Definition and performance analysis of new cooperative protocols

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Thesis report

Définition et analyse des performances de nouveaux protocoles coopératifs

Rapport de thèse

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A mon père et mon grand-père,
qui m'ont transmis leur curiosité
mathématique.

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English abstract

Cooperative techniques are a recent way to improve reliability of transmission in a wireless channel. Multiple nodes in the network cooperate together to form a virtual antenna array and exploit space-time or cooperative diversity. Cooperation is an attractive technique for industrials and a first relay has already been introduced in the LTE Advanced communication standard. In this thesis cooperative protocols are proposed and analyzed in terms of performance (outage probability and simulation results) and diversity-multiplexing gain tradeoff (DMT).

The relay channel is first investigated. In order to solve the problem of poor performance at low SNRs, an adaptive strategy is proposed for both amplify-and-forward (AF) and decode-and-forward (DF) protocols. This strategy allows to use the best transmission scheme in term of mutual information at each channel realization.

AF protocols have been extensively studied due to their low relaying complexity. On the contrary, defining an easily implementable and efficient DF protocol was still an open issue. In this thesis, based on an incomplete decoding at relays, we propose the Incomplete DF (IDF) protocol which provides both full rate and full diversity. Its DMT is similar to the one of the well-known non-orthogonal AF (NAF). In order to reduce the decoding complexity at relays, we also propose two decoding methods based on the space-time block code (STBC) structure and diophantine approximation respectively.

When several sources need to transmit simultaneously, the relay channel is generalized to the cooperative multiple access (CMA) channel. A practical implementation and two modifications of the CMA-NAF protocol defined by Azarian *et al.* are proposed, as well as a DF variant, the CMA-IDF. These protocols provide better asymptotic performance and their DMT is closer to the MISO bound.

Finally, if the direct link between source and destination is nonexistent or experiences too much fading, the relay channel is generalized to a multihop channel. In this thesis, a protocol is proposed for the K-parallel-path (KPP) network, based on path selection combined with a small STBC. This protocol achieves the optimal DMT of the KPP channel with a limited complexity.

French abstract

La coopération est une technique récente pour améliorer la fiabilité de transmission dans un canal sans-fil. Différents noeuds du réseau coopèrent afin de créer un réseau d'antennes virtuel et d'exploiter la diversité spatio-temporelle ou coopérative. Dans cette thèse, de nouveaux protocoles coopératifs sont proposés et leurs performances sont analysées en terme de probabilité de coupure, de simulations et de compromis diversité-gain de multiplexage (DMT).

On s'intéresse dans un premier temps au canal à relai. Afin de résoudre le problème des faibles performances à bas rapport signal-à-bruit (SNR), on propose une stratégie adaptative pour les protocoles "amplify-and-forward" (AF) et "decode-and-forward" (DF). Cette stratégie permet d'utiliser le meilleur système de transmission en terme d'information mutuelle à chaque nouvelle réalisation du canal.

Les protocoles AF ont été largement étudiés pour leur simplicité au relai. Au contraire, définir un protocole DF performant et facile à implémenter reste un problème non-résolu aujourd'hui. Dans cette thèse, on propose le protocole "Incomplete DF" (IDF) basé sur un décodage incomplet aux relais. Ce protocole permet d'atteindre les gains de multiplexage et de diversité maximaux, son DMT est le même que celui du protocol AF non-orthogonal (NAF). Afin de réduire la complexité aux relais, deux nouvelles méthodes de décodage sont proposées, basées pour l'une sur la structure des codes TAST, et pour l'autre sur une approximation diophantienne.

Si plusieurs sources veulent transmettre en même temps, le canal à relai se généralise au canal coopératif à accès multiple (CMA). Une implémentation pratique ainsi que deux modifications du protocole CMA-NAF défini par Azarian et al. sont proposées, ainsi qu'une variante utilisant une stratégie DF, le CMA-IDF. Ces protocoles permettent d'atteindre de meilleures performances asymptotiques et leur DMT est plus proche de la borne MISO.

Enfin, si le lien direct entre source et destination est inexistant ou soumis à un évanouissement trop important, le canal à relai se généralise au canal multi-saut. Dans cette thèse, on propose un protocole pour le canal "K-parallel-path" (KPP), basé sur la sélection de chemin combinée avec un code spatio-temporel de faible dimension. Ce protocole permet d'atteindre les limites théoriques du canal calculées par Sreeram et al, tout en ayant une complexité limitée.

Contents

French summary

Introduction

Au cours des dernières années, les communications sans-fil ont connu des progrès spectaculaires dus à l'émergence de nouvelles applications multimedia interactives et numériques, ainsi qu'à de rapides avancées dans le domaine des systèmes intégrés. Ces technologies requièrent des communications rapides et robustes. Malheureusement, les communications sans-fil sont soumises par nature à des limitations physiques telles que l'évanouissement.

Des techniques de diversité permettant de limiter et même d'exploiter le phénomène d'évanouissement sont utilisées pour améliorer les performances des communications sans-fil. Les techniques multi-antennaires (MIMO) en sont un example efficace qui permet d'exploiter les diversités spatiales et temporelles. Ces systèmes sont implémentés avec des antennes multiples à la source ainsi qu'à la destination.

Cependant, du à des limites de taille, de coût ou technologiques, la plupart des terminaux sans-fil ne peuvent pas supporter plusieurs antennes. Ce problème peut être résolu par l'exploitation d'un nouveau type de diversité connu comme la diversité de coopération [1–3]. Dans un système multi-utilisateurs, les techniques de coopération permettent aux mobiles de partager leurs antennes afin de créer un réseau d'antennes virtuel. Les terminaux peuvent alors partager leur ressources comme la puissance ou la largeur de bande, en relayant l'information d'autres utilisateurs.

Les protocoles coopératifs peuvent être répartis dans différentes familles selon le traitement effectué aux relais. Dans cette thèse, on s'intéresse à deux d'entre elles: 1) la stratégie "amplify-and-forward" (AF) consiste pour les relais à retransmettre une version amplifiée du signal précédemment reçu; 2) la stratégie "decode-and-forward" (DF) introduit plus de complexité puisque les relais doivent décoder les signaux reçus avant de les retransmettre.

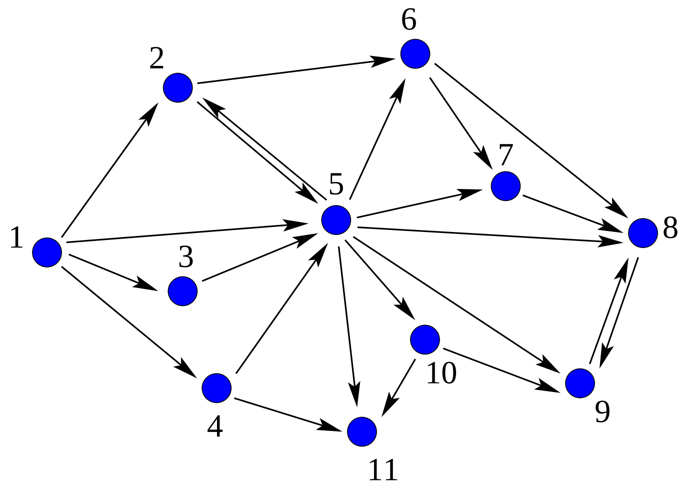


Figure 1: Exemple d'un réseau sans-fil

Un réseau sans-fil peut être représenté comme un graphe comportant des noeuds et des liens (cf Figure 1). Les noeuds représentent par exemple des terminaux mobiles, des stations de base ou des capteurs. Les liens représentent les canaux sans-fil entre deux noeuds. Le réseau peut être décomposé en éléments plus petits, comme les canaux suivants:

- le canal à relai : par exemple, le noeud 1 communique avec le noeud 5, aidé par les noeuds 2, 3 et 4;

- le canal à accès multiple coopératif (CMA) en présence ou non d'un relai dédié : par exemple, les noeuds 5 et 6 communiquent avec le noeud 8, aidés par le noeud 7, ou les noeuds 2 et 5 communiquent avec le noeud 6;

- le canal "broadcast" coopératif (CBC) en présence ou non d'un relai dédié : par exemple le noeud 5 transmet vers les noeuds 9 et 11, aidé par le noeud 10, ou le noeud 5 communique directement avec les noeuds 8 et 9.

On peut aussi définir des réseaux multi-sauts comme le canal K-parallel-path (KPP) ou le canal multi-sauts en couche. Les limites théoriques de la plupart de ces canaux ont déjà été établies, et des protocoles coopératifs permettant d'atteindre ces limites ont été proposés. Cependant, ils induisent souvent une forte complexité.

L'objectif de cette thèse est de proposer de nouveaux protocoles coopératifs AF et DF permettant d'atteindre les limites théoriques des canaux à relai, CMA et KPP, tout en ayant une complexité réduite.

Contributions et organisation de la thèse

Dans le Chapitre 1, on introduit le canal coopératif et on montre qu'il peut être modélisé comme un système MIMO. Les mêmes outils d'évaluation et les même techniques de diversité peuvent donc être utilisés. Les critères de construction des codes spatio-temporels en bloc (STBC) sont rappelés et les STBC utilisés dans cette thèse sont présentés.

L'état de l'art des protocoles AF et DF pour le canal à relai est présenté dans le Chapitre 2. Quelques problèmes d'implémentation des protocoles coopératifs y sont également étudiés comme la position du relai, l'allocation de puissance et l'influence d'une désynchronisation des signaux reçus à la destination. Cette étude montre que même pour des positions du relai et distribution de puissance optimales, et une synchronisation parfaite, les protocoles coopératifs ont de mauvaises performances à bas rapport signal à bruit (SNR). Pour résoudre ce problème, on propose une stratégie adaptative qui peut être utilisée avec les protocoles AF et DF. Cette nouvelle stratégie est basée sur les capacités instantanées de tous les schémas de transmission possibles.

Les protocoles AF ont été largement étudiés pour leur simplicité aux relais. Au contraire, les protocoles DF nécessitent un traitement plus lourd puisque les signaux doivent être décodés par les relais, et il n'existe pas à ce jour de protocole DF simple et efficace. Dans le Chapitre 3, on explore l'introduction de STBC distribués afin de définir un protocole DF pratique et ayant de bonnes performances. On propose d'abord deux protocoles DF aux rendements limités, le Alamouti DF ($\frac{1}{2}$ symbole par utilisation canal (symb. pcu)) et le "Asymmetric DF" (ADF) ($\frac{2}{3}$ symb. pcu). On définit ensuite le "Incomplete DF" (IDF) qui permet d'atteindre les même performances théoriques que le protocole AF non-orthogonal (NAF), meilleur protocole AF connu pour le canal à un relai. Ce nouveau protocole est basé sur un décodage incomplet aux relais qui peut être effectué par une recherche exhaustive. On propose également deux nouvelles méthodes de décodage permettant de réduire la complexité de la recherche exhaustive : une première méthode est basée sur la structure des codes TAST, et la seconde est basée sur une approximation diophantienne.

Le modèle du canal à relai s'appuie sur deux hypothèses : premièrement, une seule source transmet à la fois, et deuxièmement, il existe un lien direct entre cette source et la destination. Si on relâche la première hypothèse, le canal à relai devient un canal à accès multiple coopératif (CMA). On considère souvent une stratégie "time-division multiple-access" (TDMA). Cependant, si plusieurs sources veulent transmettre en même temps, de nouveaux protocoles coopératifs doivent être définis. Dans le Chapitre 4, on étudie le protocole CMA-NAF proposé par Azarian *et al* et on en donne une implémentation pratique. On propose également deux modifications de ce protocole permettant d'améliorer ses performances et son compromis diversité-gain de multiplexage (DMT). Enfin, on propose une variante de ce protocole utilisant une stratégie DF. Si on relâche la deuxième hypothèse, le cananl à relai devient un canal multi-sauts, qui peut être plus réaliste dans le contexte d'un réseau plus large. Les limites théoriques des canaux multi-sauts ont été étudiées et des protocoles permettant de les atteindre on été proposés. Cependant, souvent basés sur l'utilisation de STBC de grande dimension, ils induisent une forte complexité. Dans cette thèse, on propose un nouveau protocole multi-sauts pour le canal KPP, basé sur une sélection de chemin combinée avec un STBC de faible dimension. Ce protocole permet d'atteindre les limites théoriques du canal tout en n'introduisant qu'une faible complexité.

Chapter 1 : Diversité de coopération et codes spatio-temporels distribués

La diversité d'un système joue un rôle majeur dans la gestion de l'évanouissement du canal. Le principe est de transmettre un même message via différents chemins. La fiabilité de la transmission est alors augmentée, puisque des canaux indépendants sont soumis à des niveaux d'évanouissement indépendants. Il existe plusieurs types de diversité: 1) la diversité temporelle (un même signal est retransmis à différents instants, suffisamment éloignés dans le temps pour que les canaux utilisés soient indépendants), 2) la diversité fréquentielle (le signal est transmis à différentes fréquences, une nouvelle fois assez éloignées pour que les canaux soient indépendants), 3) la diversité spatiale (un même signal est transmis via différents trajets, c'est par exemple le cas pour les systèmes multi-antennaires) et 4) la diversité de polarisation (le signal est transmis et recu par des antennes de polarisations différentes). Dans cette thèse, on s'intéresse à une généralisation de la diversité spatiale appelée diversité coopérative, car plusieurs terminaux du réseau aident à transmettre l'information.

Les communications coopératives sont une nouvelle technique pour améliorer la fiabilité des tranmissions dans un canal sans-fil. Plusieurs terminaux échangent leurs informations pour former un système multi-antennaire virtuel et exploiter la diversité inhérente aux réseaux sasn-fil. Chaque source envoie un message qui est récu non seulement par la destination, mais aussi par tous les autres terminaux disponibles pour aider à la transmission. Après traitement du signal recu, ces terminaux (appelés dans la suite "relais") renvoient l'information traitée à travers le réseau. Cette stratégie permet de réduire le taux d'erreur à la réception, puisque la probabilité que tous les chemins utilisés pour transmettre l'information soient soumis à un fort évanouissement est très faible. Cette stratégie augmente l'ordre de diversité du système, défini comme le nombre de chemins indépendants de la source vers la destination.

Un protocole coopératif dépend fortement du type de relais utilisés et du traitement du signal effectués à ces relais. Deux types de relais peuvent être considérés: 1) des relais "full-duplex" sont capables de recevoir et transmettre des signaux simultanément, 2) des relais "half-duplex" au contraire doivent effectuer ces deux actions l'une à la suite de l'autre. Ces derniers sont généralement utilisés pour leur simplicité d'implémentation. Il existe une variété de traitements pouvant être effectués aux relais: "amplify-and-forward" (AF), "decode-and-forward" (DF), "compress-and-forward" (CF), "quantize-and-forward" (QF) etc... Dans cette thèse, nous n'étudions que les deux premiers. La stratégie AF est souvent utilisée pour sa simplicité. En effet, le signal recu est simplement retransmis après une normalisation de puissance. L'inconvénient est que le bruit est également retransmis et s'accumule donc au fil de la transmission. La stratégie DF implique une complexité plus importante puisque les signaux doivent être décodés aux relais avant d'être retransmis. Le challenge est alors de ne pas retransmettre des erreurs de décodage.

Un réseau coopératif peut être modélisé comme un système multi-antennaire ("multiple-input multiple-output", MIMO) dont la matrice du canal équivalent a une structure particulière (de forme triangulaire inférieure). Par conséquent, les techniques de codage spatio-temporel développées pour les systèmes MIMO peuvent également être utilisées dans les réseaux coopératifs, de facon distribuée.

Les principaux outils pour évaluer les performances d'un système MIMO peuvent donc également être utilisés pour évaluer celles d'un réseau coopératif: 1) la capacité du canal, qui est la quantité d'information maximale qui peut être transmise à travers le canal avec aussi peu d'erreurs de décodage que voulu, 2) la probabilité de coupure, qui est la probabilité que la capacité pour une réalisation donnée du canal soit inférieure au taux de transmission, et 3) le compromis de diversité-gain de multiplexage (DMT). Ces différentes mesures des performances de la transmission peuvent être optimisées par l'utilisation de techniques spatio-temporelles adéquates. L'étude de la probabilité d'erreur par paire de deux mots d'un code spatio-temporel (STBC) permet d'établir des critères de construction de ces codes, et en particulier: 1) le critère du rang, qui permet d'optimiser la diversité du système et 2) le critère du déterminant, qui permet d'optimiser le gain de codage. Ces critères ont servi à la définition d'un certains nombre de STBCs permettant d'atteindre des performances et des complexités variées.

Une première catégorie de STBCs sont les codes orthogonaux. Leur construction ne leur permet pas d'atteindre un rendement plein, mais assure un décodage linéaire donc avec une complexité très faible. Les codes algébriques (DAST and TAST) ont été développés pour résoudre le problème de faible rendement des codes orthogonaux. Les codes TAST atteignent un rendement et une diversité pleine, mais leurs performances décroissent avec la taille de la modulation. Les codes parfaits, dont le cas particulier du "Golden code" sont également des codes algébriques. Leur construction leur garantit une diversité et un rendement pleins, ainsi qu'un gain de codage minimum, quelle que soit la modulation utilisée. Contrairement aux codes orthogonaux, les codes algébriques impliquent une complexité de décodage élevée. Pour exploiter totalement la diversité du système, un décodage "maximum-likelyhood" (ML) doit être effectué. La complexité d'une recherche exhaustive peut être réduite à celle d'une recherche dans un espace fini, généralement une sphère, mais reste importante. Les deux principaux algorithmes utilisés sont le décodeur par sphère et l'algorithme de Schnorr-Euchner.

Chapter 2 : Performance des protocoles coopératifs pour le canal à relai

Les chapitres 2 et 3 sont consacrés à l'étude du canal à relai. Le canal à relai est la plus petite décomposition d'un canal coopératif. Une source unique veut transmettre un message à une destination unique, et peut pour cela être aidée par un ou plusieurs relais dans le réseau, qui sont assez proches de la source et de la destination.

Dans le cas de terminaux uni-antennaires, certains protocoles coopératifs proposés dans la littérature permettent d'atteindre un rendement plein de 1 symbole par utilisation canal, et une diversité pleine d'ordre $N+1$, où N est le nombre de relais utilisés. Le challenge est d'atteindre le DMT maximale, qui est celui d'un système MISO $(N+1)\times 1$. Quand les terminaux sont équippés de plusieurs antennes, les rendements et ordres de diversité optimaux, ainsi que les expressions du DMT, sont un peu plus complexes, mais le challenge reste le même: comment atteindre le DMT maximal (ou au moins en être le plus proche possible) sans introduire une trop grande complexité des protocoles coopératifs.

Avant de se consacrer à la définition de nouveaux protocoles coopératifs, on rappelle ici les principaux protocoles AF et DF existants dans la litérature. Du à sa simplicité, c'est la stratégie AF qui est le plus souvent considérée. Les principaux protocoles AF sont 1) le protocole AF orthogonal (OAF), 2) le protocole AF non-orthogonal (NAF) et 3) le protocole "slotted AF" (SAF). Le protocole OAF doit son nom à son orthogonalité, i.e. à un instant donné, un seul terminal est autorisé à transmettre. Cette propriété permet d'éviter des interférences entre les différents transmetteurs et donc de diminuer la complexité du système, et d'allouer la totalité de la puissance disponible à un seul émetteur. Cependant la source ne transmet alors qu'une utilisation canal sur deux, et le rendement est diminué de moitié. L'étude théorique du protocole OAF montre qu'il peut atteindre un DMT de $d^*(r) = (N+1)(1-2r)^+$. En particulier, il atteint une diversité optimale de $N+1$, mais un rendement réduit de $\frac{1}{2}$ symbole par utilisation canal. Contrairement au protocole OAF, le protocole NAF autorise la source et un relai à transmettre simultanément. Cela permet de résoudre le problème de faible rendement puisque la source peut alors transmettre en continu. L'étude du protocole NAF montre qu'il peut atteindre un DMT de $d^*(r) = (1-r)^+ + N(1-2r)^+$. En particulier, la diversité optimale de $N+1$ est atteinte, ainsi que le rendement plein de 1 symbole par utilisation canal. Néanmoins, le DMT n'est pas optimal puisqu'il n'atteint pas la borne correspondant à un système MISO $(N+1)\times 1$. Ces performances théoriques peuvent être atteinte en pratique en implémentant le protocole NAF avec des STBCs optimaux de facon distribuée. Le protocole SAF permet de réduire la différence entre le DMT du NAF et celui d'un système MISO $(N+1)\times 1$. Il s'agit également d'un protocole non-orthogonal. La différence avec le protocole NAF réside dans le fait qu'il autorise également les relais à écouter les signaux émis non seulement par la source mais aussi par les autres relais. Cela rend le système beaucoup plus complexe, mais permet aussi d'atteindre de meilleures performances. L'étude théorique montre que le protocole SAF permet d'atteindre le DMT de $d^*(r) = (1-r)^+ + N\left(1 - \frac{M+1}{M}r\right)^+$, où M

est la longueur temporelle de la transmission. En particulier, si la longueur de la transmission tend vers l'infini, le DMT du protocole SAF tend vers le DMT optimal d'un système MISO $(N+1) \times 1$. Il est cependant utile de noter qu'une implémentation pratique de ce protocole est complexe et que les auteurs utilisent en pratique le "naive" SAF qui néglige les interfŕences entre relais.

Un certain nombre de protocoles DF ont également été proposés, et en particulier les protocoles LTW, NBK et DDF. Un stratégie DF imposant d'avoir correctement décodé au relai pour ne pas retransmettre d'erreurs, les protocoles LTW et NBK sont d'abord soumis à une sélection dépendant de la qualité du lien source-relai. En pratique, si un relai est capable de décoder le message venant de la source, il le retransmet. Si au contraire un relai est incapable de décoder sans erreur, il reste muet. Dans le cas où aucun relai n'est capable de décoder le message provenant de la source, seul le lien direct entre source et destination est utilisé pour la transmission. Cette sélection se base dans le cas théorique sur la probabilité de coupure, et en pratique peut se baser sur une probabilité d'erreur cible. Le protocole proposé par Laneman, Tse et Wornell (LTW) est un protocole orthogonal très semblable au protocole OAF et permet d'atteindre le même DMT de $d^*(r) = (N+1)(1-2r)^+$. Le protocole proposé par Nabar, Bolcskei et Kneubuhler (NBK) pour résoudre le problème de faible rendement du LTW est très semblable au protocole NAF. La différence réside dans le fait que les signaux devant être décodés aux relais, un STBC distribué ne peut pas être utilisé, et la diversité est donc réduite. Son étude théorique montre que le protocole NBK permet d'atteindre un DMT de $d^*(r) = 1$ si $0 \leq r \leq \frac{N}{2(N+1)}$ et $d^*(r) = \frac{2(N+1)}{N+2}(1-r)^+$ si $\frac{N}{2(N+1)} \leq r \leq 1$. En particulier, il ne permet pas d'exploiter la diversité inhérente aux transmissions sans-fil. Enfin, le protocole DF dynamique (DDF) a été proposé pour permettre d'atteindre les diversité et rendement optimaux. Le principe est de laisser les relais écouter les signaux transmis par la source et les potentiels autres relais jusqu'à ce qu'ils soient capables de décoder. Ils peuvent alors commencer à retransmettre. Chaque relai peut donc commencer à transmettre à un instant différent, et la définition de codes pour ce protocole reste un problème complexe mais prometteur, car ils permettraient d'atteindre un DMT supérieur à ceux du LTW et du NBK.

Dans une seconde partie de ce chapitre, nous présentons quelques problèmes pratiques liés à l'implémentation des communications coopératives. Un premier paramètre est la localisation des relais. En étudiant les performances théoriques des protocoles OAF et NAF dans un canal à un relai, on peut montrer que le placement optimal du relai dépend de la propriété d'orthogonalité du protocole. Pour le protocole OAF, les performances sont améliorées si le relai est placé proche de la source ou proche de la destination. Pour le protocole NAF, les performances sont améliorées si le relai est placé proche de la source, mais dégradées si le relai est proche de la destination. Pour une stratégie AF, on peut donc conclure que les performances sont meilleures si les relais sont proches de la source. Un second paramètre à optimiser est la distribution de puissance entre la source et les relais. Si le protocole est orthogonal, la puissance totale doit en effet être partagée entre les différents transmetteurs. L'étude des performances du protocole NAF permet de montrer que l'influence de la distribution de puissance est plus importante à faible SNR, et que la puissance dédiée au relai doit alors être faible. Il retransmet alors en effet plus de bruit que d'information utile. A fort SNR, l'influence de la distribution de puissance est limitée, et dans la suite, on considèrera dont toujours une distribution de puissance uniforme entre la source et les relais. Enfin, un des problèmes pratiques majeurs des communications coopératives est une possible desynchronisation des signaux recus à la destination. On montre ici que si on utilise le lien direct entre la source et la destination et des terminaux “half-duplex”, la diversité est préservée même en cas de désynchronisation. Il n'est donc pas nécessaire d'utiliser des codes spécifiques qui tolèrent les délais.

La troisième et dernière partie de ce chapitre est consacrée à la première contribution de cette thèse, qui est la définition d'une stratégie adaptative permettant d'améliorer les performances de tout protocole coopératif pour toute la gamme de SNRs. L'étude des protocoles coopératifs montrent en effet qu'à faible SNR, leurs performances sont décevantes, et en particuler inférieures à celles de la transmission directe entre la source et la destination. Cela est du au fait qu'un partie de la puissance de tranmission est donnée au relai. Afin de résoudre ce problème, on propose donc de sélectionner pour chaque réalisation du canal la stratégie de transmission la plus efficace parmi la transmission directe source-destination, les stratégies coopératives avec différents nombres de relais, et les transmissions utilisant uniquement les liens source-relai-destination. Le critère de sélection développé dans cette thèse est basé sur l'étude des capacités instantanées ou informations mutuelles de ces différentes stratégies de transmission. Celle permettant d'atteindre la capacité instantanée la plus haute est choisie. Cette technique qui ne contribue que très peu à la complexité du traitement de l'information aux relais et à la destination permet cependant des gains importants allant jusqu'à 5 dB.

Chapter 3 : Vers un protocole DF pratique pour le canal à relai

Dans ce chapitre, on s'intéresse aux protocoles DF pour le canal à relai. Les protocoles DF nécessitent un traitement du signal plus complexe au niveau des relais que les protocoles AF, puisque les signaux recus doivent être décodés avant d'être retransmis. Cependant, si les signaux sont correctement décodés aux relais, les protocoles DF ont l'avantage de supprimer le bruit aux relais, au lieu de l'accumuler et de l'amplifier. Cette propriété à un impact d'autant plus important pour des sytèmes multi-sauts où les signaux sont relayés plusieurs fois avant d'atteindre la destination. Par conséquent il est nécessaire de définir des protocoles DF efficaces et facilement implémentables. Les protocoles DF existants ne permettent pas d'atteindre une diversité et un rendement pleins avec une complexité réduite. Nous nous proposons donc dans ce chapitre de définir un nouveau protocole DF permettant de résoudre ce problème, basé sur l'utilisation de codes spatio-temporels

(STBC) distribués.

En première approche, on considère un des STBCs les plus simples, le code de Alamouti, et un unique relai. Les symboles modulés sont d'abord envoyés par la source et recus par le relai et la destination. Dans un second temps, le relai décode puis répète ces symbboles pendant que la source émet leurs conjugués selon le code de Alamouti. Ce protocole permet d'obtenir de bonnes performances, en particulier pour des petites modulations, avec une complexité limitée, puisque le décodage du code de Alamouti est linéaire. L'étude théorique du Alamouti DF montre que son DMT est $d^*(r) = 2(1-2r)^+$. En particulier, le protocole permet d'atteindre l'ordre de diversité maximal 2, mais le rendement est réduit à $\frac{1}{2}$.

Afin de résoudre le problème de faible rendement du Alamouti DF, en seconde approche, on veut utiliser des codes spatio-temporels plus avancés comme les codes algébriques. On propose un nouveau protocole DF qu'on appelle dans la suite DF asymétrique (ADF) car les phases de réception et de transmission des relais sont de tailles différentes. Pendant une première phase donc, la source transmet la totalité de l'information, soit $2N$ symboles, sous la forme de $2N$ signaux codés. A la fin de cette phase, les relais tentent de décoder l'information, et s'ils en sont capables, retransmettent les N premiers signaux codés, pendant que la source retransmet les N derniers signaux codés. Le rendement de ce protocole n'est donc pas optimal ($R = \frac{2}{3} < 1$), mais supérieur à celui du Alamouti DF. L'étude des performances théoriques de l'ADF montre que son DMT est $d^*(r) = (1 - \frac{3}{2}r)^+ + N(1 - 3\frac{2N-1}{2N}r)^+$. En particulier, l'ADF permet d'atteindre l'ordre de diversité maximal de $N + 1$, et le rendement réduit de $\frac{2}{3}$ est confirmé. Ce DMT est atteignable en implémentant l'ADF avec un STBC parfait.

Le rendement de l'ADF n'est pas optimal du à la longueur de sa première phase. On cherche donc à réduire la longueur de cette première phase, ce qui nous conduit à la définition d'un nouveau protocole DF qu'on appelle le DF incomplet (IDF). Il est basé sur un décodage partiel aux relais. En effet, si on réduit la taille de la phase de réception des relais, ils recoivent moins de signaux que de symboles à décoder. L'idée est donc pour les relais de décoder une combinaison de symboles, sans décoder les symboles eux-même. Comme pour l'ADF, la transmission est divisée en deux phases, mais cette fois-ci de longueurs égales. Durant la première phase, la source envoie la totalité de l'information, soit $2N$ symboles, sous la forme de N signaux codés. Les relais détectent ces combinaisons de symboles et les retransmettent pendant la seconde phase, pendant que le source transmet N nouveaux signaux codés contenant également la totalité de l'information. Le rendement est donc bien de 1 symbole par ulisation canal. L'étude théorique de l'IDF monte que son DMT est le même que celui du protocole NAF, soit $d^*(r) = (1-r)^+ + N(1-2r)^+$. En particulier les ordre de diversité plein de $N + 1$ et rendement plein de 1 sont bien atteint. Une nouvelle fois, ces performances théoriques peuvent être atteintes en implémentant le protocole IDF avec des STBC parfaits.

Le principal challenge du protocole IDF reste donc le décodage aux relais. Une recherche exhaustive donne des résultats optimaux, mais implique également une complexité importante. On propose donc deux méthodes pour réduire cette complexité.

- Une première méthode est basée sur l'analogie entre le décodage au relai d'une combinaison de deux symboles et une approximation diophantienne. Les algorithmes développés pour résoudre ce problème dans la litérature peuvent être adaptés pour décoder aux relais. On montre par simulation que la complexité est alors de l'ordre de $\sqrt[4]{M}$, où M est la taille de la constellation utilisée par la source. Cette complexité est bien inférieure à celle d'un décodage exhaustif qui est de l'ordre de M^2. Les algorithmes d'approximation diophantienne ne sont pas optimaux. On montre cependant par simulation que les pertes de performances sont inférieures à 1 dB dans le cas d'un système à un relai.
- Une seconde méthode est basée sur l'utilisation des codes TAST et de leur structure particulière. En effet, les coefficients du codes TAST sont associés deux à deux, ce qui permet de décomposer le décodage en deux étapes successives, et de se ramener à un nombre fini de recherches dans un alphabet de M^2 éléments. Ainsi, dans le cas de deux relais, on passe d'une complexité de l'ordre de M^4 pour une recherche exhaustive, à une complexité de l'ordre de M^2. Cet décroissance de la complexité à un prix: les codes TAST n'étant pas optimaux, les performances sont dégradées par rapport à celles obtenues en utilisant un code parfait. Cependant, la différence est inférieure à 1 dB, ce qui reste un très bon compromis.

Ces deux méthodes peuvent être combinées pour réduire d'autant plus la complexité. En particulier, on peut montrer que dans le cas de deux relais, l'ordre de complexité est réduit de M^4 pour une recherche exhaustive à $\sqrt[4]{M}$ pour la combinaison des deux méthodes de réduction de complexité. Les performances, elles, ne sont réduites que de 1 dB.

Chapter 4 : Généralisation du canal à relai

Dans les deux chapitres prédédents, on s'est limité à l'étude d'un canal à relai avec une unique source, un certain nombre de relais dédiés et une unique destination. Ce modèle impose deux hypothèses qui ne s'adaptent pas au contexte d'un réseau plus large et plus complexe. Une première limitation est liée à l'usage d'une stratégie "time-division multiple-access" (TDMA). A un instant donné, un terminal du réseau joue le rôle de la source, tandis que les autres aident à la transmission en tant que relais. Les rôles sont ensuite échangés pour permettre à tous de transmettre. Cependant, ils est possible pour certaines applications en temps réel par exemple que tous les utilisateurs aient besoin de transmettre simultanément. Pour exploiter la diversité de coopération, on doit alors considérer un nouveau modèle et développer de nouvelles stratégies adaptées à ce modèle.

Dans la première partie de ce chapitre, on étudie un réseau composé de deux sources qui veulent chacune transmettre leur message à la même destination. Dans le cadre des communications multi-accès classiques, une simple stratégie TDMA est utilisée, et chaque source communique avec la destination à tour de rôle. Les techniques coopératives peuvent ici apporter deux avantages: 1) l'utilisation des deux sources de facon coopérative permet d'exploiter la diversité du système, et 2) en coopérant, les deux sources peuvent transmettre simultanément, permettant ainsi des applications en temps réel. Dans ce modèle, il n'y a pas de relai dédié; chaque source transmet son propre signal et relaie celui de l'autre source. En pratique, les deux sources écoutent et transmettent de facon orthogonale, i.e. si au temps t la première source transmet une combinaison de son propre message et de celui de l'autre source, la seconde source écoute. Au temps suivant $t+1$, c'est au tour de la seconde source de transmettre une combonaison des deux messages. Les performances théoriques de ce protocole appelé CMA-NAF ont été étudiées pour le cas d'une stratégie AF. Son DMT est alors $d^*(r) = (1-r)^+ + (1 - \frac{M}{M-1}r)^+$ qui tend vers le DMT optimal du système $d^*(r) = N(1-r)^+$ quand la longueur de la transmission M tend vers l'infini. Dans un premier temps, on propose d'implémenter ce protocole avec un code parfait afin d'atteindre ces performances théoriques en pratique. Afin d'améliorer les performances, on propose dans un second temps deux améliorations du protocole:

- le CMA-NAF original est asymétrique. L'information transmise par le premier utilisateur à transmettre est mieux protégée que celle du second. En effet, la totalité des signaux transmis par le premier utilisateur sont répétés par le second. Au contraire, le dernier signal transmis pas le second utilisateur n'est pas retransmis. Afin de pallier à ce deséquilibre, on propose d'échanger le rôle des deux sources à chaque transmission. Le DMT devient $d^*(r) = (1-r)^+ + (1 - \frac{2M}{2M-1}r)^+$ qui tend deux fois plus vite vers le DMT optimal.

- une seconde amélioration est liée à la répartition de l'énergie entre les différents signaux transmis. Dans le cas du CMA-NAF original, la totalité des signaux recus à chaque source est retransmise. La puissance totale utilisée pour transmettre les premiers signaux est donc plus importante que celle utilisée par les derniers signaux. Pour atténuer cet effet, on propose d'éliminer à chaque source la partie du signal provenant des précédentes transmissions. Les signaux transmis ne sont ainsi la combinaison que de deux symboles, d'où une meilleure distribution de l'énergie. Cette amélioration permet un meilleur gain de codage que l'on peut observer sur les courbes de probabilité de coupure et celles de taux d'erreur.

On propose également une version DF de ce protocole. La stratégie de transmission est identique, excepté pour le traitement des signaux recus à chaque source. Comme dans le cas du canal à relai étudié précedemment, le protocole DF n'est utile que si les signaux peuvent être correctement décodés à chaque source. Si ce n'est pas le cas, les signaux ne sont pas relayés, et une simple stratégie TDMA est utilisée. A cause de cette sélection nécessaire aux protocoles DF, le DMT de ce protocole est inférieur à celui du cas AF. On prouve que le DMT est identique à celui du NAF et de l'IDF dans un canal à relai:

$d^(r) = (1-r)^+ + (1-2r)^+$.

Une seconde limitation est liée à l'existence du lien direct entre la source et la destination. Dans un réseau plus large, ce lien peut être inexistant ou soumis à un évanouissement trop important pour être utile. Des relais sont alors utilisés en série pour transmettre l'information. On parle de communications multi-sauts. Depuis quelques années, ce modèle de canal est de plus en plus présent dans la litérature du à de prometteuses applications comme les réseaux de capteurs ou les réseaux ad-hoc.

Dans la seconde partie de ce chapitre, on s'intéresse à un modèle particulier de communications multi-sauts basé sur l'utilisation de K chemins parallèles (KPP). Les performances théoriques de ce modèle ont été calculées par Sreeram *et al.* [4, 5]. Un DMT de $d^*(r) = K(1-r)^+$ peut être atteint, où K est le nombre de chemins parallèles (et donc indépendants) utlisés. Les auteurs prouvent également que dans le cas d'interférences entre les chemins, ce DMT est toujours atteignable. Afin d'atteindre les performances théoriques de ce modèle, ils proposent un protocole basé sur la théorie des graphes. Ce protocole permet en effet d'obtenir d'excellentes performances, mais son implémentation pratique est complexe. Dans cette thèse, on propose une implémentation basée sur la sélection de chemin et un STBC de petite dimension, qui malgré sa faible complexité, permet également d'atteindre le DMT optimal du système. Le principe de ce protocole et de choisir un petit nombre S de chemins à utiliser. Le critère de sélection est le SNR de ces chemins à la destination. Afin d'obtenir un rendement plein dans le cas de terminaux half-duplex, on peut montrer que le nombre de chamins sélectionnés $S \geq 2$. Du à la nature des transmissions sans-fil, le signal peut également se propager à reculon, en créant ainsi des interférences supplémentaires. Afin d'éviter ces interférences qui peuvent avoir un impact négatif important sur les performances du système, quand un terminal transmet, on empêche le terminal précédent dans un même chemin d'écouter. Le nombre minimal de chemin sélectionnés est alors $S \geq 3$. Afin de limiter la perte de gain de codage et la complexité qui augmentent avec le nombre de chemins choisis, on se limite dans la suite à $S = 3$. Ce protocole peut être utilisé avec des stratégies AF ou DF aux relais, et des longueurs de chemins différentes. Avant de conclure, on étudie quelques problèmes liés à l'implémentation de ce protocole:

- le rendement plein de ce protocole est obtenu pour une trame de transmission de longueur infinie. Ceci est impossible en pratique, et pour un trame de longueur M, le rendement n'est que de $\frac{M}{n+M-1}$, où n est la longueur du plus long chemin. On montre cependant par l'étude de la probabilité de coupure du protocole que une trame de longueur $M = 33$ est suffisante pour obtenir des performances à moins de 1 dB des performances optimales.

- enfin, on étudie l'influence des interférences entre chemins sur les performances, et on observe pour l'exemple de deux chemins à un relai, que pour des interférences de même puissance que les signaux, une perte de 5 dB. Pour des interférences de puissance plus faible, les pertes sont limitées. Ce modèle est donc particulièrement adpaté au cas où les chemins sont physiquement séparés (e.g. des couloirs ou des rues parallèles).

Conclusion et perspectives

Dans cette thèse, de nouveaux protocoles "amplify-and-forward" (AF) et "decode-and-forward" (DF) ont été proposés pour différents canaux sans-fil.

Une étude du comportement des protocoles coopératifs pour différentes positione du relai et différentes distributions de puissance a montré que les performances à bas SNR sont toujours décevantes, puisque la simple tranmission directe de la source à la destination donne de meilleurs résultats. Afin de résoudre ce problème, une nouvelle stratégie adaptative, pouvant s'appliquer aux protocoles AF comme aux protocoles DF, a été proposée. Le principe de cette stratégie est de comparer les capacités instantanées de tous les schémas de transmission possibles (coopération, non-coopération, relais en série). Il est prouvé que cette stratégie permet d'optimiser les performances des protocoles coopératifs considérés en terme de probabilité de coupure et de probabilité d'erreur, sans affecter le DMT.

Dans le cas du canal à relai, la stratégie AF a été largement étudiée et des protocoles permettant d'atteindre les limites théoriques du canal ont été proposés. Ce n'est cependant pas le cas pour la stratégie DF, dont la complexité est plus importante du au décodage aux relais. Dans cette thèse, de nouveaux protocoles DF ont été proposés, en commençant par le Alamouti DF. Ce premier example est néanmoins limité à l'usage d'un seul relai et son taux de transmission maximum est de $\frac{1}{2}$ par utilisation canal (symb. pcu). Afin de contrecarrer ces limites, un nouveau protocole utilisant des codes spatio-temporels plus évolués est proposé, dont l'appelation "Asymmetric" DF est due à ses deux phases (transmission et restransmission) de longueurs différentes. Le taux de transmission de ce protocole est cependant lui aussi limité, ne pouvant dépasser $\frac{2}{3}$ symb. pcu. Enfin, un dernier protocole, appelé "Incomplete" DF, a été proposé, basé sur un décodage partiel aux relais. Ce protocole permet d'atteindre un taux de transmission et une diversité maximaux, et a le même DMT que le protocole NAF. Un décodage exhaustif aux relais imposant une forte complexité, deux méthodes permettant de la réduire ont été proposées: l'une d'elles est basée sur la structure des codes TAST, et l'autre sur une approximation diophantienne. Cette seconde méthode est sous-optimale en terme de décodage, mais les résultats de simulation montrent que la perte de performance est inférieure à 1 dB, pour une complexité considérablement réduite. Ces deux méthodes de décodage peuvent être combinées pour obtenir une complexité encore plus basse.

Quand plusieurs sources veulent transmettre simultanément vers la même destination, une stratégie de multi-accès coopérative (CMA) est considérée. Dans cette thèse, une implémentation pratique, ainsi que deux modifications du protocole CMA-NAF, sont proposées, qui permettent d'améliorer le DMT et les performances en terme de probabilité de coupure et de probabilité d'erreur. Une version DF du protocole CMA est aussi proposée, basée sur le même principe que le protocole IDF précédemment présenté. Son DMT est limité à cause de la sélection initiale avec la non-coopération, nécessaire pour guarantir le décodage aux relais, mais ses performances sont similaires à celles du cas AF.

Enfin, le cas du canal multi-saut à K trajets parallèle est étudié. Un protocole à complexité réduite est proposé, basé sur la combinaison de la sélection de trajet et l'utilisation d'un code spatio-temporel de faible dimension. Ce protocole permet d'atteindre les limites théoriques du canal. Il peut être utilisé avec une stratégie AF, ou une combinaison des stratégies AF et DF afin d'améliorer les performances, en particulier quand de nombreux sauts sont considérés.

De possible directions pour des travaux futurs sont:

Approximation diophantienne pour le décodage MIMO. Dans cette thèse, des algorithmes permettant de trouver une bonne approximation diophantienne ont été utilisés avec succès pour décoder un système à déficience de rang aux relais, tout en réduisant la complexité. Ces algorithmes pouraient être utilisés dans d'autres systèmes à déficience de rang (le canal multi-accès par example), ou même adaptés au décodage MIMO, afin de proposer un décodeur permettant d'atteindre de bonnes (mais pas optimales) performances pour une faible complexité.

Coopération dans le cas de plusieurs utilisateurs. La plupart des protocoles coopératifs proposés ont été développés pour des systèmes à une seule source et une destination. Cependant, dans un réseau de capteurs, plusieurs noeuds veulent transmettre leurs données vers un noeud central ou tous les noeuds du réseau. Le cas de deux sources transmettant vers la même destination a déjà été considéré dans cette thèse. Il serait intéressant de généraliser cette approche à un plus grand nombre de sources, et d'étudier le canal dual, i.e. le canal "broadcast".

Mauvaise estimation du canal ou absence d'information sur le canal à la destination. Tout au long de cette thèse et dans la plupart des études existantes, une estimation parfaite du canal est considérée. Cependant, cette hypothèse idéale est rarement vérifiée, et en général, des erreurs ont lieu lors de l'estimation du canal. Il serait donc intéressant d'étudier l'influence de ces erreurs sur les performances des protocoles coopératifs, et de développer des techniques pour pallier aux erreurs d'estimation du canal.

On peut aussi considérer le cas ou aucune information sur le canal n'est disponible à la destination. Il faut alors utiliser des codes spatio-temporels et des algorithmes de décodages non-cohérents. L'étude des performances de ces techniques appliquées au canal coopératif et en particulier aux protocoles proposés dans cette thèse est encore à venir.

Codage réseau san fil. Le codage réseau est une généralisation prometteuse du routage. Les neouds intermédiaires du réseau non seulement relaient l'information, mais l'encode également. Ils peuvent combiner différents signaux reçus de manière à ce qu'ils soient facilement décodables. Ils permettent donc de transmettre une plus grande quantité d'information et donc d'améliorer le taux de transmission. Mais ceci n'est pas le seul avantage du codage réseau, cette nouvelle stratégie permet également de mieux partager les resources et donc d'économiser la puissance et la bande de fréquence. Enfin, cette stratégie est plus robuste dans le cas de sytèmes dynamiques.

Le codage réseau peut être utiliser dans des réseaux cablés ou sans-fil. Néanmoins, les réseaux sans-fil semblent être un environnement plus naturel. En effet, les propriétés des communications sans-fil qui complexifient le routage (évanouissement et "broadcast") sont intéressantes dans le cas du codage.

