Codage Espace Temps pour les canaux MIMO à accès multiple

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Abstract

Extensive research has been carried out these last few years on the single-user MIMO Space-Time block code (STBC) design using advanced algebraic tools, namely cyclic division algebra. Families of single-user codes have been carefully constructed to achieve the diversity-multiplexing tradeoff (DMT) of the MIMO channel. Motivated by the promising results obtained in the single-user scenario, the aim of this thesis is to construct new families of multi-user STBCs. Multi-user MIMO (MU-MIMO) channels have recently attracted considerable attention because of its essential practical implication in today's and future communication systems, *i.e.,* IEEE 802.11n, 3GPP Long Term Evolution Advanced (LTE-A), IEEE 802.16m, etc.

The current study focuses on the uplink multiuser MIMO channel, equivalently, the MIMO multiple-access channel (MAC). In a MIMO-MAC, multiple users equipped with multiple transmit antennas communicate with one multiple-antenna receiver. A coherent communication system is considered, where the receiver has a perfect channel state information (CSI) while the transmitters do not have any CSI, but are aware of the channel statistics. The construction of the proposed multiple-access codes is based on an in-depth understanding of the information theoretic aspects of the MAC that give insight on the behavior of the channel. In order to simplify the problem, a MAC with single-antenna at the transmitters and an arbitrary number of antennas at the receiver is first considered. Next, the general MIMO-MAC with multiple-antenna at both the transmitters and the receiver is investigated. Throughout this thesis, all the presented results deal with multiple-access systems under the assumption of synchronized users which may not be practical for several reasons: the users do not necessarily share the same timing reference, they access the channel randomly, they have different geometrical locations, etc. As a practical application, the effect of asynchronism on the multiple-access code performance is studied in the single-antenna scenario. The proposed code turns out to be delay tolerant, *i.e.,* does not lose its performance gain due to asynchronism.

Finally, the multiple-access relay channel (MARC) is considered, where one or more relays help the users communicate with the destination while the cooperation between the users is not allowed. Relaying techniques exploit the spatial diversity in a wireless network and, thus, significantly improve the performance of the system. The multi-access amplify-and-forward (MAF) strategy, characterized by its low relaying complexity and its linear nature, is considered. The MAF relay channel is shown to be equivalent to a *virtual* MIMO-MAC for which, the code constructed for the MIMO-MAC can be applied in a distributed way.

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Résumé de la Thèse en Français

Introduction

Multiples travaux de recherche ont été munis ces denirères années pour construire des codes espace-temps optimaux pour les communications sans fil de point à point (*i.e.,* mono-utilisateur). Plusieurs familles de codes, dont la construction utilise des outils algébriques avancés et qui atteignent le compromis Diversité-Multiplexage (DMT), ont été proposées dans la litérature. Ce compromis a été développé par Zheng et Tse [1] pour capturer les avantages duels d'un canal à évanouissement pour les SNR élevés : l'augmentation du débit (augmentation du gain de multiplexage) et l'augementation de la fiabilité (augmentation du gain de diversité). Depuis, le DMT devint un outil puissant de la théorie de l'information utilisé pour évaluer la performance et comme critère de construction des codes espace-temps.

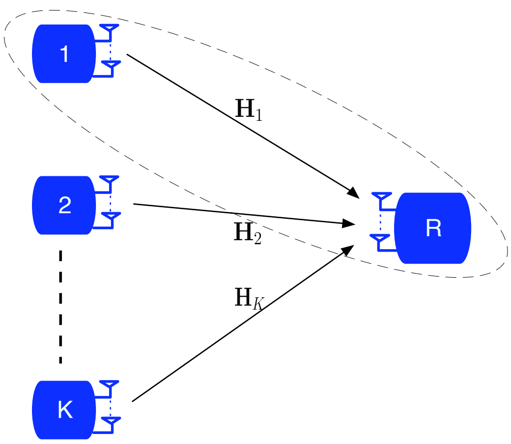


Figure 1: Le Canal MIMO à Accès Multiple.

Motivés par les résultats promettant obtenus dans le cas des canaux mono-utilisateur, notre but dans cette thèse est de proposer des nouvelles constructions de codes espace-temps pour les canaux à accès multiple (*i.e.,* multi-utilisateur) en se basant sur une profonde compréhension de ce dernier du point de vue de la théorie de l'information.

Le Canal à Accès Multiple

Dans une communication à accès multiple, plusieurs utilisateurs communiquent avec un seul récepteur (voir Fig. 1.1). Ce canal est appelé canal MIMO-AM dans ce qui suit. Les approches traditionnelles de codage pour ce scénario consistaient à utiliser des schémas de communication orthogonaux ou, plus récemment, à utiliser des schémas de codages construits pour les canaux point à point. Gärtner et Bölcskei [2] ont introduit l'idée de codage "*joint*" pour les canaux MIMO-AM prenant en consideration les caractéristiques de ces derniers. Les auteurs ont proposé un schéma de codage qui consiste à adapter le code d'Alamouti au scénario multi-utilisateur et dont les performances mettent en évidence l'importance de ce type de codage *joint* comparé aux techniques de communication orthogonales et à l'utilisation des codes mono-utilisateur.

Dans cette thèse, nous présentons une analyse détaillée des aspects théoriques du canal à accès multiple et nous proposons des nouveaux schémas de codage adapté à ce canal. Nos codes espace-temps exploitent les avantages du canal en question d'une meilleure façon comparé aux autres schémas de codage existant dans la litérature.

Hypothèses

Nous nous intéressons dans cette thèse au canal multi-utilisateur. D'où, sauf autrement spécifié, il existe plusieurs utilisateurs et un seul récepteur dans le réseau étudié. Nous supposons que les utilisateurs ne coopèrent pas entre eux de façon que l'information transmise par l'un est indépendante de celle transmise par l'autre. Tous les canaux (entre chaque utilisateur et le recepteur ainsi que le canal équivalent) sont à évanouissement lent (slowly faded channels), *i.e.*, malgré le fait qu'ils soient aléatoires, ils restent constants durant toute la communication. Nous supposons que le recepteur a une connaissance parfaite et précise de l'état de tous les canaux, tandis que les utilisateurs à l'émission n'ont aucune connaissance des canaux.

Quand la technique de coopération est considérée, le canal à accès multiple est appelé MARC (multiple access relay channel) et le protocole de relayage que nous adoptons dans notre travail est l'*Amplify-and-Forward* (AF et dans le cas multi-utilisateur MAF). Ce protocole est caractérisé, d'une part, par sa compléxité de relayage réduite impliquant la possibilité de son implémentation en pratique et d'autre part, par sa linéarité nous permettant de modéliser le MARC comme un canal MIMO-AM *virtuel*. Nous montrons que, grace à cette modélisation, il est possible d'utiliser les codes espace-temps construits pour les canaux MIMO-MA d'une façon distribuée.

Nous utilisons la probabilité de coupure (outage probability) ainsi que son approximation pour des grands SNR, *i.e.*, le compromis diversité-multiplexage (DMT), comme outils théoriques d'analyse de performance. Le DMT a été introduit par Zheng et Tse dans [1] pour le canal point à point et généralisé par Tse et *al.* dans [3] au canal à accès multiple. Concernant le MARC, une borne supérieure du DMT a été développée dans [4] et fut récemment utilisée par Chen et *al.* dans [5] pour évaluer la performance du protocole MAF qu'ils ont proposé.

D'un autre côté, nous utilisons le taux d'erreur par mot pour évaluer la performance d'un schéma de codage. Au récepteur, nous utilisons un égaliseur *"minimum mean square error decision feedback MMSE-DFE"* [6] suivi d'un décodage de réseaux de points (Schnorr-Euchner, SE) pour résoudre le problème de déficience de rang.

Plan et Contributions de la thèse

Cette thèse est organisée comme suit.

Chapitres 1 et 2 : Analogie entre les communications de point à point et celles à accès multiple

Les deux premiers chapitres de cette thèse sont dédiés à l'analyse de l'analogie entre les communications de point à point et à accès multiple. Chapitre 1 montre comment les outils de la théorie de l'information, que nous commençons par définir dans le cas mono-utilisateur, peuvent être adaptés au cas multi-utilisateur. Plus précisement, une analyse détaillée des notions élémentaires de la théorie de l'information, telles que la capacité du canal, la probabilité de coupure et le compromis Diversité-Multiplexage (DMT), est présentée dans les deux cas mono et multi-utilisateur.

Dans le Chapitre 2, une étude de l'état de l'art sur la construction des codes espace-temps est présentée. Dans la première partie de ce chapitre, le cas mono-utilisateur est considéré. Les critères de construction des codes dans ce cas sont rappelés suivis par quelques constructions de schémas de codage qui peuvent être trouvées dans la litérature et qui nous seront utiles dans la suite de la thèse. Dans la deuxème partie du chapitre, le cas multi-utilisateur est considéré et les critères de construction de codes espace-temps dans ce cas sont présentés. L'idée de l'extension des codes espace-temps à ce dernier est par la suite introduite suivie par la description et l'étude de la performance du premier code *joint* proposé par Gärtner et Bölcskei dans [2] pour le canal à accès multiple avec deux utilisateurs et $n_t = n_r = 2$. Ce code, qu'on note code GB , est de longueur 4. Il est construit en concatenant deux codes Alamouti tel que, la matrice de mot de code de l'utilisateur k peut être écrite sous la forme suivante

$$\boldsymbol{X}_k = \begin{bmatrix} s_{k,1} & s_{k,2} & s_{k,3} & s_{k,4} \\ -s_{k,2}^* & s_{k,1}^* & -s_{k,4}^* & s_{k,3}^* \end{bmatrix} \quad (1)$$

où $s_{k,j}$, $k = 1, \ldots, 2$, $j = 1, \ldots, 4$ correspondent aux quatre symboles d'information des utilisateurs 1 et 2 qui sont indépendemment choisis d'une constellation QAM. Afin de garantir un rang minimal égal à 3, une des matrices de mot de code d'un des utilisateurs,

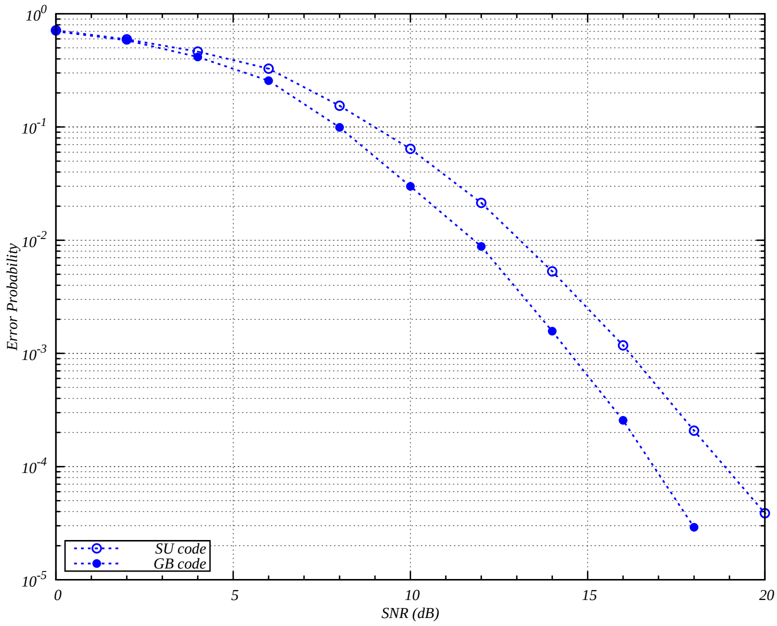


Figure 2: $P_{\mathcal{E}_2}$ du code mono-utilisateur et du code GB, $K = n_t = n_r = 2$, 4-QAM.

disons l'utilisateur 2, est modifiée comme suit

$$\boldsymbol{X}_2 = \begin{bmatrix} s_{2,1} & s_{2,3} & s_{2,2} & s_{2,4} \\ -s_{2,2}^* & -s_{2,4}^* & s_{2,1}^* & s_{2,3}^* \end{bmatrix} \quad (2)$$

Dans la figure 2.1, nous comparons la performance du code GB à celle correspondante à l'utilisation du code optimal pour une communication de point à point. Nous considérons la probabilité de l'évènement où les deux utilisateurs sont en erreur, qu'on note évènement 2, en fonction du SNR reçu. Nous remarquons une amélioration claire des performances (2.5) qui s'explique par le rang de la matrice de mot de code équivalente qui est égal à 3 (rank($\boldsymbol{X}$) = 3) pour le code GB tandis qu'il est égal à 2 pour le code mon-utilisateur (le Golden code).

Cette construction montre l'importance de l'utilisation de codes spécialement construits pour les canaux à accès multiple, confirme la sous-optimalité des codes mono-utilisateur dans un scénario multi-utilisateur et motive notre travail que nous présentons dans les prochains chapitres.

Chapitre 3 : Construction de Codes Espace-Temps pour les Canaux à Accès Multiple avec une Antenne à l'Émission

Dans ce chapitre, nous considérons un scénario multi-utilisateur relativement simple dans lequel tous les utilisateurs sont équipés d'une seul antenne (voir Fig. 3.1). Nous supposons, dans une première partie, que les utilisateurs sont synchronisés à l'émission et nous pré-

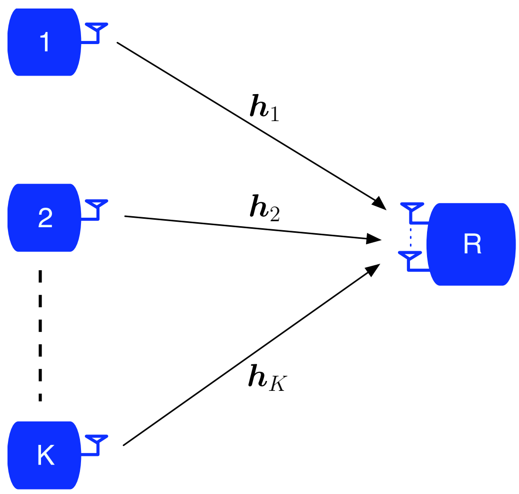


Figure 3: Un canal à accès multiple, K utilisateurs, $n_t = 1$ antenne par utilisateur.

sontons une analyse théorique basée sur la probabilité de coupure du canal en question qui permet d'évaluer sa limite théorique.

Dans la figure suivante (Fig. 3.2), nous avons tracé les probabilités de coupure d'un canal MA avec deux utilisateurs et nous avons considéré deux techniques d'accès: la technique d'accès orthogonal ("time sharing") où les utilisateurs accèdent au canal successivement, et la technique d'accés simultané où les deux utilisateurs accèdent au canal simultanément ("multiple-access"). Dans ce dernier scénario, la probabilité totale de coupure ainsi que celle de l'évènement où les canaux des deux utilisateurs sont simultanément en coupure ("outage event 2"), sont considérées.

Le même ordre de diversité est atteint avec les deux techniques d'accès mais le "time sharing" est sous optimal. Cela s'explique par la mauvaise exploitation des degrés de liberté du canal. Intuitivement, les deux utilisateurs doivent accéder au canal afin d'exploiter au mieux ses degrés de liberté. Suite à cette analyse, une question s'est naturellement posée: et si les codes espace-temps mono-utilisateur sont utilisés dans un scénario multi-utilisateur ? La figure 3.4 montre que l'utilisation du code espace-temps mono-utilisateur optimal (simple modulation QAM dans ce cas) offre une bonne performance (comparé au time sharing) dans le cas où $n_r = 2$ tandis qu'elle est sous optimale (même performance que le time sharing) dans le cas où $n_r = 1$. Ce comportement qui s'explique facilement en analysant le DMT dans les deux cas, nous a motivé à proposer notre nouvelle construction brièvement décrite dans ce qui suit.

Le code que nous proposons est de longueur K de façon que la matrice du mot de code équivalente est carrée. Le vecteur comprenant les symboles d'information de l'utilisateur k est noté $\boldsymbol{s}_k$ et est exprimé comme suit:

$$\boldsymbol{s}_k = \begin{bmatrix} s_{k,1} & s_{k,2} & \ldots & s_{k,K} \end{bmatrix}^\top$$

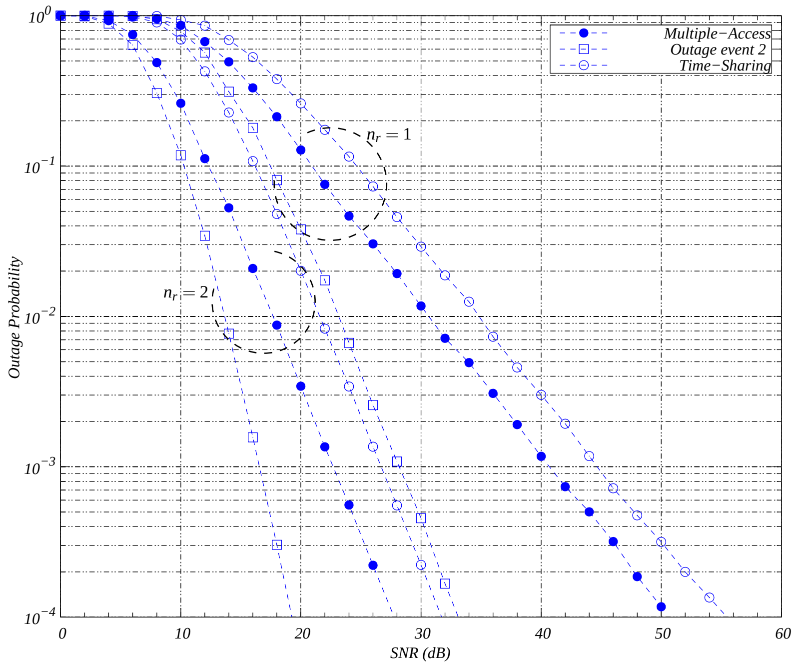


Figure 4: Probabilités de coupure d'un canal à accès multiple avec deux utilisateurs et différentes techniques d'accès, $n_t = 1$, $R = 2$ bits/puc.

où $s_{k,l}$ correspond au symbol d'information transmis par l'utilisateur k durant la l^{ieme} utilisation canal, $k, l = 1, 2, \ldots, K$. La matrice de mot de code de chaque utilisateur k est construite en multipliant le vecteur $\boldsymbol{s}_k$ par la matrice unitaire de rotation que nous notons $\boldsymbol{U}$ comme suit:

$$\boldsymbol{x}_k = \boldsymbol{U}\boldsymbol{s}_k = \begin{bmatrix} \gamma x_k & \gamma\sigma(x_k) & \ldots & \sigma^{K-1}(x_k) \end{bmatrix} \tag{3}$$

La matrice de mot de code équivalente est par la suite construite,

$$\boldsymbol{X} = \begin{bmatrix} x_1 & \sigma(x_1) & \sigma^2(x_1) & \ldots & \sigma^{K-1}(x_1) \\ \gamma x_2 & \sigma(x_2) & \sigma^2(x_2) & \ldots & \sigma^{K-1}(x_2) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \gamma x_K & \gamma\sigma(x_K) & \ldots & \ldots & \sigma^{K-1}(x_K) \end{bmatrix} \tag{4}$$

Le coefficient γ est ajouté pour garantir un déterminant non nul et donc, un ordre de diversité maximal. Nous avons montré que, si γ est un nombre transcendental, notre code a un rang plein.

Dans le cas d'un canal AM à deux utilisateurs, nous utilisons la matrice $\boldsymbol{U}$ utilisée pour la construction du Golden code:

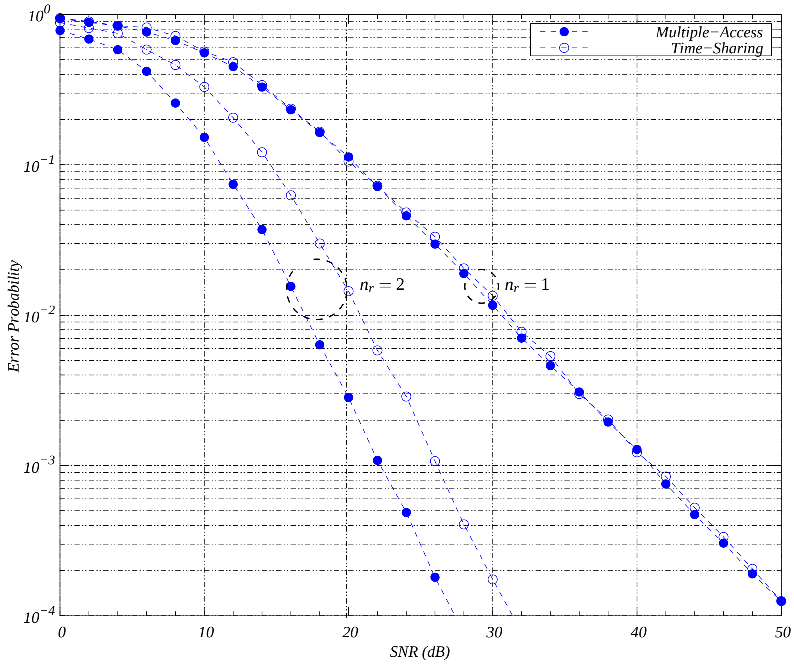


Figure 5: Probabilité d'erreur du code mono-utilisateur (modulation QAM) comparée à celle du Time-Sharing, $K = 2, n_t = 1$, 4-QAM.

$$\boldsymbol{U} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha & \alpha\theta \\ \bar{\alpha} & \bar{\alpha}\bar{\theta} \end{bmatrix}$$

où $\alpha = 1 + i - i\theta$ et $\theta = \frac{1+\sqrt{5}}{2}$. Nous avons,

$$\boldsymbol{x}_k^\top = \boldsymbol{U}\boldsymbol{s}_k = \begin{pmatrix} x_k \\ \sigma(x_k) \end{pmatrix} \tag{5}$$

La matrice de mot de code équivalente est construite comme suit:

$$\begin{aligned} \boldsymbol{X} &= \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 & \sigma(x_1) \\ \gamma x_2 & \sigma(x_2) \end{bmatrix} \\ &= \begin{bmatrix} \alpha(s_{11} + s_{12}\theta) & \bar{\alpha}(s_{11} + s_{12}\bar{\theta}) \\ i\alpha(s_{21} + s_{22}\theta) & \bar{\alpha}(s_{21} + s_{22}\bar{\theta}) \end{bmatrix} \end{aligned}$$

La performance du code proposé (BB-code) est étudiée dans la figure 3.6 où le taux d'erreur par mot correspondant est tracé en fonction du SNR. La supériorité du nouveau code est claire et celle-çi quelque soit le nombre d'antenne à la réception.

Une étude théorique déterminant le DMT atteint par le nouveau schéma de codage que nous avons proposé pour le canal AM avec une seul antenne par utilisateur a été présentée suivie d'une analyse de l'optimalité de ce dernier en terme du DMT.

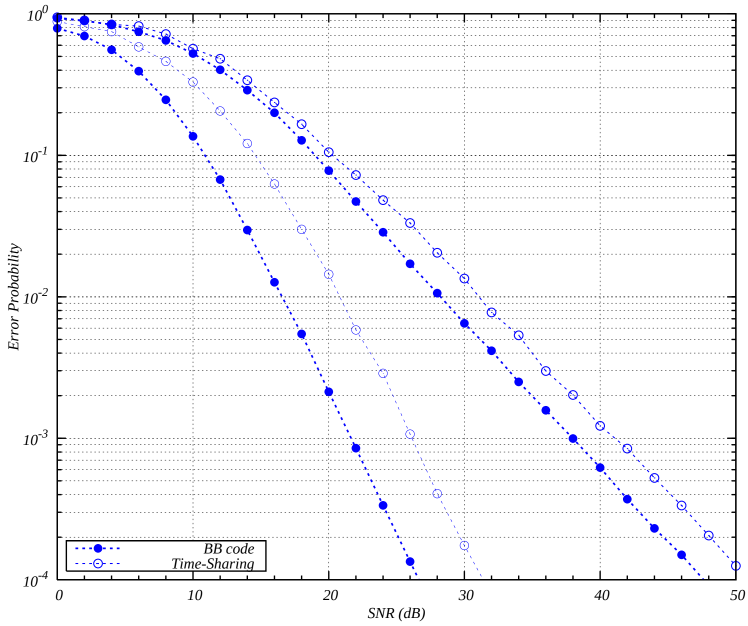


Figure 6: Performance du BB-code, $n_t = 1$, 4-QAM.

Dans la deuxième partie de ce chapitre, un scénario plus pratique dans lequel l'hypothèse de synchronisation à la transmission est omise, est considéré. Nous avons montré que, malgré un éventuel asynchronisme entre les utilisateurs, la performance et plus précisement le gain offert par notre schéma de codage est conservé (voir Fig. 3.12 dans laquelle la probabilité de l'évènement où les deux utilisateurs sont en erreur est tracée).

Chapitre 4 : Construction de Codes Espace-Temps pour les Canaux à Accès Multiple à Antennes Multiples

Motivé par le gain de performance important obtenu dans le cas mono-antenne et armé par la compréhension des aspects théoriques du canal à accès multiple, nous avons considéré dans ce chapitre 4 ce même canal mais avec des utilisateurs multi-antennaire (le canal MIMO-AM). Le premier code espace-temps connu pour ce scénario est le code GB [2] étudié précédemment. Hong et Viterbo ont considéré dans [7] le canal MIMO-AM et ont construit un nouveau code que nous notons HV. Le code HV offre des meilleures performances comparé au code GB (voir Fig. 4.3).

Indépendamment du code HV, nous avons proposé un nouveau code (code MIMO-BB) dont les performances sont meilleures que les codes précédents (voir Fig. 9). Un mot de code du code MIMO-BB est sous la forme suivante:

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\Xi}_1 & \tau(\boldsymbol{\Xi}_1) & \dots & \tau^{K-1}(\boldsymbol{\Xi}_1) \\ \boldsymbol{\Gamma}\boldsymbol{\Xi}_2 & \tau(\boldsymbol{\Xi}_2) & \dots & \tau^{K-1}(\boldsymbol{\Xi}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Gamma}\boldsymbol{\Xi}_K & \boldsymbol{\Gamma}\tau(\boldsymbol{\Xi}_K) & \dots & \tau^{K-1}(\boldsymbol{\Xi}_K) \end{bmatrix} \tag{6}$$

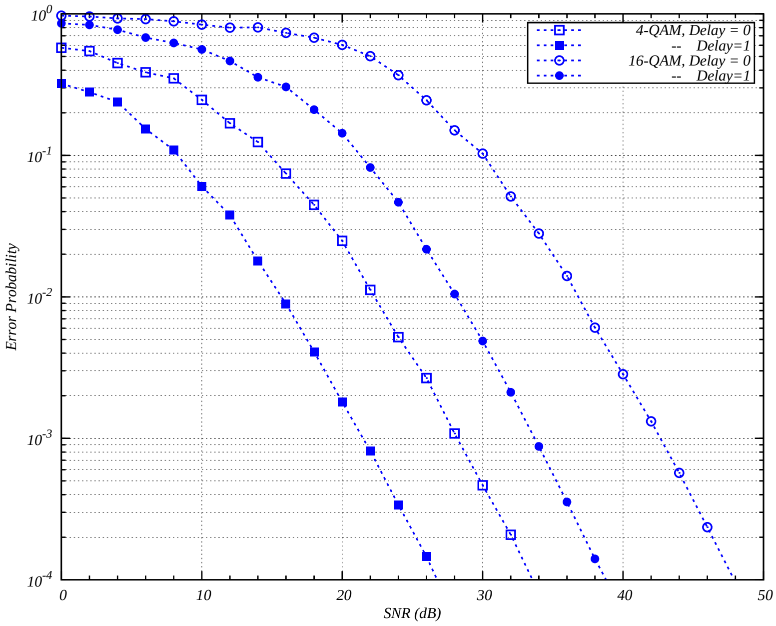


Figure 7: $P_{\mathcal{E}_2}$ pour des constellations 4 et 16-QAM. Cas synchrone et asynchrone, $K = 2$, $n_t = n_r = 1$.

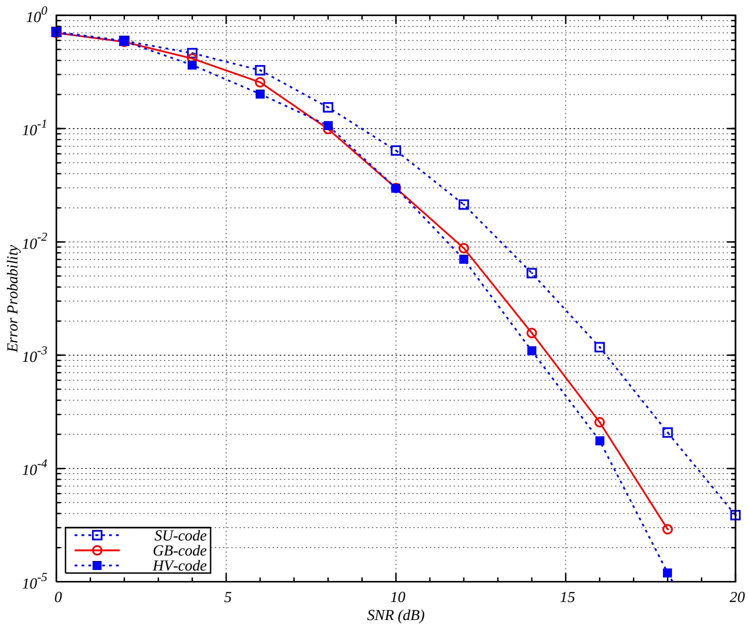


Figure 8: $P_{\mathcal{E}_2}$ des codes HV, GB et du code mono-utilisateur, $K = n_t = n_r = 2$, 4-QAM (2 bits puc).

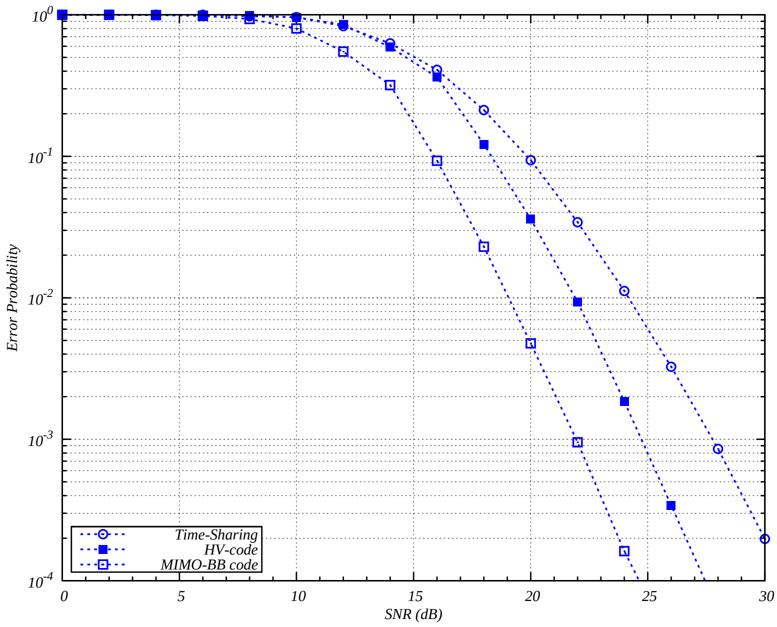


Figure 9: Probabilités d erreur des codes HV, MIMO-BB et du time sharing, $K = n_t = n_r = 2$, 4-QAM.

où $\boldsymbol{\Xi}$ correspond à la représentation matricielle d'un élément de l'algèbre cyclique de division sur laquelle le code est construit, *i.e.*,

$$\boldsymbol{\Xi} = \begin{bmatrix} x_1 & x_2 & \dots & x_{n_t} \\ \eta\sigma(x_{n_t}) & \sigma(x_1) & \dots & \sigma(x_{n_t-1}) \\ \vdots & \vdots & \ddots & \vdots \\ \eta\sigma^{n_t-1}(x_2) & \eta\sigma^{n_t-1}(x_3) & \dots & \sigma^{n_t-1}(x_1) \end{bmatrix} \tag{7}$$

Chapitre 5 : Le Canal à Accès Multiple avec Relais

Le canal à accès multiple avec relais (MARC) est un canal à accès multiple dans lequel N relais aident les utilisateurs à communiquer avec la destination (voir Fig. 5.3). Il est important de noter que les utilisateurs ne coopèrent pas entre eux. Dans ce chapitre, n_t, n_r and n_d correspondent au nombre d'antenne par utilisateur, par relais et à la destination, respectivement.

Le protocole de relayage que nous adoptons dans notre travail est le "Multi-Access Amplify-and-Forward" (MAF) qui a été récemment introduit par Chen et *al.* [5] pour le canal MARC avec un seul relais auquel nous nous intéressons dans notre travail. Nous supposons que le relais est "half-duplex", *i.e.*, le relais ne peut pas transmettre et recevoir en même temps. Un des principaux avantages du protocole MAF appartenant à la classe de protocole AF, est le bon compromis entre compléxité et performance qu'il offre comparé à d'autre protocoles tels que le "Dynamic Decode and Forward" (DDF) et le "Compress and Forward" (CF). Une comparaison de ces différents protocoles de coopération, en terme du compromis diversité-multiplexage, a été présentée dans [5] montrant la supériorité du MAF dans quelques régions de gain de multiplexage.

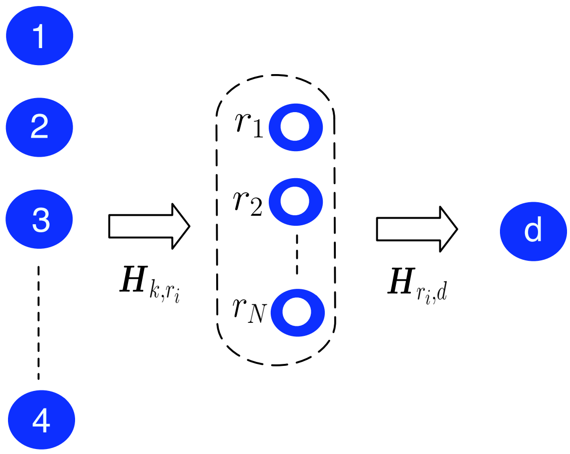


Figure 10: Le canal à accès multiple avec relais ("multiple-access relay channel").

La structure de la trame de coopération du protocole MAF est illustrée dans la figure 5.4. Dans les deux phases de la trame, les deux utilisateurs transmettent simultanément leurs informations. Le relais, due à la contrainte half-duplex qui lui est imposée, écoute dans le première phase et transmet, dans la deuxième phase, une version amplifiée du message qu'il a reçu durant la première phase. Dans la figure 5.4, les traits pleins correspondent au mode de transmission et les traits pointillés au mode de réception.

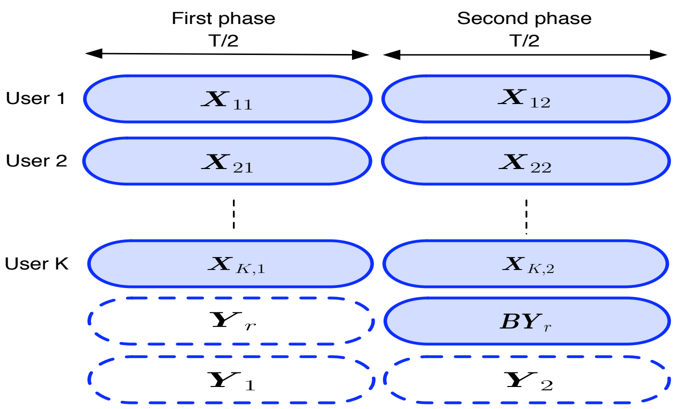


Figure 11: La structure de la trame de cooprération avec le protocole MAF.

Nous avons montré que, grace à la linéarité du protocole MAF, le canal à accès multiple avec relais peut être modélsier comme un canal MIMO-AM virtuel auquel nous avons appliqué le schéma de codage que nous avons déjà construit pour le canal MIMO-AM en une façon distribuée.

Dans la figure 5.9, nous avons tracé la probabilité d'erreur du nouveau code espace-temps distribué proposé que nous comparons à celle du time-sharing et du code mono-utilisateur distribué. Ces courbes montrent le gain offert par le nouveau code.

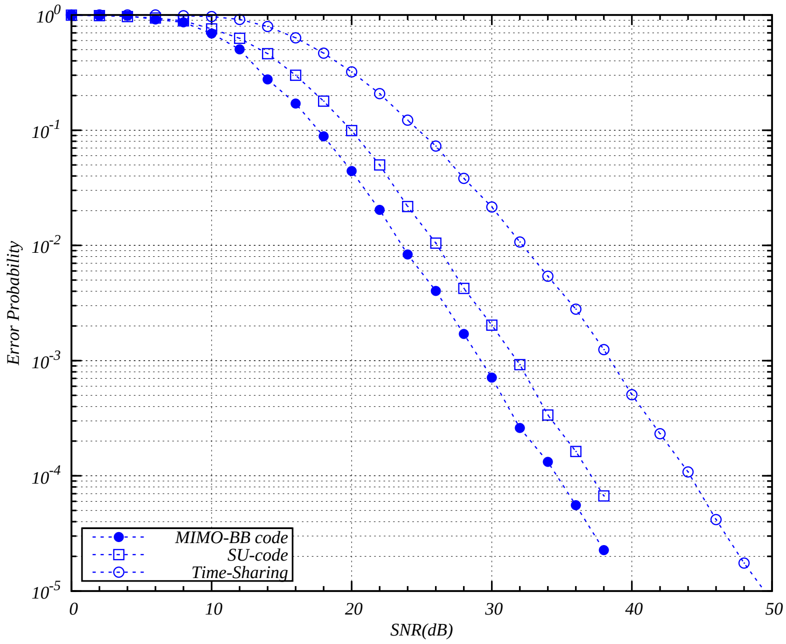


Figure 12: Performance du code MIMO-BB distribué, $n_d = 1$, 4-QAM.

Conclusions et Perspectives

Dans cette thèse, nous nous sommes intéressés au canal à accès multiple qui peut être considéré comme canal élémentaire d'un réseau de communication sans fil large. Plus précisement, nous avons proposé de nouveaux codes espace-temps adaptés au canal en question qui offrent des gains de performance remarquables. Les outils de la théorie de l'information, la probabilité de coupure et le compromis diversité-multiplexage, ont été utilisés pour l'évaluation des performances théoriques.

Nous avons commencé par un scénario simple dans lequel tous les utilisateurs sont équipés d'une seule antenne. Le code que nous avons proposé dans ce cas est le premier code espace-temps pour ce type de canal. Deux scénarios, communication synchrone et asynchrone, ont été considérés et l'optimalité du code proposé a été démontrée dans les deux cas. Une étude théorique de ce nouveau schéma de codage a été présentée montrant son optimalité, dans le sens du DMT, pour quelques régions de gain de multiplexage. Le cas plus général multi-antennaire a été considéré par la suite. Nous avons proposé un nouveau code espace-temps que nous avons comparé à d'autre codes existant dans la litérature pour montrer sa supériorité dans la plupart des cas. Une première étape vers l'analyse de ce code en terme du DMT a été présentée, la compléter constitue une importante perspective.

Dans une dernière partie, nous avons considéré la coopération dans un scénario multi-utilisateur. Les utilisateurs ne coopèrent pas entre eux mais utilisent un relais pour atteindre leur destination. Nous avons montré que ce canal est équivalent à un canal à accès multiple multi-antennaire et donc, nous avons utilisé le code espace-temps construit pour ce dernier pour proposer un nouveau code espace-temps distribué pour le canal à accès multiple avec relais. Nous avons étudié le cas mono-antenne avec un seul relais, la généralisation de ce travail pour des canaux à accès multiple plus larges est une perspective intéressante.

Plus généralement, l'étude de canaux plus complexes comme le canal à intérférence est une des perspectives majeures permettant la compréhension générale d'un large réseau de communication sans fil.

