Four essays on macroeconomic volatility and instability under alternative exchange rate regimes

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FOREWORD

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DEDICATION

To my parents
Many thanks to Daniel Cohen, Guy Laroque, Philippe Martin and Hélène Rey for their useful suggestions following their careful reading of all or part of this dissertation.
"A partir d’un certain volume romanesque encore jamais obtenu avant lui, Balzac a dû avoir le pressentiment (il en donne la preuve en immergeant après coup chacun de ses ouvrages isolés dans l’ensemble de la Comédie Humaine) que toutes les données stylistiques changeaient de poids, comme un caillou qu’on plonge dans une rivière, et de nature, comme une détonation que l’écho d’une caverne à la fois étale et amplifie. Car l’interconnexion romanesque généralisée que réalise pour la première fois le coup de génie de la Comédie Humaine ne permet pas seulement un effet de mise en écho, le jeu d’un clavier multiplié de correspondances: elle permet, tout comme l’interconnexion d’un réseau électrique, de mobiliser le potentiel d’un secteur romanesque éloigné au service d’un récit qui languit ou qui blanche, et, de fait, le miracle de cette œuvre formellement si inégale est que tout sentiment de passage à vide y disparaît le plus souvent à la lecture: les réserves romanesques affluent d’elles-mêmes comme par un jeu de vases communicants; le tout ici ne commande pas seulement à la partie, il vient colmater ses déficiences, instantanément.

De ces vertus de la mise en relations globalisées, Balzac a fini par être parfaitement conscient, et par jouer quelquefois avec une subtilité prémonitoire. [...] Ce qui était d’abord simple articulation romanesque est devenu avec le temps, dans la conception de la Comédie Humaine, osmose et même circulation du sang. Le livre finit par enfonder des racines vives dans le mur auquel il s’était d’abord seulement agrafé."

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General presentation in French

The École Polytechnique allows its PhD students to write their dissertation in English, but requires that a general presentation in French of about twenty pages should then be included at the onset of the dissertation. The general presentation in French which follows accordingly is essentially made of a translation of the general introduction and the general conclusion.

Cette présentation générale offre une vue d’ensemble, précise l’arrière-plan et énonce les principaux résultats de la thèse.

Vue d’ensemble

La présente thèse de doctorat est intitulée “Quatre essais sur la volatilité et l’instabilité macroéconomiques sous différents régimes de change”. Comme le suggère ce titre, l’objet de la thèse est d’éclairer d’un jour nouveau les liens entre volatilité macroéconomique, instabilité macroéconomique et régime de change. La question au cœur de cette thèse est plus précisément: quelles volatilité et instabilité macroéconomiques pour quel régime de change? Pour bien comprendre de quoi il s’agit, considérons les différentes parties du titre tour à tour.


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La "volatilité macroéconomique" est définie comme la variabilité d'agrégats macroéconomiques clefs due à l'occurrence de chocs fondamentaux, par opposition aux chocs sunspot. L'identité des chocs fondamentaux considérés est détaillée dans le tableau 2. Ces chocs peuvent avoir une origine (structurale) microéconomique et une forme (réduite) macroéconomique, ou bien directement une origine (ad hoc) macroéconomique. La plupart d'entre eux affectent la demande globale -comme les chocs IS et les chocs de politique monétaire- ou l'offre globale -comme les chocs cost-push.

Les chocs fondamentaux peuvent survenir de façon asymétrique entre les pays, et c'est là une condition nécessaire pour que le taux de change puisse jouer un quelconque rôle. Cette asymétrie entre les chocs nationaux1 est modélisée soit de façon exogène soit de façon endogène. Dans le premier cas, définissant un pays domestique et un pays étranger, nous considérons soit les chocs domestiques seulement, soit à la fois les chocs domestiques et les chocs étrangers avec une structure de corrélation exogène implicite ou explicite. Dans le second cas, cette corrélation entre chocs domestiques et étrangers est endogène.

En l'absence de politique monétaire, ou plutôt en présence d'une politique monétaire passive, les variables macroéconomiques sont affectées par les chocs fondamentaux. Plus précisément, chaque variable réelle ou nominale est affectée à la fois par les chocs réels et par les chocs nominaux du fait de l'existence d'une rigidité nominale, portant typiquement sur les salaires ou sur les prix. La nature et la spécification des rigidités nominales considérées sont présentées dans le tableau 3. Nous appelons "volatilité macroéconomique ex ante" la variabilité des agrégats macroéconomiques correspondante.

Or la politique monétaire a prise non seulement sur les variables nominales, mais aussi sur les variables réelles du fait de l'existence de cette rigidité nominale. Il y a donc place pour une réaction de politique monétaire à ces chocs fondamentaux, de façon à contrer leurs effets sur les variables réelles et nominales. Comme l'indique le tableau 4, lorsqu'elle n'est pas exclusivement consacrée à la défense d'un change fixe, la politique monétaire peut avoir pour but de maximiser le niveau d'utilité du ménage représentatif qui est affecté par les chocs fondamentaux, mais tel n'est pas nécessairement le cas dans tous les chapitres.

1 De nombreuses appellations parsèment la littérature, qui qualifient la nature des chocs survenant au sein d'un groupe de pays: chocs symétriques, communs, asymétriques, anti-symétriques, spécifiques, idiosyncratisques, etc. Les nuances de sens entre ces différentes appellations se revêtent parfois être difficilement saisissables. Sous la terminologie d'Evans (1997), par exemple, qui n'est pas la nôtre, les chocs asymétriques sont ceux "dont les conséquences ne sont pas similaires dans tous les pays membres, et qui sont donc susceptibles d'appeler des réponses de politique économique différentes (en nature ou en ampleur)", tandis que selon Mundell (2005, p. 199) "all shocks are asymmetric in that they affect countries differently".

Nous appelons "volatilité macroéconomique ex post" la variabilité des agrégats macroéconomiques due à la fois à l’occurrence de chocs fondamentaux et à la réaction de politique monétaire à ces chocs. Lorsque la politique monétaire a pour but de maximiser le bien-être des ménages (ainsi qu’il devrait idéalement en être), la politique monétaire optimale revient en quelque sorte à minimiser cette volatilité macroéconomique ex post. Nous parlons alors du rôle d’ajustement de la politique monétaire, ce par quoi nous entendons que la politique monétaire devrait aider l’économie à s’ajuster optimalement aux chocs fondamentaux.

Nous venons de mentionner le terme "politique monétaire" à plusieurs reprises: il est sans doute temps de clarifier ce que nous entendons par ce terme. La politique monétaire est définie ici dans un sens large, qui inclut ce qui est communément appelé politique de change dans le cas d’un régime de change fixe mais ajustable. Comme le montre le tableau 4, les instruments de politique monétaire considérés sont variés: taux d’intérêt nominal, masse monétaire, taux de change nominal. Par ailleurs, un problème de crédibilité peut apparaître dans certains cas, dû au caractère temporellement incohérent de la politique monétaire optimale, de telle sorte que l’implémentation de l’équilibre first-best nécessite alors l’existence d’une technologie de commitment à la disposition de la banque centrale. L’absence d’une telle technologie constitue l’une des quelques imperfections de la politique monétaire considérées dans cette thèse.

L’"instabilité macroéconomique" est définie comme la variabilité d’agrégrats macroéconomiques clefs due à l’occurrence de chocs sunspot, par opposition aux chocs fondamentaux. Qu’appelons-nous chocs sunspot? Ce sont des chocs, c’est-à-dire des variables stochastiques exogènes, dont la réalisation conditionne l’issue du modèle considéré bien qu’ils ne soient pas spécifiés par ce modèle. En d’autres termes, les chocs sunspot sont les chocs qui peuvent être tenus responsables de la sélection d’un équilibre donné parmi plusieurs équilibres possibles\(^2\). Le tableau 5 donne un aperçu des chocs sunspot rencontrés dans cette thèse.

Bien entendu, une condition nécessaire et suffisante pour qu’apparaisse l’instabilité macroéconomique est l’existence d’équilibres multiples dans le modèle considéré\(^3\). Les chocs sunspot seront typiquement quantitatifs dans le

\(^3\) Une autre formulation consiste à dire que les chocs sunspot sont ceux à l’origine de ce que Burmeister, Flood et Garber (1985) appellent des bulles, c’est-à-dire des composantes qui apparaissent à l’équilibre en plus de la composante reflétant les fondamentaux.

\(^3\) Batini et Pearson (2002) utilisent le terme "instabilité" lorsqu’il existe plusieurs équilibres possibles et le terme "indétermination" lorsqu’il n’en existe aucun. Nous adoptons donc leur terminologie en ce qui concerne le terme "instabilité" (macroéconomique), mais nous utiliserons le terme "indétermination" (de l’équilibre) également dans le cas d’équilibres
cas d’un continuum d’équilibres possibles, qualitatifs dans le cas d’un nombre fini d’équilibres possibles. Notons par ailleurs que l’instabilité macroéconomique est habituellement indépendante de la volatilité macroéconomique, puisque les chocs sunspot peuvent survenir en l’absence de chocs fondamentaux - et vice versa. La variabilité de chaque agrégat macroéconomique peut donc être attribuée à deux composantes indépendantes: une composante intra-équilibre correspondant à la volatilité macroéconomique et une composante inter-équilibre correspondant à l’instabilité macroéconomique.

L’instabilité macroéconomique peut être de court ou de long terme dans cette thèse. L’instabilité macroéconomique de court terme est inextricablement liée aux anticipations auto-réalisatrices des agents privés, qui peuvent être promptes à sauter d’un équilibre à l’autre. Les choses se révèlent être moins claires pour l’instabilité macroéconomique de long terme, qui peut être le résultat d’un lent processus aveugle et tétanisant. Nous ne pouvons guère en dire plus à ce propos puisque nous n’examinons pas le cheminement d’un équilibre à l’autre.

Finalement, de la même façon que pour la volatilité macroéconomique ex ante et ex post respectivement, nous définissons l’"instabilité macroéconomique de court terme ex ante" comme l’instabilité macroéconomique de court terme apparaissant en présence d’une politique monétaire passive, et l’"instabilité macroéconomique de court terme ex post" comme l’instabilité macroéconomique de court terme apparaissant en présence d’une politique monétaire active. Tout comme la volatilité macroéconomique et contrairement à l’instabilité macroéconomique de long terme, l’instabilité macroéconomique de court terme réduit le bien-être de façon non ambiguë, de telle sorte que lorsqu’elle a pour but de maximiser le bien-être des ménages, la politique monétaire a un rôle de stabilisation, ce par quoi nous entendons qu’elle devrait réagir aux chocs sunspot de façon à réduire autant que possible, et idéalement complètement éliminer, cette instabilité macroéconomique de court terme ex post.4

Trois principaux "RÉGIMES DE CHANGE" sont considérés tout au long de cette thèse, comme en témoigne le tableau 6: le régime de change flexible, le régime de change fixe mais ajustable et le régime de change irrévocablement fixe. Sous un régime de change flexible, soit le taux de change nominal est déterminé par la parité de taux d'intérêt non couverte et la parité de pouvoir d'achat relative de long terme, soit il s'ajuste de façon à équilibrer la balance commercielle multiples plutôt que dans le cas d'absence d'équilibre. De notre point de vue en effet, l'on devrait parler de modèle inadéquat plutôt que d'équilibre indéterminé lorsqu'il n'existe aucun équilibre.

4L’élimination complète de cette instabilité macroéconomique de court terme ex post est requise pour assurer la sélection de l’unique équilibre exempt de bulles.
ciale. La banque centrale peut alors librement choisir sa politique monétaire, ou plutôt nous pouvons librement spécifier le but de la politique monétaire. Comme le montre le tableau 4, ce but peut être la maximisation du bien-être des ménages par exemple. Mais la politique monétaire peut aussi n’avoir aucun but spécifié. Elle peut même se voir spécifier aucun but: dans un tel cas, la politique monétaire est passive et le régime de change flexible est alors appelé de préférence “régime de change flottant”.

Sous le régime de change fixe mais ajustable, la banque centrale a pour charge la fixité du taux de change nominal. En d’autres termes, elle doit réagir aux chocs fondamentaux et aux chocs sunspot de façon à maintenir le taux de change nominal fixe ex post. Nous utilisons le terme “ajustable” parce que la banque centrale (ou plutôt, dirions-nous dire, le gouvernement) est autorisée à dévaluer ou révaluer la monnaie au prix d’un certain coût en terme de bien-être. Lorsqu’elle n’est pas autorisée à dévaluer ou révaluer, c’est-à-dire lorsque le coût de dévaluation ou réévaluation est infini, nous utilisons aussi le terme “ajustable” simplement par opposition à “irrévocablement fixe”, pour rappeler au lecteur qu’il existe une autorité monétaire dans les coulisses responsable de la fixité du taux de change nominal.

Sous le régime de change irrévocablement fixe, précisément, il n’existe plus de banque centrale dans les coulisses, et le taux de change est fixe ex ante. En d’autres termes, le régime de change irrévocablement fixe revient à une union monétaire. Comme l’indique le tableau 6, le régime de change irrévocablement fixe peut être bilatéral, lorsque deux grandes économies décident ensemble d’abandonner leurs banques centrales nationales, d’adopter une monnaie commune unique et de mettre en place une banque centrale supranationale en charge de la politique monétaire dans toute l’union monétaire. Il peut aussi être unilatéral, lorsqu’une petite économie ouverte accède à cette d’une grande économie, et la banque centrale supranationale coïncide alors avec la banque centrale de la grande économie.

Arrière-plan

Cette section présente succinctement la littérature et les hypothèses communes à tous les chapitres de cette thèse.

Les littératures concernées diffèrent substantiellement d’un chapitre à l’autre, comme le suggère le tableau 7, mais elles partagent tout de même quelques traits communs. Les deux principaux traits communs sont de nature keynésienne: il s’agit de l’existence de rigidités nominales et d’“esprits animaux”. La présence
de rigidités nominales (décrites dans le tableau 3) dans les modèles considérés est à l’origine de la non-neutralité de la politique monétaire. Quant aux “esprits animaux” de Keynes (1936), ils correspondent aux chocs suspot (caractérisés par le tableau 5) dans notre thèse et sont donc à l’origine de ce que nous appelons l’instabilité macroéconomique.


Nous choisissons de faire une distinction entre deux branches de la littérature issue d’Obstfeld et Rogoff (1995). La première est ce qu’on appelle l’économie nouveau-keynésienne, sur laquelle s’appuie la première partie de notre thèse, constituée des deux premiers chapitres. La seconde est ce qu’on appelle la New Open Economy Macroeconomics, sur laquelle s’appuie la seconde partie de notre thèse, constituée des deux derniers chapitres. Comme l’indique le tableau 8, ce qui distingue ces deux littératures dans le cadre de notre thèse est le nombre de périodes considérées, la nature de la rigidité nominale, les déterminants du taux de change nominal lorsqu’il est flexible, l’identité de l’instrument de politique monétaire et le rôle des anticipations passées et présentes.

Une différence supplémentaire entre les deux parties de notre thèse est que la première partie examine une petite économie ouverte, tandis que la seconde traite de deux grandes économies ouvertes. La première partie repose en effet principalement sur le modèle nouveau-keynésien d’une petite économie ouverte bâti par Galí et Monacelli (2002). Si nous avions choisi de considérer plusieurs grandes économies ouvertes dans notre première partie, nous aurions utilisé un modèle nouveau-keynésien à N pays, obtenu par exemple à partir de la version canonique proposée par Clarida, Galí et Gertler (2002).
Notons que notre seule hypothèse de rigidité nominale (des prix ou des salaires) ne suffit pas à donner un rôle au régime de change dans l'ajustement des économies aux chocs fondamentaux. Comme en témoigne le tableau 3, nous supposons de plus tout au long de la thèse qu'il n'y a pas local currency pricing (LCP), de telle sorte que le taux de change nominal puisse jouer son rôle keynésien traditionnel d'expansion-switching, les variations du taux de change étant entièrement reportées sur le prix des biens importés. Cette hypothèse d'un exchange rate pass-through égal à un est soutenue par Obstfeld et Rogoff (2000), qui se montrent très critiques vis-à-vis de l'approche alternative combinant pricing to market et local currency pricing.

Les arguments d'Obstfeld et Rogoff (2000) sont les suivants. Premièrement, le lien entre le taux de change nominal et les déviations observées de la loi du prix unique peuvent être dus à l'incorporation de biens non échangeables dans les indices de prix à la consommation supposés concerner uniquement les biens échangeables. Deuxièmement, l'horizon temporel sur lequel le trade invoicing rend les prix rigides semble trop court pour avoir un impact significatif sur les interactions macroéconomiques aux fréquences des cycles économiques. Troisièmement, les observations directes de currency invoicing sont incompatibles avec le point de vue selon lequel les exportateurs fixent le plus souvent leurs prix en la monnaie de l'importateur. Et quatrièmement, les observations internationales sur les taux de marge sont compatibles avec le point de vue selon lequel les exportateurs fixent le plus souvent leurs prix en leur propre monnaie. Leur point de vue est contesté par Devereux et Engel (2002), mais ces derniers doivent recourir à des hypothèses fortes en plus de l'hypothèse de local currency pricing pour pouvoir reproduire la variabilité observée du taux de change.

Précisons finalement les limites de notre champ opératoire. Quelles questions posons-nous et lesquelles ne posons-nous pas dans cette thèse? Quels sujets abordons-nous et lesquels ignorons-nous? Une première réponse à ces questions est que la politique monétaire (encore une fois, définie au sens large) est la seule politique économique considérée dans notre thèse. En particulier, nous ne considérons pas de politique budgétaire (endogène). Cette restriction est principalement justifiée par l'existence d'un délai d'implémentation qui fait de la politique budgétaire un outil d'ajustement inadéquat. Par la suite, nous utiliserons le terme « gouvernement », à la place de « banque centrale », lorsque la politique monétaire consistera en une politique de change.

La politique budgétaire n'est pas la seule grande absence de notre thèse. En effet, notre cadre d'analyse comporte de nombreuses autres hypothèses simplificatrices, pour ne pas dire de nombreuses autres limitations. Par exemple, nous
ne considérons aucun bien non échangeable et nous ignorons tout investissement (endogène) en capital, simplement parce que nous n’en avons pas besoin pour faire passer notre message. Leur prise en compte rendrait notre cadre d’analyse plus réaliste mais aussi plus complexe, probablement sans affecter nos résultats. Nous choisissons de nous limiter au cadre d’analyse le plus simple possible afin de rendre notre message le plus clair possible.

Trois derniers points sont à noter. Premièrement, nous considérons un unique ménage représentatif dans tous les chapitres, de telle sorte que nous ne traitons pas du sujet des inégalités et de la redistribution. Deuxièmement, nous faisons l’hypothèse tout au long de la thèse que les agents partagent tous les mêmes anticipations rationnelles à chaque date, de telle sorte que nous ignorons le problème de coordination qui peut notamment se poser en présence d’équilibres multiples. En particulier, parce que les agents sont atomistiques, l’équilibre socialement optimal n’est pas plus probable que les autres en présence d’équilibres multiples. Troisièmement, tous les modèles considérés sont notamment basés sur l’hypothèse de concurrence monopolistique, qui convient aux pays développés davantage qu’aux pays en voie de développement. Notre discours porte donc résolument sur ce qui pourrait s’appeler des “pays développés homogènes”.

Résultats

Quelles conclusions tirer de notre thèse? Le tableau 9 résume les principaux résultats obtenus chapitre par chapitre et leurs implications directes en matière de politique économique. Nous ne commentons pas plus avant ces résultats, quelque nouveau et prometteurs qu’ils soient, parce qu’une telle vue kaleidoscopique n’a pas sa place dans la présentation générale d’une thèse. Nous nous intéressons plutôt aux leçons générales à tirer de notre thèse.

Cette thèse éclaire d’un jour nouveau les liens entre volatilité macroéconomique, instabilité macroéconomique et régime de change, à la fois d’un point de vue positif et d’un point de vue normatif. Elle clarifie notamment la forme que peut prendre la volatilité macroéconomique dans un cadre nouveau-keynésien, à la fois théoriquement et empiriquement, pour une petite économie ouverte sous différents régimes de change. Mais le plus important sans doute, c’est que d’une part elle dévoile des sources jusqu’alors inconnues d’instabilité macroéconomique sous différents régimes de change, et que d’autre part elle propose de nouveaux remèdes à l’instabilité macroéconomique sous différents régimes de change. Là réside sans doute notre contribution originale.
la plus significative.

Ces sources de et ces remèdes à l'instabilité macroéconomique nouvellement identifiés sont présentés dans le tableau 10. Bien que leur nature puisse substantiellement différer d'un chapitre à l'autre, ces sources et ces remèdes sont tous inextricablement liés au régime de change. Le tableau 11 montre l'instabilité macroéconomique de court terme correspondante, avec ou sans remède administré, et le tableau 12 montre de façon similaire l'instabilité macroéconomique de long terme, ainsi que l'instabilité macroéconomique de court terme sous un régime de change irrévocablement fixe, toutes deux sans remède administré. Notons qu'il n'existe habituellement pas de remède miracle à l'instabilité macroéconomique. Comme l'indique le tableau 11 en effet, les remèdes proposés auront souvent pour effet de réduire l'ensemble des équilibres multiples, plutôt que d'éliminer complètement la possibilité d'équilibres multiples.

Notre thèse débouche également sur des considérations sur le régime de change optimal. En effet, qu'elle soit optimale ou non, la politique monétaire est considérée dans tous les chapitres comme une politique conditionnelle au régime de change en vigueur, mais rien ne nous interdit naturellement de nous poser la question (située en amont) du régime de change optimal. Ce régime de change optimal est défini comme celui qui maximise le bien-être des ménages ou qui minimise la fonction de perte du gouvernement, et correspond plus ou moins à celui associé aux moindres volatilité et instabilité macroéconomiques ex post.

Le tableau 19 classe les régimes de change selon le critère de volatilité macroéconomique ex post, conditionnellement soit à l'absence d'instabilité macroéconomique ex post, soit à la comparabilité des ensembles d'équilibres multiples considérés. Sans surprise, le régime de change flexible avec commitment est classé en première position dans tous les cas sauf un, parce que la banque centrale a alors toute liberté pour réagir aux chocs fondamentaux. Les deux régimes de change fixes sont le plus souvent classés ex aequo, puisqu'ils entraînent la même volatilité macroéconomique ex post. Notons qu'ils peuvent être préférables au régime de change flexible sans commitment. Tous ces résultats présentés dans le tableau 19 sont plus ou moins en accord avec les résultats conventionnels.

Le tableau 14 classe les régimes de change selon le critère d'instabilité macroéconomique de long terme, ou bien à l'instabilité macroéconomique de court terme sous un régime de change irrévocablement fixe, ne sont pas considérés dans le tableau 12 simplement parce que leur imputation soulève des difficultés pratiques, puisqu'ils correspondent à des politiques structurelles.

L'unique exception concerne le chapitre 4, dans le cas particulier où la volatilité macroéconomique augmente le bien-être en réalité.
La nouveauté réside ici à la fois dans le principe d’un tel classement et dans les résultats de ce classement. Dans son principe, parce qu’à notre connaissance un tel classement n’a jamais été dressé dans la littérature. Dans ses résultats, parce que ce classement réhabilite le régime de change irrévocablement fixe. En effet, ce régime est maintenant classé premier dans tous les cas sauf un, ce qui le place au-dessus du régime de change flexible avec commitment. Le régime de change fixe mais ajustable est classé dernier dans tous les cas, tandis que les deux régimes de change flexible sont classés ex aequo entre les deux régimes de change fixe.

Le régime de change irrévocablement fixe, qu’il s’agisse d’une union monétaire ou d’une dollarisation, est classé premier selon le critère d’instabilité macroéconomique ex post grâce à sa capacité à ancrer les anticipations des agents privés. Dans le modèle nouveau-keynésien d’une petite économie ouverte, cette propriété assure que les équilibres divergents sont bel et bien exclus a priori sous un régime de change irrévocablement fixe, parce que les agents privés savent qu’il n’y aura pas de banque centrale nationale pour réagir à ces équilibres divergents. Sous un régime de change flexible, les équilibres divergents peuvent survenir au contraire si la règle de politique monétaire suivie ne les exclut pas, et il se pourrait bien fâcheusement que n’existent aucune règle de politique monétaire excluant les équilibres divergents sous certaines spécifications. Le seul cas où le régime de change flexible est préférable au régime de change irrévocablement fixe, selon le critère d’instabilité macroéconomique ex post, est le cas où des règles de politique monétaire adéquates peuvent être trouvées qui excluent les équilibres multiples sous le régime de change flexible, que ces équilibres soient convergents ou divergents, tandis que le régime de change irrévocablement fixe se révèle être compatible avec plusieurs équilibres convergents.

Finalement, quel point de vue offrons-nous sur la question de la désirabilité de l’Union Monétaire Européenne (UME)? Avant tout, il convient de reconnaître que notre thèse porte essentiellement sur ce qui est habituellement considéré comme les coûts d’ajustement et de stabilisation associés à l’Euro. En d’autres termes, notre champ opératoire exclut ce qui est habituellement considéré comme les bénéfices structurels associés à l’Euro. Notre thèse aura donc a priori tendance à offrir un point de vue euroscéptique biaisé. Cela dit, si elle se trouve en réalité qu’elle se révèle être plutôt en faveur de l’UME. En effet, bien que nous reconnaissons la possibilité que l’UME puisse favoriser endogènment les chocs asymétriques et ainsi augmenter la volatilité macroéconomique dans le long terme, nous soutenons qu’une telle issue ne décroît pas nécessairement le bien-être. Et en ce qui concerne les effets de l’UME à court terme, nous préférons
sans ambigüité l'UME à une quelconque variante du Système Monétaire Européen (SME), que nous considérons comme fondamentalement instable. Si l'alternative à l'UME est un régime de change flexible cependant, alors nous n'offrons aucune conclusion tranchée de façon inconditionnelle, et recommandons simplement d'examiner de plus près pays par pays ce qu'il adviendrait non seulement de la volatilité macroéconomique, mais aussi de l'instabilité macroéconomique en UME. Les dés sont jetés pour douze pays, mais c'est maintenant au tour du Danemark, du Royaume-Uni et de la Suède de décider d'adopter l'Euro ou non. Nous ne pouvons que leur souhaiter un débat éclairé sur le volet économique préalablement à leur décision. De ce point de vue, la récente parution par le gouvernement britannique de diz-huit études de qualité sur les conséquences de l'adhésion du Royaume-Uni à l'UME donne matière à l'optimisme.
<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Date</th>
<th>Co-auteur</th>
<th>Présentations</th>
<th>Soumission ou publication</th>
<th>Commentaires reçus</th>
</tr>
</thead>
</table>
| 1        | Septembre 2002  | -                   | • séminaire interne CREST-LMA, Malakoff, France, 11/10/2002  
  • 3èmes Doctoriales d'Économie et Finances Internationales organisées par le THEMA, le GDR 877 du CNRS et le GRIFI, Nanterre, France, 18-19/12/2002  
  • séminaire interne du CEPII, Paris, France, 11/03/2003 | -                                                 | Gilbert Abraham-Frois, Agnès Bénassy-Quéré, Martine Carré, Daniel Cohen, Guy Laroque, Philippe Martin, Hélène Rey |
| 2        | Juin 2003       | -                   | • séminaire interne du CEPII, Paris, France, 11/03/2003  
  • séminaire interne CREST-LMA, Malakoff, France, 06/06/2003 | -                                                 | Daniel Cohen, Hélène Rey                           |
  • Journées de l’Association Française de Sciences Économiques "Croissance, Convergences et Intégration Européennes", Lille, 26-27/05/2003 | -                                                 | Daniel Cohen, Philippe Martin, Hélène Rey          |
**Tableau 2: Chocs fondamentaux.**

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Chocs microéconomiques</th>
<th>Chocs macroéconomiques</th>
<th>Chocs résultants</th>
<th>Nationalité des chocs</th>
<th>Asymétrie des chocs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>choc sur le paramètre mesurant la préférence pour le présent du ménage représentatif</td>
<td>choc de dépense publique, choc sur le terme de risk-premium dans la parité non couverte des taux d'intérêt</td>
<td>choc sur l'équation IS (choc IS)</td>
<td>domestique</td>
<td>exogène</td>
</tr>
<tr>
<td></td>
<td>choc de productivité</td>
<td>-</td>
<td>choc sur la courbe de Phillips (choc cost-push)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>choc sur le paramètre mesurant la préférence pour le présent du ménage représentatif</td>
<td>choc de dépense publique, choc sur le terme de risk-premium dans la parité non couverte des taux d'intérêt</td>
<td>choc sur l'équation IS (choc IS)</td>
<td>domestique et étrangère</td>
<td>exogène</td>
</tr>
<tr>
<td></td>
<td>choc de productivité</td>
<td>-</td>
<td>choc sur la courbe de Phillips (choc cost-push)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>choc de taux d'intérêt nominal dû à la &quot;main tremblante&quot; de la banque centrale</td>
<td>choc sur la règle de politique monétaire (choc de politique monétaire)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>choc sur le coût politique à quitter le régime de change fixe</td>
<td>choc sur le coût fixe de dévaluation</td>
<td>domestique et étrangère</td>
<td>exogène</td>
</tr>
<tr>
<td>4</td>
<td>choc sur la préférence relative du ménage représentatif pour les différents biens</td>
<td>-</td>
<td>choc de demande spécifique à une industrie</td>
<td>domestique et étrangère</td>
<td>endogène</td>
</tr>
</tbody>
</table>
### Tableau 3: rigidités nominales.

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Salaires</th>
<th>Prix</th>
<th>Nombre $N$ de périodes</th>
<th>Loi du prix unique</th>
<th>Pricing to market</th>
<th>Local currency pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>flexibles (s’ajustent de façon à équilibrer le marché du travail)</td>
<td>rigides dans le court terme (fixation de prix forward-looking à la Calvo)</td>
<td>$N = \infty$</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>flexibles (s’ajustent de façon à équilibrer le marché du travail)</td>
<td>rigides dans le court terme (fixation de prix partiellement forward-looking à la Calvo, partiellement backward-looking)</td>
<td>$N = \infty$</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>rigidès (fixation des salaires forward-looking)</td>
<td>flexibles (s’ajustent de façon à équilibrer le marché des biens)</td>
<td>$N = 1$</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>rigidès dans le court terme, flexibles dans le long terme (s’ajustent de façon à rendre les ménages indifférents entre travailler dans l'industries 1 ou dans l'industrie 2)</td>
<td>flexibles (s’ajustent de façon à équilibrer le marché des biens)</td>
<td>$N = 1$</td>
<td>-</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Chapitre</td>
<td>Instrument de la politique monétaire</td>
<td>But de la politique monétaire (lorsqu’il n’est pas exclusivement le maintien d’un change fixe)</td>
<td>Arguments de la fonction d’utilité du ménage représentatif</td>
<td>Dispositif de commitment</td>
<td>Imperfections de la politique monétaire (selon le critère du bien-être des ménages)</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------</td>
<td>------------------------</td>
<td>----------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>R (M absent ou déterminé résiduellement, E déterminé résiduellement)</td>
<td>maximisation de la fonction d’utilité du ménage représentatif</td>
<td>consommation, travail, potentiellement monnaie</td>
<td>nécessaire sous le régime de change fixe mais ajustable, préférable sous le régime de change flexible</td>
<td>• potentiellement sans dispositif de commitment</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>R (M absent ou déterminé résiduellement, E déterminé résiduellement)</td>
<td>non spécifié</td>
<td>consommation, travail, potentiellement monnaie</td>
<td>non nécessaire</td>
<td>• potentiellement non consacrée à la maximisation du bien-être des ménages • potentiellement sans dispositif de commitment • source de perturbations exogènes</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>E (M et R absents)</td>
<td>maximisation de la fonction d’utilité du gouvernement</td>
<td>consommation, travail</td>
<td>nécessaire dans le cas de coopération internationale</td>
<td>• non consacrée à la maximisation du bien-être des ménages • potentiellement non coopérative • potentiellement non coordonnée lorsqu’elle n’est pas coopérative</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>M (R absent, E déterminé résiduellement)</td>
<td>aucun</td>
<td>consommation, monnaie</td>
<td>non nécessaire</td>
<td>• non consacrée à la maximisation du bien-être des ménages</td>
<td></td>
</tr>
</tbody>
</table>

---

7 R: taux d’intérêt nominal; M: masse monétaire; E: taux de change nominal.
Tableau 5: chocs sunspot.

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Multiplicité d'équilibres ex ante</th>
<th>Chocs sunspot</th>
<th>Instabilité macroéconomique</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>infinie</td>
<td>quantitatifs</td>
<td>de court terme</td>
</tr>
<tr>
<td></td>
<td>finie</td>
<td>qualitatifs</td>
<td>de long terme</td>
</tr>
<tr>
<td>1</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
### Tableau 6: régimes de change.

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Nombre, taille et ouverture des économies</th>
<th>Régime de change flexible</th>
<th>Régime de change fixe mais ajustable</th>
<th>Coût de dévaluation ou de révaluation</th>
<th>Régime de change irrévocablement fixe (unilatéral)</th>
<th>Régime de change irrévocablement fixe (bilateral)</th>
<th>Taux de change nominal endogène (lorsqu'il est flexible ou ajustable)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 petite économie ouverte (incluant le cas particulier de l'économie fermée)</td>
<td>✓</td>
<td>✓</td>
<td>infini</td>
<td>✓</td>
<td>-</td>
<td>déterminé par la PNCTI et la PPA relative de long terme</td>
</tr>
<tr>
<td>2</td>
<td>1 petite économie ouverte</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td>déterminé par la PNCTI et la PPA relative de long terme</td>
</tr>
<tr>
<td>3</td>
<td>2 petites économies ouvertes</td>
<td>-</td>
<td>✓</td>
<td>fini</td>
<td>-</td>
<td>-</td>
<td>choisi optimalement par le gouvernement</td>
</tr>
<tr>
<td>4</td>
<td>2 grandes économies ouvertes</td>
<td>✓</td>
<td>✓</td>
<td>infini</td>
<td>-</td>
<td>✓</td>
<td>s'ajuste de façon à équilibrer la balance commerciale</td>
</tr>
</tbody>
</table>

*PNCTI: parité non couverte des taux d'intérêt; PPA: parité de pouvoir d'achat.*
<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Littératures</th>
<th>Études les plus proches</th>
<th>Classement JEL</th>
<th>Mots-clefs</th>
<th>Théorique et/ou empirique</th>
<th>Logiciels utilisés (sauf traitement de texte)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>• New Economic Geography • New Open Economy Macroeconomics</td>
<td>Fujita, Krugman et Venables (1999), Ricci (1997, 1998)</td>
<td>F12, F15, F33, F41, R12, R13</td>
<td>chocs asymétriques, équilibres multiples, New Economic Geography, régime de change, spécialisation, zone monétaire optimale</td>
<td>théorique</td>
<td>Mathematica 4.2.0.0</td>
</tr>
</tbody>
</table>
**Tableau 8: parties.**

<table>
<thead>
<tr>
<th>Partie</th>
<th>Titre de la partie</th>
<th>Chapitre</th>
<th>Titre du chapitre</th>
<th>Littérature</th>
<th>Économies</th>
<th>Nombre N de périodes</th>
<th>Rigidité nominale</th>
<th>Taux de change endogène (lorsqu'il est flexible ou ajustable)</th>
<th>Instrument de la politique monétaire</th>
<th>La situation présente dépend de...</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>“une perspective nouvelle-keynésienne sur une petite économie ouverte”</td>
<td>1</td>
<td>“Règles de politique monétaire forward-looking pour exclure les équilibres multiples”</td>
<td>littérature nouvelle-keynésienne</td>
<td>petite économie ouverte</td>
<td>$N = \infty$</td>
<td>salaires flexibles, prix rigides</td>
<td>déterminé par la parité non couverte des taux d'intérêt et par la parité de pouvoir d'achat relative de long terme</td>
<td>taux d'intérêt nominal</td>
<td>...l'anticipation présente de la situation future</td>
</tr>
<tr>
<td>II</td>
<td>“une perspective de New Open Economy Macroeconomics sur deux économies ouvertes”</td>
<td>3</td>
<td>“Coordination, coopération, contagion et crises de change”</td>
<td>littérature de New Open Economy Macroeconomics</td>
<td>économies ouvertes</td>
<td>$N = 1$</td>
<td>salaires rigides, prix flexibles</td>
<td>s'ajuste de façon à équilibrer la balance commerciale, ou bien choisi optimalement par le gouvernement</td>
<td>masse monétaire ou taux de change nominal</td>
<td>...l'anticipation passée de la situation présente</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>“Chocs de demande endogènements asymétriques sous différents régimes de change”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Tableau 9: principaux résultats et implications en termes de politique économique.

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Principaux résultats</th>
<th>Implications en termes de politique économique</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>• quel que soit le régime de change, les règles de politique monétaire optimales (i.e. les règles assurant l'implémentation de l'équilibre optimal, i.e. les règles éliminant l'instabilité macroéconomique et minimisant la volatilité macroéconomique) sont nécessairement forward-looking</td>
<td>• quels que soient le régime de change en vigueur et la crédibilité de la banque centrale, la règle de politique monétaire suivie doit être forward-looking</td>
</tr>
<tr>
<td>2</td>
<td>• si le Royaume-Uni adoptait l'€uro aujourd'hui, il échapperait à l'instabilité macroéconomique mais pourrait bien faire l'expérience d'une volatilité macroéconomique accrue</td>
<td>• le Royaume-Uni ne devrait pas adopter l'€uro sans adapter d'abord son économie structurellement</td>
</tr>
<tr>
<td>3</td>
<td>• un régime de change fixe mais ajustable est d'autant plus vulnérable aux crises de change que le degré de concurrence commerciale monopolistique entre les pays considérés est élevé • la coopération internationale est préférable à la coordination internationale (elle-même préférable à l'absence de coopération et de coordination) parce qu'elle réduit davantage le risque de crise de change • la coordination introduit un nouveau canal de transmission des crises de change</td>
<td>• lorsque la coopération internationale n'est pas crédible aux yeux des agents privés (par exemple en l'absence d'institution internationale faisant respecter les accords), les gouvernements devraient se coordonner pour choisir le meilleur équilibre non coopératif</td>
</tr>
<tr>
<td>4</td>
<td>• en présence de chocs sectoriels, les régimes de change fixes favorisent la spécialisation nationale (et par conséquent les chocs asymétriques entre les pays) davantage que le régime de change flexible</td>
<td>• bien que les régimes de change fixe entraînent une plus grande volatilité macroéconomique que le régime de change flexible, il n'existe pas de régime de change inconditionnellement préférable aux autres, de telle sorte que le choix du régime de change devrait être fait au cas par cas</td>
</tr>
</tbody>
</table>
### Tableau 10: sources de et remèdes à l’instabilité macroéconomique.

