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Télécom ParisTech (ENST)
EURECOM

THESE

Présentée pour obtenir le Grade de Docteur
de l'Ecole Nationale Supérieure
des Télécommunications

Spécialité: Communications et Electronique

Gaoning He

**Strategie d'Allocation de Ressources sans Fils à
Base de la Théorie des Jeux**

Soutenue le 18 Janvier 2010 devant le jury composé de:

Président	J.C.Belfiore, ENST (Paris,France)
Rapporteurs	P. Duhamel, Supélec (Gif-sur-Yvette, France) B. Tuffin, IRISA-INRIA (Rennes, France)
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Directeur de Thèse	L. Cottatellucci, Eurecom (Sophia-Antipolis, France)

September 30th 2009

Télécom ParisTech (ENST)
EURECOM

THESIS

In Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
from Ecole Nationale Supérieure
des Télécommunications

Specialization: Communications and Electronics

Gaoning He

**A Game-Theoretical Approach to Resource
Allocation in Wireless Networks**

Defended on January 18, 2010. The jury committee is formed of:

President	J.C.Belfiore, ENST (Paris,France)
Reviewers	P. Duhamel, Supélec (Gif-sur-Yvette, France) B. Tuffin, IRISA-INRIA (Rennes, France)
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September 30th 2009

Résumé

Un environnement de communication multi-utilisateurs sans fils forme un système hautement structuré dans lequel compétition et coopération coexistent. L'objectif de cette thèse est d'illustrer le rôle et les applications de la théorie des jeux au problème d'allocation de ressources dans les réseaux auto-organisant sans fils et multi-utilisateurs. Divers outils issus de la théorie des jeux seront considérés, parmi lesquels les jeux stratégiques avec information complète, les jeux de potentiels, les jeux bayésiens, les jeux avec les contraintes couplé et les jeux de négociations de Nash.

Cette thèse comporte quatre résultats principaux. Premièrement, les jeux non-coopératifs avec information complète sont introduits dans le cadre de l'étude du problème d'allocation de ressources dans un réseau sans fils formé de petites cellules. Dans ce modèle, les équipements sans fils sont supposés connaître parfaitement le status du réseau. Il est démontré que ce modèle peut être vu comme un jeux de potentiel. Ainsi, l'existence et la convergence de l'équilibre peuvent être adressés. Deuxièmement, des jeux non-coopératifs avec information incomplète sont introduits pour étudier un problème d'allocation de ressources distribué dans le cadre de canaux à accès multiples. Ce cas est traité sous la forme d'un jeu bayésien de waterfilling dans lequel les équipements sans fils sont supposés avoir une connaissance locale de l'état des canaux. Cette formulation est important d'un point de vue pratique, puisque les instruments sans fils peuvent avoir une information locale et rarement globale sur le statut des canaux de communication. D'autre part, des jeux avec les contraintes couplé ainsi que divers concepts d'équité sont introduits pour étudier le problème d'allocation de débits de communication dans la région de capacité de canaux à accès multiples. Le concept d'équilibre normalisé est adopté pour considérer le problème de sélection d'équilibre. De manière assez remarquable, lorsque le processus de décision est uniquement basé sur l'information statistique, tous les concepts d'équités coïncident avec l'unique équilibre normalisé. Finalement, des

jeux de négociations de Nash sont introduits pour augmenter les performances des systèmes non coopératifs dans les réseaux sans fils à petites cellules.

Abstract

A multiuser wireless environment is a highly structured system where competition and cooperation coexist. The aim of this thesis is to illustrate the role and the applications of game theory to the resource allocation problem in self-organizing multiuser wireless networks. Various game-theoretical tools will be considered, including strategic form games with complete information, potential games, Bayesian games, coupled constraints games, and Nash bargaining games.

There are four main results in this thesis. First, non-cooperative games with complete information are introduced to study the resource allocation problem in the context of small-cell wireless networks. In this game-theoretical setup, the wireless devices are assumed to have complete information about the global network status. It is shown that this game can be viewed as a potential game. Thus, the existence and convergence of equilibrium can be readily addressed. Second, non-cooperative games with incomplete information are introduced to study a distributed resource allocation problem in the context of fading multiple access channels. This case is formulated as a Bayesian waterfilling game, in which the wireless devices are assumed to have only local information about the fading channel states. This Bayesian game formulation is important from a practical point of view, as wireless devices can have local information but can barely access to global information on the network status. Further, coupled constraints games as well as various fairness concepts are introduced to study the rate allocation problem in the capacity region of multiple access channels. The concept of normalized equilibrium is adopted to address the equilibrium selection problem. Remarkably, when the decision making is based only on statistical information, all fairness concepts coincide with the unique normalized equilibrium. Finally, Nash bargaining games are introduced to improve the non-cooperative system performance of the small-cell wireless networks.

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Notation

The notations used in this thesis are listed in this section. We use boldface upper (e.g., \mathbf{X}) and lower case (e.g., \mathbf{x}) letters for matrices and column vectors, respectively. Plain letters are used for scalars and uppercase calligraphic letters (e.g., \mathcal{X}) denote sets.

\mathbb{R}	Real numbers.
$\mathbb{R}_+, \mathbb{R}_{++}$	Nonnegative, positive real numbers.
\mathbb{R}^n	Real n -vectors ($n \times 1$ matrices).
$\mathbb{R}^{m \times n}$	Real $m \times n$ matrices.
\mathbb{C}	Complex numbers.
\mathbb{E}^n	n -dimensional Euclidean space.
\cap, \cup	Intersection and union operators.
$\mathbf{dom} f$	Domain of function f .
$f : A \rightarrow B$	f is a function on the set $\mathbf{dom} f \subseteq A$ into the set B .
∇f	Gradient of function f .
$ x $	Absolute value of a scalar.
$ \mathcal{X} $	Cardinality of set \mathcal{X} , i.e., the number of elements in \mathcal{X} .
$\mathbb{E}[\cdot]$	Expectation operator.
$(\cdot)^T$	Transpose operator.
\mathbf{X}^{-1}	Inverse of matrix \mathbf{X} .
\mathbf{I}	Identity matrix.
\mathbf{e}_i	Unit vector with all zero entries except the i^{th} one which is equal to one.
$\mathbf{x} \succeq \mathbf{y}$	\mathbf{x} majorizes \mathbf{y} or, equivalently, \mathbf{y} is majorized by \mathbf{x} .
\triangleq	Defined as.
$\mathcal{N}(\nu, \sigma^2)$	Gaussian distribution with mean ν and variance σ^2 .
$\log(\cdot)$	Natural logarithm.
$\log_2(\cdot)$	Base 2 logarithm.
$(x)^+$	Positive part of x , i.e., $\max(0, x)$.

Acronyms

The acronyms used throughout this thesis are summarized here. The meaning of an acronym is usually indicated once, when it first occurs in the text.

3GPP	Third Generation Partnership Project
AP	Access Point
AWGN	Additive White Gaussian Noise
BC	Broadcast Channel
BE	Bayesian Equilibrium
bps	Bits per second
BS	Base Station
CDMA	Code Division Multiple Access
CP	Cyclic Prefix
CSI	Channel State Information
CSIR	Channel State Information at Receiver
CSIT	Channel State Information at Transmitter
DSL	Digital Subscriber Line
FDMA	Frequency Division Multiple Access
FFT	Fast Fourier Transform
GP	Geometric Programming
GSM	Global System for Mobile Communications
IFC	Interference Channel
i.i.d.	Independent and Identically Distributed
KKT	Karush-Kuhn-Tucker Optimality Conditions
LTE	Long Term Evolution
MAC	Multiple Access Channel
MIMO	Multiple-Input Multiple-Output
MT	Mobile Terminal
NE	Nash Equilibrium
OFDM	Orthogonal Frequency Division Multiplexing

QoS	Quality of Service
SDMA	Space Division Multiple Access
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
s.t.	Subject to
TDMA	Time Division Multiple Access
TTS	Transmission Time Interval
WiMAX	Worldwide Interoperability for Microwave Access
WloG	Without loss of Generality

Résumé en Fransais

Introduction

Contexte et Motivation

Il a été près d'un demi-siècle et depuis l'invention de la transmission sans fil utilisant le spectre électromagnétique. De l'utilisation précoce de la télégraphie à étincelles (1888, H. Hertz) à la première transmission claire de la parole humaine dans l'air (1919) et de l'invention de la modulation de fréquence (FM) jusqu'à ce que la première forme de réseaux de téléphonie mobile cellulaire (1947, Bell Labs), les technologies sans fil et les applications ont été considérablement modifiées. Cependant, l'interférence est toujours une boîte de Pandore dans la conception et l'analyse des multi-utilisateur d'aujourd'hui les systèmes sans fil.

Dans les communications sans fil, de transmission à accès multiple se révèle être un scénario très commun, puisque le spectre radioélectrique est une ressource naturelle partagée dans le sens où chaque utilisateur peut accéder à la même liaison physique et de transmettre celui-ci. Interférence se pose au niveau du récepteur lorsque plusieurs émetteurs d'accéder simultanément au même lien physique. Les collisions message peut réduire considérablement la qualité de la communication, voire de prévenir la réception du signal approprié. Dans ce cas, le contrôle et la réduction des interférences joue un rôle important dans les communications sans fil. Traditionnellement, la conception de systèmes sans fil a mis l'accent sur l'accroissement de la fiabilité de la liaison physique. l'accent s'est déplacé de plus récentes visant à accroître l'efficacité spectrale. En vertu de ce contexte, de nombreuses technologies d'accès aux médias ont été introduits, tels que l'accès multiple par répartition en fréquence (AMRF), l'accès multiple par répartition temporelle (AMRT), Division de code d'accès multiples (CDMA), et l'accès division de l'espace multiple (SDMA).

Avec la demande croissante de la voix sans fil et des services de données, le spectre de fréquence est devenue une ressource naturelle rare dont tout le monde est en concurrence. En fait, l'efficacité spectrale (mesure de l'efficacité

de l'utilisation du spectre de fréquences) peuvent encore être améliorées par des techniques de gestion des ressources radio telles que l'allocation efficace voie fixe ou dynamique, contrôle de puissance, adaptation de lien et de la diversité des régimes. Au cours des deux dernières décennies, l'allocation des ressources, basée sur la théorie de l'information [1] et l'optimisation théorie [2, 3] a considérablement amélioré l'efficacité dans l'utilisation des ressources de fréquences et a joué un rôle central à cet égard. Par exemple, waterfilling algorithm de [1, 2] est l'une des techniques les plus célèbres qui considèrent la façon de maximiser la capacité du canal. Toutefois, afin d'atteindre les limites de la théorie des informations, par exemple, région de capacité, il faut une entité centrale (scheduler) à l'échelle mondiale organiser et répartir les ressources du système. Un ordonnanceur est nécessaire d'avoir la capacité de tracer les canaux sans fil dans le réseau.

De toute évidence, un ordonnanceur centralisé avec une connaissance complète et préfet de tous les canaux sans fil permet d'optimiser l'affectation des ressources réseau mondial. Cependant, cette approche implique généralement des techniques d'optimisation sophistiquée et une charge de signalisation qui grandit avec le nombre d'émetteurs et de récepteurs dans le réseau. En raison de cette complexité inhérente à l'optimisation et la signalisation exigences généraux, les infrastructures centralisées peut devenir le plus important goulot d'étranglement pour le développement des futurs réseaux sans fil. Et en fait, il a déjà exposé ses propres faiblesses dans certains aspects, par exemple, de reconfiguration lente et coûteuse contre l'environnement divers, des difficultés dans la centralisation des informations sur le réseau, l'augmentation de la complexité des calculs, etc En outre, la planification, de déploiement et d'exploitation d'un réseau cellulaire sont des processus coûteux. Les caractéristiques de propagation de la zone doivent être estimés, les stations de base doivent être soigneusement sélectionnés, et les équipements doivent être installés. Une fois le réseau opérationnel, des ajustements doivent souvent être faites. Ils peuvent même inclure la mise en place de stations de base supplémentaires pour éliminer les points morts. Au total, il s'agit d'une série de temps et de main-d'œuvre à forte intensité de tâches.

Il s'avère qu'en raison de la nature différente des appareils mobiles, ainsi que la forte mobilité du réseau, l'avenir des réseaux sans fil vont vers plus de systèmes décentralisés d'allocation des ressources dans lequel chaque station mobile ou appareil de base peut exploiter les informations locales de prendre leurs propres processus d'optimisation, et donc décider de leurs propres ressources d'utilisation [4]. En fait, les fournisseurs de services sans fil ont été chercher

pour les infrastructures qui sont auto-configuration, auto-exploitation, et l'auto-optimisation [5]. Plus précisément, les fournisseurs souhaitent stations de base pour être déployé rapidement, sans techniciens spécialisés, en mesure de découvrir automatiquement leurs voisins, reconfigure automatiquement en cas de défaillance du réseau, et d'optimiser automatiquement les paramètres radio. En plus de cela, la liaison terrestre devrait être configuré automatiquement, et la qualité de service (QoS) doit être optimisée en autonome. Ces capacités, avec beaucoup d'autres, sont la vision de l'avenir de l'auto-organisation des réseaux sans

Au cours de la dernière décennie, il a déjà été une explosion d'intérêt pour l'auto-organisation des réseaux sans fil. En fait, les gens de plus en plus de la communauté des communications sans fil font appel concepts comme l'intelligence artificielle, la radio cognitive, et des réseaux distribués dans leurs recherches. En particulier la théorie des jeux [6], comme un puissant outil mathématique, a été de plus en plus appliquée aux communications sans fil. La théorie des jeux a été introduit, dans sa forme actuelle, en 1944 par J. V. Neumann et O. Morgenstern dans leur ouvrage de référence "Théorie de la Games and Economic Behavior" [7] et a été appliquée avec succès dans divers domaines tels que l'économie, la biologie (évolution stable stratégie mise en place par JM Smith en 1982), la guerre (la guerre froide, la guerre contre le terrorisme), la philosophie (la morale de l'intérêt), les sciences sociales (explication de la paix démocratique), l'informatique (modélisation des calculs interactifs), etc.

En effet, les réseaux sans fil contient trois ingrédients qui font de la théorie des jeux une théorie féconde pour leur analyse et de conception: (1) les terminaux mobiles sont équipés d'un certain degré d'intelligence ou de la cognition, (2) les appareils mobiles partager des ressources communes qui implique une interaction naturelle entre eux, et (3) les réseaux sans fil sont très structurés. Cela favorise la réussite de l'analyse théorique jeu. Un cadre de la théorie des jeux peut s'adapter très bien le concept de réseaux auto-organisés, où l'intelligence et la prise de décision est distribué. Un tel système a des avantages apparents en termes de charge de rétroaction. Avec une configuration de la théorie des jeux, les appareils mobiles n'ont besoin que d'informations incomplètes ou une connaissance partielle de l'environnement, et pas de calculs plus lourds algorithme d'optimisation est nécessaire pour résoudre à l'ordonnanceur centralisé. Mais d'autre part, puisque aucune recherche clairement le point optimal est réalisée, lorsque chaque appareil tente de maximiser son intérêt, par exemple, le taux de données, sous certaines conditions il peut arriver que les performances du système qui en résulte est loin d'être optimale au niveau mondial [6, 8].

Thèse Objectifs

Les principaux objectifs de cette thèse sont:

- Comprendre les fondements de la théorie des jeux ainsi que la théorie d'optimisation, telles que la définition mathématique d'un problème de jeu et un problème d'optimisation, la différence entre un jeu et un problème d'optimisation, la classification des jeux et des optimisations, etc.
- Étudier les méthodes d'analyse pour étudier le résultat de problèmes de théorie des jeux, comme l'existence de l'équilibre, l'unicité de l'équilibre, l'équilibre et la sélection.
- Modèle des problèmes d'allocation des ressources en communications sans fil multi-utilisateurs comme des problèmes de jeu théorique. Classification des jeux pertinents pour l'allocation des ressources dans les réseaux sans fil.
- Évaluer la performance du réseau sans fil provenant de la théorie des jeux solutions en termes d'efficacité du système et l'équité.
- Les deux points de vue et de l'ingénieur théoricien seront considérés. En particulier, une cartographie claire entre les problèmes pratiques dans les réseaux sans fil et des approches théorie des jeux seront mis en place.

Hypothèses de Base

Dans cette section, nous discutons et le détail des hypothèses qui sont faites tout au long de cette thèse.

- L'allocation des ressources par intervalle de temps de transmission (TTS): Nous supposons que l'état du canal sans fil reste constante sur au moins un TTS, qui se réfère à la longueur d'une transmission indépendante décodables sur le lien sans fil. L'allocation des ressources doit être mis à jour tous les TTS.
- Constellation du signal: la constellation du signal est supposé être gaussienne. Le niveau de modulation est suffisamment élevée pour que l'information mutuelle est d'environ la capacité du canal. Dans notre analyse et des résultats numériques, l'influence de la modulation peut être considéré en ajoutant le rapport signal-tonoise (SNR) lacune Γ dans la capacité de

la formule: $C = \log(1 + \text{SNR}/\Gamma)$, où Γ est fonction de la taux d'erreur binaire (BER) de la modulation [9].

- Channel state information (CSI): Dans tous les chapitres, nous supposons que complète et parfaite CSI est disponible au niveau des récepteurs. Dans le chapitre 4, 6 et 7, on suppose que complet et parfait CSI est également disponible sur les émetteurs. Considérant que, dans le chapitre 5, nous supposons que les émetteurs ont incomplète CSI, c'est à dire, chaque émetteur est conscient de ses états propre canal, mais pas au courant des états-canal de tous les autres émetteurs.
- Mono-utilisateur et de décodage multi-utilisateurs: Dans le chapitre 4 et 5, nous considérons une région taux réalisables contraint à l'utilisation d'un seul utilisateur de décodage au niveau du récepteur. Cette région est très pertinent réalisables dans la pratique des systèmes sans fil avec des limitations sur la complexité du décodeur et la simplicité du système. Dans ce contexte, nous cherchons à trouver la répartition de puissance optimale pour les utilisateurs du réseau selon une certaine mesure la performance. Dans le chapitre 6 et 7, nous considérons la région de capacité correspondant à l'utilisation du multi-utilisateur de décodage au niveau du récepteur.
- La rationalité: une des hypothèses les plus courantes commises dans la théorie des jeux est la rationalité [10, 11]. Cela signifie que chaque joueur maximise son profit toujours, étant ainsi en mesure de calculer le résultat parfaitement probabiliste de chaque action. Cependant, en réalité, cette hypothèse ne peut être raisonnablement approchée depuis la rationalité des individus est limitée par l'information qu'ils ont, les limites cognitives de leur esprit, et le temps limité qu'ils ont à prendre des décisions.

Thèse Aperçu

En termes généraux, l'objet de cette thèse est sur les applications de jeu théorique à la conception de l'allocation des ressources distribuées dans les canaux sans fil. Les grandes lignes de tous les chapitres est la suivante.

Chapitre 1, du présent chapitre, donne de la motivation, le contour et les contributions de cette thèse.

Chapitre 2 décrit la formation technique dans les systèmes sans fil pris en compte dans ce mémoire, tels que les caractéristiques du canal sans fil, deux

modèles fondamentaux canal sans fil, à savoir, les canaux d'accès multiples et des canaux de l'ingérence, et la division orthogonale de la fréquence de multiplexage (OFDM) la technologie.

Le chapitre 3 introduit deux théories fondamentales: la théorie de l'optimisation et la théorie des jeux, en fournissant les principaux outils mathématiques pour cette thèse.

Le chapitre 4 considère une approche non-coopérative de théorie des jeux pour l'allocation des ressources distribuées dans les communications sans fil, en supposant que (1) complète et parfaite CSI est disponible à la fois au transmetteur et le récepteur (2) seul utilisateur de décodage stratégie est appliquée sur le côté du récepteur. Ce chapitre résume d'abord quelques approches existantes actuellement la théorie des jeux pour l'attribution de ressources sans fil dans les multiples voies d'accès et de canaux d'interférence. Puis, nous nous concentrons sur un problème particulier de répartition des ressources sans fil dans le contexte des réseaux à petites cellules en utilisant OFDM air-interface, qui est étroitement liés à l'introduction des approches existantes de la théorie des jeux. Dans ce cadre, nous montrons qu'une unique équilibre de Nash en stratégies pures existe en vertu d'une condition commune de canaux sans fil à la décoloration. En compléments, nous montrons que le problème des ressources considérées comme allocation peut être étudié comme un jeu de potentiel, et donc efficacement résolu.

Le chapitre 5 étend le cadre la théorie des jeux du chapitre 4 à la situation de façon plus réaliste incomplète CSI à l'émetteur. À cet égard, nous considérons une approche bayésienne la théorie des jeux pour le problème de l'affectation des ressources dans le contexte de la décoloration de canaux d'accès multiples. Nous établissons l'existence et l'unicité de l'équilibre bayésien. L'inefficacité de l'équilibre est évaluée par des expériences numériques.

Chapitre 6 études de jeux non-coopératifs avec les contraintes corrélés, connu sous le nom des jeux Nash généralisée dans le cadre de multiples voies d'accès. Différemment du chapitre 4 et 5, dans ce chapitre, nous considérons la région de capacité de multiples voies d'accès multi-utilisateur en supposant que le décodage est du côté récepteur. Dans ce problème de jeu théorique, la question majeure est de choisir parmi un grand nombre d'équilibres. Nous proposons l'utilisation de l'équilibre normalisé avec quelques notions d'équité, par exemple, l'équité max-min, de l'équité proportionnelle, α -équité. Dans le cas d'accès multiples canaux Gaussien orthogonal avec la région nonpolytope taux convexe, nous montrons que les concepts d'équité pouvez sélectionner différents équilibres différents. Dans le cas de voie d'accès multiples Gaussien avec la région poly-

tope convexe taux, de manière surprenante, nous montrons que tous les points d'équité coïncident avec l'équilibre unique normalisée.

Le chapitre 7 étend les résultats du chapitre 4 dans un cadre de coopération théorie des jeux, nommément jeu de négociation de Nash. Le problème de négociation de Nash est établi à partir de théorie de la dualité et de jeux possibles. Un algorithme récursif stochastique est proposée pour trouver la solution de marchandage de Nash.

Le chapitre 8 résume les principaux résultats de cette thèse et se pencher sur les perspectives de recherches futures.

Caractéristiques du Canal Sans Fil

Aujourd'hui, la communication sans fil est l'un des plus attrayantes et productives des zones dans le domaine de la communication. Il a été un sujet d'étude depuis les années 1960, et les trois dernières décennies ont vu une forte augmentation des activités de recherche dans ce domaine. Cela est principalement dû à l'influence de la Very-Large-Scale Integration (VLSI) la technologie et la demande croissante de connectivité sans fil. En particulier, la pénétration sur le marché de la téléphonie cellulaire/téléphone mobile a été spectaculaire. D'une présence négligeable dans le début des années 1980, la pénétration de la technologie dans le monde cellulaire en 2002 est estimé à 1 milliards d'abonnements mobiles. Ce chiffre est dépensé à environ 4,6 milliards d'ici la fin de 2009 [12]. Depuis le succès de la 2G (deuxième génération) numériques standards sans fil, l'orientation de la recherche au cours des trois dernières décennies a conduit à un ensemble beaucoup plus riche de perspectives sur la façon de communiquer sur les canaux sans fil. Sur la base de ces perspectives, de nombreuses nouvelles normes telles que 3G/HSPA, CDMA, Wi-Fi, WiMAX et LTE 3GPP. ont été développés pour répondre à la demande explosive pour la connectivité sans fil. Or, cette évolution est toujours en cours.

Il y a deux aspects fondamentaux de la communication sans fil qui rendent le problème est essentiellement intéressante et stimulante. Le premier est le phénomène de fading, c'est à dire, la variation temporelle des forces de canal en raison de l'effet à petite échelle d'évanouissement par trajets multiples, ainsi que les effets à plus grande échelle tels que la perte par voie d'atténuation et d'accompagnement à distance par des obstacles. D'une manière générale, les canaux sans fil changer à la fois en temps et en fréquence. La cohérence temporelle montre avec quelle rapidité le canal change dans le temps, et de même, la cohérence de fréquence montre avec quelle rapidité il change de fréquence.

- Lorsque le temps est préoccupé par la cohérence, les canaux sans fil sont classées comme évanouissement rapide et lent à la décoloration. Evanouissement rapide se produit lorsque le temps de cohérence du canal est beaucoup plus courte que l'exigence de délai de la demande. Ralentissent la décoloration se pose si le temps de cohérence du canal est plus long. Dans un canal évanouissement rapide, on peut transmettre les symboles codés sur plusieurs faciès de la chaîne, tandis que dans un canal de ralentir la décoloration, le canal est constante au cours de la transmission d'un mot de code.
- Lors de la cohérence de fréquence est concerné, les canaux sans fil sont classées comme fading sélectifs en fréquence et fading plat. Lorsque la bande passante du signal d'entrée est beaucoup plus grande que la bande de cohérence, le canal est dit sélectif en fréquence. Lorsque la bande passante est considérablement inférieure à la bande de cohérence, le canal est dit que la fréquence à plat, car elle affecte toutes les fréquences du signal dans presque de la même manière.

Notez que si un canal est rapide ou lent évanouissement, plat ou sélectifs en fréquence décoloration dépend non seulement de l'environnement sans fil, mais aussi le signal d'entrée et de ses applications, à savoir, l'exigence de délai de la demande, la bande passante du signal d'entrée.

Le contrôle et la réduction des interférences multi-utilisateur est un deuxième aspect fondamental dans les communications sans fil. Contrairement aux communications filaires, où chaque paire émetteur-récepteur peut souvent être considéré comme un maillon isolé point à point, dans les communications sans fil il ya une interférence significative entre les communications des utilisateurs sans fil par voie hertzienne. Comment faire face à l'interférence est l'une des questions centrales dans la conception des systèmes de communication sans fil et sera un thème important de cette thèse. Traditionnellement, la conception de systèmes sans fil a mis l'accent sur l'accroissement de la fiabilité de l'interface air. L'accent s'est déplacé de plus récentes visant à accroître l'efficacité spectrale. En vertu de ce contexte, de nombreuses technologies d'accès aux médias ainsi que des algorithmes efficaces d'allocation des ressources ont été introduites. Toutes ces technologies peuvent être résumées brièvement du point de vue de degrés de liberté [13]:

- **Fréquence**

Frequency Division Multiple Access (FDMA) est une méthode d'accès au

canal qui donne à chaque utilisateur une attribution individuelle d'un ou de plusieurs sous-bandes de fréquences (ou sous-canaux).

Modulation de fréquence (FM) de diffusion peut être considérée comme une simple exemple de FDMA.

- **Temps**

Time Division Multiple Access (TDMA) est une méthode d'accès au canal qui permet à plusieurs utilisateurs de partager la même fréquence, en leur permettant de transmettre à intervalles de temps différents.

La technologie TDMA est largement utilisé dans les systèmes cellulaires numériques 2G, par exemple, IS-54 (1990) et GSM (1991).

- **Code**

Code Division Multiple Access (CDMA) est une méthode d'accès au canal utilisant la technologie à étalement de spectre et d'un régime spécial de codage (où chaque émetteur est attribué un code) afin de permettre à plusieurs utilisateurs d'être multiplexés sur la même liaison physique.

La technologie CDMA permet la migration du GSM vers la 3G, par exemple, IS-95B (1999), 2,5 G et la technologie CDMA2000 (2000) que la technologie 3G.

- **L'espace**

Space Division Multiple Access (SDMA) est une entrée et plusieurs sorties multiples (MIMO) basé sur la technologie de communication sans fil. Afin d'assurer la couverture radio dans les traditionnels systèmes mobiles de réseau cellulaire, la station de base émet le signal dans toutes les directions, car il n'a pas d'informations sur la position des appareils mobiles au sein de la cellule. SDMA est une méthode avancée de canal d'accès qui permet d'accéder à un canal de communication en identifiant l'emplacement des utilisateurs et établir une correspondance un-à-tête entre la division de fréquence du réseau et l'emplacement identifié spatiale.

La technologie SDMA est considéré comme un élément clé dans les systèmes cellulaires 4G.

Lorsque le nombre d'utilisateurs, les stations de base, et de cellules dans le réseau augmente, interférence devient le goulot d'étranglement. Il s'avère qu'en raison de la nature différente des appareils mobiles, ainsi que la forte mobilité du réseau, l'avenir des réseaux sans fil vont vers plus d'auto-organisation allocations

de ressources dans le cadre desquels les appareils mobiles intelligemment allouer des ressources d'une manière décentralisée [4].

Préliminaires Mathématiques

Eléments d'Optimisation Convexe

Minimisation convexe a des applications dans un large éventail de disciplines, telles que les systèmes de contrôle automatique, l'estimation et le traitement du signal, communications et réseaux, conception de circuits électroniques, l'analyse des données et la modélisation, les statistiques et les finances. Pour la plupart de ces applications, l'optimisation mathématique est utilisé comme une aide à un décideur de l'homme, concepteur du système, ou l'exploitant du système, qui supervise le processus, les résultats des contrôles, et modifie le problème.

Problèmes d'optimisation convexe ont des propriétés attrayantes, qui les rend fondamentalement plus dociles que les problèmes d'optimisation non-convexe génériques:

1. Tout optimum local est également un optimum global.
2. Détection infaisabilité: algorithmes sont faciles à initialiser via théorie de la dualité.
3. Des méthodes efficaces solution numérique qui peut traiter des problèmes très importants.

Dans cette section, nous détaillons certaines propriétés de problèmes d'optimisation convexe, ainsi que les principales méthodes qui seront utilisées tout au long de cette thèse. Nous avons d'abord commencer par des définitions et des formulations de base. Puis, nous étudions la théorie de la dualité, qui prévoit une "amélioration de bas/borne supérieure" méthode d'approximation de la solution du problème d'optimisation d'origine via Lagrange. Nous montrons les conditions dans lesquelles le bas/borne supérieure est serré. Nous introduisons la Kuhn-Tucker (KKT) les conditions qui fournissent des méthodes

simples d'analyse pour résoudre des problèmes d'optimisation convexe. Enfin, nous montrons comment résoudre la programmation géométrique (qui est non-convexe dans sa forme originale) en transformant lui des problèmes d'optimisation convexe. Ces méthodes d'optimisation jouent un rôle important dans l'analyse et de résoudre certains problèmes théorie des jeux.

Eléments de Théorie des Jeux

La théorie des jeux est une branche des mathématiques appliquées. Le cadre de base de la théorie des jeux a été introduite dans le livre "Théorie de la Games and Economic Behavior" par John von Neumann et Oskar Morgenstern (1944). Jusqu'à présent, il est largement utilisé dans les nombreux domaines, principalement dans l'économie (en vue de la concurrence entre les entreprises du modèle), ainsi que l'informatique, la biologie, la politique et de nombreux autres domaines. La théorie des jeux est une description efficace des interactions stratégiques. Il tente de saisir mathématiquement le comportement des individus dans des situations stratégiques, dans lequel la réussite d'un individu à faire des choix dépend des choix des autres. La théorie des jeux améliore la compréhension des conflits en élaborant des théories, des modèles mathématiques et des abstractions qui servent à expliquer la nature et les résultats des conflits.

Dans la théorie des jeux, il ya certaines hypothèses de base qui sont souvent utilisés pour faciliter la construction de modèles maniable pour des situations réelles. Premièrement, on suppose que chaque individu dans le jeu a un ordre défini de préférences sur tous les résultats d'une situation donnée. Ces préférences prennent la forme d'une fonction d'utilité ou de paiement. Deuxièmement, les participants dans les jeux sont parfois considérés comme rationnels. Cela signifie qu'ils agissent toujours de façon à maximiser leurs gains, ils seront toujours capables de penser par tous les résultats possibles et en choisissant l'action qui se traduira par la outcome1 mieux possible. Dans cette section, nous présentons brièvement certains concepts et définitions de la théorie des jeux: (1) Jeux statique avec des informations complètes, (2) jeux statique avec des informations incomplètes, (3) jeux possibles, et (4) jeux de négociation de Nash.

Jeux Sous Forme Stratégique

Un jeu de stratégie est un modèle d'interaction des décideurs. En reconnaissance de l'interaction, nous nous référons aux décideurs comme des acteurs. Le modèle tient compte l'interaction entre les joueurs en permettant à chaque joueur d'être affectés par les actions de tous les acteurs, non seulement le joueur, AOS propre action. Chaque joueur a (1) un ensemble de stratégies possibles et (2) une préférence sur le profil d'action dont les caractéristiques sont capturés par une fonction de paiement.

Plus précisément, un jeu de stratégie se compose de trois éléments:

- Un ensemble joueur $\mathcal{N} = \{1, \dots, N\}$
- Une ensemble stratégie $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \times \mathcal{S}_N$
- Un ensemble gain $u = \{u_1, \dots, u_N\}$

où \mathcal{S}_n est la stratégie définie pour le joueur n , $u_n : \mathcal{S} \rightarrow \mathbb{R}$ est le remboursement ou la fonction d'utilité pour le joueur n . Soit $\mathbf{s} = \{s_1, \dots, s_N\}$ le profil de la stratégie, dans lequel la stratégie de chaque joueur est $s_n \in \mathcal{S}_n$. Alors, $\{s'_n, \mathbf{s}_{-n}\}$ désigne le profil de stratégie dans lequel chaque joueur j , sauf n choisit son action s_j tel que spécifié par \mathbf{s} , alors que le joueur n choisit s'_n . L'indice $-n$ sur \mathbf{s} signifie "sauf n ". Autrement dit, $u_n(s'_n, \mathbf{s}_{-n})$ est le gain de n joueur quand tous les autres joueurs à n adhérer à tout joueur n \mathbf{s} s'écarte s'_n . Avec l'aide de ces notions, maintenant nous pouvons dire la définition de l'équilibre de Nash (NE) avec précision.

Equilibre de Nash

Equilibre de Nash, du nom de John Forbes Nash, est un ensemble de stratégies, une pour chaque joueur, de sorte que aucun joueur n'a intérêt à changer unilatéralement sa stratégie. C'est un concept fondamental dans la non-coopération des jeux stratégiques.

defn (Equilibre de Nash)

Un profil de la stratégie \mathbf{s}^ est une stratégie pure équilibre de Nash, si pour chaque joueur $n \in \mathcal{N}$,*

$$u_n(s_n^*, \mathbf{s}_{-n}^*) \geq u_n(s_n, \mathbf{s}_{-n}^*) \quad (0.1)$$

pour chaque $s_n \in \mathcal{S}_n$. defn D'en haut, il est clair que NE représente tout simplement un particulier, "steady" etat pour un système, en ce sens que, une fois atteinte, aucun joueur n'a aucune motivation pour unilatéralement déroger. Dans

de nombreux cas, NE représente le résultat de l'apprentissage et l'évolution de tous les acteurs dans un système. Il est donc important de prévoir et caractériser ces points dans la perspective de la conception du système.

Méthodologies d'Analyse de l'Équilibre

En général, pour analyser l'équilibre de Nash, il faut tenir compte de trois aspects principaux, qui sont "existence", "l'unicité" et "la sélection d'équilibre", c'est à dire,

1. Existence - Est-ce un équilibre existent?
2. Unicité - Est-ce qu'il existe un équilibre unique ou les multiples?
3. Sélection Equilibrium - Comment sélectionner un équilibre de ceux multiples?

"Existence" est la toute première question qui vient naturellement dans notre esprit, car il est connu que, en général, un point d'équilibre n'existe pas forcément. Mathématiquement parlant, ce qui prouve l'existence d'un équilibre est équivalent à prouver l'existence d'une solution à un problème de point fixe [21]. Depuis l'existence des conseils de point fixe de l'existence d'un certain ensemble la stratégie qui est une meilleure réponse à lui-même, aucun joueur ne peut augmenter son gain en déviant, et il est donc un équilibre. Dans la littérature, il existe plusieurs théorèmes fournissant des conditions suffisantes pour l'existence d'un équilibre. Il existe de nombreux scénarios en supposant que d'habitude modèles de canaux sans fil et les mesures de performance où les théorèmes existants peuvent être appliqués, par exemple, la capacité de transmission a des propriétés de convexité souhaitable remplissant les conditions du théorème bien connu [22].

Theorem 0.1. (*Condition suffisante de Debreu*)

Si la stratégie définit S_n sont non vide, compact et convexes d'un espace euclidien, et si les fonctions de paiement des Nations u_n sont continues dans s et quasi-concave en s_n , il existe une stratégie pure équilibre de Nash.

"Unicité" de l'équilibre de Nash est le deuxième problème fondamental que nous devons aborder au moment de l'existence est assurée. L'unicité de l'équilibre est une propriété très souhaitable, si l'on veut prédire quel sera le comportement du réseau. Malheureusement, il n'y a pas beaucoup de résultats généraux pour l'analyse unique. Pour *les jeux N-personne concave*, utile des conditions suffisantes pour l'unicité de l'équilibre de Nash sont fournis dans [23]. Il est

montré que l'unicité est garantie si les fonctions de paiement satisfait que l'on appelle *diagonale strictement concave* condition. Toutefois, il existe de nombreux scénarios important lorsque l'équilibre n'est pas unique, par exemple, le routage des jeux [24], jeux de coordination [25], les jeux non-coopératifs avec des contraintes en corrélation avec la notion de "équilibre de Nash généralisée" [26], etc. Questions naturelles qui se posent concernent la sélection d'un équilibre approprié, c'est à dire, ce qu'on peut faire quand un jeu a des équilibres multiples? Y at-il équilibres "dominant" d'autres? Y at-il plus juste équilibre que d'autres?

"L'équilibre de sélection" est en fait une théorie d'âge mûr en lui-même [27]. De toute évidence, la règle de sélection est fortement liée à des critères d'équité, par exemple, l'équité max-min [28], l'équité proportionnelle [29, 30], optimisation globale, etc Dans cette thèse, nous ne fournissons pas la théorie générale de l'équilibre de sélection, au lieu nous sommes intéressés à ses applications aux jeux concaves. Plus précisément, nous allons utiliser la notion de "équilibres normalisés", introduite par J. B. Rosen [23] qui montre de manière très soignée pour s'attaquer à ce problème. Une discussion plus générale sur les méthodologies d'analyse d'équilibre dans les jeux sans fil peut être trouvée dans [31].

Jeux Statique avec Informations Incomplètes

La notion "d'information incomplète" signifie que certains joueurs ne sont pas tout connaître la structure de la partie. En particulier, certains joueurs ne savent pas les préférences des autres joueurs. Une hypothèse importante derrière la notion d'équilibre de Nash est que les joueurs ont des informations complètes sur la structure du jeu. Toutefois, cette hypothèse n'est guère rencontré dans de nombreux scénarios pratiques. Par exemple: les entreprises ne connaissent pas les coûts de production les uns des autres, joueurs d'échecs ne peut pas savoir styles les uns des autres en jouant aux échecs, etc.

Comment peut-on argumenter sur le jeu lorsque certains joueurs sont incertains sur les caractéristiques des autres joueurs? Un cadre de début pour les jeux statiques avec une information incomplète a été créé par John Harsanyi (1967) [32]. À la suite de ce cadre, nous pouvons l'étudier comme un jeu avec "bayésienne" joueurs. Ces joueurs ont des croyances initiales sur le "type" de chaque joueur. La croyance est définie analytiquement par une distribution de probabilité sur l'ensemble des possibles "types" d'un joueur.

Description des Jeux Bayésienne

Nous avons maintenant de décrire brièvement le modèle des jeux de Bayes, qui généralise la notion de jeu de stratégie à un framework informations.

Definition 0.2. *Un jeu bayésien \mathcal{G} est un jeu stratégique formulaire avec les informations incomplètes, ce qui peut être complètement décrit comme suit:*

$$\mathcal{G} = \langle \mathcal{N}, \{T_n, A_n, q_n, u_n\}_{n \in \mathcal{N}} \rangle.$$

Il se compose de

- Un ensemble **joueur**: $\mathcal{N} = \{1, \dots, N\}$,
et pour chaque joueur $n \in \mathcal{N}$
- Un ensemble **type**: T_n ($\mathcal{T} = T_1 \times T_2 \times \dots \times T_N$)
- Un ensemble **action set**: A_n ($\mathcal{A} = A_1 \times A_2 \times \dots \times A_N$)
- Un ensemble **fonction de probabilité**:

$$\rho_n : T_n \rightarrow f(T_{-n})$$

- Un ensemble **gain**:

$$u_n : \mathcal{A} \times \mathcal{T} \rightarrow \mathbb{R}$$

Nous clarifier les notions de type, de la stratégie, la probabilité et la récompense comme suit: (1) de type n de joueur $\tau_n \in T_n$ représente tout type d'information privée, plus précisément, toute information qui n'est pas la connaissance commune à tous les joueurs, et il est pertinent de leurs prises de décision. (2) la stratégie pure n joueur s_n est une fonction de mappage de type série T_n de n joueur dans son action mis en A_n , c'est à dire, $s_n : T_n \rightarrow A_n$. Nous désignons par \mathcal{S}_n la stratégie de jeu de joueur n . (3) La fonction de probabilité ρ_n mesure quel joueur n croit sur les types des autres joueurs, compte tenu de son propre type τ_n . Il prend la forme d'une probabilité conditionnelle, c'est à dire, $\rho_n(\tau_{-n}|\tau_n)$. (4) u_n , gain du joueur n , est une fonction du profil de la stratégie $\mathbf{s} = \{s_1, \dots, s_N\}$ et $\tau = \{\tau_1, \dots, \tau_N\}$ du profil type de l'ensemble des acteurs dans le jeu,

$$u_n(\mathbf{s}(\tau), \tau) = u_n(s_1(\tau_1), \dots, s_N(\tau_N), \tau_1, \dots, \tau_N)$$

Equilibre Bayésien

Notez que dans un jeu de stratégie-formulaire avec les informations complètes, chaque joueur choisit une action ou une stratégie, alors que dans un jeu bayésien, chaque joueur n est confrontée au problème du choix d'un ensemble de stratégies $s_n(\cdot)$, un pour chaque type τ_n , elle peut rencontrer.

Permettez $\{s'_n(\cdot), \mathbf{s}_{-n}(\cdot)\}$ désigner le profil de stratégie où tous les joueurs jouent $\mathbf{s}(\cdot)$, sauf n joueur joue $s'_n(\cdot)$. Ensuite, on peut écrire le gain du joueur n comme suit:

$$u_n(s'_n(\tau_n), \mathbf{s}_{-n}(\tau_{-n}), \tau) = u_n(s_1(\tau_1), \dots, s'_n(\tau_n), \dots, s_N(\tau_N), \tau)$$

Definition 0.3. (*Equilibre bayésien*)

Le profil de la stratégie de $s^*(\cdot) = \{s_n^*(\cdot)\}_{n \in \mathcal{N}}$ est une stratégie d'équilibre bayésien pure, si pour tout $n \in \mathcal{N}$, tout $s_n \in \mathcal{S}_n$ et $\mathbf{s}_{-n} \in \mathcal{S}_{-n}$

$$\mathbb{E}_{\tau_{-n}} [u_n(s_n^*(\tau_n), \mathbf{s}_{-n}^*(\tau_{-n}), \tau)] \geq \mathbb{E}_{\tau_{-n}} [u_n(s_n(\tau_n), \mathbf{s}_{-n}(\tau_{-n}), \tau)]$$

où

$$\mathbb{E}_{\tau_{-n}} [u_n(s_n(\tau_n), \mathbf{s}_{-n}(\tau_{-n}), \tau)] \triangleq \sum_{\tau_{-n} \in T_{-n}} \rho_n(\tau_{-n} | \tau_n) u_n(s_n(\tau_n), \mathbf{s}_{-n}(\tau_{-n}), \tau)$$

est défini comme le paiement espéré du joueur n .

Equilibre bayésien est parfois aussi appelé équilibre de Nash bayésien. Notez que la définition de l'équilibre bayésien est similaire à celle de l'équilibre de Nash, dans le sens où aucun joueur ne peut gagner à modifier unilatéralement sa propre stratégie. En fait, un équilibre bayésien de la jeu \mathcal{G} peut être considéré comme un équilibre de Nash en stratégies mixtes du jeu transformé $\hat{\mathcal{G}} = \langle \mathcal{N}, \hat{A} = S_1 \times S_2 \dots S_N, \hat{u} = u \rangle$. Par conséquent, nous pourrions simplement adopter la méthode d'analyse de l'équilibre de Nash pour étudier l'équilibre bayésien ensemble [31].

Jeux Potentiels

La notation des jeux potentiel a été d'abord utilisé pour les jeux sous forme stratégique par Rosenthal (1973) [33], et plus tard généralisé et résumées par Monderer et Shapley (1996) [34]. Ces jeux ont reçu une attention croissante récemment, en raison de leurs propriétés attrayantes (dont il sera question plus

loin). jeux possibles sont sous-classe de jeux sous forme stratégique. Un jeu est un jeu stratégique potentiel s'il existe une fonction appelée *potentiel*¹.

L'idée de base de jeux de potentiel est de trouver une fonction de potentiel, qui quantifie la différence dans les gains due à une déviation unilatérale de chaque joueur, soit exactement (exact jeux potentiels), en signe (potentiels ordinaux jeux), ou l'écart au mieux-réponse (jeux meilleur potentiel de réponse). Fonction potentiel peut être interprétée comme une mesure du désaccord entre les acteurs, ou, ce qui revient comme la dérive vers l'équilibre. Dans ce cas, l'ensemble des équilibres de Nash pure se trouve tout simplement en installant les optima locaux de la fonction potentielle.

Description des Jeux Potentiels

Ici, nous fournissons les définitions de base et quelques résultats utiles dans les jeux possibles. En effet, comme mentionné ci-dessus, il existe différents concepts de jeux possibles. Ceux de base entre tous les jeux possibles sont les possibilités exactes, le potentiel ordinal et le meilleur potentiel de réponse. Tous les jeux potentiels ont des définitions légèrement différentes qui peuvent conduire à des propriétés différentes de l'existence et la convergence vers un équilibre.