Notation

Scalars

K	number of users
n_t, n_r	number of transmit and receive antenna
n_r, n_d	number of antennas at the relay and the destination in a MAR channel
T	codeword block length
s	cardinality of $\mathcal{S}$
SNR	signal to noise ratio
P_{out}	outage probability
$P_{\mathcal{E}}$	error probability
ζ_n	n^{th} root of unity

Sets

$\mathcal{S}$	subset of users $\subseteq \{1, \ldots, K\}$
$\overline{\mathcal{S}}$	complement of $\mathcal{S}$ in $\{1, \ldots, K\}$
$\mathbb{Z}$	Set of integers
$\mathbb{R}$	Set of reals
$\mathbb{C}$	Set of complex numbers
$\mathbb{Q}$	Set of rational numbers

Operations

$\mathbb{E}[x]$	Expectation of x
$Q(a)$	$\int_a^\infty (1/\sqrt{2\pi}) \exp^{-x^2/2} dx$
$\mathbb{P}\{\cdot\}$	probability measure
$\|\mathcal{A}\|$	Cardianality of a set
$f(x) \doteq x^b$	exponential equality , $\lim_{x\to\infty} \frac{\log f(x)}{\log x} = b$
$\dot{\geq}, \dot{\leq}$	exponential inequality

$\log(\cdot)$	logarithme to base 2
&	and operator
$\wedge$	exclusive or operator

Matrices and vectors

$\boldsymbol{v}$	vector
$\boldsymbol{M}$	matrix
$\boldsymbol{M}_s$	horizontal concatenation of s matrices
$M_{i,j}$	element of the j-th column and i-th row of the matrix $\boldsymbol{M}$
$\boldsymbol{v}^\top, \boldsymbol{M}^T$	transpose of $\boldsymbol{v}$ and $\boldsymbol{M}$
$\boldsymbol{v}^\dagger, \boldsymbol{M}^\dagger$	conjugate transpose of $\boldsymbol{v}$ and $\boldsymbol{M}$
$[\cdot]^*$	element conjugate
$\odot$	component wise product
$\mathrm{Tr}(\boldsymbol{M})$	trace of $\boldsymbol{M}$, $\sum_i M_{i,i}$
$\det \boldsymbol{M}$	determinant of $\boldsymbol{M}$
$\mathrm{rank}(\boldsymbol{M})$	rank of $\boldsymbol{M}$
$\\|\boldsymbol{M}\\|_{\mathrm{F}}$	squared frobenius norm of $\boldsymbol{M}$, $\mathrm{Tr}(\boldsymbol{M}\boldsymbol{M}^\dagger)$
$\boldsymbol{I}_n$	$n \times n$ identity matrix
$\mathbf{0}$	all zeros matrix
$\boldsymbol{M} \otimes \boldsymbol{M}^{'}$	Kronecker product between matrices $\boldsymbol{M}$ and $\boldsymbol{M}^{'}$
$\lambda(\boldsymbol{M})$	Eigenvalue of matrix $\boldsymbol{M}$
$\mathcal{CN}(0, \sigma^2)$	Complex Gaussian random variable with zero mean and variance σ^2
$\boldsymbol{X}$	transmitted signal
$\boldsymbol{Y}$	received signal
$\boldsymbol{Z}$	additive gaussian noise
$\boldsymbol{H}_k$	channel matrix of user k
$\boldsymbol{H}$	equivalent channel matrix

Acronyms

AF	amplify-and forward
AWGN	additive white gaussian noise
BB code	Badr and Belfiore code
CDA	cyclic division algebra
CF	compress-and-forward
CSI	instantaneous channel state information
dB	deciBel
DDF	dynamic decode-and-forward
DMC	discrete memoryless channel
DMT	diversity-multiplexing tradeoff
GB code	Gärtner and Bölskei code [2]
HV code	Hong and Viterbo code [7]
iid	independent and identically distributed
LAST	lattice space-time
MAC	multiple-access channel
MIMO	multiple input multiple output
MISO	multiple input single output
MAC	Multiple Access Channel
MIMO-MAC	Multiple Input Multiple Output MAC
ML	maximum likelihood
MMSE	minimum mean square error
NAF	non-orthogonal amplify-and forward
NVD	non vanishing determinant
pcu	per chanel use
PEP	pairwise error probability
QAM	quadrature amplitude modulation
SIMO	single input multiple output
SISO	single input single output
SNR	signal to noise ratio

STBC	space time block code
SU	single user

Introduction

Motivation

Extensive research has been carried out in the last few years on the single-user multiple-input multiple-output (MIMO) space-time block code (STBC) design using advanced algebraic tools, namely cyclic division algebra (CDA) ([8], [9], [10]). Families of single-user CDA codes have been carefully constructed to achieve the outage diversity-multiplexing tradeoff (outage-DMT) of the MIMO channel. The DMT framework introduced by Zheng and Tse in [1] to evaluate the theoretical performance limit of a channel has been proven to be an important tool to construct optimal codes ([11], [12], [13], [14]). Motivated by the promising results obtained in the single-user scenario and the extension of the DMT framework to the multi-user scenario [3], the aim of this thesis is to construct optimal multi-user STBCs.

Using the multi-user information theoretic background, the goal here is to design STBCs for MIMO multi-user channels that recently became a challenging topic due to its different potential applications. We focus on the uplink multi-user MIMO channel, equivalently, the MIMO multiple-access channel (MIMO-MAC). In a MIMO-MAC multiple users (transmitters) equipped with multiple transmit antennas communicate with a single multiple-antenna receiver. Traditional approaches consisted of using orthogonal communication schemes or, more recently, employing single-user code for each user both leading to suboptimum performance. Gärtner and Bölsckei introduced in [2] the idea of *joint code design* for MIMO-MAC taking into account the multiple-access nature of the channel. They also presented a code construction based on the Alamouti code [15]. This code is not optimal but highlights the performance improvement resulting from a joint code design. This work motivates the construction of optimal codes for MIMO-MAC.

This thesis attempts to fill this gap by investigating the fundamental performance benefits of the MAC and presenting new families of multiple-access STBCs. These codes better exploit the channel's advantages compared to previous constructions. While we start our work with a "simple" single-antenna scenario, we generalize our construction to the MIMO-MAC. A wide body of results followed the introduction of our codes ([7], [16], [17], [18] and references therin) leading to some divergent points of view. Therefore, we try throughout this thesis to offer a unified view of the main research results on this topic obtained in the last few years.

Assumptions

This thesis focuses on the uplink multi-user communication. Thus, unless stated otherwise, there are multiple users and a single receiver in the network. This assumption is without loss of generality when time-sharing among users is considered. Moreover, it is assumed that the users are not allowed to cooperate with each other so that the information transmitted by each user is independant from that of the other users. All users' channels, and therefore the equivalent multi-user channel, are assumed to be slowly faded, *i.e.,* although they are random, they stay constant over the duration of communication. Since the channel varies slowly, it is assumed that the receiver has sufficient time to estimate it and thus, has an accurate knowledge of all users' channels, *i.e.*, full channel state information is available at the receiver CSIR, whereas the transmitters do not have any CSI.

When relaying is considered in the multiple-access scenario, the amplify-and-forward (AF) protocol is considered. This protocol is known in this case as the multi-access amplify-and-forward (MAF) protocol. This protocol is characterized by its low relaying complexity which makes the cooperation implementable in practice. In addition, the linearity of the AF protocol transforms the MARC into a *virtual* MIMO-MAC where it is possible to apply the results obtained for the MIMO-MACs in a distributed way.

The outage probability and its approximation for high signal-to-noise ratio (SNR), *i.e.,* the diversity-multiplexing tradeoff (DMT), are used as theoretical analysis tools. The DMT was introduced by Zheng and Tse in [1] for the point-to-point MIMO channel to characterize, in the high SNR regime, the fundamental tradeoff between the throughput increase and the reliability improvement offered by MIMO systems. This tradeoff has been extended to the symmetric multiple-access channel in [3]. Moreover, an upper bound on the achievable DMT for the multiple-access relay (MAR) channel has been derived in [4] and recently used in [5] to study the performance of the proposed MAF protocol.

On the other hand, the word error rate is adopted as a performance measure to analyze the performance of a code as a function of the SNR. At the receiver side, a minimum mean-square error decision feedback equalizer MMSE-DFE preprocessing combined with lattice decoding is used as a way to mitigate the problem of the rank deficiency resulting from n_r being smaller than $K \times n_t$. It has been shown in [6], that an appropriate combination of left, right preprocessing and lattice decoding, yields significant saving in complexity with very small degradation with respect to the ML performance. More precisely, left preprocessing modifies the channel matrix and the noise vector such that the resulting closest lattice point search has a much better conditioned channel matrix. Moreover, right preprocessing is used to change the lattice basis such that it becomes more convenient for the searching stage.

Outline and Contributions

This thesis is organized as follows.

From point-to-point to Multiple-Access Channel (Chapter 1 and 2)

As a starting point of this thesis, the transition from single-user to multi-user communication is investigated from an information theoretic point of view in Chapter 1 and a coding point of view in Chapter 2. In the former, a comprehensive vision on elementary notions of information theory is presented in both the single-user and multiple-access cases. In the latter, a state of the art study of the Space-Time block coding design is presented. The first part of the chapter is dedicated to the single-user STBCs. The corresponding design criteria are derived and different single-user STBC constructions, that will be used repeatedly in this thesis, are presented.

In the second part of the chapter, the idea of extending the Space-Time code design from point-to-point to multiple-access is introduced. The multiple-access code design criteria derived in [2] are presented. The first *joint* code construction based on the Alamouti code and motivating the multiple-access code contruction is studied. Based on the information theoretic background, DMT-optimal design criteria are derived in both point-to-point and multiple-access scenarios.

Space-Time code construction for Single-Antenna Multiple-Access Channels (Chapter 3)

In Chapter 3, a simple multiple-access scenario, where the users are equipped with a single transmit antenna, is considered. An information theoretic analysis of the channel, based on the outgae probability, helps evaluate the performance limits of the channel and gives insights on the way an optimal coding scheme should be constructed. A new family of full-rank Space-Time codes is proposed for the single-antenna MAC. The users are assumed to transmit synchronously their information. The proposed code is shown to outperform the single-user scheme where the users transmit *simultaneously* their QAM information symbols and the time-sharing scheme where the users transmit *orthogonally* their QAM information symbols. A DMT-oriented analysis of the proposed code is then presented. The single-antenna STBC is shown to be optimal, in the sense of the DMT, for a special range of mutliplexing gains.

In practice, the assumption of perfect synchronization between the users at the transmission is not easily justified. This fact motivates the study of code design for the asynchronous MAC. Asynchronism entails a change in the STBC's structure and may induce a degradation of the corresponding performance. Fortunately, the proposed code is shown to be delay-tolerant, *i.e.,*, its overall diversity order is preserved despite asynchronism among users. Numerical results evaluating the error probability of the code confirm its delay-tolerance.

Space-Time code construction for MIMO Multiple-Access Channels (Chapter 4)

Motivated by the promising gain obtained in the single antenna scenario and using the information theoretical knowledge gained to this end, the MIMO-MAC is considered in

Chapter 4. The goal is to design new space-time coding schemes that optimally exploit the capabilities of the channel. This problem has interested other researchers ([2], [7], [19]) and different coding schemes were designed. These schemes are studied in this chapter and compared to the new proposed coding scheme. The new code is shown to outperform or at least offer the same performance as all other schemes.

The performance analysis presented in this chapter follows the same footsteps as in Chapter 3. Time-sharing among users is shown to be sub-optimal while the simultaneous transmission using the single-user code is shown to offer a performance gain. This gain is more significant when a special jointly designed coding scheme is employed. A DMT-oriented analysis of the proposed code is finally presented using the DMT-optimal design criteria presented in Chapter 2. Nevertheless, verifying whether the proposed code is DMT-optimal in the sense of the latter criteria remains an open problem.

The Multiple-access relay channel (Chapter 5)

Simple cooperation techniques such as relaying are well known to improve both the reliability and the throughput in a wireless network. In this chapter, we are interested in applying cooperative techniques in a multiple-access scenario. The users are not allowed to cooperate with each other but can benefit from a common terminal, the relay, helping them reach their destination. The channel is in this case termed Multiple-Access Relay (MAR) channel and was first introduced in [20]. In contrast with other cooperation strategies requiring coordination among terminals, the relaying technique considered here has a significantly low cost and complexity. The complexity is further reduced due to the use of an Amplify-and Forward (AF) cooperation protocol where the half-duplex relay simply scales the information it receives (from all the users) and forwards it to the destination. The considered MAF cooperation protocol was recently introduced by Chen et *al.* in [5]. In addition to its low complexity, the MAF protocol was shown to achieve the optimal DMT for some multiplexing gain range. In this chapter, the MAF relay channel is modelled as an equivalent virtual MIMO-MAC in which case, applying the code constructed for the MIMO-MAC in Chapter 4 in a distributed fashion is shown to offer significant gain.

Finally, we conclude this thesis and provide some general perspectives.

Chapter 1

From Point-to-Point to Multiple-Access Channel: An Information Theoretical Perspective

In multiple-access communication, or equivalently uplink multi-user communication, the information flows from multiple transmitting terminals to one receiver (base station). If each user is considered separately, the corresponding channel is a point-to-point (single-user) channel. In order to evaluate the performance limits of a multiple-access network, different information theoretic tools are used. As may be expected, the derivation of such tools follows the same footsteps as in the single-user case with some additional constraints and properties resulting from the multiple-access nature of the channel that should be taken into account.

As a starting point of this thesis, this chapter aims to provide a review of different principles of information theory that yield important theoretical tools. These tools determine the ultimate performance that can be achieved over a given channel. They are mainly used as a basis to develop optimal coding schemes and to evaluate their performance. A comprehensive vision on the capacity, the outage probability and the diversity-multiplexing tradeoff is presented for MIMO channels in both single-user and multiple-access cases.

The rest of the chapter is organized as follows. In Section 1.1, the channel model is presented. A detailed information theoretic study of the single-user case as well as the extension to the multi-user case is then developed. In Section 1.2, the capacity of a point-to-point channel is defined. This notion is then generalized to the capacity region of a multiple-access channel. These definitions are then used in Section 1.3 to analyze the outage behavior of the channel in both scenarios. The diversity-multiplexing tradeoff is then characterized and interpreted in Section 1.4 in the point-to-point and the multiple-access cases. Different examples are given to illustrate and analyze the DMT. Finally, the achievability of the DMT is discussed in Section 1.5. The proofs are deferred to Appendix 1.A at the end of the chapter.

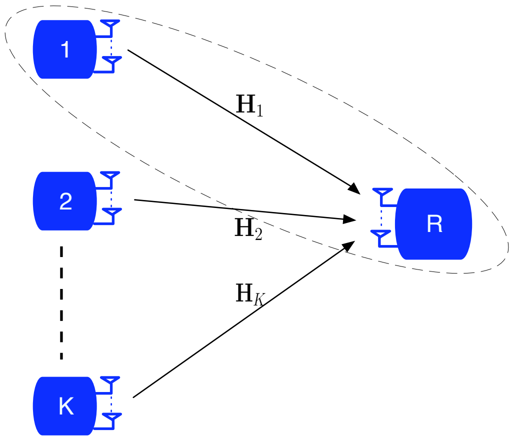


Figure 1.1: A K-user Multiple-Access Channel.

1.1 System Model

Consider a multiple-access system consisting of K users communicating with a single receiver (Fig. 1.1). Let n_t denote the number of transmit antennas per user and n_r the number of receive antennas.

Let $\boldsymbol{X}_k \in \mathbb{C}^{n_t \times T}$ denote the codeword matrix transmitted by user k (equivalently, the k-th channel input) chosen from the individual codebook $\mathcal{X}_k$ of user k and satisfying the following per-user power constraint

$$\|\boldsymbol{X}_k\|_{\mathrm{F}}^2 \leq n_t T, \quad \forall \boldsymbol{X}_k \in \mathcal{X}_k. \tag{1.1}$$

where T denotes the codeword block length. The previous constraint should be satisfied for all the users.

Remark 1.1.1 *In all this thesis, we limit our study to MACs where the users have the same number of transmit antennas n_t and the same transmit power SNR/n_t. SNR is the (per user) signal-to-noise ratio at the receiver.*

Let $\boldsymbol{H}_k \in \mathbb{C}^{(n_r \times n_t)}$ denote the random matrix modeling the channel between user k and the receiver. Each user's channel is a $n_r \times n_t$ point-to-point MIMO channel with the following input-output relation

$$\boldsymbol{Y} = \sqrt{\frac{\mathsf{SNR}}{n_t}} \boldsymbol{H}_k \boldsymbol{X}_k + \boldsymbol{Z} \tag{1.2}$$

where $\boldsymbol{Z} \in \mathbb{C}^{n_r \times T}$ the Additive White Gaussian Noise (AWGN) at the receiver with independent and identically distributed unit variance entries, *i.e.*, $\mathcal{CN}(0, \boldsymbol{I})$.

Remark 1.1.2 *The statistics of the random channel coefficients are modeled using the Rayleigh fading model. Hence, the channel matrices have independent and identically dis-*

tributed entries with zero mean, unit variance, circularly symmetric, complex Gaussian density, i.e., $\mathcal{CN}(0, \boldsymbol{I})$.

Multiple-access techniques

Throughout this thesis, two different multiple-access techniques are investigated repeatedly: the *simultaneous* multiple-access technique and the *orthogonal* multiple-access technique, more specifically *time-sharing* among users. In the former, all users transmit simultaneously their information while in the latter, only one user's information is transmitted at the same channel use.

Let $\boldsymbol{Y} \in \mathbb{C}^{n_r \times T}$ denote the received signal (channel output) when all the users are transmitting simultaneously:

$$\boldsymbol{Y} = \sqrt{\frac{\mathsf{SNR}}{n_t}} \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{X}_k + \boldsymbol{Z} \tag{1.3}$$

For the sake of clarity, the following notation is defined:

Definition 1.1.1 *The horizontal concatenation of* s *matrices with the same number of rows is defined as*

$$\boldsymbol{M}^s := \begin{bmatrix} \boldsymbol{M}_1 & \dots & \boldsymbol{M}_s \end{bmatrix} \tag{1.4}$$

Similarly, the vertical concatenation of multiple matrices with the same number of columns is given by $\boldsymbol{M}^{s\dagger}$. *For* $s = K$, $\boldsymbol{M}^K$ *is denoted* $\boldsymbol{M}$.

With the above notation, let $\boldsymbol{H}$ denote the equivalent MAC modeled by a $n_r \times Kn_t$ matrix as follows

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_2 & \dots & \boldsymbol{H}_K \end{bmatrix}. \tag{1.5}$$

Moreover, the concatenation of all users' codeword matrices $\boldsymbol{X}_k$ yields the *joint codeword matrix* $\boldsymbol{X} \in \mathbb{C}^{Kn_t \times T}$, element of the *joint codebook* $\mathcal{X}$

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}_1 \\ \vdots \\ \boldsymbol{X}_K \end{bmatrix} \tag{1.6}$$

Therefore, the received signal can be written as

$$\boldsymbol{Y} = \sqrt{\frac{\mathsf{SNR}}{n_t}} \boldsymbol{H}\boldsymbol{X} + \boldsymbol{Z} \tag{1.7}$$

Remark 1.1.3 *(Joint Coding) While it will be shown that the users' codebooks* $\mathcal{X}_k,\ k = 1,\ldots,K$ *have to be jointly designed leading to the joint codebook* $\mathcal{X}$*, these are independent from each other. This is due to the fact that there is no cooperation among the users at the transmission side.*

1.2 Channel Capacity

1.2.1 Capacity of a Point-to-Point Channel

In a point-to-point scenario, the theoretical limit of the amount of data that can be reliably transmitted over a channel is its *capacity*, C. In his pioneering work [21], Shannon showed that the transmission error probability over a channel can be made arbitrarily small as long as the transmission rate is below the channel capacity C.

For a single-user MIMO channel with input-output relation (1.2), the instantaneous capacity is given by

$$C(\boldsymbol{H}_k) = \log\left(\det\left(\boldsymbol{I} + \frac{\mathsf{SNR}}{n_t}\boldsymbol{H}_k\boldsymbol{H}_k^{\dagger}\right)\right) \tag{1.8}$$

in bits per channel use (bits/pcu), for a given channel realization $\boldsymbol{H}_k$.

Ergodic Capacity

For a *fast fading* channel, *i.e.,* when coding is performed over an infinite number of independent channel realizations, the maximum rate that can be reliably transmitted is obtained by averaging the instantaneous capacity (1.8) over all channel realizations. The latter is known as *ergodic capacity*,

$$C(\mathsf{SNR}) = \mathbb{E}_{\boldsymbol{H}_k}\left\{C(\boldsymbol{H}_k)\right\}. \tag{1.9}$$

It is shown in [22] that, asymptotically in SNR, this expression satisfies

$$C(\mathsf{SNR}) = \min(n_t, n_r)\log\mathsf{SNR} + o(1) \tag{1.10}$$

where $\min(n_t, n_r)$ is the *multiplexing gain* denoted r and representing the maximum number of degrees of freedom available for data transmission. The additional spatial degrees of freedom offered by a MIMO system, with respect to a single-antenna system, allow the transmission of multiple independent data flows and their separation at the receiver side. In the high SNR regime, the multiplexing gain determines how the data rate increases with SNR. Indeed, r additional bits can be reliably transmitted over the channel the SNR increases by 3 dB.

1.2.2 Capacity Region of a Multi-User Channel

In a multi-user scenario, the concept of channel capacity is extended to the *capacity region.* This region determines the set of all rate tuples $(R_1, \ldots, R_K)$ that all users can reliably communicate at, simultaneously. R_k denotes the data rate of user k in bits/pcu.

The capacity region is a K-dimensional polyhedron containing the set of all rates verifying the following $2^K - 1$ constraints, one for each possible subset $\mathcal{S}$ of users [23, Chapter 10]

$$\sum_{k \in \mathcal{S}} R_k \leq \log\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\sum_{k\in\mathcal{S}}\boldsymbol{H}_k\boldsymbol{H}_k^\dagger\right), \quad \forall \mathcal{S} \subseteq \{1, \ldots, K\}. \tag{1.11}$$

It is very important to point out that due to the sum rate constraint, the MAC cannot be decomposed into a set of isolated point-to-point MIMO channels but should rather be viewed as a whole system.

To understand the systems implications of the capacity region, consider a two-user MIMO-MAC. The rates R_1, R_2 achieved by the two users satisfy the following constraints

$$R_k \leq \log\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\boldsymbol{H}_k\boldsymbol{H}_k^\dagger\right), \quad k = 1, 2 \tag{1.12}$$

$$R_1 + R_2 \leq \log\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\boldsymbol{H}_1\boldsymbol{H}_1^\dagger + \frac{\mathsf{SNR}}{n_t}\boldsymbol{H}_2\boldsymbol{H}_2^\dagger\right) \tag{1.13}$$

The rate region defined by these constraints is the pentagon illustrated in Fig. 1.2. The individual rate constraints (1.12) correspond to the rate at which each user can transmit at, as if it has the entire channel for itself, *i.e.*, the rate of the $n_t \times n_r$ MIMO channel. The sum rate constraint (1.13) corresponds to the rate of a point-to-point channel with the two users acting as a single-user with Kn_t antennas, but sending independent signals at the antennas.

The same interpretation can be extended to a K-user MAC where $K!$ corner points limit the boundary region of the achievable rate region.

Ergodic capacity region

When over the transmission duration of a codeword, many channel realizations occur, the set of achievable data rates is bounded by the following ergodic capacity region:

$$\mathbb{E}_{\boldsymbol{H}_k}\left\{\log\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\sum_{k\in\mathcal{S}}\boldsymbol{H}_k\boldsymbol{H}_k^\dagger\right)\right\} \tag{1.14}$$

As in the point-to-point case, the ergodic capacity region in the high SNR regime is characterized by:

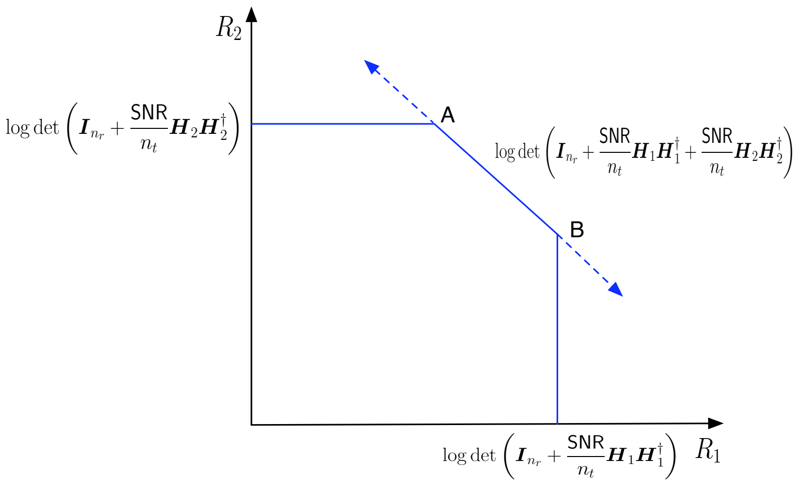


Figure 1.2: Capacity region of the two-user MIMO-MAC.

$$\sum_{k\in\mathcal{S}} R_k \leq \min(n_t, \frac{n_r}{K}) \log \mathsf{SNR}, \quad \forall \mathcal{S} \subseteq \{1,\ldots,K\}. \tag{1.15}$$

where $\min(n_t, \frac{n_r}{K})$ denotes the (per user) multiplexing gain representing the maximum number of degrees of freedom available for each user's data transmission.

1.3 Outage analysis

For a block fading channel, *i.e.,* when coding is performed over one channel realization, capacity is not Representative of the maximum rate that can be reliably transmitted. Indeed, the channel in this case is a random matrix and its capacity is a random variable $C(\boldsymbol{H})$ that can be arbitrarily small. Therefore, there is a non-zero probability that the channel cannot guarantee a reliable transmission at a given data rate. This event is known as the *outage event* and the corresponding probability, the *outage probability*, is the fundamental parameter that evaluates the channel's limit rather than its capacity.

1.3.1 Point-to-Point Outage Probability

For a point-to-point channel, an outage event occurs when the channel capacity falls below the data rate R, *i.e.,*

$$P_{\text{out}}(R) = \mathbb{P}\left\{C(\boldsymbol{H}) < R\right\}$$

Definition 1.3.1 *(Single-User MIMO Channel) For a single-user MIMO channel, whose capacity is given by (1.8), the outage probability is*

$$P_{out}^{MIMO}(R) \;=\; \mathbb{P}\left\{\log\left(\det\left(\boldsymbol{I}_{n_r}+\frac{\mathsf{SNR}}{n_t}\boldsymbol{H}_k\boldsymbol{H}_k^\dagger\right)\right) < R\right\} \tag{1.16}$$

Definition 1.3.2 *The diversity order of a channel is defined as the* SNR *exponent of the asymptotic expression of the outage probability. It corresponds to the negative slope of the outage probability plotted in a* $\log - \log$ *scale in the high* SNR *regime.*

Theorem 1.3.1 *The outage probability of a single-user MIMO channel decays, asymptotically in* SNR, *as* $1/\mathsf{SNR}^{n_t n_r}$ *where* $n_t n_r$ *is the maximum diversity order of the MIMO channel.*

Proof: The proof is given in Appendix 1.A. ■

In this context, multiple-antennas provide additional reliability compared to single-antenna systems. Indeed, the diversity technique consists in transmitting different independently faded replicas of the information symbols over independent channels leading to a higher protection against fading.

1.3.2 Multiple-Access Outage Probability

The notion of outage used to evaluate the ultimate performance limit over a slow fading channel in the high SNR regime, can be extended from point-to-point to multiple-access scenario.

Recall that for a given channel realization, the set of achievable rate tuples satisfies the following boundary constraints

$$\sum_{k \in \mathcal{S}} R_k \leq \log\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\sum_{k\in\mathcal{S}} \boldsymbol{H}_k \boldsymbol{H}_k^\dagger\right), \quad \forall \mathcal{S} \subseteq \{1, \dots, K\}.$$

Assume that all the users have a common data rate R, *i.e.,* $R_k = R, \forall k$. An outage event $\mathcal{O}$ occurs whenever (at least) one of these constraints is not satisfied. Denote $s = |\mathcal{S}|$ and $\mathcal{O}_s$ the corresponding outage event, also referred to as outage event $\boxed{s}$. The overall outage probability is then given by

$$P_{\text{out}}(R) = \mathbb{P}(\mathcal{O}) = \mathbb{P}\left(\bigcup_{\mathcal{S}\subseteq\{1,\dots,K\}} \mathcal{O}_s\right) \tag{1.17}$$

where the outage probability of $\mathcal{O}_s$ is given by

$$\mathbb{P}(\mathcal{O}_s) = \mathbb{P}\left\{\log\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\sum_{k\in\mathcal{S}} \boldsymbol{H}_k \boldsymbol{H}_k^\dagger\right) < sR\right\} \tag{1.18}$$

that corresponds to the outage probability of a point-to-point MIMO channel with sn_t transmit antennas and n_r receive antennas.

Asymptotically in SNR, $P_{\text{out}}(R)$ decays as $1/\mathsf{SNR}^{n_t n_r}$ and, thus, the maximum diversity order of the MIMO-MAC is $n_t n_r$. The proof simply follows from the single-user case by noting from (1.17) that

$$\mathbb{P}(\mathcal{O}) \leq \sum_s \mathbb{P}(\mathcal{O}_s) = \sum_s 1/\mathsf{SNR}^{s n_t n_r} \doteq 1/\mathsf{SNR}^{n_t n_r} \tag{1.19}$$

and

$$\mathbb{P}(\mathcal{O}) \geq \mathbb{P}(\mathcal{O}_s) \doteq 1/\mathsf{SNR}^{n_t n_r} \tag{1.20}$$

1.4 The Diversity-Multiplexing Tradeoff

1.4.1 DMT of a Coding Scheme

Consider a code $\mathcal{X}$ of data rate R bits/pcu. The family of codes $\mathcal{X}(\mathsf{SNR})$ at data rate $R(\mathsf{SNR})$ bits/pcu each, is called a coding scheme. In this context, the data rate scales with SNR as $R(\mathsf{SNR}) = r \log \mathsf{SNR}$ such that, at a given SNR, the corresponding codebook $\mathcal{X}(\mathsf{SNR})$ contains SNR^{Tr} codewords.

Definition 1.4.1 *([1]): A coding scheme $\mathcal{X}(\mathsf{SNR})$ is said to achieve multiplexing gain r and diversity gain $d_{\mathcal{X}}$ if*

$$\lim_{\mathsf{SNR}\to\infty} \frac{R(\mathsf{SNR})}{\log \mathsf{SNR}} = r$$

and

$$\lim_{\mathsf{SNR}\to\infty} \frac{P_{e,\mathcal{X}}(\mathsf{SNR})}{\log \mathsf{SNR}} = -d_{\mathcal{X}}(r)$$

$P_{e,\mathcal{X}}(\mathsf{SNR})$ is the average error probability of $\mathcal{X}(\mathsf{SNR})$ with a Maximum Likelihood (ML) decoder. We have

$$P_{e,\mathcal{X}}(\mathsf{SNR}) \doteq \mathsf{SNR}^{-d_{\mathcal{X}}(r)}$$

where $d_{\mathcal{X}}(r)$ is called the Diversity-Multiplexing Tradeoff (DMT) of $\mathcal{X}(\mathsf{SNR})$.

The multiplexing gain, r, is a fraction of the maximum multiplexing gain $\min(n_t, n_r)$. It determines the fraction of the ergodic capacity (1.10) that the family of code $\mathcal{X}(\mathsf{SNR})$ achieves when SNR increases. Note that, for any fixed-rate code, the multiplexing gain is zero.

Remark 1.4.1 *The DMT $d_{\mathcal{X}}(r)$ evaluating the performance of the entire family of codes $\mathcal{X}(\mathsf{SNR})$ should not be confused with the traditional concept of diversity gain that characterizes the performance of a single codebook for high SNR.*

Remark 1.4.2 *In a multiple-access scenario, $P_{e,\mathcal{X}}(\mathsf{SNR})$ is the total error probability obtained through joint ML detection, that is, the probability for the receiver to make a detection error for at least one user.*

As shall be seen next, there is a fundamental limit to the optimal $d_{\mathcal{X}}(r)$ dictated by the fading channel.

1.4.2 DMT of a fading Channel

The characterization of the DMT of a channel results from an approximation of the outage probability in the high SNR regime and is referred to as outage-DMT.

Definition 1.4.2 *The multiplexing gain r and diversity gain d of a fading channel are defined as follows*

$$\lim_{\mathsf{SNR}\to\infty} \frac{R(\mathsf{SNR})}{\log \mathsf{SNR}} = r$$

and

$$\lim_{\mathsf{SNR}\to\infty} \frac{P_{out}(r \log \mathsf{SNR})}{\log \mathsf{SNR}} = -d(r)$$

$d(r)$ is the outage-DMT of the channel that can be found as the exponent of the outage probability in the high SNR regime, i.e.,

$$P_{out}(r \log \mathsf{SNR}) \doteq \mathsf{SNR}^{-d(r)} \tag{1.21}$$

The importance of the outage-DMT of the channel is that it provides, asymptotically in SNR, an upper-bound on the DMT that can be achieved by any family of code $\mathcal{X}$.

Theorem 1.4.1 *(Converse of the DMT [1]) Assuming a ML receiver, the error probability of any coding scheme satisfies*

$$P_{e,\mathcal{X}}(\mathsf{SNR}) \geq P_{out}(r \log \mathsf{SNR}), \tag{1.22}$$

equivalently, the DMT of any coding scheme is dominated by the outage-DMT, i.e.,

$$d_{\mathcal{X}}(r) \leq d(r), \quad \forall r \tag{1.23}$$

1.4.3 Outage-DMT of SIMO and MISO channels

In a Single-Input Multiple-Output (SIMO) channel, the transmitter is equipped with one antenna while the receiver is equipped with multiple antennas, n_r, so that the spatial diversity order is increased. The outage probability is given by:

$$\begin{aligned} P_{\text{out}}^{\text{SIMO}}(R) &= \mathbb{P}\left\{\log(1 + \mathsf{SNR}\|\boldsymbol{h}\|^2) < R\right\} \\ &= \mathbb{P}\left\{\|\boldsymbol{h}\|^2 < \frac{2^R - 1}{\mathsf{SNR}}\right\} \end{aligned} \tag{1.24}$$

The entries of $\boldsymbol{h}$ being Gaussian zero-mean and spatially uncorrelated, the outage probability (1.24) can be approximated, for high SNR, as follows

$$P_{\text{out}}^{\text{SIMO}}(R) \doteq \frac{\left(2^R - 1\right)^{n_r}}{n_r!\mathsf{SNR}^{n_r}} \tag{1.25}$$

The outage-DMT of the SIMO channel can be derived by replacing R with $r \log \mathsf{SNR}$ in Eq. 1.25,

$$P_{\text{out}}^{\text{SIMO}}(r \log \mathsf{SNR}) \doteq \frac{\mathsf{SNR}^{r n_r}}{n_r!\mathsf{SNR}^{n_r}}$$

Thus,

$$d_{1\times n_r}(r) = n_r(1-r), \quad 0 \leq r \leq 1 \tag{1.26}$$

Similarly, for a Multiple-Input Single-Output (MISO) channel, where multiple antennas are available at the transmitter and a single antenna is available at the receiver, the outage probability can be approximated as

$$P_{\text{out}}^{\text{MISO}}(R) \doteq \frac{n_t^{n_t}\left(2^R - 1\right)^{n_t}}{n_t!\mathsf{SNR}^{n_t}} \tag{1.27}$$

leading to the corresponding outage-DMT

$$d_{n_t\times 1}(r) = n_t(1-r) \tag{1.28}$$

1.4.4 Outage-DMT of a Single-User MIMO Channel

Theorem 1.4.2 *The outage-DMT of a $n_t \times n_r$ Rayleigh point-to-point channel is a piecewise-linear function connecting the points $(r, d_{n_t,n_r}(r))$ for*

$$r = 0, 1, \ldots, \min(n_r, n_t) \tag{1.29}$$

and

$$d_{n_t,n_r}(r) = (n_r - r)(n_t - r) \tag{1.30}$$

Proof: An intuitive sketch of the derivation of the outage-DMT in the MIMO case is given in Appendix 1.A ■

$d_{n_t,n_r}(r)$ is plotted in Fig. 1.3. The maximum diversity gain $d_{\max}(0) = n_t n_r$ is achieved for $r = 0$. Thus, the optimal error performance can be only obtained at fixed rates. At maximum spatial multiplexing gain $r_{\max} = \min(n_t, n_r)$, the diversity gain is $d(r_{\max}) = 0$. Between these extreme points, the outage-DMT curve $d(r)$ is a decreasing function of the multiplexing gain r, *i.e.,* increasing the data rate comes at the expense of diversity.

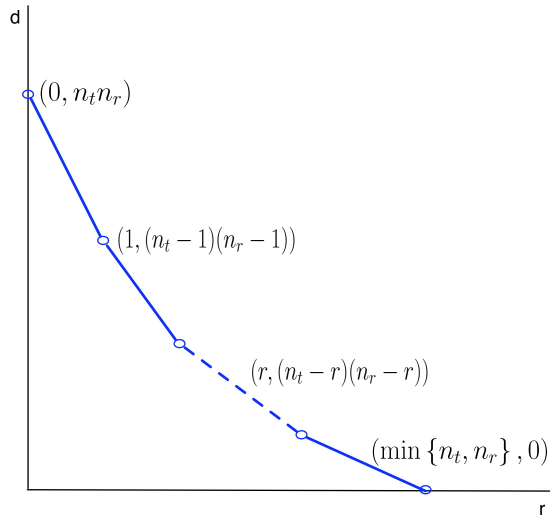


Figure 1.3: DMT of a point-to-point MIMO channel.

1.4.5 Outage-DMT of a Multiple-Access Channel

The fundamental tradeoff between diversity and multiplexing gain was extended by Tse et *al.* [3] to *symmetric* multiple-access channels where the diversity orders and the multiplexing gains of all the users are equal (to say r and d).

Theorem 1.4.3 *The largest achievable symmetric diversity gain for fixed symmetric multiplexing gain is given by*

$$d_{MAC}(r) = \min_{s=1,\ldots,K} d_{sn_t,n_r}(sr) \tag{1.31}$$

Another way to describe this outage-DMT is ([3, Theorem 3])

$$d_{MAC}(r) = \begin{cases} d_{n_t,n_r}(r), & r \leq \min(n_t, \frac{n_r}{K+1}) \\ \\ d_{Kn_t,n_r}(Kr), & r \geq \min(n_t, \frac{n_r}{K+1}) \end{cases} \tag{1.32}$$

Proof: See Appendix 1.A. ■

Interpretation of the Outage-DMT of the MAC

$d_{\text{MAC}}(r)$ is illustrated in Fig.1.4. Two fundamental parameters characterize the performance of the MAC:

1. $\min(n_t, \frac{n_r}{K})$ that represents the maximum multiplexing gain achievable by each user.
2. $\min(n_t, \frac{n_r}{K+1})$ the threshold on the multiplexing gain delimiting two regimes: the lightly loaded regime, *i.e.*, $r \leq \min(n_t, \frac{n_r}{K+1})$, and the heavily loaded regime, *i.e.*, $r \geq \min(n_t, \frac{n_r}{K+1})$. In the first regime, the single-user regime, the single-user performance

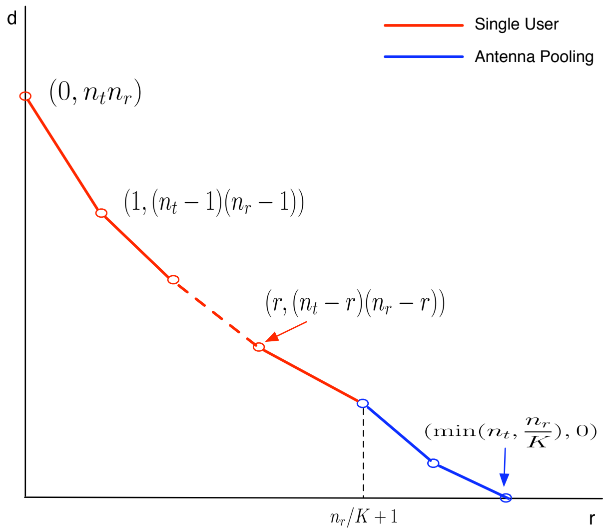


Figure 1.4: Outage-DMT of a multiple-access channel with K users with n_t transmit antennas each and a single receiver with n_r antennas.

is achieved and the highest diversity gain is that of a $n_t \times n_r$ MIMO channel, $n_t n_r$. In this regime, the presence of multiple users does not compromise the performance of each user individually. In the second regime, the antenna-pooling regime, the system is equivalent to a MIMO system with Kn_t transmit and n_r receive antennas as if the users pool up their antennas together. The highest diversity gain in this regime is $Kn_t n_r$.

Note that, for $n_r \geq (K+1)n_t$, the two parameters described above coincide, and the single-user performance extends over the entire range of multiplexing gains. However, if $n_r < (K+1)n_t$, both the single user regime and the antenna pooling regime occur.

From a coding point of view, in the first case, using optimal space-time codes designed for single-user MIMO channels is optimal in terms of the outage-DMT. While in the second case, a *joint code design* is required to guarantee optimality.

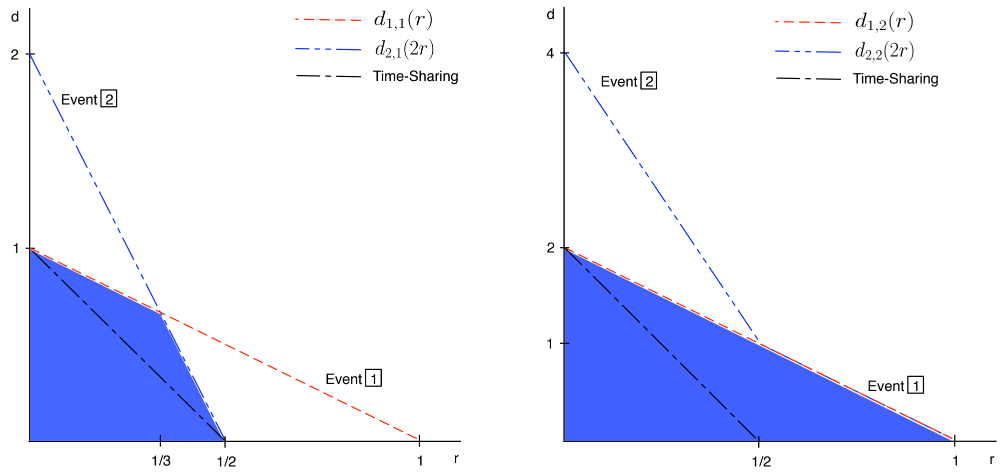
Remark 1.4.3 *Increasing the number of antennas at the destination results in a reduction of the influence of the antenna pooling regime on the overall outage-DMT. In other words, increasing the number of receive antennas decreases the importance of the joint code design as compared to single-user code design.*

1.4.6 Example: A Two-User MAC

Single transmit antenna

Consider a two-user MAC with single-transmit antenna per user. Using (1.31), the outage-DMT of this channel is given by

$$d_{\text{MAC}}(r) = \min\{d_{1,n_r}(r), d_{2,n_r}(2r)\} \tag{1.33}$$



(a) $n_r = 1$ (b) $n_r = 2$

Figure 1.5: Outage-DMT of a two-user MAC: single transmit antenna case.

and is plotted in Fig. 1.5, for $n_r = 1$ and 2.