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Régime de change concerné</th>
<th>Sources nouvellement identifiées de l’instabilité macroéconomique</th>
<th>Remèdes nouvellement identifiés à l’instabilité macroéconomique</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>régime de change flexible (avec ou sans commitment), régime de change fixe mais ajustable</td>
<td>impossibilité d’exclure tous les équilibres non optimaux, convergents ou divergents, par une règle de politique monétaire purement backward-looking</td>
<td>adoption d’une règle de politique monétaire adéquate forward-looking</td>
</tr>
<tr>
<td>2</td>
<td>régime de change irrévocablement fixe (unilatéral)</td>
<td>existence d’équilibres multiples (convergents) en l’absence de politique monétaire</td>
<td>réformes structurelles</td>
</tr>
<tr>
<td>3</td>
<td>régime de change fixe mais ajustable</td>
<td>existence de spillovers commerciaux, responsables de la contagion des crises de change</td>
<td>coordination internationale, coopération internationale</td>
</tr>
<tr>
<td>4</td>
<td>régime de change flexible, régime de change irrévocablement fixe (bilatéral)</td>
<td>existence de volatilité macroéconomique (due à des chocs de demande industriels) en présence de coûts de transport et de biens intermédiaires</td>
<td>partage des risques entre les pays</td>
</tr>
<tr>
<td>Chapitre</td>
<td>Signification de &quot;ex ante&quot;</td>
<td>Multiplicité ( M ) d'équilibres ( \text{ex ante} )</td>
<td>Signification de &quot;ex post&quot;</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>régime de change flexible (avec ou sans commitment) avec une règle de politique monétaire arbitraire</td>
<td>( M = \infty )</td>
<td>régime de change flexible (avec ou sans commitment) avec une règle de politique monétaire optimale</td>
</tr>
<tr>
<td>1</td>
<td>régime de change fixe mais ajustable avec une règle de politique monétaire arbitraire</td>
<td>( M = \infty )</td>
<td>régime de change fixe mais ajustable avec une règle de politique monétaire optimale</td>
</tr>
<tr>
<td>2</td>
<td>régime de change flexible (avec ou sans commitment) avec une règle de politique monétaire arbitraire</td>
<td>( M = \infty )</td>
<td>régime de change flexible (avec ou sans commitment) avec une règle de politique monétaire optimale</td>
</tr>
<tr>
<td>2</td>
<td>régime de change fixe mais ajustable avec une règle de politique monétaire arbitraire</td>
<td>( M = \infty )</td>
<td>régime de change fixe mais ajustable avec une règle de politique monétaire optimale</td>
</tr>
<tr>
<td>3</td>
<td>régime de change fixe mais ajustable sans coordination internationale ni coopération internationale</td>
<td>( M = 1 )</td>
<td>régime de change fixe mais ajustable avec coordination internationale</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( M = 2 )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( M = 4 )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>régime de change fixe mais ajustable sans coordination internationale ni coopération internationale (équilibres symétriques entre les pays)</td>
<td>( M = 1 )</td>
<td>régime de change fixe mais ajustable avec coopération internationale (équilibres symétriques entre les pays)</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>( M = 2 )</td>
<td></td>
</tr>
</tbody>
</table>
### Tableau 12: instabilité macroéconomique de court terme et de long terme.

| Chapitre | Régime de change | Ensemble $S$ des multiplicités d'équilibres possibles
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>régime de change irrévocablement fixe (unilatéral)</td>
<td>$S = {1}$</td>
</tr>
<tr>
<td>2</td>
<td>régime de change irrévocablement fixe (unilatéral)</td>
<td>$S \in {{0}, {1}, {\infty}}$</td>
</tr>
<tr>
<td>4</td>
<td>cas de référence</td>
<td>$S \in {{0, 1}, {1, 2}}$</td>
</tr>
<tr>
<td></td>
<td>régime de change flexible</td>
<td>$S \in {{1, 2}, {0, 1, 2}}$</td>
</tr>
<tr>
<td></td>
<td>régime de change irrévocablement fixe (bilatéral)</td>
<td>$S \in {{0, 1}, {0, 1, 2}}$</td>
</tr>
</tbody>
</table>

---

9 Parce que nous limitons notre attention à deux équilibres dégénérés dans le chapitre 4 (l'équilibre de spécialisation nationale complète et l'équilibre de dispersion industrielle parfaite), la multiplicité des équilibres ne peut pas excéder deux.
Tableau 13: classement des régimes de change selon le critère de volatilité macroéconomique ex post.

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Régime de change flexible avec commitment</th>
<th>Régime de change flexible sans commitment</th>
<th>Régime de change fixe mais ajustable</th>
<th>Régime de change irrévocablement fixe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1er</td>
<td>2ème</td>
<td>3ème ex aequo</td>
<td>3ème ex aequo</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1er</td>
<td></td>
<td></td>
<td>2ème</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>2ème</td>
<td>1er</td>
</tr>
<tr>
<td>4</td>
<td>1er ex aequo</td>
<td>1er ex aequo</td>
<td></td>
<td>2ème</td>
</tr>
<tr>
<td></td>
<td>2ème ex aequo</td>
<td>2ème ex aequo</td>
<td></td>
<td>1er</td>
</tr>
</tbody>
</table>

Le critère de classement est plus précisément le bien-être des ménages (chapitres 1 et 4), la fonction de perte du gouvernement (chapitre 3) ou la variance de l'inflation et de la production (chapitre 2), sous l'hypothèse de politique monétaire optimale lorsque cette hypothèse est pertinente (chapitre 1). Le régime de change classé 1er est celui associé au bien-être des ménages le plus élevé, à la fonction de perte du gouvernement la plus basse ou à la variance de l'inflation et de la production la plus basse. En l'absence d'instabilité macroéconomique, le classement de deux régimes de change donnés ne pose aucune difficulté, puisque un équilibre unique est alors comparé à un autre équilibre unique. En présence d'instabilité macroéconomique, le classement de deux régimes de change donnés est réalisé seulement dans le cas où tous les équilibres possibles sous un régime de change sont préférables à tous les équilibres possibles sous l'autre régime de change. Notons finalement qu'à un chapitre donné peuvent correspondre plusieurs classements, selon la valeur des paramètres dans le modèle correspondant.
**Tableau 14:** classement des régimes de change selon le critère d'instabilité macroéconomique ex post.\(^{11}\)

<table>
<thead>
<tr>
<th>Chapitre</th>
<th>Régime de change flexible avec commitment</th>
<th>Régime de change flexible sans commitment</th>
<th>Régime de change fixe mais ajustable</th>
<th>Régime de change irrévocablement fixe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1(^{er}) ex aequo</td>
<td>1(^{er}) ex aequo</td>
<td>1(^{er}) ex aequo</td>
<td>1(^{er}) ex aequo</td>
</tr>
<tr>
<td>2</td>
<td>1(^{er})</td>
<td></td>
<td>2(^{ime})</td>
<td>2(^{ime})</td>
</tr>
<tr>
<td></td>
<td>2(^{ime})</td>
<td></td>
<td>2(^{ime})</td>
<td>1(^{er})</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>2(^{ime})</td>
<td>1(^{er})</td>
</tr>
<tr>
<td>4</td>
<td>2(^{ime}) ex aequo</td>
<td>2(^{ime}) ex aequo</td>
<td></td>
<td>1(^{er})</td>
</tr>
</tbody>
</table>


\(^{11}\) Le critère de classement est simplement la multiplicité des équilibres. Le régime de change classé 1\(^{er}\) est celui associé à la moindre multiplicité d'équilibres. Notons qu'à un chapitre donné peuvent correspondre plusieurs classements, selon la valeur des paramètres dans le modèle correspondant.
General introduction

This introduction gives an overview and outlines the general background of the dissertation.

Overview

The present PhD dissertation is entitled "Four essays on macroeconomic volatility and instability under alternative exchange rate regimes". As suggested by this title, the object of the dissertation is to shed a new light on the links between macroeconomic volatility, macroeconomic instability and the exchange rate regime. The question at the core of this dissertation is more precisely: how much macroeconomic volatility and macroeconomic instability does a given exchange rate regime entail? To fully understand what it is all about, let us pay attention to the title and consider its different parts in turn.

"FOUR ESSAYS", corresponding to as many chapters, make up this dissertation. Their statuses are presented in table 1. One of them has been written with Professor Philippe Martin and published in the Journal of International Economics. I am the sole author of the others, which are yet to be submitted to any journal. All of them have been written between July 1998 and July 2003. Although as a whole these chapters form something coherent, into which a common problematics breathes life, each of them tackles the dissertation topic from its own point of view, represents individually an original contribution and as such, can be read independently of the others. Needless to say, we view this diversity as one of our dissertation's strong points.

"MACROECONOMIC VOLATILITY" is defined as the variability of key macroeconomic aggregates due to the occurrence of fundamental shocks, as opposed to sunspot shocks. The identity of the fundamental shocks considered is detailed in table 2. These shocks may have either a microeconomic (structural) origin and a macroeconomic (reduced) form, or directly a macroeconomic (ad hoc) origin. Most of them end up having an effect either on aggregate demand - e.g.
IS shocks and monetary policy shocks - or aggregate supply - e.g. cost-push shocks.

Fundamental shocks may occur asymmetrically across countries, which is a necessary condition for the exchange rate regime to play any role. This asymmetry between national shocks\(^1\) is modelized either as exogenous or as endogenous. In the former case, defining a domestic country and a foreign country, we may consider either domestic shocks only, or both domestic and foreign shocks with an implicit or explicit exogenous correlation structure. In the latter case, this correlation between domestic and foreign shocks is endogenous.

In the absence of monetary policy, or rather in the presence of a passive monetary policy, macroeconomic variables are affected by the fundamental shocks. More precisely, each real or nominal variable is affected by both real and nominal shocks because of the existence of a nominal rigidity, typically some sort of wage or price stickiness. The nature and the specification of the nominal rigidities considered are displayed in table 3. The resulting variability in macroeconomic aggregates we call "ex ante macroeconomic volatility".

Now, monetary policy can affect not only nominal variables, but also real variables because of the existence of this nominal rigidity. Hence there is room for a monetary policy reaction to the fundamental shocks, so as to counter their effects on real and nominal variables. As indicated in table 4, when not exclusively directed towards the defence of a currency peg, monetary policy may be aimed at maximizing the utility level of the representative household which is affected by the fundamental shocks, but such is not necessarily the case in some chapters.

The variability of macroeconomic aggregates due to both the occurrence of fundamental shocks and the monetary policy reaction to these shocks we call "ex post macroeconomic volatility". When monetary policy is aimed at maximizing household welfare (as it should ideally be), the optimal monetary policy amounts somehow to minimize this ex post macroeconomic volatility. We then say that monetary policy has an "adjustment role", by which we mean that it should help the economy to adjust optimally to the fundamental shocks.

We have just mentioned the term "monetary policy" on several occasions:

\(^1\)Many names can be found in the literature, which describe the nature of the shocks occurring within a group of countries: symmetric, common, asymmetric, antisymmetric, country-specific, idiosyncratic shocks, etc. The shades of meaning between these labels prove sometimes uneasy to be grasped. Under Erkel-Rousse’s (1997) terminology for instance, which is not ours, so-called asymmetric shocks are those "dont les consequences ne sont pas similaires dans tous les pays membres, et qui sont donc susceptibles d’appeler des reponses de politique economique differentes (en nature ou en ampleur)", while according to Mundell (2003, p. 199) “all shocks are asymmetric in that they affect countries differently".
it may be time to clarify what we mean by this term. Well then, monetary policy is defined here in a broad sense, which includes what is commonly called exchange rate policy in the case of a fixed but adjustable exchange rate regime. As shown in Table 4, the monetary policy instruments considered are various: nominal interest rate, money stock, nominal exchange rate. Besides, a credibility problem may arise in some cases, due to the time inconsistency of the optimal monetary policy, so that the implementation of the first-best equilibrium then requires the existence of a commitment technology at the disposal of the central bank. The lack of such a commitment technology is one of the few monetary policy imperfections considered in our dissertation.

"MACROECONOMIC INSTABILITY" is defined as the variability of key macroeconomic aggregates due to the occurrence of sunspot shocks, as opposed to fundamental shocks. What do we call sunspot shocks? Well, they are shocks, i.e. exogenous stochastic variables, whose realization conditions the outcome of the model considered though they are not specified by this model. In other words, sunspot shocks are the shocks which can be held responsible for the selection of one given equilibrium out of several possible equilibria. Table 5 provides an outlook of the sunspot shocks to be encountered in this dissertation.

Naturally, a necessary and sufficient condition for macroeconomic instability to arise is the existence of multiple equilibria in the model considered. Sunspot shocks will typically be quantitative in the case of a continuum of possible equilibria, qualitative in the case of a finite number of possible equilibria. Note also that macroeconomic instability is usually independent of macroeconomic volatility, as sunspot shocks may occur in the absence of fundamental shocks - and vice versa. The variability of each macroeconomic aggregate can therefore be divided into two independent components: an intra-equilibrium component, which corresponds to macroeconomic volatility, and an inter-equilibrium component, which corresponds to macroeconomic instability.

We may deal either with short-run macroeconomic instability or with long-run macroeconomic instability. Short-run macroeconomic instability is inex-

---

2Put differently, sunspot shocks are responsible for what Burneister, Flood and Garber (1983) call bubbles, that is to say extra components which arise at the equilibrium in addition to the component reflecting market fundamentals.

3Batini and Pearlman (2002) use the term "instability" when there are more than one possible equilibrium and the term "indeterminacy" when there is none. We therefore adopt their terminology as far as (macroeconomic) "instability" is concerned, but we shall use (equilibrium) "indeterminacy" as well in the case of multiple equilibria rather than in the case of no equilibrium. In our view indeed, something is wrong with the model considered when there is no possible equilibrium, as actually there seems to be an equilibrium outside whenever we have a look through the window, so that one should then speak of an inadequate model rather than of an indeterminate equilibrium.
tricably linked in our various frameworks to the private agents' self-fulfilling expectations, which may be quick to jump from one equilibrium to another. Things prove less clear for long-run macroeconomic instability, which may be the result of a slow blind process, but we can say little actually thereupon, as we do not examine the way from one equilibrium to the other.

Finally, similarly as for *ex ante* and *ex post* macroeconomic volatility respectively, we define "*ex ante* short-run macroeconomic instability" as the short-run macroeconomic instability arising in the presence of a passive monetary policy and "*ex post* short-run macroeconomic instability" as the short-run macroeconomic volatility arising in the presence of an active monetary policy. Just like macroeconomic volatility and contrary to long-run macroeconomic instability, short-run macroeconomic instability is unambiguously welfare-reducing so that when aimed at maximizing household welfare, monetary policy has a "stabilization" role, by which we mean that it should react to sunspot shocks in order to reduce as much as possible, and ideally completely eliminate, this *ex post* short-run macroeconomic instability.

Three main "ALTERNATIVE EXCHANGE RATE REGIMES" are considered throughout this dissertation, as indicated in table 6: the flexible exchange rate regime, the fixed but adjustable exchange rate regime and the irrevocably fixed exchange rate regime. Under a flexible exchange rate regime, the nominal exchange rate may either be determined by the uncovered interest rate parity and the long-run relative purchasing power parity, or adjust so as to balance international trade. The central bank can then freely choose its monetary policy, or rather we can freely specify the goal of monetary policy. As shown in table 4, this goal may be for instance the maximization of household welfare. But monetary policy may also happen to have no specified goal. It may even happen to be specified no goal: in such a case, monetary policy is passive and the flexible exchange rate regime is preferentially labelled "floating exchange rate regime".

Under the fixed but adjustable exchange rate regime, the central bank has charge of the fixity of the nominal exchange rate. In other words, it has to react to fundamental and sunspot shocks so as to keep the nominal exchange rate fixed *ex post*. We may use the term "adjustable" because the central bank (or rather, shall we say, the government) is allowed to devalue or revalue the currency at a cost. When it is not allowed to do so, that is to say when the devaluation or revaluation cost is infinite, we also use the term "adjustable"

---

4The complete elimination of the *ex post* short-run macroeconomic instability is needed to ensure the selection of the unique bubble-free equilibrium.
simply as opposed to "irrevocably fixed", to remind the reader that there is a monetary authority working behind the scenes and responsible for the fixity of the nominal exchange rate.

Under the irrevocably fixed exchange rate regime, precisely, there is no longer a central bank working behind the scenes, and the nominal exchange rate is fixed ex ante. In other words, the irrevocably fixed exchange rate regime amounts to a monetary union. As indicated in table 6, the irrevocably fixed exchange rate regime may be bilateral, when two large economies decide together to drop their national central banks, to adopt a single common currency and to establish a supranational central bank in charge of monetary policy in the monetary union as a whole. It may also be unilateral, when a small economy pegs its currency to the currency of a large economy, and the supranational central bank then coincides with the central bank of the large economy.

Background

This section shortly presents the literature and the assumptions common to all chapters of this dissertation.

Each chapter has its own bibliography, on which table 7 gives a glimpse. But these bibliographies share a few common features. These common features are mostly of a Keynesian nature. The main two of them are the existence of nominal rigidities and that of "animal spirits". The presence of nominal rigidities (detailed in table 3) in the models considered in this dissertation is what gives rise to monetary policy non-neutrality. As for Keynes' (1936) "animal spirits", they correspond to sunspot shocks (characterized in table 5) in our framework and are thus responsible for what we call macroeconomic instability.

It is not necessary however to go back in time as far ago to find a common ancestor to our four essays. All of them belong indeed at least partly, if not fully, to the literature pioneered by Obstfeld and Rogoff (1996). The models built and used in this literature are distinguished by the following few key features. First, they specify sticky prices and/or wages, and are usually based on monopolistic competition. Second, they are dynamic general equilibrium models whose equations derive from the optimizations of the agents, namely here a representative household, firms and a central bank. Third, they base their evaluation of monetary policy explicitly on household welfare. And fourth, they incorporate stochastic shocks. These four points are considered in tables 2, 3 and 4. Note that we will not develop truly new models in our dissertation. We will rather use existing ones, and possibly build a bridge between them, to make
our point.

We choose to make a distinction of our own between two branches of the literature pioneered by Obstfeld and Rogoff (1995). The first one is the so-called New Keynesian economics literature, on which the first part of this dissertation, made of the first two chapters, is based. The second one is the so-called New Open Economy Macroeconomics literature, on which the second part of this dissertation, made of the last two chapters, is based. As indicated in Table 8, what distinguishes these two literatures from each other within our framework is the number of periods considered, the nature of the nominal rigidity, the determinants of the nominal exchange rate (when flexible), the identity of the monetary policy instrument and the role of past and present expectations.

A further difference between the two parts of our dissertation is that the first one focuses on one small open economy, while the second one deals with two large open economies. This first part is mainly based indeed on the small open economy New Keynesian model built by Galf and Monacelli (2002). Had we instead chosen to consider several large open economies in our first part, we would have used a N-country New Keynesian model, derived for instance from the canonical version built by Clarida, Galf and Gertler (2002).

Note that our nominal (wage or price) rigidity assumption alone does not ensure that the exchange rate regime does play a role in the adjustment of the economies to the fundamental shocks. As shown in Table 3, we will indeed further assume throughout the dissertation that there is no local currency pricing (LCP), so that the nominal exchange rate can play its traditional Keynesian expenditure-switching role, the exchange rate variations being entirely transferred on the price of imported goods. This assumption of an exchange rate pass-through equal to one is backed by Obstfeld and Rogoff (2000), who are very critical of the alternative approach combining pricing to market and local currency pricing.

Obstfeld and Rogoff’s (2000) arguments are the following. First, the link between the nominal exchange rate and the measured deviations from the law of one price may be due to the incorporation of non-tradable components in consumer price indexes for supposedly tradable goods. Second, the time horizon over which trade invoicing induces price stickiness appears too brief to have a large impact on macroeconomic interactions at business-cycle frequencies. Third, the direct evidence on currency invoicing is largely inconsistent with the view that exporters set prices predominantly in importers’ currencies. And fourth, international evidence on mark-ups also seems consistent with a predominance of invoicing in exporters’ home currencies. Their view is challenged
by Devereux and Engel (2002), but the latter need to make strong assumptions in addition to local currency pricing to match the observed exchange rate variability.

Let us finally broadly outline the borders of our operative field. Which questions do we ask and which ones do we not ask in this dissertation? Which issues do we tackle and which ones do we disregard? Well, monetary policy (again, defined in a broad sense) is the only economic policy considered in this dissertation. In particular, we do not consider any (endogenous) fiscal policy. This restriction is mainly justified by the existence of an implementation delay which makes fiscal policy the wrong macroeconomic adjustment tool. In what follows, we will use the term “government”, instead of “central bank”, when monetary policy actually amounts to exchange rate policy.

Fiscal policy however is not the only notable absentee in our dissertation. Indeed, our framework entails many more simplifying assumptions, not to say many more limitations. For instance, we do not consider any non-tradable goods and we ignore (endogenous) capital investment, simply because we do not need them to make our point. Introducing them into our framework would make it more realistic but also more complex, probably without altering our results. We choose to stick to the simplest possible framework to keep our message as clear as possible.

Three last points are worth noting. First, we consider one single representative household in all chapters, so that we do not address the issues of inequality and redistribution. Second, we stick throughout our dissertation to the assumption that all agents share the same rational expectations at each date, so that we ignore the coordination problem which may notably arise in the presence of multiple equilibria. In particular, because we deal with atomistic agents, the socially optimal equilibrium is no more likely to emerge than others in the presence of multiple equilibria. Third, all the models considered are notably based on the assumption of monopolistic competition, which fits developed countries much better than developing countries. Our focus is therefore resolutely on what might be called “homogenous developed countries".
**Table 1: statuses.**

<table>
<thead>
<tr>
<th>Chapter</th>
<th>First draft</th>
<th>Co-author</th>
<th>Presentations</th>
<th>Submission or publication</th>
<th>Comments received</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>June 2003</td>
<td>-</td>
<td>• CEPII internal seminar, Paris, France, 11/03/2003 • CREST-LMA internal seminar, Malakoff, France, 06/06/2003</td>
<td>-</td>
<td>Daniel Cohen, Hélène Rey</td>
</tr>
</tbody>
</table>
Table 2: fundamental shocks.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Possible microeconomic shocks</th>
<th>Possible macroeconomic shocks</th>
<th>Resulting shocks</th>
<th>Nationality of shocks</th>
<th>Asymmetry of shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>shock on the parameter measuring the representative household’s preference for the present</td>
<td>public spending shock, shock on the risk-premium term in the uncovered interest rate parity equation</td>
<td>shock on the IS equation (IS shock)</td>
<td>domestic</td>
<td>exogenous</td>
</tr>
<tr>
<td></td>
<td>productivity shock</td>
<td></td>
<td>shock on the Phillips curve (cost-push shock)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>shock on the parameter measuring the representative household’s preference for the present</td>
<td>public spending shock, shock on the risk-premium term in the uncovered interest rate parity equation</td>
<td>shock on the IS equation (IS shock)</td>
<td>domestic and foreign</td>
<td>exogenous</td>
</tr>
<tr>
<td></td>
<td>productivity shock</td>
<td></td>
<td>shock on the Phillips curve (cost-push shock)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>shock in setting the nominal interest rate, due to the central bank’s shaking hand</td>
<td>shock on the monetary policy rule (monetary policy shock)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>shock on the political cost of opting out the fixed exchange rate system</td>
<td>shock on the fixed cost of devaluation</td>
<td>domestic and foreign</td>
<td>exogenous</td>
</tr>
<tr>
<td>4</td>
<td>shock on the preferences of the representative household for industrial goods</td>
<td>-</td>
<td>industry-specific demand shock</td>
<td>domestic and foreign</td>
<td>endogenous</td>
</tr>
</tbody>
</table>
Table 3: nominal rigidities.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Wages</th>
<th>Prices</th>
<th>Number N of periods</th>
<th>Law of one price</th>
<th>Pricing to market</th>
<th>Local currency pricing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>flexible (adjust so as to clear the labour market)</td>
<td>rigid in the short run (forward-looking price-setting mechanism à la Calvo)</td>
<td>N = ∞</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>flexible (adjust so as to clear the labour market)</td>
<td>rigid in the short run (price-setting mechanism partly forward-looking à la Calvo, partly backward-looking)</td>
<td>N = ∞</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>rigid (forward-looking wage-setting)</td>
<td>flexible (adjust so as to clear the goods market)</td>
<td>N = 1</td>
<td>✓</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>rigid in the short-run, flexible in the long run (adjust so as to make workers indifferent between working in industry 1 or in industry 2)</td>
<td>flexible (adjust so as to clear the goods market)</td>
<td>N = 1</td>
<td>-</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>Chapter</td>
<td>Monetary policy instrument(^3)</td>
<td>Monetary policy goal (when not exclusively maintaining a currency peg)</td>
<td>Arguments of the representative household’s utility function</td>
<td>Commitment technology</td>
<td>Monetary policy imperfections (when gauged with the household welfare criterion)</td>
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</tr>
<tr>
<td>---------</td>
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<td>---------------------------------------------------------------------</td>
<td>-------------------------------------------------------------</td>
<td>----------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>R (M absent or residually determined, E residually determined)</td>
<td>maximization of the representative household’s utility function</td>
<td>consumption, labour, possibly money</td>
<td>necessary under the fixed but adjustable exchange rate regime, preferable under the flexible exchange rate regime</td>
<td>• possibly lacking a commitment technology</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>R (M absent or residually determined, E residually determined)</td>
<td>not specified</td>
<td>consumption, labour, possibly money</td>
<td>not necessary</td>
<td>• possibly not aimed at maximizing household welfare • possibly lacking a commitment technology • source of exogenous perturbations</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>E (M and R absent)</td>
<td>maximization of the government’s utility function</td>
<td>consumption, labour</td>
<td>necessary in the case of international cooperation</td>
<td>• not aimed at maximizing household welfare • possibly not cooperative • possibly not coordinated when not cooperative</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>M (R absent, E residually determined)</td>
<td>none</td>
<td>consumption, money</td>
<td>not necessary</td>
<td>• not aimed at maximizing household welfare</td>
<td></td>
</tr>
</tbody>
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---

\(^3\) R: nominal interest rate; M: money stock; E: nominal exchange rate.
Table 5: sunspot shocks.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Ex ante multiplicity of equilibria</th>
<th>Sunspot shocks</th>
<th>Macroeconomic instability</th>
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<tr>
<td></td>
<td>infinite</td>
<td>finite</td>
<td>quantitative</td>
</tr>
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<td>1</td>
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<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>2</td>
<td>✓</td>
<td>-</td>
<td>✓</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>✓</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>✓</td>
<td>-</td>
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Table 6: exchange rate regimes.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Number, size and openness of the economies</th>
<th>Flexible exchange rate regime</th>
<th>Fixed but adjustable exchange rate regime</th>
<th>Devaluation or revaluation cost</th>
<th>Irrevocably fixed exchange rate regime (unilateral)</th>
<th>Irrevocably fixed exchange rate regime (bilateral)</th>
<th>Endogenous exchange rate (when flexible or adjustable)</th>
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<tbody>
<tr>
<td>1</td>
<td>1 small open economy (closed economy nested as a special case)</td>
<td>✓</td>
<td>✓</td>
<td>infinite</td>
<td>✓</td>
<td>-</td>
<td>determined by the UIP and the long-run relative PPP</td>
</tr>
<tr>
<td>2</td>
<td>1 small open economy</td>
<td>✓</td>
<td>-</td>
<td>-</td>
<td>✓</td>
<td>-</td>
<td>determined by the UIP and the long-run relative PPP</td>
</tr>
<tr>
<td>3</td>
<td>2 small open economies</td>
<td>-</td>
<td>✓</td>
<td>finite</td>
<td>-</td>
<td>-</td>
<td>optimally chosen by the government</td>
</tr>
<tr>
<td>4</td>
<td>2 large open economies</td>
<td>✓</td>
<td>✓</td>
<td>infinite</td>
<td>-</td>
<td>✓</td>
<td>adjusts so as to balance current international trade</td>
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</tbody>
</table>

* UIP: uncovered interest rate parity; PPP: purchasing power parity.*
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Literatures</th>
<th>Closest studies</th>
<th>JEL classification</th>
<th>Key-words</th>
<th>Theoretical and/or empirical</th>
<th>Softwares used (except word-processing)</th>
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<td>Clarida, Gali and Gertler (1999, 2001), Gali and Monacelli (2002), Woodford (2003)</td>
<td>E31, E52, E58, E61, F33</td>
<td>canonical New Keynesian model, fixed exchange rate regime, flexible exchange rate regime, multiple equilibria, optimal monetary policy, time inconsistency</td>
<td>theoretical</td>
<td>Mathematica 4.2.0.0</td>
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<tr>
<td>3</td>
<td>• 2\textsuperscript{nd} generation currency crises models • New Open Economy Macroeconomics • literature on international monetary cooperation and coordination</td>
<td>Bui\textsuperscript{e}t, Corsetti and Pesenti (1998), Canzoneri and Henderson (1991), Obstfeld (1996, 1997)</td>
<td>F33, F41, F42</td>
<td>contagion, cooperation, coordination, exchange rate crises, fixed exchange rates, multiple equilibria, trade competition</td>
<td>theoretical</td>
<td>Excel 2000</td>
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<tr>
<td>4</td>
<td>• New Economic Geography • New Open Economy Macroeconomics</td>
<td>Fujita, Krugman and Venables (1999), Ricci (1997, 1998)</td>
<td>F12, F15, F33, F41, R12, R13</td>
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<td>theoretical</td>
<td>Mathematica 4.2.0.0</td>
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<td>Part’s title</td>
<td>Chapter</td>
<td>Chapter’s title</td>
<td>Literature</td>
<td>Economies</td>
<td>Number N of periods</td>
</tr>
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<td>“a New Keynesian perspective on one small open economy”</td>
<td>1</td>
<td>“Forward-looking monetary policy rules to preclude multiple equilibria”</td>
<td>New Keynesian literature</td>
<td>1 small open economy</td>
<td>$N = \infty$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>“Simulation of the UK business cycle under EMU-membership”</td>
<td></td>
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<td>II</td>
<td>“a New Open Economy Macroeconomics perspective on two open economies”</td>
<td>3</td>
<td>“Coordination, cooperation, contagion and currency crises”</td>
<td>New Open Economy Macroeconomics literature</td>
<td>2 open economies</td>
<td>$N = 1$</td>
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<tr>
<td></td>
<td></td>
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<td>“Endogenously asymmetric demand shocks under alternative exchange rate regimes”</td>
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Part I

A New Keynesian perspective on one small open economy
Chapter 1

Forward-looking monetary policy rules to preclude multiple equilibria

Abstract

Chapter 1, entitled "Forward-looking monetary policy rules to preclude multiple equilibria", examines the issue of the design of monetary policy rules within the canonical New Keynesian model of a small open economy, with the closed economy nested as a special case. Unlike the existing literature, we argue that in order to ensure the implementation of the unique optimal equilibrium, the monetary policy rule should preclude not only all non-optimal convergent equilibria, but also all divergent equilibria. We characterize analytically the set of such adequate monetary policy rules, in a flexible exchange rate regime (depending on whether a commitment technology is available or not) and in a fixed exchange rate regime. We show in particular that these rules are necessarily forward-looking so as to insulate the current inflation rate from the private agents' sunspot-prone expectations about the future situation. This result is robust to natural extensions of the canonical New Keynesian framework.

Abstract in French

Le chapitre 1, intitulé "Règles de politique monétaire forward-looking pour exclure les équilibres multiples", s'intéresse au design des règles de politique monétaire dans le cadre du modèle nouveau-keynésien canonique d'une petite
économie ouverte, incluant le cas particulier d’une économie fermée. Contrairement à la littérature existante, nous soutenons qu’afin d’assurer la mise en œuvre de l’unique équilibre optimal, la règle de politique monétaire doit exclure la possibilité non seulement de tout équilibre convergent non optimal, mais aussi de tout équilibre divergent. Nous caractérisons analytiquement l’ensemble de telles règles de politique monétaire adéquates, sous un régime de change flexible (selon la présence ou l’absence d’un dispositif de commitment) et sous un régime de change fixe. Nous montrons en particulier que ces règles sont nécessairement forward-looking de façon à isoler le taux d’inflation présent des anticipations des agents privés (potentiellement sujettes à des sunspots). Ce résultat est robuste aux extensions naturelles du modèle nouveau-keynésien canonique.

1.1 Introduction

As stressed by McCallum (1999a), who relates the evolution of monetary policy theory and practice since the early 70’s, New Keynesian economics has recently come out as the most celebrated framework for monetary policy analysis. Within this framework, much attention has been paid in particular to the issue of how to design a monetary policy rule so as to avoid (undesirable) multiple equilibria. This issue is arguably of practical importance: according to Clarida, Galí and Gertler (2000) for instance, the American macroeconomic variability during the pre-Volcker era may be explained by the fact that the monetary policy rule followed by the Fed was compatible with multiple equilibria and hence made way to endogenous fluctuations, born from self-fulfilling expectations.

This chapter aims at giving a new insight into the design of optimal monetary policy rules, which we define as the monetary policy rules ensuring the implementation of the unique optimal equilibrium. In our opinion, the definition of multiple equilibria usually adopted by the existing literature is too restrictive, as only convergent equilibria (i.e. dynamically stable or stationary equilibria) are considered. We argue that the optimal monetary policy rules should rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria (i.e. dynamically unstable or non-stationary equilibria).

Two alternative justifications for disregarding divergent equilibria have been put forward by the literature. The first justification relies on the fact that the log-linear approximation of the New Keynesian model enables us to consider only small macroeconomic fluctuations around the steady state. However, if a divergent path starts to develop in the neighbourhood of the steady state (so that we can at least appreciate its initial development before losing sight of it),
then the central bank will sooner or later act as a "stabilizer of last resort", that is to say eventually abandon its monetary policy rule in order to bring all diverging variables back to their steady state values. In the end, what we call a divergent path may therefore actually remain constantly in the neighbourhood of the steady state, so that this first justification need not hold.

The second justification, put forward only by Clarida, Galí and Gertler (1999, p. 1701), rightly argues that this credible threat to act as a "stabilizer of last resort" is enough to nip any divergent equilibrium in the bud. Indeed, divergent equilibria will be precluded if the private agents expect the central bank to successfully bring any diverging variable back to its steady state value in a finite time horizon. However, the central bank could theoretically preclude not only all divergent equilibria, but also all non-optimal convergent equilibria by issuing a similar credible threat for the future, which would remove any need to follow a well-defined monetary policy rule in the present. Moreover, how exactly to react in the future to a non-optimal convergent or divergent path remains in the dark, as Clarida, Galí and Gertler (1999) do not specify to which monetary policy rule the central bank should switch, should a non-optimal convergent or divergent path start to develop.

We shall argue that the question of which monetary policy rule to switch to in the future to react to an undesirable path is very much similar to the question of which monetary policy rule to follow in the present to preclude undesirable equilibria. We show in particular that both these monetary policy rules are necessarily forward-looking so as to insulate the current inflation rate from the private agents' sunspot-prone expectations about the future situation. To our knowledge, we thus provide a new theoretical justification for the forward-looking behaviour of central banks. Indeed, apart from Clarida, Galí and Gertler (1999), the literature has been interested up to now exclusively in monetary policy rules precluding only all convergent equilibria other than the optimal one, and forward-lookingness is not a necessary condition for a monetary policy rule to preclude only convergent equilibria.

To make our point, we resort to what we call the canonical New Keynesian model, that is to say the New Keynesian model reduced to its simplest form, which has received much attention in the past few years. Its closed economy version is composed of an IS equation, a Phillips curve\(^1\) and a central bank's loss function. Its small open economy version has a very similar structure, as

\(^1\)The New Keynesian model differs from its New Classical counterpart in particular in that its Phillips curve involves the present anticipation of the future inflation rate, due to a price-setting specification à la Calvo (1983), and not the past anticipation of the present inflation rate (Lucas' supply curve).
it is composed of the same (in reduced form terms) IS equation, Phillips curve
and loss function, to which are added the uncovered interest rate parity, the law
of one price and the long-run Purchasing Power Parity (PPP). This intertemporal
general equilibrium model manages to combine a highly tractable reduced
form with sound microfoundations, as the IS equation and the Phillips curve
are derived from the optimal behaviour of the representative household and firm
respectively, and the central bank's loss function from the representative house-
hold's utility function. At the end of the chapter, we shortly point to the fact
that natural extensions to this canonical framework, making the model more
realistic but resting on more or less arbitrary assumptions, would not alter our
results qualitatively speaking.

We follow a two-step approach. First, we fully derive the model's analytical
results, which describe the optimal macroeconomic adjustment process to de-
mand and cost-push shocks, for a small open economy (with the closed economy
nested as a special case) in four alternative configurations: a flexible exchange
rate regime without commitment (FL1), a flexible exchange rate regime with
commitment (FL2), a(n ex post) fixed exchange rate regime with commitment
(FI1) and an irrevocably (ex ante) fixed exchange rate regime with commitment
(FI2). In so doing, we fill a gap in the literature, as these analytical results are
absent from all existing studies. Second, we characterize the set of monetary
policy rules ensuring the implementation of this optimal adjustment process, in
each of the relevant cases considered (FL1, FL2 and FI1). By contrast, existing
studies do it only in the FL2 case. Most importantly, unlike the existing lit-
erature, we look for monetary policy rules which ensure the implementation of
the unique optimal equilibrium by ruling out not only all convergent equilibria
other than the optimal one, but also all divergent equilibria.

The remaining of the chapter is organized as follows: section 1.2 presents
both the closed economy and the small open economy versions of the canonical
New Keynesian model. Section 1.3 determines analytically the optimal equilib-
rium, in each of the cases considered (FL1, FL2, FI1 and FI2). Section 1.4 shows
how a monetary policy rule can be chosen which ensures the implementation
of this optimal equilibrium. Section 1.5 characterizes the set of such adequate
monetary policy rules. We then conclude and provide a technical appendix.

1.2 Presentation of the model

This section presents the canonical New Keynesian model of a small open eco-
omy, with the closed economy nested as a special case.
The canonical New Keynesian model of a closed economy has been used notably by Bernanke and Woodford (1997), Clarida, Gali and Gertler (1999, 2000), Woodford (2003). Other works, listed by Woodford (2003, chap. 7), adopt a very similar, if not identical framework. McCallum (1999a) assesses and discusses the recent popularity of this model.

The canonical New Keynesian model of a small economy has been laid out by Clarida, Gali and Gertler (2001), as well as Gali and Monacelli (2002)\(^2\) from whom we borrow our presentation. (A few other works use slightly different versions of this model.)

1.2.1 Main assumptions

We focus here on the main assumptions of the model, essentially in order to introduce the parameters featuring in the closed form, and refer the reader to Gali and Monacelli (2002) for a more detailed presentation.

The representative household in the small open economy maximizes the following utility at date \(t\):

\[
U_t \equiv E_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[ \frac{C_{t+k}^{1-\sigma} - 1}{1 - \sigma} - \frac{N_{t+k}^{1+\varphi} - 1}{1 + \varphi} \right] \right\},
\]

where \(N_{t+k}\) represents hours of labour and \(C_{t+k}\) a composite consumption index at date \(t + k\), while \(E_t\) stands for the expectation operator conditionally on the information available at date \(t\). We assume \(0 < \beta < 1, \sigma > 0\) and \(\varphi > 0\).

Note that money does not enter the utility function and will be disregarded thereafter. Woodford (2003, chap. 2) gives three alternative justifications for this Wicksellian specification. First, we may deal with a genuinely cashless economy, with the implication that money (the unit of account) must earn the same rate of return as other riskless assets. Second, there may be some monetary frictions, so that money does actually enter the utility function, but if preferences are additively separable between consumption and real balances, then money is residually determined by an LM equation and plays no role in what follows, except for its direct contribution to the utility level which we assume is negligible. Third, even if it enters the utility function in a non-separable way, money will not matter in the case of what Woodford (2003, chap. 2) calls a "cashless limiting economy".