List of Figures

List of notation

Sets

$\mathbb{R}$	Field of real numbers
$\mathbb{C}$	Field of complex numbers
$\mathbb{Q}$	Field of rational numbers
$\mathbb{Z}$	Ring of real integers
$\mathbb{K}^*$	$\mathbb{K} - 0$
$\mathcal{O}_{\mathbb{K}}$	Ring of integers of the number field $\mathbb{K}$

Basic operations

$E(x)$	Expected value of the random variable x
$\Pr\{\mathcal{O}\}$	Probability of the event $\mathcal{O}$
$\mathcal{R}(x)$	Real part of x
$\mathcal{I}(x)$	Imaginary part of x
$(x)^+$	$\max(0, x)$

Vector and matrices

$\mathbf{v}$	Vector
$\mathbf{M}$	Matrix
$\mathbf{I}$	Identity matrix
$\mathbf{I}_n$	Identity matrix of size $n \times n$
$\det(\mathbf{M})$	Determinant of square matrix $\mathbf{M}$
$\mathbf{M}^\dagger$	Conjugate-transpose of matrix $\mathbf{M}$

Relations

Symbols $\doteq$, $\dot{\leq}$ and $\dot{\geq}$ denote the asymptotic behavior of the considered variable when the signal-to-noise ratio (SNR) grows to infinity.

Acronyms

ADF	Asymmetric Decode-and-Forward
AF	Amplify-and-Forward
AWGN	Additive White Gaussian Noise
BS	Base Station
CC	Coded Cooperation
CF	Compress-and-Forward
CMA	Cooperative Multiple Acces
DAST	Diagonal Algebraic Space-Time code
dB	deciBel
DDF	Dynamic Decode-and-Forward
DF	Decode-and-Forward
DMC	Discrete Memoryless Channel
DMT	Diversity-Multiplexing gain Trade-off
EEAS	End-to-End Antenna Selection
FER	Frame Error Rate
GC	Golden Code
IDF	Incomplete Decode-and-Forward
iid	identical and independantly distributed
KPP	K-Parallel Paths
LOS	Line-Of-Sight
LTW	Laneman Tse and Wornell protocol
MAC	Multiple Access Channel
MARC	Multiple Access Relay Channel
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
MMSE	Minimum Mean Square Error
NAF	Non-orthogonal Amplify-and-Forward
NBK	Nabar Bölcskei and Kneubühler protocol
NLOS	Non Line-Of-Sight
NVD	Non-Vanishing Determinant
OAF	Orthogonal Amplify-and-Forward

PC	Perfect Code
pcu	per channel use
PEP	Pairwise Error Probability
QAM	Quadratic Amplitude Modulation
QF	Quantize-and-Forward
QPSK	Quadratic Phase Shift Keying
SAF	Slotted Amplify-and-Forward
SD	Sphere Decoder
SE	Schnorr-Euchner
SNR	Signal to Noise Ratio
STBC	Space-Time Block Codes
TAST	Threaded Algebraic Space-Time code
TDMA	Time Division Multiple Access
ZF	Zero Forcing

Introduction

During the last few years, wireless technologies have been experiencing spectacular developments, due to the emergence of new interactive and digital multimedia applications as well as rapid advances in the highly integrated systems. A key requirement in such systems is the availability of high-speed and robust communication links. Unfortunately, communications over wireless channels inherently suffer from a number of fundamental physical limitations, such as multipath fading.

Diversity techniques for mitigating, and even exploiting, multipath fading are central to improve the performance of wireless communication systems and networks. Multiple-input multiple-output (MIMO) techniques are as successful example that can provide both spatial and temporal diversity. This system is implemented with antenna arrays at both the transmitter and the receiver.

However, due to size, cost or hardware limitations, most wireless terminals may not be able to support multiple antennas. This problem is solved by a new form of space-time diversity, referred to as cooperative diversity [1–3]. In a multi-user scenario, the idea of cooperation techniques is that single-antenna mobiles can share their antennas so as to create a virtual MIMO array. Based on this virtual array, the terminals are able to share their distributed resources such as bandwidth and power by using cooperative protocols.

Cooperative protocols can be divided in several families depending on the processing performed at relaying nodes. In this thesis, we are interested in two of them: 1) the amplify-and-forward (AF) strategy consists for the relays in forwarding an amplified version of what they received; 2) the decode-and-forward (DF) strategy requires a more complex processing since relays have to decode and re-encode the signals before forwarding them.

A wireless communication network can be represented by a graph with nodes and edges

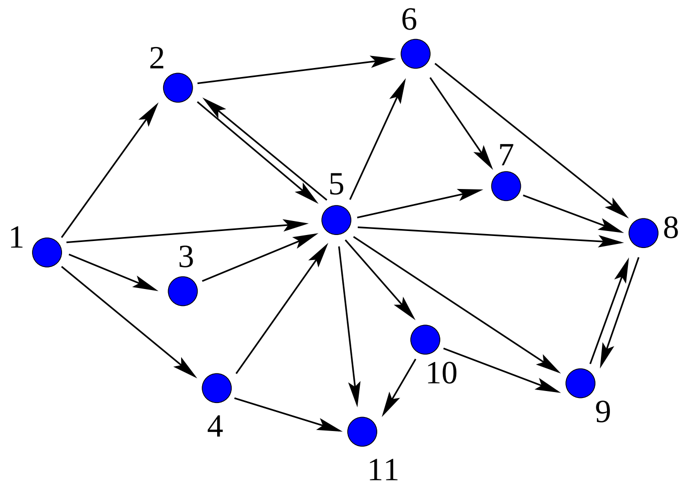


Figure 2: Example of a wireless communication network

(see Figure 1). Nodes are for example mobile terminals, base stations or sensors. Edges represent wireless communication links or channels. The network can be decomposed in smaller channels, such as the following single-hop models:

- the relay channel: for example, node 1 communicating with node 5, helped by nodes 2, 3 and 4;
- the cooperative multiple access (CMA) channel with or without the help of a dedicated relay: for example nodes 5 and 6 transmitting to node 8, helped by node 7, or nodes 2 and 5 sending a message to node 6;
- the cooperative broadcast channel (CBC) with or without a dedicated relay: for example node 5 sending a message to nodes 9 and 11, helped by node 10, or node 5 communicating with nodes 8 and 9.

We can also define larger multihop networks such as the K-parallel-path (KPP) channel or the layered multihop channel. For most of these channels, theoretical limits have been established, and cooperative protocols achieving these limits have been proposed. However, they often imply a high complexity.

In this thesis, we are interested in providing new cooperative AF and DF protocols that achieve the theoretical limits of the relay, CMA and KPP channels, while inducing a lower complexity than existing ones.

Contributions and outline of the thesis

In **Chapter 1**, we introduce the cooperative channel and show that it can be modeled as a MIMO system. Thus, the same evaluation tools and diversity techniques can be used.

The design criteria of space-time block codes (STBC) are recalled and the STBC that will be used in this thesis are presented.

State-of-the-art of amplify-and-forward (AF) and decode-and-forward (DF) protocols for the relay channel are presented in **Chapter 2**. Some implementation issues of cooperative protocols are studied such as relay location, power distribution and the impact of asynchronism at reception. This study shows that even with optimized relay location and power allocation, and a perfect synchronization, cooperative protocols still provide poor performance at low signal-to-noise ratio (SNR). To solve this problem, an adaptive strategy is proposed that can be applied either to AF or DF protocols. This new selection is based on the instantaneous capacities of all possible transmission schemes.

AF protocols have been studied extensively due to their simple processing at relays. On the contrary, DF protocols require more processing as signals have to be decoded at relays, and no simple and efficient protocol has been proposed yet. In **Chapter 3**, the definition of a practical DF protocol providing good performance is explored, based on the introduction of distributed STBCs. Two DF protocols with limited rates, namely the Alamouti DF with $\frac{1}{2}$ symbol per channel use (symb. pcu) and the Asymmetric DF (ADF) with $\frac{2}{3}$ symb. pcu, are first proposed. The Incomplete DF (IDF) protocol is then defined that achieves the same theoretical performance as the non-orthogonal AF (NAF), which is the best AF protocol for the one-relay channel. It is based on an incomplete decoding at relays that can be performed using an exhaustive search. We also propose two decoding methods to reduce the high complexity of the exhaustive search: a two step method based on the structure of TAST codes, and a second method based on diophantine approximation.

The considered relay channel model lies on two main assumptions: first, a single source is transmitting, and second, there is a direct link between this source and the destination. If we relax the first assumption, the relay channel becomes a cooperative multiple-access channel (CMA). A time-division multiple-access (TDMA) strategy is usually assumed. However, if multiple sources need to transmit simultaneously, new cooperative protocols have to be defined. In **Chapter 4**, we have studied the CMA-NAF protocol proposed by Azarian *et al.* and given a practical implementation. We have also proposed two modifications of this protocol allowing to improve both its performance and its diversity-multiplexing gain tradeoff (DMT). Finally, we have defined a variant of this protocol using a DF strategy. If we relax now the second assumption, the relay channel becomes a multihop channel, which can be more realistic in a large network context. The theoretical limits of multihop channels have been studied and protocols achieving these limits have been proposed. However, based on the use of large distributed STBC, they imply a very high complexity. In this thesis, a new multihop protocol for the KPP channel is proposed, based on path selection combined with a small STBC. This protocol achieves the theoretical limits of the channel while inducing only a small complexity and feedback.

Chapter 1

Cooperative diversity and distributed space-time coding

Cooperation is a recent technique to improve reliability of transmission in a wireless channel. Multiple terminals communicate together to form a virtual multiple antenna array and exploit space-time or cooperative diversity.

A cooperative network can be modeled as a multiple-input multiple-output (MIMO) system whose channel matrix has a particular structure. Thus the space-time coding techniques developed for MIMO systems can be used for cooperative networks also, in a distributed manner. Before studying cooperation, it is then necessary to recall the main principles of MIMO systems and the design criteria of space-time block codes (STBC), and to present the most known and used ones.

In this chapter we first introduce cooperation in section 1.1 and define the cooperative diversity, relaying nodes, relaying strategies and cooperative protocols. We then show the similitude between a cooperative network and a MIMO system in section 1.2. This likeliness justifies the use of the same evaluation tools presented in section 1.3 and the same diversity techniques, and in particular space-time coding, whose design criteria and state-of-the-art are presented in section 1.4. The last section 1.5 deals with the decoding for multi-antenna systems.

1.1 Cooperative communications

1.1.1 Cooperative diversity

Diversity plays an important role in combating fading. The idea is to sent a message over different channels. The reliability of transmission is then improved, since individual channels experience independent levels of fading.

Diversity can be exploited by using different techniques, resulting in different types of diversity. Some of them are:

- time diversity: the signal is transmitted several times at different time instants;
- frequency diversity: the signal is transmitted using several frequency channels;
- space diversity: the signal is transmitted over several propagation paths, it can be achieved by antenna diversity using multiple transmitter antennas and/or multiple receiver antennas;
- polarization diversity: a signal is transmitted and received via antennas with different polarization.

In this thesis, we are interested in a generalization of space diversity called cooperative diversity, because several nodes in the network help each other to transmit their messages.

Let's consider a simple example where two sources S_1 and S_2 want to transmit messages w_1 and w_2 respectively to the same destination D. Several transmission schemes can be used:

(a) The straightforward solution is that both sources send their message to the destination simultaneously (see figure 1.1(a)). This is the multiple-access channel (MAC). The message can be sent using either a time-division multiple-access (TDMA) strategy, or space-time codes designed for the MAC.

However, if the channel between source S_1 and destination D is subject to a strong fading and on contrary the channel between source S_2 and D is good, message w_1 will be lost while message w_2 will be correctly decoded. To avoid this loss of information, source S_1 could benefit from the high quality of the S_2-D link.

(b) A second solution is thus that sources cooperate together to send both messages w_1 and w_2 to the destination (see figure 1.1(b)). We then have to use cooperative protocols, such as the ones developed in this thesis.

(a) Non-cooperative network (b) Cooperative network

Figure 1.1: Example of a network with two sources S_1 and S_2 and one destination D

In cooperative networks, each source sends a signal which is received not only by the destination, but also by the other nodes in the network. After processing the received signal, these nodes broadcast the information again. Using this strategy, the error rate decreases, since the probability that all network paths are bad is very small. This strategy improves the diversity of the system, defined as the number of independent paths from source to destination.

For the considered example, the diversity order using the first scenario is limited to 1 (only one path from each source to destination). On the contrary, it is equal to 2 using the second scenario, since there are two independent paths from S_1 to D: the direct path and the one using S_2 as a relaying node.

1.1.2 Relaying nodes

Cooperation protocols strongly depend on the type of relaying nodes used in the network. The nature of these nodes imposes some constraints on the transmission. In particular, their half-duplex or full-duplex nature has an important impact on the data rate of the protocols.

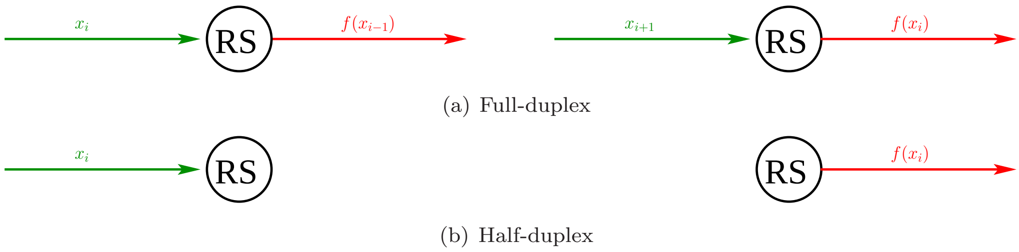


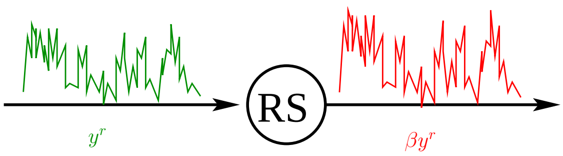
Figure 1.2: Full or half-duplex terminals

- **Full-duplex** nodes can receive and transmit at the same time (see figure 1.2(a)). This property allows all nodes to transmit continuously and so the data rate is maximized. However this implies complex terminals equipped with two sets of antennas processing at two different frequencies f_r and f_t for reception and transmission respectively.
- **Half-duplex** nodes cannot receive and transmit at the same time (see figure 1.2(b)). Consequently one antenna is sufficient and half-duplex nodes are less complex, which is more realistic for small terminals such as sensors. Unfortunately, this constraint makes the data rate drop. Indeed, relaying nodes need twice the same amount of time to transmit the information.

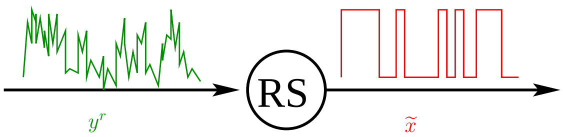
Until now, in most studies on cooperative communications as well as in industrial applications, half-duplex nodes are considered. Thus in this thesis, we will work with half-duplex nodes also.

1.1.3 Relaying strategies

To define a protocol, we have to specify the type of processing performed at relaying nodes. There are several possible strategies:



(a) Amplify-and-forward



(b) Decode-and-forward

Figure 1.3: Different relaying strategies

- The **amplify-and-forward** (AF) strategy consists for the relaying node in broadcasting the received signal multiplied by an amplifying factor β (see Figure 1.3(a)).

  Let $y^r = \sqrt{\rho} h x^s + v$ be the received signal, where ρ is the signal-to-noise ratio, h models the fading of the wireless channel, x^s is the sent signal and v is some additive white Gaussian noise (AWGN). The forwarded signal is then

  $$x^r = \beta y^r,$$

  where the amplifying factor β is chosen so that the forwarded signal power is lower than the symbol energy E_x.

  $$\beta \leq \beta_{opt} = \sqrt{\frac{E_x}{1 + \rho |h|^2 E_x}}$$

  In this thesis, in order to analyze the optimal performance of the proposed protocols, we take $\beta = \beta_{opt}$.

  Amplifying corresponds to a linear transformation at the relaying node and thus does not induce a high complexity. This is the reason why this strategy has been the most studied in literature. Moreover, thanks to this low complexity, small and cheap terminals can be used, which is interesting for industrials.

  However, in some contexts, this strategy can be inefficient. For example, in a multi-hop channel, several hops are necessary to transmit information. In this case, using an AF strategy could be fatal as some noise would be accumulated at each hop and the resulting signal could not be decoded without error.

- The **decode-and-forward** (DF) strategy consists for the relaying node in decoding the received signal, re-encoding it, and broadcasting the re-encoded signal (see Figure 1.3(b)).

  Let $y^r = \sqrt{\rho} h x^s + v$ be the received the signal. The forwarded signal is

  $$x^r = \widetilde{x^s}.$$

  Decoding at the relaying node requires more processing since it is a non-linear transformation of the received signal. Thus more power is required and small nodes cannot be used. Let us consider the example of a mobile terminal who wants to transmit to a base station (BS). Instead of using only the direct link between the mobile and the BS, another BS can help and serve as as a relay. This BS has obviously enough processing capacity to perform the decoding and thus a DF strategy can be used.

  Because of the high complexity induced by decoding at the relaying node, the DF strategy has been less studied in literature. However, in some particular contexts (such as multihop systems), regenerating the signals using a DF strategy is necessary to avoid the accumulated noise.

In the following of this thesis, we will study and propose protocol which use these two strategies. However, many other strategies have been proposed in literature such as:

- compress-and-forward (CF) [6, 7]
- quantize-and-forward (QF) [8, 9]

1.1.4 Cooperative protocols

Definition of a cooperative protocol

Once the nature of the nodes and the relaying strategy are chosen, the cooperative protocol can be defined. For a given network topology (number and role of the cooperative nodes), a cooperative protocol (in a physical layer sense) is described by its transmission frame, i.e. which nodes are listening/transmitting and what the transmitting nodes send at each time slot.

In literature, we usually consider networks where all cooperative nodes use the same processing. The defined protocols are then said to be amplify-and-forward or decode-and-forward according to the strategy used at relays.

Orthogonality of a protocol

Depending on their frame structure, we can distinguish two classes of protocols:

- In **orthogonal protocols** a single node is transmitting in each time slot. This strategy lets the protocol avoid interferences between different terminals and makes the decoding easier. However, as the source transmits only part of the time, the data rate is lower than 1 symb. pcu.

- In **non-orthogonal protocols** several terminals are allowed to transmit at the same time. In particular, the source can transmit during the whole frame, whether the relays are listening or transmitting, which increases the data rate.

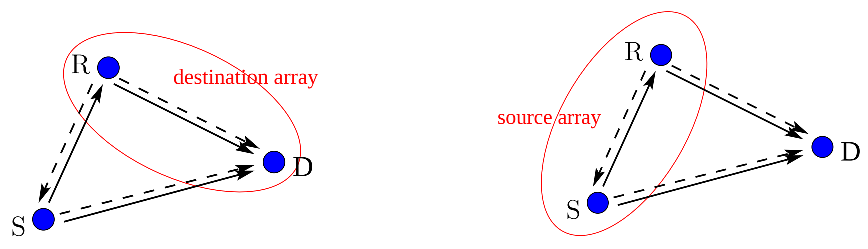
1.2 A virtual MIMO system

1.2.1 Modeling of the cooperative network as a MIMO system

During the last ten years, a huge interest has been given to multi-antenna systems. Indeed a smart transmission strategy (space-time coding) allows to increase considerably performance in terms of data rate and reliability. Numerous STBCs have been proposed. However, even though base stations can be equipped with as many antennas as necessary, this is not the case of mobile terminals due to size, cost or hardware constraints.

When cooperation is used, several nodes in the network transmit the same information to the destination. These nodes form a virtual antenna array, which allows to exploit diversity even with single-antenna mobile terminals.

Example 1. *Let us take the example of a source S transmitting a message w to the destination D. This source is helped by a relay R. Let us first consider S transmitting to both R and D: relay and destination can be considered as a virtual MIMO array with two receive antennas (see figure 1.4(a)). We can also consider D receiving information from both S and R: source and relay can be considered as a virtual MIMO array with two transmit antennas (see figure 1.4(b)).*



(a) Destination array (b) Source array

Figure 1.4: Cooperation as a virtual MIMO

The transmission takes place in two phases:

- first, the source broadcasts the message to the destination and the relays;
- second, the relays (and the source if the protocol is non-orthogonal) transmit the

message to the destination.

We call these two phases the listening and forwarding phases respectively, in relation to the status of the relays.

Let's show how to obtain an equivalent MIMO system for the one-relay channel example.

Example 2. *During the listening phase, signals at relay and destination are:*

$$y_r = \sqrt{\rho} h x_1 + w_r,$$
$$y_1 = \sqrt{\rho} g_0 x_1 + w_1.$$

During the forwarding phase, received signal at destination is:

$$y_2 = \sqrt{\rho} g_O x_2 + \sqrt{\rho} g_1 x_r + w_2,$$

with $x_r = \begin{cases} \beta y_r \text{ if an AF strategy is used;} \\ \widetilde{x_1} \text{ if a DF strategy is used.} \end{cases}$
If the protocol is orthogonal, $x_2 = 0$.

In both cases, we can write these equations as a MIMO system:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ \widetilde{w_2} \end{bmatrix},$$

with $\mathbf{H} = \begin{cases} \begin{bmatrix} \sqrt{\rho} g_0 & 0 \\ \rho h \beta g_1 & \sqrt{\rho} g_0 \end{bmatrix} \text{ if an AF strategy is used;} \\ \begin{bmatrix} \sqrt{\rho} g_0 & 0 \\ \sqrt{\rho} g_1 & \sqrt{\rho} g_0 \end{bmatrix} \text{ if a DF strategy is used,} \end{cases}$
and $\widetilde{w_2} = \begin{cases} \beta \sqrt{\rho} g_1 w_r + w_2 \text{ if an AF strategy is used;} \\ w_2 \text{ if a DF strategy is used.} \end{cases}$

In the general case, the system of equations can also be written as a MIMO system:

$$\mathbf{Y} = \mathbf{HX} + \mathbf{W}.$$

Cooperation systems have been generalized to the case of terminals with multiple antennas. The advantages of both strategies can then be combined.

1.2.2 Distributed space-time block codes

The representation of cooperative networks as MIMO systems justifies the use of the same multi-antenna techniques, and in particular space-time block codes.

Since all transmit antennas are not located on the same terminal, STBC have to be implemented in a distributed manner between the sources and relays:

- Initially relays have no information about the message that has to be transmitted to the destination. Thus, in a first step, this information has to be sent from sources to relays.
- In a second step, relay(s) (and sources if the protocol is non-orthogonal) transmit the reconstructed space-time codeword to the destination.

Either the same STBCs that have been defined for MIMO networks can be used, or new codes constructed to better answer the specific cooperation constraints, such as desynchronization at the destination.

Decoding is performed as in a MIMO system with a maximum likelihood (ML) algorithm (ML decoding will be detailed in the last section of this chapter).

1.3 Evaluation tools for multi-antenna systems

In this section, we present some information theory tools that have been developed to evaluate the performance of MIMO systems. Since a cooperative network can be modeled as a MIMO system, the same tools will be used in this thesis to analyze cooperative protocols.

1.3.1 MIMO channel model

We consider a multiple-antenna system with n_t transmit antennas and n_r receive antennas. Between each transmitting and receiving antennas, the transmission suffers from a independent fading. Multiple-antenna strategies such as space-time block codes are designed to exploit this particular property.

The transmission scheme is represented in Figure 1.5. The binary sequence is first modulated: each modulated symbol corresponds to a given number of bits depending on the chosen constellation. In this work, we will consider only M-QAM constellations (see Figure 1.6), whose modulated symbols carry $\log_2 M$ bits. The mapping is done according to a Gray coding so as to minimize the Hamming distance between neighbor symbols. In a second step, these modulated symbols are coded with the chosen STBC. Finally the resulting codeword is sent through the channel. At the destination side, the received signal is first decoded and the resulting symbols are demodulated to generate the estimated binary sequence.

Figure 1.5: Transmission chain of a MIMO system

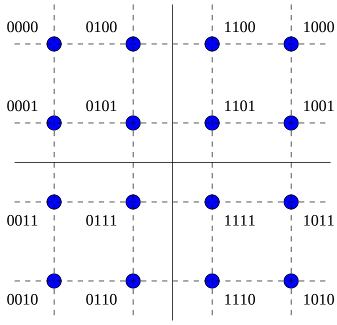


Figure 1.6: Example of the 16-QAM constellation

The received signal can be expressed as a function of the transmitted codeword, depending on the channel realization.

$$\mathbf{Y}_{n_r \times T} = \mathbf{H}_{n_r \times n_t} \mathbf{X}_{n_t \times T} + \mathbf{W}_{n_r \times T} \tag{1.1}$$

where

- T is the temporal code length (the transmission of a codeword $\mathbf{X}_{n_t \times T}$ lasts T channel uses),
- $\mathbf{Y}_{n_r \times T}$ is the matrix of received signals,
- $\mathbf{H}_{n_r \times n_t}$ is the channel matrix: each h_{ij} represents the fading between transmitting antenna i and receiving antenna j and follows a Rayleigh distribution (real and imaginary parts are independent and follow a Gaussian distribution with zero mean and variance $\frac{1}{2}$),
- $\mathbf{X}_{n_t \times T}$ is the transmitted codeword of the STBC,
- and $\mathbf{W}_{n_r \times T}$ is a matrix of additive white Gaussian noise (AWGN) with zero mean and variance σ_W^2.

If the channel is fast-fading, i.e. its coefficients vary quickly, the channel is assumed to be ergodic.

On the contrary, if the channel is slow-fading, then its coefficients can be considered constant during the transmission of at least one codeword.

1.3.2 Information theory tools

Necessary elements of information theory can be found in the book by Cover and Thomas [10].

a) Capacity

Discrete memoryless channel. Let us consider the following discrete memoryless channel (DMC) with a finite input alphabet $\mathcal{X}$ and output alphabet $\mathcal{Y}$. X and Y represents the encoded input and output of the channel.

Figure 1.7: Communication system

Definition 1. *The **capacity** of a DMC is defined as the maximized mutual information between the output Y and the input X over all possible input distribution $p(X)$.*

$$C = \max_{p(X)} I(X;Y).$$

The capacity, in bits per channel use (bits pcu) represents the maximum quantity of information that can be sent through the channel. It depends only on the characteristics of the channel.

MIMO AWGN channel. Let us suppose the considered channel is a MIMO AWGN channel and its channel matrix $\mathbf{H}$ respects the assumptions in subsection 1.3.1. Then the instantaneous capacity for a specific channel realization is given by

$$C(\mathbf{H}) = \log_2 \left(\det \left(\mathbf{I}_{n_r} + \frac{\rho}{n_t} \mathbf{H}\mathbf{H}^\dagger \right) \right).$$

If the channel is ergodic, we can average this expression for all channel realizations to obtain the ergodic capacity:

$$C = E_{\mathbf{H}} \left\{ C(\mathbf{H}) \right\}.$$

When the signal-to-noise ratio grows to infinity, this expression becomes

$$C = \min(n_t, n_r) \log_2(\rho) + \mathcal{O}(1).$$

$r^* = \min(n_t, n_r)$ is the maximum number of degrees of freedom or multiplexing gain.

b) The slow-fading case: outage probability

Under the assumtions of subsection 1.3.1, if the channel is slow-fading, the ergodic capacity is always zero.

In this case, in order to study the theoretical limits of the transmission over the channel, we consider the instantaneous capacity as a random variable and study the probability that it is less than some threshold.

Definition 2. *The* ***outage probability*** *of a MIMO AWGN channel characterized by its channel matrix* $\mathbf{H}$ *is*

$$p_{out}(R) = \Pr\{C(\mathbf{H}) < R\},$$

where R *stands for the data rate in bits pcu.*

If the instantaneous capacity is less than the data rate, then the error probability cannot be made arbitrarily small and the system is said to be in outage. On the contrary, if the instantaneous capacity is more than the data rate, then, according to Shannon's theorem, there exists a code that allows to obtain an error probability as small as desired and the channel is said not in outage.

When the SNR grows to infinity, outage probability is shown to be

$$p_{out} = \mathcal{O}\left(\frac{1}{\rho^{n_t n_r}}\right).$$

The outage probability achieves the maximal diversity order of a MIMO system $d^* = n_t n_r$ in the high SNR regime.

c) Diversity-Multiplexing gain Tradeoff

A MIMO system has a diversity order d and a multiplexing gain r that have both to be maximized. However, maximizing one of them does not imply maximizing the other one. Thus a tradeoff between these two quantities has been defined by Zheng and Tse in [11,12].

Definition 3. *The diversity gain* $d^*(r)$ *is achieved at a multiplexing gain* r *if*

$$R = r \log \rho$$

and

$$\lim_{\rho \to \infty} \frac{\log p_{out}(r \log \rho)}{\log \rho} = -d^*(r).$$

The curve representing $d^*(r)$ *is the* ***diversity-multiplexing gain tradeoff*** *(DMT).*

This new tool allows to study the theoretical limit of the system asymptotically.

In [11, 12], the authors prove that, if the channel can be considered as constant during at least $T \geq n_t + n_r - 1$ time slots, the DMT of a MIMO slow fading channel is

$$d^*(r) = (n_t - r)^+ (n_r - r)^+,$$

where $(x)^+ = \max(0, x)$.

In [13], the authors prove that a space-time block code providing a full rate and a non-vanishing determinant (NVD) achieves the DMT of a MIMO channel. The non-vanishing determinant property comes from the the STBC design criteria and will be explained in the next section.

1.4 Space-Time Block Codes

1.4.1 Design criteria

In order for the space-time block codes (STBC) to exploit all the degrees of freedom of the channel, some design criteria have been developed.

a) Pairwise error probability derivation

Design criteria can be derived from the expression of the pairwise error probability. Indeed, a good STBC is designed to minimize this probability.

Let us suppose that codeword $\mathbf{X}$ has been sent. The received signal is:

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}.$$

We want to compute the probability that an error occurs at the receiver side, and the codeword $\mathbf{T}$ is detected instead of codeword $\mathbf{X}$. This event occurs if $\mathbf{T}$ is closer to the received signal than $\mathbf{X}$. The pairwise error probability (PEP) for a given channel realization $\mathbf{H}$ is:

$$\begin{aligned}\Pr(\mathbf{X} \to \mathbf{T}|\mathbf{H}) &= \Pr\left\{||\mathbf{Y} - \mathbf{H}\mathbf{T}||^2 \leq ||\mathbf{Y} - \mathbf{H}\mathbf{X}||^2\right\} \\ &= \Pr\left\{||\mathbf{H}(\mathbf{X} - \mathbf{T}) + \mathbf{W}||^2 \leq ||\mathbf{W}||^2\right\} \\ &= \Pr\left\{\underbrace{||\mathbf{H}(\mathbf{X} - \mathbf{T})||^2 + 2\Re(\mathbf{H}(\mathbf{X} - \mathbf{T})\mathbf{W}^\dagger)}_{V} \leq 0\right\}.\end{aligned}$$

V is a Gaussian variable with mean $m_V = ||\mathbf{H}(\mathbf{X}-\mathbf{T})||^2$ and variance $\sigma_V^2 = 4||\mathbf{H}(\mathbf{X} - \mathbf{T})||^2\sigma_W^2$. The PEP is thus given by:

$$\Pr(\mathbf{X} \to \mathbf{T}|\mathbf{H}) = Q\left(\frac{m_V}{\sigma_V}\right) = Q\left(\frac{||\mathbf{H}(\mathbf{X} - \mathbf{T})||}{2\sigma_W}\right).$$

By averaging with respect to the channel realization $\mathbf{H}$ and using the exponential bound, we obtain:

$$\Pr(\mathbf{X} \to \mathbf{T}) \leq E_{\mathbf{H}}\left[\exp\left(-\frac{||\mathbf{H}(\mathbf{X} - \mathbf{T})||^2}{8\sigma_W^2}\right)\right].$$

Let us define $\mathbf{A} = (\mathbf{X} - \mathbf{T})(\mathbf{X} - \mathbf{T})^\dagger$. We can decompose $\mathbf{A}$ in the form $\mathbf{A} = \mathbf{V}\mathbf{D}\mathbf{V}^\dagger$ where matrix $\mathbf{V}$ is unitary and $\mathbf{D}$ diagonal.

$$\begin{aligned}
\Pr(\mathbf{X} \to \mathbf{T}) &\leq E_{\mathbf{H}}\left[\exp\left(-\frac{\mathbf{H}(\mathbf{X}-\mathbf{T})(\mathbf{X}-\mathbf{T})^{\dagger}\mathbf{H}^{\dagger}}{8\sigma_W^2}\right)\right] \\
&\leq E_{\mathbf{H}}\left[\exp\left(-\frac{\mathbf{HVDV}^{\dagger}\mathbf{H}^{\dagger}}{8\sigma_W^2}\right)\right] \\
&\leq E_{\mathbf{H}}\left[\exp\left(-\frac{\sum_{i=1}^{n_t}\sum_{j=1}^{n_r}\lambda_i|\beta_{ij}|^2}{8\sigma_W^2}\right)\right],
\end{aligned}$$

where the λ_i are the eigenvalues of $\mathbf{A}$ (and the diagonal elements of $\mathbf{D}$) and the β_{ij} are the elements of the matrix product $\mathbf{HV}$. Columns of $\mathbf{V}$ are eigenvectors of $\mathbf{A}$ and form an orthonormal basis of $\mathbb{C}$. Elements of $\mathbf{H}$ are Gaussian variables with zero mean and unitary variance. Thus the β_{ij} are also Gaussian variables with zero mean and unitary variance.

$$\begin{aligned}
\Pr(\mathbf{X} \to \mathbf{T}) &\leq \exp\left(-\frac{\sum_{i=1}^{n_t}\sum_{j=1}^{n_r}\lambda_i E_{\mathbf{H}}\left[|\beta_{ij}|^2\right]}{8\sigma_W^2}\right) \\
&\leq \prod_{j=1}^{n_r}\prod_{i=1}^{n_t}\exp\left(-\frac{\lambda_i}{8\sigma_W^2}\right) \\
&\leq \left(\prod_{i=1}^{n_t}\frac{1}{1+\frac{\lambda_i}{8\sigma_W^2}}\right)^{n_r}
\end{aligned}$$

Let us define r the rank of matrix $\mathbf{A}$. Then $\mathbf{A}$ has r non-zero eigenvalues. Thus, for high SNRs, the PEP can be upper bounded by:

$$\begin{aligned}
\Pr(\mathbf{X} \to \mathbf{T}) &\leq \left(\prod_{i=1}^{n_t}\lambda_i\right)^{-n_r}\left(\frac{1}{8\sigma_W^2}\right)^{-rn_r} \\
&\leq (\det \mathbf{A})^{-n_r}\left(\frac{1}{8\sigma_W^2}\right)^{-rn_r}. \qquad (1.2)
\end{aligned}$$

b) Rank criterion [14]

From the upperbound (1.2) of the PEP, we can derive the first design criteria. The diversity is given by the power of the inverse of the SNR $\left(\frac{1}{8\sigma_W^2}\right)$. We can observe that receive diversity is straightforward. As soon as n_r antennas are used at destination, a diversity order of n_r is reached. The transmit diversity is more complex to obtain.

Design criterion 1. *To reach the maximum diversity order $n_t n_r$, matrix $\mathbf{A}$ has to be full* *rank for any pair of different codewords* $(\mathbf{X}, \mathbf{T})$.

$$\forall \mathbf{X} \neq \mathbf{T}, \quad r = rank(\mathbf{A}) = n_t$$

Otherwise, diversity order is only $r_{min} n_r$, *where* r_{min} *is the minimum rank of matrix* $\mathbf{A}$.

c) Determinant criterion [14]

The second term in the upper bound of the PEP corresponds to the coding gain.

Design criterion 2. *In order to reach the maximum coding gain* $\min(n_t, n_r)$, *the minimum determinant of matrix* $\mathbf{A}$ *has to be maximized.*

$$Maximize\ \delta(\mathbf{A}) = \min_{\mathbf{X} \neq \mathbf{T}} \det(\mathbf{A})$$

The determinant of $\mathbf{A}$ depends on the constellation. In order to keep a good coding gain whatever the spectral efficiency is, it has been shown in [13] that the determinant of $\mathbf{A}$ has to be non-vanishing when the constellation size grows.

Design criterion 3. *In order to guarantee a non-decreasing coding gain when spectral efficiency grows, the minimum determinant of* $\mathbf{A}$ *has to be lower bounded by a constant* $C \neq 0$ *independent of the constellation size.*

$$\delta(\mathbf{A}) = \min_{\mathbf{X} \neq \mathbf{T}} \det(\mathbf{A}) \geq C$$

The STBC is then said to have a non-vanishing determinant (NVD).

d) Mutual information criterion

Another criterion has been introduced in [15] based on the mutual information of the sent and received signals. This criterion is independent of the rank and determinant criteria. It does not guarantee an optimized diversity, but it provides a better energy distribution among the codewords.

Design criterion 4. *An STBC is called information-lossless if its structure is such that the maximum mutual information of the resulting equivalent channel is equal to the capacity of the channel.*

e) Optimal codes

Definition 4. *A STBC is said to be optimal if it provides both full rate and full diversity, has a non-vanishing determinant and is information lossless.*

Thus a STBC is optimal if it respects criteria 1, 2, 3 and 4.

1.4.2 State of the art of space-time block codes

a) Orthogonal codes

Orthogonal codes are so called due to the orthogonality of the codeword. The simplest and most famous example of this family is the Alamouti code.

Alamouti code. Alamouti code has been proposed in [16] as one of the first STBC. This code is designed for 2 transmit and 1 receive antennas. It has a full diversity order of 2 and a full rate of 1 symb. pcu. It also verifies the NVD criterion and is information lossless. Thus it is optimal.

The transmitted codeword is

$$\mathbf{X} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$

where s_1 and s_2 are information symbols. Transmission lasts two time slots and received signals are

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \tag{1.3}$$

The second received signal is conjugated and equation (1.3) can be rewritten in the form $\mathbf{y} = \mathbf{H}_{eq}\mathbf{s} + \mathbf{w}$, where the equivalent channel matrix $\mathbf{H}_{eq}$ is orthogonal and $\mathbf{w}$ is an AWGN array.

$$\underbrace{\begin{bmatrix} y_1 \\ {y_2}^* \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ {h_2}^* & -{h_1}^* \end{bmatrix}}_{\mathbf{H}_{eq}} \underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} w_1 \\ {w_2}^* \end{bmatrix}}_{\mathbf{w}}$$

Decoding is then very easy. By multiplying the received array $\mathbf{y}$ by $\mathbf{H}_{eq}^{\dagger}$ we get the sent array $\mathbf{s}$ back, multiplied by a positive constant which depends on the channel realization.

$$\begin{aligned} \mathbf{H}_{eq}^{\dagger}\mathbf{y} &= \mathbf{H}_{eq}^{\dagger}\mathbf{H}_{eq}\mathbf{s} + \mathbf{H}_{eq}^{\dagger}\mathbf{w} \\ &= (|h_1|^2 + |h_2|^2)\mathbf{s} + \widetilde{\mathbf{w}} \end{aligned}$$

We can remark that in this case, maximum likelihood (ML) decoding is equivalent to zero-forcing (ZF).

Larger orthogonal codes. The Alamouti code principle has been generalized to higher dimensions [17–19]. $n_t \times 1$ codes has been defined, whose codewords are orthogonal. Consequently, their equivalent channel matrices are also orthogonal and they can be optimally decoded with a ZF decoder. This linear decoding induces a very low complexity, which is the main advantage of these codes.

Orthogonal codes provide full diversity. Unfortunately, their data rate drops with the number of antennas. The only code with full rate, and thus the only optimal orthogonal code is the Alamouti code.

Example 3. *In [19], authors proposed a rate $\frac{3}{4}$ orthogonal code whose codeword is:*

$$\mathbf{X} = \begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ -s_3^* & 0 & s_1^* & -s_2 \\ 0 & -s_3^* & s_2^* & s_1 \end{bmatrix}.$$

b) DAST and TAST codes

More sophisticated algebraic tools were used afterwards to construct optimal STBC for any number of transmit antennas.

Diagonal algebraic STBC. Diagonal algebraic space-time (DAST) codes have been proposed in [20] for the $n_t \times 1$ MISO channel. They provide full diversity and a symbol rate of 1 symb. pcu. Their construction is based on the use of a rotated constellation.

In order to have coded symbols better distributed among the codeword, coded symbols are multiplied by an Hadamard matrix.

Example 4. *We detail here the example of a 2×2 DAST code. Let us define $\mathbb{K} = \mathbb{Q}(i, \theta)$ a number field of degree 2 over $\mathbb{Q}(i)$, with $\theta = e^{i\frac{\pi}{4}}$. Let us define s_1 and s_2 information symbols carved in a QAM constellation. We use the rotation matrix*

$$\mathbf{M} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & \theta \\ 1 & -\theta \end{bmatrix}$$

to define the new coded symbols

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{M} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 + \theta s_2 \\ s_1 - \theta s_2 \end{bmatrix}.$$

Finally, multiplying by an Hadamard matrix, we obtain the DAST codeword:

$$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_1 & -x_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 + \theta s_2 & s_1 - \theta s_2 \\ s_1 + \theta s_2 & -s_1 + \theta s_2 \end{bmatrix}.$$

Threaded algebraic STBC. Threaded algebraic space-time (TAST) codes, introduced in [21], are layered space-time codes based on algebraic theory. Each layer is associated with a different algebraic subspace so that interferences between the layers are perfectly deleted. For example, each layer can be coded with a DAST code. Moreover each layer is constructed so that it can exploit all the degrees of freedom of the MIMO channel. Some

1	2	3	4
4	1	2	3
3	4	1	2
2	3	4	1

1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

Figure 1.8: Examples of possible layered constructions for a 4×4 layered STBC

possible layered constructions are given in Figure 1.8.

For symmetric TAST codes, the same rotation is used for each layer. All elements of the codeword belong to the same number field $\mathbb{K}$. A parameter ϕ_k is introduced to separate the different layers. The choice of ϕ_k has a big influence on the coding gain. In [21], authors proved that for L layers, choosing $(\phi_k = \phi^{\frac{k-1}{n_t}})_{k\in\{1,L\}}$ is sufficient if $(\phi^{k-1})_{k\in\{1,\ldots,L\}}$ are algebraically independent in $\mathbb{K}$ or $\phi = e^{i\lambda}$ where $\lambda \neq 0$ is an algebraic number (thus ϕ is transcendent).

TAST codes are decodable with maximum-likelihood (ML) decoders such as the sphere decoder or the Schnorr-Euchner algorithm (these algorithms will be detailed in next section). TAST codes provide full rate and full diversity, but unfortunately do not have the NVD property.

Example 5. *We detail here the example of a 2×2 TAST code. In this case, there is only one possible layer distribution. Each layer is coded with the DAST code of Example 4. Parameter $\phi = \sqrt{\theta}$ is used to separate the two layers. The codeword is then*

$$\mathbf{X} = \frac{1}{\sqrt{2}} \begin{bmatrix} s_1 + \theta s_2 & \phi(s_3 + \theta s_4) \\ \phi(s_3 - \theta s_4) & s_1 - \theta s_2 \end{bmatrix},$$

where s_1, s_2, s_3 and s_4 are information symbols carved in a QAM constellation.

Let $\mathbf{T}$ be another codeword. Elements $(d_1 + \theta d_2)$ and $(d_3 + \theta d_4)$ of $(\mathbf{X} - \mathbf{T})$ also belong to $\mathbb{K} = \mathbb{Q}(i, \theta)$. $\forall x = a + \theta b \in \mathbb{K}$, let's note $\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(x) = (a + \theta b)(a - \theta b)$ its norm in $\mathbb{K}/\mathbb{Q}(i)$. Then

$$\begin{aligned} \det(\mathbf{X}-\mathbf{T})(\mathbf{X}-\mathbf{T})^\dagger &= \left| \frac{1}{2}\left(\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_1 + \theta d_2) - \phi^2 \mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_3 + \theta d_4)\right) \right|^2 \\ &= \left(\frac{1}{2}\left(\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_1 + \theta d_2) - \theta \mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_3 + \theta d_4)\right) \right)^2 \qquad \in \mathbb{K}. \end{aligned}$$

The determinant would be zero if and only if $\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_1 + \theta d_2) = 0$ and $\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_3 + \theta d_4) = 0$, which means that codewords $\mathbf{X}$ and $\mathbf{T}$ would be the same. So the minimum determinant is non-zero but still depends on the constellation size.

c) Golden code and Perfect codes

Perfect codes [13, 22, 23] are algebraic space-time codes constructed using a cyclic division algebra $\mathcal{A} = (\mathbb{K}/\mathbb{L}, \sigma, \gamma)$, where $\mathbb{K}$ is a cyclic extension of $\mathbb{L} = \mathbb{Q}(i)$ or $\mathbb{Q}(j)$ (the number field of degree 2 over $\mathbb{Q}$ with $j = e^{i\frac{\pi}{3}}$), σ denotes the conjugation function and γ is not a norm in $\mathbb{K}^*$. Their construction leads to a layered structure: on each layer is sent a rotated version of $\mathbb{Z}[i]^{n_t}$ or $\mathbb{Z}[j]^{n_t}$ depending on the considered constellation. Layers are separated by factor γ which is chosen so that the perfect codes have a non-vanishing determinant and each element of the codeword has the same energy.

Thus, by construction, perfect codes are optimal: they provide full rate and full diversity, have the NVD property which guarantees to reach the DMT and are information lossless.