For $n_r = 2$ (Fig. 1.5-b), the outage-DMT of the channel is given by:

$$d_{\text{MAC}}(r) = \begin{cases} d_{1,2}(r) = 2(1-r) & r \leq \frac{2}{3} \\ \\ d_{2,2}(2r) = (2-2r)(2-2r) & r \geq \frac{2}{3} \end{cases} \tag{1.34}$$

and is equivalent to the outage-DMT of a 2×1 SIMO channel. Thus, the use of a coding scheme that is outage-DMT optimal for SIMO channels, *i.e.*, QAM, is outage-DMT optimal in this case.

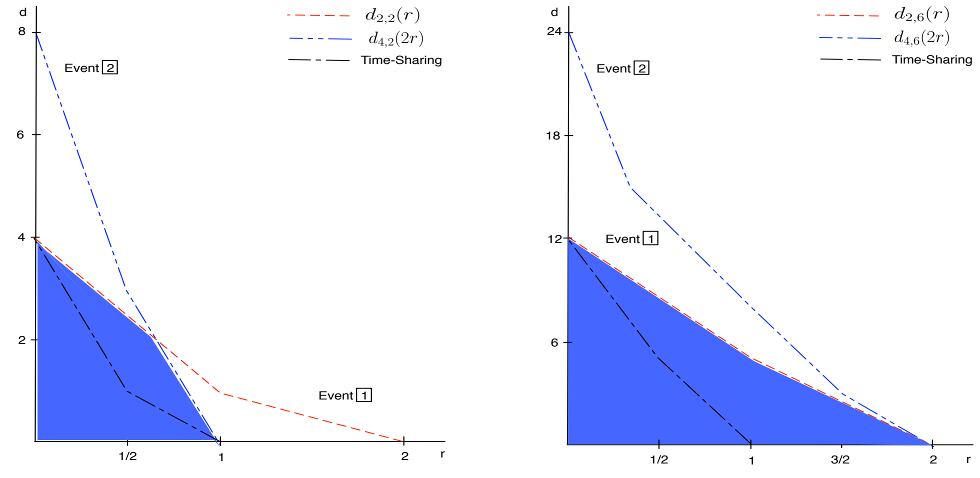
For $n_r = 1$ (Fig. 1.5-a), a joint code design is imperative since both the single-user and the antenna pooling regimes should be taken into account, *i.e.*,

$$d_{\text{MAC}}(r) = \begin{cases} d_{1,1}(r) = (1-r) & r \leq \frac{1}{3} \\ \\ d_{2,1}(2r) = (2-2r)(1-2r) & r \geq \frac{1}{3} \end{cases} \tag{1.35}$$

Note that for $r \geq \frac{1}{3}$ (antenna-pooling regime), when r increases $d_{\text{MAC}}(r)$ decays much faster than it does in the single-user regime. This is a direct effect of the truncated DMT.

Multiple transmit antennas

Consider the same channel but with two transmit antennas per user. Using (1.31), the outage MAC-DMT in this scenario is given by



(a) $n_r = 2$ (b) $n_r = 6$

Figure 1.6: Outage-DMT of a two-user MAC: multiple transmit antennas case.

$$\begin{aligned} d_{\text{MAC}}(r) &= \min\{d_{2,n_r}(r), d_{4,n_r}(2r)\} \\ &= \begin{cases} d_{2,n_r}(r) = (2-r)(n_r - r) & r \leq \frac{n_r}{3} \\ \\ d_{4,n_r}(2r) = (4-2r)(n_r - 2r) & r \geq \frac{n_r}{3} \end{cases} \end{aligned} \tag{1.36}$$

Plotting this DMT in Fig. 1.6 for $n_r = 3$ and $n_r = 6$ leads to similar conclusions. For $n_r = 3$, a jointly designed code is required, while for $n_r = 6$ using codes that are outage-DMT optimal for a 2×6 MIMO channel, is optimal.

For comparison purpose, the DMT of the time-sharing strategy is plotted in Figures 1.5 and 1.6,

$$d_{\text{TS}}(r') = n_r(1 - r') \quad 0 \leq r' = 2r \leq 1 \tag{1.37}$$

This DMT analysis highlights the sub-optimality of the time-sharing technique, that increases with the number of receive antennas. Indeed, the orthogonal multiple-access scheme makes very poor use of the increasing number of degrees of freedom. It is once again clear that, the optimal multiple-access strategy is for the users to transmit simultaneously so that all the degrees of freedom of the channel can be exploited.

1.4.7 Visualizing the MAC-DMT

Fig. 1.7 shows the relationship between the SNR, the data rate and the outage probability. Each curve shows the way the outage probability decays with SNR for a given data rate R. When the data rate R increases, the probability of the channel being in outage increases.

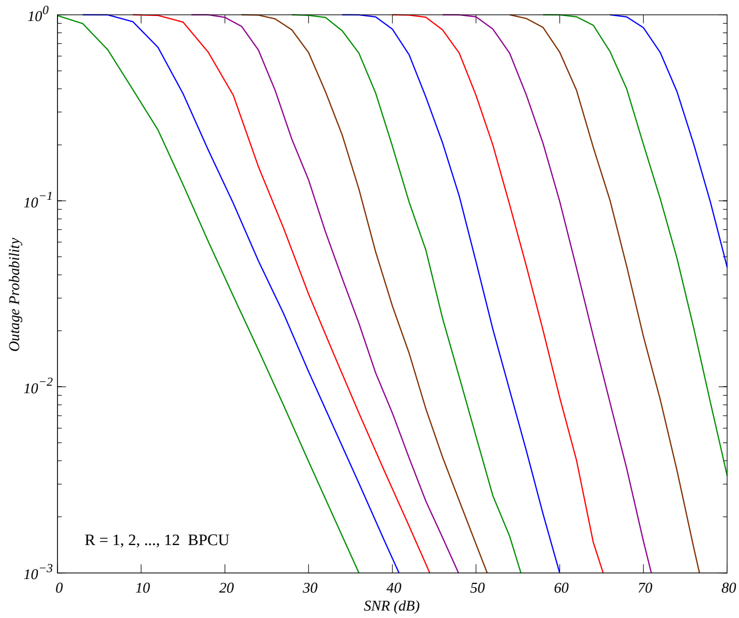


Figure 1.7: Family of outage probability curves as a function of SNR for various target rates R for a two-user MAC with $n_t = n_r = 1$.

Next, the outage probability is plotted as a function of SNR and $R = r \log \mathsf{SNR}$ for different values of r. In Fig. 1.8, $P_{\text{out}}(\mathsf{SNR}, R)$ is evaluated and each curve is labeled with the corresponding r. Fig. 1.7 is overlaid as gray lines for comparison purpose. According to (1.32), the slope of $P_{\text{out}}(r \log \mathsf{SNR})$ should asymptotically be equal to $d(r)$ for a fixed r. Indeed if lines of slopes $d(r)$ are plotted, one can see that the slopes of the outage probability curves and those of these lines match quite well at high SNR.

For a multiplexing rate that equals the maximum multiplexing gain $r = 0.5$, the diversity order is $d = 0$ which means that an increase in SNR does not improve the outage probability but just yields an increase in data rate. In contrast, for $r = 0$, increasing the SNR, increases the reliability but not the data rate.

More generally, it can be noted that, when R increases faster with SNR (r increases), the corresponding outage probability decays slower ($d(r)$ decreases). Moreover, in the lightly loaded regime, the diversity gain decreases much slower than it does in the heavily loaded regime, when the multiplexing gain increases. This is the fundamental diversity-multiplexing tradeoff truncated at $r \geq \min(n_t, \frac{n_r}{K+1})$.

1.5 Achievability of the DMT

The lower-bound of the optimal error probability given in Theorem 1.4.1 is shown to be tight in both the point-to-point and the multiple-access scenario, for a sufficiently long coding length $T \geq sn_t + n_r - 1$ ($s = 1$ for the point-to-point case). The DMT achieved by a Gaussian random coding scheme was derived in [1] for point-to-point Rayleigh channels and in [3] for multiple-access Rayleigh channels and was shown to coincide with the outage-

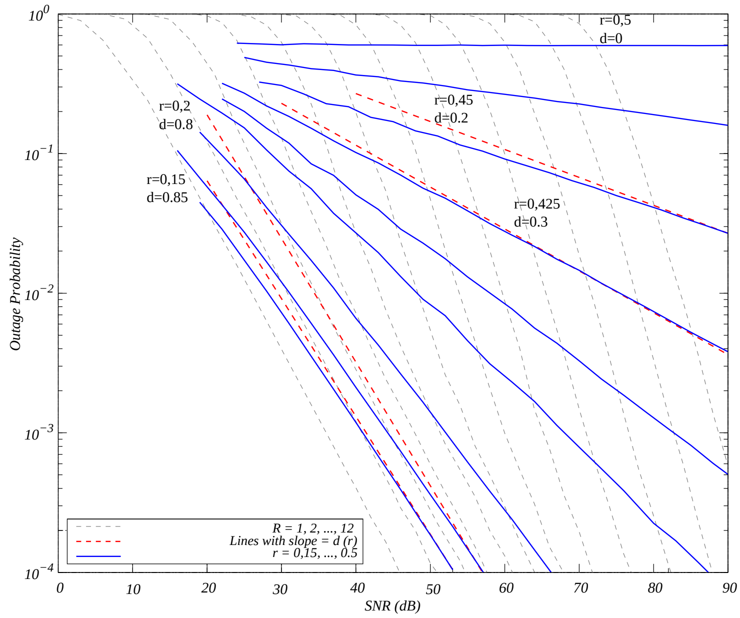


Figure 1.8: Outage probability $P_{\text{out}}(\mathsf{SNR}, R)$ of a two-user MAC for $R = r \log \mathsf{SNR}$, $n_t = n_r = 1$.

DMT $d(r)$ of the channel. Codebooks of data rate R scaling as $r \log \mathsf{SNR}$ and containing $2^{TR} = \mathsf{SNR}^{Tr}$ codewords are generated. Each codeword is a $n_t \times T$ Gaussian matrix with i.i.d. $\mathcal{CN}(0, 1)$ entries.

The achievability of the outage-DMT is proved by upper-bounding the average error probability over the ensemble of Gaussian codes. These codes don't have any structure which makes their efficient encoding or decoding impractical.

A step towards structured coding was presented by El Gamal et *al.* in [24] for single user channels. Authors showed the existence of a structured coding scheme, called the *lattice space-time (LAST)* code, that achieves the outage-DMT. This scheme was later generalized to the multiple-access case by Nam and El Gamal in [25]. They proved that the multiple-access LAST scheme, based on lattice decoding, achieves the outage-DMT of the MAC but they did not give any constructive example. These results were based on random coding arguments and an explicit construction of a single-user or a multiple-access LAST codes remains unknown. In the rest of this thesis, more practical coding schemes are used: the space-time block codes (STBCs). These schemes have a structure and can consequently be explicitly constructed.

1.A Proofs

Proof of Theorem 1.3.1. In the single-user MIMO case, the channel matrix $\boldsymbol{H}_k$ is a $n_r \times n_t$ matrix with zero-mean Gaussian *i.i.d* entries. Denote $q = \min(n_t, n_r)$. The outage probability can be written as

$$\begin{aligned} P_{\text{out}}^{\text{MIMO}}(R) &= \mathbb{P}\left\{\log\left(\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\boldsymbol{H}_k\boldsymbol{H}_k^{\dagger}\right)\right) < R\right\} \\ &= \mathbb{P}\left\{\sum_{i=1}^{q}\log\left(1 + \frac{\mathsf{SNR}}{n_t}\mu_i^2\right) < R\right\} \end{aligned} \tag{1.38}$$

where μ_i's are the singular values of $\boldsymbol{H}_k$. Depending on the instantaneous SNR, a mode of transmission can be effective, *i.e.,* $(\mathsf{SNR}\mu_i^2)/n_t$ of order SNR or not effective, *i.e.,* $(\mathsf{SNR}\mu_i^2)/n_t$ of order 1.

An outage event occurs when none of the q channel modes is effective which means that all μ_i^2 are of order $1/\mathsf{SNR}$ or less. Since

$$\sum_{i=1}^{q}\mu_i^2 = \text{Tr}(\boldsymbol{H}_k\boldsymbol{H}_k^{\dagger}) = \sum_{i,j}|h_{i,j}|^2$$

an outage event can be equivalently defined as the event where each $|h_{i,j}|^2$ is of order $1/\mathsf{SNR}$ or less. The corresponding outage probability is

$$\begin{aligned} P_{\text{out}}^{\text{MIMO}}(R) &= \mathbb{P}\left\{\bigcap_{i,j}\left(|h_{i,j}|^2 < 1/\mathsf{SNR}\right)\right\} \\ &\doteq \frac{1}{\mathsf{SNR}^{n_t n_r}} \end{aligned} \tag{1.39}$$

resulting from the fact that all $|h_{i,j}|^2$ are independent and that

$$\mathbb{P}\left\{|h_{i,j}|^2 < 1/\mathsf{SNR}\right\} \approx 1/\mathsf{SNR}.$$

□

Proof of Theorem 1.4.2. A similar procedure to that in [26] is followed. For high SNR and with $R = r\log\mathsf{SNR}$, (1.38) implies:

$$P_{\text{out}}^{\text{MIMO}}(r\log\mathsf{SNR}) = \mathbb{P}\underbrace{\left\{\log\left(\det\left(\boldsymbol{I}_{n_r} + \mathsf{SNR}\boldsymbol{H}_k\boldsymbol{H}_k^{\dagger}\right)\right) < r\log\mathsf{SNR}\right\}}_{\mathcal{O}_{\boldsymbol{H}_k}(r,\mathsf{SNR})} \tag{1.40}$$

$$\doteq \quad \underbrace{\mathbb{P}\left\{\sum_{i=1}^{q} \log\left(1+\mathsf{SNR}\lambda_i\right) < r\log\mathsf{SNR}\right\}}_{\mathcal{O}_{\boldsymbol{\lambda}(r,\mathsf{SNR})}} \tag{1.41}$$

$$\doteq \quad \mathbb{P}\left\{\sum_{i=1}^{q}(1-\alpha_i)^+ < r\right\} \tag{1.42}$$

$$\doteq \quad \underbrace{\mathbb{P}\left\{\sum_{i=1}^{k}\alpha_i > k-r,\ \forall k=1,\ldots,q\right\}}_{\mathcal{O}_{\boldsymbol{\alpha}}(r,\mathsf{SNR})} \tag{1.43}$$

where $\lambda_i = \mathsf{SNR}^{-\alpha_i}$ are the eigenvalues of the matrix $\boldsymbol{H}_k\boldsymbol{H}_k^\dagger$ in a decreasing order; $\boldsymbol{\alpha}$ the vector of the eigen-exponents corresponding to the vector of eigenvalues $\boldsymbol{\lambda}$, *i.e.,* $\alpha_i \triangleq \frac{\log\lambda_i}{\log\mathsf{SNR}}$; $\mathcal{O}_{\boldsymbol{H}_k}$, $\mathcal{O}_{\boldsymbol{\lambda}}$ and $\mathcal{O}_{\boldsymbol{\alpha}}$ are three representations of the outage region of the channel. It can be shown that

$$p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) \doteq \mathsf{SNR}^{-E(\boldsymbol{\alpha})} \tag{1.44}$$

where, for a $n_t \times n_r$ Rayleigh channel, $E(\boldsymbol{\alpha})$ is given by

$$E(\boldsymbol{\alpha}) = \sum_{i=1}^{q}(2k-1+|n_t-n_r|)\alpha_i \tag{1.45}$$

Thus,

$$\begin{aligned} P_{\text{out}}^{\text{MIMO}}(r\log\mathsf{SNR}) &= \mathbb{P}\left\{\mathcal{O}_{\alpha}(r,\mathsf{SNR})\right\} \\ &\doteq \mathsf{SNR}^{-\inf_{\mathcal{O}} E(\boldsymbol{\alpha})} \end{aligned} \tag{1.46}$$

which gives

$$d(r) = \inf_{\mathcal{O}_{\boldsymbol{\alpha}(r)}} E(\boldsymbol{\alpha})$$

whose solution (via linear programming approach) gives rise to Eq. 1.30 and concludes the proof.

□

Proof of Theorem 1.4.3. Recall the outage event definition in a multiple-access scenario given in Subsection 1.3.2:

$$\mathcal{O} \triangleq \bigcup_{s}\mathcal{O}_s, \quad \forall s = \{1,\ldots,K\} \tag{1.47}$$

with :

$$\mathbb{P}(\mathcal{O}_s) = \mathbb{P}\left\{\log\det\left(\boldsymbol{I}_{n_r} + \frac{\mathsf{SNR}}{n_t}\sum_{k\in\mathcal{S}}\boldsymbol{H}_k\boldsymbol{H}_k^{\dagger}\right) < sR\right\}. \tag{1.48}$$

$\mathbb{P}(\mathcal{O}_s)$ is equivalent to the outage probability of a point-to-point MIMO channel with sn_t transmit antennas and n_r receive antennas. Let the target data rate be $R = r\log\mathsf{SNR}$. Therefore:

$$\begin{aligned}\mathbb{P}(\mathcal{O}_s) &\doteq \mathsf{SNR}^{-d_{sn_t,n_r}(sr)} && (1.49)\\ &\doteq \mathsf{SNR}^{-(sn_t-sr)(n_r-sr)} && (1.50)\end{aligned}$$

where (1.50) results from (1.49) using (1.30). Moreover, noting that (1.17) implies that

$$\mathbb{P}(\mathcal{O}) \geq \mathbb{P}(\mathcal{O}_s) \tag{1.51}$$

for any $1 \leq s \leq K$, leads to $2^K - 1$ lower bounds on $\mathbb{P}(\mathcal{O})$. Exponentially in SNR, the tightest lower bound corresponds to the subset $\mathcal{S}$ that yields the largest outage probability, or equivalently, the smallest SNR exponent $d_{sn_t,n_r}(sr)$ is:

$$\mathbb{P}(\mathcal{O}) \dot{\geq} \mathsf{SNR}^{-\min_s d_{sn_t,n_r}(sr)} = \mathsf{SNR}^{-d_{s^*n_t,n_r}(s^*r)} \tag{1.52}$$

$s^* = \arg\min_s d_{sn_t,n_r}(sr)$ is the cardinality of dominant outage set $\mathcal{S}^*$. On the other hand:

$$\begin{aligned}\mathbb{P}(\mathcal{O}) &= \mathbb{P}\left(\bigcup_{\mathcal{S}}\mathcal{O}_s\right) \leq \sum_{\mathcal{S}}\mathbb{P}(\mathcal{O}_s)\\ &\doteq \mathsf{SNR}^{-\min_s d_{sn_t,n_r}(sr)} && (1.53)\end{aligned}$$

By combining (1.52) and (1.53), one gets

$$\mathbb{P}(\mathcal{O}) \doteq \mathsf{SNR}^{-\min_s d_{sn_t,n_r}(sr)} \tag{1.54}$$

which proves Eq. 1.31.

To complete the proof of Theorem 1.4.3, note that, as shown in [3, Section VIII], $d_{n_t,n_r}(r)$ is the smallest DMT among the K different tradeoffs for $r \leq \min(n_t, \frac{n_r}{K+1})$ and $d_{Kn_t,n_r}(Kr)$ is the smallest, otherwise.

□

Chapter 2

From Point-to-Point to Multiple-Access Channel: A Coding Perspective

Information theory treats the question of how much data can be reliably transmitted over a given channel and gives an accurate understanding of the fundamental limits of the channel. Using insights gained by the theoretical analysis presented in Chapter 1, suitable codes can be designed such that, in practice, the capabilities of the channel predicted by theory can be optimally exploited.

This chapter serves a double purpose. First, it provides a state of the art study of the Space-Time block code (STBC) design for point-to-point channels that has been studied in great details in the literature. Single-user code design criteria minimizing the pairwise error probability are presented and the DMT-optimal design criterion: the *non-vanishing* determinant criterion is discussed. Some single-user codes constructions based on cyclic division algebras are then presented. These codes are optimal in the sense that, they have a performance that is very close to theoretical limits.

Next, the problem of Space-Time code design for multiple-access channels is introduced. This is the main goal of this thesis. The typical error events that can be encountered in a MAC are first analyzed. Understanding the way an error occurs in the system helps deriving the multiple-access code design criteria as in [2]. Based on the Alamouti code, authors in [2] constructed the first multiple-access code taking into account the multiple-access nature of the channel. This construction is presented and its performance studied, showing the gain this multiple-access code offers as compared to the single-user code. Finally, the DMT-optimal code design criteria derived in [27] as well as their achievability are studied in details.

2.1 Space-Time Block Coding: The Single-User Case

The construction of STBCs for single-user MIMO channels was largely studied in the literature [15, 28, 29, 30, 31, 13]. Cyclic division algebras have been used as a tool to construct these codes. Their algebraic properties are exploited to design optimal codes satisfying the code design criteria presented in the following.

2.1.1 Design Criteria

Consider a STBC $\mathcal{X}$ with $n_t \times T$ codeword matrices denoted $\boldsymbol{X}$. The focus here is on the error event occurring when, at the receiver side, a codeword $\boldsymbol{X}^{'} \neq \boldsymbol{X}$ is detected when the codeword $\boldsymbol{X} \in \mathcal{X}$ is transmitted. An estimate of the error probability, when maximum likelihood (ML) decoding is applied, can be obtained using the *union bound*

$$P_e \leq \frac{1}{|\mathcal{X}|} \sum_{\boldsymbol{X} \in \mathcal{X}} \sum_{\boldsymbol{X} \neq \boldsymbol{X}^{'}} \mathbb{P}\left\{\boldsymbol{X} \rightarrow \boldsymbol{X}^{'}\right\} \tag{2.1}$$

where $\mathbb{P}\left\{\boldsymbol{X} \rightarrow \boldsymbol{X}^{'}\right\}$ is the pairwise error probability (PEP). For simplicity, it is assumed that the codewords are normalized so that $\mathsf{SNR} = 1/\sigma_{\boldsymbol{Z}}$. The PEP conditioned on a channel realization $\boldsymbol{H}$, is given by

$$\begin{aligned}
\mathbb{P}\left\{\boldsymbol{X} \rightarrow \boldsymbol{X}^{'} | \boldsymbol{H}\right\} &= \mathbb{P}\left\{\|\boldsymbol{Y} - \boldsymbol{H}\boldsymbol{X}^{'}\|^2 \leq \|\boldsymbol{Y} - \boldsymbol{H}\boldsymbol{X}\|^2\right\} \\
&= \mathbb{P}\left\{\|\boldsymbol{H}\left(\boldsymbol{X} - \boldsymbol{X}^{'}\right) + \boldsymbol{Z}\|^2 \leq \|\boldsymbol{Z}\|^2\right\} \\
&= \mathbb{P}\left\{\underbrace{\|\boldsymbol{H}\left(\boldsymbol{X} - \boldsymbol{X}^{'}\right)\|^2 + 2\mathcal{R}\left(\boldsymbol{H}\left(\boldsymbol{X} - \boldsymbol{X}^{'}\right)\boldsymbol{Z}^{\dagger}\right)}_{V} \leq 0\right\}
\end{aligned} \tag{2.2}$$

where V is a Gaussian variable with mean $\|\boldsymbol{H}(\boldsymbol{X} - \boldsymbol{X}^{'})\|^2$ and variance

$$4\|\boldsymbol{H}(\boldsymbol{X} - \boldsymbol{X}^{'}\|^2\sigma_{\boldsymbol{Z}}^2.$$

Therefore,

$$\mathbb{P}\left\{\boldsymbol{X} \rightarrow \boldsymbol{X}^{'} | \boldsymbol{H}\right\} = \mathcal{Q}\left(\frac{\|\boldsymbol{H}\left(\boldsymbol{X} - \boldsymbol{X}^{'}\right)\|}{2\sigma_{\boldsymbol{Z}}}\right) \tag{2.3}$$

The PEP averaged over the channel statistics is given by

$$\mathbb{P}\left\{\boldsymbol{X} \to \boldsymbol{X}'\right\} = \mathbb{E}_{\boldsymbol{H}}\left[\mathcal{Q}\left(\sqrt{\frac{\mathsf{SNR}\|\boldsymbol{H}\left(\boldsymbol{X}-\boldsymbol{X}'\right)\|^2}{2}}\right)\right] \tag{2.4}$$

Let's denote the codeword difference matrix $\boldsymbol{\Delta} = \boldsymbol{X} - \boldsymbol{X}'$. $\boldsymbol{\Delta}\boldsymbol{\Delta}^\dagger$ is diagonalizable by a unitary transformation, so one can write

$$\boldsymbol{\Delta}\boldsymbol{\Delta}^\dagger = \boldsymbol{U}\boldsymbol{D}\boldsymbol{U}^\dagger \tag{2.5}$$

where $\boldsymbol{U}$ is unitary and $\boldsymbol{D}$ is diagonal with elements $\lambda_i, i = 1, \ldots, n_t$ $(T \geq n_t)$ corresponding to the singular values of the codeword difference matrix $\boldsymbol{\Delta}$. Therefore, the PEP in Eq. 2.4 can be written as:

$$\begin{aligned}\mathbb{P}\left\{\boldsymbol{X} \to \boldsymbol{X}'\right\} &= \mathbb{E}_{\boldsymbol{H}}\left[\mathcal{Q}\left(\sqrt{\frac{\mathsf{SNR}\boldsymbol{H}\boldsymbol{U}\boldsymbol{D}\boldsymbol{U}^\dagger\boldsymbol{H}^\dagger}{2}}\right)\right] \\ &\leq \mathbb{E}_{\boldsymbol{H}}\left[\mathcal{Q}\left(\sqrt{\frac{\mathsf{SNR}\sum_{i=1}^{n_t}\sum_{j=1}^{n_r}\lambda_i^2 \mid \tilde{h}_{i,j}\mid^2}{2}}\right)\right],\end{aligned} \tag{2.6}$$

where $\tilde{h}_{i,j}$ are elements of $\tilde{\boldsymbol{H}} = \boldsymbol{H}\boldsymbol{U}$ that has the same distribution as $\boldsymbol{H}$, *i.e.,* $\tilde{h}_{i,j}$ are Gaussian variables with zero mean and unit variance. Thus, the average PEP can be bounded as follows

$$\begin{aligned}\mathbb{P}\left\{\boldsymbol{X} \to \boldsymbol{X}'\right\} &\leq \mathbb{E}_{\boldsymbol{H}}\left[\exp\left(-\frac{\mathsf{SNR}\sum_{i=1}^{n_t}\sum_{j=1}^{n_r}\lambda_i^2 \mid \tilde{h}_{i,j}\mid^2}{4}\right)\right] \\ &\leq \prod_{j=1}^{n_r}\prod_{i=1}^{n_t}\exp\left(-\frac{\mathsf{SNR}\lambda_i}{4}\right) \\ &\leq \left(\prod_{i=1}^{n_t}\frac{1}{1+\mathsf{SNR}\lambda_i^2/4}\right)^{n_r}\end{aligned} \tag{2.7}$$

Let r denote the rank of the codeword difference matrix $\boldsymbol{\Delta}$, *i.e.,* the number of non zero λ_i. Asymptotically in SNR, the PEP can be written as

$$\mathbb{P}\left\{\boldsymbol{X} \to \boldsymbol{X}'\right\} \leq \left[\det\left(\boldsymbol{\Delta}\boldsymbol{\Delta}^\dagger\right)\right]^{-n_r}\left(\frac{\mathsf{SNR}}{4}\right)^{-rn_r} \tag{2.8}$$

The diversity is given by the power of $1/\mathsf{SNR}$, hence, a diversity order of rn_r is achieved. The coding gain is determined by the minimum of the determinant over all codeword pairs, *i.e.,*

$$\delta_{\min} = \min_{\boldsymbol{X} \neq \boldsymbol{X}'} \det\left(\boldsymbol{\Delta}\boldsymbol{\Delta}^{\dagger}\right) \tag{2.9}$$

Minimizing the PEP expression (2.8) leads to the following rank and determinant criteria for the space-time code design:

Rank criterion: In order to exploit the $n_t n_r$ independent channels available in the system and therefore achieve the maximum *diversity order*, the rank of the difference of every pair of distinct codewords $\boldsymbol{X}, \boldsymbol{X}^{'}$ has to be equal to n_t. In other words, the determinant of the difference of any codewords is non-zero

$$\det \boldsymbol{\Delta} \neq 0, \quad \forall \boldsymbol{X} \neq \boldsymbol{X}^{'} \tag{2.10}$$

Determinant criterion: For a fully diverse code, the *coding gain* $\delta_{\min}$ has to be maximized.

Information preserving criterion: A STBC is *information lossless* if the mutual information of the channel does not change due to the coding. If $\boldsymbol{\Phi}$ denotes the matrix used to define the linear dependency between the elements of a codeword and the information symbols, the use of a unitary matrix $\boldsymbol{\Phi}$ preserves the information. In this case, the cubic shaping constraint, required to have a uniform average transmitted energy among coded symbols, is verified.

2.1.2 DMT-optimal Design Criterion

In order to further improve the performance of STBCs, one more property was introduced to guarantee a coding gain that does not vanish when the constellation size grows. This property, first introduced in [32], is called the *non-vanishing determinant* property.

Consider a *linear dispersion* coding scheme $\mathcal{X}(\mathsf{SNR})$ with data rate $R(\mathsf{SNR})$ bits pcu, *i.e.,* the entries of any codeword matrix in $\mathcal{X}(\mathsf{SNR})$ are linear combinations of information symbols. When the transmitting rate scales as $r \log \mathsf{SNR}$, the power constraint that any codeword matrix $\boldsymbol{X} \in \mathcal{X}$ should satisfy is defined by:

$$\|\boldsymbol{X}\|_{\mathrm{F}}^{2} \leq T\mathsf{SNR}. \tag{2.11}$$

Definition 2.1.1 *(Non-Vanishing Determinant) $\mathcal{X}(\mathsf{SNR})$ is a NVD code if the difference codeword matrix $\boldsymbol{\Delta}$ satisfies*

$$\min_{\boldsymbol{\Delta}=\boldsymbol{X}-\boldsymbol{X}'} \det\left(\frac{\boldsymbol{\Delta}\boldsymbol{\Delta}^{\dagger}}{\mathsf{SNR}\, 2^{-R(\mathsf{SNR})/n_t}}\right) \dot{\geq} \mathsf{SNR}^{0} \tag{2.12}$$

for any pair of distinct codewords. $\mathsf{SNR}2^{-R(\mathsf{SNR})/n_t}$ is a normalization factor with respect to the minimum distance in the symbols constellation. The NVD condition can be equivalently written as ([26])

$$\tilde{\upsilon}_1^2\tilde{\upsilon}_2^2\ldots\tilde{\upsilon}_{n_t}^2 \quad \geq \quad \frac{1}{2^{R(\mathsf{SNR})+o(\log \mathsf{SNR})}} \tag{2.13}$$

where $\tilde{\upsilon}_1,\ldots,\tilde{\upsilon}_{n_t}$ are the smallest n_t singular values of the normalized (by $\sqrt{\mathsf{SNR}}$) difference codeword matrix $\boldsymbol{\Delta}$ (in ascending order).

Elia and *al.* showed in [13] that the NVD property is a necessary and sufficient condition for a full-rate STBC to achieve the outage-DMT of the channel.

Theorem 2.1.1 *The achievable DMT of a rate-n_t NVD scheme $\{\mathcal{X}(\mathsf{SNR})\}$ is lower bounded by*

$$d_{\mathcal{X}}(r) \quad \geq \quad d(r), \quad \forall r. \tag{2.14}$$

Proof: See [13]. ■

As a conclusion, NVD codes are a very important class of codes that preserves the coding gain of the entire coding scheme and, from an information theoretic perspective, guarantees the outage-DMT achievability.

2.1.3 Optimal Codes: SIMO/MISO Channels

Recall the outage-DMT of the SIMO channel given in (1.26)

$$d_{1\times n_r}(r) = n_r(1-r)$$

that is achieved by transmitting QAM ([33, Chapter 3]), *i.e.*,

$$P_{e,\mathrm{QAM}}(\mathsf{SNR}) \doteq \mathsf{SNR}^{n_r(1-r)} \tag{2.15}$$

For a MISO channel, the outage-DMT is given in (1.28):

$$d_{n_r\times 1}(r) = n_t(1-r)$$

A code with diversity order n_t is clearly needed in this scenario in order to benefit from the transmit diversity. For $n_t = 2$, the Alamouti coding scheme is known to be outage-DMT achieving.

Alamouti Code

The Alamouti code is an orthogonal code proposed in [15] as one of the first STBCs. Two QAM information symbols, s_1 and s_2, are encoded into a 2×2 codeword matrix as follows:

$$\boldsymbol{X} = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \quad (2.16)$$

This code is fully diverse since $\det \boldsymbol{X}$ is non-zero for any non-zero symbols. At the receiver, one has:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad (2.17)$$

that can be rewritten as

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2^* \end{bmatrix} \quad (2.18)$$

leading to the equivalent received vector

$$\boldsymbol{y} = \boldsymbol{H}^{'} \boldsymbol{s} + \boldsymbol{z} \quad (2.19)$$

It is clear that the received vector can be easily decoded due to the orthogonality of the equivalent channel vectors $[h_1 \ h_2^*]^T$ and $[h_2 \ -h_1^*]^T$. For high SNR, the error probability is approximately given by

$$\mathbb{P}_{e,\text{Alamouti}}(\mathsf{SNR}) \doteq \mathsf{SNR}^{2(1-r)}. \quad (2.20)$$

Low decoding complexity is one of the major advantages of the Alamouti code that is optimal for MISO channel with $n_t = 2$ and $n_r = 1$. This construction has been extended to higher dimensions, $n_t \geq 2$. The full diversity is achieved by the data rate decrease when the number of antennas increases.

2.1.4 MIMO Channel: Perfect Codes

Perfect Space-Time codes [8, 9] were introduced as the class of linear dispersion having full rate, non-vanishing determinant and uniform average transmitted energy per antenna. Perfect codes are constructed using cyclic division algebras as follows.

Consider a cyclic extension $\mathbb{L}$ of degree n_t over $\mathbb{Q}(i)$ and σ the generator of its Galois group. $\mathcal{O}_{\mathbb{L}}$ denotes the ring of integers of $\mathbb{L}$. Let $\mathcal{A} = (\mathbb{L}/\mathbb{Q}(i), \sigma, \gamma)$ be a cyclic division algebra of degree n_t with γ verifying $\gamma \in \mathbb{Z}[i]$ and $\gamma, \gamma^2, \ldots, \gamma^{n_t-1}$ non-norm elements in $\mathbb{L}$. Each element in $\mathcal{A}$ has the following matrix representation (*c.f.,* A.4 in Appendix A)

$$\boldsymbol{X} = \begin{pmatrix} x_0 & x_1 & x_2 & \dots & x_{n_t-1} \\ \gamma\sigma(x_{n_t-1}) & \sigma(x_0) & \sigma(x_1) & \dots & \sigma(x_{n_t-2}) \\ \vdots & \ddots & & & \vdots \\ \gamma\sigma^{n_t-2}(x_2) & \gamma\sigma^{n_t-2}(x_3) & \gamma\sigma(x_4) & \dots & \sigma^{n_t-2}(x_1) \\ \gamma\sigma^{n_t-1}(x_1) & \gamma\sigma^{n_t-1}(x_2) & \gamma\sigma^{n_t-1}(x_3) & \dots & \sigma^{n_t-1}(x_0) \end{pmatrix} \quad (2.21)$$

where $x_i \in \mathbb{O}_{\mathbb{L}}$. $\mathcal{A}$ being a CDA implies that

$$\det \boldsymbol{X} \in \mathbb{Z}[i].$$

This determinant is non-zero for a non-zero matrix $\boldsymbol{X}$, yielding a NVD since the difference matrix of each pair of codewords is an element of the CDA.

Finally, in order to get a STBC with good shaping, elements x_i of the codeword matrix should belong to a properly chosen ideal $\mathbb{I} \subseteq \mathbb{O}_{\mathbb{L}}$ [34] and $\mid \gamma \mid = 1$. Each layer of the code can be written as $\boldsymbol{Us}$, where $\boldsymbol{s}$ is the information symbols vector and $\boldsymbol{U}$ is a *unitary* matrix that encodes the symbols into each layer.

Example 2.1.1 *(Golden Code) The Golden code is a 2×2 perfect code built on the cyclic algebra*

$$\mathcal{A} = (\mathbb{L} = \mathbb{Q}(i, \sqrt{5})/\mathbb{Q}(i), \sigma, i)$$

with $\sigma : \sqrt{5} \mapsto -\sqrt{5}$. Let $\theta = \frac{1+\sqrt{5}}{2}$ be the golden number and $\bar{\theta} = \frac{1-\sqrt{5}}{2}$ be its conjugate. Define $\alpha = 1 + i - i\theta$ and its conjugate $\bar{\alpha} = 1 + i - i\bar{\theta} \in \mathbb{I}$.

The unitary transform, used to code the QAM information symbols $s_{i,j}$, is given by

$$\boldsymbol{U} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha & \alpha\theta \\ \bar{\alpha} & \bar{\alpha}\bar{\theta} \end{bmatrix} \quad (2.22)$$

Thus, a codeword $\boldsymbol{X}$ of the Golden code has the form

$$\boldsymbol{X} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha\,(s_{1,1} + s_{1,2}\theta) & \alpha\,(s_{2,1} + s_{2,2}\theta) \\ i\bar{\alpha}\left(s_{2,1} + s_{2,2}\bar{\theta}\right) & \bar{\alpha}\left(s_{1,1} + s_{1,2}\bar{\theta}\right) \end{bmatrix} \quad (2.23)$$

where $\theta = \frac{1+\sqrt{5}}{2}$.

Since i is not a norm in $\mathbb{L}$, $\mathcal{A}$ is a CDA and the Golden code is a full rate and fully diverse code with a good shaping and a codeword determinant that does not vanish when the spectral efficiency increases.

2.1.5 Parallel MIMO Channel

Consider a set of N parallel $n_t \times n_r$ MIMO channels. The channel matrix denoted $\boldsymbol{H}$ is given by

$$\boldsymbol{H} = \text{diag}\left(\boldsymbol{H}_1, \boldsymbol{H}_2, \ldots, \boldsymbol{H}_N\right)$$

where $\boldsymbol{H}_i$ is a $n_r \times n_t$ matrix corresponding to channel i.

The construction of perfect STBCs achieving the DMT of a parallel MIMO channel was presented in [35]. This construction follows the same footsteps as the construction of codes for MIMO channels. A CDA is first constructed by

1. replacing the base field $\mathbb{Q}(i)$ by a Galois extension of degree N over $\mathbb{Q}(i)$ denoted $\mathbb{F}$ with Galois group:

$$\text{Gal}(\mathbb{F}/\mathbb{Q}(i)) = \{\tau_1, \tau_2, \ldots, \tau_N\}$$

2. replacing the field $\mathbb{L}$ by $\mathbb{K} = \mathbb{F}(\theta)$, a cyclic extension of degree n_t over $\mathbb{F}$ with the same Galois group as $\text{Gal}(\mathbb{L}/\mathbb{Q}(i))$

3. choosing η verifying $\eta, \eta^2, \ldots, \eta^{n_t-1}$ are not norms in $\mathbb{K}$.

The constructed cyclic division algebra is $\mathcal{A} = (\mathbb{K}/\mathbb{F}, \sigma, \eta)$. Let $\boldsymbol{\Xi}$ be the matrix representation of some element of $\mathcal{A}$ which is a $n_t \times n_t$ matrix. Since the channel is block diagonal, the constructed code has a block diagonal structure with $n_t \times n_t$ square blocks on the diagonal

$$\boldsymbol{X} = \begin{bmatrix} \tau_1(\boldsymbol{\Xi}) & & & \\ & \tau_2(\boldsymbol{\Xi}) & & \\ & & \ddots & \\ & & & \tau_N(\boldsymbol{\Xi}) \end{bmatrix} \tag{2.24}$$

This code is full-rate, *i.e.* each codeword corresponds to $N.n_t^2$ information symbols. We have

$$\begin{aligned} \prod_{k=1}^{N} \det\left(\tau_k(\boldsymbol{\Xi})\right) &= \prod_{k=1}^{N} \tau_k(\det \boldsymbol{\Xi}) \\ &= N_{\mathbb{F}/\mathbb{Q}(i)}(\det \boldsymbol{\Xi}) \end{aligned} \tag{2.25}$$

that is in $\mathbb{Z}(i)$ leading to a NVD code. An explicit code construction is provided in the following example.

Example 2.1.2 *($N = 2^m$ sub-channels and $n_t = 2$ transmit antennas ([35])) Define $\zeta_{2^{m+1}} = e^{\frac{i\pi}{2^m}}$. Consider the base field $\mathbb{F} = \mathbb{Q}(\zeta_{2^{m+2}})$ an extension of degree 2^m of $\mathbb{Q}(i)$ and $\mathbb{K} = \mathbb{F}(\sqrt{5}) = \mathbb{Q}(\zeta_{2^{m+2}}, \sqrt{5})$. $\eta = \zeta_{2^{m+2}}$ can be shown to be a non-norm element in $\mathbb{K}$. Let $\theta = \frac{1+\sqrt{5}}{2}$, $\sigma : \theta \mapsto \bar{\theta} = \frac{1-\sqrt{5}}{2}$ and the ring of integers of $\mathbb{K}$, $\mathbb{O}_{\mathbb{K}}$, given by*

$$\mathbb{O}_{\mathbb{K}} = \{a + b\theta \mid a, b \in \mathbb{Z}[\zeta_{2^{m+2}}]\}$$

the chosen ideal is principle, i.e., $\mathbb{I} = (\alpha)\mathbb{O}_{\mathbb{K}}$ with $\alpha = 1 + i - i\theta$ and $\bar{\alpha} = 1 + i - i\bar{\theta}$. The matrix $\boldsymbol{\Xi}$ is given by:

$$\boldsymbol{\Xi} = \begin{bmatrix} \alpha(s_{1,1} + s_{1,2}\theta) & \alpha(s_{2,1} + s_{2,2}\theta) \\ \eta\bar{\alpha}(s_{2,1} + s_{2,2}\bar{\theta}) & \bar{\alpha}(s_{1,1} + s_{1,2}\bar{\theta}) \end{bmatrix} \tag{2.26}$$

where $s_{i,j} \in \mathbb{Z}[\zeta_{2^{m+2}}]$.