The composite consumption index is defined by:

---

\(^2\)According to McCallum and Nelson (2000), "the GM [Gali and Monacelli (2002)] model has a strong claim to be viewed as a canonical NOEM [New Open Economy Macroeconomics] model, owing to its elegance and tractability".
\[
C_{t+k} = \left[ \alpha \frac{1}{\alpha - 1} C_{H,t+k}^{\frac{\alpha - 1}{\alpha}} + (1 - \alpha) \frac{1}{\alpha - 1} C_{F,t+k}^{\frac{\alpha - 1}{\alpha}} \right]^{\frac{\alpha}{\alpha - 1}},
\]

where \( C_{H,t+k} \) and \( C_{F,t+k} \) are CES indices of domestic and foreign goods consumption:

\[
C_{H,t+k} \equiv \left[ \int_0^t C_{H,t+k} (i)^{\frac{1}{1 - \epsilon}} \, di \right]^{\frac{1}{\epsilon}}, \quad C_{F,t+k} \equiv \left[ \int_0^t C_{F,t+k} (i)^{\frac{1}{1 - \epsilon}} \, di \right]^{\frac{1}{\epsilon}}.
\]

Parameter \( \mu \) measures the elasticity of substitution between domestic and foreign goods, whereas parameter \( \epsilon \) measures the elasticity of substitution between the varieties of the differentiated good produced in a given country. We assume \( \mu > 0 \) and \( \epsilon > 1 \). The degree of openness of our small open economy is measured by \( 1 - \alpha \), with \( \alpha \) in-between 0 and 1.

The utility maximization is subject to a sequence of intertemporal budget constraints of the form

\[
\int_0^t \left[ P_{H,t+k} (i) C_{H,t+k} (i) + P_{F,t+k} (i) C_{F,t+k} (i) \right] \, di + E_t \{ Q_{t+k,t+k+1,1} D_{t+k+1} \} \leq D_{t+k} + W_{t+k} N_{t+k} + T_{t+k}
\]

for \( k = 0, 1, 2, \ldots \), where \( P_{H,t+k} (i) \) and \( P_{F,t+k} (i) \) denote the prices of domestic and foreign good \( i \) respectively, \( W_{t+k} \) the nominal wage and \( T_{t+k} \) lump-sum transfers or taxes at date \( t + k \), while \( D_{t+k+1} \) the nominal payoff at date \( t + k + 1 \) of the portfolio held at the end of period \( t + k \) (which includes shares in firms). All the previous variables are expressed in units of domestic currency. \( Q_{t+k,t+k+1,1} \) represents the stochastic discount factor for nominal payoffs. We assume that households have access to a complete set of contingent claims, traded internationally.

Each firm produces a variety \( i \) of the differentiated good with a linear technology described by the following production function:

\[
Y_t (i) = A_t N_t (i),
\]

with \( \ln A_t = \bar{\alpha} + \epsilon_t^\alpha \), where \( \bar{\alpha} \neq 0 \) and where \( \epsilon_t^\alpha \) is an exogenous technology shock with zero mean. We thus disregard investment dynamics: private spending has no effect upon the economy's productive capacity, as we deal with non-durable consumption expenditure. Woodford (2003, chap. 5) finds that relaxing this assumption leads to (to some extent) qualitatively similar results.
We assume the existence of an employment subsidy, whose role is to offset the monopolistic distortions at the steady state. Firms set prices in a staggered fashion, à la Calvo (1983): each firm can modify its price at date $t$ only with probability $(1 - \theta)$ strictly comprised between 0 and 1. (This time-dependent price-setting rule may seem less realistic than state-dependent ones, but proves more convenient to handle analytically.) The model thus incorporates a temporary nominal rigidity which will result in a short-run trade-off for the central bank between inflation and output gap deviations from their targets. Of course, each firm sets its price, when allowed to change it, so as to maximize the discounted value of its profits.

We also assume that there is no local currency pricing, that is to say that the price of each variety of the differentiated good is denominated in the producer's currency, not in the consumer's. This assumption ensures that the variations in the nominal exchange rate impact on aggregate demand by modifying the price of the goods produced in one country and consumed in the other country. Besides, even though we do not rule out pricing to market, that is to say even though each producer can make its price depend on whether its good is sold on the domestic market or on the foreign market, each producer ends up choosing the same price on both markets, as she faces the same elasticity of substitution here and there. As a consequence, the law of one price holds.

Contrary to prices, wages are assumed to be perfectly flexible. This assumption enables us to analyze inflation and output gap dynamics without any reference to the labour market. Woodford (2003, chap. 3), who relaxes this assumption, finds that wage vs. price stickiness (more precisely staggered wage-setting vs. staggered price-setting) matters essentially for the loss function.

The foreign economy is modeled in the same way as the domestic one. The corresponding parameters are signalled by an asterisk. As the foreign economy is large compared to the domestic one, $\alpha^*$ is close to zero and domestic fluctuations have therefore no impact on the foreign economy. To keep things simple, we assume that the foreign economy remains constantly at its steady state, experiencing no fluctuations.

### 1.2.2 Closed form

The closed form of the model, log-linearized around its steady state, is essentially composed of an IS equation, derived from the representative household's utility maximization; of a Phillips curve, derived from the producers' price-setting decisions; and of a loss function, which derives from the representative household's utility function and which the central bank seeks to minimize. We
refer the reader to Gali and Monacelli (2002) for a detailed derivation of this closed form.

Let us note $R_t$ the gross return of a riskless one-period bond denominated in domestic currency, $Y_t$ the aggregate output index, $P_{H,t}$ the producer price index (PPI), $P_{F,t}$ the price index for imported goods and $P_t$ the consumer price index (CPI):

$$R_t = \frac{1}{E_t(\bar{Q}_{t,t+1})}, \quad Y_t = \left[ \int_0^1 Y_t(t) \frac{dt}{\bar{Q}_t} \right]^{\frac{1}{\gamma - 1}},$$

$$P_{H,t} = \left[ \int_0^1 P_{H,t}(t)^{\frac{1}{1-\gamma}} dt \right]^{\frac{1}{1-\gamma}}, \quad P_{F,t} = \left[ \int_0^1 P_{F,t}(t)^{\frac{1}{1-\gamma}} dt \right]^{\frac{1}{1-\gamma}},$$

$$P_t = \left[ \alpha P_{H,t}^{1-\mu} + (1 - \alpha) P_{F,t}^{1-\mu} \right]^{\frac{1}{1-\mu}}.$$

Let us also note $\bar{Y}_t$ the level of production obtained at date $t$ when prices are perfectly flexible ($\theta = 0$); $\bar{y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$ the rate of deviation of $Y_t$ from this level, more concisely called the output gap; and $r_t = \frac{R_t - \bar{R}}{\bar{R}}$ the rate of deviation of $R_t$ from its non-zero stationary value $\bar{R} = \frac{1}{\gamma}$. Assuming that $y_t$ and $r_t$ are close to zero, we can approximate $\ln(1 + y_t)$ by $y_t$ and $\ln(1 + r_t)$ by $r_t$. Besides, if sufficiently close to zero, the CPI inflation rate between dates $t$ and $t + 1$ can be written $\Delta P_{t+1}$, where $p_t \equiv \ln P_t$ and where $\Delta$ is the first difference operator, as the first-order approximation $\Delta P_{t+1} = \frac{P_{t+1} - P_t}{P_t}$ then holds. Similarly, $\Delta P_{H,t+1}$ (where $p_{H,t} \equiv \ln P_{H,t}$) represents the PPI inflation rate.

The law of one price implies the following first-order approximation:

$$\Delta p_t = \alpha \Delta p_{H,t} + (1 - \alpha) \Delta e_t, \quad (1.2.1)$$

where $e_t$ denotes the log of the nominal exchange rate at date $t$ (value of one foreign currency unit expressed in domestic currency). Under the assumption of complete international financial markets, the dynamics of the nominal exchange rate is described by the uncovered interest rate parity relationship, which holds up to a first-order approximation too. The nominal interest rate being constantly equal to its stationary value in the foreign country, this UIP relationship is written:

$$E_t \{ \Delta e_{t+1} \} = r_t. \quad (1.2.2)$$

The Euler equation and the goods market clearing condition, together with equations (1.2.1) and (1.2.2), lead to the following equation:
\[ y_t = E_t \{ y_{t+1} \} - \eta (r_t - E_t \{ \Delta p_{H,t+1} \}) + \epsilon_t^{\phi}, \quad (1.2.3) \]

where \( \eta \equiv \frac{1 + (1 - \phi)(1 + \phi)}{\phi} \) and where \( \epsilon_t^{\phi} \) represents an exogenous shock with mean zero occurring at date \( t \). Equation (1.2.3) corresponds to the standard IS equation of the New Keynesian model. The shock \( \epsilon_t^{\phi} \), which has been added in an ad hoc fashion, can be interpreted as a temporary demand shock, corresponding for instance to an unexpected exogenous public spending. Alternatively, it could derive from an adequately specified preference shock \( \xi_t \) entering the utility function (so that the factor \( \beta \hat{h} \) is replaced by \( \beta \xi_t \), as shown by Ireland (2002)). For simplicity, we assume it is not autocorrelated.

Equation (1.2.3) directly derives from the Euler equation in the closed economy case, where \( c_t = y_t \) and \( \Delta p_{H,t} = \Delta p_t \) at each date \( t \). Its interpretation is then straightforward: the present output gap is expressed as an increasing function of the expected future output gap and a decreasing function of the ex ante real interest rate, due to income and substitution effects. Two points are worth noting in the small open economy case. First, the deflator in the expression of the real interest rate is the PPI inflation rate, not the CPI inflation rate; but \( r_t - E_t \{ \Delta p_{H,t+1} \} \) and \( r_t - E_t \{ \Delta p_{t+1} \} \) are proportional to each other, due to the law of one price (1.2.1) and the uncovered interest rate parity (1.2.2) relationships. Second, the equation involves \( E_t \{ \Delta y_{t+1} \} \), rather than \( E_t \{ \Delta c_{t+1} \} \) as in the Euler equation; but \( E_t \{ \Delta c_{t+1} \} \) is proportional to the variation in the terms of trade \( E_t \{ \Delta \epsilon_{t+1} - \Delta p_{H,t+1} \} \) as a first-order approximation, due to our CES consumption index assumption, and \( E_t \{ \Delta \epsilon_{t+1} - \Delta p_{H,t+1} \} \) is itself proportional to \( r_t - E_t \{ \Delta p_{H,t+1} \} \), due to the UIP relationship.

The optimization programme of the representative household does not only lead to the IS equation (via the Euler equation). Indeed, as in all frameworks with infinitely-lived utility-maximizing agents, there is also a transversality condition attached to this programme. In what follows, this transversality condition will be satisfied in all cases - even along what we call "divergent paths", as made clear by subsection 1.4.3, because these paths are actually bounded, as the central bank eventually reacts to them so as to bring them back to the neighbourhood of the stationary state.

The price-setting decisions of firms lead to the following equation:

\[ \Delta p_{H,t} = \beta E_t \{ \Delta p_{H,t+1} \} + \gamma y_t + \epsilon_t^{p}, \quad (1.2.4) \]

where \( \gamma \equiv \frac{(1 - \phi)(1 - \beta \phi)}{\phi} \) and \( \epsilon_t^{p} \equiv -\frac{(1 - \phi)(1 - \beta \phi)(1 + \phi)}{\phi} \epsilon_t^{\phi} \). Equation (1.2.4) corresponds to the standard Phillips curve of the New Keynesian model.
It is forward-looking because firms know that the price they choose today will remain effective for a (random) number of periods. Like the demand shock $\varepsilon^d_t$, the cost-push shock $\varepsilon^c_t$ is assumed not to be autocorrelated: in the same way as Woodford (2003, chap. 7), we will thus focus on monetary policy inertia which does not stem from any lagged variables in the structural equations, nor from any serial correlation in the exogenous disturbances.

As no lagged (hence pre-determined) variable enters equation (1.2.4) at first sight, the inflation rate appears as a jump variable. As a consequence, the New Keynesian Phillips curve has been criticized for failing to provide enough inflation inertia: one had to appeal - so was it argued - to adaptive expectations to reconcile this equation with the data. However, lagged variables can enter the equation through the output gap term, if the monetary policy rule is backward-looking, so that this criticism need not hold. (As next section makes clear, the first-best monetary policy does actually involve nominal interest rate and inflation rate inertia.) Moreover, the empirical investigations of Gali and Gertler (1999), Sbordone (2002) indicate that forward-looking behaviour matters more than backward-looking behaviour in the price-setting process. (What these authors question, however, is the empirical relevance of the theoretical link between real marginal costs and the output gap, so that they estimate equation (1.2.4) with real marginal costs instead of the output gap.)

Clarida, Gali and Gertler (2001), Gali and Monacelli (2002) show that the quadratic approximation of the representative household’s utility function, taken in the neighbourhood of the stationary equilibrium where a system of lump-sum transfers or taxes exactly offsets the monopolistic distortions, leads to the following social loss function in the special case $\mu = \sigma = 1$:

$$L_t^S = E_t \left\{ \sum_{k=0}^{+\infty} \delta^S_k \left[ (\Delta p_{t+k})^2 + \lambda^S (\nu_{t+k})^2 \right] \right\},$$

where $(\delta^S, \lambda^S) = \left( \beta, \frac{1-\delta(1-\delta \theta)}{\theta^2} \right)$, and Woodford (2002; 2003, chap. 6) derives the equivalent social loss function in the closed economy case, corresponding to $\alpha = 1$. We assume the existence of such an optimal subsidy scheme so as to focus on the welfare losses associated with price stickiness and imperfect stabilization of shocks, because monetary policy is not aimed at addressing first-order distortions. Under this optimal subsidy scheme, first-order effects disappear, only second-order effects remain (in this second order approximation). There is no apparent “terms of trade gap” term in $L_t^S$ because this gap turns out to be proportionate to the output gap (in what can be interpreted as a goods market clearing condition) and can therefore be included in the output...
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gap term.

The presence of a PPI inflation term in $L^S_t$ comes from the fact that variability in the general level of prices $p_t$ creates discrepancies between relative prices, due to the absence of synchronization in the adjustment of the prices of different goods, and these relative price distortions lead in turn to an inefficient sectoral allocation of labour, even when the aggregate level of output is correct, i.e. even when the output gap is nil. These distortions matter all the more that the elasticity of substitution between goods is large and that the frequency of price adjustment is low, hence $\lambda_S$ depends negatively on $\varepsilon$ and $\theta$. Besides, $\lambda_S$ depends positively on $\varphi$, as the welfare costs of fluctuations in the output gap increase with the elasticity of the utility function with respect to labour.

Of course, $L^S_t$ arises as the natural choice for the central bank’s loss function in the case $(\mu, \sigma) = (1, 1)$. Now in order to handle other cases as well, we assume more generally that the central bank chooses the nominal interest rate $r_t$ so as to minimize the following quadratic loss function$^3$:

$$L_t = E_t \left\{ \sum_{k=0}^{+\infty} \delta^k \left[ (\Delta p_{t+k})^2 + \lambda (y_{t+k})^2 \right] \right\}, \quad (1.2.5)$$

where $(\delta, \lambda)$ is a pair of positive parameters, possibly different from $(\delta_S, \lambda_S)$, even if the specific case $(\mu, \sigma) = (1, 1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$ will naturally be examined at regular intervals in the following. The monetary authorities seek therefore anyway to maintain the PPI inflation rate and the output gap as close as possible from their respective values at the stationary state.

Finally, as shown by Galí and Monacelli (2002), the initial conditions can be chosen for the sake of convenience and without any loss of generality so that the condition that PPP should hold (or equivalently here that trade should be balanced) in the long run can be written: $(p_{t+k} - e_{t+k}) \rightarrow 0$ as $k \rightarrow +\infty$. As a consequence, we get, if the infinite sum in the right-hand side does converge:

$$\Delta e_t = \frac{p_{t-1} - e_{t-1}}{\alpha} + \Delta p_{t:t} + \sum_{k=1}^{+\infty} (E_t \{ \Delta p_{t+k} \} - E_t \{ \Delta e_{t+k} \}). \quad (1.2.6)$$

With a flexible exchange rate regime, the closed form of our small open economy model is made of equations (1.2.1), (1.2.2), (1.2.3), (1.2.4), (1.2.5) and (1.2.6). (With a fixed exchange rate regime, (1.2.5) should be replaced by the condition $\Delta e_{t+k} = 0$ for $k \geq 0$.) Note that the structure of the system is block-recursive: $y$, $\Delta p_t$ and $r$ are derived from equations (1.2.3), (1.2.4) and (1.2.5) only, with $\Delta p$ and $\Delta e$ being residually determined with the help

$^3$This loss function, though admittedly ad hoc, is widely used in the literature.
of equations (1.2.1), (1.2.2) and (1.2.6). As for the closed form of the closed economy model, it is made of equations (1.2.3), (1.2.4) and (1.2.5), with $\alpha = 1$. In both the closed economy and the small open economy versions of the canonical New Keynesian model, $y_t, \Delta p_H$ and $r_t$ are therefore derived from the same (qualitatively speaking) IS equation, Phillips curve and central bank’s loss function

The stationary state of the small open economy, obtained in the absence of shocks $\epsilon_t^f$ and $\epsilon_t^T$, is characterized by $y_t = \Delta p_t = \Delta p_H,t = \Delta q_t = r_t = 0$ at each date $t$\footnote{Clarida, Galí and Gertler (2001) were the first to show this isomorphism between the reduced forms of the closed economy and the small open economy versions of the canonical New Keynesian model.}. Note that the model provides no inflationary bias à la Barro and Gordon (1983a, 1983b), since the output gap and inflation objectives of the central bank coincide with the stationary values of these variables; still, the first-best monetary policy will be temporally inconsistent, as will be seen below.

Of course, this stylized model is too simple to be realistic. In particular, the absence of inertial terms in the structural equations can be criticized. As stressed by Woodford (1999; 2003, chap. 7) however, what matters is that “it incorporates forward-looking private sector behavior in three respects, each of which is surely of considerable importance in reality”.

1.3 Analytical resolution of the model

This section determines analytically and comments on the optimal equilibrium of the model, depending on whether the exchange rate is flexible or fixed, (when flexible) on whether a commitment technology is available or not\footnote{The stationary value of the gross nominal interest rate $R_t$ is $\frac{1}{\beta}$, and the net nominal interest rate $i_t = R_t - 1$ thus fluctuates around $\frac{1-\beta}{\beta}$. For small enough fluctuations, it does not reach therefore its lower bound 0.}, and (when fixed) on whether this commitment applies to a monetary policy rule or to the fixity of the exchange rate. The results obtained are summarized in table 1.1.

We thus consider four alternative configurations for our small open economy: a flexible exchange rate regime without commitment (FL1), a flexible exchange rate regime with commitment (FL2), a(n ex post) fixed exchange rate regime with commitment (FL1) and an irrevocably (ex ante) fixed exchange rate regime with commitment (FL2). When $\alpha = 1$, the FL1 and FL2 cases respectively correspond to that of a closed economy without commitment (CE1) and a closed economy with commitment (CE2).

\footnote{McCullum (1999b) discusses this distinction and reviews the corresponding literature.}
To our best knowledge, most of the analytical results displayed in this section are new, in the sense that they have not been obtained by the existing literature. The primary reason for that is that shocks $\varepsilon^{sc}$ and $\varepsilon^{se}$ are serially correlated in most of the existing studies, so that the analytical resolution of the model then requires the more difficult analytical determination of the roots of a polynomial whose degree is strictly higher than two\(^7\). Still, a few studies consider serially uncorrelated shocks, but they stop at the first-order conditions of the optimization problem, without going the whole way and expressing each variable as a function of the exogenous shocks only, as shown in table 1.2.

The only impulse-response functions already known are those of $\Delta p_H$, $y$ and $r$ in the FL1 case. All the others, namely the impulse-response functions of $\Delta c$ and $\Delta p$ in the FL1 case, those of $\Delta p_H$, $y$, $r$, $\Delta c$ and $\Delta p$ in the FL2, FI1 and FI2 cases, have been incompletely characterized by some studies, but fully derived by none, as shown in table 1.2 which makes a (to our knowledge exhaustive) inventory of existing studies based on the canonical New Keynesian model.

Before solving the minimization problem faced by the central bank, we need to specify the model timing. We suppose that the private agents form their (rational) expectations and the monetary authorities choose the nominal interest rate after the realization and the observation of shocks $\varepsilon^{fr}_t$ and $\varepsilon^{pc}_t$. There is therefore no informational asymmetry between the private agents on the one hand and the monetary authorities on the other hand.

### 1.3.1 Flexible exchange rate regime without commitment (FL1)

This first subsection examines the case (labelled FL1) of a small open economy with a flexible exchange rate regime and without commitment, which corresponds to the case (labelled CE1) of a closed economy without commitment when $\alpha = 1$. By “without commitment”, we mean that only time-consistent monetary policies are credible for the private agents. When no commitment technology is available, the private agents expect the central bank to re-optimize at each period, that is to say to choose $r_{t+n}$ (for each $n \geq 0$) only after the realization of shocks $\varepsilon^{fr}_{t+n}$ and $\varepsilon^{pc}_{t+n}$. As a consequence, their expectations about the future situation in our purely forward-looking framework do not depend on the present monetary policy decision, and the central bank takes therefore these expectations as given when choosing $r_t$.

\(^7\)For instance, Gali and Monacelli (2002) derive analytically the optimal equilibrium in the FL2, FI1 and FI2 cases, and so does Monacelli (2003) in the FL1, FL2, FI1 and FI2 cases, but not as a function of the exogenous shocks only, i.e. not in the form of impulse-response functions, because they consider serially correlated shocks.
The resulting outcome, usually named discretionary equilibrium, or time-consistent plan, or non-reputational solution, is easily determined. Because the central bank takes expectations as given when choosing $r_t$ at date $t$, the first-order condition of the minimisation of $L_t$ (which corresponds to the derivative of $L_t$ with respect to $r_t$ being zero) is written $\lambda y_t + \gamma \Delta p_{H,t} = 0$. Facing the same optimization programme in the future, the central bank will behave in a similar way and the private agents expect therefore: $\lambda E_t \{y_{t+n}\} + \gamma E_t \{\Delta p_{H,t+n}\} = 0$ for $n \geq 1$. Using (1.2.4) from date $t+1$ onwards, we then get $E_t \{\Delta p_{H,t+n+1}\} = \frac{\gamma^2 + \lambda}{\gamma^2 + \lambda} E_t \{\Delta p_{H,t+n}\}$ for $n \geq 1$.

Let us now assume that $\delta \left( \frac{\gamma^2 + \lambda}{\beta \lambda} \right)^2 \geq 1$. (Note that this inequality is indeed satisfied at point $(\delta, \lambda) = (\delta_3, \lambda_3)$, as well as, by continuity, in the neighbourhood of this point.) Under this assumption, the solution to the optimization problem satisfies $E_t \{\Delta p_{H,t+1}\} = 0$, because $L_t$ takes an infinite value if $E_t \{\Delta p_{H,t+1}\} \neq 0$. This implies in turn $E_t \{\Delta p_{H,t+n}\} = 0$ for $n \geq 1$. Using (1.2.4) at date $t$ together with the condition $\lambda y_t + \gamma \Delta p_{H,t} = 0$, we then get $\Delta p_{H,t} = \frac{\lambda}{\gamma^2 + \lambda} \epsilon_t^{pc}$. The impulse-response functions of $y$ and $r$ are finally residually determined from those of $\Delta p_{H}$ with the help of the Phillips curve and the IS equation. We obtain the following results:

$$\Delta p_{H,t} = \frac{\lambda}{\gamma^2 + \lambda} \epsilon_t^{pc} \quad \text{and} \quad \Delta p_{H,t+n} = 0 \quad \text{for} \quad n \geq 1,$$

$$y_t = \frac{-\gamma}{\gamma^2 + \lambda} \epsilon_t^{pc} \quad \text{and} \quad y_{t+n} = 0 \quad \text{for} \quad n \geq 1,$$

$$r_t = \frac{1}{\eta} \epsilon_t^{is} + \frac{\gamma}{(\gamma^2 + \lambda) \eta} \epsilon_t^{pc} \quad \text{and} \quad r_{t+n} = 0 \quad \text{for} \quad n \geq 1.$$ 

Note that we choose in this section to express all the results in the form of impulse-response functions. These impulse-response functions characterize the effect of shocks $\epsilon_t^{is}$ and $\epsilon_t^{pc}$ (at the exclusion of any other shock) on the paths followed by the different variables. In other words, they isolate the effect of the present shocks on the dynamics of the economy. This restriction takes place without any loss of generality, as past, present and future shocks are orthogonal to each other.

These impulse-response functions for $\Delta p_H$, $y$ and $r$ characterize completely the optimal equilibrium in the CE1 case, and incompletely the optimal equilibrium in the FL1 case. (In the latter case, they will be completed by the impulse-response functions of $\Delta e$ and $\Delta p$.) They are discussed in details by Clarida, Galí and Gertler (1999). In brief, they indicate that demand shocks
\( \varepsilon^{is} \) are entirely countered by monetary policy and have therefore no impact on the output gap and the inflation rate. (In other words, output gap stabilisation and inflation stabilization are then mutually compatible.) On the contrary, cost-push shocks \( \varepsilon^{pc} \) are not entirely countered, and the central bank faces a trade-off between a higher inflation rate and a lower output gap following such a shock. In both cases (\( \varepsilon^{is} \) or \( \varepsilon^{pc} \)), the effect of the shock is one-shot, that is to say that the variations in \( \Delta p_H, y \) and \( \tau \) display no inertia.

Besides, equation (1.2.6) holds as the infinite sum in its right-hand side does converge. Acknowledging that the past term \( \frac{\Delta e_{i-1}}{\alpha} \) cannot depend on present shocks and using the non covered interest rate parity equation, we then obtain the following impulse-response functions:

\[
\Delta e_t = -\frac{1}{\eta} \varepsilon_t^{is} + \frac{\eta \lambda - \gamma}{(\gamma^2 + \lambda) \eta} \varepsilon_t^{pc},
\]

\[
\Delta e_{t+1} = -\frac{1}{\eta} \varepsilon_t^{is} + \frac{\gamma}{(\gamma^2 + \lambda) \eta} \varepsilon_t^{pc} \quad \text{and} \quad \Delta e_{t+n} = 0 \quad \text{for} \quad n \geq 2,
\]

\[
\Delta p_t = -\frac{(1 - \alpha)}{\eta} \varepsilon_t^{is} + \frac{\eta \lambda - \gamma + \gamma \alpha}{(\gamma^2 + \lambda) \eta} \varepsilon_t^{pc},
\]

\[
\Delta p_{t+1} = -\frac{1 - \alpha}{\eta} \varepsilon_t^{is} + \frac{(1 - \alpha) \gamma}{(\gamma^2 + \lambda) \eta} \varepsilon_t^{pc} \quad \text{and} \quad \Delta p_{t+n} = 0 \quad \text{for} \quad n \geq 2.
\]

These results indicate that the effect of the shocks \( \varepsilon_t^{is} \) and \( \varepsilon_t^{pc} \) on \( \Delta p \) and \( \Delta e \) is spread on dates \( t \) and \( t + 1 \). It is therefore more prolonged than the effect of the same shocks on \( y \), \( \Delta p_H \) and \( \tau \), due to the non covered interest rate parity equation.

Following a positive \( \varepsilon_t^{is} \) shock, the nominal exchange rate appreciates at date \( t \), then depreciates at date \( t + 1 \) to go back to its initial value. This depreciation at date \( t + 1 \) is the consequence (via the non covered interest rate parity equation) of the increase in the nominal interest rate at date \( t \). The producers price level being left unchanged by the shock \( \varepsilon_t^{is} \), PPP holds in the long run if and only if the final value of the nominal exchange rate equals its initial value: the nominal exchange rate must therefore appreciate at date \( t \) to offset its depreciation at date \( t + 1 \). The evolution of the consumers price level follows then accurately that of the nominal exchange rate with the multiplicative factor \( (1 - \alpha) \), since the producer price level remains unchanged.

Following a positive \( \varepsilon_t^{pc} \) shock, the nominal exchange rate depreciates in a two-period time: \( e_{t+1} = e_{t+1} - e_{t-1} = \frac{\lambda}{\gamma^2 + \lambda} \varepsilon_t^{pc} \), in order to compensate the effect of a higher producer price level on the long run real exchange rate.
This overall depreciation is unevenly spread on each of the two periods: at date $t + 1$, a depreciation occurs which results from the increase in the nominal interest rate at date $t$, via the non covered interest rate parity equation; but at date $t$, what occurs is either a depreciation (if $\eta \lambda > \gamma$) or an appreciation (if $\eta \lambda < \gamma$). Note that in the special case $(\sigma, \mu) = (1,1)$ and $(\delta, \lambda) = (\delta, \lambda)$ considered above, the condition $\eta \lambda < \gamma$ is necessarily satisfied, as it is equivalent to $\varepsilon > 1$, so that the nominal exchange rate appreciates at date $t$.

The nominal exchange rate is the more likely to appreciate at date $t$, the lower is the elasticity $\eta$ of the output gap with respect to the nominal interest rate (because then the increase in the nominal interest rate at date $t$ is large, and therefore so is the nominal exchange rate depreciation at date $t + 1$), or the lower is the relative weight $\lambda$ of the central bank output gap objective (because then the increase in the producer price level is small, and therefore so is the nominal exchange rate depreciation required to satisfy the long run PPP).

As for the evolution of the consumer price index, it is explained by that of the nominal exchange rate and the producer price index: $p$ increases therefore during the two periods considered as a whole ($p_{t+1} - p_{t-1} = p_{t+1} - p_{t-1} = \gamma \frac{p_{t+1}^c}{p_t^c}$), since so does $p_H$ and since $e$ depreciates; $p$ increases at date $t + 1$ too, since $p_H$ remains unchanged at this date and since $e$ depreciates; finally, $p$ can either increase or decrease at date $t$, depending on the sign of $(\eta \lambda - \gamma) + \gamma \alpha$, and decreases only if $e$ appreciates sufficiently to do more than compensate the effect of the increase in $p_H$ on $p$.

Let $L^{FL1}$ denote the mean $E \{L_4\}$ of the loss function in the FL1 case. Because shock $\varepsilon^pc$ is serially uncorrelated, we obtain:

$$L^{FL1} = \frac{\lambda}{(1 - \delta)(\gamma^2 + \lambda)} V(\varepsilon^pc),$$

where $V(\varepsilon^pc)$ denotes the variance of $\varepsilon^pc$.

### 1.3.2 Flexible exchange rate regime with commitment (FL2)

This second subsection considers the case (labelled FL2) of a small open economy with a flexible exchange rate regime and with commitment, which corresponds to the case (labelled CE2) of a closed economy with commitment when $\alpha = 1$. By "with commitment", we mean that the central bank can (credibly) commit itself to following a time-inconsistent monetary policy rule.

When no commitment technology is available, the central bank cannot conduct the first-best monetary policy, because this policy does not fulfill the tem-
poral consistency requirement, as will be seen below: the central bank will face the incentive not to act tomorrow according to what it announces today. Announcing that the first-best monetary policy will be conducted is therefore not credible.

The existence of a commitment technology enables the central bank to avoid the trap of discretionary optimization by tying its hands: announcing that the first-best monetary policy will be conducted is then credible, because the central bank will be compelled to meet its obligations. In this case, it does not re-optimize at each period, but only implements the policy decided beforehand.

By first-best monetary policy, we mean the unique impulse-response function for variable $r$ which is compatible (via the IS equation) with the first-best equilibrium. And by first-best equilibrium, we mean the unique impulse-response functions for variables $\Delta p_H$ and $y$ which minimize the loss function $L_t$ subject to the constraint represented by the Phillips curve.

In other words, we specify the variables as (possibly not time-invariant) linear combinations of the complete history of the exogenous disturbances, from date $t$ onwards, up through the current date $t + n$: 

$$\Delta p_{H,t+n} = \sum_{k=0}^{n} (\eta_{n-k} p_{t+n-k} + \psi_{n-k} e_{t+n-k}), \quad y_{t+n} = \sum_{k=0}^{n} (\alpha_{n-k} p_{t+n-k} + \theta_{n-k} e_{t+n-k}),$$

$$r_{t+n} = \sum_{k=0}^{n} (\phi_{n-k} p_{t+n-k} + \varphi_{n-k} e_{t+n-k}) \text{ for } n \geq 0,$$

and we determine these linear combinations which minimize the loss function subject to the constraints represented by the structural equations.

Note that there are two steps in our approach. The first step, involving (1.2.4) and (1.2.5), determines the optimal impulse-response functions (that is to say the optimal patterns of responses to disturbances, or equivalently the optimal state-contingent paths) for $\Delta p_H$ and $y$. For either variable, the impulse-response function thus defined turns out to be unique. The second step, using (1.2.3), residually determines the (here again unique) impulse-response function for $r$ associated with the ones obtained for $\Delta p_H$ and $y$.

In so doing, we leave temporarily aside the question of whether the impulse-response function obtained for $r$ is compatible only with the (optimal) impulse-response functions obtained for $\Delta p_H$ and $y$, or with other (non-optimal) impulse-response functions for these two variables as well. This question obviously matters, as it amounts to ask whether or not the central bank should express its instrument $r \text{ ex ante}$ in the form of this impulse-response function. This question matters so much actually that we choose to devote the next two sections to answering it. (The answer will be negative.)

Note also that unlike many studies, we are not optimizing over a low-dimensional parametric family of monetary policy rules (usually Taylor-type
rules). We are not even following the approach adopted (to our knowledge only) by Clarida, Gali and Gertler (1999), Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), which consists in seeking what Clarida, Gali and Gertler (1999) call the "unconstrained optimal rule", that is to say the optimum within the class of rules which are time-invariant linear combinations of the entire history of shocks. Indeed, we are optimizing over the family of rules which are (possibly not time-invariant) linear combinations of the entire history of shocks. As shown in the appendix, the optimal rule turns out to be time-invariant anyway.

Before turning to the results, let us consider the optimum within the class of rules which specify the nominal interest rate \( r_t \) as a linear combination of the current shocks \( e_t^\nu \) and \( e_t^{pc} \) only. As noted by Clarida, Gali and Gertler (1999), Woodford (2003, chap. 7), the consideration of this (arbitrarily) restricted family of rules has a pedagogical virtue, as it shows whether what matters is commitment with inertia or commitment without inertia. The resolution of the optimization problem is then very simple: it amounts to follow the procedure presented above while imposing the restriction \( \forall n \geq 1, \forall k \in \{1, \ldots, n\}, \alpha_n^{n-k} = \beta_n^{n-k} = c_n^{n-k} = d_n^{n-k} = f_n^{n-k} = g_n^{n-k} = 0 \). Because shocks are assumed to be serially uncorrelated, we find that the corresponding optimum coincides with the optimal solution in the absence of any commitment technology. In other words, commitment to a non-inertial behaviour is not welfare-improving (relatively to no commitment) in our framework.

The resolution of the model (in the general case) is given in the appendix. We obtain the following impulse-response functions:

\[
\Delta p_{t+n} = \frac{\delta}{\beta} e_t^{pc} \quad \text{and} \quad \Delta p_{t+t+n} = \frac{-\gamma^2 \delta z^{n+1}}{\beta \lambda (1 - \beta z)} e_t^{pc} \quad \text{for} \quad n \geq 1,
\]

\[
y_{t+n} = \frac{-\gamma \delta z^{n+1}}{\beta \lambda} e_t^{pc} \quad \text{for} \quad n \geq 0,
\]

\[
r_t = \frac{1}{\eta} e_t^\nu + \frac{\gamma \delta z \left[ \beta z^2 - (1 + \beta + \gamma \eta) z + 1 \right]}{\beta \eta \lambda (1 - \beta z)} e_t^{pc},
\]

\[
r_{t+n} = \frac{\gamma \delta \left[ \beta z^2 - (1 + \beta + \gamma \eta) z + 1 \right] z^{n+1}}{\beta \eta \lambda (1 - \beta z)} e_t^{pc} \quad \text{for} \quad n \geq 1,
\]

where \( z \) is a constant, expressed in the appendix as a function of the parameters. These results characterize completely the optimal equilibrium in the CE2 case, and need to be completed by the impulse-response functions of \( \Delta e \) and \( \Delta p \) to characterize completely the optimal equilibrium in the FL2 case. As in the
previous subsection, monetary policy insulate the output gap and the inflation rate from the effects of demand shocks $\varepsilon^d$ (by adopting a “leaning against the wind” attitude), but not from those of cost-push shocks $\varepsilon^{pc}$.

The main difference between these results and those of the previous subsection is that the effect of $\varepsilon^{pc}$ is more prolonged here. The shock $\varepsilon^{pc}$ is one-shot, but the variations in $\Delta p_H$, $y$ and $r$ display some inertia. This is because the central bank can now trade off not only between a higher inflation rate and a lower output gap at a given date, but also between the present and the future situations. In other words, the commitment technology enables it to spread the burden of the adjustment to the shock over several periods. Note that commitment (which enables the central bank to credibly choose the entire future state-contingent evolution of the nominal interest rate, once and for all, at date $t$) does matter here, because the central bank faces no actual incentive to go on reacting to bygone shocks.

This inertial feature of the first-best monetary policy is interpreted by Woodford (2003, chap. 7) in the following way: as implicitly stated by the (iterative) IS equation, the effect of monetary policy goes through the long term interest rate, which is determined by market expectations of future short-term interest rates, so that the central bank must make the private sector expect future short term interest rates maintained at given levels to substantially affect the current output gap and inflation rate. To support this interpretation, Woodford (2003, chap. 7) reports the results of empirical studies providing evidence that the variations in long-term interest rates are contemporaneously affected by those in short-term interest rates.

Let us focus on the impulse-response functions of the different variables to the cost-push shock. Following a positive shock $\varepsilon^{pc}$, the price level increases at date $t$, then decreases and tends exponentially towards its long run value. The latter, noted $p_{H,+\infty}$, is characterized by $p_{H,+\infty} - p_{H,-1} = \frac{(\delta - \beta)\varepsilon^{pc}}{\beta(1 - \beta)}$: the final value of the price level is therefore higher than its initial value if and only if $\delta > \beta$, that is to say if and only if the monetary authorities are more patient than the private agents. In the meantime, the output gap decreases at date $t$, then increases and tends exponentially towards its stationary value ($y_{t+\infty} = 0$).

The central bank reacts to the initial positive shock $\varepsilon^{pc}$ either by increasing or decreasing the nominal interest rate, depending on $z$ being respectively lower or higher than the unique real root\(^8\) in-between 0 and 1, noted $z$, of the polynomial $P(q) = \beta q^2 - (1 + \beta + \gamma\eta) q + 1$. This value $z$ corresponds indeed to what could be called a natural harmonic of the system, that is to say a root of the

\(^8\)The analytical expression of this root is given in subsection 1.3.3.
characteristic polynomial of the system's recurrence equation when monetary policy is passive ($r_{t+n} = 0$ for $n \geq 0$).

When $z < x$, the central bank wants the different variables to tend towards their long run values more rapidly than allowed by the economic system left by itself: it increases then the nominal interest rate in order to speed up this convergence process. On the contrary, when $x > z$, the central bank wants to slow down the convergence of the different variables towards their long run values, which makes it decrease the nominal interest rate following a positive cost-push shock.

Note that if $\delta = \delta_S$, the condition $z < x$ is equivalent to $\eta\lambda < \gamma$: we find again here the inequality obtained in the previous subsection. Thus, in the special case $(\sigma, \mu) = (1, 1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$ considered above, this condition is necessarily satisfied, as it is equivalent to $\varepsilon > 1$, so that monetary policy is always tightening in reaction to a positive cost-push shock (i.e. a negative productivity shock), all the more so that the elasticity of substitution $\varepsilon$ between the varieties of the differentiated good is large. (Indeed, a larger $\varepsilon$ implies a larger welfare cost of inflation and therefore a larger weight on the inflation objective of the central bank.)

This outcome, under our preferred specification $(\sigma, \mu) = (1, 1)$ and $(\delta, \lambda) = (\delta_S, \lambda_S)$, proves in accordance with the conventional wisdom, which states that monetary policy, when aimed at stabilizing aggregate output, should react procyclically in the case of productivity shocks and countercyclically in the case of demand shocks, because in doing so it replicates the behaviour of the (real) economy under flexible prices. The mechanisms at work are different here, as the inflation rate enters the loss function in our framework, but the conclusion is the same.

