Stability and Receptivity of the Swept Wing Attachment-Line Boundary Layer: a Multigrid Numerical Approach

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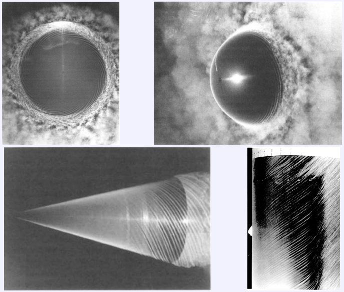
Motivation

- Gain a better understanding of the coherent flow structures and their dynamics in the leading-edge region of swept wings
- Identify the regions of maximum receptivity and sensitivity

Industrial research

Academic research



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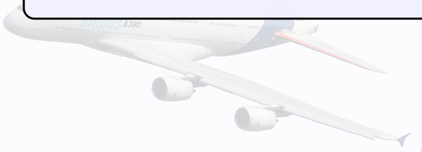
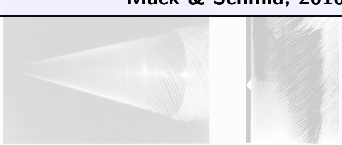
“The goal of any scientific study of a fluid-dynamical process is not in the reproduction of its physical features by direct numerical simulations but in the extraction of the governing underlying mechanisms from the data the DNS produces. In other words, we are interested in the intrinsic flow behavior captured by the dynamics of coherent structures.”

Mack & Schmid, 2010





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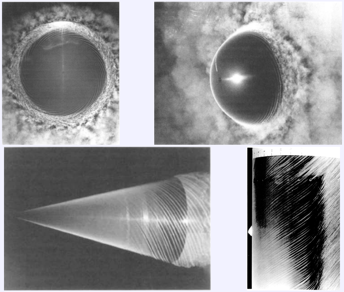

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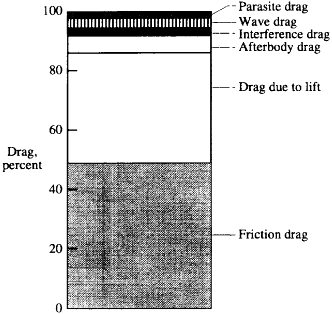
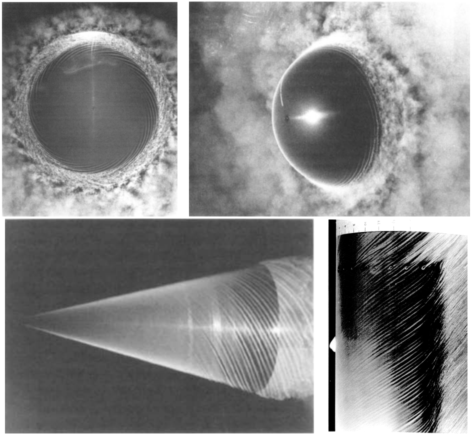


Drag components for a subsonic aircraft; Thibert, Reneaux & Schmitt, 1990

- Friction drag $\approx$ 50% of total drag for a subsonic aircraft
- Laminar friction drag $\approx 1/10$ of turbulent friction drag
- Large savings are possible by extending the laminar flow region
- An understanding of the instabilities leading to turbulence is required

Academic research



Reed and Saric, 1989; Poll, 1978

Non-alignment between the velocity vector and the pressure gradient is at the origin of three dimensional boundary layers

Outline

- Swept-wing instabilities
- Procedure and tools
- Multigrid
- Base flow
- Perturbations: a modal description
- Receptivity and sensitivity
- Conclusions and future work

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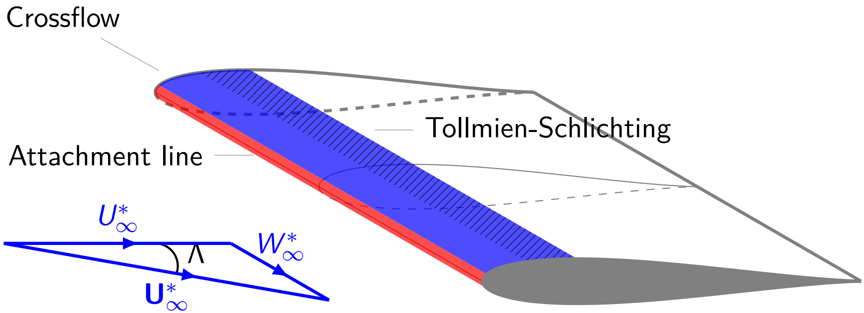
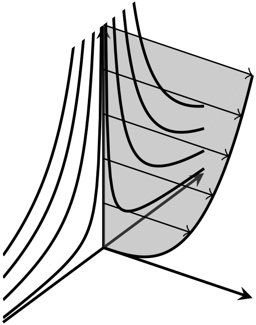
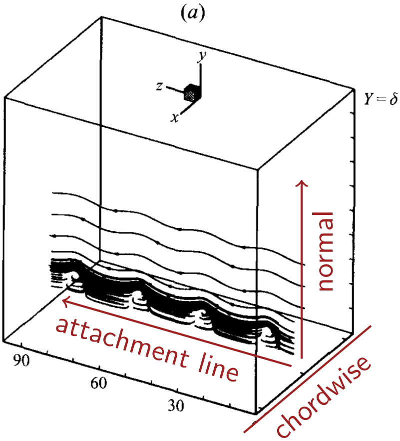
Swept-wing instabilities



Local analysis based on simplified flow models provides us with different instability mechanisms for different regions

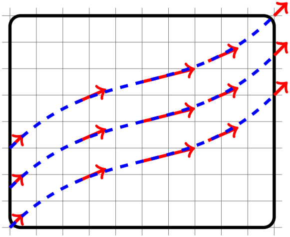
Swept-wing instabilities — Attachment line

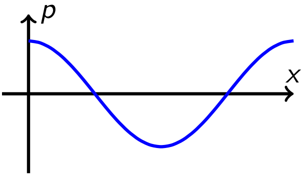




Attachment-line instability: streamlines under the influence of the most unstable modes; Lin & Malik 1996.

Swept-wing instabilities — Crossflow vortices





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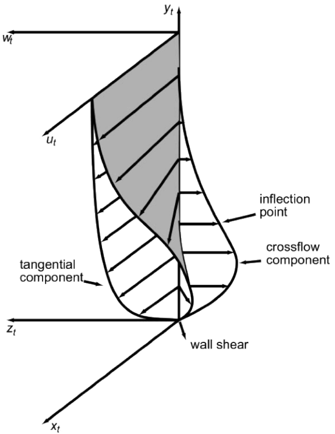

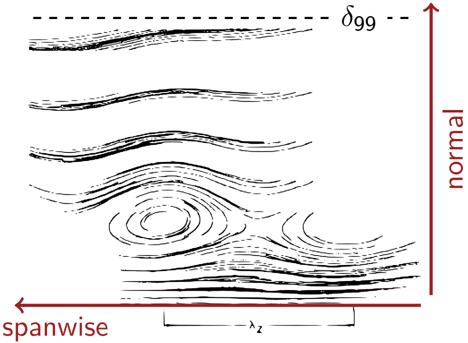

Crossflow instability: streamlines of crossflow vortices obtained from experimental results; Reed, 1988

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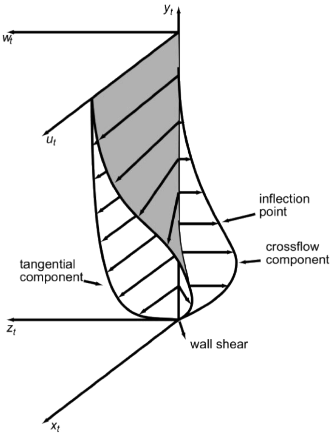
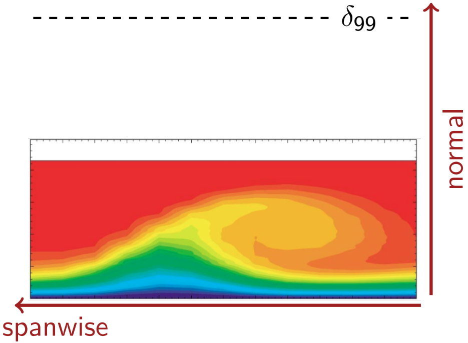
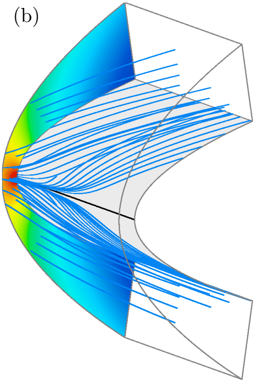
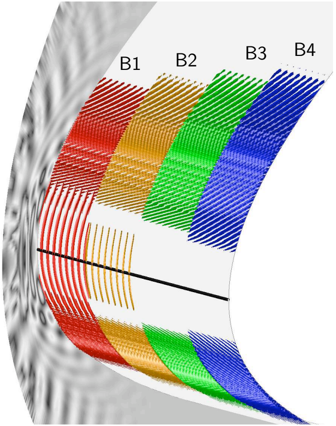




Streamwise velocity contours under the influence of crossflow vortices, from experimental results; Haynes & Reed, 2000