Example 6. *The famous Golden code [24] is a* 2×2 *perfect code so called because it uses the Golden number* $\theta = \frac{1+\sqrt{5}}{2}$*. Its construction is based on the algebra* $\mathcal{A} = (\mathbb{Q}(i,\theta)/\mathbb{Q}(i), \sigma, \gamma)$*, with* $\sigma : \theta = \frac{1+\sqrt{5}}{2} \longmapsto \overline{\theta} = 1 - \theta = \frac{1-\sqrt{5}}{2}$ *and* $\gamma = i$*. Its codeword is*

$$\mathbf{X} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha(s_1 + \theta s_2) & \alpha(s_3 + \theta s_4) \\ i\overline{\alpha}(s_3 + \overline{\theta} s_4) & \overline{\alpha}(s_1 + \overline{\theta} s_2) \end{bmatrix}, \tag{1.4}$$

where s_1*,* s_2*,* s_3 *and* s_4 *are information symbols taken in a M-QAM,* $\alpha = 1 + i - i\theta$*.*

Let $\mathbf{T}$ *be another codeword. The determinant of* $(\mathbf{X}-\mathbf{T})(\mathbf{X}-\mathbf{T})^\dagger$ *is a constant:*

$$\begin{aligned} \det(\mathbf{X}-\mathbf{T})(\mathbf{X}-\mathbf{T})^\dagger &= \left| \frac{1}{5}\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(\alpha)\left(\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_1 + \theta d_2) - i\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_3 + \theta d_4)\right)\right|^2 \\ &= \frac{1}{25}\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(\alpha)^2\left(\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_1 + \theta d_2)^2 + \mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(d_3 + \theta d_4)^2\right) = \frac{1}{5} \end{aligned}$$

where $\forall x = a + \theta b \in \mathbb{K}$*, the norm is* $\mathcal{N}_{\mathbb{K}/\mathbb{Q}(i)}(x) = (a+\theta)(a+\overline{\theta}b)$*.*

The Golden code respects the NVD property and thus reaches the optimal DMT

$$d^*(r) = (2-r)^+(2-r)^+$$

of the 2×2 *MIMO system.*

Perfect codes are designed for MIMO systems where the number of transmit antennas n_t is equal to the coherence time T_C. However they can be easily generalized to the case where $n_t < T_C$. Rectangular perfect codes are $n_t \times T_C$ space-time codes which have the same construction as perfect codes (based on the same algebra $\mathcal{A}$), but a higher number of layers (T_C layers) [25].

Rectangular perfect codes inherit from the properties of squared perfect codes. They provide full rate and full diversity, reach the optimal DMT and have a uniform energy distribution.

Example 7. *Using the same parameters as for the Golden code in Example 6, we can* *construct a* 2×3 *perfect rectangular code whose codeword is:*

$$\mathbf{X} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha(s_1 + \theta s_2) & \alpha(s_3 + \theta s_4) & \alpha(s_5 + \theta s_6) \\ i\overline{\alpha}(s_5 + \overline{\theta} s_6) & \overline{\alpha}(s_1 + \overline{\theta} s_2) & \overline{\alpha}(s_3 + \overline{\theta} s_4) \end{bmatrix}.$$

d) Delay-tolerant STBC: Code D

Previous codes have been designed for multi-antenna systems where all antennas are collocated on the same terminal. Synchronization at the receiver side is thus perfect. However, in this thesis, we are interested in cooperative networks where antennas are distributed over different nodes of the network. Synchronization of the received signals at destination is then a more complex problem.

In this context, it can be interesting to use STBCs that have a new property, i.e. delay tolerance. A STBC is said to be delay-tolerant if a delay of a fixed number of time slots between the signals received from different antennas does not reduce the diversity order.

Example 8. *In [26], Damen proposes a* 2×2 *algebraic STBC called code D that turns out to be delay-tolerant [27]. Its codeword is given by*

$$\mathbf{X} = \begin{bmatrix} as_1 + bs_2 - cs_3 - ds_4 & -cs_1 - ds_2 - as_3 - bs_4 \\ -bs_1 + as_2 + ds_3 - cs_4 & -ds_1 + cs_2 - bs_3 + as_4 \end{bmatrix}, \tag{1.5}$$

where s_1*,* s_2*,* s_3 *and* s_4 *are information symbols and coefficients are* $a = \frac{1}{\sqrt{(5+\sqrt{5})(2+\sqrt{2})}}$, $b = \frac{1}{\sqrt{(5-\sqrt{5})(2+\sqrt{2})}}$, $c = \frac{1}{\sqrt{(5+\sqrt{5})(2-\sqrt{2})}}$ *and* $d = \frac{1}{\sqrt{(5-\sqrt{5})(2-\sqrt{2})}}$. *This code also provides full rate and full diversity.*

When the signals sent by the second antenna are received with a delay of one time slot, the codeword becomes

$$\mathbf{X} = \begin{bmatrix} as_1 + bs_2 - cs_3 - ds_4 & -cs_1 - ds_2 - as_3 - bs_4 & 0 \\ 0 & -bs_1 + as_2 + ds_3 - cs_4 & -ds_1 + cs_2 - bs_3 + as_4 \end{bmatrix}. \tag{1.6}$$

The symbol rate drops ($R = \frac{2}{3}$*) as the same amount of information is sent in* 3 *time slots instead of* 2*. The interesting property is that, despite the delay, diversity is preserved because all* 2×2 *submatrices of the new codeword are invertible and thus the rank criterion (see Design criterion 1) is respected.*

Recently a delay-tolerant code based on the Golden code has also been proposed in [28]. It provides the same performance as the Golden code when the system is synchronized, and the same performance as code D when reception from an antenna is delayed by one time slot.

1.5 Lattice decoding

In order to benefit from the advantages brought by the space-time codes construction, and in particular the diversity order, an efficient decoding has to be applied.

1.5.1 Lattice representation

To perform the decoding, we first have to vectorize the received signal. For linear STBC, such as the codes previously presented, the vectorized codeword can be written as a function of the information symbols vector: $\mathbf{x} = \mathbf{G}\mathbf{s}$, where the code generating matrix $\mathbf{G}$ is unitary.

Example 9. *The vectorized Golden codeword for example is generated from matrix* $\mathbf{G}$*:*

$$\mathbf{x} = \underbrace{\frac{1}{\sqrt{5}} \begin{bmatrix} \alpha & \alpha\theta & 0 & 0 \\ 0 & 0 & \alpha & \alpha\theta \\ 0 & 0 & \overline{\alpha} & i\overline{\alpha}\overline{\theta} \\ \overline{\alpha} & \overline{\alpha}\overline{\theta} & 0 & 0 \end{bmatrix}}_{\mathbf{G}} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

The received signal then can be written:

$$\mathbf{y}_{n_rT\times 1} = \mathbf{H}_{n_rT\times n_tT}\mathbf{G}_{n_tT\times n_tT}\mathbf{s}_{n_tT\times 1} + \mathbf{w}_{n_rT\times 1}. \tag{1.7}$$

After separation of the real and imaginary parts of 1.7, we obtain

$$\underbrace{\begin{bmatrix} \Re(\mathbf{y}) \\ \Im(\mathbf{y}) \end{bmatrix}}_{\mathbf{y}_{\mathbb{R}}} = \underbrace{\begin{bmatrix} \Re(\mathbf{H}) & -\Im(\mathbf{H}) \\ \Im(\mathbf{H}) & \Re(\mathbf{H}) \end{bmatrix}}_{\mathbf{H}_{\mathbb{R}}} \underbrace{\begin{bmatrix} \Re(\mathbf{G}) & -\Im(\mathbf{G}) \\ \Im(\mathbf{G}) & \Re(\mathbf{G}) \end{bmatrix}}_{\mathbf{G}_{\mathbb{R}}} \underbrace{\begin{bmatrix} \Re(\mathbf{s}) \\ \Im(\mathbf{s}) \end{bmatrix}}_{\mathbf{s}_{\mathbb{R}}} + \underbrace{\begin{bmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{bmatrix}}_{\mathbf{w}_{\mathbb{R}}}, \tag{1.8}$$

and finally, after transposing equation (1.8), we obtain a lattice representation of the system

$$\mathbf{y}_{\mathbb{R}}^{\dagger} = \mathbf{s}_{\mathbb{R}}^{\dagger}\mathbf{M} + \mathbf{w}_{\mathbb{R}}^{\dagger}, \tag{1.9}$$

where $\mathbf{M}_{2n_tT\times 2n_rT} = \mathbf{G}_{\mathbb{R}}^{\dagger}\mathbf{H}_{\mathbb{R}}^{\dagger}$ is the generator matrix of lattice Λ.

1.5.2 Maximum Likelihood decoding

In order to exploit all the diversity orders provided by the STBC, a maximum likelihood (ML) decoder has to be used. Indeed, a sub-optimal decoder such as a zero-forcing (ZF) or minimum mean square error (MMSE) recovers only part of the diversity (a diversity order of $n_r - n_t + 1$ is achieved). If the transmitter and the receiver have the same number of antennas, these sub-optimal decoders do not even get diversity at all.

Considering the lattice representation of the system (1.8), the ML decoder minimizes the distance between the received point and $\mathbf{x}$ a point of the lattice Λ:

$$\min_{\mathbf{x}\in\Lambda} ||\mathbf{y}_{\mathbb{R}}^{\dagger} - \mathbf{x}||.$$

This problem can be solved by using an exhaustive search over all possible symbol vectors of the constellation. However, the complexity of this decoder increases exponentially with the lattice dimension, making it impossible to implement for large array sizes and high spectral efficiencies.

Lattice decoding algorithms have been used to reduce the computational complexity of the ML detector by only searching over the noiseless received signals that lie within an hypersphere centered on the received signal (see Figure 1.9).

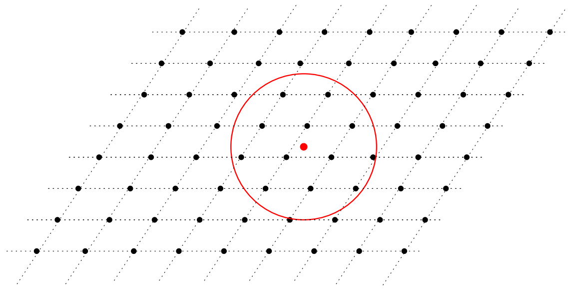


Figure 1.9: Search in an hypersphere centered on the received signal

Sphere decoder (SD)

The sphere decoder algorithm [29] searches for the closest point among all lattice points inside a sphere of a given radius R centered at the received point.

For each component u_i of the lattice point $\mathbf{x} = \mathbf{uM}$, lower and upper bounds are computed taking into account the initial radius R and the considered constellation, which defines an interval I_i. The search is then performed in these intervals. For each components combination, the distance d^2 to the received signal is computed. If this distance is lower than the radius R, the point is stored and the radius is updated.

The choice of the initial radius R is fundamental for the convergence of the algorithm. Indeed, a large enough radius is necessary to guarantee that at least one point is found inside the sphere. On the other side, a lower radius brings a lower complexity. Several methods are proposed in literature to calculate the sphere radius as a function of the signal-to-noise ratio and/or the channel realization.

A variant of the sphere-decoder has been proposed in [30] by Agrell *et al* based on a previous algorithm by Schnorr and Euchner (SE). This algorithm has the same principle as the SD, but points are enumerated inside the sphere in a different order.

In the following of this thesis, the SE is used at the destination node so as to reach and analyze the optimal performance of the proposed protocols.

1.5.3 A tradeoff between performance and complexity: Sequential decoders

Sub-optimal decoding algorithms such as ZF or MMSE have a low complexity but poor performance. On the contrary, ML decoding algorithms such as SD or SE allow to reach the optimal performance of a MIMO system, but have a high complexity.

Sequential decoders like Fano and stack allows to obtain a range of performance from ML to ZF-DFE, with proportional complexity.

A bias is added to the cost function (Euclidean distance) to accelerate the search by favoring the long paths on the search tree. If the bias is equal to 0, the ML performance is obtained, and if the bias is high, ZF-DFE performance is obtained. Depending on the application constraints, the bias is chosen so as to provide the desired performance level.

1.6 Conclusion of the chapter

In this chapter, we have introduced cooperation and defined cooperative protocols. We have shown that the cooperative network can be modeled as a MIMO system. Thus, the same multi-antenna techniques can be used in a distributed manner. We have presented some information theory tools, that will be used all along this thesis to evaluate the proposed cooperative protocols: capacity, outage probability and diversity-multiplexing gain tradeoff. A method to reach the theoretical limits of a multi-antenna channel (optimum rate and diversity) is to use space-time coding. We have recalled the design criteria of an optimal space-time code, and presented the main STBC that will be used in the following chapters.

Chapter 2

Relay channel I: performance of cooperative protocols

The two following chapters focus on the relay channel. The relay channel is the smallest decomposition of a wireless network. A single source wants to transmit information to a single destination. This transmission can be helped by one or several relays in the network that are directly linked to both source and destination. This model has been the most studied and numerous protocols have been proposed.

In the case of single-antenna terminals, some of the proposed protocols succeed to provide full rate of 1 symb. pcu and full diversity order of $N+1$, where N is the number of relays. The challenge is to achieve the DMT upper-bound, which is the one of the $(N+1) \times 1$ MISO channel.

When terminals are equipped with several antennas, the optimum rate and diversity orders, as well as the DMT upper-bound expressions are slightly more complex, but the challenge remains the same: how to achieve the DMT upper-bound (or at least to approach it in the tightest way) without adding too much complexity to the transmission protocols.

We will begin this chapter by defining the relay channel model in section 2.1 and presenting the different existing protocols for such a system in section 2.2. The performance of these protocols strongly depends on the context of the study. In section 2.3, we will show interest in the influence of several implementation constraints such as relay location, power distribution, or the influence of an asynchronism between received signals. This study shows that cooperation brings high gains asymptotically, thanks to the cooperative diversity, but performance at low SNRs is worse than with non-cooperative transmission, whatever the implementation constraints are. We address this problem in section 2.4 where a new adaptive transmission strategy is designed to optimize the performance of cooperative protocols. Finally conclusions are given in section 2.5.

2.1 Relay channel model

We consider a wireless network composed of one source, N relays and one destination.

The channel links are assumed to be Rayleigh distributed and slow fading, so their coefficients can be considered constant during the transmission of at least one frame. Besides, we suppose a symmetric scenario, i.e. all the channel links are subject to the same average SNR.

In each time slot, the total power of transmitted signals is set to $P_{tot} = 1$. When several nodes are transmitting simultaneously, this power has to be shared.

The terminals we consider are half-duplex; they cannot receive and transmit at the same time. They are equipped with only one antenna; the MIMO case is not considered in this thesis.

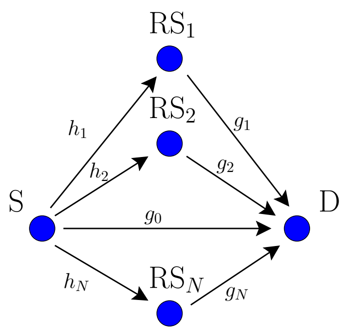


Figure 2.1: Relay channel model with one source, N relays and one destination

In the next sections, the notation of Figure 2.1 will be used. The channel coefficient of the source-destination link is g_0. $\forall n \in \{1, \ldots, N\}$, the channel coefficients of the source-relay RS$_n$ and the relay RS$_n$-destination links are h_n and g_n respectively.

We assume that the source has no channel state information (CSI), and that each relay RS$_n$ knows its corresponding source-relay channel coefficient h_n. Depending on the relaying strategy (i.e. amplify-and-forward or decode-and-forward), the channel knowledge at the destination side is different. If an amplify-and-forward strategy is used, the destination is supposed to know all the channel products $g_n \beta_n h_n$, where β_n is the amplifying factor. If a decode-and-forward strategy is used, the destination is supposed to know only the channel coefficients g_n necessary for the decoding.

2.2 Existing protocols: state-of-the-art

A multitude of protocols have been proposed in the past years, most of them using an amplify-and-forward strategy. In this section, we will detail the main existing AF and DF protocols in literature.

2.2.1 Amplify-and-forward (AF) protocols

Amplify-and-forward protocols have been the most studied due to their simplicity. Indeed, relays simply forward the signals they received in a previous time slot, multiplied by an amplifying factor.

The orthogonal amplify-and-forward (OAF)

One-relay case. The protocol proposed in [31] for the one-relay case is an orthogonal protocol (where source and relay do not transmit simultaneously).

Its transmission frame can be divided in two phases due to the half-duplex constraint: the listening and the forwarding phase. During the first phase the source broadcasts the message to the relay and destination. In the second one, the relay forwards an amplified version of the previously received signal while the source remains silent. The transmission frame is described in Figure 2.2.

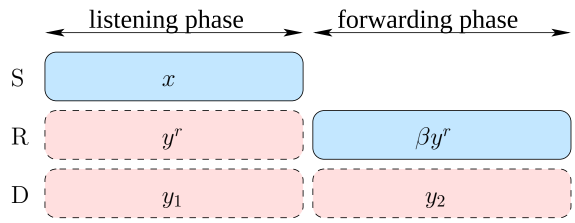


Figure 2.2: OAF protocol for 1 relay

Received signals at relay and destination are:

$$\begin{aligned} y_1 &= \sqrt{\rho} g_0 x + w_1, \\ y_r &= \sqrt{\rho} h x + v, \\ y_2 &= \sqrt{\rho} g_1 \beta y_r + w_2 = \rho g_1 \beta h x + \sqrt{\rho} g_1 \beta v + w. \end{aligned} \tag{2.1}$$

The optimum value of the amplifying factor β is obtained by normalizing the power of the forwarded signal:

$$\beta = \frac{1}{\sqrt{1 + \rho |h|^2}}. \tag{2.2}$$

This protocol provides a full diversity order of 2, but the data rate is limited to $\frac{1}{2}$ symb.

pcu. The DMT of this protocol is

$$d^*(r) = 2(1-2r)^+.$$

N-relay case. The OAF protocol has been generalized to several relays in [32]. The new transmission frame is then given in Figure 2.3 where:

- $(x_n)_{n\in\{1,\ldots,N\}}$ are the signals to be transmitted,
- $(y^r_n)_{n\in\{1,\ldots,N\}}$ are the received signals at each relay,
- $(y_k)_{k\in\{1,\ldots,2N\}}$ are the received signals at destination,
- and $(\beta_n)_{n\in\{1,\ldots,N\}}$ are the scale factors at each relay. The optimum value of each scale factor β_n is

$$\beta_n = \frac{1}{\sqrt{1+\rho|h_n|^2}}. \tag{2.3}$$

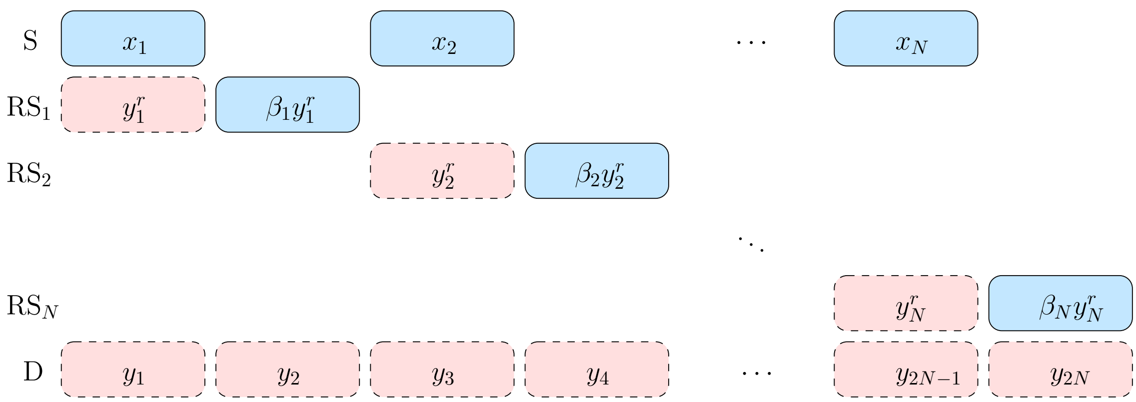


Figure 2.3: OAF protocol for N relay

Full diversity order of $N+1$ is provided, but the data rate is still limited to $\frac{1}{2}$ symb. pcu. The DMT becomes

$$d^*(r) = (N+1)(1-2r)^+.$$

This DMT can be achieved using a distributed space-time code.

The non-orthogonal amplify-and-forward (NAF)

The non-orthogonal AF (NAF) protocol is the best known cooperative protocol for the one-relay cooperative channel. That is why we will consider it as a reference in the next chapters.

The NAF protocol was proposed in [33] for the one-relay case and generalized in [34] to $N > 1$ relays.

One-relay case. Once again, the transmission frame can be divided in the listening and forwarding phases. It has been proven in [34] that the source should transmit during both phases in order to maximize the multiplexing gain. Moreover, it has also been proven in [34] that having both phases of same length is optimal. The destination obviously listens during the whole transmission. The transmission frame is described in Figure 2.4.

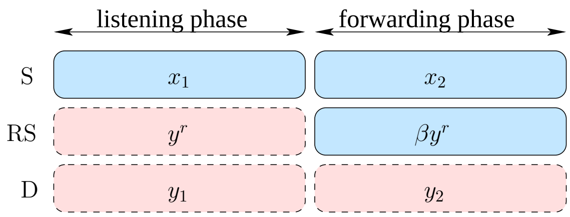


Figure 2.4: NAF frame for one relay

Received signals during the listening phase are:

$$y^r = \sqrt{\rho} h x_1 + v$$
$$y_1 = \sqrt{\rho} g_0 x_1 + w_1$$

and during the forwarding phase:

$$y_2 = \sqrt{\frac{\rho}{2}} g_1 \beta y^r + \sqrt{\frac{\rho}{2}} g_0 x_2 + w_2 = \sqrt{\frac{\rho}{2}} (g_1 \beta \sqrt{\rho} g_0 x_1 + g_0 x_2) + \left(\sqrt{\frac{\rho}{2}} g_1 \beta v + w_2 \right).$$

The amplifying factor β is the same as in the orthogonal case (see equation (2.2)) and the $\frac{1}{\sqrt{2}}$ factor comes from the total power constraint as an uniform energy distribution is assumed between source and relay.

By normalizing the noise, these equations can be rewritten in the usual form of a MIMO system: $\mathbf{y} = \sqrt{\rho} \mathbf{H}_{eq} \mathbf{x} + \mathbf{w}$, where $\mathbf{w}$ is an array of AWGN.

$$\underbrace{\begin{bmatrix} y_1 \\ \sqrt{\frac{2}{2+\rho\beta^2|g_1|^2}} y_2 \end{bmatrix}}_{\mathbf{y}} = \sqrt{\rho} \underbrace{\begin{bmatrix} g_0 & 0 \\ \sqrt{\frac{\rho}{2+\rho\beta^2|g_1|^2}} g_1 \beta h & \frac{1}{\sqrt{2+\rho\beta^2|g_1|^2}} g_0 \end{bmatrix}}_{\mathbf{H}_{eq}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} w_1 \\ \widetilde{w_2} \end{bmatrix}}_{\mathbf{w}} \tag{2.4}$$

After vectorization and separation of real and imaginary parts of complex expressions, we obtain a lattice representation of the system. So decoding can be performed by using ML lattice decoders, such as the sphere decoder [29] or the Schnorr-Euchner algorithm [30].

Theoretical limits of this protocol has been studied when the SNR grows to infinity. In [34] the DMT of such a scheme is proved to be

$$d^*(r) = (1-r)^+ + (1-2r)^+.$$

This DMT can be achieved using an optimal 2×2 STBC such as the Golden code [35] in a distributed manner. The Golden codeword is given in equation (1.4). The first line of this matrix is sent by the source in the first phase. During the second phase, the source sends the second line of the matrix and the relay retransmits the received signals corresponding to the first line (see Figure 2.5).

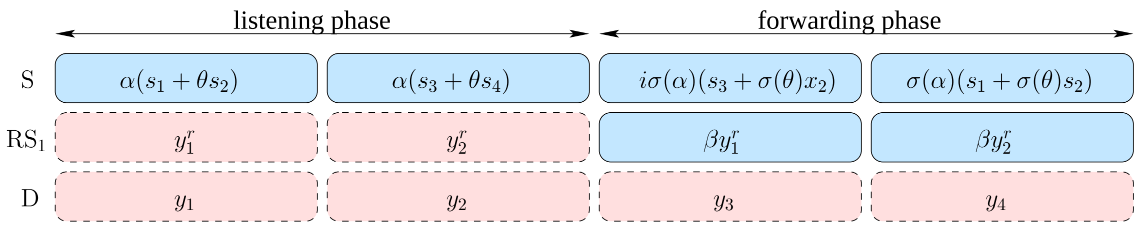


Figure 2.5: NAF frame implemented with the distributed Golden code

Remark 1. *If the relay has no memory, time slots 2 and 3 can be switched. The relay thus can retransmit signals immediately without having to stock them.*

N-relay case. Generalization of this protocol to several relays is straightforward. The source keeps transmitting during the whole frame. Each relay listens once, and forwards the amplified received signal in the next time slot.

The transmission frame is schematized in Figure 2.6.

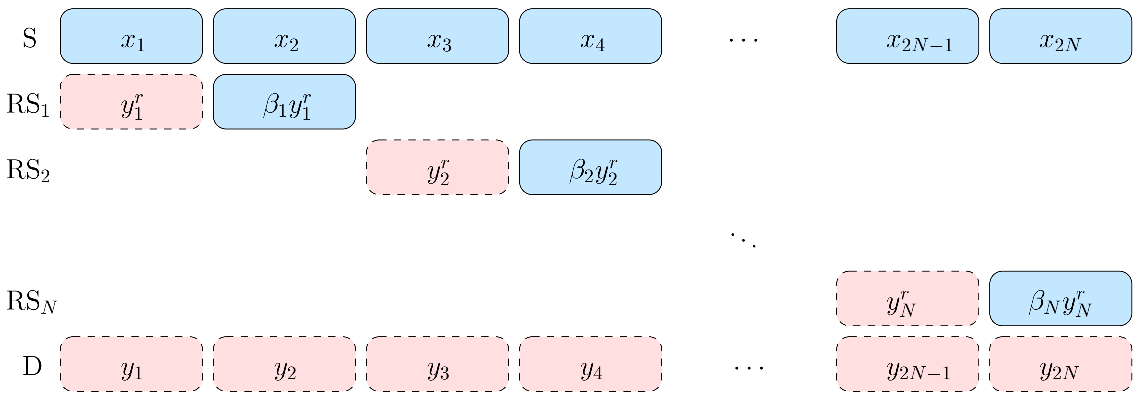


Figure 2.6: NAF frame for N relays

An equivalent model of the MIMO form $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$ can be calculated for any number of relays and decoding can be performed using ML lattice decoders.

The DMT of the NAF protocol using N relays is

$$d^*(r) = (1-r)^+ + N(1-2r)^+.$$

It has been proven in [35] that this protocol is optimal when used with a distributed $2N \times 2N$ perfect code [23].

The slotted amplify-and-forward (SAF)

Slotted amplify-and-forward (SAF) has been proposed in [36] in order to protect a larger part of the signal.

For one relay this protocol is equivalent to the NAF.

In the N-relay case, during each time slot (except for the first one), the source and one of the relay are transmitting, while another relay is listening. Thus the message is divided into $N+1$ parts and N of them are protected by relays. The transmission frame is schematized in Figure 2.7.

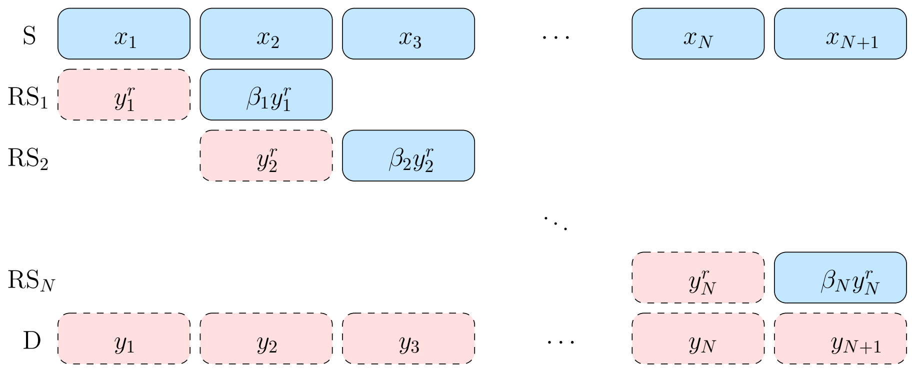


Figure 2.7: SAF frame for N relays

This protocol is more complex than the NAF as interferences are created between relays. Indeed, relay RS_n not only listens to the coded symbol x_n transmitted by the source, but also to the signal $\beta_{n-1}y_{n-1}^r$ forwarded by relay RS_{n-1}.

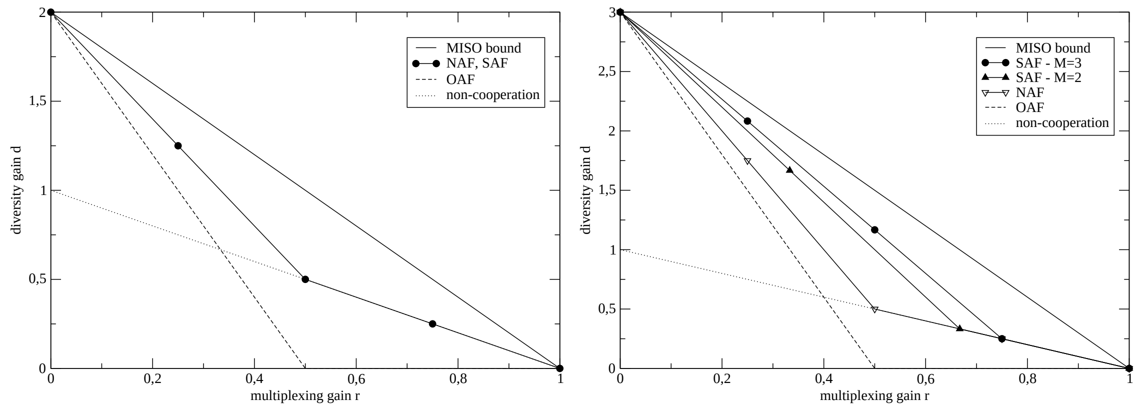
However, it provides a much better DMT as a greater part of the signal is protected. In [36], Yang *et al.* even proved that for a number of relays $N > 1$, the DMT reaches the MISO bound when the frame length M grows to infinity.

$$d^*(r) = (1-r)^+ + N\left(1 - \frac{M+1}{M}r\right)^+$$

$$\lim_{M\to\infty} d^*(r) = (N+1)(1-r)^+$$

This DMT is achieved by the Naive SAF where relays are assumed isolated, i.e. interferences between relays are negligible.

In Figures 2.8(a) and 2.8(b) are represented the DMT curves of the presented AF protocols for the one-relay and two-relay channels respectively. The protocol having the best asymptotic limit is the SAF, followed by the NAF. The OAF is the one providing the worse performance because of its low symbol rate.



(a) One relay (b) Two relays

Figure 2.8: DMTs of existing AF protocols

2.2.2 Decode-and-forward (DF) protocols

This subsection is dedicated to the state-of-the-art of decode-and-forward protocols. They have been less studied than AF ones because they require much complexity at relays. Indeed relaying terminals have to decode received signals before retransmitting them.

This complexity is counterbalanced by their high performance in some specific context, such as multihop channels.

Initial selection between decode-and-forward and non-cooperation

DF protocols assume that signals are correctly decoded at relays during the listening phase, which is obviously not always the case. That is why we have to guarantee the first phase of transmission. In literature, a selection based on the source-relay links was proposed [31]. The used criterion is the outage probability.

Indeed, according to Shannon theorem, if the link between source and relay RS_n, $n \in \{1, \ldots, N\}$, is in outage, no detection is possible at this relay without error. Thus this relay cannot be used in a decode-and-forward strategy. In the other case, the source-relay RS_n link is not in outage, a correct detection is possible. We can assume that the used STBC is powerful enough so that no error occurs and we use this relay in the cooperative protocol.

The outage event of a source-relay RS_n link is defined by

$$\mathcal{O} = \left\{ \log\left(1 + \rho_n |h_n|^2\right) < R_n \right\},$$

where ρ_n is the signal to noise ratio and R_n is the spectral efficiency of the source-relay RS_n link.

Definition 5. *If its link with source is not in outage, a relay is said to be active.*

Only active relays are selected. If there are $N_u \geq 1$ active relays, a DF protocol with N_u relays is used, and if there is no active relay, we use a non-cooperative strategy.

In practice, each relay can determine whether it should be active or not and send this information to the destination with a single bit. The destination then knows how many relays can be used and so the scheme to be applied. The destination broadcasts this information to the other nodes of the network. This implementation aspect (channel estimation and feedback) will not be detailed further in this thesis as we are interested mainly in studying existing cooperative protocols and proposing new ones.

The Laneman-Tse-Wornell decode-and-forward (LTW DF)

The LTW DF is an orthogonal protocol proposed in [31] and named after its authors Laneman, Tse and Wornell.

Its transmission frame is very similar to the one of the OAF in Figure 2.3 except for the processing at relay: after receiving a signal, the relay decodes it and forwards this decoded version. However, because of the decoding at relays:

- the source first sends all the coded symbols which are listened to by all relays. Indeed, relays need all these signals to decode without a rank deficiency.
- the use of this decode-and-forward strategy is restricted by the existence of active relays (see previous subsection). Otherwise non-cooperation is used.

For a system with N relays, a data rate of $\frac{1}{2}$ symb. pcu is achieved and a diversity order of $(N+1)$. The DMT is

$$d^*(r) = (N+1)(1-2r)^+.$$

The Nabar-Bölcskei-Kneubühler decode-and-forward (NBK DF)

NBK DF protocol has been proposed in [33] to solve the problem of low rate of the LTW DF. It is called after its authors Nabar, Bölcskei and Kneubühler. In order to increase the transmission rate, the source transmits information during the whole frame. Thus, only half of the information is repeated by the relays. The transmission frame is very similar to the one of the NAF protocol (see Figure 2.6) except that the received signals at relays are decoded before being retransmitted. Moreover, only active relays are used.

This protocol has a full data rate of 1 symb. pcu, but unfortunately an order of diversity limited to 1. Indeed, unlike in the amplify-and-forward case, a distributed STBC cannot be used because it would introduce a rank deficiency for the decoding at relays. Thus only information symbols can be sent. Its DMT, calculated in [37], is

$$d^*(r) = \begin{cases} 1 & \text{if } \frac{N}{2(N+1)} \geq r \geq 0 \\ \frac{2(N+1)}{N+2}(1-r) & \text{if } 1 \geq r \geq \frac{N}{2(N+1)} \end{cases}.$$

The dynamic decode-and-forward (DDF)

The dynamic decode-and-forward (DDF) was proposed in [34] by Azarian *et al.*

One-relay case. In the one-relay case, the source transmits during the whole transmission frame. The relay listens till it is able to decode the message. It then retransmits this decoded version. The length of the first phase is thus variable and changes dynamically depending on the channel realization and the noise.

Studying the capacity of the source-relay link, authors show that the length T_1 of the first phase is

$$T_1 = \min\left\{T, \left\lceil \frac{TR}{\log(1+\rho|h|^2)} \right\rceil\right\},$$

where T is the whole transmission duration and R is the spectral efficiency in bits pcu.

The DMT of the DDF is calculated in [34] and proved to be better than any known DF protocol, and even optimal for $r \leq \frac{1}{2}$.

$$d^*(r) = \begin{cases} 2(1-r) & \text{if } 0 \leq r \leq \frac{1}{2} \\ \frac{1-r}{r} & \text{if } \frac{1}{2} \leq r \leq 1 \end{cases}$$

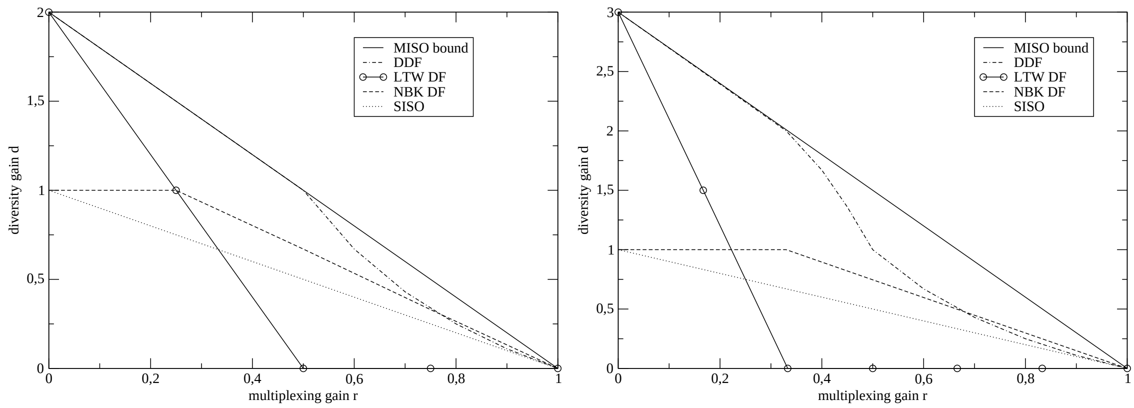
N-relay case. The DDF protocol was generalized to the N-relay case. The protocol is then more complex since each relay needs a different time to decode the message. Except for the first relay, to decode, relays have to take into account not only the source signal, but also the other relays signals. Thus they have to know the decoding time of all other relays.

The DMT in the N-relay case provided in [34] is optimal for $r \leq \frac{1}{N+1}$.

$$d^*(r) = \begin{cases} (N+1)(1-r) & \text{if } 0 \leq r \leq \frac{1}{N+1} \\ 1 + \frac{N(1-2r)}{1-r} & \text{if } \frac{1}{N+1} \leq r \leq \frac{1}{2} \\ \frac{1-r}{r} & \text{if } \frac{1}{2} \leq r \leq 1 \end{cases}$$

This protocol is quite hard to implement in practice: an implementation has been proposed by Kumar and Caire in [38], but does not succeed to reach the performance expected from the outage probability curves. For the one-relay case a variant of the DDF is proposed in [39] to reduce its decoding complexity by using the Alamouti code.

In Figures 2.9(a) and 2.9(b) are represented the DMT curves of the presented DF protocols for the one-relay and two-relay channels respectively. The DDF has the best asymptotic limit, even better than the best AF protocol in the one-relay case. Both LTW and NBK



(a) One relay

(b) Two relays

Figure 2.9: DMTs of existing DF protocols

DF have bad DMTs: the first one because of its low data rate and the second one because it does not provide diversity.

2.3 Implementation constraints

The performance of the relay channel depends on numerous parameters such as relay location or power allocation. Moreover, when several terminals transmit simultaneously, a desynchronization at destination between signals from source and relays could have a disastrous impact on system performance. In this section, we study the influence of these parameters through simulation results for the case of the one-relay channel using an amplify-and-forward strategy.

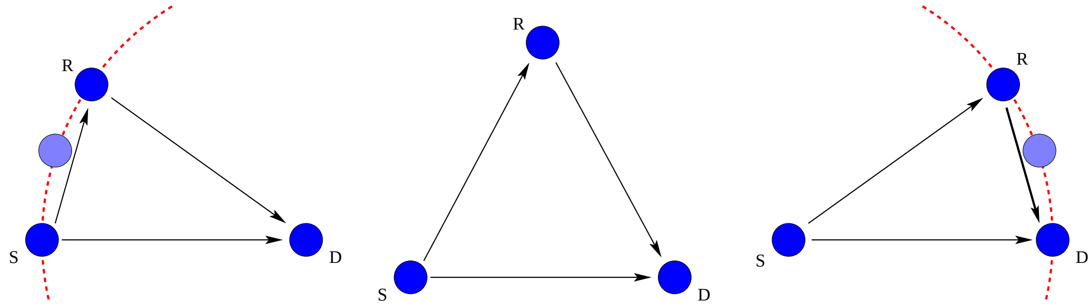
2.3.1 Influence of relay location

Most papers dealing with the relay channel consider a symmetric scenario: the signal-to-noise ratios of all links in the network are the same, which physically corresponds to a relay located at equal distance from source and destination (see Figure 2.10(b)). This case represents the less interesting one to use cooperation. In this paragraph, we study the influence of relay location by considering different gains for the channel links. We base our study on simulation results obtained for different scenarios.

We distinguish two cases whether the protocol is orthogonal or not.

The orthogonal AF (OAF)

In Figure 2.11 is represented the performance of the OAF with different channel gains on the source-relay and relay-destination links.



(a) near the source (b) at equal distance from source and destination (c) near the destination

Figure 2.10: Relay location

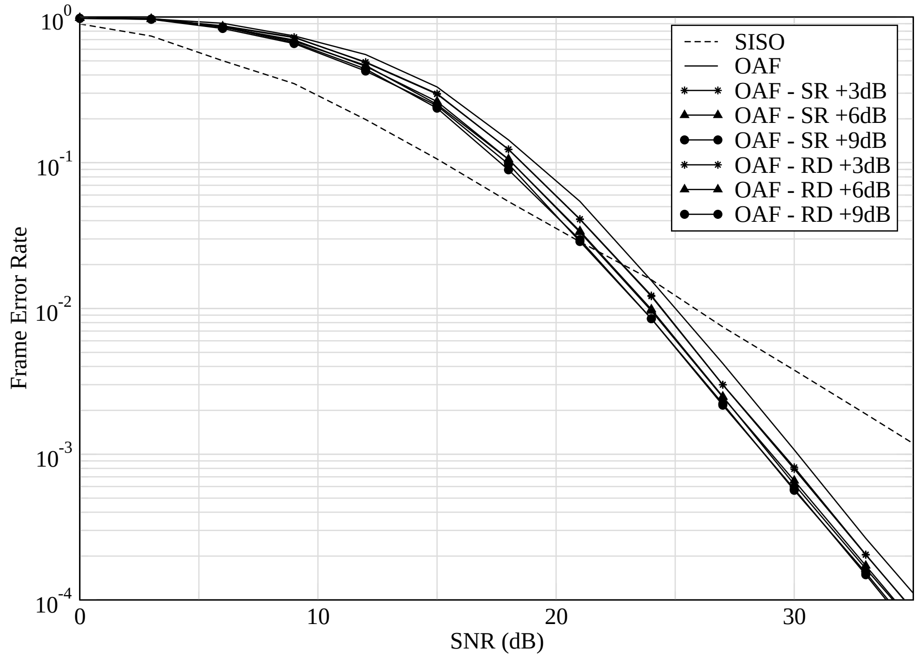


Figure 2.11: Influence of the relay location on the OAF performance

Obviously, if we set the relay closer to the source or closer to the destination (see Figures 2.10(a) and 2.10(c) respectively), performance are improved since th overall path is shorter. Curves obtained for the same gain on the source-relay link or the relay-destination one are perfectly superimposed. We can conclude that setting the relay rather closer to the source or to the destination has no influence. Only the channel product impacts the performance.

Using equations (2.1), we can derive the capacity of the system:

$$C(\mathbf{H}) = \log\left(\left(1+\rho|g_0|^2\right)\left(1+\frac{\rho^2|g_1|^2\beta^2|h|^2}{1+\rho|g_1|^2\beta^2}\right)\right),$$

which, replacing β by its expression, becomes

$$C(\mathbf{H}) = \log\left(\left(1+\rho|g_0|^2\right)\left(1+\frac{\rho^2|g_1|^2|h|^2}{1+\rho|h|^2+\rho|g_1|^2}\right)\right).$$

This last expression shows that the source-relay and relay-destination links have similar roles, and so similar influence on the performance.

The non-orthogonal AF (NAF)

In Figure 2.12(a) and 2.12(b) is represented the performance of the NAF protocol implemented with the distributed Golden code for different channel gains on the source-relay and relay-destination links respectively.

Obviously, if we set the relay closer to the source but keeping the distance between relay and destination constant, performance are improved. But this is not the case if the relay is set closer to the destination.

