed in the previous section, the decision of the firm that has delayed investment in stage 3 depends on the access charge level.

If $w \geq \tilde{w}$, the firm that has delayed investment chooses between "seeking access" or "bypassing" while having full information of the state of the world. In a good state it bypasses and in a bad state it seeks access. Its expected profits at stage 2 are then $\mu\Pi^C(\theta_H) + (1-\mu)\Pi^S(w, \theta_L)$. The expected profits for the firm that invests in stage 2 are $\mu\Pi^C(\theta_H) + (1-\mu)\Pi^S(w, \theta_L)$.

If on the other hand the access charge is low $c \leq w < \tilde{w}$, then in either state of the world the firm that delays investment bypasses. Its expected profits are then $\mu\Pi^S(w, \theta_H) + (1-\mu)\Pi^S(w, \theta_L)$, and consequently the firm that invests at stage 2 is the access provider for either case, expecting profits $\mu\Pi^P(w, \theta_H) + (1-\mu)\Pi^P(w, \theta_L)$.

In sum, the expected payoffs for the firms that invests first are

$$X_I(w) = \begin{cases} \mu\Pi^P(w, \theta_H) + (1-\mu)\Pi^P(w, \theta_L), & c \leq w < \tilde{w} \\ \mu\Pi^C(\theta_H) + (1-\mu)\Pi^P(w, \theta_L), & \tilde{w} \leq w \leq w_m \end{cases} \tag{2.6}$$

and for the firm that delays investment:

$$X_D(w) = \begin{cases} \mu\Pi^S(w, \theta_H) + (1-\mu)\Pi^S(w, \theta_L), & c \leq w < \tilde{w} \\ \mu\Pi^C(\theta_H) + (1-\mu)\Pi^S(w, \theta_L), & \tilde{w} \leq w \leq w_m \end{cases} \tag{2.7}$$

Considering this, the game in stage 2 can be rewritten in normal form

	Invest	Delay investment
Invest	$E[\Pi^C(\theta)]$, $E[\Pi^C(\theta)]$	$X_I(w)$, $X_D(w)$
Delay investment	$X_D(w)$, $X_I(w)$	0 , 0

where $E[\Pi^C(\theta)] = \mu\Pi^C(\theta_H) + (1-\mu)\Pi^C(\theta_L)$ is the expected gain in facilities-based competition.

The strategic form representation of the game allows us to determine the effect of the access terms on the equilibrium outcome of this investment game.

Observe first that in this simplified framework, firms face a second-mover advantage. In other words, the best response to *"Invest"* is to *"Delay investment"*. In effect, after a firm has rolled out its network, it is mandated to open it for firms that delay investment. This gives, as Guthrie (2006) calls, "investment flexibility" to coming entrants. It is profitable for firms to wait for the other firm to commit to invest and to have industry profits revealed before deciding how to enter the market. Consequently, firms facing uncertainty will unlikely invest simultaneously. Proposition 2.1 follows.

Proposition 2.1. $E[\Pi^C(\theta)] < X_D(w)$ *for all* $w \leq w_m$. *In other words, if facilities-based competition is only feasible in an optimal state of the world then* {Invest, Invest} *is not equilibrium for any access charge.*

Proof. See the appendix for all proofs. □

If the construction of at least one infrastructure is desired, the regulator needs to encourage one of the firms to lead investment and obtain a {*Invest, Delay investment*} equilibrium. Formally, in this model, the access charge that guarantees some form of investment must set the expected profit of the investing firm higher than its delaying option $X_I(w) \geq 0$.

2.5 Access charge levels and the role of commitment

This section analyses in more detail the equilibrium of the investment game. It characterizes the access charges that encourage investment, and most importantly, it argues that in some cases a commitment clause between the investing firm and its rival is the only way to have market competition and investment.

2.5.1 Access tariffs that induce investment

As suggested in the previous section, there are two possible equilibria where the market develops and some form of competition is preserved.

In the first one, with a low access charge, only one firm invests having service-based competition in any state of the world, call it SBC-equilibrium. The access charge that induces it must guarantee an expected non-negative profit for the network provider:

$$w_{SBC} = \min\left\{w \in [c, \tilde{w}) \mid X_I(w) = E[\Pi^P(w,\theta)] \geq 0\right\}$$

Fix μ the probability of a good state, and write $E[\theta] = \mu\theta_H + (1-\mu)\theta_L$ the mean of the stochastic part of profit. Our simple framework allows for an explicit computation of the SBC-equilibrium tariff:

$$w_{SBC} = \begin{cases} c, & I < \tilde{I}_{SBC} \\ w_m - 3\sqrt{\frac{E[\theta]\pi_m - I}{5E[\theta]}} & \tilde{I}_{SBC} < I \leq \bar{I}_{SBC} \end{cases}$$

where

$$\tilde{I}_{SBC} = \frac{4}{9}E[\theta]\pi_m \quad \text{and} \quad \bar{I}_{SBC} = \frac{16}{9}\left(\frac{\theta_H E[\theta]}{4\theta_H - 5E[\theta]}\right)\pi_m.$$

The second equilibria can go either way, FBC in a realized good state and SBC in a bad state. Write it xBC-equilibrium, the wholesale tariff that induces it is the solution of

$$w_{xBC} = \min\left\{w \in [\tilde{w}, w_m) \mid X_I(w) = E[\Pi^P(w,\theta)] - \mu\left(\Pi^P(w,\theta_H) - \Pi^C(\theta_H)\right) \geq 0\right\}$$

Observe that w_{xBC} must account for the wholesale profit reduction in good states. This gives a mark-up to the wholesale price that the literature has called “risk premium”. In other words, it is necessary to set an access charge that compensates the “leading” firm for the opportunistic behavior of rivals. As Guthrie (2006) and Pindyck (2007) point out, the “leading” firm bears all of the downside risk and it must be compensated.

The xBC access tariff can be explicitly calculated:

$$w_{xBC} = \begin{cases} \tilde{w}, & I < \tilde{I}_{xBC} \\ w_m - 3\sqrt{\frac{1}{5}\left(\pi_m - \frac{I - \mu\theta_H\pi_c}{(1-\mu)\theta_L}\right)}, & \tilde{I}_{xBC}(\mu) < I \leq \bar{I}_{xBC} \end{cases}$$

where

$$\tilde{I}_{xBC} = \frac{16}{9}\left(\frac{\theta_H E[\theta]}{4\theta_H - 5(1-\mu)\theta_L}\right)\pi_m \quad \text{and} \quad \bar{I}_{xBC} = \frac{1}{9}\left(4\mu\theta_H + 9(1-\mu)\theta_L\right)\pi_m.$$

Naturally, access charges increase with the cost of the network. For low infrastructure costs, they take the lowest value. In the SBC equilibrium retail profits cover fixed costs and it is sufficient to set the wholesale tariff equal to marginal costs $w_{SBC} = c$. Whereas for the xBC equilibrium, the "risk premium" access charge exceeds marginal costs $w_{xBC} = \tilde{w} > c$. See Figure 2.2 for an illustration of the access charges.

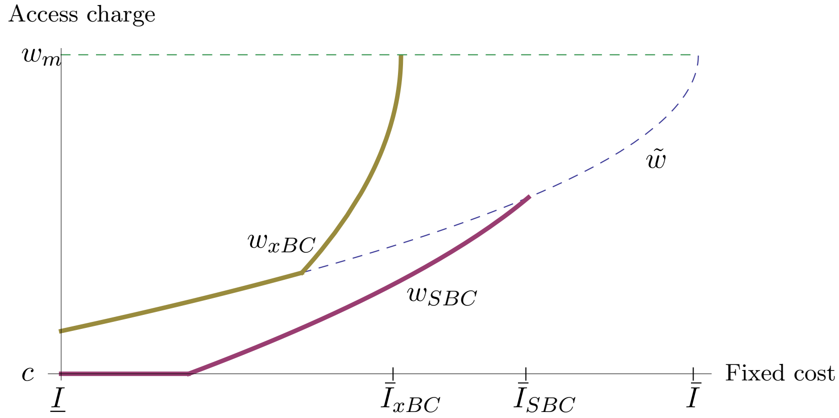

Figure 2.2: Access charges in function of the fixed cost. Parameters are $c = 0$, $\mu = 0.2$, $\theta_H = 2$, $\theta_L = 0.5$.

With low sunk costs, both firms make some profits. For higher costs the budget condition is saturated for the firm that invests first, whereas the other firm makes some profits. For even higher costs, it might be that investment is not feasible.

When firms do not invest The highest cost at which a firm invests in a xBC-equilibrium is $\bar{I}_{xBC}$. In effect, $w_{xBC}(\bar{I}_{xBC}) = w_m$, in a bad state the network provider is in monopoly. For higher investment costs $I > \bar{I}_{xBC}$, the expected profit for leading firm is negative, $X_I < 0$.

At the SBC-equilibrium, the maximal feasible network cost $\bar{I}_{SBC}$ is defined by the access charge threshold $\tilde{w}$. Whenever $w \geq \tilde{w}$, we have an xBC-equilibrium and the expected profits are negative for the firm that invests first if $w < w_{xBC}$. This is illustrated in Figure 2.2.

Firms invest (for all $\mu \in [0,1]$) in the interval $[\underline{I}, \bar{I}]$ whenever the impact of the stochastic part on profits is not important and the following condition does not hold:

$$\frac{\theta_H}{\theta_L} > \frac{\pi_p}{\pi_c} = \frac{9}{4} \tag{2.8}$$

When this condition holds the investment upper limit is bellow $\bar{I}$ for low values of μ. Then we have, as illustrated in Figure 2.2, that the xBC-equilibrium is not feasible in the interval $[\bar{I}_{xBC}, \bar{I}]$ and the SBC-equilibrium is not feasible in the interval $[\bar{I}_{xBC}, \bar{I}]$.

Monopoly investment We are interested to compare the regulated access charge scenario with the natural monopoly[9] framework. This gives us a benchmark to the upper limit infrastructure costs to which a private investment is possible without public funding.

A natural monopoly has expected profits equal to $E[\Pi^M] = \mu(\theta_H \pi_m - I) + (1 - \mu)(\theta_L \pi_m - I)$. Then, a monopoly invests if $I \leq \bar{I}_M = E[\theta]\pi_m$. Observe that if condition (2.8) holds, then $\bar{I}_M < \bar{I}$ for low values of μ.

We conclude this section with an important remark. For costs in the interval $[\bar{I}_{SBC}, \bar{I}_M]$, the expected payoff of a firm that serves as access provider in all states of nature is positive only for access charges exceeding the threshold $\tilde{w}$. Nevertheless, no firm is willing to invest because it anticipates that at this access charge, the competitor would bypass instead of buying access in a good state. Access contracts with commitment clauses are, in this situation, virtuous.

2.5.2 The role of commitment

Suppose that for a given μ the market configuration is adverse (a relatively expensive network compared to expected profits) such that no investing equilibrium is feasible but a firm in monopoly would invest,

$$\bar{I}_M \geq I > \max\left\{\bar{I}_{SBC}, \bar{I}_{xBC}\right\} \tag{2.9}$$

[9]That is, a firm that has exclusive rights over the territory to serve the market.

Such configurations are particularly challenging. Conciliating private investment and market competition seems incompatible. The lack of investment is closely related to the hold-up problem for suboptimal investment. In general, it can be solved with ex ante commitment contracts.

In particular, if firms could contract before investment decisions are made, then a firm committing to another to buy access during good and bad states allows to extends the range of feasible costs up to the maximal feasible private investment level $\bar{I}_M$. In effect, consider commitment contracts of the form $\{w, F\}$, where w is the linear access payed by the access seeker in any state of the world, and F is a compensation fixed payment in case of non-fulfillment of the commitment clause.

Commitment contract when no network is deployed When the cost of investment is high and condition (2.9) holds neither firm is willing to invest in a regulated market: {Delay investment, Delay investment} is the equilibrium of the game. They both have reservation profits equal to zero.

If firms were to agree *ex ante* to an access charge, it must be set at a level that both firms obtain non-negative profits. Write $\underline{w}_{ic}$ the minimal access charge that the network provider is willing to accept $E[\Pi^P(\underline{w}_{ic}, \theta)] = E[\theta]\pi_p(\underline{w}_{ic}) - I = 0$, and w_m. The maximal access charge that the network seeker is willing to pay is the monopolistic one $E[\Pi^S(w_m, \theta)] = E[\theta]\pi_s(w_m) = 0$.

We show that there exists a range of access charges, $[\underline{w}_{ic}, w_m]$, for which both firms expect positive profits. In effect,

$$\frac{\pi_p(\bar{w}_{ic}) - \pi_c}{\pi_c - \pi_s(w_m)} = \frac{\frac{I}{E[\theta]} - \pi_c}{\pi_c - 0} = \frac{I}{\frac{4}{9}E[\theta]\pi_m} - 1 < \frac{5}{4}$$

which holds as long as monopoly investment is feasible. Given relation (2.3) and that π_p is increasing we have that $\underline{w}_{ic} < w_m$.

The non-fulfillment fixed payment is set to avoid opportunistic behavior. For example, for the wholesale tariff chosen, it is sufficient to set $F > \Pi^C(\theta_H) - \Pi^S(w, \theta_H)$.

The level of the access charge in a contract with commitment clauses in $[\underline{w}_{ic}, w_m]$ can be negotiated between firms (as discussed in the next section) or it can be determined by the regulator. To be precise, we should no longer refer to a commitment contract but to an infrastructure monopoly franchise if the regulator sets the access charge level and forbids the duplication of the infrastructure immediately after industry profits are revealed. In this case, the regulated wholesale tariff can be exactly $\underline{w}_{ic}$. In any case, the tariff will be lower than the monopoly access charge that leads monopoly retail prices.

The proposition summarizes the discussion.

Proposition 2.2. *Access contracts with commitment clauses between firms are one answer to the dilemma of preserving market competition and encouraging investment in adverse market configurations.*

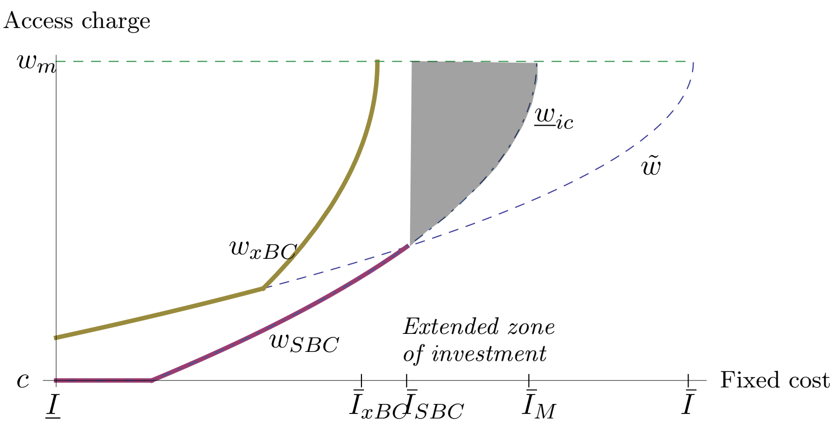

Figure 2.3: All access charges in the gray zone $[\underline{w}_{ic}, w_m]$ extend the zone of possible investment. Parameters are $c = 0$, $\mu = 0.15$, $\theta_H = 2$, $\theta_L = 0.5$.