Swept-wing instabilities — Global analysis



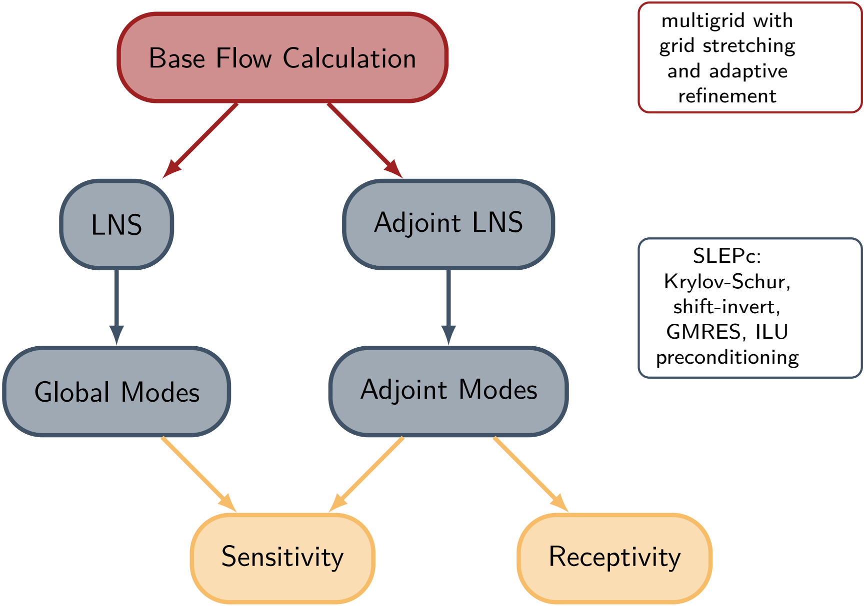


Isosurfaces of the normal velocity component of the disturbance

Hypersonic flow with bow-shock inflow; Mack, 2009

Outline

Procedure and tools



Procedures and tools — Governing equations

Base Flow

$$\mathcal{R}(\mathbf{q}) \equiv \begin{cases} \nabla\mathbf{u}\,\mathbf{u} - \nu\Delta\mathbf{u} + \nabla p & = & 0 \\ \nabla\cdot\mathbf{u} & = & 0 \end{cases}$$

Linearized equations for the perturbations $\mathbf{q}'$

$$\mathcal{L}\,\mathbf{q}' = \left(B\,\partial_t + \left.\frac{\partial\mathcal{R}}{\partial\mathbf{q}}\right|_{\mathbf{Q}} \right)\mathbf{q}' \qquad \Longrightarrow \qquad \mathcal{L}(\mathbf{Q})\,\mathbf{q}' = \mathbf{f}'$$

Modal decomposition $\hat{\mathbf{q}}'$

$$\hat{\mathbf{q}}' = \int_0^\infty \mathbf{q}' e^{-\sigma t} dt \qquad \Longrightarrow \qquad \hat{\mathcal{L}}\,\hat{\mathbf{q}}' = (\sigma B + A)\,\hat{\mathbf{q}}' = \hat{\mathbf{f}}' + \mathbf{q}_0$$

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Procedures and tools — Receptivity and sensitivity

Lagrangian functional

$$\mathcal{I}\left(obj, \hat{\mathbf{q}}, \hat{\mathbf{q}}^{+}, \hat{\mathbf{f}}, A\right) = obj - \langle \hat{\mathbf{q}}^{+}, (\sigma B + A)\, \hat{\mathbf{q}} - \hat{\mathbf{f}} \rangle$$

$$\langle \mathbf{a}, \mathbf{b} \rangle = \int_{\Omega} \mathbf{a}^{H}\, \mathbf{b}\, d\Omega$$

Receptivity

What is the variation of the objective functional given a variation in the forcing?

$$\frac{\partial \mathcal{I}}{\partial \hat{\mathbf{f}}} \delta \hat{\mathbf{f}} = 0$$

$$\delta\,(obj) = -\langle \hat{\mathbf{q}}^{+}, \delta \hat{\mathbf{f}} \rangle$$

Sensitivity

What is the variation of the objective functional given a variation in the operator A?

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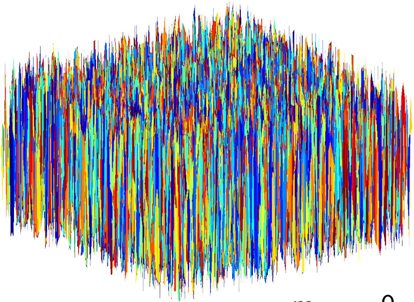

$m = 0$

Poisson problem: error $e^m = \bar{q} - q^m$ of the approximate solution at iteration m

$m = 0$ $m = 10$ $m = 100$

The Gauss-Seidel algorithm

- Simple and memory efficient algorithm
- Fast convergence for high wavenumbers
- Most of the computational cost is related to the reduction of the low wavenumber error components

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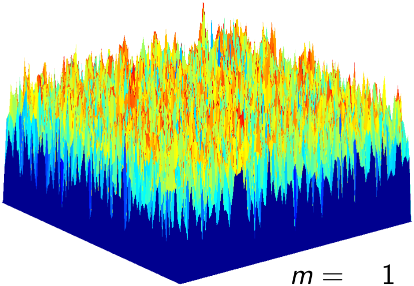

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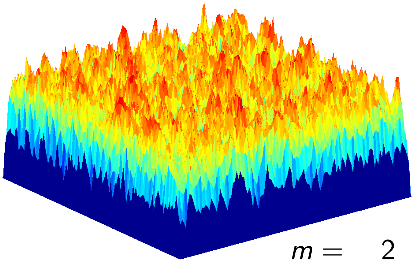

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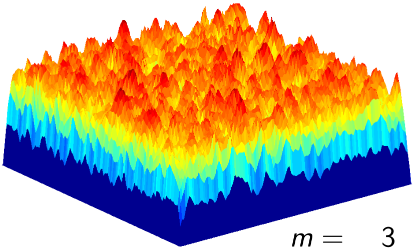

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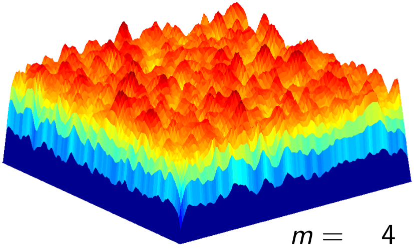

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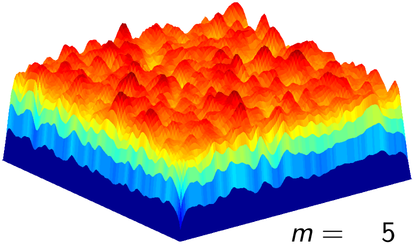

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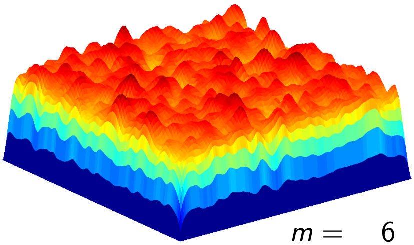

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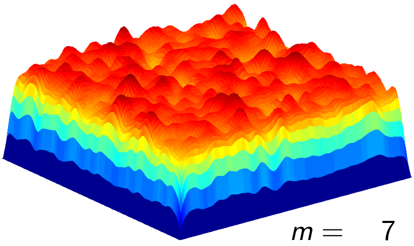

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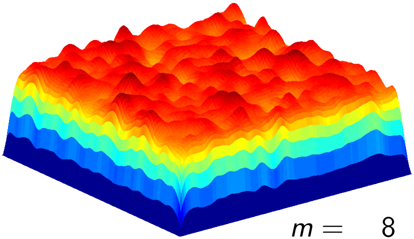

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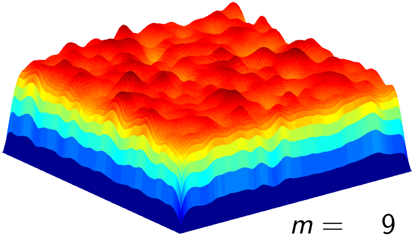

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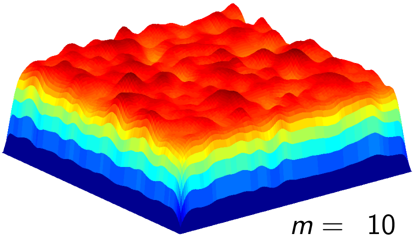

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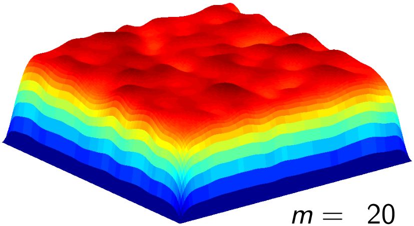

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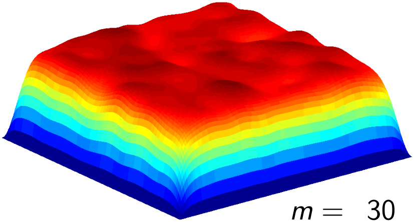

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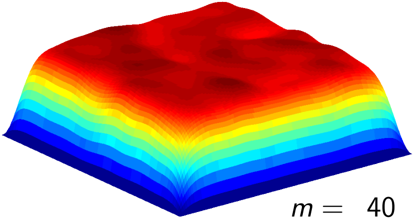

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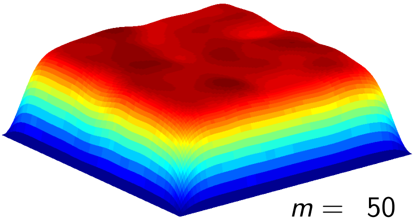

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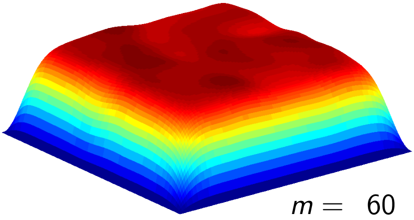