Definition 0.4. (*Jeux potentiels*)

Un jeu de stratégie \mathcal{G} est appelé *exacte/ordinaire/meilleur jeu de potentiel* si il existe une fonction $v : \mathcal{S} \mapsto \mathbb{R}$ satisfaisant

$$\mathbf{Exact:} \quad v(s_n, \mathbf{s}_{-n}) - v(s'_n, \mathbf{s}_{-n}) = u_n(s_n, \mathbf{s}_{-n}) - u_n(s'_n, \mathbf{s}_{-n})$$

$$\mathbf{Ordinal:} \quad v(s_n, \mathbf{s}_{-n}) - v(s'_n, \mathbf{s}_{-n}) > 0 \iff u_n(s_n, \mathbf{s}_{-n}) - u_n(s'_n, \mathbf{s}_{-n}) > 0$$

$$\mathbf{Best-response:} \quad \arg \max_{s_n \in \mathcal{S}_n} v(s_n, \mathbf{s}_{-n}) = \arg \max_{s_n \in \mathcal{S}_n} u_n(s_n, \mathbf{s}_{-n})$$

pour tous les $n \in \mathcal{N}$, et $(s_n, \mathbf{s}_{-n}), (s'_n, \mathbf{s}_{-n}) \in \mathcal{S}$, où $s_n \neq s'_n$.

D'après la définition ci-dessus, il peut être démontré que l'une des conditions exactes/ordinaire/mieux-réponse garantie que l'équilibre de l'ensemble $\mathcal{G} = \langle \mathcal{N}, \mathcal{S}, u \rangle$ jeu de stratégie originale coïncide avec l'équilibre ensemble de la jeu potentiel $\mathcal{G}^p = \langle \mathcal{N}, \mathcal{S}, v \rangle$. En outre, la relation suivante pour les différentes

¹En physique, un potentiel se réfère à un domaine défini dans l'espace, à partir de laquelle de nombreuses propriétés physiques importantes peuvent être tirées. Par exemple, dans le domaine de vitesse d'un fluide ou de gaz, la définition du potentiel implique que la direction de l'écoulement à tout moment coïncide avec la direction de la plus forte baisse du potentiel à ce point.

catégories de jeux possibles²:

Exact potential \subset Ordinal potential \subset Meilleure réponse potential

Des discussions approfondies sur divers jeux potentiels, les lecteurs intéressés sont appelés à [35] pour les jeux potentiel ordinal, [36] pour les jeux meilleur potentiel de réponse, et la [37] pour les pseudo jeux potentiel.

Propriétés des Jeux Potentiels

Jeux potentiels sont connus pour avoir des propriétés intéressantes, telles que l'existence d'un équilibre et la convergence des algorithmes de réponse optimale à l'équilibre.

Existence d'un équilibre de Nash: Il est connu que tous les stratégie $s \in \mathcal{S}$ qui maximise fonction potentielle $v(s)$ est une stratégie pure équilibre de Nash du jeu original \mathcal{G} [34]. Mais la réciproque n'est pas vraie en général, c'est à dire, il pourrait y avoir quelques NE stratégie pure ou mixte qui sont juste des points maximum local de $v(s)$. Néanmoins, on pourrait trouver de nombreux scénarios dans lesquels les résultats suivants sont utiles: (1) si la stratégie définie \mathcal{S} est convexe et v est continûment différentiable sur \mathcal{S} , puis tous les équilibres de Nash de \mathcal{G} est un point stationnaire de v , (2) si v est concave, puis tous les équilibres de Nash de \mathcal{G} est un point maximal de v , et d'ailleurs, (3) Si v est strictement concave, alors l'équilibre de Nash est unique.

La convergence vers l'équilibre de Nash: Il a été montré dans [34] que tous les jeux potentiels bornés a la propriété *approximate finite improvement property* (AFIP), c'est à dire pour tous les arbitraires $\epsilon > 0$ petits, tous les chemins ϵ -amélioration³ est finie. En d'autres termes, la convergence séquentielle mieux répondre à la ϵ -équilibre indépendamment du point initial. Notez que cette déclaration donne une condition très flexible pour la convergence, depuis l'ordre de lecture peut être déterministe ou aléatoire et ne doivent pas être synchronisés. Il est la propriété la plus intéressante de jeux possibles, en particulier pour distributive trouver un équilibre dans les systèmes d'auto-organisation.

²Il ya beaucoup d'autres notions de jeux potentiels, tels que: le potentiel pondérée généralisée potentiel ordinal, généralisée ϵ -potentiel, quasi-potentiel et pseudo-potentiel.

³Le chemin est défini comme une séquence d'étapes, à chaque étape, un seul joueur est autorisé à s'écarter. Un chemin $\phi = \{s^0, s^1, \dots\}$ est un chemin de ϵ -amélioration par rapport à \mathcal{G} , si pour tous les $t \geq 0$, $u_n(s^{t+1}) > u_n(s^t) + \epsilon$, où n représente l'indice d'un acteur unique au $t + 1$ étape.

Jeux de marchandage de Nash

Le modèle standard de la négociation de Nash a été présenté par J. Nash (1953) [38]. E. Kalai (1977) étendu le modèle de Nash sur le cas de priorité joueur non symétrique. Pour une discussion plus détaillée sur le modèle Nash symétriques et asymétriques ainsi que d'autres modèles de problèmes de négociation, les lecteurs intéressés peuvent se référer à l'article de A. Roth (1979) [39].

La négociation mot se réfère à une situation dans laquelle un groupe de deux ou plusieurs joueurs choisissent leurs paiements à partir d'un ensemble d'alternatives possibles, dont un seul peut être le résultat d'une négociation convenus par tous les négociateurs. Dans le cas où aucun accord unanime est établi, les joueurs se retrouvent avec des résultats pré-spécifié de désaccord, connu comme *point de menace*. Un jeu de négociation est généralement décrite par un ensemble $\mathcal{N} = \{1, \dots, N\}$ d'joueurs et d'une paire $(\mathcal{S}, \mathbf{d})$, où $\mathcal{S} \subset \mathbb{R}^N$ est un ensemble convexe compact, ce qui représente l'ensemble des *retombées possibles* pour les joueurs, $\mathbf{d} = \{d_1, \dots, d_N\}$ est le point de menace.

Allocation des ressources aux Jeux complet d'information

Récemment, il ya eu un intérêt croissant pour les petites cellules de réseaux sans fil, où les gens peuvent accéder à Internet via de nombreux points d'accès différents (APS) ou de petites stations de base, qui sont également connus comme les porte-femto-cellules, voir [4, 42]. Typiquement, dans un tel réseau sans fil, plusieurs points d'accès sont installés en plein air sur un réseau de base donnée (avec des liens hétérogènes que la fibre, ADSL, lignes électriques) pour assurer une couverture du signal dans des environnements denses. L'idée générale est d'exploiter les infrastructures hétérogènes filaire sans aucun nouveau câblage et de fournir sans fil très haut débit aux utilisateurs grâce à des mécanismes auto-organisés. Malheureusement, si les utilisateurs sont connectés à un seul point d'accès, ils peuvent souffrir de faible débit de temps à autre en raison de la capacité limitée backhaul (certains haute vitesse sans fil femto-cellules accéder à Internet via DSL ou des liens de faible capacité des lignes électriques, par exemple, 1 Mbps), malgré la présence d'un lien haute vitesse sans fil. En conséquence, les utilisateurs ont besoin d'accéder à différents points d'accès dans les environs femto-cellules, afin de regrouper l'ensemble des capacités des liaisons terrestres. Un concept intéressant est de considérer la proximité femto-cellules comme un groupe virtuel femto-cellule, dont la capacité de backhaul est la somme des capacités d'accès de tous les proches femto-cellules. La question de l'équilibrage de charge [43] dans le réseau filaire (et comment les différents paquets sont répartis à l'égard de la capacité de backhaul à partir d'un planificateur principal décentralisée), bien qu'importante, n'est pas traitée dans ce chapitre et nous allons supposer que l'équilibrage de charge parfaite détient.

Nous nous concentrons sur le problème d'allocation des ressources pour le scénario de liaison descendante, c'est à dire, du groupe femto-cellule pour terminaux mobiles (MT), utilisant la division de fréquence orthogonale (multi-

plexage OFDM) air-interface [44] sur un certain nombre de sous-canaux dédiés. Nous supposons que le réseau de codage est appliqué à la source Internet, c'est à dire, tous ces femto-cellules sont codées indépendamment des paquets depuis Internet via leurs retours à, et de les envoyer physiquement à chaque MT de manière distribuée. Habituellement, dans cette situation chaque point d'accès doit décider comment répartir le total disponible sous la puissance d'émission plus descendante N-canaux (sous-porteuses ou des groupes de sous-porteuses), c'est à dire, devrait renvoyer l'ensemble de ses pouvoirs à un seul sous-canal, la propagation du pouvoir sur l'ensemble des sous-canaux, ou de choisir un sous-ensemble des sous-canaux pour la transmission?

Dans les réseaux sans fil traditionnels, comme un problème d'allocation des ressources est considérée comme un problème d'optimisation globale. Le principal objectif est généralement d'atteindre des limites des performances théoriques, à savoir, région de capacité, vitesse maximale réalisable, sous réserve de quelques contraintes de ressources, par exemple, la puissance, la fréquence, etc. Toutefois, cette approche nécessite une entité centrale de calcul (scheduler) avec une connaissance complète des états du réseau, impliquant des commentaires généraux et de la communication dont la charge échelles linéairement avec le nombre d'émetteurs et de récepteurs dans le réseau. Cela est particulièrement important pour certains systèmes modernes sans fil comme femto-cellule et à l'extérieur du réseau à petites cellules, où il est assez difficile de centraliser les informations en raison d'une liaison terrestre capacité limitée. En outre, la forte densité de points d'accès ou de stations de base augmenterait le coût de centraliser les informations.

Il est certainement possible d'améliorer les transmissions de données utiles en réduisant les transmissions d'informations en retour insignifiants ou inutiles. En ce sens, un algorithme de la diversité sélective multi-utilisateurs a été introduit dans [45]. L'idée clé est de trouver un compromis approprié entre les performances du réseau et la charge de rétroaction. Néanmoins, cette approche partielle de rétroaction a encore son auto-limitation des problèmes de mise à l'échelle du réseau. Comme les réseaux sans fil semblent être de plus en plus dense, l'approche d'optimisation globale devient de plus en plus insuffisante pour répondre aux besoins de développement de la communication sans fil de demain.

Au cours des dix dernières années, un intérêt accru pour la recherche a été accordée à la radio cognitive [46] et d'auto-organisation des réseaux mobiles flexibles [4] dans lequel les périphériques sans fil intelligente sens et allouer les ressources de manière décentralisée. La théorie des jeux [21, 6], fournit une

méthode alternative d'analyse en considérant chaque noeud sans fil comme un joueur qui joue le jeu par adaptative et rationnellement le choix de ses stratégies de transmission, par exemple, transmettre des niveaux de puissance.

Nous passons en revue et d'enquêter sur l'utilisation de la théorie des jeux non-coopératifs avec information complète au problème de l'affectation des ressources sans fil, plus précisément la répartition de puissance. Pour un tel problème, le taux de transmission de chaque utilisateur sans fil est de plus en plus son propre niveau de transmission de puissance et diminue dans d'autres utilisateurs de transmettre le niveau de puissance. Dans la théorie des jeux, ce compromis peut être capturée avec un gain ou d'une fonction d'utilité et d'une structure de jeu bien conçu. Si les niveaux de puissance tous les autres utilisateurs ont été fixés, l'accroissement de la seule puissance augmenterait son signal-to-noise-plus-interference ratio (SINR) ainsi que ainsi que le taux de transmission. D'autre part, une telle action augmente l'interférence perçue par les autres utilisateurs, de conduire leur SINR et à les inciter à accroître leur transmettre propres niveaux de puissance.

Ensuite, nous présentons un cadre général en théorie des jeux pour le problème de l'affectation des ressources dans le sens descendant de réseaux de petites cellules sans fil, où plusieurs points d'accès ou stations de base indépendante envoyer des informations réseau crypté à MTS plusieurs par la voie OFDM. Dans cette étude la théorie des jeux, il est important d'étudier l'équilibre de Nash (NE), car il représente un résultat prévisible d'un réseau d'auto-organisation. La question centrale est de savoir si l'on considère A NE existe, et si oui, si le réseau fonctionne de façon efficace à la NE. Nous montrons que pour des canaux à évanouissements indépendants, la NE est toujours unique. En outre, nous montrons que ce problème d'allocation des ressources peut être étudié comme un jeu exact potentiel, et donc peut être efficacement résolu. Nous montrons la convergence des waterfilling meilleur algorithme de base-réponse, et nous discutons de quelques propriétés intéressantes de jeux possibles. Enfin, les résultats numériques sont fournis pour enquêter sur l'inefficacité de la NE.

Résultat Général

Nous avons présenté une approche non-coopérative théorie des jeux pour résoudre le problème d'allocation des ressources dans un multi-OFDM à petites cellules de réseau sans fil. Chaque émetteur est modélisé comme un joueur qui décide, de manière distribuée, la stratégie de la manière de répartir la puissance totale à travers plusieurs canaux indépendants à la décoloration. Nous

avons analysé l'existence et l'unicité de l'équilibre de Nash qui représente un résultat naturel de la partie. Pour toute réalisation de canaux indépendants décoloration, nous avons prouvé que l'équilibre de Nash est unique, avec une probabilité unitaire. En outre, nous avons introduit une approche potentielle théorie des jeux pour étudier la convergence de l'équilibre. Nous avons montré que notre jeu peut être étudiée comme un jeu potentiel. Afin d'obtenir le point d'équilibre, nous avons proposé un algorithme itératif simple basée sur les propriétés de jeux possibles. Nous avons montré comment la performance du réseau moyenne se comporter à l'équilibre de Nash unique, et nous l'avons comparé avec l'approche optimale centralisée. Des études numériques ont montré que, avec une probabilité très élevée, l'algorithme proposé itératif converge à 99% du taux final de moins de 5 itérations.

Allocation des Ressources aux Jeux Information Incomplète

En règle générale, les modèles de la théorie des jeux utilisés dans de nombreux travaux antérieurs [47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58] supposent que l'information complète et parfaite est connue de tous les appareils sans fil. Toutefois, cette hypothèse n'est guère respectée dans la pratique. Dans de nombreux systèmes actuels de communication sans fil, les appareils sans fil peuvent contenir des informations locales, mais à peine l'accès à l'information mondiale sur l'état du réseau. Le cas de la connaissance complète est une hypothèse simplificatrice que peut être une bonne approximation dans certains cas.

Ainsi, dans des contextes pratiques, ces jeux d'allocation des ressources doivent être réexaminés avec des hypothèses réalistes quant au niveau des connaissances des appareils sans fil. En vertu de cette considération, il est d'un grand intérêt pour étudier des scénarios dans lesquels les dispositifs ont des "informations incomplètes" sur leurs composants, par exemple, un dispositif est aware de son gain du canal, mais ignorent le gain des canaux d'autres dispositifs. Dans la théorie des jeux, un jeu de stratégie avec des informations incomplètes est appelé un jeu bayésien, et la notion d'"information incomplète" signifie que certains joueurs ne sont pas tout connaître les caractéristiques des autres joueurs dans le jeu, qui comprennent:

- Ensemble de joueurs (combien de dispositifs concernés? Qui sont-ils?)
- Ensemble d'actions des autres joueurs (ce sont leurs comportements?)
- Autres joueurs fonctions de paiement (ce sont les objets?)

Par conséquent, la motivation de ce chapitre est d'étudier comment les outils théoriques du jeu bayésien peuvent être mis en œuvre en pratique les paramètres sans fil.

En particulier, un jeu statique non-coopératifs avec information complète a été introduite dans le cadre de deux utilisateurs MAC la décoloration, connu sous le nom “waterfilling jeu” [47]. En supposant que les utilisateurs en concurrence avec les taux de transmission que l’utilité et de transmettre les pouvoirs que se déplace, les auteurs montrent qu’il existe un équilibre de Nash unique qui correspond à la somme maximum de point des taux de la région de capacité. Cette affirmation est quelque peu surprenant, étant donné que dans l’équilibre général Nash est inefficace comparant à l’optimalité de Pareto. Cependant, leurs résultats s’appuient sur le fait que les deux émetteurs ont la connaissance complète de CSI, en particulier, parfait CSI de tous les émetteurs dans le réseau.

Nous introduisons un modèle bayésien la théorie des jeux pour concevoir et analyser le problème d’allocation des ressources dans un des canaux à évanouissement d’accès multiples, où les utilisateurs sont supposés maximiser leur égoïsme les taux moyens réalisables avec des informations incomplètes sur les gains de canal à évanouissements. Dans cette étude la théorie des jeux, la question centrale est de savoir si un équilibre bayésien existe, et si oui, si le réseau fonctionne de façon efficace au point d’équilibre. Nous montrons qu’il existe exactement un équilibre dans notre jeu bayésien. En outre, nous étudions le problème de maximisation du réseau la somme des taux en supposant que les utilisateurs de coordonner la stratégie de profil symétrique. Ce résultat est aussi une limite supérieure pour l’unique équilibre bayésien. Enfin, les résultats de simulation sont fournis pour montrer l’efficacité du réseau à l’équilibre, et de le comparer avec d’autres stratégies.

Résultat Général

Nous avons présenté un cadre bayésien la théorie des jeux distribués pour l’allocation des ressources dans la décoloration MAC, où les utilisateurs sont supposés avoir que des informations locales sur le gain du canal du réseau Etats. En introduisant la prise en charge des Etats chaîne finie, nous avons réussi à trouvé un moyen d’analyse pour caractériser l’ensemble d’équilibre bayésien. Tout d’abord, nous avons prouvé l’existence et l’unicité de l’équilibre dans notre jeu bayésien. Deuxièmement, nous avons analysé la stratégie optimale de la puissance symétrique basé sur des préoccupations concrètes de la conception d’allocation des ressources. Cette méthode peut être également considérée comme une mesure de la limite supérieure pour l’unique équilibre bayésien. Enfin, la limite supérieure, la limite inférieure, et l’inefficacité de l’équilibre bayésien ont montré des résultats numériques.

Sélection des équilibres multiples dans les régions où le taux d'accès

Dans le chapitre 4 et le chapitre 5, nous présentons les approches non-coopératives théorie des jeux pour étudier le problème d'allocation des ressources aux différentes hypothèses faites sur la connaissance des dispositifs sans fil. Plus précisément, dans le chapitre 4, nous supposons que tous les dispositifs ont des informations complètes sur le gain du canal de chacun, dans le chapitre 5, nous supposons que les dispositifs ont des informations incomplètes sur le gain des canaux d'autres dispositifs.

Toutefois, dans les deux chapitres, nous avons seulement supposé que le récepteur utilise un décodage mono-utilisateur, donc, ce que nous avons examiné jusqu'à présent n'est que le taux de la région réalisable. On pourrait facilement améliorer les taux des utilisateurs par l'application multi-utilisateur de décodage au niveau du récepteur. Nous présentons une étude complète la théorie des jeux de l'allocation de débit pour les systèmes sans fil à accès multiple adoption de décodage multi-utilisateur au niveau du récepteur.

La région de la capacité d'un MAC N -utilisateur est présenté en détail au chapitre 2. Typiquement, pour une région N -dimension, on a un large choix de points de fonctionnement (allocations de taux). Toutefois, les points de fonctionnement différents possèdent des propriétés différentes en termes d'efficacité, l'équité, la stabilité, etc Nous classons MAC par les caractéristiques géométriques de la capacité de leurs régions. Plus précisément, nous étudions deux types de régions la capacité, c'est à dire, et les régions polytope convexe nonpolytope:

- Un exemple de MAC avec une région de capacité polytope est le général invariant Gaussien MAC [1].

- Deux exemples de MAC avec une région de capacité non-polytope convexe sont TDMA et FDMA MAC [1].

En général, on peut considérer deux approches méthodologiques pour sélectionner des points de fonctionnement dans la région de capacité: (1) l'approche théorie des jeux qui conduit à un point d'équilibre, et (2) approche de la fonction d'équité qui conduit à point de fonctionnement d'un système optimal. Pour l'approche théorie des jeux, nous considérons les jeux avec les contraintes découlant de corrélation avec les régions MAC capacité polytope. Dans ce chapitre, nous introduisons le concept d'équilibre normalisé pour régler le problème de sélection d'équilibre. Pour l'approche de la fonction d'équité, nous considérons des critères variés tels que l'équité max-min l'équité, l'équité proportionnelle, α équité, etc Nous montrons que dans le cas de la région de capacité polytope certains points spécifiques optimale juste coïncident avec l'équilibre points obtenus dans certains jeux. Ce dernier permet d'utiliser des algorithmes décentralisés pour atteindre ces points de fonctionnement.

Résultat Général

Nous avons étudié les voies d'accès multi-utilisateur dans le contexte des jeux non-coopératifs avec les contraintes liées. Dans les jeux non-coopératifs avec des contraintes en corrélation les actions disponibles pour un joueur dépendent de celles utilisées par les autres. Une caractéristique typique de ces jeux est qu'ils possèdent souvent infiniment plusieurs équilibres. Pour sélectionner parmi les équilibres que nous avons proposé d'utiliser l'équilibre de Nash normalisé et les concepts d'équité différentes telles que l'équité max-min, de l'équité proportionnelle, et α -équité. Nous avons examiné deux principaux types de canaux d'accès multi-utilisateur: les régions polymatroid et les nonpolytope régions convexes. Dans le cas de voie d'accès multiples Gaussien (un exemple d'un canal avec la région taux polymatroid réalisables), l'équilibre de Nash normalisé et tous les concepts d'équité sélectionner le même équilibre. Dans le cas d'accès multiples canaux Gaussien orthogonal (un exemple d'un canal avec convexe région taux de non-polytope possible) des concepts d'équité pouvez sélectionner équilibres différents.

Jeux Coopératifs Sans Fil via La Négociation de Nash

Dans le chapitre 4, nous avons établi l'unicité de l'équilibre d'un réseau OFDM à petites cellules en fonction lorsque le gain du canal matrice est supposée être une matrice aléatoire indépendantes et identiquement distribuée entrées en continu. Toutefois, à partir des résultats numériques, nous avons montré que l'inefficacité des performances des approches jeu non coopératif peut être importante par rapport aux approches centralisées, en particulier lorsque le nombre d'émetteurs se développe. En outre, le déséquilibre des performances au niveau du récepteur peut être un problème grave, car il n'y a absolument aucun contrôle sur la question de l'équité.

Les limites mentionnées et les inconvénients de l'affectation des ressources basée sur des jeux non-coopératifs peuvent être atténués par des approches de coopération. En fait, il existe deux types fondamentalement différents des jeux dans la théorie des jeux: jeux non-coopératifs et coopératifs. La mise en jeu non coopératif est celui dans lequel chaque joueur agit individuellement pour optimiser son rendement, sans égard à la performance des autres joueurs. Une telle procédure conduit à un point d'équilibre de Nash dans le réseau. Ce paramètre est important pour les joueurs agir sur la base uniquement sur des informations locales. En revanche, dans le cadre des jeux coopératifs, les joueurs sont autorisés à communiquer avant de choisir leurs stratégies et de jouer le jeu. Ils peuvent convenir, mais aussi en désaccord sur une stratégie commune. Si les joueurs jouent un jeu coopératif, puis les résultats de rendement est généralement meilleur que celui de l'équilibre de Nash. C'est parce que l'équilibre de Nash est Pareto-efficace sous certaines conditions [8]. A titre d'exemple, dans [83] il a été démontré qu'une approche centralisée peut surpasser l'itératif distribué waterfilling algorithme jusqu'à 135%.

Ce chapitre étend le cadre des jeux non-coopératifs dans le chapitre 4 dans un

cadre des jeux coopératifs, et que le réseau fonctionne à points Pareto-optimale. La coopération est plus probable que certains points d'accès à proximité des stations de base ou appartiennent à la même fournisseur de réseau. Dans cette situation, le fournisseur de réseau peuvent préférer laisser ces appareils sans fil coopérer pour fonctionner de manière plus efficace et plus équitable. Une des questions principales en théorie des jeux coopératifs est de trouver un compromis raisonnable, ou point de fonctionnement entre tous les acteurs. Dans ce chapitre, nous adoptons le concept de la négociation de Nash [84], car il fournit un point tel fonctionnement équitable de l'environnement de communication coopérative. la négociation de Nash est une des notions clés de répartition efficace et équitable dans la théorie des jeux coopératifs. Nous étudions le problème de négociation en utilisant des conditions KKT taux et le potentiel des jeux. Nous développons un algorithme stochastique pour obtenir la solution de négociation taux de Nash, sous la contrainte des exigences des joueurs taux minimal et maximal de transmettre des pouvoirs. Enfin, les résultats numériques sont fournis afin de confirmer nos prétentions théoriques.

Résultat Général

Dans cet article nous décrit le scénario de petites cellules sans fil où plusieurs émetteurs de communiquer avec plusieurs récepteurs par Independent OFDM sous-canaux comme un jeu coopératif. Nous étudions principalement le jeu de marchandage de Nash à l'aide de conditions KKT et pur potentiel. Un algorithme récursif stochastique est proposée. Il converge vers un point en tant que BNS sur la frontière de Pareto. Enfin, dans nos résultats de simulation, nous montrons la BNS ne fournit pas seulement une répartition équitable de puissance pour les joueurs, mais aussi mieux que l'efficacité du système spectrale par rapport à la NE et Optima solutions (sans tenir compte de l'équité) des jeux non-coopératifs. travaux futurs seront axés sur les jeux de la coalition et dynamique de négociation de Nash.

Conclusions et Perspectives

Conclusion

Cette thèse illustre le rôle important de la théorie des jeux dans la résolution des conflits sur les ressources existantes dans la répartition des ressources radio dans les multi-réseaux sans fil actuels.

Tout d'abord, le modèle de jeu statique est considérée comme une étape fondamentale pour mettre la théorie des jeux comme un outil pour la conception de l'affectation des ressources distribuées dans le cadre de réseaux sans fil. En effet, il nous aide à comprendre la motivation initiale, concept intuitif et la formulation commune d'un problème de jeu, et surtout, de se familiariser avec la méthodologie pour analyser ses résultats. Dans cette partie, deux modèles de base de canal sans fil sont discutés en détail, à savoir, les canaux d'accès multiples et des canaux d'interférence. Sur la base de la discussion précédente, dans [103], nous présentons une approche non-coopérative théorie des jeux pour le problème de l'affectation des ressources distribuées dans le contexte de multi-utilisateurs division orthogonale de la fréquence de multiplexage réseaux hot-spot où plusieurs émetteurs doivent décider comment partager leur puissance d'émission total sur plusieurs canaux parallèles et indépendantes à la décoloration. Nous avons prouvé qu'il existe presque sûrement un équilibre de Nash unique. Afin d'obtenir le point d'équilibre, nous avons proposé un algorithme itératif simple, à savoir Distributed potentiels itératif WaterFilling (DPIWF).

Deuxièmement, l'approche bayésienne théorie des jeux est introduit pour étudier un scénario plus pratique d'allocation des ressources, où les terminaux sans fil sont supposés avoir des informations incomplètes sur l'état du réseau. Plus précisément, nous supposons que chaque terminal est seulement conscient de l'information locale, par exemple, l'état de son propre canal, mais pas au courant des canaux d'autres terminaux. Dans [104, 105], nous nous concentrons sur le jeu de répartition de puissance dans les canaux d'accès multiples

avec un nombre arbitraire d'utilisateurs. Nous reformulons le jeu waterfilling comme une version d'un jeu à information incomplète, et nous avons réussi à prouver l'existence et l'unicité de l'équilibre bayésien. L'unicité de l'équilibre est tout à fait une propriété souhaitable, si l'on veut prédire quel sera le comportement du réseau. Cela est particulièrement important dans le contexte de l'administration réseau et la gestion, où les fournisseurs de services sans fil sont intéressées de façon optimale le réglage des paramètres de conception de réseau, en tenant compte de leur impact sur la performance à l'équilibre. La mesure d'efficacité de Pareto est abordée à travers des résultats de simulation. Les œuvres futures sont envisagées pour améliorer l'efficacité de l'équilibre bayésien grâce à la tarification ou les approches coopératives théorie des jeux.

Troisièmement, étant donné que les deux approches ci-dessus ne considère une région taux réalisables contraint à l'utilisation de décodage mono-utilisateur, nous avons étendu notre étude à multiples voies d'accès multi-utilisateur avec le décodage au niveau du récepteur. Le problème d'allocation des ressources est effectuée en utilisant les jeux non-coopératifs avec des contraintes en corrélation les actions disponibles pour un joueur dépendent de celles utilisées par les autres. Une caractéristique typique de ces jeux est qu'ils possèdent souvent plusieurs équilibres infiniment. Pour sélectionner parmi les équilibres que nous avons proposé d'utiliser l'équilibre de Nash normalisé et les concepts d'équité différentes telles que l'équité maxmin, l'équité lexicographique, α -équité et de justice Jain. Nous avons examiné deux principaux types de canaux d'accès multi-utilisateur: les régions polymatroids et les nonpolytope régions convexes. Dans le cas de canaux avec la région de la capacité polymatroid, l'équilibre de Nash normalisé et tous les concepts d'équité sélectionner le même équilibre. Dans le cas de canaux avec convexe région taux de non-polytope réalisables, les concepts d'équité différents du rendement de la sélection des équilibres différents. Nous avons caractérisé ces équilibres. Il s'avère que dans certains canaux avec les non-convexe régions où le taux réalisable l'allocation max-min juste pourrait même ne pas exister. Une voie de recherche intéressante est d'étudier l'avenir de sélection des équilibres dans le cas de plusieurs stations de base.

Enfin, un concept de répartition équitable des ressources, la négociation de Nash, est introduit pour étudier le problème d'allocation des ressources Orthogonal Frequency Division Multiplexing (OFDM) systèmes. L'objectif est de maximiser le taux de l'ensemble du système compte tenu mesures d'équité entre les utilisateurs. Nous proposons une solution de marchandage de Nash comme un outil qui permet d'obtenir un bon point sur la frontière de Pareto du jeu région taux théorique dans des conditions asymétriques. En outre, nous four-

nissons un algorithme pratique stochastique qui peuvent converger vers l'un des points d'équité sur la frontière de Pareto. Les résultats numériques montrent que la solution de marchandage de Nash non seulement maintient l'allocation des ressources équitable pour tous les utilisateurs, mais aussi une efficacité spectrale souhaitable pour le système OFDM.

Future Works

- Jeux de Coalition

En général, la théorie des jeux peut être divisé en deux branches: la théorie des jeux non coopératifs et coopératifs. Non coopératifs étudies théorie des jeux les choix stratégiques découlant de l'interaction entre les joueurs en compétition, où chaque joueur choisit sa propre stratégie pour améliorer ses propres performances (utilité) ou de réduire ses pertes (coûts). Alors que les scénarios non coopératifs étudies théorie des jeux compétitifs, théorie des jeux coopératifs fournit des outils d'analyse pour étudier le comportement des acteurs rationnels quand ils coopèrent. La branche principale des jeux coopératifs décrit la formation de la coopération des groupes de joueurs, dénommé coalitions qui peuvent renforcer les positions des joueurs dans un match. Jeux de Coalition ont été largement étudiées dans différentes disciplines comme l'économie ou la science politique. Récemment, la coopération est apparue comme un paradigme de mise en réseau de nouvelles qui a un effet dramatique de l'amélioration des performances de la couche physique jusqu'à la couche réseau. Jeux de Coalition se révéler un outil très puissant pour la conception juste, les stratégies de coopération robuste, pratique et efficace dans les réseaux de communication.

- Jeux de Stackelberg

Jeux de Stackelberg sont des outils puissants pour étudier les réseaux sans fil avec la hiérarchie, ce qui signifie prises de décision des utilisateurs sont asymétriques, par exemple, dans des contextes où il ya des primaires (licence) et les utilisateurs secondaires (sans licence) les utilisateurs qui peuvent sentir leur environnement, car il sont équipés d'un cognitive radio. Il est démontré que l'introduction d'une certaine hiérarchie dans les jeux non-coopératifs de contrôle de puissance peut améliorer la performance individuelle de tous les utilisateurs, et peut atteindre un désirée compromis entre la performance du réseau mondial à l'équilibre et le montant

demandé des signalisations de les réseaux.

- Apprentissage et jeux dynamiques

Stratégie jeux modéliser l'interaction entre les décideurs en termes de résultat d'équilibre, mais ne précisent pas comment les joueurs atteignent un équilibre. Un problème de sensibilisation dans les communications sans fil est liée à l'algorithme itératif de l'eau de remplissage de type qui convergent vers l'état d'équilibre. Afin d'appliquer ces algorithmes, les émetteurs sont supposées être strictement rationnelle joueurs qui connaissent la structure de la partie. Certaines hypothèses peuvent ne pas être réaliste, par exemple, des renseignements complets, ensembles discrets action finie, etc Une autre manière d'expliquer comment les joueurs d'un jeu peut converger vers une NE est la théorie de l'apprentissage. algorithmes d'apprentissage sont à long terme des processus dans lequel les joueurs avec une connaissance très peu de contraintes et de la rationalité essayer d'optimiser leurs prestations. L'apprentissage et la dynamique sont très importants dans des environnements dynamiques tels que les canaux sans fil, de la présence d'observations imparfaites, des délais et des mesures de bruit.

Chapter 1

Introduction

1.1 Background and Motivation

It has been nearly one and half century since the invention of wireless transmission using electromagnetic spectrum. From the early use of spark-gap telegraphy (1888, H. Hertz) to the first clear transmission of human speech in air (1919) and to the invention of frequency modulation (FM) until the early form of mobile phone cellular networks (1947, Bell Labs), wireless technologies and applications have been greatly changed. However, interference is still a Pandora's box in the design and analysis of today's multi-user wireless systems.

In wireless communications, multiple access transmission turns out to be a very common scenario, since the radio spectrum is a shared natural resource in the sense that every user can access the same physical link and transmit over it. Interference arises at the receiver when several transmitters simultaneously access the same physical link. The message collisions can significantly reduce the communication quality or even prevent proper signal reception. In this case, the control and reduction of interference plays an important role in wireless communications. Traditionally the design of wireless systems has focused on increasing the reliability of the physical link. Recent focus has shifted more towards increasing the spectral efficiency. Under this background, many media access technologies have been introduced, such as frequency division multiple

access (FDMA), time division multiple access (TDMA), code division multiple access (CDMA), and space division multiple access (SDMA).

With the increasing demand of wireless voice and data services, frequency spectrum has become a scarce natural resource for which everyone competes. In fact, the spectral efficiency (efficiency measurement of using frequency spectrum) can still be improved by radio resource management techniques such as efficient fixed or dynamic channel allocation, power control, link adaptation and diversity schemes. Over the past two decades, resource allocation based on information theory [1] and optimization theory [2, 3] has significantly improved the efficiency in the utilization of frequency resource and played a central role in this respect. For instance, waterfilling algorithm [1, 2] is one of the most famous techniques that consider how to maximize the channel capacity. However, in order to achieve the information-theoretic limits, e.g., capacity region, it requires a central entity (scheduler) to globally schedule and allocate the system resources. A scheduler is required to have the capability of tracing the wireless channels in the network.

Obviously, a centralized scheduler with complete and perfect knowledge of all the wireless channels can optimize the global network resource assignment. However, this approach usually involves sophisticated optimization techniques and a signaling load that grows with the number of transmitters and receivers in the network. Due to this inherent optimization complexity and signaling overhead requirements, the centralized infrastructure may become the largest bottleneck for the development of future wireless networks. And in fact, it has already exposed its own weakness in some aspects, e.g., slow and expensive reconfiguration against varying environment, difficulty in centralizing network information, increased computational complexity, etc. Moreover, planning, deployment and operations of a cellular network are expensive processes. The propagation characteristics of the area need to be estimated, the base station sites need to be carefully selected, and the equipments have to be installed. Once the network is operational, adjustments must often be made. They may even include the placement of additional base stations to eliminate dead spots. Altogether this is a time-consuming and manpower-intensive series of tasks.

It turns out that due to the different nature of the mobile devices as well as the high mobility of the network, future wireless networks are going towards more decentralized resource allocation schemes in which each mobile device or base station can exploit local information to make their own optimization processes, and therefore decide of their own resources usage [4]. As a matter of fact, wireless service providers have been seeking for infrastructures that are self-

configuring, self-operating, and self-optimizing [5]. Specifically, the providers long for base stations to deploy quickly with no specialized technicians, be able to automatically discover their neighbors, automatically reconfigure in case of network failures, and automatically optimize their radio parameters. In addition to this, backhaul should be automatically configured, and quality of service (QoS) should be autonomously optimized. These capabilities, along with many others, are the vision of future self-organizing wireless networks.

During the past decade there has already been an explosion of interest in self-organizing wireless networks. As a matter of fact, more and more people from the community of wireless communications are using concepts like artificial intelligence, cognitive radio, and distributed networks in their research. In particular, game theory [6], as a powerful mathematical tool, has been increasingly applied to wireless communications. Game theory was introduced, in its actual form, in 1944 by J. V. Neumann and O. Morgenstern in their landmark book “Theory of Games and Economic Behavior” [7] and has been applied successfully in various fields such as economics¹, biology (evolutionary stable strategy introduced by J. M. Smith in 1982), war (cold war, war on terrorism), philosophy (morality from self-interest), social science (explanation for the democratic peace), computer science (modeling interactive computations), etc.

Indeed, wireless networks contain three ingredients that make game theory a fruitful theory for their analysis and design: (1) mobile terminals are equipped with a certain degree of intelligence or cognition, (2) mobile devices share some common resources which implies natural interaction between them, and (3) wireless networks are highly structured. This favors the success of game theoretic analysis. A game-theoretical framework can fit very well the concept of self-organizing networks, where the intelligence and decision making is distributed. Such a scheme has apparent benefits in terms of feedback load. With a game-theoretical setup, mobile devices only need incomplete information or partial knowledge of the environment, and no more computationally heavy optimization algorithm is required to solve at the centralized scheduler. But on the other hand, since no clear search of the optimal point is performed, when each device attempts to maximize its interest, e.g., data rate, under certain conditions it may happen that the resulting system performance is far from being globally optimal [6, 8].

¹In the famous literature *The Wealth of Nations*, Adam Smith (A Scottish moral philosopher, pioneer of political economy, father of modern economics) expounded how rational self-interest and competition can lead to economic prosperity and well-being through macroeconomic adjustments.

1.2 Thesis Objectives

The main objectives of this thesis are:

- Understand the fundamentals of game theory as well as optimization theory, such as mathematical definition of a game problem and an optimization problem, difference between a game and an optimization problem, classification of games and optimizations, etc.
- Investigate the analytical methods for studying the outcome of game-theoretical problems, such as existence of equilibrium, uniqueness of equilibrium, and equilibrium selection.
- Model the resource allocation problems in multiuser wireless communications as game-theoretical problems. Classification of games relevant for resource allocation in wireless networks.
- Evaluate the wireless network performance derived from game-theoretic solutions in terms of system efficiency and fairness.
- Both the engineer and theorist standpoints will be considered. In particular a clear mapping between practical problems in wireless networks and game-theoretic approaches will be established.

1.3 Basic Assumptions

In this section, we discuss and detail some assumptions that are made throughout this dissertation.

- Resource allocation per transmission time slot (TTS): We assume that the wireless channel state stays constant over at least one TTS, which refers to the length of an independently decodable transmission on the wireless link. Resource allocation needs to be updated every TTS.
- Signal constellation: The signal constellation is assumed to be Gaussian. The modulation level is high enough so that the mutual information is approximately the channel capacity. In our analysis and numerical results, the influence of the modulation can be considered by adding the signal-to-noise ratio (SNR) gap Γ in the capacity formula: $C = \log(1 + \text{SNR}/\Gamma)$, where Γ is a function of the bit error rate (BER) of the modulation [9].

- Channel state information (CSI): In all chapters we assume that complete and perfect CSI is available at the receivers. In Chapter 4, 6, and 7, we assume that complete and perfect CSI is also available at the transmitters. Whereas in Chapter 5, we assume the transmitters have incomplete CSI, i.e., each transmitter is aware of her own channel states, but unaware of the channel states of all other transmitters.
- Single user and multi-user decoding: In Chapter 4 and 5, we consider an achievable rate region constrained to the use of single user decoding at the receiver. This achievable region is very relevant in practical wireless systems with limitations on the decoder complexity and simplicity of the system. In this context, we aim at finding the optimal power allocation for the network users according to some performance measure. In Chapter 6 and 7, we consider the capacity region corresponding to the use of multi-user decoding at the receiver.
- Rationality: One of the most common assumptions made in game theory is rationality [10, 11]. It means that every player always maximizes her payoff, thus being able to perfectly calculate the probabilistic result of every action. However, in reality this assumption can only be reasonably approximated since rationality of individuals is limited by the information they have, the cognitive limitations of their minds, and the finite amount of time they have to make decisions.

1.4 Thesis Outline

In general terms, the focus of this dissertation is on the game-theoretical applications to the design of distributed resource allocation in wireless channels. The outline of all chapters is as follows.

Chapter 1, the present chapter, gives the motivation, outline, and contributions of this dissertation.

Chapter 2 overviews the technical background in wireless systems considered in this dissertation, such as wireless channel characteristics, two fundamental wireless channel models, i.e., multiple access channels and interference channels, and orthogonal frequency division multiplexing (OFDM) technology.

Chapter 3 introduces two fundamental theories: optimization theory and game theory, providing the main mathematical tools for this dissertation.

Chapter 4 considers a non-cooperative game-theoretic approach for distributed resource allocation in wireless communications, assuming that (1) complete and

perfect CSI is available both at transmitter and receiver (2) single user decoding strategy is applied at the receiver side. This chapter first summarizes some current existing game-theoretic approaches for allocating wireless resource in multiple access channels and interference channels. Then, we focus on a particular wireless resource allocation problem in the context of small-cell networks using OFDM air-interface, which is closely relevant to the introduced existing game-theoretic approaches. In this framework, we show that a unique pure strategy Nash equilibrium exists under a common condition of wireless fading channels. In additions, we show that the considered resource allocation problem can be studied as a potential game, and hence efficiently solved.

Chapter 5 extends the game-theoretical framework of Chapter 4 to the more realistic situation of incomplete CSI at the transmitter. In this respect, we consider a Bayesian game-theoretic approach for the resource allocation problem in the context of fading multiple access channels. We establish the existence and uniqueness of the Bayesian equilibrium. The inefficiency of equilibrium is evaluated through numerical experiments.

Chapter 6 studies non-cooperative games with correlated constraints, known as generalized Nash games in the context of multiple access channels. Differently from Chapter 4 and 5, in this chapter we consider the capacity region of multiple access channels assuming multiuser decoding at the receiver side. In this game-theoretical problem, the major issue is to select among a large number of equilibria. We propose the use of normalized equilibrium together with some fairness concepts, e.g., max-min fairness, proportional fairness, α -fairness. In the case of Gaussian multiple access orthogonal channel with convex non-polytope rate region, we show that different fairness concepts can select different equilibria. In the case of Gaussian multiple access channel with convex polytope rate region, surprisingly, we show that all fairness points coincide with the unique normalized equilibrium.

Chapter 7 extends the results of Chapter 4 to a cooperative game-theoretical setting, namely Nash bargaining game. The Nash bargaining problem is established using duality theory and potential games. A recursive stochastic algorithm is proposed to find the Nash bargaining solution.

Chapter 8 summarizes the main results of this dissertation and look into the perspectives of future researches.

The dependence among the chapters is illustrated in Figure 1.1.

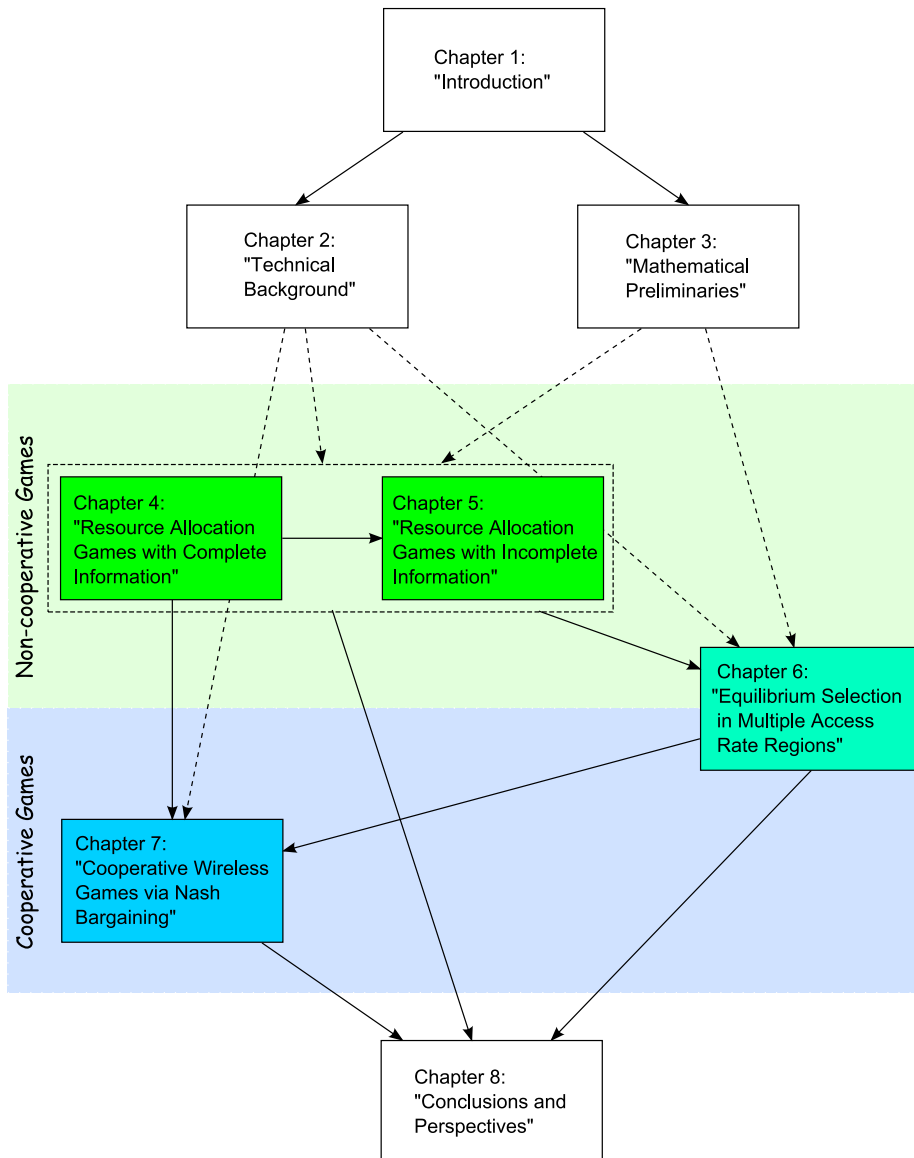


Figure 1.1 Structure chart of chapters

1.5 List of Publications

Book Chapter

- G. He, M. Debbah, S. Lasaulce, and E. Altman, “Wireless Games with Incomplete Information”, accepted by *Game Theory for Wireless Communications and Networking*, Auerbach Publications, Taylor and Francis Group, CRC Press, July 2009.
- T. Chahed, L. Cottatelluci, R. Elazouzi, S. Gault, G. He, “Information Theoretic Capacity of WiMAX”, published in *Radio resources management in WiMAX: from theoretical capacity to system simulations*, Wiley-Iste/Hermes Science Publisher, March 2009.