In both cases ($z < x$ or $z > x$), the nominal interest rate, after its reaction at date $t$, tends exponentially towards its initial value ($r_{t,\infty} = 0$). In the intermediate case $z = x$, the nominal interest rate keeps equal to zero ($r_{t+n} = 0$ for $n \geq 0$): the central bank remains passive ex post, but active ex ante, since it follows a monetary policy rule, as indicated in sections 1.4 and 1.5.

The other impulse-response functions are obtained as previously:

$$\Delta e_t = \frac{-1}{\eta} e_t^{zs} + \frac{\delta (\eta\lambda - \gamma) z}{\beta\eta\lambda} e_t^{pc},$$

$$\Delta e_{t+1} = \frac{1}{\eta} e_t^{zs} + \frac{\gamma\delta [\beta z^2 - (1 + \beta + \gamma\eta) z + 1] z}{\beta\eta\lambda (1 - \beta z)} e_t^{pc},$$
\[
\Delta e_{t+n} = \frac{\gamma \delta [\beta z^2 - (1 + \beta + \gamma \eta) z + 1]}{\beta \eta \lambda (1 - \beta z)} z^n e_{t}^{pc} \text{ for } n \geq 2,
\]
\[
\Delta p_t = -\frac{(1 + \alpha)}{\eta} e_{t}^{is} + \frac{\delta [(\eta \lambda - \gamma) + \gamma \alpha]}{\beta \eta \lambda} z e_{t}^{pc},
\]
\[
\Delta p_{t+1} = \frac{1 - \alpha}{\eta} e_{t}^{is} + \frac{\gamma \delta [(1 - \alpha)(1 - z)(1 - \beta z) - \eta \eta z]}{\beta \eta \lambda (1 - \beta z)} z e_{t}^{pc},
\]
\[
\Delta p_{t+n} = \frac{\gamma \delta [(1 - \alpha)(1 - z)(1 - \beta z) - \eta \eta z]}{\beta \eta \lambda (1 - \beta z)} z^n e_{t}^{pc} \text{ for } n \geq 2.
\]

The impulse-response functions of \(\Delta e\) and \(\Delta p\) to the shock \(e_{t}^{is}\) are identical to those described in the previous subsection, since the central bank reacts in the same way to the shock \(e_{t}^{is}\) with or without commitment technology. The model therefore predicts in particular that no matter whether a commitment technology is available or not, the nominal exchange rate appreciates at date \(t\) and depreciates at date \(t + 1\) following a positive \(e_{t}^{is}\) shock, to go back to its initial value.

Following a positive \(e_{t}^{is}\) shock, the nominal exchange rate depreciates (if \(\delta > \beta\)) or appreciates (if \(\delta < \beta\)) in the long run: \(e_{+\infty} - e_{-1} = (\delta - \beta) z e_{t}^{pc}\), in order to offset the increase or decrease in the producer price level. It depreciates from date \(t + 1\) if and only if \(z < \eta\): we find again naturally the distinction made above. And it depreciates at date \(t\) if and only if \(\eta \lambda > \gamma\): we find again here, more unexpectedly, the distinction made in the previous subsection.

Thus, in the special case \((\sigma, \mu) = (1, 1)\) and \((\delta, \lambda) = (\delta_{S}, \lambda_{S})\) examined above, where both conditions \(z < \eta\) and \(\eta \lambda > \gamma\) are equivalent to \(\varepsilon > 1\), the nominal exchange rate appreciates instantaneously and depreciates thereafter, following a positive cost-push shock, to go back in the long run to its initial value. Its volatility depends positively on the elasticity of substitution \(z\) between the varieties of the differentiated good, because so does the volatility of the nominal interest rate, as seen above. In the limit case \(\varepsilon = 1\), the nominal exchange rate remains fixed, whatever the shocks \(e_{t}^{pc}\) affecting the small open economy.

As for the evolution of the consumer price index, it is explained by that of the nominal exchange rate and the producer price index, in the same way as in the previous subsection.

Let \(L^{FL2}\) denote the mean \(E\{L_{t}\}\) of the loss function in the FL2 case. Because shock \(e_{t}^{pc}\) is serially uncorrelated, we obtain:

\[
L^{FL2} = \frac{\delta z}{\beta (1 - \delta)} V(e_{t}^{pc}) .
\]
In a flexible exchange rate regime, the existence of a commitment technology is of course beneficial:

\[ L_{PL1} - L_{PL2} = \frac{\gamma^2 \delta z^2}{(1 - \delta)(\gamma^2 + \lambda)(1 - \beta z)} V(e^{pe}) > 0. \]

This is because with the help of a commitment technology, the central bank is able to trade off not only between a higher inflation rate and a lower output gap at a given date, but also between the present and future situations.

1.3.3 Fixed exchange rate regimes with commitment (FI1 and FI2)

This third subsection focuses on the case of fixed exchange rate regimes with commitment. By "with commitment", we mean either commitment to a (time-inconsistent) monetary policy rule ensuring the fixity of the exchange rate, or adoption of an irrevocably fixed exchange rate regime. In the former case, labelled FI1, there are still a national central bank, still a monetary policy rule. In the latter case, labelled FI2, there are no more national central bank, no more monetary policy rule. The fully-fledged dollarization of some small South American economies and the EMU-membership of some small European economies fall into the latter case.

The distinction between FI1 and FI2 obviously matters in terms of monetary policy rules: in the former case, the central bank remains passive ex post, but active ex ante, since it follows a monetary policy rule, as indicated in sections 1.4 and 1.5; in the latter case, the (shadow) central bank is passive ex ante. As far as the resolution of the model (i.e. the outcome) is concerned however, there is no difference between FI1 and FI2, and we shall speak of the general FI case in the present subsection.

For the nominal exchange rate to remain fixed, we need two (straightforward) conditions to be satisfied: \( \Delta e_t = 0 \) and \( \Delta e_{t+n} = 0 \) for \( n \geq 1 \). The second condition implies, via the non covered interest rate parity, that the nominal interest rate should keep constantly equal to its stationary value: \( r_{t+n} = 0 \) for \( n \geq 0 \). That \( r_{t+n} = 0 \) for \( n \geq 1 \) in particular implies in turn, together with the IS equation and the Phillips curve, that the inflation rates \( \Delta p_{H,t+n} \) for \( n \geq 1 \) (we drop the operator \( E_t \{ \cdot \} \) to simplify the notations) follow a recurrence equation whose second-order characteristic polynomial has \( x \) and \( x' \) for roots, where:

\[ x = \frac{1 + \beta + \gamma \eta - \sqrt{(1 + \beta + \gamma \eta)^2 - 4 \beta}}{2 \beta}, \]
\[ x' = \frac{1 + \beta + \gamma \eta + \sqrt{(1 + \beta + \gamma \eta)^2 - 4\beta}}{2\beta}. \]

We easily check that \( 0 < x < 1 \) and \( x' > 1 \). The general form of the solution is the following: \( \Delta p_{H,t+n} = \phi x^n + \phi' x'^n \) for \( n \geq 1 \), where \( \phi \) and \( \phi' \) are two real numbers. Now, the conditions \( \Delta \epsilon_t = 0 \) and \( \Delta \epsilon_{t+n} = 0 \) for \( n \geq 1 \), together with equation (1.2.6), imply \( \phi' = 0 \), if we reasonably assume that \( \Delta p_{H,t} \neq +\infty \), in other words if we assume that the inflation rate has a finite value at each date, though allowing this value to become arbitrarily large and to tend towards infinity as time passes.

We thus get: \( \Delta p_{H,t+n+1} = x \Delta p_{H,t+n} \) for \( n \geq 1 \). Then the condition \( \gamma_t = 0 \), together with the IS equation and the Phillips curve, implies \( \Delta p_{H,t} - \frac{1}{\gamma} \Delta \epsilon_{t+1} = x^n \epsilon_{t}^{is} + x^n \epsilon_{t}^{pc} \), and the long-run condition (1.2.6) becomes \( \Delta p_{H,t} + \frac{1}{1-\alpha} \Delta p_{H,t+1} = 0 \). These three equations enable us to get \( \Delta p_{H,t+n} \) for \( n \geq 0 \), from which we recover \( y_{t+n} \) for \( n \geq 0 \) with the help of the Phillips curve. We thus obtain the following impulse-response functions:

\[ \Delta p_{H,t} = \gamma x \epsilon_{t}^{is} + x \epsilon_{t}^{pc}, \]

\[ \Delta p_{H,t+n} = -\gamma (1-x) x^n \epsilon_{t}^{is} - (1-x) x^n \epsilon_{t}^{pc} \text{ for } n \geq 1, \]

\[ y_t = x (1 + \beta - \beta x) \epsilon_{t}^{is} - \frac{(1 - \beta x)(1 - x)}{\gamma} \epsilon_{t}^{pc}, \]

\[ y_{t+n} = -(1 - \beta x)(1 - x) x^n \epsilon_{t}^{is} - \frac{(1 - \beta x)(1 - x) x^n \epsilon_{t}^{pc}}{\gamma} \text{ for } n \geq 1. \]

Note that these impulse-response functions have been obtained without any optimization of the loss function: they actually characterize the only possible equilibrium in a fixed exchange rate regime.

One result contrasts with those of the previous subsections: the output gap and the inflation rate are no longer insulated from the effects of demand shocks \( \epsilon_{t}^{is} \). This is because a "leaning against the wind" monetary policy reaction to these shocks would be incompatible with the fixity of the exchange rate. Following a positive shock \( \epsilon_{t}^{is} \), the price level increases at date \( t \), then decreases and tends exponentially towards its initial value \( (p_{H,+\infty} = p_{H,-1}) \), so that PPP holds in the long run. In the meantime, the output gap first increases (at date \( t \)), then decreases by more (at date \( t+1 \)), before eventually increasing and tending exponentially towards its stationary value \( y_{+\infty} = 0 \).
Following a positive shock $\varepsilon_{p}^{t}$, the price level increases at date $t$, then decreases and tends exponentially towards its initial value ($p_{t+\infty} = p_{t-1}$), so that once again PPP holds in the long run. In the meantime, the output gap decreases at date $t$, then increases and tends exponentially towards its stationary value ($\mu_{t+\infty} = 0$). The speed of convergence of the variables is measured by parameter $x$, which corresponds to what we have called a natural harmonic of the system. The higher $x$, the slower the convergence of these variables towards their long run values.

Note that commitment (whether to a monetary policy rule in F1 or to the fixity of the exchange rate in F2) does matter here, because without it the central bank would seek to react to the shocks. Note finally that we also have, of course, $\Delta p_{t} = \alpha \Delta p_{H,t}$ and $\Delta p_{t+n} = \alpha \Delta p_{H,t+n}$ for $n \geq 1$.

Let us note $L^{FI}$ the mean $E\{L_{t}\}$ of the loss function. Given that shocks $\varepsilon_{p}^{t}$ and $\varepsilon_{z}^{t}$ are serially uncorrelated and orthogonal to each other, we obtain:

$$L^{FI} = \frac{x^{2} \left[ (1 + 2 \beta + \delta + \beta^{2}) - 2 \left( \beta + \delta + \beta \delta + \beta^{2} \right) x + \beta (\beta + 2 \delta) x^{2} \right]}{(1 - \delta)(1 - \delta x^{2})} V(\varepsilon_{p}^{t})$$

$$+ \frac{\gamma^{2} \left( 1 - \delta - 2 \delta x^{2} \right) x^{2} + \lambda (1 - \beta)^{2} x^{2} (1 - x)^{2}}{\gamma^{2} (1 - \delta)(1 - \delta x^{2})} V(\varepsilon_{z}^{t}),$$

where $V(\varepsilon_{p}^{t})$ denotes the variance of $\varepsilon_{p}^{t}$. The comparison between $L^{FI}$ and $L^{FQL}$ proves easier than that between $L^{FI}$ and $L^{FQL}$. Indeed, since a non constraint optimization is more performing than a constraint one, we naturally have: $L^{FI} \geq L^{FQL}$. Moreover, we can show that $L^{FI} = L^{FQL} \iff V(\varepsilon_{p}^{t}) = 0$, $\delta = \beta$ and $z = x$. In this case indeed, the optimal monetary policy in the FL2 case is passive (ex post) and therefore coincides with the necessary monetary policy reaction in the FL1 case.

Thus, in the absence of demand shocks and in the special case $(\sigma, \mu) = (1, 1)$ and $(\delta, \lambda) = (\delta_{0}, \lambda_{0})$ examined above, where $z = x \iff \varepsilon = 1$, the fixed exchange rate regime is close to the optimal regime if the elasticity of substitution $\sigma$ between the varieties of differentiated good is close to one. As $\varepsilon$ increases (from $\varepsilon = 1$), the welfare cost of inflation increases as well and therefore so does the relative weight of the central bank's inflation objective, so that the optimal monetary policy reaction to a positive cost-push shock in the FL2 case is no longer passivity, but a rise in the nominal interest rate, which is incompatible with the fixity of the exchange rate.

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Monacelli (2003) finds in a quasi-identical framework, through calibration and simulation, that $L^{FI}$ can indeed be lower than $L^{FQL}$ when $\lambda$ is high or when $\mu$ is high while $\alpha$ is low.
At first sight, the canonical New Keynesian model considered here takes into account none of the advantages usually attributed to the fixed exchange rate regime, because it focuses on its stabilization properties: the flexibility of the nominal exchange rate (FL1 and FL2 vs. FI) enables the central bank to trade off between $\Delta p_{H,t}$ and $y_t$ in very much the same way as the existence of a commitment technology (FL2 vs. FL1) enabled it to trade off between the present and the future situations.

As a consequence, not only does our framework offer a biased point of view on the fixed exchange rate regime, but it also provides no rationale for the adoption of such a regime. Indeed, either no commitment technology is available, and the central bank will not be able to escape the FL1 equilibrium; or a commitment technology is available, and the central bank will prefer to stick to a (time-inconsistent) monetary policy rule implementing the FL2 equilibrium rather than to stick to a (time-inconsistent) monetary policy rule implementing the FI1 equilibrium.

In order to tip the scales towards the fixed exchange rate regime, the consideration of an exogenous shock $\epsilon^*$ affecting the nominal exchange rate under a flexible exchange rate regime, specified as a variable risk-premium added to the non covered interest rate parity equation, would not fit the bill. Indeed, this shock would merely end up as a component of the aggregate demand shock, whose effect on the target variables is completely countered by monetary policy in a flexible exchange rate regime.

What we would need instead is an exogenous shock $\epsilon^*$, added to the monetary policy rule and representing the involuntary and non-systematic deviations of the nominal interest rate from its value prescribed by the monetary policy rule, so as to account for the central bank's shaking hand. The introduction of such a shock would create a non-degenerated trade-off, depending on the variance $V(\epsilon^*)$, between the FL2 and the FI2 regimes. Besides, we could say that one (less easily quantifiable) advantage of the FI2 regime over the FI1 regime, which will become apparent in section 1.5, is that the FI2 regime does not make the implementation of the desired equilibrium rest on the perilous application (by the central bank) and the improbable understanding (by the private agents) of a rather complicated monetary policy rule. This result would tend to be in accordance with conventional wisdom, which advocates the choice of a "corner solution" for the exchange rate regime.
1.4 Adoption of a monetary policy rule

In this section, we first illustrate the well-known fact that the adoption of a monetary policy rule expressing the nominal interest rate as a function only of the exogenous shocks leads to multiple equilibria. We then indicate how the adoption of a monetary policy rule expressing the nominal interest rate as a well-chosen function of the endogenous variables enables the central bank to select the desired equilibrium among these multiple equilibria. Finally, we lay emphasis on the importance of ruling out divergent equilibria in particular.

1.4.1 Existence of multiple equilibria

Section 1.3 shows that a necessary condition for the minimisation of the loss function (in the FL1 and FL2 cases) or for the fixity of the exchange rate (in the FT1 case) is that the nominal interest rate should follow a well-defined state-contingent (i.e. expressed as a function of the exogenous shocks) path, which of course depends on the case considered (FL1, FL2 or FT1). This condition is necessary, but not sufficient. Indeed, in the FL1 and FL2 cases, this path \((r_{t+n})_{n \geq 0}\) proves compatible not only with the optimal paths \((\Delta p_{H,t+n})_{n \geq 0}\) and \((y_{t+n})_{n \geq 0}\) obtained in subsections 1.3.1 and 1.3.2, but also with an infinity of other paths which do not minimize \(L_t\). Similarly, in the FT1 case, the path \(r_{t+n} = 0\) for \(n \geq 0\) does imply \(E_t \{\Delta e_{t+n}\} = 0\) for \(n \geq 1\), but not \(\Delta e_t = 0\), so that the fixity of the exchange rate is not ensured.

As an illustration, let us assume that the central bank pledges in a credible way to choose a nominal interest rate following the path obtained in the FL2 case. Equations (1.2.3) and (1.2.4) then imply that the expected inflation rates \(E_t \{\Delta p_{H,t+n}\}_{n \geq 1}\) satisfy a recurrence equation of order three, whose characteristic polynomial has \(x, x'\) and \(x''\) for roots. The general form of the solution is the following: \(E_t \{\Delta p_{H,t+n}\} = az^n + bx'^n + cx''^n\) for \(n \geq 1\), where \(a, b\) and \(c\) are three real numbers.

We therefore have four unknowns \((a, b, c, \Delta p_{H,t})\), which must be determined by the initial condition(s). (Once the current and expected future inflation rates are determined, the current and expected future output gaps are residually obtained through the IS equation.) Now, we only have one initial condition, namely a mix of the IS equation, the Phillips curve and the monetary policy rule taken at date \(t\), involving \(E_t \{\Delta p_{H,t+1}\}, E_t \{\Delta p_{H,t+2}\}, E_t \{\Delta p_{H,t+3}\}, \Delta p_{H,t}, \varepsilon_t^{in}\) and \(\varepsilon_t^{in}\). The results obtained in subsection 1.3.2 (corresponding in particular to \(b = c = 0\)) represent of course one possible solution, but there exist an infinity of other solutions, either convergent or divergent, which are usually called "sunspot
equilibria" as they do not depend only on the fundamentals.

This multiplicity of equilibria comes from the fact that the present values of the inflation rate and the output gap depend in particular on their expected future values, via the IS equation and the Phillips curve. Now, these expected future values cannot be controlled by the central bank: the model says how the private sector’s expectations influence the current situation, not the other way round, as Woodford (2003, chap. 2) makes clear\textsuperscript{10}. Our framework is thus one in which the current situation depends on expectations about the indefinite future, hence the indeterminacy of the equilibrium.

Kerr and King (1996), Bernanke and Woodford (1997), Clarida, Gali and Gertler (2000), McCallum (1999c) identify and discuss this nominal and real indeterminacy of the equilibrium in the canonical New Keynesian model\textsuperscript{11}. Their framework is that of a closed economy with a commitment technology available to the central bank, and the following literature about indeterminacy in the canonical New Keynesian model has stuck to this framework, which corresponds more or less to our CE2 case (i.e. our FL2 case with $\alpha = 1$), more or less do we say because most of the existing studies consider specific (usually Taylor-type) monetary policy rules rather than the ones implementing the optimal equilibrium. We argue that this indeterminacy problem arises not only in the FL2 case, but also in the FL1 and FI1 cases\textsuperscript{12}.

1.4.2 Selection of a unique equilibrium

The remedy advocated by the existing literature to remove (at least partially) this indeterminacy consists in choosing an adequate monetary policy rule expressing the nominal interest rate $r_t$ as a function of past, present or expected future endogenous variables, rather than as a function of the exogenous shocks $\epsilon^{\text{in}}$ and $\epsilon^{\text{pc}}$ having occurred in the past and occuring in the present (as implicitly

\textsuperscript{10}In Woodford’s own terms (2003, chap. 2): “Such reasoning involves a serious misunderstanding of the causal logic of [the] difference equation [(1.2.4)] [...]. The equation does not indicate how the equilibrium inflation rate in period $t+1$ is determined by the inflation that happens to have occurred in the previous period. [...] But instead, the equation indicates how the equilibrium inflation rate in period $t$ is determined by expectations regarding inflation in the following period. These expectations determine the real interest rate, and hence the incentive for spending [...]."

\textsuperscript{11}This indeterminacy was first identified by Woodford (1994), but in a framework different from that of the canonical New Keynesian model. It is also mentioned by Svensson (2000) in a more general framework. Besides, McCallum (1999b) discusses the fact that this is no more nominal indeterminacy. In another context, Sargent and Wallace (1975) were the first to point to the (nominal) indeterminacy of the equilibrium when the monetary policy instrument is the nominal interest rate, rather than the money stock.

\textsuperscript{12}Actually, Benigno, Benigno and Ghironi (2002) do also acknowledge this indeterminacy problem in the case of a fixed exchange rate regime, though not within the framework of the canonical New Keynesian model.
done in section 1.3). Besides, another rationale put forward in the literature for adopting such a monetary policy rule, rather than specifying the nominal interest rate as a function of the complete history of the exogenous disturbances, is that this kind of rule typically requires the knowledge of no more than a few lagged, current and expected future endogenous variables.

In the previous example corresponding to the FL2 case, if \( r_t \) is expressed as a function of \( \Delta p_{H,t} \) and \( y_t \), or of \( E_t \{ \Delta p_{H,t+n} \} \) and \( E_t \{ y_{t+n} \} \) for \( n \geq 1 \), then the number and the values of the roots of the characteristic polynomial of the recurrence equation followed by the expected inflation rates \( E_t \{ \Delta p_{H,t+n} \}_{n \geq 1} \) are \textit{a priori} modified, as well as the expression of the initial condition. If \( r_t \) is expressed as a function of \( \Delta p_{H,t-n} \) and \( y_{t-n} \) for \( n \geq 1 \), it is then not only the number and the values of the roots of the characteristic polynomial, as well as the expression of the initial condition, which are \textit{a priori} affected, but also the number of initial conditions.

Actually, we can independently control the number of roots of the characteristic polynomial and the number of initial conditions. For instance, adding a term \( \omega (\Delta p_{H,t-1} - \beta \Delta p_{H,t} - \gamma y_{t-1}) \), where \( \omega \neq 0 \), to an otherwise non backward-looking monetary policy rule, provides one more initial condition without affecting the degree of the characteristic polynomial. Indeed, this additional term becomes \( \omega \tau \) in the expression of \( E_t \{ r_{t+1} \} \) and 0 in the expression of \( E_t \{ r_{t+n} \} \) for \( n \geq 2 \), because it corresponds to the deterministic part of the Phillips curve. Adding this term amounts therefore somehow to postpone the starting date of the recurrence equation, without affecting this recurrence equation.

To decrease the number of roots of the characteristic polynomial amounts to decrease the number of unknowns. To increase the number of initial conditions amounts to increase the number of equations. An adequate choice of monetary policy rule can therefore reduce the indeterminacy, and possibly remove it completely.

For instance, we only need one root (equal to \( z \) in the FL2 case and to \( z \) in the FL1 case) and two initial conditions to ensure the implementation of the results obtained in the FL2 and FL1 cases. Indeed, in each of these two cases, the results can be summarized by the value for \( \Delta p_{H,t} \), that for \( \Delta p_{H,t+1} \), and the recurrence equation \( \Delta p_{H,t+n} = \chi \Delta p_{H,t+n-1} \) for \( n \geq 2 \), where \( \chi = z \) in the FL2 case and \( \chi = x \) in the FL1 case. (From the impulse-response function of the inflation rate can then be recovered those of the other variables.) Similarly, no root and one initial condition are enough to ensure the implementation of the results obtained in the FL1 case.
A whole branch of the New Keynesian literature, whose most representative authors are Bernanke and Woodford (1997), Woodford (1999; 2003, chap. 4, 7 and 8), Giannoni and Woodford (2003a, 2003b), aims at characterizing the monetary policy rules ensuring the implementation of the unique optimal equilibrium. As already said, these studies focus on the CE2 case (i.e. the FL2 case with $\alpha = 1$), while we flush the indeterminacy problem not only in the FL2 case, but also in the FL1 and FI1 cases. More importantly, the literature has been concerned only about the possible existence of multiple convergent equilibria\(^\text{13}\), which entail endogenous fluctuations, and has disregarded divergent equilibria so far. We do not.

### 1.4.3 Ruling out divergent equilibria

Almost all the existing studies\(^\text{14}\) concerned about the possible indeterminacy of the equilibrium restrict their attention to bounded paths, and thus are satisfied with obtaining the unicity of the path of each variable conditionally on its boundedness. In other words, they characterize monetary policy rules which rule out all convergent equilibria other than the optimal one, but which do not a priori rule out divergent equilibria. In the example of subsection 1.4.1, with $0 < \beta < 1 < \beta'$ and $\beta < 1$ (the appendix provides a sufficient condition for the latter inequality to be satisfied), this would amount to let parameter $b$ be free, rather than constrain it to be nil. The reason usually put forward to justify this restriction is that the linearization of the model is acceptable only for small macroeconomic fluctuations around the steady state, and therefore is not adapted to the study of non-bounded paths; as a consequence, the latter should be ignored. We disagree with this justification for two (alternative) reasons.

First, we may wish to rule out divergent equilibria as a precautionary measure. Indeed, suppose that the central bank adopts a monetary policy rule which does not preclude the development of divergent paths. If we consider

\(^{13}\)Rather an isolated voice, McCallum (1999c, 2000) expresses doubts on the empirical relevance of these multiple equilibria, and thinks that the fundamental (or bubble-free) solution is the most likely to emerge in the economy.

\(^{14}\)Apart from Clarida, Gali and Gertler (1999), we know of only three exceptions which do not disregard unbounded solutions among all these possible saddle-point equilibria. First, Christiano and Gust (1999) distinguish between determinate, indeterminate and explosive equilibria, but their work hinges on numerical simulations, not analytical results, within the framework of a limited participation model, not the framework of the canonical New Keynesian model. Second, Batini and Haldane (1999) distinguish between explosive and non-explosive (simulated, not analytical) solutions to a New Keynesian model close to our canonical version, but fail to acknowledge the possible existence of multiple (non-explosive) equilibria. Third, in a more general framework than ours, Currie and Levine (1993, chap. 4, section 5) consider "overstable feedback rules" which remove all unstable roots from the system, but these rules do not remove undesired stable roots and hence do not rule out multiple (bounded) equilibria.
an ad hoc exogenous commitment technology (thus forbidding the central bank to abandon its monetary policy rule, whatever the welfare costs caused by the divergent equilibria), then we lose sight of these divergent paths as soon as the variables are sufficiently far away from their stationary values, not only because these paths then invalidate our log-linear approximation of the model, but also because they invalidate the model itself, and in particular our price-setting specification à la Calvo (1983), our CES modelization of the domestic consumption basket, or our assumption on the currency in which prices are quoted. In particular, we have no clue about whether these paths eventually violate the transversality condition\(^{15}\) and (in the case of a small open economy) the long-run PPP condition (1.2.6), that is to say about whether these paths actually correspond to equilibria of the model. In the end, we know very little about divergent equilibria, not even whether they exist or not, only that they are likely to be welfare-reducing if they exist, so that it seems more prudent to us to seek to rule them out.

Second, and more importantly, there exists actually no such thing as an exogenous commitment technology, by which the central bank commits itself to sticking to its monetary policy rule. Only in the FI2 case should the commitment technology be considered as exogenous, as the commitment then applies to the fixity of the exchange rate, not to a monetary policy rule. In the FL2 and FI1 cases, it seems more relevant to deal with an endogenous commitment technology, coming from reputation effects for instance\(^{16}\), so that the central bank weighs the pro and contra before deciding whether to stick to its monetary policy rule. Suppose then that a divergent path starts to develop in the neighbourhood of the steady state. The central bank will therefore sooner or later abandon its monetary policy rule in order to bring the divergent variables back to their steady state values. In the end, what we call a divergent path may actually remain constantly in the neighbourhood of the steady state, thus violating neither the transversality condition nor (in the case of a small open economy) the long-run PPP condition (1.2.6).

All these considerations point to the fact that divergent equilibria do indeed matter in our context. Now the question arises of how exactly to rule out these divergent equilibria. To this question the literature provides one answer,

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\(^{15}\) As Blanchard and Fischer (1989, p. 78) put it: "Of course, the proof that the transversality condition is violated on all but the saddle point path in the linearized system does not establish the fact that the paths of the original system that are not saddle point paths explode [...] A complete proof requires a characterisation of the dynamics of the original nonlinear system."

\(^{16}\) Loisel (2003) represents a first attempt at endogenizing the commitment technology in the canonical New Keynesian model through reputation effects.
put forward by Clarida, Galí and Gertler (1999, p. 1701), who rightly argue that the credible threat of the central bank to act eventually as a "stabilizer of last resort" is enough to nip any divergent equilibrium in the bud: "To avoid global indeterminacy, the central bank may have to commit to deviate from a simple interest rule if the economy were to get sufficiently off track. This threat to deviate can be stabilizing, much the way off the equilibrium path threats induce uniqueness in game theory. Because the threat is sufficient to preclude indeterminate behavior, further, it may never have to be implemented in practice."

Indeed, divergent equilibria will be precluded if the private agents expect the central bank to successfully bring any diverging variable back to its steady state value in a finite time horizon. However, such a reasoning could be applied to non-optimal convergent equilibria as well\(^\text{17}\) and would consequently remove any need for the central bank to follow a well-defined monetary policy rule. Ultimately, the optimal monetary policy could then be unspecifically defined in these general terms: react in the future in a dissuasive way to whatever path is not optimal so as to ensure the implementation of the optimal equilibrium. How exactly to react to each non-optimal path remains in the dark, as Clarida, Galí and Gertler (1999) do not specify to which monetary policy rule the central bank should then switch\(^\text{18}\).

We argue that the problem of how to stop a developing undesirable path is very much similar to the problem of how to preclude the development of an undesirable path. In other words, the question of which monetary policy rule to switch to in the future to react to an undesirable path amounts to the question

\(^{17}\) Admittedly, this assertion rests implicitly on the assumption that the central bank can change its monetary policy rule at no cost. Indeed, were the change of monetary policy rule costly (e.g. in terms of reputation), the central bank would certainly still face the incentive to react to a divergent path because the cost of letting a divergent path develop would naturally outweigh any benefit of sticking to the initial rule, but this result would not necessarily hold in the case of a (non-optimal) convergent path. Now, this assumption of non-costly change of monetary policy rule may prove quite strong, especially so when the issue of commitment to the time-inconsistent first-best monetary policy is considered (as in the FLI case), whether this commitment is exogenous or comes from reputation effects.

\(^{18}\) The central bank may actually even switch to a monetary policy not defined by an interest rate rule. As an illustration, the following quotation details Benigno, Benigno and Ghironi's (2002, p. 7) proposition to rule out divergent equilibria in the FI1 case: "To summarize, we have shown that either the exchange rate is fixed or there is a positive probability that the exchange rate will explode or implode in an infinite time. How can we rule out those explosive equilibria and determine a unique rational expectations equilibrium with a fixed exchange rate under rule [...]? We can eliminate the explosive solutions by assuming that authorities are committed to drastic policy actions if the exchange rate settles on an explosive path. An example of such commitment is the assumption that each monetary authority can back its currency with a fraction, even small, of some assets (such as gold) or with goods, in the case the currency becomes too devalued."
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of which monetary policy rule to follow in the present to preclude undesirable equilibria. Indeed, as will be clear later, both these monetary policy rules are necessarily forward-looking so as to insulate the current inflation rate from the private agents' sunspot-prone expectations about the future situation. Instead of two monetary policy rules, say one for the present to preclude all non-optimal convergent equilibria and the other for the future to react in a dissuasive way to divergent paths, we propose therefore a single monetary policy rule which ensures the implementation of the unique optimal equilibrium by precluding all non-optimal convergent and divergent equilibria.

1.5 Characterization of the adequate monetary policy rules

This section characterizes the monetary policy rules ensuring the implementation of the optimal equilibrium determined in section 1.3, in each of the relevant cases considered (FL1, FL2 and FI1), while the existing literature attempts to do it only in the CE2 case (i.e. the FL2 case with $\alpha = 1$). As made clear by section 1.4, we require (unlike the existing literature) from a monetary policy rule the property to rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria. Table 1.2 shows that of all the existing studies based on the canonical New Keynesian model, only Giannoni and Woodford (2003a, 2003b) as well as Woodford (2003, chap. 8) do consider a class of monetary policy rules which includes what we call adequate monetary policy rules.

Besides, in both the closed economy and the small open economy cases, we restrict our attention, like Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), to the monetary policy rules which express the nominal interest rate as a function only of the (past and present) exogenous shocks, the (past) nominal interest rates and the (past, present and expected future) target variables. This restriction is merely to keep things as simple and our message as clear as possible: of course, the same reasoning and the same qualitative results would hold, were this assumption to be relaxed. In effect, it amounts to forbid $p_H$ in the closed economy case, $p_H$, $p$, $e$, $\Delta p$ and $\Delta e$ in the small open economy case to enter the monetary policy rules considered, as none of these variables is a target variable in our framework (i.e. none of them enters the loss function).

This section is divided into two subsections, which examine respectively the forward-looking part and the backward-looking part of the adequate monetary policy rules.
1.5.1 Forward- lookingness

The appendix shows that whatever the case considered (FL1, FL2, FL1, as well as in particular CE1 and CE2), the finite linear monetary policy rules ensuring the implementation of the desired equilibrium are necessarily forward-looking\textsuperscript{19} in a well-defined manner, so as to control the effect of the expected future values of the inflation rate and the output gap on their present values.

More precisely, the only way to remove indeterminacy consists in choosing a monetary policy rule whose forward-looking part counters exactly the effect of expected future values of the inflation rate and the output gap on the present value of the inflation rate, that is to say that it opposes this effect so as to cancel it. This effect is described by the IS equation in which \( y_t \) and \( E_t \{ y_{t+1} \} \) are expressed as a function of \( E_t \{ \Delta p_{H,t+2} \} \), \( E_t \{ \Delta p_{H,t+1} \} \), \( \Delta p_{H,t} \) and \( \epsilon_t^{p,c} \) with the help of the Phillips curve:

\[
\Delta p_{H,t} = \epsilon_t^{p,c} + \gamma \epsilon_t^{c} - \gamma \eta y_t + (1 + \beta + \beta \eta) E_t \{ \Delta p_{H,t+1} \} - \beta E_t \{ \Delta p_{H,t+2} \}.
\]

The forward-looking part of \( r_t \) should therefore amount to the term \( \frac{1}{\gamma} \left[ (1 + \beta + \beta \eta) E_t \{ \Delta p_{H,t+1} \} - \beta E_t \{ \Delta p_{H,t+2} \} \right] \textsuperscript{20}. \) Only at this condition will the present value of the inflation rate be uniquely pinned down. Each expected future value of the inflation rate is then determined in a similar way, using the equation corresponding to the expected application of the monetary policy rule in the future\textsuperscript{21}, while the present and expected future output gaps are residually determined by the Phillips curve.

This result can be interpreted in the following way. In the canonical New Keynesian model, current variables depend on expected future variables, so that in order to pin down current variables, monetary policy should first pin down expected future variables. But these expected future variables depend in turn on still further expected future variables, and so on, so that a possible indeterminacy problem arises in this framework. The only way to remove indeterminacy

\textsuperscript{19} Forward-looking monetary policy rules in our framework correspond to "implicit instrument rules" in the terminology of Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b): "an implicit instrument rule [...] is a formula for setting the policy instrument as a function of other variables, some of which must be projected by the central bank in order to implement the rule, with the projections themselves being conditional upon (and affected by) the instrument setting". (Authors' emphasis.)

\textsuperscript{20} Complete examples of adequate monetary policy rules, including both the forward- and the backward-looking parts of these rules, will be given in the next subsection.

\textsuperscript{21}In other words, \( \Delta p_{H,t} \) is determined by the application of the monetary policy rule at date \( t \), while \( E_t \{ \Delta p_{H,t+k} \} \) for \( k \geq 1 \) is determined by the expected application of the monetary policy rule at date \( t + k \).
is for monetary policy to be forward-looking so as to disconnect current variables from expected future variables, more precisely to disconnect the current inflation rate from expectations about the future situation. In so doing, the central bank kills two birds with one stone: not only does it insulate the current inflation rate from the sunspot-prone expectations about the future situation, but it does also insulate these expectations from sunspots, as they are similarly disconnected from expectations about the further future situation.

It is well-known that the efficiency of monetary policy in the canonical New Keynesian model mainly depends on the central bank’s ability to influence the private agents’ expectations. What we argue is that the central bank should actually react to (and in so doing influence) these expectations so as to cancel their effects on the current inflation rate. Of course, such forward-looking rules require from the central bank precise knowledge of the current situation as well as accurate observation of the private agents’ expectations (conditional on the monetary policy chosen) about the future situation, not to mention perfect information about the true values of the parameters, which is unlikely to be the case in practice, as argued notably by McCallum (1999b). But nobody said central banking was easy.

Note that expectations of future variables can be expressed in a backward-looking form in equilibrium (ex post), but not out of equilibrium (ex ante). It is therefore essential that the monetary policy rule should be explicitly forward-looking. In saying so, we agree with Evans and Honkapohja (2002, 2003), who insist on ruling out indeterminacy by basing monetary policy on observed private expectations, but we disagree with Batini and Haldane (1999, p. 161), according to whom “any forward-looking rule can be given a backward-looking representation and respecified in terms of current and previously-dated variables”; similarly with Taylor (1999a, 1999b), who dismisses the very idea of forward-looking monetary policy rules as of little relevance, on the ground that forecasts are based on current and lagged data; again with Levin, Wieland and Williams (2001, p. 3), who argue that “since every forecast can be expressed in terms of current and lagged state variables, a forecast-based rule cannot yield

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22In Woodford’s (2003) own terms, “markets can to a large extent do the central bank’s work for it” (chap. 1), or, more precisely, “the bond market does the Fed’s work for it” (chap. 7).

23Of all these powers ascribed to the monetary authorities, the ability to observe the private agents’ expectations seems to us the less far-fetched for two reasons. First, readily available business and households surveys are more often than not at the disposal of the central bank. Second, much can be derived from the yield curve about the private agents’ expectations of future inflation rates and nominal interest rates.

24Their point differs from ours however, as they reach this conclusion under adaptative learning by private agents.
any improvement in macroeconomic stability relative to the fully optimal policy rule (which incorporates all of the relevant state variables)"; and even with Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), for whom "if a forecast-based policy rule can be found that is consistent with the desired equilibrium, one can necessarily also obtain a purely backward-looking rule [...] by substituting for the forecast the particular function of predetermined and exogenous variables that represents the rational forecast".

As stressed by Bernanke and Woodford (1997) indeed, what actually matters is not so much the central bank's forecasts as the private sector's expectations, which can be affected by sunspots. Once again, the model says how the private sector's expectations influence the current situation, not the other way round. The necessity for forward-looking monetary policy rules in our framework comes directly from the fact that monetary policy should aim at disconnecting the current situation from the private sector's sunspot-prone expectations about the future situation. In our view, the explicit (and in some cases published) forecasts on which the central banks of Canada, New Zealand and the UK for instance base their monetary policy should therefore ideally be made conditional on the private sector's expectations.