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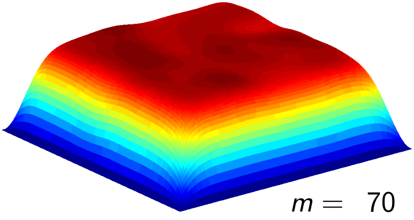

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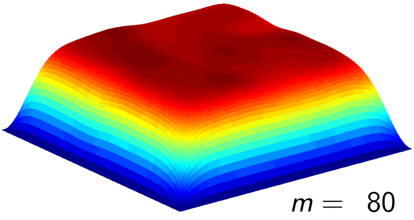

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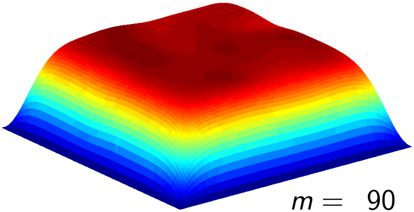
The Gauss-Seidel algorithm

- Simple and memory efficient algorithm
- Fast convergence for high wavenumbers
- Most of the computational cost is related to the reduction of the low wavenumber error components

Multigrid — Gauss-Seidel $\Delta q = f$



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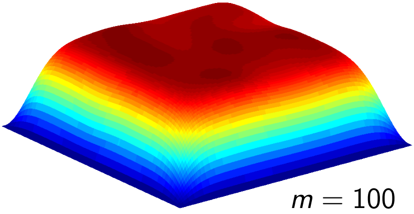

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- Simple and memory efficient algorithm
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Multigrid — Gauss-Seidel $\Delta q = f$

Error definition

$$e^m = \bar{q} - q^m = \sum_i \varepsilon_i^m \psi_i(x) \qquad \underbrace{\psi_i(x) = \exp\left(i\boldsymbol{\theta}^T \boldsymbol{x}/\boldsymbol{h}\right)}_{\text{eigenvectors decomposition}}$$

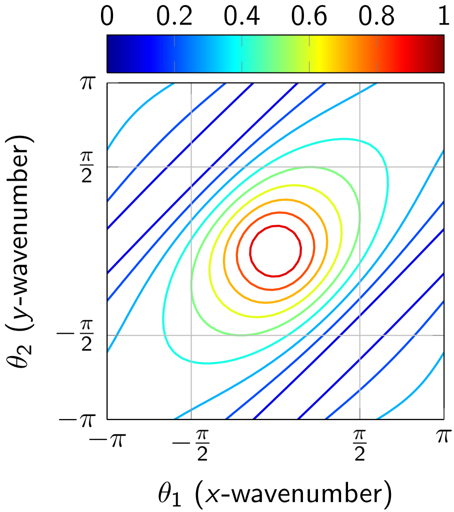
Error evolution

$$e^{m+1} = Me^m \qquad \Longrightarrow \qquad |\varepsilon_{\boldsymbol{\theta}}^{m+1}| = |\lambda_{\boldsymbol{\theta}}|\,|\varepsilon_{\boldsymbol{\theta}}^m|$$

$$|\lambda_{\boldsymbol{\theta}}| > 1 \quad \rightarrow \quad \text{divergence}$$

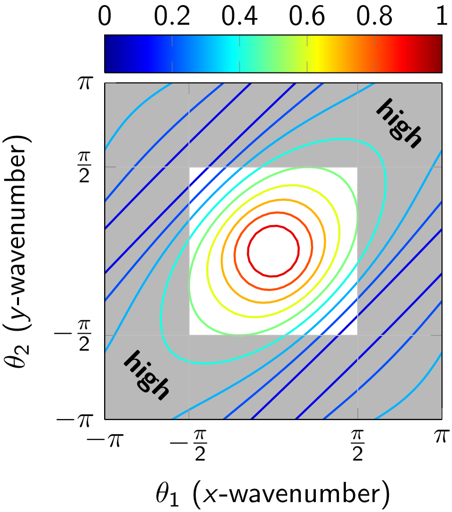
$$|\lambda_{\boldsymbol{\theta}}| < 1 \quad \rightarrow \quad \text{convergence}$$

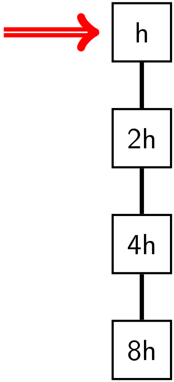
Multigrid — Gauss-Seidel $\Delta q = f$



amplification factor $\rho = |\lambda_\theta|$ **for the error amplitude:** $|\varepsilon_\theta^{m+1}| = |\lambda_\theta|\,|\varepsilon_\theta^m|$

Multigrid — Gauss-Seidel $\Delta q = f$





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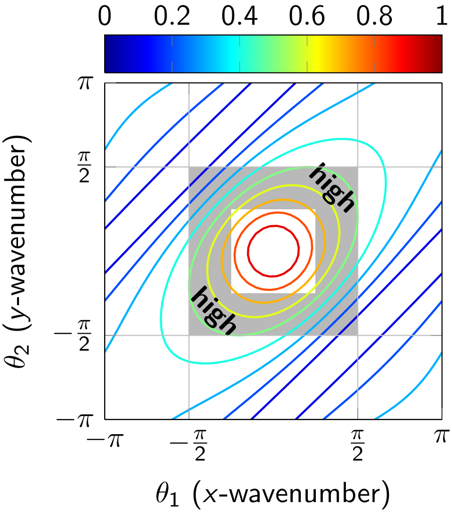

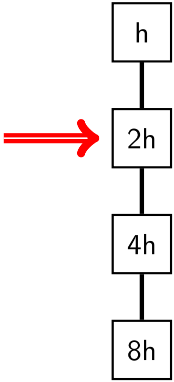

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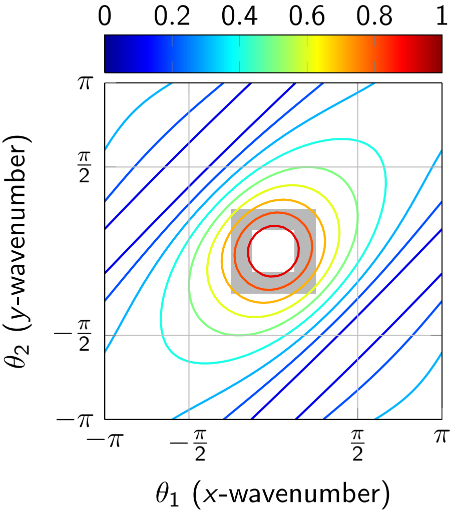
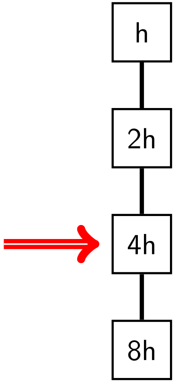
Multigrid — Gauss-Seidel $\Delta q = f$





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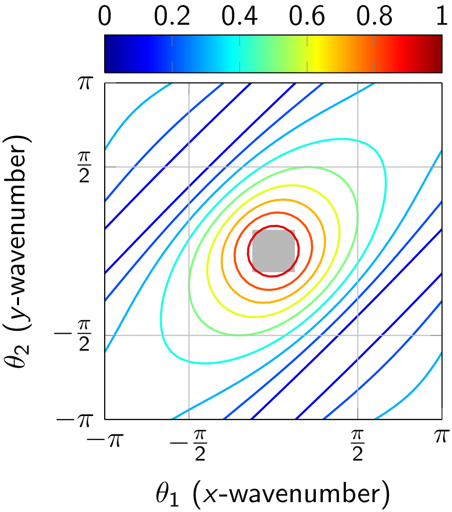

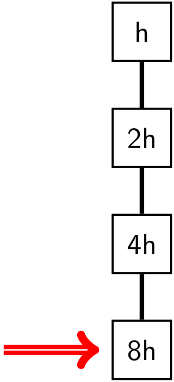

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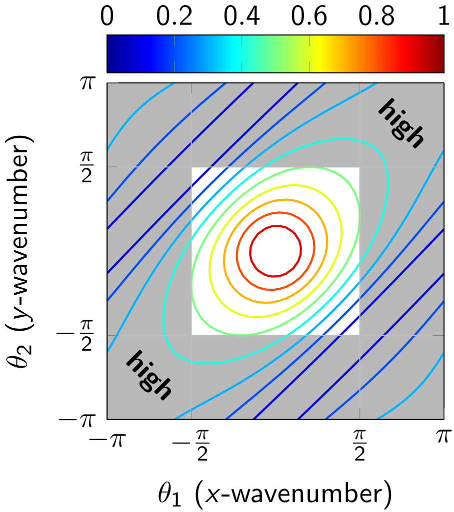