Journal

- G. He, L. Cottatellucci and M. Debbah, “The Waterfilling Game-Theoretical Framework for Distributed Wireless Network Information Flow”, *submitted to EURASIP Journal on Wireless Communications and Networking, 2nd Revision*, 2009.
- G. He, M. Debbah and E. Altman, “A Bayesian Game-Theoretic Approach for Distributed Resource Allocation in Fading Multiple Access Channels”, *accepted to EURASIP Journal on Wireless Communications and Networking*, 2009.

Conference

- G. He, Y. Hayel, S. Lasaulce and M. Debbah, “A Hierarchical Energy Game in Wireless Networks”, *5th International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CrownCom)*, Cannes, France, June 2010.
- G. He, M. Debbah and E. Altman, “Game-Theoretic Techniques for Intelligent Wireless Networks”, *Cognitive Systems with Interactive Sensors*, Paris, France, Nov. 2009.
- G. He, M. Debbah, and S. Lasaulce: “N-player Bayesian Waterfilling Game for Multiple Access Channel”, *The Third International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, Aruba, Dutch Antilles, Nov. 2009.

- G. He, S. Betz, and M. Debbah, “Game-Theoretic Deployment Design of Small-Cell OFDM Networks”, *3rd Workshop on Game Theory in Communication Networks*, Pisa, Italy, Oct. 2009.
- G. He, H. Tembine and M. Debbah, “Deployment Analysis of Cooperative OFDM Base Stations”, *IEEE International Conference on Game Theory for Networks (GameNets)*, Turkey, June 2009.
- E. Altman, L. Cottatellucci, M. Debbah, G. He and A. Suarez, “Operating Point Selection in Multiple Access Rate Regions”, *International Teletraffic Congress*, Paris, France, June 2009.
- G. He, S. Gault, M. Debbah and E. Altman, “Iterative Mercury/waterfilling for Parallel Multiple Access Channels”, *IEEE International Conference on Communications (ICC)*, Beijing China, 19-23 May 2008.
- G. He, S. Gault, M. Debbah and E. Altman, “Distributed Power Allocation Game for Uplink OFDM Systems”, *2nd International Workshop on Wireless Networks: Communication, Cooperation and Competition*, Berlin Germany, April 2008.

Patent

- G. He, S. Gault and M. Muck, “An IEEE 802.11 VHT OFDMA Frame: Proposal of a Novel Uplink Signaling Field and Suitable Sub-carrier Allocation Mechanisms”, August 2007.

Chapter 2

Technical Background

2.1 Wireless Channel Characteristics

Nowadays, wireless communication is one of the most attractive and productive areas in the communication field. It has been a topic of study since the 1960s, and the past three decades have seen a surge of research activities in this area. This is mostly due to the influence of Very-Large-Scale Integration (VLSI) technology and the increasing demand for wireless connectivity. In particular, the penetration in the market of cellular/mobile phones has been spectacular. From a negligible presence in the early 1980s, the worldwide cellular technology penetration in 2002 is estimated at 1 billion mobile subscriptions. This figure is expended to be about 4.6 billion by the end of 2009 [12]. Since the success of 2G (second generation) digital wireless standards, the research thrust in the past three decades has led to a much richer set of perspectives on how to communicate over wireless channels. Based on these perspectives, many new standards such as 3G/HSPA, CDMA, Wi-Fi, WiMAX and 3GPP LTE. have been developed to cope with the explosive demand for wireless connectivity. Now, this evolution is still going on.

There are two fundamental aspects of wireless communications that make the problem essentially challenging and interesting. The first one is the phenomenon of *fading*, i.e., the time variation of the channel strengths due to the small-scale

effect of multipath fading, as well as larger-scale effects such as path loss via distance attenuation and shadowing by obstacles. Generally speaking, wireless channels change both in time and frequency. The time coherence¹ shows how quickly the channel changes in time, and similarly, the frequency coherence² shows how quickly it changes in frequency.

- When time coherence is concerned, wireless channels are categorized as *fast fading* and *slow fading*. Fast fading occurs when the channel coherence time is much shorter than the delay requirement of the application. Slow fading arises if the channel coherence time is longer. In a fast fading channel, one can transmit the coded symbols over multiple fades of the channel, while in a slow fading channel, the channel is constant during the transmission of a codeword.
- When frequency coherence is concerned, wireless channels are categorized as *frequency-selective fading* and *flat fading*. When the bandwidth of the input signal is much larger than the coherence bandwidth, the channel is said to be frequency-selective. When the bandwidth is considerably less than the coherence bandwidth, the channel is said to be frequency-flat, since it affects all signal frequencies in almost the same manner.

Note that whether a channel is fast or slow fading, flat or frequency-selective fading depends not only on the wireless environment but also on the input signal and its applications, i.e., the delay requirement of the application, the bandwidth of the input signal.

The control and reduction of multiuser *interference* is a second fundamental aspect in wireless communications. In contrast to wireline communications, where each transmitter-receiver pair can often be considered as an isolated point-to-point link, in wireless communications there is a significant interference between wireless users' communications over the air. How to deal with interference is one of the central questions in the design of wireless communication systems and will be an important theme of this dissertation. Traditionally the design of wireless systems has focused on increasing the reliability of the air interface. Recent focus has shifted more towards increasing the spectral efficiency. Under this background, many media access technologies together with efficient

¹The major effect in determining time coherence is the Doppler spread. The larger the Doppler spread, the smaller the time coherence.

²The delay spread of the channel dictates its frequency coherence. Delay spread is a type of distortion that is caused when an identical signal arrives at different times at its destination due to multipath effects.

resource allocation algorithms have been introduced. All these technologies can be briefly summarized from the viewpoints of *degrees of freedom* [13]:

- **Frequency**

Frequency Division Multiple Access (FDMA) is a channel access method which gives each user an individual allocation of one or several frequency sub-bands (or sub-channels).

Frequency Modulation (FM) broadcasting can be considered as a simple example of FDMA.

- **Time**

Time Division Multiple Access (TDMA) is a channel access method which allows several users to share the same frequency channel by allowing them to transmit in different time slots.

TDMA technology is widely used in the digital 2G cellular systems, e.g., IS-54 (1990) and GSM (1991).

- **Code**

Code Division Multiple Access (CDMA) is a channel access method employing spread-spectrum technology and a special coding scheme (where each transmitter is assigned a code) to allow multiple users to be multiplexed over the same physical link.

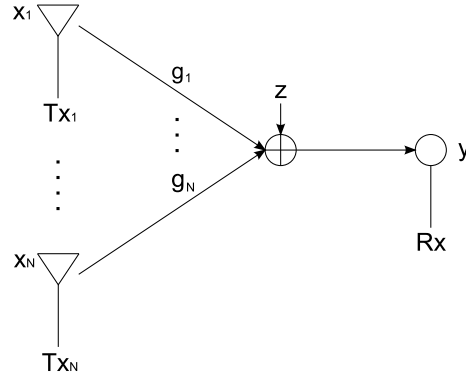
CDMA technology leads the migration from GSM to 3G, e.g., IS-95B (1999) as 2.5G technology and CDMA2000 (2000) as 3G technology.

- **Space**

Space Division Multiple Access (SDMA) is a multiple input and multiple output (MIMO) based wireless communication technology. In order to provide radio coverage in traditional mobile cellular network systems, the base station radiates the signal in all directions, since it has no information on the position of the mobile devices within the cell. SDMA is an advanced channel access method which enables access to a communication channel by identifying the user location and establishing a one-to-one mapping between the network frequency division and the identified spatial location.

SDMA technology is considered as a key feature in 4G cellular systems.

When the number of users, base stations, and cells in the network increases, interference becomes the bottleneck. It turns out that due to the different nature

Figure 2.1 N -user multiple access channel

of the mobile devices as well as the high mobility of the network, future wireless networks are going towards more self-organizing resource allocations schemes in which mobile devices intelligently allocate resource in a decentralized manner [4].

2.2 Multiple Access Channels

The Mathematical Model

In wireless communications, multiple access channel (MAC) is a basic channel model that allows several terminals connected to the same transmission medium to transmit through it and to share its capacity (Figure 2.1). The input-output equation for a N -user MAC can be written as

$$y(t) = \sum_{n=1}^N \sqrt{g_n(t)} x_n(t) + z(t)$$

where $x_n(t)$ and $g_n(t)$ are the input signal and fading channel gain of the n^{th} user, respectively. The input signal $x_n(t)$ can be further written as $x_n(t) = \sqrt{p_n(t)} s_n(t)$ where $p_n(t)$ and $s_n(t)$ are user n 's transmit power and data with normalized power, respectively. User n is subject to an average transmit power constraint P_n^{max} . $z(t)$ is assumed to be zero-mean additive white Gaussian noise (AWGN) with variance σ^2 . Note that in this basic model, we consider fading effects which are frequency flat.

Capacity Region of Multiple Access Channels

We consider first the simple situation of Multiple access channel with time-invariant channels, and the signal of user n is attenuated by a constant factor

of g_n at the receiver, i.e., $g_n(t) = g_n$ for all time t . The capacity region of a N -user time-invariant MAC is well known [14, 15]. It is the set of all rate vectors $\mathbf{r} = \{r_1, \dots, r_N\}$ satisfying

$$\mathcal{R} = \left\{ (r_1, \dots, r_N) : 0 \leq \sum_{n \in \mathcal{X}} r_n \leq \mathcal{I}(\mathcal{X}), \forall \mathcal{X} \subseteq \{1, 2, \dots, N\} \right\}$$

where $\mathcal{I}(\mathcal{X})$ is defined as $\mathcal{I}(\mathcal{X}) := I(X_{\mathcal{X}}; Y)$, the mutual information between the input variables $X_{\mathcal{X}} = \{X_n\}_{n \in \mathcal{X}}$ and the output variable Y . Note that \mathcal{X} is any subset of users in $\{1, 2, \dots, N\}$. The channel capacity [16], denoted by $C(\mathcal{X})$, is obtained by maximizing the mutual information over all possible input distributions $\Pr(X_{\mathcal{X}})$, i.e.,

$$C(\mathcal{X}) = \max_{\Pr(X_{\mathcal{X}})} I(X_{\mathcal{X}}; Y) = \log \left(1 + \frac{\sum_{n \in \mathcal{X}} g_n p_n}{\sigma^2} \right) \quad (2.1)$$

where the maximum is achieved when all the inputs X_1, \dots, X_N are independent Gaussian variables. Throughout this dissertation, if not differently stated, we assume that the transmitted symbols are independent from user to user and Gaussian distributed. Under this assumption, we can derive the capacity region as

$$\mathcal{C} = \left\{ (r_1, \dots, r_N) : 0 \leq \sum_{n \in \mathcal{X}} r_n \leq C(\mathcal{X}), \forall \mathcal{X} \subseteq \{1, 2, \dots, N\} \right\} \quad (2.2)$$

which is known to be a convex polytope. A front face of \mathcal{C} related to set \mathcal{X} is defined as

$$\mathcal{F}_{\mathcal{X}} = \mathcal{C} \cap \left\{ (r_1, \dots, r_N) : \sum_{n \in \mathcal{X}} r_n = C(\mathcal{X}) \right\}$$

Note that this capacity region consists of $2^N - 1$ front faces, each corresponding to a sum-rate constraint with a different nonempty subset of users \mathcal{X} . There is one front face of \mathcal{C} that stands out from the others. It is the dominant face $\mathcal{F}_{\{1, \dots, N\}}$. All points of the dominant face have correspond to rate allocation with maximum sum-rate (or total rate). See [17] for more details on the structure of the dominant face. In addition, this capacity region has precisely $N!$ vertices in the positive quadrant, each is achievable by a successive decoding using one of the $N!$ possible decoding orderings. Successive decoding consists in decoding the users sequentially by single user decoding and treating the users not yet decoded as noise. At each iteration, the decoded user signal is subtracted from the sum signal.

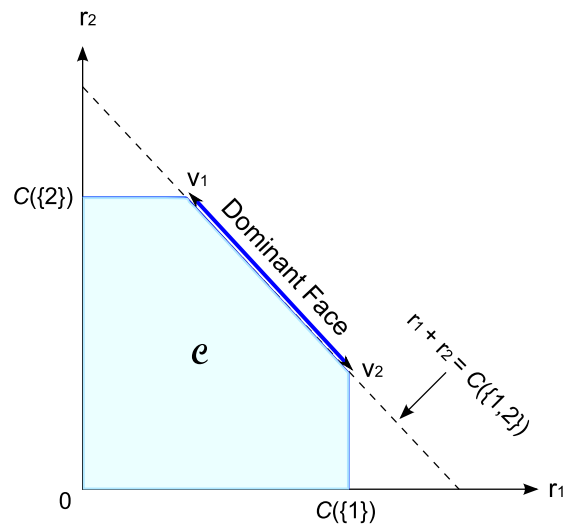


Figure 2.2 Capacity region of two-user multiple access time-invariant channel

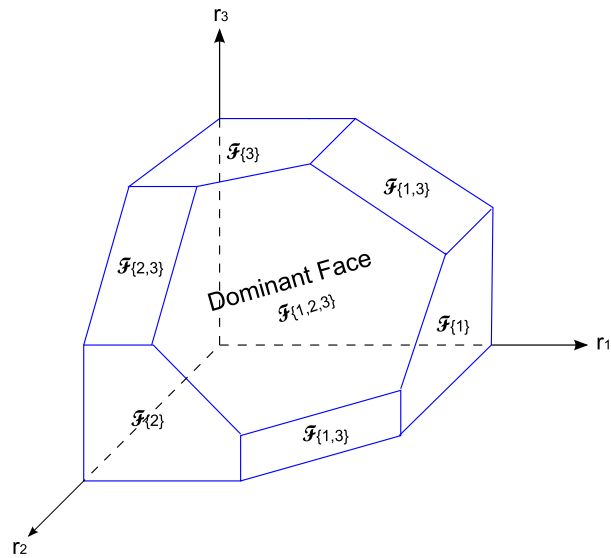


Figure 2.3 Capacity region of three-user multiple access time-invariant channel

Example 1: The capacity region of a two-user time-invariant MAC is the pentagon (Figure 2.2), given by:

$$\begin{aligned} 0 \leq r_n &\leq \log \left(1 + \frac{g_n p_n}{\sigma^2} \right), \quad n = 1, 2 \\ r_1 + r_2 &\leq \log \left(1 + \frac{g_1 p_1 + g_2 p_2}{\sigma^2} \right) \end{aligned}$$

Note that the two vertices v_1 and v_2 can be achieved by two different successive decoding orders $x_1 \rightarrow x_2$ (decode x_1 before x_2) and $x_2 \rightarrow x_1$ (decode x_2 before x_1), respectively. The dominant face is the line segment $\overline{v_1 v_2}$.

Example 2: The capacity region of a three-user MAC (Figure 2.3) is given by:

$$\begin{aligned} 0 \leq r_n &\leq \log \left(1 + \frac{g_n p_n}{\sigma^2} \right), \quad n = 1, 2, 3 \\ r_i + r_j &\leq \log \left(1 + \frac{g_i p_i + g_j p_j}{\sigma^2} \right), \quad (i, j) = (1, 2), (1, 3), (2, 3) \\ r_1 + r_2 + r_3 &\leq \log \left(1 + \frac{g_1 p_1 + g_2 p_2 + g_3 p_3}{\sigma^2} \right) \end{aligned}$$

In this case, there are six vertices, which leads to a hexagon dominant face.

When a simple single user decoding strategy is applied at the receiver side, the decoder detects user n 's signal x_n while treating the interference from other users as noise. Under this assumption, the transmission rate of user n is upper-bounded by a maximum achievable rate, which can be expressed as

$$r_n \leq \log \left(1 + \frac{g_n p_n}{\sigma^2 + \sum_{j=1, j \neq n}^N g_j p_j} \right),$$

and similarly for other users. For simplicity of expressions, we will denote by r_n the maximum achievable rate of user n , and use the short term ‘‘achievable rate’’ to refer to the maximum achievable rate with single user decoding.

Capacity Region of Multiple Access Fading Channels

Now, we consider the multiple access fading channel. In general, there are two notions of capacity for the fading channel depending on how fast the channel varies and the delay requirement of the application.

The first is the classic notion of Shannon capacity directly applied to the fast fading channel. In the case of fast fading, the channel is assumed to be stationary and ergodic during the transmission of a codeword. Then, the codeword length has to be sufficiently long such that during the codeword transmission

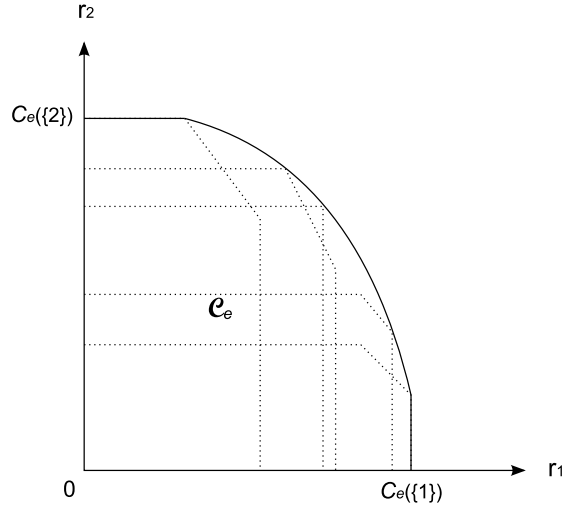


Figure 2.4 Capacity region of two-user fast-fading MAC with CSIT

all the channel states happen with “normalized frequency” equal to the one of the channel distribution characterization. Fast fading multiple access channels have been deeply investigated in the two cases as both transmit and receiver have knowledge of the channel state and when the receiver has knowledge of the channel state but not the transmitter. When channel state information (CSI) is perfectly observed at the receiver but the transmitters have no such information, the codewords cannot be chosen as a function of the CSI but the decoding can take advantage of such information. The capacity region of multiple access fast-fading channel is known [18, 19], and is given by

$$\mathcal{C}_e = \left\{ (r_1, \dots, r_N) : 0 \leq \sum_{n \in \mathcal{X}} r_n \leq C_e(\mathcal{X}), \forall \mathcal{X} \subseteq \{1, 2, \dots, N\} \right\} \quad (2.3)$$

Here, $C_e(\mathcal{X})$ is the ergodic capacity

$$C_e(\mathcal{X}) = \mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{\sum_{n \in \mathcal{X}} g_n p_n}{\sigma^2} \right) \right]$$

where $\mathbf{g} = \{g_1, \dots, g_N\}$ is a random vector having the stationary distribution of the joint fading process. Note that in this dissertation, the term “ergodic capacity” is used to refer the ergodic capacity with channel state information at the receiver (CSIR) only. When channel state information at the transmitter (CSIT) is also available, the capacity region is a union of capacity regions (Figure 2.4), each corresponding to a feasible transmit power strategy [20].

A transmit power policy $\mathbf{p}(\cdot)$ is a mapping from the fading state space to \mathbb{R}_+^N . Given a joint fading state \mathbf{g} , $p_n(\mathbf{g})$ can be interpreted as the transmit power of user n . For a given power policy \mathbf{p} , we can write the set of rates in (2.3) as a function of \mathbf{p} , i.e., $\mathcal{C}(\mathbf{p})$. Thus, the capacity region for multiple access fast-fading channel with both CSIR and CSIT can be written as

$$\tilde{\mathcal{C}}(\mathbf{p}) = \bigcup_{\mathbf{p} \in \mathcal{P}} \mathcal{C}(\mathbf{p})$$

where \mathcal{P} is the set of all feasible transmit power strategies satisfying

$$\mathcal{P} = \{\mathbf{p} : \mathbb{E}_{\mathbf{g}} [p(\mathbf{g})] \leq P_n^{\max}, \forall n\}.$$

The second is the notion of *outage capacity*. When the time delay requirements for the transmission of a codeword is shorter than the time scale for channel ergodicity the channel is referred to as slow fading. In this context, a major role is played by the outage capacity. It is known that the Shannon capacity is in general zero for a slow-fading channel with CSIR only [1]. This is because with a strict delay constraint, the channel may remain in deep fading over the whole transmission duration of the codeword, and a nonzero possibility of error exists for any positive rate target. Therefore, it is reasonable to allow a certain percentage of outage and try to achieve a rate target for the remainder of the time. Interested readers are referred to [13] for a complete characterization of outage capacity region of the multiple access slow-fading channel and the associated optimal resource allocation schemes.

2.3 Interference Channels

Interference is a central phenomenon in wireless communication when multiple uncoordinated links share a common communication medium. An interference channel is a communication medium shared by multiple transmitter-to-receiver pairs. Signal transmission from each transmitter-to-receiver pair interferes with the communications between the other transmitter-to-receiver pairs.

Mathematical Model

An important information-theoretical model is the two-user interference channel (Figure 2.5), in which two transmitter-to-receiver pairs with AWGN interfere with each other. The input-output equations can be written as:

$$\begin{aligned} y_1 &= \sqrt{g_{1,1}}x_1 + \sqrt{g_{2,1}}x_2 + z_1 \\ y_2 &= \sqrt{g_{2,2}}x_2 + \sqrt{g_{1,2}}x_1 + z_2 \end{aligned}$$

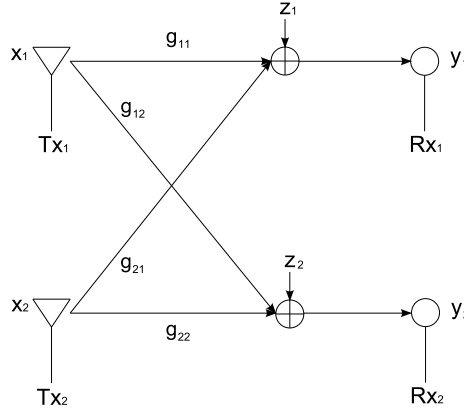


Figure 2.5 Two-user interference channel

where x_n and y_n are the input and output signals of user n , $g_{n,m}$ is the channel gain from transmitter n to receiver m , $z_n \sim \mathcal{N}(0, \sigma^2)$ is usually assumed to be independent and identically distributed (*i.i.d.*) zero-mean AWGN with variance σ^2 . The input signal x_n can be further written as $x_n = \sqrt{p_n} s_n$, where p_n and s_n are the transmit power and coded symbol of user n , respectively.

The major difference between the interference channel and the multiple access channel is that each receiver n is only interested to capture the information from transmitter n , and there is no information sharing between receivers.

Capacity Region and Maximum Achievable Rate

The general capacity region of the interference channel is still unknown even for the two-user Gaussian case, and it has been an open issue for more than thirty years, see [1]. For some extreme cases, for example when the interference is weak or strong, respectively, the capacity region is known. In the case of weak interference, the interference can simply be treated as additional noise. Remarkably, in the case of strong interference, it can be shown that the capacity region is the same as if there were no interference whatsoever.

When single user decoding is used at the receiver and the channel is constant and known at the transmitter, the achievable rates of user 1 and 2 are

$$r_1 \leq \log \left(1 + \frac{g_{1,1} p_{1,1}}{\sigma^2 + g_{2,1} p_{2,1}} \right)$$

$$r_2 \leq \log \left(1 + \frac{g_{2,2} p_{2,2}}{\sigma^2 + g_{1,2} p_{1,2}} \right)$$

As previously mentioned, we denote by r_n the achievable rate of user n .

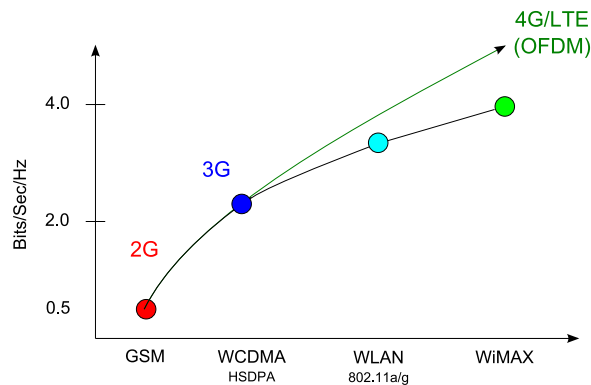


Figure 2.6 The spectrum efficiency of OFDM

2.4 Orthogonal Frequency Division Multiplexing

Orthogonal frequency division multiplexing (OFDM) is a worldwide emerging technology for high data rates. OFDM has developed into a popular scheme for wideband digital wireline and wireless communication, used in applications such as digital television and audio broadcasting, wireless networking and broadband Internet access. In wireless cellular networks, OFDM is known to have high spectral efficiency (Figure 2.6). It is the chosen physical layer for many standards, such as IEEE 802.11a, 802.11g, 802.20, and for both major emerging cellular network standards: WiMAX and 3GPP LTE.

OFDM is a particular form of multi-carrier transmission and is suited for frequency-selective channels and high data rates. Based on the fact that circular convolution can be diagonalized in a fast Fourier transform (FFT) basis, the frequency-selective wideband channel is transformed into a set of parallel flat-fading narrowband channels, i.e., subcarriers. More precisely, the orthogonal transmission is obtained by using an invert fast Fourier transform (IFFT) at transmission and a FFT at reception (Figure 2.7). The transmitted data are first divided into L_{FFT} parallel groups which are independently modulated. Let \mathbf{x}_ℓ be the complex subsymbols at the output of the ℓ^{th} modulator that corresponds to the ℓ^{th} subcarrier. The IFFT module transforms all subsymbols into L_{FFT} time samples X_m , i.e.,

$$X_m = \frac{1}{\sqrt{L_{\text{FFT}}}} \sum_{\ell=0}^{L_{\text{FFT}}-1} \mathbf{x}_\ell e^{2\pi j \frac{m\ell}{L_{\text{FFT}}}}$$

A cyclic prefix length of L_{CP} time is appended to the L_{FFT} time samples. It simply copies the L_{CP} last samples of each OFDM symbol, and is inserted at

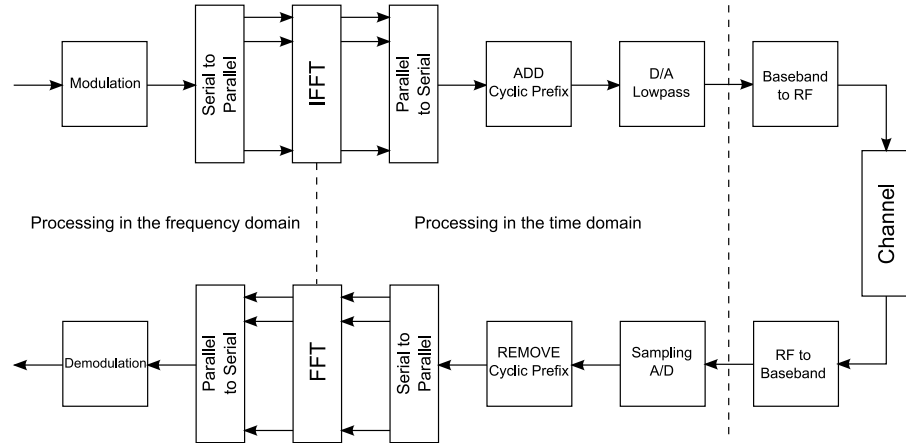


Figure 2.7 Implementation of an OFDM system

the beginning of the symbol. L_{CP} must be chosen greater than the maximum multipath delay spread, otherwise the multipath delay of one symbol will effect the cyclic prefix of next symbol. At the receiver, after the cyclic prefix is removed, \mathbf{x}_ℓ is retrieved by FFT

$$\hat{\mathbf{x}}_\ell = \mathbf{h}_\ell \mathbf{x}_\ell + \mathbf{n}_\ell$$

where \mathbf{h}_ℓ and \mathbf{n}_ℓ are complex channel transfer factor and additive channel noise for subcarrier ℓ , respectively.

These features make OFDM robust against large delay spreads by preserving orthogonality in the frequency domain. Moreover, the ingenious introduction of cyclic redundancy at the transmitter reduces the complexity to only FFT processing and one tap scalar equalization at the receiver. The spectral efficiency is increased by allowing frequency overlapping of the different subcarriers. However, the OFDM system also exhibits several weaknesses. Its simplicity is achieved at a cost of underutilizing two resources, resulting in a possible loss of performance. First, the cyclic prefix occupies an amount of time which cannot be used for useful data transmission. The second loss is in the transmit power. A fraction of the average power is allocated to the cyclic prefix and cannot be used towards communicating useful data. OFDM is also more vulnerable to frequency off-set as well as synchronization problems. In the first case, a frequency off-set yields inter-carrier interference and destroys the orthogonality between subcarriers. In the second case, synchronization errors incurs a phase shift on the estimated symbols.

Chapter 3

Mathematical Preliminaries

3.1 Basic Mathematical Concepts

First of all, we clarify some fundamental mathematical definitions, which will be used throughout this dissertation.

Definition 3.1. (*Bounded set*)

In an Euclidean space \mathbb{E}^n , a set \mathcal{C} is said to be bounded if and only if it is contained inside some hypersphere of finite radius.

Definition 3.2. (*Closed set*)

In an Euclidean space \mathbb{E}^n , a set \mathcal{C} is said to be closed if and only if it contains all points that are arbitrarily close to it.

Definition 3.3. (*Compact set*)

A subset of an Euclidean space is called compact if it is closed and bounded.

Definition 3.4. (*Convex set*)

In an Euclidean space \mathbb{E}^n , a set \mathcal{C} is convex if for every pair of points within \mathcal{C} , every point on the straight line segment that joins them is also within \mathcal{C} , i.e., if for any $x_1, x_2 \in \mathcal{C}$ and any $\theta \in [0, 1]$, we have

$$\theta x_1 + (1 - \theta)x_2 \in \mathcal{C}$$

Definition 3.5. (Concave function)

A function $f : \mathcal{X} \rightarrow \mathbb{R}$ is concave if the domain of the function f is a convex set and if for any $x_1, x_2 \in \mathcal{X}$ and any $\theta \in [0, 1]$, we have

$$f(\theta x_1 + (1 - \theta)x_2) \geq \theta f(x_1) + (1 - \theta) f(x_2)$$

Definition 3.6. (Quasi-concave function)

A function $f : \mathcal{X} \rightarrow \mathbb{R}$ is quasi-concave if the domain of the function f is a convex set and if for any $x_1, x_2 \in \mathcal{X}$ and any $\theta \in [0, 1]$, we have

$$f(\theta x_1 + (1 - \theta)x_2) \geq \min \{f(x_1), f(x_2)\}$$

Note that a concave function is quasi-concave, since we have

$$f(\theta x_1 + (1 - \theta)x_2) \geq \theta f(x_1) + (1 - \theta) f(x_2) \geq \min \{f(x_1), f(x_2)\}$$

Definition 3.7. (Diagonally strictly concave function)

A weighted nonnegative sum function $f(\mathbf{x}, \gamma) = \sum_{i=1}^n \gamma_i \varphi_i(\mathbf{x})$ is called diagonally strictly concave for any vector $\mathbf{x} \in \mathbb{R}^n$ and fixed vector $\gamma \in \mathbb{R}_{++}^n$, if for any two different vectors $\mathbf{x}^0, \mathbf{x}^1$, we have

$$\Omega(\mathbf{x}^0, \mathbf{x}^1, \gamma) \triangleq (\mathbf{x}^1 - \mathbf{x}^0)^T \delta(\mathbf{x}^0, \gamma) + (\mathbf{x}^0 - \mathbf{x}^1)^T \delta(\mathbf{x}^1, \gamma) > 0 \quad (3.1)$$

where $\delta(\mathbf{x}, \gamma)$ is the pseudo-gradient of $f(\mathbf{x}, \gamma)$, defined as

$$\delta(\mathbf{x}, \gamma) \triangleq \begin{bmatrix} \gamma_1 \frac{\partial \varphi_1}{\partial x_1} \\ \vdots \\ \gamma_n \frac{\partial \varphi_n}{\partial x_n} \end{bmatrix}. \quad (3.2)$$

Definition 3.8. (Monomial function)

A monomial is a function of the form

$$f(\mathbf{x}) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$$

where $\mathbf{x} = \{x_1, \dots, x_n\}$ and the constant parameters are $c \in \mathbb{R}_{++}$ and $a_i \in \mathbb{R}$.

Definition 3.9. (Posynomial function)

A posynomial is a function of the form

$$f(\mathbf{x}) = \sum_{k=1}^K c_k x_1^{a_{k1}} x_2^{a_{k2}} \cdots x_n^{a_{kn}}$$

where $\mathbf{x} = \{x_1, \dots, x_n\}$ and the constant parameters are $c_k \in \mathbb{R}_{++}$ and $a_{ik} \in \mathbb{R}$.

3.2 Elements of Convex Optimization

Convex minimization has applications in a wide range of disciplines, such as automatic control systems, estimation and signal processing, communications and networks, electronic circuit design, data analysis and modeling, statistics, and finance. For most of these applications, mathematical optimization is used as an aid to a human decision maker, system designer, or system operator, who supervises the process, checks the results, and modifies the problem.

Convex optimization problems have appealing properties that makes them fundamentally more tractable than generic nonconvex optimization problems:

1. Any local optimum is also a global optimum.
2. Infeasibility detection: algorithms are easy to initialize via duality theory.
3. Efficient numerical solution methods that can handle very large problems.

In this section, we detail some properties of convex optimization problems, as well as the main methods that will be used throughout this dissertation. We first start with some definitions and basic formulations. Then, we investigate the duality theory, which provides an “improving lower/upper bound” method to approximate the solution of the original optimization problem via Lagrangian. We show the conditions under which the lower/upper bound is tight. We introduce the Karush-Kuhn-Tucker (KKT) conditions which provide simple analytical methods to solve convex optimization problems. Finally, we show how to solve geometric programming (which is non-convex in its original form) by transforming it to convex optimization problems. These optimization methods play an important role in analyzing and addressing some game-theoretical problems.

3.2.1 Convex Optimization Problems

Optimization in Standard Form

We first consider the formulation of optimization problems at a general level. The standard form of an optimization problem is the following

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m \\ & h_i(\mathbf{x}) = 0, \quad i = 1, \dots, \ell \end{aligned} \tag{3.3}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the *optimization variable*, function $f_0 : \mathbb{R}^n \rightarrow \mathbb{R}$ is the *objective function*, functions $f_i(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$ are the *inequality constraint functions*, functions $h_i(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$ are the *equality constraint functions*. We define the domain of (3.3) as

$$\mathcal{D} = \left(\bigcap_{i=0}^m \text{dom} f_i \right) \cap \left(\bigcap_{i=1}^{\ell} \text{dom} h_i \right)$$

Therefore, the problem (3.3) is to find an optimal solution \mathbf{x}^* that minimizes $f_0(\mathbf{x})$ among all $\mathbf{x} \in \mathcal{D}$.

Convex Minimization Problems

An optimization problem in standard form is a *convex optimization problem*, if (1) the objective function f_0 and the inequality functions f_1, \dots, f_m are convex, and (2) the equality functions h_1, \dots, h_ℓ are affine, i.e., $h_i(\mathbf{x}) = \mathbf{a}_i^T \mathbf{x} - b_i$. It is often written in the minimization form

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m \\ & \mathbf{a}_i^T \mathbf{x} = b_i, \quad i = 1, \dots, \ell \end{aligned}$$

where $\mathbf{a}_i \in \mathbb{R}^n$ and $b_i \in \mathbb{R}$. The equality constraints can be written in the matrix form: $\mathbf{A}\mathbf{x} = \mathbf{b}$, where $\mathbf{A} \in \mathbb{R}^{\ell \times n}$ (\mathbf{a}_i are the rows of \mathbf{A}) and $\mathbf{b} \in \mathbb{R}^\ell$.

Concave Maximization Problems

An optimization problem in standard form can also be considered as a *concave maximization problem*, if (1) the objective function f_0 is concave, (2) the inequality functions f_1, f_2, \dots, f_m are convex, and (3) the equality functions h_1, \dots, h_ℓ are affine

$$\begin{aligned} \max_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m \\ & \mathbf{a}_i^T \mathbf{x} = b_i, \quad i = 1, \dots, \ell \end{aligned}$$

A concave maximization problem is equivalent to a convex minimization problem, if we replace the “max f_0 ” by “min $-f_0$ ” subject to the constraints.

3.2.2 Duality Theory and KKT Conditions

Duality theory arises in the general optimization problems (3.3), which will be called the *primal problem*. It is a useful tool in many aspects, such as: to prove the optimality of a primal solution by constructing a dual function, to bound the values of the primal solutions, to analyze the convergence of algorithms, etc.

Duality Theory

The basic idea in Lagrangian duality is to take the constraints in (3.3) into account by augmenting the objective function with a weighted sum of the constraint functions. We define the *Lagrangian* $\mathcal{L} : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^\ell \rightarrow \mathbb{R}$ associated with the optimization problem (3.3) as

$$\mathcal{L}(\mathbf{x}, \lambda, \nu) = f_0(\mathbf{x}) + \sum_{i=1}^m \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{\ell} \nu_i h_i(\mathbf{x}) \leq f_0(\mathbf{x}) \quad (3.4)$$

where λ_i and ν_i are called *Lagrange multipliers* or *dual variables* associated with the i^{th} inequality constraint $f_i(\mathbf{x}) \leq 0$ and the i^{th} equality constraint $h_i(\mathbf{x}) = 0$, respectively. Let $\lambda = \{\lambda_1, \dots, \lambda_m\}$ and $\nu = \{\nu_1, \dots, \nu_\ell\}$.

We define the *Lagrange dual function* (or simply *dual function*) $g : \mathbb{R}^m \times \mathbb{R}^\ell \rightarrow \mathbb{R}$ as the infimum of \mathcal{L} over $\mathbf{x} \in \mathcal{D}$

$$g(\lambda, \nu) = \inf_{\mathbf{x} \in \mathcal{D}} \mathcal{L}(\mathbf{x}, \lambda, \nu) = \inf_{\mathbf{x} \in \mathcal{D}} \left(f_0(\mathbf{x}) + \sum_{i=1}^m \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{\ell} \nu_i h_i(\mathbf{x}) \right) \quad (3.5)$$

From (3.4) and (3.5), for any feasible point $\mathbf{x}' \in \mathcal{D}$, we have

$$g(\lambda, \nu) = \inf_{\mathbf{x} \in \mathcal{D}} \mathcal{L}(\mathbf{x}, \lambda, \nu) \leq \mathcal{L}(\mathbf{x}', \lambda, \nu) \leq f_0(\mathbf{x}') \leq f^*$$

where f^* is the maximum value of the primal problem (3.3). The dual function $g(\lambda, \nu)$ provides a certain lower bound on f^* for any pair λ, ν . A natural question is therefore how to find the best lower bound that can be obtained from $g(\lambda, \nu)$. This leads to the *dual problem*

$$\begin{aligned} \max_{\lambda} \quad & g(\lambda, \nu) \\ \text{s.t.} \quad & \lambda \geq 0 \end{aligned} \quad (3.6)$$

Let g^* be the optimal value of $g(\lambda, \nu)$. We define $\Delta^* = f^* - g^*$ as the *duality gap* of the primal problem. We say that weak duality holds, if $\Delta^* \geq 0$, and that strong duality holds, if $\Delta^* = 0$, i.e., the optimal solution of the primal problem (3.3) coincides with the optimal solution of the dual problem (3.6). If

the primal problem is a convex optimization, we usually (but not always) have strong duality. A simple sufficient condition for the strong duality of convex optimization problems is the Slater's condition

$$(1) \exists \mathbf{x} \in \mathcal{D} : f_i(\mathbf{x}) < 0, \quad i = 1, \dots, m \quad (2) \mathbf{Ax} = \mathbf{b}$$

Note that the Slater's conditions hold as soon as the feasible set of $\mathbf{Ax} = \mathbf{b}$ is nonempty. In this sense, strong duality always holds for any feasible convex optimization problem.

KKT Optimality Conditions

Here, we assume that f_0, \dots, f_m and h_1, \dots, h_ℓ are differentiable. The Karush-Kuhn-Tucker (KKT) conditions are

$$\begin{aligned} f_i(\mathbf{x}^*) &\leq 0, \quad i = 1, \dots, m \\ h_i(\mathbf{x}^*) &= 0, \quad i = 1, \dots, \ell \\ \lambda_i^* &\geq 0, \quad i = 1, \dots, m \\ \lambda_i^* f_i(\mathbf{x}^*) &= 0, \quad i = 1, \dots, m \\ \nabla f_0(\mathbf{x}^*) + \sum_{i=1}^m \lambda_i^* \nabla f_i(\mathbf{x}^*) + \sum_{i=1}^{\ell} \nu_i^* \nabla h_i(\mathbf{x}^*) &= 0 \end{aligned} \tag{3.7}$$

The usage of KKT conditions should be distinguished as: (1) For general optimization problems, KKT conditions provide some necessary conditions for the optimality. (2) For convex optimization problems, however, KKT conditions completely characterize the optimal solutions, i.e., necessary and sufficient conditions for the optimality.

The KKT conditions play a crucial role in optimization, especially in convex optimization. Many algorithms for convex optimization can be interpreted as methods for solving the KKT conditions analytically, e.g., waterfilling [1, 2]. We will use KKT conditions throughout this dissertation, for solving single-user or multi-user resource allocation problems having convex structure.

3.2.3 Geometric Programming

Minimizing a posynomial subject to posynomial upper bound inequality constraints and monomial equality constraints is called a geometric program (GP)

in standard form:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f_0(\mathbf{x}) \\ \text{s.t.} \quad & f_i(\mathbf{x}) \leq 1, \quad i = 1, \dots, m \\ & h_i(\mathbf{x}) = 1, \quad i = 1, \dots, \ell \end{aligned} \quad (3.8)$$

where f_i , $i = 0, 1, \dots, m$ are posynomials:

$$f_i(\mathbf{x}) = \sum_{k=1}^{K_i} c_{ik} x_1^{a_{ik}^1} x_2^{a_{ik}^2} \cdots x_n^{a_{ik}^n},$$

and h_i , $i = 1, \dots, \ell$ are monomials:

$$h_i(\mathbf{x}) = c_i x_1^{a_i^1} x_2^{a_i^2} \cdots x_n^{a_i^n}.$$

with constant parameters $c_{ik}, c_i \in \mathbb{R}_{++}$ and $a_{ik}^j, a_i^j \in \mathbb{R}$, and $\mathbf{x} = \{x_1, \dots, x_n\}$.

GP in standard form is not a convex optimization problem, because posynomials are not convex functions. However, with a logarithmic change of all the variables and multiplicative constants: $\tilde{c}_{ik} = \log c_{ik}$, $\tilde{c}_i = \log c_i$ and $\tilde{x}_j = \log x_j$, we can turn it into the following problem:

$$\begin{aligned} \min_{\tilde{\mathbf{x}}} \quad & \sum_{k=1}^{K_0} \exp(\mathbf{a}_{0k}^T \tilde{\mathbf{x}} + \tilde{c}_{0k}) \\ \text{s.t.} \quad & \sum_{k=1}^{K_i} \exp(\mathbf{a}_{ik}^T \tilde{\mathbf{x}} + \tilde{c}_{ik}) \leq 1, \quad i = 1, \dots, m \\ & \mathbf{a}_i^T \tilde{\mathbf{x}} + \tilde{c}_i = 0, \quad i = 1, \dots, \ell \end{aligned} \quad (3.9)$$

where $\mathbf{a}_{ik} = [a_{ik}^1, a_{ik}^2, \dots, a_{ik}^n]^T$, which is equivalent to:

$$\begin{aligned} \min_{\tilde{\mathbf{x}}} \quad & \log \sum_{k=1}^{K_0} \exp(\mathbf{a}_{0k}^T \tilde{\mathbf{x}} + \tilde{c}_{0k}) \\ \text{s.t.} \quad & \log \sum_{k=1}^{K_i} \exp(\mathbf{a}_{ik}^T \tilde{\mathbf{x}} + \tilde{c}_{ik}) \leq 0, \quad i = 1, \dots, m \\ & \mathbf{a}_i^T \tilde{\mathbf{x}} + \tilde{c}_i = 0, \quad i = 1, \dots, \ell \end{aligned} \quad (3.10)$$

Since the log-sum-exp function is convex, the proposed change of variables turns the nonconvex optimization problem (3.8) into the convex optimization (3.10). Therefore, we can apply Lagrange duality theory and use KKT conditions to find the optimal solution \tilde{x}_j^* . The optimal solution to (3.8) is obtained via the inverse mapping $x_j^* = e^{\tilde{x}_j^*}$.

Over the last decade, GP has been applied to study a variety of problems in the analysis and design of communication systems, e.g., information theory, queuing theory, network protocols and signal processing. In this dissertation, we use GP to solve the multi-user resource allocation problem in Chapter 5.4.

3.3 Elements of Game Theory

Game theory is a branch of applied mathematics. The basic framework of game theory was introduced in the book “*Theory of Games and Economic Behavior*” by John von Neumann and Oskar Morgenstern (1944). Until now it is widely used in the many fields, primarily in economics (in order to model competition between companies), as well as computer science, biology, politics and many other areas. Game theory is an effective description of strategic interactions. It attempts to mathematically capture behavior of individuals in strategic situations, in which an individual’s success in making choices depends on the choices of others. Game theory enhances the understanding of conflict by devising theories, mathematical models and abstractions that serve to explain nature and results of conflicts.

In game theory, there are some basic assumptions which are often utilized to facilitate the construction of tractable models for real situations. First, it is assumed that each individual in the game has a definite ordering of preferences over all outcomes of a given situation. These preferences take the form of a utility function or payoff. Second, the participants in games are sometimes considered to be *rational*. This means that they always act in a way that maximizes their payoffs, they will always be capable of thinking through all possible outcomes and choosing the action which will result in the best possible outcome¹. In this section, we briefly introduce some concepts and definitions of game theory: (1) Static games with complete information, (2) Static games with incomplete information, (3) Potential games, and (4) Nash bargaining games.