Commitment contracts that reduce access charges Another adverse market configuration is given when, with excessively high access prices, the only possible outcome is the xBC-equilibrium. It could be that,

$$\bar{I}_{xBC} \geq I > \bar{I}_{SBC} \tag{2.10}$$

High wholesale prices result in high detail prices, which significantly reduce consumer surplus. At the expense of facilities-based competition, a commitment contract could be then more efficient given that access charges can be considerably reduced. With a lower access price, the profit for the network seeker increase in bad states. If they are sufficiently lowered, its expected profit exceeds that in the xBC-equilibrium. For the network provider, gains come in good states at the expense of lower upstream profits. Actually, there is room for gain for both firms. The proposition follows.

Proposition 2.3. *If the only possible investment equilibrium is xBC, and if it requires a high wholesale price, then commitment clauses allow to lower access charges improving static efficiency. In particular, there exists a range of wholesale prices that are incentive compatible for both firms.*

Proof. See the appendix. □

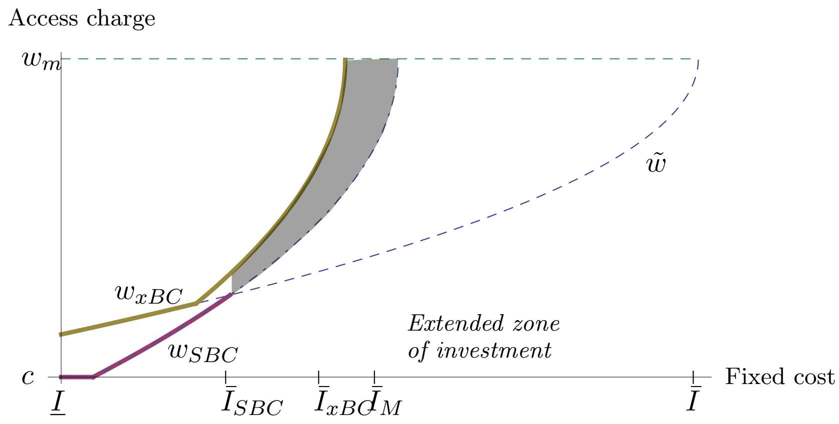

Figure 2.4: All access charges in the gray zone $[\underline{w}_{ic}, w_{xBC}]$ are lower than the access charge w_{xBC}, which increases static surplus. Parameters are $c = 0$, $\mu = 0.05$, $\theta_H = 2$, $\theta_L = 0.5$.

The trade-off between static efficiency (lower access charges) and dynamic efficiency (investment, facilities-based competition in good states) will be analyzed in section 2.6. The message of this section is clear: access contracts with commitment clauses can be beneficial in adverse market configurations as they either encourage investment where otherwise would have not been made, and they reduce wholesale price.

Figure 2.5 does some comparative statics to identify the regions where commitment clauses are necessary to lower access charges, or to create investment incentives.

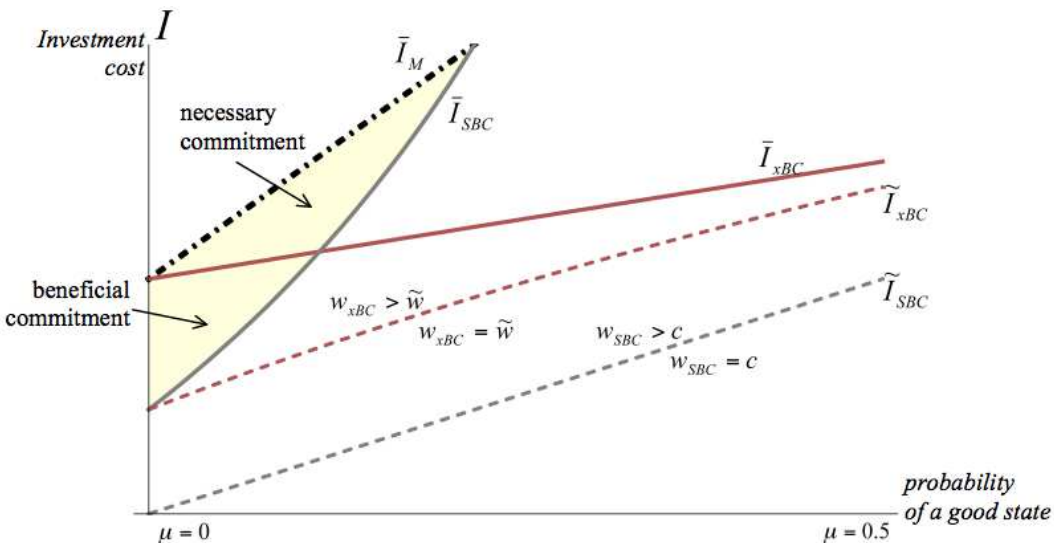

Figure 2.5: Comparative statics. If condition (2.10) holds, a commitment is necessary for investment. For inequality (2.9) a commitment contract allows a lower access charge.

2.5.3 Firms negotiate the access contract

The previous section showed that commitment clauses are necessary to encourage infrastructure investment and to lower access charges. This section focuses on describing how such access contracts, in particular the level of the wholesale price, can be determined between firms.

Linear tariffs It is common to use the Nash bargaining solution[10] as the outcome of the negotiation process between firms. The Nash bargaining solution has some nice properties. For example, it is Pareto efficient and it is somehow robust to renegotiation.[11]

[10]See Binmore et al (1986) for an implementation guide of the cooperative Nash solution to economic modeling.

[11]In the sense of independence of irrelevant alternatives

It has been largely used in the economics literature to model negotiation of wages and contracts in vertical industries, among others. A negotiation problem is defined by a set of possible outcomes and a negotiation breaking point (or as called by the literature, the outside option). The set of possible outcomes is defined in our context by all the possible payoffs firms might obtain according to the access charge level. The outside option is the payoff firms would normally obtain if no agreement were reached. The breaking point is defined here as the non-cooperative equilibrium payoffs firms obtain with the access charge set by the regulator. As illustrated by Figure 2.5, they depend on the market configuration, i.e. the cost of investment and the probability of a good state.

Suppose first that for a given probability, no investment is possible with market competition but it is feasible for a monopoly, inequality (2.9) holds. Then, given that the regulator does not grant monopoly exploitation to the firms, the outside option for both firms is to delay investment, and profits are zero.

The program that solves for the negotiated access charge is

$$\max_{w} \left\{E[\Pi^P(w,\theta)] - 0\right\}^{1/2} \left\{E[\Pi^S(w,\theta)] - 0\right\}^{1/2}$$

The solution is of this program is

$$w_{N_1} = w_m - 3\sqrt{\frac{E[\theta]\pi_m - I}{10E[\theta]}}$$

Observe that as long as investment is feasible for a monopoly, the access charge is lower than the foreclosing one, but higher than the otherwise regulated one. Therefore, prices for consumers are not at the monopoly level. The expected profits that firms obtain are $\frac{1}{2}E[\Pi^M(\theta)]$ for the firm that invests, and $\frac{2}{5}E[\Pi^M(\theta)]$ for the firm that is only active on the service market. Industry profits are lower than expected profits in monopoly.

This shows that an access contract with commitment clauses where firms enter into private agreements[12] are still a more efficient than the access policy that considers simple access contracts (either no investment, or a monopoly franchise).

[12]Notice that this agreement maximizes firms joint profits. The bargaining program is equivalent to maximizing the sum of the log of each firms expected profit.

Similarly, if the xBC-equilibrium is the only equilibrium possible, firms can agree to a lower access charge level with a commitment clause. In this case, the outside option should be taken as the profit firms would obtain with the xBC-equilibrium access charge. Actually, if inequality (2.9) holds, then the expected budget condition binds for the network provider $X_I(w_{xBC}) = 0$ (observe that $\bar{I}_{SBC} \geq \tilde{I}_{xBC}$). The program to solve is

$$\max_{w} \left\{E[\Pi^P(w,\theta)] - 0\right\}^{1/2} \left\{E[\Pi^S(w,\theta)] - X_D(w_{xBC})\right\}^{1/2}$$

Taking into account that the outside option for the delaying firm can be reduced to $X_D(w_{xBC}) = \frac{4}{5}E[\Pi^M(\theta)] - \mu I$, the solution is given by

$$w_{N_2} = w_m - \frac{3}{\sqrt{5}}\sqrt{\pi_m - \left(1 + \frac{5}{8}\mu\right)\frac{I}{E[\theta]}}$$

By construction, the Nash bargaining solution has access charges lower than w_{xBC}, and expected profits for the firms are higher than their outside option.

2.5.4 General commitment contracts

In what follows, three general types of agreements are analyzed and compared in a general setting.

Linear tariff in a general setting Write L^* the outside option payoff for the firm that invests first, and F^* the payoff for the follower. The Nash bargaining solution is of the form

$$w_N = w_m - 3\sqrt{\frac{E[\theta]\pi_m - I - L^* + (5/4)F^*}{10E[\theta]}}$$

Two-part access tariffs Instead of agreeing on a linear access charge, payment for the access to the network can be made in two-part tariffs $\{w, T\}$.

The program that solves for the Nash solution is

$$\max_{w,T} \left\{E[\Pi^P(w,\theta)] + T - L^*\right\}^{1/2} \left\{E[\Pi^S(w,\theta)] - T - F^*)\right\}^{1/2}$$

By standard computation, we find that the fixed part of the tariff is

$$T = \frac{1}{2}(E[\Pi^S(w,\theta)] - F^*) - \frac{1}{2}(E[\Pi^P(w,\theta)] - L^*)$$

The variable tariff solves

$$\max_w E[\theta](\pi_p(w) + \pi_s(w)) - I - L^* - F^*$$

The tariff that maximizes this problem is $w = \frac{1+c}{2} = w_m$, the access charge that leaves the access seeker out of the market. The fixed fee is then

$$T = -\frac{1}{2}(E[\theta]\pi_m - I - L^* + F^*) < 0$$

In other words, when the two firms negotiate a two-part tariff, the access seeker stays out of the market so the access provider can profit from monopoly power. In compensation, the fixed part compensates the access seeker for cooperating.

This solution in general seems to go on the opposite direction of the linear access commitment contract. Welfare is not enhanced.

Access charges contingent to the revealed state of the world We inspect now contracts of the form $\{w_l, w_h\}$ where w_l is the access charge to be used in a bad state of the world and w_h in a good state. Such access charges must at least improve firms' expected profits:

$$\begin{aligned} \mu\theta_H\pi_s(w_h) + (1-\mu)\theta_L\pi_s(\theta_l) &\geq F^* \\ \mu\theta_H\pi_p(w_h) + (1-\mu)\theta_L\pi_p(\theta_l) - I &\geq L^* \end{aligned}$$

By using equation 2.3 and combining these two inequalities, one can see that there exists such a feasible solution if

$$I \leq E[\theta]\pi_m - L^* - \frac{5}{4}F^*$$

Then, by doing some algebra, we find that for w_l given, there exists an interval of access charges for a good state that raise expected profits for both firms.

$$w_h \in \left(w_m - 3\sqrt{\frac{E[\theta]\pi_m - I - L^*}{5\mu\theta_H} - \frac{(1-\mu)\theta_L\pi_s(w_l)}{4\mu\theta_h}}, w_m - 3\sqrt{\frac{F^*}{4\mu\theta_H} - \frac{(1-\mu)\theta_L\pi_s(w_l)}{4\mu\theta_h}} \right)$$

2.6 Welfare implications

The choice the regulator makes between the two equilibrium depends on the dynamic benefits of facilities-based competition and the probability of a good state. If they are null, $\phi = 0$, the Cournot market configuration leads to higher industry profits in service-based competition than in facilities-based competition. For range of parameters taken $[\underline{I}, \bar{I}]$, service-based competition will mostly lead to higher welfare. Then, we feel that as in Nitsche and Wiethaus (2010) it is more interesting to study consumer surplus instead of social welfare.

The welfare for consumers of commitment contracts when only a monopoly would have invested are clear. This section focuses on the second case.

The net consumer surplus is higher with facilities-based competition than with service-based competition $CS_c = \frac{2(1-c)^2}{9} \geq \frac{(2-c-w)^2}{18} = CS(w)$ whenever wholesale price is above marginal cost. An access contract with commitment clauses benefit consumers if the retail price reduction driven by the lower access charge is sufficiently high to compensate the facilities-based competition surplus in a good state.

Supposing that xBC is the only equilibrium outcome, firms negotiating an access contract is beneficial for consumers if the surplus gain exceeds the opportunity cost of facilities-based competition

$$CS(w_N) - CS(w_{xBC}) > \mu \left(CS_c - CS(w_{xBC}) \right)$$

In fact, this is the case. Fixing the stochastic part of profits such that it satisfies condition (2.8) and fixing the marginal cost, it can be numerically verified that the inequality above holds. Figure 2.6 represents the difference in consumer surplus for the relevant area (such that condition (2.10) holds). In general, it can be established numerically that as long as xBC is the only equilibrium access contracts with commitment clauses outperform the non-cooperative equilibrium for a range of parameters $c \in [0, 1)$, $\frac{\theta_H}{\theta_L} \in (\frac{9}{4}, 20)$.

The intuition is simple. If the xBC equilibrium is the only equilibrium, it is because there is a small chance of having a good state. The right hand of the inequality above, the opportunity cost of having facilities-based competition, is then reduced by the small μ. A decrease of the access charge is then valuable compared to this "small" opportunity cost.

Naturally, when firms negotiate the access charge they have higher profits than their outside option. Therefore, social welfare is as well higher.