2.2 Space-Time Block Coding: The Multiple-Access Case

In this section, the typical error events occurring in a multiple-access scenario are first analyzed. Similar error events analysis was first presented by Gallager in [36] for the AWGN-MAC and revisited in [3] and [2] for the fading MAC.

2.2.1 Error Event Analysis

In a multiple-access scenario, the error can occur in different ways, depending on the number of users that are decoded erroneously. In this context, the overall error event can be decomposed into multiple disjoint error events $\mathcal{E}_s$, $s = 1, \ldots, K$, (also referred to as error event $\boxed{s}$) that occurs when s users are decoded in error and $(K - s)$ users are decoded correctly. Recall that $\mathcal{S} \subseteq \{1, \ldots, K\}$ is a set of all the users that are decoded erroneously, $\overline{\mathcal{S}}$ its complement, $s = |\mathcal{S}|$ and $1 \leq s \leq K$.

For $k = 1, \ldots, K$, let $\boldsymbol{X}_k$ denote the codeword matrix transmitted by user k and $\hat{\boldsymbol{X}}_k$ the joint ML decision at the receiver. We have:

$$\mathcal{E}_s \triangleq \left\{\hat{\boldsymbol{X}}_k \neq \boldsymbol{X}_k,\ \forall k \in \mathcal{S} \text{ and } \hat{\boldsymbol{X}}_k = \boldsymbol{X}_k,\ \forall k \in \overline{\mathcal{S}}\right\} \tag{2.27}$$

Define $\mathbb{P}(\mathcal{E}_s \mid \boldsymbol{H})$ the probability of $\mathcal{E}_s$ for a given channel realization. Therefore, the total error probability is the sum of the K disjoint error events and:

$$P_e = \sum_{s=1}^{K} \mathbb{P}(\mathcal{E}_s) \tag{2.28}$$

where

$$\mathbb{P}(\mathcal{E}_s) \;=\; \mathbb{E}_{\boldsymbol{H}}[\mathbb{P}(\mathcal{E}_s \mid \boldsymbol{H})] \tag{2.29}$$

Example 2.2.1 *Consider a two-user MAC with an arbitrary number of transmit and receive antennas. This channel entails two different types of error events;* $\mathcal{E}_1$ *where one user is in error, i.e.,* $\mathcal{S} = \{1\}$ *or* $\{2\}$ *and* $\mathcal{E}_2$ *where both users are in error, i.e.,* $\mathcal{S} = \{1, 2\}$*.*

More generally, an error occurs if

$$(\hat{\boldsymbol{X}}_1, \hat{\boldsymbol{X}}_2) \;\neq\; (\boldsymbol{X}_1, \boldsymbol{X}_2)$$

Thus,

$$\begin{aligned} \mathcal{E}_1 \;&\triangleq\; \Big\{(\hat{\boldsymbol{X}}_1 \neq \boldsymbol{X}_1 \;\&\; \hat{\boldsymbol{X}}_2 = \boldsymbol{X}_2) \;\wedge\; (\hat{\boldsymbol{X}}_2 \neq \boldsymbol{X}_2 \;\&\; \hat{\boldsymbol{X}}_1 = \boldsymbol{X}_1)\Big\} \\ \mathcal{E}_2 \;&\triangleq\; \Big\{(\hat{\boldsymbol{X}}_1 \neq \boldsymbol{X}_1) \;\&\; (\hat{\boldsymbol{X}}_2 \neq \boldsymbol{X}_2)\Big\} \end{aligned}$$

The overall error probability is given by

$$P_e \;=\; \mathbb{P}(\mathcal{E}_1) + \mathbb{P}(\mathcal{E}_2) \tag{2.30}$$

Depending on the transmission rate, one of the K error probabilities in (2.28) dominates the total error probability. This dependency was first characterized in [36] for the two-user AWGN-MAC and then in [2] for the fading MAC using the standard upper bound in terms of error exponents

$$\mathbb{P}(\mathcal{E}_s \mid \boldsymbol{H}) \;\;\leq\;\; \exp^{-TE_s(sR,\boldsymbol{H})}, \;\; \mathcal{S} \subseteq \{1, \ldots, K\} \tag{2.31}$$

where the random coding exponent $E_s(sR, \boldsymbol{H})$ is given in [36, Th. 2].

Once related to the transmission data rates, the probability of each error event brings a *numerical* evaluation of the dominant error event over the achievable finite rate region (cf. [2] for details).

2.2.2 Design Criteria

There is a main conceptual difference between a multiple-access and point-to-point scheme due to the fact that each user's own codeword is limited to its transmitting antenna. Hence, the design criteria originally derived for point-to-point space-time coding are not sufficient to construct good multiple-access coding schemes.

The understanding of the nature of error events occurring in a MAC enables the development of criteria for optimal space-time codes design derived in [2]. These criteria essentially aim at minimizing the overall error probability.

Define $\boldsymbol{X}^s$ the codeword resulting from the concatenation of the codewords of the s users that are in error. In the high SNR regime, the space-time code design criteria minimizing the error probability, P_e given in (2.28), consists first in determining the type of the dominant error event depending on the rate tuple, and then minimizing the corresponding dominant error probability $\mathbb{P}(\mathcal{E}_s)$ where $s \in \{1, 2, \ldots, K\}$ using codes that fulfill the following criteria

1. *Rank criterion*: For every codeword pair $(\boldsymbol{X}^s, \boldsymbol{X}'^s)$ with $\boldsymbol{X}^s \neq \boldsymbol{X}'^s$ the rank of the corresponding codeword difference matrix should be maximized.
2. *Eigenvalue criterion*: For every codeword pair $(\boldsymbol{X}^s, \boldsymbol{X}'^s)$ with $\boldsymbol{X}^s \neq \boldsymbol{X}'^s$ the product of the nonzero eigenvalues of the corresponding codeword difference matrix should be maximized.

Interestingly, when the single-user error event dominates, *i.e.,* $P_{\mathcal{E}_1}$ dominates, these design criteria are equivalent to those of the single-user MIMO codes. This scenario can be dealt by using well known space-time codes designed for the single-user case. The same observation is made when the event where more than one user are in error dominates, but these criteria apply to the special overall STBC that is required in this case.

Remark 2.2.1 *Although these design criteria mimic those of the conventional co-located MIMO STBCs, there is an important conceptual difference in the multiple-access scenario. When ranks (and determinants) of the differences of distinct codewords are considered for the joint error events, none of the rows are identical in* $\boldsymbol{X}^s$ *and* $\boldsymbol{X}'^s$*.*

2.2.3 Multiple-Access Codes Construction

Little was known about space-time block coding for MACs before the work of Gärtner and Bölcskei [2] where the authors proposed the first code taking into account the multi-user nature of the channel. Traditional approaches are mainly based on the use of orthogonal transmission schemes, such as TDMA and CDMA. More recent works [37, 38] consisted in using single-user STBCs for each user.

In [37], Naguib et *al.* presented a MMSE interference cancellation technique combined with a ML decoding in order to suppress interference in a multi-user scenario. As an example, the two-user MAC was considered. The number of transmit and receive antennas are $n_t = 2$ and $n_r \geq 2$, respectively. Each user's information is transmitted using a length two Alamouti code (*c.f.* 2.1.3). While this technique effectively suppresses the co-channel interference, using single-user coding schemes leads to sub-optimal performance.

In [38], the authors proposed a technique aiming to separate the users in signal space using what they called an *interference-resistant modulation* (IRM). This technique is based on the multi-dimensional rotated constellations previously addressed in [39] for AWGN-MAC. Single-user STBC combined with IRM was shown to enhance each user's performance but suffers from a reduced transmission rate. This is the unavoidable drawback of this coding scheme which significance increases with the number of users.

Gärtner and Bölcskei (GB) Code

Gärtner and Bölcskei considered in [2] a two-user MAC with $n_t = n_r = 2$. They proposed the first multiple-access STBC taking into account the multiple-access nature of the channel. This code, referred to as GB code, is of length 4 and results from a concatenation of two Alamouti codes. The codeword matrix transmitted by user k is given by

$$\boldsymbol{X}_k = \begin{bmatrix} s_{k,1} & s_{k,2} & s_{k,3} & s_{k,4} \\ -s_{k,2}^* & s_{k,1}^* & -s_{k,4}^* & s_{k,3}^* \end{bmatrix} \tag{2.32}$$

where $s_{k,j}$, $k = 1, \ldots, 2$, $j = 1, \ldots, 4$ denote the four information symbols of user 1 and 2, independently chosen from a QAM constellation.

In order to take the error event $\boxed{2}$ into account, a STBC guaranteeing a minimum rank of three, is proposed. This joint design consists in swapping two columns of the codeword matrix of one user, say user 2, as follows:

$$\boldsymbol{X}_2 = \begin{bmatrix} s_{2,1} & s_{2,3} & s_{2,2} & s_{2,4} \\ -s_{2,2}^* & -s_{2,4}^* & s_{2,1}^* & s_{2,3}^* \end{bmatrix} \tag{2.33}$$

Fig. 2.1 illustrates the performance of GB code in terms of the probability of the error event $\boxed{2}$ versus the received SNR. Compared to the single-user code, the proposed code clearly exhibits better performance when both users are in error (2.5 dB better). While the rank of each user's codeword is the same for the two codes ($\text{rank}(\boldsymbol{X}_k) = 2$), the rank of the overall codeword matrix is 2 for the SU code and 3 for the GB code. The higher diversity order achieved by GB code, when both users are in error, explains the observed gain.

This construction simply highlights the importance of the multiple-access code design and shows the sub-optimality of the use of a single-user code in a multiple-access scenario.

2.2.4 DMT-Optimal Design Criteria

Coronel et *al.* established in [27] the outage-DMT of selective-fading MIMO-MACs and derived sufficient conditions on the users' codebooks that, if verified, guarantee that the coding scheme is outage-DMT optimal. In other words, the error probability of the code behaves, exponentially in SNR, as $P_{\text{out}}(R)$. This work highlights the relation between the dominant error event regions and the outage-DMT. It also shows that, exploiting the DMT framework helps in rigorously characterizing the error behavior of the MAC.

We focus on the DMT optimal code design criteria derived in ([27]) using a standard PEP-driven space-time analysis. In [16], Coronel et *al.* revisited the derivation of their criteria and provided a less stringent criteria that is presented in the following theorems for a Rayleigh block fading channel that can be seen as a particular frequency selective

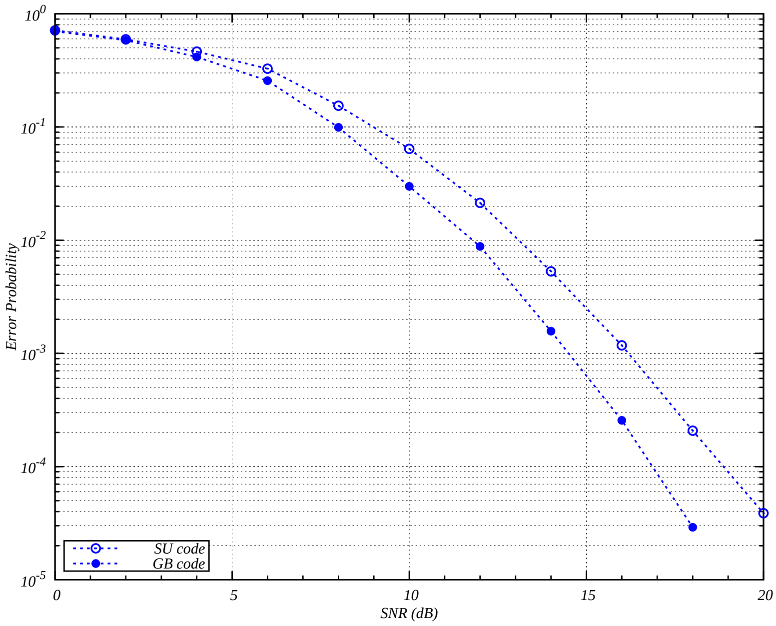


Figure 2.1: $P_{\mathcal{E}_2}$ for Single-User code and GB code (joint code design), two-user MAC, $n_t = n_r = 2$, 4-QAM.

fading channel with one multipath.

Let $\mathcal{X}_k(\mathsf{SNR})$ denote the space-time coding scheme of user k with data rate $R(\mathsf{SNR})$ bits pcu scaling as $r \log \mathsf{SNR}$ and assume that a codeword matrix $\boldsymbol{X}_k \in \mathcal{X}_k$ satisfies

$$\|\boldsymbol{X}_k\|_{\mathrm{F}}^2 \;\leq\; T\mathsf{SNR}. \tag{2.34}$$

The non-vanishing determinant condition defined in Definition 2.1.1 in the single-user scenario can be applied to the multiple-access case as follows.

Definition 2.2.1 *(Joint Non-Vanishing Determinant) A multiple-access coding scheme $\mathcal{X}$ is said to be a joint NVD code, if the difference codeword matrix $\boldsymbol{\Delta}^s = \boldsymbol{X}^s - \boldsymbol{X}'^s$, for any pair of distinct codewords, satisfies*

$$\min_{\{\boldsymbol{\Delta}^s\}_{s=|\mathcal{S}|}} \det\left(\frac{\boldsymbol{\Delta}^s \boldsymbol{\Delta}^{s\dagger}}{\mathsf{SNR}\, 2^{-R(\mathsf{SNR})/n_t}}\right) \dot{\geq} \mathsf{SNR}^0 \tag{2.35}$$

for all values of $1 \leq s \leq K$. $\mathsf{SNR}\, 2^{-R(\mathsf{SNR})/n_t}$ is the normalization factor with respect to the minimum distance in the symbols constellation.

Theorem 2.2.1 *([27]) For every $s \in \{1, 2, \ldots, K\}$, T satisfies $T \geq sn_t$ and m_s is defined as*

$$m_s \;\triangleq\; \min(sn_t, n_r).$$

Denote λ_l, $l = 1, \ldots, m_s$ the m_s non-zero eigenvalues (in ascending order) of the normalized *(by* SNR*)* $\mathbf{\Delta}^s\mathbf{\Delta}^{s\dagger}$ *and*

$$\Psi_{m_s}^{sn_t}(\mathsf{SNR}) \triangleq \min_{\{\mathbf{\Delta}^s\}_{s=|\mathcal{S}|}} \prod_{l=1}^{m_s} \lambda_l. \tag{2.36}$$

If the following condition is satisfied

$$\Psi_{m_s}^{sn_t}(\mathsf{SNR}) \geq \frac{1}{2^{sR(\mathsf{SNR})+o(\log \mathsf{SNR})}} \tag{2.37}$$

then, the probability of the error event $\boxed{s}$ *is upper bounded by*

$$P_{\mathcal{E}_s} \dot{\leq} \mathsf{SNR}^{-d_{sn_t,n_r}(sr)} \tag{2.38}$$

Theorem 2.2.2 *([16]) If the family of code* $\mathcal{X}(\mathsf{SNR})$ *satisfies (2.37) for the dominant outage set, i.e., for* $s = s^*$ *and for every other set, i.e., for* $s \neq s^*$*, the following condition is satisfied*

$$\Psi_{m_s}^{sn_t}(\mathsf{SNR}) \geq \frac{1}{\mathsf{SNR}^{\rho_s(r)+o(\log \mathsf{SNR})}} \tag{2.39}$$

where

$$0 \leq \rho_s(r) \leq r_{sn_t,n_r}\left(d_{s^*n_t,n_r}(s^*r)\right) \tag{2.40}$$

then $\mathcal{X}(\mathsf{SNR})$ *achieves the optimal DMT, i.e.,*

$$d_{\mathcal{X}}(r) = d_{MAC}(r).$$

Proof: Using (2.28), one gets:

$$P_{e,\mathcal{X}}(\mathsf{SNR}) \doteq \mathbb{P}(\mathcal{E}_{s^*}) + \sum_{s\neq s^*} \mathbb{P}(\mathcal{E}_s) \tag{2.41}$$

For $s = s^*$, (2.37) is satisfied and it follows from Theorem 2.2.1 that

$$\mathbb{P}(\mathcal{E}_{s^*}) \dot{\leq} \mathsf{SNR}^{-d_{s^*n_t,n_r}(s^*r)} \tag{2.42}$$

For $s \neq s^*$, (2.39) implies

$$\mathbb{P}(\mathcal{E}_s) \dot{\leq} \mathsf{SNR}^{-d_{sn_t,n_r}(\rho_s(r))} \tag{2.43}$$

Inserting (2.42) and (2.43) in (2.41) and noting that

$$\mathsf{SNR}^{-d_{sn_t,n_r}(\rho_s(r))} \leq \mathsf{SNR}^{-d_{s^*n_t,n_r}(s^*r)}, \quad \forall s \neq s^*$$

yields

$$
\begin{aligned}
P_{e,\mathcal{X}}(\mathsf{SNR}) &\dot{\leq} \mathsf{SNR}^{-d_{s^*n_t,n_r}(s^*r)} + \sum_{s\neq s^*} \mathsf{SNR}^{-d_{sn_t,n_r}(\rho_s(r))} \\
&\doteq \mathsf{SNR}^{-d_{s^*n_t,n_r}(s^*r)}
\end{aligned}
$$

Finally, since $P_{e,\mathcal{X}}(\mathsf{SNR}) \dot{\geq} \mathbb{P}(\mathcal{O})$, one concludes that

$$
P_{e,\mathcal{X}}(\mathsf{SNR}) \doteq \mathbb{P}(\mathcal{O}) \doteq \mathsf{SNR}^{-d_{s^*n_t,n_r}(s^*r)}. \tag{2.44}
$$

■

Theorem 2.2.2 implies that, if (2.37) and (2.39) are satisfied for s^* and $s \neq s^*$, respectively,

$$
P_{e,\mathcal{X}}(\mathsf{SNR}) \doteq \mathsf{SNR}^{-\min_{s=1,\ldots,K} d_{sn_t,n_r}(sr)} \tag{2.45}
$$

which, combined with Theorem 1.4.3 leads to the following conclusion: depending on the rates of users, the typical way errors occur is either one of the users is in error or all the users are in error.

Remark 2.2.2 *In their first result stated in [27], Coronel et al. claimed that, in order to achieve the outage-DMT, a coding scheme should satisfy the joint NVD criterion. In other words, condition (2.37) should be satisfied for all outage events, i.e.,* $\rho_s(r) = sr$*, which is stronger than condition (2.39). Obviously, for a dominant outage event* $\boxed{s^*}$*,*

$$
sr \leq r_{sn_t,n_r}\left(d_{s^*n_t,n_r}(s^*r)\right), \quad \forall s
$$

showing that condition (2.39) is more relaxed. The first criterion [27] is sufficient but is too strict.

For $n_r \geq Kn_t$, criterion (2.37) in Theorem 2.2.2 is equivalent to the NVD criterion (2.35) and thus, helps determine the way the following normalized minimum determinant behaves at high SNR

$$
\delta^s_{\min} = \min_{\{\mathbf{\Delta}^s\}_{s=|\mathcal{S}|}} \det\left(\frac{\mathbf{\Delta}^s\mathbf{\Delta}^{s\dagger}}{\mathsf{SNR}\,2^{-R(\mathsf{SNR})/n_t}}\right). \tag{2.46}
$$

Since $m_s = sn_t$, then (2.36) can be rewritten as

$$
\begin{aligned}
\Psi^{sn_t}_{sn_t}(\mathsf{SNR}) &= \min_{\{\mathbf{\Delta}^s\}_{s=|\mathcal{S}|}} \prod_{l=1}^{sn_t} \lambda_l \\
&= \min_{\{\mathbf{\Delta}^s\}_{s=|\mathcal{S}|}} \det\left(\frac{\mathbf{\Delta}^s\mathbf{\Delta}^{s\dagger}}{\mathsf{SNR}}\right)
\end{aligned} \tag{2.47}
$$

If condition (2.37) is satisfied, then

$$\min_{\{\boldsymbol{\Delta}^s\}_{s=|\mathcal{S}|}} \det\left(\frac{\boldsymbol{\Delta}^s \boldsymbol{\Delta}^{s\dagger}}{\mathsf{SNR}}\right) \;\dot{\geq}\; \mathsf{SNR}^{-sr}$$

$$\delta^s_{\min} \;\dot{\geq}\; \mathsf{SNR}^0 \tag{2.48}$$

which is clearly equivalent to (2.35). On the other hand, if condition (2.39) is satisfied, we get

$$\min_{\{\boldsymbol{\Delta}^s\}_{s=|\mathcal{S}|}} \det\left(\frac{\boldsymbol{\Delta}^s \boldsymbol{\Delta}^{s\dagger}}{\mathsf{SNR}}\right) \;\dot{\geq}\; \mathsf{SNR}^{-\rho_s(r)}$$

$$\delta^s_{\min} \;\dot{\geq}\; \mathsf{SNR}^{-(\rho_s(r)-sr)} \tag{2.49}$$

where

$$0 \leq \rho_s(r) \leq r_{sn_t,n_r}\left(d_{s^* n_t,n_r}(s^* r)\right)$$

Theorem 2.2.3 *(Pigeon Hole bound [40]) Consider a full-rate multiple-access lattice code $\mathcal{X}$ for a K-user MAC with n_t transmit antennas per user. The minimum determinant (2.46) can be upper bounded as follows*

$$\delta^K_{\min} \;=\; \min_{\boldsymbol{\Delta}} \det\left(\frac{\boldsymbol{\Delta}\boldsymbol{\Delta}^\dagger}{\mathsf{SNR}\, 2^{-R(\mathsf{SNR})/n_t}}\right) \dot{\leq} \mathsf{SNR}^{-(K-1)r} \tag{2.50}$$

Next, the implications of the DMT-optimal design criteria stated in Theorem 1.4.3 are investigated for both the single-user and the antenna pooling regimes.

Single-user regime

In the single-user regime, $0 \leq r \leq \frac{n_r}{K+1}$. The dominant outage event is event $\boxed{1}$, *i.e.,* $s^* = 1$. As shown in (2.48), condition (2.37) implies:

$$\delta^1_{\min} \;\dot{\geq}\; \mathsf{SNR}^0 \tag{2.51}$$

i.e., the NVD condition should be satisfied by each individual code. For the non-dominant outage event of interest, event $\boxed{K}$, condition (2.39) implies

$$\delta^K_{\min} \;\dot{\geq}\; \mathsf{SNR}^{-(\rho_K(r)-Kr)} \tag{2.52}$$

where

$$0 \;\leq\; \rho_K(r) \;\leq\; r_{Kn_t,n_r}\left(d_{n_t,n_r}(r)\right).$$

Combining (2.52) and (2.50) leads to

$$\mathsf{SNR}^{-(\rho_K(r)-Kr)} \ \dot{\leq} \ \delta_{\min}^{K} \ \dot{\leq} \ \mathsf{SNR}^{-(K-1)r} \tag{2.53}$$

which leads to the conclusion that, in the single-user regime, condition (2.39) can be satisfied only if

$$\rho_K(r) \ \geq \ (2K-1)r \tag{2.54}$$

Antenna pooling regime

The system operates in the antenna pooling regime when the multiplexing gain satisfies $\frac{n_r}{K+1} \leq r \leq \min(n_t, \frac{n_r}{K})$. Outage event $\boxed{K}$ dominates in this regime, *i.e.*, $s^* = K$. Using (2.37), the overall code should satisfy

$$\delta_{\min}^{K} \ \dot{\geq} \ \mathsf{SNR}^{0} \tag{2.55}$$

which combined with (2.50), shows the non-existence of a multiple-access code satisfying the DMT-optimal design criteria [16] in the antenna pooling regime.

Chapter 3

Space-Time Block Coding for Single-Antenna Multiple-Access Channels

As a first approach to the multiple-access Space-Time code design, we consider in this chapter a simple multiple-access system in which the number of transmit antenna (per user) is fixed to one. The destination is equipped with an arbitrary number of receive antennas. Two different scenarios are distinguished, the synchronous and the asynchronous multiple-access system.

In a synchronous multiple-access system (studied in Section 3.1), the users are assumed to be synchronized at the transmission. Based on an outage analysis, the theoretical limits of the channel is evaluated and the sub-optimality of the time-sharing technique as compared to the multiple-access technique is highlighted. Obviously, the users should transmit simultaneously in order to exploit the degrees of freedom of the channel. Moreover, it can be shown that the use of the best single-user Space-Time block coding schemes in a multiple-access scenario is sub-optimal. Motivated by these statements that are proven numerically for the two-user MAC in Subsection 3.1.1, we propose a new family of single-antenna Space-Time block codes. The algebraic tools originally proposed for constructing STBCs for single-user MIMO channels are used (*c.f.,* Chapter 2). The new construction, termed single-antenna BB-code, better exploits the MAC's capabilities and consequently, offers a significant gain compared to both the time-sharing and the single-user coding schemes. The general construction for K users is presented in Subsection 3.1.2 followed by an explicit construction for a two-user MAC in Subsection 3.1.3. This first part is concluded by a DMT analysis of the new coding scheme presented in Subsection 3.1.5.

In the second part of this chapter, a more practical scenario without the assumption of synchronization at the transmission is considered. Asynchronism among users leads to shifted codeword matrices and thus, change the structure of the code. The main result here, is the proof of the delay-tolerance of the code proposed for the single-antenna MAC. Indeed, despite the asynchronism and the resulting structural changes, the full diversity

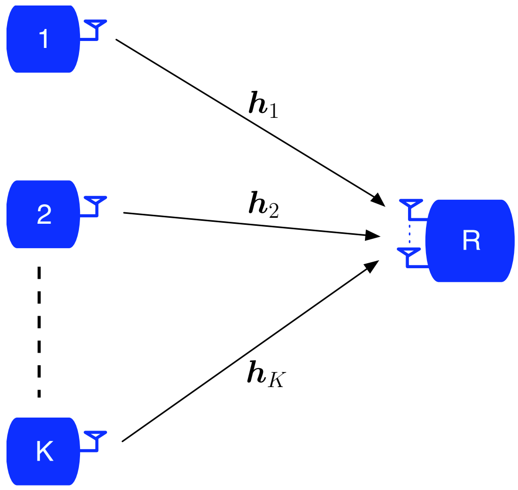


Figure 3.1: A K-user Multiple-Access Channel with single transmit antenna.

of single-antenna BB-code is shown to be preserved. Section 3.2 of this chapter, deals with asynchronism and its effect on the STBC's performance. First, the asynchronous MAC is modeled in Subsection 3.2.1. In Subsection 3.2.2, the code design criteria is derived such that a full diversity is achieved at the receiver. The single-antenna BB-code is then shown to be delay-tolerant in Subsection 3.2.3. Numerical results presented in Subsection 3.2.5 finally confirm that the diversity of the coding scheme is preserved despite the asynchronism.

3.1 Synchronous Multiple-Access Scenario

Consider the MAC illustrated in Fig. 3.1 where K users with single antenna each are communicating with one receiver with an arbitrary number of antennas. The users are not allowed to cooperate together. Each user's channel is modeled by the following vector $\boldsymbol{h}_k \in \mathbb{C}^{n_r}$

$$\boldsymbol{h}_k = \begin{bmatrix} h_{k,1} & h_{k,2} & \dots & h_{k,n_r} \end{bmatrix} \tag{3.1}$$

A perfect synchronization is assumed between the users, such that, at the receiver side, all users' informations are received without any delay. In this case, the received signal is given by

$$\begin{aligned} \boldsymbol{Y} &= \begin{bmatrix} \boldsymbol{h}_1^\top & \boldsymbol{h}_2^\top & \dots & \boldsymbol{h}_K^\top \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_K \end{bmatrix} + \boldsymbol{Z} \\ &= \boldsymbol{H}\boldsymbol{X} + \boldsymbol{Z} \end{aligned} \tag{3.2}$$

where $\boldsymbol{Y} \in \mathbb{C}^{n_r \times T}$ denotes the received matrix, $\boldsymbol{x}_k \in \mathbb{C}^T$ the coded information vector

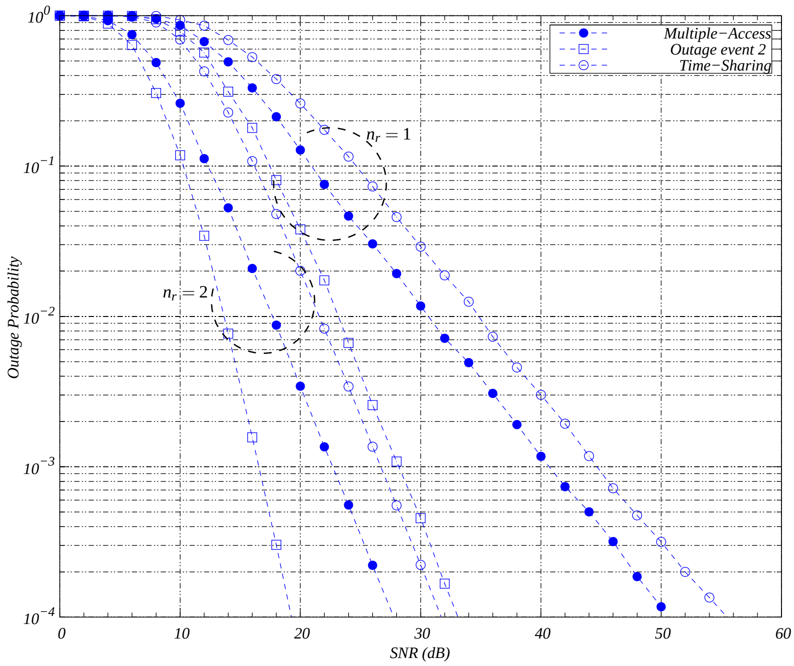


Figure 3.2: Outage performance of a two-user MAC, $n_t = 1$, $R = 2$ bits/pcu.

transmitted by user k and $\boldsymbol{Z} \in \mathbb{C}^{n_r \times T}$ the AWGN.

3.1.1 Motivation

What is the best transmission strategy and the optimal performance that can be achieved over the single-antenna MAC ?

An answer to the first part of the question was already given in Subsection 1.4.6, where the outage-DMT of the single-antenna MAC is analyzed. In Fig. 1.5-a and b, the DMT of the multiple-access technique, where both users transmit simultaneously their information, is compared to the DMT of the time-sharing technique. These figures show the sub-optimality of time-sharing in terms of the achievable DMT.

Here, the question is answered *numerically* by analyzing the outage behavior of the channel with different multiple-access techniques. Recall that, asymptotically with SNR, the outage probability constitutes an upper-bound on the error probability of any coding scheme. Consider a two-user MAC. In Fig. 3.2, the corresponding outage probability is plotted for $n_r = 1$ and 2. The probability of the outage event $\boxed{2}$, *i.e.*, $\mathcal{O}_2$, is also plotted. The spectral efficiency is $R = 2$ bits/pcu. These curves show that the same diversity order is achieved with the two techniques, $d = n_r$ and $d = 2n_r$ for $\mathcal{O}_1$ and $\mathcal{O}_2$, respectively. However, time-sharing technique is clearly sub-optimal. This is due to the fact that orthogonal transmission makes very poor use of the available spatial degrees of freedom. Intuitively, to exploit the available degrees of freedom, both users must access the channel simultaneously. A natural question arises from this conclusion:

What if each user employs the optimal single-user STBC?

For a two-user MAC, each user's channel considered separately is a SISO channel when $n_r = 1$ and a SIMO channel when $n_r = 2$. In this case, simply transmitting QAM symbols is optimal in the sense of the outage-DMT (*c.f.,* 2.1.3). To evaluate the performance of this single-user scheme in the multiple-access scenario, we consider that both users transmit simultaneously their QAM symbols (equivalent to an uncoded scheme). The received signal is given by

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{Z} \tag{3.3}$$

where $\boldsymbol{x}$ is the vector of transmitted signals with entries drawn independently from some QAM constellation.

Theoretical performance analysis

In Fig. 3.3 the DMT achieved by the uncoded and the time-sharing schemes, *i.e.,*

$$d_{\text{QAM}} = n_r(1-r) \quad \text{and} \quad d_{\text{TS}} = n_r(1-2r)$$

are illustrated. The outage-DMT $d_{\text{MAC}}(r)$ is plotted for comparison purpose,

$$d_{\text{MAC}}(r) = \begin{cases} d_{1,n_r}(r) = n_r(1-r) & r \leq \frac{n_r}{3} \\ \\ d_{2,n_r}(2r) = (2-2r)(n_r-2r) & r \geq \frac{n_r}{3} \end{cases}$$

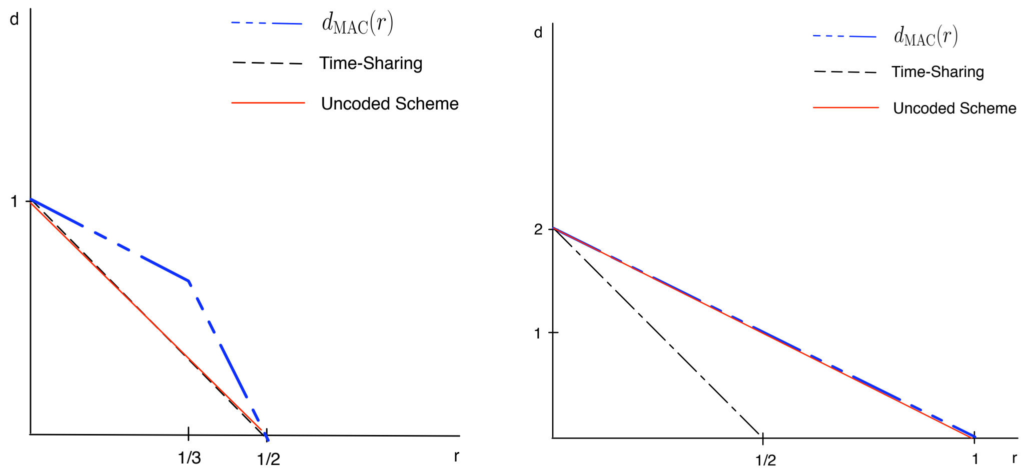
The QAM modulation achieves the outage-DMT of the channel with two receive antennas since the outage-DMT corresponds in this case to that of a single-user MIMO channel with one transmit antenna and two receive antennas. Contrarily, for $n_r = 1$, the uncoded scheme is sub-optimal and achieves the same DMT as time-sharing.

Numerical analysis

Numerical results illustrated in Fig. 1.5 confirm the previous analysis. With two receive antennas, the uncoded scheme outperforms time-sharing and offers a significant gain that is comparable to that observed on the outage probability curves. Indeed, for $n_r = 2$, the event of one user being in error is the dominant event and the only one to be considered. This explains the optimality of the uncoded scheme.

For $n_r = 1$, the uncoded scheme and the time-sharing scheme have (roughly) the same error behavior. This result shows that, using the optimal single-user coding scheme is sub-optimal since it does not exploit the multiple-access nature of the channel required in this case.

A closer examination of the error behavior is illustrated in Fig. 3.5 where the probability of each error event is considered separately: $\mathbb{P}(\mathcal{E}_1)$, the probability of the error event $\boxed{1}$



(a) $n_r = 1$ (b) $n_r = 2$

Figure 3.3: DMT achieved by the uncoded scheme (QAM modulation) and time-sharing scheme in a single transmit antenna MAC.

where, at the receiver, the information of only one user is decoded erroneously; $\mathbb{P}(\mathcal{E}_2)$, the probability of the error event $\boxed{2}$ where both users are decoded erroneously; and P_e the total error probability:

$$P_e = \mathbb{P}(\mathcal{E}_1) + \mathbb{P}(\mathcal{E}_2).$$

With two receive antennas, the error probability of event $\boxed{2}$ is smaller than that of event $\boxed{1}$ which dominates the overall error probability. Even though QAM is sub-optimal when both users are in error, it offers optimal error performance when one user is in error, which is the dominant error event.

With a single receive antenna, both error events have a significant effect on the overall probability (they occur with the same probability) and the uncoded scheme is no longer optimal. QAM leads to a bad error performance when both users are in error which significantly affects the overall error probability. A joint code design offering a higher diversity order ($d = 2$) is needed in this case.

The generalization of this analysis to the K-user MAC is straightforward. As a conclusion, for $n_r \leq (K+1)n_t$, a joint code design dealing with the different error events is of major importance. The idea behind this joint coding is to focus on the dominating error event while making sure that the other events do not have a much worse performance.

Remark 3.1.1 *(Joint Code) We call the joint code, the code resulting from the concatenation of the individual codes of all the users that are independent from each other because there is no cooperation between the users at the transmission.*

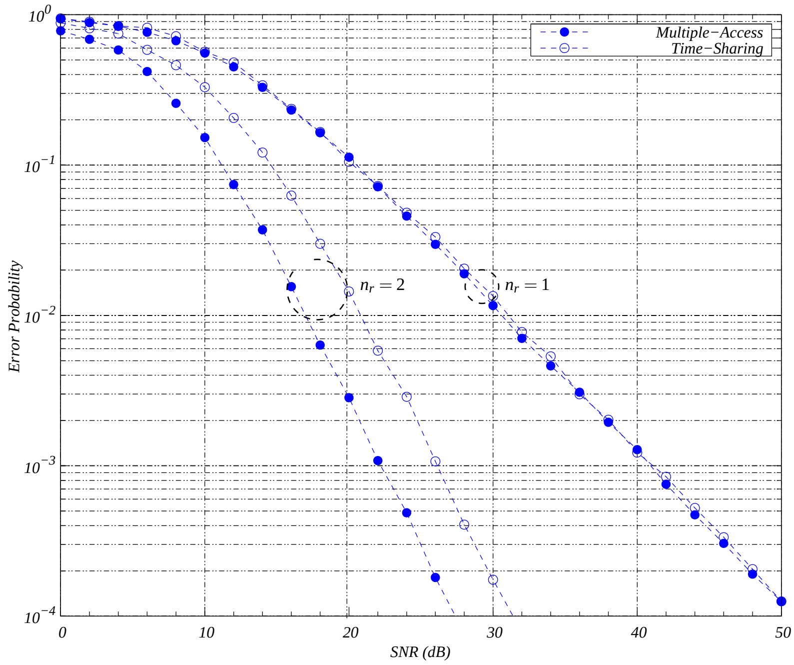


Figure 3.4: Error performance of an Uncoded scheme *vs* Time-Sharing, $K = 2, n_t = 1$, 4-QAM.

3.1.2 New Construction: The Single-Antenna BB-Code

The proposed code spans K slots, *i.e.* $T = K$, such that the joint codeword matrix is a square matrix. The vector of information symbols of each user is given by:

$$\boldsymbol{s}_k = \left[\begin{array}{cccc} s_{k,1} & s_{k,2} & \dots & s_{k,K} \end{array} \right]^\top$$

where $s_{k,l}$ denotes the QAM information symbol of user k at l^{th} channel use, $k, l = 1, 2, \dots, K$.

Let $\mathbb{L}$ be a cyclic extension over $\mathbb{Q}(i)$ of degree K and $\mathcal{O}_{\mathbb{L}}$ its ring of integers. Denote σ the generator of the Galois group of $\mathbb{L}$. From this extension, a unitary matrix is extracted [41, 42], that associates to the vector of information symbols the following vector of length K, transmitted by each user k

$$\boldsymbol{x}_k = \boldsymbol{U}\boldsymbol{s}_k = \left[\begin{array}{cccc} \gamma x_k & \gamma\sigma(x_k) & \dots & \sigma^{K-1}(x_k) \end{array} \right] \tag{3.4}$$

where $x_k \in \mathcal{O}_{\mathbb{L}}$ is a linear combination of K information symbols per user and $\gamma \in \mathbb{Q}(i)$ is a carefully chosen multiplication factor for the $k-1$ first components of $\boldsymbol{x}_k$, added to guarantee a full diversity.