3.3.1 Static Games with Complete Information

When all players make decisions simultaneously², without knowledge of the strategies chosen by other players, the game is static.

¹There are some game-theoretical topics in which perfect rationality is not assumed, e.g., games with bounded rationality. These topics are not covered in the scope of this dissertation

²A game can still be considered static, if the decisions are taken at different time instances but each player does not have any information about the decisions of other players.

Games in Strategic Form

A strategic game is a model of interacting decision-makers. In recognition of the interaction, we refer to the decision-makers as *players*. The model captures interaction between the players by allowing each player to be affected by the actions of all players, not only the player's own action. Each player has (1) a set of possible *strategies* and (2) a preference about the action profile whose characteristics are captured by a *payoff* function.

More precisely, a strategic game consists of three elements:

- A player set $\mathcal{N} = \{1, \dots, N\}$
- A strategy set $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \dots \times \mathcal{S}_N$
- A payoff set $u = \{u_1, \dots, u_N\}$

where \mathcal{S}_n is the strategy set for player n , $u_n : \mathcal{S} \rightarrow \mathbb{R}$ is the payoff or utility function for player n . Let $\mathbf{s} = \{s_1, \dots, s_N\}$ be the strategy profile, in which the strategy of each player is $s_n \in \mathcal{S}_n$. Then, $\{s'_n, \mathbf{s}_{-n}\}$ denotes the strategy profile in which every player j except n chooses her action s_j as specified by \mathbf{s} , whereas player n chooses s'_n . The $-n$ subscript on \mathbf{s} stands for “except n ”. That is, $u_n(s'_n, \mathbf{s}_{-n})$ is the payoff of player n when all the players other than n adhere to \mathbf{s} while player n deviates to s'_n . With the aid of these notions, now we can state the definition of Nash equilibrium (NE) precisely.

Nash Equilibrium

Nash equilibrium, named after John Forbes Nash, is a set of strategies, one for each player, such that no player has incentive to unilaterally change her strategy. It is a fundamental concept in non-cooperative strategic games.

Definition 3.10. (*Nash equilibrium*)

A strategy profile \mathbf{s}^* is a pure strategy Nash equilibrium if for every player $n \in \mathcal{N}$,

$$u_n(s_n^*, \mathbf{s}_{-n}^*) \geq u_n(s_n, \mathbf{s}_{-n}^*) \quad (3.11)$$

for all $s_n \in \mathcal{S}_n$.

From above, it is clear that a NE simply represents a particular “steady” state for a system, in the sense that, once reached, no player has any motivation to unilaterally deviate from it. In many cases, NE represents the result of learning and evolution of all players in a system. It is therefore important to predict and characterize such points from the perspective of system design.

An Example: Wireless Version of Prisoner's Dilemma

One of the most well-known strategic games is the Prisoner's Dilemma. The game models a situation in which there are gains from cooperation but each player has an incentive to "free ride" whatever the other player chooses. This model is important because many other situations have similar structures.

Consider a two-user MAC, in which two users (transmitters) compete to send information towards a single base station (receiver). Suppose that users can only transmit with one of the two power levels, i.e., normal power (denote by "Normal") or very high power (denote by "High"). They must decide simultaneously (without communication) which power level to choose.

We can model this problem as a static game, in which the player set is $\mathcal{N} = \{1, 2\}$, and each player n has the same action set $\mathcal{S}_m = \{\text{Normal}, \text{High}\}$. Typically, the payoff set \mathbf{u} can be generalized to the following matrix³

	Normal	High
Normal	(win, win)	(lose much, win much)
High	(win much, lose much)	(lose, lose)

In each entry (a, b) , the values a and b represent the payoff of player 1 and 2, respectively. Intuitively, we have the following observations:

- If both users transmit with high power, they will suffer from the increased interference caused by the other, which results in a "lose-lose" situation.
- If one user transmits with normal power and the other transmits with high power, compared to the "lose-lose" case, the former will get a worse performance (denote by "lose much") and the latter will benefit from the reduced interference and enjoy a better performance (denote by "win much").
- If both users transmit with normal power, the result is "win-win".

Obviously, to find the solution of this problem is beyond the capability of optimization theory, since user 1's best strategy depends on the strategy chosen by user 2, which user 1 does not know, and reciprocally for user 2. One may guess that both users must strictly prefer to transmit with normal power, which results in "win-win". However, it is not the "solution" of this game, i.e., it is not a natural outcome of selfish and rational players. It might be quite surprising that the only "solution", i.e., NE, of this game is the "lose-lose" situation.

The reason is the following: From player 1's standpoint, she strictly prefers to choose "High", because she is always better off regardless of player 1's choice,

³Here, we do not give specific values for the payoffs, but show it in a "win-lose" terminology.

since "win much" > "win" and "lose" > "lose much". And similarly for player 2. From definition (3.11), we know that when both players choosing "High", the result "lose-lose" is a NE, which is the only pure strategy NE in this game.

Methodologies for Analyzing Equilibrium

In general, to analyze Nash equilibrium, one needs to consider three main aspects, which are "existence", "uniqueness" and "equilibrium selection", i.e.,

1. Existence - Does an equilibrium exist?
2. Uniqueness - Does there exist a unique equilibrium or multiple ones?
3. Equilibrium selection - How to select an equilibrium from multiple ones?

"Existence" is the very first question that naturally comes into our mind, since it is known that, in general, an equilibrium point does not necessarily exist. Mathematically speaking, proving the existence of an equilibrium is equivalent to prove the existence of a solution to a fixed-point problem [21]. Since the existence of the fixed-point hints the existence of some strategy set which is a best response to itself, no player could increase her payoff by deviating, and so it is an equilibrium. In literature, there exist several theorems providing sufficient conditions for the existence of an equilibrium. There are many scenarios assuming usual wireless channel models and performance metrics where the existing theorems can be applied, e.g., channel capacity (2.1) has desirable convexity properties satisfying the conditions of the following well-known theorem [22].

Theorem 3.11. (*Debreu's sufficient condition*)

If the strategy sets \mathcal{S}_n are nonempty, compact, and convex subsets of an Euclidean space, and if the payoff functions u_n are continuous in \mathbf{s} and quasi-concave in s_n , there exists a pure strategy Nash equilibrium.

"Uniqueness" of Nash equilibrium is the second fundamental problem that we need to address when the existence is ensured. The uniqueness of an equilibrium is a very desirable property, if we wish to predict what will be the network behavior. Unfortunately, there are not many general results for the uniqueness analysis. For *concave N -person games*, useful sufficient conditions for the uniqueness of Nash equilibrium are provided in [23]. It is shown that the uniqueness is guaranteed if the payoff functions satisfy the so called *diagonally strictly concave* condition, see Definition 3.7. However, there are many important scenarios where the equilibrium is not unique, e.g., routing games [24], coordination games [25], non-cooperative games with correlated constraints together with the

concept of “generalized Nash equilibrium” [26], etc. Natural questions that arise concern the selection of an appropriate equilibrium, i.e., what can be done when a game has multiple equilibria? are there some equilibria “dominating” others? are there some equilibria more “fair” than others?

“Equilibrium selection” is in fact a mature theory in itself [27]. Obviously, the selection rule is strongly related to the fairness criteria, e.g., max-min fairness [28], proportional fairness [29, 30], global optimization, etc. In this dissertation, we do not provide the general theory for equilibrium selection, instead we are interested in its applications to concave games. More specifically, we will use the concept of “normalized equilibria”, introduced by J. B. Rosen [23] that shows a very neat way to tackle this problem. A more general discussion about equilibrium analysis methodologies in wireless games can be found in [31].

3.3.2 Static Games with Incomplete Information

The notion of “incomplete information” means that some players do not completely know the structure of the game. In particular, some players do not know the preferences of other players. An important assumption behind the notion of Nash equilibrium is that players have complete information about the game structure. However, this assumption is hardly met in many practical scenarios. For example: companies may not know each others’ production costs, chess players may not know each others’ chess playing styles, etc.

How can we argue about the game when some players are uncertain about the characteristics of other players? An early framework for static games with incomplete information was established by John Harsanyi (1967) [32]. Following this framework, we can study it as a game with “Bayesian” players. Such players have initial beliefs about the “type” of each player. The belief is analytically defined by a probability distribution over all the possible “types” for a player.

Description of Bayesian Games

We now briefly describe the model of Bayesian games, which generalizes the notion of strategic game to an incomplete information framework.

Definition 3.12. (*Bayesian games*)

A Bayesian game \mathcal{G} is a strategic-form game with incomplete information, which can be completely described as follows:

$$\mathcal{G} = \langle \mathcal{N}, \{T_n, A_n, q_n, u_n\}_{n \in \mathcal{N}} \rangle.$$

It consists of

- A **player set**: $\mathcal{N} = \{1, \dots, N\}$,
and for each player $n \in \mathcal{N}$
- A **type set**: T_n ($\mathcal{T} = T_1 \times T_2 \times \dots \times T_N$)
- An **action set**: A_n ($\mathcal{A} = A_1 \times A_2 \times \dots \times A_N$)
- A **probability function set**:

$$\rho_n : T_n \rightarrow f(T_{-n})$$

- A **payoff function set**:

$$u_n : \mathcal{A} \times \mathcal{T} \rightarrow \mathbb{R}$$

We clarify the notions of type, strategy, probability and payoff, as follows: (1) Player n 's type $\tau_n \in T_n$ represents any kind of private information, more precisely, any information that is not common knowledge to all players, and it is relevant to their decision makings. (2) Player n 's pure strategy s_n is a function mapping the type set T_n of player n into her action set A_n , i.e., $s_n : T_n \rightarrow A_n$. We denote by \mathcal{S}_n the strategy set of player n . (3) The probability function ρ_n measures what player n believes about the types of the other players, given her own type τ_n . It takes the form of a conditional probability, i.e., $\rho_n(\tau_{-n}|\tau_n)$. (4) Player n 's payoff u_n is a function of the strategy profile $\mathbf{s} = \{s_1, \dots, s_N\}$ and the type profile $\tau = \{\tau_1, \dots, \tau_N\}$ of all players in the game \mathcal{G} ,

$$u_n(\mathbf{s}(\tau), \tau) = u_n(s_1(\tau_1), \dots, s_N(\tau_N), \tau_1, \dots, \tau_N)$$

Bayesian Equilibrium

Note that in a strategic-form game with complete information, each player chooses an action or strategy, whereas in a Bayesian game, each player n faces the problem of choosing a set of strategies $s_n(\cdot)$, one for each type τ_n she may encounter.

Let $\{s'_n(\cdot), \mathbf{s}_{-n}(\cdot)\}$ denote the strategy profile where all players play $\mathbf{s}(\cdot)$ except player n plays $s'_n(\cdot)$. Then, we can write the payoff of player n as:

$$u_n(s'_n(\tau_n), \mathbf{s}_{-n}(\tau_{-n}), \tau) = u_n(s_1(\tau_1), \dots, s'_n(\tau_n), \dots, s_N(\tau_N), \tau)$$

Definition 3.13. (*Bayesian equilibrium*) The strategy profile $s^*(\cdot) = \{s_n^*(\cdot)\}_{n \in \mathcal{N}}$ is a pure strategy Bayesian equilibrium, if for all $n \in \mathcal{N}$, all $s_n \in \mathcal{S}_n$ and $s_{-n} \in \mathcal{S}_{-n}$

$$\mathbb{E}_{\tau_{-n}} [u_n(s_n^*(\tau_n), s_{-n}^*(\tau_{-n}), \tau)] \geq \mathbb{E}_{\tau_{-n}} [u_n(s_n(\tau_n), s_{-n}^*(\tau_{-n}), \tau)]$$

where

$$\mathbb{E}_{\tau_{-n}} [u_n (s_n(\tau_n), \mathbf{s}_{-n}(\tau_{-n}), \tau)] \triangleq \sum_{\tau_{-n} \in T_{-n}} \rho_n(\tau_{-n} | \tau_n) u_n (s_n(\tau_n), \mathbf{s}_{-n}(\tau_{-n}), \tau)$$

is defined as the expected payoff of player n .

Bayesian equilibrium is sometimes also called Bayesian Nash equilibrium. Note that the definition of Bayesian equilibrium is similar to that of Nash equilibrium, in the sense that no player can gain by unilaterally changing her own strategy. In fact, a Bayesian equilibrium of the game \mathcal{G} can be considered as a mixed strategy Nash equilibrium of the transformed game $\hat{\mathcal{G}} = \langle \mathcal{N}, \hat{A} = S_1 \times S_2 \dots S_N, \hat{u} = u \rangle$. Therefore, we could simply adopt the analytical method of Nash equilibrium to study the Bayesian equilibrium set [31].

3.3.3 Potential Games

The notation of potential games was firstly used for games in strategic form by Rosenthal (1973) [33], and later generalized and summarized by Monderer and Shapley (1996) [34]. These games received increasing attention recently, due to their appealing properties (which will be mentioned later on). Potential games are subclass of games in strategic form. A strategic game is a potential game if there exists a function called *potential*⁴.

The basic idea of potential games is to find a potential function, which quantifies the difference in the payoffs due to a unilaterally deviation of each player either exactly (exact potential games), in sign (ordinal potential games), or deviation to the best-response (best-response potential games). Potential function can be interpreted as a measure of the disagreement among players, or, equivalently as the drift towards the equilibrium. In this case, the set of pure Nash equilibria can be found by simply locating the local optima of the potential function.

Description of Potential Games

Here, we provide the basic definitions and some useful results in potential games. In fact, as mentioned above, there are various concepts of potential games. The basic ones among all the potential games are the exact potential, the ordinal

⁴In physics, a potential refers to a field defined in space, from which many important physical properties may be derived. For example, in velocity field of a fluid or gas flow, the definition of the potential implies that the direction of the flow at any point coincides with the direction of the steepest decrease of the potential at that point.

potential and the best-response potential. All potential games have slightly different definitions which may lead to different properties for the existence and convergence to an equilibrium.

Definition 3.14. (*Potential games*)

A strategic game $\mathcal{G}^p = \langle \mathcal{N}, \mathcal{S}, v \rangle$ is called an exact/ordinal/best-response potential game if there exists a function $v : \mathcal{S} \mapsto \mathbb{R}$ satisfying

$$\textbf{Exact:} \quad v(s_n, \mathbf{s}_{-n}) - v(s'_n, \mathbf{s}_{-n}) = u_n(s_n, \mathbf{s}_{-n}) - u_n(s'_n, \mathbf{s}_{-n})$$

$$\textbf{Ordinal:} \quad v(s_n, \mathbf{s}_{-n}) - v(s'_n, \mathbf{s}_{-n}) > 0 \iff u_n(s_n, \mathbf{s}_{-n}) - u_n(s'_n, \mathbf{s}_{-n}) > 0$$

$$\textbf{Best-response:} \quad \arg \max_{s_n \in \mathcal{S}_n} v(s_n, \mathbf{s}_{-n}) = \arg \max_{s_n \in \mathcal{S}_n} u_n(s_n, \mathbf{s}_{-n})$$

for all $n \in \mathcal{N}$, and $(s_n, \mathbf{s}_{-n}), (s'_n, \mathbf{s}_{-n}) \in \mathcal{S}$, where $s_n \neq s'_n$.

From the definition above, it can be shown that any of the exact/ordinal/best-response conditions guarantee that the equilibrium set of the original strategic game $\mathcal{G} = \langle \mathcal{N}, \mathcal{S}, u \rangle$ coincides with the equilibrium set of the potential game $\mathcal{G}^p = \langle \mathcal{N}, \mathcal{S}, v \rangle$. Additionally, the following relationship holds for different classes of potential games⁵:

$$\text{Exact potential} \subset \text{Ordinal potential} \subset \text{Best response potential}$$

For thorough discussions of various potential games, the interested readers are referred to [35] for ordinal potential games, [36] for best-response potential games, and [37] for pseudo potential games.

Properties of Potential Games

Potential games are known to have interesting properties, such as existence of an equilibrium and convergence of best-response algorithms to the equilibrium.

Existence of Nash equilibrium: It is known that every strategy $s \in \mathcal{S}$ that maximizes potential function $v(s)$ is a pure strategy Nash equilibrium of the original game \mathcal{G} [34]. But the converse is not in general true, i.e., there might be some pure or mixed strategy NE that are just local maximum points of $v(s)$. Nevertheless, one could find many scenarios in which the following results are useful: (1) if the strategy set \mathcal{S} is convex and v is continuously differentiable on \mathcal{S} , then every Nash equilibrium of \mathcal{G} is a stationary point of v , (2) if v is concave, then every Nash equilibrium of \mathcal{G} is a maximum point of v , and moreover, (3) if v is strictly concave, then the Nash equilibrium is unique.

⁵There are many other notions of potential games, such as: weighted potential, generalized ordinal potential, generalized ϵ -potential, quasi-potential and pseudo-potential.

Convergence to Nash equilibrium: It was shown in [34] that every bounded potential game⁶ has *approximate finite improvement property* (AFIP), i.e., for every arbitrary small $\epsilon > 0$, every ϵ -improvement path⁷ is finite. In other words, the sequential best-response converges to the ϵ -equilibrium independently of the initial point. Note that this statement gives a very flexible condition for the convergence, since the order of playing can be deterministic or random and need not to be synchronized. It is the most interesting property of potential games, especially in order to distributively find an equilibrium in self-organizing systems.

3.3.4 Nash Bargaining Games

The standard model of Nash bargaining was introduced by J. Nash (1953) [38]. E. Kalai (1977) extended the Nash's model to the case of nonsymmetric player priority. For a more detailed discussion about symmetric and asymmetric Nash model as well as other models of bargaining problems, the interested readers can refer to A. Roth's article (1979) [39].

The word "bargaining" refers to a situation in which a group of two or more players choose their payoffs from a set of feasible alternatives (payoff allocations), any one of which can be the outcome of a bargaining agreed by all the bargainers. In the case that no unanimous agreement is established, the players end up with some pre-specified disagreement outcome, known as *threat point*. A bargaining game is usually described by a set $\mathcal{N} = \{1, \dots, N\}$ of players and a pair $(\mathcal{S}, \mathbf{d})$, where $\mathcal{S} \subset \mathbb{R}^N$ is a compact convex set, representing the set of *feasible payoffs* to the players, $\mathbf{d} = \{d_1, \dots, d_N\}$ represents the threat point.

Nash Bargaining Solution

A bargaining solution on a class \mathcal{B} of bargaining problems is defined as a function $f : \mathcal{B} \rightarrow \mathbb{R}^N$ such that it assigns an allocation $f(\mathcal{S}, \mathbf{d}) = \{f_1, \dots, f_N\} \in \mathcal{S}$ to each bargaining problem $(\mathcal{S}, \mathbf{d})$ in \mathcal{B} . In [38], Nash proposed that such a solution should satisfy the following four properties:

- **Pareto optimality:** For any bargaining game $(\mathcal{S}, \mathbf{d}) \in \mathcal{B}$, if $x, y \in \mathcal{S}$ and $y > x$, we have $f(\mathcal{S}, \mathbf{d}) \neq x$.

⁶A game is called a bounded game if the payoff functions are bounded.

⁷The path is defined as a sequence of steps, in each step, only one player is allowed to deviate. A path $\phi = \{s^0, s^1, \dots\}$ is an ϵ -improvement path with respect to \mathcal{G} if for all $t \geq 0$, $u_n(s^{t+1}) > u_n(s^t) + \epsilon$, where n represents index of a unique player at step $t + 1$.

- **Symmetry:** For any bargaining game $(\mathcal{S}, \mathbf{d}) \in \mathcal{B}$, $d_1 = d_2 = \dots = d_n$ implies $f_1(\mathcal{S}, \mathbf{d}) = f_2(\mathcal{S}, \mathbf{d}) = \dots = f_n(\mathcal{S}, \mathbf{d})$.
- **Invariant to affine transformations:** For any bargaining game $(\mathcal{S}, \mathbf{d}) \in \mathcal{B}$ and for any positive affine transformation, e.g., $g : \mathbb{R}^N \rightarrow \mathbb{R}^N$, we have $f(g(\mathcal{S}), g(\mathbf{d})) = g(f(\mathcal{S}, \mathbf{d}))$.
- **Independence of irrelevant alternatives:** For any two bargaining games $(\mathcal{S}, \mathbf{d}), (\mathcal{S}', \mathbf{d}) \in \mathcal{B}$ with $\mathcal{S} \subseteq \mathcal{S}'$, $f(\mathcal{S}', \mathbf{d}) \in \mathcal{S}$ implies $f(\mathcal{S}', \mathbf{d}) = f(\mathcal{S}, \mathbf{d})$.

Nash showed that these properties could actually characterize a unique solution to the bargaining problem. More precisely, Nash's solution is a function which selects the unique outcome which satisfies the four properties stated above. This function can be written as

$$\begin{aligned} \max_{\mathbf{u} \in \mathcal{S}} \quad & \prod_{n=1}^N (u_n - d_n) \\ \text{s.t.} \quad & \mathbf{u} \geq \mathbf{d} \end{aligned}$$

where $\mathbf{u} = \{u_1, \dots, u_N\}$ is defined as players' payoff set.

The symmetry property in the standard Nash bargaining assumes that all players in the cooperative game have the same priority. When the symmetric property is removed, E. Kalai [40] showed that the bargaining solution is still unique, and can be characterized by

$$\begin{aligned} \max_{\mathbf{u} \in \mathcal{S}} \quad & \prod_{n=1}^N (u_n - d_n)^{\omega_n} \\ \text{s.t.} \quad & \mathbf{u} \geq \mathbf{d} \end{aligned} \tag{3.12}$$

where the weight ω_n can be interpreted as "bargaining power" [41] (which is related to the relative abilities of players in a situation to exert influence over each other) for player n . The unique solution to (3.12) is known as the asymmetric Nash bargaining solution with bargaining powers $\omega_1, \dots, \omega_N$. However, in order to find the Nash bargaining solution, we need to solve the non-convex optimization (3.12), which can actually be transformed into the following convex optimization problem

$$\begin{aligned} \max_{\mathbf{u} \in \mathcal{S}} \quad & \sum_{n=1}^N \omega_n \log(u_n - d_n) \\ \text{s.t.} \quad & \mathbf{u} \geq \mathbf{d} \end{aligned}$$

where the objective function is concave in \mathbf{u} , and the constraint set \mathcal{S} is compact and convex. Thus, we could simply apply the methods of convex optimization introduced in section 3.2, e.g., duality theory and KKT conditions, to analyze and solve it in an efficient way.

Chapter 4

Resource Allocation Games with Complete Information

4.1 Introduction

Recently, there has been an increasing interest for small-cell wireless networks, where people can access Internet over many different access points (APs) or small base stations, which are also known as out-door femto-cells, see [4, 42]. Typically, in such a wireless network, several APs are installed out-door on a given backbone network (with heterogeneous links as fiber, ADSL, power line) to provide signal coverage in dense environments. The general idea is to exploit the heterogeneous wired infrastructure without any new cabling and provide wireless high data rate to the users through self-organized mechanisms. Unfortunately, if users are connected to a single AP, they may suffer from low throughput from time to time due to the limited-backhaul capacity (some wireless high speed femto-cells access the Internet through low capacity DSL or power line links, e.g., 1Mbps), despite the presence of a high speed wireless link. As a result, users need to access to different APs in the nearby femto-cells in order to aggregate the sum capacity of the backhaul links. An interesting concept is to consider the nearby femto-cells as a virtual femto-cell group, whose backhaul capacity is the sum of the access capacities of all the nearby femto-cells (Figure 4.1). The issue

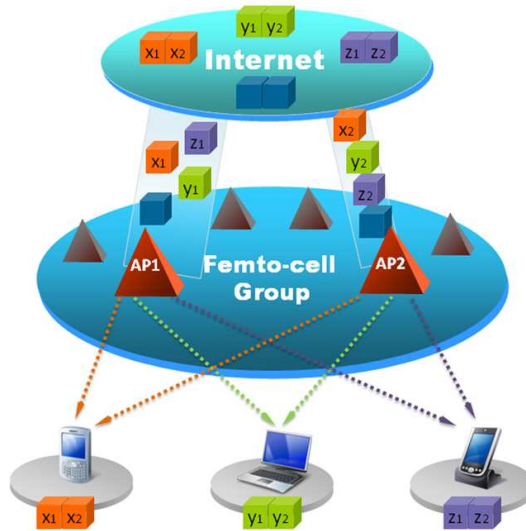


Figure 4.1 A femto-cell group with distributed network information flow

of load balancing [43] in the wired network (and how the different packets are split with respect to the backhaul capacity from a main decentralized scheduler), although important, is not dealt with in this chapter and we will suppose that perfect load balancing holds.

We focus on the resource allocation problem for the downlink scenario, i.e., from femto-cell group to mobile terminals (MTs), using orthogonal frequency division multiplexing (OFDM) air-interface [44] over a number of dedicated sub-channels. We assume that network coding is applied at the Internet source, i.e., all these femto-cells get independently coded packets from the Internet via their backhauls, and send them physically to each MT in a distributed manner. Usually, in this situation each AP needs to decide how to distribute the total available transmit power over N downlink sub-channels (subcarriers or clusters of subcarriers), i.e., should it allocate all its power to a single sub-channel, spread the power over all the sub-channels, or choose some subset of sub-channels on which to transmit?

In traditional wireless networks, such a resource allocation problem is considered as a global optimization problem. The major goal is usually to achieve some theoretical performance limits, i.e., capacity region, maximum achievable rate, subject to some resource constraints, e.g., power, frequency, etc. However,

this approach requires a central computing entity (scheduler) with complete knowledge of the network states, involving feedback and overhead communication whose load scales linearly with the number of transmitters and receivers in the network. This is especially crucial for some modern wireless systems like femto-cell and outdoor small-cell network, where it is quite difficult to centralize the information due to a limited capacity backhaul. Moreover, the high density of APs or base stations would increase the cost of centralizing the information.

It is certainly possible to improve the useful data transmissions by reducing transmissions of insignificant or unnecessary feedback information. In this direction, a selective multi-user diversity algorithm has been introduced in [45]. The key idea is to find a suitable trade-off between the network performance and the feedback load. Nevertheless, this partial feedback approach still has its self-limitation in network scaling problems. As wireless networks appear to be more and more dense, the global optimization approach becomes more and more inadequate to meet the needs of future wireless communication development.

Within the recent ten years, increased research interest has been given to cognitive radio [46] and self-organizing mobile flexible networks [4] in which wireless devices intelligently sense and allocate resources in a decentralized manner. Games theory [21, 6], provides an alternative analytical method by considering every wireless node as a player who “plays” the game by adaptively and rationally choosing her transmission strategies, e.g., transmit power levels.

In this chapter, we first review and investigate the usage of non-cooperative game theory with complete information to the wireless resource allocation problem, more precisely, power allocation. For such a problem, each wireless user’s transmission rate is increasing in her own transmit power level and decreasing in other user’s transmit power level. In game theory, this trade-off can be captured with a payoff or utility function and a well designed game structure. If all other users’ power levels were fixed, increasing one’s power would increase her signal-to-noise-plus-interference ratio (SINR) as well as transmission rate. On the other hand, such an action increases the interference seen by the other users, driving their SINR down and inducing them to increase their own transmit power levels.

Then we present a general game-theoretical framework for the resource allocation problem in the downlink of wireless small-cell networks, where several APs or base stations send independent coded network information to several MTs through OFDM channels. In such a game-theoretic study, it is important to study the Nash equilibrium (NE) because it represents a predictable outcome for a self-organizing network. The central question we consider is whether a NE

exists, and if so, whether the network operates efficiently at the NE. We prove that for independent fading channels, the NE is always unique. Furthermore, we show that this resource allocation problem can be studied as an exact potential game, and hence can be efficiently solved. We show the convergence of waterfilling based best-response algorithm, and we discuss some interesting properties of potential games. Finally, numerical results are provided to investigate the inefficiency of NE.

4.2 State-of-the-Art

Recently, there has been a significant amount of work based on game theory and applied to wireless networks. Non-cooperative games have been widely applied in various types of wireless channel models to analyze and solve the resource allocation and power control problems, e.g., multiple access channels (MAC) [47], interference channels (IFC) [48, 49, 50, 51, 52] and some applications in code division multiple access (CDMA) networks [53, 54, 55], multiple input and multiple output (MIMO) networks [56, 57, 58], etc.

In this section, we will focus on investigating the existing applications of non-cooperative games with complete information in the mentioned two fundamental wireless channel models: MAC and IFC. These approaches are closely related to our small-cell channel model to be introduced in the following section.

4.2.1 Flat-fading Multiple Access Channels

Let us first consider a time invariant two-user MAC. Assume that a simple single user decoding strategy is applied at the receiver side. A natural objective of user 1 is to maximize her achievable rate r_1 , under a certain constraint on the transmit power, i.e., $p_1 \leq P_1^{\max}$. Then, this problem can be mathematically formulated as

$$\begin{aligned} \max_{p_1} r_1 &= \max_{p_1} \log \left(1 + \frac{g_1 p_1}{\sigma^2 + g_2 p_2} \right) \\ \text{s.t. } p_1 &\leq P_1^{\max} \\ p_1 &\geq 0 \end{aligned} \quad (4.1)$$

where the channel gains g_1, g_2 and the Gaussian noise variance σ^2 are treated as constants. Note that for a fixed transmit power of user 2, p_2 , the problem (4.1) is a concave maximization problem whose solution is given by $p_1 = P_1^{\max}$. This is because that r_1 is a strictly increasing function in p_1 . Now, if we consider a two-user static game with r_k as user k 's payoff and the transmit power $p_k \in [0, P_k^{\max}]$

as her action, it is easy to see that a unique pure Nash equilibrium exists in this game, i.e., $p_1^* = P_1^{\max}$, $p_2^* = P_2^{\max}$. As a result, both users always transmit with full power, regardless of which power level the other user chooses to transmit. This model is an example of systems where the decision making of one player has no influence on the decision making of the other player. However, as we will see in the following part, the fading scenario can be actually formulated into a meaningful game problem, in which interaction exists between players' decision makings.

In [47], the waterfilling game framework was introduced to study the resource allocation problem in a flat-fading MAC, under the assumption of complete information of the channel state information at both the transmitter and the receiver. To explain the concept of channel state information at the transmitter, we consider the two-user MAC as an example. To have complete channel state information, in this context complete information means that both users are aware of the realizations of both fading channel gains g_1 and g_2 . Under this assumption, a user's transmit power can be modeled as a function of the complete set of the fading channel gains. Denote by $p_1(g_1, g_2)$ and $p_2(g_1, g_2)$ the transmit power strategies of user 1 and 2, respectively. Now, for a fixed strategy of user 2, $p_2(g_1, g_2)$, finding the optimal strategy $p_1(g_1, g_2)$ for user 1 requires the solution of the following optimization problem which maximizes her average achievable rate \bar{r}_1

$$\begin{aligned} \max \bar{r}_1 = & \max_{p_1(g_1, g_2)} \mathbb{E}_{g_1, g_2} \left[\log_2 \left(1 + \frac{p_1(g_1, g_2)g_1}{\sigma^2 + p_2(g_1, g_2)g_2} \right) \right] \\ \text{s.t.} & \mathbb{E}_{g_1, g_2} [p_1(g_1, g_2)] \leq P_1^{\max} \\ & p_1(g_1, g_2) \geq 0 \end{aligned} \quad (4.2)$$

and similarly for user 2. The problem (4.2) is a convex optimization, whose solution is given by the waterfilling algorithm [1]

$$p_1^*(g_1, g_2) = \left(\lambda_1 - \frac{\sigma^2 + p_2(g_1, g_2)g_2}{g_1} \right)^+ \quad (4.3)$$

with $(x)^+ = \max\{x, 0\}$ and λ_1 chosen such that the average power constraint in (4.2) is satisfied with equality

$$\mathbb{E}_{g_1, g_2} \left(\lambda_1 - \frac{\sigma^2 + p_2(g_1, g_2)g_2}{g_1} \right)^+ = P_1^{\max}. \quad (4.4)$$

Note that the solution (4.3) depends on user 2's strategy $p_2(\cdot)$, which user 1 does not know, and reciprocally for user 2. However, given the game model, each user is able to adjust his strategy adaptively to her guess of the strategy of the other user.

It is shown in [47] that there exists a unique Nash equilibrium in this two-user waterfilling game, i.e.,

$$\begin{cases} p_1^*(g_1, g_2) = \left(\lambda_1 - \frac{\sigma^2}{g_1}\right)^+, & \text{when } g_1 \geq \frac{\lambda_2}{\lambda_1} g_2 \\ p_2^*(g_1, g_2) = \left(\lambda_2 - \frac{\sigma^2}{g_2}\right)^+, & \text{when } g_2 \geq \frac{\lambda_1}{\lambda_2} g_1 \end{cases} \quad (4.5)$$

with $p_1^*(g_1, g_2) = 0, p_2^*(g_1, g_2) = 0$ in other cases. The dual variables λ_1 and λ_2 are given by

$$\begin{cases} \mathbb{E}_{g_1, g_2} \left[\left(\lambda_1 - \frac{\sigma^2}{g_1}\right)^+ \middle| g_1 \geq \frac{\lambda_2}{\lambda_1} g_2 \right] = P_1^{\max} \\ \mathbb{E}_{g_1, g_2} \left[\left(\lambda_2 - \frac{\sigma^2}{g_2}\right)^+ \middle| g_2 \geq \frac{\lambda_1}{\lambda_2} g_1 \right] = P_2^{\max} \end{cases}$$

Since the transmit power $p_1^*(g_1, g_2)$ and $p_2^*(g_1, g_2)$ in (4.5) can not be simultaneously strictly positive, this Nash equilibrium power strategy corresponds to the time-sharing transmission scheme, i.e., at a certain time slot, only one user is allowed to transmit. This result shows that, for this problem, the Nash equilibrium corresponds to an opportunistic TDMA scheme. Moreover, this unique Nash equilibrium (p_1^*, p_2^*) is shown to coincide with a maximum sum-rate point of the capacity region. This is an interesting property, since in general Nash equilibrium is not Pareto-efficient [8, 6]. Thus, an interesting conclusion is that the selfish behavior of the users will result in the joint optimization of the global channel capacity. Finally, we should emphasize that the above results also hold for arbitrary number of users, as shown in [47].

4.2.2 Frequency-selective Interference Channels

Although the capacity region of the interference channel is still unknown, even for the simplest Gaussian two-user case (see a recent result in [59]), many works consider that a single user decoding strategy is applied at the receiver, which leads to an achievable rate region. This achievable rate region is very relevant in practical systems with limitations on the decoder complexity and simplicity of the system. Write user n 's achievable rate as

$$r_n(\underbrace{\mathbf{p}_1, \dots, \mathbf{p}_N}_{\text{power allocation}}) = \sum_{i=1}^L \log \left(1 + \frac{g_{n,m}^{(i)} p_n^{(i)}}{\sigma^2 + \underbrace{\sum_{j=1, j \neq n}^M g_{j,m}^{(i)} p_j^{(i)}}_{\text{multiuser interference}}} \right) \quad (4.6)$$

where $g_{n,m}^{(i)}$ represents the i^{th} sub-channel's channel gain from user n to user m , $p_m^{(i)}$ is user n 's transmit power on sub-channel i , N is the total number of users, L is the total number of sub-channels, and \mathbf{p}_n is considered as the strategy of user n , i.e.,

$$\mathbf{p}_n = \{p_n^{(1)}, p_n^{(2)}, \dots, p_n^{(L)}\} \quad (4.7)$$

subject to

$$\sum_{i=1}^L p_n^{(i)} \leq P_n^{\max}$$

where $p_n^{(i)} \geq 0$ for all n, i .

In this case, each user faces a sum-rate maximization problem

$$\begin{aligned} \max_{\mathbf{p}_n} \quad & r_n(\mathbf{p}_1, \dots, \mathbf{p}_N) \\ \text{s.t.} \quad & \sum_{i=1}^L p_n^{(i)} \leq P_n^{\max} \\ & p_n^{(i)} \geq 0 \end{aligned} \quad (4.8)$$

Intuitively, for user n , this optimization problem is simply a question of how to spread her total available power across all the frequency sub-channels, while being aware of the possible interference caused by other users, who face a similar power allocation problem. When the power strategy profile of other users $\mathbf{p}_{-n} \triangleq \{\mathbf{p}_1, \dots, \mathbf{p}_{n-1}, \mathbf{p}_{n+1}, \dots, \mathbf{p}_N\}$ is known by user n , the single user sum-rate maximization problem (4.8) is a convex optimization problem, whose solution is given by waterfilling algorithm. Therefore, this competitive sum-rate maximization game problem can be considered in the framework of *waterfilling game*.

Here, we provide an overview of the current results, mainly focused on the characterization of the Nash equilibrium set. Many works in this direction have been separately studied in [48, 49, 50, 51, 52] with slightly different assumptions. In [48], the interference channel is modeled as a non-cooperative game, and the conditions for the existence and uniqueness of Nash equilibrium are established for a two-player version of the game. The similar conditions for the existence and uniqueness have been extended to the multiuser case in [49] and later in its journal version [50], in which the authors also focus on the practical design of distributed algorithms to compute the Nash equilibrium point, namely asynchronous iterative waterfilling algorithm. In [51], the so called symmetric waterfilling game was studied, in which the authors assume that for a set of sub-channels and receivers the channel gains from all transmitters are the same. The results show that an infinite number of Nash equilibrium exist in the game.

In summary, IFC game usually has a pure strategy Nash equilibrium, since it belongs to the concave N -persons game [23], but the uniqueness of Nash equilibrium does not hold in general. However, even if the uniqueness of the Nash equilibrium is established under some conditions, it does not imply a stable convergence to the equilibrium point. The interested readers are referred to [49, 52, 60, 61] for the topic of equilibrium convergence in IFC games.

4.2.3 Some Comments

One may notice that the way of modeling the strategy \mathbf{p}_n as a power vector (4.7) in the IFC game is similar to the strategy $p(g_1, g_2)$ as a function (4.2) modeled in the MAC game. In fact, to consider the strategy $p(g_1, g_2)$ as a function of fading channel gains is, in some sense, equivalent to consider the problem of how to spread the total power in the time domain. Both ways of modeling the strategy have a common feature, i.e., user's total energy resource is spread as a vector in a certain domain. Note that multiuser interference enables game theory to be a fruitful tool to study users' interaction. Such a strategy, in a more general sense, could always take the form of a finite or infinite nonnegative vector, which indicates a user how to spread her total available resource in certain domain, e.g., time, frequency, space, etc.

4.3 Resource Allocation game for OFDM Small-cell networks

In this section, we present a non-cooperative game-theoretic approach to solve the problem of distributed resource allocation in the context of multiuser OFDM small-cell networks. Multiple transmitters need to decide how to share their total transmit power over several independent frequency-selective fading channels. We consider games with complete information.

This work originates from the wide usage of wireless hot-spot networks, in particular Wi-Fi hot-spots, where people can access Internet over a wireless local area network (LAN). Typically, in such a wireless network, several APs are installed to provide signal coverage for a small area, e.g., airports, military bases, restaurants, hotels, hospitals, libraries, supermarkets, etc. Here, we consider that these APs belong to different network service providers, using OFDM over a number of dedicated sub-channels. This wireless channel can be modeled as the case where multiple transmitters simultaneously send information to their receivers through several independent frequency-selective fading channels.

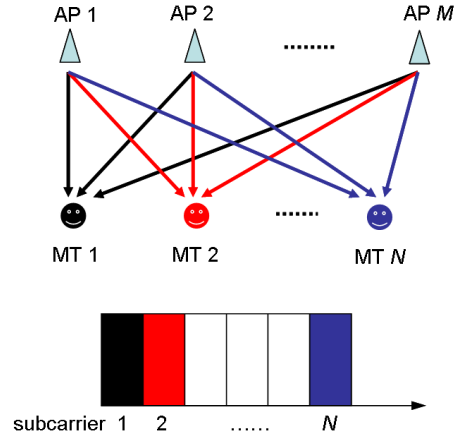


Figure 4.2 Multi-user OFDM model for small-cell wireless networks

Usually, in this situation each AP needs to decide how to distribute the total available transmit power over N downlink sub-channels, i.e., should it allocate all its power to a single sub-channel, spread the power over all the sub-channels, or choose some subset of sub-channels on which to transmit? Note that a similar power allocation problem can be considered for the uplink transmission where MTs are the transmitters who decide their transmit power.

4.3.1 Channel Model

Consider a Gaussian interference channel with N APs simultaneously sending information to M MTs over M independent frequency-selective sub-channels. Figure 4.2 shows the multi-user OFDM network mode. Assume that each sub-channel is pre-assigned to a different MT by a scheduler, therefore, each MT detects the signals only on the assigned sub-channel. We also assume that the sub-channels have block fading so that the channel fading coefficients are constant during the period of each transmission block. Within a given transmission block, let $\mathbf{G} \in \mathbb{R}_{++}^{N \times M}$ be the channel gain matrix whose (n, m) entry $g_{n,m}$ is the channel power gain of the link from the n^{th} AP to the m^{th} MT on the pre-assigned sub-channel m . We assume that \mathbf{G} is a random $N \times M$ matrix with *i.i.d.* continuous entries due to independent fading. We also assume that the information flows from different APs are independently coded via network coding (due to heterogeneous wired infrastructure), and single user decoding is applied at all the receivers, i.e., the signals from other APs are treated as interference.

Therefore, the signal-to-interference plus noise ratio (SINR) of the signal from the n^{th} AP to the m^{th} MT is

$$\gamma_{n,m} = \frac{g_{n,m} p_{n,m}}{\sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m}} \quad (4.9)$$

where $p_{n,m} \geq 0$ represents the power transmitted by the AP n on sub-channel m . For the sake of simplicity, we assume that the white Gaussian noise variance σ^2 is the same for each sub-channel m . The maximum achievable sum-rate for AP n is given by [1]

$$r_n = \sum_{m=1}^M \log(1 + \gamma_{n,m}), \quad \forall n \quad (4.10)$$

Each AP n has the power constraint

$$\sum_{m=1}^M p_{n,m} \leq P_n^{\max}, \quad \forall n \quad (4.11)$$

where $P_n^{\max} > 0$ is the maximum power constraint of user n .

4.3.2 Game Formulation

Here, we consider the described channel model as a non-cooperative strategic game. In this game, the goal of each AP (player) n is to choose its own power vector $\mathbf{p}_n = [p_{n,1} \dots p_{n,M}]^T$, subject to its total power constraint (4.11), in order to selfishly maximize the sum-rate r_n (4.10). Let the power vector $\mathbf{p} = [\mathbf{p}_1^T, \dots, \mathbf{p}_N^T]^T$ denote the outcome of the game in terms of transmission power levels of all N players on M sub-channels. We can completely describe this non-cooperative game as

$$\mathcal{G} \triangleq \langle \mathcal{N}, \mathcal{P}, \mathcal{U} \rangle, \quad (4.12)$$

where the elements of the game are

- *The player set:* $\mathcal{N} = \{1, \dots, N\}$.
- *The strategy set:* $\mathcal{P} = \mathcal{P}_1 \times \mathcal{P}_2 \times \dots \times \mathcal{P}_N$, where the strategy of player n is

$$\mathcal{P}_n = \left\{ \mathbf{p}_n : p_{n,m} \geq 0, \forall m, \sum_{m=1}^M p_{n,m} = P_n^{\max} \right\}. \quad (4.13)$$

- The utility or payoff function set: $\mathcal{U} = \{u_1, \dots, u_N\}$, where the payoff of player n is

$$u_n(\mathbf{p}_n, \mathbf{p}_{-n}) = \sum_{m=1}^M \log \left(1 + \frac{g_{n,m} p_{n,m}}{\sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m}} \right) = r_n \quad (4.14)$$

and \mathbf{p}_{-n} denotes the power vector of length $(N-1)M$ consisting of elements of \mathbf{p} other than the n^{th} element, i.e.,

$$\mathbf{p}_{-n} = [\mathbf{p}_1^T, \dots, \mathbf{p}_{n-1}^T, \mathbf{p}_{n+1}^T, \dots, \mathbf{p}_N^T]^T$$

4.3.3 Characterization of Nash Equilibrium Set

Existence and Uniqueness of Nash Equilibrium

It is known that in general a Nash equilibrium point does not necessarily exist. Therefore, our first interest turns to investigate the existence of equilibrium. We introduce the following theorem:

Theorem 4.1. *A Nash equilibrium exists in game \mathcal{G} (4.12).*

Proof. Since \mathcal{P}_n is convex, closed, and bounded for each n ; $u_n(\mathbf{p}_n, \mathbf{p}_{-n})$ is continuous in both \mathbf{p}_n and \mathbf{p}_{-n} ; and $u_n(\mathbf{p}_n, \mathbf{p}_{-n})$ is concave in \mathbf{p}_n for any set \mathbf{p}_{-n} , at least one Nash equilibrium point exists for \mathcal{G} [23], [6]. \square

Once existence is established, it is natural to consider the characterization of the equilibrium set. Ideally, we would prefer to have a unique equilibrium, but unfortunately this is not true for many game problems [6]. A special case of our game \mathcal{G} , namely the *symmetric waterfilling game* is studied in [51], where the authors show an infinite number of Nash equilibria existing under the assumption of equal crosstalk channel coefficients. This implies that in general our game \mathcal{G} does not have a unique equilibrium point. Nevertheless, we will show in the following context that, under the assumption that \mathbf{G} has *i.i.d.* continuous entries, the probability of having a unique Nash equilibrium is equal to 1.