As shown in table 1.2, the existing literature about forward-looking monetary policy rules within the canonical New Keynesian framework mainly focuses on simple specific families of monetary policy rules, for instance to address the question of the optimal forecast horizon, like Batini and Haldane (1999), Levin, Wieland and Williams (2001), or the question of the equilibrium (in)determinacy, like Clarida, Gali and Gertler (2000), Batini and Pearlman (2002). Of course, their results depend on the class of rules considered. We adopt the more general approach of Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), and consider a much broader class of forward-looking monetary policy rules. While these authors require from their monetary policy rules the (robustness) property that they should rule out all convergent

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25 Some justifications for forward-looking monetary policy rules put forward in the literature are questionable in our opinion. For instance, Batini and Haldane's (1999) as well as Batini and Pearlman's (2002) monetary policy transmission lags, which require preemptive strikes from the central bank, should be perfectly compatible with purely backward-looking monetary policy rules.

26 Table 1.2 focuses on the studies based on the canonical New Keynesian model, but things are hardly different for other studies. De Fiore and Liu (2002) for instance, who use a small open economy model which is no New Keynesian model, consider a very specific forward-looking monetary policy rule to address the issue of equilibrium (in)determinacy. They derive analytical results when possible, calibrate and simulate their model otherwise. Naturally, they consider only convergent equilibria, like all other studies. However, they happen to have a particular reason of their own to do so. Indeed, no divergent equilibrium can arise in their specific model.
equilibria other than the optimal one whatever the statistical properties of the exogenous disturbances, we require from ours the (stability) property that they should rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria. What we then find is that this requirement is enough to entirely pin down (modulo the Phillips curve, as made clear by the appendix) the forward-looking part of our monetary policy rules.

1.5.2 Backward-lookingness

Flexible exchange rate regime without commitment (FL1)

The appendix shows that in the FL1 case (which includes the CEI case), the finite linear monetary policy rules ensuring the implementation of the desired equilibrium can be backward-looking \((N_1 > 0)\) or not \((N_1 = 0)\), and that the set of these rules of “size” \(N_1\) is a \(3N_1 + 2\)-dimensional vectorial space.

Let us take an example. The set of adequate finite linear monetary policy rules of size \(N_1 = 0\) is a 2-dimensional vectorial space. Among these “minimally history-dependent rules”, in the terminology of Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), there is only one which satisfies to the double constraint \((c_0, d_0) = \left(\frac{-E_t^2(\beta gamma^2)}{\beta gamma}, 0\right)\).

It is written in the following way:

\[
\tau_t = \frac{1}{\eta} E_t \{ y_{t+1} \} - \frac{\beta + \gamma}{\beta \eta} y_t - \frac{\gamma^2}{\beta \lambda} \Delta p_{H,t} + \frac{1}{\eta} e_t^s.
\]

This rule is (by definition) applied at each date. The private agents will find it credible, in spite of the absence of commitment technology, precisely because it implements the optimal solution in the absence of commitment technology. In other words, this rule is temporally consistent: if the private agents expect it to be followed in the future, then the central bank will have no incentive to deviate from it.

Let us check that this rule does really implement the desired equilibrium. Suppose this rule is applied at date \(t\). Using the IS equation and the Phillips curve at date \(t\), we then easily obtain \(\Delta p_{H,t}\) identical to the result of the subsection 1.3.1. Suppose moreover that the private agents expect the rule to be applied in the future: using the IS equation and the Phillips curve at these dates, we then get \(E_t \{ \Delta p_{H,t+n} \} = 0\) for \(n \geq 1\), which does correspond to the desired result. The present and expected future output gaps are then determined with the help of the Phillips curve. The present and expected future interest rates

\footnote{We choose this constraint on \(c_0\) throughout the whole subsection because it enables us to express the forward-looking part of the monetary policy rule in a very simple way, more precisely to limit this forward-looking part to the single term \(\frac{1}{\eta} E_t \{ y_{t+1} \}\).}
are eventually obtained with the help of the IS equation. All these results are identical to those obtained in subsection 1.3.1.

Flexible exchange rate regime with commitment (FL2)

The appendix shows that in the FL2 case (which includes the CE2 case), finite linear monetary policy rules ensuring the implementation of the desired equilibrium are necessarily backward-looking ($N_1 \geq 1$), and that the set of these rules of size $N_1$ is a $3N_1 + 1$-dimensional vectorial space. The (partially) backward-looking nature of these rules offsets the purely forward-looking nature of the IS equation and the Phillips curve, where no lagged variable features. It amounts to introduce at each period predetermined variables which can play a anchoring role and thus provide additional initial conditions.

Let us take an example. The set of adequate finite linear monetary policy rules of size $N_1 = 1$ is a 4-dimensional vectorial space. Among these minimally history-dependent rules, there is only one\(^{28}\) which satisfies to the quadruple constraint $(b_{-1}, c_0, d_0, d_{-1}) = \left(0, -\frac{\beta + \eta}{\beta}, 0, 0\right)$. It is written in the following way:

$$
\tau_t = \frac{1}{\eta} E_t \{y_{t+1}\} - \frac{\beta + \gamma \eta}{\beta \eta} y_t + Ay_{t-1} - \frac{\gamma^2}{\beta \lambda (1 - \beta \varepsilon)} \Delta p_{H,t} + B \Delta p_{H,t-1} + \frac{1}{\eta} \varepsilon_t^e,
$$

with

\[
A = \frac{\gamma \left[\beta \lambda^2 (1 - \beta \varepsilon)^2 + \gamma^4 \varepsilon^2\right]}{\delta \left[\beta \lambda^3 (1 - \beta \varepsilon)^3 - \beta^2 \gamma^4 \varepsilon^2\right]},
\]

\[
B = \frac{-\gamma^4 \varepsilon}{\beta \lambda^2 (1 - \beta \varepsilon)^3 - \beta^2 \gamma^4 \varepsilon^2}.
\]

There also exist infinite linear monetary policy rules implementing the desired equilibrium. Indeed, the unique adequate linear monetary policy rule featuring only the past, present and/or expected future inflation rate (as far as endogenous variables are concerned) is an infinite rule which is written in the

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\(^{28}\)Given our requirements, this monetary policy rule happens to be a “direct rule” in the terminology of Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), that is to say a rule which involves only (lags and leads of) target variables. As should be clear from the appendix however, there also exist adequate monetary policy rules which are not “direct” in the sense that they involve the lagged nominal interest rate.
following way:

\[ \begin{align*}
    r_t &= -\beta E_t \{ \Delta p_{H,t+2} \} + (1 + \beta + \gamma \eta) E_t \{ \Delta p_{H,t+1} \} \\
    &+ \frac{\beta - \delta_x}{\gamma \delta x} \sum_{i=0}^{\infty} \left( \frac{\beta}{\delta} \right)^i \Delta p_{H,t-i} + \frac{1}{\eta} \varepsilon_{t+1}.
\end{align*} \]

Fixed exchange rate regime with commitment (FI1)

Subsection 1.3.3 has shown that the canonical New Keynesian model as such provides no clear and direct rationale for the adoption of a fixed exchange rate regime of the FI1 type. Let us nonetheless suppose that the small economy embraces such a fixed exchange rate regime. Even though it keeps the nominal interest rate constantly equal to its stationary value, the central bank is not passive: as section 1.4 makes clear, it has to follow \textit{ex ante} a monetary policy rule in order to ensure the \textit{ex post} fixity of the exchange rate.

The appendix shows that in this FI1 case, finite linear monetary policy rules ensuring the implementation of the desired equilibrium are necessarily backward-looking (\(N_1 \geq 1\)), and that the set of these rules of size \(N_1\) is a 3\(N_1+1\)-dimensional vectorial space. Let us take an example. The set of adequate finite linear monetary policy rules of size \(N_1 = 1\) is a 4-dimensional vectorial space. Among these minimally history-dependent rules, there is only one which satisfies to the quadruple constraint (\(b_{-1}, a_0, d_0, d_{-1}\) = \(0, -\frac{\beta + \gamma}{\beta \eta}, 0, 0\)). It is written in the following way:

\[ \begin{align*}
    r_t &= \frac{1}{\eta} E_t \{ y_{t+1} \} - \frac{\beta + \gamma \eta}{\beta \eta} y_t - \frac{1 - x}{\beta x} \Delta p_t + \frac{\gamma}{\beta (1 - \beta x)} y_{t-1} \\
    &+ \frac{\beta + \gamma \eta}{\beta \eta} \varepsilon_{t+1} - \frac{\gamma}{\beta (1 - \beta x)} \varepsilon_{t+1}.
\end{align*} \]

1.6 Conclusion

Given how successful the New Keynesian model is nowadays, we found it opportune to examine its canonical version in order to give a new insight into the design of optimal monetary policy rules. Our original contribution is actually twofold, as we first determine analytically the optimal equilibrium and then characterize the monetary policy rules ensuring the implementation of this equilibrium, but we view the latter contribution as much more significant than the former one.

Our first (and minor) contribution thus consists in fully deriving the model's analytical results, which describe the optimal macroeconomic adjustment pro-
cess to demand and cost-push shocks, for a small open economy (with the closed economy nested as a special case) in four alternative configurations: a flexible exchange rate regime without commitment (FL1), a flexible exchange rate regime with commitment (FL2), a (n ex post) fixed exchange rate regime with commitment (FI1) and an irrevocably (ex ante) fixed exchange rate regime with commitment (FI2). Only in a special case (CE1) of the first configuration (FL1) had these results been fully derived in the existing literature. Moreover, we optimize in the FL2 case over a class of possibly not time-invariant solutions, while on the contrary all existing studies consider only time-invariant solutions.

These results notably indicate that the optimal monetary policy reaction to a cost-push shock, in the FL2 case, can be to raise or to lower the nominal interest rate, depending on the value of the various parameters. Under our preferred specification however, monetary policy should be tightened in response to a positive cost-push shock (i.e. a negative productivity shock), in accordance with conventional wisdom. As the elasticity of substitution $\varepsilon$ between the varieties of the differentiated good gets closer to one (thus decreasing the welfare cost of inflation and hence the relative weight of the central bank's inflation objective), the optimal monetary policy reaction to a cost-push shock becomes passivity under this specification, so that a fixed exchange rate regime (FI1 or FI2) provides in the limit case $\varepsilon = 1$ the same welfare level as the flexible exchange rate regime (FL2) in the absence of demand shocks.

These results also indicate that all variables (among which the inflation rate, the output gap and the nominal interest rate) are stationary, whatever the (demand or cost-push) shock and the (FL1, FL2, FI1 or FI2) case considered, except the price level and the nominal exchange rate following a cost-push shock in the FL1 case, as well as in the FL2 case when the central bank's degree of patience differs from the society's ($\delta \neq \beta$). This non-stationarity is not obtained by Galí and Monacelli (2002), who disregard the FL1 case and consider the FL2 case only for $\delta = \beta$, and only partially obtained by Monacelli (2003), who examines the FL1 case but considers the FL2 case only for $\delta = \beta$.

Our second (and major) contribution consists in characterizing the set of monetary policy rules ensuring the implementation of this optimal adjustment process, in each of the relevant cases considered (FL1, FL2 and FI1). By contrast, the existing literature does it only in the CE2 case (i.e. the FL2 case with $\alpha = 1$). Most importantly, unlike the existing literature, we look for stabilizing feedback rules which rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria.

We show that these adequate rules are necessarily forward-looking so as to
insulate the current inflation rate from the private agents' sunspot-prone expectations about the future situation. This result provides what is to our knowledge a new theoretical justification for the observed forward-looking behaviour of central banks. Indeed, even though the literature has shown that some forward-looking monetary policy rules could rule out all convergent equilibria other than the optimal one, forward-lookingness is not a necessary condition to rule out only convergent equilibria, that is to say that purely backward-looking monetary policy rules can do the job just as well.

All these conclusions have been reached within the specific context of the canonical New Keynesian model, which has been chosen to illustrate our point in a simple way. This very simple framework can be extended in many ways. For instance, many authors specify the shocks as autoregressive processes of order one. This extension would certainly alter the analytical expression of the optimal equilibrium, but would not fundamentally question or invalidate (qualitatively speaking) our conclusions on the optimal implementation of monetary policy. We could also consider other sources of exogenous disturbances, for instance take into account foreign macroeconomic fluctuations in the small open economy model, or introduce a risk-premium shock in the uncovered interest rate parity equation. These extensions would simply add new terms to the analytical expression of the optimal equilibrium, under the natural assumption that all shocks are orthogonal to each other. In our opinion, there should exist monetary policy rules ensuring the implementation of this new optimal equilibrium, especially so if we allow the new disturbances to enter the rules considered, and our results on the qualitative properties of adequate monetary policy rules (such as forward-lookingness) should remain robust.

Other natural extensions to the canonical New Keynesian model aim at addressing a criticism often formulated about the purely forward-looking nature of its structural equations. Indeed, it is now widely agreed that some form of costly adjustment or habit formation needs to be introduced into this model in order to match the inertia or the lagged responses which are apparent in the data. In our view, such extensions should dramatically alter the analytical expression of the optimal equilibrium, but would not fundamentally question or invalidate (qualitatively speaking) our conclusions on the optimal implementation of monetary policy, provided that they amount to adding only lagged variables in the Phillips curve and the IS equation, in the same (more or less arbitrary) way for instance as Clarida, Galí and Gertler (1999), Woodford (1999; 2003, chap. 3 and 8), which is the case of most extensions to be encountered in the literature.

Now, some of these extensions introduce additional expected leads of the
endogenous variables into the Phillips curve and the IS equation. Such is notably the case of habit formation in consumer preferences. Whenever habit formation simply amounts to introducing additional expected leads of the output gap into the IS equation\textsuperscript{29}, adequate monetary policy rules will still exist. Indeed, in a similar way as in the canonical version of the New Keynesian model, monetary policy rules can then be found which pin down the inflation rate uniquely, while the output gap is residually determined by the Phillips curve. However, Amato and Laubach (2003a) argue that the consideration of habit formation should also make expected leads of the output gap enter the Phillips curve. In this case, the output gap will not be residually determined by the Phillips curve if the monetary policy rule is chosen so as to pin down the inflation rate uniquely. It proves therefore not clear at first sight whether adequate monetary policy rules would then exist. In other words, there may well be a particular relevant leads structure of the Phillips curve and the IS equation for which no single monetary policy rule can ensure the uniqueness of the equilibrium implemented.

1.7 Appendix

1.7.1 Analytical resolution of the model (FL2)

The method followed by the literature and in particular by Clarida, Gali and Gertler (1999) as well as Woodford (2003), inspired by Curris and Levine (1993), leads directly to the solution in the form of impulse-response functions, \textit{i.e.} to the values of $\Delta p_{H,t+k}$, $y_{t+k}$, $r_{t+k}$ (for $n \geq 0$) as functions of the current shocks $\varepsilon^e_t$ and $e^s_t$. This method consists in choosing the inflation rates and the output gaps so as to minimize the loss function under the constraint imposed by the Phillips curve, where all operators $E^t \{ \}$ have been arbitrarily dropped. The nominal interest rates are then residually determined by the IS equation. If $\mu_k$ denotes the coefficient corresponding to the constraint represented by the Phillips curve at date $t+k$, then this method consists in determining the values of $\Delta p_{H,t+k}$ and $y_{t+k}$ for $k \geq 0$ which minimize the following Lagrangian:

\[
\sum_{k=0}^{+\infty} \beta^k \left[ (\Delta p_{H,t+k})^2 + \lambda (y_{t+k})^2 \right] - \mu_0 (\Delta p_{H,t} - \beta \Delta p_{H,t+1} - \gamma y_t - \varepsilon^e_t) - \sum_{k=1}^{+\infty} \mu_k (\Delta p_{H,t+k} - \beta \Delta p_{H,t+k+1} - \gamma y_{t+k}).
\]

\textsuperscript{29}Such is typically the case, as attested by Bouakes, Cardia and Ruge-Murcia (2002), Christiano, Eichenbaum and Evans (2003), Edge (2000), Fuhrer (2000), McCallum and Nelson (1999a).
Instead of adopting this standard approach to solve analytically the central bank's optimization problem, we follow another method which to our knowledge has never been used in a New Keynesian context. This method, which belongs to the class of so-called undetermined coefficients methods, is more general than the method followed in the literature, as we optimize over a class of solutions which are possibly not time-invariant, but happens to point to the same solution. We therefore show that the time-invariant linear solution which existing studies find is optimal among all time-invariant linear solutions, is also optimal among all linear solutions. In more concrete terms, writing \( y_{t+k} = \sum_{j=0}^{k} (a_{k-j} e_{t+k-j} + b_{k-j} e_{t+k-j}) \) and \( \Delta y_{t+k} = \sum_{j=0}^{k} (f_{k-j} e_{t+k-j} + g_{k-j} e_{t+k-j}) \) for \( k \geq 0 \), we look for the coefficients \( a_{k-j} \), \( b_{k-j} \), \( k-j \), \( b_{k-j} \), and \( e_{t+k-j} \) for \( k \geq 0 \) and \( 0 \leq j \leq k \) which minimize \( L_{t+k} \) subject to the constraints represented by the Phillips curve considered at all dates, i.e. which minimize the following Lagrangian:

\[
E_{t} \left\{ \sum_{k=0}^{+\infty} \delta^{k} \left[ (\Delta y_{t+k})^{2} + \lambda (y_{t+k})^{2} \right] \right\} - \sum_{k=0}^{+\infty} \beta^{k} (\Delta y_{t+k} - \beta E_{t+k}(\Delta y_{t+k+1} - \gamma y_{t+k} - e_{t+k}^p))
\]

The coefficients \( f_{k-j} \) and \( g_{k-j} \) for \( k \geq 0 \) and \( 0 \leq j \leq k \) characterizing the nominal interest rate \( r_{t+k} = \sum_{j=0}^{k} (f_{k-j} e_{t+k-j} + g_{k-j} e_{t+k-j}) \) for \( k \geq 0 \) are then residually determined with the help of the IS equation. Note that we choose not to allow for retroactivity. The commitment, which is announced at date \( t \) and takes place from that date onwards\(^{30}\), involves indeed no shock having occurred before that date. Allowing for (or rather actually imposing) retroactivity would require considering instead the following linear combinations: \( y_{t+k} = \sum_{j=0}^{+\infty} (a_{k-j} e_{t+k-j} + b_{k-j} e_{t+k-j}) \), \( \Delta y_{t+k} = \sum_{j=0}^{+\infty} (f_{k-j} e_{t+k-j} + g_{k-j} e_{t+k-j}) \) and \( r_{t+k} = \sum_{j=0}^{+\infty} (f_{k-j} e_{t+k-j} + g_{k-j} e_{t+k-j}) \) for \( k \geq 0 \). Had we imposed retroactivity, the commitment chosen would then have depended on date \( t \) (assuming that the shocks having occurred before that date have been observed), because the central bank would take advantage of the fact that expectations formed before date \( t \) (i.e. before the time when the commitment is both announced

\(^{30}\)In order to simplify notations and without any loss in generality, we choose the same starting date (namely date \( t \)) for both the commitment technology considered here in this appendix and the impulse-response functions presented there in subsection 1.3.2.

\(^{31}\)Retroactivity does not matter obviously if the economy was at its stationary state until date \( t-1 \) included (i.e. \( e_{t-1}^p = e_{t-1}^p = 0 \) for \( k \geq 1 \)), or if the economy starts from scratch at date \( t \) (with \( k-1 \) and \( e_{t-1} \) being exogenously given).
and implemented) are given. That the optimal solution should depend on date \( t \) is little satisfactory, and we choose therefore, like Clarida, Gali and Gertler (1999), Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), to adopt a timeless perspective, which in effect amounts to rule out retroactivity\(^{32}\).

In a straightforward manner, we find that \( \forall k \geq 0 \) and \( \forall j \in \{0, \ldots, k\} \), \( c_k^{k-j} = a_k^{k-j} = g_k^{k-j} = 0 \). Now, the first-order conditions of the Lagrangian's minimization with respect to \( b_k^0, b_k^j \) for \( k \geq 1 \), \( b_k^{k-j} \) for \( k \geq 2 \) and \( j \in \{1, \ldots, k-1\} \), \( b_k^j \) for \( k \geq 1 \), \( a_k^{k-j} \) for \( k \geq 1 \) and \( j \in \{0, \ldots, k-1\} \), \( a_k^0 \) for \( k \geq 0 \) can be respectively written in the following way\(^{33}\):

\[
\begin{align*}
2 \left( e_t^{pc} \right)^2 b_0^0 - \mu_0 e_t^{pc} & = 0, \\
2^k V \left( e_t^{pc} \right) b_k^0 - \mu_k e_t^{pc} & = 0 \text{ for } k \geq 1, \\
2^k V \left( e_t^{pc} \right) b_k^{k-j} - \mu_k e_t^{pc} - \beta \mu_{k-1} e_{t+k-j} & = 0 \text{ for } k \geq 2 \text{ and } j \in \{1, \ldots, k-1\}, \\
2^k \left( e_t^{pc} \right)^2 b_k^j - \mu_k e_t^{pc} + \beta \mu_{k-1} e_t^{pc} & = 0 \text{ for } k \geq 1, \\
2^k \lambda V \left( e_t^{pc} \right) a_k^{k-j} + \mu_k \gamma e_t^{pc} & = 0 \text{ for } k \geq 1 \text{ and } j \in \{0, \ldots, k-1\}, \\
2^k \lambda \left( e_t^{pc} \right)^2 a_k^0 + \mu_k \gamma e_t^{pc} & = 0 \text{ for } k \geq 0.
\end{align*}
\]

Moreover, the Phillips curve considered at all dates leads to the following two additional equations:

\[
\begin{align*}
\gamma a_k^k + \beta b_{k+1}^k - b_k^k + 1 & = 0 \text{ for } k \geq 0, \\
\gamma a_k^{k-j} + \beta b_{k+1}^{k-j} - b_k^{k-j} & = 0 \text{ for } k \geq 1 \text{ and } j \in \{1, \ldots, k\}.
\end{align*}
\]

Let us note \( u \equiv k-j \), \( v \equiv j \), \( A_{u,v} \equiv a_k^{k-j} \) and \( B_{u,v} \equiv b_k^{k-j} \), so that \( A_{u,v} \) and \( B_{u,v} \) characterize respectively the reactions of \( y_{t+u+v} \) and \( \Delta p_{t+u+v} \) to \( e_t^{pc} \). Our eight equations are then equivalent to the following system:

\[
\begin{align*}
\lambda A_{u,0} + \gamma B_{u,0} & = 0 \text{ for } u \geq 0, \\
\delta \lambda A_{u,v+1} - \beta \lambda A_{u,v} + \gamma \delta B_{u,v+1} & = 0 \text{ for } u \geq 0 \text{ and } v \geq 0, \\
\gamma A_{u,v} + \beta B_{u,v+1} - B_{u,v} & = 0 \text{ for } u \geq 0 \text{ and } v \geq 1, \\
\gamma A_{u,v} + \beta B_{u,1} - B_{u,v} + 1 & = 0 \text{ for } u \geq 0.
\end{align*}
\]

\(^{32}\)If we imposed retroactivity while assuming that the shocks having occurred before date \( t \) have not been observed, then the optimal solution would not depend on parameter \( \delta \), which is also unsatisfactory.

\(^{33}\)Naturally, we assume as throughout the chapter that the central bank has observed \( e_t^{pc} \) when it forms its expectation \( E_t \{ \cdot \} \). It is worth noting however that relaxing this assumption would not affect the results.
Since the coefficients of this system do not depend on $u$, the solution will be time-invariant. For the sake of simplicity, we consider therefore a given $u \geq 0$ in the following. Let us first determine the coefficients $B_{u,v}$ for $v \geq 0$, and then residually obtain the coefficients $A_{u,v}$ for $v \geq 0$. From the system above, the coefficients $B_{u,v}$ are found to satisfy the following system:

\[
\begin{align*}
\beta \lambda B_{u,1} - (\gamma^2 + \lambda) B_{u,0} &= 0, \\
\beta \delta \lambda B_{u,v+2} - (\gamma^2 \delta + \beta^2 \lambda + \delta \lambda) B_{u,v+1} + \beta \lambda B_{u,v} &= 0 \text{ for } v \geq 0.
\end{align*}
\]

The latter equation corresponds to a recurrence equation on the $B_{u,v}$ for $v \geq 0$. The corresponding (second-order) characteristic polynomial has two positive real roots, one noted $z$ potentially lower than one, the other noted $z'$ strictly higher than one:

\[
\begin{align*}
z &= \frac{(\beta^2 \lambda + \gamma^2 \delta + \delta \lambda) - \sqrt{(\beta^2 \lambda + \gamma^2 \delta + \delta \lambda)^2 - 4 \beta^2 \delta \lambda^2}}{2 \beta \delta \lambda}, \\
z' &= \frac{(\beta^2 \lambda + \gamma^2 \delta + \delta \lambda) + \sqrt{(\beta^2 \lambda + \gamma^2 \delta + \delta \lambda)^2 - 4 \beta^2 \delta \lambda^2}}{2 \beta \delta \lambda},
\end{align*}
\]

where $z < 1$ if and only if $\gamma^2 \delta + \beta^2 \lambda + \delta \lambda > \beta \delta \lambda + \beta \lambda$. We assume this inequality satisfied in the following. (Note that it is indeed satisfied at point $(\delta, \lambda) = (\delta_0, \lambda_0)$, as well as, by continuity, in the neighbourhood of this point.)

The general form of the solution to the recurrence equation is therefore $B_{u,v} = k z^v + k' z'^v$ for $v \geq 0$, where $k$ and $k'$ are two real numbers. Two equations are then needed to determine $k$ and $k'$. The first one is provided by the initial condition $\beta \lambda B_{u,1} - (\gamma^2 + \lambda) B_{u,0} = 0$. The second one is simply $k' = 0$ and comes from the fact that $\delta z'^2 \geq 1$, as can be readily checked, so that no solution with $k' \neq 0$ would fit the bill as $L_1$ would then be infinite. At the end of the day, we thus obtain $B_{u,v}$ for $v \geq 0$ and therefore $A_{u,v}$ for $v \geq 0$, from which we derive the impulse-response functions of $\Delta P_T$ and $y$ displayed in subsection 1.3.2. The impulse-response functions of $r$ are eventually residually determined with the help of the IS equation. Note that the certainty equivalence property holds here, as in all linear quadratic optimization problems. In other words, the solution will not depend on the variances of the exogenous shocks.
1.7.2 Characterization of the adequate monetary policy rules

If, as in Woodford (2003, chap. 8), Giannoni and Woodford (2003a, 2003b), the only endogenous variables allowed to enter the monetary policy rules are the nominal interest rate \( r \) and the target variables \( \Delta p_H \) and \( y \) in our framework, then the general form of finite linear monetary policy rules is the following, no matter whether we deal with a closed economy or a small open economy:

\[
r_t = \sum_{t=0}^{N_1} a_t \Delta p_{H,t-1} + \sum_{t=0}^{N_1} b_t r_{t-1} + \sum_{t=0}^{N_1} c_t y_{t-1} + \sum_{t=0}^{N_2} d_t \varepsilon_{t-1} + \sum_{t=0}^{N_2} f_t \varepsilon_{t} + \sum_{t=1}^{N_2} a_t E_t \{ \Delta p_{H,t+1} \} + \sum_{t=1}^{N_2} b_t E_t \{ r_{t+1} \} + \sum_{t=1}^{N_2} c_t E_t \{ y_{t+1} \} \tag{1.7.7}
\]

where \( N_1 \geq 0 \) and \( N_2 \geq 0 \). Without any loss of generality, we impose \( b_0 = 0 \) and \( (a_{-N_1}, b_{-N_1}, c_{-N_1}, d_{-N_1}, f_{-N_1}) \neq (0, 0, 0, 0, 0) \). The private agents expect the monetary policy rule (1.7.7) to be applied in the future: for \( k \geq N_1 + 1 \), we obtain therefore, with the Phillips curve (1.2.4):

\[
E_t \{ r_{t+k} \} = \sum_{i=-N_1}^{N_2} a_i E_t \{ \Delta p_{H,t+k} \} + \sum_{i=-N_1}^{N_2} b_i E_t \{ r_{t+k} \} + \sum_{i=-N_1}^{N_2} c_i E_t \{ \Delta p_{H,t+k} \} - \beta E_t \{ \Delta p_{H,t+k+1} \} \tag{1.7.8}
\]

Besides, using the IS equation (1.2.3) and the Phillips curve (1.2.4), we obtain the condition \( C_0 \):

\[
\beta E_t \{ \Delta p_{H,t+2} \} - (1 + \beta + \gamma \eta) E_t \{ \Delta p_{H,t+1} \} + \Delta p_{H,t} + \gamma \eta r_t - (\gamma \varepsilon_t^p + \varepsilon_t^p) = 0,
\]

and the conditions \( C_k \) for \( k \geq 1 \):

\[
\beta E_t \{ \Delta p_{H,t+k+2} \} - (1 + \beta + \gamma \eta) E_t \{ \Delta p_{H,t+k+1} \} + \Delta p_{H,t+k} + \gamma \eta E_t \{ r_{t+k} \} = 0.
\]

These conditions enable us to rewrite equation (1.7.8) as a recurrence equation on the expected future inflation rates:

\[
\forall k \geq N_1 + 1, \quad \sum_{t=-N}^{N} g_t E_t \{ \Delta p_{H,t+k} \} = 0,
\]

where \( N \geq 0 \). This recurrence equation holds at least from \( k = N_1 + 1 \), and potentially before.
Let us note $M = \max\{i \in \{-N_1, ..., N_1\}, \quad g_i \neq 0\}$. The monetary policy rule must be chosen such that $M$ exists; indeed, if $\forall i \in \{-N_1, ..., N_1\}, g_i = 0$, then the expected future inflation rate proves undetermined from a certain date onwards, which is incompatible with the desired results.

This recurrence equation necessitates $N_1 + M + 1$ initial conditions, in order to determine $\Delta pH_{t,t}, E_t\{\Delta pH_{t,t+1}\}, ..., E_t\{\Delta pH_{t,t+N_1+M}\}$. Now, we have only $N_1 + 1$ initial conditions at our disposal, corresponding to the monetary policy rule taken at dates $t, ..., t + N_1$, rewritten with the help of conditions $C_k$ for $k \geq 0$. We must therefore have $M \leq 0$, that is to say that the monetary policy rule must be forward-looking so as to exactly counter the effect of the expected future values of the inflation rate and the output gap on the present value of the inflation rate.

Note that the forward-looking part of the monetary policy rule is thus uniquely defined modulo the Phillips curve, by which we mean that there are an infinity of (distinct though equivalent) expressions for this forward-looking part, which are linked to each other through the Phillips curve. Note also that these expressions depend on the choice of $c_0$ (once again through the Phillips curve): with $c_0 = -\frac{\beta}{\gamma}$, the forward-looking part of the monetary policy rule can be written $\frac{1}{\eta} E_t\{y_{t+1}\}$, or equivalently written $\frac{1}{\gamma} E_t\{\Delta pH_{t,t+1}\} - \frac{\beta}{\gamma} E_t\{\Delta pH_{t,t+2}\}$, or still equivalently written as any convex linear combination of these two expressions.

Having characterized its forward-looking part, we now turn to the backward-looking part of the monetary policy rule. We have $5N_1 + 5$ coefficients: $a_{-1}, b_{-1}, c_{-1}, d_{-1}$ and $f_{-1}$ for $i \in \{0, ..., N_1\}$, on which are imposed a certain number of linear constraints. One of these constraints corresponds to the normalization $b_0 = 0$. A number $2(N_1 + 1)$ of other constraints come from the initial conditions.

Indeed, these $N_1 + 1$ initial conditions, which correspond to the application of the monetary policy rule at date $t$ and its expected application at dates $t + k$ for $k \in \{1, ..., N_1\}$, should determine $\Delta pH_{t,t}, E_t\{\Delta pH_{t,t+1}\}, ..., E_t\{\Delta pH_{t,t+N_1}\}$. In other words, the coefficients $a_{-1}, b_{-1}, c_{-1}, d_{-1}$ and $f_{-1}$ for $i \in \{0, ..., N_1\}$ should ensure that each of these $N_1 + 1$ inflation rates depend on the two shocks $\varepsilon_{a}^{\text{PS}}$ and $\varepsilon_{b}^{\text{PS}}$ in the way described in section 1.3, which effectively corresponds to $2(N_1 + 1)$ constraints whatever the case considered (be it FL1, FL2 or FT1).

Finally, in the FL2 and FT1 cases, one additional constraint comes from...
the fact that \( z \) or \( x \) must be a root of the characteristic polynomial of the recurrence equation on the expected future inflation rates, given the desired results (described in subsections 1.3.2 and 1.3.3). This requirement implies moreover that \( N_1 \geq 1 \), whereas \( N_1 \) can be nil in the FL1 case.

Consequently, the set of adequate finite linear monetary policy rules, that is to say rules described by equation (1.7.7) and ensuring the implementation of the optimal equilibrium, is a \( 3N_1 + 2 \)-dimensional vectorial space\(^{35} \) (where \( N_1 \geq 0 \)) in the FL1 case, and a \( 3N_1 + 1 \)-dimensional vectorial space (where \( N_1 \geq 1 \)) in the FL2 and FF1 cases.

\(^{35}\)Actually, the vectorial space in question is not the set \( \{ r \} \) of adequate monetary policy rules per se, but the set \( \{ r - r_0 \} \) of adequate monetary policy rules relatively to a given benchmark adequate monetary policy rule \( r_0 \).
Table 1.1: *ex post* macroeconomic volatility\(^{36}\).

<table>
<thead>
<tr>
<th>Exchange rate regime</th>
<th>Equilibrium</th>
<th>Inertial effect of shocks</th>
<th>Convergence</th>
<th>Short-run real effects of shock $\varepsilon^{he}$</th>
<th>Long-run nominal effects of shock $\varepsilon^{ne}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL1</td>
<td>optimal</td>
<td>0</td>
<td>immediate</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>FL2</td>
<td>optimal</td>
<td>+ or -</td>
<td>in $z'$</td>
<td>0</td>
<td>+ or -</td>
</tr>
<tr>
<td>FL1 and FL2</td>
<td>unique</td>
<td>+ or -</td>
<td>in $x'$</td>
<td>+</td>
<td>0</td>
</tr>
</tbody>
</table>

\(^{36}\) By *ex post* macroeconomic volatility, we mean the macroeconomic volatility arising in the presence of the optimal monetary policy.
Table 1.2: *ex post* macroeconomic instability
in the literature based on the canonical New Keynesian model.

<table>
<thead>
<tr>
<th>Study</th>
<th>Economy</th>
<th>$\varepsilon^{ie}$ and $\varepsilon^{sc}$</th>
<th>Results</th>
<th>Class of monetary policy rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batini and Pearlman (2002)</td>
<td>CE</td>
<td>i.i.d.</td>
<td>SR</td>
<td>restricted class of BL and/or FL rules, including no optimal rule</td>
</tr>
<tr>
<td>Bernanke and Woodford (1997)</td>
<td>CE</td>
<td>AR(1)</td>
<td>IAR</td>
<td>restricted class of BL rules, including no optimal rule</td>
</tr>
<tr>
<td>Clarida, Gali and Gertler (1999)</td>
<td>CE</td>
<td>AR(1)</td>
<td>IAR</td>
<td>restricted class of BL and/or FL rules, including no optimal rule</td>
</tr>
<tr>
<td>Clarida, Gali and Gertler (2000)</td>
<td>CE</td>
<td>AR(1)</td>
<td>SR</td>
<td>restricted class of BL and/or FL rules, including no optimal rule</td>
</tr>
<tr>
<td>Gali and Monacelli (2002)</td>
<td>SOE</td>
<td>AR(1)</td>
<td>IAR and SR</td>
<td>-</td>
</tr>
<tr>
<td>Giannoni and Woodford (2003a, 2003b), Woodford (2003, chap. 8)</td>
<td>CE</td>
<td>not specified</td>
<td>IAR</td>
<td>large class of BL and/or FL rules, including all optimal rules</td>
</tr>
<tr>
<td>Kerr and King (1996)</td>
<td>CE</td>
<td>-</td>
<td>IAR</td>
<td>restricted class of BL rules, including no optimal rule</td>
</tr>
<tr>
<td>Levin, Wieland and Williams (2001)</td>
<td>CE</td>
<td>i.i.d.</td>
<td>SR</td>
<td>restricted class of BL and/or FL rules, including no optimal rule</td>
</tr>
<tr>
<td>Monacelli (2003)</td>
<td>SOE</td>
<td>AR(1)</td>
<td>IAR and SR</td>
<td>-</td>
</tr>
<tr>
<td>Woodford (1999), Woodford (2003, chap. 7)</td>
<td>CE</td>
<td>AR(1)</td>
<td>IAR</td>
<td>large class of BL rules, including no optimal rule</td>
</tr>
<tr>
<td>Woodford (2003, chap. 4)</td>
<td>CE</td>
<td>not specified</td>
<td>IAR</td>
<td>restricted class of neither BL nor FL rules, including no optimal rule</td>
</tr>
<tr>
<td>Chapter 1</td>
<td>CE and SOE</td>
<td>i.i.d.</td>
<td>CAR</td>
<td>large class of BL and/or FL rules, including all optimal rules</td>
</tr>
</tbody>
</table>

37 CE: closed economy; SOE: small open economy.
38 CAR: complete analytical results; IAR: incomplete analytical results (*i.e.* endogenous variables not expressed as functions of exogenous shocks only); SR: simulation results.
39 BL: backward-looking; FL: forward-looking; the optimal monetary policy rules in question are optimal according to our definition of optimality, that is to say that these rules rule out not only all convergent equilibria other than the optimal one, but also all divergent equilibria.
Chapter 2

Simulation of the UK business cycle under EMU-membership

Abstract

Chapter 2, entitled “Simulation of the UK business cycle under EMU-membership”, outlines a method of simulation of the business cycle of a small open economy joining a monetary union, which is applied to the case of the UK adopting the Euro. This simulation method is based on the predictions of a New Keynesian model estimated on pre-EMU data and overcomes the Lucas critique in the sense that it takes into account the impact of the regime change on the formation of the private agents’ rational expectations. The simulation results suggest that the euroized UK would escape macroeconomic instability, which arises in the presence of multiple equilibria, but could nonetheless experience a higher macroeconomic volatility.

Abstract in French

Le chapitre 2, intitulé “Simulation du cycle macroéconomique du Royaume-Uni en Eurozone”, décrit une méthode de simulation du cycle macroéconomique d’une petite économie ouverte rejoignant une union monétaire. Cette méthode de simulation, qui est appliquée au cas du Royaume-Uni adoptant l’Euro, est basée sur les prédictions d’un modèle nouveau-keynésien estimé sur données pré-Euro et s’affranchit de la critique de Lucas dans le sens où elle prend en compte l’impact du changement de régime sur la formation des anticipations.
rationnelles des agents privés. Les résultats suggèrent que le Royaume-Uni euroisé échapperait à l'instabilité macroéconomique, qui survient en présence d'équilibres multiples, mais pourrait néanmoins faire l'expérience d'une volatilité macroéconomique accrue.

2.1 Introduction

The case for the UK adopting the Euro officially rests on the assessment of five economic tests set by HM Treasury in October 1997. The first of these tests deals with the impact of membership of Economic and Monetary Union (EMU) on the UK business cycle. More precisely, it is about whether there is sustainable convergence between the UK and the Eurozone business cycles. In other words, can the UK live comfortably with the one-size-fits-all single interest rate set in Frankfurt-am-Mein?

This chapter aims at assessing this test by simulating the business cycle of the UK within the Eurozone. Our simulation method is based on the predictions of a small structural model which derives its key relationships from explicit optimizing problems for consumers and firms, rather than from ad hoc behavioural assumptions, and incorporates explicit rational expectations behaviour. This theoretical underpinning based on sound microeconomic foundations enables us to overcome the Lucas critique by acknowledging and taking into account the shift in reduced-form parameters caused by the impact of the regime change on the private agents' expectations.