Let $\mathcal{X}$ denote the joint codebook. A codeword $\boldsymbol{X}$ of $\mathcal{X}$ results from the (vertical) concatenation of all users codewords as follows

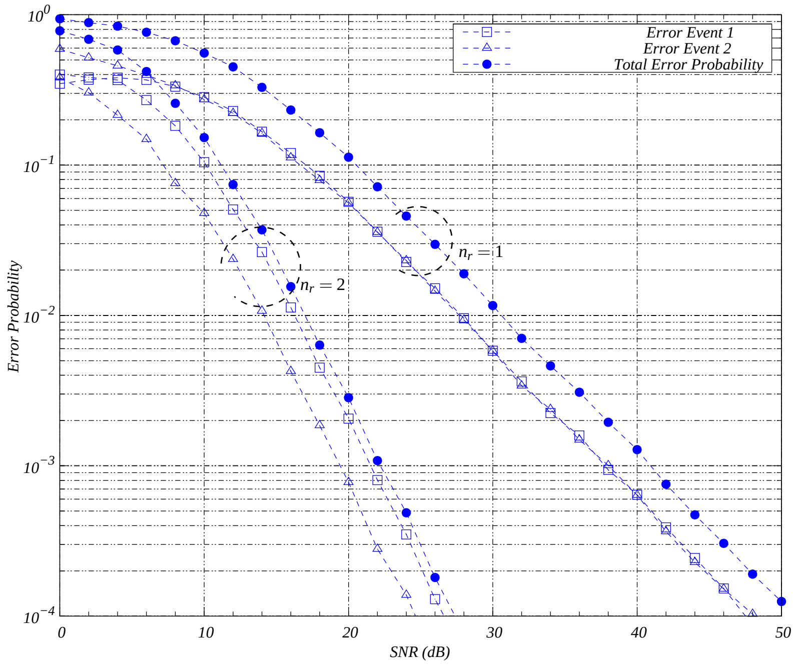


Figure 3.5: Different error events of an Uncoded scheme, $K = 2, n_t = 1$, 4-QAM.

$$\boldsymbol{X} = \begin{bmatrix} x_1 & \sigma(x_1) & \sigma^2(x_1) & \dots & \sigma^{K-1}(x_1) \\ \gamma x_2 & \sigma(x_2) & \sigma^2(x_2) & \dots & \sigma^{K-1}(x_2) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \gamma x_K & \gamma\sigma(x_K) & \dots & \dots & \sigma^{K-1}(x_K) \end{bmatrix} \tag{3.5}$$

Remark 3.1.2 *Compared to the perfect code (Eq. A.4), the proposed code does not have the same algebraic properties. Hence, the conditions that* γ *should verify in order to get a fully diverse code should be derived with respect to the new construction.*

Theorem 3.1.1 *If* γ *is transcendental, the proposed coding scheme* $\mathcal{X}$ *is fully diverse.*

Proof: The determinant of the codeword matrix (3.5) is a polynomial function of γ, *i.e.*,

$$\det \boldsymbol{X} = P_{\mathbf{X}}(\gamma) = \sum_{\pi \in \mathcal{S}_K} \operatorname{sgn}(\pi) \prod_{i=1}^{K} x_{\pi(i),i} \tag{3.6}$$

where $sgn(\cdot)$ is the sign function of a permutation in the permutations group $\mathcal{S}_K$, that returns $+1$ and -1 for even and odd permutations, respectively [43]. $x_{\pi(i),i}$, $i = 1, \dots, K$ denote the codeword's elements (row $\pi(i)$, column i). This polynomial has a degree $K - 1$ since the coefficient of the term γ^{K-1} given by

$$\sigma^{K-1}(x_1)\,.\,x_2\,.\,\sigma(x_3)\ldots\sigma^{K-2}(x_K)$$

is non zero. Indeed, the full diversity constraint is required when event $\boxed{K}$ dominates, in other words, when all users are in error, *i.e.*, $x_{\pi(i),i} \neq 0, \forall i$.

Since all the coefficients of $P_{\boldsymbol{X}}(\gamma)$ are in $\mathbb{L}$, the roots of the polynomial equation $P_{\boldsymbol{X}}(\gamma) = 0$ are in some algebraic extension of $\mathbb{L}$ of degree at most $K-1$ [44]. Therefore, a sufficient condition that guarantees a non-zero determinant is to choose γ transcendental. Hence, by the linearity of the mapping (3.4), the determinant of any codeword difference matrix is also non-zero and the full-rank property is satisfied.

■

Remark 3.1.3 *The above Lemma gives a necessary condition on the value of γ in order to guarantee the full-rank property. Other non transcendental values of γ can also yield a full-rank code. This is shown in the following explicit construction.*

3.1.3 Explicit Construction: Two-user MAC

In the two-user case, the unitary transformation matrix $\boldsymbol{U}$ initially defined in (2.22) for the construction of the Golden code is used,

$$\boldsymbol{U} = \frac{1}{\sqrt{5}}\begin{bmatrix} \alpha & \alpha\theta \\ \bar{\alpha} & \bar{\alpha}\bar{\theta} \end{bmatrix}$$

where $\alpha = 1 + i - i\theta$ and $\theta = \frac{1+\sqrt{5}}{2}$. $\boldsymbol{U}$ associates to the vector of information QAM symbols $\boldsymbol{s}_k = [s_{k1}\ s_{k2}]^\top$ the vector

$$\boldsymbol{x}_k^\top = \boldsymbol{U}\boldsymbol{s}_k = \begin{pmatrix} x_k \\ \sigma(x_k) \end{pmatrix} \tag{3.7}$$

One of the users, say user 2, multiplies the symbol transmitted in the first slot by a constant $\gamma \in \mathbb{Q}(i)$ yielding the equivalent matrix

$$\begin{aligned} \boldsymbol{X} &= \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 & \sigma(x_1) \\ \gamma x_2 & \sigma(x_2) \end{bmatrix} \\ &= \begin{bmatrix} \alpha(s_{11} + s_{12}\theta) & \bar{\alpha}(s_{11} + s_{12}\bar{\theta}) \\ \gamma\alpha(s_{21} + s_{22}\theta) & \bar{\alpha}(s_{21} + s_{22}\bar{\theta}) \end{bmatrix} \end{aligned}$$

Theorem 3.1.2 *For $\gamma \neq \pm 1$, the codeword matrix $\boldsymbol{X}$ is full rank and hence, $\mathcal{X}$ is fully-diverse.*

Proof: The idea is to find the condition on the value of γ for which the two columns of the codeword $\boldsymbol{X}$ are proportional leading to a zero determinant, and hence, consider

$$\lambda_0 \begin{bmatrix} x_1 \\ \gamma x_2 \end{bmatrix} + \lambda_1 \begin{bmatrix} \sigma(x_1) \\ \sigma(x_2) \end{bmatrix} = 0 \tag{3.8}$$

where $\lambda_0, \lambda_1 \in \mathbb{Q}(i)$. Assuming that $\lambda_0 = 1$ and $\lambda_1 = \lambda$, the following system is obtained:

$$x_1 = \lambda\sigma(x_1) \tag{3.9}$$

$$\gamma x_2 = \lambda\sigma(x_2) \tag{3.10}$$

(3.9) implies that $\lambda = +1$ or -1 for $x_j \in \mathbb{Q}(i)$ $(x_j = \sigma(x_j))$ or $x_j \in \sqrt{5}\mathbb{Q}(i)$ $(x_j = -\sigma(x_j))$, respectively. Using (3.10), we conclude that

$$\det \boldsymbol{X} = 0 \Longleftrightarrow \gamma = \pm 1$$

Hence, any $\gamma \in \mathbb{Q}(i) \setminus \{\pm 1\}$ yields a non zero determinant. Hence, let $\gamma = i$, giving rise to the fully-diverse code $\mathcal{X}$. ■

3.1.4 Error Performance

The performance of the new coding scheme is studied in terms of its error probability illustrated in Fig. 3.6 and 3.7 for 4-QAM and 16-QAM information symbols, respectively. At the receiver side, a joint ML decoding is used.

Compared to time-sharing, the proposed code achieves the same diversity order, $d = n_t n_r$, but offers a significant performance gain. For a 4-QAM constellation and at an error rate of 10^{-4}, a coding gain of 3.5 dB is observed for $n_r = 1$ and 5 dB for $n_r = 2$. When the spectral efficiency is increased (16-QAM), the coding gain increases to 8 dB for $n_r = 1$ and 9.5 dB for $n_r = 2$.

Moreover, we notice that the sub-optimality of the time-sharing scheme increases when the number of receive antennas increases. This is due to the fact that time-sharing does not exploit the available spatial degrees of freedom. Interestingly, if the outage probability curves of the channel (Fig. 3.2) are compared to the error probability curves of the new code (Fig. 3.6), similar behaviors are "roughly" observed. This result shows (numerically) the superiority of the proposed scheme.

Remark 3.1.4 *For the time-sharing scheme, the information symbols are carved from a 16- and 256-QAM constellations, respectively, in order to take into consideration the code length and thus to guarantee a fair comparison.*

In order to further analyze the error behavior of the new coding scheme, the probability of each error event is considered separately and the corresponding error probabilities, $\mathbb{P}(\mathcal{E}_1)$ and $\mathbb{P}(\mathcal{E}_2)$, are plotted in Fig. 3.8.

Comparing these error probabilities to those of the uncoded scheme (Fig. 3.5) explains the gain offered by the new code. We notice that error event $\boxed{1}$ has the same probability in

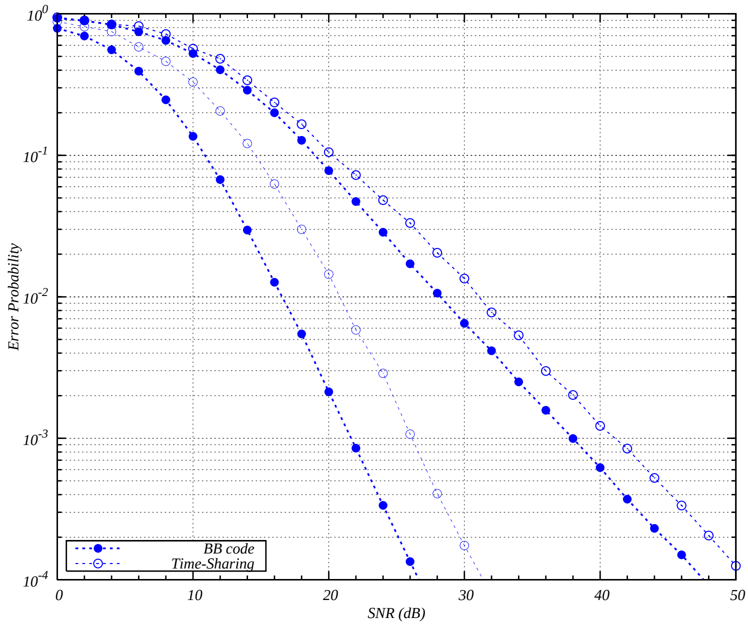


Figure 3.6: Performance of the proposed code, $n_t = 1$, 4-QAM.

both cases. Indeed, when one user is in error, we are only interested in the codeword of this user given in 3.7 which is simply equivalent to rotated QAM symbols. The importance of the new code mainly results from its higher diversity order that improves the performance of the system when both users are in error by decreasing the probability of error event $\boxed{2}$. While this improvement has a marginal effect for $n_r = 2$, it explains the gain obtained when $n_r = 1$.

The structure of the code, as well as the careful choice of γ, guarantee the full rank property of the code and thus the previously discussed performance improvement. The impact of the value of γ on the performance of the code is studied in Fig. 3.9. We see that the performance of the proposed code is degraded, *i.e.,* the performance gain is lost, for $\gamma = 1$ which is mainly due to the loss of the diversity order (2) when both users are in error.

3.1.5 DMT Analysis

Up to this point, we have proposed a new coding scheme for single transmit antenna MACs that has been shown to satisfy the overall full rank criterion *i.e.,* the determinant of any joint codeword $\boldsymbol{X}$ is non-zero whenever all the users' codewords $\boldsymbol{X}_k$, $k = 1, \ldots, K$ are non-zero. Simulation results evaluated the performance gain obtained using the proposed construction and showed (numerically) that the corresponding error probability and the outage probability of the channel have the same behavior.

In this section, we present a DMT-oriented analysis of the proposed single-antenna BB-code. In the DMT framework, the common data rate R of the system scales with SNR as $R(\mathsf{SNR}) = r \log \mathsf{SNR}$. In this case, we are interested in studying the performance of the family of codebooks $\mathcal{X}_k(\mathsf{SNR})$, one for each SNR with block length $T = K$. The overall

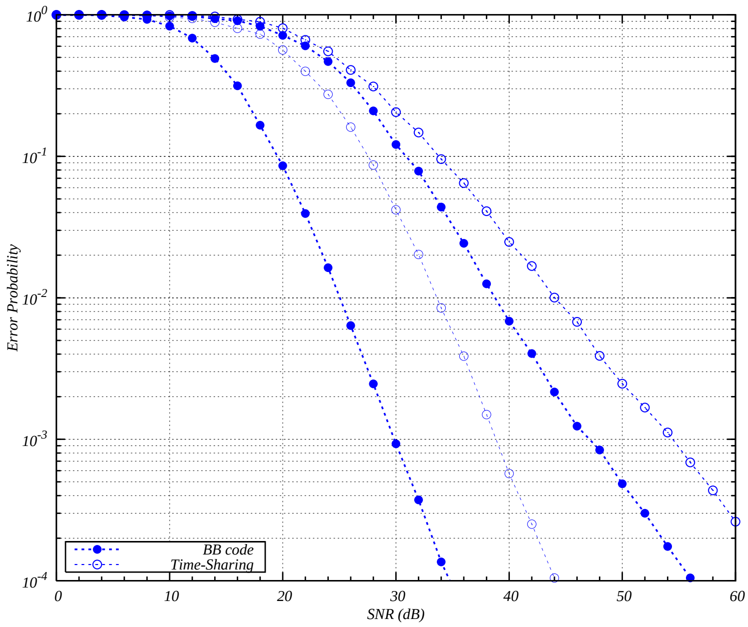


Figure 3.7: Performance of the proposed code, $n_t = 1$, 16-QAM.

family of codes $\mathcal{X}(\mathsf{SNR})$, results from the concatenation of all individual coding schemes $\mathcal{X}_k(\mathsf{SNR})$. It consists of $(K \times T)$ code matrices and, thus, contains SNR^{TKr} codewords, at a given SNR.

The best DMT a coding scheme is expected to achieve is the outage-DMT of the channel given, as in (1.32), by:

$$d_{\text{SA-MAC}}(r) = \begin{cases} d_{1,n_r}(r), & r \leq \min(1, \frac{n_r}{K+1}) \\ \\ d_{K,n_r}(Kr), & r \geq \min(1, \frac{n_r}{K+1}) \end{cases} \tag{3.11}$$

DMT optimal design criteria: Single-Antenna case

According to the DMT optimal code design criteria derived in [16] and presented in Theorem 2.2.2, a coding scheme achieves the outage-DMT of a MAC with single transmit antenna per user, *i.e.*, $P_{e,\mathcal{X}} = d_{\text{SA-MAC}}(r)$, if the following inequality resulting from (2.37) is satisfied

$$\Psi^{s^*}_{m_{s^*}}(\mathsf{SNR}) \quad \dot{\geq} \quad \mathsf{SNR}^{-(s^* r - \epsilon)} \tag{3.12}$$

for the dominant outage event $\boxed{s^*}$ and for all the other outage events, there exists $\epsilon > 0$ such that

$$\Psi^{s}_{m_s}(\mathsf{SNR}) \quad \dot{\geq} \quad \mathsf{SNR}^{-(\rho_s - \epsilon)} \tag{3.13}$$

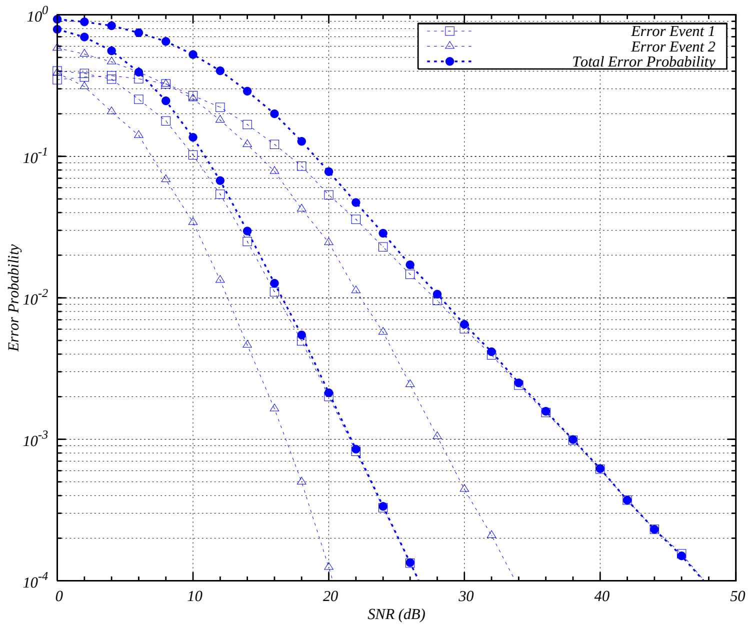


Figure 3.8: Different error events of the proposed code, $n_t = 1$, 4-QAM.

where $m_s = \min(s, n_r)$ and ρ_s satisfies (2.40) for $n_t = 1$, *i.e.*,

$$0 \leq \rho_s \leq r_{s,n_r}\left(d_{s^*,n_r}(s^* r)\right) \tag{3.14}$$

BB-coding scheme

We assume that each user k transmits K information symbols, $s_{k,l}$, chosen from a QAM constellation denoted $\mathcal{C}_k$ with $2^{R(\mathsf{SNR})}$ points carved from $\mathbb{Z}[i] = \{m + in : m, n \in \mathbb{Z}\}$, *i.e.*,

$$\mathcal{C}_k(\mathsf{SNR}) = \left\{ (m + in) : \frac{-2^{R(\mathsf{SNR})/2}}{2} \leq m, n \leq \frac{2^{R(\mathsf{SNR})/2}}{2} \right\}, \quad m, n \in \mathbb{Z} \tag{3.15}$$

The symbol vector $\boldsymbol{s}_k$ is then coded using a unitary transform matrix leading to user's k codeword

$$\boldsymbol{x}_k^\top = \boldsymbol{U}\boldsymbol{s}_k. \tag{3.16}$$

We have

$$\max_{\boldsymbol{x}_k^\top = \boldsymbol{U}\boldsymbol{s}_k} \|\boldsymbol{x}_k\|^2 = \max_{s_{k,l} \in \mathcal{C}_k(\mathsf{SNR})} \|\boldsymbol{s}_k\|^2 = T\left(\frac{2^{R(\mathsf{SNR})}}{2}\right) \tag{3.17}$$

Define $\kappa^2 = \frac{2^{R(\mathsf{SNR})}}{2}$, used to scale the transmit vector as follows

$$\tilde{\boldsymbol{x}}_k = \frac{1}{\kappa}\boldsymbol{x}_k \tag{3.18}$$

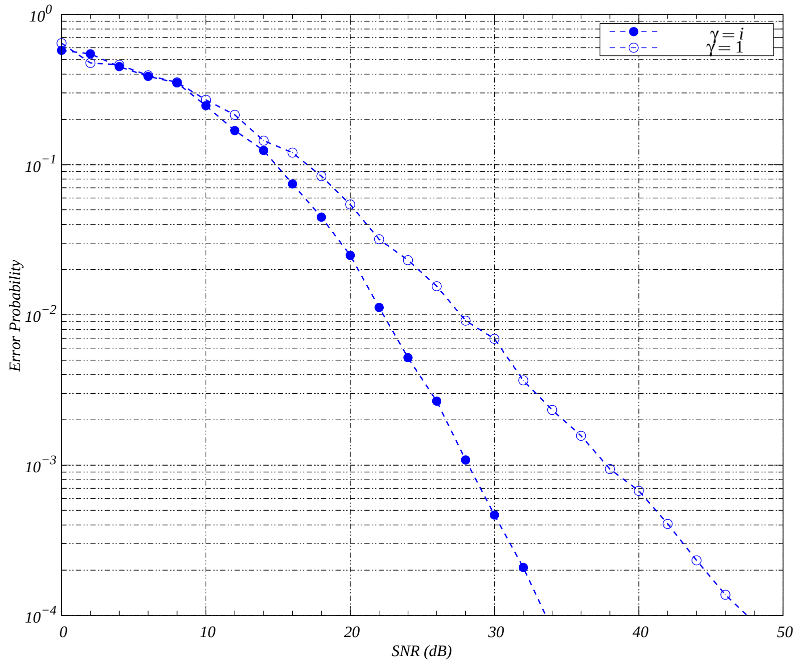


Figure 3.9: The impact of the value of γ on the error probability of event 2, $K = 2$, $n_r = n_t = 1$, 4-QAM.

such that the power constraint (1.1), $\|\boldsymbol{X}_k\|_{\mathrm{F}}^2 \leq n_t T$, is satisfied for all the users ($|\gamma| = 1$)

$$\max_{\tilde{\boldsymbol{x}}_k \in \mathcal{X}_k(\mathsf{SNR})} \|\tilde{\boldsymbol{x}}_k\|^2 = \frac{1}{\kappa^2} \max_{\boldsymbol{x}_k^\top = \boldsymbol{U}\boldsymbol{s}_k} \|\boldsymbol{x}_k\|^2 = T \tag{3.19}$$

We are now ready to study the DMT of the proposed code using (3.12) and (3.13). As in [27], $R(\mathsf{SNR})$ is assumed to scale with SNR as $(r - \epsilon)\log \mathsf{SNR}$ for some $\epsilon > 0$. For $s = 1$, we have

$$\begin{aligned}
\Psi_1^1(\mathsf{SNR}) &= \min_{\tilde{\boldsymbol{x}}_1, \tilde{\boldsymbol{x}}'_1 \in \mathcal{X}_1(\mathsf{SNR})} \|\tilde{\boldsymbol{x}}_1 - \tilde{\boldsymbol{x}}'_1\|^2 \\
&= 2^{1-R(\mathsf{SNR})} \min_{\boldsymbol{x}_1^\top = \boldsymbol{U}\boldsymbol{s}_1;\, \boldsymbol{x}_1'^\top = \boldsymbol{U}\boldsymbol{s}_1'} \|\boldsymbol{x}_1 - \boldsymbol{x}_1'\|^2 && (3.20) \\
&= 2^{1-R(\mathsf{SNR})} \min_{s_{1,l}, s'_{1,l} \in \mathcal{C}_1(\mathsf{SNR})} \underbrace{\|\boldsymbol{s}_1 - \boldsymbol{s}_1'\|^2}_{\geq d_{\min}^2} && (3.21) \\
&\doteq \mathsf{SNR}^{-(r-\epsilon)} && (3.22)
\end{aligned}$$

where $d_{\min} = 1$ is independent of SNR. For $s = K$, the overall transmitted codeword is given by:

$$\tilde{\boldsymbol{X}} = \kappa^{-1}\boldsymbol{X} = \kappa^{-1}\begin{bmatrix} \tilde{x}_1 & \sigma(\tilde{x}_1) & \dots & \sigma^{K-1}(\tilde{x}_1) \\ \gamma\tilde{x}_2 & \sigma(\tilde{x}_2) & \dots & \sigma^{K-1}(\tilde{x}_2) \\ \vdots & \ddots & \ddots & \vdots \\ \gamma\tilde{x}_K & \gamma\sigma(\tilde{x}_K) & \dots & \sigma^{K-1}(\tilde{x}_K) \end{bmatrix} \tag{3.23}$$

From the linearity of the mapping σ over $\mathbb{L}$, the codeword difference matrix is obtained as

$$\tilde{\boldsymbol{D}} = \begin{bmatrix} \tilde{d}_1 & \sigma(\tilde{d}_1) & \dots & \sigma^{K-1}(d_1) \\ \gamma\tilde{d}_2 & \sigma(\tilde{d}_2) & \dots & \sigma^{K-1}(\tilde{d}_2) \\ \vdots & \ddots & \ddots & \vdots \\ \gamma\tilde{d}_K & \gamma\sigma\left(\tilde{d}_K\right) & \dots & \sigma^{K-1}\left(\tilde{d}_K\right) \end{bmatrix} \tag{3.24}$$

where $\tilde{d}_k = \kappa^{-1}\left(\tilde{x}_k - \tilde{x}_k^{'}\right) \in \mathbb{L}$ for $k = 1, \dots, K$.

$$\Psi^K_{\min(K,n_r)}(\mathsf{SNR}) = \min_{\tilde{\boldsymbol{D}}} \prod_{l=1}^{\min(K,n_r)} \lambda_l. \tag{3.25}$$

For a sufficient number of receive antennas, $n_r \geq K$, $m_s = K$ such that, from (3.25), we get

$$\begin{aligned} \Psi^K_K(\mathsf{SNR}) &= \min_{\tilde{\boldsymbol{D}}=\tilde{\boldsymbol{X}}-\tilde{\boldsymbol{X}}^{'}} \prod_{l=1}^{K} \lambda_l \\ &= \min_{\tilde{\boldsymbol{D}}=\tilde{\boldsymbol{X}}-\tilde{\boldsymbol{X}}^{'}} |\det \tilde{\boldsymbol{D}}|^2 \\ &= \kappa^{-2K} \min_{\boldsymbol{\Delta}=\boldsymbol{X}-\boldsymbol{X}^{'}} |\det \boldsymbol{\Delta}|^2, \end{aligned}$$

hence,

$$\Psi^K_K(\mathsf{SNR}) = 2^{K(1-R(\mathsf{SNR}))} \min_{\boldsymbol{\Delta}=\boldsymbol{X}-\boldsymbol{X}^{'}} |\det \boldsymbol{\Delta}|^2. \tag{3.26}$$

Recall that $\det \boldsymbol{\Delta} \neq 0$ since the multiple-access coding scheme is full rank. Let $\omega(\mathsf{SNR})$ denote

$$\omega(\mathsf{SNR}) = \min_{\boldsymbol{\Delta}=\boldsymbol{X}-\boldsymbol{X}^{'}} |\det \boldsymbol{\Delta}|^2. \tag{3.27}$$

For high SNR regime, replacing $R(\mathsf{SNR})$ by $(r-\epsilon)\log \mathsf{SNR}$, yields

$$\Psi_K^K(\mathsf{SNR}) \quad \doteq \quad \mathsf{SNR}^{-(Kr-\epsilon^{'})}\omega(\mathsf{SNR}) \tag{3.28}$$

Definition 3.1.1 *The determinant of a coding scheme is said to decay with exponent β if*

$$\lim_{\mathsf{SNR}\to\infty} -\frac{\log \omega(\mathsf{SNR})}{\log \mathsf{SNR}} = \beta. \tag{3.29}$$

Remark 3.1.5 *It is clear that the smaller this decay exponent is, the better the corresponding code behaves asymptotically.*

Using (3.29), we get

$$\Psi_K^K(\mathsf{SNR}) \quad \doteq \quad \mathsf{SNR}^{-(Kr+\beta-\epsilon^{'})} \tag{3.30}$$

In the antenna pooling regime, $r \geq \min(1, \frac{n_r}{K+1})$, (3.30) should satisfy the code design criterion (3.12) for $s^* = K$. This is only possible in the case of a sub-polynomial decay of $\omega(\mathsf{SNR})$, $\beta = 0$. Otherwise, for $\beta > 0$, the quantity $\Psi_K^K(\mathsf{SNR})$ decays faster than required in (3.12). In other words, the code is not DMT optimal in the sense of Theorem 2.2.2 in the antenna pooling regime.

In the single-user regime, $r \leq \min(1, \frac{n_r}{K+1})$, the dominant outage event is event $\boxed{1}$ and $s^* = 1$. The first criterion (3.12) with $s^* = 1$ is shown to be satisfied in (3.22). On the other hand, in order to satisfy (3.14) for $s \neq 1$, the following inequality should be satisfied

$$Kr + \beta \leq r_{K,n_r}\left(d_{1,n_r}(r)\right) \tag{3.31}$$

Note that

$$d_{1,n_r}(x) = n_r(1-x), \quad 0 \leq x \leq 1 \tag{3.32}$$

and

$$d_{K,n_r}(x) = (K-x)(n_r-x), \quad 0 \leq x \leq K \tag{3.33}$$

As a conclusion, the achievability of the DMT optimal design criteria depends on the decay function of the considered coding scheme.

Decay function of the Two-user BB-code

Consider a two-user MAC, $K = n_r = 2$ and $n_t = 1$. Following the same reasoning as above, we have:

$$\Psi_1^1(\mathsf{SNR}) \doteq \mathsf{SNR}^{-(r-\epsilon)} \tag{3.34}$$

and

$$\Psi_2^2(\mathsf{SNR}) \doteq \mathsf{SNR}^{-(2r+\beta-\epsilon^{'})} \tag{3.35}$$

When the system is operating in the antenna pooling regime, *i.e.,* $r \geq 2/3$ and $s^* = 2$, (3.35) clearly decays faster than $\mathsf{SNR}^{-(2r-\epsilon)}$ required in condition (3.12).

In the single-user regime, *i.e.,* $r \leq 2/3$ and $s^* = 1$, (3.34) satisfies (3.12). However, for the non-dominant event $\boxed{2}$, $s = 2$, (3.35) implies that, in order to satisfy (2.39) and using (2.40), the decay exponent of the code should satisfy

$$2r + \beta \leq r_{2,2}\left(d_{1,2}(r)\right) \tag{3.36}$$

with

$$d_{2,2}(x) = \begin{cases} 4 - 3x, & 0 \leq x \leq 1 \\ \\ 2 - 2x, & 1 \leq x \leq 2 \end{cases} \tag{3.37}$$

and $d_{1,2}(r) = 2 - 2r$, thus

$$r_{2,2}\left(d_{1,2}(r)\right) = \begin{cases} \frac{2+2r}{3}, & 0 \leq r \leq \frac{1}{2} \\ \\ r, & \frac{1}{2} \leq x \leq 1 \end{cases} . \tag{3.38}$$

Lahtonen et *al.,* studied in [18] the way $\omega(\mathsf{SNR})$ decays with SNR for the two-user BB-code, $K = n_r = 2$ and $n_t = 1$. They showed that

$$\omega(\mathsf{SNR}) \doteq \mathsf{SNR}^{-2r} \tag{3.39}$$

and thus, $\beta = 2r$. Combining this result with (3.38) implies that the BB-code is DMT optimal in the sense of Theorem 2.2.2 for $r \leq \frac{1}{5}$.

3.2 Asynchronous Multiple-Access Scenario

So far, we dealt with multiple-access systems under the assumption of synchronized users which may not be practical for many reasons. In practice, the users do not necessarily share the same timing reference, they access the channel randomly, they have different geometrical locations and their transmitted messages may have different lengths. Therefore, the assumption of perfect synchronization between the users at the transmission is not easily justified.

Asynchronism in cooperative communication has recently received significant interest. Distributed space-time coding schemes previously designed for synchronous relays were shown to be suboptimal for asynchronous relays. This fact motivated the design of delay-tolerant distributed space-time codes preserving the full cooperative diversity order without the synchronization assumption. For more details on the design of space-time codes for asynchronous cooperative communication schemes, the reader is referred to [45, 46, 47] and references therein.

Even though the MAC is inherently asynchronous in practice, to the best of our knowledge, there is no study of code design for the asynchronous MAC. In this section, a practical multiple-access system is considered where the (single transmit antenna) users are not synchronized. The design criteria for the different users are developed such that full diversity is achieved at the receiver. Asynchronism entails the introduction of delays between the users leading to shifted codeword matrices. Such structural change may induce a degradation of the performance of the code. Following the same reasoning as in [47], we show that, even with shifted codeword matrices, the MAC Space-Time code proposed in Subsection 3.1.2 for synchronous MAC, is delay-tolerant. Numerical results presented at the end of this section confirm the delay-tolerance of the code.

3.2.1 Asynchronous Signal Model

Consider the MAC with K single-antenna users ($n_t = 1$) and one receiver equipped with n_r receive antennas illustrated in Fig. 3.1. When the users are synchronous, the channel is modeled as in Eq. 1.7 where, at each channel use, the signals of all the users reach the destination simultaneously. The $K \times n_t$ joint codeword matrix $\boldsymbol{X}$ in this synchronous scenario can be written as

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1T} \\ x_{21} & x_{22} & \dots & x_{2T} \\ \vdots & \vdots & \dots & \vdots \\ x_{K1} & x_{K2} & \dots & x_{KT} \end{bmatrix} \tag{3.40}$$

where x_{kj} is the j-th transmitted symbol from the k-th user.

When the users are asynchronous, the transmitted codeword spans over more than T symbol intervals due to the delays. We assume that the users are synchronized at the frame level such that the beginning and the end of each codeword can be aligned for different users by transmitting zero symbols. In this case, there is no interference between the transmitted codewords. The fractional delays are absorbed in multi path, so asynchronous delays are integer factors of the symbol interval. Furthermore, it is assumed that the delays are unknown at the users, but, along with the channel matrix, are known at the base station receiver.

To model the asynchronous MAC, denote δ_k the relative transmission delay of user k, which corresponds to the number of symbol periods before the arrival of the signal of user k at the receiver relative to the earliest received user and

$$\Delta \triangleq (\delta_1, \delta_2, \dots, \delta_K)$$

the delay profile associated with the set of K users. If the asynchronous transmission of one single codeword is considered, the shifted codeword matrix is as follows

$$\boldsymbol{X}_{\Delta} = \begin{bmatrix} \mathbf{0}^{\delta_1} & x_{11} & \dots & x_{1T} & \mathbf{0}^{\delta_{\max}-\delta_1} \\ \mathbf{0}^{\delta_2} & x_{21} & \dots & x_{2T} & \mathbf{0}^{\delta_{\max}-\delta_1} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \mathbf{0}^{\delta_K} & x_{K1} & \dots & x_{KT} & \mathbf{0}^{\delta_{\max}-\delta_1} \end{bmatrix} \tag{3.41}$$

where $\mathbf{0}^l$ denote an all-zero vector of length l and $\delta_{\max}$ is the maximum relative delay.

The received signal in the asynchronous case where the previous $K \times (T + \delta_{\max})$ codeword matrix is transmitted rather than the $K \times T$ matrix given in Eq. 3.40 is

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X}_{\Delta} + \boldsymbol{Z} \tag{3.42}$$

Example 3.2.1 *Consider a two-user MAC with single transmit antenna. Let the following matrix of length* 2 *denote the transmitted codeword in the synchronous scenario*

$$\boldsymbol{X} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$$

Now, if the first user is considered to be delayed by one symbol instant with respect to the second user, i.e., $\Delta = (1, 0)$*, the new length* 3 *asynchronous codeword matrix is*

$$\boldsymbol{X}_{\Delta} = \begin{bmatrix} 0 & x_{11} & x_{12} \\ x_{21} & x_{22} & 0 \end{bmatrix}$$

At the receiver side, the received signal is given by

$$\begin{bmatrix} \boldsymbol{y}_1 & \boldsymbol{y}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_1 & \boldsymbol{h}_2 \end{bmatrix} \begin{bmatrix} 0 & x_{11} & x_{12} \\ x_{21} & x_{22} & 0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{z}_1 & \boldsymbol{z}_2 \end{bmatrix} \tag{3.43}$$

The main goal here is to design a coding scheme that, even with asynchronous users, does not loose its diversity order. We start by deriving the code design criteria for an asynchronous MAC before investigating a delay-tolerant Space-Time construction for this channel.

3.2.2 Code Design Criteria

Code design criteria for synchronous MACs were provided in Subsection 3.2.2 based on an error event analysis. Recall that rate regions where single-user error events dominate can be treated by using well known space-time codes designed for the single-user case. However, the rate regions where the event of more than one user being in error dominates, require a joint code design. In both cases, the rank and eigenvalue criteria derived in [2] should be verified.

The generalization of these design criteria to the asynchronous MAC follows from the analysis presented in [45] for the asynchronous cooperative relays:

Definition 3.2.1 *A multiple-access Space-Time code $\mathcal{X}$ is τ-delay tolerant for an asynchronous MAC if for all distinct codewords $\boldsymbol{X}, \boldsymbol{X}^{'} \in \mathcal{X}$, the difference matrix $\boldsymbol{X} - \boldsymbol{X}^{'}$ is full rank when the rows of the codewords are transmitted with arbitrary delays of duration at most τ symbols.*

Definition 3.2.2 *$\mathcal{X}$ is called fully delay-tolerant if it is τ-delay tolerant for any τ.*

Remark 3.2.1 *It is very important to note that the particular error event we are interested in is the joint error event where all the users are in error. Thus, a given user k has two different messages in $\boldsymbol{X}, \boldsymbol{X}^{'}$.*

3.2.3 Delay-Tolerant Multiple-Access Space-Time Code

Consider the single-antenna MAC and let us show that the new family of STBCs proposed in Subsection 3.1.2, is delay-tolerant. In other words, we prove that the diversity order of the code, corresponding to the rank of the difference between distinct codewords, is the same when the users are transmitting asynchronously. We first consider the two-user MAC with asynchronous users. The effect of asynchronism on the coding scheme constructed in Subsection 3.1.3 is studied. This code is shown to provide full diversity despite the asynchronous transmission. The generalization of this result to the K-user MAC is then presented.

As a first approach to the problem, we do not deal with the interference resulting from either the next or the previous codewords. It is assumed that the users insert enough guard interval (*i.e.,* filling symbols) between every two consecutive codewords to ensure that the constructed space-time code is received without any interference. The case where all the users transmit continuously without any delay guard will be considered later in Section 3.2.4.

Two-user MAC

A codeword matrix of the BB-code constructed in Subsection 3.1.3 for the two-user MAC is given by

$$\begin{aligned} \boldsymbol{X} &= \begin{bmatrix} x_1 & \sigma(x_1) \\ \gamma x_2 & \sigma(x_2) \end{bmatrix} \\ &= \begin{bmatrix} \alpha(s_{11} + s_{12}\varphi) & \bar{\alpha}(s_{11} + s_{12}\bar{\varphi}) \\ \gamma\alpha(s_{21} + s_{22}\varphi) & \bar{\alpha}(s_{21} + s_{22}\bar{\varphi}) \end{bmatrix} \end{aligned} \tag{3.44}$$

Each row of this codeword matrix corresponds to the codeword transmitted by each user. Assuming that the two users do not share a common timing reference leads to an asynchronous reception of the transmitted information and equivalently, to an arbitrary shifted codeword per user. The idea is to show that, even with arbitrary shifted rows, the difference between two codewords,

$$\boldsymbol{X}_{\Delta} - \boldsymbol{X}'_{\Delta} \tag{3.45}$$

is of full rank. It is important to note that in this scenario, the full diversity, $d = 2$, becomes significant for the error event $\boxed{2}$, *i.e.*, both users are in error: $x_k \neq x'_k$, $k = 1, 2$.

Since proving delay tolerance for either one of the two delay profiles $\Delta_1 = (1, 0)$ and $\Delta_2 = (0, 1)$ requires the same reasoning, we consider the following shifted codeword matrix

$$\boldsymbol{X}_{\Delta} = \begin{bmatrix} 0 & x_1 & \sigma(x_1) \\ \gamma x_2 & \sigma(x_2) & 0 \end{bmatrix} \tag{3.46}$$

that should be of full rank when $\boldsymbol{s}_k \neq 0$. This is shown by considering the 2×2 submatrix

$$\boldsymbol{X}_{\Delta} = \begin{bmatrix} x_1 & \sigma(x_1) \\ \sigma(x_2) & 0 \end{bmatrix} \tag{3.47}$$

whose determinant $\sigma(x_1)\,\sigma(x_2) \neq 0$ if $\boldsymbol{s}_k \neq 0$, $k = 1, 2$ which is the case for the error event $\boxed{2}$, *i.e.*, when both users are in error.

K-user MAC

For the K-user MAC, the BB-code was constructed in Subsection 3.1.2. The equivalent codeword is in this case a $K \times K$ matrix

$$\boldsymbol{X} = \begin{bmatrix} x_1 & \sigma(x_1) & \sigma^2(x_1) & \dots & \sigma^{K-1}(x_1) \\ \gamma x_2 & \sigma(x_2) & \sigma^2(x_2) & \dots & \sigma^{K-1}(x_2) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \gamma x_K & \gamma\sigma(x_K) & \dots & \dots & \sigma^{K-1}(x_K) \end{bmatrix} \tag{3.48}$$

To verify the delay tolerance of this code, one needs to show that all the codeword difference matrices are full rank for all delay profiles Δ. Define the matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ as follows

$$\boldsymbol{A} \triangleq \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ \gamma & 1 & 1 & \ldots & 1 \\ \gamma & \gamma & 1 & \ldots & 1 \\ \vdots & & \ddots & & \vdots \\ \gamma & \gamma & \ldots & \gamma & 1 \end{bmatrix} \tag{3.49}$$

and

$$\boldsymbol{C} = \begin{bmatrix} x_1 & \sigma(x_1) & \ldots & \sigma^{K-1}(x_1) \\ x_2 & \sigma(x_2) & \ldots & \sigma^{K-1}(x_2) \\ \vdots & \vdots & & \vdots \\ x_K & \sigma(x_K) & \ldots & \sigma^{K-1}(x_K) \end{bmatrix} \tag{3.50}$$

The K-user codeword matrix (Eq. 3.48) can be written as multiples of $\boldsymbol{A}$

$$\boldsymbol{X} = \boldsymbol{C} \odot \boldsymbol{A}$$

such that, when delays are introduced, it is sufficient to show that $\boldsymbol{A}_\Delta$ is full rank for arbitrary delay profiles Δ since $\boldsymbol{C}$ has non-zero coefficients when considering the joint error event. Note that each row of $\boldsymbol{C}$ corresponds to the codeword transmitted by one user and is independent of all other users' codewords. Thus, all the rows of a codeword difference matrix $\boldsymbol{C} - \boldsymbol{C}'$ are independent and all the elements are nonzero when all users are in error, *i.e.*, error event $\boxed{K}$.

The idea is to find the largest square submatrix $\boldsymbol{S}$ of $\boldsymbol{A}_\Delta$ with nonzero determinant. Because of the form of the original matrix $\boldsymbol{A}$ in Eq. 3.49, $\boldsymbol{S}$ can be constructed as follows: the $(K-i-1)$-th column, $i = 0, \ldots, K-2$, is chosen such that it contains a γ at the $(K-i)$-th row and the last column contains a nonzero element, 1, at the first row. $\boldsymbol{S}$ is a $K \times K$ matrix given by

$$\boldsymbol{S} = \begin{bmatrix} \star & \star & \star & \ldots & 1 \\ \gamma & \star & \ldots & \star & \star \\ \star & \gamma & \star & \ldots & \star \\ \vdots & & \ddots & & \vdots \\ \star & \star & \ldots & \gamma & \star \end{bmatrix} \tag{3.51}$$

where $\star$ can be zero or any element of $\boldsymbol{A}_\Delta$. With such column selection, $\boldsymbol{S}$ has at least one thread with all nonzero elements containing $(K-1)$ elements equal to γ. Its determinant is a polynomial function of γ with degree $K-1$

$$\det \boldsymbol{S} = \gamma^{K-1} + P(\gamma) \tag{3.52}$$

where $P(\gamma)$ is a polynomial in γ of degree $K-2$. As the coefficients of the codeword are algebraic numbers and since γ is chosen to be a transcendental number, it follows that $\det \boldsymbol{S}$ is nonzero.