For any player n , given all other players' strategy profile \mathbf{p}_{-n} , the *best-response* power strategy \mathbf{p}_n can be found by solving the following maximization problem,

$$\begin{aligned} \max_{\mathbf{p}_n} \quad & u_n(\mathbf{p}_n, \mathbf{p}_{-n}) \\ \text{s.t.} \quad & \sum_{m=1}^M p_{n,m} \leq P_n^{\max} \\ & p_{n,m} \geq 0, \quad \forall m \end{aligned} \quad (4.15)$$

which is a classical convex optimization problem, since the objective function is concave in \mathbf{p}_n and the constraint set is convex. Therefore, the Karush-Kuhn-Tucker (KKT) condition of the optimization is sufficient and necessary for the optimality [2]. To derive the KKT conditions, we form the Lagrangian for player n ,

$$\begin{aligned} \mathcal{L}_n(\mathbf{p}, \lambda, \nu) &= \sum_{m=1}^M \log \left(1 + \frac{g_{n,m} p_{n,m}}{\sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m}} \right) - \lambda_n \left(\sum_{m=1}^M p_{n,m} - P_n^{\max} \right) \\ &\quad + \sum_{m=1}^M \nu_{n,m} p_{n,m}. \end{aligned}$$

The corresponding KKT conditions are

$$\frac{g_{n,m}}{\sigma^2 + \sum_{j=1}^N g_{j,m} p_{j,m}} - \lambda_n + \nu_{n,m} = 0, \quad \forall m \quad (4.16)$$

$$\lambda_n \left(\sum_{m=1}^M p_{n,m} - P_n^{\max} \right) = 0 \quad (4.17)$$

$$\nu_{n,m} p_{n,m} = 0, \quad \forall m \quad (4.18)$$

where $\lambda_n \geq 0$, $\forall n$ and $\nu_{n,m} \geq 0$, $\forall n \forall m$ are dual variables associated with the constraints of total transmit power and transmit power positivity, respectively. The solution to this problem is known as waterfilling:

$$p_{n,m} = \left(\frac{1}{\lambda_n} - \frac{\sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m}}{g_{n,m}} \right)^+, \quad \forall m \quad (4.19)$$

where $(x)^+ = \max\{0, x\}$ and λ_n satisfies

$$\sum_{m=1}^M \left(\frac{1}{\lambda_n} - \frac{\sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m}}{g_{n,m}} \right)^+ = P_n^{\max}. \quad (4.20)$$

Before analyzing the equilibrium set, we derive the following theorem:

Theorem 4.2. *A power strategy set $\{\mathbf{p}_1^*, \dots, \mathbf{p}_N^*\}$ is a Nash equilibrium of game \mathcal{G} (4.12) if and only if each player's power strategy \mathbf{p}_n^* is the single-player waterfilling result (4.19) while treating other players' signals as noise, and the corresponding necessary and sufficient conditions are:*

$$\frac{g_{n,m}}{\sigma^2 + \sum_{j=1}^N g_{j,m} p_{j,m}} - \lambda_n + \nu_{n,m} = 0, \quad \forall m \forall n \quad (4.21)$$

$$\lambda_n \left(\sum_{m=1}^M p_{n,m} - P_n^{\max} \right) = 0, \quad \forall n \quad (4.22)$$

$$\nu_{n,m} p_{n,m} = 0, \quad \forall m \forall n. \quad (4.23)$$

The proof can be found in Appendix 4.6.1.

From (4.21), it is easy to see that $\lambda_n > 0$, since we have $\nu_{n,m} \geq 0$ and $g_{n,m} > 0, \forall m, \forall n$. Then from (4.22), we have

$$\sum_{m=1}^M p_{n,m} = P_n^{\max}, \quad \forall n \quad (4.24)$$

Note that (4.24) has an intuitive meaning: every player at Nash equilibrium must dedicate her total available power over all the sub-channels, due to their “selfish instinct”. However, it is still difficult to find the analytical solution for (4.21)-(4.23), since the system of the solution forms (4.19) and (4.20) is nonlinear. The idea to simplify this problem is therefore to consider *linear* equations instead of nonlinear ones. The following lemma provides a key step in that direction:

Lemma 4.3. *For any realization of channel matrix \mathbf{G} , there exist unique values of the Lagrange dual variables λ and ν for any Nash equilibrium of the game \mathcal{G} . Furthermore, there is a unique vector $\mathbf{s} = [s_1, \dots, s_M]^T$ such that any vector \mathbf{p} corresponding to a Nash equilibrium of the game satisfies*

$$\sum_{n=1}^N g_{n,m} p_{n,m} \triangleq s_m, \quad \forall m. \quad (4.25)$$

The proof can be found in Appendix 4.6.2.

Now, let \mathbf{Z} be the following $(M + N)$ by MN matrix:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{I}_N & \mathbf{I}_N & \cdots & \mathbf{I}_N \\ \mathbf{g}_1^T & \mathbf{0}_N^T & \cdots & \mathbf{0}_N^T \\ \mathbf{0}_N^T & \mathbf{g}_2^T & \cdots & \mathbf{0}_N^T \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_N^T & \mathbf{0}_N^T & \cdots & \mathbf{g}_M^T \end{bmatrix}_{(M+N) \times MN}$$

where \mathbf{g}_m is the m^{th} column of \mathbf{G} , \mathbf{I}_N is the $N \times N$ identity matrix, and $\mathbf{0}_N$ is the zero vector of length N . Let \mathbf{c} be the following vector of length $M + N$

$$\mathbf{c} = [P_1^{\max} \ P_2^{\max} \ \dots \ P_N^{\max} \ s_1 \ s_2 \ \dots \ s_M]^T.$$

Then, (4.24) and (4.25) can be written in the form of a *linear* matrix equation

$$\mathbf{Z}\mathbf{p} = \mathbf{c}. \quad (4.26)$$

Define the following sets:

$$\begin{aligned}\mathcal{X} &\triangleq \{(n, m) : \nu_{n,m} = 0\}, \\ \mathcal{N} &\triangleq \{m : \exists n \text{ such that } (n, m) \in \mathcal{X}\}.\end{aligned}$$

From equation (4.23), if an index $(m, n) \notin \mathcal{X}$ we must have $p_{n,m} = 0$. Without loss of generality, we assume that $\mathcal{M} = \{1, \dots, \tilde{M}\}$ for $\tilde{M} \leq M$. Let $\tilde{\mathbf{Z}}$ be the $(\tilde{M} + N) \times (\tilde{M}N)$ matrix formed from the first $\tilde{M} + N$ rows and first $\tilde{M}N$ columns of \mathbf{Z} , $\tilde{\mathbf{p}}$ is formed from the first $\tilde{M}N$ elements of \mathbf{p} , and $\tilde{\mathbf{c}}$ is formed from the first $\tilde{M} + N$ elements of \mathbf{c} , then any Nash equilibrium solution must satisfy the following linear equation:

$$\tilde{\mathbf{Z}}\tilde{\mathbf{p}} = \tilde{\mathbf{c}}. \quad (4.27)$$

Let $|\mathcal{X}|$ denote the cardinality of set \mathcal{X} , and $\hat{\mathbf{Z}}$ be the $(\tilde{M} + N) \times |\mathcal{X}|$ matrix formed from the columns of $\tilde{\mathbf{Z}}$ that correspond to the elements of \mathcal{X} . Similarly, let $\hat{\mathbf{p}}$ be the vector of length $|\mathcal{X}|$ with elements $p_{n,m}$ such that $(n, m) \in \mathcal{X}$ in the same order as in \mathbf{p} . Then, any Nash equilibrium solution must satisfy

$$\hat{\mathbf{Z}}\hat{\mathbf{p}} = \tilde{\mathbf{c}}. \quad (4.28)$$

Lemma 4.4. *For any realization of a random $N \times M$ channel gain matrix \mathbf{G} with i.i.d. continuous entries, if $\tilde{M}N > \tilde{M} + N$, the probability that $|\mathcal{X}| \leq \tilde{M} + N$ is equal to 1.*

Lemma 4.5. *1. If $\tilde{M}N > \tilde{M} + N$ and $|\mathcal{X}| \leq \tilde{M} + N$, the probability that $\text{rank}(\hat{\mathbf{Z}}) = |\mathcal{X}|$ is equal to 1.*

2. If $\tilde{M}N \leq \tilde{M} + N$, the probability that $\text{rank}(\tilde{\mathbf{Z}}) = \tilde{M}N$ is equal to 1.

The proofs of Lemma 4.4 and 4.5 can be found in Appendix 4.6.3 and 4.6.4, respectively.

Based on the previous lemmas, now we are ready to show the uniqueness of Nash equilibrium through the following theorem:

Theorem 4.6. *For any realization of a random $N \times M$ channel gain matrix \mathbf{G} with i.i.d. entries, the game \mathcal{G} has exactly one Nash equilibrium.*

The proof can be found in Appendix 4.6.5.

Thus, we prove the existence and uniqueness of Nash equilibrium in our game \mathcal{G} .

4.3.4 Convergence to the Nash equilibrium

Equilibrium is meaningful in practice only if it is reachable from non-equilibrium states. In fact, there is no reason to expect a system to operate initially at equilibrium. The “convergence to equilibrium” is in general a much harder problem which is usually related to the analysis of synchronous or asynchronous update mechanisms. See some references for interference channels [61], [50].

Potential game approaches

Fortunately, our game \mathcal{G} (4.12) can be studied as a potential game. Potential games are known to have nice properties for the convergence of greedy algorithms to the equilibrium. Such games have received increasing attention recently in wireless networks [62, 63, 64], since the existence of potential function enables the design of fully distributed algorithms for resource allocation problems.

Taking inspiration from the result derived in the single channel case [63], it is not difficult to see that in our multi-channel case, \mathcal{G} is an exact potential game with potential function

$$\begin{aligned} v^*(\mathbf{p}_n, \mathbf{p}_{-n}) &= \sum_{m=1}^M \log \left(\sigma^2 + \sum_{n=1}^N g_{n,m} p_{n,m} \right) \\ &= \sum_{m=1}^M \log \left[g_{n,m} p_{n,m} + \underbrace{\left(\sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m} \right)}_{\text{aggregate interference + noise}} \right] \end{aligned} \quad (4.29)$$

Let us denote by $\zeta_{n,m}$ the term $\left(\sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m} \right)$, which represents the aggregate interference plus noise to player n 's signal on sub-channel m . Now, v^* is a *common payoff* to all players in the potential game.

From a single player point of view, e.g., player n , in order to maximize her own payoff, player n needs to solve the following maximization problem:

$$\begin{aligned} \max_{\mathbf{p}_n} v^*(\mathbf{p}_n, \mathbf{p}_{-n}) &\Leftrightarrow \max_{\mathbf{p}_n} \sum_{m=1}^M \log (\zeta_{n,m} + g_{n,m} p_{n,m}) \\ &s.t. \quad \sum_{m=1}^M p_{n,m} \leq P_n^{\max} \\ &\quad p_{n,m} \geq 0, \quad \forall m \end{aligned} \quad (4.30)$$

When the coefficients $g_{n,m}$ (private channel gain) and $\zeta_{n,m}$ (aggregate interference plus noise) are known to player n for all sub-channel m , the single-user best response solution to (4.30) can be considered as a convex optimization problem. In fact, the objective function v^* is concave in \mathbf{p}_n and the constraint set is convex. It can be further verified that the KKT conditions (4.30) coincide with (4.16)-(4.18), which are known to be the KKT conditions of the single-player best-response in the original game \mathcal{G} . Therefore, the solution to this problem is still the waterfilling procedure.

Note that if each AP has complete knowledge of the CSI, i.e., knowledge about the whole channel matrix \mathbf{G} , the uniqueness of the Nash equilibrium guaranties that each AP can determine independently in a decentralized way the power allocation at the Nash equilibrium. In order to acquire information about the whole channel matrix \mathbf{G} is typically necessary a feedback channel from MTs to APs to transmit the channel estimations. In fact, in this case each AP can perform locally the best-response algorithm and based on repeated maximization of problem (4.30) by starting from a random initial point \mathbf{p}_{-n} . However, the structure of the problem (4.30) suggests an alternative approach to reduce eventually the signaling on the feedback channel. In fact, the repeated optimization of problem (4.30) could be performed in a distributed way feeding back at the APs only the private channel gain g_n and the aggregate interference plus noise ζ_n . Nevertheless, note that such a distributed implementation of the algorithm would lead to a temporary phase where the APs are not transmitting at an equilibrium point. In our numerical results, i.e., Section 4.4, we will ignore the cost of feedback, and we focus on analyzing the theoretic upper-bound.

Distributed Potential Iterative WaterFilling (DPIWF)

A general discussion about the convergence properties of potential games can be found in [34], where it shows that every bounded potential game¹ has the *approximate finite improvement property* (AFIP), i.e., for every $\epsilon > 0$, every ϵ -improvement path is finite. Then, it is obvious that every such finite improvement path of the exact potential games terminates in an ϵ -equilibrium² point. In other words, the *sequential best-response* (players move in turn and always choose a best-response) converges to the ϵ -equilibrium independent of the initial point. Note that this is a very flexible condition for the convergence, since order of playing can be deterministic or random and need not to be synchronized. It is one of the most interesting properties of the potential games, especially in

¹A game is called a bounded game if the payoff functions are bounded.

²An ϵ -equilibrium is a strategy profile that approximately satisfies the condition of NE.

order to distributively find the equilibrium in self-organizing systems.

It is not difficult to find that the *simultaneous best-response* (at each iteration, all the players choose their best-responses simultaneously) does not necessarily converges, due to the “ping-pong” effect generated by myopic players. However, [37] has shown that for infinite pseudo-potential games (a general case of exact potential games) with convex strategy space and single-valued best-response³, the sequence of simultaneous best-response (reminiscent of fictitious play) also converges to the equilibrium.

It is interesting to note that for many practical systems with finite transmit power states, similar results still hold for the convergence of the sequential best-response. The only difference is that, in the finite case, the existence of exact potential function implies the *finite improvement property* (FIP), and therefore, the sequential best-response converges to the exact Nash equilibrium.

Based on the discussion above, we provide an algorithm called DPIWF that each player can follow to reach a NE of the network. From the previous discussion, this algorithm enables:

- To reach the unique Nash equilibrium, if there exactly exists a unique one, as in the case of independent continuous fading channels.
- To reach at least one Nash equilibrium, if there exist multiple ones.

Algorithm 1 DPIWF algorithm

```

initialize  $t = 0$ ,  $p_{n,m}^{(0)} = 0$ ,  $\forall m \forall n$ 
repeat
   $t = t + 1$ 
  for  $n = 1$  to  $N$  do
    for  $m = 1$  to  $M$  do
       $\zeta_{n,m}^{(t)} = \sigma^2 + \sum_{j \neq n} g_{j,m} p_{j,m}^{(t)}$ 
    end for
     $\left[ p_{n,1}^{(t+1)}, \dots, p_{n,M}^{(t+1)} \right] = \arg \max_{\substack{p_n \geq 0 \\ \sum_m p_{n,m} \leq P_n^{\max}}} \sum_n \log \left( \zeta_{n,m}^{(t)} + g_{n,m} p_{n,m} \right)$ 
  end for
until convergence
  
```

In this algorithm, we assume that the same game could be myopically played repeatedly: in each round, every player has neither memory of past game-rounds

³Games with strictly multi-concave potential (concave in each players’ unilateral deviation) have single-valued best-response

nor speculation of future events, but it chooses her own power strategy according to the single-player waterfilling that depends on the current state of the game. More precisely, each round of the algorithm DPIWF consists in finding the basic single-user waterfilling solution for each player while regarding the aggregate signals from all other players as noise.

Theorem 4.7. *The DPIWF algorithm converges to a Nash equilibrium of the game \mathcal{G} (4.12).*

The proof can be found in Appendix 4.6.6.

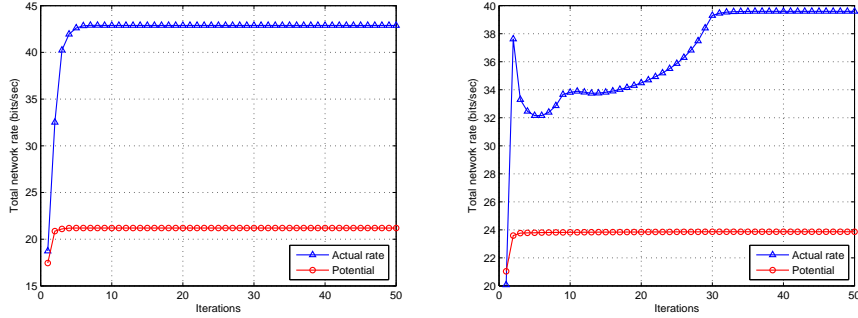
4.4 Numerical Evaluation

The next essential question which comes to our mind is: from the global resource allocation point of view, is it efficient for the network to operate at the Nash equilibrium? In order to give a satisfactory answer to this question, we will briefly introduce in Section 4.4 the social optimality in centralized approaches which represents a target upper-bound for our decentralized approach.

In this part, numerical results are provided to validate our theoretical claims. We consider frequency-selective fading channels with channel matrix \mathbf{G} of size $N \times M$, where N is the total number of transmitters (players) and M is the total number of receivers. We assume the Rayleigh fading channel gain $g_{n,m}$ are *i.i.d.* among players and for different sub-channels. The maximum power constraint for each player n is assumed to be identical and normalized as $P^{\max} = 1$.

In Figure 4.3, we show the convergence behaviors of potential function and the actual total network rate (we will use the short term “actual rate”) by using the proposed DPIWF algorithm for a random channel realization. We set the number of transmitters to $N = 10$ and the number of receivers to $M = 10$. As expected, in both Figure 4.3a and Figure 4.3b the potential function converges rapidly at the 4th iteration. In Figure 4.3a, the actual rate converges slightly slower at the 6th iteration and maintains the monotonically increasing slope. On the contrary, in Figure 4.3b, the actual rate finally converges, but unfortunately it neither monotonically increases nor rapidly converges (at the 34th iteration) comparing to the convergence speed of its potential function. Note that we use this example in order to show that a “defective” convergence of the actual rate may happen during the iteration steps of DPIWF algorithm.

In order to measure the performance efficiency of distributed networks operating at the unique NE, we provide here the optimal power allocation strategy in centralized approaches as a target upper-bound for the total network rate,



(a) An example of “ideal” convergence (b) An example of “defective” convergence

Figure 4.3 Convergence behavior of potential function for DPIWF algorithm. Total number of transmitters $N = 10$, total number of receivers $M = 10$, noise variance $\sigma^2 = 0.1$, power constraint $P_n^{\max} = 1, \forall n$.

i.e., the transmit sum-rate of all players in the network. We will ignore the performance loss due to the necessary uplink and downlink signaling transmission. The total network rate maximization problem can be formulated as

$$\begin{aligned}
 \max_{\mathbf{p}} \quad & \sum_{n=1}^N u_n(\mathbf{p}) \\
 \text{s.t.} \quad & \sum_m p_{n,m} \leq P_n^{\max}, \quad \forall n \\
 & p_{n,m} \geq 0, \quad \forall n \forall m
 \end{aligned} \tag{4.31}$$

which unfortunately is a difficult problem, since the objective function is *non-convex* in \mathbf{p} . However, a relaxation of this optimization problem (see in [65]) can be considered as a geometric programming problem [3], and thus can be transformed into a convex optimization problem. A low complexity algorithm was proposed in [65] to solve the dual problem by updating dual variables through a gradient descent. Note that the algorithm always converges, but may converge to a local maximum point in a few cases. We will use this algorithm in our simulations.

In the following part, we will address two main practical questions through numerical results:

1. How does the network performance behave at the unique NE (decentralized optimality) in comparison to the global optimal solution (centralized optimality)? More precisely, we are interested in comparing the *average*

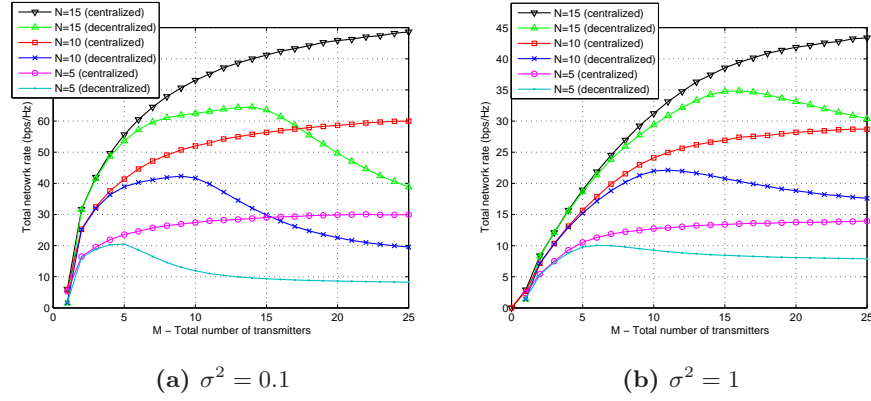


Figure 4.4 Average total network rate, decentralized vs. centralized optimality. Total number of transmitters $N = [1, 25]$, total number of receivers $M = 5, 10, 15$, power constraint $P_n^{\max} = 1, \forall n$.

total network rate instead of the *instantaneous* total network rate, i.e. $\bar{u}(N, M)$ is the average total network rate for a N transmitters and M receivers system,

$$\bar{u}(N, M) = \mathbb{E}_{\mathbf{G}} \left[\sum_{n=1}^N \sum_{m=1}^M \log \left(1 + \frac{p_{n,m} g_{n,m}}{\sigma^2 + \sum_{j \neq n} p_{j,m} g_{j,m}} \right) \right].$$

2. What about the convergence behavior for the actual total network rate when using DPIWF algorithm? Does it converge rapidly (as in Figure 4.3a) for most cases?

Let's consider the first question. In Figure 4.4, we compare the average total network rate of both *decentralized* and *centralized* networks for two different channel noise levels $\sigma^2 = 0.1$ and 1 , respectively. Network parameters are selected as follows: the number of transmitters $N \in [1, 25]$, the number of receivers M takes several representative values, such as $5, 10$ and 15 . The plots are obtained through Monte-Carlo simulations over 10^4 realizations of the random channel gain matrix \mathbf{G} . First, in both Figure 4.4a and Figure 4.4b, the centralized optimum approach always outperforms the decentralized optimum algorithm, as expected. Second, for a fixed number of transmitters M , when we increase the number of receivers N , the performance loss of decentralized systems (compare to centralized systems) becomes more and more apparent. In fact, this phenomenon can be intuitively understood as follows: *when there are*

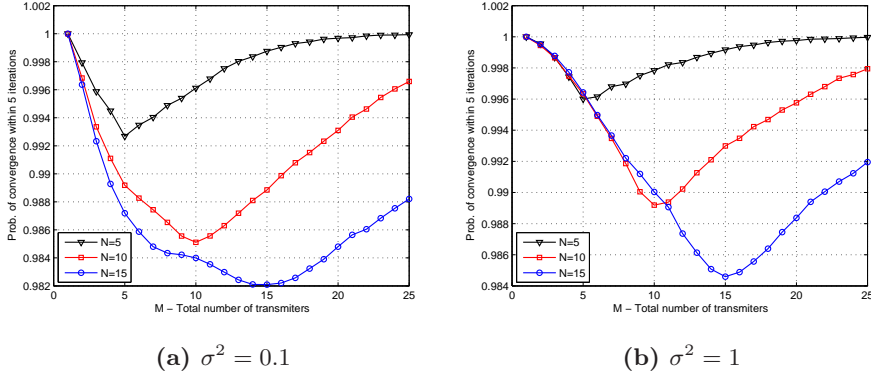


Figure 4.5 Probability of convergence within 5 iterations.

Total number of transmitters $N = [1, 25]$, total number of receivers $M = 5, 10, 15$, power constraint $P_n^{\max} = 1, \forall n$.

a great number of *selfish* players, the hostile competition turns the multi-user communication system into an interference-limited environment, where interference begins to dominate the performance efficiency.

Moreover, we note that in Figure 4.4 the average performance of centralized systems is an increasing function of N (for a fixed value of M), and the average performance of decentralized systems corresponding to NE show an increasing slope before diminishing and reaching convergence. For the typical values of M , i.e., $M = 5, 10$ and 15 , in Figure 4.4a, when $\sigma^2 = 0.1$ the average performance of decentralized systems are maximized approximately at $N = 4, 9, 14$, respectively. In Figure 4.4b, when $\sigma^2 = 1$ the average performance of decentralized systems are maximized approximately at $N = 6, 11, 16$, respectively. This comparison simply shows that different noise variances have a different impact on the decentralized system performance. This observation is fundamentally important for improving the spectral efficiency of a distributed multi-user OFDM small-cell network: *for a given area, a given number of receivers N and given current channel condition, there exists an optimal number of access points, denoted as M^* , to be put in the network. Roughly speaking: when $N > N^*$, the system is **overloaded** due to the increase of competition over limited resources; when $N < N^*$, the system is operated at the **unsaturated** state, since system resources are not fully exploited.*

Let's now consider the second question. In Figure 4.5, we show the probability of convergence to the decentralized optimality (Nash equilibrium) within 5 iterations for $\sigma^2 = 0.1$ and 1 , respectively. To be more precise, we say that the

algorithm converges within 5 iterations if the total network rate exceeds 99% performance of the final rate. We find that the probability of convergence is quite satisfactory (more than 98.2% in all cases), and this convergence probability tends to 1 when $N \gg M$ and $N \ll M$. An interesting observation is that the minimal convergence probability always occurs when $N = M$, regardless of the noise variance value σ^2 .

4.5 Conclusion

In the state-of-the-art of this chapter, two basic wireless channel models are discussed in detail, i.e., flat-fading multiple access channels and frequency-selective interference channels. Based on the discussion, we presented a non-cooperative game-theoretical approach to tackle the resource allocation problem in a multiuser OFDM small-cell wireless network. Each transmitter (AP) is modeled as a player who decides, in a distributed way, the strategy of how to spread the total power through several independent fading channels. We analyzed the existence and uniqueness of Nash equilibrium which represents a natural outcome of the game. For any realization of independent fading channels, we proved that Nash equilibrium is unique with unitary probability. Furthermore, we introduced a potential game-theoretical approach to study the convergence of equilibrium. In order to obtain the equilibrium point, we proposed a simple iterative algorithm based on the properties of potential games. We showed how does the average network performance behave at the unique Nash equilibrium, and we compared it with the optimal centralized approach. Numerical studies have shown that, with very high probability, the proposed iterative algorithm converges to 99% of the final rate under 5 iterations.

4.6 Appendix

4.6.1 Proof of Theorem 4.2

Proof. We prove the necessary and sufficient parts separately.

1. *Proof of necessary condition (the only if part):* From the definition of Nash equilibrium (Definition 3.10), if a power set $\{\mathbf{p}_n\}$ is a Nash equilibrium, it must satisfy all the best-response conditions in (3.11) simultaneously. Suppose that all the players' power except player n 's power reaches the Nash equilibrium point: $\{p_1^*, \dots, p_{n-1}^*, p_n, p_{n+1}^*, \dots, p_N^*\}$. In this case when all other players' powers are fixed, as shown in (4.15), the best-response of player n is to set her power according to (4.19), which is exactly given by the single-player waterfilling treating all other players' signals as noise.
2. *Proof of sufficient condition (the if part):* From convex optimization theory [2], we know that the KKT conditions of the convex optimization problem are necessary and sufficient conditions for optimality. Therefore, we can say that a power strategy \mathbf{p}_n satisfies the best response condition if and only if it satisfies the single-player KKT conditions (4.16)-(4.18). Then collectively, we say a set $\{\mathbf{p}_n\}$ satisfies all the best-response conditions simultaneously if and only if it satisfies (4.21)-(4.23). From Definition 3.10, if a set $\{\mathbf{p}_n\}$ satisfies all the best-response conditions, it must be a Nash equilibrium.

This completes the proof. \square

4.6.2 Proof of Lemma 4.3

Proof. Consider a Nash equilibrium $\mathbf{p} \in \mathbb{R}^{MN \times 1}$, from Theorem 4.2, the following equation is true

$$\phi(\mathbf{p}) + \nu - \lambda = \mathbf{0}$$

where

$$\phi(\mathbf{p}) = \begin{bmatrix} \frac{g_{1,1}}{\sigma^2 + \sum_j p_{j,1} g_{j,1}} \\ \frac{g_{1,2}}{\sigma^2 + \sum_j p_{j,1} g_{j,1}} \\ \vdots \\ \frac{g_{N,M}}{\sigma^2 + \sum_j p_{j,M} g_{j,M}} \end{bmatrix}_{MN \times 1} \quad \nu = \begin{bmatrix} \nu_{1,1} \\ \nu_{1,2} \\ \vdots \\ \nu_{N,M} \end{bmatrix}_{MN \times 1} \quad \lambda = \begin{bmatrix} (\lambda_1)_{M \times 1} \\ (\lambda_2)_{M \times 1} \\ \vdots \\ (\lambda_N)_{M \times 1} \end{bmatrix}_{MN \times 1}$$

Now, assume there exist two different Nash equilibria, e.g. $\mathbf{p}^0, \mathbf{p}^1$ ($\mathbf{p}^0 \neq \mathbf{p}^1$), the following equation must also hold

$$\underbrace{\left[(\mathbf{p}^1 - \mathbf{p}^0)^T (\mathbf{p}^0 - \mathbf{p}^1)^T \right]}_{\alpha^T} \left(\underbrace{\begin{bmatrix} \phi(\mathbf{p}^0) \\ \phi(\mathbf{p}^1) \end{bmatrix}}_{\beta} + \underbrace{\begin{bmatrix} \nu^0 - \lambda^0 \\ \nu^1 - \lambda^1 \end{bmatrix}}_{\gamma} \right) = 0 \quad (4.32)$$

from where we have

$$\begin{aligned} \alpha^T \beta &= (\mathbf{p}^1 - \mathbf{p}^0)^T \phi(\mathbf{p}^0) + (\mathbf{p}^0 - \mathbf{p}^1)^T \phi(\mathbf{p}^1) \\ &= \sum_{m=1}^M \sum_{n=1}^N \left[(p_{n,m}^1 - p_{n,m}^0) \frac{g_{n,m}}{\sigma^2 + \sum_{j=1}^N p_{j,m}^0 g_{j,m}} + (p_{n,m}^0 - p_{n,m}^1) \frac{g_{n,m}}{\sigma^2 + \sum_{j=1}^N p_{j,m}^1 g_{j,m}} \right] \\ &= \sum_{m=1}^M \sum_{n=1}^N \frac{g_{n,m} (p_{n,m}^0 - p_{n,m}^1) \sum_{j=1}^N [g_{j,m} (p_{j,m}^0 - p_{j,m}^1)]}{\left(\sigma^2 + \sum_{j=1}^N p_{j,m}^0 g_{j,m} \right) \left(\sigma^2 + \sum_{j=1}^N p_{j,m}^1 g_{j,m} \right)} \\ &= \sum_{m=1}^M \frac{\left[\sum_{j=1}^N g_{j,m} (p_{j,m}^0 - p_{j,m}^1) \right]^2}{\left(\sigma^2 + \sum_{j=1}^N p_{j,m}^0 g_{j,m} \right) \left(\sigma^2 + \sum_{j=1}^N p_{j,m}^1 g_{j,m} \right)} \geq 0 \end{aligned}$$

and

$$\begin{aligned} \alpha^T \gamma &= (\mathbf{p}^1 - \mathbf{p}^0)^T (\nu^0 - \lambda^0) + (\mathbf{p}^0 - \mathbf{p}^1)^T (\nu^1 - \lambda^1) \\ &= \sum_{m=1}^M \sum_{n=1}^N [(p_{n,m}^1 - p_{n,m}^0)(\nu_{n,m}^0 - \lambda_n^0) + (p_{n,m}^0 - p_{n,m}^1)(\nu_{n,m}^1 - \lambda_n^1)] \\ &= \sum_{n=1}^N \left[\underbrace{\left(\sum_{m=1}^M p_{n,m}^1 - \sum_{m=1}^M p_{n,m}^0 \right)}_{P_n^{\max} - P_n^{\max} = 0} (\lambda_n^1 - \lambda_n^0) \right] + \sum_{m=1}^M \sum_{n=1}^N (p_{n,m}^0 \nu_{n,m}^1 + p_{n,m}^1 \nu_{n,m}^0) \\ &= \sum_{m=1}^M \sum_{n=1}^N (p_{n,m}^0 \nu_{n,m}^1 + p_{n,m}^1 \nu_{n,m}^0) \geq 0 \end{aligned}$$

From above, it is easy to see that (4.32) holds if and only if we have $\alpha^T \beta = 0$ and $\alpha^T \gamma = 0$, which are equivalent to the following two equations, respectively,

$$\sum_{n=1}^N g_{n,m} p_{n,m}^0 - \sum_{n=1}^N g_{n,m} p_{n,m}^1 = 0, \quad \forall m \quad (4.33)$$

$$p_{n,m}^0 \nu_{n,m}^1 = p_{n,m}^1 \nu_{n,m}^0 = 0, \quad \forall m \quad \forall n \quad (4.34)$$

First, from (4.33), we observe that the value of $s_n (= \sum_n g_{n,m} p_{n,m})$ is fixed for any Nash equilibrium point. Second, for a specific positive power coefficient, e.g. $p_{n^*,m^*}^0 > 0$, we must have $\nu_{n^*,m^*}^0 = 0$ due to (4.18), therefore, from (4.34) we must also have $\nu_{n^*,m^*}^1 = 0$. This implies $\lambda_{n^*}^1 = \lambda_{n^*}^0$ because of (4.16). Finally, we obtain $\nu_{n^*,m}^0 = \nu_{n^*,m}^1 \forall m$, since we have shown that s_m is fixed for any Nash equilibrium point. The same proof holds for any other index n^* . \square

4.6.3 Proof of Lemma 4.4

Proof. When $\nu_{n,m} = 0$, from (4.16) we have

$$\lambda_n - g_{n,m} d_m = 0, \quad \forall (n, m) \in \mathcal{X} \quad (4.35)$$

where $d_m \triangleq \frac{1}{\sigma^2 + s_m}$. From Lemma 4.3, we know that all the Nash equilibria must satisfy (4.35), with the same λ_n and d_m . In (4.35), the number of independent linear equations is $|\mathcal{X}|$, while the number of unknown parameters is $\tilde{M} + N$ (since the rest of d_m , $m \notin \mathcal{M}$ is known to be $d_m = \frac{1}{\sigma^2}$). It is well known that the solution to the system of linear equations is the empty set, if the number of independent equations is larger than the number of variables [66]. Since each entry $g_{n,m}$ is i.i.d. random, the probability that equations (4.35) are independent from each other is equal to 1, therefore, we must have $|\mathcal{X}| \leq \tilde{M} + N$. \square

4.6.4 Proof of Lemma 4.5

Proof. We only give the proof for the case as $\tilde{M}N > \tilde{M} + N$. The case as $\tilde{M}N \leq \tilde{M} + N$ can be proved in a similar way. Matrix $\hat{\mathbf{Z}}$ can be transformed into a 2×2 block matrices, by applying some elementary column and row operations, as follows,

$$\hat{\mathbf{Z}} \xrightarrow{\text{column}} \begin{bmatrix} \mathbf{I}_\tau & \mathbf{A}_{\tau \times \xi_2} \\ \mathbf{B}_{\xi_1 \times \tau} & \mathbf{C}_{\xi_1 \times \xi_2} \end{bmatrix} \xrightarrow{\text{column}} \begin{bmatrix} \mathbf{I}_\tau & \mathbf{0}_{\tau \times \xi_2} \\ \mathbf{B}_{\xi_1 \times \tau} & \hat{\mathbf{C}}_{\xi_1 \times \xi_2} \end{bmatrix} \xrightarrow{\text{row}} \begin{bmatrix} \mathbf{I}_\tau & \mathbf{0}_{\tau \times \xi_2} \\ \mathbf{0}_{\xi_1 \times \tau} & \hat{\mathbf{C}}_{\xi_1 \times \xi_2} \end{bmatrix}$$

where $\tau = \min\{\tilde{M}, N\}$, $\xi_1 = \tilde{M} + N - \tau \geq \tau$, $\xi_2 = |\mathcal{X}| - \tau$. $\hat{\mathbf{C}}$ is a $\xi_1 \times \xi_2$ matrix, where each column contains one or two random variables, and each row contains at least one random variable. Again we can transform $\hat{\mathbf{C}}$ in row echelon form, denoted as $\hat{\mathbf{C}}_r$. Note that the probability of $\text{rank}(\hat{\mathbf{C}}_r) = \xi_2$ is equal to 1, since each leading coefficient of a row is a random variable or the linear combination of two i.i.d. random variables. So, the probability that any leading coefficient takes the value of 0 is equal to 0. Therefore, we have the probability that $\text{rank}(\hat{\mathbf{Z}}) = \tau + \xi_2 = |\mathcal{X}|$ is equal to 1. \square

4.6.5 Proof of Theorem 4.6

Proof. If $\tilde{M}N > \tilde{M} + N$, from Lemma 4.4, we have that the probability of $\text{rank}(\hat{\mathbf{Z}}) = |\mathcal{X}|$ is equal to 1. Any Nash equilibrium must satisfy (4.28); assume that two different power strategies $\hat{\mathbf{p}}$ and $\hat{\mathbf{p}}'$ are both solutions to (4.28). Then $\hat{\mathbf{Z}}(\hat{\mathbf{p}} - \hat{\mathbf{p}}') = 0$. By the rank-nullity theorem [66], since the rank of $\hat{\mathbf{Z}}$ is equal to the number of its columns, this implies $\hat{\mathbf{p}} - \hat{\mathbf{p}}' = 0$, which means there must be exactly one Nash equilibrium.

If $\tilde{M}N \leq \tilde{M} + N$, from Lemma 4.5, we have that the probability of at most one solution existing for (4.27) is equal to 1. Since any Nash equilibrium must satisfy (4.27) and we know that there is at least one Nash equilibrium solution, so the Nash equilibrium must be unique. \square

4.6.6 Proof of Theorem 4.7

Proof. We prove this theorem in two steps.

1. *Algorithm convergence:* It is easy to see that the potential function $v^*(\mathbf{p})$ is non-decreasing within each round of the single user waterfilling. Moreover, since each player's transmit power is limited by a maximum but finite power constraint, there must exist an upper-bound for the potential function $v^*(\mathbf{p})$. This confirms the convergence.
2. *Converge to Nash equilibrium:* From the discussions above, we directly have that the KKT condition of the potential game coincide with the KKT condition of the game \mathcal{G} , due to the property of potential function (3.12). By using the sufficient part of Theorem 4.2, we know that if every player's power allocation \mathbf{p}_1 is given by the single user waterfilling while treating other players' signals as noise, the set $\{\mathbf{p}_n\}$ must be a Nash equilibrium of the game \mathcal{G} . Therefore, we can conclude that if the algorithm DPIWF converges (through the process of iterating single user waterfilling), it must converge to a Nash equilibrium point.

This completes the proof. \square

Chapter 5

Resource Allocation Games with Incomplete Information

5.1 Introduction

Typically, the game-theoretic models used in many previous works [47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58] assume that the complete and perfect information is known to all wireless devices. However, this assumption is hardly met in practice. In many current wireless communication systems, wireless devices can have local information but can barely access to global information on the network status. The case of complete knowledge is a simplifying assumption that may be a good approximation in some cases.

Thus, in practical settings, these resource allocation games need to be reconsidered with some realistic assumptions made on the knowledge level of wireless devices. Under this consideration, it is of great interest to investigate scenarios in which devices have “incomplete information” about their components, e.g., a device is aware of her own channel gain, but unaware of the channel gains of other devices. In game theory, a strategic game with incomplete information is called a “Bayesian game”, and the notion of “incomplete information” means that some players do not completely know the characteristics of other players in the game, which include:

- Player set (how many devices involved? who are they?)
- Other player’s action set (what are their behaviors?)
- Other players’ payoff functions (what are their objects?)

Therefore, the motivation of this chapter is to study how Bayesian game theoretic tools can be implemented in practical wireless settings.

In particular, a static non-cooperative game with complete information has been introduced in the context of two-user fading MAC, known as “waterfilling game” [47]. By assuming that users compete with transmission rates as utility and transmit powers as moves, the authors show that there exists a unique Nash equilibrium which corresponds to the maximum sum-rate point of the capacity region. This claim is somewhat surprising, since in general Nash equilibrium is inefficient comparing to the Pareto optimality. However, their results rely on the fact that both transmitters have the complete knowledge of CSI, and in particular, perfect CSI of all transmitters in the network. As we previously pointed out, this assumption is rarely possible in practice.

In this chapter, we introduce a Bayesian game-theoretic model to design and analyze the resource allocation problem in a fading multiple access channels, where users are assumed to selfishly maximize their average achievable rates with incomplete information about the fading channel gains. In such a game-theoretic study, the central question is whether a Bayesian equilibrium exists, and if so, whether the network operates efficiently at the equilibrium point. We prove that there exists exactly one Bayesian equilibrium in our game. Furthermore, we study the network sum-rate maximization problem by assuming that users coordinate to the symmetric strategy profile. This result also serves as an upper bound for the unique Bayesian equilibrium. Finally, simulation results are provided to show the network efficiency at the equilibrium, and compare it with other strategies.

5.2 State-of-the-Art

Before discussing about games with incomplete information, we need to clarify two other similar notions: “imperfect information” and “imperfect/partial CSI”. The former “imperfect information” is a common notion used in the sequential games [6]. In contrast to games with perfect information, a game is said to have imperfect information if not all players know all moves that have taken place (or the history of the game is not available to all players). The latter “imperfect/partial CSI” is a specific term widely used in the wireless communication

society, it means that the channel state is not perfectly estimated/observed at the transmitter/receiver side. It is important to note that the assumption of incomplete information is closely related to the notions of “imperfect information” and “imperfect CSI” mentioned above. For example, we can consider the following communication scenario with imperfect CSI as a game of incomplete information: a device has perfect CSI about its own channel, but it has imperfect CSI about any other device’s channel. Note that this is a common situation that usually happens in a wireless network, since it may be too “expensive” for every device to keep track of the time-variant channels of all other devices. Especially when there is a large number of devices in the network. Although selective multi-user diversity algorithms (imperfect/partial feedback based on threshold) was introduced in [45] to reduce the feedback load, it is designed in the centralized framework where a central computing resource is needed, involving feedback and overhead communication whose load scales linearly with the total number of transmitters and receivers in the network.

The available literature is poor of results on using Bayesian games to model and solve the resource allocation problems in wireless communications. In [67], a static Bayesian game-theoretic model is applied to the problem of transmit power allocation in the uplink of a self-organizing CDMA wireless network. In another work [68], Bayesian game model is applied to study the two-user multi-carrier Gaussian interference channels, where each mobile device selects a power profile over the entire available bandwidth to maximize her data rate. The receiver is considered to use a simple single-user decoding strategy. It is shown that there exists a unique Bayesian equilibrium under the assumption of finite power actions.

5.2.1 A Bayesian Game Model for Interference Channels

For a two-user multi-channel interference channel, the input-output equations can be written as:

$$\begin{aligned} y_1^{(i)} &= \sqrt{g_{1,1}^{(i)}}x_1^{(i)} + \sqrt{g_{2,1}^{(i)}}x_2^{(i)} + z_1^{(i)} \\ y_2^{(i)} &= \sqrt{g_{2,2}^{(i)}}x_2^{(i)} + \sqrt{g_{1,2}^{(i)}}x_1^{(i)} + z_2^{(i)} \end{aligned}$$

where $x_n^{(i)}$ and $y_n^{(i)}$ represent user n ’s input and output signals on sub-channel i , respectively. $g_{n,m}^{(i)}$ represents the channel gain from transmitter Tx_n to receiver Rx_m . It is assumed that the channel is frequency flat, i.e., $g_{n,m}^{(k)} = g_{n,m}^{(i)}$ for all $i = 1, \dots, L$, where L is the total number of sub-channels. $z_n^{(i)} \sim \mathcal{N}(0, \sigma^2)$ is *i.i.d.* zero mean AWGN with variance σ^2 .

In this game, both users are considered to have “incomplete information” about the channel gain matrix \mathbf{G} . More precisely, each user n is only aware of her channel gains $\mathbf{g}_n = \{g_{n,n}, g_{-n,n}\}$, but unaware of the channel gains of the other user \mathbf{g}_{-n} . It is assumed that the objective of each user n is to maximize the following sum-capacity, i.e.,

$$r_n = \sum_{i=1}^L \log \left(1 + \frac{g_{n,n}^{(i)} p_n^{(i)}}{\sigma^2 + g_{-n,n}^{(i)} p_{-n}^{(i)}} \right), \quad n = 1, 2 \quad (5.1)$$

assuming that each user treats other users’ signals as noise and no interference cancellation techniques are applied. In this case, the strategy of player n is to choose vector $\mathbf{p}_n = \{p_n^{(1)}, \dots, p_n^{(L)}\} \in \mathcal{P}_n$, subject to the power constraint $\sum_{i=1}^L p_n^{(i)} \leq P_n^{\max}$. For simplicity, assume $P^{\max} = P_1^{\max} = \dots = P_N^{\max}$.

The symmetric Bayesian equilibrium was investigated in [68], i.e., both players use the same strategy in the equilibrium, restricting to the following finite action set:

$$\mathcal{P} = \mathcal{P}_1 = \dots = \mathcal{P}_N = \{\mathbf{e}_1 P^{\max}, \dots, \mathbf{e}_L P^{\max}, \mathbf{1}(P^{\max}/L)\} \quad (5.2)$$

where \mathbf{e}_i represents a unit vector with all zero entries except a “1” in the i^{th} position, $\mathbf{1}$ represents a vector with all entries being equal to 1, and P^{\max} represents the total power constraint. These specifically selected actions (5.2) have the following intuitive meanings:

- $\mathbf{e}_i P^{\max}$ - player concentrate the transmit power on the i^{th} sub-channel.
- $\mathbf{1}(P^{\max}/L)$ - player spreads the total available transmit power uniformly across sub-channels.

Thus, if each user selfishly maximizes her own rate r_n and if we consider the value of channel gain \mathbf{g}_n as user n ’s “private information” or “type”, this is a Bayesian game with finite actions. It is shown that there exists a unique pure strategy symmetric Bayesian equilibrium in this game, which is both users choose the uniform power allocation, i.e., $\mathbf{1}(P^{\max}/L)$.

It is known that, in the described model, when users have absolutely no knowledge about the channel gain matrix \mathbf{G} , the best strategy for each user is to choose the uniform power allocation $\mathbf{1}(P^{\max}/L)$ [13]. This result is surprising to some extent, because it shows that more information does not make any difference to players’ decision makings, nor help to improve the system performance.

5.3 Bayesian Games for Multiuser MAC

5.3.1 System Model

We consider the uplink of a single-cell network, in which N users are simultaneously sending information to one base station. This corresponds to a fading MAC, in which users are the transmitters and the base station is the receiver. The signal received by the base station can be mathematically expressed as

$$y(t) = \sum_{n=1}^N \sqrt{g_n(t)} x_n(t) + z(t)$$

where $x_n(t)$ and $g_n(t)$ are the input signal and fading channel gain of user n , $z(t)$ is a zero-mean AWGN with variance σ^2 . The input signal $x_n(t)$ can be further written as

$$x_n(t) = \sqrt{p_n(t)} s_n(t)$$

where $p_n(t)$ and $s_n(t)$ are the transmitted power and data of user n at time t .