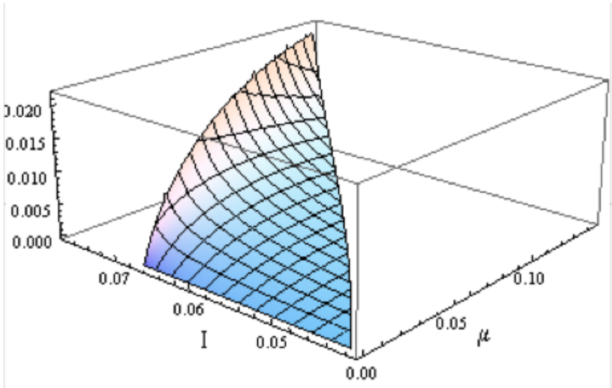

Figure 2.6: Expected consumer surplus difference. Graphic shows the difference between the consumer surplus: $CS_{SBC}(w_N) - (\mu CS_{FBC} + (1-\mu)CS_{SBC}(w_{xBC}))$ for $\bar{I}_{xBC} \geq I > \bar{I}_{SBC}$.

When both equilibria are possible, the regulatory authority must decide on one of them. It is out of the scope of the note to give further considerations about his choice. Facilities-based competition, when probable, seems to bring dynamic efficiencies in the long term at the cost of high access retail prices today.

2.7 Conclusion

This paper has built a simple game that examines investment incentives when firms are subject to access regulation. The model puts at the centre of the analysis the incertitude firms have on future profits. In doing so, it formalizes the idea that access regulation gives firms a second-mover advantage. It is more profitable to wait until demand is revealed by letting the opponent invest and then either seek access or bypass it. This opportunistic behavior allocates asymmetrically the risk on the firm that invests first.

The note has characterized the investment equilibrium the regulator can induce by using the regulated access charge. In order to induce investment, the regulatory authority has two options. To set a low access charge that discourages opportunistic bypassing or to set a high access price that compensates the leading investing firm for loss revenues in good states. However, when investment costs are high and only a monopoly would invest, a firm anticipating the bypass would not invest. In such situations, access contracts with commitment clauses between firms allow some market competition and make investment feasible. In addition, commitment clauses allow reducing the wholesale price in adverse market situations where the probability of high profits is low but the fixed network cost is substantial.

By numerical means, we have highlighted that access contracts with commitment clauses can be more efficient as they conciliate static efficiency gains from lower access charges with investment even if firms agree privately on the access charge level. In absolute terms, discussions about efficient regulation policies should not be restricted to the level of access charges; they should encompass richer forms of access contracts.

2.8 *Appendix:* Proofs of Propositions

The following technical lemma will be of use later on.

Lemma 2.1. *Let $w_1, w_2 > c$ be two different levels of access charge. Then,*

$$\frac{\pi_p(w_1) - \pi_c}{\pi_c - \pi_s(w_2)} < \frac{5}{4} \quad \Rightarrow \quad w_1 < w_2$$

Proof of lemma 2.1. Observe that

$$\frac{\pi_p(w_2) - \pi_c}{\pi_c - \pi_s(w_2)} = \frac{5}{4} > \frac{\pi_p(w_1) - \pi_c}{\pi_c - \pi_s(w_2)}$$

Then, as π_p is increasing the lemma follows. □

Proof of proposition 2.1. We show that for any access charge "delay investment" is best response to "invest today". First, observe that by the industry viability assumption given by (2.4), and given that the access seeker is active in the market: $\Pi^C(\theta_L) \leq 0 <$

$\Pi^S(w, \theta_L)$ for any access charge. Then, the inequality follows for a high access charge $w_m > w \geq \tilde{w}$.

$$X_D(w) - E[\Pi^C(\theta)] = \mu \left(\Pi^C(\theta_H) - \Pi^C(\theta_H)\right) + (1-\mu) \underbrace{\left(\Pi^S(w, \theta_L) - \Pi^C(\theta_L)\right)}_{>0}$$

For a low access charge, $w < \tilde{w}$, it is also immediate by definition of $\tilde{w}$ and given that profits for the access seeker are decreasing

$$X_D(w) - E[\Pi^C(\theta)] = \mu \underbrace{\left(\Pi^S(w, \theta_H) - \Pi^C(\theta_H)\right)}_{>0} + (1-\mu) \underbrace{\left(\Pi^S(w, \theta_L) - \Pi^C(\theta_L)\right)}_{>0}$$

□

Proof of proposition 2.3. Suppose that for a given probability μ of a good state condition (2.10) holds, xBC-equilibrium is the only possible one. Further, if condition (2.10) holds, $I > \tilde{I}_{xBC}$, so the xBC-equilibrium sets zero rents for the leading investing firm, and

$$X_D(w_{xBC}) = \mu\left(\theta_H \pi_c - I\right) + (1-\mu)\theta_L \left(\pi_c - \frac{4}{5}\left(\frac{I - \mu\theta_H\pi_c}{(1-\mu)\theta_L} - \pi_c\right)\right)$$

Again, we can use lemma 2.1. The maximum wholesale tariff that the access seeker is willing to pay is $\bar{w}_{ic}$, defined as $E[\Pi^S(\bar{w}_{ic}, \theta)] = X_D(w_{xBC})$. Given that the access seekers profit decreases with the charges level, necessarily $\bar{w}_{ic} < w_{xBC}$. The access charge that guarantees investment is the same as in the Proposition 2.2, $\bar{w}_{ic}$, with $E[\Pi^P(\bar{w}_{ic}, \theta)] = 0$. It is then sufficient to show that $\underline{w}_{ic} < \bar{w}_{ic}$. In effect,

$$\frac{\pi_p(\bar{w}_{ic}) - \pi_c}{\pi_c - \pi_s(\underline{w}_{ic})} = \frac{\frac{I}{E[\theta]} - \pi_c}{\pi_c - \frac{X_D(w_{xBC})}{E[\theta]}} = \frac{I - E[\theta]\pi_c}{\mu I + \frac{4}{5}(I - E[\theta]\pi_c)} < \frac{5}{4}$$

Then $\underline{w}_{ic} < \bar{w}_{ic}$, and the interval of access charges that improve expected profits for both firms exist. □

2.9 *Appendix:* Graphical representation of the game tree

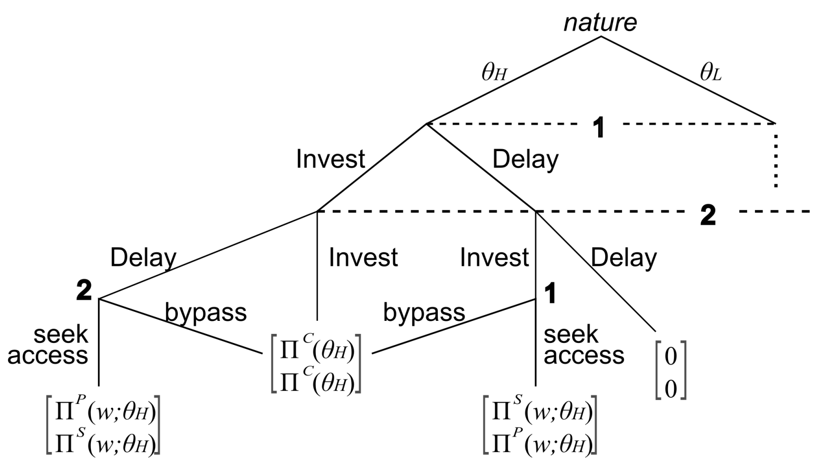

Figure 2.7: The game tree

Bibliography

Bessen, J., and Maskin, E. S. (2009). Sequential Innovation, Patents, and Imitation. *The RAND Journal of Economics*, 40(4), pp. 611-635.

Binmore, K., Rubinstein, A., and Wolinsky, A. (1986). The nash bargaining solution in economic modelling. *RAND Journal of Economics*, 17(2) pp. 176-188.

Bourreau, M., and Doğan, P. (2005). Unbundling the local loop. *European Economic Review*, 49(1) pp. 173-199.

Camacho, F. T., and Menezes, F. M. (2009). Access pricing and investment: a real options approach. *Journal of Regulatory Economics*, 36(2), pp. 107-126.

Cambini, C., and Jiang, Y. (2009) Broadband investment and regulation: A literature review *Telecommunications Policy*, 33, pp. 559-574.

Cave, M. (2006). Encouraging infrastructure competition via the ladder of investment *Telecommunications Policy*, 30, pp. 223237.

Gans, J. S. (2001). Regulating Private Infrastructure Investment: Optimal Pricing for Access to Essential Facilities. *Journal of Regulatory Economics*, 20(2), pp. 167-189.

Guthrie, J. (2006). Regulating infrastructure: The impact on risk and investment. *Journal of Economic Literature*, 44(4), pp. 925-972.

Hori, K., and Mizuno, K. (2006). Access pricing and investment with stochastically growing demand. *Journal of Industrial Organization*, 24(4), pp. 795-808.

Hori, K., and Mizuno, K. (2009). Competition schemes and investment in network infrastructure under uncertainty. *Journal of Regulatory Economics*, 35(2), pp. 179-200.

Katz, M. L., and Shapiro, C. (1987). R&D rivalry with licensing or imitation. *American Economic Association*, 77(3), pp. 402-420.

Kotakorpi, K. (2006). Access price regulation, investment and entry in telecommunications *International Journal of Industrial Organization*, 24, pp. 1013-1020.

Nitsche, R., and Wiethaus, L. (2010). Access regulation and investment in Next Generation Networks - a ranking of regulatory regimes. Forthcoming in *International Journal of Industrial Organization.*

Pindyck, R. S. (2007). Mandatory Unbundling and Irreversible Investment in Telecom Networks. *Review of Network Economics*, 6(3), pp. 274-298.

Onofri, L. (2003). Contracts, Investment Incentives and Efficiency in the Restructured Electricity Market. *European Journal of Law and Economics*, 16(1), pp. 23-38.

Schutz, N., and Trégouët, T. (2008). The Ladder of Investment in Telecoms: Stairway to Heaven or Highway to Hell? *Mimeo*. Available at `http://sites.google.com/site/nicolasschutz/research`.

Vareda, J., and Hoernig, S. (2009). Racing for Investment under Mandatory Access. *Working paper*. Available at personal webpage `http://docentes.fe.unl.pt/~shoernig/`.

Chapter 3

Market structure and technological progress, a differential games approach

3.1 Introduction

In many industries, entry is limited by public authorities. By limiting access to an activity, a social planner has the ability to artificially create scarcity which increases market prices. By the same mechanism, a social planner can increase the number of firms in the market and induce a reduction of prices redistributing resources from firms to consumers.

Economic theory affirms that perfect competition is the optimal market performance mechanism in situations where technological advance is unaffected by resource allocation. Once technological progress is considered to be an economic variable, it is widely accepted that pure perfect competition is no longer the most efficient mechanism. In effect, market profits are invested in process and product innovation. Innovation is ultimately passed to the benefit of consumers.

The question of market performance accompanied by technological progress is particularly relevant in the telecommunications and information technology industries. Koh and Magee (2006) assess technological progress in the IT industry by building a 100 plus years database to find that the annual progress rate in this industry ranges from 20% to 40%.[1] Amaya and Magee (2009) have recently performed the same exercise to quantify the progress in wireless data transport. They find that since the introduction of the cellular technology, the progress has followed annual increases greater than 50%. They predict that wireless interfaces are to become the dominant mode for connecting to the Internet.

As the use of radio frequency is regulated by governments in most countries, entry in the mobile telephony industry is determined by a social planner. It is then relevant to understand which is the degree of competition, and in particular the number of competing firms in the market, that fosters continual innovation and that maximizes welfare.

The question of market structure and innovation has been long treated by economists going back to Schumpeter (1950) and Arrow (1962). A large body of work has studied the institutional design that fosters disruptive innovation.[2] A smaller body of literature focuses on innovation as an incremental gradual process. Vives (2008) builds a general framework to obtain results on the effect of indicators of competitive pressure on process innovation. He finds that increasing the number of firms tends to decrease cost reduction expenditure per firm, whereas increasing the degree of product substitutability increases it.

This article adds to the existing literature by adopting an explicit dynamic approach - modeled as a differential game - to describe firms' efforts aimed at process innovation. In this framework, innovation consists in a continual cost reduction resulting from recurring

[1]The progress rate is defined as the percent increase in performance per year. The rates of technological progress in the IT industry were measured for three functional categories: storage, transformation and transportation.

[2]This literature includes the design intellectual property such as patent, copyrights, etc. See for an overview of the state of the art literature Scotchmer (2004).

investment on technology upgrades and in-house research and development. The main aport of using a dynamic model is that our results can be compared with the technological progress observed in empirical studies.[3]

We analyze the evolution of a market composed of n product differentiated firms, where the number of firms in the market is once and for all set at $t = 0$ by a social planner, and where firms set at each instant of time market prices and their innovational effort. In doing so, we determine the market structure in terms of the number of firms in the market that brings more innovation and that maximizes welfare.

Our main results are summarized as follows. On the one hand, we find that increasing the number of firms reduces process innovation. Given that firms are imperfect substitutes and that the number of them is supposed constant in time, firms anticipate future efficiency rents and thus have incentives to become efficient as early as possible. Then, a firm in a concentrated market invests more as it anticipates a greater portion of future profits. On the other hand, we find that social welfare is not necessarily correlated with process innovation. Even if a monopoly is the most efficient market structure it is also the one able to set the highest price markup. That being so, a market with more firms results in lower prices given that competition reduces markups compensating for the efficiency loss. The dynamic approach of differential games allows to separate static effects from dynamic effects and it suggests the number of firms that optimally combines both.

Furthermore, the paper analyses the role of the degree of product substitutability on innovation. We find that increasing the degree of product substitutability increases product innovation. As a consequence – provided that products are not too differentiated – the optimal number of firms decreases with the degree of product differentiation. A duopoly is then the best market configuration when price rivalry is fierce among firms.

The use of differential games to study dynamic processes has been applied to diverse areas of economics and management science such as growth and capital accumulation,

[3]See Figure 3.7 in pg. 105.

marketing, or environmental problems of resources extraction and pollution. Applications of differential games to process innovation focus on diverse questions. For example, Gaimon (1989) constitutes an early attempt that concentrates on comparing two solution concepts on the investment outcome. Navas and Kort (2006), Cellini and Lambertini (2005, 2009) investigate process innovation in the presence of spillovers and the benefits of R&D cooperation among firms. This later article is the closest to our modeling approach. Yet, the main contribution of our work resides in the analysis of the relationship between market structure and process innovation by the introduction of a demand function that allows a proper comparative statics on the number of firms in the market.