Using equation (2.4), we can derive the capacity of the system:

$$\begin{aligned}C(\mathbf{H}_{eq}) &= \log\det\left(\mathbf{I}_2+\mathbf{H}_{eq}\mathbf{H}_{eq}^{\dagger}\right)\\ &= \log\left(1+\rho|g_0|^2+\rho\frac{|g_0|^2+\rho\beta^2|h|^2|g_1|^2}{2+\rho\beta^2|g_1|^2}+\rho^2\frac{|g_0|^4}{2+\rho\beta^2|g_1|^2}\right),\end{aligned}$$

which, replacing β by its expression, becomes

$$C(\mathbf{H}_{eq}) = \log\left(1+\rho|g_0|^2+\rho\frac{|g_0|^2(1+\rho|h|^2)+\rho|h|^2|g_1|^2}{2(1+\rho|h|^2)+\rho|g_1|^2}+\rho^2\frac{|g_0|^4(1+\rho|h|^2)}{2(1+\rho|h|^2)+\rho|g_1|^2}\right). \tag{2.5}$$

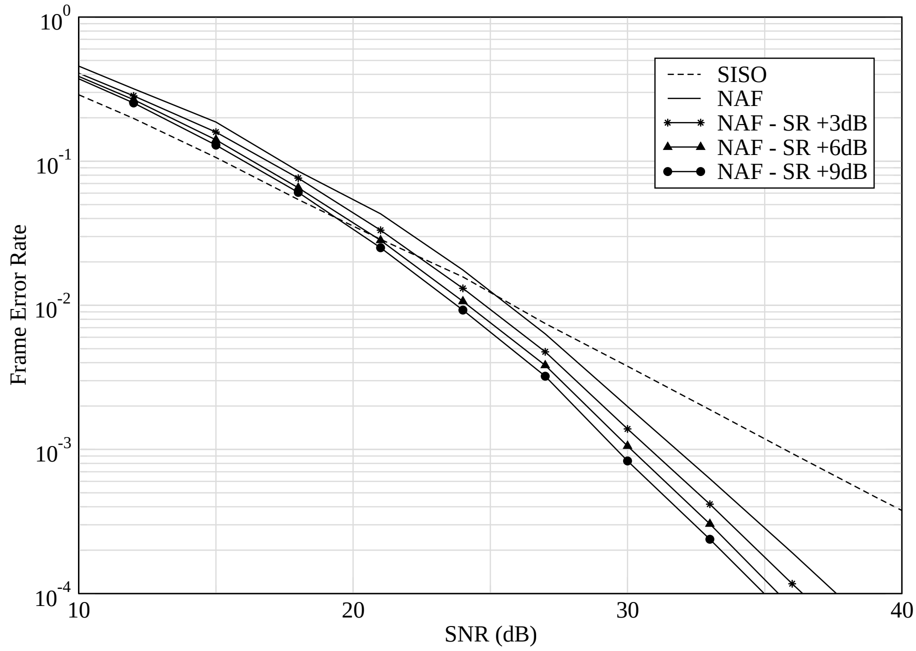
Source-relay and relay-destination have no longer similar roles. Let's differentiate expression (2.5) with respect to $|h|^2$:

$$\begin{aligned}\frac{d\left(2^{C(\mathbf{H}_{eq})}\right)}{d|h|^2} &= \frac{\rho^2(|g_0|^2+|g_1|^2)+\rho^3|g_0|^4}{2(1+\rho|h|^2)+\rho|g_1|^2} - 2\rho\frac{\rho|g_0|^2(1+\rho|h|^2)+\rho^2|h|^2|g_1|^2+\rho^2|g_0|^4(1+\rho|h|^2)}{(2(1+\rho|h|^2)+\rho|g_1|^2)^2}\\ &= \frac{\rho^2(\rho|g_0|^4+|g_1|^2)(1+\rho|h|^2)+\rho^2|g_1|^2+\rho^3|g_0|^2(|g_0|^2+|g_1|^2)+\rho^4|g_0|^6}{(2(1+\rho|h|^2)+\rho|g_1|^2)^2}\geq 0\end{aligned}$$

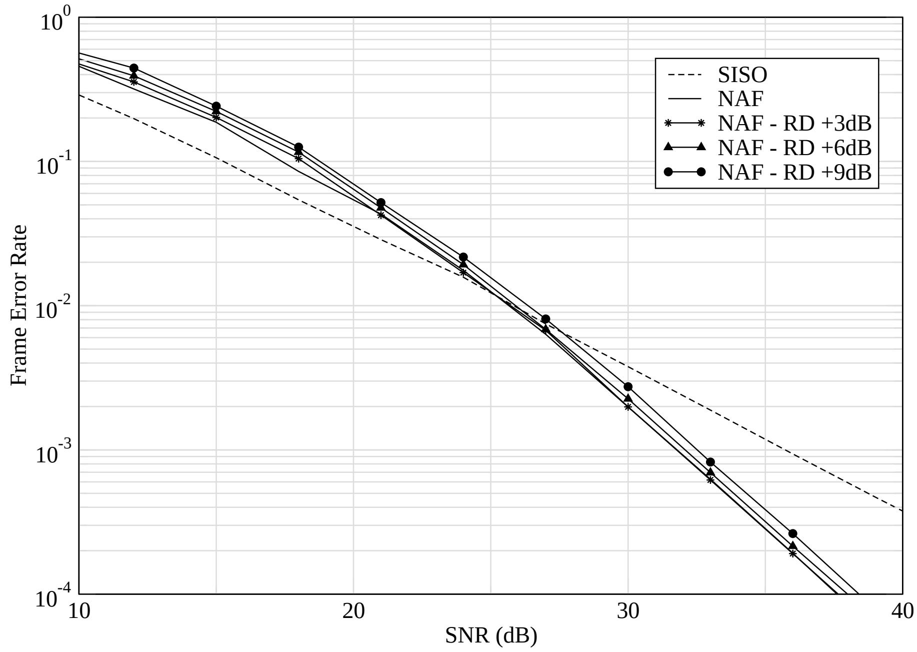
The capacity is always increasing with the source-relay link $|h|^2$.

Let's now differentiate expression (2.5) with respect to $|g_1|^2$:

$$\begin{aligned}\frac{d\left(2^{C(\mathbf{H}_{eq})}\right)}{d|g_1|^2} &= \frac{\rho^2|h|^2}{2(1+\rho|h|^2)+\rho|g_1|^2} - \rho\frac{\rho|g_0|^2(1+\rho|h|^2)+\rho^2|h|^2|g_1|^2+\rho^2|g_0|^4(1+\rho|h|^2)}{(2(1+\rho|h|^2)+\rho|g_1|^2)^2}\\ &= \frac{\rho(1+\rho|h|^2)(2|h|^2-\rho|g_0|^2(1+\rho|g_0|^2))}{(2(1+\rho|h|^2)+\rho|g_1|^2)^2}\end{aligned}$$



(a) Relay close to the source



(b) Relay close to the destination

Figure 2.12: Influence of the relay location on the NAF performance

Thus, if $2|h|^2 < |g_0|^2(1+\rho|g_0|^2)$, which is very likely and even more if the SNR is high, capacity is decreasing with the relay-destination link $|g_1|^2$. This explains the simulation results.

2.3.2 Impact of power allocation

We assume the total power in each time slot is constant and set to $P_{tot} = 1$. In the case of a non-orthogonal protocol, the transmit power during the second phase of transmission has then to be shared between source and relay. Let's call P_s and P_r the transmit powers at source and relay respectively. The transmit power during the first phase of the transmission is 1 and during the second phase $P_s + P_r = 1$.

In Figure 2.13 is represented the performance of the NAF protocol with different power distributions. Transmit powers at source and relay are given by $P_s = 1 - a$ and $P_r = a$.

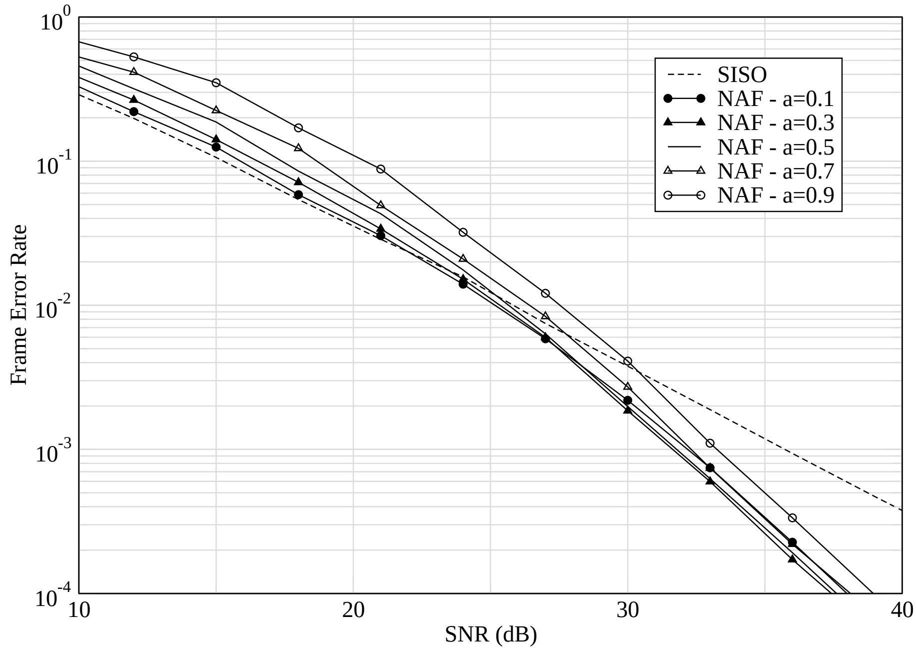


Figure 2.13: Influence of the power distribution on the NAF performance

When the relay transmits with a higher power than the source, performance are decreased, which was foreseeable since the noise is then amplified. So a should not be chosen greater than $\frac{1}{2}$. When the source transmits with a higher power than the relay, a tradeoff has to be found. Indeed, the protocol does not have the same behavior for low and high SNRs and two regimes can be defined:

- for low SNRs, performance is better as a is small ($a = 0.1$),
- for high SNRs (SNR$>$ 27.5 dB), performance is better for $a = 0.3$.

The capacity of the system is given by

$$C(\mathbf{H}_{eq}) = \log\left(1+\rho|g_0|^2 + \frac{1}{1+\frac{\rho|g_1|^2\beta^2}{a}}\left(\frac{\rho|g_0|^2}{1-a}(1+\rho|g_0|^2) + \frac{1}{a}\rho^2|g_1|^2\beta^2|h|^2\right)\right).$$

If we consider the two last terms of this expression, the first one is increasing with a, while the other one is decreasing. That is why a tradeoff is needed.

But the choice of $a \in]0, \frac{1}{2}]$ has only a small influence on performance. Whatever the transmit power at relay is, diversity is preserved, and performance losses are lower than 1 dB.

To find the optimal power distribution we have to use optimization algorithms for power allocation. Coefficient a is then a function of the signal-to-noise ratio and channel realization.

In the following, for simplicity, we will always consider a uniform energy distribution.

2.3.3 Effect of a desynchronization

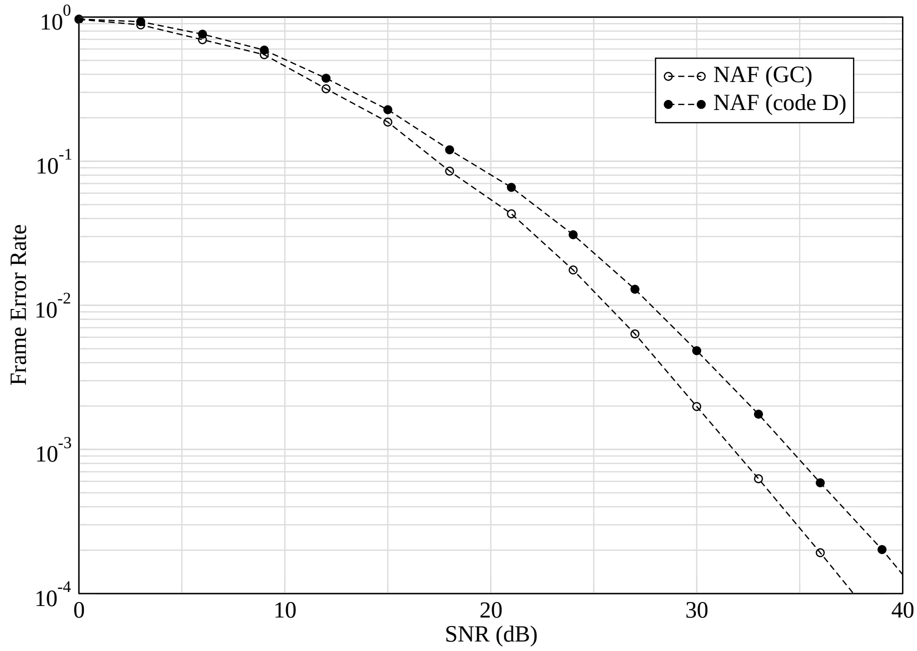
We have assumed in previous paragraphs that the destination receives signals transmitted by source and relay without any delay, in a synchronized manner. Indeed, this is necessary to preserve the space-time code structure and exploit diversity. However, in practice, a desynchronization can occur. We will consider now the case of a desynchronization of one time slot and study its influence on the performance of the NAF protocol. The same protocol is implemented with two different STBC: the Golden code and code D especially designed to be delay-tolerant.

Figures 2.14(a) and 2.14(b) represent the performance of the NAF protocol implemented with distributed Golden code and code D. In the first graph, perfect synchronization is assumed, while in the second one, a desynchronization of one time slot happens. When a desynchronization occurs, the data rate is decreased, so the two graphs should not be compared in terms of performance. The interesting point is the behavior of the different STBC compared to each other.

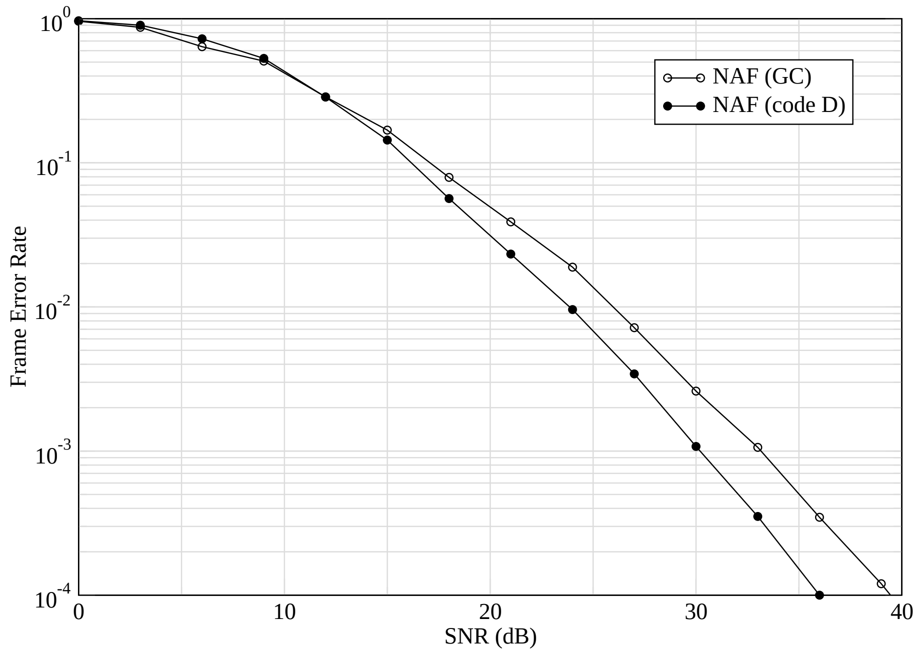
When the system is perfectly synchronized, the distributed Golden code has better performance than code D (3 dB gain). However when there is a desynchronization of one time slot, the more efficient code is code D, with a 3 dB gain over the Golden code. Diversity is preserved thanks to the first phase of the transmission (see Figure 2.15(a)), but the coding gain is not the same since desynchronization breaks the code structure.

In order to choose efficiently the distributed STBC to use, we have to study the probability of a desynchronization:

- if perfect synchronization is more likely, a distributed Golden code is implemented;
- if reception is desynchronized more than half of transmissions, a distributed code D



(a) Perfect synchronization



(b) Desynchronization of 1 time slot

Figure 2.14: Performance of the NAF protocol with the Golden code and code D

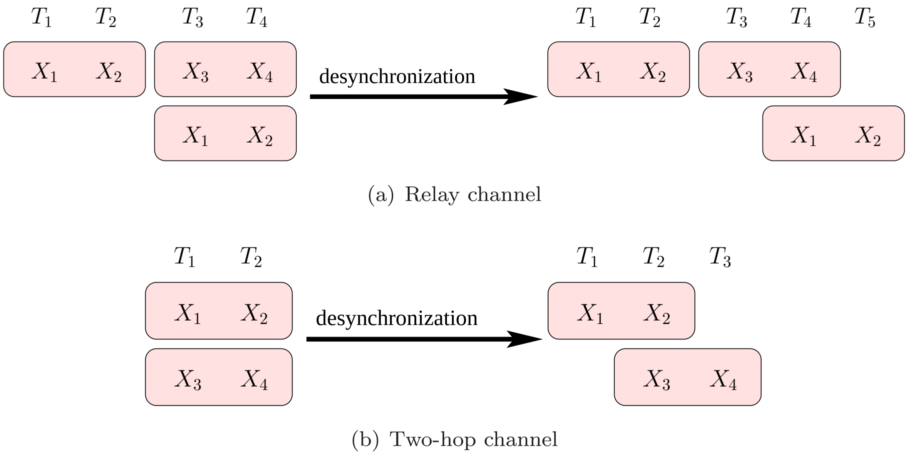


Figure 2.15: Desynchronization of one slot applied to a relay channel and a two-hop channel

provides better performance.

If no statistic is known about the delay event, the delay-tolerant code recently proposed in [28] can be used. This code provides good performance in both cases.

Remark 2. *In [40], the authors show that code D preserves diversity when a desynchronization happens, while the Golden code does not. That is because the authors consider the two-hop channel with two parallel single-antenna relays and a double-antenna destination (and not a relay channel). The source transmits information to two different relays, and these two relays retransmit this information to the destination (see Figure 2.15(b)).*

2.4 Adaptive protocols

We have seen in previous section that even with optimized relay location and/or power allocation, cooperation provides poor performance at low SNRs. It is even more efficient to use non-cooperation. So the question is why to add complexity through cooperative protocols, when these do not provide better performance.

We propose in this section a new strategy named Adaptive Cooperation which can be applied either to AF or DF protocols and solves the problem of bad performance at low SNRs. This new strategy consists in choosing the best transmission scheme, based on the instantaneous capacities of all possible transmission schemes. In subsection 2.2.2, we have already presented a selection between cooperation and non-cooperation which has been proposed in literature for DF protocols [31, 34] in order to guarantee good performance. Relay selection has also been considered in [41] to provide diversity. But in literature, no selection has ever been proposed for AF protocols. Moreover the usual selection criterion of DF protocol is based only on the source-relay outage probability, we propose in this thesis a selection taking into account all the channel links.

2.4.1 Adaptive AF strategy

Adaptive AF for the one-relay case

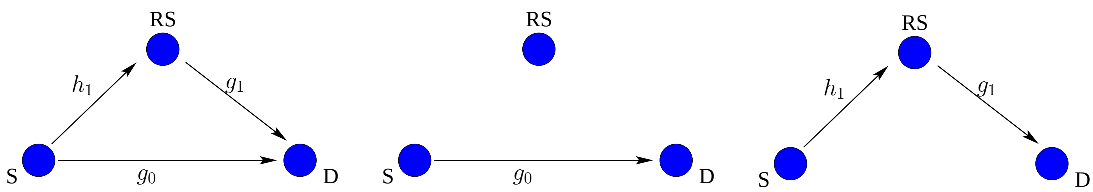
The idea leading to the definition of the Adaptive AF strategy is to consider all possible transmission schemes and decide which one to select. In order to better understand this strategy, the one-relay case is detailed, before the generalization to the N-relay case.

There are only three possible transmission schemes (Figure 2.16):

(a) AF case: a fully cooperative scheme is used, symbols are sent using the AF protocol. In case of a full rate protocol such as the NAF [35], the symbol rate is 1 symb. pcu;

(b) SISO case: only the direct link is used, symbols are sent over the source-destination link in a non-coded manner, at a rate of 1 symb. pcu;

(c) NLOS case: only the non-line-of-sight (NLOS) link is used. In a first phase symbols are sent over the source-relay link in a non-coded manner and forwarded by the relay in a second phase. The rate is then $\frac{1}{2}$ symb. pcu. Therefore in order to have the same spectral efficiency of 1 symb. pcu as in the other cases, we need to use a larger constellation. For example, if the other protocols use a 16-QAM constellation, the NLOS scheme must use a 256-QAM.



(a) AF scheme (b) SISO scheme (c) NLOS scheme

Figure 2.16: 3 possible cooperation schemes in the 1-relay case

When these three schemes are in outage and so errors occur inevitably (Shannon's theorem), a 4^{th} case could be added with non-transmission. We have not considered this case however as it will induce a rate decrease. Moreover the use of error correcting codes can improve the decoding of the message.

The principle of this new Adaptive AF strategy is to evaluate the qualities of the three schemes (SISO, AF and NLOS) and to select the best of them.

Selection criterion

In the previous subsection we have listed all the possible transmission cases. The question is now which criterion to use to select the best one.

We propose to study all these schemes and to select the one which has the largest instantaneous capacity.

Let the possible transmission schemes be numbered from 1 to N_S and note $C_i(\mathbf{H})$ the instantaneous capacity of the i^{th} scheme. The selected transmission scheme is the one offering the maximum instantaneous capacity:

$$\arg \max_{i \in \{1,\ldots,N_S\}} \{C_i(\mathbf{H})\} \tag{2.6}$$

with

$$C_i(\mathbf{H}) = \log_2(1 + \rho \mathbf{H}^{\dagger}\mathbf{H}). \tag{2.7}$$

Generalization to the N-relay case

This selection can be generalized quite easily to a higher number of relays.

For example, for 2 relays R_1 and R_2, there are 7 possible transmission schemes:

1. full cooperation: symbols are sent using the AF protocol for 2 relays. With a full rate protocol, the symbol rate is 1 symb. pcu;
2. cooperation with only relay R_1: symbols are sent using the AF protocol for only 1 relay. With a full rate protocol, the symbol rate is still 1 symb. pcu;
3. cooperation with only relay R_2;
4. non-cooperation: symbols are sent in a non-coded manner over the direct link: the symbol rate is 1 symb. pcu again;
5. NLOS link using only relay R_1: symbols are sent in a non-coded manner and the symbol rate is $\frac{1}{2}$ symb pcu;
6. NLOS link using only relay R_2;
7. both NLOS links: the symbol rate is $\frac{1}{2}$ symb. pcu.

The 7^{th} case can be ignored since its instantaneous capacity is always lower than the one of the best NLOS using one relay.

Proof. Without loss of generality we can assume that this relay is the first one. Then

$$\begin{gathered}
\log\left(1 + \rho^2 \frac{|g_1|^2\beta_1^2|h_1|^2}{1+\rho|g_1|^2\beta_1^2}\right) > \log\left(1 + \rho^2 \frac{|g_2|^2\beta_2^2|h_2|^2}{1+\rho|g_2|^2\beta_2^2}\right) \\
\frac{|g_1|^2\beta_1^2|h_1|^2}{1+\rho|g_1|^2\beta_1^2} > \frac{|g_2|^2\beta_2^2|h_2|^2}{1+\rho|g_2|^2\beta_2^2} \\
(|g_1|^2\beta_1^2|h_1|^2)(1+\rho|g_2|^2\beta_2^2) > (|g_2|^2\beta_2^2|h_2|^2)(1+\rho|g_1|^2\beta_1^2).
\end{gathered}$$

Let's compare the instantaneous capacity of the 7^{th} case with the one of the 5^{th} one:

$$\begin{aligned}
&sign\left[\log\left(1+\rho^2\frac{|g_1|^2\beta_1^2|h_1|^2+|g_2|^2\beta_2^2|h_2|^2}{2+\rho|g_1|^2\beta_1^2+\rho|g_2|^2\beta_2^2}\right)-\log\left(1+\rho^2\frac{|g_1|^2\beta_1^2|h_1|^2}{1+\rho|g_1|^2\beta_1^2}\right)\right]\\
&=sign\left[\frac{|g_1|^2\beta_1^2|h_1|^2+|g_2|^2\beta_2^2|h_2|^2}{2+\rho|g_1|^2\beta_1^2+\rho|g_2|^2\beta_2^2}-\frac{|g_1|^2\beta_1^2|h_1|^2}{1+\rho|g_1|^2\beta_1^2}\right]\\
&=sign\left[(|g_1|^2\beta_1^2|h_1|^2+|g_2|^2\beta_2^2|h_2|^2)(1+\rho|g_1|^2\beta_1^2)-(|g_1|^2\beta_1^2|h_1|^2)(2+\rho|g_1|^2\beta_1^2+\rho|g_2|^2\beta_2^2)\right]\\
&=sign\left[-|g_1|^2\beta_1^2|h_1|^2(1+\rho|g_2|^2\beta_2^2)+|g_2|^2\beta_2^2|h_2|^2(1+\rho|g_1|^2\beta_1^2)\right]<0
\end{aligned}$$

□

The number of possible transmission schemes grows with the number of relays. In the N-relay case, there are $\sum_{k=0}^{N}\binom{N}{k}=2^N$ different cooperation cases from the non-cooperative one (no relay, $k=0$) to the full cooperation one (N relays, $k=N$). We also consider the N NLOS cases using one relay. So finally, there are 2^N+N different transmission schemes to consider.

However, this high number of cases does not increase complexity that much. Indeed, only a simple test is necessary to determine the best one. As some schemes are identical except for exchanging coefficients (for example, NLOS with relay R_1 or relay R_2), the decoding complexity reduces to only $N_S=(N+1)+1=N+2$ different algorithms. So the complexity of this new selection protocol increases linearly with the number of relays, which is quite reasonable.

Moreover, we will show in the example of section 2.4.2 that depending on the chosen AF scheme, some cases can be omitted, which reduces the complexity even more.

Implementation constraints

To implement the new Adaptive AF strategy, a node in the network has to decide which transmission scheme to be used. We suppose that this node is the destination. So it has to estimate the channel coefficient g_0 of the direct link and the product channels $g_i\beta_ih_i$ for each relay R_i, calculate the instantaneous capacity of each possible transmission scheme and determine the one to be used. Then it broadcasts no more than $\lceil\log_2\left(2^N+N\right)\rceil=N+1$ bits at both source and relays in order to inform them about its decision.

As we consider a slow fading channel, an estimation is made for several frames and so the transmission strategy remains the same. When a new estimation is made and if the strategy has to be changed, it is effective after a delay of one frame during which the strategy will not be optimal.

2.4.2 Example of the Adaptive NAF

In order to better understand this new selection strategy and its possible simplifications, we develop in this section the example of the Adaptive NAF protocol.

As can be seen immediately in Figure 2.6 in section 2.2.1, the NAF scheme is a parallel protocol. Indeed, the N relays of the NAF scheme play exactly the same role and are never used simultaneously. By studying the instantaneous capacities of the cooperation schemes using NAF protocol with different number of relays, we can see easily that the greatest instantaneous capacity will be associated to a one-relay case.

So we can avoid to study all the NAF strategies with several relays, which reduces considerably the complexity. Indeed, the Adaptive NAF protocol is then the result of the selection of the best transmission scheme between the SISO scheme and the NAF schemes and the NLOS schemes using only one relay. Finally we have only $1 + N + N = 1 + 2N$ possible transmission cases to study and 3 corresponding decoding algorithms. And so, we can remark that the decoding complexity does not increase with the number of relays.

Outage Probability Analysis

The outage probability can be expressed as a function of the instantaneous capacity. For each scheme numbered from 1 to $N_S = 2^N + N$ as in subsection 2.4.1:

$$P_{\text{out}}^{(i)} = P\left\{C_i(\mathbf{H}) < R\right\} \tag{2.8}$$

where R is the spectral efficiency in bits per channel use (bits pcu).

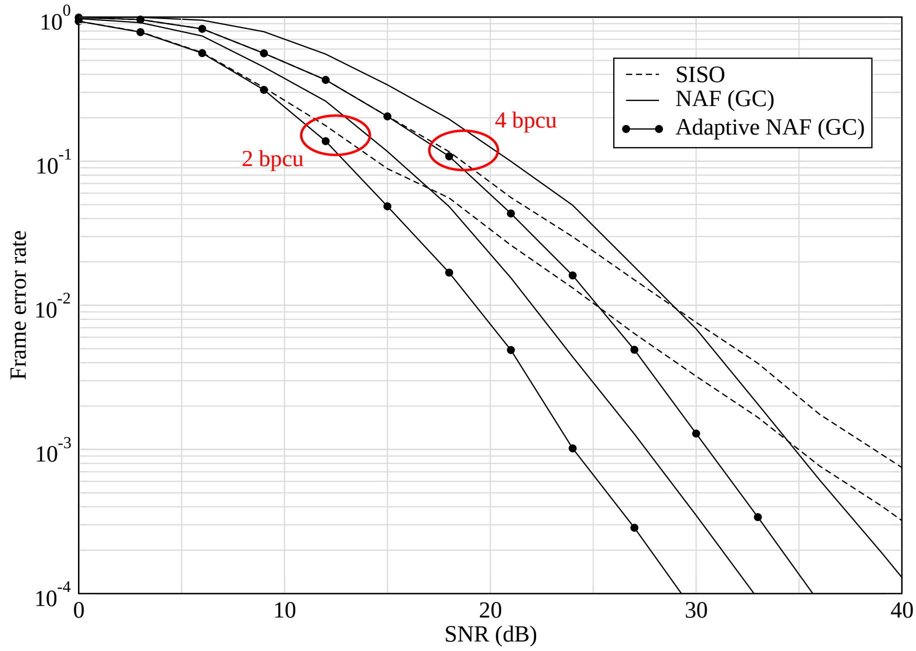
The principle of the Adaptive AF protocol is to choose the transmission scheme that maximizes the instantaneous capacity $C_i(\mathbf{H})$ over i. So the instantaneous capacity of the new Adaptive AF protocol is larger than each $C_i(\mathbf{H})$ for a fixed channel realization $\mathbf{H}$. Thus, the selection scheme is in outage if and only if (iff) the N_S possible transmission schemes are all in outage. So we get,

$$P_{\text{out}}^{(\text{AAF})} \leq P_{\text{out}}^{(i)} \qquad i \in \{1, \dots, N_S\}\,. \tag{2.9}$$

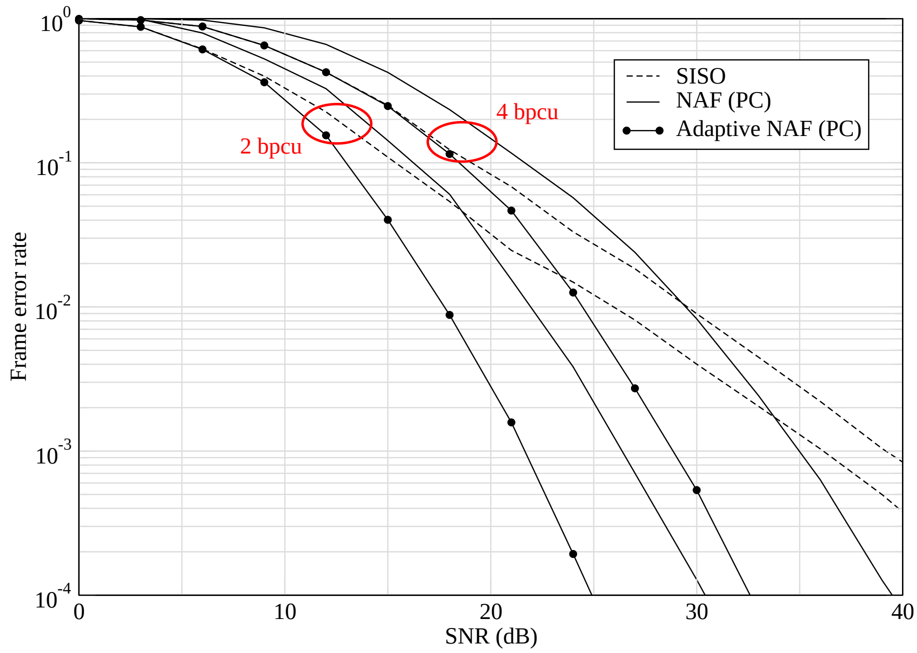
We can calculate and plot the outage probabilities of these different protocols as functions of the SNR thanks to Monte Carlo simulations.

In Figure 2.17(a), we plot the outage probabilities of the SISO, NAF and Adaptive NAF protocols for a one-relay scheme and spectral efficiencies of 2 and 4 bits pcu. We can note that the Adaptive NAF performs better than the NAF protocol, with 3 and 4 dB asymptotic gains for 2 and 4 bits pcu respectively. Even more interesting is the fact that the Adaptive NAF always performs better than SISO, even at low SNR, which was the main weakness of the NAF protocol without selection.

In Figure 2.17(b), we plot the outage probabilities of the SISO, NAF and Adaptive NAF protocols for a two-relay scheme and spectral efficiencies of 2 and 4 bits pcu. Here again,



(a) 1-relay scheme



(b) 2-relay scheme

Figure 2.17: Adaptive NAF outage probability

the enhancement of the Adaptive NAF over the NAF protocol is verified, as we obtain 5 and 6 dB asymptotic gains for 2 and 4 bits pcu respectively and solve the problem of bad performance at low SNR.

Simulation results

In Figures 2.18(a) and 2.18(b) we plot the frame error rates of the SISO, NAF and Adaptive NAF protocols as functions of the SNR for spectral efficiencies of 2 and 4 bits pcu.

In Figure 2.18(a) are represented the curves for the one-relay scheme. The NAF protocol is implemented with a distributed Golden code [24] and a Schnorr-Euchner decoding. Simulation results confirm theoretical ones obtained by outage probability calculations. We can observe that the Adaptive NAF performs better asymptotically than the NAF protocol, with 3 and 4 dB gains. Moreover we can check that it solves the problem of bad performance at low SNR.

In Figure 2.18(b) are represented the curves for the two-relay scheme. The NAF protocol is implemented with a distributed 4×4 perfect code [23] and a Schnorr-Euchner decoding. The improved performances of the Adaptive NAF are here again confirmed with 5 and 6 dB gains over the NAF protocol. Besides, the problem of bad performance of the NAF at low SNR is solved with two relays too, since the Adaptive NAF curve is always under the SISO curve.

2.4.3 Adaptive DF strategy

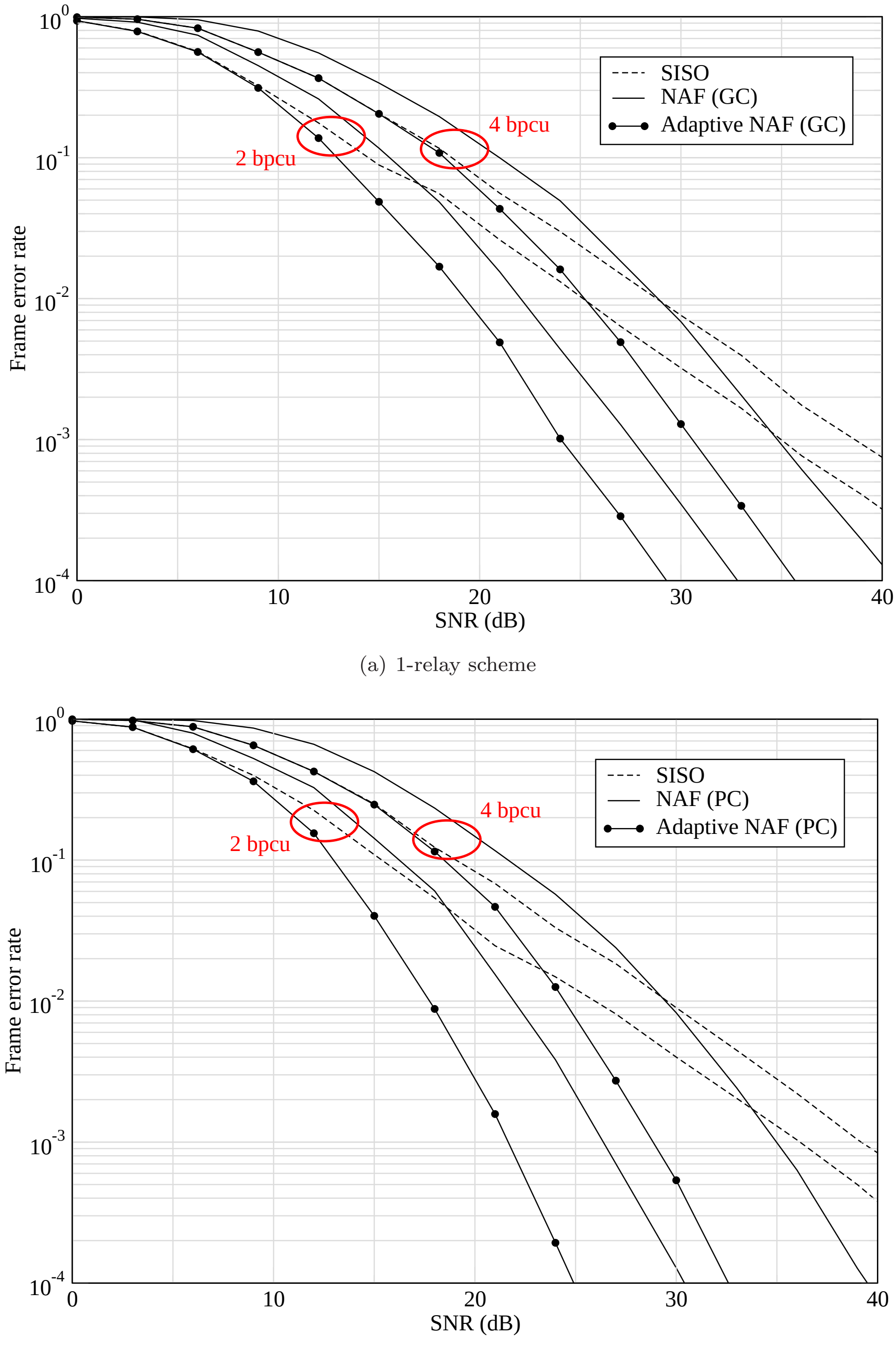
This new selection working quite efficiently on AF protocols, we propose to adapt it to DF protocols, which have the same problem as AF protocols: poorer performance at low SNR than non-cooperation.

Presentation of the Adaptive DF

The Adaptive DF strategy is based on the same principle than the Adaptive AF strategy. However relays do not amplify the signals but decode them for both DF and NLOS protocols. In order to guarantee an error-free decoding at relays, the selection presented in subsection 2.2.2 has to be added to the protocol. The scheme selection is then made in two steps:

1. first, we select only the N_u active relays;

2. second, we select the best transmission scheme in the $2^{N_u} + N_u$ possible ones in term of instantaneous capacity.



(a) 1-relay scheme

(b) 2-relay scheme

Figure 2.18: Adaptive NAF frame error rate

Implementation constraints

As in the Adaptive AF strategy, it is the destination which selects the best transmission scheme. However, before considering the possible transmission schemes, it has to know which relays are active, i.e. which source-relay links are not in outage. We propose that each relay estimates its own source-relay link and transmits a single bit to the destination indicating whether it is in outage or not.

Then, the steps are the same as for the Adaptive AF: the destination estimates the direct link g_0 and the relay-destination links g_i for all K relays which are not in outage. Estimations of the source-relay links are not necessary as the relays decode the signals. Thanks to these estimations, it can calculate the instantaneous capacities of all possible transmission schemes and determine the best one. $N+1$ bits are then necessary to broadcast the information on the chosen scheme to the source and relays.

The adaptive DF strategy will be applied in next chapter to the proposed DF protocol in order to confirm that performance is also improved in this case.

2.5 Conclusion of the chapter

In this chapter, we have presented the state-of-the-art of both amplify-and-forward and decode-and-forward protocols.

We have explored the influence of practical parameters, such as relay location, power allocation and desynchronization, on the performance of a relay channel using an amplify-and-forward strategy. This study has showed the difference between orthogonal and non-orthogonal protocols even if in both cases, choosing a relay closer to the source improves performance. Power allocation is a more complex problem since the optimal power distribution depends also on the SNR. Low power should be given to the relay at low SNR, and much power when the SNR grows. Desynchronization of one time slot has not such a bad impact since diversity is preserved and only the coding gain is decreased.

This study has also showed that a big drawback of cooperation is its bad performance at low SNRs. In the last section, we have proposed Adaptive amplify-and-forward and decode-and-forward protocols based on a new selection criterion derived from the calculations of the instantaneous capacities of all possible transmission schemes (SISO, cooperative schemes, NLOS schemes). For the Adaptive DF protocol, an additional selection on the source-relay links is necessary to guarantee an efficient decoding at relays. Both outage probability and performance from simulation results prove that the Adaptive cooperation enhances the performance of the initial cooperation schemes at high SNRs, and solves the problem of poor performance at low SNRs.

Chapter 3

Relay channel II: toward a practical decode-and-forward protocol

In this chapter, we are interested in decode-and-forward (DF) protocols for the relay channel. DF protocols require more processing than amplify-and-forward (AF) ones, as the signals have to be decoded at relay before being forwarded. However, if signals are correctly decoded at relays, the performance is better than that of AF protocols, as noise is deleted.

Moreover, in this chapter, our work is motivated by the potential advantages of DF protocols over AF protocols in some scenarios. For example, it has been proven in [42, Proposition 2.2] that in a clustered multihop context it is necessary to use a DF protocol at some relays to regenerate the signals. Indeed a full AF strategy would add more noise at each hop, which makes signals no longer decodable.

DF protocols in literature usually do not succeed at bringing both full diversity and full symbol rate (the LTW DF has a rate of $\frac{1}{2}$ symb. pcu, the NBK DF provides no diversity). The only proposed solution to this problem is the DDF protocol, but its implementation is quite complex and a practical implementation of the DDF has not been proposed.

In order to provide both full rate and full diversity, we explore the introduction of distributed space-time codes in DF protocols, in the same way they have been successfully used in AF protocols, and in particular the NAF protocol [34, 35]. As a first approach, we propose to use the famous Alamouti code in DF protocols. This solution provides full diversity, but the rate is limited to $\frac{1}{2}$ symb. pcu. As a second approach, we propose to introduce more sophisticated space-time codes. We propose a DF protocol with asymmetric listening and forwarding phases, which brings full diversity, but a rate of only $\frac{2}{3}$ symb. pcu. In order to solve this problem of low symbol rate, we propose a third DF protocol, the Incomplete DF, based on an incomplete decoding at the relay, which provides full rate and full diversity.

3.1 Alamouti Decode-and-Forward

In this section we propose a new DF protocol based on the Alamouti space-time code [16], chosen because of its decoding simplicity. In the following, we call this new protocol the Alamouti DF. The corresponding AF protocol is the NAF implemented with a distributed Alamouti code. A DF protocol using the Alamouti code has already been proposed for a full-duplex channel in [43], but never in the half-duplex case.

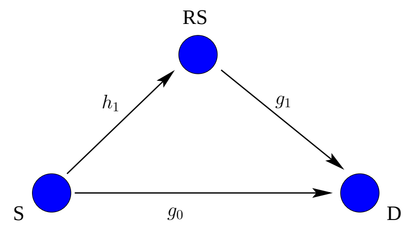


Figure 3.1: One-relay channel model

As we use the Alamouti code, which is designed for two transmit antennas, we consider the one-relay channel. The other assumptions on the channel are the same as in section 2.1.

3.1.1 Transmission scheme of the Alamouti DF

The transmission of the Alamouti DF protocol requires 4 channel uses to send 2 information symbols: the symbol rate is $\frac{1}{2}$ symb. pcu.

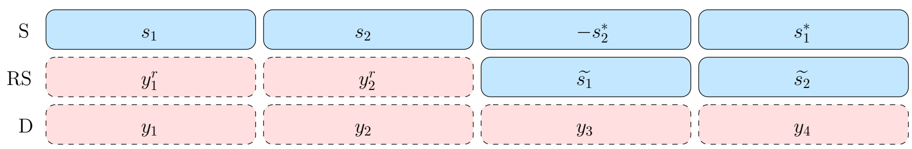


Figure 3.2: Transmission frames of the Alamouti DF protocol

As schematized in Figure 3.2, the source sends the first line of the Alamouti codeword (s_1 and s_2) during the listening phase, while the relay listens. In the forwarding phase, the relay sends a decoded version of the received signal, while the source sends the second line of the Alamouti codeword ($-s_2^*$ and s_1^*).

To have an efficient DF protocol we have to assume an initial selection between the decode-and-forward strategy and non-cooperation as defined in subsection 2.2.2. This selection considers the outage event of the source-relay link:

$$\mathcal{O} = \left\{ \log\left(1 + \rho |h_1|^2\right) < 2R \right\}$$

where R is the global spectral efficiency and $2R$ is the one of the source-relay link. The corresponding outage probability is $p_{\mathcal{O}}(R)$ and the probability of the link not being in outage is given by $p_{\overline{\mathcal{O}}}(R) = 1 - p_{\mathcal{O}}(R)$.

Received signals at relay are

$$\begin{aligned} y_{r1} &= \sqrt{\rho} h_1 x_1 + v_1 \\ y_{r2} &= \sqrt{\rho} h_1 x_2 + v_2 \end{aligned}$$

and received signals at destination

$$\begin{aligned} y_1 &= \sqrt{\rho} g_0 x_1 + w_1 \\ y_2 &= \sqrt{\rho} g_0 x_2 + w_2 \\ y_3 &= \sqrt{\frac{\rho}{2}} (-g_0 x_2^* + g_1 \widetilde{x_1}) + w_3 \\ y_4 &= \sqrt{\frac{\rho}{2}} (g_0 x_1^* + g_1 \widetilde{x_2}) + w_4 \end{aligned}$$

where $\widetilde{x_1}$ and $\widetilde{x_2}$ are are the decoded signals at the relay.

Assuming that x_1 and x_2 have been correctly decoded at relay, this system of equations can be rewritten as a MIMO system:

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \\ y_3 \\ y_4^* \end{bmatrix}}_{\mathbf{y}} = \sqrt{\rho} \underbrace{\begin{bmatrix} g_0 & 0 \\ 0 & g_0^* \\ \frac{g_1}{\sqrt{2}} & -\frac{g_0}{\sqrt{2}} \\ \frac{g_0^*}{\sqrt{2}} & \frac{g_1^*}{\sqrt{2}} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} w_1 \\ w_2^* \\ w_3 \\ w_4^* \end{bmatrix}}_{\mathbf{w}}. \tag{3.1}$$

The equivalent channel matrix $\mathbf{H}$ is orthogonal. Thus linear decoding can be performed.

3.1.2 Theoretical performance of the Alamouti DF

Outage Probability Analysis

To illustrate the performance of the proposed Alamouti DF protocol we begin by analyzing its outage probability.