The model in question belongs to the class of so-called New Keynesian models, which have been extensively used in the past few years for monetary policy analysis. We choose to resort to this class of models precisely because of their structural features, which we view as a prerequisite for safely handling such an important institutional change as the adoption of the Euro, that is to say for escaping the Lucas critique. Unfortunately, New Keynesian models tend to fit the data poorly, as if there were a trade-off between theoretical consistency and empirical relevance. In this trade-off we take care not to favour too much theoretical consistency to the detriment of empirical relevance, and use accordingly the extended version of a New Keynesian model, rather than its canonical version, so as to fit the data better. At the end of the day however, there seems admittedly to be still room for improvement as far as the estimation results are concerned, even though these results match both qualitatively and

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1 As stressed by McCallum (1999a), who relates the evolution of monetary policy theory and practice since the early 70's, New Keynesian economics has recently come out as the most celebrated framework for monetary policy analysis.
quantitatively those of the existing literature. We therefore end this chapter by considering a further extended (this time non-structural) model which fits the data better.

Our method of simulation of the UK business cycle under EMU-membership derives from the predictions of the New Keynesian model considered, namely an extended version of Gali and Monacelli's (2002) model. These predictions, which are also those of Gali and Monacelli’s (2002) canonical version, state notably that under EMU-membership the UK nominal interest rate should be constantly equal to the nominal interest rate in the rest of the Eurozone, and that relative purchasing power parity (PPP) should hold in the long-run. To our knowledge actually, these two points are jointly captured by only three of all the existing simulations of business cycles under irrevocably fixed exchange rates, whether these simulations are based on structural or reduced-form, estimated or calibrated small macroeconomic models². The characteristics of these three simulations are presented in table 2.1, together with those of this chapter.

The first of these three studies is Gali and Monacelli’s (2002) itself, which derives analytically and simulates the effect of irrevocably fixed exchange rates on the business cycle of a small open economy in the absence of foreign fluctuations. Our contribution actually amounts to adapt their simulation method to a more general framework, which accounts for foreign fluctuations (by acknowledging the presence of foreign variables in the structural equations) and allows for richer dynamics (by including additional lags in the structural equations). A further difference (no less important) between their study and the present chapter is that they calibrate their model for a generic small open economy, while we estimate our extended version of their model for the UK.

The second of these three studies is Westaway’s (2003), which calibrates a reduced-form New Keynesian model in order to simulate the business cycle of the UK under EMU-membership, this time with a richer dynamic structure and in the presence of foreign fluctuations. Taking foreign fluctuations into account is obviously welcome in our context, as the desirability of EMU-membership for the UK depends both on the correlation between the shocks occurring on either side of, and on the similarity of the mechanisms of propagation of these shocks in place on either side of, well, the English Channel, the Irish Sea and the North Sea. However, because Westaway’s (2003) model is calibrated rather than

²We will review the VAR literature thereupon. Of course, these two predictions are routinely captured by large macroeconometric models, but the latter usually fail to bring to the fore the few key effects or mechanisms at work because of their complexity, so that it proves still worth using small macroeconomic models.
estimated, the statistical properties of the shocks are set more or less arbitrarily\(^3\) in his framework. By contrast, we avoid this shortcoming by directly estimating our small open economy model. A further difference between his study and the present chapter lies in the fact that he uses a reduced-form model, while we mainly use a structural model - as such less subject to the Lucas critique.

The third of these three studies is Driver and Wren-Lewis' (1999), extended by Driver (2000), which calibrates a reduced-form two-country New Keynesian model in order to simulate the effect of a monetary union on the business cycle of its member countries. Now, our aim is to simulate the business cycle of the UK under EMU-membership. From this point of view, Driver and Wren-Lewis' (1999) or similarly Driver's (2000) study suffers from the same shortcomings as Gali and Monacelli's (2002), namely poor domestic dynamics, no account of the correlation between domestic and foreign shocks, and calibration for a generic country. In addition, it is based on a questionable reduced form, contrary to Gali and Monacelli's (2002). For instance indeed, its IS equation features no lead of output and consequently cannot derive from intertemporal optimisation by consumers. By contrast, this chapter is exempt from all these shortcomings.

Perhaps most importantly of all however, this chapter differs from Gali and Monacelli's (2002), Westaway's (2003) as well as from Driver and Wren-Lewis' (1999) or Driver's (2000) in that we acknowledge the possibility of macroeconomic instability for the UK under EMU-membership. Macroeconomic instability we simply define as the existence of multiple equilibria. Clarida, Gali and Gertler (2000) have famously argued that the observed difference in the variability of key US macroeconomic variables between the pre- and post-1979 periods may have been due to the adoption from that date onwards of a monetary policy rule which does rule out multiple equilibria, contrary to the monetary policy rule followed before. Well, we argue that Clarida, Gali and Gertler's (2000) point can readily be extended from the choice of a monetary policy rule to the choice of an exchange rate regime. In other words, we argue that the adoption of the Euro by a small open economy may have this undesirable feature of being compatible with multiple equilibria. Now, to change a monetary policy rule seems to be as easy and painless as seems to be difficult or painful to change the structural parameters of an economy or to withdraw from such an irrevocably exchange rate regime as a monetary union. Hence the usefulness for a

\(^3\)More precisely, the statistical properties of the shocks are derived in his framework from a structural VAR which does not distinguish between the reaction to a symmetric shock and the reaction to an asymmetric shock. As a consequence, the identification of the shocks is questionable, to say the least of it. A proper identification scheme would actually be all the more difficult to be found within his framework as he does not resort to the small open economy assumption.
small open economy of thinking twice about macroeconomic instability before
deciding to join a monetary union.

The central question addressed by this chapter is: what would become of
the UK business cycle under EMU-membership? More precisely, how would the
British economy react to idiosyncratic and common shocks within the Eurozone?
And also, could EMU-membership inherently be a source of macroeconomic
instability for the UK? The remaining of the chapter is organized as follows:
we present the main model considered and derive our simulation method therefrom
in section 2.2, we then estimate and simulate this model as well as (shortly)
another model in section 2.3, and we conclude thereafter.

2.2 Theoretical underpinning

This section gives a general outline of the main model considered in this chapter,
called model A, and derives the simulation method from the predictions of this
model.

2.2.1 Model overview

Notations are borrowed from Gali and Monacelli (2002) as well as from Gali
and Gertler (1999). We refer the reader to these two studies for a detailed
presentation of the model.

Closed form of the benchmark model

The benchmark model is the canonical New Keynesian model of a small open
economy, built by Clarida, Gali and Gertler (2001) as well as Gali and Monacelli
(2002)\(^4\). The closed form of this dynamic stochastic general equilibrium model is
composed of five equations holding at each date, i.e. the uncovered interest rate
parity, the law of one price\(^5\), an IS equation, a Phillips curve and a monetary
policy rule, to which should be added one terminal condition, namely the long-
run PPP\(^6\). The corresponding five endogenous domestic variables are the PPI

---

\(^4\)According to McCallum and Nelson (2000), "the GM [Gali and Monacelli (2002)] model has
a strong claim to be viewed as a canonical NOEM [New Open Economy Macroeconomics]
model, owing to its elegance and tractability".

\(^5\)The law of one price, hardly established empirically, is used only in first-difference terms
in this the model, so that our theoretical framework proves relevant even in the presence of
deviations from the law of one price, provided that these deviations are not time-varying.

\(^6\)PPP does not hold in the short run because the domestic and the foreign consumption
baskets differ from each other. More precisely, in this model where monopolistically produced
tradable goods are the only goods considered, the small open economy assumption for the
domestic country implies that foreign consumers value much more foreign goods than domestic
goods, while domestic consumers value both domestic and foreign goods significantly.
inflation rate $\Delta p_H$, the logarithm of the real output level $y$, the short term nominal interest rate $r$, the CPI inflation rate $\Delta p$ and the first difference of the logarithm of the nominal exchange rate $\Delta e$.

If the monetary policy rule does not make $r_t$ depend explicitly on the past, present and expected future values of $\Delta e$ and $\Delta p$, then the model has a block-recursive structure, in the sense that $\Delta p_H$, $y$ and $r$ can be derived (i.e. expressed as functions of the sole exogenous variables) from three equations only, namely the Phillips curve, the IS equation and the monetary policy rule, while $\Delta e$ and $\Delta p$ are residually determined by the uncovered interest rate parity, the law of one price and the long-run PPP. As shown by Clarida, Gali and Gertler (2001), this three-variable three-equation setting is then isomorphic to the closed form of the closed economy version of the canonical New Keynesian model.

But we shall assume more generally that $\Delta e$ and $\Delta p$ may well feature explicitly in the monetary policy rule, and furthermore that this monetary policy rule may not be stable over time. In so doing, we merely acknowledge the fact that our estimation period includes the EMS period\(^8\). For these reasons, we do not further specify and will not seek to estimate the monetary policy rule.

We will therefore focus in the following on three endogenous variables ($\Delta p_H$, $y$ and $r$) and two equations (the Phillips curve and the IS equation), for which we have no reason to suspect that they may have changed during the estimation period. In our simulation of the UK business cycle under EMU-membership, $r$ will turn exogenous and we will be left with two equations for two endogenous variables.

The IS equation, which is derived from the representative household’s optimal behaviour, is written in the following way:

$$ y_t = E_t \{ y_{t+1} \} - \frac{1}{\sigma} \left( \frac{\alpha (2 - \alpha) \sigma - 1}{\sigma} \right) (r_t - E_t \{ \Delta p_{H,t+1} \}) $$

$$ + \alpha (2 - \alpha) (\sigma - 1) E_t \{ \Delta y^*_{t+1} \} + e_t^s, $$

where $y^*$ represents the logarithm of the foreign real output level. Variables $y$, $y^*$ and $r$ are expressed as deviations from their respective steady state values. Index $t$ or $t+1$ for a given variable refers to the date at which this variable is considered, and $E_t$ denotes the expectation operator based on the information set available at date $t$, which includes the exogenous shocks occurring at this

\(^7\)More accurately, $\Delta$ denotes the first difference operator while $p_H$, $p$ and $e$ represent the logarithms of the PPI, the CPI and the nominal exchange rate respectively.

\(^8\)For a perfectly successful peg admitted, what can be measured is the dependence of $r^*$ on $r^*$, not its dependence on the (constant) nominal exchange rate, but the UK participation in the EMS can hardly be considered as a perfectly successful peg.
date, so that $E_t \{ x_{t-k} \} = x_{t-k} \forall k \geq 0$ for any variable $x$. The exogenous shock $\varepsilon^{es}$ represents a temporary aggregate demand shock, for instance a government spending shock, or comes from a shock on the discount rate. It is assumed to follow an autoregressive process of order one: $\varepsilon^{es}_t = \rho_{es} \varepsilon^{es}_{t-1} + \eta^{es}_t$ with $0 \leq \rho_{es} \leq 1$, where $\eta^{es}_t$ is identically and independently distributed with mean zero. Finally, parameter $\alpha \in [0, 1]$ represents the degree of openness, $\beta \in [0, 1]$ the discount rate, $\eta > 0$ the elasticity of substitution between domestic and foreign goods, $\sigma > 0$ the intertemporal elasticity of consumption.

The Phillips curve, which is derived from the firms' optimal behaviour, is written in the following way:

$$
\Delta p_{H,t} = \beta E_t \{ \Delta p_{H,t+1} \} + \frac{(1- \theta)(1- \beta \theta)}{\theta} \left[ \frac{\sigma}{1 + \alpha (2- \alpha) (\sigma \eta - 1)} + \varphi \mu_t \right]
$$

$$
+ \left[ \frac{\alpha (2- \alpha) (\sigma \eta - 1)}{1 + \alpha (2- \alpha) (\sigma \eta - 1)} \mu_t \right] + \varepsilon^{pc}_t,
$$

where parameter $\varphi > 0$ measures the disutility of labour and parameter $\theta \in [0, 1]$ represents the probability for a given firm to keep its price unchanged at a given date. The exogenous shock $\varepsilon^{pc}_t$, which comes from an aggregate technological shock, is assumed to follow an autoregressive process of order one\footnote{This assumption is justified by the fact that the effects of real shocks on potential GDP are spread over time, according to Neiss and Nelson (2002).}: $\varepsilon^{pc}_t = \rho_{pc} \varepsilon^{pc}_{t-1} + \eta^{pc}_t$ with $0 \leq \rho_{pc} \leq 1$, where $\eta^{pc}_t$ is identically and independently distributed with mean zero. The two shocks $\varepsilon^{es}$ and $\varepsilon^{pc}$ are assumed to be uncorrelated with each other: $\text{cov} (\eta^{es}_t, \eta^{pc}_t) = 0$\footnote{This assumption matters only in the simulation step, not in the estimation step.}.

This canonical New Keynesian Phillips curve, which ensures that monetary policy has real effects, stems from the existence of a nominal rigidity modeled as a staggered price-setting à la Calvo (1983), a time-dependent mechanism which has the advantage over state-dependent mechanisms of leading to a simple closed-form solution\footnote{Another popular price-setting mechanism is Rotemberg's (1982), which is neither time-dependent nor state-dependent, but leads to a similar reduced-form New Keynesian Phillips curve.}. The choice of a time-dependent (rather than state-dependent) price-setting mechanism is moreover backed by Bergen, Dutta, Levy, Ritson and Zbaracki (2000), who provide microeconomic evidence that the costs of changing prices associated with reoptimization (namely information gathering, decision making, negotiation and communication costs) far outweigh menu costs, i.e. the physical costs of changing prices.

Finally, the large foreign country is considered as a closed economy, whose macroeconomic fluctuations are exogenous from the point of view of our small
open economy. The reduced form of the closed economy version of the canonical New Keynesian model, used notably by Clarida, Galí and Gertler (1999) and Woodford (2003), is composed of an IS equation, a Phillips curve and a monetary policy rule. We note \( p^* \) the foreign CPI, or equivalently the foreign PPI, and \( r^* \) the foreign short term nominal interest rate. For simplicity, we assume that the foreign economy can be modeled by a three-dimension VAR whose endogenous variables are \( \Delta p^*, y^* \) and \( r^* \), so that we do not need to specify and estimate a closed economy New Keynesian model. This assumption is actually made of two assumptions, the first one being that the foreign monetary policy rule has ruled out multiple equilibria over the estimation period\(^{12}\), the second one being that this foreign monetary policy rule has been stable over the estimation period\(^{13}\).

Let us note \( X_t^* \equiv [\Delta p_t^*, y_t^*, r_t^*]' \). This foreign VAR is written \( X_t^* = Q^*(L)X_{t-1}^* + \nu_t^* \) with \( Q^*(L) \equiv \sum_{i=0}^{n^*} Q_i^* L^i \), where \( L \) is the lag operator, \( Q_i^* (0 \leq i \leq n^*) \) are 3 x 3 matrices and \( \nu_t^* \) is an independently and identically distributed Gaussian vector with mean zero. We choose not to consider a so-called "structural" VAR, that is to say that we do not assume the existence of a 3 x 3 matrix \( S^* \) such that \( \nu_t^* = S^* \eta_t^* \), where \( \eta_t^* \equiv [\eta_t^{p*,*} \ \eta_t^{y*,*} \ \eta_t^{r*,*}]' \), simply because the New Keynesian model of a closed economy does not naturally provide the restrictions needed to identify \( S^* \). Indeed, the output variable \( y^* \) considered is the detrended production level, which is assumed to be stationary, and not the first difference of the production level, so that the usual restrictions identifying the supply shock \( \eta^{p*,*} \) as the only shock to have a long-run effect on the production level are not available. Moreover, the model implies that the monetary policy instrument \( r^* \) should contemporaneously affect the economy, so that we cannot resort to the usual restriction disentangling the two demand shocks \( \eta^{d*,*} \) and \( \eta^{m*,*} \) from each other by imposing no contemporaneous effect of the monetary policy shock on the output variable.

\(^{12}\)Note however that we do not assume that the unique equilibrium implemented is optimal in any sense. Chapter 1 characterizes the monetary policy rules, in the framework of the canonical New Keynesian model, which ensure the implementation of a unique equilibrium out of all possible equilibria, be they convergent or divergent.

\(^{13}\)A Chow test will be conducted to check whether the foreign monetary policy rule has indeed proved stable over the estimation period. In any case, this stability assumption seems to us more acceptable for the foreign economy than for the domestic economy, as "the monetary policy rule of the future Eurozone countries as a whole" may be stabilised by the procedure of aggregation of the national monetary policy rules. Moreover, the foreign country happens to be only Germany in our framework for reasons of data availability, and the German monetary policy rule is likely to be the most stable of all EMS members' monetary policy rules, as the Bundesbank has most probably behaved as a Stackelberg leader.
Closed form of the extended model

This IS equation and this Phillips curve prove very simple and stylistic. In particular, they are purely forward-looking. As such, they have been subject to much criticism in the literature, most notably from Estrella and Fuhrer (2002), for being at odds with the data. Indeed, their purely forward-looking nature predicts that $\Delta p_H$ and $y$ should jump in response to the various shocks rather than gradually react to them, no matter whether these shocks are specified as i.i.d. or AR(1) processes. Hence these two equations dramatically fail to capture the observed hump-shaped response of both variables to the various shocks.\(^{14}\)

As a consequence, a better specification for the IS equation would include lagged values of $y$. Clarida, Galí and Gertler (1999) interpret these lagged values in the IS equation as by-products of adjustment costs. More precisely, the best-known justification for the presence of these lagged values, or should we say the currently most popular solution to this excess smoothness puzzle, is the existence of habit formation in consumer preferences, first considered by Fuhrer (2000).\(^{15}\)

Similarly, a better specification for the Phillips curve would feature $\Delta p_{H,t-1}$ in addition to $E_t \{\Delta p_{H,t+1}\}$. This specification, usually known as the hybrid New Keynesian Phillips curve, can be justified by the existence of backward-looking firms, as in Galí and Gertler (1999), or by the existence of a subset of price-setters forming adaptative (rather than rational) expectations, as in Roberts (1998). But still other justifications can be encountered in the literature: Fuhrer and Moore (1995), for instance, appeal to Butler and Jewitt’s (1981) relative wage hypothesis.

Table 2.8 lists the various possible extensions provided by the literature to the IS equation and the Phillips curve of (either the closed economy version or the small open economy version of) the canonical New Keynesian model. More precisely, table 2.8 aims at making an exhaustive inventory of existing studies estimating a least one of the following three well-defined equations: a Phillips curve expressing $\Delta p_H$ or $\Delta p$ as a function of $E \{\Delta p_H\}$ or $E \{\Delta p\}$ among other variables, an IS equation expressing $y$ as a function of $E \{y\}$ among other variables\(^{16}\), and a monetary policy rule expressing $r$ as a function of $E \{\Delta p_H\}$.

\(^{14}\)Demery and Duz (2002, p. 1) even claim on these grounds that “the empirical failure of the NK model [read: the canonical New Keynesian Phillips curve] is now widely recognized”.


\(^{16}\)In these IS equations, $y$ may actually represent the logarithm either of real output or of
As Table 2.8 makes clear, there are many different ways to extend the IS equation and the Phillips curve of our benchmark model so as to fit the data better. Some of these extensions we choose to rule out a priori, as they would prevent us from conducting our simulation method. Such is for instance the case for these extensions, actually of a neo-classical rather than New Keynesian obedience, justified by decision lags in consumption or delays in price-setting, which amount to replace $E_t \{ \Delta p_{H,t+1} \}$ and $E_t \{ y_{t+1} \}$ by $E_{t-1} \{ \Delta p_{H,t} \}$ and $E_{t-1} \{ y_t \}$ respectively. Indeed, these extensions would remove the rationale for multiple equilibria and consequently make our simulation impossible.

In our opinion, the ideal specification would combine the existence of habit formation in consumer preferences with that of backward-looking firms, so as to allow for rich enough dynamics without too much weakening the structural feature of the model. Habit formation in particular seems to us especially appealing with respect to these two points. Indeed, not only is it unlikely that the regime change should affect the specification of habit formation, but it may also be the case, as argued by Amato and Laubach (2003a), that the consideration of habit formation should amount to the introduction of additional lags and leads of $y$ both in the IS equation and in the Phillips curve, thereby providing the basis for substantially rich dynamics.

Unfortunately, every rose has its thorn and this strong point of habit formation makes it not so easily adaptable to our open economy framework. Moreover, it may be the case that the introduction of $\Delta p_{H,t-1}$ in the Phillips curve as the sole extension to the benchmark model be enough to get a hump-shaped response of $y$ to the shocks considered. At the end of the day, we therefore choose to keep the canonical IS equation and to consider the following hybrid New Keynesian Phillips curve:\textsuperscript{18}:

\textsuperscript{17}In particular, the list does not include studies calibrating (rather than estimating) these equations, like Amato and Laubach (2003a) or Svensson (2000), however interesting these studies may be in other respects - and they may be indeed, as Svensson (2000) for instance proves one of the few studies to use an extended version of the canonical New Keynesian model of a small open economy. Note also that Ireland (2001) is excluded from the list because its Phillips curve and IS equation fail to be explicitly specified, while its monetary policy rule does not make it eligible on its own.

\textsuperscript{18}The consideration of an hybrid New Keynesian Phillips curve is all the more welcome as the canonical New Keynesian Phillips curve with a serially correlated error term is rejected by Roberts (2001) - and of all the estimation methods used by the studies inventoried in tables 2.8 and 2.9, Roberts' (2001) proves the one most similar to ours - even though his estimation is conducted on US data in a univariate reduced-form setting, while ours is conducted on UK data in a multivariate structural setting.
\[
\Delta p_{t+1} = \frac{\beta \theta}{\theta + \omega [1 - \theta (1 - \beta)]} E_t \{ \Delta p_{t+1} \} + \frac{\omega}{\theta + \omega [1 - \theta (1 - \beta)]} \Delta p_{t-1} \\
+ \frac{(1 - \omega) (1 - \theta) (1 - \beta \theta)}{\theta + \omega [1 - \theta (1 - \beta)]} \left[ \left( \frac{\sigma}{1 + \alpha (2 - \alpha) (\sigma \eta - 1)} + \varphi \right) y_t \right. \\
\left. + \frac{\alpha (2 - \alpha) (\sigma \eta - 1) \sigma}{1 + \alpha (2 - \alpha) (\sigma \eta - 1)} y_t^i \right] + \epsilon_t^{pe},
\]

where \( \omega \in [0,1] \) represents the proportion of backward-looking firms and hence measures the degree of backward-lookingness in price-setting. This extension to Gali and Monacelli's (2002) canonical New Keynesian Phillips curve can be easily derived from Gali and Gertler's (1999) specification\(^{19}\). In the following, the term "model A" refers to the model made of the canonical IS equation and this extended Phillips curve\(^{20}\).

Predictions

What are the predictions of model A concerning the impact of an irrevocably fixed exchange rate regime on the business cycle of our small economy, disregarding the period of transition from one regime to the other? First, foreign dynamics is left unchanged, as the large foreign economy is unaffected by the currency peg of the small domestic economy. Second, the domestic monetary policy rule is removed, as it no longer exists under the monetary union regime. Third, three constraints, labelled (C1), (C2) and (C3), should be imposed on the Phillips curve and the IS equation.

The short-run constraint (C1) imposes \( r_t = r^*_t \) at each date \( t \), i.e. \( r_t = 0 \) in the absence of foreign fluctuations. This constraint stems from the fixity of the expected future nominal exchange rate from date \( t + 1 \) onwards, via the uncovered interest rate parity relationship. The double long-run constraint (C2) imposes \( \Delta p_{t+\infty} = y_{t+\infty} = 0 \). This constraint corresponds to a basic convergence requirement. Under a flexible exchange rate regime, divergent equilibria may exist if the monetary policy rule followed does not preclude them, forcing the central bank sooner or later to abandon its rule in order to bring the economy back in the neighbourhood of its steady state\(^{21}\). Under the monetary

\(^{19}\)Note that this Phillips curve features only one lag. According to Cho and Moreno (2002, p. 12), "as Clarida, Gali and Gertler (1999) point out, it is a daunting task to justify macroeconomic models which include more than one lag".

\(^{20}\)Of course, the foreign IS equation and Phillips curve can be extended in the same way as their domestic counterparts so as to allow for more inertia. Our VAR specification of the foreign economy does take these extensions into account.

\(^{21}\)A more detailed discussion on this point can be found in chapter 1. Note that we
union regime, there is no such "stabilizer of last resort". The private agents acknowledge this change and accordingly no longer expect divergent equilibria.

The long-run constraint (C3) imposes \( p_{H,t+\infty} - p_{H,t-1} = p^*_{t+\infty} - p^*_{t-1} \) following a shock occurring at date \( t \). This constraint corresponds to the stationarity of \( p_{H} - p^* \), that is to say to the long-run relative PPP under an irrevocably fixed exchange rate regime. Gali and Monacelli's (2002) model actually predicts that even absolute PPP should hold in the long run\(^{22}\), whatever the exchange rate regime in force, but our simulation method rests on the weaker condition that only relative PPP needs to hold in the long run. Under a flexible exchange rate regime, the nominal exchange rate can be considered as a residually determined variable which adjusts at each date so that the long-run relative PPP is satisfied. Under the monetary union regime, the nominal exchange rate can no longer vary and the long-run relative PPP constraint is transferred to the prices \( p_{H} \) and \( p^* \)\(^ {23} \).

These considerations echo those of Bec, Ben Salem and Rabbek (2003), who show in a multivariate (but non-linear) framework that the burden of adjustment towards PPP within Europe under the EMS has been borne mostly by the nominal exchange rate and little by prices. According to the authors, these findings suggest that those ultimately responsible for PPP may be (currency realignment) policies rather than markets. We favour another interpretation however, more in accordance with our model's predictions, namely that prices did little to adjust towards PPP only because of the EMS' lack of credibility, as agents (by which we do not necessarily mean only price-setters) rightly expected the nominal exchange rate to be ultimately adjusted. This outcome should naturally be ruled out under such an irrevocably fixed exchange rate regime as the European monetary union.

Technically speaking, the imposition of constraint (C1) leaves us with two equations (the Phillips curve and the IS equation) for two endogenous variables (\( \Delta p_H \) and \( y \)). In these conditions, one may wonder at first sight how the ad-

\(^{22}\)Indeed, under both the assumption of the stationarity of the model's driving forces and the assumption of complete financial markets at the international level, international risk sharing implies that the terms of trade and consequently the real exchange rate (via the law of one price) should converge towards unity, that is to say that absolute PPP should hold in the long run. Under a particular assumption about the initial distribution of wealth across countries, this result amounts to the long-run balanced trade condition.

\(^{23}\)HM Treasury (2003, pp. 22-23) finds that although variations in the nominal exchange rate between the Pound and the European Union currencies have more contributed than UK commodity price changes to movements in the real exchange rate between the UK and the European Union over the past decade, these (annually observed) contributions prove of the same order for most of the years considered, so that the frequency and the magnitude of commodity price changes need not increase substantially to stand in for the variations in the nominal exchange rate as much (in)efficiently in their role of adjustment towards PPP.
ditional constraints (C2) and (C3) can be imposed on the system. The answer lies in the hidden degree of freedom of this system, as the number of unknowns outweighs the number of endogenous variables. Indeed, we count four unknowns in the Phillips curve and the IS equation, namely $\Delta p_{H,t}$, $E_t \{ \Delta p_{H,t+1} \}$, $y_t$, and $E_t \{ y_{t+1} \}$, so that in the absence of constraints (C2) and (C3), these two equations prove obviously too few to uniquely pin down these four unknowns. Rational expectations models can notoriously lead to multiple equilibria, and ours is no exception to this rule - the reason being that the current variables depend on the private agents' (possibly sunspot prone) expectations about the future variables. The imposition of constraints (C2) and (C3) simply amounts to the selection of one equilibrium or several equilibria (if any) out of these multiple equilibria.

As already said in the introduction, apart from Galí and Monacelli (2002), Westaway (2003), Driver and Wren-Lewis (1999), Driver (2000), we do not know of any other monetary union simulation, based on a small economic model, which would jointly capture constraints (C1) and (C3). Within the class of so-called New Keynesian studies and apart from these four studies, the closest study we can actually think of would be Rotemberg and Woodford's (1997) counterfactual analysis, which examines the effect of alternative monetary policy rules in the United States within a closed economy New Keynesian framework. The main difference between their study and this chapter is that, well, this chapter focuses on the effect of irrevocably fixed exchange rates on the United Kingdom in a small open economy New Keynesian framework. In particular, the long-run relative PPP constraint, one of our simulation’s cornerstones, is naturally absent from their study. Technically speaking, in their work, the constraint (C1) is replaced by the imposition of another (non-degenerated) monetary policy rule, and the constraint (C3), proper to an open economy, does not exist. Moreover, they manage to circumvent Blanchard and Kahn’s (1980) requirement, as the existence of decision lags in consumption and delays in price-setting makes $\Delta p$ and $y$ depend on $E_{t-1} \{ \Delta p_t \}$ and $E_{t-1} \{ y_t \}$ rather than on $E_t \{ \Delta p_{t+1} \}$ and $E_t \{ y_{t+1} \}$ in their framework. As a consequence, such a long-run constraint as (C3) could not be implemented anyway under their specification.

Finally, our model’s predictions are not subject to the Lucas critique to the extent that the so-called structural parameters are not affected by the regime change. In particular, we assume the robustness to the regime change of both the price adjustment frequency and the proportion of backward-looking firms24.

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24 Denney and Duck (2002, p. 12) would probably disagree, who argue that “in the absence of any explanation of why some firms are backward-looking and some forward-looking, this
This assumption will be all the more satisfactory that the regime change will have a limited impact on the UK business cycle.

2.2.2 Simulation method

This subsection indicates how to implement our simulation method when the values of the parameters are known.

Impulse-response functions

Our simulation results will be displayed in the form of the impulse-response functions of $\Delta p_H$ and $y$, either to the domestic shocks $\eta^{pc}$ and $\eta^{ls}$, or to the foreign innovations $\nu^* = [\nu^{ls}_1, \nu^{ls}_2, \nu^{ls}_3]$. In other words, we seek to determine $E_t(\Delta p_{H,t+k})$ and $E_t(y_{t+k})$ for $k \geq 0$ as functions of $\eta_{t}^{pc}$, $\eta_{t}^{ls}$, $\nu_{t}^{ls}$, $\nu_{t}^{ls}$ and $\nu_{t}^{ls}$. On account of the nature of constraints (C1), (C2) and (C3), these impulse-response functions are expected to show a substantial effect of the regime change not only on the propagation mechanism (impact of past shocks on current variables), but also on the impulsion mechanism (impact of current shocks on current variables). Of course, we will further need to know the $5 \times 5$ variance-covariance matrix of $\eta^{pc}$, $\eta^{ls}$, $\nu^{ls}$, $\nu^{ls}$ and $\nu^{ls}$ to compute the implied variances of $\Delta p_H$ and $y$.

For the sake of the argument, we assume in this subsection that $\rho_{pc} = \rho_{ls} = 0$ and that $\text{cov}(\eta_{t}^{pc}, \nu_{t}^{ls}) = \text{cov}(\eta_{t}^{ls}, \nu_{t}^{ls}) = \text{cov}(\nu_{t}^{ls}, \nu_{t}^{ls}) = 0$ for $(i,j) \in \{1, 2, 3\}$ with $i \neq j$. These assumptions ensure that shocks $\varepsilon^{pc}$, $\varepsilon^{ls}$, $\nu^{ls}$, $\nu^{ls}$ and $\nu^{ls}$ are serially uncorrelated and independent from each other, which enables us to focus on the effect of an isolated one-off shock $\varepsilon^{pc}$, $\varepsilon^{ls}$, $\nu^{ls}$ and $\nu^{ls}$ on $\Delta p_H$ and $y$. Relaxing these assumptions thereafter will not raise any difficulty, as it amounts for one to consider one additional eigenvalue ($\rho_{pc}$ or $\rho_{ls}$, both strictly lower than one) in the simulation method described below, and for the other to simulate the joint effect of one-off shocks. We will thus eventually be able to conduct the simulation taking into account the serial correlation of domestic shocks as well as the correlation between domestic shocks and foreign innovations.

Let us write model A’s IS equation and Phillips curve in the following form:

$$y_t = E_t(y_{t+1}) + g_1(r_t - E_t(\Delta p_{H,t+1})) + g_2 E_t(\Delta y_{t+1}) + \varepsilon_{t}^{ls};$$

$$\Delta p_{H,t} = h_1 E_t(\Delta p_{H,t+1}) + h_2 \Delta p_{H,t-1} + h_3 y_t + h_4 y_t^{pc} + \varepsilon_{t}^{pc},$$

modification to the NK model [read: the canonical New Keynesian model] inevitably appears somewhat contrived."
where $g_1$, $g_2$, $h_1$, $h_2$, $h_3$ and $h_4$ are the reduced-form parameters of the model. Whether we consider the reaction of our small open economy to domestic shocks or to foreign innovations, the first point to be noted is that the domestic monetary policy rule drops out under the monetary union regime, together with the monetary policy shock.

Let us focus in the first place on the reaction of our small open economy to the domestic shocks alone under the monetary union regime. In the absence of foreign fluctuations, the Phillips curve and the IS equation have a very simple form, because $y^*$ is constantly equal to zero and because constraint (C1) amounts to keeping $r$ constantly equal to zero. We thus get a system of two (cross-)recurrence equations in the two variables $\Delta p_H$ and $y$, so that the impulse-response functions of these variables will be entirely determined once their initial values are known.

We have therefore four unknowns, namely $\Delta p_{Ht}$, $E_t \{ \Delta p_{Ht+1} \}$, $y_t$ and $E_t \{ y_{t+1} \}$, and we need as many independent conditions on these unknowns. Two of them are provided by the Phillips curve and the IS equation considered at date $t$, which correspond to linear equations in the unknowns. The last two conditions are provided by constraints (C2) and (C3). Indeed, let us note $Z_t = \begin{bmatrix} \Delta p_{Ht} & y_t & \Delta p_{Ht-1} \end{bmatrix}$. Our system is then equivalent to $AE_t \{ Z_{t+1} \} = BZ_t - \begin{bmatrix} \eta_{p}^c & \eta_{q}^e & 0 \end{bmatrix}$, where

$$A = \begin{bmatrix} h_1 & 0 & 0 \\ -g_1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -h_3 & -h_2 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$ 

Since $h_1$ cannot be nil in our model, $A$ is invertible and the system can therefore be written in the following Blanchard and Kahn's (1980) form: $E_t \{ Z_{t+1} \} = A^{-1}BZ_t - A^{-1} \begin{bmatrix} \eta_{p}^c & \eta_{q}^e & 0 \end{bmatrix}$. This implies in turn that $\forall k \geq 1$, $E_t \{ Z_{t+k} \} = (A^{-1}B)^{k-1} \left( A^{-1}BZ_t - A^{-1} \begin{bmatrix} \eta_{p}^c & \eta_{q}^e & 0 \end{bmatrix} \right)$. Let $m \in \{ 0, 1, 2, 3 \}$

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35We call "reduced-form parameter" every parameter which does not belong to the list of pre-defined structural parameters $\alpha, \beta, \eta, \theta, \rho_{p}, \rho_{q}, \sigma, \rho, \omega, V (\eta_{p}^c)$ and $V (\eta_{q}^e)$. A reduced-form parameter will naturally be unaffected by the regime change if it is expressed as a given function of the structural parameters only. In that sense, it could be called "semi-structural parameter".

36The most significant change may be the removal of the rule, not that of the shock. Indeed, Rotemberg and Woodford (1998, pp. 31-32) find that "in the simulation of the <historical> policy regime, the monetary policy shocks are responsible, over the long run, for only 5.0% of the variance of deviations of real output from trend, and (perhaps more surprisingly) for only 1.3% of the variance of inflation. But these results do not imply that monetary policy is unimportant. Nor do they necessarily absolve the Fed from any blame for the instability of output or inflation. What they mean is that it is the systematic part of recent monetary policy that has been of significance for recent economic performance, not the stochastic variation in Fed policy (which, according to our estimates, has been minimal)." (Authors' emphasis.)
denote the number of (possibly complex) eigenvalues of \(A^{-1}B\) whose modulus is larger than or equal to one.

Constraint (C2) then imposes \(m\) constraints on \(A^{-1}BZ = \begin{bmatrix} \eta^p & \eta^s \end{bmatrix}^T\), which correspond to as many linear equations in the unknowns. If \(m = 2\) in particular, then we get as many independent linear equations as we have unknowns, that is to say that Blanchard and Kahn's (1980) condition is satisfied, which states that there should be as many explosive roots as non-predetermined variables in the model. Now, constraint (C3) provides an additional linear equation in the unknowns, namely that the first element of the vector \(Z + (I - A^{-1}B)^{-1} \left( A^{-1}BZ - A^{-1} \begin{bmatrix} \eta^p & \eta^s \end{bmatrix} \right)\) should be nil, because (C3) amounts to the terminal condition \(\rho_{H,+\infty} = \rho_{H,-1}\) in the absence of foreign fluctuations\(^{27}\).

We thus eventually get a system of \(3 + m\) linear equations, both independent from and compatible with each other, for \(4\) unknowns. The corresponding degree of freedom is equal to \(1 - m\). As a consequence, there are three possibilities for the number \(N_s\) of solutions to our simulation, i.e. the number of impulse-response functions satisfying the constraints (C1), (C2) and (C3): either \(N_s = 0\) if \(m \geq 1\), or \(N_s = 1\) if \(m = 1\), or \(N_s = +\infty\) if \(m \leq 1\).\(^{28}\) The case \(N_s = 1\) is the ideal case and deserves little further discussion\(^{29}\). In the case \(N_s = 0\), the simulation is impossible. We can then infer from our model that some structural parameters will have to change under the monetary union regime. In the meantime, the economy is likely to experience some variability which may cause substantial damage.

In the case \(N_s = +\infty\), our simulation leads to multiple equilibria. We are then able to simulate the effects of sunspot shocks, like Clarida, Gali and Gertler (2000), but unable to simulate those of the fundamental shocks \(\eta^p\) and \(\eta^s\), as there are an infinity of solutions. Various criteria have been put forward in the literature to select what their proposer's usually claim is the economically relevant solution, by which they mean the solution on which the agents are the most likely to coordinate\(^{30}\). But we will not need to resort to any of these conditions.

\(^{27}\)Gali and Monacelli's (2002) presentation suggests another way to impose constraint (C3), which amounts to consider Blanchard and Kahn's (1980) condition on the system \((\rho_H, y, r)\) rather than on the system \((\Delta p_H, y, r)\), thus ensuring the stationarity of \(p_H\). This way of doing would be simpler (than ours) to determine the impulse-response functions to domestic shocks, but would be useless (unlike ours) to determine the impulse-response functions to foreign innovations.

\(^{28}\)The cases \(N_s = 0\) and \(N_s = +\infty\) would respectively correspond to "instability" and "indeterminacy" in Batin and Parchman's (2002) terminology.

\(^{29}\)The (theoretical) canonical New Keynesian model corresponds to this case, as shown by Gali and Monacelli (2002).

\(^{30}\)A list of these more or less ad hoc criteria is drawn up by McCallum (1999c), who reviews the corresponding literature. To this list should be added Cho and Moreno's (2002) criterion.
criteria anyway as our estimation results will point to \( m = 1 \) and consequently to \( N_q = 1 \).

Let us now turn to the reaction of our small open economy to the foreign innovations under the monetary union regime. We focus on the reaction of \( \Delta p_H \) and \( y \) to the innovations \( \nu_1^*, \nu_2^* \) and \( \nu_3^* \), not to the shocks \( \eta_1^{ex}, \eta_2^{ex} \) and \( \eta_3^{ex} \). As already said indeed, these shocks remain unidentified as we cannot think of any relevant (i.e. model-consistent) identifying restriction\(^{31}\). The recovery of these shocks would require the estimation of a closed economy New Keynesian model for the foreign country, consisting in a Phillips curve, an IS equation and a monetary policy rule. Then indeed, the shocks could be recovered from these equations by using the values of the expectations predicted by the non-structural VAR. But we choose, at least as a first try, not to estimate this closed economy New Keynesian model for the foreign country.