The paper is organized as follows: Section 3.2 introduces the microeconomic bases of the model in a static setting. Then section 3.3 presents the dynamic model and it solves for the steady state equilibrium. Section 3.4 performs comparative statics on the equilibria. By doing so, we characterize the optimal number of firms in a market. Section 3.5 explores the evolution the solution in time, it computes the time paths converging to the steady state equilibria. Finally, technological spillovers are incorporated in a simple extension of the model in section 3.6. Section 3.7 summarizes the main insights of the paper.

3.2 Microeconomic foundations and the static model

Consider a market where N single-product firms produce goods that are symmetrically related substitutes. The social planner sets $n \leq N$ the number licenses for the market. Suppose that among the candidates, firms are randomly chosen to enter the market, for simplicity the cost of the license is normalized to zero.

The representative consumer has preferences following Shubik and Levitan (1980)'s quadratic utility function:

$$U = \sum_{j=1}^{n} q_j - \frac{1}{2}\left(\sum_{j=1}^{n} q_j\right)^2 - \frac{N}{2(1+\gamma)}\left\{\sum_{j=1}^{n} q_j^2 - \frac{1}{N}\left(\sum_{j=1}^{n} q_j\right)^2\right\}$$

where $q_j \geq 0$ denotes the consumption of firm j's good. Firms $n+1, \dots, N$ without a license cannot enter the market thus have null production.[4] The parameter $\gamma \geq 0$ measures the substitutability of the goods.

For a given set of prices $\{p_i\}$ the representative consumer determines for each one of the available products the level of consumption that maximizes its net utility: $\max_{q_1,\ldots,q_n} U - \sum_{j=1}^{n} p_j q_j$. Supposing that all n license-holding firms are active in the market, the solution of this program yields the set of linear demand functions

$$q_i = \nu \left\{ 1 - p_i - \gamma \frac{n}{N} (p_i - \bar{p}) \right\}, \quad i = 1, \ldots, n \tag{3.1}$$

where $\bar{p} = \frac{1}{n} \sum_{j=1}^{n} p_j$ is the price average and $\nu \equiv \nu(n, \gamma) = \frac{1+\gamma}{N+\gamma n}$. The formulation has an intuitive interpretation: demand decreases with firm's own price and it further decreases if the price is higher than the market average price. The expression given by ν reflects the ability of a firm to substitute the products absent from the market. It increases when goods are closer substitutes, $\frac{\partial \nu}{\partial \gamma} = \frac{N-n}{(N+\gamma n)^2} > 0$, and decreases with the number of competing firms, $\frac{\partial \nu}{\partial n} = -\frac{\gamma(1+\gamma)}{(N+\gamma n)^2} < 0$.

On the production side, suppose that all firms in this market face a constant marginal costs $c \in [0, 1]$.[5] The profit for firm i is then $\pi_i = (p_i - c) q_i$. We look at the one-shot (static) game where all firms set simultaneously and non-cooperatively their prices. The symmetric equilibrium price is

$$p^s = \frac{N + (N + \gamma(n-1))c}{2N + \gamma(n-1)} \tag{3.2}$$

which yields individual output

$$q^s = \frac{(1+\gamma)(N + \gamma(n-1))(1-c)}{(N+\gamma n)(2N + \gamma(n-1))} \tag{3.3}$$

Define $\phi \equiv \phi(n, \gamma) = \frac{N+\gamma(n-1)}{2N+\gamma(n-1)}$, we can thus re-write the one-shot equilibrium price and individual demand as

$$p^s = c + (1 - \phi)(1 - c), \quad q^s = \nu \phi (1 - c) \tag{3.4}$$

[4]For $n = N$ one obtains the usual Shubik-Levitan demand system. This formulation, proposed by Höffler (2008), allows for a consistent welfare analysis of the number of available varieties in the market.

[5]The results described bellow can be generalized to a situation where cost are different for firms (as long as they are efficient enough to be active in the market).

The term $(1-\phi)(1-c)$ constitutes the price markup where ϕ regroups the effect of both parameters – the substitutability of goods and the number of active firms – in one to measure effective competition in the market. Competition increases with both parameters $\frac{\partial \phi}{\partial \gamma} = \frac{N(n-1)}{(2N+\gamma(n-1))^2} > 0$ and $\frac{\partial \phi}{\partial n} = \frac{\gamma N}{(2N+\gamma(n-1))^2} > 0$. For $\gamma = 0$, $\phi = \frac{1}{2}$ and the equilibrium price is the monopolistic one $p^S = \frac{1+c}{2}$ even in a market with many firms. In contrast, $\phi \to 1$ when $\gamma \to \infty$, then $p^S \to c$, ie. prices approach the static efficient equilibrium for goods that are perfect substitutes.

This formulation allows to clearly illustrate the markup reduction at equilibrium whenever competition is intensified by either making firms closer substitutes or by introducing additional competitors to the market. Then, as the proposition bellow shows, welfare implications are clear in the static game. It is in the interest of consumers and society to have a maximum number of firms, as closer substitutes as possible, active in the market.

Proposition 3.1. *The symmetric Nash equilibrium of the one-shot game given by (3.2).*

(i) The equilibrium price strictly decreases with γ *and* n*:* $\frac{\partial p^s}{\partial \gamma} < 0$ *and* $\frac{\partial p^s}{\partial n} < 0$.

(ii) Individual output increases with γ *and decreases with* n*:* $\frac{\partial q^s}{\partial \gamma} > 0$ *and* $\frac{\partial q^s}{\partial n} < 0$.

(iii) Consumer surplus and social welfare at the equilibrium price are

$$CS^s = \frac{1}{2} n \nu \phi^2 (1-c)^2, \qquad W^s = \frac{1}{2} n \nu (2-\phi)\phi(1-c)^2,$$

They both increase with γ *and* n.

Proof. See the appendix for all proofs. □

In this static game technological advance is unaffected by resource allocation. Once technological progress is considered to be an economic variable, it is accepted that pure perfect competition is no longer the most efficient mechanism. The reasoning above would be the point of view of a myopic social planner. The next section introduces the differential game, the central object of the model, in order to incorporate technological progress to the analysis.

3.3 The dynamic model

Consider now a situation where $n \leq N$ firms compete in the market over time $t \in [0, \infty)$, where the number of firms is exogenously set at $t = 0$ by the social planner. For tractability, the model supposes that the number of active firms remains fixed through the entire period of time. In every instant each firm i sets its price $p_i(t)$ which yields an instantaneous demand given by (3.1) as in the static model: $q_i(t) = \nu \left\{1 - p_i(t) - \gamma \frac{n}{N} \left(p_i(t) - \frac{1}{n} \sum_{j=1}^{n} p_j(t)\right)\right\}$.

The marginal cost supported by each firm i can evolve over time as described by the following dynamic equation

$$\frac{\mathrm{d}c_i(t)}{\mathrm{d}t} = -\rho\psi(k_i(t))c_i(t)$$

where the parameter $\rho > 0$ denotes the potential technological progress characteristic to the industry,[6] also referred as "technological opportunity", which can be attained if firms make what the literature calls in general terms an *effort*. In the model the effort is denoted by $k_i(t) \geq 0$. It can be thought in terms of a firm renewing equipments to more efficient ones, expanding capacities, adopting improved technologies, or investing in R&D for process innovation. Throughout the paper k is interchangeably called investment or effort. Given that spillovers are not the main concern of the model this first approach supposes that each firm's effort impacts only its own cost and not its competitors' costs. Section 3.6 extends the analysis to describe the effect of positive spillovers in the industry.

The function $\psi(\cdot)$ is a measure of the efficiency of the effort in reducing costs. We impose natural properties to the efficiency function: it must be increasing with the effort $\psi'(k) > 0$, but with decreasing returns to scale $\psi''(k) < 0$, zero if no effort is made $\psi(0) = 0$, and bounded by one such that costs decrease no faster than at the industry's potential technological progress rate. To simplify the model firm's effort is taken to deterministically reduce costs.

[6] For example, information technology-based industries or energy-based industries are usually sectors with a potential technological development higher labour and capital intensive industries.

Suppose that firms, consumers and the social planner discount future payoffs at the same discount rate $r \geq 0$. All the elements for the differential game have been stated, we can now proceed to describe the objective of each player.

The problem of firm i consists in setting at each instant the market price and the level of effort in order to maximize its discounted net present profit

$$\max_{p_i(\cdot),k_i(\cdot)} \Pi_i = \int_{t=0}^{\infty} \pi_i(t) \mathrm{e}^{-rt} \mathrm{d}t \tag{3.5}$$

where $\pi_i(t) = \{p_i(t) - c_i(t)\}q_i(t) - k_i(t)$. Each firm is subject to the set of dynamics

$$c_j'(t) = -\rho\psi(k_j(t))c_j(t), \qquad j = 1, \ldots, i, \ldots, n \tag{3.6}$$

and the set of initial conditions $\{c_j(0)\} = \{c_{j,0}\}$.

Two solution concepts are mainly used for solving differential games.[7] Open-loop solutions are functions that depend only on time. For this problem, an open-loop equilibrium is a profile of price and effort time functions that is a Nash equilibrium. Closed-loop solutions are functions that depend on the current state variables, for this problem the marginal costs. Closed-loop equilibria has the property to be subgame perfect, whereas open-loop equilibria might not be. The counterpart is that open-loop equilibria is easy to find, whereas closed-loop equilibria can be characterized for very specific cases. As it will be showed, the simple formulation of our problem allows for a closed-loop solution that collapses into an open-loop form, it is degenerate as it depends only on time.

As a general roadmap, the derivation of the equilibrium is described. First, we state the necessary optimality conditions. From this conditions we obtain a system of ordinary differential equations that describe the dynamics of the equilibrium in time. The objective is then to look for stationary points, ie. steady states to which the solution converges. Finally, we find the equilibrium time paths that converge to the steady states.

[7] The reader not familiar with differential games can refer to Dockner et al (2000) for an introduction and extensive treatment of differential games and its applications to economics.

Derivation of equilibrium

To find the equilibrium we follow standard procedures employed in the literature, in particular the one used by Cellini and Lambertini (2009). The (current value) Hamiltonian of firm i is

$$\mathcal{H}_i = \{p_i(t) - c_i(t)\}q_i(t) - k_i(t) + \sum_{j=1}^{n} \lambda_{ij}(t)\{-\rho\psi(k_j(t))c_j(t)\} \tag{3.7}$$

where $\lambda_{ij}(t)$ is the co-state or adjoint variable associated with the state variable $c_j(t)$.

To simplify notation the time is omitted. The relevant first order conditions for the optimum are

$$\frac{\partial \mathcal{H}_i}{\partial p_i} = 0 \quad \Rightarrow \quad q_i + (p_i - c_i)\frac{\partial q_i}{\partial p_i} = 0, \qquad i = 1, \dots, n \tag{3.8}$$

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = 0 \quad \Rightarrow \quad -1 - \lambda_{ii}\rho\psi'(k_i)c_i = 0, \qquad i = 1, \dots, n \tag{3.9}$$

Actually, there is no crossed feedback effect in the price or effort choice. Indeed, equations (3.8) and (3.9) contain only the state variable of firm i, hence firm i chooses the optimal output at any time disregarding the current cost or efficiency of their rivals. This implies that the open-loop equilibrium is strongly time consistent, or equivalently, subgame perfect.[8]

Lemma 3.1. *The open-loop Nash equilibrium of the game is subgame perfect.*

We can now focus on the open-loop equilibrium of the game. We proceed by laying out the rest of the optimality conditions and, as it is customary, we look for steady states. Given that firms are symmetric, the focus is set on the symmetric open-loop equilibrium, which greatly simplifies the task of finding analytical solutions. As an additional step section 3.5 finds the time path that converges to the steady state.

The co-state or adjoint equations for the open-loop solution are

$$\lambda'_{ii} = r\lambda_{ii} - \frac{\partial \mathcal{H}_i}{\partial c_i} \quad \Rightarrow \quad \lambda'_{ii} = q_i + \lambda_{ii}\left(r + \rho\psi(k_i)\right), \quad i = 1, \dots, n \tag{3.10}$$

$$\lambda'_{ij} = r\lambda_{ij} - \frac{\partial \mathcal{H}_i}{\partial c_j} \quad \Rightarrow \quad \lambda'_{ij} = \lambda_{ij}\left(r + \rho\psi(k_j)\right), \qquad i \neq j = 1, \dots, n \tag{3.11}$$

[8]Fershtman (1987) characterizes the types of games for which a feedback equilibrium is degenerate and falls into the open-loop solution.

and the transversality conditions are

$$\lim_{t\to\infty} \lambda_{ij}(t)c_j(t)\mathrm{e}^{-rt} = 0, \qquad i,j = 1,\dots,n \tag{3.12}$$

Remark first that the co-state equations admit as solution $\lambda_{ij}(t) \equiv 0$ for all $j \neq i$, this further justifies the fact that no crossed feedback is present in the equilibrium. Second, we derive the adjoint variable from (3.9) $\lambda_{ii} = -\frac{1}{\rho\psi'(k_i)c_i}$. Now, totally differentiating equation (3.9) respect to time and then using equations (3.6), (3.10) and λ_{ii} as above, allows us to obtain the dynamic equation of the investment

$$\frac{\mathrm{d}k_i}{\mathrm{d}t} = -\frac{\rho\psi'(k_i)}{\psi''(k_i)}\left(\frac{r}{\rho} - \psi'(k_i)c_i q_i\right) \tag{3.13}$$

To characterize the equilibrium price we solve the system (3.8). Actually, the equilibrium price of the dynamic game is exactly as the equilibrium price of the static game, with the exception that costs vary in time. As the focus is set on the symmetric outcome, marginal costs are equal for all firms at all times $c_i(t) = c(t)$, including the initial states $c_{i,0} = c_0$, and equilibrium prices and individual demands are exactly as in (3.4):

$$p(t) = c(t) + (1 - \phi(n,\gamma))(1 - c(t)), \quad q(t) = \nu(n,\gamma)\phi(n,\gamma)\,(1 - c(t))$$

Finally, the dynamics of the investment and efficiency are characterized by the system of ordinary differential equations given by equations (3.6) and (3.13):

$$\begin{cases} c'(t) &= -\rho\psi(k(t))c(t) \\ k'(t) &= -\frac{\rho\psi'(k(t))}{\psi''(k(t))}\left(\frac{r}{\rho} - \nu(n,\gamma)\phi(n,\gamma)\psi'(k(t))(1 - c(t))c(t)\right) \end{cases} \tag{3.14}$$

The system above admits two steady states or stationary points, ie. the solution of the system $c'(t) = 0, k'(t) = 0$ for all t, illustrated by the (c,k) phase space in Figure 3.1.