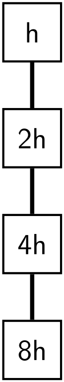

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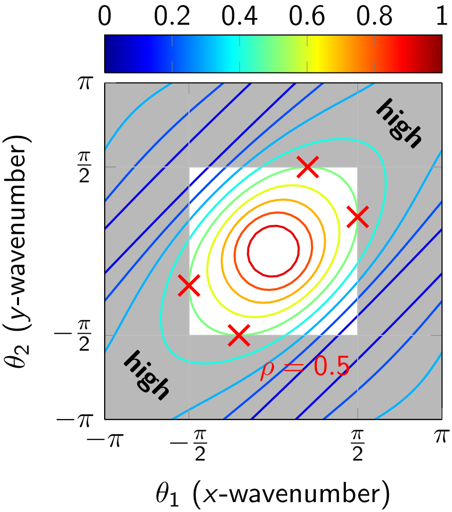
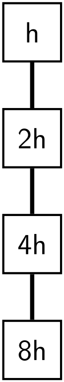
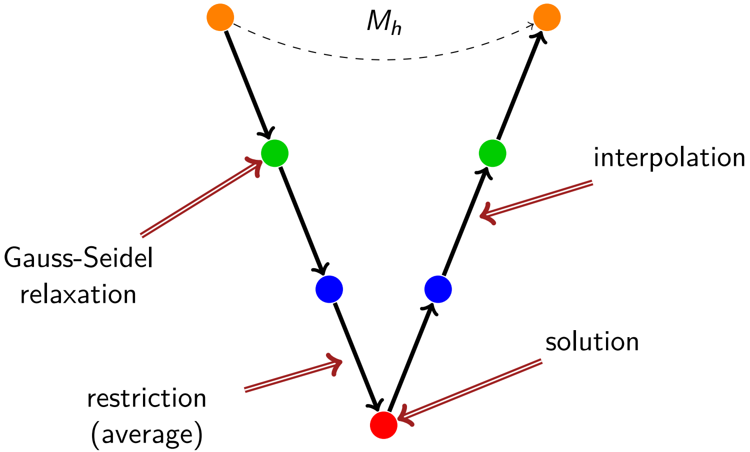
Multigrid — Gauss-Seidel $\Delta q = f$



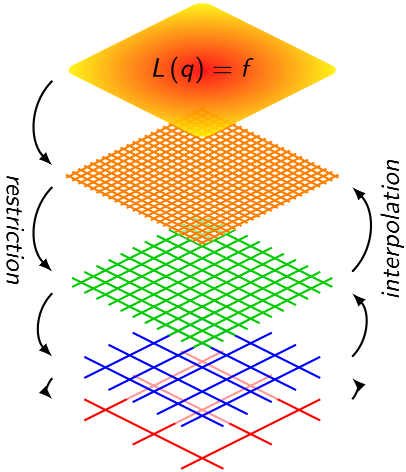


amplification factor $\rho = |\lambda_\theta|$ **for the error amplitude:** $|\varepsilon_\theta^{m+1}| = |\lambda_\theta|\,|\varepsilon_\theta^m|$

Multigrid — V-cycle



Multigrid — Full approximation scheme



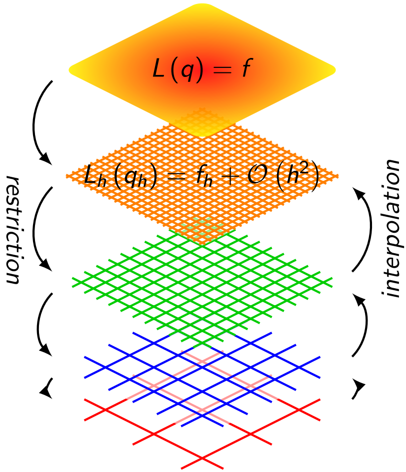
Ideas

- Represent the full solution an all grids
- τ_h^H: forcing of coarser grids' equations guarantees the same solution on all grids

Advantages

- Treatment of non-linear equations
- Natural approach to adaptive grid refinement

Multigrid — Full approximation scheme



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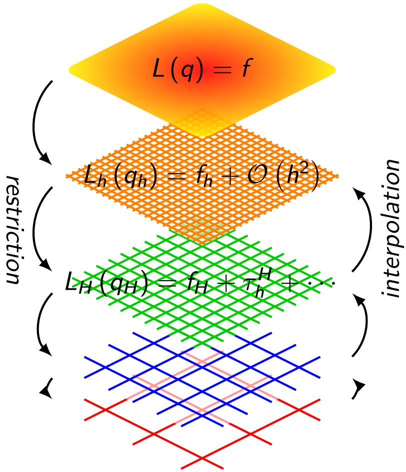

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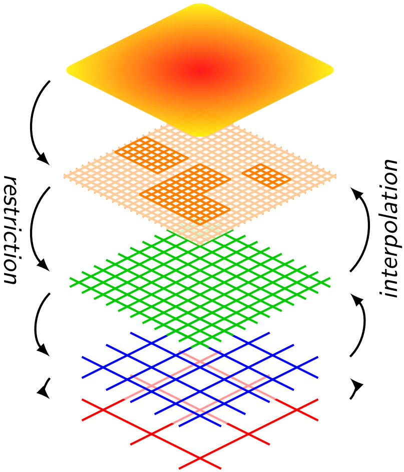

Ideas

- Represent the full solution an all grids
- τ_h^H: forcing of coarser grids' equations guarantees the same solution on all grids

Advantages

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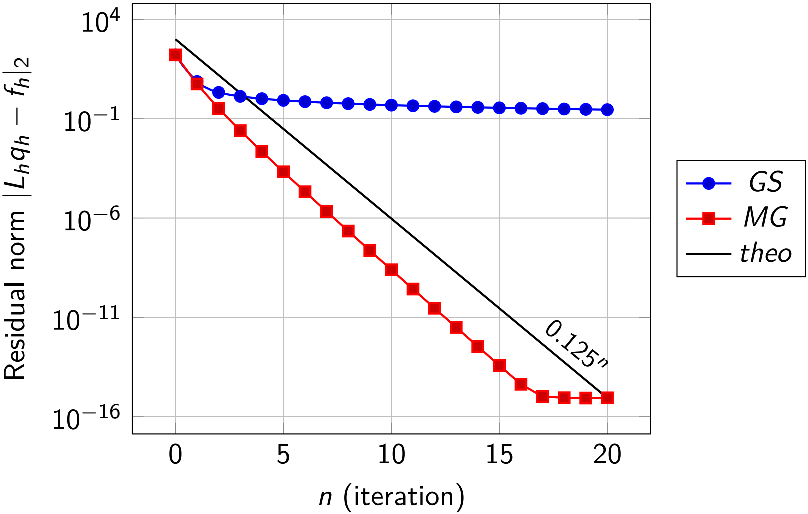
Multigrid — Test case $\Delta q = f$

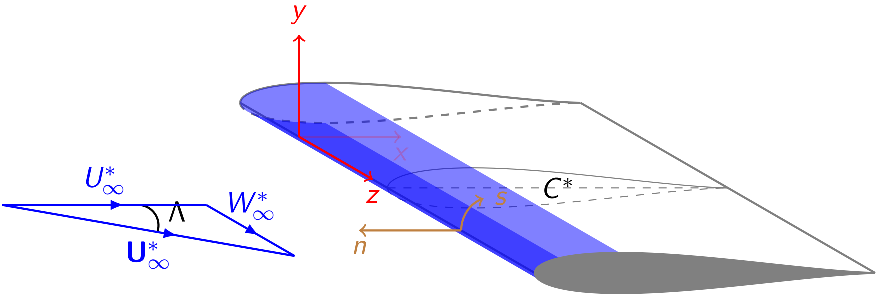


Convergence rate of the residual

Outline

- Swept-wing instabilities
- Procedure and tools
- Multigrid
- Base flow
- Perturbations: a modal description
- Receptivity and sensitivity
- Conclusions and future work