In the presence of foreign fluctuations and in the absence of domestic shocks, the Phillips curve and the IS equation still form a system of two (cross-)recurrence equations in the two variables \( \Delta p_H \) and \( y \), but this system now includes the exogenous foreign variables \( E_t \{ \Delta y_{t+k+1}^* \} \) for \( k \geq 0 \) and, due to constraint (C1), \( E_t \{ r_{t+k}^* \} \) for \( k \geq 0 \). Now the impulse-reponse functions of \( r^* \) and \( \Delta y^* \) to \( \nu_1^*, \nu_2^* \) and \( \nu_3^* \) are known from the non-structural VAR characterizing the foreign dynamics, so that as previously the impulse-response functions of \( \Delta p_H \) and \( y \) will be entirely determined once their initial values are known. We can therefore proceed in a similar way as previously. Only, we first need to introduce the foreign dynamics into our Blanchard and Kahn’s (1980) form. Because this foreign dynamics typically brings only eigenvalues of modulus strictly lower than one into the system, without altering the value of the “domestic eigenvalues”, we will face the same three possible cases as previously. Note also that constraints (C2) and (C3) will be significantly affected, and that the latter will involve in particular the term \( E_t \{ p_{t+\infty} \} \), which we naturally derive as a function of \( \nu_1^*, \nu_2^* \) and \( \nu_3^* \) from the foreign non-structural VAR.

Comparison with VAR simulations

As indicated in table 2.2, there exist a few simulations of national business cycles within the Eurozone based on VAR estimations on pre-Euro data. However, \(^{31}\)To our knowledge, Giordani (2002) is the only study to provide restrictions derived from a New Keynesian theoretical framework for the identification of foreign structural shocks in a non-constrained VAR. These (short-term) restrictions are however useless in our context, as they are based on Svensson (2000)’s particular specification whose structure of lags and leads does not correspond to ours.
none of these simulations rests on a structural model with rational expectations, and this is problematic for two reasons. First, these studies may not impose the right constraint(s) on the VAR(s) estimated: Mélitz and Weber (1996) and Nikolakaki (1997), for instance, modelize the monetary union as the sole equalization of the national monetary policy shocks, which is little satisfactory. Second and more importantly, these studies assume that the VAR coefficients are not affected by the regime change, which is questionable, to say the least of it. They prove therefore highly vulnerable to the Lucas critique.

By contrast, we make clear that in our three-variable \((\Delta p_H, y, \tau)\) framework, the irrevocable fixity of the nominal exchange rate imposes all three constraints (C1), (C2) and (C3). In particular, imposing constraint (C1) alone, i.e. setting only \(r = r^*\) at all dates, would leave the current nominal exchange rate free to move. Another way to impose these three constraints would be to set the nominal exchange rate to zero in a four-dimension VAR specifying PPP as a cointegration relationship\(^{33}\), the VAR's fourth variable, besides \(\Delta p_H\), \(y\) and \(r\), being precisely the nominal exchange rate \(e\). But this would be an ad hoc manner to impose the right constraints, as there is no structural model to support this simulation method.

Actually, we can easily show that a VAR simulation (of the UK business cycle under EMU-membership) would be simply irrelevant within our New Keynesian context, because our model implies that all the coefficients (not just those of the monetary policy rule) of a three-dimension VAR with \(\Delta p_H\), \(y\) and \(r\) as endogenous variables would be affected by the regime change, and that the corresponding simulation would consequently be subject to the Lucas critique.

To that aim, let us assume for a brief moment that the monetary policy rule followed during the estimation period has both been stable and ruled out multiple equilibria. This assumption enables us to modelize our small open economy by a three-dimension VAR, whose endogenous variables are \(\Delta p_H, y\) and \(r\). Let us note \(X_t = [\Delta p_{H,t} \quad y_t \quad r_t]^\prime\). This VAR is written \(X_t = Q(L) X_{t-1} + R(L) \tau_{t-1} + \nu_t\) with \(Q(L) = \sum_{i=0}^{n} Q_i L^i\) and \(R(L) = \sum_{i=0}^{n} R_i L^i\), where \(Q_i\) (\(0 \leq i \leq n\)) and \(R_j\) (\(0 \leq j \leq n\)) are \(3 \times 3\) matrices and \(\nu_t\) is an independently and identically distributed Gaussian vector with mean zero.

Such a VAR would lead to the following system: \(E_t \{X_{t+1}\} = Q(L) X_t + R(L) \tau_t^*,\) whose first two lines correspond more or less to linear combinations of the Phillips curve and the IS equation. Now let us assume, as usual in structural VAR studies, that \(\eta^M\), \(\eta^y\) and \(\eta^r\) are serially uncorrelated and independent from

\(^{33}\)Hénin and Nicolaud's (1999) study, featuring in table 2.2, adopts a similar PPP-cointegrated VAR approach to simulate the UK business cycle under EMU-membership.
each other, where $\eta^*$ denotes the monetary policy shock, and that there exists a $3 \times 3$ matrix $S$ such that $\nu_t = S\eta_t$, where $\eta_t = [\eta_t^P, \eta_t^q, \eta_t^f]^\prime$.

On the one hand, given the Phillips curve and the IS equation, we need at the very least $S_{13} = S_{23} = S_{31} = S_{32} = 0$ to make the first two lines of $Q(L)$ independent of the monetary policy rule. On the other hand, as shown by chapter 1, the absence of multiple equilibria (be they convergent or divergent, a necessary condition for the economy to be represented by a VAR) requires a forward-looking monetary policy rule having immediate effects, which implies $S_{13} \neq 0$, $S_{23} \neq 0$, $S_{31} \neq 0$ and $S_{32} \neq 0$.

At the end of the day, the general model presented at the onset of this subsection predicts that the coefficients of an economically meaningful VAR do necessarily depend on the monetary policy rule followed by the central bank. We can think of no explicit assumption under which this result would not hold. In other words, a VAR simulation is simply impossible within our theoretical framework, as it provides no means to disentangle in each VAR parameter the part due to the structural parameters, which will be unaffected by the regime change, from the part due to the coefficients of the monetary policy rule.

2.3 Empirical application

This section estimates model A for the UK and applies the simulation method to the UK joining the Eurozone. Because the estimation results prove not fully satisfying, we shortly point to directions for future research at the end of the section. In particular, we specify (directly in reduced-form terms) an extended model, labelled model B, which performs better as far as the estimation results are concerned, so that one direction for future research could be to base this model on microeconomic foundations.

Our model we do want to estimate, rather than calibrate, for two reasons. First, estimation should be seen as a way to gauge the consistency of the model with the data. Second, estimation provides values for some parameters which happen to be neither directly observable nor easily obtained by other means, such as typically the correlation between domestic and foreign shocks.

\footnote{A third reason could even be put forward, namely that estimation enables one to accompany her simulations by confidence intervals, but we do not make use of this third advantage of estimation over calibration.}
2.3.1 Data selection

Even though we choose here to focus on the UK, our simulation method could be interestingly applied to one country "already in" and two other countries "still out" the Eurozone, namely Ireland, Denmark and Sweden. Like the UK, each of these three countries can indeed be considered as a small open economy with respect to the (rest of the) Eurozone, has relatively close commercial links with the (rest of the) Eurozone, and has a development level similar to that of the (rest of the) Eurozone.

Considering Ireland would be interesting because it has had ever since the launch of the Euro a business cycle of its own, not to say a business cycle at odds with that of the rest of the Eurozone. Another member country, Portugal, has experienced quite an idiosyncratic business cycle as well, but this may be due with a productivity catch-up process which is not captured by our model. Considering Denmark and Sweden would also be interesting in terms of economic policy advice because these countries, just like the UK, are currently contemplating the possibility of adopting the Euro. The simulation of their euroized business cycle would therefore give an indication about whether or not they should join the EMU.

The CEECs may also adopt the Euro one day, but our model is less well adapted to these small open economies which are catching up and are mostly price-takers. The same difficulty arises for other countries which have recently adopted the US Dollar as their unique currency, like Ecuador and El Salvador\(^\text{34}\). On the contrary, Obstfeld and Rogoff (2000) argue that the no local currency pricing assumption (one of our model's cornerstones) fits the UK rather well, as 62% of British exports and 43% of British imports are denominated in Sterling, according to the ECU Institute (1995)\(^\text{35}\).

One underlying assumption in our simulation method is that foreign dynam-

\(^{34}\)But we could still on these grounds consider Argentina, which has recently contemplated dollarization.

\(^{35}\)This view is challenged by Neise and Nelson (2002, pp. 15-18), "An alternative view is that closed-economy NKPCs [New Keynesian Phillips curves] should not be used to analyze CPI inflation. On this view, import prices behave dissimilarly from other elements of the CPI and are largely associated with exchange rate and world price changes, leading to a strong influence on the CPI from open-economy factors. But this view does not have much support in the data (see e.g. Stock and Watson, 2001), even for highly open economies like the UK, and so we proceed with an analysis that does not have explicit open-economy elements. [...] For the UK, while both inflation measures [CPI inflation and GDP deflator inflation] are positively correlated with unit labor costs, the relation is noticeably stronger for CPI inflation. This may appear surprising from the perspective of models that stress domestic-goods inflation as being driven by marginal cost, and import price inflation as determined by a different set of factors. But we have argued that there are grounds for regarding import price inflation, and so total CPI inflation, as driven heavily by domestic-economy factors." We proved however unable to find any reference thereupon in Stock and Watson (2001).
ics is left unchanged by the regime change. This assumption would be relevant in the case of a dollarizing Argentina, because the US would not be affected this dollarization, but it is problematic in the case of the UK (as it would be in the case of Denmark, Ireland or Sweden) adopting the Euro, because things have changed in the Eurozone since the estimation period: indeed, the national monetary policy rules have been replaced by the ECB monetary policy rule. Our simulation of the effects of foreign shocks rests therefore on the (strong) assumption that the national central banks have fully cooperated with each other during the estimation period, so that they have collectively behaved like a unique central bank with regard to common shocks.\footnote{An alternative way to proceed would be to infer a calibrated monetary policy rule from the few years of existence of the ECB, and to use the New Keynesian model of a closed economy to simulate the effect of this new rule on the Eurozone's business cycle, taking into account the impact of this change in the monetary policy rule on the private agents' rational expectations.}

We actually choose to assimilate the Eurozone to Germany for reasons of data availability. Adding France, Italy and Spain to Germany would enable us to consider up to 79% and 80% of the Eurozone in terms of population and real GDP respectively, on the basis of Aten, Heston and Summers' (2002) dataset for the year 1996, instead of 27% and 29% respectively when Germany is considered alone. But our French, Italian and Spanish data do not go back in time as far as our British and German data, as shown by table 2.3, so that we decide to leave them aside.

Table 2.3 indicates the precise nature of the data chosen and table 2.9 provides a comparison with the data used in all the studies inventoried in table 2.8. We use FPI indices for the price index \( P_H \), in accordance with our model and in contrast with all other studies, including those having an open-economy setting. We choose industrial output for the production level \( Y \), unlike all other studies but Clarida, Gali and Gertler's (1998), because our model focuses on tradable goods exclusively. And we use the three-month Treasury bill rate for the nominal interest rate \( R \). The \( \Delta p_H \) and \( r \) series, respectively corresponding to \( \Delta \log (P_H) \) and \( R \), are de-meaned, de-seasonalized and annualized, i.e. expressed in per cent per annum, while \( y \equiv \log (Y) \), already seasonally adjusted, is linearly detrended so as to be expressed in per cent of deviation of \( Y \) from its steady state value.

We resort to linear detrending for \( y \) as we assume that the deterministic trend in the logarithm of potential output is linear. Note that we do acknowledge that the logarithm of potential output may have and has indeed a stochastic component in addition to its deterministic linear trend. This stochastic component
we include in the shocks $\varepsilon^{ne}$ and $\varepsilon^{fr}$. In other words, our variable $y$ is not supposed to represent the output gap, defined as the percentage difference between the actual output level and the flexible-price, flexible-wage output level, i.e. the output level which would prevail, were there no nominal rigidity in the economy. If the choice of the detrending method for $y$ has been so much discussed in the literature, it is because in contrast to our variable, $y$ is typically supposed to represent the output gap in a large body of the literature which includes most of the studies featuring in table 2.9. And this output gap, which captures the portion of output variation due to the presence of nominal rigidities, is typically very volatile, so that the choice of a smooth detrending method seems irrelevant.\footnote{This difficulty is mentioned in Amato and Laubach (2003b), Lindé (2002), Nelson (2002) and Woodford (2003) among others. Besides, Galí and Gertler (1999), Neiss and Nelson (2001), Woodford (2001) and Galí (2002) all provide empirical evidence on the fact that the theoretically consistent output gap (or real marginal cost) is not closely - and may even be negatively - correlated with the detrended output level. To our knowledge, Neiss and Nelson (2002) is the only study to use a theory-consistent output gap measure. Note finally that oddly enough, Cho and Moreno (2002, p. 9) advocate the choice of a linear or quadratic trend rather than a Hodrick- Prescott trend, so as to be allowed to interpret $\varepsilon^{fr}$ as an aggregate demand shock.}

Two last points are worth noting. First, we choose like most other studies to consider the quarterly frequency, although because we use industrial output rather than GDP data, we could like Clarida, Galí and Gertler (1998) consider the monthly frequency, which provides three times as many observations. Second, we need to assume that $\Delta p_H$, $y$ and $r$ are stationary variables, as implied by the model. Clarida, Galí and Gertler (1998, 2000) justify this assumption more or less convincingly by putting forward the “well-known low power of unit root tests” for short samples.

2.3.2 Estimation method

Before applying our simulation method, we need to estimate the parameters of our model. We choose to estimate the structural parameters rather than the reduced-form parameters, so as to take full account of the theoretical cross-equation restrictions on the reduced-form parameters, which implies a joint estimation of the Phillips curve and the IS equation.\footnote{Oddly enough, this advantage of multivariate estimation over univariate estimation within the framework of the New Keynesian model was first pointed out only recently, by Leeper and Zha (2001). Note that in our framework these cross-equation restrictions take the form of inequalities rather than equalities, given the licit range of the structural parameters.} The estimation of these structural parameters involves non-linear estimation techniques, as these parameters enter non-linearly the two equations.
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We therefore choose GMM as our estimation method. As reminded by Roberts (2001), full-information estimation techniques may well have greater econometric efficiency when the correct specification of the model is known, but limited information techniques, as for them, are robust to incorrect model specifications. According to this author, GMM also has the advantage of handling the possible correlation between the error term and the explanatory variables, as well as the advantage of protecting against biases stemming from data measurement errors.

But GMM does not only have general advantages and shortcomings. Indeed, the use of GMM to estimate or to test New Keynesian models in particular has recently come under criticism from two studies for two different reasons. First, as argued by Rudd and Whelan (2001), GMM cannot distinguish between backward-looking and forward-looking Phillips curves because inflation is highly autocorrelated. According to these authors, GMM estimates are biased if the true Phillips curve is backward-looking and if lagged inflation rates are used as instruments, while Hansen's test of overidentifying restrictions is then likely to fail to detect this misspecification.

Our answer to this criticism is simply that the present chapter is not aimed at testing the New Keynesian specification of the Phillips curve against the alternative specification without any expectations term. In other words, we do not take part to the currently raging forward-looking versus backward-looking debate. Instead, we assume the existence of a forward-looking Phillips curve which we seek to estimate. As should be clear to the reader by now, our simulation method does rest on the forward-looking nature of the model, by which we mean that we could not conduct our simulation within a backward-looking framework.

Second, Lindé (2002) expresses serious doubts on the performance of univariate GMM to estimate New Keynesian Phillips curves. Using simulated data, he finds that in the absence of measurement errors, univariate GMM perform badly compared to univariate NLLS when \( \gamma \) is persistent and when the true Phillips curve is more forward-looking than backward-looking, while in the presence of measurement errors (specified as white noises incorporated into \( p \) and \( y \)), neither univariate GMM nor univariate NLLS works well. He also shows still on simulated data that univariate GMM estimates of New Keynesian Phillips curves are biased if both \( y \) and \( \gamma \) are persistent, or if the monetary policy rule is unstable.

To this criticism we have little to answer, except that, well, we do actu-

\[^{42}\text{Cho and Moreno (2002) put forward this advantage of FIML over GMM.}\]
The joint estimation of the Phillips curve and the IS equation has the additional advantage of taking into account the correlation between the two corresponding error terms. Given our estimation method (detailed below), these error terms are written as linear combinations of expectation errors, and they are likely to be correlated to each other because both of them involve in particular \( \Delta p_{H,t+2} - E_t \{ \Delta p_{H,t+2} \}, \Delta p_{H,t+1} - E_t \{ \Delta p_{H,t+1} \}, y_{t+1} - E_t \{ y_{t+1} \} \) and \( y_{t+1}^* - E_t \{ y_{t+1}^* \} \).

As can be seen in Table 2.9, four main alternative methods can be used to estimate the structural parameters. The first one is the Generalized Method of Moments (GMM), which consists in using adequate instruments to estimate the IS equation and the Phillips curve where the expectations \( E_t \{ \Delta p_{H,t+1} \}, E_t \{ y_{t+1} \} \) and \( E_t \{ \Delta y_{t+1}^* \} \) have been replaced by the actual values \( \Delta p_{H,t+1}, y_{t+1} \) and \( \Delta y_{t+1}^* \). The second one is the Minimum Distance Estimation (MDE) method, which consists in choosing the values of the parameters which minimize the difference (in the sense of a judiciously chosen measure) between the predictions of the model and those of a non-structural VAR. The third one is the well-known Full Information Maximum Likelihood (FIML) method. And the fourth one is simply the Non-Linear Least Squares (NLLS) method, whose use is relevant in our context when survey data are used as a proxy for expected variables.

Now, the NLLS method can be ruled out as we do not use survey data, and we cannot resort to the MDE and FIML methods as we stick to a partial equilibrium framework by not specifying the domestic monetary policy rule. As already said, we choose not to estimate any domestic monetary policy rule mainly because we happen to have good reasons to suspect that this rule might well not have been stable over the estimation period, which includes the EMS period. Finally, we also view the MDE and FIML methods as problematic in the context of rational expectations models characterized (as often) by multiple equilibria, simply because they rest on the arbitrary selection of one equilibrium out of all possible equilibria.

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39 In Cho and Moreno’s (2002) terms, “the estimation of the full system has the advantage of allowing for the interaction among the different economic agents: consumers, firms and the central bank” (p. 1), or in other words, “the joint estimation has the advantage that it accounts for the simultaneous effect of all the structural shocks on each of the variables in estimation” (p. 5).

40 Note that the estimation of the domestic monetary policy rule would be a constrained estimation if this rule is assumed to ensure the implementation of a unique equilibrium in the sense of chapter 1, because some of its coefficients are then theoretically linked to the coefficients of the Phillips curve and the IS equation.

41 For instance, Cho and Moreno (2002) have not only to assume the existence of a solution in the form of a VAR of order one, but also to choose one stationary solution out of several at each iteration of their FIML estimation procedure, with the help of a criterion which selects
ally perform multivariate GMM, not univariate GMM, as we jointly estimate the Phillips curve and the IS equation. Our approach is thus intermediate between a single equation estimation approach and the three-equation full-system estimation approach advocated by Lindé (2002). True, our justification for disregarding the third equation (namely the monetary policy rule) is the suspected instability of the latter, and Lindé’s (2002) last criticism about univariate GMM estimates of the Phillips curve applies precisely in the case of an unstable monetary policy rule. But we just hope that his results are not robust to the joint estimation of the Phillips curve and the IS equation.

2.3.3 Estimation procedure

Our GMM estimation procedure is very similar to Roberts’ (2001), even though ours is applied to the estimation of the structural parameters of two equations, while his is applied to the estimation of the reduced-form parameters of one single equation. Let us first remove the AR(1) shocks from our IS equation and Phillips curve by quasi-differentiation:

\[ E_t \{ y_{t+1} \} - \rho y_t = E_t \{ y_{t+2} \} - \rho y_t E_t \{ y_{t+1} \} + g_1 \{ E_t \{ r_{t+1} - \Delta P_{H,t+2} \} - \rho y_t \} \]
\[ + g_2 \{ E_t \{ \Delta y_{t+2}^* \} - \rho y_t E_t \{ \Delta y_{t+1}^* \} \}, \]

\[ E_t \{ \Delta P_{H,t+1} \} - \rho_{pc} \Delta P_{H,t} = h_1 \{ E_t \{ \Delta P_{H,t+2} \} - \rho_{pc} E_t \{ \Delta P_{H,t+1} \} \} \]
\[ + h_2 \{ \Delta P_{H,t} - \rho_{pc} \Delta P_{H,t-1} \} + h_3 \{ E_t \{ y_{t+1} \} \} - \rho_{pc} y_t \}
\[ + h_4 \{ E_t \{ y_{t+1}^* \} - \rho_{pc} y_t^* \} \].

These two equations can then be re-written in terms of the observed variables and of expectations error terms \( u_t^e \) and \( u_t^{pc} \), which gives the two estimated equations:

\[ y_{t+1} - \rho y_t = y_{t+2} - \rho y_{t+1} + g_1 \{ \{ r_{t+1} - \Delta P_{H,t+2} \} - \rho y_t \} \]
\[ - \Delta P_{H,t+1} \} + g_2 \{ \Delta y_{t+2}^* - \rho y_t \Delta y_{t+1}^* \} + u_t^e, \]

\[ \Delta P_{H,t+1} - \rho_{pc} \Delta P_{H,t} = h_1 \{ \Delta P_{H,t+2} - \rho_{pc} \Delta P_{H,t+1} \} + h_2 \{ \Delta P_{H,t} - \rho_{pc} \Delta P_{H,t-1} \} \]
\[ + h_3 \{ E_t \{ y_{t+1} \} - \rho_{pc} y_t \} \]
\[ + h_4 \{ E_t \{ y_{t+1}^* \} - \rho_{pc} y_t^* \} + u_t^{pc}. \]
where

\[ u^s_t = (1 + \rho_s) (y_{t+1} - E_t \{y_{t+1}\}) - (y_{t+2} - E_t \{y_{t+2}\}) \]
\[ - g_1 (r_{t+1} - E_t \{r_{t+1}\}) - g_1 (\Delta p_{H,t+2} - E_t \{\Delta p_{H,t+2}\}) \]
\[ - g_1 \rho_u (\Delta p_{H,t+1} - E_t \{\Delta p_{H,t+1}\}) \]
\[ - g_2 (\Delta y^*_t - E_t \{\Delta y^*_t\}) + g_2 \rho_y (\Delta y^*_t - E_t \{\Delta y^*_t\}), \]

\[ u^{PC}_t = (1 + h_1 \rho_{PC}) (\Delta p_{H,t+1} - E_t \{\Delta p_{H,t+1}\}) - h_1 \Delta p_{H,t+2} \]
\[ - E_t \{\Delta p_{H,t+2}\}) - h_3 (y_{t+1} - E_t \{y_{t+1}\}) - h_4 (\Delta y^*_t - E_t \{\Delta y^*_t\}). \]

Under the assumption of rational expectations, \( u^s_t \) and \( u^{PC}_t \) are orthogonal to current and past variables, which can consequently be used as instruments. Under our preferred specification, we will choose \( y_{t-k}, r_{t-k}, y^*_t, \) and \( r^*_t \) for \( 1 \leq k \leq 3 \) as our instruments, thus leaving aside the lagged inflation rates. This choice is advocated by Roberts (2001), who warns of possibly serially correlated measurement errors in \( \Delta p_H \). Moreover, this author finds that the coefficient associated to \( y_t \) in the hybrid Phillips curve has the right sign (alas without being significant) only when lagged inflation rates are not used as instruments.

Because we estimate our two equations jointly, the GMM estimator is a two-stage least squares estimator, the first step corresponding to traditional two-stage least squares, from which an optimal weighting matrix is constructed. Acknowledging the serial correlation in both our instruments and the error terms \( u^s \) and \( u^{PC} \), we resort to the weighting matrix advocated by Hansen (1982), which provides a more efficient estimator.\footnote{This weighting matrix is used by RATS 4.31 with the options LAGS and DAMP of the-NLSYSTEM procedure.} Note indeed that our error terms \( u^s_t \) and \( u^{PC}_t \) happen to be MA(2) by construction, just like the n-step forecast error in Clarida, Galí and Gertler's (1998, 2000) framework is MA(n) by construction and calls similarly for the use of an optimal weighting matrix.

How do we recover the structural shocks \( \epsilon^{PC} \) and \( \epsilon^s \), or equivalently \( \eta^{PC} \) and \( \eta^s \), after estimating the structural parameters of the model? We know of only one method, namely Rotemberg and Woodford's (1997), which consists in using the private agents' expectations derived from a non-structural VAR to residually obtain shocks \( \epsilon^{PC} \) and \( \epsilon^s \) from the IS equation and the Phillips curve. Unfortunately, this method is unavailable to us as we rule out the possibility of modeling our small open economy by a VAR.
Instead, we proceed as follows to estimate the variance-covariance matrix $\Omega$ of the (serially uncorrelated) vector of exogenous perturbances $[\eta_t^p, \eta_t^p, \nu_t^a, \nu_t^a, \nu_t^a]^T$. Let $\nu_t^a$ and $\nu_t^{pc}$ denote the residuals of the non quasi-differentiated equations, which can be readily estimated:

$$\nu_t^a \equiv y_t - y_{t+1} - g_1 (r_t - \Delta p_{H,t+1}) - g_2 \Delta y_{t+1}^c,$$

$$\nu_t^{pc} \equiv \Delta p_{H,t} - h_1 \Delta p_{H,t+1} - h_2 \Delta p_{H,t-1} - h_3 y_t - h_4 y_t^c.$$

These residuals can be expressed as functions of the structural shocks and some forecast errors:

$$\nu_t^a = -(y_{t+1} - E_t \{y_{t+1}\}) + g_1 (\Delta p_{H,t+1} - E_t \{\Delta p_{H,t+1}\}) - g_2 (\Delta y_{t+1}^c - E_t \{\Delta y_{t+1}^c\}) + \varepsilon_t^a,$$

$$\nu_t^{pc} = -h_1 (\Delta p_{H,t+1} - E_t \{\Delta p_{H,t+1}\}) + \varepsilon_t^{pc}.$$

We know that $\text{cov} (\eta_t^p, \eta_t^p) = 0$ by assumption and we can directly estimate the variances and covariances of $\nu_t^a$, $\nu_t^a$ and $\nu_t^{pc}$ from the foreign VAR. Because both forecast errors and past shocks are orthogonal to the current foreign innovations, we also know that $\text{cov} (\nu_t^a, \nu_t^a) = \text{cov} (\eta_t^p, \nu_t^a)$ and $\text{cov} (\nu_t^{pc}, \nu_t^a) = \text{cov} (\eta_t^p, \nu_t^{pc})$ for $i \in \{1, 2, 3\}$, which provides us with estimates of $\text{cov} (\eta_t^p, \nu_t^a)$ and $\text{cov} (\eta_t^p, \nu_t^{pc})$ for $i \in \{1, 2, 3\}$. We thus get estimates for all the coefficients of $\Omega$, but $\Omega (1,1)$ and $\Omega (2,2)$.

We will not be able to get direct estimates for these last two coefficients, but rather a lower bound and an upper bound for each of them. Indeed, because forecast errors are orthogonal to current shocks, we get $\text{var} (\eta_t^p) \leq (1 - \rho_p^2) \text{var} (\nu_t^a)$ and $\text{var} (\eta_t^p) \leq (1 - \rho_p^2) \text{var} (\nu_t^{pc})$. Similarly, because forecast errors are orthogonal to present and past variables, we get:

$$\text{var} (\eta_t^p) \geq \max_{x \in (\Delta p_{H,t}, \Delta p_{H,t+1}, \Delta p_{H,t-1})} \left[ \frac{(1 - \rho_p^2) \text{cov}^2 (\nu_t^a, x_{t-k})}{\text{var} (x_{t-k})} \right],$$

$$\text{var} (\eta_t^p) \geq \max_{x \in (\Delta p_{H,t}, \Delta p_{H,t+1}, \Delta p_{H,t-1})} \left[ \frac{(1 - \rho_p^2) \text{cov}^2 (\nu_t^{pc}, x_{t-k})}{\text{var} (x_{t-k})} \right].$$

Finally, $\text{var} (\eta_t^p)$ and $\text{var} (\eta_t^p)$ must be such that the variance-covariance matrix of $[\eta_t^p, \eta_t^p, \nu_t^a, \nu_t^a, \nu_t^a]^T$ is semi-definite positive, which imposes an additional constraint on $\Omega (1,1)$ and $\Omega (2,2)$. 

2.3.4 Estimation results

All results have been obtained with the RATS 4.31 software. As already said, we seek to estimate the structural parameters so as to take full account of the cross-equation restrictions between the reduced-form parameters \( g_1, g_2, h_1, h_2, h_3 \) and \( h_4 \). But we also directly estimate these reduced-form parameters in order to appreciate how much constraining these restrictions prove to be. Moreover, these structural and reduced-form estimations are also carried out for each equation separately, again so as to provide a useful benchmark to our results. The corresponding estimates are displayed in tables 2.4 and 2.5.

It should first be mentioned that the calibration of several structural parameters is required for the estimation algorithm to converge towards a stable solution to the minimization problem. We have to set these structural parameters prior to estimation for two reasons. First, because they may not be jointly identifiable. Such is the case of parameters \( \alpha \) and \( \eta \), whose separate effects cannot be disentangled in the expression \( \alpha (2 - \alpha) (\sigma \eta - 1) \). Second, because the estimation algorithm may not converge even though the parameters are theoretically identifiable. Leith and Malley (2001) meet the same difficulty in their non-linear GMM estimation procedure. Like them, we have to condition our estimation on these calibrated parameters.

We choose to set \( \beta = 0.99 \), which given our quarterly frequency implies a riskless annual return of about 4% in the steady state, and (admittedly somewhat arbitrarily) \( \eta = 0.55 \). As indicated in table 2.4, the estimation of the IS equation alone then provides \( \hat{\sigma} = 0.50 \) and \( \hat{\theta} = 0.15 \). We thereafter calibrate these four parameters at these values to estimate either the Phillips curve alone, or both the IS equation and the Phillips curve. At the end of the day, some of our estimates prove in accordance with existing studies, for instance \( \hat{\sigma}_{pc} = 0.81 \) and \( \hat{\theta} = 0.92 \) which happen to be the most significant of all (together with \( \hat{\rho}_{1} \)), while others are clearly unreasonable but happen to have a confidence interval large enough to include more reasonable values, like \( \hat{\sigma} = 0.07 \) whose confidence interval at the 95% level (under the normality assumption) includes the value \( \theta = 0.75 \) consistent with an average period of one year between price adjustments. Finally, we suspect that our highly significant \( \hat{\rho}_{1} = 1.01 \) signals the existence of omitted lags of \( y \) in the IS equation.

Note that the coefficients \( \gamma_1 \) and \( \gamma_3 \) of the two forcing variables in the reduced-form estimations have always the right sign but are usually not significantly different from zero. This disappointing estimation result proves rather common in the literature. Cho and Moreno (2002, p. 21) say that "this bias [...] seems to be related to the measurement error contained in the detrended out-
put measure", but this culprit we can rule out in our framework, as we reckon to have satisfactorily dealt with the issue of detrending. As argued by Lindé (2002), the non-significance or the wrong sign of the $y$-coefficient in the estimated Phillips curve would tend to support the old Phillips curve rather than the New Keynesian Phillips curve. But as already said, we do not take part to this debate.

Hansen's specification J-test of the overidentifying restrictions validates our orthogonality conditions in all cases, though much less so for the estimation of the structural parameters. We suspect that this result comes from the fact that the additional lags of the endogenous variables which have to be used as instruments for the joint estimation of the Phillips curve and the IS equation (as more parameters are to be estimated) prove not so good instruments. Finally, the $R^2$ ranges from 0.77 to 0.84 for the IS equation and from 0.86 to 0.89 for the Phillips curve, which is pretty high, while the Durbin-Watson statistic ranges from 2.92 to 2.98 for the IS equation and from 3.02 to 3.15 for the Phillips curve, which suggests a negative serial correlation (at the first order) for $u_t$ and $u_t^P$, precisely what should be expected at first sight (say, if the forecast errors on two different variables are not correlated with each other).

We have carried out a robustness analysis to check whether our estimation results depend on our particular estimation procedure. We have thus considered other detrending methods for $y$ and found that the estimators are little affected by the use of a quadratic or a Hodrick-Prescott trend rather than a linear trend. Similarly, the choice of various initial values for the parameters in the estimation algorithm does not impact on the final estimators when the algorithm converges, though it may precisely impact on whether the algorithm does or does not converge. We have used an alternative normalization of the orthogonality conditions, just like Galí and Gertler (1999), as GMM estimators are sometimes criticized (e.g. by Demery and Duck, 2002) for being sensitive to the normalization of orthogonality conditions in the context of a non-linear estimation over a small sample, but our results proved altogether robust to this alternative specification.

We have also considered various specifications for the serial correlation of the Kronecker product of our instruments by $u_t^s$ and $u_t^P$, in the form of MA($k$) processes for $2 \leq k \leq 20$, and found that our results may be affected by the specification chosen. As the true specification may well be AR rather than MA, but our econometric software does not enable us to specify an AR process, we eventually chose a large degree for the MA process, namely 20. Finally, we have used different sets of instruments alternatively and found that our results do
significantly depend on the set chosen, especially on whether this set does or does not include lagged inflation rates. As advocated by Roberts (2001), we chose to leave the lagged inflation rates out of our instrument set.

We resort to Akaike's, Hannan and Quinn's as well as Schwartz's criteria to choose the order of the foreign VAR. These criteria advocate the choices of 8, 2 and 1 lag(s) respectively. We decide to meet halfway by adopting a VAR(2) framework. Moreover, the order 2 is precisely the one which minimizes the unweighted sum of the functions associated to the three criteria considered. Besides, we carry out a Chow test to assess the stability of the VAR, as the data refer to unified Germany from 1991.

2.3.5 Simulation results

Having estimated the first- and second-order parameters of our model, we are able to simulate the macroeconomic fluctuations of the UK within the Eurozone (disregarding the period of transition from one regime to the other) along the lines of the simulation method described above. Our simulation is based more precisely on the results of the joint structural estimation of the Phillips curve and the IS equation, that is to say on the following system:

\[ y_t = E_t \{ y_{t+1} \} - 2,08 \times (r_t - E_t \{ \Delta p_H, t+1 \}) - 0,69 \times E_t \{ \Delta y^*_t \} + \varepsilon^y_t, \]

\[ \Delta p_H, t = 0,07 \times E_t \{ \Delta p_H, t+1 \} + 0,93 \times \Delta p_H, t-1 + 0,06 \times y_t - 0,02 \times y^*_t + \varepsilon^p_t, \]

where \( \varepsilon^p_t = 0,81 \times \varepsilon^p_{t-1} + \eta^p_t \) and (admittedly somewhat arbitrarily\(^\text{44}\)) \( \varepsilon^y_t = 0,83 \times \varepsilon^y_{t-1} + \eta^y_t \), with a variance-covariance matrix of \( [\eta^y_t \eta^p_t \nu^y_t \nu^p_t \nu^*_t \nu^*_p] \) estimated by

\[
\hat{\Omega} = \begin{bmatrix}
1,05 & 0 & -0,39 & -0,06 & 0,08 \\
0 & 0,01 & -0,00 & -0,04 & 0,00 \\
-0,39 & -0,00 & 0,20 & 0,13 & 0,02 \\
-0,06 & -0,04 & 0,13 & 2,56 & 0,01 \\
0,08 & 0,00 & 0,02 & 0,01 & 0,06
\end{bmatrix}.
\]

This matrix \( \hat{\Omega} \) is obtained as previously indicated. Concerning \( \hat{\Omega}(1,1) \) and \( \hat{\Omega}(2,2) \) in particular, the computations lead to \( 0,70 \leq \hat{V} (\hat{\eta}^p) \leq 2,87 \) and

\(^{44}\)We cannot retain \( \rho_\mu = 1,01 \), as implied by our estimation results, for obvious reasons of convergence. We use instead the alternative and more realistic value 0,83 for \( \rho_\mu \), which fall into the bilateral confidence interval at the 95% level under the normality assumption. The system should ideally be estimated again conditionally on \( \rho_\mu = 0,83 \), but we leave it unchanged so as to avoid the overabundance of estimation results.
\(0.01 \leq \hat{\mathcal{V}}(\hat{\eta}^{PC}) \leq 0.05\). However, \(\hat{\Omega}\) does not prove semi-definite positive for all these admissible values of \(\hat{\mathcal{V}}(\hat{\eta}^{PC})\) and \(\hat{\mathcal{V}}(\hat{\eta}^{PC})\). Actually, \(\hat{\Omega}\) is found to be positive semi-definite if and only if \(\hat{\mathcal{V}}(\hat{\eta}^{PC}) \geq 1.05\), whatever the value of \(\hat{\mathcal{V}}(\hat{\eta}^{PC})\) in-between 0.01 and 0.05. At the end of the day, we choose to use the lower estimates \(\hat{\Omega}(1,1) = 1.05\) and \(\hat{\Omega}(2,2) = 0.01\), which we view as more relevant than the upper estimates \(\hat{\Omega}(1,1) = 2.87\) and \(\hat{\Omega}(2,2) = 0.06\), simply because the latter are based on the doubtful assumption of zero forecast errors, contrary to the former. If anything, our simulation results will therefore probably tend to underestimate macroeconomic volatility for the UK under EMU-membership.

Let us now turn for a brief moment to the issue of macroeconomic instability. Of the three eigenvalues associated to this system, one is real and higher than one (15.33), while the two others are complex and of modulus lower than one (0.93). This result ensures the existence of one single equilibrium under the monetary union regime, and we therefore escape the case of no equilibrium as well as the case of multiple equilibria. Because we are left with two complex eigenvalues, we get an oscillatory system for our simulation of the UK business cycle under EMU-membership.

Figures 2.1 and 2.2 display the simulation graphs characterizing the UK business cycle under EMU-membership. More precisely, figure 2.1 displays the impulse-response functions of \(\Delta p_H\), \(p_H\) and \(y\) to the domestic shocks \(\eta^x\) and \(\eta^{PC}\) in the absence of foreign fluctuations, while figure 2.2 displays the impulse-response functions of \(\Delta p_H\), \(p_H\), \(y\), \(\Delta p^*\), \(p^*\), \(y^*\) and \(r = r^*\) to the foreign innovations \(\nu^L_1\), \(\nu^L_2\) and \(\nu^R_2\) in the absence of domestic shocks. These initial shocks and innovations are one-off, positive and equal to their standard error.

For the sake of clarity, they are considered separately in these simulation graphs even though they are actually correlated with each other.

The simulation graphs of figure 2.1 indicate that the immediate effect of a positive cost-push shock \(\eta^{PC}\), that is to say a negative productivity shock, is a fall in the production level and a rise in the inflation rate as well as the price level, while the immediate effect of a positive IS shock \(\eta^x\), i.e. a positive aggregate demand shock, is an increase in the production level, the inflation rate and the price level. These results are naturally in accordance with conventional wisdom. Note however that the opposite results are a priori conceivable in our four-equation four-unknown framework.