Two cases have to be considered, whether cooperation is used or not. If the source-relay link is not in outage, cooperation is used. Then, the instantaneous capacity can be calculated from the expression of $\mathbf{H}$ defined in equation (3.1).

$$\begin{aligned} C^{AlDF}(\mathbf{H}) &= \frac{1}{4}\log\left(\det\left(I+\rho\mathbf{H}\mathbf{H}^{\dagger}\right)\right) \\ &= \frac{1}{2}\log\left(1+\rho\left(|g_0|^2+\frac{|g_0|^2+|g_1|^2}{2}\right)\right) \end{aligned}$$

and

$$p_{out}^{AlDF|\overline{\mathcal{O}}}(R) = \Pr\left\{C^{AlDF}(\mathbf{H}) < R\right\} \tag{3.2}$$

If the source-relay link is in outage, only the direct link from source to destination is used. The instantaneous capacity is then:

$$p_{out}^{SISO}(R) = \Pr\left\{\log\left(1+\rho|g_0|^2\right) < R\right\}$$

Finally, we can write

$$p_{out}^{AlDF}(R) = p_{out}^{AlDF|\overline{\mathcal{O}}}(R)p_{\overline{\mathcal{O}}}(R) + p_{out}^{SISO}(R)p_{\mathcal{O}}(R). \tag{3.3}$$

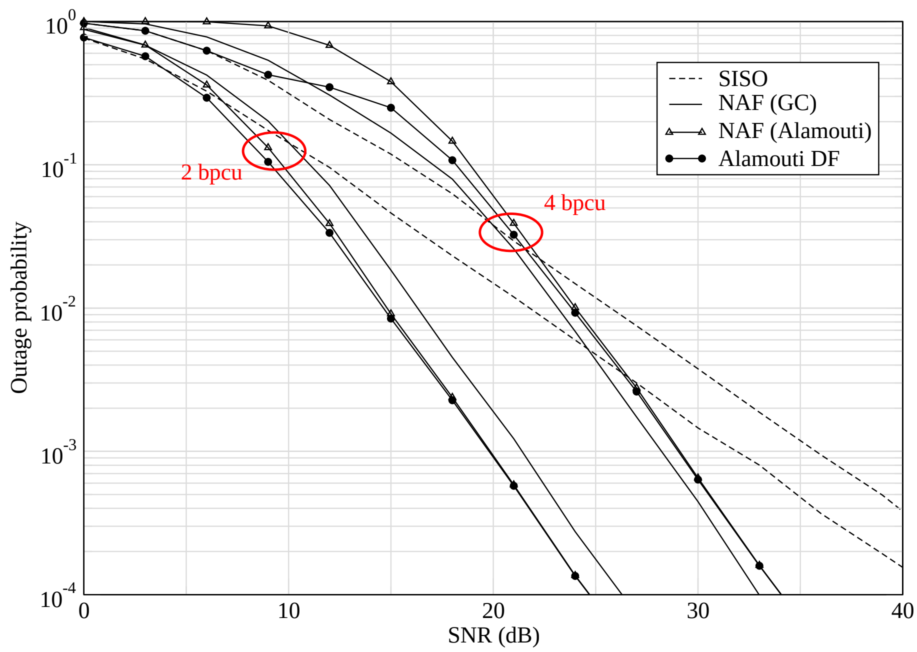


Figure 3.3: Alamouti DF outage probability

We plotted the outage probabilities of the Alamouti DF thanks to Monte Carlo simulations. In a real application such as a sensor network, several relays are available. Thus, to be more close to a real context, a relay selection has been added to all cooperative schemes: the relay is chosen as the best of 3 available relays in terms of SNR at destination for both

AF and DF protocols.

In Figure 3.3 is plotted the performance of the Alamouti DF protocol compared with those of the SISO and NAF protocol implemented with the distributed Alamouti code or an optimal STBC such as the distributed Golden code. The outage probabilities are plotted as functions of the SNR for spectral efficiencies of 2 and 4 bits pcu.

At low spectral efficiency (2 bits pcu), in spite of their rate of $\frac{1}{2}$ symb. pcu, the Alamouti DF and the NAF implemented with the distributed Alamouti code outperforms the NAF protocol implemented with an optimal STBC, providing a 2 dB gain. This is explained by the high coding gain of the Alamouti code. However, at high spectral efficiency, the situation is reverse and the NAF implemented with an optimal STBC provides 1 dB gain compared to the Alamouti DF. This is explained by the rate of $\frac{1}{2}$ symb. pcu of the Alamouti protocol compared to the full rate of 1 symb. pcu for the NAF with an optimal STBC.

Diversity-Multiplexing Gain Tradeoff (DMT) Analysis

Let's study the asymptotic behavior of $p_{out}^{AlDF}(r\log\rho)$ when the SNR grows to infinity. Let u_0, u_1 and v_1 be the exponentials orders of $\frac{1}{|g_0|^2}$, $\frac{1}{|g_1|^2}$ and $\frac{1}{|h_1|^2}$ respectively. Then the outage probability in (3.2) becomes:

$$\begin{aligned} p_{out}^{AlDF|\overline{\mathcal{O}}}(r\log\rho) &\doteq \Pr\left\{\log\left(1+\tfrac{3}{2}\rho^{1-u_0}+\tfrac{1}{2}\rho^{1-u_1}\right)<2r\log\rho\right\} \\ &\doteq \Pr\left\{\max(0,1-u_0,1-u_1)<2r\right\}, \end{aligned}$$

where $\doteq$ denotes an asymptotic behavior when $\rho\to\infty$.

Using the results in Appendix A.1, we deduce the corresponding DMT:

$$d_{out}^{AlDF|\overline{\mathcal{O}}}(r)=\inf(u_0+u_1)=2(1-2r)^+.$$

In the same way, we can express the outage probability of the direct link:

$$p_{out}^{SISO}(r\log\rho)\doteq\Pr\left\{\log\left(1+\rho^{1-u_0}\right)<r\log\rho\right\}\doteq\Pr\left\{\max(0,1-u_0)<r\right\}$$

and its corresponding DMT

$$d_{out}^{SISO}(r)=\inf(u_0)=(1-r)^+,$$

as well as the outage probability of the source-relay link

$$\begin{aligned} p_{\mathcal{O}}(r\log\rho) &\doteq \Pr\left\{\log\left(1+\rho^{1-v_1}\right)<2r\log\rho\right\} \\ &\doteq \Pr\left\{\max(0,1-v_1)<2r\right\} \end{aligned}$$

and its DMT

$$d_{\mathcal{O}}(r)=\inf(v_1)=(1-2r)^+.$$

Finally, the asymptotic behavior of the outage probability of the Alamouti DF protocol is

$$p_{out}^{AlDF}(r \log \rho) \doteq \rho^{-d^{AlDF}}$$

with

$$d^{AlDF}(r) = \min\left(d_{out}^{AlDF|\overline{\mathcal{O}}}(r), d_{out}^{SISO}(r) + d_{\mathcal{O}}(r)\right) = 2(1-2r)^{+}. \tag{3.4}$$

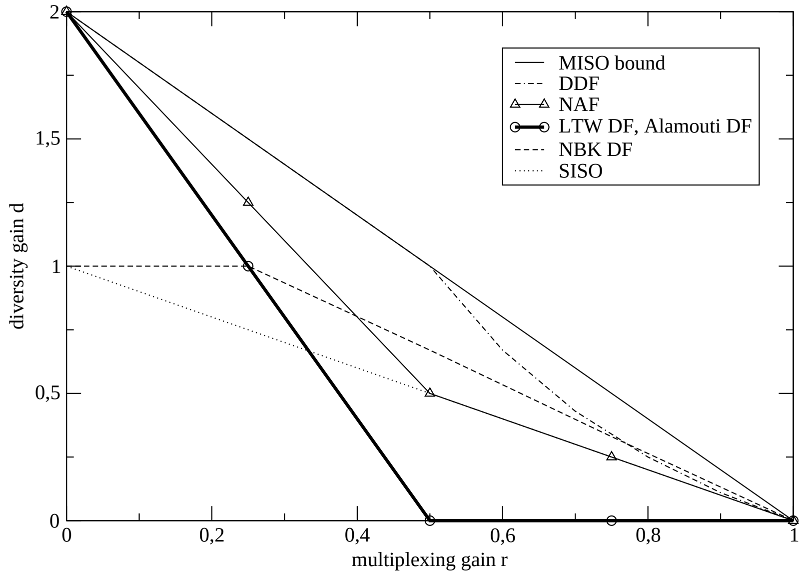


Figure 3.4: Alamouti DF DMT

We can notice in Figure 3.4 that the Alamouti DF has the same DMT as the LTW DF, which is lower than the one of the NAF protocol, even if it provides better performance at low spectral efficiency.

The Alamouti DF protocol provides good performance for 2 bits pcu, but its low symbol rate is a big drawback for higher spectral efficiencies. Moreover, if we try to generalize the Alamouti DF to a higher number of relays, using other orthogonal codes, the symbol rate will decrease dramatically.

3.2 Asymmetric Decode-and-Forward (ADF)

The Asymmetric DF is our second approach to the introduction of distributed space-time codes in DF protocols. It is composed of 2 phases of unequal lengths. Only half of the symbols sent during the first phase are repeated by the relays. The other half are repeated by the source itself. This protocol is to be implemented with a distributed $2N \times 2N$ algebraic STBC.

3.2.1 Transmission scheme of the Asymmetric DF

Transmission frame

The transmission frame for the N-relay channel is described in Figure 3.5. $2N$ symbols are sent in $3N$ time slots, thus the symbol rate is $\frac{2N}{3N} = \frac{2}{3}$ symb. pcu. The transmission frame is divided in two phases:

- during the listening phase, which lasts $2N$ channel uses, the source sends $2N$ symbols in succession and the N relays listen to all of them. Thus, each relay is able to decode the $2N$ symbols without a rank deficiency.

- during the forwarding phase, which lasts N channel uses, the source repeats the N last symbols, while the N relays send the recoded version of the N first ones. $\forall n \in \{1, \ldots, N\}$, relay RS_n sends the n^{th} recoded symbol $\widetilde{x_n}$ while source sends the $(N+n)^{\text{th}}$ symbol x_{N+n}.

The destination keeps listening during the whole transmission.

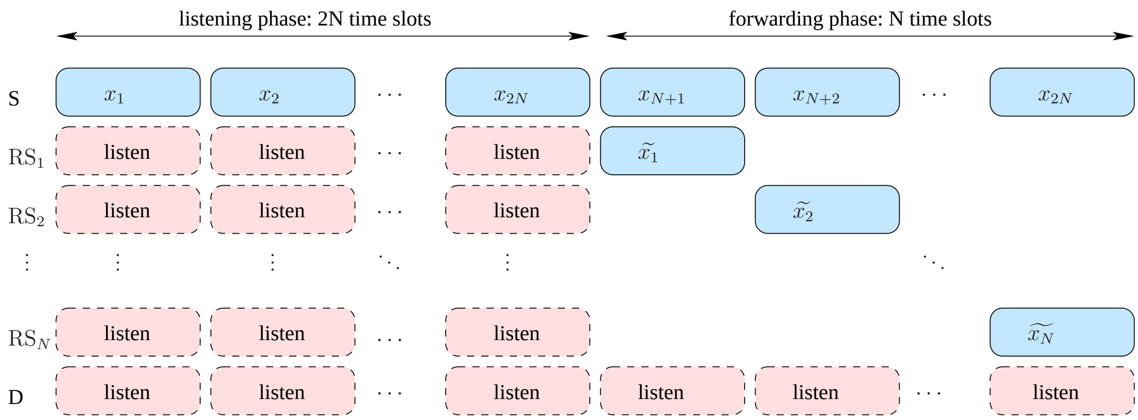


Figure 3.5: Transmission frame of the Asymmetric DF protocol

Received signals at destination can be expressed as a MIMO system:

$$\begin{bmatrix} y_1 \\ \vdots \\ \vdots \\ \vdots \\ y_{2N} \\ y_{2N+1} \\ \vdots \\ y_{3N} \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} g_0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & g_0 \\ \frac{g_1}{\sqrt{2}} & \cdots & 0 & \frac{g_0}{\sqrt{2}} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{g_N}{\sqrt{2}} & 0 & \cdots & \frac{g_0}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \\ x_{N+1} \\ \vdots \\ x_{2N} \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ \vdots \\ \vdots \\ w_{2N} \\ w_{2N+1} \\ \vdots \\ w_{3N} \end{bmatrix},$$

where $\forall k \in \{1, \ldots, 2N\}$

- y_k is the k^{th} received signal at destination,
- x_k is the k^{th} sent symbol from source,
- w_k is an AWGN.

The factor $\frac{1}{\sqrt{2}}$ in the channel matrix comes from the power normalization during the second transmission phase. As two terminals send simultaneously in each time slot, they have to share the resources.

Reordering the received signals at destination we obtain the equivalent expression:

$$\mathbf{y}_{eq} = \sqrt{\rho}\mathbf{H}_{eq}\mathbf{x}_{eq} + \mathbf{w}_{eq} \tag{3.5}$$

with

$$\mathbf{H}_{eq} = \begin{bmatrix} \mathbf{H}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{H}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H}_N \end{bmatrix} \tag{3.6}$$

and $\forall n \in \{1, \ldots, N\}$

$$\mathbf{H}_n = \begin{bmatrix} g_0 & 0 \\ 0 & g_0 \\ \frac{g_n}{\sqrt{2}} & \frac{g_0}{\sqrt{2}} \end{bmatrix}. \tag{3.7}$$

Selection between the Asymmetric DF protocol and the non-cooperative case

The selection strategy described in subsection 2.2.2 is used to guarantee the decoding at the relays. In our case, the outage event of a source-relay RS_n link is defined by

$$\mathcal{O} = \left\{ \log\left(1 + \rho|h_n|^2\right) < \frac{3}{2}R \right\} \tag{3.8}$$

where R is the global spectral efficiency, and so $R_n = \frac{3}{2}R$ is the spectral efficiency of the source-relay RS_n link since the same number of symbols is sent in 2 time slots instead of 3.

Only active relays (whose source-relay link is not in outage) can be used. If there are $N_u \geq 1$ active relays, the Asymmetric DF protocol with N_u relays is used, and if there is no active relay, we use the non-cooperative strategy.

3.2.2 Implementation: example of the one-relay case

The Asymmetric DF protocol can be implemented with a distributed $2N \times 2N$ STBC, such as a TAST code or a perfect code. In this section we present the example of the one-relay case.

Let's consider a network with one source, one relay and one destination. We propose to implement the Incomplete DF with the distributed Golden code whose codeword is

$$\mathbf{X} = \begin{bmatrix} \alpha x_1 & \alpha x_2 \\ i\sigma(\alpha)\sigma(x_2) & \sigma(\alpha)\sigma(x_1) \end{bmatrix}, \tag{3.9}$$

where $x_1 = s_1 + \theta s_2$ and $x_2 = s_3 + \theta s_4$, the s_j, $j \in \{1, \ldots, 4\}$ are the information symbols taken in a QAM constellation, $\theta = \frac{1+\sqrt{5}}{2}$ is the Golden number and $\alpha = 1 + i - i\theta$. The elements of the code matrix are in $\mathcal{O}_{\mathbb{K}}$ the ring of integers of the number field $\mathbb{K} = \mathbb{Q}(i, \theta)$.

The transmission frame is described in Figure 3.6.

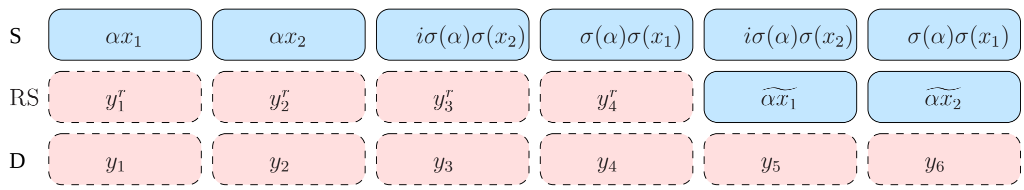


Figure 3.6: Transmission frame of the Asymmetric DF protocol implemented with a distributed Golden code in the one-relay case

The relay listens to the entire codeword. Thus it can decode the message by using ML lattice decoders such as a sphere decoder or a Schnorr-Euchner algorithm. Decoding at destination is also performed by using ML lattice decoders.

3.2.3 Theoretical performance of the Asymmetric DF protocol

Outage probability analysis

Considering the equivalent channel expressed in (3.6) and (3.7), we can provide the outage probability of the Asymmetric DF.

Theorem 1. *The outage probability of the Asymmetric DF protocol is*

$$p_{out} = \sum_{N_u=0}^{N} \binom{N}{N_u} p_{out,N_u} p_{\mathcal{O},N-N_u}. \tag{3.10}$$

where N is the number of relays in the network, N_u is the number of relays whose source-relay link is not in outage, p_{out,N_u} is the outage probability of the DF strategy with N_u decoding relays

$$\begin{aligned} p_{out,N_u} &= \Pr\left\{\sum_{i=1}^{N_u} \log\left(1+\tfrac{5\rho}{2}|g_O|^2+\tfrac{\rho}{2}|g_1|^2+\tfrac{3\rho^2}{2}|g_O|^4+\tfrac{\rho^2}{2}|g_O|^2|g_1|^2\right) < 3N_uR\right\} \quad \textit{if} \quad N_u \neq 0 \\ &= \Pr\left\{\log\left(1+\rho|g_O|^2\right) < R\right\} \quad \textit{if} \quad N_u = 0 \end{aligned} \tag{3.11}$$

and $p_{\mathcal{O},N_{N-N_u}}$ is the probability that the source-relay links of the $N-N_u$ other relays are in outage

$$p_{\mathcal{O},N_{N-N_u}} = \prod_{i=1}^{N_u} \Pr\left\{\log\left(1+\rho|h_i|^2\right) > \frac{3}{2}R\right\} \prod_{i=N_u+1}^{N} \Pr\left\{\log\left(1+\rho|h_i|^2\right) < \frac{3}{2}R\right\}. \tag{3.12}$$

Proof. The proof of this theorem is developed in Appendix A.2. □

Figure 3.7(a) represents the performance of the Asymmetric DF protocol compared with those of the SISO and NAF protocol for the one-relay case. The outage probabilities are plotted as functions of the SNR (obtained numerically by Monte Carlo simulations) for spectral efficiencies of 2 and 4 bits pcu.

At low spectral efficiency (2 bits pcu), the Asymmetric DF protocol provides the same performance than the NAF protocol. However for a spectral efficiency of 4 bits pcu, asymptotically, the NAF has 1 dB gain over the ADF. This is due to the $\frac{2}{3}$ symbol rate of the protocol.

In the two-relay case (see Figure 3.7(b)), the Asymmetric DF protocol have lower performance than the NAF protocol, even at 2 bits pcu.

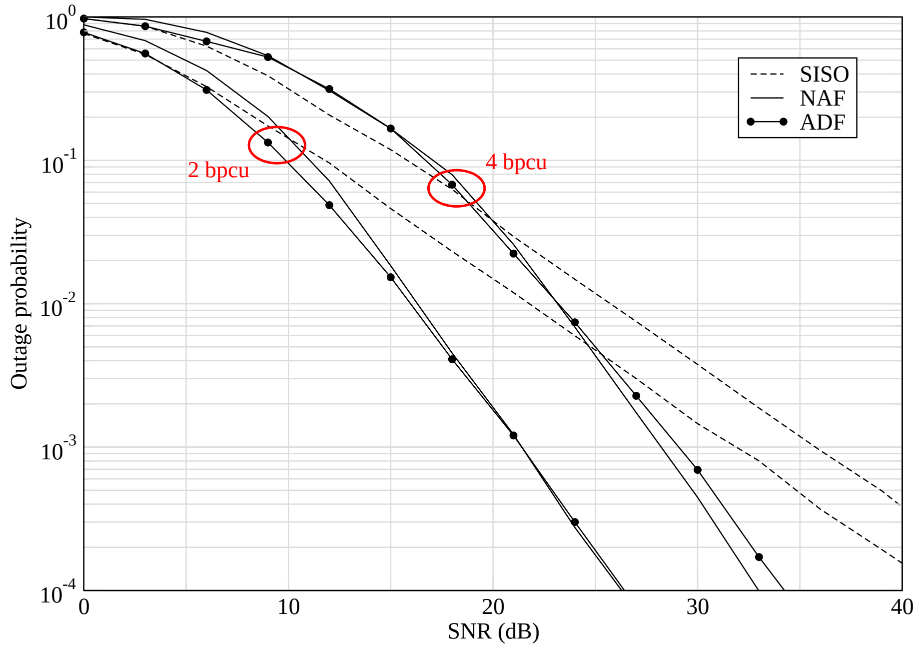
Diversity-Multiplexing gain Tradeoff

Studying the asymptotic behavior of the outage probability when the SNR grows to infinity, we can provide the DMT of the Asymmetric DF protocol.

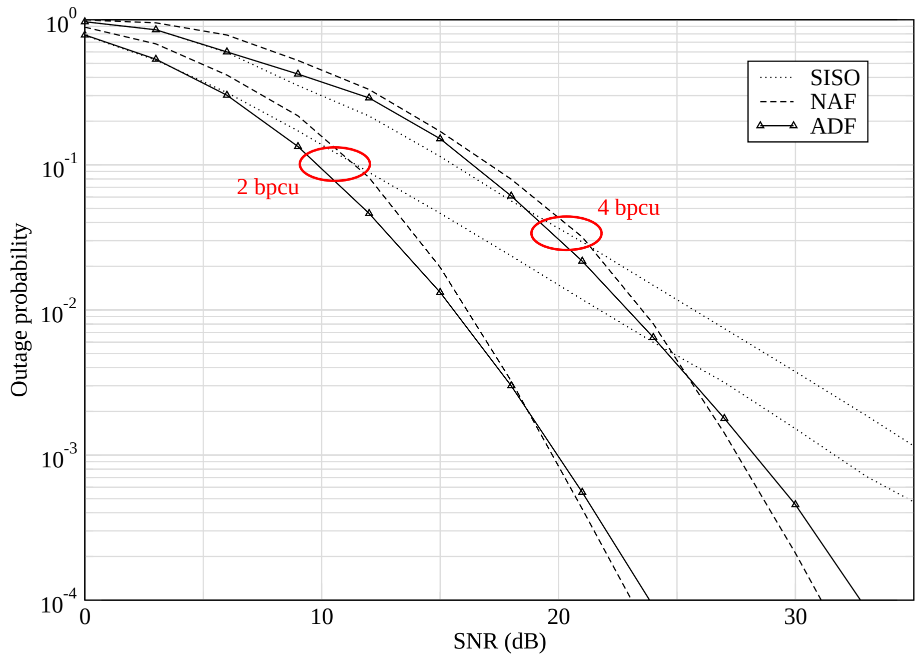
Theorem 2. *The DMT of the Asymmetric DF is*

$$d^*(r) = \left(1-\frac{3}{2}r\right)^+ + N\left(1-3\frac{2N-1}{2N}r\right)^+, \tag{3.13}$$

where N is the number of relays in the network.



(a) One relay



(b) Two relays

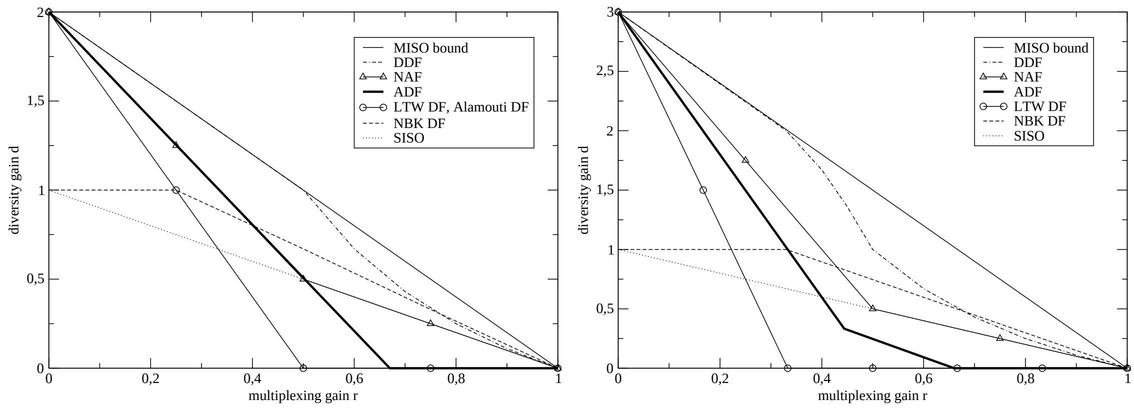
Figure 3.7: Asymmetric DF outage probability

We can notice that the DMT of the Asymmetric DF tends to

$$d^*(r) = \left(1 - \frac{3}{2}r\right)^+ + N\left(1 - 3r\right)^+$$

when the number of relays N tends to infinity.

Proof. The proof of this theorem is developed in Appendix A.3. □



(a) One relay

(b) Two relays

Figure 3.8: Asymmetric DF DMT

In Figures 3.8(a) and 3.8(b) are represented the DMTs of various cooperative protocols for the one and two-relay cases. In the one relay case, the DMT of the Asymmetric DF protocol is the same as the one of the NAF protocol for multiplexing gains lower than $\frac{1}{2}$, but is worse for multiplexing gains between $\frac{1}{2}$ and 1. Because of the low rate of the protocol, the diversity gain is even zero for multiplexing gains higher than $\frac{2}{3}$.

In the two relay case, the DMT of the Asymmetric protocol is always worse than the one of the NAF, but better than the one of the LTW DF.

In both cases, the DMT of the DDF is much better, however, practical implementation is much more easier with this new protocol.

We can finally remark that like the Alamouti DF protocol, the Asymmetric DF provides good performance at low spectral efficiencies, but because of its low rate, it is less efficient for spectral efficiencies greater than 4 bits pcu.

3.3 Incomplete Decode-and-Forward (IDF)

In order to solve the problem of low rates of the Asymmetric DF protocol, we define a new protocol named Incomplete DF. To increase the rate, the first phase of the transmission is shortened and the second phase is kept the same. The Incomplete DF protocol is also designed to be used with a $2N \times 2N$ algebraic ST code, with N the number of relays.

3.3.1 Transmission scheme of the Incomplete DF

Transmission scheme

The transmission frame is described in Figure 3.9. $2N$ symbols are sent in $2N$ time slots, thus the symbol rate is then $\frac{2N}{2N} = 1$ symb. pcu. The transmission frame lasts $2N$ channel uses and is divided in two phases:

- during the first one, which lasts N channel uses, the source sends N coded symbols in succession and the N relays listen;

- during the second phase, which also lasts N channel uses, the source sends N new coded symbols, while the N relays send the decoded version of the N first ones. Relay RS_n, $n \in \{1, \dots, N\}$, sends the decoded symbol $\widetilde{x_n}$ while source sends symbol x_{N+n}.

The destination keeps listening during the whole transmission.

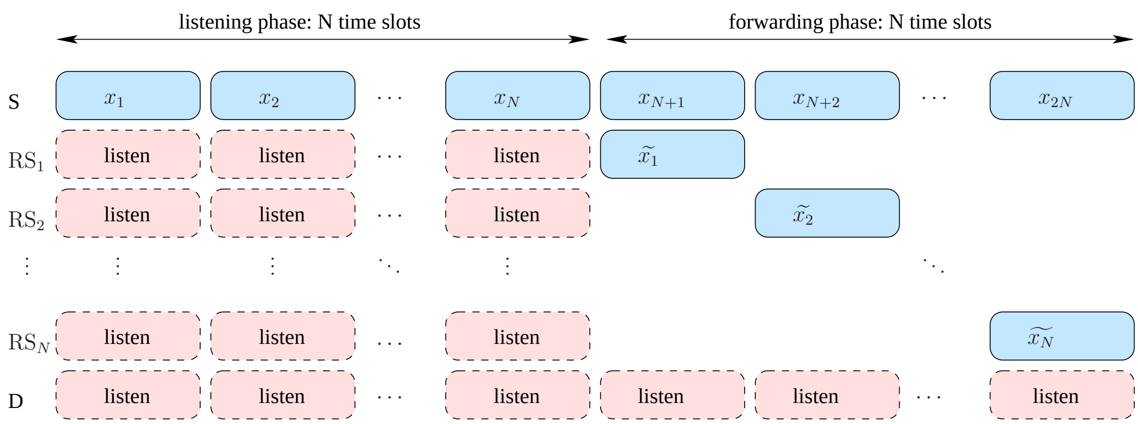


Figure 3.9: Transmission frame of the Incomplete DF protocol

In the same way as for the Asymmetric DF in previous section, reordered received signals at destination can be expressed as a MIMO system:

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{w} \tag{3.14}$$

where $\mathbf{H}$ is a block diagonal matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{H}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{H}_N \end{bmatrix} \tag{3.15}$$

and $\forall n \in \{1, \ldots, N\}$

$$\mathbf{H}_n = \begin{bmatrix} g_0 & 0 \\ \frac{g_n}{\sqrt{2}} & \frac{g_0}{\sqrt{2}} \end{bmatrix}. \tag{3.16}$$

The Incomplete DF protocol can be implemented with the same distributed $2N \times 2N$ STBC as the Asymmetric DF. Decoding at destination is also performed by using ML lattice decoders such as a sphere decoder or a Schnorr-Euchner algorithm.

Incomplete decoding at the relays

The challenge of this new transmission scheme is decoding at relays. Indeed, in order to provide a rate of 1 symb. pcu and full diversity, $2N$ information symbols have to be sent during the first phase of transmission. Thus, a full decode-and-forward strategy would imply that relays decode every information symbol s_j, $j \in \{1, \ldots, 2N\}$ of our original constellation from only N received signals.

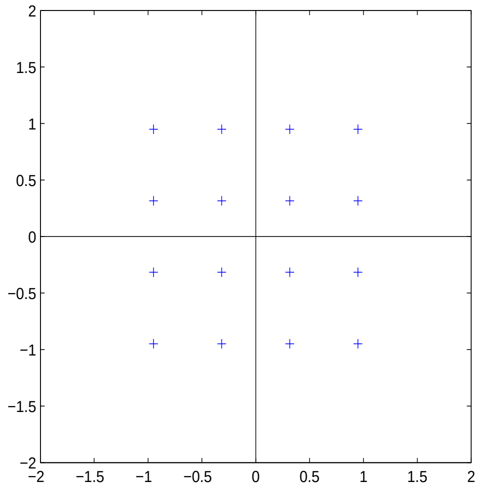
The idea of the Incomplete DF is to estimate received signals by elements $x_k, k \in \{1, \ldots, N\}$ of the the ring of integers $\mathcal{O}_{\mathbb{K}}$ of the number field $\mathbb{K}$ used for the construction of the considered algebraic STBC (the x_k are linear combinations of the $2N$ information symbols), without stating definitely about the information symbols s_j. Indeed, the knowledge of the s_j is not necessary at relays, as soon as they know the signals x_k that have to be forwarded. Incomplete decoding at relays is sufficient.

To study the theoretical limits of the IDF protocol and its optimal performance, we will compute an exhaustive decoding at the relays. In the next sections we propose two methods to reduce the decoding complexity at relays. The first one is based on the structure of TAST codes and the second one on diophantine approximation.

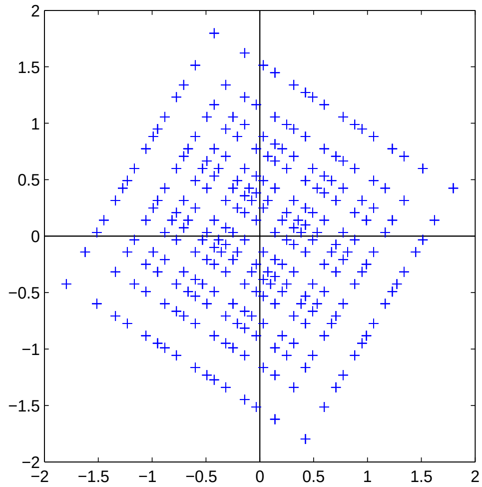
One-relay case Let us assume the information symbols s_1 and s_2 belong to a constellation C. We can define a new constellation C' to which the coded symbol x_1 belongs to, which is a finite subset of $\mathcal{O}_{\mathbb{K}}$.

Example 10. *We consider information symbols taken in a 16-QAM (see Figure 3.10(a)). If $x_1 = \alpha(s_1 + \theta s_2)$ with θ the Golden number $\frac{1+\sqrt{5}}{2}$ and $\alpha = 1 + i(1 - \theta)$, then x_1 belongs to a new "Golden" constellation represented in Figure 3.10(b).*

An exhaustive search is performed in this new constellation. $\widetilde{x_1}$ is obtained by looking for



(a) 16-QAM



(b) “Golden” constellation

Figure 3.10: 16-QAM and “Golden” constellations

the element $x \in C'$ that minimizes the distance between the received signal at relay y_1^r and $\sqrt{\rho}h_1 x$:

$$\widetilde{x_1} = \arg\min_{x \in C'} \{|y_1^r - \sqrt{\rho}h_1 x|^2\}.$$

The complexity of the exhaustive search grows with the size of the constellation. If information symbols are taken in an M-QAM, the complexity of the exhaustive search is M^2.

Two-relay case x_1 and x_2 can be decoded at relays by an exhaustive search as in the one-relay case. The difference is that they cannot be decoded separately as they are both linear combinations of the same information symbols.

Let's assume the s_j, $j \in \{1, \ldots, 4\}$, belong to a constellation C. We can define two constellations C_a and C_b to which x_1 and x_2 belong respectively.

Let y_1^r and y_2^r be the received signals at relay. Decoded versions of x_1 and x_2 are obtained by looking for elements $x_a \in C_a$ and $x_b \in C_b$ minimizing the distance between $\sqrt{\rho}h_1 x_a$ and y_1^r and the distance between $\sqrt{\rho}h_1 x_b$ and y_2^r. We minimize the sum of these two distances. At the each relay k, $k \in \{1, 2\}$:

$$\{\widetilde{x_1}, \widetilde{x_2}\} = \arg\min_{(x_a, x_b) \in (C_a, C_b)} \left\{ |y_1^{r_k} - \sqrt{\rho}h_k x_a|^2 + |y_2^{r_k} - \sqrt{\rho}h_k x_b)|^2 \right\}.$$

However, this exhaustive decoding can be quite complex if a high constellation size is considered. Indeed, if the information symbols s_j belong to an M-QAM constellation, the received signals have to be decoded in a new constellation of M^4 elements. The decoding complexity is M^4.

Selection between the Incomplete DF protocols and the non-cooperative case

The same selection strategy as for the Asymmetric DF (described in subsection 2.2.2) is used. Only the expressions of the outage probabilities of the source-relay links change.

Here the outage event of a source-relay RS_n link is defined by

$$\mathcal{O} = \left\{\log\left(1+\rho|h_n|^2\right) < 2R\right\},$$

where R is the global spectral efficiency, and $R_n = 2R$ the spectral efficiency of the source-relay link is twice since the same information is sent in two times less channel uses.

3.3.2 Theoretical and simulation performance of the Incomplete DF

Outage probability

From the system model of equations (3.14), (3.15) and (3.16), we can compute the theoretical performance of the Incomplete DF.

Theorem 3. *The outage probability of the Asymmetric DF protocol is*

$$p_{out} = \sum_{N_u=0}^{N} \binom{N}{N_u} p_{out,N_u} p_{\mathcal{O},N-N_u}. \tag{3.17}$$

where N is the number of relays in the network and N_u is the number of active relays, p_{out,N_u} is the outage probability of the DF strategy using N_u active relays

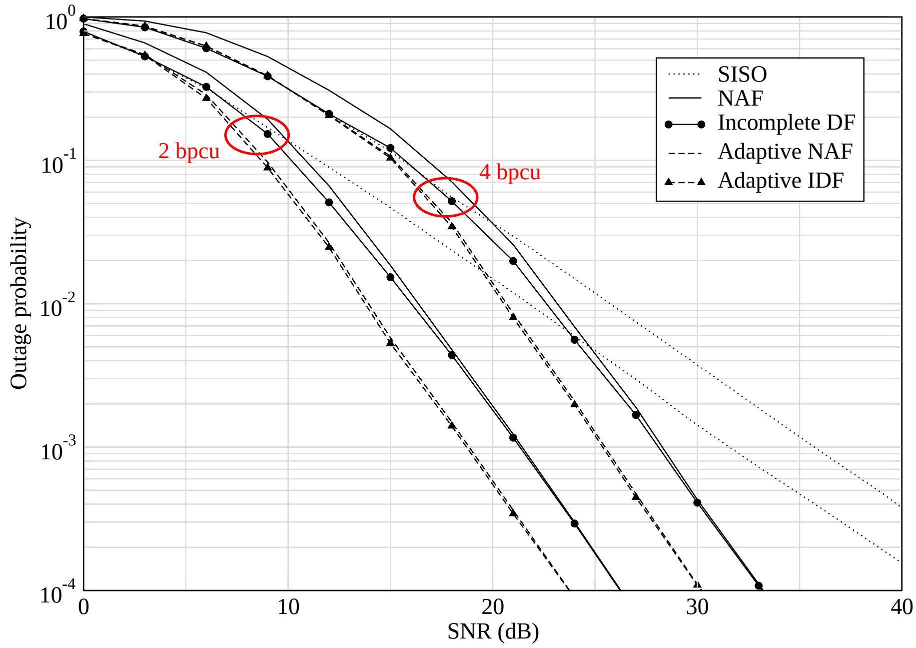
$$\begin{aligned} p_{out,N_u} &= \Pr\left\{\textstyle\sum_{i=1}^{N_u}\log\left(1+\frac{3\rho}{2}|g_0|^2+\frac{\rho}{2}|g_1|^2+\frac{\rho^2}{2}|g_0|^4\right) < 2N_uR\right\} && \text{if } N_u \neq 0 \\ &= \Pr\left\{\log\left(1+\rho|g_0|^2\right) < R\right\} && \text{if } N_u = 0 \end{aligned} \tag{3.18}$$

and $p_{\mathcal{O},N_{N-N_u}}$ is the probability that the source-relay links of the $N-N_u$ other relays are in outage

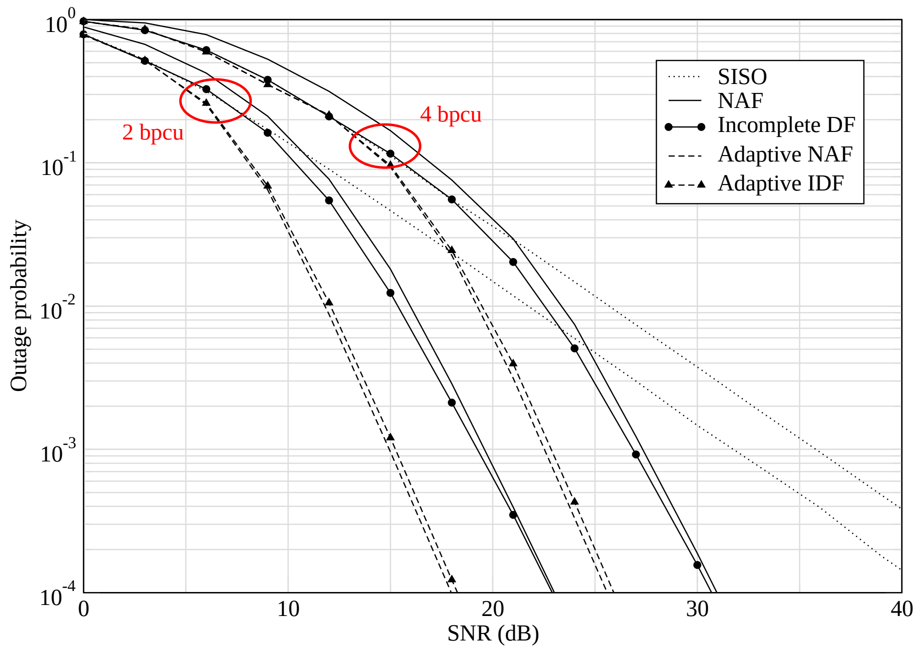
$$p_{\mathcal{O},N_{N-N_u}} = \prod_{i=1}^{N_u}\Pr\left\{\log\left(1+\rho|h_i|^2\right) > 2R\right\} \prod_{i=N_u+1}^{N} \Pr\left\{\log\left(1+\rho|h_i|^2\right) < 2R\right\}. \tag{3.19}$$

Proof. The outage probability of the Incomplete DF can be proven in the same way as Theorem 1 in Appendix A.2. □

Figures 3.11(a) and 3.11(b) represent the outage probabilities of the SISO, NAF and



(a) One relay case



(b) Two relay case

Figure 3.11: Incomplete DF outage probability

Incomplete DF protocols as functions of the SNR. Spectral efficiencies of 2 and 4 bits pcu are considered for both the one-relay and two-relay cases.

In all cases, the Incomplete DF provides the same asymptotic performance as the NAF protocol, but it has better performance at low SNR due to the initial selection.

Performance of the adaptive NAF and adaptive IDF are also plotted in these graphs. The outage probabilities expected in previous chapter are confirmed. Both adaptive strategies provide the same performance, always outperforming non-cooperation, even at low SNRs.

Diversity-Multiplexing gain Tradeoff (DMT)

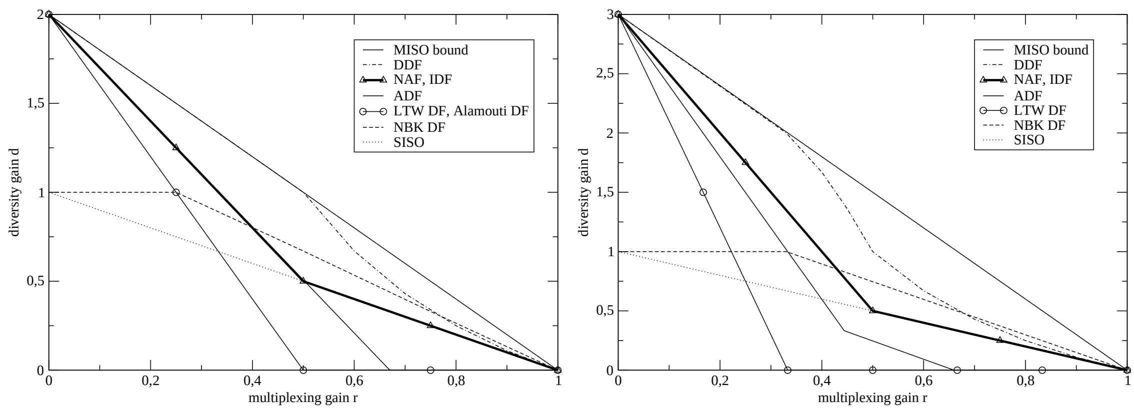
The asymptotic behavior of the outage probability (in equation (3.17)) allows to derive the DMT of the Incomplete DF protocol.

Theorem 4. *The DMT of the Incomplete DF is*

$$d^*(r) = (1-r)^+ + N(1-2r)^+, \tag{3.20}$$

where N is the number of relays in the network.

Proof. The proof is detailed in Appendix A.4. □



(a) One relay (b) Two relays

Figure 3.12: Incomplete DF DMT

In Figures 3.12(a) and 3.12(b) are represented the DMT of several cooperative protocols in the one-relay and two-relay cases. One can remark that the DMT of the Incomplete DF protocol is exactly the same as the one of the NAF protocol, outperforming the ones of the NBK, LTW, Alamouti and Asymmetric DF protocols. The DMT of the DDF protocol is still better, but Incomplete DF implementation is much easier, especially using one or both of the two following decoding methods at relays.

3.3.3 Reducing the decoding complexity at relays for a higher number of relays

The decoding complexity at relays using the exhaustive search increases considerably when the number of used relays becomes high. In this section, we propose a method based on the particular structure of the TAST codes (or codes with a similar construction) to reduce this decoding complexity without any loss in performance.

To illustrate this method we take the example of a network composed of one source, 2 relays and one destination. We propose to implement the Incomplete DF with the distributed 4×4 TAST code [21] constructed using the cyclotomic field $\mathbb{K} = \mathbb{Q}(i, \theta)$, where $\theta = e^{i\frac{\pi}{8}}$, the generator of the Gallois group $\sigma : \theta \mapsto i\theta$ and $\phi = e^{i\frac{\pi}{8}}$. Its codeword is

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ \phi\sigma(x_4) & \sigma(x_1) & \sigma(x_2) & \sigma(x_3) \\ \phi\sigma^2(x_3) & \phi\sigma^2(x_4) & \sigma^2(x_1) & \sigma^2(x_2) \\ \phi\sigma^3(x_2) & \phi\sigma^3(x_3) & \phi\sigma^3(x_4) & \sigma^3(x_1) \end{bmatrix},$$

where, $\forall k \in \{1, \dots, 4\}$, $x_k = s_{4k-3} + \theta s_{4k-2} + \theta^2 s_{4k-1} + \theta^3 s_{4k}$.

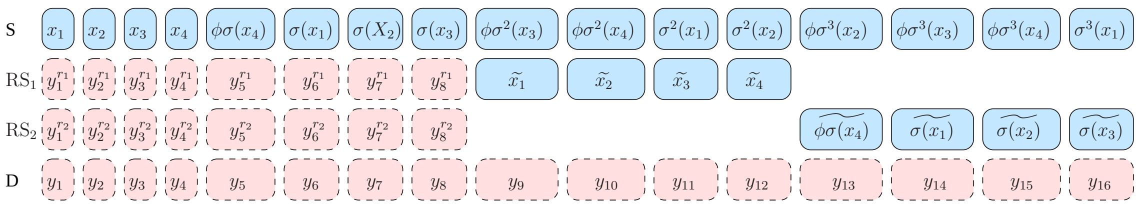


Figure 3.13: Transmission frame of the Incomplete DF protocol in the 2-relay case implemented with a distributed 4×4 TAST code

The transmission scheme is schematized in Figure 3.13.