As $\psi(0) = 0$ and $\psi'(k) > 0$ for all $k \geq 0$, the steady state points are given by $k^* = 0$ and $c^*_- = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{r}{\rho\psi'(0)\nu(n,\gamma)\phi(n,\gamma)}}$ or $c^*_+ = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{rn}{\rho\psi'(0)\nu(n,\gamma)\phi(n,\gamma)}}$. The point $\{c^*_+, k^*\}$ will be proved to be unstable. Then denote $c^*_n = c^*_-$ the steady state of interest in the game with n firms in the market.

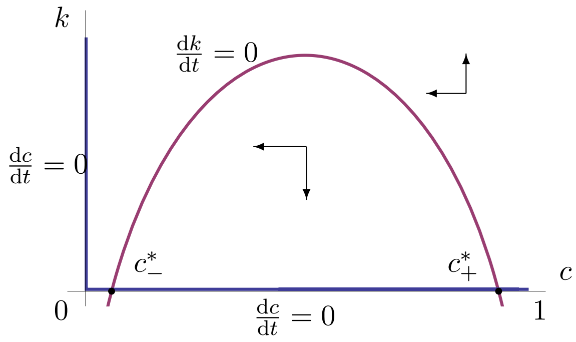

Figure 3.1: Dynamics in the space (c, k).

Proposition 3.2. *Provided that* $\rho \geq \frac{4r}{\psi'(0)\nu(n,\gamma)\phi(n,\gamma)}$, *the steady state point* $\{c_n^*, k^*\}$ *is the unique saddle point equilibrium of the game, where*

$$c_n^* = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{r}{\rho\psi'(0)\nu(n,\gamma)\phi(n,\gamma)}}, \quad k^* = 0.$$

The interest of studying stable points of the system is directly related to the solution of the game. The steady state points are the points to which a solution of the game, ie. the optimal time path functions $c(t)$ and $k(t)$ converge in time. Thus, this solution suggests that the marginal cost will eventually converge to a value between $\left[0, \frac{1}{2}\right]$, whereas efforts converge to zero when the potential technological progress is saturated.

Existence of steady states The existence of a steady state is subject to the expression inside the radical of c_n^* to be positive. If the expression inside the radical is negative, then there exists no steady solution. The condition of existence can be interpreted in terms of a minimal technological opportunity. Define $\rho_n^{min} = \frac{4r}{\psi'(0)\nu(n,\gamma)\phi(n,\gamma)}$. The technological opportunity of the industry must exceed the threshold ρ_n^{min} for investments to be realized.

3.4 Comparative statics analysis

This section examines the effect different parameters of the model – notably the market structure – have on the long term cost and price. The focus is set on costs and prices and not on firms' effort in process innovation given that it is null in the long term. Section 3.5 covers the question of firm's investment.

Comparative statics on cost

We start by deriving the following natural properties of the steady state cost respect to the parameters of the model:

- The higher the industry's potential technological progress, the smaller long term costs, the more efficient the industry: $\frac{\partial c_n^*}{\partial \rho} = -\frac{r}{2\rho^2 \psi'(0)\nu\phi}\left(\frac{1}{4} - \frac{r}{\rho\psi'(0)\nu\phi}\right)^{-1/2} < 0$.
- The more impatient firms, more they value present profits, the less investments will be made to reduce costs: $\frac{\partial c_n^*}{\partial r} = \frac{1}{2\rho\psi'(0)\nu\phi}\left(\frac{1}{4} - \frac{r}{\rho\psi'(0)\nu\phi}\right)^{-1/2} > 0$.

Regarding the effect of indicators of competitive pressure on cost, we state the following:

Proposition 3.3. *(i) The steady state marginal cost decreases with the degree of product substitution.*

(ii) A monopoly is the most efficient market structure in the sense of cost reduction. For all $\gamma > 0$, the larger the number of firms, the higher the steady state marginal cost $c_n^* < c_{n+1}^*$.

If firms' products are completely differentiated, $\gamma = 0$, steady state marginal costs are the same for all market configurations: $c^* = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{2Nr}{\rho\psi'(0)}}$.

In order to fix ideas and illustrate the results, Figure 3.2 provides a graphical representation of the paths followed by the cost in time. These paths will be explicitly calculated in Section 3.5. The focus here is on the steady states to which they converge. The figure bellow depicts the trajectories for three market structures (a monopoly, a duopoly and a market with four firms) converging to their respective steady states.

Proposition 3.3 follows this intuition. Observe from the linear structure of the instantaneous profit function that the incentive to reduce cost is larger when the firm

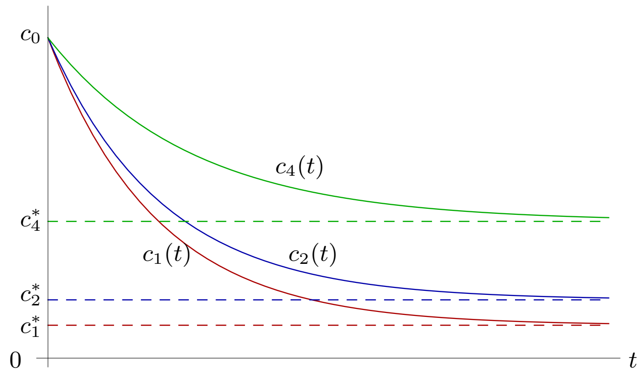

Figure 3.2: The evolution of marginal cost in time for a market with 1, 2 and 4 firms.

produces a larger output. However, as showed in proposition 3.1, individual output at equilibrium prices decreases with the number of firms in the market. Consequently, a monopoly has the largest incentives to reduce costs and costs increase with the number of active firms.

On the contrary costs decrease with γ. Remark that individual output at equilibrium increases with γ due to two effects. First, there is a direct effect on output as a result of higher replacement between products. And second, there is an indirect effect on prices. As showed in proposition 3.1, larger substitutability reduces the equilibrium price which at the same time pushes individual output up.[9]

Proposition 3.3 joins Schumpeter (1950). He was among the first to examine the relation between market structure and innovation. He suggests that there exists a positive relation between innovation and market power, ie. large firms in a concentrated market invest and innovate more than small ones because they have greater means to invest and cope with risk by diversifying it. But he also pointed out that this is a two-way relationship given that these large firms can, at the same time, monopolize innovations through patents resulting in a concentrated market configuration. In this model the entry barriers guarantee future profits, a monopolist makes more profits, so it can invest more in

[9]Both effects are also present on the variation of the number of active firms. The direct effect on demand dominates the indirect effect of price.

cost reducing technologies knowing that future profits will be easily appropriated.

Equally important, our comparative statics results are in accordance with Arrow (1962). He argues that firms have more incentives to innovate if they are in a competitive market than in a monopolistic situation. This is because a firm that becomes more efficient in a competitive environment will sell more units, and as a consequence, it will have a greater returns to innovation than the firm facing reduced competition. In our model, the industry is more efficient in the long term when firms are closer substitutes. This captures the Arrowian position, more intense price competition among firms oblige them to further invest in order to remain efficient in the market.

In sum, our results show that by decomposing both drivers of competition: the market concentration and the substitutability between products, we can somehow conciliate both positions. Firms facing intense price competition have more incentives to become efficient but they also need to have the means to do so.

Comparative statics on price

That being said, a social planner often cares about market prices. This section focuses on the steady state price. The objective is to characterize the market structure (in terms of the number of active firms) that minimizes the steady state price. As it will be pointed out, the market structure that maximizes consumer surplus or social welfare shares the same qualitative properties than the one that minimizes the steady state price.

The steady state price is simplify of the form $p_n^* = c_n^* + (1-\phi)(1-c_n^*)$. Then comparative statics analysis can be easily done as we already inspected the steady state marginal cost. The steady state price decreases with the potential technological progress $\frac{\partial p_n^*}{\partial \rho} = \phi \frac{\partial c_n^*}{\partial \rho} < 0$, but it increases with the discount factor $\frac{\partial p_n^*}{\partial r} = \phi \frac{\partial c_n^*}{\partial r} > 0$, and it decreases if goods are closer substitutes $\frac{\partial p_n^*}{\partial \gamma} = -\frac{\partial \phi}{\partial \gamma}(1-c_n^*) + \phi \frac{\partial c_n^*}{\partial \gamma} < 0$. Increasing goods substitutability has two positive effects on prices. First, it increases static price competition, which reduces the markup, and second, the dynamic efficiency effect reduces long term marginal costs.

Now, let's inspect the steady state price in two limit market configurations, first in a market with goods completely differentiated and second for perfect substitutes. If products are totally differentiated all market configurations result in the same price, $p_n^* = p_{n+1}^* = \frac{1+c^*}{2}$. Monopoly pricing with the highest marginal costs.

If firms are perfect substitutes, $\gamma \to +\infty$, a market with at least two firms in competition results in prices close to the marginal cost $p_n^* \to c_n^*$. Additionally, observe that $c_2^* \to c_1^*$ when $\gamma \to +\infty$, a duopoly is as efficient as a monopoly in this limit case. But for larger market structures the marginal costs are always above the monopoly one. Hence a duopoly becomes the market structure that minimizes the steady state price if firms are perfect substitutes.

In general and for intermediate values of γ, one can see that the addition of a marginal firm to the market has two effects:

$$\frac{\partial p_n^*}{\partial n} = \underbrace{\frac{\partial c_n^*}{\partial n}}_{\substack{>0 \\ \text{efficiency loss}}} + \underbrace{\left(-\frac{\partial \phi}{\partial n}(1-c_n^*) - (1-\phi)\frac{\partial c_n^*}{\partial n}\right)}_{\substack{<0 \\ \text{markup reduction}}}$$

A positive effect on prices, as seen in the one-shot equilibrium analysis, that reduces price markups, and a negative effect on the long term price due to the loss of dynamic efficiency as stated in Proposition 3.3. Actually, by solving $\frac{\partial p_n^*}{\partial n} = 0$ for n, one can approximate the number of firms that optimally combine the static and dynamic effects. The following proposition states some properties respect to this optimal number of firms in the market.

Proposition 3.4. *The comparative statics regarding the market structure have the following properties:*

(i) There exists an optimal number of firms $\hat{n} \leq N$ that minimizes the steady state price.

(ii) The optimal number of firms in the market increases with the technological opportunity factor ρ.

(iii) The optimal number of firms is non-monotonous with respect to the substitutability factor: it is increasing for low values of γ, and it decreases for larger values.

(iv) If goods are perfect substitutes, then a duopoly is the optimal market configuration.

Figure 3.3 illustrates Proposition 3.4, it partitions the (γ, ρ) space determining the number of firms that minimize the steady state price. The small gray area at the bottom of the figure corresponds to $\rho < \rho_1^{min}$ where potential progress is not high enough and investments are not made.

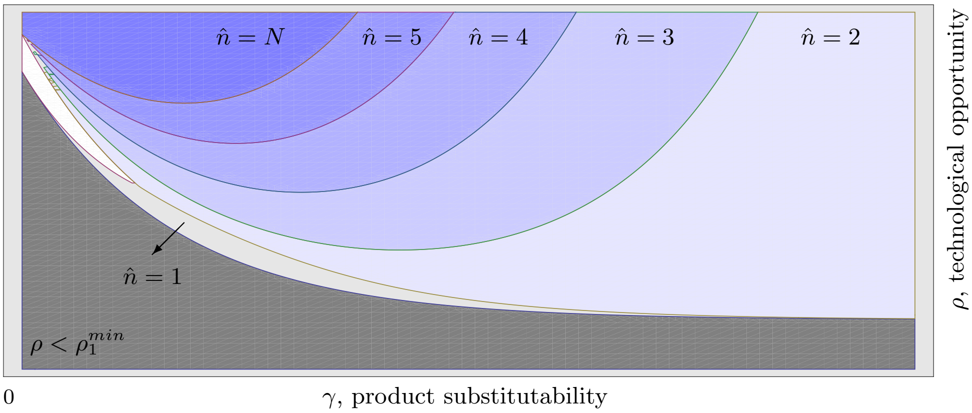

Figure 3.3: Comparative statics on the number of firms that minimize p_n^*. The figure has been properly rescaled.

This section concludes commenting Proposition 3.4. The monotonicity of $\hat{n}$ respect to ρ is motivated by the following intuition. Higher potential technological opportunity allows firms to attain efficiency levels with less effort. Hence, an additional firm in the market will, ceteris paribus, bring prices down as the static competition effect dominates the dynamic efficiency loss.

The non-monotonicity respect to γ results from the complementarity role of the substitutability factor and the number of firms in intensifying market competition. In order to see this, it is necessary to understand the contribution of a marginal increase of n to competition. In a market with highly differentiated goods, the impact of an additional firm to competition is rather low because firms are mainly local monopolies.

As substitutability among products increases, the impact of an additional firm becomes more important. But only up to a certain degree. If firms are perfect substitutes, the impact of an additional firm to market competition is again limited because intense price rivalry leads already to equilibrium prices close to marginal costs. For this reason, the dynamic efficiency effect dominates the static markup reduction in limit market configurations (γ small or large) making thus optimal to have markets with a reduced number of firms. Whereas outside the two extremes, if firms are moderately differentiated it is optimal to have more firms competing in the market.

Welfare consideration

This paragraph briefly covers the welfare to considerations for the solution. Consumer surplus and social welfare in the steady state are $CS^* = \frac{n}{2}\nu\phi^2(1-c_n^*)^2$ and $W^* = \frac{n}{2}\nu(2-\phi)\phi(1-c_n^*)^2$.

It can be readily seen that these measures of welfare are no longer monotonous in n. We find again the tradeoff between instantaneous welfare maximization and the long term efficiency loss. In effect,

$$\frac{\partial CS^*}{\partial n} = \underbrace{\frac{\partial(n\nu\phi^2)}{\partial n}\frac{(1-c_n^*)^2}{2}}_{>0\text{, static effect}} + \underbrace{n\nu\phi^2(1-c_n^*)\left(-\frac{\partial c_n^*}{\partial n}\right)}_{<0\text{, dynamic effect}}$$

With the two effects in play, the mechanism of price minimization applies to the maximization of welfare. We limit the analysis to perform the same comparative statics on the number of firms that maximize consumer surplus and social welfare respect to the same parameters on Fig. 3.3.[10] This allows us to verify that the optimal market structure has the same qualitative properties than the market structure that minimizes prices. Figure 3.4 computes the number of firms that maximize consumer surplus. Figure 3.5 computes the number of firms that maximize social welfare. Observe that consumer

[10] A Mathematica file with the simulation is available at `http://sites.google.com/site/claudiasaavedra/research`

surplus in general requires a higher number of firms in the market compared to social welfare.