In this chapter, we consider the wireless transmission in fast fading environments, i.e., the coherence time of the channel is small relative to the delay constraint of the application. When the receiver can perfectly track the channel but the transmitters have no such information, the codewords cannot be chosen as a function of the state of the channel but the decoding can make use of such information. We have previously shown that when the fading process is assumed to be stationary and ergodic, the channel capacity in fast fading channel corresponds to the notion of ergodic capacity, i.e.,

$$\mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{g_n p_n}{\sigma^2 + \sum_{j=1, j \neq n}^N g_j p_j} \right) \right] \quad (5.3)$$

where $\mathbf{g} = \{g_1, \dots, g_N\}$ is a vector of channel gain variables. Note that in (5.3) we assume the receiver applies a single user decoding and there is no sophisticated successive decoding to be used. An intuitive understanding of this result can be obtained by viewing capacities in terms of time averages of mutual information, see [18].

Before introducing the Bayesian game model, we clarify a prior assumption.

Assumption 5.1. *We assume that each user's channel gain g_n is i.i.d. from two discrete values: g_- and g_+ with probability ρ_- and ρ_+ , respectively. Wlog, we assume $g_- < g_+$.*

On the one hand, this assumption is closely related to the way how feedback information are signaled to the transmitters. In order to get the channel information $g_n(t)$ at the transmitter side, the base station needs to estimate it and then feedbacks to user n at a given precision. Since in digital wireless communications, any information is represented by a finite number of bits, channels gains are mapped into a set with a finite number of states, 2^x states, if the channel gains are quantified by x bits.

On the other hand, this is a necessary assumption for analytical tractability, since in principle the functional strategic form of a player can be quite complex with infinite actions and infinite states. In [68] the authors successfully modeled a multi-carrier Gaussian interference channel as a Bayesian game with finite actions and infinite states. Inspired from [68], in this chapter, we model the fading MAC as a Bayesian game under the assumption of infinite actions and finite states.

5.3.2 Game Formulation

Here, we model the N -user fading MAC as a Bayesian game, in which users do not have complete information. In a N -user MAC, to have “complete information” means that, at each time t , the channel gain realizations $g_1(t), \dots, g_N(t)$ are known at all the transmitters $\text{Tx}_1, \dots, \text{Tx}_N$. Any other condition corresponds to a situation of incomplete information. In this chapter, the “incomplete information” refers to a context where each transmitter Tx_n only knows her own channel gain realization $g_n(t)$, but does not know the channel gains of other transmitters $\mathbf{g}_{-n}(t) = \{g_1(t), \dots, g_{n-1}(t), g_{n+1}(t), \dots, g_N(t)\}$. In the following context, we will denote by g_n user n 's channel gain variable whose distribution is assumed to be stationary and ergodic.

In such a communication system, a natural object of each user is to maximize her ergodic capacity subject to an average power constraint, i.e.,

$$\begin{aligned} \max_{p_n} \quad & \mathbb{E}_{\mathbf{g}} \left[\log \left(1 + \frac{g_n p_n(g_n)}{\sigma^2 + \sum_{j \neq n} g_j p_j(g_j)} \right) \right] \\ \text{s.t.} \quad & \mathbb{E}_{g_n} [p_n(g_n)] \leq P_n^{\max} \\ & p_n(g_n) \geq 0 \end{aligned} \tag{5.4}$$

where $p_n(\cdot)$ is the transmit power strategy of user n , P_n^{\max} is the average power constraint for user n . Under the assumption that each user has incomplete information about the fading channel gains, user n 's strategy $p_n(\cdot)$ is defined as a function of her own channel gain g_n , i.e., $p_n(g_n)$. Note that (5.4) implies

the fact that user n should know at least the statistics of other users' channels.

For a given set of power strategies $\mathbf{p}_{-n} = \{p_1, \dots, p_{n-1}, p_{n+1}, \dots, p_N\}$, the single-user maximization problem (5.4) is a convex optimization problem. Via Lagrangian duality, the solution is given by the following equation

$$\mathbb{E}_{\mathbf{g}_{-n}} \left[\frac{g_n}{\sigma^2 + g_n p_n(g_n) + \sum_{j \neq n} g_j p_j(g_j)} \right] = \lambda_n \quad (5.5)$$

where the dual variable λ_n is chosen such that the power constraint in (5.4) is satisfied with equality. However, the solution of (5.5) depends on the strategies of other users $\mathbf{p}_{-n}(\cdot)$ which user n does not know, and similarly for other users. Thus, in order to obtain the optimal power allocation, each user must adjust her power level based on the guess of all other users' strategies. Now, given the following game model, each user is able to adjust her strategy according to the belief it has on the strategy of the other user.

The N -player MAC Bayesian game can be completely characterized as:

$$\mathcal{G}_{mac} \triangleq \langle \mathcal{N}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, \mathcal{U} \rangle$$

- Player set: $\mathcal{N} = \{1, \dots, N\}$.
- Type set: $\mathcal{T} = \mathcal{T}_1 \times \dots \times \mathcal{T}_N$
where $\mathcal{T}_n = \{g_-, g_+\}$. A player's type is defined as her channel gain, i.e., $g_n \in \mathcal{T}_n$.
- Action set: $\mathcal{P} = \mathcal{P}_1 \times \dots \times \mathcal{P}_N$
where $\mathcal{P}_n = [0, P_n^{\max}]$. A player's action is defined as her transmit power, i.e., $p_n \in \mathcal{P}_n$.
- Probability set: $\mathcal{Q} = \mathcal{Q}_1 \times \dots \times \mathcal{Q}_N$
where $\mathcal{Q}_n = \{\rho_-, \rho_+\}$. We have $\rho_+ = \Pr(g_n = g_+)$ and $\rho_- = \Pr(g_n = g_-)$.
- Payoff function set: $\mathcal{U} = \{u_1, \dots, u_N\}$
where u_n is chosen as player n 's achievable rate.

$$u_n(p_1, \dots, p_N) = \log \left(1 + \frac{g_n p_n(g_n)}{\sigma^2 + \sum_{j=1, j \neq n}^N g_j p_j(g_j)} \right) \quad (5.6)$$

In games of incomplete information, a player's type represents any kind of private information that is relevant to her decision making. In our context, the fading channel gain g_n is naturally considered as the type of user n , since her

decision (in terms of power) can only rely on g_n . Note that this is a *continuous game*¹ with discrete states, since each player's action p_n can take any value satisfying the power constraint $p_n \in [0, P_n^{\max}]$ and the channel state g_n is finite $g_n \in \{g_-, g_+\}$.

5.3.3 Characterization of Bayesian Equilibrium Set

From Definition 3.13, we know that \mathbf{p}^* is a pure strategy Bayesian equilibrium, if for all $n \in \mathcal{N}$, and for all $p_n \in \mathcal{P}_n$

$$\bar{u}_n(p_n^*, \mathbf{p}_{-n}^*) \geq \bar{u}_n(p_n, \mathbf{p}_{-n}^*)$$

where we defined $\bar{u}_n \triangleq \mathbb{E}_{\mathbf{g}}[u_n]$ as the ergodic capacity (5.3) of user n . It is clear that in a Bayesian equilibrium no player can benefit by changing her strategy while the other players keep theirs unchanged.

It is well known that, in general, an equilibrium point does not necessarily exist [6]. Therefore, our primary interest in this chapter is to investigate the *existence* and *uniqueness* of Bayesian equilibrium in \mathcal{G}_{mac} . To this aim, we introduce the following lemma,

Lemma 5.2. *The weighted nonnegative sum of the average payoffs \bar{u}_n in \mathcal{G}_{mac} is diagonally strictly concave.*

The proof of this lemma is given in Appendix 5.7.

Then, we can directly state our main result:

Theorem 5.3. *There exists a unique Bayesian equilibrium in the N -user MAC game \mathcal{G}_{mac} .*

Proof. It is easy to prove the existence of equilibrium from Debreu's sufficient condition [22], since the strategy space p_n is convex, compact and nonempty for each n ; the payoff function u_n is continuous in both p_n and \mathbf{p}_{-n} ; and u_n is concave in p_n for any \mathbf{p}_{-n} .

In order to prove the uniqueness of equilibrium, we should rely on a sufficient condition given in [23]: a non-cooperative game has a unique equilibrium, if the nonnegative weighted sum of the payoff functions is *diagonally strictly concave*, see its definition in Section 3.1.

By appealing to Lemma 5.2, the sum-payoff $f^u(\mathbf{p}, \gamma) = \sum_{n=1}^N \gamma_n \bar{u}_n(\mathbf{p})$ is diagonally strictly concave. From Theorem 2 in [23], the uniqueness of Nash equilibrium in our game \mathcal{G}_{mac} follows. \square

¹A continuous game extends the notion of a discrete game (where players choose from a finite set of pure strategies), it allow players to choose a strategy from a continuous pure strategy set

5.4 Optimal Symmetric Strategies

Bayesian game-theoretic approach provides us a better understanding of the wireless resource competition existing in the fading MAC when every wireless device acts as a selfish and rational decision maker. In such a non-cooperative game, there is no action restriction rule nor action priority policy to influence the decision making process of each player. In other words, wireless devices are entirely “free” to make their own choices.

However, from the global system performance perspective, it is usually inefficient to give complete “freedom” to wireless devices and let them take decisions without any policy control over the network. It is very interesting to note that a similar situation happens in the market economy, where consumers can be modeled as players to compete for the market resources. In the famous literature *The Wealth of Nations*, Adam Smith² expounded how rational self-interest and competition can lead to economic prosperity and well-being through macroeconomic adjustments. For example, all states today have some form of macroeconomic control over the market that removes the free and unrestricted direction of resources from consumers and prices such as tariffs and corporate subsidies.

Moreover, in the introduced Bayesian game model some basic assumptions may not be satisfied in the current wireless systems. One of the crucial questions is related to the acceptance level of the distributed computational complexity, i.e., a modern wireless device may not be “intelligent” and “powerful” enough to act like a rational player (to find the best response within limited time under complex situations). In this case, it may be better to assign them the pre-analyzed strategies, guiding them how to react under all kinds of different situations. Thus, there is no surprise why the wireless networks are often designed in a manner such that all identical wireless devices follow the same policy and strategy, that is they react in the same way when they face the same situation.

Typically, wireless service providers would like to design an appropriate policy to efficiently manage the wireless resource so that the global network performance can be optimized or enhanced to a certain theoretical limit. Based on the Bayesian game settings, we provide in this part a special discussion on the practical concerns of resource allocation design, i.e., the optimal symmetric resource allocation strategy. This result can be also treated as a theoretical upper-bound for the performance measurement of Bayesian equilibrium.

We now introduce an necessary assumption, as follows

²A Scottish moral philosopher, pioneer of political economy, father of modern economics.

Assumption 5.4. *Wireless devices are designed to use the same power transmission strategies, i.e., they apply the same strategy if their observations on the channel states are symmetric. And we assume the wireless devices have the same average power constraint, i.e., $P_1^{\max} = P_2^{\max} \triangleq P^{\max}$.*

5.4.1 Two Channel States

For the sake of simplicity, we first consider a two-user MAC with two channel states, i.e., g_n can be either g_- or g_+ with probability ρ_- and ρ_+ , respectively. In fact, the analysis of multi-user case can be extended in a similar way. According to Assumption 5.4, we define

$$\begin{aligned} p_- &\triangleq p_1(g_-) = p_2(g_-) \\ p_+ &\triangleq p_1(g_+) = p_2(g_+) \end{aligned}$$

and we have $\rho_- p_- + \rho_+ p_+ = P^{\max}$. In the following context, we consider user 1 as the reference user, since the problem is symmetric for user 2. Then, user 1's average payoff is

$$\begin{aligned} \bar{u}_1 &= \mathbb{E}_{g_1, g_2} \left[\log_2 \left(1 + \frac{g_1 p_1(g_1)}{\sigma^2 + g_2 p_2(g_2)} \right) \right] \\ &= \rho_-^2 \log_2 \left(1 + \frac{g_- p_-}{\sigma^2 + g_- p_-} \right) + \rho_- \rho_+ \log_2 \left(1 + \frac{g_- p_-}{\sigma^2 + g_+ (P^{\max} - \rho_- p_-)} \right) + \\ &\quad \rho_- \rho_+ \log_2 \left(1 + \frac{g_+ (P^{\max} - \rho_- p_-)}{\sigma^2 + g_- p_-} \right) + \rho_+^2 \log_2 \left(1 + \frac{g_+ (P^{\max} - \rho_- p_-)}{\sigma^2 + g_+ (P^{\max} - \rho_- p_-)} \right) \end{aligned}$$

Now, \bar{u}_1 is transformed into a function of p_- , i.e., $\bar{u}_1(p_-)$. To maximize the average achievable rate, as mentioned in (5.4), user 1 needs to solve the following optimization problem,

$$\begin{aligned} \max_{p_-} \quad & \bar{u}_1(p_-) \\ \text{s.t.} \quad & p_- \leq P^{\max} \\ & p_- \geq 0 \end{aligned}$$

Due to the symmetric property made in Assumption 5.4, it can be shown that the outcome of the above single-user maximization problems is equivalent to the outcome of global sum-rate maximization problem, i.e., $\max(\bar{u}_1 + \bar{u}_2)$, which is our object in this section.

However, function \bar{u}_1 is not in general convex, so the single-user problem may not be a convex optimization problem. It can be further verified that \bar{u}_1 is convex under some special conditions, depending on the following parameters

$g_-, g_+, \rho_-, \rho_+, P^{\max}$ and σ^2 . Here, we will not discuss all the convex cases, but only focus on a high SNR regime, i.e., when the noise is negligible compared to the signal strength. In this case, we have

$$\lim_{\sigma^2 \rightarrow 0} u_1 = \rho_- \rho_+ \left[\log_2 \left(1 + \frac{g_- \rho_-}{g_+ (P^{\max} - \rho_- \rho_-)} \right) + \log_2 \left(1 + \frac{g_+ (P^{\max} - \rho_- \rho_-)}{g_- \rho_-} \right) \right] + \rho_-^2 + \rho_+^2$$

which is strictly convex. More precisely, it is decreasing on $\left[0, \frac{g_+ P^{\max}}{g_- \rho_+ + g_+ \rho_-}\right)$ and increasing on $\left(\frac{g_+ P^{\max}}{g_- \rho_+ + g_+ \rho_-}, \frac{P^{\max}}{\rho_-}\right]$, and the solution is given by

$$\{p_-^*, p_+^*\} = \begin{cases} \left\{ 0, \frac{P^{\max}}{\rho_+} \right\}, & \frac{g_+}{\rho_+} \geq \frac{g_-}{\rho_-} \\ \left\{ \frac{P^{\max}}{\rho_-}, 0 \right\}, & \frac{g_+}{\rho_+} < \frac{g_-}{\rho_-} \end{cases}$$

Note that in this setting the choice of the optimal symmetric strategy is to concentrate the full available transmit power on a single channel state. The selection of the state on which to transmit depends not only on the channel conditions but also on the probability of the channel states. This result implies that, in the high SNR regime, the optimal symmetric power strategy is to transmit information in an ‘‘opportunistic’’ way, see some related references in [69].

5.4.2 Multiple Channel States

In this subsection, we discuss the extension to arbitrary L ($L > 2$) channel states.

Assumption 5.5. *Each user’s channel gain g_n has L positive states, which are a_1, \dots, a_L with probability ρ_1, \dots, ρ_L respectively. Without loss of generality $a_1 < \dots < a_L$ and $\sum_{\ell=1}^L \rho_\ell = 1$.*

Based on Assumption 5.4, we define $p_\ell \triangleq p_1(g_\ell) = p_2(g_\ell)$, $\ell = 1, \dots, L$, the transmit power when a user’s channel gain is g_ℓ . As previously mentioned, in this part, our object is to maximize the sum average rate of the system, i.e., $\max \sum_n \bar{u}_n$. Under the symmetric assumption, this is equivalent to the following single-user maximization problem

$$\begin{aligned} \max_{\mathbf{P}} \quad & \sum_i \sum_j \rho_i \rho_j \log_2 \left(1 + \frac{g_i p_i}{\sigma^2 + g_j p_j} \right) \\ \text{s.t.} \quad & \sum_i \rho_i p_i = P^{\max} \\ & p_i \geq 0, \quad i = 1, \dots, L \end{aligned} \tag{5.7}$$

where \mathbf{p} is now defined as $\mathbf{p} = \{p_1, \dots, p_L\}$. This optimization problem is difficult, since the objective function is again nonconvex in \mathbf{p} . However, we can consider a relaxation of the optimization by introducing a lower bound [65]

$$\alpha \log z + \beta \leq \log(1 + z) \quad (5.8)$$

where α and β are specified below

$$\begin{cases} \alpha = \frac{z_0}{1+z_0} \\ \beta = \log(1 + z_0) - \frac{z_0}{1+z_0} \log z_0 \end{cases} \quad (5.9)$$

It is worth noticing that the lower bound (5.8) is tight with equality at a chosen value z_0 .

Let us consider the lower bound, ξ , by using (5.8) to the objective function expressed in (5.7)

$$\xi(\mathbf{p}) \triangleq \sum_i \sum_j \rho_i \rho_j \left[\alpha_{i,j} \log_2 \left(\frac{g_i p_i}{\sigma^2 + g_j p_j} \right) + \beta_{i,j} \right] \quad (5.10)$$

which is still nonconvex, and so it is not concave in \mathbf{p} . However, by using Geometric Programming (see Section 3.2.3) we can transform (5.10) into a convex optimization problem. Define a mapping operation \tilde{x} as

$$\tilde{x} \triangleq \log_2 x$$

Then (5.10) can be rewritten as

$$\xi(\tilde{\mathbf{p}}) = \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} (\tilde{g}_i + \tilde{p}_i) - \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} \log_2 (\sigma^2 + 2^{(\tilde{g}_j + \tilde{p}_j)}) + \sum_i \sum_j \rho_i \rho_j \beta_{i,j} \quad (5.11)$$

Now, it is easy to verify that the lower bound ξ is concave in the transformed set $\tilde{\mathbf{p}}$, since the log-sum-exp function is convex. The corresponding constraint set is convex. So, the KKT condition of the optimization is sufficient and necessary for the optimality. Given the Lagrangian \mathcal{L} :

$$\begin{aligned} \mathcal{L}(\tilde{\mathbf{p}}, \nu, \lambda) &= \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} (\tilde{a}_i + \tilde{p}(a_i)) - \sum_i \sum_j \rho_i \rho_j \alpha_{i,j} \log_2 (\sigma^2 + 2^{(\tilde{a}_j + \tilde{p}(a_j))}) + \\ &\quad + \sum_i \sum_j \rho_i \rho_j \beta_{i,j} - \nu \left(\sum_i 2^{(\tilde{a}_i + \tilde{p}(a_i))} - P^{\max} \right) + \sum_i \lambda_i 2^{(\tilde{a}_i + \tilde{p}(a_i))} \end{aligned} \quad (5.12)$$

the KKT conditions are

$$\begin{aligned} \rho_\ell \sum_j \rho_j \alpha_{\ell,j} - \rho_\ell \left(\frac{2^{(\tilde{a}_\ell + \tilde{p}(a_\ell))}}{\sigma^2 + 2^{(\tilde{a}_\ell + \tilde{p}(a_\ell))}} \right) \sum_i \rho_i \alpha_{i,\ell} + (\ln 2)(\lambda_\ell - \nu) 2^{(\tilde{a}_\ell + \tilde{p}(a_\ell))} &= 0, \quad \forall \ell \\ \lambda_\ell 2^{(\tilde{a}_\ell + \tilde{p}(a_\ell))} &= 0, \quad \forall \ell \end{aligned}$$

and $\nu, \lambda_\ell \geq 0, \forall \ell$, where ν and λ_ℓ are dual variables associated with the power constraints and positive constraint, respectively. From the second equation above, one can find $\lambda_\ell = 0, \ell = 1, \dots, L$, since $2^{(\tilde{a}_\ell + \tilde{p}(a_\ell))}$ is a nonzero term. Define $x_\ell \triangleq 2^{(\tilde{a}_\ell + \tilde{p}(a_\ell))}, \ell = 1, \dots, L$, the equivalent KKT conditions can be simply written as a quadratic equation

$$A_\ell x_\ell^2 + B_\ell x_\ell + C_\ell = 0, \quad \forall \ell$$

where the parameters are

$$\begin{cases} A_\ell &= \nu \ln 2, \quad \forall \ell \\ B_\ell &= \rho_\ell \sum_i \rho_i (\alpha_{i,\ell} - \alpha_{\ell,i}) + \sigma^2 \nu \ln 2, \quad \forall \ell \\ C_\ell &= -\rho_\ell \sigma^2 \sum_i \rho_i \alpha_{\ell,i}, \quad \forall \ell \end{cases} \quad (5.13)$$

and $\nu \geq 0$. Note that A_ℓ and B_ℓ are functions of ν . We can write them as $A_\ell(\nu)$ and $B_\ell(\nu)$. Since $x_\ell \geq 0$, the solution to the KKT conditions can only be one of the roots to the quadratic equation, i.e.,

$$p_\ell^* = \frac{-B_\ell(\nu) + \sqrt{B_\ell^2(\nu) - 4A_\ell(\nu)C_\ell}}{2A_\ell(\nu)}, \quad \forall \ell \quad (5.14)$$

where ν is chosen such that $\sum_\ell \rho_\ell p_\ell^* = P^{\max}$. Thus, for some fixed value of α, β , we can maximize the lower-bound ξ (5.10) by directly applying (5.14). Then, it is natural to improve the bound periodically. Bases on the previous results, we propose a Lower Bound Tightening (LBT) algorithm.

The algorithm convergence can be easily proved, since the objective is monotonically increasing at each iteration. However, the global optimum is not always guaranteed, due to the nonconvex property.

5.5 Numerical Results

In this section, numerical results are presented to validate our theoretical claims. From Figure 5.1 to Figure 5.4, the network parameters are chosen as $\rho_- = \rho_+ = 0.5, P^{\max} = 1$ and $\sigma^2 = 0.1$.

First, we show the existence and uniqueness of Bayesian equilibrium in the scenario of two-user fading MAC. In Figure 5.3, we assume the channel gains are $g_- = 1, g_+ = 3$, while in Figure 5.4, we assume the channel gains are $g_- = 1, g_+ = 10$. In both X and Y axis, the p_1 and p_2 represent the power allocated by user 1 and user 2 when the channel gain is g_- . The curves $r_1(p_2)$ and $r_2(p_1)$ represent the best-response functions of user 1 and user 2, respectively. As

Algorithm 2 Lower Bound Tightening (LBT)

Initialize $t = 0$; $\nu = 0$; $\alpha_{i,j}^{(t)} = 1$, for $i = 1, \dots, L$, $j = 1, \dots, L$.
repeat
 repeat
 $\nu = \nu + \Delta\nu$
 for $i = 1$ to L **do**
 update A_i, B_i, C_i using (5.13)
 $p^*(a_i) = \frac{-B_i + \sqrt{B_i^2 - 4A_i C_i}}{2a_i A_i}$
 end for
 until $\sum_i \rho_i p^*(a_i) = P^{\max}$
 for $i = 1$ to L and $j = 1$ to L **do**
 $z_{i,j}^{(t)} = \frac{a_i p^*(a_i)}{\sigma^2 + a_j p^*(a_j)}$; $\alpha_{i,j}^{(t+1)} = \frac{z_{i,j}^{(t)}}{1 + z_{i,j}^{(t)}}$.
 end for
 $t = t + 1$
until converge

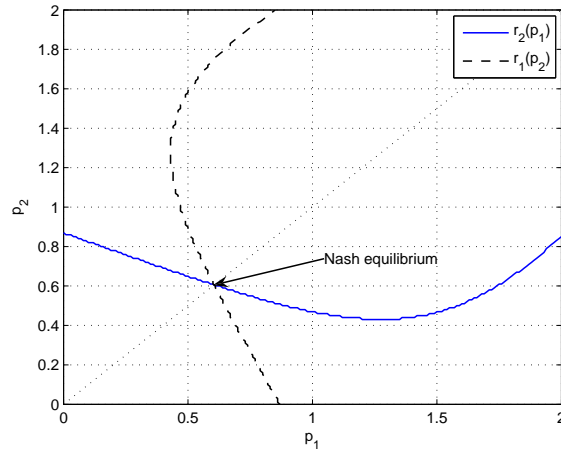


Figure 5.1 The uniqueness of Bayesian equilibrium ($g_- = 1, g_+ = 3$).

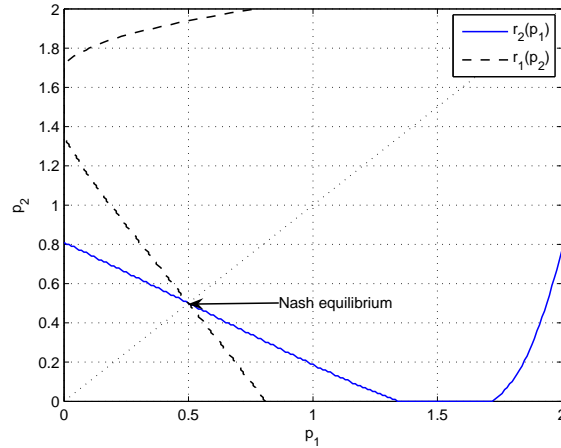


Figure 5.2 The uniqueness of Bayesian equilibrium ($g_- = 1, g_+ = 10$).

expected, the Bayesian equilibrium is unique in both cases, i.e., (0.6,0.6) and (0.5,0.5). Also note that the performance of the equilibrium point varies with different channel statistics g_-, g_+ . when the values of g_- and g_+ are relatively close/far, the Nash equilibrium is close/far to the time-domain uniform power allocation strategy, which is represented by the point (1,1) in this setting.

Second, we investigate the efficiency of Bayesian equilibrium from the viewpoint of global average network performance. On the X axis, SNR is defined as the ratio between the power constraint P^{\max} and the noise variance σ^2 . In Figure 5.3, we assume $g_- = 1, g_+ = 3$, while in Figure 5.4, we assume $g_- = 1, g_+ = 10$. The curve ‘‘Pareto’’ represents the Nash equilibrium in the waterfilling game, in which users have complete information. This gives the upper bound for the Bayesian equilibrium, since it is also the Pareto optimal solution [47]. The curve ‘‘Uniform’’ represents the time-domain uniform power allocation. This is the strategy when users have no information about the channel gains, obviously it corresponds to a lower bound. The curve ‘‘Symmetric’’ represents the optimal symmetric strategy we presented in section 5.4. This can be treated as a weaker upper bound (inferior to the Pareto optimality) for the Bayesian equilibrium. The Bayesian equilibrium is plotted using exhaustive search. From the slopes of these curves, we can clearly observe the inefficiency of Bayesian equilibrium, especially in high SNR regime. We can conclude that in our game \mathcal{G}_{mac} , users with ‘‘incomplete information’’ does improve the global network performance compared to the scenario in which users has no informa-

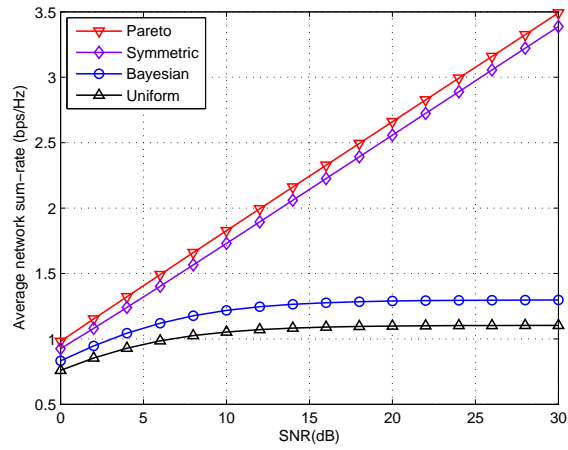


Figure 5.3 Average network sum-rate ($g_- = 1, g_+ = 3$).

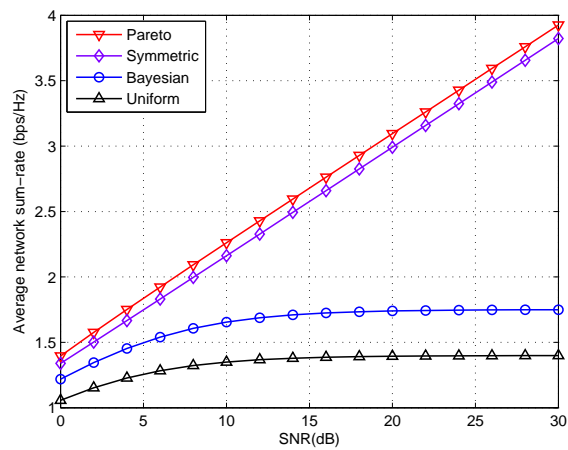


Figure 5.4 Average network sum-rate ($g_- = 1, g_+ = 10$).

tion. However, additional information does not improve the performance slope.

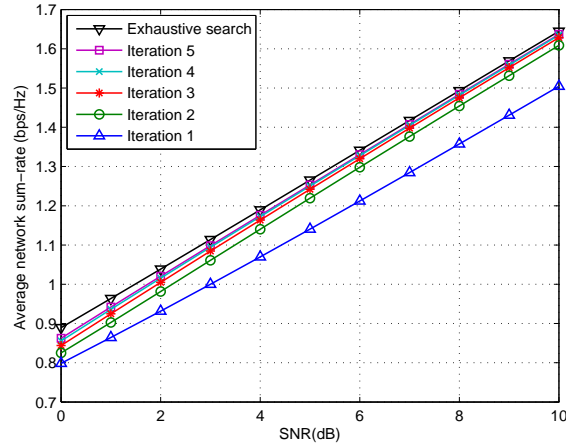


Figure 5.5 The convergence of the lower bound tightening (LBT) algorithm.

Finally, we show the convergence behavior of the lower bound tightening (LBT) algorithm. In Figure 5.5, we choose the parameters as $L = 3, g_1 = 1, g_2 = 2, g_3 = 3, \rho_1 = \rho_2 = \rho_3 = 1/3$. The sum capacity versus the SNR are plotted for five iterations, and the upper bound is achieved by exhaustive search. As expected, one can easily observe the convergence behavior. In the low SNR regime, we can find that the algorithm converges to a local maximum instead of the global maximum. However, we also find that the performance of the local optimum is improved while the SNR increases.

5.6 Conclusion

We presented a Bayesian game-theoretic framework for distributed resource allocation in fading MAC, where users are assumed to have only local information about the network channel gain states. By introducing the assumption of finite channel states, we successfully found an analytical way to characterize the Bayesian equilibrium set. First, we proved existence and uniqueness of the Bayesian equilibrium in our game. Second, we analyzed the optimal symmetric power strategy based on a practical concern of the resource allocation design. This method can be also viewed as an upper-bound measurement for the unique Bayesian equilibrium. Finally, the upper-bound, lower bound, and the inefficiency of Bayesian equilibrium were shown from numerical results.

APPENDIX

5.7 Proof of Lemma 5.4

Proof. Write the weighted nonnegative sum of the average payoffs as:

$$f^u(\mathbf{p}, \gamma) \triangleq \sum_{n=1}^N \gamma_n \bar{u}_n(\mathbf{p}), \quad (5.15)$$

where $\mathbf{p} = [p_1 \dots p_N]^T$ is the transmit power vector, $\gamma = [\gamma_1 \dots \gamma_N]^T$ is a non-negative vector assigning weights $\gamma_1, \dots, \gamma_N$ to the average payoffs $\bar{u}_1, \dots, \bar{u}_N$, respectively. Similar to (3.2), we let $\delta^u(\mathbf{p}, \gamma) \triangleq [\gamma_1 \frac{\partial \bar{u}_1}{\partial p_1} \dots \gamma_N \frac{\partial \bar{u}_N}{\partial p_N}]^T$ be the pseudo-gradient of $f^u(\mathbf{p}, \gamma)$. Now, we define

$$p_n \triangleq p_n(g_-) \quad \forall n,$$

the transmit power of player n when her channel gain is g_- . Since we have shown from Lagrangian (5.5) that, at the equilibrium, the power constraint is satisfied with equality, i.e., $\sum_{g_n} p_n(g_n) = P_n^{\max}$, we can write $P_n^{\max} - p_n = p_n(g_+) \quad \forall n$, as the transmit power when her channel gain is g_+ . Therefore, it is easy to find that the average payoff \bar{u}_n can be actually transformed into a weighted sum-log function, as follows

$$\bar{u}_n(p_n) = \sum_i \omega_i \log \left[1 + \frac{\alpha_n^i + \beta_n^i p_n}{\sigma^2 + \sum_{j \neq n} (\alpha_j^i + \beta_j^i p_j)} \right]$$

where i represents the index for different jointly probability events, ω_i represents the corresponding probability for event i that are related to the probabilities $\{\rho_-, \rho_+\}$, α_n^i and β_n^i represent some positive and non-zero real numbers that are related to the channel gains $\{g_-, g_+\}$. Note that the following conditions hold for all i, n

$$\alpha_n^i + \beta_n^i p_n \geq 0, \quad \alpha_n^i > 0, \quad \beta_n^i \neq 0, \quad \sigma^2 > 0$$

Now, we can write the pseudo-gradient δ^u as

$$\delta^u(\mathbf{p}, \gamma) = \begin{bmatrix} \gamma_1 \frac{\partial \bar{u}_1}{\partial p_1} \\ \vdots \\ \gamma_N \frac{\partial \bar{u}_N}{\partial p_N} \end{bmatrix} = \begin{bmatrix} \gamma_1 \sum_i \beta_1^i \phi_i^{-1}(\mathbf{p}) \\ \vdots \\ \gamma_N \sum_i \beta_N^i \phi_i^{-1}(\mathbf{p}) \end{bmatrix} = \sum_i \begin{bmatrix} \gamma_1 \beta_1^i \phi_i^{-1}(\mathbf{p}) \\ \vdots \\ \gamma_N \beta_N^i \phi_i^{-1}(\mathbf{p}) \end{bmatrix}$$

where function $\phi_i(\mathbf{x})$ is defined as

$$\phi_i(\mathbf{x}) \triangleq \sigma^2 + \sum_{n=1}^N (\alpha_n^i + \beta_n^i x_n)$$

To check the diagonally strictly concave condition (3.1), we let $\mathbf{p}^0, \mathbf{p}^1$ be two different vectors satisfying the power constraint, and define

$$\begin{aligned}
 \Omega^u(\mathbf{p}^0, \mathbf{p}^1, \gamma) &\triangleq (\mathbf{p}^1 - \mathbf{p}^0)^T \delta^u(\mathbf{p}^0, \gamma) + (\mathbf{p}^0 - \mathbf{p}^1)^T \delta^u(\mathbf{p}^1, \gamma) \\
 &= (\mathbf{p}^1 - \mathbf{p}^0)^T [\delta^u(\mathbf{p}^0, \gamma) - \delta^u(\mathbf{p}^1, \gamma)] \\
 &= [\Delta p_1 \ \cdots \ \Delta p_N] \begin{bmatrix} \gamma_1 \sum_i \beta_1^i (\phi_i^{-1}(\mathbf{p}^0) - \phi_i^{-1}(\mathbf{p}^1)) \\ \vdots \\ \gamma_N \sum_i \beta_N^i (\phi_i^{-1}(\mathbf{p}^0) - \phi_i^{-1}(\mathbf{p}^1)) \end{bmatrix} \\
 &= \sum_i [\phi_i^{-1}(\mathbf{p}^0) - \phi_i^{-1}(\mathbf{p}^1)] \zeta_i \\
 &= \sum_i \phi_i^{-1}(\mathbf{p}^0) \phi_i^{-1}(\mathbf{p}^1) \zeta_i^2
 \end{aligned}$$

where Δp_n and ζ_i are defined as

$$\begin{aligned}
 \Delta p_n &\triangleq p_n^1 - p_n^0 \\
 \zeta_i &\triangleq \sum_{n=1}^N \gamma_n \beta_n^i \Delta p_n.
 \end{aligned}$$

Since $\mathbf{p}^0, \mathbf{p}^1$ are assumed to be two different vectors, we must have $\Delta \mathbf{p} = [\Delta p_1 \ \cdots \ \Delta p_N]^T \neq \mathbf{0}$. Now, we can draw a conclusion from the equation above: $\Omega^u(\mathbf{p}^0, \mathbf{p}^1, \gamma) > 0$. This is because: (1) the first part $\phi_i^{-1}(\mathbf{p}^0) \phi_i^{-1}(\mathbf{p}^1) > 0$ for all i , since $\sigma^2 > 0$ and $\alpha_n^i + \beta_n^i p_n \geq 0$ for all i, n ; (2) the second part $\zeta_i^2 \geq 0$ for all i , and there exists at least one nonzero term ζ_i^2 , due to $\Delta \mathbf{p} \neq 0$ and $\gamma_n \neq 0, \beta_n^i \neq 0$ for all i, n . Therefore, the summation of all the products of the first and the second terms must be positive. From Definition 3.7, the sum-payoff function $f^u(\mathbf{p}, \gamma)$ satisfies the condition of diagonally strictly concave. This completes the proof of this lemma. \square

Chapter 6

Equilibrium Selection in Multiple Access Rate Regions

6.1 Introduction

In Chapter 4 and Chapter 5, we introduce non-cooperative game-theoretical approaches to study the resource allocation problem with different assumptions made on the knowledge levels of wireless devices. More precisely, in Chapter 4 we assume that all devices have complete information about the channel gain of everyone, in Chapter 5 we assume that devices have incomplete information about the channel gains of other devices. However, in both chapters we only assume that the receiver uses a single user decoding, therefore, what we have considered so far is just the achievable rate region. One could readily improve users' rates by applying multiuser decoding¹ at the receiver side. In this chapter, we provide a complete game-theoretic study of the rate allocation for wireless multiple access systems adopting multiuser decoding at the receiver.

¹Certainly, multiuser decoders have higher signal processing complexity compared to single-user decoders. However, in this chapter, we will ignore this complexity problem.

The capacity region of a N -user MAC is introduced in detail in Chapter 2. Typically, for such a N -dimension region, one has a broad choice of operating points (rate allocations). However, different operating points possess different properties in terms of efficiency, fairness, stability, etc. We classify MAC by geometrical characteristics of their capacity regions. Namely, we investigate two kinds of capacity regions, i.e., polytope and convex nonpolytope regions:

- An example of MAC with a polytope capacity region is the general time-invariant Gaussian MAC [1].
- Two examples of MAC with a convex non-polytope capacity region are TDMA and FDMA MAC [1].

In general, one can consider two methodological approaches to select operating points in the capacity region: (1) game-theoretical approach which leads to an equilibrium point, and (2) fairness function approach which leads to an optimal system operating point. For the game-theoretical approach, we consider games with correlated constraints arising in MAC with polytope capacity regions. In this chapter, we introduce the concept of normalized equilibrium to address the equilibrium selection problem. For the fairness function approach, we consider various fairness criteria such as max-min fairness, proportional fairness, α fairness, etc. We show that in the case of the polytope capacity region some specific optimal fair points coincide with the equilibrium points attained in some games. The latter allows one to use decentralized algorithms to achieve those operating points.

6.2 State-of-the-Art

First, in constrained games, each player is faced with a constrained optimization problem rather than a simple non-constrained optimization. The constraints may be independent of actions of other players. Then, they are called “orthogonal constrained” [23]. A more complex situation arises when the actions available to one player depend on those used by the others. Such games are called “games with correlated constraints”. Games with correlated constraints exhibit various characteristics that are very different than those without constraints or with orthogonal constraints. A central feature in these games is that they often possess a large number of equilibria. Natural questions that arise concern selection of an equilibrium, e.g., can we identify ones that are more fair or more stable than others?

Second, we review some relevant fairness criteria that are often used in designing a fair communication system. In this respect, max-min fairness is probably the most well-known fairness criterion. It was introduced as a design objective for communication networks by Bertsekas and Gallager [28]. The max-min fairness criterion relaxes equity in symmetric fairness (every user obtains the same rate) and allows increasing the rates of some users without lowering the minimum rate in the system. In many cases, the max-min fairness is considered quite stringent: it gives an absolute priority to the users, in the sense that if there is a user x , which has less payoff than the other user y , then no increase in y can compensate for any decrease in x , regardless of how large the increase in y is and how small the decrease in x is. To overcome this drawback, Kelly [29] introduced the concept of proportional fairness. Proportional fairness can be viewed as a compromise between max-min fairness and global optimization. Roughly speaking, a rate allocation is proportionally fair if any adjustment will decrease the sum of percentage change over all users. The concept of α -fairness was introduced by Mo and Walrand [70]. Interestingly, it is shown that the α -fairness allocation converges to: (1) max-min fairness allocation, when $\alpha \rightarrow \infty$, (2) proportional fairness allocation, when $\alpha \rightarrow 1$, and (3) globally optimal allocation, when $\alpha \rightarrow 0$. In this sense, α -fairness generalizes the concept of proportional fairness and includes arbitrarily close approximations of max-min fairness.

Third, regarding resource allocation games considering capacity region, we show some existing works using these fairness concepts to address the problem of fair rate allocation. In fact, picking a particular point in the rate region means assigning data rates to users according to some fairness criteria. For many kinds of MAC, the capacity region is a polymatroid. It is well-known that the sum capacity at the corner points can be achieved by successive decoding methods. In this respect, Stackelberg games can be used as a game theory tool to achieve any desired corner point on the dominant face of the capacity region. In such games, the base station is introduced as a game leader who announces its strategy in the first stage and then the remaining players react accordingly, see an example in [47]. In a more general framework, the polymatroid structure [20] of the capacity region of Gaussian MAC and the problem of fair rate assignment is investigated in [71]. For such kind of regions, there is a strong notion of fairness, namely there is a point on the dominant face that is majorized by all other points on the dominant face, and is both max-min and proportional fair [72]. In the next section, we will give a review of the definitions for some fairness criteria, majorization and Schur concavity, which will be useful for the following discussion in this chapter.

6.3 Fairness, Majorization and Schur Concavity

6.3.1 Various Fairness Criteria

Here, we give the formal definition for some fairness criteria that will be used in this chapter. Before introducing various fairness concepts, we would like to show an “unfair” criterion of maximizing the global sum-rate, which can be treated as a basis for comparison.

Definition 6.1. (*Globally optimal*)

A rate allocation $\mathbf{r} = \{r_n\}_{n \in \mathcal{N}}$ is globally optimal if it maximizes sum-rate $\sum_{n \in \mathcal{N}} r_n$ or, in general, the sum of some utility functions of r_n .

A major drawback of this criterion is that it can lead to situations in which the rate is zero for one or more users. An “opposite” approach is the max-min criterion that seeks to allocate the resources in the most homogeneous way.

Definition 6.2. (*Max-min fair*)

A rate allocation $\mathbf{r} = \{r_n\}_{n \in \mathcal{N}}$ is said to be max-min fair if each r_n cannot be increased, while maintaining feasibility, without decreasing r_j ($j \neq n, j \in \mathcal{N}$) which is already smaller than or equal to r_n .

Definition 6.3. (*Proportional fair*)

A rate allocation $\mathbf{r} = \{r_n\}_{n \in \mathcal{N}}$ is said to be proportionally fair if for any other feasible vector \mathbf{x}' , the aggregate of proportional changes is zero or negative (while maintaining feasibility)

$$\sum_{n \in \mathcal{N}} \frac{x'_n - x_n}{x_n} \leq 0$$

Another characterization of the proportional fair allocation is as follows. A rate allocation $\mathbf{r} = \{r_n\}_{n \in \mathcal{N}}$ is said to be proportionally fair if it maximizes $\sum_{n \in \mathcal{N}} \log r_n$.

Definition 6.4. (*α -fair*)

A rate allocation \mathbf{r} is α -fair if it maximizes $\sum_{n \in \mathcal{N}} V_\alpha(r_n)$, where $V_\alpha(x)$ represents a family of payoff function indexed by a parameter $\alpha \in \mathbb{R}_+$ ($\alpha \neq 1$)

$$V_\alpha(x) = \frac{x^{1-\alpha}}{1-\alpha}$$

The idea of generalized α -fairness is to assign fairly some utility function of the rate vector \mathbf{r} , instead of \mathbf{r} itself. The utility function can therefore generally represent the satisfaction of a user from its allocation.

Definition 6.5. (*Generalized α -fair*)

A rate allocation \mathbf{r} is generalized α -fair if it maximizes $\sum_{n \in \mathcal{N}} W_\alpha(r_n)$, where $W_\alpha(x)$ is defined as a function of utility function $U(x)$

$$W_\alpha(x) = \frac{U(x)^{1-\alpha}}{1-\alpha}$$

6.3.2 Majorization and Schur Concavity

In order to mathematically capture the characteristics of any fair rate allocation and establish the relation between different fairness criteria, we shall use the properties of Schur concave. We first define the majorization order, as follows,

Definition 6.6. (*Majorization*)

For two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}_+^N$ with decreasing ordered components $x_1 \geq \dots \geq x_N$ and $y_1 \geq \dots \geq y_N$, we say \mathbf{x} is majorized by \mathbf{y} (written as $\mathbf{x} \preceq \mathbf{y}$) if

$$\sum_{n=1}^k x_n \leq \sum_{n=1}^k y_n$$

for $k = 1, \dots, N-1$, and

$$\sum_{n=1}^N x_n = \sum_{n=1}^N y_n$$

Roughly speaking, $\mathbf{x} \preceq \mathbf{y}$ means that the components of \mathbf{x} are less spread out than those of \mathbf{y} . A simple example is that the uniform vector $\mathbf{x} = \mathbf{1} \in \mathbb{R}^N$ is majorized by any vector $\mathbf{y} \in \mathbb{R}_+^N$ whose components sum to N . In some sense, one could consider that the allocation corresponding to \mathbf{x} is fairer than \mathbf{y} . There are other interpretations and characterizations of majorization, see [73] for details.

Second, we introduce the definition of Schur concavity. The notion of Schur concavity was first introduced by Schur in 1923 [74]. It has many important applications in many fields such as analytic inequalities, combinatorial optimization, isoperimetric problem for polytopes, linear regression, graphs and matrices, gamma and digamma functions, reliability and availability, etc. The following definition for Schur concave or convex can be found in [74, 75] and references therein.