3.2.4 Delay-Tolerant MAC Codes with Overlapped Codewords

The assumption made on the guard intervals in the previous section, limits the transmission rate and restricts the applications of the scheme. To avoid these drawbacks, a more practical scenario where the users are assumed to transmit continuously without any delay guard between codewords is considered in this section. In other words, the interference resulting from either the previous or the next transmitted codewords is taken into consideration.

We show that, when the maximum delay between the users is confined to the delay length of the code, the overlapped code is delay-tolerant for every delay profile. For the sake of simplicity, the two-user scenario is only considered. The generalization for the K-user case is straightforward.

For a two-user MAC, the resulted overlapped codeword constructed by concatenating the consecutive transmitted codewords $\boldsymbol{X}^{(i)}$ (as in Eq. 3.47) without any guard interval, has the following form

$$\boldsymbol{X} = \left[\begin{array}{cc|cc|cc|c} x_1^{(1)} & \sigma\left(x_1^{(1)}\right) & \ldots & \ldots & x_1^{(i)} & \sigma\left(x_1^{(i)}\right) & \ldots \\ \gamma x_2^{(1)} & \sigma\left(x_2^{(1)}\right) & \ldots & \ldots & \gamma x_2^{(i)} & \sigma\left(x_2^{(i)}\right) & \ldots \end{array}\right] \tag{3.53}$$

The event of interest is the error event $\boxed{2}$ for which, preserving the diversity order 2 of the code is mandatory. In this case, $\exists x_1^{(i)} \neq 0$ and $x_2^{(k)} \neq 0$ such that the following submatrix of $\boldsymbol{X}_\Delta$ has a nonzero determinant

$$\begin{bmatrix} x_1^{(i)} & \sigma\left(x_1^{(i)}\right) \\ \star & \gamma x_2^{(k)} \end{bmatrix} \tag{3.54}$$

where $\star$ can be zero or any element of $\boldsymbol{X}$. If $\star$ is zero, the determinant of the previous matrix is $\gamma x_1^{(i)} x_2^{(k)} \neq 0$. For any other value of $\star$, the structure of the code (*c.f.* 3.2.3) guarantees a nonzero determinant.

3.2.5 Numerical Results

In this section, we present some performance results of the discussed codes in terms of the codeword error rate versus the received signal-to-noise ratio (SNR) in dB. A two-user MAC

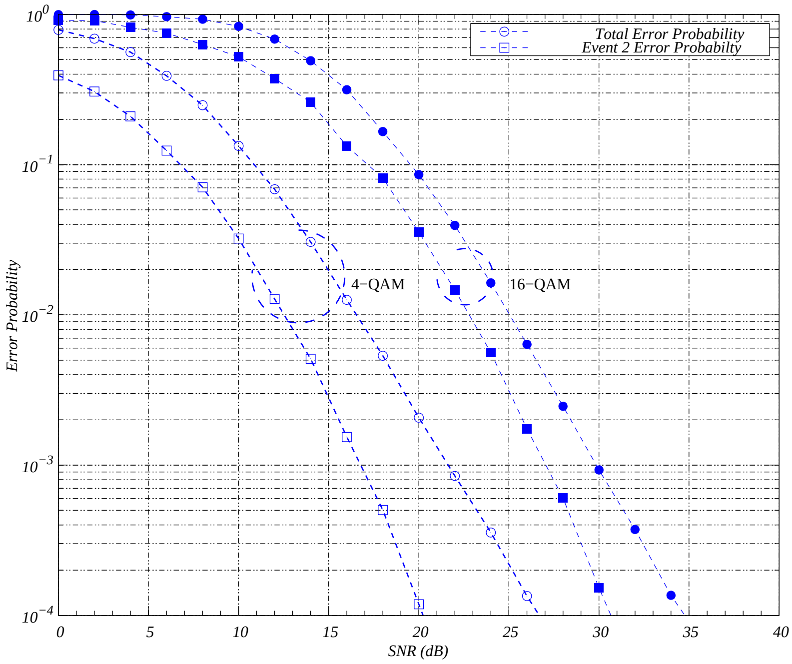


Figure 3.10: Error Performance of the two-user BB-code, synchronous scenario, $n_r = n_t = 1$.

is considered with single transmit antenna and $n_r = 1$ and 2 receive antennas. Joint ML decoding is performed at the receiver.

The performance of the BB-coding scheme for the two-user MAC is shown in Fig. 3.10 for $n_r = 1$ and in Fig. 3.11 for $n_r = 2$. The users are assumed to transmit their information synchronously. Similar numerical results were presented in Subsection 3.1.4 and are plotted again here for comparison purposes.

The total error probability, as well as the probability of the error event $\boxed{2}$ are plotted. For the former, the diversity order d equals 1 while for the latter that corresponds to the event where both users are in error, a higher diversity order, $2 \times n_r$, is observed. This diversity order is the one we are interested in preserving when the users are asynchronous.

The error behavior of the BB-coding scheme in an asynchronous scenario is shown in Fig. 3.12 for $n_r = 1$ and in Fig. 3.13 for $n_r = 2$. We assume that one of the users, say user 1, is delayed by one symbol instant with respect to the other user. These curves show that the code does not lose its diversity order with asynchronism and thus, confirm (numerically) its delay-tolerance.

Remark 3.2.2 *A sub-optimality in terms of the coding gain is observed compared to the synchronous scenario. This is due to the fact that, the rate of the code is reduced to* $\frac{K}{T+\delta_{\max}}$ *symbols per channel use in the asynchronous case while it is* $\frac{K}{T}$ *symbols per channel use in the synchronous case.*

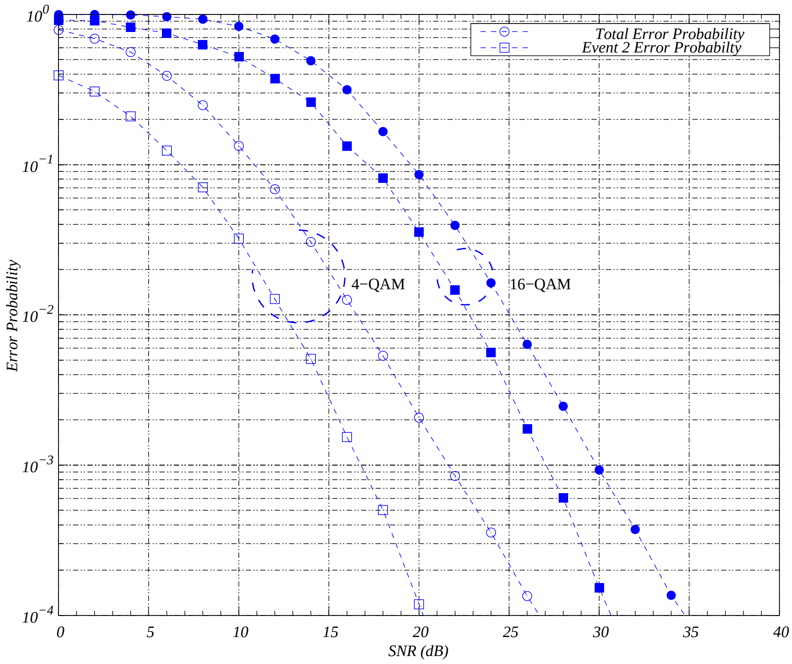


Figure 3.11: Error Performance of the two-user BB-code, synchronous scenario, $n_r = 2$.

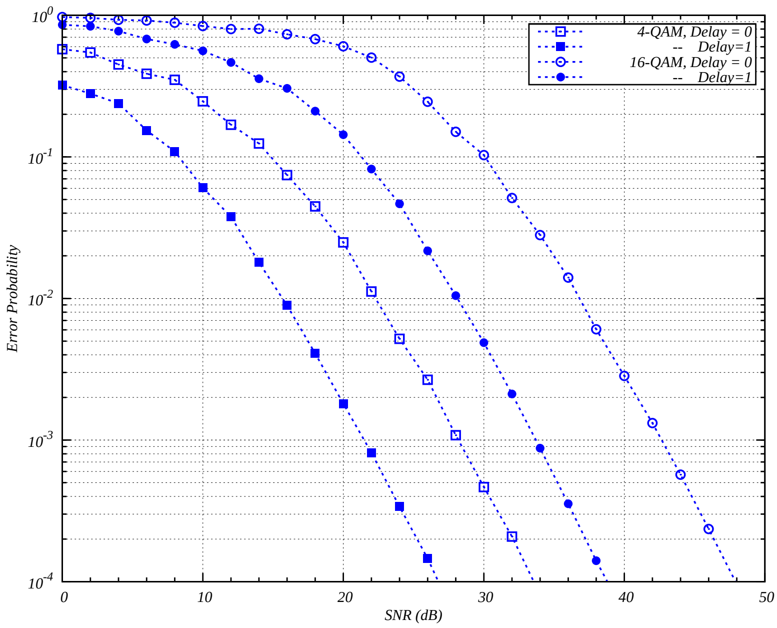


Figure 3.12: $P_{\mathcal{E}_2}$ for 4 and 16-QAM constellations. Synchronous *vs* asynchronous scenario, $K = 2$, $n_t = n_r = 1$.

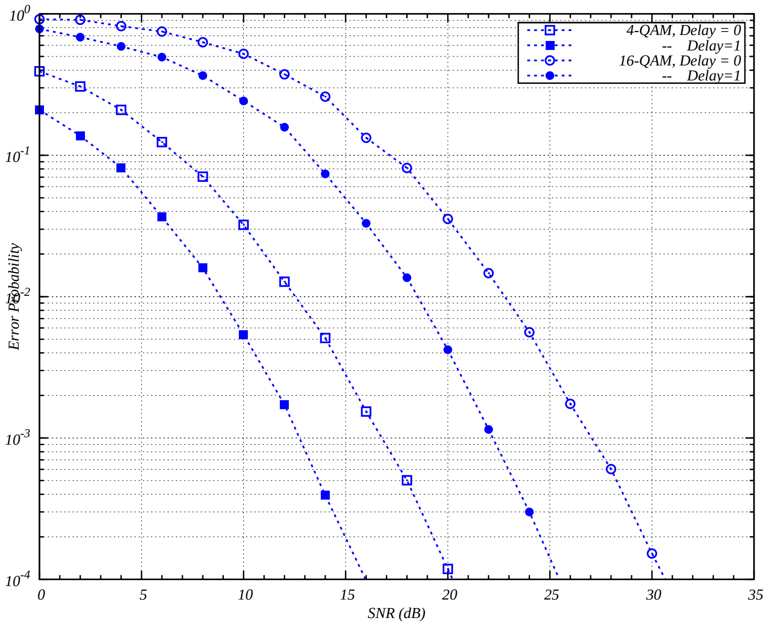


Figure 3.13: $P_{\mathcal{E}_2}$ for 4 and 16-QAM constellations. Synchronous *vs* asynchronous scenario, $K = 2$, $n_r = 2$.

Chapter 4

Space-Time Block Coding for MIMO Multiple-Access Channels

Multiple-Input Multiple-Output (MIMO) technique that uses multiple antennas at both the transmitter and the receiver sides, is well known to significantly improve the reliability of a transmission by increasing the spatial diversity and to offer higher data rate by providing spatial multiplexing. These advantages naturally hold in a multiple-access scenario where multiple antennas are deployed at the transmitters and at the receiver. Motivated by the promising gain obtained by the multiple-access coding scheme constructed in Chapter 3, the multiple-antenna multiple-access channel (MIMO-MAC) is considered in this chapter. A new family of multiple-access STBCs is proposed for this channel and different coding schemes constructed in some related works [2, 7, 19] are discussed.

After recalling the channel model in Section 4.1, a preliminary analysis is presented in Section 4.2 where the first jointly designed code, proposed by Gärtner and Bölcskei in [2], is compared to the best single-user code in a two-user multiple-access scenario. This comparison highlights the importance of the joint design and motivates the construction of new codes that exploit the channel capabilities in a better way. In Section 4.3, the coding scheme proposed by Hong and Viterbo in [7] (HV-code) for a K-user MAC with two transmit and two receive antennas is presented. The joint HV-code achieves the full diversity order, $d = Kn_t$, and therefore, outperforms the GB-code in the two-user case.

A new family of codes (MIMO-BB code) for a general K-user MAC is proposed in Section 4.4 followed by an explicit example for the two-user MAC with two transmit antennas. A similar construction proposed by Lu et *al.* in [19] is then presented in Section 4.5. Section 4.6 is dedicated for different numerical results. The theoretical limit of the channel is first analyzed based on its outage behavior. The use of the best single-user coding scheme is then shown to improve the performance of the system as compared to time-sharing. The MIMO-BB code and Lu et *al.*'s code both exploiting the multiuser nature of the channel are shown to have comparable error behavior and to offer a significant gain. Finally, a DMT-analysis of the proposed MIMO-BB code is presented in Section 4.7.

4.1 Channel Model

The MIMO-MAC consists of K users equipped with n_t transmit antennas communicating with one base station with n_r receive antennas (Fig. 1.1). The users are not allowed to cooperate together and are assumed to transmit synchronously.

Recall the signal model presented in Section 1.1 where the received signal is given by

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{Z} \tag{4.1}$$

$\boldsymbol{H}$ denotes the equivalent MIMO-MAC resulting from the (horizontal) concatenation of all the users' channels matrices, *i.e.,*

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_2 & \dots & \boldsymbol{H}_K \end{bmatrix}$$

and $\boldsymbol{X}$ the $Kn_t \times T$ joint codeword matrix, *i.e.,*

$$\boldsymbol{X}^T = \begin{bmatrix} \boldsymbol{X}_1 & \boldsymbol{X}_2 & \dots & \boldsymbol{X}_K \end{bmatrix}$$

The goal here is to optimally design the joint codebook $\mathcal{X}$, $\boldsymbol{X} \in \mathcal{X}$ in order to exploit all the capabilities of the MIMO-MAC.

4.2 Preliminary Analysis and Motivation

In this section a two-user MIMO-MAC with two transmit antennas per user is considered. Gärtner and Bölcskei proposed in [2] the first first joint coding scheme (GB-code) for the two-user MIMO-MAC with $n_t = n_r = 2$, that shows the importance of the joint code design as compared to the single-user code (SU-code).

The construction of the GB-code is based on the Alamouti structure that is extended to the multiple-access scenario in a way to achieve a diversity order of $d = 3$ for the error event $\boxed{2}$ (Subsection 2.2.3). The joint codeword matrix is as follows:

$$\boldsymbol{X} = \begin{bmatrix} s_{1,1} & s_{1,2} & s_{1,3} & s_{1,4} \\ -s_{1,2}^* & s_{1,1}^* & -s_{1,4}^* & s_{1,3}^* \\ s_{2,1} & s_{2,3} & s_{2,2} & s_{2,4} \\ -s_{2,2}^* & -s_{2,4}^* & s_{2,1}^* & s_{2,3}^* \end{bmatrix} \tag{4.2}$$

In order to highlight the performance gain offered by the GB-code, its performance is compared to that of the optimal single-user STBC. The single-user channel in this scenario is a $2 \times n_r$ MIMO channel, in which case, using the golden code is optimal (*c.f.* Subsection 2.1.4). Hence, to study the performance of the single-user code in the multiple-access scenario, it is assumed that the users are transmitting simultaneously their information using the golden code. Each user's codeword matrix is thus given by:

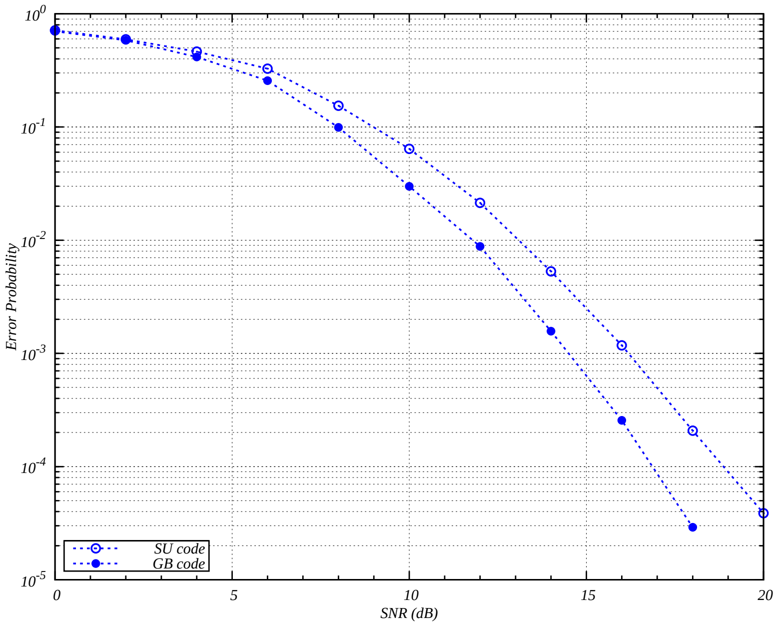


Figure 4.1: $P_{\mathcal{E}_2}$ of the SU-code and GB-code, $K = n_t = n_r = 2$, 4-QAM (2 bits pcu).

$$\begin{aligned} \boldsymbol{X}_k &= \begin{bmatrix} x_{k,1} & x_{k,2} \\ i\sigma(x_{k,2}) & \sigma(x_{k,1}) \end{bmatrix} \\ &= \begin{bmatrix} \alpha\left(s_{k,1} + s_{k,2}\theta\right) & \alpha\left(s_{k,3} + s_{k,4}\theta\right) \\ i\bar{\alpha}\left(s_{k,3} + s_{k,4}\bar{\theta}\right) & \bar{\alpha}\left(s_{k,1} + s_{k,2}\bar{\theta}\right) \end{bmatrix} \end{aligned} \tag{4.3}$$

Fig. 4.1 recalls the performance of GB-code in terms of the probability of the error event $\boxed{2}$ as compared to the SU-code, previously presented in Chapter 2.2.4. The higher diversity order achieved by the GB-code when both users are in error ($d = 3$), explains the performance gain (2 dB at $P_e = 10^{-4}$) compared to the SU-code ($d = 2$).

4.3 Hong and Viterbo (HV) Code

Hong and Viterbo derived in [7, 48] design criteria using a truncated union-bound approximation of the total error probability. Based on these criteria, they proposed a code (HV-code) for the general K-users MAC with $n_t = 2$ transmit antennas per user and $n_r = 2$ receive antennas that outperforms the GB-code. The construction of the HV-code is based on full-diversity algebraic rotations used for the construction of Perfect codes (*c.f.*, Subsection 2.1.4).

For the general K-user MAC, an ideal $\mathbb{I}$ of the ring of integers $\mathbb{O}_{\mathbb{L}}$ of an algebraic field extension $\mathbb{L}$ of degree $N = 2K$ over $\mathbb{Q}(i)$ is considered. From the canonical embedding of an integral basis of this ideal, a unitary transform matrix is extracted and applied to each user's information symbol vector, generating the following vector:

$$\boldsymbol{x}_k = \begin{bmatrix} x_k & \sigma(x_k) & \dots & \sigma^{N-1}(x_k) \end{bmatrix} \quad (4.4)$$

and the users' codewords are constructed as in (4.5). The x_k's are elements of the ideal $\mathbb{I}$ and hence, are linear combinations of N QAM symbols.

$$\begin{aligned}
\boldsymbol{X}_1 &= \begin{bmatrix} x_1 & \sigma(x_1) & \dots & & \sigma^{N-1}(x_1) \\ \gamma\sigma^{N-1}(x_1) & x_1 & & \dots & \sigma^{N-2}(x_1) \end{bmatrix} \\
\boldsymbol{X}_2 &= \begin{bmatrix} \gamma\sigma^{N-2}(x_2) & \gamma\sigma^{N-1}(x_2) & \dots & & \sigma^{N-3}(x_2) \\ \gamma\sigma^{N-3}(x_2) & \gamma\sigma^{N-2}(x_2) & \sigma^{N-1}(x_2) & \dots & \sigma^{N-4}(x_2) \end{bmatrix} \\
\vdots & \quad \ddots \\
\boldsymbol{X}_k &= \begin{bmatrix} \gamma\sigma^{N-(n_tk-n_t)}(x_k) & & \dots & \gamma x_k & & \sigma^{N-3}(x_k) \\ \gamma\sigma^{N-(n_tk-n_t)-1}(x_k) & \gamma\sigma^{N-(n_tk-n_t)}(x_k) & \dots & \gamma x_k & \dots & \sigma^{N-(n_tk-n_t)-2}(x_k) \end{bmatrix}
\end{aligned} \quad (4.5)$$

where $\gamma \neq 1$ is chosen to be transcendental in order to guarantee the full-rank property ([7, Lemma 1 and 2]).

4.3.1 Explicit HV-code Construction

Let us consider a design example to study the performance of this code. Consider the case where $K = 2$ users each equipped with two transmit antennas ($n_t = 2$). At the receiver, the number of antennas $n_r = 2$. In this scenario, the HV-code spans $N = 4$ channel uses and is constructed as follows.

Consider $\mathbb{L} = \mathbb{Q}(i, \zeta_{15} + \zeta_{15}^{-1})$ a field extension of degree $N = 4$ over $\mathbb{Q}(i)$ with cyclic Galois group $\mathrm{Gal}(\mathbb{L}/\mathbb{Q}(i)) = \langle\sigma\rangle$ and the ideal $\mathbb{I} \subseteq \mathbb{O}_{\mathbb{L}}$. The following unitary transform matrix can be extracted

$$\boldsymbol{M} = \begin{bmatrix} 0.26 - 0.31i & 0.35 - 0.42i & -0.42 + 0.51i & -0.21 + 0.26i \\ 0.26 + 0.09i & 0.47 + 0.16i & 0.16 + 0.05i & 0.76 + 0.26i \\ 0.26 + 0.21i & -0.51 + 0.16i & -0.42 - 0.36i & 0.31 + 0.26i \\ 0.26 - 0.76i & -0.05 + 0.16i & 0.16 - 0.47i & -0.09 + 0.26i \end{bmatrix}$$

and applied to the 4-QAM symbols of each user, leading to the following codeword matrices:

$$\boldsymbol{X}_1 = \begin{bmatrix} x_{11} & \sigma(x_{11}) & \sigma^2(x_{11}) & \sigma^3(x_{11}) \\ \gamma\sigma^3(x_{11}) & x_{11} & \sigma(x_{11}) & \sigma^2(x_{11}) \end{bmatrix}$$

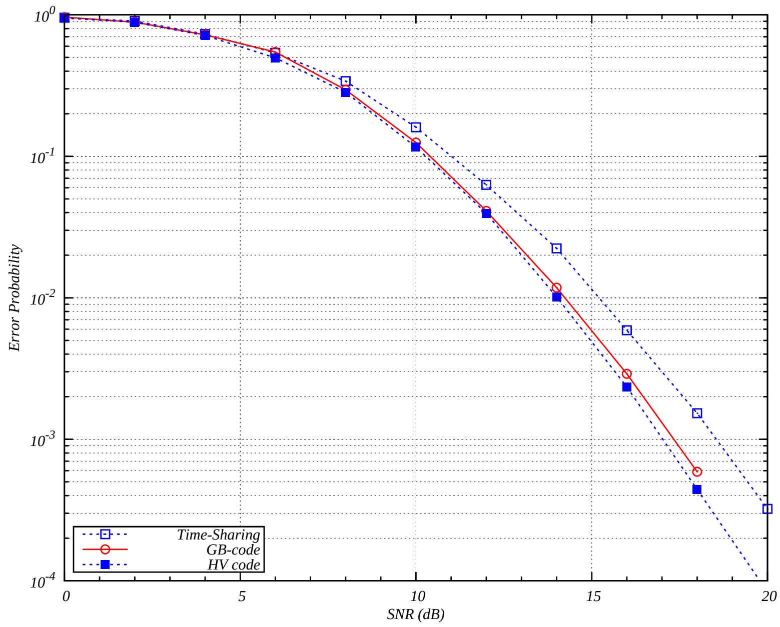


Figure 4.2: Total error probability P_e for HV-code, GB-code and Time-Sharing. Two-user MAC, $n_t = n_r = 2$, 4-QAM (2 bits pcu).

$$\boldsymbol{X}_2 = \begin{bmatrix} \gamma\sigma^2(x_{21}) & \gamma\sigma^3(x_{21}) & x_{21} & \sigma(x_{21}) \\ \gamma\sigma(x_{21}) & \gamma\sigma^2(x_{21}) & \gamma\sigma^3(x_{21}) & x_{21} \end{bmatrix}$$

The choice of the transcendental number γ was numerically optimized in [7] based on the criteria the authors derived: $\gamma = \exp(i\lambda)$ with $\lambda = 3$ for 4-QAM signaling. The rank of each user codeword matrix is 2 while the joint codeword matrix has rank 4.

4.3.2 Error Performance

The performance of the HV-code is compared to that of the GB-code in Fig. 4.2. The error performance of the time-sharing, where the users access the channel orthogonally and transmit their information using the golden code, is plotted for comparison purpose. The total error probability, taking into account all the error events, is plotted.

The same diversity order, $d = n_t n_r = 4$, is achieved for all the schemes. As expected, the time-sharing scheme has the worst performance while HV-code slightly outperforms GB-code. This can be explained by analyzing the probability of the error event $\boxed{2}$ with the different schemes, plotted in Fig. 4.3 where the single-user code performance is plotted for comparison purpose. Note that, the rank of the joint codeword matrices is 3 for the GB-code and 4 for the HV-code, whereas it is 2 for the SU-code. These higher diversity orders reduce the probability of error event $\boxed{2}$ for the jointly designed codes, HV and GB codes, and thus, explain the observed gain (0.3 dB at $P_e = 10^{-4}$).

Remark 4.3.1 *Lu et al. considered in [19] the problem of designing STBCs for the two-user MAC with $n_t = 2$ that are sphere decodable in order to preserve ML performance.*

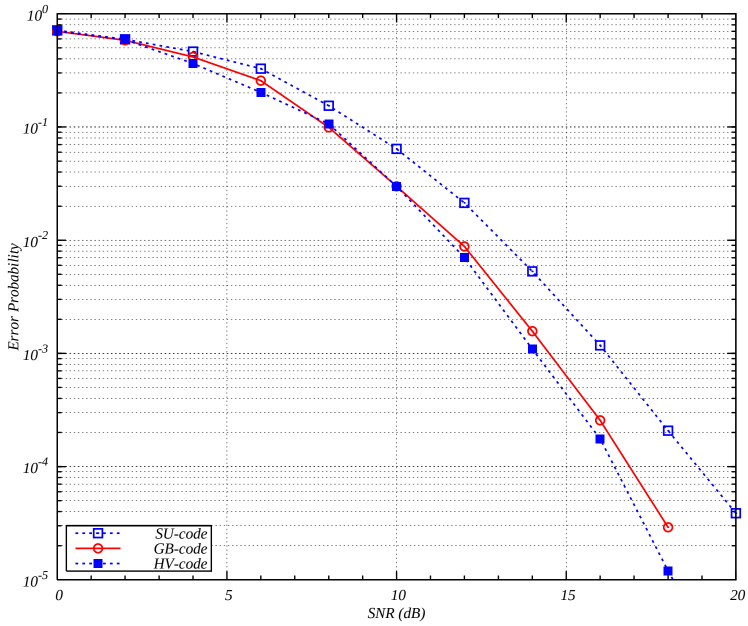


Figure 4.3: $P_{\mathcal{E}_2}$ of HV-code, GB-code, SU-code. Two-user MAC, $n_t = n_r = 2$, 4-QAM (2 bits pcu).

With two receive antennas, $n_r = 2$, each user's code should be sphere decodable with a single receive antenna, or equivalently, $r_{\max} = \min\{n_t, n_r/K\} = 1$ symbol pcu per user. Note that GB-code as well as HV-code are sphere decodable in a two-user scenario with $n_t = n_r = 2$. Lu et al. proposed a coding scheme for the same scenario. They considered the following algebra

$$\mathcal{A} = \left(\mathbb{Q}(i, \sqrt{(}2))/\mathbb{Q}(\sqrt{(}2)), \sigma, -1\right) \tag{4.6}$$

such that each user's code is optimized, i.e., has a better coding gain. Lu et al.'s code takes advantage of the multi-block structure, offers a diversity 4 and slightly outperforms HV-code (0.8 dB at $P_e = 10^{-4}$ for a 4-QAM constellation [19]) thanks to the better coding gain of the single-user code. The code construction and the corresponding performance are not presented in this thesis for the case of $n_r = 2$ but will be discussed in the following sections for different scenarios.

4.4 Proposed Code: The MIMO-BB Code

Independently from HV's construction [7], we propose the following STBC for the MIMO-MAC based on the construction of STBCs for parallel MIMO channels presented in Section 2.1.5.

Remark 4.4.1 *Despite the problem of rank deficiency that we tackle using the MMSE-DFE combined with lattice decoding if $n_r < Kn_t$, we will focus in the rest of this chapter on a different construction approach, valid for an arbitrary number of receive antennas.*

4.4.1 General K-user MIMO-MAC

Let $\mathbb{F}$ denote a Galois extension of degree K, the number of users, over the base number field $\mathbb{Q}(i)$, with Galois group:

$$\mathrm{Gal}(\mathbb{F}/\mathbb{Q}(i)) = \{1, \tau, \tau^2, \ldots, \tau^{K-1}\}$$

and $\mathbb{K}$ a cyclic extension of degree n_t, the number of transmit antennas per user, on $\mathbb{F}$ with σ denoting the generator of its Galois group. Note that the extension $\mathbb{K}/\mathbb{F}$ remains cyclic with the same Galois group as $\mathrm{Gal}(\mathbb{L}/\mathbb{Q}(i))$ where $\mathbb{L} = \mathbb{Q}(i, \theta)$ is a cyclic extension of degree n_t on $\mathbb{Q}(i)$ satisfying $\mathbb{F} \cap \mathbb{L} = \mathbb{Q}(i)$ (Fig. 4.4). There exists a suitable non-norm element $\eta \in \mathbb{F}$ such that $\mathcal{A} = (\mathbb{K}/\mathbb{F}, \sigma, \eta)$ is a CDA.

Denote $\mathbf{\Xi}$ the matrix representation of an element of this algebra, given by:

$$\mathbf{\Xi} = \begin{bmatrix} x_1 & x_2 & \ldots & x_{n_t} \\ \eta\sigma(x_{n_t}) & \sigma(x_1) & \ldots & \sigma(x_{n_t-1}) \\ \vdots & \vdots & \ddots & \vdots \\ \eta\sigma^{n_t-1}(x_2) & \eta\sigma^{n_t-1}(x_3) & \ldots & \sigma^{n_t-1}(x_1) \end{bmatrix} \tag{4.7}$$

Each user transmits a multi-block matrix $\boldsymbol{X}_k$ with information symbols of user k, given by

$$\boldsymbol{X}_k = \begin{bmatrix} \mathbf{\Xi}_k & \tau(\mathbf{\Xi}_k) & \ldots & \tau^{K-1}(\mathbf{\Xi}_k) \end{bmatrix} \tag{4.8}$$

The equivalent joint codeword matrix, using Kn_t^2 information symbols per user, is constructed as follows

$$\boldsymbol{X} = \begin{bmatrix} \mathbf{\Xi}_1 & \tau(\mathbf{\Xi}_1) & \ldots & \tau^{K-1}(\mathbf{\Xi}_1) \\ \mathbf{\Gamma\Xi}_2 & \tau(\mathbf{\Xi}_2) & \ldots & \tau^{K-1}(\mathbf{\Xi}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Gamma\Xi}_K & \mathbf{\Gamma}\tau(\mathbf{\Xi}_K) & \ldots & \tau^{K-1}(\mathbf{\Xi}_K) \end{bmatrix} \tag{4.9}$$

where $\mathbf{\Gamma}$ is a multiplication matrix for the $k-1$ first matrices of $\boldsymbol{X}_k$ added to guarantee the full-rank criteria, *i.e.,* $\det \boldsymbol{X} \neq 0$.

Theorem 4.4.1 *Let $\Gamma = \gamma \boldsymbol{I}_{n_t}$. Choosing γ transcendental guarantees that MIMO-BB code is full rank.*

Proof: The reasoning is similar to that in the single-antenna case. The codeword matrix defined in (4.9) is a $K \times K$ multi-block matrix (with K^2 blocks). It can be easily verified that the codeword determinant is a polynomial function of $\mathbf{\Gamma} = \gamma \boldsymbol{I}_{n_t}$ denoted $P_{\boldsymbol{X}}(\gamma)$. If the degree of $P_{\boldsymbol{X}}(\gamma)$ is zero, *i.e.* for $\gamma = 0$, we have:

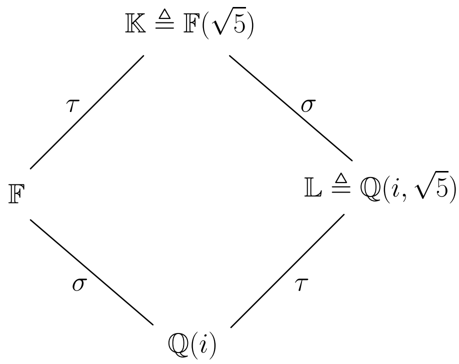


Figure 4.4: Field extension tower.

$$\begin{aligned} P_{\boldsymbol{X}}(\gamma) = \det \boldsymbol{X} &= \prod_{k=0}^{K} \det\left(\tau^{k-1}(\boldsymbol{\Xi}_k)\right) \\ &= \prod_{k=0}^{K} \tau^{k-1}\left(\det \boldsymbol{\Xi}_k\right) \end{aligned} \tag{4.10}$$

that is a non-zero element of $\mathbb{Z}(i)$ for $\Xi_k \neq 0$. Otherwise, choosing γ transcendental guarantees a non zero determinant. ■

4.4.2 Explicit Two-User MIMO-BB Code

Consider a two-user MAC with two transmit antennas per user, *i.e.*, $K = 2, n_t = 2$. Each user's codeword is in this case a 2×4 multi-block matrix and the equivalent joint codeword matrix is a 4×4 matrix constructed as follows.

Let $\mathbb{F} = \mathbb{Q}(\zeta_8)$ be an extension of $\mathbb{Q}(i)$ of degree $K = 2$, with $\zeta_8 = e^{\frac{i\pi}{4}}$ and $\mathbb{K} = \mathbb{F}(\sqrt{5}) = \mathbb{Q}(\zeta_8, \sqrt{5})$. Denote

$$\mathbb{O}_{\mathbb{K}} = \{a + b\theta \,|\, a, b \in \mathbb{Z}[\zeta_8]\}$$

the ring of integers of $\mathbb{K}$. Let $\theta = \frac{1+\sqrt{5}}{2}$, $\alpha = 1 + i - i\theta$ and $\sigma : \theta \mapsto \bar{\theta} = \frac{1-\sqrt{5}}{2}$. The chosen ideal is principle, *i.e.,* $\mathbb{I} = (\alpha)\mathbb{O}_{\mathbb{K}}$ with $\alpha = 1 + i - i\theta$. User's k codeword $\boldsymbol{X}_k$ is:

$$\boldsymbol{X}_k = \begin{bmatrix} \boldsymbol{\Xi}_k & \tau(\boldsymbol{\Xi}_k) \end{bmatrix} \tag{4.11}$$

where τ maps ζ_8 into $-\zeta_8$ and $\boldsymbol{\Xi}_k$ is the matrix representation of an element of $\mathcal{A}$ and is defined as:

$$\boldsymbol{\Xi}_k =$$

$$\frac{1}{\sqrt{5}} \begin{bmatrix} \alpha(s_{k,1} + s_{k,2}\zeta_8 + s_{k,3}\theta + s_{k,4}\zeta_8\theta) & \alpha(s_{k,5} + s_{k,6}\zeta_8 + s_{k,7}\theta + s_{k,8}\zeta_8\theta) \\ \zeta_8\bar{\alpha}(s_{k,5} + s_{k,6}\zeta_8 + s_{k,7}\bar{\theta} + s_{k,8}\zeta_8\bar{\theta}) & \bar{\alpha}(s_{k,1} + s_{k,2}\zeta_8 + s_{k,3}\bar{\theta} + s_{k,4}\zeta_8\bar{\theta}) \end{bmatrix} \tag{4.12}$$

and

$$\tau(\mathbf{\Xi}_k) =$$

$$\frac{1}{\sqrt{5}}\begin{bmatrix} \alpha(s_{k,1} - s_{k,2}\zeta_8 + s_{k,3}\theta - s_{k,4}\zeta_8\theta) & \alpha(s_{k,5} - s_{k,6}\zeta_8 + s_{k,7}\theta - s_{k,8}\zeta_8\theta) \\ -\zeta_8\bar{\alpha}(s_{k,5} - s_{k,6}\zeta_8 + s_{k,7}\bar{\theta} - s_{k,8}\zeta_8\bar{\theta}) & \bar{\alpha}(s_{k,1} - s_{k,2}\zeta_8 + s_{k,3}\bar{\theta} - s_{k,4}\zeta_8\bar{\theta}) \end{bmatrix} \quad (4.13)$$

where s_{kj} denotes the j-th QAM information symbol of user k. $\eta = \zeta_8$ has been proven, in [35], to be a non-norm element in $\mathbb{K}$. This guarantees that $\det \mathbf{\Xi}_k$, element of $\mathcal{O}_{\mathbb{F}}$, is non-zero. The equivalent joint codeword matrix of the proposed code is:

$$\boldsymbol{X} = \begin{bmatrix} \mathbf{\Xi}_1 & \tau(\mathbf{\Xi}_1) \\ \mathbf{\Gamma}\mathbf{\Xi}_2 & \tau(\mathbf{\Xi}_2) \end{bmatrix} \quad (4.14)$$

Choosing $\mathbf{\Gamma} = \gamma \boldsymbol{I}_{n_t}$ with γ transcendental guarantees a fully diverse code. However, in this case the codeword determinant can be analytically derived leading to necessary conditions that the matrix $\mathbf{\Gamma}$ should verify.

Theorem 4.4.2 *Any matrix* $\mathbf{\Gamma}$ *satisfying* ± 1 *not an eigenvalue of* $\mathbf{\Gamma}$ *yields non-zero determinant.*

Proof: The determinant of the transmitted codeword (4.14) is given by:

$$\det \boldsymbol{X} = \det\left(\tau(\mathbf{\Xi}_2) - \mathbf{\Gamma}\mathbf{\Xi}_2\mathbf{\Xi}_1^{-1}\tau(\mathbf{\Xi}_1)\right)\det \mathbf{\Xi}_1$$

$\det \boldsymbol{X} = 0$ if

$$\tau(\mathbf{\Xi}_2) - \mathbf{\Gamma}\mathbf{\Xi}_2\mathbf{\Xi}_1^{-1}\tau(\mathbf{\Xi}_1) = 0$$

One can rewrite the previous expression as:

$$\mathbf{\Gamma}\mathbf{\Theta} = \tau(\mathbf{\Theta})$$

or equivalently

$$(\mathbf{\Gamma} + \boldsymbol{I}_{n_t})\,\mathbf{\Theta} = \mathrm{Tr}\,(\mathbf{\Theta})$$

for some $\mathbf{\Theta} = \mathbf{\Xi}_2\mathbf{\Xi}_1^{-1}$ verifying $\mathbf{\Theta} \in \mathbb{Q}(i, \sqrt{5})$ or $\mathbf{\Theta} \in \zeta_8\mathbb{Q}(i, \sqrt{5})$. These conditions yield:

$$(\mathbf{\Gamma} \pm \boldsymbol{I}_{n_t}) = \mathbf{0}$$

As a conclusion,

$$\det \boldsymbol{X} = 0 \Leftrightarrow \det(\mathbf{\Gamma} \pm \boldsymbol{I}_{n_t}) = 0 \quad (4.15)$$

We choose the following matrix $\mathbf{\Gamma}$:

$$\mathbf{\Gamma} = \begin{bmatrix} 0 & 1 \\ i & 0 \end{bmatrix} \tag{4.16}$$

■

Before studying the performance of the proposed code, in the next section a similar code designed by Lu et *al.* in [19] for the two-user MIMO-MAC is presented.

4.5 Lu et *al.*'s Code

Lu et *al* focused in their work [19] on sphere decodable codes for multiple-access scenarios in order to preserve ML performance. Therefore, the code of each user must be sphere decodable with $\frac{n_r}{2}$ receive antennas.