Base Flow — Configuration

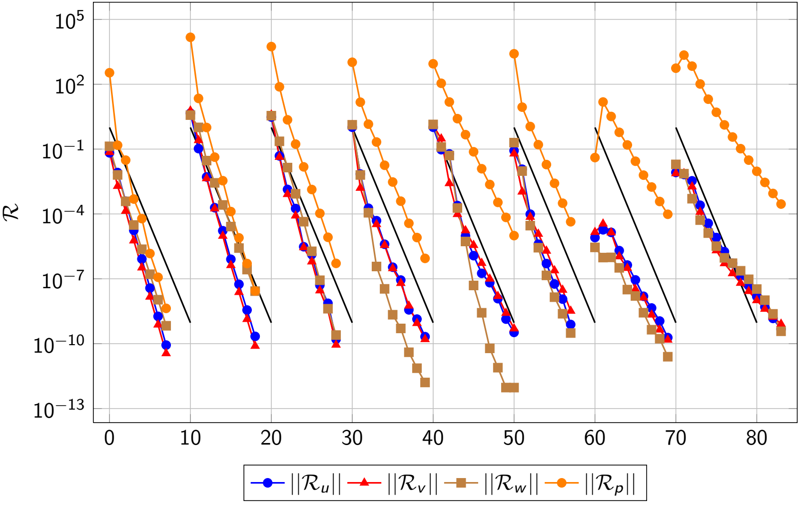


$$Re_C = 10^6 \qquad \Lambda = 45\,^\circ$$

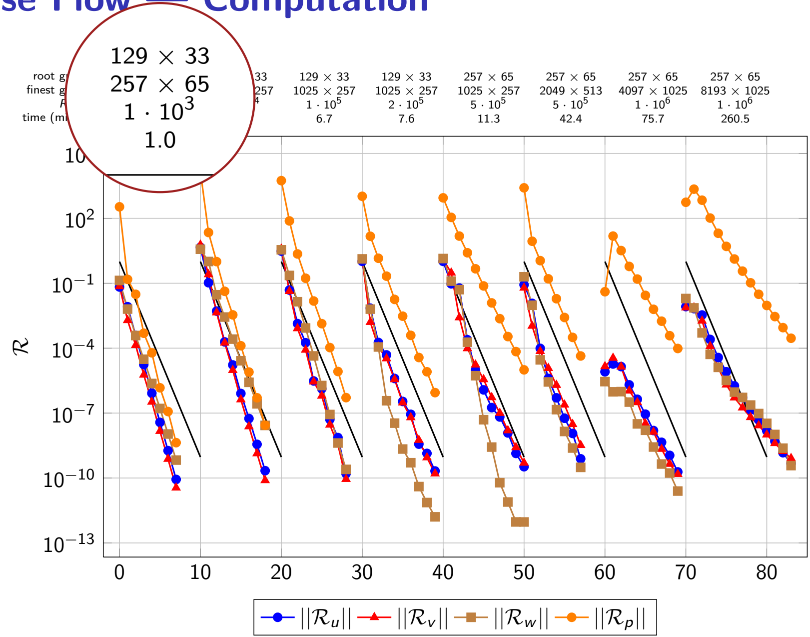
$$Re_r = 16000 \qquad Re_s = 126$$

Base Flow — Computation

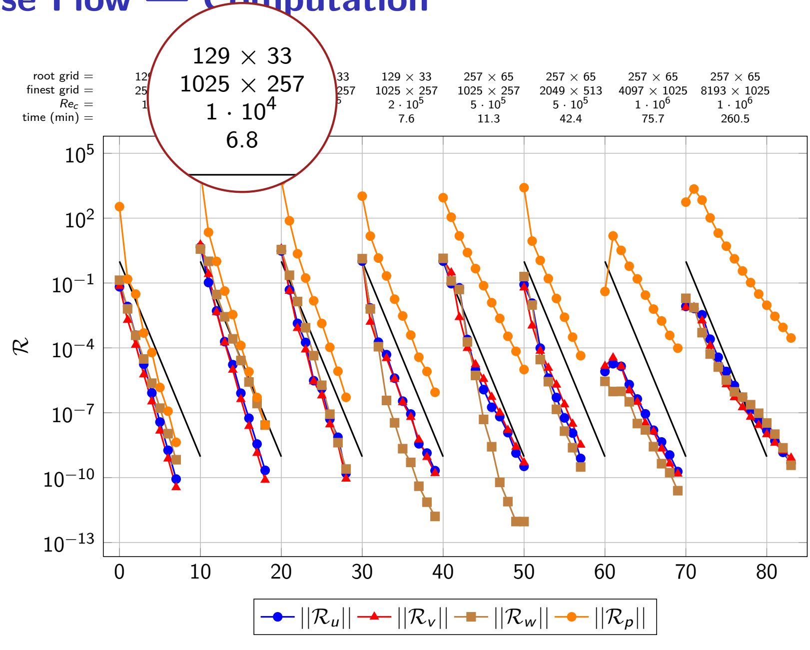
root grid =	129 × 33	129 × 33	129 × 33	129 × 33	257 × 65	257 × 65	257 × 65	257 × 65
finest grid =	257 × 65	1025 × 257	1025 × 257	1025 × 257	1025 × 257	2049 × 513	4097 × 1025	8193 × 1025
Re_c =	$1 \cdot 10^3$	$1 \cdot 10^4$	$1 \cdot 10^5$	$2 \cdot 10^5$	$5 \cdot 10^5$	$5 \cdot 10^5$	$1 \cdot 10^6$	$1 \cdot 10^6$
time (min) =	1.0	6.8	6.7	7.6	11.3	42.4	75.7	260.5



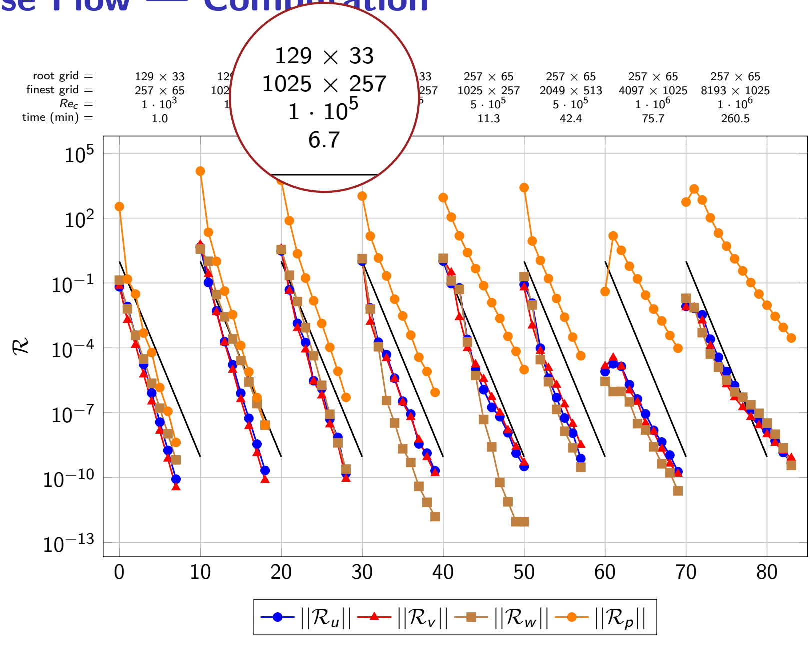
Base Flow — Computation



Base Flow — Computation



Base Flow — Computation



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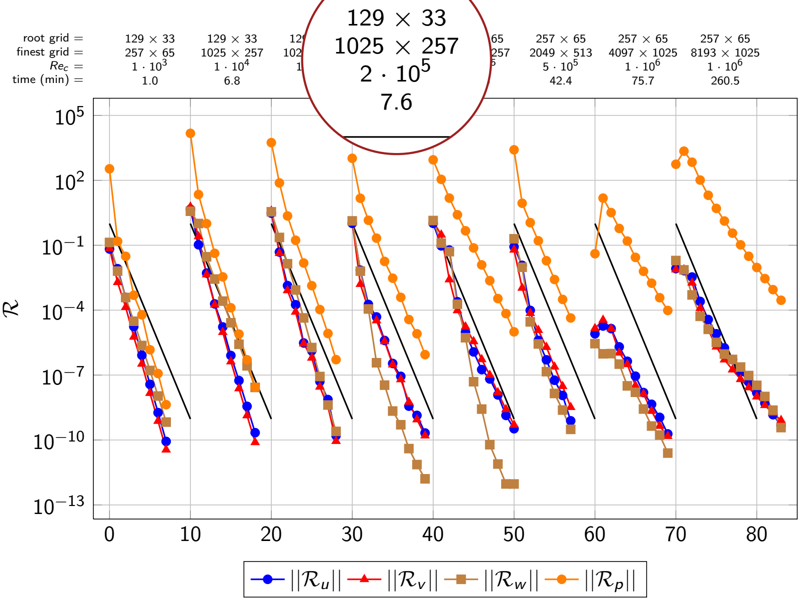

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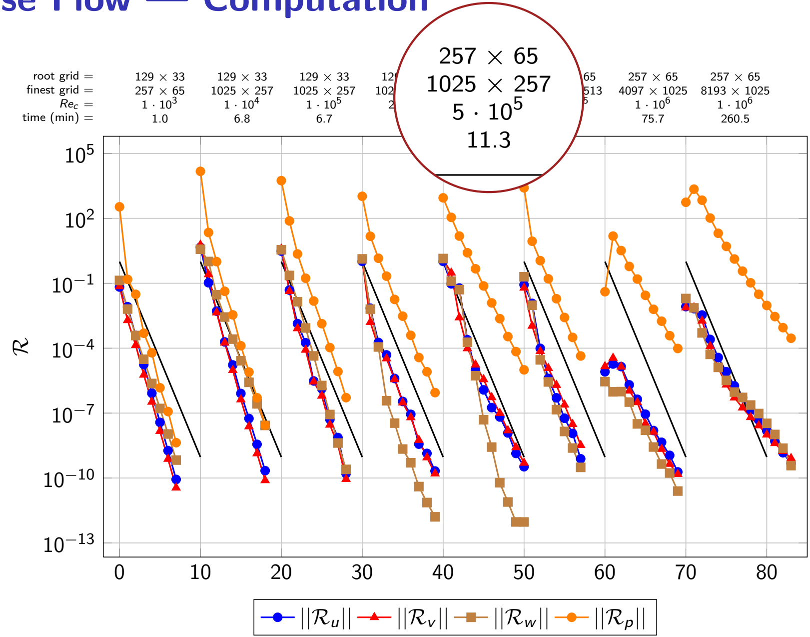
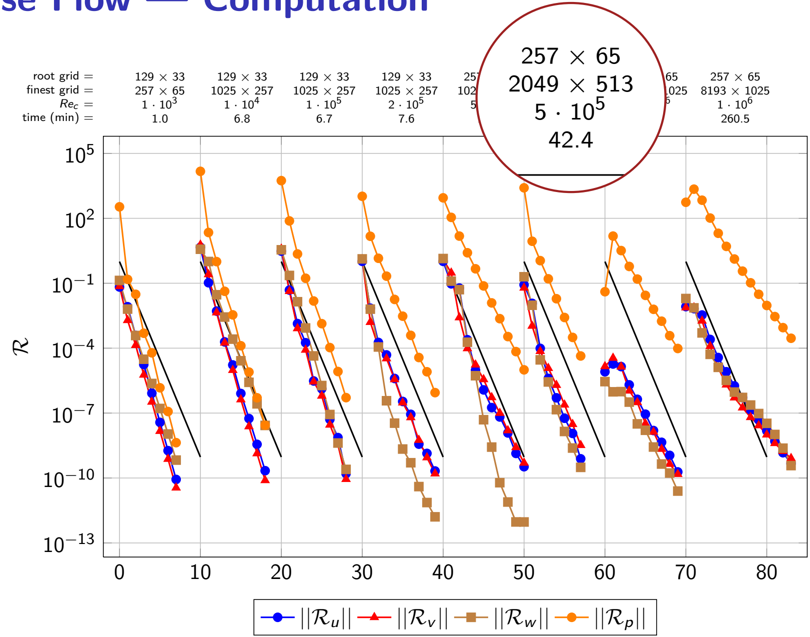
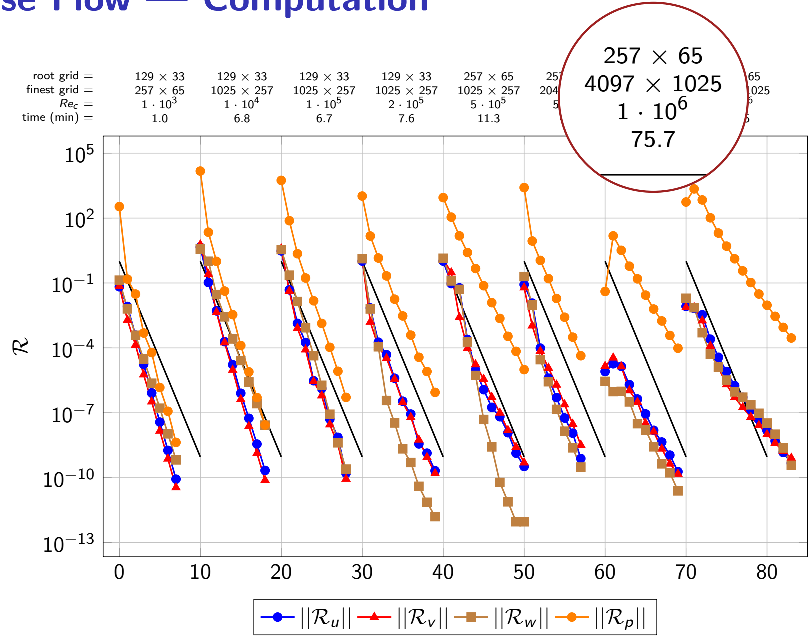