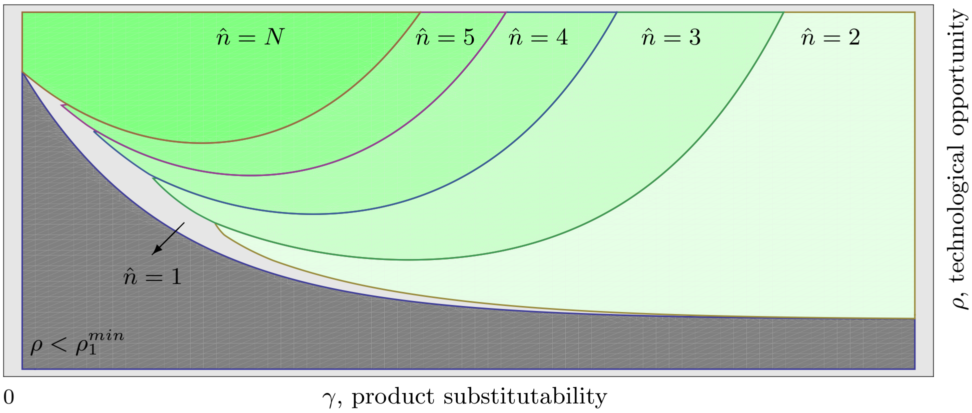

Figure 3.4: Comparative statics on the number of firms that maximizes CS^*.

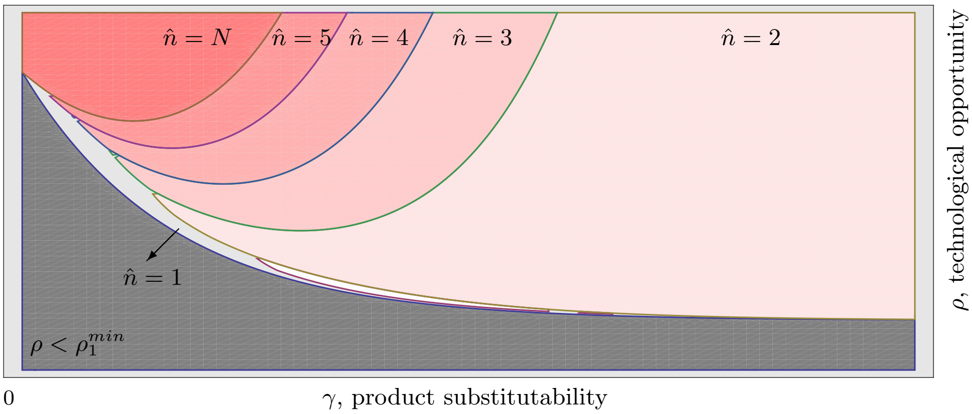

Figure 3.5: Comparative statics on the number of firms that maximizes W^*.

3.5 The time path

So far the analysis has focused on the steady states of cost and price. This section explores the evolution of the system (3.14) in time. In order to obtain an explicit analytical expression that characterizes the time paths of the system we consider a linear approximation around $\{c_n^*, 0\}$ to obtain the (saddle) trajectories that converge to the steady state. This results in the following:

Proposition 3.5. *Provided that the steady state exists, the saddle path dynamics of cost and investment effort over $t \in [0, \infty)$ are*

$$c_n(t) = c_n^* + (c_0 - c_n^*) \cdot \exp(wt) \tag{3.15}$$

$$k_n(t) = \rho \frac{\psi'(0)^2}{-\psi''(0)} \nu(n, \gamma) \phi(n, \gamma) \frac{1 - 2c_n^*}{r - \nu} (c_0 - c_n^*) \cdot \exp(wt) \tag{3.16}$$

where ν denotes the negative eigenvalue associated to the appropriate dynamical system, this is

$$w = \frac{r}{2} - \sqrt{\left(\frac{r}{2}\right)^2 + \rho^2 \frac{\psi'(0)^3}{-\psi''(0)} \nu(n, \gamma) \phi(n, \gamma)(1 - 2c_n^*) c_n^*} \tag{3.17}$$

The analysis had focused exclusively on cost and prices given that the steady state of effort, $k^* = 0$, did not convey any information. The trajectory of the effort given by expression (3.16) allows for further insides on the solution. Figure 3.6.a provides a graphical representation of the trajectories of effort in time for firms in three market configuration: a monopoly, duopoly and a market with four firms. As suggested by the resulting steady state cost characterization given in Proposition 3.3, individual effort decreases with the number of firms in the market. Conversely, individual effort increases for all t if competing firms are more substitutable.

Corollary 3.1. *For all $t \geq 0$ and for all ranges of acceptable parameters $k_n(t) \geq k_{n+1}(t)$ and $\frac{\partial k_n(t)}{\partial \gamma} > 0$.*

Equilibrium prices at instant t are the static equilibrium prices evaluated at the instant cost, $p_n(t) = c_n(t) + (1 - \phi)(1 - c_n(t))$, as well as individual demands $q_n(t) = \nu\phi(1 - c_n(t))$. Then, we can directly compute individual profits in time:

$$\pi_n(t) = \nu(1 - \phi)\phi\,(1 - c_n(t))^2 - k_n(t)$$

Figure 3.2.b provides the representation of profits paths. Remark that a firm in monopoly invests more in early stages because it anticipates that it will be able to recover future rents of efficiency gain as it is protected by the entry barriers. For this same reason firms in all market configurations start the game investing more than current revenues

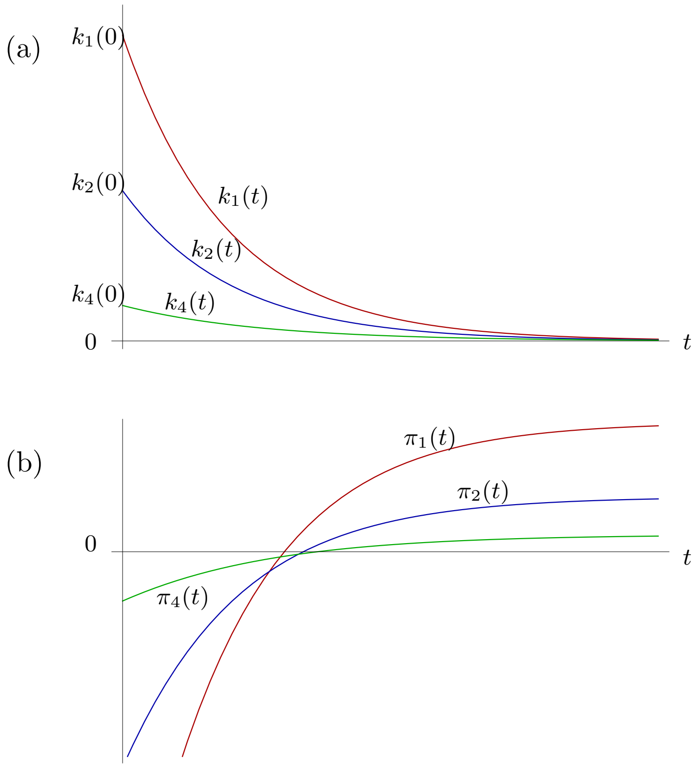

Figure 3.6: Effort paths (a) and profit paths (b) in a market with 1, 2 and 4 firms.

as future gains are secured.

This paper was initially inspired by the empirical evidence of strong technological progress in the information technology and telecommunications sector. In particular, outstanding work by Koh and Magee (2006) build a 100 years daba base to document the performance improvement of three categories of the information technology sector: storage, transport and transformation.[11] The added value of their work consists in using a unifying metric able to measure the performance of the different technologies dominating at different times. As a result, they find that in the long run progress is relatively continuous and stable. However, when focusing on single underlying technologies over

[11]For example, the storage category inspects the performance of punch cards, magnetic tapes, magnetic discs and optical disks. Data transport measure the performance of submarine coaxial and optical cables. And more recently, Amaya and Magee (2009) extend this approach to wireless data transport.

relatively short periods of time, researchers advocate for the use of s-curves to explain the development of a particular technology. This is because a technology will in general be limited by economic and population constraints or by calculable physical limits that bounds its performance in time. Yet, the constant progress documented by Amaya and Magee (2009) in the long run and the s-curves observed in the short run are explained by the liked s-curve theory represented in figure 3.7. Researchers observe that when the s-curve has passed its inflection point a disruptive innovation replaces the current technology forming the dashed linked s-curve path.

The use of differential games to model technological progress allows for a solution coherent with real-world technological observations. Define the performance of the current technology as $\Omega_n(t) = 1/c_n(t)$ consumption per unit cost, then we show that $\Omega(t)$ follows an s-curve in time. First observe that since cost decreases over time the performance of the technology improves over time: $\Omega_n'(t) = \frac{-\nu(c_0-c_n^*)e^{\nu t}}{(c_n^*+(c_0-c_n^*)e^{\nu t})^2} > 0$. Second, if the market structure allows for a significant reduction of costs, $c_n^* < \frac{c_0}{2}$, the performance follows an s-path with the inflection point at $\hat{t} = (-\nu)\log\left(\frac{c_0-c_n^*}{c_n^*}\right)$. In effect, $\Omega_n(t)$ is concave for $t < \hat{t}$ and convex otherwise: $\Omega_n''(t) = \frac{\nu^2(c_0-c_n^*)e^{\nu t}}{(c_n^*+(c_0-c_n^*)e^{\nu t})^3}(-c_n^* + (c_0 - c_n^*)e^{\nu t}) > 0$ only if $t < \hat{t}$. Then, our model represents the evolution of a technology that evolves, as it was defined, with incremental progress excluding disruptive innovations.

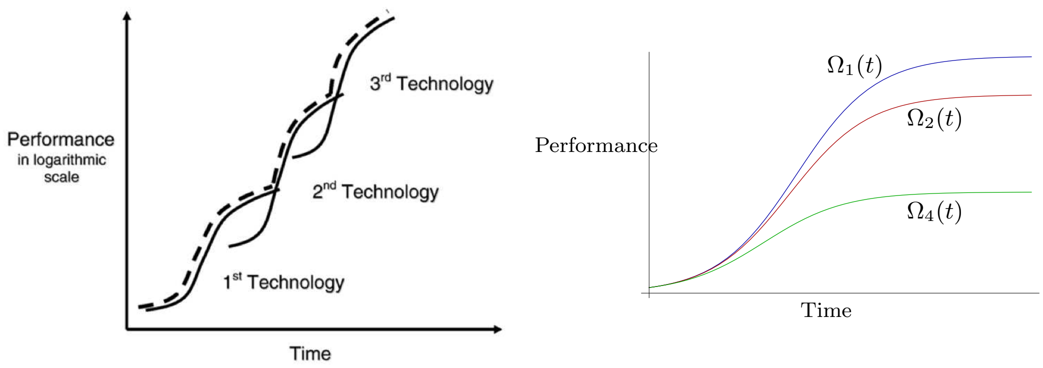

Figure 3.7: Left: Growth of technology in the form of individual linked s-curves: Christensen (1992). Right: Technology performance in a market with 1, 2 and 4 firms.

3.6 Spillovers

This section extends the model to consider a well studied externality of the R&D process: *technological spillovers*. Spillovers is the term used in the economics literature that refers to the involuntary leakage of innovative information. First stated by d'Aspremont and Jacquemin (1988) and now widely accepted,[12] the leakage of information causes firms to recover only partially the benefits of R&D efforts leading them to invest at suboptimal levels. Yet, the diffusion of knowledge is socially desirable, so the introduction of public policies to reduce spillovers is not the ideal solution. Economists have highlighted that cooperation between firms in the upstream research process can be a more efficient alternative. In effect, when spillovers are strong, R&D cooperation allows firms to internalize global industry gains from research which results in higher levels of investment. The positive effect of cooperation has also been proved to hold in the dynamic situation by Cellini and Lambertini (2009) adaptation of the d'Aspremont and Jacquemin (1988) model to a differential game. Then, this section limits itself to show that technological spillovers reduce firm's innovative effort by a simple extension of our model.

The dynamic equation describing the evolution of the marginal cost for firm i is

$$\frac{\mathrm{d}c_i(t)}{\mathrm{d}t} = -\rho\psi\left(k_i(t) + \beta\sum_{j\neq i} k_j(t)\right)c_i(t) \tag{3.18}$$

where $\beta \in [0, 1]$ measures the intensity of the spillover. With $\beta = 0$, like equation (3.6) in the main model, there are no spillovers and firm i's effort impact only its efficiency regardless of the rivals' investment. Whereas with $\beta = 1$ spillovers are perfect as firm j's effort reduces its cost in the same proportion as its rivals' marginal cost. In order to simplify notation write $\psi_i = \psi(k_i + \beta\sum_{j\neq i} k_j)$ and $\psi_i' = \psi'(k_i + \beta\sum_{j\neq i} k_j)$ avoiding the time specification.

We proceed as in section 3.3 by writing the Hamiltonian for firm i of the problem (3.5) subject to the dynamics (3.18) and the set of initial conditions. The Hamiltonian is: $\mathcal{H}_i = \{p_i - c_i\}q_i - k_i + \sum_{j=1}^{n} \lambda_{ij}\{-\rho\psi_j c_j\}$. The optimality conditions are: $\frac{\partial \mathcal{H}_i}{\partial p_i} =$

[12] See for reference Kamien et al. (1992) or Hinloopen (2000) among many others.

$q_i - (p_i - c_i)\frac{\partial q_i}{p_i} = 0$ and $\frac{\partial \mathcal{H}_i}{\partial k_i} = -1 - \lambda_{ii}\rho\psi_i' c_i - \sum_{j\neq i}\lambda_{ij}\rho\beta\psi_j' c_j = 0$. Again, the price a firm sets does not depend on the state variable of its rivals, however the effort seems to do so.

When searching for the open-loop equilibria we find that the crossed feedback is null. In effect, the co-state condition for $j \neq i$ is $\lambda_{ij}' = r\lambda_{ij} - \frac{\partial \mathcal{H}_i}{\partial c_j} = \lambda_{ij}(r + \rho\psi_j)$, then $\lambda_{ij} \equiv 0$ is a feasible solution of the problem. Hence, the optimal effort of firm i depends only on its efficiency level as the second optimality condition reduces to: $-1 - \lambda_{ii}\rho\psi_i' c_i = 0$. This allows us to obtain, by proceeding exactly as in section 3.3, the dynamic equation of firms' effort for the symmetric open-loop equilibria of the game:

$$\frac{\mathrm{d}k}{\mathrm{d}t} = -\frac{\rho\psi'\left(k + (n-1)\beta k\right)}{(1 + (n-1)\beta)\psi''\left(k + (n-1)\beta k\right)}\left(\frac{r}{\rho} - \psi'\left(k + (n-1)\beta k\right)\nu\phi(1-c)c\right) \quad (3.19)$$

This expression is the generalization of expression (3.13) for $\beta = 0$. Dynamic equations (3.18)-(3.19) characterize the solution of the investment problem with spillovers resulting in the following:

Proposition 3.6. *Provided that $\rho \geq \rho_n^{min}$,*

(i) Spillovers do not affect stable steady states of the open-loop solution: the cost converges to c_n^*, the optimal effort converges to 0, and the optimal pricing converges to $p_n^* = c_n^* + (1-\phi)(1-c_n^*)$.