Lu et *al.*'s construction in the two-transmit antennas multiple-access scenario is based on the following algebra:

$$\mathcal{A} = \left(\mathbb{Q}(\xi)/\mathbb{Q}(\zeta_8), \sigma, \frac{2+i}{2-i} \right)$$

where $\xi = \zeta_{16} = e^{\pi i/8}$, $\zeta_8 = \frac{1+i}{\sqrt{2}}$ and σ maps ζ_{16} into $-\zeta_{16}$. Note that this algebra has been already used in [49, 50]. Each user codeword is constructed as in (4.11),

$$\boldsymbol{X}_k = \begin{bmatrix} \mathbf{\Xi}_k & \tau(\mathbf{\Xi}_k) \end{bmatrix} \tag{4.17}$$

where

$$\begin{aligned} \tau : \zeta_{16} &\mapsto i\zeta_{16} \\ \zeta_8 &\mapsto -\zeta_8 \end{aligned}$$

and $\mathbf{\Xi}_k$ is the matrix representation of an element of $\mathcal{A}$, given by

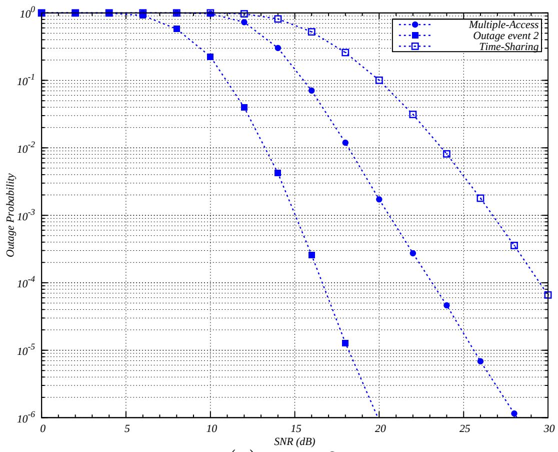
$$\mathbf{\Xi}_k = \begin{bmatrix} s_{k,1} + s_{k,2}\zeta_8 + s_{k,3}\xi + s_{k,4}\zeta_8\xi & \frac{2+i}{2-i}(s_{k,5} + s_{k,6}\zeta_8 - s_{k,7}\xi - s_{k,8}\zeta_8\xi) \\ s_{k,5} + s_{k,6}\zeta_8 + s_{k,7}\xi + s_{k,8}\zeta_8\xi & s_{k,1} + s_{k,2}\zeta_8 - s_{k,3}\xi - s_{k,4}\zeta_8\xi \end{bmatrix} \tag{4.18}$$

The equivalent joint codeword matrix resulting from the concatenation of the codeword matrices of both users is as follows:

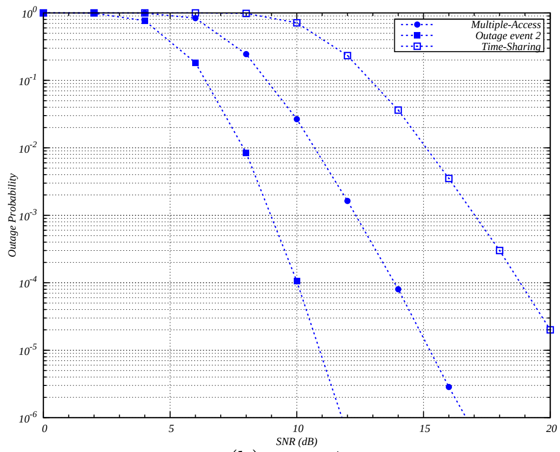
$$\begin{bmatrix} \mathbf{\Xi}_1 & \tau(\mathbf{\Xi}_1) \\ \mathbf{\Xi}_2 & \tau(\mathbf{\Xi}_2) \end{bmatrix} \tag{4.19}$$

In order to get a fully diverse code, *i.e.,* non-singular codeword matrix $\boldsymbol{X}$, authors in [19] suggested to add the following matrix:

$$\mathbf{\Gamma} = \begin{bmatrix} \zeta_7 & 0 \\ 0 & \zeta_7 \end{bmatrix} \tag{4.20}$$



(a) $n_r = 2$



(b) $n_r = 4$

Figure 4.5: Outage performance of a two-user MAC with $n_t = 2$, $R = 4$ bits pcu.

yielding the following joint codeword matrix

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{\Gamma}\boldsymbol{\Xi}_1 & \tau(\boldsymbol{\Xi}_1) \\ \boldsymbol{\Xi}_2 & \boldsymbol{\Gamma}\tau(\boldsymbol{\Xi}_2) \end{bmatrix} \tag{4.21}$$

4.6 Numerical Results

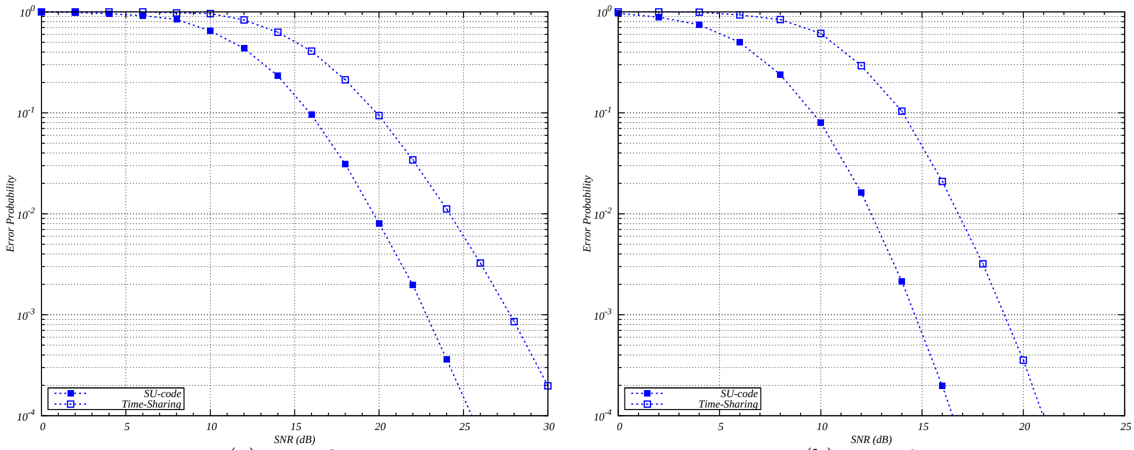
In this section, a two-user MAC with two transmit antennas and an arbitrary number of receive antennas is considered. A preliminary analysis based on the outage behavior of the channel is first presented giving insights on the expected achievable performance.

The error performance of the proposed MIMO-BB code as well as Lu et *al.*'s code is then presented, in terms of the total error probability and the the probability of the error event $\boxed{2}$ that highlights the advantages of the joint code design.

4.6.1 Outage Probability

Fig. 4.5(a) and 4.5(b) illustrate the outage behavior of the two-user MAC with $n_r = 2$ and $n_r = 4$, respectively. The total outage probability, *i.e.,* the probability of the outage event $\mathcal{O}$ where at least one user is in outage, as well as the probability of the outage event $\mathcal{O}_2$ where both users are in outage are shown. For comparison purposes, the outage probability of the channel when the time-sharing technique is considered is also shown. The spectral efficiency is fixed to $R = 4$ bits pcu.

These outage curves show that the two schemes (multiple-access and time-sharing) achieve the same diversity order ($d = 2n_r$) but time-sharing is significantly sub-optimal. This observation numerically confirms the DMT analysis presented in Subsection 1.4.6. This is explained by the fact that the time-sharing scheme does not optimally exploit the spatial degrees of freedom of the channel. It is obvious that, to do so, the users should transmit simultaneously. This outage analysis shows that, with $n_r = 2$, a gain of 6.5 dB



(a) $n_r = 2$ (b) $n_r = 4$

Figure 4.6: Error probability of the Single-User (SU) code and Time-Sharing, $K = n_t = 2$, 4-QAM (4 bits pcu).

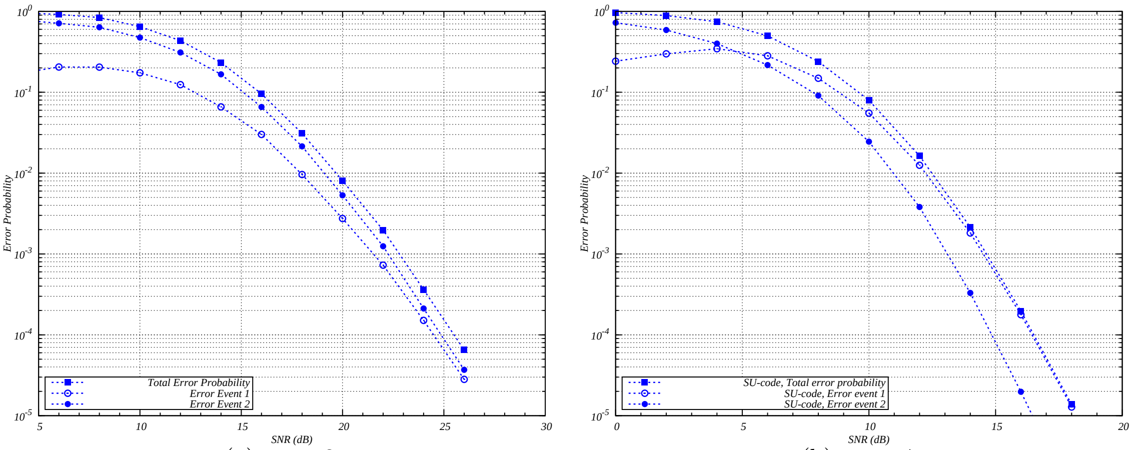
is to be expected if the users are transmitting simultaneously using an optimal coding scheme. This gain equals 5 dB for $n_r = 4$.

4.6.2 Single-User Code

What if the users transmit simultaneously the optimal single-user STBC?

Let us study the performance of the single-user code in the multiple-access scenario. Assume that the users are transmitting simultaneously using the golden code. The corresponding error probability is shown in Fig. 4.6(a) and (b) together with the time-sharing scheme for $n_r = 2$ and 4, respectively.

The two schemes achieve the same diversity order $d = 2$ but the SU-code offers a significant coding gain. For $n_r = 2$, this gain is equal to 5 dB at $P_e = 10^{-4}$ while for $n_r = 4$, it is equal to 4.4 dB. The use of the optimal single-user code at the transmission allows the users to simultaneously exploit their individual channel's degrees of freedom which explains the performance improvement. However, as observed in the outage behaviors analysis, a better gain can be obtained using a more convenient coding scheme, especially for a small number of receive antennas. Indeed, as already explained in Subsection 1.4.6, when the number of receive antennas increases, the impact of the antenna pooling regime on the achievable DMT region decreases. In terms of error probability, this means that the impact of the probability of error event $\boxed{2}$ on the total error probability decreases when the number of receive antennas increases. This is confirmed numerically in Fig. 4.7(a) and 4.7(b) that illustrate the different error events probabilities for $n_r = 2$ and $n_r = 4$, respectively.



(a) $n_r = 2$ (b) $n_r = 4$

Figure 4.7: Different error events of the Single-User (SU) code, $K = n_t$, 4-QAM (4 bits pcu).

4.6.3 Error Probability

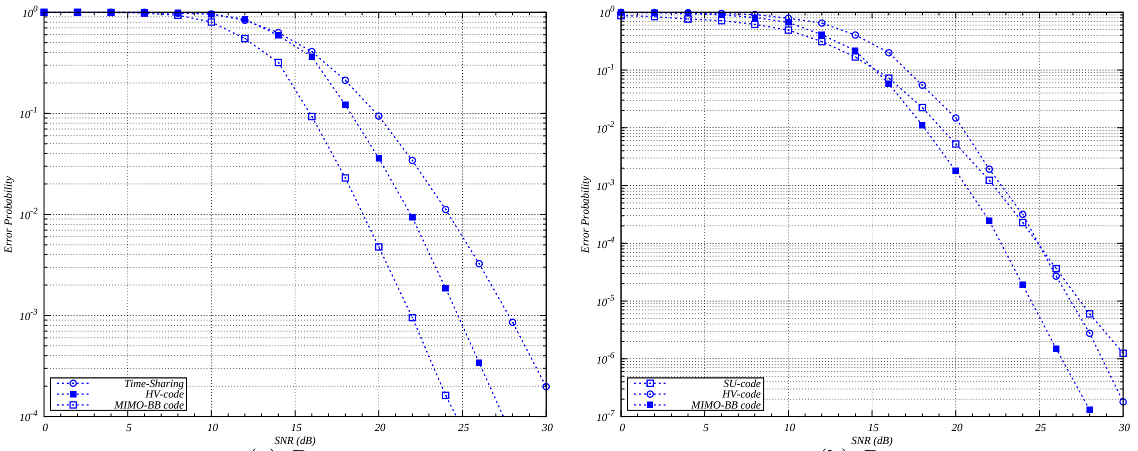
Fig. 4.8(a) shows a comparison between the performance of the MIMO-BB code, the HV-code and time-sharing. The three schemes achieve the same diversity order, $n_t n_r = 4$. Both HV-code and MIMO-BB code offer a significant performance gain (3.2 dB and 6 dB, respectively, at $P_e = 10^{-4}$) compared to the time-sharing scheme that makes a very poor use of the available degrees of freedom. Moreover, it can be noticed that the MIMO-BB code outperforms the HV-code (by 2.8 dB). Interestingly, compared to the outage performance of the channel (4.5(a)), the MIMO-BB code seems to have similar behaviors. This proves (numerically) the optimality of the proposed coding scheme.

The probability of error event $\boxed{2}$ is plotted in Fig. 4.8(b) where the SU-code performance is also considered. It is clear that both the MIMO-BB code and the HV-code have an equal diversity order while the SU-code has a smaller diversity order. Nevertheless, the SU-code outperforms HV-code for a large range of SNR. For high SNR, the higher diversity order of HV-code explains its better performance. On the other hand, the MIMO-BB code outperforms the two other codes for a large SNR range.

The MIMO-BB code is finally compared to Lu et *al.*'s code in Fig. 4.9 and 4.10 for $n_r = 2$ and $n_r = 4$, respectively. For $n_r = 2$, the two codes have the same error behavior while for $n_r = 4$, Lu et *al.*'s code slightly outperforms the MIMO-BB code.

4.6.4 The influence of Γ

The importance of the matrix $\mathbf{\Gamma}$ that has been added to guarantee the full rank property, is studied in Fig. 4.11 and 4.12 in terms of the total error probability and the probability of event $\boxed{2}$, respectively. Fig. 4.11 shows that it is irrelevant whether the matrix Γ is added or not. In other words, verifying the full-rank property of the code appears to be optional in the SNR range of the simulations. Indeed, in the considered SNR range, even the error



(a) P_e (b) $P_{\mathcal{E}_2}$

Figure 4.8: Error performance, MIMO-BB code, $K = n_t = 2$, 4-QAM.

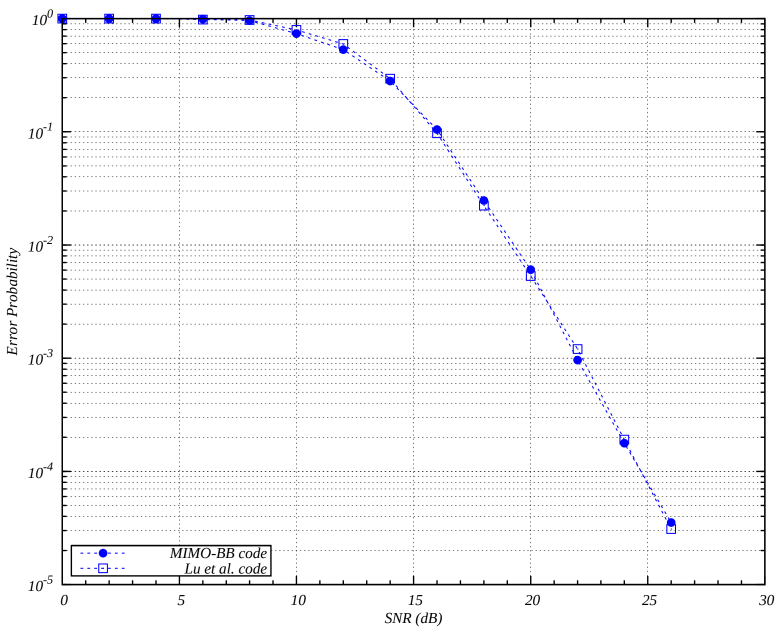


Figure 4.9: Error probability of the MIMO-BB code and Lu et $al.$'s code, $K = n_t = n_r = 2$, 4-QAM.

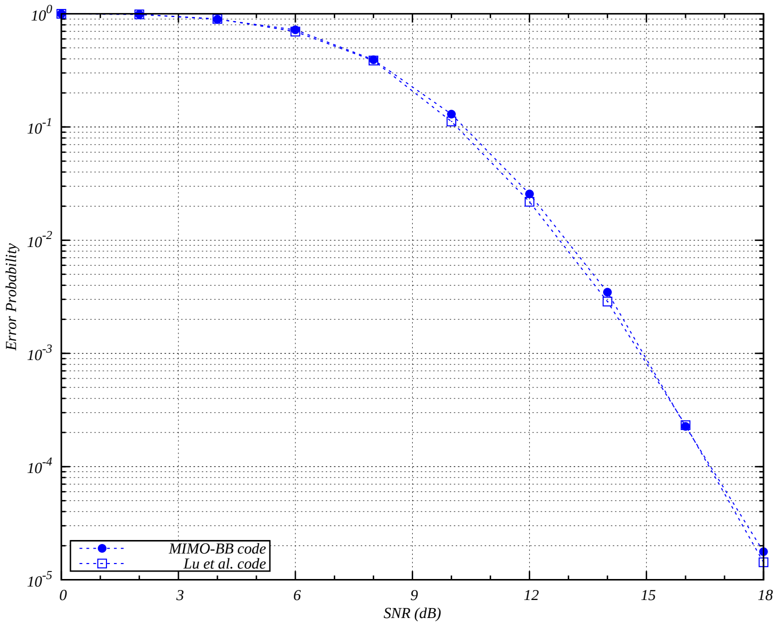


Figure 4.10: Error probability of the MIMO-BB code and Lu et *al.*'s code, $K = n_t = 2$, $n_r = 4$, 4-QAM.

probability of the event where both users are in error is not affected by the presence of matrix $\mathbf{\Gamma}$.

Remark 4.6.1 *This observation is in line with the results presented by Lu et al. in [19] that drew the attention on the irrelevance of* $\mathbf{\Gamma}$*. The authors derived in this paper a relaxed DMT-optimal design criteria allowing the codeword matrix to be singular, i.e., omitting* $\mathbf{\Gamma}$*. The probability of this event was shown numerically to be extremely small and to get closer to zero when the size of the constellation increases. Therefore, even without the matrix* $\mathbf{\Gamma}$*, the code is of full rank with a probability close to 1.*

4.7 DMT Analysis

The best DMT that a coding scheme can achieve in the multiple-antenna scenario, is given in Theorem 1.4.3 by

$$d_{\text{MA-MAC}}(r) = \begin{cases} d_{n_t,n_r}(r), & r \leq \min(1, \frac{n_r}{K+1}) \\ \\ d_{Kn_t,n_r}(Kr), & r \geq \min(1, \frac{n_r}{K+1}) \end{cases} \tag{4.22}$$

According to Theorem 2.2.2, a family of codes $\mathcal{X}(\mathsf{SNR})$ is optimal in the sense of the DMT if, for the dominant outage set, *i.e.*, for $s = s^*$, it satisfies (2.37),

$$\Psi_{m_{s^*}}^{s^* n_t}(\mathsf{SNR}) \quad \dot{\geq} \quad \mathsf{SNR}^{-(s^* r - \epsilon)} \tag{4.23}$$

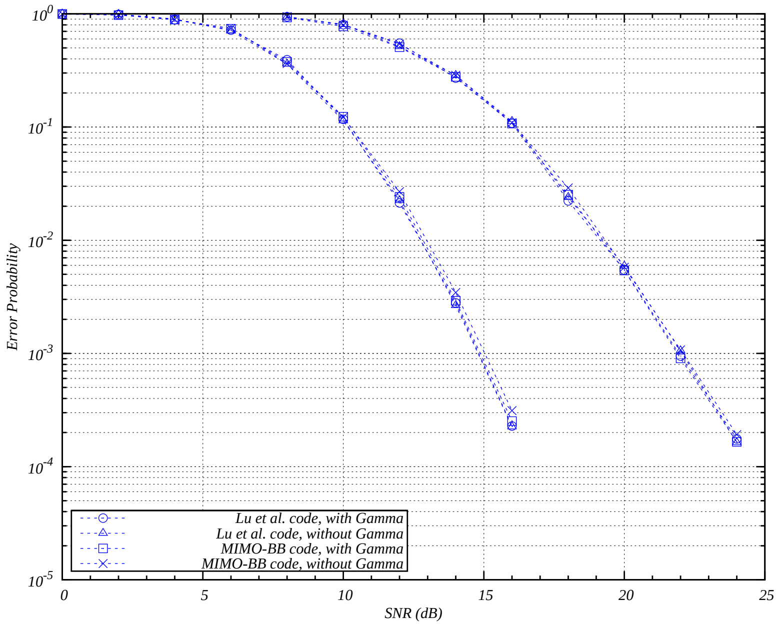


Figure 4.11: The influence of Γ, P_e of the MIMO-BB code and Lu et *al.*'s code, $K = n_t = 2$, 4-QAM.

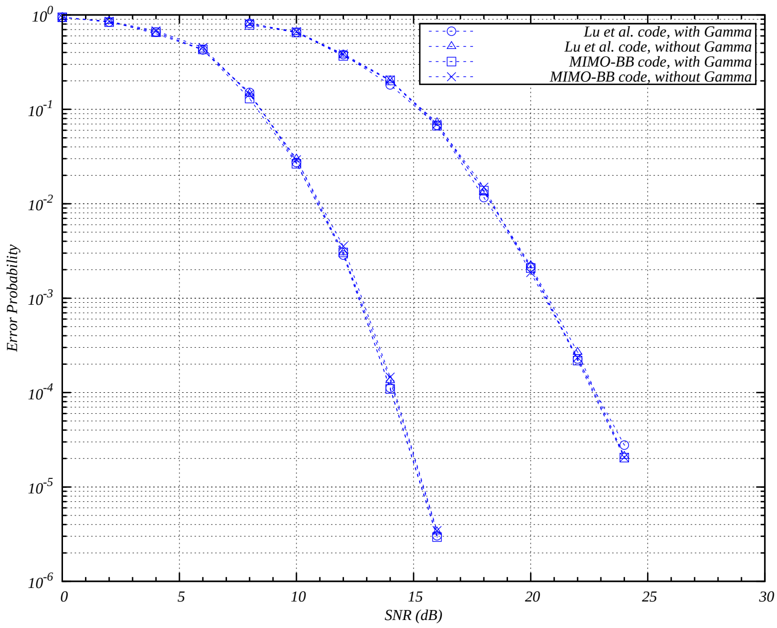


Figure 4.12: The influence of Γ, $P_{\mathcal{E}_2}$ of the MIMO-BB code and Lu et *al.*'s code, $K = n_t = 2$, 4-QAM.

and for every other set, *i.e.,* $s \neq s^*$, the following condition is satisfied:

$$\Psi_{m_s}^{sn_t}(\mathsf{SNR}) \ \dot{\geq} \ \mathsf{SNR}^{-(\rho_s(r)-\epsilon)} \tag{4.24}$$

where

$$0 \leq \rho_s(r) \leq r_{sn_t,n_r}\left(d_{s^*n_t,n_r}(s^*r)\right) \tag{4.25}$$

Lemma 4.7.1 *Consider a $n \times nm$ matrix $\boldsymbol{A}$ resulting form the horizontal concatenation of m $n \times n$ matrices $\boldsymbol{A}_i$, i.e.,*

$$\boldsymbol{A} = \left[\begin{array}{cccc} \boldsymbol{A}_1 & \boldsymbol{A}_2 & \ldots & \boldsymbol{A}_m \end{array}\right].$$

It can be verified that:

$$\det\left(\boldsymbol{A}\boldsymbol{A}^\dagger\right) \ \geq \ m^n \left(\prod_{i=1}^{m} |\det(\boldsymbol{A}_i)|\right)^{2/m}.$$

MIMO-BB coding scheme

Consider the coding scheme proposed in Section 4.4 and let the data rate $R(\mathsf{SNR})$ scale as $r\log\mathsf{SNR}$ and assume $n_r \geq Kn_t$. Each user k transmits Kn_t^2 information symbols, $s_{k,l}$, chosen from the following QAM constellation:

$$\mathcal{C}_k(\mathsf{SNR}) = \left\{(m+in) : \frac{-\mathsf{SNR}^{\frac{r}{2n_t}}}{2} \leq m,n \leq \frac{\mathsf{SNR}^{\frac{r}{2n_t}}}{2}\right\}, \quad m,n \in \mathbb{Z}. \tag{4.26}$$

These information symbols are then coded into the following codeword matrix:

$$\boldsymbol{X}_k = \left[\begin{array}{cccc} \boldsymbol{\Xi}_k & \tau(\boldsymbol{\Xi}_k) & \ldots & \tau^{K-1}(\boldsymbol{\Xi}_k) \end{array}\right]$$

that satisfies:

$$\begin{aligned}
\max\|\boldsymbol{X}_k\|_{\mathrm{F}}^2 &= \max\left(\sum_{i=1}^{K}\|\tau^{i-1}(\boldsymbol{\Xi}_k)\|_{\mathrm{F}}^2\right) \\
&= \sum_{i=1}^{K} T_i n_t \left(\frac{\mathsf{SNR}^{\frac{r}{n_t}}}{2}\right) \\
&= T n_t \left(\frac{\mathsf{SNR}^{\frac{r}{n_t}}}{2}\right)
\end{aligned} \tag{4.27}$$

where T_i is the length of each block $\boldsymbol{\Xi}_k$ of the codeword matrix $\boldsymbol{X}_k$. Define $\kappa^2 = \frac{\mathsf{SNR}^{\frac{r}{n_t}}}{2}$, used to scale the transmit codeword as follows:

$$\tilde{\boldsymbol{X}}_k = \frac{1}{\kappa}\boldsymbol{X}_k$$

$\tilde{\boldsymbol{X}}_k$ satisfies the power constraint (1.1), *i.e.,* $\max \|\tilde{\boldsymbol{X}}_k\|_{\mathrm{F}}^2 = Tn_t$. Let $\tilde{\boldsymbol{D}}_k$ denote the per user difference matrix:

$$\begin{aligned} \tilde{\boldsymbol{D}}_k &= \tilde{\boldsymbol{X}}_{\boldsymbol{k}} - \tilde{\boldsymbol{X}}'_{\boldsymbol{k}} \\ &= \frac{1}{\kappa}\left[\begin{array}{cccc} \boldsymbol{\Xi}_k - \boldsymbol{\Xi}'_k & \tau(\boldsymbol{\Xi}_k - \boldsymbol{\Xi}'_k) & \ldots & \tau^{K-1}(\boldsymbol{\Xi}_k - \boldsymbol{\Xi}'_k) \end{array}\right] \end{aligned} \tag{4.28}$$

For $s = 1$ and $m_s = \min(sn_t, n_r) = n_t$, one can write:

$$\begin{aligned} \Psi_{n_t}^{n_t}(\mathsf{SNR}) &= \min_{\tilde{\boldsymbol{D}}_k = \tilde{\boldsymbol{X}}_k - \tilde{\boldsymbol{X}}'_k} \prod_{l=1}^{n_t} \lambda_l = \min_{\tilde{\boldsymbol{D}}_k = \tilde{\boldsymbol{X}}_k - \tilde{\boldsymbol{X}}'_k} |\det \tilde{\boldsymbol{D}}_{\boldsymbol{k}}|^2 \\ &= \kappa^{-2n_t} \min_{\boldsymbol{D} = \boldsymbol{X} - \boldsymbol{X}'} |\det \boldsymbol{D}_k|^2 \qquad (4.29) \\ &\geq \kappa^{-2n_t} K^{n_t} \left(\prod_{k=1}^{K} \det\left(\tau^{k-1}(\boldsymbol{\Xi}_k - \boldsymbol{\Xi}'_k)\right) \right)^{2/K} \qquad (4.30) \\ &\geq \kappa^{-2n_t} K^{n_t} \left(\prod_{k=1}^{K} \tau^{k-1} \left(\det\left(\boldsymbol{\Xi}_k - \boldsymbol{\Xi}'_k\right)\right) \right)^{2/K} \end{aligned}$$

where (4.30) follows from (4.29) using Lemma 4.7.1. Hence,

$$\Psi_{n_t}^{n_t}(\mathsf{SNR}) \ \dot{\geq} \ \mathsf{SNR}^{-(r-\epsilon)} \tag{4.31}$$

resulting from the non-vanishing determinant of each block $\boldsymbol{\Xi}_k$. For $s = K$, the overall codeword matrix $\boldsymbol{X}$ is given in (4.9) and is scaled as follows:

$$\tilde{\boldsymbol{X}} = \frac{1}{\kappa}\boldsymbol{X}$$

Let $\tilde{\boldsymbol{D}}$ denote the overall difference matrix, $\tilde{\boldsymbol{X}} - \tilde{\boldsymbol{X}}'$. $m_s = Kn_t$ and

$$\begin{aligned} \Psi_{Kn_t}^{Kn_t}(\mathsf{SNR}) &= \min_{\tilde{\boldsymbol{D}} = \tilde{\boldsymbol{X}} - \tilde{\boldsymbol{X}}'} \prod_{l=1}^{Kn_t} \lambda_l = \min_{\tilde{\boldsymbol{D}} = \tilde{\boldsymbol{X}} - \tilde{\boldsymbol{X}}'} |\det \tilde{\boldsymbol{D}}|^2 \\ &= \kappa^{-2Kn_t} \min_{\boldsymbol{D} = \boldsymbol{X} - \boldsymbol{X}'} |\det \boldsymbol{D}|^2 \\ &\doteq \mathsf{SNR}^{-(Kr-\epsilon')}\, \Omega(\mathsf{SNR}) \qquad (4.32) \end{aligned}$$

where

$$\Omega(\mathsf{SNR}) = \min_{\boldsymbol{D} = \boldsymbol{X} - \boldsymbol{X}'} |\det \boldsymbol{D}|^2 \tag{4.33}$$

Let β denote the decay exponent of the determinant of the considered coding scheme, *i.e.,*

$$\lim_{\mathsf{SNR}\to\infty} -\frac{\log \Omega(\mathsf{SNR})}{\log \mathsf{SNR}} = \beta$$

such that (4.32) can be rewritten as

$$\Psi_{Kn_t}^{Kn_t}(\mathsf{SNR}) \;=\; \mathsf{SNR}^{-(Kr+\beta-\epsilon^{'})} \tag{4.34}$$

Example: Two-user MAC

Consider the two-user MIMO-MAC with $n_t = 2$, $n_r = 4$ and $m_s = \min(2s, n_r) = 2s$. (4.31) and (4.34) are in this case given by:

$$\Psi_2^2(\mathsf{SNR}) \;\dot{\geq}\; \mathsf{SNR}^{-(r-\epsilon)} \tag{4.35}$$

$$\Psi_4^4(\mathsf{SNR}) \;\geq\; \mathsf{SNR}^{-(2r+\beta-\epsilon)} \tag{4.36}$$

Consider the case where the dominant outage set is $\mathcal{S}^* = \{1, 2\}$, or equivalently, when the system is operating in the antenna pooling regime, $r \geq 4/3$. In this regime, $\Psi_4^4(\mathsf{SNR})$ should satisfy condition (4.23) for $s^* = 2$. This is only possible in case of a sub-polynomial decay of $\Omega(\mathsf{SNR})$ ($\beta = 0$). Otherwise, (4.36) clearly decays faster than required in (4.23).

In the single-user regime, *i.e.,* $r \leq 4/3$ and $s^* = 1$, $\Psi_2^2(\mathsf{SNR})$ satisfies (4.23). On the other hand, in order for $\Psi_4^4(\mathsf{SNR})$ to satisfy (4.24) for $s = 2$, the following condition should be satisfied:

$$2r + \beta \leq r_{4,4}(d_{2,4}(r)) \tag{4.37}$$

However, the decay behavior of the minimum determinant of the proposed MIMO-BB code $\Omega(\mathsf{SNR})$ is yet to be determined.

Chapter 5

Space-Time Block Codes Construction for the Multiple-Access Relay Channel

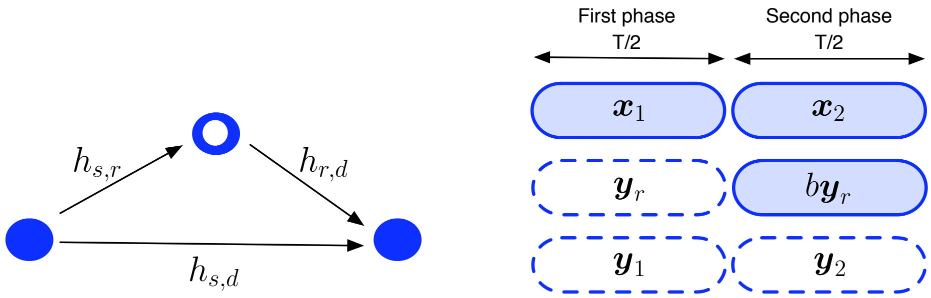
Cooperative diversity techniques such as relaying, known to significantly improve the reliability of a wireless network, have received considerable interest in the last few years. Relaying helps a source in exploiting the distributed spatial diversity in order to combat the channel fading. Since its introduction in [51, 52], this subject has been largely studied in the literature [53, 54, 55, 56, 57, 58]. Different cooperation protocols have been developed and can be categorized into two principle classes, the amplify-and-forward (AF) and the decode-and-forward (DF). In the former, the relays simply amplify the received signal before forwarding it, whereas with the DF strategy, the relays decode the signal, re-encode it and re-transmit it to the destination once decoded reliably.

In this chapter, we consider relaying in a multiple-access scenario where the terminals are all half-duplex, *i.e.,* they cannot transmit and receive simultaneously. In practice, most cooperative techniques suffer from a significantly high complexity and cost due to the coordination required among terminals. This is not the case here where an alternative scheme, the Multiple-Access Relay (MAR) channel, is considered. In a MAR channel, first introduced in [20], multiple users communicate with a single destination with the help of one or more relays. The users are not allowed to cooperate together and need not to be aware of the existence of the relays, leading to a reduced cost and complexity. We focus on the two-user single-relay MAR channel with single-antenna terminals. A practical application of this model might be a network where two terminals cannot cooperate to help each other *i.e.,* due to practical limitations, but can both send their information to another terminal of the network. Sharing this terminal helps the transmitting terminals reach their destination and significantly improves the reliability of the communication.

The cooperative MAC, where the users cooperate together, was compared to the MAR channel in [59] using a partial decode-and-forward strategy where the half-duplex relay only decodes a part of each user's transmitted message. The MAR channel is shown to achieve higher rates than the cooperative MAC. Using a full-duplex relay, Kramer et *al.* showed in [60, Theorem 10] that the decode-and-forward strategy achieves the capacity of the AWGN-MAR channel if the users and the relay are geometrically close to each other. In [4], the Dynamic Decode-Forward (DDF) protocol was applied to the MAR channel. In the DDF strategy, both users transmit their information symbols throughout an entire block. The relay decodes the information it receives only when it has sufficient information for a correct detection. Then the relay re-encodes the message and transmits it to the destination. It is shown in [4] that the DDF protocol achieves the optimal DMT for low multiplexing gains while being suboptimal for high multiplexing gains. The Compress-and-Forward (CF) relaying strategy was applied to the MAR channel in [61]. In the CF protocol, the relay uses source coding to compress its received signal and forward it to the destination. It was shown that this strategy achieves the optimal DMT for high multiplexing gains, however it suffers from a diversity loss for a low multiplexing gain. In a recent work, Azarian et *al.* [4] derived an upper bound on the achievable DMT for the MAR channel. As in the case of point-to-point and multiple-access systems, the DMT framework turns out to be an interesting theoretical tool to evaluate the behavior and compare different cooperative strategies.

This work focuses on the class of AF protocols for two reasons. First, these have a low relaying complexity. In fact, avoiding power-consuming data processing makes the cooperation practically implementable for small terminals such as the mobile handsets and sensors. Second, the linearity nature of the AF facilitates the analysis of the system performance, since an AF scheme converts the network into an equivalent multiple-input multiple-output multiple-access channel (MIMO-MAC). It is thus possible to apply the results obtained for the MIMO-MAC to the MAR channel. More precisely, we consider the Multi-Access Amplify-and-Forward (MAF) protocol introduced by Chen et *al.* in [5] assuming a half-duplex relay. The MAF protocol was shown to provide significant gains in addition to the complexity reduction it offers compared to other protocols. Moreover, authors in [5] derived the DMT of the two-user MAF relay channel and showed that their protocol achieves the optimal DMT in some cases. Based on the DMT framework, the main contribution of this chapter is the construction of a new family of distributed space-time block codes for a K-user MAF relay channel by applying the code designed for the MIMO-MAC in Chapter 4 in a distributed way.

The rest of the chapter is divided into two main sections. In section 5.1, the single-user relay channel is considered. After modeling the channel, the Amplify-and-Forward protocol is defined in the single-user scenario and it is shown that the channel is equivalent in this case to a virtual point-to-point MIMO channel in Subsections 5.1.1 and 5.1.2, respectively. In Subsection 5.1.3, it is shown how single-user STBCs can be applied to the relay channel in a distributed manner. In the second part of this chapter (Section 5.2), the multiple-access relay channel is considered. We start by modeling the system and defining the assumptions we made. The MAF protocol is defined in Subsection 5.2.1 and the MAF relay channel is shown to be equivalent to a *virtual* MIMO-MAC in Subsection 5.2.2. A DMT analysis comparing different multiple-access scenarios and showing the advantages



(a) Single-user relay channel (b) Cooperation frame structure

Figure 5.1: Cooperation in a half-duplex single-user NAF relay channel.

of relaying is presented in Subsection 5.2.3. Subsection 5.2.4 shows the construction of the proposed coding scheme followed by an explicit example and some numerical results.

5.1 The Single-User Relay Channel

In this first section, the single-user relay channel illustrated in Fig. 5.1(a) is considered. A source is communicating with its destination with the help of a single half-duplex relay. A half-duplex relay cannot transmit and receive at the same time. The variables $h_{s,r}$, $h_{r,d}$ and $h_{s,d}$ represent the source-relay, relay-destination and source-destination channel coefficients, respectively. The source transmits its signal to both the destination (via a direct link) and the relay. With such cooperation, the reliability of the communication may be significantly improved. Indeed, if the direct link is in deep fade or under shadowing, exploiting the distributed spatial diversity by transmitting the source signal by the relay, helps the destination better decode the received signal.

Different relaying strategies were considered in the literature. In order to evaluate and compare the performance of different relaying strategies, the DMT framework is used. The cut-set bound is used to upper-bound the DMT and thus, to determine the best DMT that a relaying strategy can achieve.

Proposition 5.1.1 *The optimal DMT for a single-user relay channel, and equivalently the DMT of any relaying strategy $\mathcal{R}$, is upper-bounded by*

$$d_{\mathcal{R}}(r) \leq \min\{d_{s,rd}(r), d_{sr,d}(r)\} \tag{5.1}$$

where $d_{s,rd}(r)$ and $d_{sr,d}(r)$ are the DMTs of the channel from the two possible cuts.

Example 5.1.1 *Consider a single-user relay channel with single-antenna terminals. In this case, the first possible cut leads to a 1×2 SIMO channel and the second cut leads to a 2×1 MISO channel. $d_{s,rd}(r)$ and $d_{sr,d}(r)$ are both equal to $2(1-r)^+$ in this case and the DMT of any relaying strategy $\mathcal{R}$ is upper-bounded by*

$$d_{\mathcal{R}}(r) \leq 2(1-r) \tag{5.2}$$

5.1.1 The Amplify-and-Forward Protocol

As already stated, we focus in this work on the amplify-and-forward class of relaying strategies characterized by their low complexity. Depending on whether the source is allowed to transmit simultaneously with the relay or not, leads to a non-orthogonal scheme (NAF) or an orthogonal scheme (OAF). Each cooperative protocol is described by its cooperation frame structure determining the role (transmitting/listening) of each node at each phase of the transmission. Without loss of generality, the cooperation frame consists of two phases, each composed of $T/2$ symbols times, as illustrated in Fig. 5.1 where T denotes the cooperation frame length. In the first phase, the source transmits while the destination is listening. The relay scales the received signal and re-transmits it in the second phase. If the source does not transmit during the second phase, the scheme is orthogonal (OAF). Otherwise, the scheme is non-orthogonal (NAF) and clearly achieves the maximum multiplexing gain in contrast with the OAF. The NAF is shown to be the optimal cooperative scheme for the AF single-relay single-antenna case.

Theorem 5.1.1 *The NAF protocol achieves the optimal DMT of an AF single-relay channel with single-antenna terminals:*

$$d_{NAF}(r) = (1-r) + (1-2r) \tag{5.3}$$

Proof: See [56]. ∎

5.1.2 Virtual MIMO Channel

Cooperation is known to be a solution to benefit from the advantages of MIMO systems when the number of antennas at the terminals is limited due to practical constraints. Indeed, the re-transmission of the source signal by the relay creates a virtual antenna array and the cooperative network can be modeled as a virtual MIMO channel.