Base Flow — Computation



Base Flow — Computation



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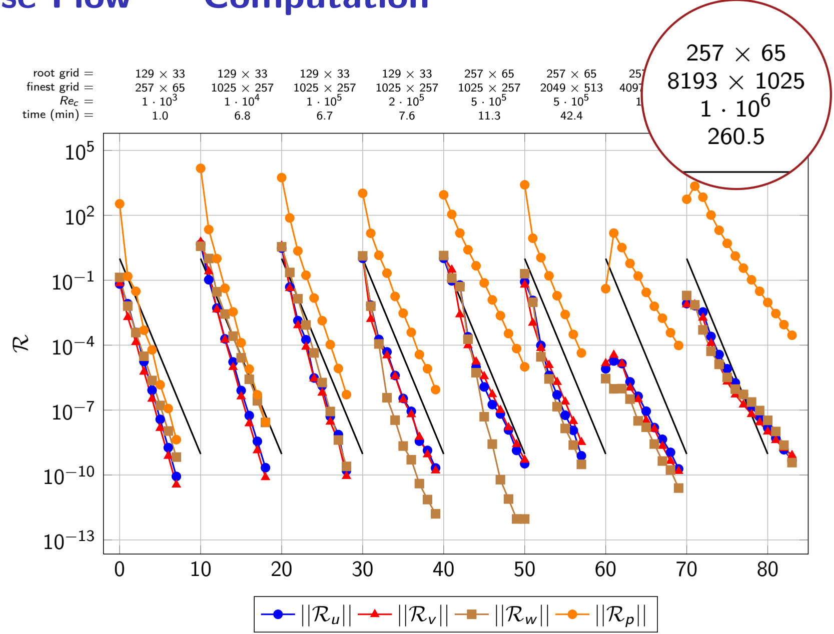
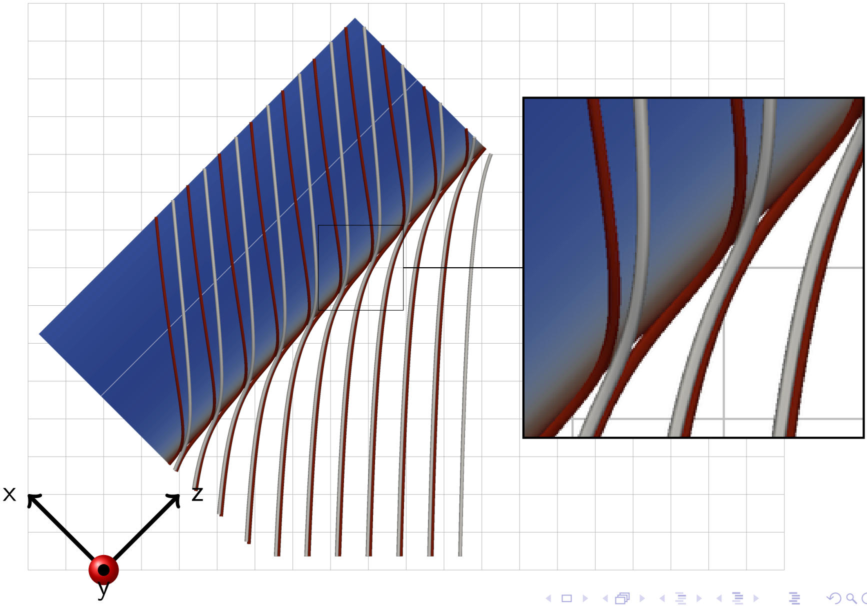


Base Flow — Streamlines



Outline

- Swept-wing instabilities
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- Multigrid
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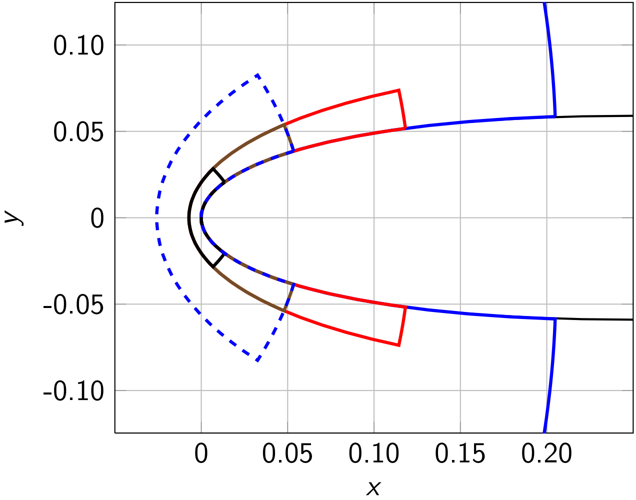
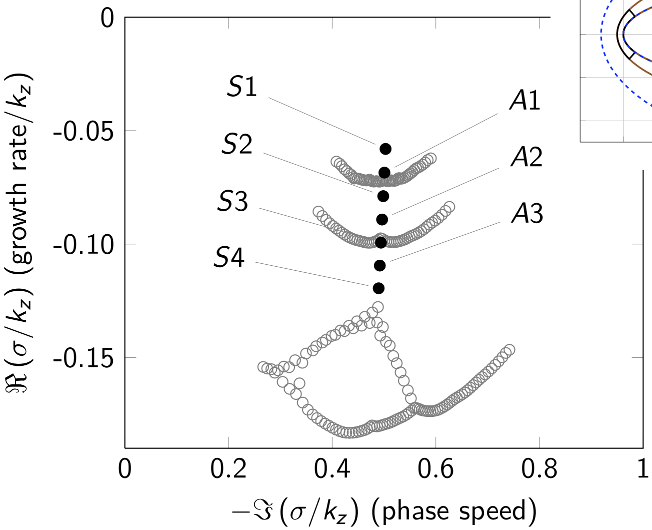
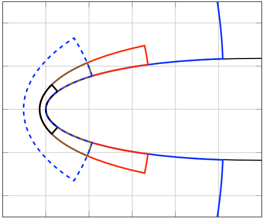
- Perturbations: a modal description
- Receptivity and sensitivity
- Conclusions and future work