Definition 6.7. (*Schur concave function*)

A function $f : \mathbb{R}_+^N \rightarrow \mathbb{R}$ is Schur concave if $\mathbf{x} \preceq \mathbf{y}$ implies $f(\mathbf{x}) \geq f(\mathbf{y})$.

Similarly, f is said to be Schur convex if $\mathbf{x} \preceq \mathbf{y}$ implies $f(\mathbf{x}) \leq f(\mathbf{y})$.

Lemma 6.8. *Function $f : \mathbb{R}^N \rightarrow \mathbb{R}$ is Schur concave, if f can be written as*

$$f(\mathbf{x}) = \sum_{n=1}^N g(x_n)$$

where $g : \mathbb{R} \rightarrow \mathbb{R}$ is a concave function.

6.4 Polymatroid Achievable Rate Region

In this section, we consider equilibrium selection problem as well as fair rate allocation problem in the multiple access systems whose capacity region can be characterized by a convex polytope. Some examples of MAC with polymatroid capacity region are listed below:

- MAC with time-invariant channels [1].
- Flat-fading MAC, when the CSI is known at the receiver but not known at the transmitters [19].
- Unbiased flat-fading MIMO MAC, when the CSI is known at the receiver but not known at the transmitters [76].

We consider the uplink communication scenario within a single-cell, where N mobile devices simultaneously send information towards a single base station through flat-fading channels. We assume the base station can perfectly track the channels, but the mobile devices have no such information. The capacity region of this system is a convex polytope \mathcal{C} given by (see Section 2.2)

$$\mathcal{C} = \left\{ (r_1, \dots, r_N) : 0 \leq \sum_{n \in \mathcal{X}} r_n \leq C(\mathcal{X}), \forall \mathcal{X} \subseteq \{1, \dots, N\} \right\} \quad (6.1)$$

where

$$C(\mathcal{X}) = \log \left(1 + \frac{\sum_{n \in \mathcal{X}} g_n p_n}{\sigma^2} \right).$$

where g_n is the uplink channel gain from mobile n to base station, σ^2 is the variance of zero-mean white Gaussian noise, p_n is mobile n 's transmit power. The transmitters send at constant power since they do not know the current state of the channel. Without loss of generality, we assume that the channel gain g_n and the transmit power p_n of mobile n are always positive. For a mobile with zero channel gain or zero transmit power, we could simply remove it from the global mobile set.

6.4.1 Coupled Constraints Game Formulation

We now model the rate allocation problem of N -user MAC as a non-cooperative game, with respect to the capacity region \mathcal{C} defined in (6.1). We assume that each mobile n wishes to maximize her own transmission rate r_n , or in a more general sense, some strictly concave increasing function $u_n(r_n)$. We also assume that no user can tolerate losses that would occur if the transmission rates, denoted by $\mathbf{r} = \{r_1, \dots, r_N\}$, were chosen outside the capacity region \mathcal{C} . This strategic-form game can be formulated as

$$\mathcal{G}_{mac}^{[\mathcal{C}]} = \langle \mathcal{N}, \mathcal{C}, \mathcal{U} \rangle$$

which consists of

- A *player set*: $\mathcal{N} = \{1, \dots, N\}$.

- A *strategy set*: $\mathcal{C} = \mathcal{C}_1 \times \mathcal{C}_2 \times \dots \times \mathcal{C}_N$,

which is characterized by (6.1), with each $r_n \in \mathcal{C}_n, n = 1, \dots, N$.

- A *payoff set*: $\mathcal{U} = \{u_1, \dots, u_N\}$,

where u_n , the payoff of player n , is strictly concave increasing in r_n .

Note that this game has coupled constraints, i.e., the choice of strategies of a player depends on the strategies chosen by other players. This type of games, namely *games with coupled constraints* or *generalized Nash game*, have a long history originating from Arrow and Debreu [77] and Mackenzie [78], which appeared slightly later than the famous work by Nash [79]. Such games have particularly attracted increasing attention of researchers from different fields, due to the fact that it has a number of important applications in, for example, liberalized energy markets, global and regional environments, and various engineering design problems.

The equilibrium in such games is called *generalized Nash equilibrium*, which can be seen as a concept extension to the famous Nash equilibrium, since in Nash's game model [21, 79] each player is restricted to a simplex in her own strategy space, therefore, players' constraint sets are actually considered to be orthogonal. Recently, it was shown in [80] that to define games with constraints it is not sufficient to know the payoff and constraints of each given player. One should also specify how does a player value the fact that constraints of another player are satisfied or violated. Two extreme cases are: (1) orthogonal constraints: a player is indifferent to satisfaction of constraints of other players, and (2) common constraints: if a constraint is violated for one player then it

is violated for all players. In this sense, the game $\mathcal{G}_{mac}^{[C]}$ we are considering is modeled as games with common constraints. See [80] for a detailed discussion on the properties of constrained games.

6.4.2 Equilibrium Characterization and Normalized Equilibrium

In game $\mathcal{G}_{mac}^{[C]}$, we consider a situation in which players aim to maximize their concave increasing payoffs, subject to a common constraint $\mathbf{r} \in \mathcal{C}$. With this formulation a generalized NE point of $\mathcal{G}_{mac}^{[C]}$ is given by a point \mathbf{r}^* such that

$$\begin{aligned} r_n^* &= \arg \max_{r_n} u_n(r_n), \quad \forall n \in \{1, \dots, N\} \\ \text{s.t. } \sum_{n \in \mathcal{X}} r_n^* &\leq C(\mathcal{X}), \quad \forall \mathcal{X} \subseteq \mathcal{N} \end{aligned} \quad (6.2)$$

At such a point no player can increase her payoff by a unilateral change in her strategy.

Lemma 6.9. *A rate allocation $\mathbf{r} \in \mathcal{C}$ is equilibrium of $\mathcal{G}_{mac}^{[C]}$ if and only if*

$$\sum_{n=1}^N r_n - C(\mathcal{N}) = 0 \quad (6.3)$$

Proof. The *if* part is quite straightforward, since if a rate allocation \mathbf{r} satisfies (6.3), it must also satisfy the generalized NE conditions (6.2). The *only if* part can be proved as follows: suppose that there exists an equilibrium, e.g., $\mathbf{r}' = \{r'_1, \dots, r'_N\}$, which does not satisfy the condition of (6.3). Then, there is at least one player n could improve her payoff from $u_n(r'_n)$ to $u_n\left(C(\mathcal{N}) - \sum_{j \neq n} r'_j\right)$ without decreasing any other player's payoff. This implies a contradiction to (6.2). This completes the proof. \square

Lemma 6.10. *All the equilibrium points \mathbf{r}^* are Pareto-efficient. In general, the game $\mathcal{G}_{mac}^{[C]}$ possesses infinite number of equilibria on the dominant face (6.3).*

Proof. To prove the Pareto-efficient, we assume that in the rate region (6.1) there exists a rate allocation $\hat{\mathbf{r}}$ that Pareto dominates the NE \mathbf{r}^* , i.e., $\hat{r}_n \geq r_n^*$, $\forall n$ and at least one $\hat{r}_j > r_j^*$. This leads to $\sum_n \hat{r}_n > \sum_n r_n^* = C(\mathcal{N})$, whereas the sum-rate constraint in (6.1) is not satisfied. This implies a contradiction to the existence of $\hat{\mathbf{r}}$, i.e., no rate allocation could Pareto dominate \mathbf{r}^* . Thus, \mathbf{r}^* is Pareto-efficient. This completes the proof. \square

From Lemma 6.9, one can observe that there exist an infinite number of Nash equilibria in game $\mathcal{G}_{mac}^{[C]}$. In view of the large number of equilibria, we address next the problem of selecting one special equilibrium that has a certain additional property: *normalized equilibrium*. Normalized equilibrium, first introduced by Rosen [23], is a special generalized NE characterized by the Lagrange multipliers. To introduce the concept of normalized equilibrium, we first write the KKT conditions equivalent to (6.2) as follows:

$$\begin{aligned} \sum_{n \in \mathcal{X}} r_n - C(\mathcal{X}) &\leq 0, \quad \forall \mathcal{X} \\ \lambda_n^{\mathcal{X}} \left(\sum_{n \in \mathcal{X}} r_n - C(\mathcal{X}) \right) &= 0, \quad \forall \mathcal{X} \forall n \\ u'_n(r_n) - \sum_{\mathcal{X} \subseteq \mathcal{N}} \lambda_n^{\mathcal{X}} &= 0, \quad \forall \mathcal{X} \forall n \end{aligned} \quad (6.4)$$

where $u'_n(x)$ is the first derivative of $u_n(x)$, $\lambda_n^{\mathcal{X}} \geq 0$ is the Lagrangian multiplier associated with player n 's sum-rate constraint for any set $\mathcal{X} \subseteq \{1, \dots, N\}$. We denote by $\lambda_n = \{\lambda_n^{\mathcal{X}}\} \in \mathbb{R}_+^{2^N - 1}$ the Lagrangian multiplier vector for player n . Note that λ_n has $2^N - 1$ components associated with all the sum-rate constraints, i.e., $\sum_{n \in \mathcal{X}} r_n \leq C(\mathcal{X})$, $\forall \mathcal{X} \subseteq \{1, \dots, N\}$. We now give the formal definition of normalized equilibrium.

Definition 6.11. (*Normalized equilibrium*)

A generalized NE is called a normalized equilibrium with some positive weights $\gamma = \{\gamma_1, \dots, \gamma_N\} \in \mathbb{R}_{++}^N$ if the Lagrangian multiplier vector $\lambda_n, n = 1, \dots, N$ satisfy, in addition to the KKT conditions (6.4), the condition

$$\gamma_1 \lambda_1 = \gamma_2 \lambda_2 = \dots = \gamma_N \lambda_N = \lambda \quad (6.5)$$

In our context, we restrict ourselves to the special case where the weights of all players are equal, i.e., $\gamma_1 = \gamma_2 = \dots = \gamma_N$. From (6.5), this implies $\lambda_1 = \lambda_2 = \dots = \lambda_N = \lambda$. Thus, a normalized equilibrium with equal weights is characterized as a generalized NE for which there exists a single Lagrangian multiplier vector λ such that the KKT conditions (6.4) can be simplified to:

$$\begin{aligned} \sum_{n \in \mathcal{X}} r_n - C(\mathcal{X}) &\leq 0, \quad \forall \mathcal{X} \\ \lambda^{\mathcal{X}} \left(\sum_{n \in \mathcal{X}} r_n - C(\mathcal{X}) \right) &= 0, \quad \forall \mathcal{X} \\ u'_n(r_n) - |\mathcal{X}| \lambda^{\mathcal{X}} &= 0, \quad \forall \mathcal{X} \forall n \end{aligned} \quad (6.6)$$

For the sake of convenience, in the remainder of this chapter we consider the dominant face as an open², bounded hyperplane characterized by (6.3) within \mathcal{C} , and we denote it by $\mathcal{F}_{\mathcal{N}}$. In this case, since all the sum-rate constraints (except the constraint for the dominant face) are orthogonal, their corresponding Lagrangian multipliers are equal to zero, i.e., $\lambda^{\mathcal{X}} = 0, \forall \mathcal{X} \subseteq \mathcal{N} (\mathcal{X} \neq \mathcal{N})$. Therefore, the normalized Lagrangian multiplier vector λ can be reduced to a scalar variable $\lambda^{\mathcal{N}}$ associated with (6.3). The KKT conditions of normalized equilibrium (6.6) can be further reduced to

$$u'_n(r_n) - \lambda^{\mathcal{N}} = 0, \forall n \quad (6.7)$$

while the global sum-rate constraint (6.3) is satisfied.

6.4.3 A Relaxed Game and Scalable Pricing

To address the equilibrium selection problem for the game $\mathcal{G}_{mac}^{[\mathcal{C}]}$, one could in fact consider a modified game $\mathcal{G}_{mac}^{[\hat{\mathcal{C}}]}$ by relaxing the original strategy set \mathcal{C} to $\hat{\mathcal{C}}$ and payoff set \mathcal{U} to \mathcal{L} , as follows

$$\mathcal{G}_{mac}^{[\hat{\mathcal{C}}]} = \langle \mathcal{N}, \hat{\mathcal{C}}, \mathcal{L} \rangle$$

which consists of

- A *player set*: $\mathcal{N} = \{1, \dots, N\}$.
- A *strategy set*: $\hat{\mathcal{C}} = \hat{\mathcal{C}}_1 \times \hat{\mathcal{C}}_2 \times \dots \times \hat{\mathcal{C}}_N$, which is characterized by

$$\hat{\mathcal{C}} = \left\{ (r_1, \dots, r_N) : 0 \leq \sum_{n \in \mathcal{X}} r_n < C(\mathcal{X}), \forall \mathcal{X} \subset \mathcal{N} \right\} \quad (6.8)$$

- A *payoff set*: $\mathcal{L} = \{\mathcal{L}_1, \dots, \mathcal{L}_N\}$, where player n 's payoff \mathcal{L}_n is given by

$$\mathcal{L}_n(\mathbf{r}, \lambda_n) = u_n(r_n) + \lambda_n \left(\sum_{n=1}^N r_n - C(\mathcal{N}) \right) \quad (6.9)$$

If the relaxed game $\mathcal{G}_{mac}^{[\hat{\mathcal{C}}]}$ has a solution, \mathcal{L}_n can be viewed as a Lagrangian function that corresponds to the constrained optimization problem faced by player n when the other players choose $\mathbf{r}_{-n} = \{r_j\}_{j \neq n}$. From KKT theorem, we know that there exists a vector of Lagrangian multipliers $\{\lambda_1, \dots, \lambda_N\}$, whose entries are not necessarily equal, such that the equilibrium of the relaxed game

²A set is open if every point in the set has a neighborhood lying in the set.

$\mathcal{G}_{mac}^{[\hat{\mathcal{C}}]}$ coincides with the equilibrium of the original game $\mathcal{G}_{mac}^{[\mathcal{C}]}$. Note that the feasible region \mathcal{C} is now relaxed to $\hat{\mathcal{C}}$ ($\mathcal{C} \subseteq \hat{\mathcal{C}}$).

We are interested in pricing mechanisms that induce equilibria strategies and that can be implemented in a scalable and decentralized way. In general, $\{\lambda_1, \dots, \lambda_N\}$ can be used to define a non-scalable pricing³, where each λ_n is considered as the per packet price for user n . Consider a constant λ and let $\lambda \mathbf{1} \in \mathbb{R}^N$ be a vector with every its entry being equal to λ . Then, we wish to find λ such that $\mathbf{r}^*(\lambda \mathbf{1})$ is an equilibrium of the original game $\mathcal{G}_{mac}^{[m]}$. If we could find such a constant λ , then it certainly defines a scalable distributed pricing since the billing per packet can be implemented without any knowledge to which mobile the packet belongs to. As a matter of fact, such an equilibrium associated with some constant λ can be considered as the normalized equilibrium we previously defined $\mathbf{r}^*(\lambda^N \mathbf{1})$ whose n^{th} component is given by the solution to the KKT condition (6.7). Rosen [23] established the existence of normalized equilibrium for concave N -person games. The following theorem shows the uniqueness.

Theorem 6.12. *There exists a unique normalized equilibrium to the original game $\mathcal{G}_{mac}^{[\mathcal{C}]}$ associated with some constant λ^N .*

Proof. Define \mathbf{X} as a N -dimensional square matrix whose ij^{th} entry is $\frac{\partial^2 \mathcal{L}_n(\mathbf{r})}{\partial r_i \partial r_j}$. Then, it is easy to see that $\mathbf{X} + \mathbf{X}^T$ is strictly negative definite, since it is a diagonal square matrix whose diagonal elements are all negative. The theorem then follows from Theorem 4 in [23]. \square

6.4.4 Relation of Fairness Concepts and Normalized Equilibrium

In this part, we review and study some fairness concepts in the convex polytope rate region \mathcal{C} (6.1) and their relation to normalized equilibrium. In particular, we assume that all players are restricted to the same utility function u , which represents the satisfaction of a user from its rate allocation r_n .

Max-min Fair Allocation

The following theorem from [72] characterizes the max-min fair rate allocation in terms of majorization.

Theorem 6.13. *The max-min fair rate assignment belongs to the dominant face $\mathcal{F}_{\mathcal{N}}$ and is majorized by any other point on the dominant face.*

³Non-scalable pricing means that the price per packet depends on user/player index n .

We would like to note that there is an algorithm with $O(N^2)$ complexity for determination of the max-min fair rate assignment [72].

Next we show an very interesting result: all the fairness allocations together with global optimization as well as normalized equilibrium coincide with max-min fair rate allocation.

Global Optimization

First, consider the global optimization problem of maximizing the sum of the utilities $\sum_{n=1}^N u(r_n)$ over the achievable rate region $\mathbf{r} \in \mathcal{C}$.

Theorem 6.14. *The max-min fair assignment maximizes the sum of utilities over the achievable rate region.*

Proof. We relax the constraint $\sum_{n=1}^N r_n \leq C(\mathcal{N})$ and write the Lagrangian of the sum-utility maximization problem as

$$\mathcal{L}(\mathbf{r}, \lambda) = \sum_{n=1}^N u(r_n) + \lambda \left(\sum_{n=1}^N r_n - C(\mathcal{N}) \right) = \sum_{n=1}^N \psi(r_i) \quad (6.10)$$

where $\psi(x) \triangleq U(x) + \lambda x - C(\bar{\mathcal{S}})/N$ is a concave function. From Lemma 6.8, we observe that \mathcal{L} is Schur concave. Since we have shown in Theorem 6.13 that the max-min fair rate assignment is majorized by any other point on the dominant face $\mathcal{F}_{\mathcal{N}}$, it maximizes any Schur concave function on $\mathcal{F}_{\mathcal{N}}$. In particular, it maximizes \mathcal{L} for any multiplier λ . From KKT theorem, we know that there exists a λ such that the sum-utility is maximized. Therefore, the max-min rate assignment maximizes the sum-utility. This completes the proof. \square

Alpha Fair Allocation

Second, we consider the relation between max-min fairness and α -fairness.

Theorem 6.15. (1) *For any $\alpha \in \mathbb{R}_+$ ($\alpha \neq 1$), the α -fair rate assignment coincides with the unique max-min rate assignment.* (2) *For some strictly concave increasing utility function u , and for any $\alpha \in \mathbb{R}_+$ ($\alpha \neq 1$), the generalized α -fair rate assignment coincides with the unique max-min rate assignment.*

Proof. Both the α -utility V_α as well as the generalized α -utility W_α are concave. It follows from Lemma 6.8 that $\sum_{n=1}^N V_\alpha(r_n)$ and $\sum_{n=1}^N W_\alpha(r_n)$ are Schur concave. Since these are concave functions defined on a convex compact set, they have a solution (which is unique for all $\alpha > 0$). The solution is thus the same for any $\alpha > 0$ and is majorized by any other point on the dominant face of the

rate region. In particular, since this solution does not depend on α , it is the limit of the α fair solutions as $\alpha \rightarrow \infty$, and therefore it follows from [70] that the solution is max-min fair. \square

Normalized Equilibrium

Finally, we show the relation between max-min and normalized equilibrium.

Theorem 6.16. *The unique normalized equilibrium coincides with the max-min fair assignment.*

Proof. From Theorem 6.12, we know that there exists a unique normalized equilibrium \mathbf{r}^* to the original game associated with some λ^N . Thus, \mathbf{r}^* is a rate vector such that for each n , its n^{th} component maximizes the Lagrangian $\mathcal{L}_n = u(r_n) + \lambda \left(\sum_{n=1}^N r_n - C(\mathcal{N}) \right)$. This implies that \mathbf{r}^* is the unique vector that maximizes \mathcal{L} (6.10) over $\hat{\mathcal{C}}$, where \mathcal{L} is the Lagrangian corresponding to the global optimization problem. Therefore, \mathbf{r}^* is the globally optimal solution. From Theorem 6.14, we conclude that the unique normalized equilibrium also coincides with the max-min fair assignment. This completes the proof. \square

It might be somewhat surprising that all fairness concepts as well as normalized equilibrium provide the same rate allocation. However, this is not the case in general fair assignment (see examples in [81]) although the utilities V_α and W_α are always Schur concave. To understand this, note that both α -fair and generalized α -fair assignments are Pareto optimal. Whenever the Pareto optimal set has the property that the sum-rate is constant then indeed any Schur concave function will have the same maximizer and thus all the fairness concepts would coincide.

6.5 Convex Nonpolytope Achievable Rate Region

In this section, we consider the Gaussian multiple access orthogonal channel. Classical example of orthogonal channels are TDMA and FDMA. In this case, the achievable rate region is given by

$$\mathcal{C} = \left\{ (r_1, \dots, r_N) : 0 \leq r_n \leq \theta_n \log \left(1 + \frac{g_n p_n}{\theta_n \sigma^2} \right), \forall n, 0 \leq \sum_{n=1}^N \theta_n \leq 1 \right\} \quad (6.11)$$

For given g_n and p_n , the achievable rate region (6.11) is convex but it is not a polytope and it is strictly contained in (6.1). All points on the capacity region border are Pareto efficient and Nash equilibria. Therefore, here again we need

to choose among many Pareto efficient points and Nash equilibria. We suggest to use α -fairness utility function to select a particular equilibrium point. In the following theorem we characterize the α -fair equilibrium selection.

Theorem 6.17. *The α -fair rate allocation in the Gaussian multiple access orthogonal channel, which maximized the utility function*

$$u(\mathbf{r}) = \sum_{n=1}^N \frac{r_n^{1-\alpha}}{1-\alpha}$$

is unique for any value of the parameter α and is given by θ which solve the following system of equations

$$\left[\theta_n \log \left(1 + \frac{g_n p_n}{\theta_n \sigma^2} \right) \right]^{-\alpha} \left[\log \left(1 + \frac{g_n p_n}{\theta_n \sigma^2} \right) - \frac{g_n p_n}{\theta_n \sigma^2 + g_n p_n} \right] = \lambda_0, \quad \forall n$$

$$\sum_{n=1}^N \theta_n = 1 \quad (6.12)$$

The above system has a solution in explicit form in the case of total rate maximization, i.e., $\alpha = 0$,

$$\theta_n^* = \frac{g_n p_n}{\sum_{n=1}^N g_n p_n} \quad (6.13)$$

When $\alpha \rightarrow \infty$, the α -fair rate allocation provides max-min fair rate allocation, i.e., $r_1^* = r_2^* = \dots = r_N^*$.

Proof. The uniqueness of the α -fair rate allocation follows from the fact that we deal with the convex optimization problem. The equations (6.12) follow from the KKT conditions for the Lagrangian

$$\mathcal{L} = \sum_{n=1}^N \frac{r_n^{1-\alpha}}{1-\alpha} + \lambda_n \left[\theta_n \log \left(1 + \frac{g_n p_n}{\theta_n \sigma^2} \right) - r_n \right] + \lambda_0 \left(1 - \sum_{n=1}^N \theta_n \right)$$

For the particular case $\alpha = 0$, it follows from system (6.12) that

$$\log \left(1 + \frac{g_n p_n}{\theta_n \sigma^2} \right) - \frac{g_n p_n}{\theta_n \sigma^2 + g_n p_n} = \log \left(1 + \frac{g_j p_j}{\theta_j \sigma^2} \right) - \frac{g_j p_j}{\theta_j \sigma^2 + g_j p_j},$$

or

$$\log \left(1 + \frac{g_n p_n}{\theta_n \sigma^2} \right) - \log \left(1 + \frac{g_j p_j}{\theta_j \sigma^2} \right) = \frac{g_n p_n}{\theta_n \sigma^2 + g_n p_n} - \frac{g_j p_j}{\theta_j \sigma^2 + g_j p_j},$$

where $n \neq j$. Now it is easy to see that the values of θ^* provided by (6.13) make zero both sides of the above equation. Also, it is clear that the values of θ^* provided by (6.13) satisfy the normalization condition. \square

In fact, there are many examples of MACs having a convex non-polytope capacity region with non linear boundary surfaces:

- Flat-fading MAC with CSI known both at the receiver and transmitters [20].
- Time invariant MAC with multiple antennas and without intersymbol interference [82].
- Time invariant MAC with single [20] or multiple antennas [21] and intersymbol interference.

Note that the approach of Theorem 6.17 equally applies to the above multiple access channels. In particular, in the case of any strictly convex capacity region, there is a unique α -fair rate allocation.

6.6 Conclusion

We have studied multiuser access channels in the context of non-cooperative games with correlated constraints. In the non-cooperative games with correlated constraints the actions available to one player depend on those used by the others. A typical feature in these games is that they often possess infinitely many equilibria. To select among the equilibria we have proposed to use the normalized Nash equilibrium and different fairness concepts such as max-min fairness, proportional fairness, and α -fairness. We have considered two main types of multiuser access channels: polymatroid regions and convex nonpolytope regions. In the case of Gaussian multiple access channel (an example of a channel with polymatroid achievable rate region), the normalized Nash equilibrium and all fairness concepts select the same equilibrium. In the case of Gaussian multiple access orthogonal channel (an example of a channel with convex non-polytope achievable rate region) different fairness concepts can select different equilibria.

Chapter 7

Cooperative Wireless Games via Nash Bargaining

7.1 Introduction

In chapter 4, we established the uniqueness of equilibrium for an OFDM based small-cell network when the channel gain matrix is assumed to be a random matrix with *i.i.d.* continuous entries. However, from the numerical results, we showed that the performance inefficiency of non-cooperative game approaches can be significant compared to the centralized approaches, especially when the number of transmitters grows. Moreover, the performance unbalance at the receiver side can be a serious problem, since there is absolutely no control on the fairness issue.

The mentioned limits and drawbacks of resource allocations based on non-cooperative games can be mitigated using cooperative approaches. As a matter of fact, there are two inherently different types of games in game theory: non-cooperative and cooperative games. The non-cooperative game setting is one in which every player acts individually to optimize her performance measure without regard to the performance of other players. Such a procedure leads to a Nash equilibrium point in the network. This setting is important when the players act based only on local information. In contrast, in the cooperative

game setting, the players are allowed to communicate before choosing their strategies and playing the game. They can agree but also disagree about a joint strategy. If the players play a cooperative game, then the performance outcome is usually better than that of Nash equilibrium. This is because the Nash equilibrium is Pareto-inefficient under certain conditions [8]. As an example, in [83] it was shown that a centralized approach can outperform the distributed iterative waterfilling algorithm by up to 135%.

This chapter extends the framework of non-cooperative games in Chapter 4 to a cooperative game setting, and let the network operate at Pareto-optimal points. Cooperation is more likely when some nearby access points or base stations belong to the same network provider. In this situation, the network provider may prefer to let these wireless devices cooperate to operate in a more efficient and fair manner. One of the main questions in cooperative game theory is to find a reasonable compromise or operation point among all players. In this chapter, we adopt the concept of Nash bargaining [84], as it provides such a fair operation point for cooperative communication environment. Nash bargaining is one of the key notions for efficient and fair allocation in cooperative game theory. We study the rate bargaining problem using KKT conditions and potential games. We develop a stochastic algorithm to achieve the rate Nash bargaining solution, under the constraints of players' minimal rate requirements and maximal transmit powers. Finally, numerical results are provided to confirm our theoretical claims.

7.2 State-of-the-Art

For over a decade, the concept of Nash Bargaining from cooperative game theory has been frequently used in networks to efficiently and fairly allocate network resources. We mention as examples the flow control in networks [85], the bandwidth allocation in broadband networks [86, 81], and the inter-domain traffic engineering [87].

Although the application of Nash Bargaining to resource allocation in multi-user wireless networks is a recent research topic, there are some results available. Han and al. in a pioneering work [88], studied the Nash Bargaining under frequency division multiplexing and time division multiplexing strategies and with total power constraint. The issue of fairness in rate allocation is addressed in [72], using several fairness criteria, including max-min fairness, proportional fairness and Nash bargaining solution. For a simple two-user interference channel, Nash bargaining game is used to provide preferred points on the boundary

of the achievable rate region [89]. From simulation results, the cooperative solutions are shown to significantly outperform the competitive equilibrium. In [90], an asymmetric weighted Nash bargaining solution is used to study the resource allocation in interference channels. The results show an interesting connection between cooperative game theory, proportional fair resource allocation, and recent results on properties of log-convex interference functions [91]. In [92], iterative sub-optimal algorithms are applied to determine Nash bargaining solution for spectrum allocation under average power constraint. For more applications, the interested readers are referred to [93] [94] [95] [96] for MIMO networks, [97] for CDMA networks, and [81] for a discussion on how to allocate the bandwidth between applications with general concave utilities.

7.3 Asymmetric Nash Bargaining for OFDM Small-cell Networks

7.3.1 Channel Model

In this chapter, we consider the multi-user OFDM model for small-cell networks as shown in Chapter 4, see Section 4.3.1. Here we briefly describe this model and introduce some notations that will be used in the following discussion.

Consider a Gaussian interference channel with N access points (APs) simultaneously sending signals to M mobile terminals (MTs) over M OFDM subcarriers or sub-channels, see Figure 4.2. We assume that each sub-channel is pre-assigned to a different MT by a scheduler, therefore, each MT detects the signals only on the assigned sub-channel.

We write the signal-to-interference plus noise ratio (SINR) of the signal from the n^{th} AP to the m^{th} MT as

$$\gamma_{n,m} = \frac{g_{n,m}p_{n,m}}{\sigma^2 + \sum_{j \neq n} g_{j,m}p_{j,m}} \quad (7.1)$$

where $g_{n,m}$ is the channel power gain of the link from the n^{th} AP to the m^{th} MT on the pre-assigned sub-channel m , $p_{n,m}$ is the power transmitted by the AP n on sub-channel m , σ^2 is the zero mean AWGN with variance σ^2 . The maximum achievable sum-rate for AP n is given by [1]

$$r_n = \sum_{m=1}^M \log(1 + \gamma_{n,m}), \quad \forall n \quad (7.2)$$

Each AP n is restricted with a power constraint

$$\sum_{m=1}^M p_{n,m} \leq P_n^{\max}, \quad \forall n \quad (7.3)$$

where P_n^{\max} is the maximum power constraint of user n .

7.3.2 Rate Bargaining

For a set of players $\mathcal{N} = \{1, \dots, N\}$ a Nash bargaining game is defined by a pair $(r_n^{\min}, \mathcal{R})$, where \mathcal{R} is a compact and convex set which defines the feasible set of utilities of all the users, and $\{r_{n,m}^{\min}\}_{n,m} \in \mathcal{R}$ is known as the disagreement point which defines the amount of utility each player will get if the bargaining process fails. Whenever the disagreement situation can be decided by a non-cooperative game, it is reasonable to assume that the disagreement rate solution is given by a Nash equilibrium of the relevant non-cooperative game. Assume that the utility of player n is the achievable rate $r_n(\mathbf{p})$ given by (7.2), and let \mathbf{p}^* be the power allocation matrix satisfying (4.21)-(4.23) at the Nash equilibrium of the non-cooperative game discussed in Chapter 4. Then, the disagreement point r_n^{\min} can be taken as

$$r_n^{\min} = r_n(\mathbf{p}^*) = \sum_{m=1}^M \log(1 + \gamma_{n,m}(\mathbf{p}_n^*)), \quad \forall n$$

where vector $\mathbf{p}_n^* = \{p_{n,1}, \dots, p_{n,M}\}$ represents the power strategy of user n . It is the n^{th} column of \mathbf{p}^* . As shown in Section 3.3.4, the asymmetric bargaining solution $\{r_n\}_n \in \mathcal{R}$ should satisfy the four conditions (refer to Section 3.3.4) which lead to the following maximization problem

$$\max_{\substack{\mathbf{r}, \\ r_{n,m} \geq r_{n,m}^{\min}}} \prod_{n=1}^N (r_n - r_n^{\min})^{\omega_n} \quad (7.4)$$

subject to

$$\mathcal{R} = \left\{ \mathbf{r} : 0 \leq \sum_{n \in \mathcal{J}} r_{n,m} \leq \log \left(1 + \frac{\sum_{n \in \mathcal{J}} p_{n,m} g_{n,m}}{\sigma^2} \right), \quad \forall \mathcal{J} \subseteq \{1, 2, \dots, N\} \right\}$$

where $\mathbf{r} = \{r_1, \dots, r_N\}$, $\omega_n \geq 0$ is the weight of “bargaining power” for player n in the interaction, which is related to the relative abilities of players in a situation to exert influence over each other.

7.3.3 Transformation to Concave Optimization Problem

In order to find the Nash bargaining solution of rate allocation, we consider the non-concave maximization problem (7.4), which can be actually transformed into a convex optimization problem

$$\begin{aligned} \max_{\mathbf{r} \in \mathcal{R}} \quad & \sum_{n \in \mathcal{N}} \omega_n \log(r_n - r_n^{\min}) \\ \text{s.t.} \quad & r_n \geq r_n^{\min} \end{aligned} \quad (7.5)$$

where the objective function is concave in \mathbf{r} and the constraint space \mathcal{R} is non-empty, convex and compact.

We regularize the Nash product (7.5) by substituting the logarithmic function with the following function, defined for all $x \in \mathbb{R}$ by

$$\phi_\epsilon(x) = \begin{cases} \log(\epsilon) + \frac{x-\epsilon}{\epsilon} & \text{if } x \leq \epsilon \\ \log(x) & \text{if } x > \epsilon \end{cases}$$

This regularization replaces the logarithm function by its Taylor first order development for small rates. Then, we have a regular function to maximize in all the feasible sum-rate set. The following Lemma shows that this regularization gives the same bargaining solutions.

Lemma 7.1. *For $\epsilon > 0$ sufficiently small, optimization problem (7.5) is equivalent to:*

$$\max_{\mathbf{r} \in \mathcal{R}} \sum_{n \in \mathcal{N}} \omega_n \phi_\epsilon(r_n - r_n^{\min}) \quad (7.6)$$

The proof can be found in Appendix 7.6.1.

Note that the expression (7.6) in Lemma 7.1 maximizes over the entire set \mathcal{R} of feasible and individual rational rate profile. This result is crucial for gradient-based algorithms to neglect scenarios where a base station currently achieves a utility lower than the disagreement point, which may happen before the algorithm converges. We will guarantee to each base station at least the worse Nash equilibrium rate at any step of the adaptive algorithm.

7.3.4 KKT Optimality Conditions

It is easy to verify that the sum-log function in (7.5) is concave and the corresponding constraint set is convex. Thus, the Karush-Kuhn-Tucker (KKT) condition of the optimization is necessary and sufficient for optimality [2]. To

derive the KKT conditions, form the Lagrangian function \mathcal{L} as:

$$\begin{aligned} \mathcal{L}(\mathbf{r}, \lambda) &= \sum_{n \in \mathcal{N}} \omega_n \log \left(\sum_{m \in \mathcal{M}} r_{n,m} - r_n^{\min} \right) \\ &\quad - \sum_{\mathcal{J} \subseteq \mathcal{N}} \sum_{m \in \mathcal{M}} \lambda_m^{\mathcal{J}} \left(\sum_{n \in \mathcal{X}_{\mathcal{J}}} r_{n,m} - \bar{r}_m^{\mathcal{J}} \right) \end{aligned}$$

where \mathcal{X} is a subset of \mathcal{M} , $\bar{r}_m^{\mathcal{X}}$ is the maximum sum-rate on subcarrier m of the subset of players \mathcal{X} , $\lambda_m^{\mathcal{X}} \geq 0$ is the dual variable for the associated sum-rate constraint, and the KKT conditions are

$$\frac{\omega_n}{\sum_{m \in \mathcal{M}} r_{n,m}^* - r_n^{\min}} - \beta_m = 0, \quad \forall n \quad \forall m \quad (7.7)$$

$$\lambda_m^{\mathcal{X}} \left(\sum_{n \in \mathcal{X}} r_{n,m}^* - \bar{r}_m^{\mathcal{X}} \right) = 0, \quad \forall n \quad \forall \mathcal{X} \quad (7.8)$$

where we define $\beta_m = \sum_{\mathcal{X}} \lambda_m^{\mathcal{X}}$, and the optimal rate set r^* represents the sum-rate bargaining solution r^{NBS} . Based on (7.7), we can observe that:

- All the β_m have the same value, and we define $\beta \triangleq \beta_1 = \dots = \beta_M$.
- $\beta > 0$, since $\omega_n > 0$, $\forall n$.

Lemma 7.2. *The Lagrangian multiplier $\lambda_m^{\mathcal{M}}$ is positive for all m , and at the optimality point, the largest rate constraint must be satisfied with equality, i.e.*

$$\sum_{n \in \mathcal{N}} r_{n,m}^* = \bar{r}_m^{\mathcal{N}} \quad (7.9)$$

The proof can be found in Appendix 7.6.2.

From KKT conditions we have the following linear equations

$$\sum_{n \in \mathcal{N}} r_{n,m}^* = \bar{r}_m^{\mathcal{N}}, \quad \forall m \quad (7.10)$$

$$\sum_{m \in \mathcal{M}} r_{n,m}^* = r_n^{\min} + \omega_n \beta^{-1}, \quad \forall n \quad (7.11)$$

and the parameter β can be calculated as

$$\beta^{-1} = \sum_{m \in \mathcal{M}} \bar{r}_m^{\mathcal{N}} - \sum_{n \in \mathcal{N}} r_n^{\min} \quad (7.12)$$

In general, the $M + N$ independent linear equations in (7.10) and (7.11) have $MN + 1$ unknowns. This implies that the optimization problem has more than one solution. From (7.11), we observe that each player's sum-rate $\sum_{m \in \mathcal{M}} r_{n,m}^*$ is unique at a bargaining solution. This confirms the uniqueness in sum-rate, but we may have multiple splitting rate solutions over the subcarriers.

7.3.5 Pure Potential

In potential games, there exists a potential function such that the incentives of all players are mapped into one global function, and the set of pure Nash equilibria can be found by simply locating the local optima of the potential function. In our problem, for a subgroup of players $\mathcal{X} \subseteq \mathcal{N}$, we define

$$v(\mathcal{X}) \triangleq \sum_{m \in \mathcal{M}} \log \left(1 + \frac{\sum_{n \in \mathcal{X}} p_{n,m} g_{n,m}}{\sigma^2} \right) \quad (7.13)$$

the maximum sum-rate that the coalition \mathcal{X} receives. Then, $v(\emptyset) = 0$ and the following inequality holds:

$$v(\{n\}) = \sum_{m \in \mathcal{M}} \log \left(1 + \frac{p_{n,m} g_{n,m}}{\sigma^2} \right) \geq r_n \quad (7.14)$$

For $\forall n \notin \mathcal{X}$, we define

$$\begin{aligned} \Delta v_{n,\mathcal{X}} &\triangleq v(\mathcal{X} \cup \{n\}) - v(\mathcal{X}) \\ &= \sum_{m \in \mathcal{M}} \log \left(1 + \frac{p_{n,m} g_{n,m} + \sum_{j \in \mathcal{X}} p_{j,m} g_{j,m}}{\sigma^2} \right) \\ &\quad - \sum_{m \in \mathcal{M}} \log \left(1 + \frac{\sum_{j \in \mathcal{X}} p_{j,m} g_{j,m}}{\sigma^2} \right) \\ &= \sum_{m \in \mathcal{M}} \log \left(1 + \frac{p_{n,m} g_{n,m}}{\sigma^2 + \sum_{j \in \mathcal{X}} p_{j,m} g_{j,m}} \right) \\ &\geq r_n. \end{aligned} \quad (7.15)$$

Hence, for any $\mathcal{X}, \mathcal{Y} \subseteq \mathcal{N}$ satisfying $\mathcal{X} \subseteq \mathcal{Y}$, we have

$$\Delta v_{n,\mathcal{X}} \geq \Delta v_{n,\mathcal{Y}} \quad (7.16)$$

Note that when $\mathcal{X} = \mathcal{N} \setminus \{n\}$, the term $\Delta v_{n,\mathcal{X}}$ is exactly the maximum sum-rate of MT n . We say that v is a pure strategy potential function. To see the link with static continuous-kernel non-cooperative potential games, consider the difference

$$\begin{aligned} \Delta r_n &\triangleq r_n(\mathbf{p}_n, \mathbf{p}_{-n}) - r_n(\mathbf{p}'_n, \mathbf{p}_{-n}) \\ &= \sum_{m \in \mathcal{M}} \log \left(\sigma^2 + \sum_{j \in \mathcal{N}} p_{j,m} g_{j,m} \right) - \\ &\quad \sum_{m \in \mathcal{M}} \log \left(\sigma^2 + p'_{n,m} g_{n,m} + \sum_{j \in \mathcal{N} \setminus \{n\}} p_{j,m} g_{j,m} \right) \\ &= w(p_n, p_{-n}) - w(\mathbf{p}'_n, p_{-n}) \end{aligned} \quad (7.17)$$

where

$$w(\mathbf{p}'_n, p_{-n}) = \sum_{m \in \mathcal{M}} \log \left(1 + \frac{p'_{n,m} g_{n,m} + \sum_{j \in \mathcal{N} \setminus \{n\}} p_{j,m} g_{j,m}}{\sigma^2} \right)$$

From (7.13), we have $w(p_n, p_{-n}) = v(\mathcal{N})$.

Next, we derive some relations between the pure potential and maximal feasible solutions. For any two subgroups of players $\mathcal{X}_1, \mathcal{X}_2 \subseteq \mathcal{N}$, the sum of the potentials is given by

$$\begin{aligned} v(\mathcal{X}_1) + v(\mathcal{X}_2) &= \sum_{m \in \mathcal{M}} \log \left[\left(1 + \frac{\sum_{n \in \mathcal{X}_1} p_{n,m} g_{n,m}}{\sigma^2} \right) \left(1 + \frac{\sum_{n \in \mathcal{X}_2} p_{n,m} g_{n,m}}{\sigma^2} \right) \right] \\ &> \sum_{m \in \mathcal{M}} \log \left(1 + \frac{\sum_{n \in (\mathcal{X}_1 \cup \mathcal{X}_2)} p_{n,m} g_{n,m}}{\sigma^2} \right) \\ &= v(\mathcal{X}_1 \cup \mathcal{X}_2) \end{aligned} \quad (7.18)$$

This implies that v is *subadditive*, i.e., $v(\mathcal{X}_1 \cup \mathcal{X}_2) < v(\mathcal{X}_1) + v(\mathcal{X}_2)$.

Using these properties, the set of feasible utilities with maximal value for \mathcal{N} , i.e., $\sum_{n \in \mathcal{N}} r_n = v(\mathcal{N})$, satisfies $\sum_{n \in \mathcal{X}} r_n \leq v(\mathcal{X})$ for $\forall \mathcal{X} \subset \mathcal{N}$. This set is sometimes referred to *core*, which is a solution concept in coalition formation [98]. The interested readers are referred to [99] for a recent application of coalitional games to wireless networks. Now, we transform the inequalities that define the core to a linear program (LP) problem:

$$(\text{LP}) : \begin{cases} \max & \sum_{n \in \mathcal{N}} r_n \\ \text{s.t.} & \mathbf{r} \in \mathcal{R} \end{cases} \quad (7.19)$$

Then, the non-emptiness of the core is exactly equivalent to the optimum value of the primal problem since (7.19) is equal to $v(\mathcal{N})$. The dual problem, through Lagrangian multipliers, can be written as:

$$\mathcal{L}(\mathbf{r}, \lambda, \mu) = \sum_{n \in \mathcal{N}} r_n - \sum_{\mathcal{X} \subseteq \mathcal{N}} \lambda_{\mathcal{X}} \left(\sum_{n \in \mathcal{X}} r_n - v(\mathcal{X}) \right) + \sum_{n \in \mathcal{N}} \mu_n r_n$$

The primal problem (7.19) can be interpreted as

$$\max_{\mathbf{r} \in \mathbb{R}_+^N} \inf_{\substack{\lambda \geq 0, \\ \mu \geq 0}} \mathcal{L}(\mathbf{r}, \lambda, \mu)$$

and the dual problem (DP) is

$$\inf_{\substack{\lambda \geq 0, \\ \mu \geq 0}} \sup_{\mathbf{r} \in \mathbb{R}_+^N} \mathcal{L}(\mathbf{r}, \lambda, \mu).$$

Note that

$$\sup_{\mathbf{r} \in \mathbb{R}_+^N} \mathcal{L}(\mathbf{r}, \lambda, \mu) = \begin{cases} \sum_{\mathcal{X}} \lambda_{\mathcal{X}} v(\mathcal{X}) & \text{if } \sum_{\mathcal{X}} \lambda_{\mathcal{X}} = 1 \\ +\infty & \text{otherwise} \end{cases}$$

Thus, the dual problem can be rewritten as

$$(\text{DP}) : \begin{cases} \min & \sum_{\mathcal{X}} \lambda_{\mathcal{X}} v(\mathcal{X}) \\ \text{s.t.} & \sum_{\mathcal{X}} \lambda_{\mathcal{X}} = 1, \forall n \in \mathcal{X} \\ & \lambda_{\mathcal{X}} \geq 0, \forall \mathcal{X} \subseteq \mathcal{N} \end{cases}$$

7.3.6 Stochastic Algorithm for Nash Bargaining

In this part, we develop a recursive method to compute a Nash bargaining solution (7.6). Denote

$$\mathcal{L}(\mathbf{r}, \lambda) = \sum_m w_n \phi_\epsilon (r_n - r_n^{\min}) - \lambda_m^{\mathcal{N}} \left(\sum_{n \in \mathcal{N}} r_{n,m} - \bar{r}_m \right).$$

A saddle point of the Lagrangian \mathcal{L} is the pair $(\mathbf{r}^*, \lambda^*)$ satisfying

$$\mathcal{L}(\mathbf{r}, \lambda^*) \leq \mathcal{L}(\mathbf{r}^*, \lambda^*) \leq \mathcal{L}(\mathbf{r}^*, \lambda), \quad \forall \mathbf{r} \forall \lambda.$$

A necessary and sufficient condition for $\mathbf{r} = \{r_{n,m}\}_{n,m}$ to be a constrained minimum is that there is a λ with nonnegative components such that (\mathbf{r}, λ) is a saddle point of $\mathcal{L}(\mathbf{r}, \lambda)$. Let us now consider a recursive algorithm for finding a saddle point. Let $r_{n,m}^{(t)}$ denote the t^{th} estimates and use the following form of the Robbins-Monro procedure. Let \mathcal{C} denote the hypercube

$$\prod_{n,m} \left[r_{n,m}^{\min}, \log \left(1 + \frac{p_{n,m} g_{n,m}}{\sigma^2} \right) \right]$$

The multipliers λ are bounded because they are non-negative and their sum is upper-bounded (see (7.12)) by

$$\frac{1}{\sum_{m \in \mathcal{M}} \log \left(1 + \frac{\sum_{n \in \mathcal{N}} p_{n,m} g_{n,m}}{\sigma^2} \right)}$$

From the KKT conditions, the constraint set can be reduced to

$$\mathcal{C} \cap \left\{ \mathbf{r}, \sum_{n=1}^N r_{n,m} \leq \bar{r}_m, \forall m \in \mathcal{M} \right\}$$

Denote

$$q_m(\mathbf{r}) = \bar{r}_m - \sum_{n=1}^N r_{n,m}.$$

The recursive algorithm is then given by

$$\begin{aligned}\lambda_m^{(t+1)} &= \max\left(0, \lambda_m^{(t)} + \epsilon^{(t)} q_m(r^{(t)}) + \epsilon^{(t)} \chi_m^{(t)}\right) \\ r_m^{(t+1)} &= \text{proj}_{\mathcal{C}} \left[r_m^{(t)} + \epsilon^{(t)} \left(\mathcal{L}_r(r^{(t)}, \lambda^{(t)}) + \xi_m^{(t)} \right) \right]\end{aligned}$$

where

$$\begin{aligned}r_m^{(t)} &= \left[r_{1,m}^{(t)}, \dots, r_{N,m}^{(t)} \right], \\ \mathcal{L}_r(r^{(t)}, \lambda^t) &= \left[\frac{\partial}{\partial r_{1,m}^{(t)}} \mathcal{L}(\cdot, \lambda_m^{(t)}), \dots, \frac{\partial}{\partial r_{N,m}^{(t)}} \mathcal{L}(\cdot, \lambda_m^{(t)}) \right],\end{aligned}$$

$\text{proj}_{\mathcal{C}}$ denotes the projection operator over the hypercube \mathcal{C} , $\xi_m^{(t)}$ is the “noise” or “error” in the estimate of $\mathcal{L}(r_m^{(t)}, \lambda_m^{(t)})$, and $\chi_m^{(t)}$ is the “noise” in the estimate of q_m . Using the regularized function ϕ_ϵ in the Lagrangian, we obtain a regular concave-convex function to maximize on the sum-rates and minimize on λ . Using the methods in [100], the algorithm can be shown to converge to a saddle point under some conditions on the learning rates $\epsilon^{(t)}$ ($\sum_t \epsilon^{(t)} = +\infty$, $\sum_t (\epsilon^{(t)})^2 < +\infty$) with a good approximation of the noises.