Elements x_1, x_2, x_3 and x_4 and their conjugates have to be recovered from the signals $y_1^{r_j}$ to $y_8^{r_j}$ received at the relay RS_j, $j \in \{1, \dots, 2\}$. The decoding can be performed using an exhaustive search. However, this would induce a high complexity.

Two-step decoding method

Studying the structure of the codeword matrix, we can notice that x_1 and its second conjugate $\sigma^2(x_1)$ can be rewritten in the form:

$$x_1 = (s_1 + \theta^2 s_3) + \theta(s_2 + \theta^2 s_4)$$
$$\sigma^2(x_1) = (s_1 + \theta^2 s_3) - \theta(s_2 + \theta^2 s_4)$$

$$\begin{bmatrix} x_1 \\ \sigma^2(x_1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \theta \\ 1 & -\theta \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \tag{3.21}$$

where $z_1 = (s_1 + \theta^2 s_3)$ and $z_2 = (s_2 + \theta^2 s_4)$ are elements of the ring of integers of the field $\mathbb{Q}(e^{i\frac{\pi}{4}})$ of dimension 2 over $\mathbb{Q}(i)$. As $\frac{1}{\sqrt{2}}\mathbf{M}$ is a rotation matrix, a simple multiplication by $\mathbf{M}^\dagger$ allows to obtain z_1 and z_2 from x_1 and $\sigma^2(x_1)$.

In order to take advantage of this property, the idea is that the source sends the first and third lines of the codeword matrix during the first phase of the transmission and the second and fourth lines during the second phase of the transmission.

The incomplete decoding at relays is then performed in two steps. First we compute the matrix product

$$\begin{bmatrix} z_1' \\ z_2' \end{bmatrix} = \frac{1}{2}\mathbf{M}^\dagger \begin{bmatrix} \frac{y_1^{r_k}}{\sqrt{\rho}h_1} \\ \frac{y_6^{r_k}}{\sqrt{\rho}h_1} \end{bmatrix}.$$

Then we decode elements z_1 and z_2 in an exhaustive way. Finally x_1 and its conjugate $\sigma^2(x_1)$ can be easily deduced from (3.21).

This method allows to decrease considerably the complexity. Indeed, the exhaustive search is now performed in a constellation of M^2 elements instead of M^4, which is more reasonable.

This second decoding method cannot be applied to 4×4 perfect codes whose structure do not have the same property. That is why we have chosen to use TAST code.

Simulation results

Simulations have been run in the two-relay case for spectral efficiencies of 2 and 4 bits pcu. Figure 3.14 represents the frame error rates of the SISO, NAF and Incomplete DF protocols as functions of the SNR. The NAF is implemented with the distributed 4×4 perfect code. The Incomplete DF is implemented with both the distributed 4×4 perfect code, which allows to provide optimal performance, and the distributed 4×4 TAST code, which allows to use the two-step decoding method at relays.

The good performance for low and high SNR noticed in subsection 3.3.2 on the outage probability curves are confirmed here by simulation results.

The distributed TAST code provides slightly worse performance than the distributed perfect code. Indeed, it does not respect the NVD property, contrary to the perfect code.

3.3.4 Reducing the decoding complexity at relays for a larger constellation

The decoding complexity at relays using exhaustive search increases with the constellation size. We propose here to use diophantine approximation to compute decoding at relays. This new method will lead to sub-optimal performance, but with a very low complexity.

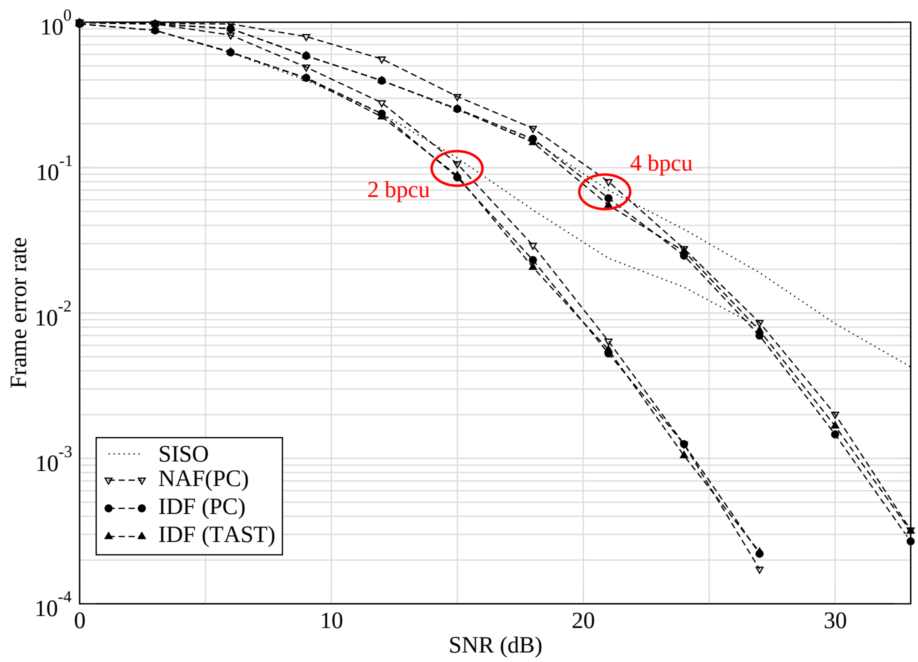


Figure 3.14: Incomplete DF frame error rate for two relays

One-relay case

System model Let us consider a network with one source, one relay and one destination. We propose to implement the Incomplete DF with the distributed Golden code whose codeword is given in (3.9). The transmission frame is described in Figure 3.15.

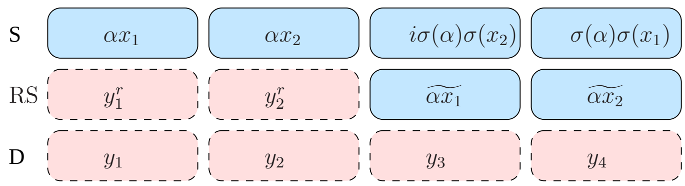


Figure 3.15: Transmission frame of the Incomplete DF protocol in the 1-relay case implemented with a distributed Golden code

Elements x_1 and x_2 both contain two information symbols. They have to be decoded at the relay from the received signals y_1^r and y_2^r respectively.

Diophantine approximation An exhaustive decoding could be used, but in order to reduce relay decoding complexity, we propose to study the analogy of this problem to a diophantine approximation. There exist two types of diophantine approximation.

Definition 6. *A homogeneous diophantine approximation of $\zeta \in \mathbb{R}$ is a fraction $\frac{p}{q} \in \mathbb{Q}$ such that $|\zeta - \frac{p}{q}|$ or $D(p,q) = |q\zeta - p|$ is small.*

Definition 7. *An inhomogeneous diophantine approximation of* $\zeta \in \mathbb{R}$*, given* $\beta \in \mathbb{R}$*, is a fraction* $\frac{p}{q} \in \mathbb{Q}$ *such that* $D(p,q) = |q\zeta - p - \beta|$ *is small.*

Definition 8. *A pair* $(p,q) \in \mathbb{N}^2$ *is a best diophantine approximation if* $\forall (p',q') \neq (p,q) \in \mathbb{N}^2$*, we have:*

$$q' \leq q \Rightarrow D(p',q') \geq D(p,q).$$

Cassels' algorithm has been proposed in [44] and explained in detail in [45]. Given $\zeta, \beta \in \mathbb{R}$, this algorithm enumerates all best inhomogeneous approximations.

Diophantine approximation only deals with real numbers. Since in this case $\theta \in \mathbb{R}$, diophantine approximation perfectly fits to the decoding of the elements of the Golden codeword. The problem is first divided into its real and imaginary parts. Let us note

$$\widetilde{y_1^r} = \frac{y_1^r}{\sqrt{\rho} h_1 \alpha} \qquad \text{and} \qquad \widetilde{y_2^r} = \frac{y_2^r}{\sqrt{\rho} h_2 \alpha}.$$

Given $(\theta, Re(\widetilde{y_1^r})) \in \mathbb{R}^2$, we want to find $(Re(s_1), Re(s_2)) \in \sqrt{M}$-PAM such that

$$d = |Re(\widetilde{y_1^r}) - Re(s_1) - \theta Re(s_2)| \tag{3.22}$$

is minimized. Using the notation of Definition 7, this problem can be identified with an inhomogeneous diophantine approximation where

$$\begin{aligned} \zeta &\leftrightarrow -\theta, \\ p &\leftrightarrow Re(s_1), \\ q &\leftrightarrow Re(s_2), \\ \beta &\leftrightarrow -Re(\widetilde{y_1^r}). \end{aligned}$$

However, $Re(s_1)$ and $Re(s_2)$ have to be decoded in a $\sqrt{M}$-PAM. Cassels' algorithm thus has to be modified (see Algorithm 1). The constraint on Q_{n-1} on line 7 allows to restrict the search to a finite set $\{1, \ldots, Z\}$. When $Q_{n-1} > Z$, the computation is stopped. A change of basis is done on line 1 and the reverse on lines 24 and 25 in order to provide (p,q) in a Z-PAM.

The same processing is performed to decode the imaginary part of the first coded symbol, as well as the real and imaginary parts of the second coded symbol.

The incomplete decoding using a diophantine approximation is not optimal. However, it allows a considerable drop of the decoding complexity at the relay side. Indeed, exhaustive decoding has a complexity M^2. When running the modified Cassels' algorithm for different sizes of constellations, the average number of iterations of the algorithm is of the order $2\sqrt{Z}$ for a Z-PAM (see Figure 3.16). The decoding complexity using diophantine approximation is thus of the order $\sqrt{Z} = \sqrt[4]{M}$.

Input: y, θ, Z
Output: $\widehat{X}$

$\beta = -(y + (Z+1)(1+\theta))/2;$
$\alpha = -\theta;$
$\eta_0 = \alpha; \eta_1 = -1; \zeta_1 = -\beta;$
$p_0 = 0; p_1 = 1; P_1 = 0;$
$q_0 = 1; q_1 = 0; Q_1 = 0;$
$n = 2;$
while $\eta_{n-1} \neq 0 \wedge \zeta_{n-1} \neq 0 \wedge Q_{n-1} \leq Z$ **do**
  $a_n = \lfloor -\frac{\eta_{n-2}}{\eta_{n-1}} \rfloor;$
  $p_n = p_{n-2} + a_n p_{n-1};$
  $q_n = q_{n-2} + a_n q_{n-1};$
  $\eta_n = \eta_{n-2} + a_n \eta_{n-1};$
  if $Q_{n-1} \leq q_{n-1}$ **then**
    $b_n = \lfloor -\frac{\zeta_{n-1} - \eta_{n-2}}{\eta_{n-1}} \rfloor;$
    $P_n = P_{n-1} + p_{n-2} + b_n p_{n-1};$
    $Q_n = Q_{n-1} + q_{n-2} + b_n q_{n-1};$
    $\zeta_n = \zeta_{n-1} + \eta_{n-2} + b_n \eta_{n-1};$
  else
    $P_n = P_{n-1} - p_{n-1};$
    $Q_n = Q_{n-1} - q_{n-1};$
    $\zeta_n = \zeta_{n-1} - \eta_{n-1};$
  end
  $n = n + 1;$
end
$P = 2P_n - (Z+1);$
$Q = 2Q_n - (Z+1);$
$\widehat{X} = P + \theta Q;$

Algorithm 1: Modified Cassels' algorithm for decoding symbols in a Z-PAM, with $\theta \in \mathbb{R}$

Two-relay case

In the two-relay case, we consider the system model described in section 3.3.3. The network is composed of one source, two relays and one destination. The Incomplete DF is implemented with the 4×4 distributed TAST code.

In order to have the lowest processing complexity at relays, we want to combine the advantages of the two-step decoding method and the diophantine approximation. The problem is that θ is not a real number in this case.

However, we can show that if $\theta = e^{i\frac{\pi}{4}}$ the decomposition in real and imaginary part is more

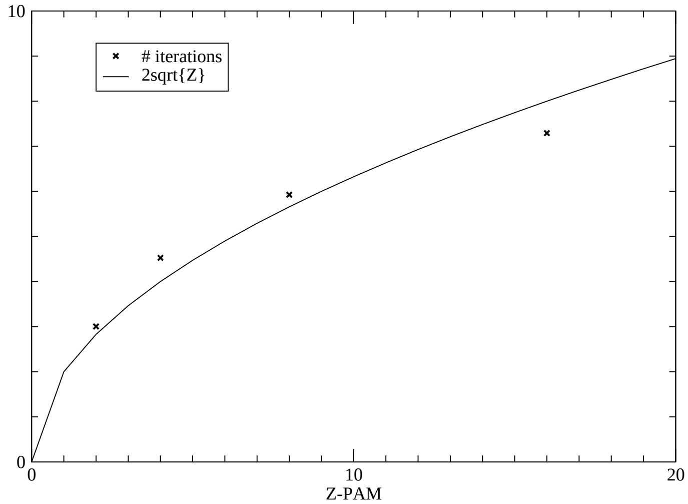


Figure 3.16: Complexity of the modified Cassels' algorithm: number of iterations of the modified Cassels' algorithm as a function of the size of the constellation

complex, but the diophantine approximation can still be used with a slight modification of the given algorithm.

A coded symbol is in the form

$$s_c = s_1 + \theta s_2,$$

where s_1 and s_2 are complex information symbols and $\theta = e^{i\frac{\pi}{4}}$.

$$\begin{aligned} s_c &= \Re(s_1) + i\Im(s_1) + \frac{1}{\sqrt{2}}(1+i)\left(\Re(s_2) + i\Im(s_2)\right) \\ &= \Re(s_1) + \frac{1}{\sqrt{2}}\left(\Re(s_2) - \Im(s_2)\right) + i\left[\Im(s_1) + \frac{1}{\sqrt{2}}\left(\Re(s_2) + \Im(s_2)\right)\right] \end{aligned}$$

The decomposition of the coded symbol into real and imaginary parts leads to numbers of the form $n = a + \frac{1}{\sqrt{2}}b$ where a belongs to the Z-PAM, but b is the sum of two elements of the Z-PAM. The Cassels' algorithm has to be modified again to take into account this new constraint (see Algorithm 2). Because of the real and imaginary decomposition, θ is replaced by $\frac{1}{\sqrt{2}}$ on lines 1, 2 and 26. Moreover, as the second element of the approximation is a sum of symbols taken in a Z-PAM, the search interval has to be changed. This is done on lines 1, 7 and 25.

The exhaustive search has a complexity M^4. The two-step decoding method allows to reduce this complexity to M^2. Finally, the combination with the diophantine approximation reduces the complexity to $\sqrt[4]{M}$.

Input: y, Z
Output: $\widehat{X}$

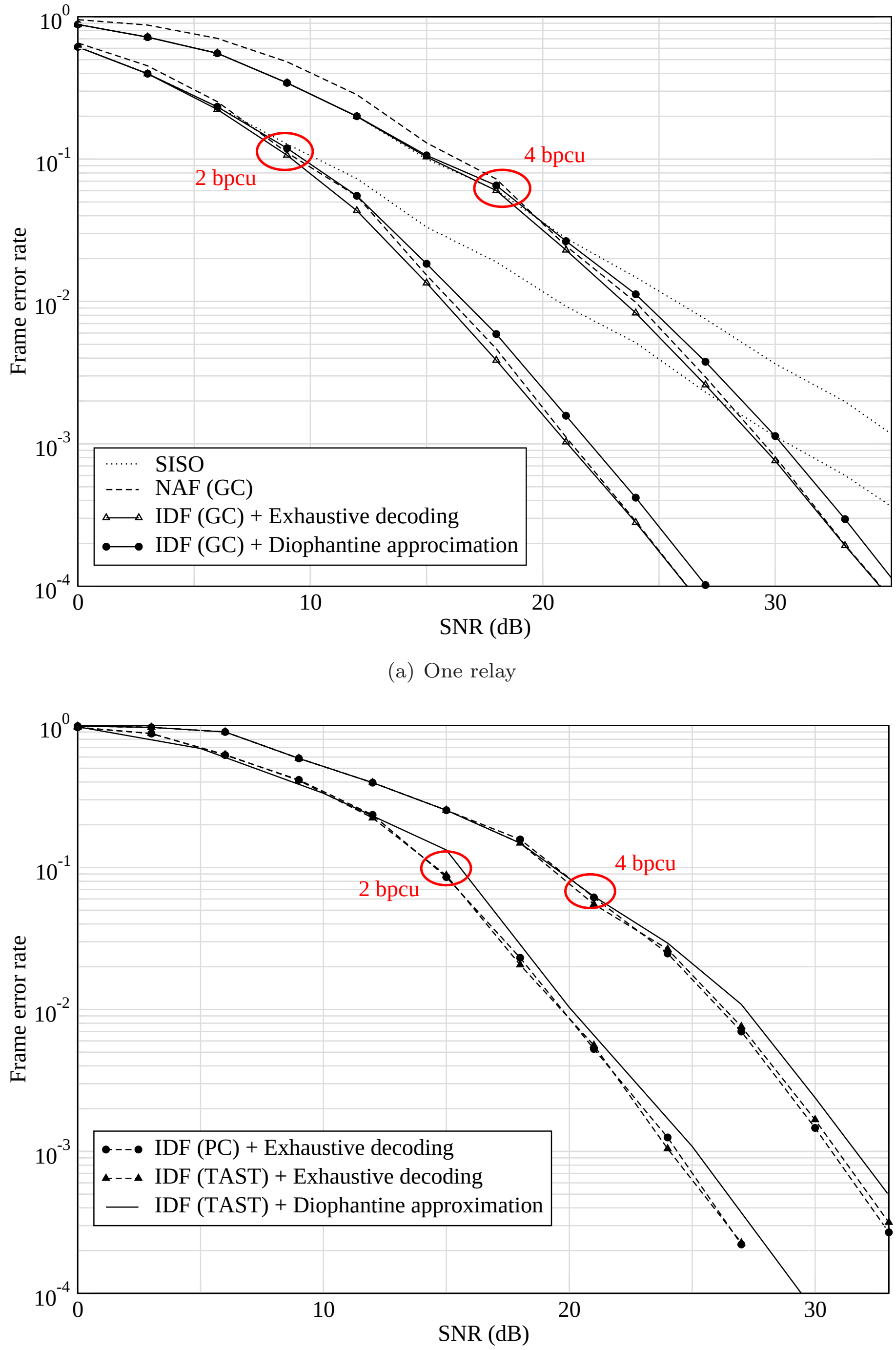
$\beta = -(y + (Z+1) + 2Z\frac{1}{\sqrt{2}}))/2;$
$\alpha = -\frac{1}{\sqrt{2}};$
$\eta_0 = \alpha; \eta_1 = -1; \zeta_1 = -\beta;$
$p_0 = 0; p_1 = 1; P_1 = 0;$
$q_0 = 1; q_1 = 0; Q_1 = 0;$
$n = 2;$
while $\eta_{n-1} \neq 0 \wedge \zeta_{n-1} \neq 0 \wedge Q_{n-1} \leq 2Z - 1$ **do**
  $a_n = \lfloor -\frac{\eta_{n-2}}{\eta_{n-1}} \rfloor;$
  $p_n = p_{n-2} + a_n p_{n-1};$
  $q_n = q_{n-2} + a_n q_{n-1};$
  $\eta_n = \eta_{n-2} + a_n \eta_{n-1};$
  if $Q_{n-1} \leq q_{n-1}$ **then**
    $b_n = \lfloor -\frac{\zeta_{n-1} - \eta_{n-2}}{\eta_{n-1}} \rfloor;$
    $P_n = P_{n-1} + p_{n-2} + b_n p_{n-1};$
    $Q_n = Q_{n-1} + q_{n-2} + b_n q_{n-1};$
    $\zeta_n = \zeta_{n-1} + \eta_{n-2} + b_n \eta_{n-1};$
  else
    $P_n = P_{n-1} - p_{n-1};$
    $Q_n = Q_{n-1} - q_{n-1};$
    $\zeta_n = \zeta_{n-1} - \eta_{n-1};$
  end
  $n = n + 1;$
end
$P = 2P_n - (Z+1);$
$Q = 2Q_n - 2Z;$
$\widehat{X} = P + \frac{1}{\sqrt{2}}Q;$

Algorithm 2: Modified Cassels' algorithm for $\theta = e^{i\frac{\pi}{4}}$

Simulation results

Simulations have been run in the one-relay case for spectral efficiencies of 2 and 4 bits pcu. Figure 3.17(a) represents the frame error rates of the SISO, NAF and Incomplete DF protocols as functions of the SNR. The NAF and the IDF are implemented with a distributed Golden code. Both exhaustive decoding and diophantine approximation are considered at the relay.

Diophantine approximation at relay performs slightly worse than the exhaustive decoding. This is explained by the fact that it is a sub-optimal decoding. However, this small loss



(a) One relay

(b) Two relays

Figure 3.17: Incomplete DF frame error rate

in performance, only 0.5 dB, is counterbalanced by a much lower decoding complexity decreasing from M^2 to $\sqrt{M}$.

In the two-relay case, the Incomplete DF protocol has been implemented with the distributed 4×4 perfect and TAST codes. When using the distributed perfect code, only exhaustive search is possible at relays. But when using the distributed TAST code, both exhaustive search and diophantine approximation are considered.

One can see in Figure 3.17(b) that the diophantine approximation causes less than 1 dB loss compared to the exhaustive search, which makes the total loss equal to 1 dB compared to the distributed perfect code with an exhaustive decoding at relays. This small loss is outweighed by the considerable drop of decoding complexity at relays: the initial complexity of M^4 is reduced to $\sqrt[4]{M}$.

3.4 Conclusion of the chapter

The study conducted through this chapter lead to the definition of the Incomplete DF protocol using distributed space-time codes that provides both full diversity and full rate, as the best known AF protocols. This new protocol is based on an incomplete decoding at relays. The received signals at relays are decoded as elements of the ring of integers of the considered number field without decoding the information symbols. Several decoding methods are proposed at relays: exhaustive search, diophantine approximation or a method based on the decomposition of the decoding in two steps according to the code structure. The two last methods allow a considerable decrease of complexity. Ideally the Incomplete DF would be the least complex and more efficient if used with an optimal distributed STBC offering both a structure allowing the two-step decoding and $\theta \in \mathbb{R}$ for a simple diophantine approximation.

The diversity-multiplexing gain tradeoff is proved to be the same as the one of the NAF protocol which is the best known AF protocol for the one-relay channel. In addition, outage probability and simulation results prove that the Incomplete DF gives slightly better performance than the NAF protocol in the high SNR regime, and initial selection provides an improvement for low SNRs.

Chapter 4

Generalization of the relay channel

In the two last chapters, we have been interested only in the relay channel with one source, one destination, and some dedicated relays. This model imposes two limitations which could be problems in a large network context.

The first limitation is related to the use of a TDMA strategy. At each frame, one source sends its information, and the other nodes play the role of relays. However, in some contexts such as real time application, several users need to transmit simultaneously. If we still want to exploit cooperative diversity, a new strategy is defined, called cooperative multiple access (CMA).

The second limitation is related to the existence of the direct link between source and destination. In a large network, this link could be inexistent or could suffer from deep fading, which makes it unusable. Serial relays are used to transmit information: a signal is first transmitted to a first relaying node, then forwarded to a second one, and so on till it reaches the destination. Since a few years, the multihop channel has been extensively studied for its interesting applications such as sensor and ad-hoc networks.

The chapter is organized as follows. Section 4.1 is devoted to the cooperative multiple-access channel. The protocol proposed by Azarian et al. [34], the CMA-NAF, is studied. A practical implementation as well as two modifications are proposed. A decode-and-forward CMA protocol is also defined based on the same incomplete decoding as in previous chapter.

In section 4.2, we study a specific model of multihop channel: the K-parallel-path channel. A low-complexity protocol is proposed based on a path selection combined with a small STBC. Several implementation issues such as the influence of path interferences on the performance are further discussed.

4.1 Cooperative Multiple Access (CMA) networks

In this section, we consider a system where two sources transmit simultaneously.

4.1.1 CMA channel model

Two CMA strategies exist in literature, considering the presence or absence of a dedicated relaying node in the network. In [6] the authors suppose the existence of a common relay which helps N sources to transmit to the same destination. This model is called the multiple access relay channel (MARC) (see Figure 4.1). Most cooperative protocols designed for a relay channel can be easily generalized to MARC.

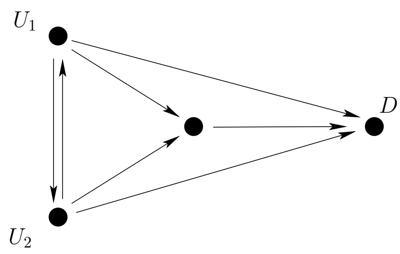


Figure 4.1: The multiple access relay channel (MARC) model for two sources

The cooperative multiple-access (CMA) channel considers only $N+1$ nodes: N sources cooperate together to communicate with the same destination. We are interested here in this last model for which very few results exist.

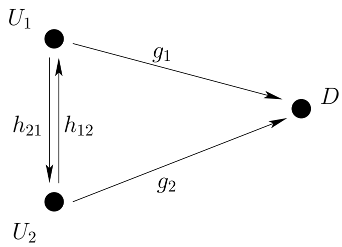


Figure 4.2: The cooperative multiple-access (CMA) channel model for two sources

In this work we consider a network with 2 users U_1 and U_2 transmitting to the same destination D. We consider single-antenna, half-duplex terminals. Channel links are supposed to be Rayleigh distributed and slow fading, thus their coefficient can be considered constant during the transmission of at least one frame.

In the following, we will use the notation given in Figure 4.2. The channel coefficient of user $U_j, j \in \{1,2\}$ and destination D link is g_j, and the channel coefficient of user U_1 and user U_2 link is h_{12} (U_2, U_1 and h_{21} respectively). We can assume that the attenuations from U_1 to U_2 and from U_2 to U_1 are the same, with opposite phases. Then we can simplify the notation and write

$$h_{12} = h \qquad \text{and} \qquad h_{21} = h^*.$$

4.1.2 Original CMA-NAF protocol and proposed implementation

For a network with two sources and one destination, a decode-and-forward (DF) CMA protocol based on superposition modulation has been proposed in [46] and further explored in [47] and [48]. However, this protocol does not use distributed space-time coding. In [34] authors propose an amplify-and-forward CMA as an extension of their NAF protocol, named CMA-NAF, and compute its DMT. However, it is a theoretical study and no code or implementation was provided.

Original CMA-NAF

The CMA-NAF is an orthogonal protocol, i.e. only one terminal transmits in each time slot. The strategy used is amplify-and-forward. The transmitted signal at each time slot is a linear combination of the signal received in the previous time slot and the information of the transmitting user. A frame is defined so that each user is helped once by all other users. If each user transmits during M time slots, the total frame lasts $N \times M$ times slots, where N is the number of users.

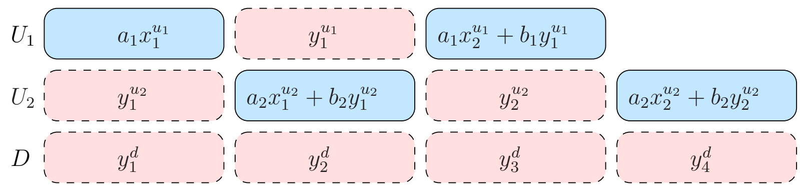


Figure 4.3: Transmission frame of the CMA-NAF protocol for 2 users and $M = 2$

For the two user case, with $M = 2$, the frame structure is described in Figure 4.3. The linear combination factors a_i and b_i are chosen so that the transmitted signal respects the total power constraint $P_{tot} \leq 1$. The total rate of this protocol is 1 symb. pcu.

It is shown in [34] that the CMA-NAF protocol asymptotically achieves the optimal DMT corresponding to the MISO bound when the frame size grows to infinity:

$$\lim_{M \to \infty} d^*(r) = N(1-r)^+. \tag{4.1}$$

Practical implementation of the CMA-NAF

We propose in the following a practical implementation of the protocol proposed by Azarian *et al.*

We first consider the two-source case. The implementation of the CMA-NAF using the rotation of the Golden code corresponds to the following coded signals for user U_1 and U_2 respectively:

$$\begin{bmatrix} x_1^{u_1} \\ x_2^{u_1} \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha & \alpha\theta \\ \overline{\alpha} & \overline{\alpha}\overline{\theta} \end{bmatrix} \begin{bmatrix} s_1^{u_1} \\ s_2^{u_1} \end{bmatrix} \tag{4.2}$$

and

$$\begin{bmatrix} x_1^{u_2} \\ x_2^{u_2} \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} \alpha & \alpha\theta \\ i\overline{\alpha} & i\overline{\alpha}\overline{\theta} \end{bmatrix} \begin{bmatrix} s_1^{u_2} \\ s_2^{u_2} \end{bmatrix}, \tag{4.3}$$

where $\theta = \frac{1+\sqrt{5}}{2}$ is the Golden number, $\overline{\theta} = \frac{1-\sqrt{5}}{2}$ is its conjugate, $\alpha = 1 + i - i\theta$ and $\overline{\alpha} = 1 + i - i\overline{\theta}$, and the $s_i^{u_j}, i \in \{1,2\}$ are the QAM information symbols of user $U_j, j \in \{1,2\}$. Each user transmits two coded symbols, thus $M = 2$ and the frame size is $2M = 4$ time slots.

In the more general case of N sources and $M \geq N$, a distributed $N \times M$ rectangular perfect code [25] can be used instead.

In order to take advantage of the space-time diversity provided by the protocol, an ML decoder has to be used at the destination, such as the sphere decoder [29] or the Schnorr-Euchner algorithm [30].

The optimization of coefficients a_i and b_i is very difficult to compute. For simplicity, we consider here for both users the same coefficients:

$$a_i = a, \qquad \text{and} \qquad b_i = b\beta.$$

The amplifying factor β and coefficients a and b are chosen so that the signal power is normalized

$$\beta = \frac{1}{\sqrt{1+\rho|h|^2}} \qquad \text{and} \qquad a^2 + b^2 = 1.$$

4.1.3 Improvements of the CMA-NAF

We propose two improvements of the original CMA-NAF. The first one consists in switching the roles of the two sources after each transmission in order to have the same performance for both users. The second one is based on the deletion of each user's own information to provide a better energy distribution.

Switching sources transmitting order

The CMA-NAF as described in [34] is asymmetric with respect to both users: as can be seen in Figure 4.3, information of user U_1 is better protected than information of user U_2. Indeed, all its coded symbols are repeated once, which is not the case for the last coded symbol of user U_2. This could affect user U_2 performance in the case of a small frame size.

This problem can be easily solved by defining a new superframe of $4M$ time slots. This superframe is divided into two subframes of $2M$ time slots each:

- the first subframe is defined as in Figure 4.3;
- in the second subframe, the roles of the 2 users are switched.

Signals can be decoded at the end of each subframe.

The new superframe is represented in Figure 4.4 for $2M = 4$. In this way, the protocol is fairer and protects the two users in the same manner (each user sends $2M$ coded symbols and has $2M-1$ of them protected).

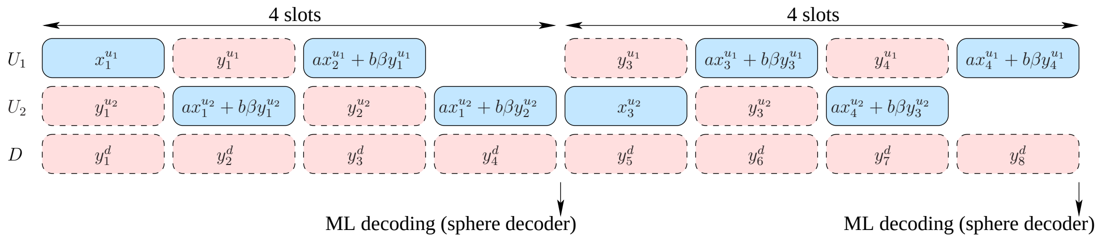


Figure 4.4: Transmission frame of the improved CMA-NAF protocol for a super frame of 8 time slots

In subsection 4.1.5, we show that this strategy improves the DMT of the protocol for a given subframe length $2M$.

Deleting each user's own information

In the CMA-NAF proposed in [34] the whole received signal is forwarded. So, at the end of the frame, the transmit power is shared between $2M$ different coded symbols. Moreover, the first coded symbol to be sent, $x_1^{u_1}$ is forwarded $2M-1$ times, while the last one to be forwarded, $x_M^{u_1}$, is retransmitted only once, and with a lower power.

This unbalanced power distribution can be avoided if each user subtracts its own information contribution from the received signals. Transmitted signals are then linear combinations of only two coded symbols (one from each user) and a better energy distribution is obtained.

Signal model of the modified CMA-NAF

The received signal at source U_2 during the first time slot is

$$y_1^{u_2} = \sqrt{\rho} h^* x_1^{u_1} + v_1^{u_2},$$

where $v_1^{u_2}$ is the Gaussian noise. The expression of the first amplifying factor can be deduced from this equation. Normalizing the forwarded signal $\beta_1 y_1^{u_2}$ gives

$$\beta_1 = \frac{1}{\sqrt{1+\rho|h|^2}}.$$

During odd time slots, user U_1 is transmitting and during even time slots, user U_2 is.

$$\begin{array}{ll} \textbf{time slot 2i:} & y_i^{u_1} = \sqrt{\rho}h\left(ax_i^{u_2} + bf_1(x_i^{u_1})\right) + v_i^{u_1}, \\ \textbf{time slot 2i+1:} & y_{i+1}^{u_2} = \sqrt{\rho}h^*\left(ax_{i+1}^{u_1} + bf_2(x_i^{u_2})\right) + v_{i+1}^{u_2}, \end{array}$$

where $f_1(x_i^{u_1})$ is a linear function of $x_i^{u_1}$ and accumulated noise ($f_2(x_i^{u_2})$ and $x_i^{u_2}$ respectively). At each time slot, noise is added to the signal. Let $P_{w,k}$ be the power of the accumulated noise at the users at time slot k. It is defined recursively as:

$$P_{w,1} = 1, \tag{4.4}$$

$$\forall k \in \{2, \dots, 2M\}, \quad P_{w,k} = 1 + \rho b^2 \beta_{k-1}^2 |h|^2 P_{w,k-1}. \tag{4.5}$$

Amplifying factors β_k are calculated so that the power of the forwarded signals are normalized. Thus they are defined recursively as

$$\beta_1 = \frac{1}{\sqrt{1+\rho|h|^2}}, \tag{4.6}$$

$$\forall k \in \{2, \dots, 2M-1\}, \quad \beta_k^2 = \frac{1}{P_{w,k} + \rho a^2 |h|^2}. \tag{4.7}$$

The total noise power at destination is given recursively by

$$P_1 = 1, \tag{4.8}$$

$$\forall k \in \{1, \dots, M\}, \quad P_{2k} = 1 + \rho \beta_{2k-1}^2 |g_2|^2 P_{w,2k-1}, \tag{4.9}$$

$$P_{2k+1} = 1 + \rho \beta_{2k}^2 |g_1|^2 P_{w,2k}. \tag{4.10}$$

Considering the first subframe of size $2M$ and normalizing the noises we can write

$$\begin{bmatrix} y_1^d \\ \frac{y_2^d}{\sqrt{P_2}} \\ \frac{y_3^d}{\sqrt{P_3}} \\ \frac{y_4^d}{\sqrt{P_4}} \\ \vdots \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} g_1 & 0 & 0 & 0 & \cdots \\ \sqrt{\frac{\rho}{P_2}} b\beta_1 g_2 h & \frac{1}{\sqrt{P_2}} a g_2 & 0 & 0 & \cdots \\ 0 & \sqrt{\frac{\rho}{P_3}} ab\beta_2 g_1 h^* & \frac{1}{\sqrt{P_3}} a g_1 & 0 & \cdots \\ 0 & 0 & \sqrt{\frac{\rho}{P_4}} ab\beta_3 g_2 h & \frac{1}{\sqrt{P_4}} a g_2 & \ddots \\ \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} x_1^{u_1} \\ x_1^{u_2} \\ x_2^{u_1} \\ x_2^{u_2} \\ \vdots \end{bmatrix} + \begin{bmatrix} w_1^d \\ \widetilde{w_2^d} \\ \widetilde{w_3^d} \\ \widetilde{w_4^d} \\ \vdots \end{bmatrix} \tag{4.11}$$

where w_1^d and $\widetilde{w_i^d}$ are Gaussian noises. $\forall i, \widetilde{w_i^d}$ is the sum of noises accumulated at each interuser transmission plus the noise at destination, normalized by the total noise power P_i. Thus the variance of $\widetilde{w_i^d}$ is 1.

In particular, if $M = 2$ and the rotation of the Golden code is used, we can write the previous system in the form:

$$\mathbf{y} = \mathbf{H}\phi\mathbf{s} + \mathbf{w},$$

where ϕ is the generating matrix of the Golden code.

Finally, considering the whole superframe of size $4M$, we can define two $2M \times M$ matrices $\mathbf{M}_1^{AF}(f, g, h)$ and $\mathbf{M}_2^{AF}(f, g, h)$, functions of three variables f, g and h. The received signal can be rewritten as:

$$\mathbf{y}^{AF} = \sqrt{\rho} \underbrace{\begin{bmatrix} \mathbf{M}_1^{AF}(g_1, g_2, h) & \mathbf{0}_{2M\times M} \\ \mathbf{0}_{2M\times M} & \mathbf{M}_2^{AF}(g_1, g_2, h) \end{bmatrix}}_{\mathbf{H}_1^{AF}} \mathbf{x}^{u_1} + \sqrt{\rho} \underbrace{\begin{bmatrix} \mathbf{M}_2^{AF}(g_2, g_1, h^*) & \mathbf{0}_{2M\times M} \\ \mathbf{0}_{2M\times M} & \mathbf{M}_1^{AF}(g_2, g_1, h^*) \end{bmatrix}}_{\mathbf{H}_2^{AF}} \mathbf{x}^{u_2} + \mathbf{w} \tag{4.12}$$

where

- $\mathbf{y}^{AF}$ is the received signal vector at destination of size $4M$;
- $\mathbf{H}_1^{AF}$ is the equivalent channel matrix for user U_1 ($\mathbf{H}_2^{AF}$ and U_2 respectively);
- $\mathbf{x}^{u_1}$ is user U_1 coded symbols vector of size $2M$ ($\mathbf{x}^{u_2}$ and U_2 respectively);
- $\mathbf{w}$ is a Gaussian noise vector of size $4M$.

We can remark that the roles of g_1 and g_2, h and h^* are switched between matrix $\mathbf{H}_1^{AF}$ and matrix $\mathbf{H}_2^{AF}$.

Matrices $\mathbf{M}_1^{AF}(f, g, h)$ and $\mathbf{M}_2^{AF}(f, g, h)$ are given by

$$\begin{aligned} \mathbf{M}_1^{AF}(f,g,h) &= \begin{bmatrix} f & 0 & \cdots \\ \sqrt{\frac{\rho}{P_2}} b\beta_1 gh & 0 & \cdots \\ 0 & \frac{1}{\sqrt{P_3}} af & \ddots \\ 0 & \sqrt{\frac{\rho}{P_4}} ab\beta_3 gh & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix} \\ \text{and } \mathbf{M}_2^{AF}(f,g,h) &= \begin{bmatrix} 0 & 0 & \cdots \\ \frac{1}{\sqrt{P_2}} af & 0 & \cdots \\ \frac{\rho}{\sqrt{P_3}} ab\beta_2 gh & 0 & \ddots \\ 0 & \frac{1}{\sqrt{P_4}} af & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix}. \end{aligned} \tag{4.13}$$

One can see in this system model that the equivalent channel matrix is no longer fully triangular such as for the original CMA-NAF, but double diagonal. This comes from the fact that each user removes its own information. Through the structure of the matrices $\mathbf{H}_1^{AF}$ and $\mathbf{H}_2^{AF}$, we can see that the power is better balanced between the coded symbols, which improves performance.

4.1.4 Decode-and-forward strategy: CMA-IDF

Based on the same idea as the IDF protocol defined in previous chapter, we propose here a CMA protocol using a DF strategy, called CMA-IDF, which has the same well-balanced frame structure as the modified CMA-NAF.

Outage selection

In the same way as for the relay channel, we have to distinguish two cases:

- if the inter-user link is in outage, DF protocol cannot be used, signals are sent in a non-cooperative manner using a TDMA strategy;
- if the inter-user link is not in outage, signals can be correctly decoded by each user, CMA-IDF protocol can be used.

The inter-user outage event is defined as:

$$\mathcal{O}^{IU} = \left\{\log(1+\rho|h|^2) < 2R\right\} \tag{4.14}$$

where R is the total spectral efficiency. The spectral efficiency of the inter-user link is twice R since there are two information symbols in each coded symbol $x_i^{u_j}$.

The CMA-IDF protocol

The CMA-IDF protocol has the same frame structure as the improved CMA-NAF (see Figure 4.4). The difference resides in the received signal processing at each user side. As in the previous section, user $U_j, j \in \{1,2\}$ removes its own information symbols from its received signals. Then it decodes the other user's information using the incomplete decoding method described in Chapter 3, section 3.3 for the decoding at the relays of the IDF protocol. The received signals are decoded as coded symbols x_i of the ring of integers of the considered number field (for example $Q(i,\sqrt{5})$ in the case of the distributed Golden code) without having to decode the information symbols s_j.

Assuming that coded symbols have been correctly detected at user U_1 and U_2, the received signals at destination can be written in the same way as for the AF case:

$$\mathbf{y}^{DF} = \sqrt{\rho}\underbrace{\begin{bmatrix} \mathbf{M}_1^{DF}(g_1,g_2) & \mathbf{0}_{2M\times M} \\ \mathbf{0}_{2M\times M} & \mathbf{M}_2^{DF}(g_1,g_2) \end{bmatrix}}_{\mathbf{H}_1^{DF}}\mathbf{x}^{u_1} + \sqrt{\rho}\underbrace{\begin{bmatrix} \mathbf{M}_2^{DF}(g_2,g_1) & \mathbf{0}_{2M\times M} \\ \mathbf{0}_{2M\times M} & \mathbf{M}_1^{DF}(g_2,g_1) \end{bmatrix}}_{\mathbf{H}_2^{DF}}\mathbf{x}^{u_2} + \mathbf{w}, \tag{4.15}$$

where $\mathbf{M}_1^{DF}(f,g)$ and $\mathbf{M}_2^{DF}(f,g)$ are $2M \times M$ matrices defined as functions of only two link coefficients f and g:

$$\mathbf{M}_1^{DF}(f,g) = \begin{bmatrix} f & 0 & \cdots \\ bg & 0 & \cdots \\ 0 & af & \ddots \\ 0 & bg & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix} \quad \text{and} \quad \mathbf{M}_2^{DF}(f,g) = \begin{bmatrix} 0 & 0 & \cdots \\ af & 0 & \cdots \\ bg & 0 & \ddots \\ 0 & af & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix}. \tag{4.16}$$

One can see in this system model that there is no noise amplification at the relaying nodes. If coded symbols are correctly decoded by both users, the destination receives a signal which is similar to the one sent by a MIMO array equipped with two antennas.

4.1.5 Performance of the proposed protocols

Outage probability derivation

The multiple access channel outage event was defined by Tse *et al.* in [49].

Definition 9. *For a multiple access channel with K users, each equipped with n_t transmit antennas, and a receiver equipped with n_r receive antennas, the outage event is*

$$\mathcal{O} = \bigcup_S \mathcal{O}_S.$$

The union includes all subsets $S \subseteq \{1, ..., K\}$. For each subset, the outage event is

$$\mathcal{O}_S = \left\{ I(\mathbf{X}_S; \mathbf{Y} | \mathbf{X}_{\overline{S}}, \mathbf{H}) < \sum_{i \in S} R_i \right\},$$

where $\mathbf{X}_S$ contains the input signals from the users in subset S and $\mathbf{X}_{\overline{S}}$ the input signals from the users not in S, $\mathbf{H} \in \mathbb{C}^{n_r \times K n_t}$ is the channel matrix.

Considering only two single-antenna users and a single-antenna destination and using Definition 9, the outage event is:

$$\mathcal{O} = \mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{O}_{1,2}, \tag{4.17}$$

where

- $\mathcal{O}_1$ is the outage event of user U_1 if the information of user U_2 is known

$$\mathcal{O}_1 = \{ I(X_1; Y | X_2, \mathbf{H} = \mathbf{H}_1) < R_1 \}, \tag{4.18}$$

  where R_1 is the spectral efficiency of user U_1.

- $\mathcal{O}_2$ is the outage event of user U_2 if the information of user U_1 is known

$$\mathcal{O}_2 = \{I\left(X_2; Y|X_1, \mathbf{H} = \mathbf{H}_2\right) < R_2\}, \tag{4.19}$$

where R_2 is the spectral efficiency of user U_2.

- and $\mathcal{O}_{1,2}$ is the outage event of both users U_1 and U_2

$$\mathcal{O}_{1,2} = \{I\left(X; Y, \mathbf{H} = \mathbf{H}\right) < R\}, \tag{4.20}$$

where $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \end{bmatrix}$ and $R = R_1 + R_2$ is the total spectral efficiency.

For all the proposed protocols in this paper (improved CMA-NAF and CMA-IDF), we have considered $R_1 = R_2 = \frac{R}{2}$.