Perturbations — Domains

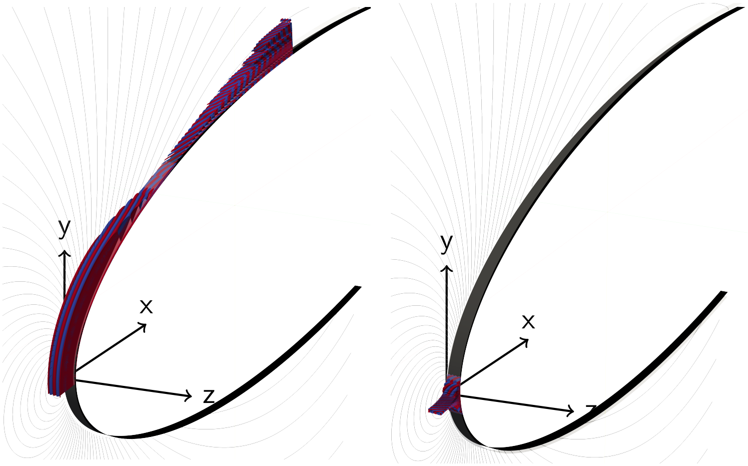


Perturbations — Spectrum



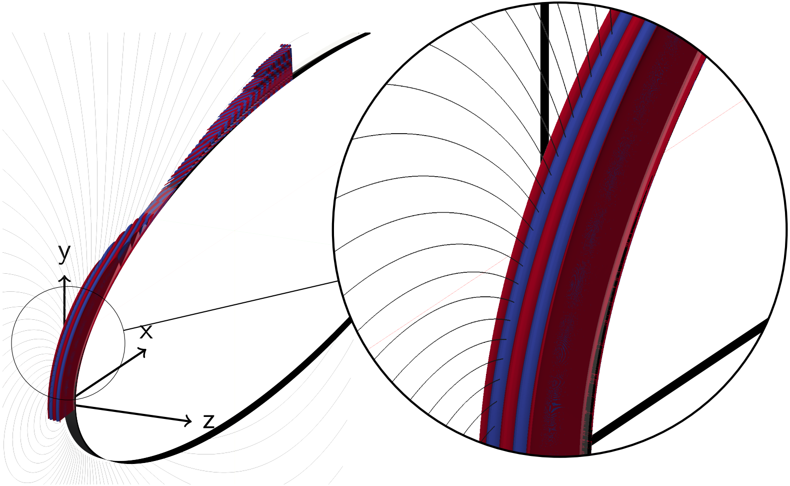


Perturbations — Direct and adjoint eigenvectors



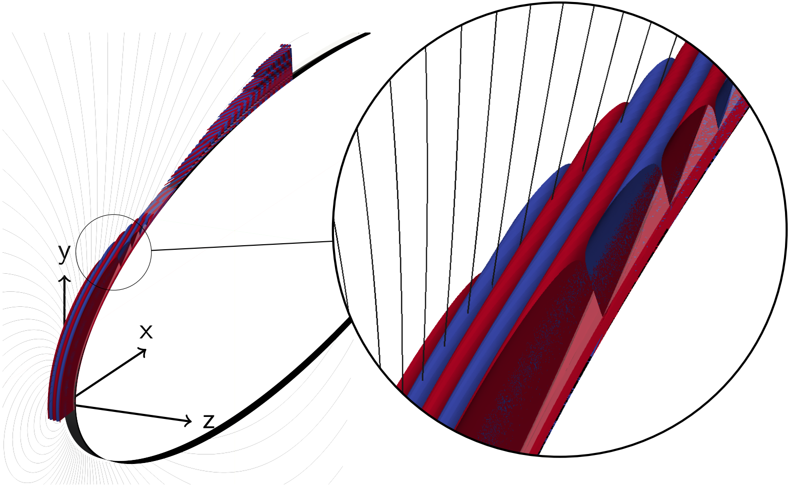
Iso-contours of the u-compoment of the perturbation. Blue: negative, red: positive

Perturbations — Direct and adjoint eigenvectors



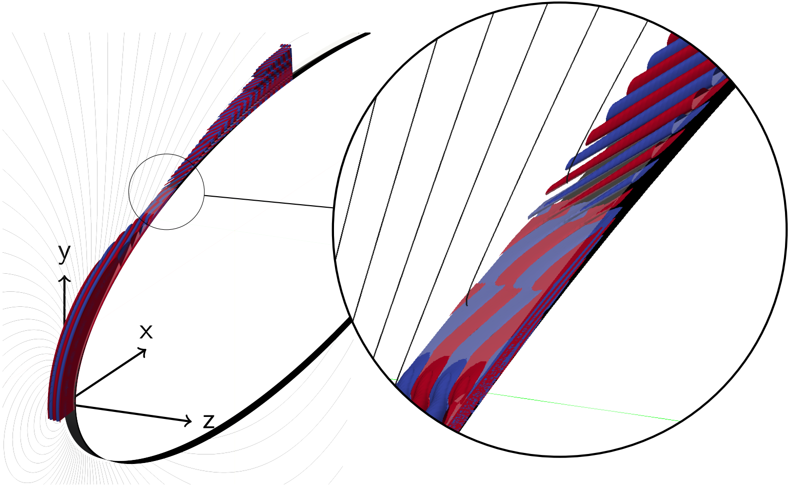
Direct eigenvector, attachment-line like structures

Perturbations — Direct and adjoint eigenvectors



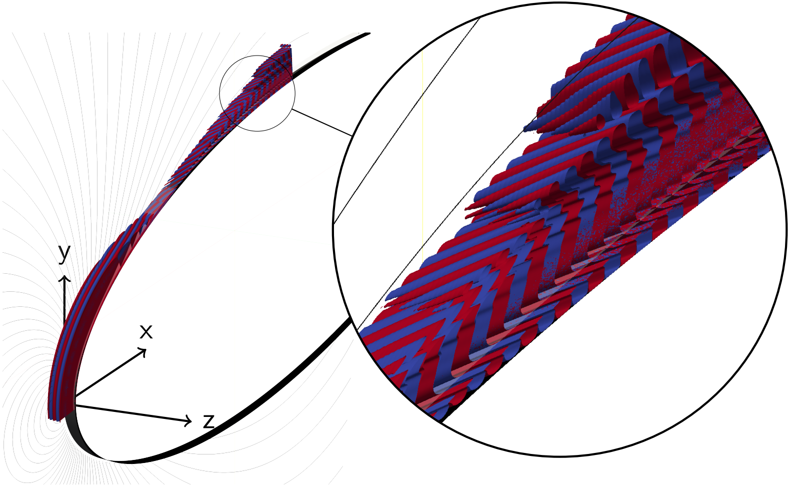
Direct eigenvector, structure orientation is against streamlines

Perturbations — Direct and adjoint eigenvectors



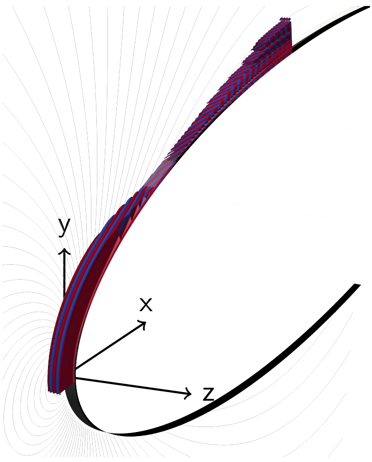
Direct eigenvector, transition from attachment line to crossflow like structures

Perturbations — Direct and adjoint eigenvectors

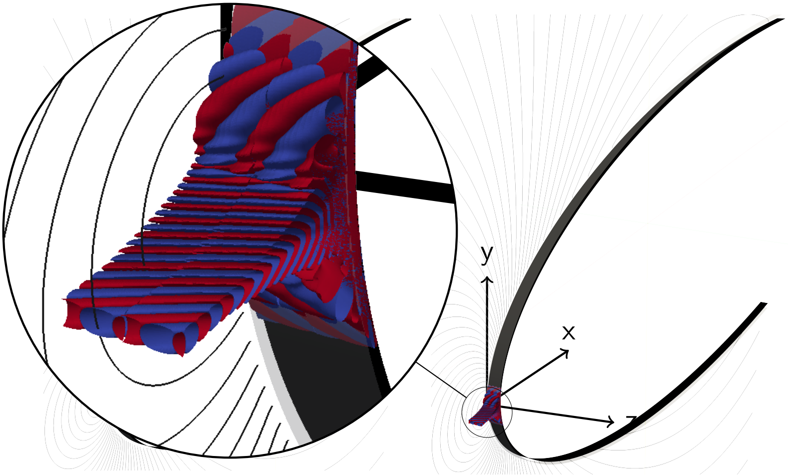


Direct eigenvector, crossflow like structures

Perturbations — Direct and adjoint eigenvectors

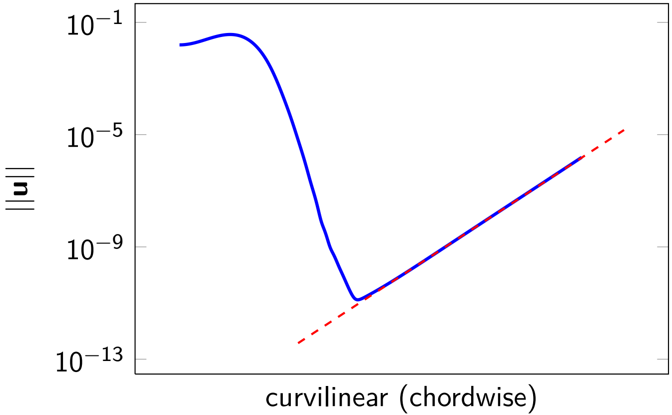


Perturbations — Direct and adjoint eigenvectors



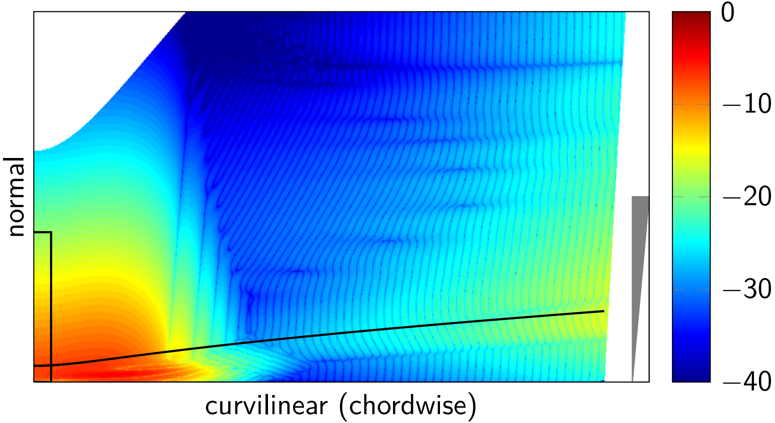
Adjoint eigenvector, localized upstream of and close to the attachment line region