(ii) The cost time path is given by (3.15), however the effort time path is

$$k_n(t) = \rho\frac{\psi'(0)^2}{-\psi''(0)}\frac{\nu(n,\gamma)\phi(n,\gamma)}{1 + (n-1)\beta}\frac{1 - 2c_n^*}{r - \nu}(c_0 - c_n^*) \quad (3.20)$$

where ν is the negative eigenvalue given by (3.17).

(iii) Individual effort decreases with spillovers. Further, spillovers exacerbate the effort reduction caused by the entry of an additional firm to the market.

It is surprising to find that spillovers do not enhance global market efficiency, the steady state of cost remains unchanged in the open-loop equilibria. This can be partially explained by inspecting the time path given by (3.20). Firms invest less at every instant anticipating that at equilibrium they will profit from rivals' effort. But this may also be so because the open-loop solution is not subgame perfect. In fact the feedback equilibria does not necessarily fall into the open-loop solution when spillovers are present.[13] Indeed, the optimality conditions stated above remain unchanged for the feedback equilibria, then effort is the only variable with crossed feedback. When looking for the feedback equilibria the adjoint equations are:

$$\lambda_{ij}' = r\lambda_{ij} - \frac{\mathrm{d}\mathcal{H}_i}{\mathrm{d}c_j} \quad \Rightarrow \quad \lambda_{ij}' = r\lambda_{ij} - \frac{\partial \mathcal{H}_i}{\partial c_j} - \sum_{\ell \neq i} \frac{\partial \mathcal{H}_i}{\partial k_\ell}\frac{\partial k_\ell^e}{\partial c_j}, \quad \text{for } j \neq i$$

where the terms $\frac{\partial k_\ell^e}{\partial c_j}$ capture the feedback effects of the equilibrium. Remark that the term $\frac{\partial \mathcal{H}_i}{\partial k_i}$ is null because of the optimality condition. For $j \neq i$ fixed, the above equation is

$$\begin{aligned} \lambda_{ij}' = \lambda_{ij}(r + \rho\psi_j) + \rho \left(\beta\lambda_{ii}\psi_i' c_i + \lambda_{ij}\psi_j' c_j + \beta \sum_{m \neq i,j} \lambda_{im}\psi_m' c_m \right) \frac{\partial k_j^e}{\partial c_j} \\ + \sum_{\ell \neq i,j} \rho \left(\beta\lambda_{ii}\psi_i' c_i + \lambda_{i\ell}\psi_\ell' c_\ell + \beta \sum_{m \neq i,\ell} \lambda_{im}\psi_m' c_m \right) \frac{\partial k_\ell^e}{\partial c_j} \end{aligned}$$

If spillovers are null the crossed feedback is also null. In effect, consider the symmetric solution of the game, this implies that $\lambda_{ij} \equiv \lambda_{i\ell}$ for $j, \ell \neq i$. Then, the co-state or adjoint condition simplifies to $\lambda_{ij}' = \lambda_{ij}\left(r - \rho\psi_j + \rho\psi_j' c_j \frac{\partial k_j^e}{\partial c_j} + \rho(n-2)\psi_\ell' c_\ell \frac{\partial k_\ell^e}{\partial c_j}\right)$ which admits $\lambda_{ij} \equiv 0$ as solution, proving that the feedback equilibria falls into the open-loop equilibria. However, for $\beta > 0$ this no longer holds unless $\frac{\partial k_\ell^e}{\partial c_j}$ and $\frac{\partial k_j^e}{\partial c_j}$ are both proportional to λ_{ij}. This is not necessarily the case for the general form of ψ. Note that by the same reasoning the open-loop equilibria might not necessarily be subgame perfect even in the particular setting of Cellini and Lambertini (2009).[14]

Limitations regarding the commitment of the open-loop equilibria are well known as well as the inherent difficulty of finding explicit characterization of the feedback solution

[13]Even without spillovers the open-loop solution differs from the feedback solution in general frameworks, see for instance Gaimon (1989).

[14]See the note Saavedra (2010).

for general formulations. Nevertheless, the open-loop solution has been accepted in the literature as a benchmark for the study of dynamic models. In this model, the open-loop solution of the extension shows first that spillovers reduce indeed firms' innovative efforts. And second, it shows that the main results respect to the optimal market structure hold for this solution concept as steady state points remain unchanged.

3.7 Discussion

This paper has developed a differential game model to study the relationship between process innovation and market structure. We have studied a market where firms compete in prices for a long period of time and where firms can continuously invest to reduce their unitary costs. The market structure was characterized by the number of firms competing in the market and by the rivalry between them.

Our results suggest that there exists an optimal number of active firms in the market that maximizes consumer welfare. This optimal market is the result of two clear effects. On the one hand, dynamic efficiency of technological progress decreases with the number of active firms in the market. On the other hand, a small number of firms grant too much market power allowing firms to extract consumer surplus.

We have further established a relationship between the level of product rivalry and the optimal number of active firms. The conclusions are as follows. A small number of firms is preferred whenever rivalry between firms is either too strong or too weak. Whereas more firms active in the market can efficiently reduce prices in intermediary situations. If a social planner could decide without other costs on the market structure parameters, he would maximize consumer welfare by having a perfectly substitutable duopoly.

Finally, the dynamic model has allowed us to find parallels between empirical observations of technological progress and the theoretical results.

3.8 *Appendix:* Proofs of propositions

Proof of Proposition 3.1. Standard computation of equilibria leads to (3.2). Having computed the partial derivatives of ϕ and ν allows to readily show the first two points.

(i) $\frac{\partial p^s}{\partial \zeta} = -\frac{\partial \phi}{\partial \zeta}(1-c) < 0$ for $\zeta = n, \gamma$.

(ii) To show that output at equilibrium, $q^s = \nu\phi(1-c)$, increases with γ observe that $\frac{\partial(\nu\phi)}{\partial \gamma} > 0$ given that both ϕ and ν increase with γ. We compute the derivative to show that individual output decreases with the number of firms. For $n \geq 2$ we have that

$$\frac{\partial(\nu\phi)}{\partial n} = -\frac{\gamma(1+\gamma)(N^2+\gamma^2(n-1)^2+\gamma(2n-3)N)}{(N+n\gamma)^2(2N+\gamma(n-1))^2} < 0 \qquad (3.21)$$

(iii) Consumer's utility with equal consumption q for all goods is: $U(q) = nq - \frac{1}{2}(nq)^2 - \frac{N-n}{2(1+\gamma)}nq^2 = nq - \frac{n}{2\nu}q^2$. Replacing equilibrium prices (and quantities) in the consumer surplus $CS = U(q) - npq$ yields $CS^s = \frac{1}{2}n\nu\phi^2(1-c)^2$. As both ν and ϕ are increasing with γ it is straightforward to see that CS increases with γ. Consumer surplus increases with the number of firms as

$$\frac{\partial(n\nu\phi^2)}{\partial n} = \frac{(1+\gamma)N(N+\gamma(n-1))\left(2N^2+\gamma(5n-3)N+\gamma^2(1+n(3n-2))\right)}{(N+\gamma n)^2(2N+\gamma(n-1))^3} > 0$$

for all $n \geq 1$.

It is straightforward to compute social welfare, $W = U(q) - ncq$, at equilibrium prices: $W^s = \frac{1}{2}n\nu(2-\phi)\phi(1-c)^2$. It is straightforward to see that W^s increases with γ, for n we compute the derivative of the relevant part of the expression:

$$\frac{\partial(n\nu(2-\phi)\phi)}{\partial n} = \frac{(1+\gamma)N\left(\gamma^3(n-1)^3+2\gamma^2(2(2n-1)(n-1)+1)N+\gamma(13n-11)N^2+6N^3\right)}{(N+\gamma n)^2(2N+\gamma(n-1))^3} > 0$$

□

Proof of Proposition 3.2. The stability properties are assessed by evaluating the determinant and the trace of the Jacobian at the steady state points. The Jacobian is of the form

$$J(c,k) = \begin{pmatrix} \frac{\partial c'}{\partial c} & \frac{\partial c'}{\partial k} \\ \frac{\partial k'}{\partial c} & \frac{\partial k'}{\partial k} \end{pmatrix},$$

where

$$
\begin{aligned}
\tfrac{\partial c'}{\partial c}(c,k) &= -\rho\psi(k) \\
\tfrac{\partial c'}{\partial k}(c,k) &= -\rho\psi'(k)c \\
\tfrac{\partial k'}{\partial c}(c,k) &= \rho\nu\phi\tfrac{\psi'(k)^2}{\psi''(k)}(1-2c) \\
\tfrac{\partial k'}{\partial k}(c,k) &= \rho\nu\phi\psi'(k)(1-c)c + \left(\tfrac{\psi'(k)\psi^{(3)}(k)}{\psi''(k)^2} - 1\right)(r - \rho\nu\phi\psi'(k)(1-c)c)
\end{aligned}
$$

The steady state $\{c_-^*, 0\}$ is analyzed first. The upper-left entry of the Jacobian is zero $\frac{\partial c'}{\partial c}(0, c_-^*) = -\rho\psi(0) = 0$. Thus the determinant is negative: $\det J(c_-^*, 0) = \rho^2\nu\phi\frac{\psi'(0)^3}{\psi''(0)}(1-2c_-^*)c_-^* < 0$ given that $c_-^* < \frac{1}{2}$ and given that $\psi''(0) < 0$. The trace of the Jacobian is significantly reduced given that the second term of $\frac{\partial k'}{\partial k}(c_-^*, 0)$ is zero and the first term is reduced, $\rho\nu\phi\psi'(0)(1-c_-^*)c_-^* = r$. Then it is readily seen that the trace is positive $\mathrm{tr}J(c_-^*, 0) = 0 + r > 0$, which confirms that $\{c_-^*, 0\}$ is a saddle point.

The analysis of stability for the fixed point $\{c_+^*, 0\}$ is similar. One can verify that the determinant is positive since $c_+^* > \frac{1}{2}$. The trace is also positive, making thus the steady state $\{c_+^*, 0\}$ unstable. □

Proof of Proposition 3.3. Suppose that existence conditions are satisfied.

(i) The more substitutes products, the more dynamic competition among firms, the more efficient the industry in the long term:

$$\frac{\partial c_n^*}{\partial \gamma} = -\frac{r}{2\rho\psi'(0)\nu^2\phi^2}\left(\nu\frac{\partial\phi}{\partial\gamma} + \frac{\partial\nu}{\partial\gamma}\phi\right)\left(\frac{1}{4} - \frac{r}{\rho\psi'(0)\nu\phi}\right)^{-1/2} < 0$$

(ii) First remark that for all $\gamma \geq 0$ the steady state marginal cost in a monopolistic configuration is lower than the one in a duopoly $c_1^* = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{r}{\rho\psi'(0)}\frac{2(\gamma+N)}{1+\gamma}} < \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{r}{\rho\psi'(0)}\frac{(N+2\gamma)(2N+\gamma)}{(1+\gamma)(N+\gamma)}} = c_2^*$ for all $\gamma > 0$.

As in the proof of Proposition 3.1, we can proceed by showing that for $n \geq 2$ a real number, c_n^* is strictly increasing, and since c_n^* is continuous and smooth, then it is increasing for n integer.

$$\frac{\partial c_n^*}{\partial n} = -\frac{r}{2\rho\psi'(0)(\nu\phi)^2}\frac{\partial(\nu\phi)}{\partial n}\left(\frac{1}{4} - \frac{r}{\rho\psi'(0)\nu\phi}\right)^{-1/2} > 0$$

given that $\frac{\partial(\nu\phi)}{\partial n} < 0$ by (3.21) whenever $n \geq 2$.

□

Proof of Proposition 3.4. Now we prove the proposition point by point, except for the last one, *(iv)*, which has been already proven in the text.

(i) To see that p_n^* has minimizer we show that p_n^* is convex in n for $n \geq 2$. By computing the second derivative of the price respect to n we have

$$\frac{\partial^2 p_n^*}{\partial n^2} = -\frac{\partial^2 \phi}{\partial n^2}(1 - c_n^*) + 2\frac{\partial \phi}{\partial n}\frac{\partial c_n^*}{\partial n} + \phi\frac{\partial^2 c_n^*}{\partial n^2}$$

The first term is positive since $\frac{\partial^2 \phi}{\partial n^2} = -\frac{4\gamma^3}{(2N+\gamma(2n-1))^3} < 0$. The second term is also positive since $\frac{\partial \phi}{\partial n} > 0$ and $\frac{\partial c_n^*}{\partial n} > 0$ for all $n \geq 2$. To see if the third term is positive, we need to verify if c_n^* is concave in n. Its first derivative can be rearranged as follows

$$\frac{\partial c_n^*}{\partial n} = \frac{r}{2\rho\psi'(0)}\frac{\partial(1/(\nu\phi))}{\partial n}\left(\frac{1}{2} - c_n^*\right)^{-1}$$

The second derivative of c_n^* is then:

$$\frac{\partial^2 c_n^*}{\partial n^2} = \frac{r}{2\rho\psi'(0)}\left(\frac{\partial^2(1/(\nu\phi))}{\partial n^2}\left(\frac{1}{2} - c_n^*\right)^{-1} + \frac{\partial(1/(\nu\phi))}{\partial n}\left(\frac{1}{2} - c_n^*\right)^{-2}\frac{\partial c_n^*}{\partial n}\right) > 0$$

The first term is positive since $\frac{\partial^2(1/(\nu\phi))}{\partial n^2} = \frac{2\gamma^3 N}{(1+\gamma)(N+\gamma(n-1))^3} > 0$. The second term is also positive $\frac{\partial(1/(\nu\phi))}{\partial n} = \frac{\gamma\left(N^2+\gamma(2n-3)N+\gamma^2(n-1)^2\right)}{(1+\gamma)(N+\gamma(n-1))^2}$. Thus we have that $\frac{\partial^2 p_n^*}{\partial n^2} > 0$.