Applying this definition to our protocols, we obtain the outage events of the modified CMA-NAF and CMA-IDF:

- In the AF case, the three possible outage events can be expressed as

$$\mathcal{O}_1 = \left\{\frac{1}{4}\log\det\left(\mathbf{I} + \rho\mathbf{H}_1^{AF}(\mathbf{H}_1^{AF})^\dagger\right) < \frac{R}{2}\right\}, \tag{4.21}$$

$$\mathcal{O}_2 = \left\{\frac{1}{4}\log\det\left(\mathbf{I} + \rho\mathbf{H}_2^{AF}(\mathbf{H}_2^{AF})^\dagger\right) < \frac{R}{2}\right\}, \tag{4.22}$$

$$\mathcal{O}_{1,2} = \left\{\frac{1}{4}\log\det\left(\mathbf{I} + \rho\mathbf{H}^{AF}(\mathbf{H}^{AF})^\dagger\right) < R\right\}. \tag{4.23}$$

where $\mathbf{H}_1^{AF}$ and $\mathbf{H}_2^{AF}$ are defined in equation (4.12), and $\mathbf{H}^{AF} = [\mathbf{H}_1^{AF}\mathbf{H}_2^{AF}]$.

- In the DF case, the expressions of the outage probabilities are more complex due to the selection between non-cooperation and the cooperative strategy which is necessary to preserve the DF protocol gain.

Then, the outage event $\mathcal{O}_1$ can be expressed as the event of user U_1 in outage using the cooperative MAC protocol with the inter-user link not in outage, or the event of user U_1 in outage using the SISO protocol with the inter-user link in outage, which can be expressed by:

$$\begin{aligned}\mathcal{O}_1 = \ & \left\{\left\{\tfrac{1}{4}\log\det\left(\mathbf{I} + \rho\mathbf{H}_1^{DF}(\mathbf{H}_1^{DF})^\dagger\right) < \tfrac{R}{2}\right\} \cap \left\{\log\det\left(1 + \rho|h|^2\right) > 2R\right\}\right\} \\ & \cup \left\{\left\{\log\det\left(1 + \rho|g_1|^2\right) < R\right\} \cap \left\{\log\det\left(1 + \rho|h|^2\right) < 2R\right\}\right\}.\end{aligned} \tag{4.24}$$

We can note that the two events in the union are independent, so the probability of O_1 will be a sum of probabilities. Moreover, the two events in each intersection are also independent, so the probability of each union term will be a product of probabilities. Finally we can write:

$$P(\mathcal{O}_1) = P(\mathcal{O}_1^{DF})(1 - P(\mathcal{O}^{IU})) + P(\mathcal{O}_1^{SISO})P(\mathcal{O}^{IU}). \tag{4.25}$$

In the same way, we can write

$$\begin{aligned}\mathcal{O}_2 = \quad & \left\{\left\{\tfrac{1}{4}\log\det\left(\mathbf{I}+\rho\mathbf{H}_2^{DF}(\mathbf{H}_2^{DF})^\dagger\right) < \tfrac{R}{2}\right\} \cap \left\{\log\det\left(1+\rho|h|^2\right) > 2R\right\}\right\} \\ & \cup \left\{\left\{\log\det\left(1+\rho|g_2|^2\right) < R\right\} \cap \left\{\log\det\left(1+\rho|h|^2\right) < 2R\right\}\right\}\end{aligned} \tag{4.26}$$

and

$$P(\mathcal{O}_2) = P(\mathcal{O}_2^{DF})(1-P(\mathcal{O}^{IU})) + P(\mathcal{O}_2^{SISO})P(\mathcal{O}^{IU}). \tag{4.27}$$

And for the last outage event, we get

$$\begin{aligned}\mathcal{O}_{1,2} = \quad & \left\{\left\{\tfrac{1}{4}\log\det\left(\mathbf{I}+\rho\mathbf{H}^{DF}(\mathbf{H}^{DF})^\dagger\right) < R\right\} \cap \left\{\log\det\left(1+\rho|h|^2\right) > 2R\right\}\right\} \\ & \cup \left\{\left\{\log\det\left(1+\rho|g_1|^2\right) < R\right\} \cap \left\{\log\det\left(1+\rho|h|^2\right) < 2R\right\}\right\} \\ & \cup \left\{\left\{\log\det\left(1+\rho|g_2|^2\right) < R\right\} \cap \left\{\log\det\left(1+\rho|h|^2\right) < 2R\right\}\right\}\end{aligned} \tag{4.28}$$

and

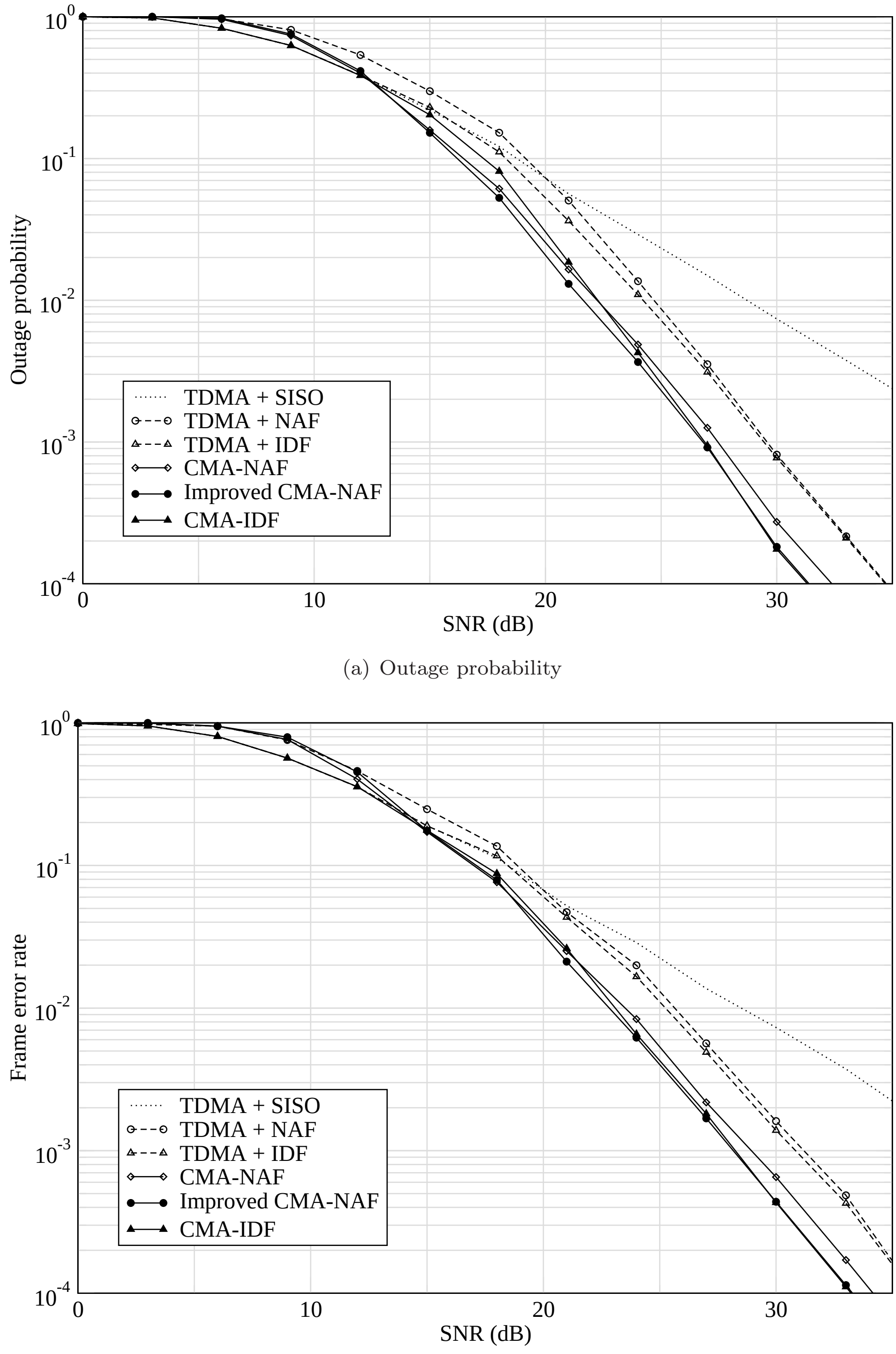
$$P(\mathcal{O}_{1,2}) = P(\mathcal{O}^{DF})(1-P(\mathcal{O}^{IU})) + P(\mathcal{O}_1^{SISO})P(\mathcal{O}^{IU}) + P(\mathcal{O}_2^{SISO})P(\mathcal{O}^{IU}). \tag{4.29}$$

Performance analysis

In Figure 4.5(a) are represented the new protocols outage probabilities obtained by Monte Carlo simulation, compared to those of the NAF and IDF protocols using TDMA strategy for two users. The total outage probability is plotted as a function of the SNR. In this example, we have chosen $a^2 = 0.4$, $M = 2$ and the total spectral efficiency is 4 bits pcu. We can remark that the CMA strategy provides better performance than the usual TDMA strategy combined with the NAF and IDF protocols (gain of 3 dB). Moreover, we can note that the improvements of the CMA-NAF bring a gain of 1 dB over the original CMA-NAF. The DF strategy gives similar results as the improved CMA-NAF because a small frame size is considered.

In Figure 4.5(b) are represented the simulation results of the new protocol compared with the NAF and IDF protocols using TDMA strategy. The total frame error rates of these protocols are plotted as functions of the SNR. The assumptions are the same: $a^2 = 0.4$, $M = 2$ and the total spectral efficiency is 4 bits pcu. The improvements observed on outage probability curves are confirmed by simulation results. Thanks to the CMA strategy, we obtain better performance than with the usual TDMA strategy associated to usual cooperative protocols. Improved CMA-NAF and CMA-IDF give similar results, outperforming the original CMA-NAF with a gain of 1 dB.

We can remark that due to the initial outage selection, the CMA-IDF shows better performance than the improved CMA-NAF at low SNR. However, by applying the same selection to the improved CMA-NAF, we could obtain the same behavior.



(a) Outage probability

(b) Frame error rate

Figure 4.5: Modified CMA-NAF and CMA-IDF performance

Diversity-Multiplexing gain Tradeoff analysis

The DMT of the two previously presented protocols are provided in the two following theorems.

Theorem 5. *The DMT of the improved CMA-NAF is*

$$d^* = (1-r) + \left(1 - \frac{2M}{2M-1}r\right)^+. \tag{4.30}$$

This DMT is better than the one of the NAF protocol and tends to the MISO bounds two times faster than the one of the original CMA-NAF which is:

$$d^* = (1-r) + \left(1 - \frac{M}{M-1}r\right)^+.$$

Proof. The outage events expressions are given in equations (4.21), (4.22) and (4.23). In order to compute the DMT of this cooperative strategy, we have to study the behavior of the outage probability in the high SNR regime.

An upper bound on the outage probability is given by

$$P(\mathcal{O}) = P\left(\mathcal{O}_1 \cup \mathcal{O}_2 \cup \mathcal{O}_{1,2}\right) \leq P(\mathcal{O}_1) + P(\mathcal{O}_2) + P(\mathcal{O}_{1,2}). \tag{4.31}$$

Let d_1, d_2 and $d_{1,2}$ be the DMTs of the outage events $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_{1,2}$ respectively. Let's define

$$d^* = \min\{d_1, d_2, d_{1,2}\} \tag{4.32}$$

and $\mathcal{O}^*$ the corresponding outage event.

By definition $P(\mathcal{O}^*) \leq P(\mathcal{O})$ since $\mathcal{O}^*$ is one of the three possible outage events. Moreover, the exponential order of $\mathcal{O}^*$ is greater than those of the other outage events, so asymptotically $P(\mathcal{O}_1) + P(\mathcal{O}_2) + P(\mathcal{O}_{1,2}) \doteq P(\mathcal{O}^*)$. Thus we get $P(\mathcal{O}^*) \leq P(\mathcal{O}) \dot{\leq} P(\mathcal{O}^*)$ and finally

$$P(\mathcal{O}) \doteq P(\mathcal{O}^*) \doteq \rho^{-d^*}. \tag{4.33}$$

Using the expressions of the outage events, we can compute the DMTs corresponding to each of them. The detailed derivation of these DMTs is given in Appendix A.5.

$$d_1 = d_2 = (1-r) + \left(1 - \frac{2M}{2M-1}r\right)^+,$$

where $2M$ is the frame size, and

$$d_{12} = 2(1-r).$$

Thus

$$d^* = \min(d_1, d_2, d_{12}) = (1-r) + \left(1 - \frac{2M}{2M-1} r\right)^+ .$$

□

When using the decode-and-forward strategy, the DMT is given by the following theorem.

Theorem 6. *The DMT of the CMA-IDF protocol is*

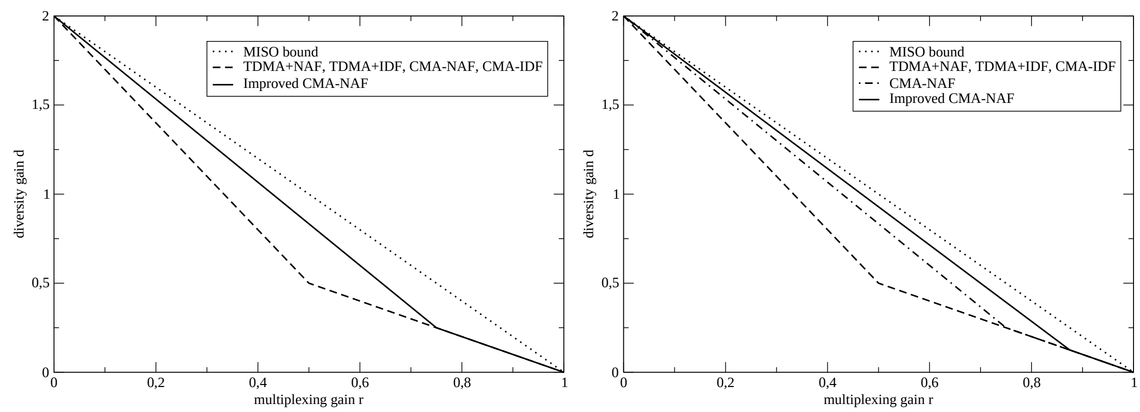
$$d^* = (1-r) + (1-2r)^+ . \tag{4.34}$$

The DMT of the CMA-IDF protocol is the same as the DMT of the Incomplete DF protocol using a TDMA strategy in the two user case.

Proof. Similar calculations as in the AF case provide the same results when the cooperating mode is selected. Unfortunately the overall DMT is limited due to the use of initial outage selection. Indeed

$$P\left\{\log(1+\rho|g_1|^2) < R\right\} P\left\{\log(1+\rho|h|^2) < 2R\right\} \doteq \rho^{-(1-r)} \rho^{-(1-2r)^+} .$$

□



(a) $4M = 4$ times slots (b) $4M = 8$ times slots

Figure 4.6: Modified CMA-NAF and CMA-IDF DMT

The DMT of the improved CMA-NAF and CMA-IDF protocols are represented in Figures 4.6(a) and 4.6(b) and compared with those of the NAF and IDF protocols using a TDMA strategy for superframe lengths of $4M = 4$ and $4M = 8$ time slots.

One can see that the improved CMA-NAF is closer to the MISO bound than the original one for a fixed superframe length.

4.2 Multi-hop networks

The theoretical performance of the multihop networks has been analyzed in literature. Multihop protocols have also been proposed to reach these theoretical performance. Diversity is usually brought using space-time coding [4,5,42,50,51]. However, these strategies induce a high decoding complexity. In order to provide diversity with limited complexity, we can use selection. Antenna selection was already used in MIMO systems [52, 53]. This idea has been generalized to relay or path selection in multihop cooperative networks [54, 55].

4.2.1 KPP channel model

We consider a network composed of one source, one destination, and several relaying nodes. The network is modeled by a K-Parallel-Paths (KPP) channel.

The length of the k^{th} path is denoted by n_k. Thus the k^{th} path contains $(n_k + 1)$ nodes.

The first node of a path is the source S and the last one is the destination D. The i^{th} relay of the k^{th} path is noted R_i^k. A node can only broadcast information to its neighbors. The channel coefficient between node i and node $i+1$ of the k^{th} path is h_i^k. We can assume that the transmission between node $i+1$ and node i is submitted to the same attenuation with an opposite phase. Consequently, the corresponding channel coefficient is $(h_i^k)^*$. All the channel coefficients follow a Rayleigh distribution and vary very slowly. Thus they can be considered constant during the transmission of at least one frame. We also suppose a symmetric scenario, i.e. all the channel links are subject to the same average SNR.

Considered terminals are half-duplex; they cannot receive and transmit simultaneously. They are equipped with a single antenna; the MIMO case is not considered in this work.

We assume that there is no interference between paths. This assumption is discussed further in section 4.2.6.

Example 11. *In the following, theoretical results are illustrated through the example of the 4-PP channel, where each path length $n_1 = n_2 = n_3 = n_4 = 3$ (see Figure 4.7).*

4.2.2 Existing works

Edge coloring strategy

In [4, 5], authors defined and studied the KPP channel from a theoretical point of view. In particular, the optimal DMT of such a channel is proven to be

$$d^*(r) = K(1-r)^+. \tag{4.35}$$

A transmission protocol is proposed to achieve these optimal performance using an edge coloring strategy. Let N be the cycle length for the protocol and $C = \{c_1, c_2, \ldots, c_N\}$

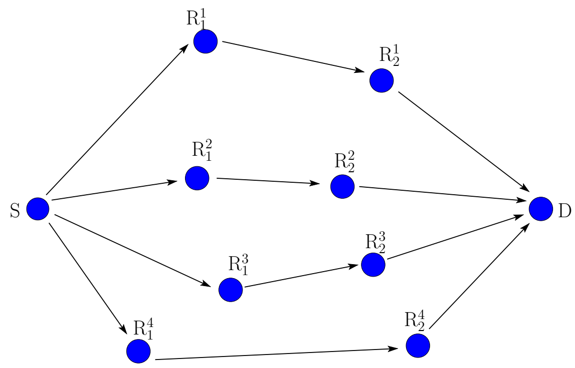


Figure 4.7: 4-PP channel with path length $n = 3$

be the set of colors. Each color in C represents a time slot. $\forall k \in \{1, \dots, K\}$ and $i \in \{1, \dots, n_k\}$, let C_i^k be the set of colors associated to the edge between relay R_i^k and relay R_{i+1}^k (the source is abusively noted R_0^k). Each color in C_i^k represents the time slots during which relay R_i^k is transmitting to relay R_{i+1}^k. The edge coloring has to respect the following constraints:

- at each time slot, the source is transmitting to only one path:
$$\forall (k, k') \in \{1, \dots, K\}^2, k \neq k', C_0^k \cap C_0^{k'} = \varnothing;$$
- at each time slot, the destination is receiving from only one path:
$$\forall (k, k') \in \{1, \dots, K\}^2, k \neq k', C_{n_k}^k \cap C_{n_{k'}}^{k'} = \varnothing;$$
- since the nodes are half-duplex, two neighbors cannot transmit simultaneously:
$$\forall k \in \{1, \dots, K\}, \forall i \in \{1, \dots, n_k - 1\}, C_i^k \cap C_{i+1}^{k'} = \varnothing;$$
- each node on a path is transmitting during the same number of time slots T_k in order for all signals to reach destination:
$$\forall k \in \{1, \dots, K\}, \forall i \in \{1, \dots, n_k\}, Card(C_i^k) = T_k.$$

Implemented with an approximately universal STBC [56], this protocol is proven to achieve the MISO bound.

End-to-end antenna selection

In [55], authors considered a layered network. Each layer l contains A_l nodes. A node in a layer l is connected to all nodes in layers $l-1$ and $l+1$. Authors proposed an end-to-end antenna selection (EEAS) designed to bring diversity with low decoding complexity. They considered both the full-duplex and half-duplex modes.

In the full-duplex mode, one path is selected so as to maximize the SNR at destination. This strategy allows to reach the optimum DMT of the system:

$$d^*(r) = \alpha(1-r)^+,$$

where $\alpha = \min_l\{A_{l-1} \times A_l\}$ is the maximum diversity.

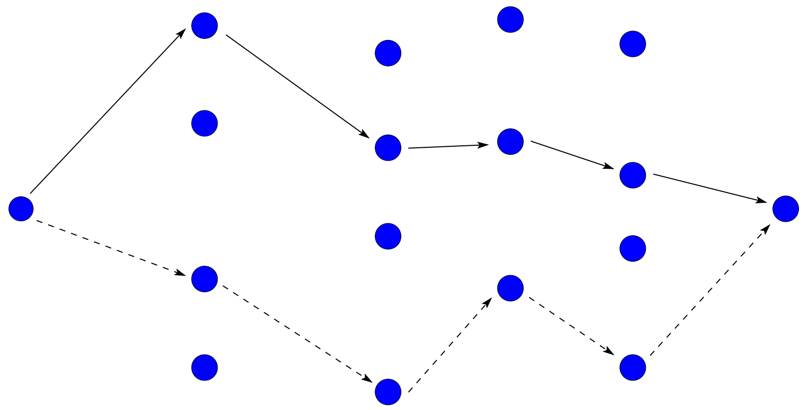


Figure 4.8: End-to-end antenna selection strategy: only the plain path is selected in the full-duplex mode; both the plain and dashed paths are selected in the half-duplex mode.

In the half-duplex mode, two paths are selected:

- the first path is chosen according to the SNR at destination;
- the second one is then selected according to the same criteria but considering only the remaining nodes.

The maximum diversity is limited by the choice of the second path to $\beta = \min_l\{(A_{l-1}-1) \times (A_l-1)\}$. Information symbols are sent alternatively on first and second paths, in order to reach a full rate of 1 symb. pcu. This strategy is proven to achieve a DMT of

$$d(r) = \beta(1-r).$$

The drawback of this strategy is that some diversity is lost, which degrades asymptotic performance.

4.2.3 Proposed protocol using an amplify-and-forward strategy

In order to achieve the theoretical limits of the KPP channel with a low complexity, we propose to combine two diversity strategy: path selection and space-time coding. First we define our protocol for the case of equal path lengths ($\forall k \in \{1, K\}, n_k = n$) and generalize it to different path lengths in the sequel.

A smart combining of path selection and space-time coding

We propose to select S paths (S is not fixed yet). At each time slot, the source sends a signal to the first node of one of the selected paths. Meanwhile, all nodes that have received a signal in the previous time slot forward this signal to the next node in the path. Using this transmission strategy, a node is transmitting only $\frac{1}{S}$ of the time.

The paths are independent and thus can be selected independently. The first path is chosen to maximize the SNR at destination. The second one is selected using the same criterion but considering only the $(K-1)$ remaining paths. And so on till the S^{th} path is selected.

Let g_k be the product of the channel coefficients composing the k^{th} path. g_k can be expressed:

$$g_k = \left(\prod_{j=1}^{n-1} \sqrt{\rho}\beta_j^k h_j^k\right) h_n^k, \tag{4.36}$$

where β_j^k are the amplifying factors chosen so that the power of the transmitted signal is normalized

$$\forall j \in \{1, ..., n-1\}, \beta_j^k = \frac{1}{\sqrt{1+\rho|h_j^k|^2}}. \tag{4.37}$$

Let the g_{k_i} be the ordered channel products:

$$\forall (i,j) \in \{1, ..., K\}^2, i > j, |g_{k_i}|^2 > |g_{k_j}|^2.$$

Then the selected paths are paths $(k_i)_{1 \leq i \leq S}$.

Once the S paths are selected, the space-time code to be used can be chosen. Since selection brings diversity, we do not need to take a large set of paths. A small value of S is sufficient and it allows to use a small STBC and thus to have a low decoding complexity.

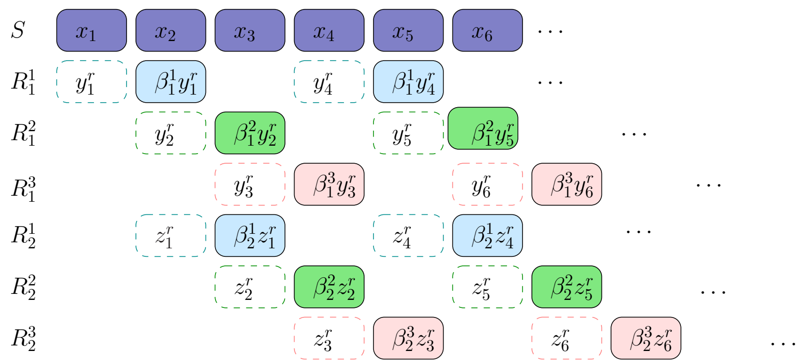


Figure 4.9: Transmission frame of the path selection multihop protocol

The protocol illustrated in Figure 4.9 for the case of $S = 3$ is an orthogonal protocol.

So the equivalent channel model is the same as the one of the MISO channel. DAST codes [20] or a diagonal of a perfect code can be used. These codes were proven to be universal in [57].

Coded symbols x_i can then be written as:

$$\mathbf{x} = \mathbf{M}\mathbf{s},$$

where $\mathbf{M}$ is a unitary matrix (rotation). $\mathbf{x}$ and $\mathbf{s}$ are the arrays of coded and information symbols respectively.

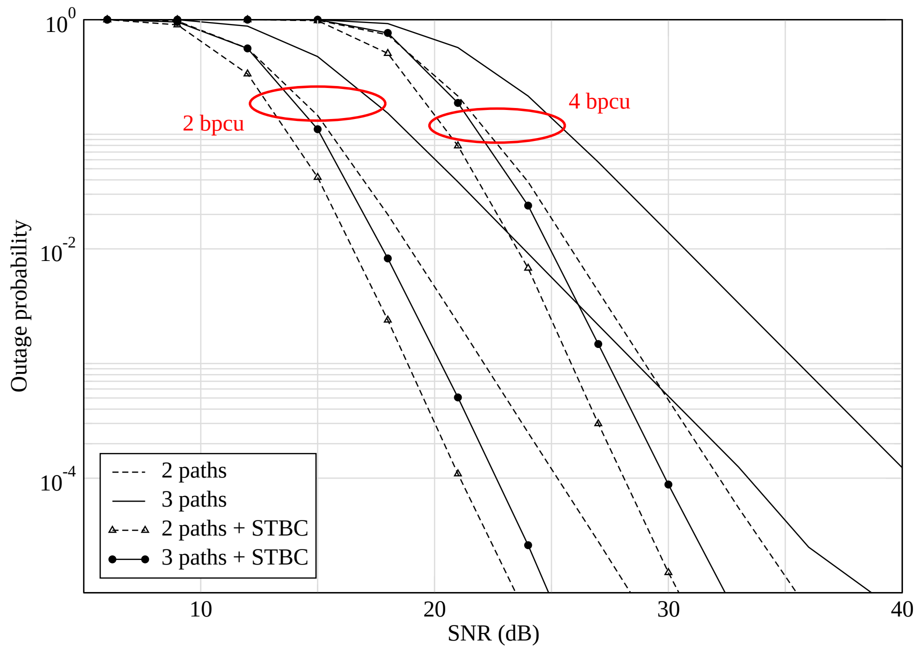


Figure 4.10: Outage probability of the path selection multihop protocol with and without a distributed STBC

We consider again the network presented in Example 11. Figure 4.10 shows the performance obtained with and without a STBC when two or three paths are selected. Several remarks can be done considering this figure:

- The more paths, the lower performance. Indeed, according to the selection criterion, the additional paths have lower SNR at destination.
- As expected, space-time coding brings diversity and thus better asymptotic performance. We can observe diversity orders of 4 when an STBC is used, while the uncoded strategies bring diversity orders of 3 and 2 only for $S = 2$ and $S = 3$ respectively.
- The higher the number of selected paths, the larger the gain between the uncoded strategy and the one using a STBC. Indeed, more diversity orders are brought by the STBC.

Diversity-multiplexing gain tradeoff analysis

In order to compute the DMT of the proposed protocol, we study the asymptotic behavior of its outage probability. The system model can be written:

$$\mathbf{y}^{(S\times1)} = \mathbf{H}^{(S\times S)}\mathbf{x}^{(S\times1)} + \mathbf{z}^{(S\times1)}, \tag{4.38}$$

where $\mathbf{y}^{(S\times1)}$ contains the set of received signals, the equivalent channel matrix $\mathbf{H}^{(S\times S)}$ is diagonal and its diagonal elements are the channel products $g_{k_i}, i \in \{1, ..., S\}$ of the S selected paths, $\mathbf{x}^{(S\times1)}$ contains the set of coded symbols and $\mathbf{z}^{(S\times1)}$ is an array of colored noise since it contains all noises accumulated at each relaying node.

It was proven in [4, Theorem 2.3] that the DMT is the same if the noise is colored or white. So the noise can be considered as white in the scale of interest and the DMT of the proposed protocol is the same as the one of the new system model:

$$\widetilde{\mathbf{y}}^{(S\times1)} = \mathbf{H}^{(S\times S)}\mathbf{x}^{(S\times1)} + \mathbf{w}^{(S\times1)}, \tag{4.39}$$

where $\mathbf{w}^{(S\times1)}$ is an array of AWGN.

The outage probability of the new system is given by:

$$p_{out}(r\log\rho) = \Pr\left\{\frac{1}{S}\log\det\left(\mathbf{I} + \mathbf{H}\mathbf{H}^{\dagger}\right) \leq r\log\rho\right\}. \tag{4.40}$$

Replacing the channel matrix by its expression, we obtain:

$$p_{out}(r\log\rho) = \Pr\left\{\log\prod_{i=1}^{S}\left(1+|g_{k_i}|^2\right) \leq Sr\log\rho \,\middle|\, \forall(i,j)\in\{1,...,K\}^2, i>j, |g_{k_i}|^2 > |g_{k_j}|^2\right\}.$$

A straight upper bound to the outage probability is:

$$p_{out}(r\log\rho) \leq \Pr\left\{\log\prod_{i=1}^{S}|g_{k_i}|^2 \leq Sr\log\rho \,\middle|\, \forall i\in\{S+1,...,K\}, \log(|g_{k_i}|^2)^S \leq Sr\log\rho\right\}.$$

The channel paths are independent. Thus the previous upper bound can be rewritten as a product of probabilities:

$$p_{out}(r\log\rho) \leq \Pr\left\{\log\prod_{i=1}^{S}|g_{k_i}|^2 \leq Sr\log\rho\right\} \times \prod_{i=S+1}^{K}\Pr\left\{\log|g_{k_i}|^2 \leq r\log\rho\right\}.$$

Replacing the g_{k_i} by their expression (4.36), we obtain:

$$p_{out}(r\log\rho) \leq \Pr\left\{\log\prod_{i=1}^{S}\prod_{j=1}^{n}\rho|\beta_j^{k_i}h_j^{k_i}|^2 \leq Sr\log\rho\right\} \times \prod_{i=S+1}^{K}\Pr\left\{\log\prod_{j=1}^{n}\rho|\beta_j^{k_i}h_j^{k_i}|^2 \leq r\log\rho\right\}.$$

$\forall j \in \{1, ..., n-1\}$, we have the asymptotic behavior:

$$\rho|\beta_j^{k_i} h_j^{k_i}|^2 = \frac{\rho|h_j^{k_i}|^2}{1+\rho|h_j^{k_i}|^2} \doteq 1.$$

Thus, the asymptotic behavior of the outage probability can be upperbounded:

$$p_{out}(r\log\rho) \dot{\leq} \Pr\left\{\log\prod_{i=1}^{S}\rho|h_n^{k_i}|^2 \leq Sr\log\rho\right\} \times \prod_{i=S+1}^{K}\Pr\left\{\log\rho|h_n^{k_i}|^2 \leq r\log\rho\right\}$$

$$p_{out}(r\log\rho) \dot{\leq} \Pr\left\{\prod_{i=1}^{S}|h_n^{k_i}|^2 \leq \rho^{-S(1-r)}\right\} \times \prod_{i=S+1}^{K}\Pr\left\{|h_n^{k_i}|^2 \leq \rho^{-(1-r)}\right\}$$

$$p_{out}(r\log\rho) \dot{\leq} \rho^{-S(1-r)^+} \times \prod_{i=S+1}^{K}\rho^{-(1-r)^+}$$

$$p_{out}(r\log\rho) \dot{\leq} \rho^{-K(1-r)^+}.$$

So finally the DMT is lower-bounded by $K(1-r)$ which is the MISO upper-bound. Thus the DMT of the proposed protocol is optimal:

$$d^*(r) = K(1-r)^+. \tag{4.41}$$

Choice of S: influence of backflow interference

In [55], authors pointed out that two paths are necessary to achieve full rate in the half-duplex case. Indeed, because of the half-duplex constraint, nodes can transmit only half of the time. Thus, to achieve a rate of 1 symb. pcu, signals have to be sent on at least two paths alternatively.

In a wireless network, nodes usually ignore other nodes positions. Thus beam antennas cannot be used and the signals are broadcasted in all directions, and in particular, the signal is also sent backward to the previous node. The presence of these backflow interferences cannot be neglected. In [4, 5], authors show that these backflow interferences do not impair the DMT.

To see the influence of backflow on performance, we have simulated the 4-PP channel of Example 11 for two selected paths. In order to simplify implementation, we assume that backflow occurs once every two transmissions. Thus it is a lower bound for the case where backflow is happening at each transmission. We can remark in Figure 4.12 that in this case the 4-PP channel experiences a 5 dB loss for a outage probability of 10^{-5} compared to the case without backflow.

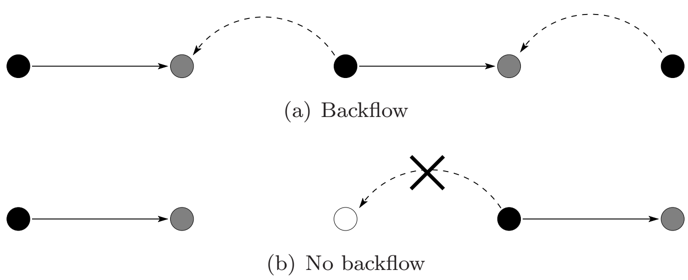


Figure 4.11: How to avoid backflow interferences: black nodes are transmitting, gray nodes are listening, and white nodes are inactive

Avoiding backflow interferences would then be a great improvement for the protocol. This can be done quite easily by leaving an inactive node (neither transmitting nor listening) between each listening and transmitting ones (see Figure 4.11). With this strategy, a node is transmitting only once in three time slot. In order to get full rate, at least three paths have then to be selected instead of two.

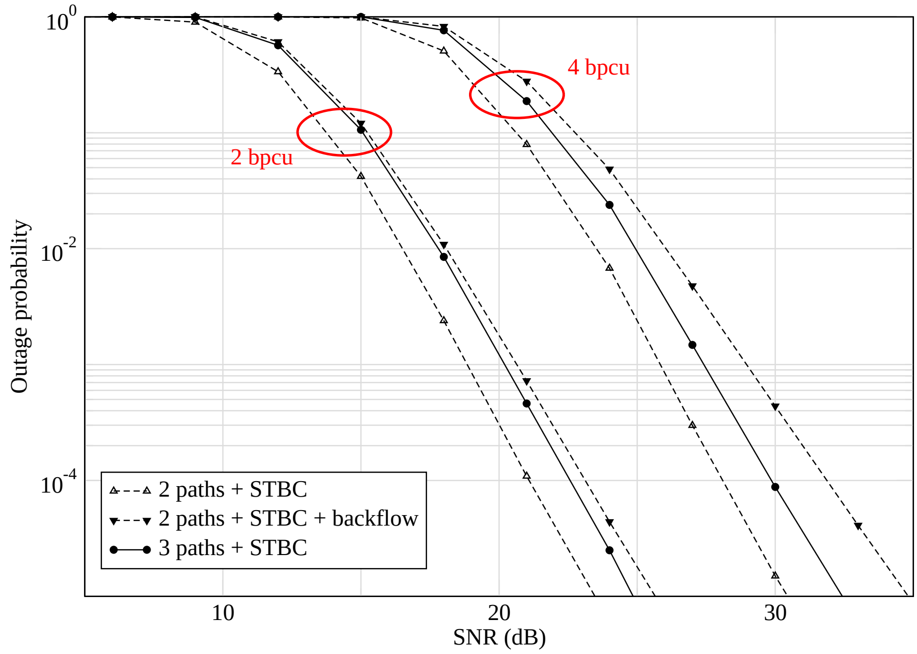


Figure 4.12: Influence of backflow on the outage probability of the path selection multihop protocol

Figure 4.12 shows that it is more efficient to use a 3 paths selection where no backflow can occur, rather than a 2 paths selection with backflow. The gain loss is twice smaller for a spectral efficiency of 4 bits pcu and is also reduced for 2 bits pcu.

It is proven in the previous section that selecting $S > 3$ paths also achieves full diversity of K. However adding some more paths decreases performance. Thus we can limit the number of selected paths to $S = 3$.

Remark 3. *For a two-hop network ($n = 2$, one relay per path), there is no possible backflow. In this case, two paths are sufficient.*

Remark 4. *Even in the full-duplex mode, three paths are necessary to avoid backflow interferences. The same protocol has to be used in both full and half-duplex modes.*

4.2.4 Generalization to the decode-and-forward strategy

We have seen in subsection 2.2.2 that a DF protocol can be used only if relaying nodes are able to correctly decode the signals. In previous chapters, when all source-relay links were in outage, non-cooperation was used, i.e. signals were sent through the source-destination link (SISO).

However, in the KPP channel, source and destination are considered too far away and there is no direct link. Thus we propose to use a selection between DF and AF strategies: a relaying node decodes the signal only when it is able to according to the outage criterion, i.e. when the equivalent channel at this node is not in outage.

By studying the transmission on a path, we can provide the following lemma.

Lemma 1. *If a relaying node is not able to decode, the following nodes on the same path (except for the destination) will not be neither.*

Proof. Let y_i^k be the received signal at relay R_i^k:

$$y_i^k = f_i^k x_k + w_i^k,$$

where f_i^k is the equivalent channel, x_k is the sent signal and w_i^k is AWGN.

Suppose relay R_i^k is not able to decode the signal, i.e. the equivalent channel is in outage:

$$\log(1 + |f_i^k|^2) < R,$$

with R the spectral efficiency of the protocol.

An AF strategy is then used and the received signal at relay R_{i+1}^k is:

$$\begin{aligned} y_{i+1}^k &= \sqrt{\rho} h_i^k \beta_i^k y_i^k + w_{i+1}^k \\ &= \sqrt{\rho} h_i^k \beta_i^k f_i^k x_k + (\sqrt{\rho} h_i^k \beta_i^k w_i^k + w_{i+1}^k), \end{aligned}$$

where the amplifying factor is chosen so that the power is normalized: $\beta_i^k = \frac{1}{\sqrt{1+|f_i^k|^2}}$.

In order to determine if it can decode, we compute its instantaneous capacity:

$$\log\left(1 + \frac{\rho |h_i^k|^2 (\beta_i^k)^2}{1 + \rho |h_i^k|^2 (\beta_i^k)^2} |f_i^k|^2\right) < \log(1 + |f_i^k|^2) < R.$$

The equivalent channel at relay R_{i+1}^k is also in outage, thus this relay is not able to decode the signal neither. □

Consequently, the selection criterion does not have to be checked at each node and the strategy can be simplified:

- as long as decoding at relays is possible, a DF strategy is used;
- once a relay is not able to decode, an AF strategy is used till the end.

This way, noise accumulated during the transmission is reduced and performance is improved.

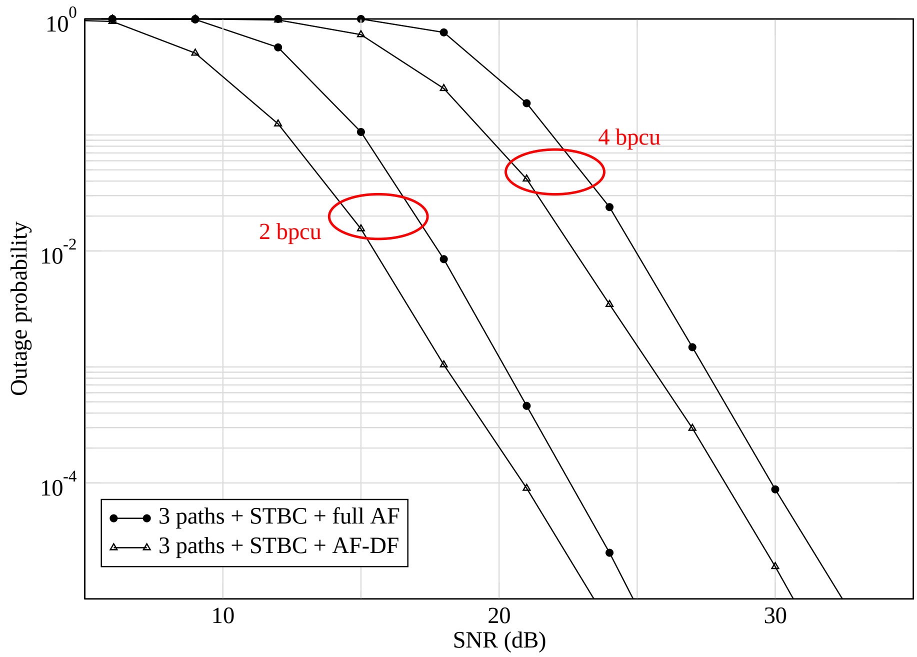


Figure 4.13: Outage probability of the path selection multihop protocol using a combination of DF and AF strategies

Figure 4.13 represents the outage probability of this DF-AF strategy compared to the full AF strategy. As expected, decoding and reencoding the signals at relaying nodes induces an improvement of performance. We can observe a 2 dB gain for both 2 and 4 bits pcu spectral efficiencies.

The selection does not change the asymptotic analysis and the DMT of this DF-AF strategy is the same as the one of the full AF strategy:

$$d^*(r) = K(1-r)^+. \tag{4.42}$$

4.2.5 Generalization to different path lengths

In previous sections, we have considered that all the paths have the same length n. In practical systems, this is not always the case.

The proposed protocol can be easily generalized to the case where the path lengths are equal modulo $S = 3$. Let $n - 3q_i$ be the length of path $k_i, i \in \{1, 2, 3\}$. For each path k_i, the transmission is delayed by $3q_i$ time slots to compensate the delay due to the longest path.

If path lengths are not equal modulo 3, a delay has to be created in the shorter paths. Without loss of generality, we can assume that the difference between the lengths of the different paths is lower than 3. The delay cannot be introduced by the source, because it is already transmitting at each time slot. Thus this has to be done at relays. As a node is transmitting once in 3 time slots, a relay cannot add more than one time slot of delay.

Example 12. *On Figure 4.14 is represented the transmission for a 3PP network with path lengths $n = n_1 = 5$, $n_2 = 4$ and $n_3 = 3$.*

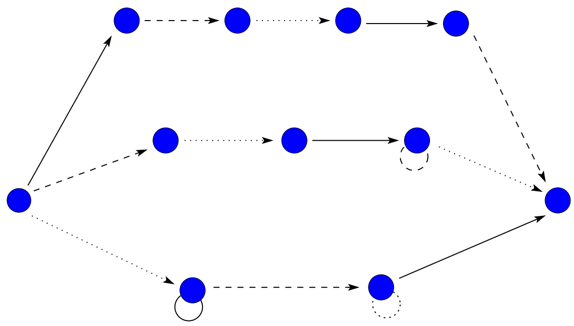


Figure 4.14: Transmission in a KPP network with different path lengths: continuous, dashed and dotted arrows represent transmissions on time slots 3T, 3T+1 and 3T+2 respectively.

4.2.6 Implementation issues

How long should the frame be?

The full rate of 1 symb. pcu is a theoretical rate that is reached when the frame length is infinite, which is obviously not the case in practical applications. The achievable rates are of the form $\frac{T}{n+S(T/S-1)+S-1} = \frac{T}{n+T-1}$ where T is the number of symbols in a frame. $n + T - 1$ is then the transmission length for $S = 3$.

Figure 4.15 shows the influence of rate on the performance for a spectral efficiency of 4 bits pcu. The network of Example 11 is once again considered ($n = 4$). Three paths are selected and a space-time code is used to achieve full diversity. One can see, that frames of at least 30 symbols have to be sent in order to limit the loss compared to the theoretical full rate to less than 1 dB.

However, transmitting a longer frame does not increase the decoding complexity. Indeed, the equivalent MIMO channel is a diagonal matrix. Thus the message can be decoded every three slots and an STBC of dimension 3×1 is sufficient to achieve full diversity. Only some coding gain would be loss.

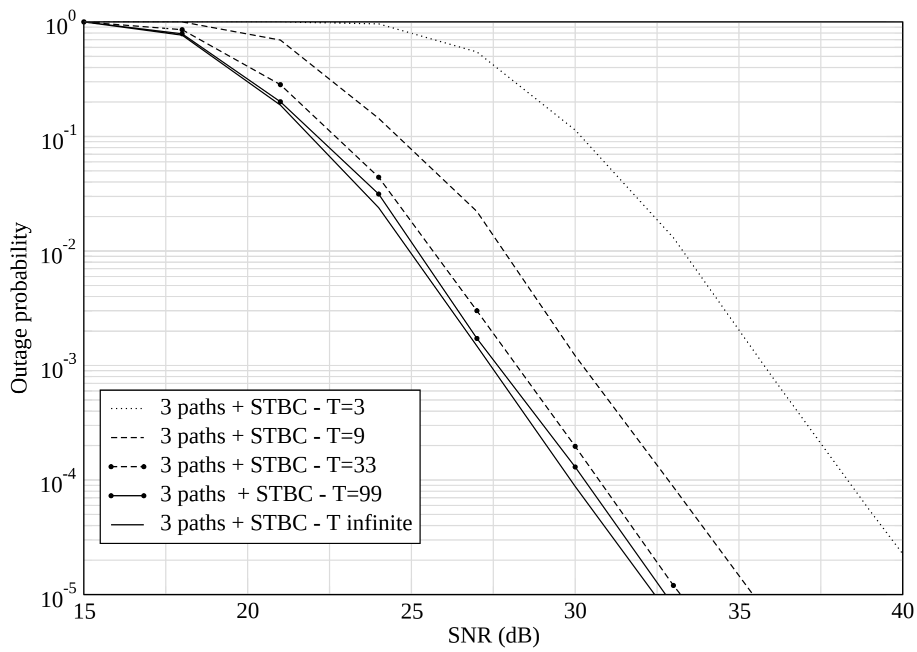


Figure 4.15: Outage probability of the path selection multihop protocol for different rate/frame length: T is the number of symbols to sent, thus the rate is $\frac{T}{n+T-1}$.