Perturbations — Direct eigenvector



Direct eigenvector, norm of the velocity as a function of the chordwise coordinate

Perturbations — Direct eigenvector



Direct eigenvector, *u* component at fixed spanwise position; logarithmic scale in color

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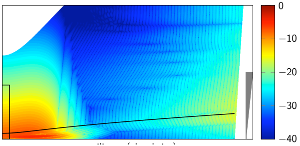
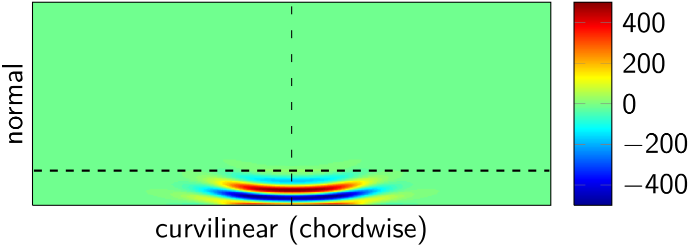
Perturbations — Receptivity $\quad \delta\,(amp) = -\langle \hat{\mathbf{q}}^{+}, \delta\hat{\mathbf{f}} \rangle$





Adjoint eigenvector, u^{+} component at fixed spanwise location

Perturbations — Wavemaker

Sensitivity

$$\mathcal{I}\left(obj, \hat{\mathbf{q}}, \hat{\mathbf{q}}^{+}, \hat{\mathbf{f}}, A\right) = obj - \langle \hat{\mathbf{q}}^{+}, (\sigma B + A)\,\hat{\mathbf{q}} - \hat{\mathbf{f}} \rangle$$

$$\frac{\partial \mathcal{I}}{\partial A}\delta A = 0 \qquad \Longrightarrow \qquad \delta(\sigma) = \langle \hat{\mathbf{q}}^{+}, \delta A\,\hat{\mathbf{q}} \rangle$$

δA — variation in the operator

- base-flow change
- boundary conditions
- feedback forcing
- generic structural change

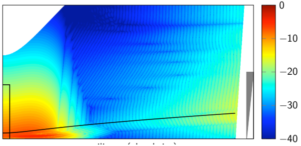
Feedback forcing

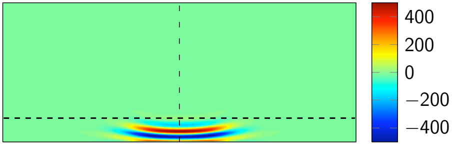
$$\delta A = C_0 \delta(x - x_0)$$

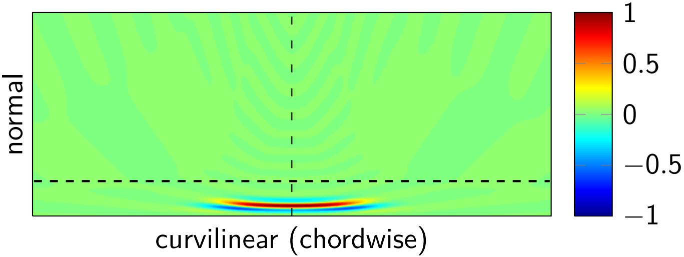
localized feedback forcing
dependent on the perturbation
e.g. small cylinder

Giannetti & Luchini, 2003, 2007

Perturbations — Wavemaker $\delta\sigma = \langle \hat{\mathbf{q}}^{+}, \delta A\, \hat{\mathbf{q}} \rangle$







Wavemaker at fixed spanwise location, $u^{+}\, u$

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Conclusions

Numerics

- Multigrid has been proven as an extremely efficient solver for the non-linear Navier-Stokes equations on stretched grids and with adaptive refinement
- The Krylov subspace method implemented in SLEPc has been used to identify the flow's coherent structures
- The location of the outflow boundary has no influence on the stability results

Physics

- Direct modes show features of both attachment line and crossflow vortices: they share the same growth rate and phase speed
- The amplitude and growth rate of the global eigenvectors are dependent on a very small region a few boundary layer thicknesses across the attachment line
- The localization of the most sensitive region suggests local stability results are valid

Future work

Numerics

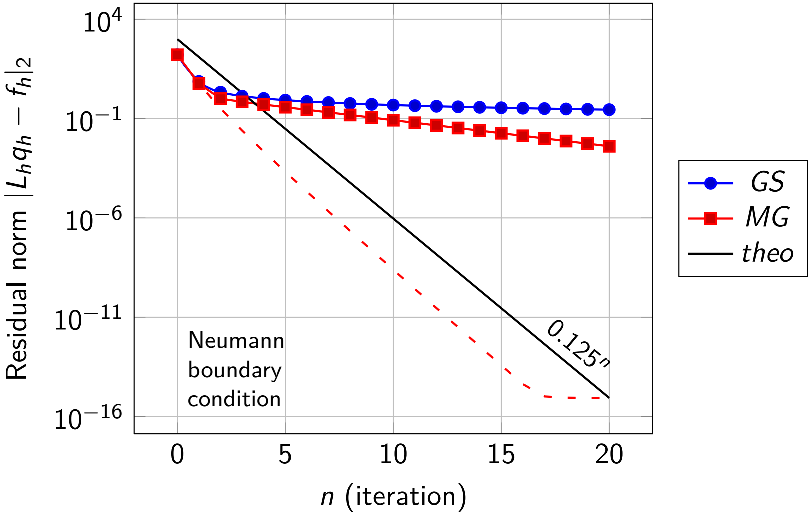
- Extension of multigrid to complex-valued linear problems will provide a fast and memory efficient solution strategy for eigenvector computations
- Multigrid refinement strategies for eigenvector computations
- Provide documentation and clean up/optimize the code (parallelization?)

Physics

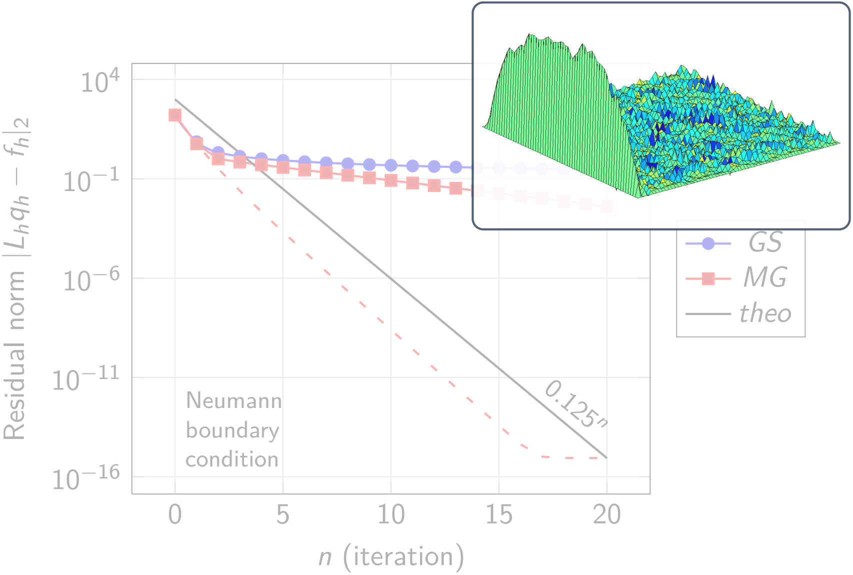
- The role of the adjoint's boundary values regarding receptivity to boundary conditions
- Identification of physical mechanisms leading to the transition between attachment line and crossflow vortices
- Parametric study and identification of the critical Reynolds number

Thanks

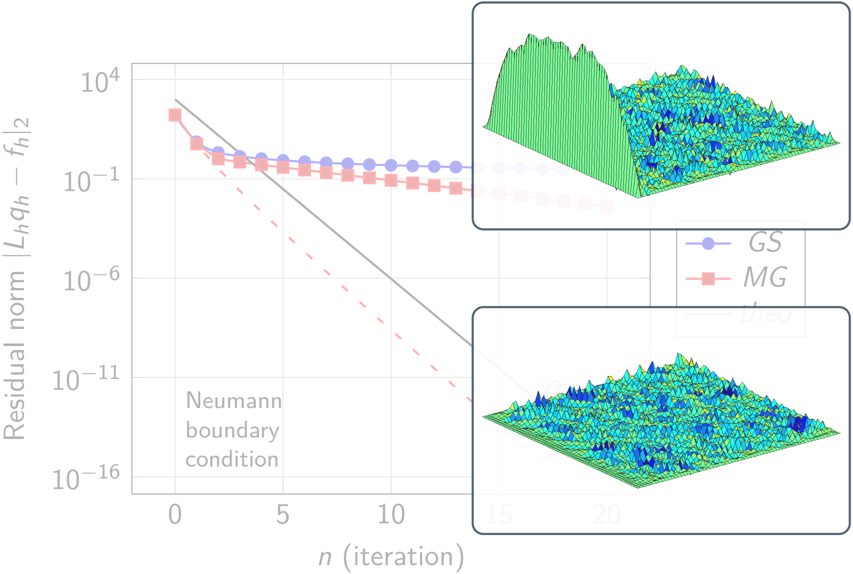
Gauss-Seidel vs. Multigrid $\quad \Delta q = f, \quad \partial_n q = 0$



Gauss-Seidel vs. Multigrid $\quad \Delta q = f, \quad \partial_n q = 0$



Gauss-Seidel vs. Multigrid $\Delta q = f, \quad \partial_n q = 0$



Gauss-Seidel vs. Multigrid $\Delta q = f, \quad \partial_n q = 0$