7.3.7 Strong Equilibria

Much of the research in computational game theory has focused on the classical Nash equilibrium as the primary solution concept. Indeed Nash equilibrium has many benefits, and most importantly it always exists (in mixed strategies). However, the solution concept of Nash equilibrium is resilient only to unilateral deviations, while in reality, players may be able to coordinate their actions. A strong equilibrium [101] is a state from which no coalition of any size can deviate and improve the utility of every member of the coalition while possibly lowering the utility of players outside the coalition. This resilience to deviations by coalitions of the players is highly attractive, and one can hope that once a strong equilibrium is reached it is highly likely to sustain. To study our problem, we will introduce the concept of strong equilibria. We first describe the constrained strategic game \mathcal{G} defined as:

- The set of players is \mathcal{N} .

- The set of rates of player n is $r_{n,m} \geq 0$ and $\sum_{m \in \mathcal{M}} r_{n,m} \leq \bar{r}_n$ such that in each subcarrier, any subgroup \mathcal{X} of players are constrained to a common set

$$\sum_{n \in \mathcal{X}} r_{n,m} \leq \log \left(1 + \sum_{n \in \mathcal{X}} \frac{p_{n,m} g_{n,m}}{\sigma^2} \right)$$

- If all the constraints are satisfied, each player n receives r_n otherwise they get zero.

A vector of rates is a k -strong equilibrium of \mathcal{G} if it is feasible and no coalition of size k can improve the sum-rate of each of its members with the respect to the constraints in \mathcal{R} . A vector of sum-rates is a strong equilibrium if it is a k -strong equilibrium for any size $k = 1, \dots, N$. A strong equilibrium is then a splitting rate from which no coalition of any size can deviate and improve the sum-rate of every member of the coalition (group of the simultaneous moves), while possibly lowering the sum-rate of players outside the coalition group. This notion of strong equilibria is very attractive because it is resilient against coalitions of players. Most of the games do not admit any strong equilibrium.

Lemma 7.3. *The Nash bargaining solution for the sum-rate problem is a strong equilibria for \mathcal{G} .*

The proof can be found in Appendix 7.6.3.

Moreover, the game \mathcal{G} has many strong equilibria, which will be described in the following theorem.

Theorem 7.4. *The set of strong equilibria of G is*

$$\left\{ (r_{n,m}^*)_{n,m} \in \mathcal{R} \mid \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} r_{n,m}^* = v(\mathcal{N}), r_{n,m}^* \geq r_{n,m} \right\}$$

The proof can be found in Appendix 7.6.4.

Note that we may not have strong equilibria in a strategic way in terms of transmission power. The following assertions are equivalent:

- $\mathbf{p}^* = (\mathbf{p}_n^*, \mathbf{p}_{-n}^*)$ is a feasible transmit power set which gives a rate profile that satisfies (7.10) and (7.11).
- $\mathbf{p}^* = (\mathbf{p}_n^*, \mathbf{p}_{-n}^*)$ is a strategic strong equilibrium.

7.3.8 Efficiency and Benefit of Bargaining

We aim to measure the gap between the worse bargaining solution by varying the disagreement value r^{\min} and the social optimum value, that is the fully cooperative solution. Define the efficiency of sum-rate bargaining as the minimum over (r^{\min}, \mathcal{J}) of the ratio between the total sum-rate at the bargaining solution and the maximum value for the potential (7.13). It is easy to see that the ratio is less than one in a sum-rate bargaining solution.

We introduce the benefit of bargaining (BoB) to measure the relative gap between the worse bargaining solution and the worse competitive Nash equilibrium.

$$BoB = \frac{\text{total-sum-rate}_{NB} - \text{total-sum-rate}_{NE}}{\text{total-sum-rate}_{NE}}$$

where total-sum-rate at NBS is defined as

$$\sum_{m \in \mathcal{M}} r_m^{NBS}.$$

Since the worse competitive Nash equilibrium is an interior point of the capacity region, there is a gap between this point and the Pareto frontier. This means that the BoB in sum-rate is at least non-negative.

7.4 Numerical Results

In this section, we provide some numerical results to demonstrate the advantage of NBS regarding the *average* system performance. As a basis of comparison, the Nash equilibrium and the social optima of the non-cooperative games will be provided and denoted by “NE” and “optima”, respectively.

We consider a Wyner linear network with 3 transmitting base stations (BSs) and 2 receiving mobile terminals (MTs), as shown in Figure 3. The 3 BSs are equally spaced with distance $d = 20$ meters. The location of MT1 is fixed at the middle of BS1 and BS2. Let L_2 be the horizontal distance between BS2 and MT2, we let MT2 moves from the right border of BS2 ($L_2 = 1$ meter far from BS2) to the middle between BS2 and BS3 ($L_2 = 10$ meters far from BS2). The height of each BS is $h_{BS} = 3$ meters; the power constraint of each BS is $P^{\max} = 1$; the variance of additive white Gaussian noise is set to $\sigma^2 = 0.01$; The propagation loss factor is set to three. The system performance (spectral efficiency in Bits/Sec/Hz) is averaged over 10^4 realizations of frequency selective fading channels.

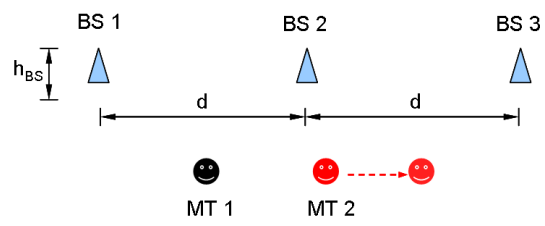
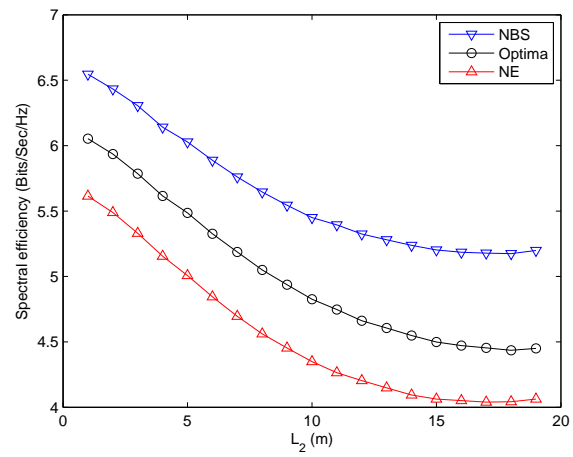


Figure 7.1 Illustration of the linear Wyner network

Figure 7.2 System spectral efficiency vs. L_2

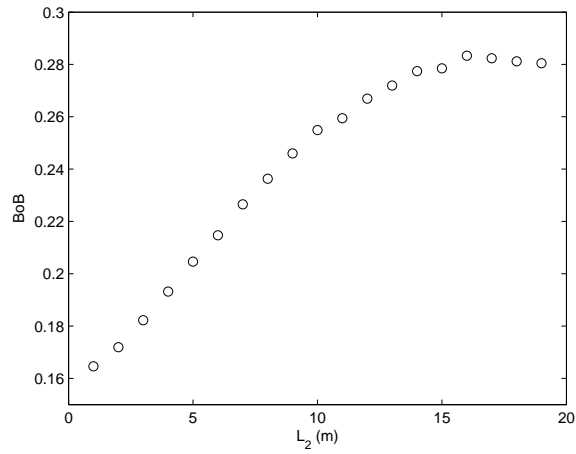
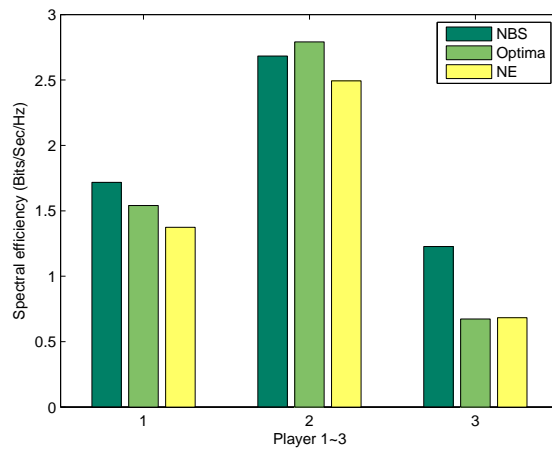
Figure 7.3 Benefit of bargaining (BoB) vs. L_2 

Figure 7.4 Fairness for NBS, Optima and NE.

Figure 4 shows the average system spectral efficiency (SE) vs. L_2 (in meters). The NBS outperforms the NE and the optima of the non-cooperative games. Figure 5 shows the benefit of bargaining (BoB) vs. L_2 . Note that by performing base stations cooperation through Nash bargaining, the overall system performance is improved around 23% (between 16.5% and 28% when L_2 varies from 1 meter to 10 meters, respectively), comparing to the Nash equilibrium in the non-cooperative game setting. In Figure 6, we compare the average spectral efficiency for the three BSs when different game solutions are applied. As expected, Nash bargaining provides a more “fair” and efficient solution compared to the non-cooperative game solutions: NE and optima. By using NBS scheme, BS1 and BS2 maintain the similar performance as in NE and optima, but BS3 obtains nearly doubled-performance compare to NE and Optima schemes.

7.5 Conclusion

In this paper we described the small-cell wireless scenario where multiple transmitters communicating with multiple receivers through independent OFDM sub-channels as a cooperative game. We mainly study the Nash bargaining game by using KKT conditions and pure potential. A recursive stochastic algorithm is proposed. It converges to one point as a NBS on the Pareto boundary. Finally, in our simulation results, we show the NBS not only provides a fair power allocation for players, but also outperform the system spectral efficiency compared to NE and Optima solutions (without considering fairness) of the non-cooperative games. Future works will focus on coalition games and dynamic Nash bargaining.

7.6 Appendix

7.6.1 Proof of Lemma 7.1

Proof. By definition of the disagreement outcome and the feasible payoffs set \mathcal{R} , suppose that there exists a feasible rate profile \mathbf{r} that component-wise dominates r^{\min} , then we must have $r_n > r_n^{\min}$, $\forall n$, which implies the existence of $\epsilon_n > 0$ such that $r_n - r_n^{\min} \geq \epsilon_n$ for all n . Let $\bar{\epsilon}$ be the minimum of the $\{\epsilon_n\}_{n \in \mathcal{N}}$. Then, $\bar{\epsilon} > 0$ is strictly positive. Let ϵ such that $0 < \epsilon < \bar{\epsilon}$. The supremum over \mathcal{R} must be in the region $r_n > r_n^{\min} + \epsilon$ for all n . Thus, $\forall n$, $\phi_\epsilon(r_n - r_n^{\min}) = \log(r_n - r_n^{\min})$ and the constrained maximization of $\sum_{n=1}^N \omega_n \phi_\epsilon(r_n - r_n^{\min})$ coincides with $\sum_{n=1}^N \omega_n \log(r_n - r_n^{\min})$. Now consider the case where the rate profile satisfies the following: there exists a mobile n for which the total rate over all the channels \bar{r}_n is strictly lower than the disagreement rate. $r_n^{\min} + \epsilon > \bar{r}_n$. Then, we show that this configuration cannot be a Nash bargaining solution.

$$\phi_\epsilon(\bar{r}'_n - r_n^{\min}) = \log(\epsilon) + \frac{\bar{r}'_n - r_n^{\min} - \epsilon}{\epsilon}$$

and the term $\frac{\bar{r}'_n - r_n^{\min} - \epsilon}{\epsilon}$ is negative. Thus,

$$cc \triangleq \sum_{n'} \omega_{n'} \phi_\epsilon(\bar{r}'_{n'} - r_{n'}^{\min}) \quad (7.20)$$

$$\leq \omega_n \log(\epsilon) + \sum_{n' \neq n} \omega_{n'} \log(\bar{r}_{n'} - r_{n'}^{\min}) \quad (7.21)$$

$$< \sum_{n'} \omega_{n'} \log(r_{n'} - r_{n'}^{\min}) \quad (7.22)$$

Therefore, $\bar{\mathbf{r}}$ is never the maximizer of (7.5). Thus, for $\bar{\mathbf{r}}$ to be a maximizer, it must satisfy $\bar{r}'_n > r_{n,\min}$ for all n , i.e., $\forall n$, $\phi_\epsilon(r_n - r_n^{\min}) = \log(r_n - r_n^{\min})$ and the constrained maximization of $\sum_{n=1}^N \omega_n \phi_\epsilon(r_n - r_n^{\min})$ over \mathcal{R} coincides with $\sum_{n=1}^N \omega_n \log(r_n - r_n^{\min})$ over $\mathcal{R} \cap \{\mathbf{r}, \bar{r}_n > r_n^{\min}\}$. This completes the proof. \square

7.6.2 Proof of Lemma 7.2

Proof. We will use the proof by contradiction. First, let us assume $\lambda_m^{\mathcal{N}} = 0$ is true, and we have the corresponding sum-rate constraint

$$\sum_{n \in \mathcal{N}} r_{n,m}^* \leq \bar{r}_m^{\mathcal{N}} \quad (7.23)$$

On the one hand, since $\lambda_m^{\mathcal{N}} = 0$, there must exist at least one player set \mathcal{J} ($\mathcal{J} \neq \mathcal{N}$) satisfying $\lambda_m^{\mathcal{J}} > 0$, due to the fact $\beta > 0$. This implies

$$\sum_{n \in \mathcal{J}} r_{n,m}^* = \bar{R}_m^{\mathcal{J}} \quad (7.24)$$

On the other hand, for the complementary player set of \mathcal{J} , i.e. $\mathcal{J}^c := \mathcal{N} - \mathcal{J}$, the following constraint must be satisfied

$$\sum_{n \in \mathcal{J}^c} r_{n,m}^* \leq \bar{r}_m^{\mathcal{J}^c} \quad (7.25)$$

From (7.24) and (7.25), we obtain

$$\sum_{n \in \mathcal{N}} r_{n,m}^* \leq \bar{r}_m^{\mathcal{J}} + \bar{r}_m^{\mathcal{J}^c} \quad (7.26)$$

which is in contradiction with (7.23), because of the subadditive property, i.e. $\bar{R}_m^{\mathcal{N}} = \bar{r}_m^{\mathcal{J} \cup \mathcal{J}^c} < \bar{r}_m^{\mathcal{J}} + \bar{r}_m^{\mathcal{J}^c}$ (This proof is provided in subsection 7.3.5). Therefore, $\lambda_m^{\mathcal{N}} = 0$ can not be true. We must have $\lambda_m^{\mathcal{N}} > 0$ and (7.9) due to (7.8). \square

7.6.3 Proof of Lemma 7.3

Proof. A sum-rate Nash bargaining solution \mathbf{r}^* is a Pareto optimal solution for the sum-rate maximization. Hence, \mathbf{r} satisfies $\sum_n r_n^* = v(\mathcal{N})$. Consider now that a \mathcal{J} of players deviate from \mathbf{r}^* and form a new coalition with rates $r'_{\mathcal{J}}$. The new rate profile is then $(r'_{\mathcal{J}}, r_{\mathcal{J}^c})$. If $r'_{\mathcal{J}}$ does not meet the capacity constraints, the payoff of each player will be zero. In the capacity region, the total sum rate is constrained to the following inequality:

$$\sum_{n \in \mathcal{J}} r'_n \leq v(\mathcal{N}) - \sum_{n \in \mathcal{J}^c} r'_n = \sum_{n \in \mathcal{J}} r_n$$

So, the optimal deviation will have the equality

$$\sum_{n \in \mathcal{J}} r'_n = \sum_{n \in \mathcal{J}} r_n$$

This means that if a sum-rate of one of the players in \mathcal{J} increases, then there exists another player in the coalition \mathcal{J} for which the sum-rate decreases after deviating. Thus, the coalition is non-profitable, and this holds for any coalition $\mathcal{J} \subseteq \mathcal{N}$. We conclude that the bargaining solutions are strong equilibria of the game G . \square

7.6.4 Proof of Theorem 7.4

Proof. First, remark that if the sum-rate profile R is not in boundary surface of the capacity region, then α is not resilient by deviation by a single player. Hence, R cannot be a coalition-proof equilibrium [102]. This says that a necessary condition for a rate profile to be a strong equilibrium for G is to be in the

subset the maximal face of the sum-rate capacity region. It is easy to see that this last condition is also sufficient: if k players deviate simultaneously from the rate profile \mathbf{r} and form a coalition then, the sum-rate constraints of the new coalition is bounded $\sum_{n \in \mathcal{J}} r'_n \leq v(\mathcal{N}) - \sum_{n \notin \mathcal{J}} r_n$. The players in the coalition with a lower rate $r'_n \leq r_n$ do not benefit to be member of the coalition (Shapley criterion of membership of coalition does not hold), and this holds for any $J \subseteq \mathcal{N}$ with cardinality of $\mathcal{J} \geq 1$. This completes the proof.

□

Chapter 8

Conclusions and Perspectives

8.1 Conclusions

This dissertation illustrates the important role of game theory in solving the resource conflicts existing in the allocation of radio resource in current multiuser wireless networks.

Firstly, Static game model is considered a fundamental step to bring game theory as a tool for distributed resource allocation design in the context of wireless networks. Indeed, it helps us to understand the initial motivation, intuitive concept and common formulation of a game problem, and more importantly, to be familiar with the methodology to analyze its outcomes. In this part, two basic wireless channel models are discussed in detail, i.e., multiple access channels and interference channels. Based on the previous discussion, in [103] we present a non-cooperative game-theoretic approach for the distributed resource allocation problem in the context of multi-user orthogonal frequency division multiplexing hot-spot networks where multiple transmitters need to decide how to share their total transmit power over several parallel independent fading channels. We proved that there exists almost surely a unique Nash equilibrium. In order to obtain the equilibrium point, we proposed a simple iterative algorithm, namely Distributed Potential Iterative WaterFilling (DPIWF).

Secondly, Bayesian game-theoretical approach is introduced to study a more

practical resource allocation scenario, where wireless terminals are assumed to have incomplete information about the network status. More precisely, we assume each terminal is only aware of local information, e.g., the condition of its own channel, but unaware of other terminals' channels. In [104, 105], we focus on the power allocation game in multiple access channels with arbitrary number of users. We reformulate the waterfilling game as a version of a game with incomplete information, and we successfully proved the existence and uniqueness of Bayesian equilibrium. The uniqueness of an equilibrium is quite a desirable property, if we wish to predict what will be the network behavior. This is particularly important in the context of network administration and management, where wireless service providers are interested in optimally setting the network design parameters, taking into account their impact on the performance in equilibrium. The Pareto-efficiency measurement is addressed through simulation results. Future works are considered to improve the efficiency of Bayesian equilibrium through pricing or cooperative game-theoretic approaches.

Thirdly, since the two approaches above only consider an achievable rate region constrained to the use of single user decoding, we extended our study to multiple access channels with multiuser decoding at the receiver. The resource allocation problem is carried out using non-cooperative games with correlated constraints the actions available to one player depend on those used by the others. A typical feature in these games is that they often possess infinitely many equilibria. To select among the equilibria we have proposed to use the Normalized Nash equilibrium and different fairness concepts such as maxmin fairness, lexicographic fairness, α -fairness and Jain's fairness. We have considered two main types of multiuser access channels: polymatroid regions and convex non polytope regions. In the case of channels with polymatroid capacity region, the normalized Nash equilibrium and all fairness concepts select the same equilibrium. In the case of channels with convex non-polytope achievable rate region, different fairness concepts yield the selection of different equilibria. We have characterized these equilibria. It turns out that in some channels with non-convex achievable rate regions the max-min fair allocation might not even exist. An interesting future research direction is to study equilibria selection in the case of several base stations.

Finally, a fair resource allocation concept, Nash bargaining, is introduced to study the resource allocation problem in Orthogonal Frequency Division Multiplexing (OFDM) systems. The goal is to maximize the overall system rate considering fairness metrics among users. We propose Nash bargaining solution as a tool that achieves a fair point on the Pareto frontier of the game theoretic-

cal rate region under asymmetric conditions. Moreover, we provide a practical stochastic algorithm that can converge to one of the fairness points on the Pareto boundary. The numerical results show that the Nash bargaining solution not only maintains fair resource allocation for all users, but also provides a desirable spectral efficiency for the OFDM system.

8.2 Future Works

- Coalitional games

In general, game theory can be divided into two branches: non-cooperative and cooperative game theory. Non-cooperative game theory studies the strategic choices resulting from the interactions among competing players, where each player chooses its strategy independently for improving its own performance (utility) or reducing its losses (costs). While non-cooperative game theory studies competitive scenarios, cooperative game theory provides analytical tools to study the behavior of rational players when they cooperate. The main branch of cooperative games describes the formation of cooperating groups of players, referred to as coalitions that can strengthen the players' positions in a game. Coalitional games have been widely explored in different disciplines such as economics or political science. Recently, cooperation has emerged as a new networking paradigm that has a dramatic effect of improving the performance from the physical layer up to the networking layers. Coalitional games prove to be a very powerful tool for designing fair, robust, practical, and efficient cooperation strategies in communication networks.

- Stackelberg games

Stackelberg games are powerful tools to study wireless networks with hierarchy, meaning users' decision makings are asymmetric, e.g., in contexts where there are primary (licensed) users and secondary (unlicensed) users who can sense their environment because there are equipped with a cognitive radio. It is shown that introducing a certain degree of hierarchy in non-cooperative power control games may improve the individual performance of all the users, and may reach a desired trade-off between the global network performance at the equilibrium and the requested amount of signalings in the networks.

- Learning and dynamic games

Strategic games model the interaction between decision-makers in terms of equilibrium outcome, but do not specify how players reach an equilibrium. A raising issue in wireless communications is related to the iterative water-filling type algorithms that converge to the equilibrium state. In order to apply these algorithms, the transmitters are assumed to be strictly rational players that know the structure of the game. Some assumptions may not be realistic, e.g., complete information, discrete finite action sets, etc. An alternative way of explaining how the players of a game can converge to an NE is the theory of learning. Learning algorithms are long run processes in which players with very little knowledge and rationality constraints try to optimize their benefits. Learning and dynamics are very important in dynamic environments such as wireless channels, in the presence of imperfect observations, time delays and noisy measurements.

Bibliography

- [1] T. M. Cover, and J. A. Thomas, *Elements of Information Theory*, Wiley, 1991. [4, 31, 32, 40, 57, 58, 66, 83, 88, 126, 130, 143]
- [2] S. Boyd, and L. Vandenberghe, “Convex Optimization”, *Cambridge University Press*, 2004. [4, 40, 66, 90, 101, 145]
- [3] M. Chiang, “Geometric Programming for Communication Systems,” *Foundations and Trends in Communications and Information Theory*, vol. 2, no. 1/2, pp. 1-153, 2005. [4, 40, 97]
- [4] M. Debbah, “Mobile Flexible Networks: The challenges ahead”, *2008 International Conference on Advanced Technologies for Communications*, invited paper, Hanoi, Vietnam. [4, 14, 25, 26, 40, 52, 79, 81]
- [5] “LTE Operations and Maintenance Strategy, Using Self-Organizing Networks to Reduce OPEX”, Motorola white paper, 2009. [5, 41]
- [6] D. Fudenberg, and J. Tirole, *Game Theory*, MIT Press, 1991. [5, 26, 41, 81, 84, 89, 106, 112]
- [7] J. V. Neumann, and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, 1944. [5, 41]
- [8] P. Dubey, “Inefficiency of Nash equilibria”, *Mathematics of Operations Research*, vol. 11, no. 1, pp. 1-8, 1986. [5, 33, 41, 84, 142]
- [9] A. Goldsmith, and S. Chua, “Variable-rate Variable-power MQAM for fading channels”, *IEEE Transactions on Communications*, vol. 45, no. 10, pp. 1218-1230, Oct. 1997. [7, 42]
- [10] H. A. Simon, “A Behavioral Model of Rational Choice”, *The Quarterly Journal of Economics*, vol. 69, no. 1, pp. 99-118, Feb. 1955. [43]

-
- [11] R. Aumann, "Backward Induction and Common Knowledge of Rationality," *Games and Economic Behavior*, vol. 8, pp. 6-19, 1995. [43]
- [12] 4.6 billion mobile subscriptions by the end of 2009, http://www.itu.int/newsroom/press_releases/2009/39.html, Oct. 2009. [11, 49]
- [13] D. Tse, and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005. [51, 57, 108]
- [14] R. Ahlswedem "Multi-way channels", em Proc. 2nd. Int. Symp. Inform. Thoery, pp.23-52, 1971. [53]
- [15] H. Liao, "Multiple access channels", Ph.D. dissertation, Dep. Elec. Eng., Univ. Hawaii, Honolulu, 1972. [53]
- [16] C. E. Shannon, "A mathematical theory of communication", *Bell System Technical Journal*, vol. 27, pp. 379-423 and 623-656, July and October, 1948. [53]
- [17] B. Rimoldi and R. Urbanke, "On the Structure of the Dominant Face of Multiple-Access Channels", in *Proc. IEEE Inf. Theory Workshop*, pp. 12-14, Jun. 1999. [53]
- [18] R. Gallager, "An inequality on the capacity region of multiaccess fading channels", in *Communications and Cryptography-Two Sides of One Tapestry*, Boston, MA: Kluwer, pp. 129-139, 1994. [56, 109]
- [19] S. Shamai, and A.D. Wyner, "Information Theoretic Considerations for Symmetric, Cellular, Multiple-access Channels-Part I", *IEEE Trans. on Information Theory*, vol. 43, pp. 1877-1894, Nov. 1997. [56, 130]
- [20] D. N. C. Tse, and S. Hanly, "Multiaccess Fading Channels-Part I: Polymatroid Structure, Optimal Resource Allocation and Throughput Capacities," *IEEE Trans. Inform. Theory*, vol. 44, no. 7, pp. 2796-2815, Nov. 1998. [56, 127, 139]
- [21] J. F. Nash, "Equilibrium points in n-points games", *Proc. of the National Academy of Science*, vol. 36, no. 1, pp. 48-49, Jan. 1950. [18, 26, 71, 81, 131]
- [22] D. Debreu, "A Social Equilibrium Existence Theorem", *Proceedings of the National Academy of Sciences*, vol. 38, pp. 886-893, 1952. [18, 71, 112]

- [23] J. B. Rosen, "Existence and Uniqueness of Equilibrium Points for Concave N-Person Games," *Econometrica*, vol. 33, pp. 520-534, July 1965. [18, 19, 71, 72, 86, 89, 112, 126, 133, 135]
- [24] T. Roughgarden, and E. Tardos, "Bounding the Inefficiency of Equilibria in Nonatomic Congestion Games", *Games and Economic Behavior*, vol. 47, pp. 389-403, 2004. [19, 71]
- [25] R. Cooper, "Coordination Games", *Cambridge University Press*, 1998. [19, 71]
- [26] E. Altman, and A. Shwartz, "Constrained Markov Games: Nash Equilibria", *Annals of Dynamic Games*, Vol. 5, pp. 213-221, 2000. [19, 72]
- [27] J. C. Harsanyi, and R. Selten, "A General Theory of Equilibrium Selection in Games", *MIT Press*, Mar. 2003. [19, 72]
- [28] D. Bertsekas, and R. Gallager, *Data Networks*, Prentice Hall, 1987. [19, 72, 127]
- [29] F.P. Kelly, "Charging and Rate Control for Elastic Traffic", *Eur. Trans. Telecom.*, vol. 8, pp. 33-37. [19, 72, 127]
- [30] F.P. Kelly, A. Maullo, and D. Tan, "Rate Control for Communication Networks: Shadow Prices, Proportional Fairness and Stability", *J. Oper. Res. Soc.* vol. 49, 1998. [19, 72]
- [31] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for Analyzing Equilibria in Wireless Games", *IEEE Signal Processing Magazine special issue on Game Theory*, vol. 26, no. 5, pp. 51-52, 2009, Sep. 2009. [19, 21, 72, 74]
- [32] J. C. Harsanyi, "Games with Incomplete Information Played by "Bayesian" Players, I-III. Part I. The Basic Model", *Management Science*, vol. 14, no. 3, pp. 159-182, Nov. 1967. [19, 72]
- [33] R. W. Rosenthal, "A Class of Games Possessing Pure-Strategy Nash Equilibria", *International Journal of Game Theory*, vol. 2, no. 1, pp. 65-67, Jan. 1973. [21, 74]
- [34] D. Monderer, and L. S. Shapley, "Potential games", *Games and Economic Behavior*, vol. 14, pp.124-143, 1996. [21, 23, 74, 75, 76, 94]

- [35] M. Voorneveld, and H. Norde, "A characterization of ordinal Potential Games", *Games and Economics Behavior*, vol. 19, pp. 235-242, 1997. [23, 75]
- [36] M. Voorneveld, "Best-response potential games", *Economics letters*, vol. 66, pp. 289-295, 2000. [23, 75]
- [37] P. Dubey, O. Haimanko, and A. Zapechelnyuk, "Strategic Complements and Substitutes, and Potential Games", *Games and Economics Behavior*, vol. 54, pp. 77-94, 2006. [23, 75, 95]
- [38] J. Nash, "Two-Person Cooperative Games", *Econometrica*, vol. 21, pp. 129-140, 1953. [24, 76]
- [39] A. Roth, *Axiomatic Models of Bargaining*, Springer-Verlag, 1979. [24, 76]
- [40] E. Kalai, "Nonsymmetric Nash Solutions and Replications of Two-Person Bargaining", *International Journal of Game Theory*, vol. 6, pp. 129-133, 1977. [77]
- [41] M. Shubik, *Strategy and Market Structure: Competition, Oligopoly, and The Theory of Games.*, New York, Wiley, 1959. [77]
- [42] <http://iosc-workshop.homeip.net/> [25, 79]
- [43] S. Kandula, D. Katabi, S. Sinha, and A. Berger, "Dynamic Load Balancing Without Packet Reordering", *ACM Journal of Computer and Communication Review (CCR)*, 2007. [25, 80]
- [44] J. Bingham, "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come", *IEEE Communications Magazine*, vol. 28, no. 5, pp. 514, May 1990. [26, 80]
- [45] D. Gesbert, and M. S. Alouini, "How Much Feedback is Multi-User Diversity Really Worth?" *IEEE International Conference on Communications (ICC)*, Paris, France, vol. 1, pp. 234-238, June 2004. [26, 81, 107]
- [46] J. Mitola III, and G. Q. Jr, "Cognitive radio: making software radios more personal", *IEEE Personal Communications*, vol. 6, no. 4, pp. 13-18, Aug. 1999. [26, 81]
- [47] L. Lai, and H. El Gamal, "The Water-Filling Game in Fading Multiple-Access Channels", *IEEE Trans. on Information Theory*, Vol. 54, No. 5, pp. 2110-2122, May 2008. [29, 30, 82, 83, 84, 105, 106, 119, 127]

- [48] W. Yu, G. Ginins, and J. M. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines", *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, June 2002. [29, 82, 85, 105]
- [49] G. Scutari, D. Palomar, and S. Barbarossa, "Asynchronous Iterative Water-filling for Gaussian Frequency-selective Interference Channels: A Unified Framework", *Information Theory and Applications Workshop*, pp. 349-358, Jan. 2007. [29, 82, 85, 86, 105]
- [50] G. Scutari, D. Palomar, and S. Barbarossa, "Asynchronous Iterative Water-filling for Gaussian Frequency-selective Interference Channels", *IEEE Trans. on Information Theory*, vol. 54, pp. 2868-2878, July 2008. [29, 82, 85, 93, 105]
- [51] E. Altman, K. Avrachenkov, and A. Garnaev, "Closed Form Solutions for Symmetric Water Filling Games", *IEEE Conference on Computer Communications (INFOCOM)*, pp. 673-681, Apr. 2008. [29, 82, 85, 89, 105]
- [52] A. Leshem and E. Zehavi, "Game theory and the frequency selective interference channel - A tutorial", <http://arxiv.org/abs/0903.2174>, submitted on 12 Mar. 2009. [29, 82, 85, 86, 105]
- [53] T. Alpcan, T. Basar, R. Srikant, and E. Altman, "CDMA Uplink Power Control as a Noncooperative game", *Wireless Networks, Springer*, vol. 8, no. 6, pp. 659-670, Nov. 2002. [29, 82, 105]
- [54] T. Alpcan, and T. Basar, "A hybrid systems model for power control in multicell wireless networks", *Proc. of Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WIOPT), INRIA Sophia-Antipolis, France*, March 2003. [29, 82, 105]
- [55] X. Fan, M. Arcak, and J. T. Wen, "A robustness analysis of game-theoretic CDMA power control", *Turkish Journal of Electrical Engineering and Computer Science*, vol. 12, no. 3, pp. 139-150, 2004. [29, 82, 105]
- [56] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Uniform Power Allocation in MIMO Channels: A Game Theoretic Approach", *IEEE Trans. on Information Theory*, vol. 49, no. 7, pp. 1707-1727, July 2003. [29, 82, 105]
- [57] E. V. Belmega, S. Lasaulce, and M. Debbah, "Power allocation games for MIMO multiple access channels with coordination", *IEEE Trans. on Wireless Communications, revised*, Sept. 2008. [29, 82, 105]

- [58] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for analyzing equilibria in wireless games", *submitted to IEEE Signal Processing Magazine, Special issue on Game Theory for Signal Processing*, Sep. 2009. [29, 82, 105]
- [59] R. H. Etkin, D. N. C. Tse, and W. Hua, "Gaussian Interference Channel Capacity to Within One Bit", *IEEE Transactions on Information Theory*, vol. 54, no. 12, pp. 5534-5562, Dec. 2008. [84]
- [60] S. T. Chung, S. J. Kim, J. Lee, and J. M. Cioffi, "A game-theoretic approach to power allocation in frequency-selective Gaussian interference channels", *IEEE International Symposium on Information Theory (ISIT 2003)*, pp. 316-316, June 2003. [86]
- [61] G. Scutari, D.R. Palomar, and S. Barbarossa, "Simultaneous Iterative Water-Filling for Gaussian Frequency-Selective Interference Channels", *IEEE International Symposium on Information Theory (ISIT 2006)*, pp. 600-604, July 2006. [86, 93]
- [62] J. Neel, J. Reed, and R. Gilles, "Convergence of cognitive radio networks", in *Proc. of Wireless Communications and Networking Conference*, Phoenix, AZ, Nov. 2004. [93]
- [63] G. Scutari, S. Barbarossa, and D. P. Palomar, "Potential games: A framework for vector power control problems with coupled constraints", in *Proc. of IEEE International Conference on Acoustic, Speech, and Signal Processing*, Toulouse, France, May 2006. [93]
- [64] T. Heikkinen, "Potential Game Approach to Distributed Power Control and Scheduling", *Computer Networks*, vol. 50, pp. 2295-2311, 2006. [93]
- [65] J. Papandriopoulos, and J. S. Evans, "Low-Complexity Distributed Algorithms for Spectrum Balancing in Multi-User DSL Networks," *IEEE International Conference on Communications 2006*, vol. 7, pp. 3270-3275, June 2006. [97, 116]
- [66] C. D. Meyer, *Matrix Analysis and Applied Linear Algebra*, Philadelphia, Society for Industrial and Applied Mathematics (SIAM), 2000. [103, 104]
- [67] S. Jean, and B. Jabbari, "Bayesian game-theoretic modeling of transmit power determination in a self-organizing CDMA wireless network," *Proc. IEEE VTC Fall 2004*, vol. 5, pp. 3496-3500, Sep. 2004. [107]

- [68] S. Adlakha, R. Johari, and A. Goldsmith, "Competition in Wireless Systems via Bayesian Interference Games", <http://arxiv.org/abs/0709.0516>, Sep. 2007. [107, 108, 110]
- [69] R. Knopp, and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc IEEE Int. Conf. Communication*, Seattle, WA, vol. 1, pp. 331-335, Jun. 1995. [115]
- [70] J. Mo, and J. Walrand, "Fair End-to-End Window-Based Congestion Control", *IEEE Transactions on Networking*, vol. 8, no. 5, Oct. 2000. [127, 137]
- [71] M. A. Maddah-Ali, A. Mobasher, and A. K. Khandani, "Using Polymatroid Structures to Provide Fairness in Multiuser Systems", *IEEE International Symposium on Information Theory*, pp.158-162, Seattle, July 2006. [127]
- [72] K. W. Shum, and C. W. Sung, "On the Fairness of Rate Allocation in Gaussian Multiple Access Channel and Broadcast Channel", <http://arxiv.org/abs/cs/0611015v1>, submitted on Nov. 2006. [127, 135, 136, 142]
- [73] A. W. Marshall, and I. Olkin, "Inequalities: Theory of Majorization and its Applications", *Academic Press*, 1979, vol. 143 of Mathematics in Science and Engineering. [129]
- [74] I. Schur, "Über eine klasse von mittelbildungen mit anwendungen auf die determinantentheorie", *Sitzungsberichte der Berliner Mathematischen Gesellschaft*, vol. 22, pp. 9-20, 1923. [129]
- [75] K.-Z. Guan, "Some properties of a class of symmetric functions", *Journal of Mathematical Analysis and Applications*, vol. 336, no. 1, pp. 70-80, 2007. [129]
- [76] W. Rhee, and J.M. Cioffi, "On the Capacity of Multiuser Wireless Channels with Multiple Antennas", *IEEE Trans. on Information Theory*, vol. 49, pp. 2580-2595, Oct. 2003. [130]
- [77] K. J. Arrow, and G. Debreu, "Existence of an Equilibrium for a Competitive Economy", *Econometrica*, pp. 265-290, 1954. [131]
- [78] O. L. Mackenzie, "On the Existence of General Equilibrium for a Competitive Market", *Econometrica*, vol. 27, pp. 54-71, 1959. [131]

- [79] J. F. John, "Non-cooperative Games", *The Annals of Mathematics*, vol. 54, no. 2, pp. 286-295, 1951. [131]
- [80] E. Altman, and E. Solan, "Constrained Games: The Impact of the Attitude to Adversary's Constraints", *IEEE Transactions on Automatic Control*, vol. 54, no. 10, pp. 2435-2440, Oct. 2009. [131, 132]
- [81] C. Touati, E. Altman, and J. Galtier, "Generalized Nash Bargaining Solution for bandwidth allocation", *Computer Networks*, vol. 50, pp. 3242-3263, 2006. [137, 142, 143]
- [82] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative Water-filling for Gaussian Vector Multiple Access Channels", *IEEE Transactions on Information Theory*, vol. 50, pp. 145-C152, Jan. 2004. [139]
- [83] R. Cendrillon, W. Yu, M. Moonen, J. Verlinden, and T. Bostoen, "Optimal Multi-user Spectrum Management for Digital Subscriber Lines", *IEEE International Conference on Communication (ICC)*, vol. 1, pp. 1-5, 2004. [33, 142]
- [84] J. F. Nash, "The bargaining problem", *Econometrica*, vol. 18, pp. 155-162, 1950. [34, 142]
- [85] R. Mazumdar, L.G. Mason, C. Douligeris, Fairness in Network Optimal Flow Control: Optimality of Product Forms, *IEEE Transactions on Communications*, vol. 39, pp. 775-782, 1991. [142]
- [86] H. Yaiche, R.R. Mazumdar, C. Rosenberg, A Game Theoretic Framework for Bandwidth Allocation and Pricing in Broadband Networks, *IEEE/ACM Transactions on Networking*, vol. 8, pp. 667-678, Oct. 2000. [142]
- [87] G. Shrimali, A. Akella, A. Mutapcic, "Cooperative Inter-domain Traffic Engineering using Nash Bargaining and Decomposition", *IEEE INFOCOM*, 2007. [142]
- [88] Z. Han, Z. Ji, and K. J. R. Liu, "Fair Multiuser Channel Allocation for OFDMA Networks Using Nash Bargaining Solutions and Coalitions", *IEEE Transactions on Communications*, vol. 53, no. 8, Aug. 2005. [142]
- [89] A. Leshem and E. Zehavi, "Bargaining over the interference channel", *Proceedings of IEEE International Symposium on Information Theory*, pp. 2225-2229, July 2006. [143]

- [90] H. Boche, M. Schubert, N. Vucic, and S. Naik, "Non-Symmetric Nash Bargaining Solution for Resource Allocation in Wireless Networks and Connection to Interference Calculus", *Proceedings of European Signal Processing Conference*, Poznan, Poland, Sept. 2007. [143]
- [91] H. Boche, M. Schubert, and M. Wiczanowski, "An algebra for log-convex interference functions", *Proceedings of International Symposium on Information Theory and its Applications (ISITA)*, Seoul, Korea, Nov. 2006. [143]
- [92] Y. Zhao, L. kang Zeng, G. Xie, Y. an Liu, and F. Xiong, "Fairness based resource allocation for uplink OFDMA systems", *Journal of China Universities of Posts and Telecommunications*, vol. 15, no. 2, pp. 50-55, 2008. [143]
- [93] M. Nokleby, A. Swindlehurst, Y. Rong, and Y. Hua, "Cooperative power scheduling for wireless MIMO networks", *IEEE Global Telecommunications Conference (GLOBECOM)*, pp. 2982-2986, Nov. 2007. [143]
- [94] E. Jorswieck and E. Larsson, "The MISO Interference Channel from a Game-Theoretic Perspective: A combination of Selfishness and Altruism Achieves Pareto Optimality", *IEEE International Conference on Acoustics, Speech and Signal Processing*, pp. 5364-5367, April 2008. [143]
- [95] J. Gao, S. A. Vorobyov, and H. Jiang, "Game Theoretic Solutions for Precoding Strategies over the Interference Channel", *IEEE Global Telecommunications Conference (GLOBECOM)*, pp. 1-5, Nov. 2008. [143]
- [96] J. Gao, S.A. Vorobyov, H. Jiang, "Cooperative Precoding/Resource Allocation Games under Spectral Mask and Total Power Constraints", <http://arxiv.org/abs/0911.2847>, Nov. 2009. [143]
- [97] A. Feiten, and R. Mathar, "Optimal Power Control for Multiuser CDMA Channels", *International Symposium on Information Theory*, pp. 1903-1907, Adelaide, SA, Sep. 2005. [143]
- [98] T. Sandholm, K. Larson, M. Anderson, O. Shehory, and F. Tohme, "Coalition structure generation with worst case guarantees", *Artificial Intelligence*, vol. 10, pp. 209-238, Jul. 1999. [148]
- [99] W. Saad, Z. Han, M. Debbah, and A. Hjørungnes: "A Distributed Coalition Formation Framework for Fair User Cooperation in Wireless Networks", *IEEE Transactions on Wireless Communications*, 2009. [148]

-
- [100] J. Harold, G. Kushner, and Y. George, *Stochastic Approximation and Recursive Algorithms and Applications*, 2003. [150]
- [101] R. J. Aumann, "Acceptable Points in General Cooperative n-Person Games", in *Contributions to the Theory of Games IV*, Annals of Mathematics Study 40, edited by A. W. Tucker and R. D. Luce, Princeton: at the University Press, pp. 287-324, 1959. [150]
- [102] D. Moreno, and J. Wooders, "Coalition-Proof Equilibrium", *Games and Economic Behavior*, vol. 17, pp. 80-112, 1996. [157]
- [103] G. He, L. Cottatellucci and M. Debbah, "The Waterfilling Game-Theoretical Framework for Distributed Wireless Network Information Flow", *submitted to EURASIP Journal on Wireless Communications and Networking*, 2009. [35, 159]
- [104] G. He, M. Debbah, and S. Lasaulce: "N-player Bayesian Waterfilling Game for Multiple Access Channel", *The Third International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, Aruba, Dutch Antilles, Nov. 2009. [35, 160]
- [105] G. He, S. Gault, M. Debbah, and E. Altman, "Bayesian Waterfilling Game for Fading Multiple Access Channels", *submitted to Journal IEEE Transactions on Wireless Communications*, 2009. [35, 160]