Proof of (ii) Suppose that $\hat{n} \geq 2$ is a solution of the first order condition. To show the second point we use the implicit function theorem to show that

$$\frac{\partial \hat{n}}{\partial \rho} = -\frac{\left(\frac{\partial \phi}{\partial n}\frac{\partial c_n^*}{\partial \rho} + \phi\frac{\partial^2 c_n^*}{\partial \rho \partial n}\right)(\hat{n})}{\frac{\partial^2 p_n^*}{\partial n^2}(\hat{n})} > 0$$

In effect, it was previously showed that $\frac{\partial c_n^*}{\partial \rho} < 0$ and $\frac{\partial^2 p_n^*}{\partial n^2} > 0$, then it remains to verify that $\frac{\partial^2 c_n^*}{\partial \rho \partial n} < 0$. So,

$$\frac{\partial}{\partial \rho}\left(\frac{\partial c_n^*}{\partial n}\right) = \frac{r}{2\rho\psi'(0)}\frac{\partial(1/(\nu\phi))}{\partial n}\left(\frac{1}{2} - c_n^*\right)^{-1}\left(-\frac{1}{\rho} + \frac{\partial c_n^*}{\partial \rho}\left(\frac{1}{2} - c_n^*\right)^{-1}\right) < 0.$$

Proof of (iii) In order to see the non-monotonicity of $\hat{n}$ respect to γ, observe that $\partial_n p_n^*(\gamma) \equiv \frac{\partial p_n^*}{\partial n}$ is a quasiconvex function in γ with a global minimum. By doing so, one can affirm that $\frac{\partial^2 p_n^*}{\partial \gamma \partial n}$ is first negative and then positive in γ for all n, in particular for $\hat{n}$. Then using the implicit function theorem, as in *(ii)*, we can conclude that $\frac{\partial \hat{n}}{\partial \gamma} = -\frac{(\partial^2 p_n^*/\partial \gamma \partial n)(\hat{n})}{(\partial^2 p_n^*/\partial n^2)(\hat{n})}$ is first positive and then negative.

So, suppose that for n fixed, the steady state solution exists. The quasiconvexity of $\partial_n p_n^*$ is mainly due to the term $\partial_n \phi(\gamma) \equiv \frac{\partial \phi(n,\gamma)}{\partial n} = \frac{\gamma N}{(2N+\gamma(n-1))^2}$, the competition intensity increase due to a marginal incorporation of an active firm in the market, which is quasiconcave. In deed, the aport of a supplementary firm to competition is very small when firms are totally differentiated, $\partial_n \phi(0) = 0$, as well as when firms are perfect substitutes, $\partial_n \phi \to 0$ for $\gamma \to \infty$. By inspecting its derivative, $\partial_n \phi'(\gamma) = \frac{N(2N+\gamma(n-1))}{(2N+\gamma(n-1))^3}$, one concludes that it attains a maximum at $\tilde{\gamma} = \frac{2N}{n-1}$. A numerical sketch of proof is verified in a Mathematica simulation available at `http://sites.google.com/site/claudiasaavedra/research`.

□

Proof of Proposition 3.5. The linear approximation of the system (3.14) at the steady state $\{c_n^*, 0\}$ is

$$\begin{pmatrix} c'(t) \\ k'(t) \end{pmatrix} = \begin{pmatrix} 0 & -\rho\psi'(0)c_n^* \\ \rho\frac{\psi'(0)^2}{\psi''(0)}\nu\phi(1-2c_n^*) & r \end{pmatrix} \begin{pmatrix} c(t) - c_n^* \\ k(t) - 0 \end{pmatrix}$$

The associated eigenvalues are

$$w_{\pm} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 - \rho^2\frac{\psi'(0)^3}{\psi''(0)}\nu\phi(1-2c_n^*)c_n^*}$$

where w_- is the eigenvalue with the negative sign, which is clearly negative given that $\psi''(0) < 0$. w_+ is the positive eigenvalue. This confirms that the steady state point is saddle. The time path that solves the system has the following general shape

$$
\begin{aligned}
c(t) - c_n^* &= \frac{C_1\left(\rho\psi'(0)c_n^*\right) + C_2 w_+}{w_+ - w_-}\cdot\exp(tw_-) + \frac{-C_1\left(\rho\psi'(0)c_n^*\right) - C_2 w_-}{w_+ - w_-}\cdot\exp(tw_+)\\
k(t) - 0 &= \frac{-C_1 w_- - C_2\left(\rho\frac{\psi'(0)^2}{\psi''(0)}\nu\phi(1-2c_n^*)\right)}{w_+ - w_-}\cdot\exp(tw_-) + \cdots\\
&\quad\cdots + \frac{C_1 w_+ + C_2\left(\rho\frac{\psi'(0)^2}{\psi''(0)}\nu\phi(1-2c_n^*)\right)}{w_+ - w_-}\cdot\exp(tw_+)
\end{aligned}
$$

to determine the constants C_1 and C_2 the use of the initial cost is needed. $c_0 = c(0) = c^* + 0\cdot C_1 + 1\cdot C_2$, so $C_2 = c_0 - c_n^*$. As the initial value for the effort is not specified there exists an infinity of solutions. It is custom to select the one that converges to the steady state. Taking $C_1 = -\frac{c_0 - c_n^*}{w_+}\left(\rho\frac{\psi'(0)^2}{\psi''(0)}\nu\phi(1-2c_n^*)\right)$ we have that $k(t)\to 0$ when $t\to\infty$. Finally, remark that $w_+ + w_- = r$, then we take only the negative eigenvalue an rename it $w = w_-$ to obtain (3.15) and (3.16). □

Proof of Corollary 3.1. To see that $k_n > k_{n+1}$ on could inspect the effect of a marginal increase in the number of firms on the expression given by (3.16), but this leads to a cumbersome expression. We rather inspect equation (3.13) the dynamic equation for the effort. When solving for k at the steady state equilibrium, $k' \equiv 0$, we obtain the steady effort respect to the cost or equivalently the feedback solution. Given that ψ is by hypothesis strictly concave ψ' is decreasing and invertible, write $\chi(\cdot) = (\psi')^{-1}(\cdot)$ its inverse function. Hence, the optimal effort is given by $k = \chi\left(\frac{r}{\rho\nu\phi(1-c)c}\right)$. Now, it is straightforward to verify that the marginal increase of a player in the market decreases individual firm's effort $\frac{\partial k}{\partial n} = -\chi'\left(\frac{r}{\rho\nu\phi(1-c)c}\right)\frac{r}{\rho(\nu\phi)^2(1-c)c}\frac{\partial(\nu\phi)}{\partial n} < 0$ given that χ is decreasing and $\frac{\partial(\nu\phi)}{\partial n} < 0$ as showed in (3.21). In the same way one can verify that the effort increases with γ: $\frac{\partial k}{\partial\gamma} = -\chi'\left(\frac{r}{\rho\nu\phi(1-c)c}\right)\frac{r}{\rho(\nu\phi)^2(1-c)c}\frac{\partial(\nu\phi)}{\partial\gamma} > 0$. □

Proof of Proposition 3.6. One can readily verify that the transversality condition $\lambda'_{ii} = q_i + \lambda_{ii}(r + \rho\psi_i)$ plus the other optimality conditions lead to equation (3.19) by proceeding exactly as in section 3.3. It is also straightforward to see that the system of ordinary differential equations (3.18)-(3.19) admits as steady state $\{c_n^*, 0\}$. The Jacobian of the system (3.18)-(3.19) evaluated at this steady state is

$$J = \begin{pmatrix} 0 & -\rho(1+(n-1)\beta)\psi'(0)c_n^* \\ \rho\frac{\psi'(0)^2}{\psi''(0)}\frac{\nu\phi}{1+(n-1)\beta}(1-2c_n^*) & r \end{pmatrix}$$

We have that its determinant is negative and that its trace is positive, then the steady state is saddle. To obtain the path that converges to the steady state one can proceed exactly as in proposition 3.5 to obtain $c_n(t)$ given by (3.15) and $k_n(t)$ given by (3.20).

To verify *(iii)*, that effort decreases with spillovers:

$$\frac{\partial k_n(t)}{\partial \beta} = -\frac{\rho\psi'(0)^2(n-1)\nu\phi(1-2c_n^*)(c_0-c_n^*)}{-\psi''(0)(1+(n-1)\beta)^2(r-w)} < 0$$

And finally as in the proof of proposition 3.1, write $\chi(\cdot) = (\psi')^{-1}(\cdot)$ to show that the optimal effort at the steady state $k = \frac{1}{1+(n-1)\beta}\chi\left(\frac{r}{\rho\nu\phi(1-c)c}\right)$ are exacerbated with the spillover effects as an additional firm competes in the market:

$$\frac{\partial k}{\partial n} = -\frac{\beta\chi\left(\frac{r}{\rho\nu\phi(1-c)c}\right)}{(1+(n-1)\beta)^2} - \frac{\chi'\left(\frac{r}{\rho\nu\phi(1-c)c}\right)}{1+(n-1)\beta}\frac{r}{\rho(\nu\phi)^2(1-c)c}\frac{\partial(\nu\phi)}{\partial n} < 0$$

□

3.9 *Appendix:* A robustness analysis

This section checks the robustness of the results obtained. We solve the model when firms compete in price à la Salop and then with firms competing in quantities. For both market structures one can follow the computation made in pg 93 of section 3.3 to find that the system characterizing the dynamics of the solution is given by

$$\begin{cases} c'(t) &= -\rho\psi(k(t))c(t) \\ k'(t) &= -\frac{\rho\psi'(k(t))}{\psi''(k(t))}\left(\frac{r}{\rho} - \psi'(k(t))c(t)q^*(t)\right) \end{cases}$$

where $q^*(t)$ is the individual output at the static equilibrium of the game.

Cournot competition With Cournot competition firms set quantities at every instant t which determine instantaneous market price $p(t) = 1 - \sum_{i=1}^{n} q_i(t)$. The equilibrium output is $q_n^* = \frac{1-c_n^*}{n+1}$, then the steady state points are

$$c_n^* = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{r(n+1)}{\rho\psi'(0)}} \quad k_n^* = 0$$

The steady state cost increases with the number of firms in the market. The steady state price is $p_n^* = c_n^* + \frac{1-c_n^*}{n+1}$. As with the main model, there exists a number of firms that minimizes long-term prices

$$\hat{n} = \sqrt{\frac{\rho\psi'(0)}{r}} - 2$$

It has also been checked by computing the cost reduction time path that it decreases with the number of firms in the market.

Spacial competition In a Salop model, n active firms in the market face a demand $q_i(t) = \frac{1}{n} - \frac{2p_i(t)-p_{i+1}(t)-p_{i-1}(t)}{2t}$, where $1/t > 0$ represents the degree of product substitutability. Market shares at equilibrium are $q_n^*(t) = \frac{1}{n}$. In this model, the inelastic demand results in outputs that at the symmetric equilibrium are constant respect to firms' efficient levels. As a consequence, the steady state costs do not depend on the degree of product substitutability.

$$c_n^* = \frac{rn}{\rho\psi'(0)}, \quad k_n^* = 0$$

However we still observe that steady state costs increase with the number of firms in the market. Equilibrium prices are $p_n(t) = c_n^* + \frac{t}{n}$. Finally, we can compute the number of firms that minimize long term prices,

$$\hat{n} = \sqrt{\frac{\rho\psi'(0)}{r}t}$$

which decreases with the degree of product substitutability.

Bibliography

Amaya, M. A., and Magee, C. L. (2009). The Progress in Wireless Data Transport and its Role in the Evolving Internet. *MIT Working Paper*, available at xx.

Arrow, K. J. (1962). Economic Welfare and the Allocation of Resources for Invention, The Rate and Direction of Industrial Activity. Edited by R. Nelson, Princeton University Press, Princeton, New Jersey.

Cellini, R., and Lambertini, L. (2005). R&D Incentives and Market Structure: Dynamic Analysis. *Journal of Optimization Theory and Applications*, 126(1), pp. 85-96.

Cellini, R. and Lambertini, L., (2009). Dynamic R&D with spillovers: Competition vs cooperation. *Journal of Economic Dynamics & Control*, 33(3), pp. 568-582.

Christensen, C. M. (1992). Exploring the limits of technology S-curve. Part 1: components technologies. *Production and Operations Management*, 4, pp. 334-357.

D'Aspremont, C., and Jacquemin, A. (1988). Cooperative and Noncooperative R&D in Duopoly with Spillovers. *The American Economic Review*, 78(5), pp. 1133-1137.

Dockner, E. et al (2000). *Differential games in economics and management science.* Cambridge University Press.

Fershtman, C. (1987). Identification of Classes of Differential Games for Which the Open Loop Is a Degenerate Feedback Nash Equilibrium. *Journal of Optimization Theory and Applications*, 55(2), pp. 217-231.

Gaimon, C. (1989). Dynamic game results of the acquisition of new technology. *Operations Research*, 37(3), pp. 410-425.

Globerman, S. (1973). Market Structure and R&D in Canadian Manufacturing Industries. *Quarterly Review of Economics & Business*, 13, pp. 59-67.

Hinloopen, J. (2000). Strategic R&D Co-operatives. *Research in Economics*, 54(2), 153-185.

Höffler(2008), F. (2008). On the consistent use of linear demand systems if not all varieties are available. *Economics Bulletin*, 4 (14), pp. 1-5.

Kamien, M. I., Muller, E., and Zang, I. (1992). Research Joint Ventures and R&D Cartels. *The American Economic Review*, 82(5), pp. 1293-1306.

Kamien, M. I., and Schwartz, N. L. (1975). Market Structure and Innovation: A Survey. *Journal of Economic Literature*, 13(1), pp. 1-37.

Kho, H., and Magee, C. L. (2006). A functional approach for studying technological progress: Application to information technology. *Technological Forecasting & Social Change*, 73(9), pp. 1061-1083.

Navas, J., and Kort, P. M. (2006). Time to complete and research joint ventures: A differential game approach. *Journal of Economic Dynamics & Control*, 31(5), pp. 16721696.

Saavedra, C. (2010) Dynamic R&D with spillovers: a note on subgame perfectness. Available at `http://sites.google.com/site/claudiasaavedra/research`.

Scotchmer, S. (2004). Innovation and Incentives. MIT Press.

Shubik, M. and Levitan, R. (1980). Market Structure and Behavior. Cambridge, Harvard Press.

Schumpeter, J. A. (1950). Capitalism, Socialism, and Democracy, Harper, New York, NY.

Vives, X. (2008) Innovation and competitive pressure *The journal of Industrial Economics*, 56(3), pp. 419-469.