In order to show how to obtain the equivalent MIMO channel, consider the single-antenna scenario. T denotes the cooperation frame length assumed to be smaller than the channel coherence time and thus, the channel is assumed to stay constant during the transmission of a frame. For the NAF scheme whose corresponding cooperation frame is illustrated in Fig. 5.1(b), the following signal model is obtained:

$$\begin{cases} \boldsymbol{y}_1 = \sqrt{P_1} h_{s,d} \boldsymbol{x}_1 + \boldsymbol{z}_1 \\ \boldsymbol{y}_r = \sqrt{P_1} h_{s,r} \boldsymbol{x}_1 + \boldsymbol{w} \\ \boldsymbol{y}_2 = \sqrt{P_r} h_{r,d} (b \boldsymbol{y}_r) + \sqrt{P_2} h_{s,d} \boldsymbol{x}_2 + \boldsymbol{z}_2 \end{cases} \tag{5.4}$$

where $\boldsymbol{x}_i, \boldsymbol{y}_i \in \mathbb{C}^{T/2}, i = 1,2$ are the transmitted signals from the source and the received signals at the destination, respectively, in the i^{th} slot. $\boldsymbol{y}_r \in \mathbb{C}^{T/2}$ is the received signal at the relay in the first slot. $\boldsymbol{z}_i$ and $\boldsymbol{w} \in \mathbb{C}^{T/2}$ are independent AWGN with normalized i.i.d. entries. b is the normalization factor satisfying $\mathbb{E}\left\{\|b\boldsymbol{y}_r\|^2\right\} \leq \frac{T}{2}\mathsf{SNR}$. P_1, P_2 and P_r denote the transmission powers of the source during the first and second slot and the transmission

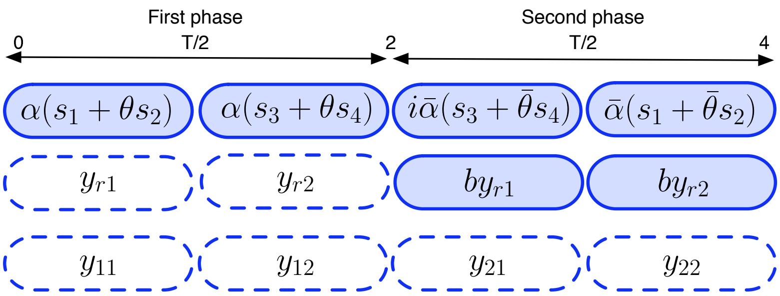


Figure 5.2: NAF cooperation frame with the distributed Golden code.

power of the relay, respectively. Each power is a fraction of the average received SNR at each time slot. The channel model (5.4) can be rewritten as

$$\boldsymbol{Y} = \begin{bmatrix} \sqrt{P_1}h_{s,d} & 0 \\ \sqrt{P_1P_3}bh_{s,r}h_{r,d} & \sqrt{P_2}h_{s,d} \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} 0 & 0 \\ \sqrt{P_3}bh_{r,d} & 0 \end{bmatrix} \boldsymbol{W} + \boldsymbol{Z} \tag{5.5}$$

where $\boldsymbol{A} \triangleq \begin{bmatrix} \boldsymbol{a}_1^\top & \boldsymbol{a}_2^\top \end{bmatrix}^\top$, $\boldsymbol{A}$ replacing $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{W}$ and $\boldsymbol{Z}$. After whitening the noise and normalizing the system, the equivalent channel model can be written as

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{H}}\boldsymbol{X} + \tilde{\boldsymbol{Z}} \tag{5.6}$$

where $\tilde{\boldsymbol{Z}}$ is the equivalent whitened noise and $\tilde{\boldsymbol{H}}$ the equivalent channel matrix

$$\tilde{\boldsymbol{H}} \triangleq \begin{bmatrix} \sqrt{P_1}h_{s,d} & 0 \\ \sqrt{\frac{P_1P_3}{1+P_r|bh_{r,d}|^2}}bh_{s,r}h_{r,d} & \sqrt{\frac{P_2}{1+P_r|bh_{r,d}|^2}}h_{s,d} \end{bmatrix} \tag{5.7}$$

5.1.3 Distributed Space-Time Block codes

Since the relay channel can be viewed as a virtual MIMO channel, STBCs already designed for point-to-point MIMO channels [13, 8, 12] can be used for the relay channel in a distributed way. In [62, 63], the authors proposed algebraic distributed STBCs that are optimal in the sense of the DMT.

Example 5.1.2 *(Distributed Golden Code) The DMT of the NAF cooperative protocol given in (5.3) can be achieved using the optimal 2×2 STBC, the Golden code, in a distributed manner [62]. The codeword matrix of the Golden code, that is denoted here $\boldsymbol{M}$ (mother codeword), is given in (2.23). When applied in a distributed way, the codeword matrix is as follows*

$$\boldsymbol{X} \triangleq [\boldsymbol{M}(1:1, 1:2) \;\; \boldsymbol{M}(2:2, 1:2)] \tag{5.8}$$

Equivalently, the first row of $\boldsymbol{M}$ is transmitted by the source during the first phase. During the second phase, the source transmits the second row of $\boldsymbol{M}$ while the relay transmits a scaled version of the first row. The corresponding cooperation frame of length $T = 4$ is

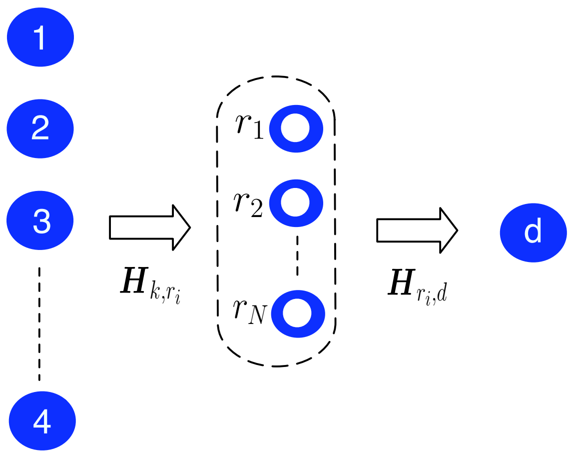


Figure 5.3: The general multiple-access relay channel.

illustrated in Fig. 5.2 where plain line slots represent transmission mode and dashed line slots represent listening mode.

5.2 The Multiple-Access Relay Channel

The general K-user MAR channel is a K-user MAC with N relays helping the users to communicate with the destination while the cooperation between the users is not allowed. Let n_t, n_r and n_d denote the number of antennas at each user, at the relay and at the destination, respectively. The channel model is illustrated in Fig. 5.3, where $\boldsymbol{H}_{k,r_i} \in \mathbb{C}^{n_d \times n_t}$, $\boldsymbol{H}_{r_i,d} \in \mathbb{C}^{n_r \times n_t}$ and $\boldsymbol{H}_{k,d} \in \mathbb{C}^{n_d \times n_r}$, $k = 1, \ldots, K$, $i = 1, \ldots, N$ are independent channel matrices modeling the k^{th} user-destination, k^{th} user-ith relay and i^{th} relay-destination MIMO channels, respectively. All these matrices have zero-mean unit variance $i.i.d.$ Gaussian entries, $i.e.$, $h_{i,j} \sim \mathcal{CN}(0, 1)$.

All the fading coefficients remain constant within one cooperation frame of length T but change independently from one frame to the other. It is assumed that the CSI can be tracked at the receiver, though it is not available at the transmitters. Note that it is assumed that the receiver has knowledge of all CSIs, including those of the user-relay links. In the following, we focus on the single-relay scenario, $N = 1$.

5.2.1 Multi-Access Amplify-and-Forward

The Multi-Access Amplify-and-Forward (MAF) protocol was recently introduced by Chen et *al.* in [5] for a MAR channel with a single relay. The main advantage of the MAF is the balance between complexity and performance it provides, compared to other protocols, such as the DDF and CF that add a significant complexity to the relaying terminal. In addition to the low complexity of the MAF, authors in [5] showed that it outperforms both the DDF in the high multiplexing regime and the CF protocol in the low multiplexing regime.

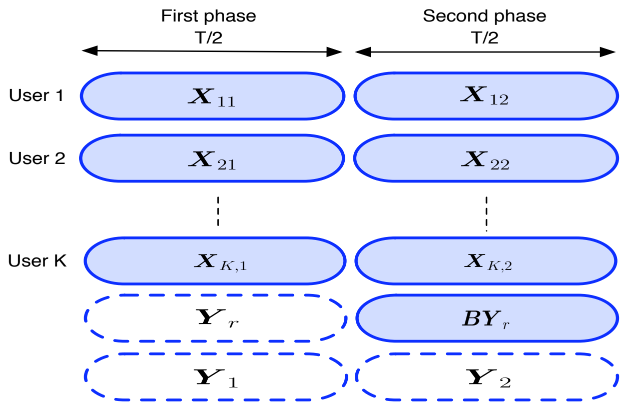


Figure 5.4: MAF cooperation frame structure.

In the MAF protocol both users transmit their informations throughout an entire cooperation frame (two phases). Due to the half-duplex constraint, the relay listens to both users during the first phase, then, during the second phase, it simply scales the source signal before forwarding it to the destination. A MAF cooperation frame can be illustrated as in the Fig. 5.4 where plain line slots represent transmission mode, whereas dashed slots represent listening mode. The matrices $\boldsymbol{X}_{kt} \in \mathbb{C}^{n_t \times \frac{T}{2}}$ with *i.i.d.* unit variance entries, represent the Space-Time codeword matrices transmitted from the k^{th} user at the t^{th} slot. $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ are the received signal at the destination during the first and second phase, respectively. $\boldsymbol{Y}_r$ is the received signal at the relay during the first phase.

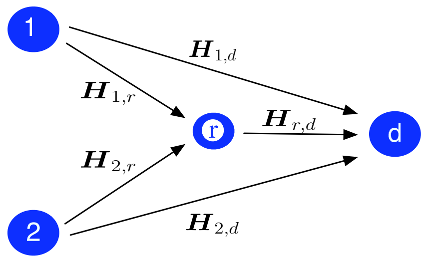
5.2.2 Signal Model: Virtual MIMO-MAC

Based on the cooperation frame structure described above, the following signal model is obtained:

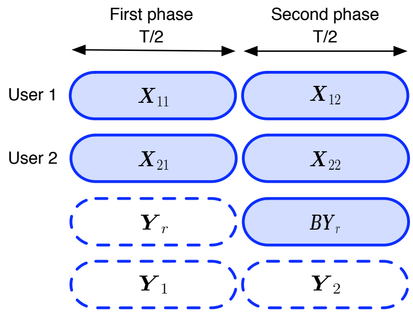
$$\begin{cases} \boldsymbol{Y}_1 = \sum_k \sqrt{P_{k1}} \boldsymbol{H}_{k,d} \boldsymbol{X}_{k1} + \boldsymbol{V}_1 \\ \boldsymbol{Y}_r = \sum_k \sqrt{P_{k1}} \boldsymbol{H}_{k,r} \boldsymbol{X}_{k1} + \boldsymbol{W} \\ \boldsymbol{Y}_2 = \sqrt{P_r} \boldsymbol{H}_{r,d} \boldsymbol{B} \boldsymbol{Y}_r + \sum_k \sqrt{P_{k2}} \boldsymbol{H}_{k,d} \boldsymbol{X}_{k2} + \boldsymbol{V}_2 \end{cases} \tag{5.9}$$

where $\boldsymbol{H}_{k,d}$ and $\boldsymbol{H}_{k,r}$, $k = 1, \ldots, K$, denote the k^{th} user-destination and the k^{th} user-relay channels, respectively. $\boldsymbol{V}_1$, $\boldsymbol{V}_2$ and $\boldsymbol{W}$ are independent AWGN matrices with normalized *i.i.d.* entries. P_{kt}, $t = 1, 2$ and P_r denote user k's transmission power at the t^{th} slot and the relay's transmission power, respectively.

Each power is a fraction (π_{kt} and π_r) of the average received SNR at the destination. The total transmit power in both time slots is $(n_t \sum_{k,t} \pi_{kt} + n_r \pi_r)$SNR. Since the channel coefficients and the AWGN are normalized, $(n_t \sum_{k,t} \pi_{kt} + n_r \pi_r)$SNR represents the average received SNR for both time slots. The values of π's are chosen such that they satisfy $n_t \sum_{k,t} \pi_{kt} + n_r \pi_r = 2$. $\boldsymbol{B}$ is an $n_r \times n_r$ normalization matrix subject to the power constraint



(a) Two-user MAF relay channel



(b) Cooperation frame structure

Figure 5.5: Cooperation in a half-duplex two-user MAF relay channel.

$$\mathsf{E}\left\{||\boldsymbol{B}\boldsymbol{Y}_r||_F^2\right\} \leq \frac{T}{2}n_r$$

As in the single-user case, the signal model (5.9) can be rewritten as a virtual MIMO-MAC,

$$\tilde{\boldsymbol{Y}} = \sum_{k=1}^{K} \tilde{\boldsymbol{H}}_k \boldsymbol{X}_k + \tilde{\boldsymbol{Z}} \tag{5.10}$$

where $\boldsymbol{X}_k \triangleq \left[\boldsymbol{X}_{k1}^{\mathsf{T}}\ \boldsymbol{X}_{k2}^{\mathsf{T}}\right]^{\mathsf{T}}$ and $\tilde{\boldsymbol{Y}} \triangleq \left[\boldsymbol{Y}_1^{\mathsf{T}}\ \boldsymbol{Y}_2^{\mathsf{T}}\right]^{\mathsf{T}}$. $\tilde{\boldsymbol{Z}}$ is the equivalent whitened AWGN, the equivalent $2n_t \times 2n_d$ channel matrix of user k is

$$\tilde{\boldsymbol{H}}_k \triangleq \begin{bmatrix} \sqrt{P_{k1}}\boldsymbol{H}_{k,d} & \boldsymbol{0}_{n_d \times n_t} \\ \sqrt{P_r P_{k1}}\boldsymbol{\Upsilon}\boldsymbol{H}_{r,d}\boldsymbol{B}\boldsymbol{H}_{k,r} & \sqrt{P_{k2}}\boldsymbol{\Upsilon}\boldsymbol{H}_{k,d} \end{bmatrix} \tag{5.11}$$

where $\boldsymbol{\Upsilon}$ is the whitening matrix satisfying

$$(\boldsymbol{\Upsilon}^\dagger\boldsymbol{\Upsilon})^{-1} = (\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}^\dagger)^{-1} = \boldsymbol{I} + P_r(\boldsymbol{H}_{r,d}\boldsymbol{B})\boldsymbol{B}^\dagger\boldsymbol{H}_{r,d}^\dagger \tag{5.12}$$

5.2.3 DMT Analysis

Using the DMT framework, we present here a theoretical analysis highlighting performance enhancement that the users can benefit from by sharing a relay that helps them reach the destination. We also compare the DMT of the MAF relay channel to the best DMT that can be achieved by any relaying protocol: the upper-bound on the DMT of the MAR channel obtained using the min-cut max-flow theorem.

Remark 5.2.1 *While the channel is modeled for an arbitrary number of antennas at all the terminals, we focus in the following DMT analysis on the scenario with single-antenna at all terminals for simplicity of presentation, i.e.* $K = 2, N = 1$. *The channel model as well as the corresponding MAF cooperation frame are illustrated in Fig. 5.5.*

Theorem 5.2.1 *The optimal diversity gain for the symmetric two-users MAR channel* *with single-antenna terminals is upper bounded by*

$$d_{MAR}(r) \leq \begin{cases} 2-r, & 0 \leq r \leq \frac{1}{2} \\ 3\,(1-r)\,, & \frac{1}{2} \leq r \leq 1 \end{cases} \tag{5.13}$$

Proof: Using a simple min-cut max-flow examination of the scheme in figure 5.5(a), one gets:

$$\begin{aligned} d_{MAR}(r) &\leq \min\left\{d_{3\times 1}(r), d_{2\times 2}(r), d_{2\times 1}(\frac{r}{2}), d_{1\times 2}(\frac{r}{2})\right\} \\ &\leq \min\left\{3(1-r), 2(2-r), 2(1-\frac{r}{2})\right\} \\ &\leq \begin{cases} 2-r, & 0 \leq r \leq \frac{1}{2} \\ 3\,(1-r)\,, & \frac{1}{2} \leq r \leq 1 \end{cases} \end{aligned}$$

■

Note that the upper-bound (5.13) also constitutes an upper-bound on the DMT of any relaying protocol. Now, what about the DMT achieved by the MAF relaying strategy? The following theorem can be deduced from from [5, Theorem 1]

Theorem 5.2.2 *For a symmetric two-user MAR channel with one relay, the DMT of the MAF protocol is given by*

$$d_{MAF}(r) = \begin{cases} 2-\frac{3}{2}r, & 0 \leq r \leq \frac{2}{3} \\ 3\,(1-r)\,, & \frac{2}{3} \leq r \leq 1 \end{cases} \tag{5.14}$$

Proof: See [5]. ■

For comparison purposes, recall the DMT achieved by a 2-user MAC (Theorem 1.4.3) with $n_t = n_r = 1$ given by

$$d_{\mathrm{MAC}}(r) = \begin{cases} 1-\frac{r}{2}, & 0 \leq r \leq \frac{2}{3} \\ 2\,(1-r)\,, & \frac{2}{3} \leq r \leq 1 \end{cases} \tag{5.15}$$

Figure 5.6 compares these DMTs given in (5.13), (5.14) and (5.15) as well as the DMT achieved by a time-sharing scheme. This comparison reveals the significant advantage that multiple users can potentially gain from a single MAF relay helping them reach the destination. The MAF protocol is shown to achieve the optimal DMT for $2/3 \leq r \leq 1$. Moreover, this comparison shows the sub-optimality of the time-sharing strategy in both multiple-access scenarios. Note that, if time-sharing is considered, the system without relaying is equivalent to a point-to-point scheme, *i.e.,*

$$d_{\mathrm{TS}} = (1-r), \ \ 0 \leq r \leq 1.$$

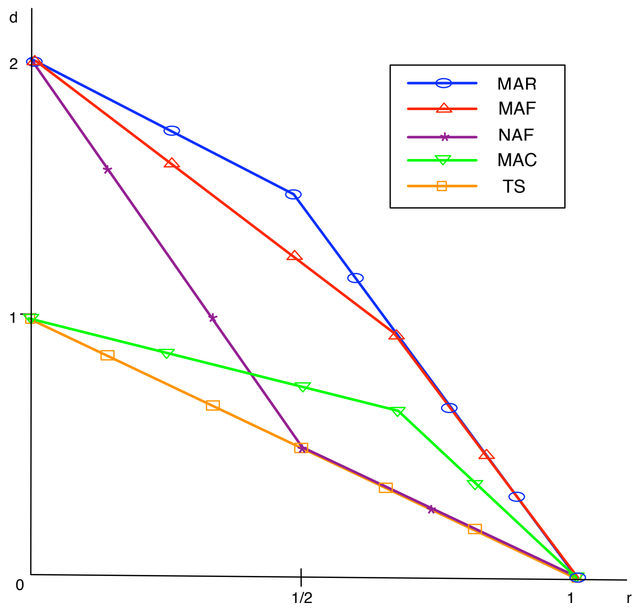


Figure 5.6: DMT of a two-user MAR, MAF relay channel and MAC, $n_t = n_r = n_d = 1$.

On the other hand, if relaying is considered, the MAF protocol is equivalent to the NAF protocol. The corresponding DMT is given in Eq. 5.3:

$$d_{\text{NAF}}(r) = (1 - r) + (1 - 2r).$$

5.2.4 General Code Construction

We propose here a new family of Space-Time block codes for the MAF relay channel shown in (5.10) to be equivalent to a virtual MIMO-MAC. Therefore, the idea is to apply the coding scheme designed in Chapter 4 for the MIMO-MAC, to the MAR channel in a distributed manner.

Codewords Structure

Let $\mathbb{F}$ be a Galois extension of degree K, the number of users, over $\mathbb{Q}(i)$ with Galois group $\text{Gal}(\mathbb{F}/\mathbb{Q}(i)) = \{\tau_1, \tau_2, \ldots, \tau_K\}$. $\mathbb{K}$ is a cyclic extension of degree $2n_t$ on $\mathbb{F}$ and σ the generator of its Galois group. Based on (4.8), define the mother codeword of user k, denoted $\boldsymbol{M}_k$, as a single-user component of a codeword of $\mathcal{X}$ given by

$$\boldsymbol{M}_k = \left[\begin{array}{cccc} \boldsymbol{\Gamma}\tau_1(\boldsymbol{\Xi}_k) & \boldsymbol{\Gamma}\tau_2(\boldsymbol{\Xi}_k) & \ldots & \tau_K(\boldsymbol{\Xi}_k) \end{array} \right]$$

where $\boldsymbol{\Gamma}$ is a multiplication matrix for the $k-1$ first matrices of $\boldsymbol{M}_k$ chosen as in Theorem 4.4.1. Now consider an equivalent code $\mathcal{C}$ whose codewords (per user) are in the form

$$\boldsymbol{C}_k \triangleq [\boldsymbol{M}_k(1 : n_t, 1 : 2Kn_t) \;\; \boldsymbol{M}_k(n_t + 1 : 2n_t, 1 : 2Kn_t)]$$

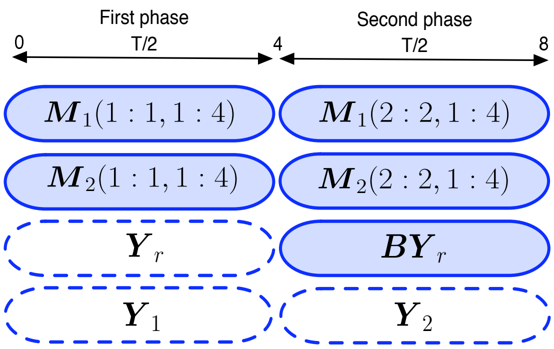


Figure 5.7: MAF cooperation frame with the distributed MIMO-BB code.

The constructed code $\mathcal{C}$ for the K-user MAF channel with n_t transmit antennas per user is of length $T = 4Kn_t$ and consists in transmitting $\boldsymbol{C}_k$ by user k, $k = 1, \ldots, K$ in each cooperation frame. For the sake of simplicity, we focus in the following on the two-user MAR channel where one single-antenna relay node is assigned to assist the two multiple-access users.

5.2.5 Explicit Construction: Two-user MAF Relay channel

First, let us design each user's mother codeword. Let $\mathbb{F} = \mathbb{Q}(\zeta_8)$ be an extension of $\mathbb{Q}(i)$ of degree $K = 2$, with $\zeta_8 = e^{\frac{i\pi}{4}}$ and $\mathbb{K} = \mathbb{F}(\sqrt{5}) = \mathbb{Q}(\zeta_8, \sqrt{5})$. Let $\sigma : \theta = \frac{1+\sqrt{5}}{2} \mapsto \bar{\theta} = \frac{1-\sqrt{5}}{2}$, $\alpha = 1 + i - i\theta$ and $\bar{\alpha} = 1 + i - i\bar{\theta}$. User k's equivalent mother codeword $\boldsymbol{X}_k$ is constructed as in (4.11):

$$\boldsymbol{M}_k = \begin{bmatrix} \boldsymbol{\Xi}_k & \tau(\boldsymbol{\Xi}_k) \end{bmatrix} \tag{5.16}$$

where τ changes ζ_8 into $-\zeta_8$. s_{kj} denotes the j^{th} information symbol of user k and $\boldsymbol{\Xi}_k$ is defined by

$$\boldsymbol{\Xi}_k = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha.(s_{k,1} + s_{k,2}\zeta_8 + s_{k,3}\theta + s_{k,4}\zeta_8\theta) & \alpha.(s_{k,5} + s_{k,6}\zeta_8 + s_{k,7}\theta + s_{k,8}\zeta_8\theta) \\ \zeta_8\bar{\alpha}.(s_{k,5} + s_{k,6}\zeta_8 + s_{k,7}\bar{\theta} + s_{k,8}\zeta_8\bar{\theta}) & \bar{\alpha}.(s_{k,1} + s_{k,2}\zeta_8 + s_{k,3}\bar{\theta} + s_{k,4}\zeta_8\bar{\theta}) \end{bmatrix} \tag{5.17}$$

This code uses 8 QAM symbols per user. Finally, the following equivalent mother codeword matrix is obtained:

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_1 \\ \boldsymbol{M}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Xi}_1 & \tau(\boldsymbol{\Xi}_1) \\ \boldsymbol{\Gamma\Xi}_2 & \tau(\boldsymbol{\Xi}_2) \end{bmatrix} \tag{5.18}$$

with

$$\boldsymbol{\Gamma} = \begin{bmatrix} 0 & 1 \\ i & 0 \end{bmatrix}. \tag{5.19}$$

The equivalent code $\mathcal{C}$ has per user codewords, of length $T = 4Kn_t = 8$, in the form

$$\boldsymbol{C}_k \triangleq [\boldsymbol{M}_k(1:1,1:4)\ \boldsymbol{M}_k(2:2,1:4)] \tag{5.20}$$

Both users transmit simultaneously the first row of their mother codewords, $\boldsymbol{M}_k$, during the first phase. During the second phase, each user transmits the second row of its mother codeword $\boldsymbol{M}_k$ while the relay transmits a scaled version of the first rows. The corresponding cooperation frame of length $T = 8$ is illustrated in Fig. 5.7 where plain line slots represent transmission mode and dashed line slots represent listening mode.

5.2.6 Numerical Results

Outage behavior

The outage behavior of the two-user MARC is illustrated in Fig. 5.8 for $R = 2$ bits pcu. These curves show the sub-optimality of the time-sharing strategy and to highlight the benefit of relaying. The outage probabilities of the channel are compared in four different scenarios: 1- the users are transmitting simultaneously with the help of the relay using the MAF protocol (labeled MAF); 2- the users are transmitting simultaneously without relaying (labeled MAC); 3- time-sharing with relaying using NAF protocol (labeled NAF); 4- time-sharing without relaying (labeled Time-Sharing). The time-sharing and the MAC outage curves achieve the same diversity order ($d = 1$ for $n_d = 1$) but time-sharing is clearly sub-optimal. A significant gain can be achieved by letting the users transmit simultaneously, and thus exploiting the multiple-access nature of the channel. The same interpretation holds for the NAF as compared to the MAF. NAF and MAF both achieve the same diversity order ($d = 2$ for $n_d = 1$). It is clear that the existence of a relay helping the users to reach the destination offers a significant gain in both multi-access and time-sharing schemes.

Code performance

The performance of the proposed scheme is compared in Fig. 5.9 to the time-sharing scheme and the single-user code (using the distributed Golden code), for $n_d = 1$ and a 4-QAM constellation. The proposed code achieves the same diversity order ($d = 2$ for $n_d = 1$) but offers a significant performance gain. At $P_e = 10^{-3}$, performance gains of 8 dB and 3 dB are observed as compared to the time-sharing scheme and the single-user code, respectively. Interestingly, if the performance of the coded scheme is compared to the outage performance of the channel, similar behaviors are observed.

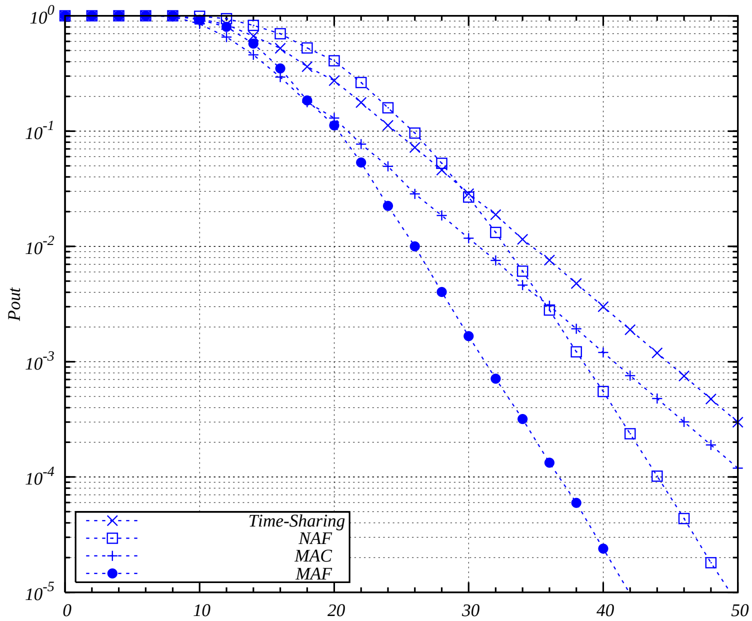


Figure 5.8: Outage performance of a two-user MAR channel, MAF protocol *vs* Time-Sharing, $n_d = 1$, R = 2 bits pcu.

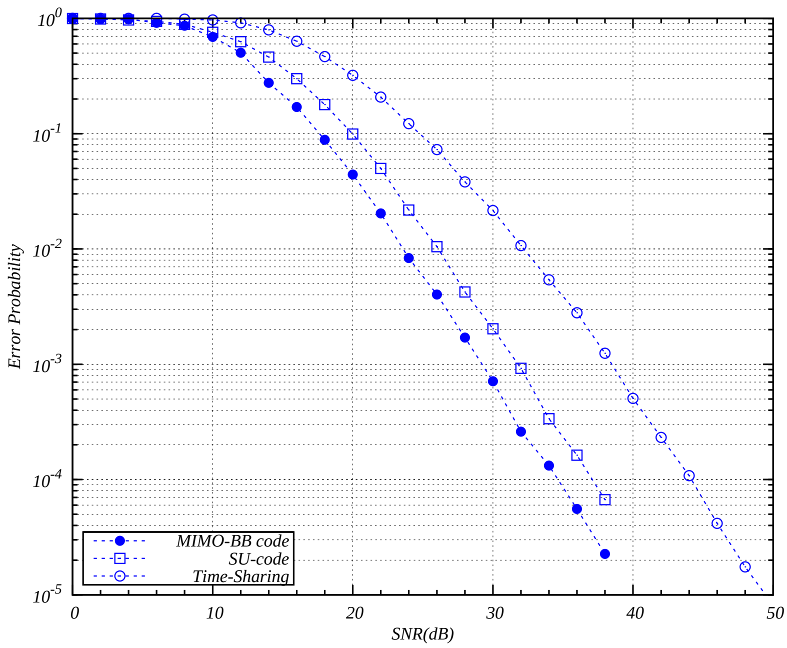
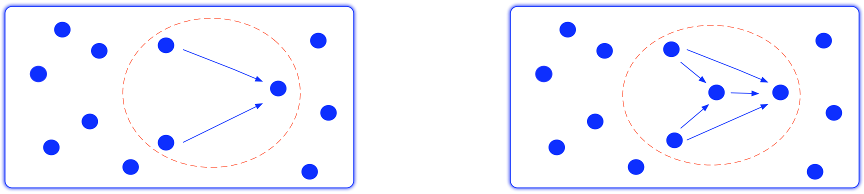


Figure 5.9: Performance of the Space-Time Code designed for the two-user MAF relay channel, $n_d = 1$, 4-QAM.

Conclusion and Perspectives

In this thesis, the Space-Time coding design for some elementary channels constituting a large wireless communication network is considered. Specifically, Multiple-Access and Multiple-Access Relay channels illustrated in Fig. 5.10 are studied. Undoubtedly, time-sharing among the users in a multiple-access scenario is sub-optimal since it makes very poor use of the degrees of freedom of the channel. New multiple-access Space-Time block codes that better exploit the channel's capabilities and offer significant performance improvements are designed. In addition to a numerical performance evaluation, a DMT-oriented analysis of the proposed coding schemes is presented.



(a) MAC

(b) MAR Channel

Figure 5.10: Decomposition of a network into elementary channels.

For multiple-access channels, the single transmit antenna users case and the multi-antenna users case were separated. In the former, two scenarios are considered: the synchronous and the asynchronous scenario. The Space-Time coding scheme proposed for the synchronous MAC offers a significant performance gain and is shown to satisfy the DMT-optimal design criteria derived in [27] for a special range of multiplexing gains. Moreover, the proposed code is shown to be delay-tolerant. In other words, the advantages of the coding scheme are preserved despite the structure change that may be induced by an asynchronism at the transmitters side. One interesting perspective resulting form this work would be a more information theoretic analysis of asynchronism in MACs, such as the characterization of the diversity-multiplexing tradeoff of the asynchronous MAC and its achievability.

For the MIMO case, a new coding scheme is proposed. It is shown to outperform or to offer at least similar performance as compared to other existing schemes. A partial DMT-analysis of the code is presented, but the DMT-optimality in this case is yet to be determined.

For multiple-access relay channels, an AF cooperation protocol was considered. This protocol known as the MAF experiences low complexity at the relay and achieves the optimal DMT for a special range of multiplexing gains. As a result of the linear nature of the relaying scheme, the MAF relay channel can be modeled as an equivalent MIMO-MAC to which the multiple-access coding scheme can be applied in a distributed way. The distributed Space-Time code, used to profit from the gain offered by the existence of the relay compared to a system without relaying, is shown to offer a significant enhancement of the performance.

The DMT-achievability of the coding scheme proposed for the MIMO-MAC and hence, that of the distributed code applied to the MAR channel, remain an open problem to solve.

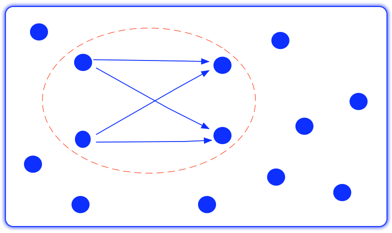


Figure 5.11: The interference channel.

More generally, the decomposition of a large wireless network into elementary channels leads to more complex channels such as the interference channel illustrated in Fig. 5.11. The interference channel models a communication between M unrelated transmitters and M independent receivers. Space-Time code construction for fading interference channel is an interesting direction for future research.

Appendix A

Algebraic Tools for STB Coding

The aim of this Appendix is to provide a (brief) overview of the most relevant algebraic tools that are used in this thesis. For the proofs of these results and for a more detailed introduction, we let the reader refer to [33].

Let $\mathbb{Z}$ be the set of rational integers, $\mathbb{Q}$ be the set of rational numbers and $\mathbb{C}$ the set of complex numbers.

Definition A.1 *(Field extension) Let $\mathbb{F}$ and $\mathbb{L}$ be two fields. $\mathbb{L}$ is said to be a field extension of $\mathbb{F}$ if $\mathbb{F} \subseteq \mathbb{L}$. Such extension is denoted $\mathbb{L}/\mathbb{F}$. The dimension of $\mathbb{L}$ as a vector space over $\mathbb{F}$ is called the degree of $\mathbb{L}$ over $\mathbb{F}$.*

Example A.1 *Consider the field $\mathbb{Q}$ and the element i such that $i^2 = -1$ that is not an element $\mathbb{Q}$. $\mathbb{Q}(i)$ is a field extension of degree 2 of $\mathbb{Q}$ built by adding i to $\mathbb{Q}$. An element of x of $\mathbb{Q}(i)$ can be written as $x = a + ib$ where $a, b \in \mathbb{Q}$. The dimension of $\mathbb{Q}(i)$ as a vector space over $\mathbb{Q}$ is 2.*

Similarly, $\mathbb{Q}(i, \sqrt{5})$ can be built by adding $\sqrt{5}$ to $\mathbb{Q}(i)$. An element w of $\mathbb{Q}(i, \sqrt{5})$ can be written as $w = x + y\sqrt{5}$ where $x, y \in \mathbb{Q}(i)$. $\mathbb{Q}(i, \sqrt{5})$ is a field extension of degree 2 over $\mathbb{Q}(i)$ or of degree 4 over $\mathbb{Q}$.

Definition A.2 *A finite field extension of $\mathbb{Q}$ is called a number field.*

Definition A.3 *(Algebraic number) An element α of $\mathbb{L}$ is algebraic over $\mathbb{F}$ if there exists a non-zero irreducible monic polynomial $p \in \mathbb{F}[x]$ (with highest coefficient equal to 1) such that $p(\alpha) = 0$.*

Definition A.4 *$\mathbb{K}$ is called an algebraic extension of $\mathbb{Q}$, if all the elements of $\mathbb{K}$ are algebraic.*

Definition A.5 *(Transcendental number) If α is not algebraic, it is said to be transcendental (does not belong to any finite algebraic extension).*

Definition A.6 *(Ring of integers) $\alpha \in \mathbb{L}$ is an algebraic integer if it is a root of a monic polynomial with coefficients in $\mathbb{Z}$. The set of algebraic integers of $\mathbb{L}$ is a ring called the ring of integers of $\mathbb{L}$ and is denoted $\mathcal{O}_{\mathbb{L}}$.*

Definition A.7 *An ideal* $\mathbb{I}$ *of a commutative ring* A *is an additive subgroup of* A *stable under multiplication by* A*, i.e.,* $a\mathbb{I} \subseteq \mathbb{I}, \forall a \in A$*.* $\mathbb{I}$ *is said to be principal if it is of the form* $\mathbb{I} = (x)A = \{xy, y \in A\}, x \in \mathbb{I}$*.*

Definition A.8 $\mathrm{Gal}(\mathbb{L}/\mathbb{F})$ *denotes the Galois group of* $\mathbb{L}/\mathbb{F}$ *defined as the group of the* $\mathbb{F}$*-automorphisms of* $\mathbb{L}$ *under compositions of maps.* $\mathbb{L}/\mathbb{F}$ *is called a Galois extension.* $\mathrm{Gal}(\mathbb{L}/\mathbb{F})$ *is said to be a cyclic group if it is generated by one element. In the sequel,* $\langle\sigma\rangle$ *will denote a cyclic Galois group with* σ *being its generator.*

Definition A.9 *(Cyclic Algebra) Consider an algebraic number field* $\mathbb{F}$ *and assume that* $\mathbb{L}/\mathbb{F}$ *is a cyclic extension of degree* n *with the Galois group* $\mathrm{Gal}(\mathbb{L}/\mathbb{F}) = \langle\sigma\rangle$*. An associative* $\mathcal{F}-$*algebra can be defined as follows*

$$\mathcal{A} = (\mathbb{L}/\mathbb{F}, \sigma, \gamma) = \mathbb{L} \oplus \mathbb{L} \cdot e \oplus \mathbb{L} \cdot e^2 \oplus \cdots \oplus \mathbb{L} \cdot e^{n-1} \tag{A.1}$$

where $e \in \mathcal{A}$ *satisfies*

$$\lambda \cdot e = e \cdot \sigma(\lambda) \quad \forall \lambda \in \mathbb{L} \;\; \textit{and} \;\; e^n = \gamma \in \mathbb{F}^*$$

where $\mathbb{F}^*$ *is the set of non-zero elements of* $\mathbb{F}$*.* $\mathcal{A}$ *is called a cyclic algebra and the field* $\mathbb{F}$ *its center.*

Definition A.10 *(Cyclic Division Algebra, CDA) A cyclic algebra is a division algebra iff all the non-zero elements of the algebra are invertible.*

Definition A.11 *Let* $\mathbb{L}/\mathbb{F}$ *be a field extension of degree* n*.* $\sigma_1, \ldots, \sigma_n$ *denote the* n *relative embeddings of* $\mathbb{L}$*. For an element* x *of* $\mathbb{L}$*,* $\sigma_1(x), \sigma_2(x), \ldots, \sigma_n(x)$ *are called the conjugates of* x*. We define the norm and trace of* x *as follows*

$$N_{\mathbb{L}/\mathbb{F}}(x) = \prod_{i=1}^{n} \sigma^i(x) \tag{A.2}$$

$$Tr_{\mathbb{L}/\mathbb{F}}(x) = \sum_{i=1}^{n} \sigma^i(x) \tag{A.3}$$

The norm and the trace of an element $x \in \mathbb{L}$ *are elements of* $\mathbb{F}$*, i.e.,* $N_{\mathbb{L}/\mathbb{F}}(x)$*,* $Tr_{\mathbb{L}/\mathbb{F}}(x) \in \mathbb{F}$*. If* $x \in \mathcal{O}(\mathbb{L})$*,* $N_{\mathbb{L}/\mathbb{F}}(x)$ *and* $Tr_{\mathbb{L}/\mathbb{F}}(x) \in \mathbb{F}$*.*

Proposition A.1 *(Non-norm Element)* $\mathcal{A} = (\mathbb{L}/\mathbb{F}, \sigma, \gamma)$ *of degree* n *is a division algebra iff the only* t *for which* γ^t *is the norm of some element of* $\mathbb{F}^*$ *is* n*, i.e.,* γ *is referred to as non-norm element.*

Definition A.12 *Consider the CDA* $\mathcal{A} = (\mathbb{L}/\mathbb{F}, \sigma, \gamma)$*. Every element* x *of* $\mathcal{A}$*, i.e.,*

$$x = x_0 + x_1 \,.\, e + \ldots, + x_{n-1} \,.\, e^{n-1}$$

has the following matrix representation

$$\begin{pmatrix} x_0 & x_1 & x_2 & \dots & x_{n-1} \\ \gamma\sigma(x_{n-1}) & \sigma(x_0) & \sigma(x_1) & \dots & \sigma(x_{n-2}) \\ \vdots & & & & \vdots \\ \gamma\sigma^{n-2}(x_2) & \gamma\sigma^{n-2}(x_3) & \gamma\sigma(x_4) & \dots & \sigma^{n-2}(x_1) \\ \gamma\sigma^{n-1}(x_1) & \gamma\sigma^{n-1}(x_2) & \gamma\sigma^{n-1}(x_3) & \dots & \sigma^{n-1}(x_0) \end{pmatrix} \tag{A.4}$$

where $x_i \in \mathbb{L}$. *Since all the elements of a cyclic division algebra are invertible, the above matrix representation has a non-zero determinant.*

Example A.2 *Let* $n = 2$ *and* $\mathbb{L} = \mathbb{Q}(i, \sqrt{5})$. *An element* $x \in \mathcal{A}$ *is written as*

$$x = x_0 + x_1 \cdot e$$

where $x_0 = a_0 + \sqrt{5}b_0$ *and* $x_1 = a_1 + \sqrt{5}b_1$ *with* $a_0, a_1, b_0, b_1 \in \mathbb{Q}(i)$. *The corresponding matrix representation is in this case as follows*

$$\begin{pmatrix} x_0 & x_1 \\ \gamma\sigma(x_1) & \sigma(x_0) \end{pmatrix}$$

Note that QAM symbols are elements of $\mathbb{Z}[i]$, *thus, they belong to the base field* $\mathbb{Q}(i)$ *and each* x_i *encodes 2 QAM symbols. In the general case, since an element* x *of* $\mathcal{A}$ *has* n *coefficients, it encodes* n^2 *information symbols.*
The determinant of the above matrix is given by

$$\det \begin{pmatrix} x_0 & x_1 \\ \gamma\sigma(x_1) & \sigma(x_0) \end{pmatrix} = x_0\sigma(x_0) - \gamma x_1\sigma(x_1) = N_{\mathbb{L}/\mathbb{Q}(i)}(x_0) - \gamma N_{\mathbb{L}/\mathbb{Q}(i)}(x_1)$$

which is clearly non-zero if $\mathcal{A}$ *is a CDA, i.e.,* γ *a non-norm element of* $\mathbb{L}$.

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