Discussion on path interferences

The DMT is proven in [4, 5] to be optimal even in presence of interference between paths. However we can show by simulation that these interferences have a strong influence on the performance.

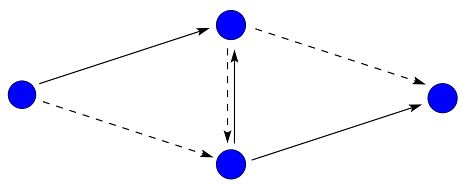


Figure 4.16: Two-hop two-relay single-antenna channel model

Let's consider the simple example of a two-hop two-relay single-antenna network, also known as the diamond channel [58, 59] (see Figure 4.16). We assume that the two paths are not isolated: there are interferences between the two relays. However, as the relaying nodes are smartly chosen so that the interferences are limited, the SNR between the two relays is lower than between the nodes of a same path.

In Figure 4.17 are represented the performance with different interference powers and a spectral efficiency of 4 bits pcu. If interferences have the same power than the signal, we observe a 3 dB loss compared to the ideal case where paths are isolated. If the power of

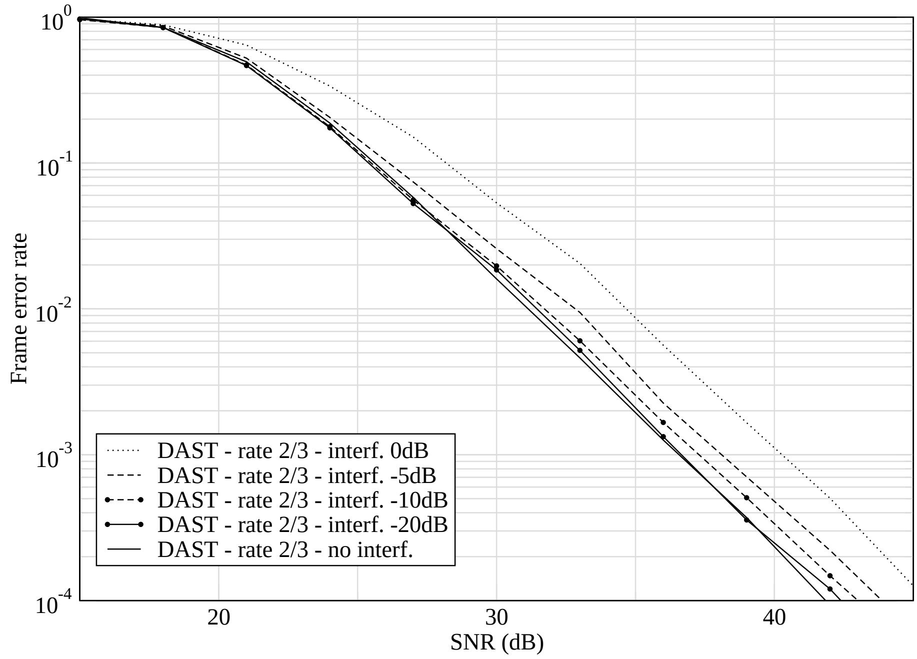


Figure 4.17: Influence of path interference on the performance of the path selection multihop protocol

the interference is 5 dB lower than the signal, this loss is reduced to 2 dB. Finally, if the power of the interference is 20 dB lower than the signal, the performance are very close to the perfect case, so paths can be considered as isolated.

This case can occur if there is an obstacle between the two relays. In an indoor environment, the two relays could be set in different rooms for example. In an outdoor environment, the two relays could be separated by a building.

4.3 Conclusion of the chapter

In this chapter we proposed a practical implementation and improvements of the CMA-NAF protocol for a cooperative network with two sources and one destination. A new CMA protocol using a DF strategy has also been proposed. Their theoretical and practical performance in terms of outage probability and simulation results show that they are more efficient than the NAF and IDF protocols using TDMA strategy. Moreover, the diversity-multiplexing gain tradeoff of the improved CMA-NAF protocol is proven to be better than that of the original CMA-NAF as it is closer to the MISO bound. The DMT of the CMA-IDF however is the same as that of the IDF using TDMA strategy.

In a second part, we provide a simple and low-complexity protocol for the KPP multihop network. All paths are first considered to have the same length. The three best paths are selected considering their SNR to destination. Symbols are coded with a 3×1 STBC and sent successively on the three selected paths. The choice of three paths allows to considerably improve the performance by avoiding backflow interferences. This strategy is proven to achieve full rate of 1 symb. pcu, as well as full diversity K. Moreover, this protocol reaches the optimum DMT $d^*(r) = K(1-r)^+$. A generalization of this protocol is provided for the case where path lengths are different.

This kind of protocol is of particular interest in sensor networks. However, in such a context, several nodes want to transmit their information. Therefore, in future work, we would like to combine these two generalizations of the relay channel in order to provide a multi-access multi-hop protocol adapted to applications in a sensor network.

Conclusion and future works

In this thesis, we have proposed amplify-and-forward (AF) and decode-and-forward protocols (DF) for various wireless channels.

A study of the behavior of cooperative protocols for different relay location and power allocation has shown that performance at low SNR are always deceiving, since non-cooperation gives better results. In order to solve this problem, we proposed a new adaptive strategy that can be applied either to AF or DF protocols. The idea is to compare the instantaneous capacities of all possible transmission schemes including cooperation, non-cooperation and serial relays. This adaptive strategy is proven to optimize the performance of the considered cooperative protocols in terms of outage probability and simulation results, without affecting the DMT.

Considering the relay channel, the AF strategy was already extensively studied and several protocols were proposed to achieve the theoretical limits of this channel. However, this is not the case of the DF strategy whose complexity is higher due to decoding at relays. In this thesis, we have proposed DF protocols beginning with the Alamouti DF. However this protocol is limited to a rate of $\frac{1}{2}$ symbol per channel use (symb. pcu) and to the use of a single relay. We then proposed a second protocol using more sophisticated STBC, called the Asymmetric DF since its two phases do not have the same lengths. Unfortunately this protocol is also limited in rate to $\frac{2}{3}$ symb. pcu. Finally, we proposed a last DF protocol, called the Incomplete DF, based on an incomplete decoding at relays. This protocol achieves full rate and full diversity and provides the same DMT as the NAF protocol. Since exhaustive decoding at relays brings a high complexity, two more methods are proposed in order to reduce this complexity: one is based on the structure of TAST code and the other one on diophantine approximation. This second method is sub-optimal but simulation results show that the loss in performance is less than 1 dB while the complexity is considerably reduced. This two decoding methods can be combined to obtain an even lower complexity.

When several sources need to transmit simultaneously to the same destination, a cooperative multiple-access (CMA) strategy is considered. In this thesis, we proposed a practical implementation as well as two modifications of the CMA-NAF protocol, which improve both the DMT and the performance in terms of outage probability and simulation results. We also proposed a DF version of the CMA protocol inspired by the previously presented IDF. Its DMT is limited because of the initial selection with non-cooperation which is necessary for DF protocols, but its performance are similar to the AF case.

Finally, we have been interested in the K-Parallel-Path (KPP) multihop network. We proposed a low-complexity protocol based on a combination of path selection and a small STBC, which achieves the theoretical limits of the channel. This protocol can be used either with an AF strategy or a combination of DF and AF strategies in order to improve performance especially when long paths are considered.

Some possible directions for future works are:

Diophantine approximation for MIMO decoding. In this thesis, diophantine approximation algorithms have been used with success to decode a rank deficient system at relay while limiting the complexity. These algorithms could be used in other systems in presence of a rank deficiency (the MAC channel for example), or even adapted to the decoding of MIMO systems, to provide a new decoder with good (but not optimal) performance and a low complexity.

Multiuser cooperation. Most cooperative protocols have been designed for a system with single source and single destination. However, in a sensor network, multiple nodes wants to transmit their data either to a central node/destination or to all other nodes in the network. We have already considered the case of two sources transmitting simultaneously to the same destination. It would be interesting to generalize this work to a higher number of sources, and to study the reverse channel, i.e. the cooperative broadcast channel.

Bad or nonexistent channel state information (CSI) at destination. All along this thesis and in most existing studies, a perfect CSI is assumed at destination. However, this is an ideal assumption, and when estimating the channel, errors can occur. Another interesting perspective would be the study of the influence of channel estimation errors at destination on the performance of cooperative protocols, and the development of techniques to mitigate this effect.

We can also consider a network where no CSI is available at destination. We then have to use and study the performance of distributed non-coherent STBCs and non-coherent decoding when applied to the protocols proposed in this thesis.

Wireless Network coding. Network coding is a promising generalization of routing. Intermediate nodes in the network not only forward but also encode the data. They can combine several received signals in a way that they are easily decodable. Thus they bring more information and improve the data rate. Increasing the throughput is not the only advantage of network coding: this new strategy also better shares resources and thus saves power and bandwidth. Moreover it is more robust in a dynamic system such as a wireless network.

Network coding can be used either in a wireline or a wireless environment. Nevertheless wireless networks seem a more natural setting for network coding. Indeed, the particular properties of wireless communications that complicate routing (fading and broadcast nature) are solved by coding. Moreover, protocols in a wireless network are subject to less constraints.

Chapter A

Appendix

A.1 Preliminaries

Definition 10. *Let g follow a Rayleigh distribution. The exponential order of $\frac{1}{|g|^2}$ is*

$$u = -\lim_{\rho\to\infty} \frac{\log |g|^2}{\log \rho}.$$

We can note $|g|^2 \doteq \rho^{-u}$ where the notation $\doteq$ denotes an asymptotic behavior when $\rho \to \infty$.

Lemma 2. *The probability density function of u is*

$$p_u = \lim_{\rho\to\infty} \log(\rho)\rho^{-u} \exp(-\rho^{-u}),$$

which satisfies

$$p_u \doteq \begin{cases} \rho^{-\infty}, & \text{for } u < 0 \\ \rho^{-u}, & \text{for } u \geq 0 \end{cases}.$$

Lemma 3. *Let $\mathcal{O}$ be a certain set and $p_{\mathcal{O}} = \Pr\{(u_1, \ldots, u_N) \in \mathcal{O}\}$, then*

$$p_{\mathcal{O}} \doteq \rho^{-d} \text{ with } d = \inf_{(u_1,\ldots,u_N)\in\mathcal{O}^+} \sum_{j=1}^{N} u_j$$

where $\mathcal{O}^+ = \mathcal{O} \cap \mathbb{R}^{N+}$.

Proof. The proof is drawn in [36, Lemma 2]. □

A.2 Proof of Theorem 1

In order to compute the outage probability of the Asymmetric DF protocol, different cases have to be distinguished depending on the number of relays whose link with source is in outage. As the outage events of the source-relay links are independent, the probability of having only the last $N - N_u$ relays in outage, is the product of the probabilities of each N_u first source-relay links not being in outage, and each of the last $N - N_u$ source-relay links being in outage. These outage events are defined in equation (3.8). So the outage probability of the last $N - N_u$ relays only can be written

$$p_{\mathcal{O},N-N_u} = \prod_{i=1}^{N_u} \Pr\left\{\log\left(1+\rho|h_i|^2\right) > \frac{3}{2}R\right\} \prod_{i=N_u+1}^{N} \Pr\left\{\log\left(1+\rho|h_i|^2\right) < \frac{3}{2}R\right\}. \quad \text{(A.1)}$$

When $N_u \geq 1$ source-relay links are not in outage ($N - N_u$ relays are in outage) , the Asymmetric DF cooperation scheme with N_u relays is used. The equivalent channel matrix in (3.6) is block diagonal, thus we can write:

$$\det(\mathbf{I} + \rho\mathbf{H}\mathbf{H}^\dagger) = \prod_{i=1}^{N_u} \det(\mathbf{I} + \rho\mathbf{H}_i\mathbf{H}_i^\dagger)$$

and using the expression of $\mathbf{H}_i$ in (3.7), the outage probability becomes:

$$p_{out,N_u} = \Pr\left\{\frac{1}{3N_u}\log\prod_{i=1}^{N_u}\left(1+\frac{5}{2}\rho|g_0|^2+\frac{1}{2}\rho|g_i|^2+\frac{3}{2}\rho^2|g_0|^4+\frac{1}{2}\rho^2|g_0|^2|g_i|^2\right) < R\right\}. \quad \text{(A.2)}$$

When all source-relay links are in outage, we use the non-cooperative scheme, whose outage probability is

$$p_{out,0} = \Pr\left\{\log\left(1+\rho|g_0|^2\right) < R\right\}. \quad \text{(A.3)}$$

Finally, as there are $\binom{N}{N_u}$ possible combinations of N_u relays in N, we can write in the general case

$$p_{out} = \sum_{N_u=0}^{N}\binom{N}{N_u} p_{out,N_u} p_{\mathcal{O},N-N_u}. \quad \text{(A.4)}$$

A.3 Proof of Theorem 2

The outage probability of the Asymmetric DF is given in Theorem 1. In order to compute the DMT of this cooperative strategy, we have to study the asymptotic behavior of this expression when ρ grows to infinity.

Let u_0, u_n and v_n, $n \in \{1, \ldots, N\}$ be the exponential orders of $\frac{1}{|g_0|^2}$, $\frac{1}{|g_n|^2}$ and $\frac{1}{|h_n|^2}$ respectively.

First, we have to determine how many relays can be used. Asymptotically, the probability that N_u relays are not in outage becomes:

$$\begin{aligned} p_{\mathcal{O},N-N_u} &\doteq \prod_{i=1}^{N_u} \Pr\left\{\log(1+\rho^{1-u_i}) > \frac{3}{2}r\log\rho\right\} \prod_{i=N_u+1}^{N} \Pr\left\{\log(1+\rho^{1-u_i}) < \frac{3}{2}r\log\rho\right\} \\ &\doteq 1 \times \prod_{i=N_u+1}^{N} \Pr\left\{\max(0, 1-u_i) < \frac{3}{2}r\right\}, \end{aligned}$$

so the corresponding diversity-multiplexing gain tradeoff is

$$d_{\mathcal{O},N-N_u}(r) = (N-N_u)\left(1-\frac{3}{2}r\right)^+.$$

If N_u relays are selected, cooperation is used and p_{out,N_u} asymptotically becomes

$$\begin{aligned} p_{out,N_u} &\doteq \Pr\left\{\sum_{i=1}^{N_u}\log\left(1+\frac{5}{2}\rho^{1-v_0}+\frac{1}{2}\rho^{1-v_i}+\frac{3}{2}\rho^{2-2v_0}+\frac{1}{2}\rho^{2-v_0-v_i}\right) < 3N_u r\log\rho\right\} \\ &\doteq \Pr\left\{\sum_{i=1}^{N_u}\max\left(0, 1-v_0, 1-v_i, 2-2v_0, 2-v_o-v_i\right) < 3N_u r\right\}. \end{aligned}$$

In particular

$$\sum_{i=1}^{N_u} 2(1-v_0) < 3N_u r$$

provides the lower bound

$$1-\frac{3}{2}r < v_0$$

and

$$\sum_{i=1}^{N_u}(2-v_0-v_i) < 3N_u r$$

leads to

$$\begin{aligned} 2N_u - N_u v_0 - \sum_{i=1}^{N_u} v_i &< 3N_u r, \\ 2N_u - (N_u-1)v_0 - 3N_u r &< v_o + \sum_{i=1}^{N_u} v_i, \\ N_u + 1 - 3N_u r = (N_u+1)\left(1 - 3\frac{N_u}{N_u+1}r\right) &< v_o + \sum_{i=1}^{N_u} v_i. \end{aligned}$$

Finally, we obtain the diversity-multiplexing gain tradeoff

$$\begin{aligned} d_{out,N_u}(r) &= \inf\left(v_0 + \sum_{i=1}^{N_u} v_i\right) = \max\left(1 - \frac{3}{2}r, (N_u+1)\left(1 - 3\frac{N_u}{N_u+1}r\right)\right) \\ &= \left(1-\frac{3}{2}r\right) + N_u\left(1 - 3\frac{2N_u - 1}{2N_u}r\right)^+ . \end{aligned}$$

In the case of all source-relay links being in outage, the probability of the direct transmission asymtotically becomes

$$\begin{aligned} p_{out,0} &\doteq \Pr\left\{\log(1+\rho^{1-v_0}) < r\log\rho\right\} \\ &\doteq \Pr\left\{\max(0, 1-v_0) < r\right\} \end{aligned}$$

and the corresponding diversity-multiplexing gain tradeoff is

$$d_{out,0}(r) = (1-r)^+.$$

Finally we can write

$$p_{out} \doteq \sum_{N_u=0}^{N} C_{N_u}^{N} \rho^{-d_{out,N_u}(r)} \rho^{-d_{\mathcal{O},N-N_u}(r)}$$

so the total diversity-multiplexing gain tradeoff is

$$d(r) = \min_{N_u \in \{0,\dots,N\}} \left(d_{out,N_u}(r) + d_{\mathcal{O},N-N_u}(r)\right) = \left(1 - \frac{3}{2}r\right) + N\left(1 - 3\frac{2N-1}{2N}r\right)^+ . \quad \text{(A.5)}$$

A.4 Proof of Theorem 4

The outage probability of the Incomplete DF is given in Theorem 3. In order to compute the DMT of this cooperative strategy, we have to study the asymptotic behavior of this expression when ρ grows to infinity.

Let u_0, u_n and v_n, $n \in \{1,\dots,N\}$ be the exponential orders of $\frac{1}{|g_0|^2}$, $\frac{1}{|g_n|^2}$ and $\frac{1}{|h_n|^2}$ respectively.

The probability that we can use N_u relays to cooperate with the source asymptotically is:

$$\begin{aligned} p_{\mathcal{O},N-N_u} &\doteq \prod_{i=1}^{N_u} \Pr\left\{\log(1+\rho^{1-u_i}) > 2r\log\rho\right\} \prod_{i=N_u+1}^{N} \Pr\left\{\log(1+\rho^{1-u_i}) < 2r\log\rho\right\} \\ &\doteq 1 \times \prod_{i=N_u+1}^{N} \Pr\left\{\max(0, 1-u_i) < 2r\right\}, \end{aligned}$$

so the diversity-multiplexing gain tradeoff is

$$d_{\mathcal{O},N-N_u}(r) = (N-N_u)(1-2r)^+.$$

In the case of cooperation with N_u relays, the outage probability tends to:

$$\begin{aligned} p_{out,N_u} &\doteq \Pr\left\{\sum_{i=1}^{N_u}\log\left(1+\frac{3}{2}\rho^{1-v_0}+\frac{1}{2}\rho^{1-v_i}+\frac{1}{2}\rho^{2-2v_0}\right) < 2N_u r\log\rho\right\} \\ &\doteq \Pr\left\{\sum_{i=1}^{N_u}\max\left(0,1-v_0,1-v_i,2-2v_0\right) < 2N_u r\right\}. \end{aligned}$$

The inequality

$$\sum_{i=1}^{N_u}(1-v_i) < 2N_u r$$

gives

$$N_u(1-2r) < \sum_{i=1}^{N_u} v_i$$

and the inequality

$$\sum_{i=1}^{N_u}(2-2v_0) < 2N_u r$$

provides

$$1-r < v_0$$

Finally we obtain the diversity-multiplexing gain tradeoff

$$d_{out,N_u}(r) = \inf\left(v_0+\sum_{i=1}^{N_u} v_i\right) = (1-r)^+ + N_u(1-2r)^+.$$

In the case of all source-relay links being in outage, the diversity-multiplexing gain tradeoff corresponding to direct transmission is:

$$d_{out,0}(r) = (1-r)^+.$$

Finally we can write:

$$p_{out} \doteq \sum_{N_u=0}^{N} C_{N_u}^{N}\rho^{-d_{out,N_u}(r)}\rho^{-d_{\mathcal{O},N-N_u}(r)}$$

so the total diversity-multiplexing gain tradeoff is

$$d(r) = \min_{N_u\in\{0,\dots,N\}}\left(d_{out,N_u}(r)+d_{\mathcal{O},N-N_u}(r)\right) = (1-r)^+ + N(1-2r)^+. \quad \text{(A.6)}$$

A.5 Proof of Theorem 5

In order to compute the DMT of the whole system, we have to consider the three DMTs d_1, d_2 and $d_{1,2}$ of the three outage events $\mathcal{O}_1$, $\mathcal{O}_2$ and $\mathcal{O}_{1,2}$ defined in equations (4.21), (4.22) and (4.23) respectively.

DMT of the outage events $\mathcal{O}_1$ and $\mathcal{O}_2$

As the protocol is symmetric, the outage events $\mathcal{O}_1$ and $\mathcal{O}_2$ obviously have the same DMT. Thus, in the following, only the DMT of $\mathcal{O}_1$ is derived.

The equivalent channel $\mathbf{H}_{1,k}$ for user U_1 and a subframe of size $2k$ is defined as

$$\mathbf{H}_{1,k} = \begin{bmatrix} \mathbf{M}_{1,k} & \mathbf{0}_{2k\times k} \\ \mathbf{0}_{2k\times k} & \mathbf{M}_{2,k} \end{bmatrix}.$$

Thus $D_k = \det\left(\mathbf{I}_{2k} + \rho \mathbf{H}_{1,k}\mathbf{H}_{1,k}^\dagger\right) = \det\left(\mathbf{I}_k + \rho \mathbf{M}_{1,k}\mathbf{M}_{1,k}^\dagger\right)\det\left(\mathbf{I}_k + \rho \mathbf{M}_{2,k}\mathbf{M}_{2,k}^\dagger\right) = D_{1,k}D_{2,k}$ and the problem can be divided in two parts.

We can notice that

$$\mathbf{M}_{1,k} = \left[\begin{array}{c|c} \mathbf{M}_{1,k-1} & \mathbf{0}_{2(k-1)\times 1} \\ \hline \mathbf{0}_{2\times(k-1)} & \begin{array}{c} \frac{1}{\sqrt{P_{2k-1}}} a g_1 \\ \sqrt{\frac{\rho}{P_{2k}}} a b \beta_{2k-1} h g_2 \end{array} \end{array}\right],$$

thus

$$\mathbf{M}_{1,k}\mathbf{M}_{1,k}^\dagger = \left[\begin{array}{c|c} \mathbf{M}_{1,k-1}\mathbf{M}_{1,k-1}^\dagger & \mathbf{0}_{2(k-1)\times 2} \\ \hline \mathbf{0}_{2\times 2(k-1)} & \begin{array}{cc} \frac{1}{P_{2k-1}} a^2 |g_1|^2 & \sqrt{\frac{\rho}{P_{2k-1}P_{2k}}} a^2 b \beta_{2k-1} g_1 h^* g_2^* \\ \sqrt{\frac{\rho}{P_{2k-1}P_{2k}}} a^2 b \beta_{2k-1} g_1^* h g_2 & \frac{\rho}{P_{2k}} a^2 b^2 \beta_{2k-1}^2 |h|^2 |g_2|^2 \end{array} \end{array}\right],$$

and

$$\begin{aligned} D_{1,k} &= D_{1,k-1}\left(\left(1+\rho\frac{1}{P_{2k-1}}a^2|g_1|^2\right)\left(1+\rho^2\frac{1}{P_{2k}}a^2b^2\beta_{2k-1}^2|h|^2|g_2|^2\right)\right. \\ &\qquad\qquad \left. -\rho^3\frac{1}{P_{2k-1}P_{2k}}a^4b^2\beta_{2k-1}^2|g_1|^2|h|^2|g_2|^2\right) \\ &= D_{1,k-1}\left(1+\rho\left(\frac{1}{P_{2k-1}}a^2|g_1|^2 + \frac{\rho}{P_{2k}}a^2b^2\beta_{2k-1}^2|h|^2|g_2|^2\right)\right). \end{aligned}$$

As $D_{1,1} = \left(1+\rho\left(|g_1|^2 + \frac{\rho}{P_2}b^2\beta_1^2|h|^2|g_2|^2\right)\right)$, we obtain

$$D_{1,k} = \left(1+\rho\left(|g_1|^2 + \frac{\rho}{P_2}b^2\beta_1^2|h|^2|g_2|^2\right)\right)\prod_{j=2}^{k}\left(1+\rho\left(\frac{1}{P_{2j-1}}a^2|g_1|^2 + \frac{\rho}{P_{2j}}a^2b^2\beta_{2j-1}^2|h|^2|g_2|^2\right)\right) \tag{A.7}$$

We can notice that

$$\mathbf{M}_{2,k} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\mathbf{M}_{2,k-1}} & \mathbf{0}_{2(k-1)\times 1} \\ \hline \mathbf{0}_{2\times(k-2)} & \begin{array}{c} \sqrt{\frac{\rho}{P_{2k-1}}} ab\beta_{2k-2}hg_2 \\ 0 \end{array} & \begin{array}{c} 0 \\ \frac{1}{\sqrt{P_{2k}}} ag_1 \end{array} \end{array}\right],$$

thus

$$\mathbf{M}_{2,k}\mathbf{M}_{2,k}^{\dagger} = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{\mathbf{M}_{2,k-1}\mathbf{M}_{2,k-1}^{\dagger}} & \multicolumn{2}{c}{\begin{array}{cc} \multicolumn{2}{c}{\mathbf{0}_{(\mathbf{2k-3})\times \mathbf{2}}} \\ \sqrt{\frac{\rho}{P_{2k-1}P_{2k-1}}} a^2 b\beta_{2k-2} g_1^* h g_2 & 0 \end{array}} \\ \hline \mathbf{0}_{2\times(2k-3)} & \begin{array}{c} \sqrt{\frac{\rho}{P_{2k-1}P_{2k-1}}} a^2 b\beta_{2k-2} g_1 h^* g_2^* \\ 0 \end{array} & \begin{array}{c} \frac{\rho}{P_{2k-1}} a^2 b^2 \beta_{2k-2}^2 |h|^2 |g_2|^2 \\ 0 \end{array} & \begin{array}{c} 0 \\ \frac{1}{P_{2k}} a^2 |g_1|^2 \end{array} \end{array}\right],$$

and again

$$D_{2,k} = D_{2,k-1} \frac{1+\rho\frac{1}{P_{2k}}a^2|g_1|^2}{1+\rho\frac{1}{P_{2k-2}}a^2|g_1|^2} \left(1+\rho\left(\frac{1}{P_{2k-2}}a^2|g_1|^2 + \frac{\rho}{P_{2k-1}}a^2b^2\beta_{2k-2}^2|h|^2|g_2|^2\right)\right).$$

As $D_{2,1} = \left(1+\rho\frac{1}{P_2}a^2|g_1|^2\right)$, we obtain

$$D_{2,k} = \left(1+\rho\frac{1}{P_2}a^2|g_1|^2\right) \prod_{j=2}^{k} \frac{1+\rho\frac{1}{P_{2j}}a^2|g_1|^2}{1+\rho\frac{1}{P_{2j-2}}a^2|g_1|^2} \left(1+\rho\left(\frac{1}{P_{2j-2}}a^2|g_1|^2 + \frac{\rho}{P_{2j-1}}a^2b^2\beta_{2j-2}^2|h|^2|g_2|^2\right)\right)$$

$$D_{2,k} = \left(1+\rho\frac{1}{P_{2k}}a^2|g_1|^2\right) \prod_{j=2}^{k} \left(1+\rho\left(\frac{1}{P_{2j-2}}a^2|g_1|^2 + \frac{\rho}{P_{2j-1}}a^2b^2\beta_{2j-2}^2|h|^2|g_2|^2\right)\right) \quad \text{(A.8)}$$

Finally

$$\begin{aligned} D_k = \; & \left(1+\rho\left(|g_1|^2 + \frac{\rho}{P_2}b^2\beta_1^2|h|^2|g_2|^2\right)\right)\left(1+\rho\frac{1}{P_{2k}}a^2|g_1|^2\right) \\ & \prod_{j=3}^{2k}\left(1+\rho\left(\frac{1}{P_{j-1}}a^2|g_1|^2 + \frac{\rho}{P_j}a^2b^2\beta_{j-1}^2|h|^2|g_2|^2\right)\right) \end{aligned} \quad \text{(A.9)}$$

Studying the asymptotic behavior of D_k when the SNR grows to infinity, we can write:

$$D_k \doteq 1 + \rho^{2k}\left(\prod_{j=2}^{2k}\frac{1}{P_j}\right)|g_1|^{2\times 2k} + \rho^{2(2k-1)+1}\left(\prod_{j=2}^{2k}\frac{\beta_{j-1}^2}{P_j}\right)\frac{1}{P_{2k}}|g_1|^2|h|^{2(2k-1)}|g_2|^{2(2k-1)}.$$

We can easily show that $P_j \geq 1$ and $\beta_j^2 \leq \frac{1}{1+\rho a^2|h|^2}$ thus

$$D_k \dot{\leq} 1 + \rho^{2k}|g_1|^{2\times 2k} + \rho^{2(2k-1)+1}\left(\frac{1}{1+\rho|h|^2}\right)^{2k-1}|g_1|^2|h|^{2(2k-1)}|g_2|^{2(2k-1)}.$$

Outage probability P_1 can then be lower bounded by

$$\begin{aligned}
P_1 &= \Pr\{\log D_k \leq 2kR\} \\
&\dot{\geq} \Pr\left\{\log\left(1+\rho^{2k}|g_1|^{2\times 2k}+\rho^{2(2k-1)+1}\left(\frac{1}{1+\rho|h|^2}\right)^{2k-1}|g_1|^2|h|^{2(2k-1)}|g_2|^{2(2k-1)}\right)\leq 2kr\log\rho\right\} \\
&\dot{\geq} \Pr\left\{\log(1+\rho^{2k(1-v_1)}+\rho^{(1-v_1)+(2k-1)(1-v_2)+(2k-1)(1-u)-(2k-1)(1-u)^+})\leq 2kr\log\rho\right\} \\
&\dot{\geq} \Pr\left\{\log(1+\rho^{2k(1-v_1)}+\rho^{(1-v_1)+(2k-1)(1-v_2)})\leq 2kr\log\rho\right\} \\
&\dot{\geq} \Pr\{\max(0,2k(1-u_1),(1-u_1)+(2k-1)(1-u_2))\leq 2kr\}
\end{aligned}$$

so the DMT is upper bounded by

$$d_1 = \inf(u_1+u_2) \leq (1-r)^+ + \left(1-\frac{2k}{2k-1}r\right)^+ \tag{A.10}$$

To prove the achievability, we consider a network where two sources are so close to each other, that $|h|^2 > |g_1|^2$ and $|h|^2 > |g_2|^2$.

Then we can upper bound the power of the signal

$$\begin{aligned}
P_{2k-1} &= 1+\rho b^2\beta_{2k-2}^2|g_1|^2P_{w,2k-2} \\
&\leq 1+\rho b^2\beta_{2k-2}^2|h|^2P_{w,2k-2} \leq P_{w,2k-1}
\end{aligned}$$

$$\begin{aligned}
P_{w,2k-1} &= 1+\rho b^2\beta_{2k-2}^2|h|^2P_{w,2k-2} \\
&= 1+\rho b^2\frac{1}{P_{w,2k-2}+\rho a^2|h|^2}|h|^2P_{w,2k-2} \\
&\leq 1+\rho b^2\frac{1}{\rho a^2|h|^2}|h|^2P_{w,2k-2} \\
&\leq \sum_{j=0}^{2k-2}\left(\frac{b^2}{a^2}\right)^j
\end{aligned}$$

so $P_{2k-1} \leq \sum_{j=0}^{2k-2}\left(\frac{b^2}{a^2}\right)^j$ and the same way $P_{2k} \leq \sum_{j=0}^{2k-1}\left(\frac{b^2}{a^2}\right)^j$. Finally

$$P_k \leq \sum_{j=0}^{k-1}\left(\frac{b^2}{a^2}\right)^j \tag{A.11}$$

and

$$\beta_k^2 = \frac{1}{P_{w,k}+\rho a^2|h|^2} \geq \frac{1}{\sum_{j=0}^{k-1}\left(\frac{b^2}{a^2}\right)^j+\rho a^2|h|^2}. \tag{A.12}$$

The asymptotic behavior of the determinant can then be lower bounded

$$D_k \dot{\geq} 1 + \rho^{2k} \left(\prod_{j=2}^{2k} \frac{1}{\sum_{l=0}^{j-1} \left(\frac{b^2}{a^2}\right)^j} \right) |g_1|^{2\times 2k} + \rho^{2(2k-1)+1} \left(\prod_{j=2}^{2k} \frac{\frac{1}{\sum_{l=0}^{j-2}\left(\frac{b^2}{a^2}\right)^l + \rho a^2 |h|^2}}{\sum_{l=0}^{j-1} \left(\frac{b^2}{a^2}\right)^j} \right) \frac{1}{P_{2k}} |g_1|^2 |h|^{2(2k-1)} |g_2|^{2(2k-1)}$$

$$\dot{\geq} 1 + \rho^{2k} |g_1|^{2\times 2k} + \rho^{2(2k-1)+1} \left(\prod_{j=2}^{2k} \frac{1}{\sum_{l=0}^{j-2} \left(\frac{b^2}{a^2}\right)^l + \rho a^2 |h|^2} \right) \frac{1}{P_{2k}} |g_1|^2 |h|^{2(2k-1)} |g_2|^{2(2k-1)}$$

Outage probability P_1 can then be upper bounded by

$$\begin{aligned}
P_1 &= \Pr\{\log D_k \leq 2kR\} \\
&\dot{\leq} \Pr\left\{ \log\left(1 + \rho^{2k}|g_1|^{2\times 2k} + \rho^{2(2k-1)+1} \left(\prod_{j=2}^{2k} \frac{1}{\sum_{l=0}^{j-2}\left(\frac{b^2}{a^2}\right)^l + \rho a^2 |h|^2} \right) \frac{1}{P_{2k}} |g_1|^2 |h|^{2(2k-1)} |g_2|^{2(2k-1)} \right) \leq 2kr \log \rho \right\} \\
&\dot{\leq} \Pr\left\{ \log(1 + \rho^{2k(1-v_1)} + \rho^{(1-v_1)+(2k-1)(1-v_2)+(2k-1)(1-u)-(2k-1)(1-u)^+}) \leq 2kr \log \rho \right\} \\
&\dot{\leq} \Pr\left\{ \log(1 + \rho^{2k(1-v_1)} + \rho^{(1-v_1)+(2k-1)(1-v_2)}) \leq 2kr \log \rho \right\} \\
&\dot{\leq} \Pr\{\max(0, 2k(1-u_1), (1-u_1) + (2k-1)(1-u_2)) \leq 2kr\}
\end{aligned}$$

so $d_1 \geq (1-r)^+ + \left(1 - \frac{2k}{2k-1} r\right)^+$ which proves the achievability.

The same way, we prove that $d_2 = (1-r)^+ + \left(1 - \frac{2k}{2k-1} r\right)^+$.

We can remark that if roles between user U_1 and user U_2 were not switched at each subframe, then:

$$P_1 = \Pr\{D_{1,k} \leq kR\} \qquad \text{and} \qquad P_2 = \Pr\{D_{2,k} \leq kR\}$$

and so

$$d_1 = 2(1-r)^+ \qquad \text{and} \qquad d_2 = (1-r)^+ + \left(1 - \frac{k}{k-1} r\right)^+$$

and the final DMT

$$d^*(r) = \min(d_1(r), d_2(r)) = d_2 = (1-r)^+ + \left(1 - \frac{k}{k-1} r\right)^+$$

is worse than the one we get by switching the roles of the two users.

DMT of the outage event $\mathcal{O}_{1,2}$

The equivalent channel $\mathbf{H_{2k-1}}$ for a subframe of size $2k-1$ is defined recursively as

$$\mathbf{H}_{2k-1} = \begin{bmatrix} & \mathbf{H}_{2k-2} & \mathbf{0}_{(2k-2)\times 1} \\ \mathbf{0}_{1\times(2k-3)} & \sqrt{\frac{\rho}{P_{2k-1}}} ab\beta_{2k-2} h^* g_1 & \frac{1}{\sqrt{P_{2k-1}}} a g_1 \end{bmatrix}$$

and $\mathbf{H}_{2k}$ for a subframe of size $2k$ is

$$\mathbf{H}_{2k} = \begin{bmatrix} & \mathbf{H}_{2k-1} & \mathbf{0}_{(2k-1)\times 1} \\ \mathbf{0}_{1\times(2k-2)} & \sqrt{\frac{\rho}{P_{2k}}}ab\beta_{2k-1}hg_2 & \frac{1}{\sqrt{P_{2k}}}ag_2 \end{bmatrix}$$

Thus

$$\mathbf{H}_{2k-1}\mathbf{H}_{2k-1}^{\dagger} = \begin{bmatrix} & \mathbf{H}_{2k-2}\mathbf{H}_{2k-2}^{\dagger} & \begin{matrix}\mathbf{0}_{(2k-3)\times 1} \\ \sqrt{\frac{\rho}{P_{2k-2}P_{2k-1}}}a^2b\beta_{2k-2}g_1^*hg_2\end{matrix} \\ \mathbf{0}_{1\times(2k-3)} & \sqrt{\frac{\rho}{P_{2k-2}P_{2k-1}}}a^2b\beta_{2k-2}g_1h^*g_2^* & \frac{1}{P_{2k-1}}a^2|g_1|^2\left(1+\rho b^2\beta_{2k-2}^2|h|^2\right) \end{bmatrix}$$

and

$$\mathbf{H}_{2k}\mathbf{H}_{2k}^{\dagger} = \begin{bmatrix} & \mathbf{H}_{2k-1}\mathbf{H}_{2k-1}^{\dagger} & \begin{matrix}\mathbf{0}_{(2k-2)\times 1} \\ \sqrt{\frac{\rho}{P_{2k-1}P_{2k}}}a^2b\beta_{2k-1}g_1h^*g_2^*\end{matrix} \\ \mathbf{0}_{1\times(2k-2)} & \sqrt{\frac{\rho}{P_{2k-1}P_{2k}}}a^2b\beta_{2k-1}g_1^*hg_2 & \frac{1}{P_{2k}}a^2|g_2|^2\left(1+\rho b^2\beta_{2k-1}^2|h|^2\right) \end{bmatrix}$$

So

$$\begin{aligned} D_{2k-1} &= \left(1+\rho\frac{1}{P_{2k-1}}a^2|g_1|^2\left(1+\rho b^2\beta_{2k-2}^2|h|^2\right)\right)D_{2k-2} - \frac{\rho^3}{P_{2k-2}P_{2k-1}}a^4b^2\beta_{2k-2}^2|g_1|^2|h|^2|g_2|^2D_{2k-3}, \\ D_{2k} &= \left(1+\rho\frac{1}{P_{2k}}a^2|g_2|^2\left(1+\rho b^2\beta_{2k-1}^2|h|^2\right)\right)D_{2k-1} - \frac{\rho^3}{P_{2k-1}P_{2k}}a^4b^2\beta_{2k-1}^2|g_1|^2|h|^2|g_2|^2D_{2k-2}. \end{aligned}$$

We have $D_1 = 1+\rho|g_1|^2$ and $D_2 = 1+\rho|g_1|^2 + \frac{\rho}{P_2}a^2|g_2|^2 + \frac{\rho^2}{P_2}b^2\beta_1^2|h|^2|g_2|^2 + \frac{\rho^2}{P_2}a^2|g_1|^2|g_2|^2$.
Let's suppose that

$$\begin{aligned} D_{2k-1} &= \frac{1}{\prod_{j=1}^{2k-1}P_j}a^{2(2k-2)}\rho^{2k-1}|g_1|^{2k}|g_2|^{2k-2} + P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k-2}, \\ D_{2k} &= \frac{1}{\prod_{j=1}^{2k}P_j}a^{2(2k-1)}\rho^{2k}|g_1|^{2k}|g_2|^{2k} + P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k-1}, \end{aligned}$$

which is true for D_1 and D_2.

Then, recursively,

$$\begin{aligned}
D_{2k+1} &= \left(1+\rho\frac{1}{P_{2k+1}}a^2|g_1|^2\left(1+\rho b^2\beta_{2k}^2|h|^2\right)\right)D_{2k} - \frac{\rho^3}{P_{2k}P_{2k+1}}a^4b^2\beta_{2k}^2|g_1|^2|h|^2|g_2|^2D_{2k-1} \\
&= \left(1+\rho\frac{1}{P_{2k+1}}a^2|g_1|^2\left(1+\rho b^2\beta_{2k}^2|h|^2\right)\right)\left(\frac{1}{\prod_{j=1}^{2k}P_j}a^{2(2k-1)}\rho^{2k}|g_1|^{2k}|g_2|^{2k}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k-1}\right) \\
&\quad - \frac{\rho^3}{P_{2k}P_{2k+1}}a^4b^2\beta_{2k}^2|g_1|^2|h|^2|g_2|^2\left(\frac{1}{\prod_{j=1}^{2k-1}P_j}a^{2(2k-2)}\rho^{2k-1}|g_1|^{2k}|g_2|^{2k-2}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k-2}\right) \\
&= \left(\frac{1}{\prod_{j=1}^{2k+1}P_j}a^{2(2k)}\rho^{2k+1}|g_1|^{2k+2}|g_2|^{2k}\left(1+\rho b^2\beta_{2k}^2|h|^2\right)+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k}\right) \\
&\quad - \left(\frac{1}{\prod_{j=1}^{2k+1}P_j}a^{2(2k)}b^2\rho^{2k+2}\beta_{2k}^2|h|^2|g_1|^{2k+2}|g_2|^{2k}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k}\right) \\
&= \frac{1}{\prod_{j=1}^{2k+1}P_j}a^{2(2k)}\rho^{2k+1}|g_1|^{2k+2}|g_2|^{2k}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k}
\end{aligned}$$

and the same way

$$\begin{aligned}
D_{2k+2} &= \left(1+\rho\frac{1}{P_{2k+2}}a^2|g_2|^2\left(1+\rho b^2\beta_{2k+1}^2|h|^2\right)\right)D_{2k+1} - \frac{\rho^3}{P_{2k+1}P_{2k+2}}a^4b^2\beta_{2k+1}^2|g_1|^2|h|^2|g_2|^2D_{2k} \\
&= \left(1+\rho\frac{1}{P_{2k+2}}a^2|g_2|^2\left(1+\rho b^2\beta_{2k+1}^2|h|^2\right)\right)\left(\frac{1}{\prod_{j=1}^{2k+1}P_j}a^{2(2k)}\rho^{2k+1}|g_1|^{2k+2}|g_2|^{2k}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k}\right) \\
&\quad - \frac{\rho^3}{P_{2k+1}P_{2k+2}}a^4b^2\beta_{2k+1}^2|g_1|^2|h|^2|g_2|^2\left(\frac{1}{\prod_{j=1}^{2k}P_j}a^{2(2k-1)}\rho^{2k}|g_1|^{2k}|g_2|^{2k}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k-1}\right) \\
&= \left(\frac{1}{\prod_{j=1}^{2k+2}P_j}a^{2(2k+1)}\rho^{2k+2}|g_1|^{2k+2}|g_2|^{2k+2}\left(1+\rho b^2\beta_{2k+1}^2|h|^2\right)+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k+1}\right) \\
&\quad - \left(\frac{1}{\prod_{j=1}^{2k+2}P_j}a^{2(2k+1)}b^2\rho^{2k+3}\beta_{2k+1}^2|h|^2|g_1|^{2k+2}|g_2|^{2k+2}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k+1}\right) \\
&= \frac{1}{\prod_{j=1}^{2k+2}P_j}a^{2(2k+1)}\rho^{2k+2}|g_1|^{2k+2}|g_2|^{2k+2}+P(|g_1|^{2i}|g_2|^{2j})_{i+j\leq 2k+1}
\end{aligned}$$

Let's consider the same case as before where the two sources are so close that $|h|^2 > |g_1|^2$ and $|h|^2 > |g_2|^2$.

Then we can lower bound the asymptotic behavior of the determinant

$$D_{2k} \doteq \frac{1}{\prod_{j=1}^{2k}P_j}a^{2(2k-1)}\rho^{2k}|g_1|^{2k}|g_2|^{2k} \quad \dot{\geq} \frac{1}{\prod_{j=1}^{2k}\sum_{l=0}^{j-1}\left(\frac{b^2}{a^2}\right)^l}a^{2(2k-1)}\rho^{2k}|g_1|^{2k}|g_2|^{2k}\dot{\geq}\rho^{2k}|g_1|^{2k}|g_2|^{2k}$$

and so upper bound the outage probability $P_{1,2}$

$$\begin{aligned} P_{1,2} &= \Pr\{\log D_{2k} \leq 2kR\} \\ &\dot{\leq} \Pr\left\{\log(\rho^{2k}|g_1|^k|g_2|^k \leq 2kr\log\rho\right\} \\ &\dot{\leq} \Pr\left\{\log(\rho^{k(2-(u_1+u_2)} \leq 2kr\log\rho\right\} \\ &\dot{\leq} \Pr\{(2-(u_1+u_2) \leq 2r\} \end{aligned}$$

so $d_{1,2} \geq \inf(u_1+u_2) = 2(1-r)^+$ which is the MISO bound. Finally

$$d_{1,2} = 2(1-r)^+ \tag{A.13}$$

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