As implied by our simulation method, the three variables (\(\Delta p_H\), \(p_H\) and \(y\)) are stationary following a domestic shock, that is to say that the corresponding impulse-response functions converge towards zero in the long term. As can be seen on the simulation graphs, the speed of convergence towards PPP is not
particularly low. Indeed, the impulse-response functions of $p_H$ to the domestic shocks under EMU-membership take around 10 quarters to cover half the distance to PPP. This speed of convergence implied by our simulation results proves somewhat higher than existing studies’ estimates\footnote{In Obstfeld and Rogoff’s (1996, p. 623) terms, “as Froot and Rogoff (1995) show, consensus estimates for the rate at which PPP shocks damp out are very slow. Consider a regression of the form $q_t = \alpha_0 + \rho q_{t-1} + \epsilon_t$, where $q$ is the real exchange rate and $\epsilon$ is a random disturbance. On annual panel data for industrialized countries, a typical estimate of $\rho$ is 0.85. This implies an average half-life of deviations from PPP of roughly 4.2 years.”}.

Besides, it is worth noting that our impulse-response functions to domestic shocks prove quite similar to Westaway’s (2003) qualitatively speaking, as they share the same oscillatory pattern. This similarity is rather encouraging. The only noticeable difference is a somewhat longer oscillatory period in our case (18 versus 12 quarters).

This oscillatory pattern is naturally also present in the impulse-response functions to the foreign innovations displayed in figure 2.2. We do not interpret these simulation graphs however, as the foreign innovations are no structural shocks and consequently have no clear economic meaning. Only, the rather high degree of international transmission is worth noting. Indeed, the UK macroeconomic fluctuations under EMU-membership caused by EMU macroeconomic fluctuations (in the absence of UK shocks) are very much comparable in magnitude to these EMU macroeconomic fluctuations themselves. This international transmission of macroeconomic fluctuations goes through three channels under EMU-membership: the presence of $y^*$ in the domestic IS equation and Phillips curve, the constraint (C1) equalizing $r$ to $r^*$ in the domestic IS equation and the long-run relative PPP constraint (C3) linking $p_{H,+\infty}$ and $p_{f,+\infty}$ to each other.

Let us now examine the effect of EMU-membership on the variances of $\Delta p_H$ and $y$, i.e. let us compare the variances implied by our simulation to those observed over the estimation period. As indicated in table 2.7, the standard error of $\Delta p_H$ is left roughly unchanged by EMU-membership, while the standard error of $y$ is increased twofold. This result for $\Delta p_H$ proves more or less in accordance with those of existing studies, contrary to the result for $y$ which proves much more pessimistic for the euroized UK than those of existing studies. Note however that our results prove compatible with those obtained by Driver and Wren-Lewis (1999) or Driver (2000) for a generic country. Indeed, these authors define a loss function as a weighted sum of the squared deviations of the variables of interest (including inflation and output) from their steady state values, and find that depending on the scenario and the shock considered, the move from free float to EMU multiplies the value taken by the loss function by
a factor contained between 0.84 and 173.\footnote{More precisely, these authors consider three different shocks occurring only in the domestic country, namely a one-year IS shock (case A), a three-year IS shock (case B) and a one-year cost-push shock (case C). Depending on the scenario considered, the move from free float to EMU multiplies the value taken by the domestic loss function by a factor contained between 2.21 and 7.56 in case A, between 2.93 and 6.64 in case B, between 0.84 and 1.35 in case C, while the value taken by the foreign loss function is multiplied by a factor contained between 12 and 39 in case A, between 19 and 96 in case B, between 17 and 173 in case C.}

Note that our results rest on the assumption that the uncovered interest rate parity holds without any error term. Had we introduced an exogenous risk-premium shock into the uncovered interest rate parity, this shock would have ended up as an additional term in the IS equation. Naturally, this term would have had to be removed from the simulation exercise, as the risk-premium shock no longer exists under the monetary union regime. The large increase in macroeconomic volatility which we find the UK would experience if it adopted the Euro may therefore reflect our overestimation of the true \( V(\eta^{1*}) \) as we do not specify any risk-premium shock.

Note furthermore that if we focus on domestic shocks only, that is to say if we ignore the effect of foreign innovations (admittedly a questionable way to proceed, given that domestic shocks are correlated to foreign innovations), then the variances of the variables of interest \( \Delta p_H \) and \( y \) under EMU-membership are increased up to twofold: the macroeconomic volatility ratios take the value 1.44 (instead of 1.06) for \( \Delta p_H \) and 2.75 (instead of 2.06) for \( y \). In other words, taking into account the EMU fluctuations does actually lower the UK macroeconomic volatility under EMU-membership. This result, which comes naturally from the correlation between the domestic and the foreign exogenous perturbations, underscores the importance of our estimated variance-covariance matrix \( \tilde{\Omega} \). More precisely, it is probably due here to the fact that \( \eta^{1*} \) and \( \nu^{1*} \) happen to be negatively correlated with each other and to have similar effects on each of the variables of interest (\( \Delta p_H \) and \( y \)). Little can be said thereabout however, as the innovation on the EMU inflation rate \( \nu^{1*} \) has no economic interpretation.

2.3.6 Further extension

The estimation results obtained for model A are not fully satisfying for three main reasons. First, the estimates are more often than not little significant, whether we deal with structural or reduced-form parameters. Second, the estimates of the structural parameters, or their values compatible with the data when we impose them, prove sometimes unreasonable. Third, the highly significant estimate of \( \rho_{1s} \) is too large and compel us to use another value in the
simulation step. One could suspect that the estimation period is partly responsible for these three shortcomings. Indeed, this estimation period includes the 70's, a singular decade of highly volatile inflation. However, things do not change fundamentally when model A is estimated over the period from 1980:1 to 1999:1.

We choose therefore to consider another model, labelled model B, whose reduced form differs from model A's in that it includes a term $y_{t-1}$ in the IS equation\(^{47}\). The motivation for this extension is primarily empirical. As already said indeed, the existence of this additional lag is suggested by the fact that our highly significant estimate of $\rho_{12}$ is very close to one. In the closed economy version of the New Keynesian model, Clarida, Gali and Gertler (1999) justify (without explicitly modelizing) the appearance of this lag in the IS equation by the existence of some form of adjustment costs. Similarly, we specify model B directly in reduced-form terms, as the derivation of a fully-fledged structural model whose reduced form would correspond to model B's is beyond the scope of this study. We actually view this subsection as an exploratory work paving the way for future research.

We also introduce as many lags and leads of $y^*$ as there are of $y$ in the IS equation, because the IS equation comes originally from the Euler equation which involves domestic consumption (of domestic and foreign goods), which in turn is expressed as a function of both the domestic and the foreign production levels. The Phillips curve is assumed to be left unaffected by this change. Finally, we impose three constraints prior to estimation, namely that the sum of the $y$-coefficients in the IS equation, the sum of the $y^*$-coefficients in the IS equation and the sum of the $\Delta p_t$-coefficients in the Phillips curve should all be nil\(^{48}\). We therefore estimate the following equations:

\(^{47}\)As shown by table 2.8, most of the canonical New Keynesian model's extensions considered in the literature amount to introduce additional lags and leads of the endogenous variables in the Phillips curve and the IS equation. We could also take this literature into account by considering a more general specification for the Phillips curve and the IS equation, which would incorporate most of the conceivable extensions (including habit formation) to Gali and Monacelli's (2002) canonical New Keynesian model of a small open economy. Our simulation method would still be applicable in this generalized framework, because this method rests on the existence of a Blanchard and Kahn's (1980) form for the Phillips curve and the IS equation, which itself rests in turn on the invertibility of a given matrix $A$ - now the particular cases where $A$ proves not invertible are of measure zero, that is to say that their probability to occur is zero. Indeed, all the parameters in the (two) equations of interest are exogenous, unlike those in the (three) equations of interest in chapter 1 (which considers the monetary policy rule in addition to the Phillips curve and the IS equation), so that there is no reason why these particular cases should specially occur.

\(^{48}\)These three constraints are predicted by most New Keynesian models - and in particular by model A even though the last constraint, namely the verticality of the Phillips curve, holds only approximatively as $\beta$ is closed but not equal to one.
\[ y_t = q_1 E_t \{ y_{t+1} \} + (1 - q_1) y_{t-1} + q_2 (r_t - E_t \{ \Delta p_{H,t+1} \}) + q_3 E_t \{ y^*_t \} + q_4 y^*_t - (q_3 + q_4) y^*_{t-1} + \epsilon^*_t, \]

\[ \Delta p_{H,t} = s_1 E_t \{ \Delta p_{H,t+1} \} + (1 - s_1) \Delta p_{H,t-1} + s_2 y_t + s_3 y^*_t + \epsilon^*_{pc}. \]

The estimation procedure is exactly the same as previously, except that we use \( r_{t-k}, y_{t-k}, r^*_{t-k} \) and \( y^*_{t-k} \) for \( k \in \{1, 2, 3, 4\} \) (instead of \( k \in \{1, 2, 3\} \)) as instruments, because there are more parameters to be estimated. For the same reason as previously, we carry out the estimation over two different periods, namely from 1963:1 to 1999:1 and from 1980:1 to 1999:1. The estimation results are displayed in table 2.6.

The first point to be noted is that whatever the estimation period and the equation estimated, the \( R^2 \), the Durbin-Watson statistic and the J-test significance level all take satisfactory values, respectively around 0.80 \( \sim 0.90, 2.5 \sim 3 \) and 0.80 \( \sim 0.90 \). For the period from 1963:1 to 1999:1, we note in particular that \( \hat{\rho}_d \) is now significantly lower than one, while \( \hat{\gamma}_t \) is rather low - and also significantly lower than one, which suggests that the constraint imposed by model A on the coefficient of \( E_t \{ y_{t+1} \} \) (equal to unity) was actually binding. Disappointingly however, only five of the nine estimates prove significant at the 10% level. Finally, the moduli of the four eigenvalues associated with this system are 2.61, 1.35, 1.35 and 0.87, which implies that \( m = 3 \). As a consequence, there are more equations than unknowns and our simulation method provides no solution for the UK business cycle under EMU-membership.

For the period from 1980:1 to 1999:1, the estimation results prove more satisfactory. Now indeed, only one (namely \( s_3 \)) out of nine first-order parameters estimates proves not significant at the 10% level. Again, \( \hat{\rho}_d \) takes a value not too close to one. Note besides that these IS equation and Phillips curve display a much higher degree of forward-lookingness than their counterpart estimated over the period from 1963:1 to 1999:1. Finally, the moduli of the four eigenvalues associated with this system are 1.62, 0.51, 0.29 and 0.15, which implies that \( m = 1 \). As a consequence, there are as many equations as unknowns and our simulation method provides one single solution for the UK business cycle under EMU-membership, if we assume (as is likely to be the case) that model A's predictions concerning the effect of irrevocably fixed exchange rates still hold for model B. We can then conduct our simulation in a similar way as previously. This simulation is now based on the following system:
\[ y_t = 0.82 \cdot E_t \{ y_{t+1} \} + 0.18 \cdot y_{t-1} - 0.65 \cdot (r_t - E_t \{ \Delta p_{H,t+1} \}) - 0.26 \cdot E_t \{ y^*_t \} + 0.33 \cdot y^*_t - 0.07 \cdot y^*_t - u_t + \varepsilon^*_t, \]

\[ \Delta p_{H,t} = 0.86 \cdot E_t \{ \Delta p_{H,t+1} \} + 0.14 \cdot \Delta p_{H,t-1} + 0.20 \cdot y_t - 0.02 \cdot y^*_t + \varepsilon^*_{pc}, \]

where \( \varepsilon^*_{pc} = 0.90 \cdot \varepsilon^*_{t-1} + \eta^*_{pc} \) and \( \varepsilon^*_{t} = 0.84 \cdot \varepsilon^*_{t-1} + \eta^*_{t} \), with a variance-covariance matrix of \([\eta^*_{t} \ \eta^*_{pc} \ \nu^*_1 \ \nu^*_2 \ \nu^*_3]' \) estimated by

\[
\hat{\Omega} = \begin{bmatrix}
0.09 & 0 & -0.08 & -0.06 & -0.02 \\
0 & 0.07 & -0.01 & -0.21 & -0.00 \\
-0.08 & -0.01 & 0.11 & 0.00 & 0.03 \\
-0.06 & -0.21 & 0.00 & 2.23 & 0.02 \\
-0.02 & -0.00 & 0.03 & 0.02 & 0.03
\end{bmatrix}.
\]

The matrix \( \hat{\Omega} \) is obtained in a similar way as previously, the foreign fluctuations being again modeled by a VAR of order two. In particular, we choose the lower estimates of \( \hat{\Omega} (1,1) \) and \( \hat{\Omega} (2,2) \) for the same reason as previously. This choice happens this time to be compatible with a positive semi-definite matrix \( \hat{\Omega} \).

Figures 2.3 and 2.4 are the counterparts of figures 2.1 and 2.2. The simulation graphs of figure 2.3 indicate that the immediate effects of shocks \( \eta^*_{pc} \) and \( \eta^*_{t} \) on the variables \( \Delta p_H, p_H \) and \( y \) are in accordance with conventional wisdom. The main difference between figures 2.1 and 2.3 lies in the absence of any oscillatory pattern in the latter figure, as all eigenvalues are real numbers here. Because the moduli of these eigenvalues are lower for model B than for model A, the speed of convergence towards PPP is also higher here than there. Figure 2.4 shows like figure 2.2 the rather high degree of international transmission of fluctuations. We do not further interpret this figure however, as the foreign innovations are no structural shocks and consequently have no clear economic meaning. Finally, the standard errors of \( y \) and \( \Delta p_H \) are found to be multiplied respectively by 1.01 and 1.81 under EMU-membership. As shown by table 2.7, this result proves more in accordance with those of other studies than the results obtained in subsection 2.3.5 with model A.

### 2.4 Conclusion

This chapter, which carries out a simulation of the UK business cycle under EMU-membership, represents an original contribution to the literature. To our
knowledge indeed, the closest studies are Westaway (2003), Galf and Monacelli (2002), Driver and Wren-Lewis (1999) as well as Driver (2000), all based on a New Keynesian model, which all things considered prove quite different from the present chapter, as shown by table 2.1. In particular, they calibrate their models and have consequently to specify a more or less ad hoc variance-covariance matrix (if any) for the exogenous domestic and foreign perturbations in order to simulate the effect of irrevocably fixed exchange rates. By contrast, we estimate our model so as to identify these perturbations and to get their variance-covariance matrix in a model-consistent way. In this respect, our simulation should therefore lead to a more relevant assessment of the quantitative impact of the regime change on the UK business cycle\textsuperscript{49}.

But our main contribution to the literature in general and to these three studies in particular may actually be that we unveil the possibility for EMU-membership to be inherently a source of macroeconomic instability, as it may indeed prove compatible with multiple equilibria. In other words, we adapt Clarida, Galf and Gertler's (2000) point to what might be called a very specific monetary policy, namely the adoption of an irrevocably exchange rate regime. This chapter thus brings to the fore a possible major drawback of the irrevocably fixed exchange rate regime for a small open economy. It is certainly well-known that the fixity of the nominal exchange rate may raise macroeconomic volatility, as there is no national central bank any longer to react to the fundamental shocks. But we argue that the \textit{ex ante} fixity of the nominal exchange rate (as opposed to its \textit{ex post} fixity, paired with an adequate monetary policy rule) may well raise macroeconomic instability too, since the national central bank can no longer play its "parapet" role. This result opposes the conventional wisdom, which advocates the choice of a "corner solution" for the exchange rate regime.

Macroeconomic instability (defined as the existence of multiple equilibria) is not the only danger to threaten our small open economy under the monetary union regime. Indeed, the case of the absence of any (convergent) equilibrium may arise in our simulation. In this case, the value of the structural parameters, or even the model itself, will probably have to change. In the case of the UK, this would correspond to HM Treasury's second test, which is about whether there is sufficient flexibility to cope with economic change. Up to now, the literature has passed in silence over the fact that the simulation of the effect of irrevocably fixed exchange rates in a rational expectations model did not

\textsuperscript{49}Qualitatively speaking however, the simulations carried out by these two studies are not subject to this criticism. Therefore, the fact that our simulation graphs (based on model A) closely resemble Westaway's (2003) qualitatively speaking lends additional support to our results.
necessarily point to a unique solution, i.e. could well lead to either several solutions or no solution. In Galf and Monacelli’s (2002) canonical model, the monetary union equilibrium happens to exist and to be unique, but in a more general framework accounting for natural extensions to this canonical model, the monetary union equilibrium might well either not exist or not be unique.

We find however that the UK would escape both these Scylla and Charybdis, as our simulation points to a unique solution for the UK business cycle under EMU-membership. But macroeconomic volatility might still prove high for the euroized UK, as our simulation results suggest that the pressure exerted by the sole remaining force to react to idiosyncratic fluctuations, namely long-run PPP, could be weak. Indeed, these simulation results indicate that the standard error of the UK inflation rate and detrended output level should either remain the same, or increase twofold under EMU-membership. If the desirability of EMU for the UK is to be judged on the variances of the inflation rate and the detrended output level, then the UK seems therefore better off outside EMU.

This possibly dramatic result should however be tempered down for two reasons. First, it rests on our assumption that the foreign exchange market is no additional source of exogenous perturbations. Indeed, should an exogenous risk-premium shock actually exist when the exchange rate is not irrevocably fixed, it would be a component of what we would then wrongly identify as a pure IS shock, so that we would overestimate the variance of the true IS shock and hence most probably the UK macroeconomic volatility under EMU-membership as well.

Second, this result rests on the predictions of two models which are not fully satisfying. As a structural model, model A is less subject to the Lucas critique than non-structural models, but fits the data not so well. Model B fits the data better, actually even better than model A in reduced-form terms, but as a non-structural model is more subject to the Lucas critique than model A. As shown in Table 2.7, model B’s simulation results (which indicate that the standard error of the UK detrended output level should roughly be unaffected by EMU-membership, while the standard error of the UK inflation rate should increase nearly twofold) prove more in accordance with those of other studies than model A’s. One natural extension to our work would be therefore to ground model B on as sound as possible microeconomic foundations. We choose to leave this extension for future research.

50Only when model B is estimated over the period from 1963:1 to 1999:1 do we reach a different conclusion. In that case indeed, we find that no monetary union equilibrium exists. This result can be legitimately questioned however, as it rests on the predictions of a non-structural model which fits the data not so well.
Our work could be extended in other directions as well. First, sticking to our current theoretical framework, we could use monthly instead of quarterly, pan-European instead of German data. We could also apply our simulation method to Denmark, Ireland and Sweden. Second, our simple model could naturally be extended (so as to fit the data better) along the lines of the studies inventoried in tables 2.8 and 2.9 - and we did say how highly the habit persistence extension in particular should be praised in this respect. What is essential here, if we are to follow the steps of these studies and adapt our theoretical framework accordingly, is to base these extensions on microeconomic foundations, mainly in order to circumvent the Lucas critique, but also in order to be able to address the issue of structural policies: what would be the effect of a change in the value of this or that structural parameter? In particular, how would the UK business cycle under EMU-membership depend on such structural parameters as the degree of price-setting rigidity?

Applied to already eurized countries, our contribution would make a point in today's lively debate about whether the ECB one-size-fits-all monetary policy, acting as a centrifugal force, tends to exacerbate the heterogeneity or even the divergence of the national business cycles within the Eurozone, for instance currently pushing Germany into deflation while fuelling a housing bubble in Spain. Indeed, our framework would then enable us to determine a lower threshold for the ECB inflation target such that deflation would be ruled out with a reasonable probability for all member countries. No doubt that the answer would depend on the estimated strength of the centripetal force, i.e. on the estimated speed of adjustment towards PPP.

---

51 We could also adapt the model so as to account for (endogenous) investment dynamics, as does Woodford (2003) within the closed economy framework. In Rotemberg and Woodford's (1996, p. 41) own terms, "in this paper we have worked with a minimal model, both to show how this method can be applied and to show that even very simple optimizing models can fit the data rather well. Even so, it would be desirable to have a model that deals explicitly with investment and the resulting capital accumulation as well as with labor market variables."

52 The issue of deflation has been recently addressed within the framework of the canonical New Keynesian model by Eggertsson and Woodford (2003), though in the context of a closed economy.
Table 2.1: comparison with the closest studies.

<table>
<thead>
<tr>
<th>Study</th>
<th>Model</th>
<th>Structural or reduced form</th>
<th>Dynamics</th>
<th>Foreign shocks</th>
<th>Calibration or estimation</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driver and Wren-Lewis (1999)</td>
<td>New Keynesian model</td>
<td>reduced form</td>
<td>+</td>
<td>not correlated with domestic shocks</td>
<td>calibration</td>
<td>generic country</td>
</tr>
<tr>
<td>Gali and Monacelli (2002)</td>
<td>canonical New Keynesian model</td>
<td>structural form</td>
<td>++</td>
<td>absent</td>
<td>calibration</td>
<td>generic country</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>New Keynesian model</td>
<td>structural form (model A)</td>
<td>+++</td>
<td>correlated with domestic shocks</td>
<td>estimation</td>
<td>UK</td>
</tr>
<tr>
<td></td>
<td></td>
<td>reduced form (model B)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.2: comparison with the VAR literature.

<table>
<thead>
<tr>
<th>Study</th>
<th>Countries</th>
<th>Number of VARs</th>
<th>Dimension of the VARs</th>
<th>Variables</th>
<th>Structural shocks</th>
<th>Constraint(s) imposed by the monetary union</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mélitz and Weber (1996)</td>
<td>FRA, GER</td>
<td>1 by country</td>
<td>6</td>
<td>$\Delta y, r, \Delta p, \Delta p, \Delta (r-r^<em>)$, $\Delta \chi, \Delta (e+P^</em>-P)$</td>
<td>$e^<em>, e^d, e^s, e^m, e^{m</em>}, e^r$</td>
<td>$e^m = e^{m*}$</td>
</tr>
<tr>
<td>Nikolakaki (1997)</td>
<td>AUS, FRA, GER, ITA, NET, SPA, UK</td>
<td>1 by country</td>
<td>3</td>
<td>$\Delta y, \Delta p, \Delta (e+P^*-P)$</td>
<td>$e^*, e^s, e^m$</td>
<td>$e^m = e^{m*}$</td>
</tr>
<tr>
<td>Dornbusch, Favero and Giavazzi (1998)</td>
<td>FRA, GER, ITA, SPA, SWE, UK</td>
<td>1 by country</td>
<td>1</td>
<td>$\Delta y$ (regressed variable), $\Delta y^*, r, e$</td>
<td>none</td>
<td>$e = 0$</td>
</tr>
<tr>
<td>Hénin and Nicoulaud (1999)</td>
<td>FRA, UK</td>
<td>1 for all countries</td>
<td>5</td>
<td>$y, y^<em>, p, p^</em>, e$</td>
<td>$e^*, e^d, e^s, e^r, e^d$</td>
<td>$e^d = 0$ et $e = 0$</td>
</tr>
<tr>
<td>Kontolemis and Samiei (2000)</td>
<td>UK</td>
<td>1 by country</td>
<td>6</td>
<td>$(m-p), y, r, r^*, \Delta p$</td>
<td>none</td>
<td>$g \rightarrow 1$ in equation $\Delta r_t = -g (r-r^*)k + ...$</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>UK and possibly ARG, DEN, IRL, SWE</td>
<td>1 by country</td>
<td>3</td>
<td>$y, \Delta P_H, r, y^<em>, \Delta p^</em>, r^*$</td>
<td>$e^{\hat{r}}, e^d, e^s, e^r, e^{r*}, e^s$</td>
<td>$e^{\hat{r}} = 0, r = r^<em>, P_H = p^</em> \alpha$</td>
</tr>
</tbody>
</table>

53 ARG: Argentina; AUS: Austria; DEN: Denmark; FRA: France; GER: Germany; IRL: Ireland; ITA: Italy; NET: Netherlands; SPA: Spain; SWE: Sweden; UK: United Kingdom.

54 $\Delta$: first difference operator; $y$: domestic real production or output gap; $y^*$: foreign real production or output gap; $r$: domestic nominal interest rate; $r^*$: foreign nominal interest rate; $p$: domestic CPI; $p^*$: foreign CPI; $p$*: domestic PPI; $e$: nominal exchange rate; $m$: domestic money stock; $m^*$: foreign money stock; $\chi$: ratio between the current account and the domestic production level. All variables, except nominal interest rates, are expressed in logarithm.

Table 2.3: data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Country</th>
<th>Label</th>
<th>Description</th>
<th>Source</th>
<th>Starting date (month or quarter)</th>
<th>Unit</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>FRA</td>
<td>FRA.PPIAMP01.IXOB</td>
<td>PPI intermediate goods - proxy</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GER</td>
<td>DEU.PPIAMP01.IXOB*</td>
<td>PPI all items - proxy</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>ITA</td>
<td>ITA.PPIAMP01.IXOB</td>
<td>PPI total – proxy</td>
<td>MEI</td>
<td>1981:1</td>
<td>1990=100</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SPA</td>
<td>ESP.PPIAMP01.IXOB</td>
<td>PPI all items - proxy</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>UK</td>
<td>GBR.PPIAMP01.IXOB*</td>
<td>PPI MFG output all products</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>-</td>
</tr>
<tr>
<td>y</td>
<td>FRA</td>
<td>FRA.PRPEIN01.IXOSA</td>
<td>IIP total</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>GER</td>
<td>DEU.PRPEIN01.IXOSA*</td>
<td>IIP total</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>ITA</td>
<td>ITA.PRPEIN01.IXOSA</td>
<td>IIP total</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>SPA</td>
<td>ESP.PRPEIN01.IXOSA</td>
<td>IIP total</td>
<td>MEI</td>
<td>1965:1</td>
<td>1990=100</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>UK</td>
<td>GBR.PRPEIN01.IXOSA*</td>
<td>IIP total</td>
<td>MEI</td>
<td>1960:1</td>
<td>1990=100</td>
<td>✓</td>
</tr>
<tr>
<td>r</td>
<td>FRA</td>
<td>FRA.IRT31B01.ST</td>
<td>3-month PIBOR</td>
<td>MEI</td>
<td>1970:1</td>
<td>% per annum</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GER</td>
<td>DEU.IRT31B01.ST*</td>
<td>3-month FIBOR</td>
<td>MEI</td>
<td>1960:1</td>
<td>% per annum</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>ITA</td>
<td>ITA.IRT31B01.ST</td>
<td>3-month interbank deposits</td>
<td>MEI</td>
<td>1978:4</td>
<td>% per annum</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>SPA</td>
<td>ESP.IRT31B01.ST</td>
<td>3-month interbank loans</td>
<td>MEI</td>
<td>1977:1</td>
<td>% per annum</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>UK</td>
<td>GBR.IRT31B01.ST</td>
<td>3-month interbank loans</td>
<td>MEI</td>
<td>1978:1</td>
<td>% per annum</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AJNB*</td>
<td>3-month Treasury bills</td>
<td>NS</td>
<td>1963:1</td>
<td>% per annum</td>
<td>-</td>
</tr>
</tbody>
</table>

56 FRA: France; GER: Germany; ITA: Italy; SPA: Spain; UK: United Kingdom. German data from 1991 refer to unified Germany for variables p and y.
57 The chosen series are signalled by a star. The ending date for these series (and hence for our estimation period) is 1999:1.
58 MEI: OECD Main Economic Indicators; NS: National Statistics (http://www.statistics.gov.uk/).
59 SA: Seasonally Adjusted.
Table 2.4: estimation results for model A (structural parameters)\textsuperscript{60}.

<table>
<thead>
<tr>
<th>Estimation\textsuperscript{61}</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\eta$</th>
<th>$\theta$</th>
<th>$\rho_{IS}$</th>
<th>$\rho_{PC}$</th>
<th>$\sigma$</th>
<th>$\phi$</th>
<th>$\omega$</th>
<th>Hansen’s J-test significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>S estimation of IS</td>
<td>0.50**</td>
<td>-</td>
<td>0.55</td>
<td>-</td>
<td>1.04***</td>
<td>-</td>
<td>0.15</td>
<td>-</td>
<td>-</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td></td>
<td>(0.58)</td>
<td></td>
<td>(0.04)</td>
<td></td>
<td>(0.11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S estimation of PC</td>
<td>0.50</td>
<td>0.99</td>
<td>0.55</td>
<td>0.22**</td>
<td>0.78*</td>
<td>0.15</td>
<td>0.27</td>
<td>0.95***</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.40)</td>
<td>(1.64)</td>
<td>(0.17)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>joint S estimation of IS and PC</td>
<td>0.50</td>
<td>0.99</td>
<td>0.55</td>
<td>0.07</td>
<td>1.01***</td>
<td>0.15</td>
<td>0.40</td>
<td>0.92***</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.38)</td>
<td>(0.09)</td>
<td>(0.87)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.16)</td>
<td></td>
<td>(0.13)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{60} When the structural parameter considered is estimated rather than calibrated, the corresponding standard errors (based on asymptotic theory) are then displayed in parentheses, while *, ** and *** denote significance at the 10%, 5% and 1% levels respectively.

\textsuperscript{61} IS: IS equation; PC: Phillips curve; S: structural.
Table 2.5: estimation results for model A (reduced-form parameters)\(^{62}\).

<table>
<thead>
<tr>
<th>Estimation(^{63})</th>
<th>IS equation</th>
<th>Phillips curve</th>
<th>Hansen's J-test significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>g₁</td>
<td>g₂</td>
<td>ρₚₜ</td>
</tr>
<tr>
<td>RF estimation of IS</td>
<td>-2,09** (0,94)</td>
<td>-0,69*** (0,24)</td>
<td>1,04*** (0,04)</td>
</tr>
<tr>
<td>S estimation of IS</td>
<td>-2,08</td>
<td>-0,69</td>
<td>1,04*** (0,04)</td>
</tr>
<tr>
<td>RF estimation of PC</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S estimation of PC</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>joint RF estimation of IS and PC</td>
<td>-2,58 (2,36)</td>
<td>-1,16* (0,60)</td>
<td>1,01*** (0,09)</td>
</tr>
<tr>
<td>joint S estimation of IS and PC</td>
<td>-2,08</td>
<td>-0,69</td>
<td>1,01*** (0,09)</td>
</tr>
</tbody>
</table>

\(^{62}\) When the reduced-form parameter considered is directly estimated, the corresponding standard errors (based on asymptotic theory) are then displayed in parentheses, while *, ** and *** denote significance at the 10%, 5% and 1% levels respectively.

\(^{63}\) IS: IS equation; PC: Phillips curve; RF: reduced form; S: structural.
Table 2.6: estimation results for model B.

<table>
<thead>
<tr>
<th>Estimation period</th>
<th>IS equation</th>
<th>R²</th>
<th>Durbin-Watson statistic</th>
<th>Hansen’s J-test significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q₁</td>
<td>q₂</td>
<td>q₃</td>
<td>q₄</td>
</tr>
<tr>
<td>1963:1 to 1999:1</td>
<td>0.35***</td>
<td>-0.59 (0.47)</td>
<td>0.27 (0.25)</td>
<td>0.65**</td>
</tr>
<tr>
<td></td>
<td>0.82***</td>
<td>-0.65* (0.38)</td>
<td>-0.26**</td>
<td>0.33*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phillips curve</th>
<th>R²</th>
<th>Durbin-Watson statistic</th>
<th>Hansen’s J-test significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s₁</td>
<td>0.30* (0.17)</td>
<td>0.006 (0.009)</td>
<td>-0.001 (0.003)</td>
</tr>
<tr>
<td>s₂</td>
<td>0.06*** (0.06)</td>
<td>-0.02 (0.04)</td>
<td>0.90*** (0.08)</td>
</tr>
</tbody>
</table>

The standard errors (based on asymptotic theory) are displayed in parentheses, while *, ** and *** denote significance at the 10%, 5% and 1% levels respectively.
Table 2.7: UK macroeconomic volatility ratios\textsuperscript{65}.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NiGEM</td>
<td>NiGEM</td>
<td>Liverpool model</td>
<td>Three Bears (New Keynesian) model</td>
<td>New Keynesian model</td>
<td></td>
</tr>
<tr>
<td>Ratio for $\Delta p_m$</td>
<td>2</td>
<td>from 0.75 to 0.78</td>
<td>from 2.92 to 3.12</td>
<td>from 0.84 to 1.59</td>
<td>1.06 (model A)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.81 (model B)</td>
</tr>
<tr>
<td>Ratio for $y$</td>
<td>1.10</td>
<td>from 1.23 to 1.73</td>
<td>from 1.09 to 1.12</td>
<td>from 0.92 to 1.27</td>
<td>2.06 (model A)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.01 (model B)</td>
</tr>
</tbody>
</table>

\textsuperscript{65} The UK macroeconomic volatility ratio is defined for a given UK variable as the standard deviation of this variable for the UK inside EMU over the standard deviation of this variable for the UK outside EMU.
Figure 2.1: UK-in-EMU impulse-response functions (model A)
To UK shocks in the absence of EMU innovations
Figure 2.2: UK-in-EMU and EMU impulse-response functions (model A)

To EMU innovations in the absence of UK shocks
Figure 2.2 (continued): UK-in-EMU and EMU impulse-response functions (model A)
To EMU innovations in the absence of UK shocks
Figure 2.3: UK-IN-EMU impulse-response functions (model B)
To UK shocks in the absence of EMU innovations
Figure 2.4: UK-in-EMU and EMU impulse-response functions (model B)
To EMU innovations in the absence of UK shocks
Figure 2.4 (continued): UK-in-EMU and EMU impulse-response functions (model B)
To EMU innovations in the absence of UK shocks
<table>
<thead>
<tr>
<th>Study</th>
<th>Phillips curve</th>
<th>IS equation</th>
<th>(Nominal interest rate) monetary policy rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta p_i$ as a linear function of...</td>
<td>I or $J^{66}$ shock</td>
<td>I or $J^{67}$ $y_i$ as a linear function of...</td>
</tr>
<tr>
<td>Adam and Padula (2003)</td>
<td>$E_{t-2} { \Delta p_{t+1} }, y_i$ and possibly $\Delta p_{t-1}$</td>
<td>DPUS</td>
<td>i.i.d.</td>
</tr>
<tr>
<td>Amato and Laubach (1999)</td>
<td>$E_{t-4} { \Delta p_{t+1} }, E_{t-2} { \Delta p_t }, y_i$ and other variables</td>
<td>DPS</td>
<td>not specified</td>
</tr>
</tbody>
</table>

---

$^{66}$ Interpretation or Justification of the (possible) extension(s) to the canonical New Keynesian Phillips curve, expressing $\Delta p_i$ as a linear function of $E_{t-1} \{ \Delta p_{t+1} \}$ and $y_i$. "Interpretation" means that the extension is introduced in an ad hoc manner and interpreted afterwards, while "Justification" means that the extension is explicitly modeled. AESP: Adaptive Expectations for a Subset of Price-Setters; DPS: Delayed Price-Setting (when the firm can reoptimize its price); DPUS: Dynamic Price-Updating Scheme (when the firm cannot reoptimize its price); EQAC: Extended Quadratic Adjustment Cost; IIG: Imported Intermediate Goods; IRPAL: Real-world Information, Recognition, Processing and Adjustment Lags; PBM: P-Bar Model; RWC: Real Wage Contracting à la Fuhrer and Moore (1995); SBLPS: Subset of Backward-Looking Price-Setters; SPUS: Static Price-Updating Scheme (when the firm cannot reoptimize its price); TSCM: Taylor staggered contracting model; TVMU: Time-Varying Mark-Ups.

$^{67}$ Interpretation or Justification of the (possible) shock in the Phillips curve. "Interpretation" means that the shock is introduced in an ad hoc manner and interpreted afterwards, while "Justification" means that the shock is explicitly modeled. CPS: Cost-Push Shock (i.e. price-level shock); IDP: Inadequate Detrending Procedures; ME: Measurement Errors; PE: Pricing Error; PS: Productivity Shock; RWC: Real Wage Contracting à la Fuhrer and Moore (1995); SDPO: Shock on the Disutility ofProducing Output; SPO: Shock on Potential Output; TSCM: Taylor staggered contracting model; UDW: Unobserved Determinants of Wages.

$^{68}$ Interpretation or Justification of the (possible) extension(s) to the canonical IS equation, expressing $y_i$ as a linear function of $E_{t} \{ y_{t+1} \}$ and $r_{t} - E_{t} \{ \Delta p_{t+1} \}$. DLC: Decision Lag for Consumption; EHP: External Habit Persistence à la Fuhrer (2000).

$^{69}$ Interpretation or Justification of the (possible) shock in the IS equation. ADS: Aggregate Demand Shock; CTS: Consumption Taste Shock (affecting the utility of consumption); GSP: General Shock to Preferences; GSS: Government Spending Shock; IS: Investment Shock; NES: Net Exports Shock; SMUC: Shock on the Disutility of Producing Output.

$^{70}$ Interpretation of the form taken by the monetary policy rule. ORQLF: Optimal Rule for a Quadratic Loss Function; OTR: Operational Taylor Rule; TESIRC: Traditional Explanations for Smoothing Interest Rate Changes.

$^{71}$ Interpretation of the (possible) shock in the monetary policy rule. IAMSDM: Imperfect Adjustment of Money Supply to Money Demand; MPIE: Monetary Policy Implementation Error; PSIO: Persistent Shock on the Inflation Objective; TIRS: Temporary Shock on the Interest Rate; UCMP: Unsystematic Component of Monetary Policy.
<table>
<thead>
<tr>
<th>Authors</th>
<th>Specification</th>
<th>Methodology</th>
<th>Estimation</th>
<th>Identification</th>
<th>ARMA</th>
<th>Publications</th>
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<tbody>
<tr>
<td>Børdsen, Jansen and Nymoen (2002)</td>
<td>$E_t(\Delta p_{t-1})$, $y_t$ and possibly $\Delta p_{t-1}$, $\Delta p_{t-4}$, $y_{t-1}$, open-economy variables</td>
<td>SBLPS, IIG, TVMU</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Balakrishnan and López-Salido (2002)</td>
<td>$E_t(\Delta p_{t-1})$, $y_t$ and possibly $\Delta p_{t-1}$, open-economy variables</td>
<td>SBLPS, IIG, TVMU</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Boivin and Giannoni (2002)</td>
<td>$E_t(\Delta p_{t-1})$, $E_{t-1}(\Delta p_{t-1})$, $\Delta p_{t-1}$, $E_{t-1}(\Delta p_{t-1})$, $E_{t-1}(\Delta p_{t-1})$, $E_{t-1}(\Delta p_{t-1})$</td>
<td>SBLPS, AR(1), SDPO</td>
<td>E_{t-2}(y_{t-2}), E_{t-2}(\Delta p_{t-2}), E_{t-2}(\Delta p_{t-2})</td>
<td>DLC and EHP</td>
<td>AR(2)</td>
<td>SMUC, TESIRC, i.i.d.</td>
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<tr>
<td>Cho and Moreno (2002)</td>
<td>$E_t(\Delta p_{t-1})$, $\Delta p_{t-1}$, $y_{t-1}$</td>
<td>RWC, AR(1), SDPO</td>
<td>E_{t-2}(y_{t-2}), E_{t-2}(\Delta p_{t-2}), E_{t-2}(\Delta p_{t-2})</td>
<td>DLC and EHP</td>
<td>AR(2)</td>
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<td>Christiano, Eichenbaum and Evans (2001)</td>
<td>$E_t(\Delta p_{t-1})$, $E_{t-1}(\Delta p_{t-1})$, $E_{t-1}(\Delta p_{t-1})$, $E_{t-1}(\Delta p_{t-1})$</td>
<td>DPUS, SPUS</td>
<td>E_{t-2}(y_{t-2}), E_{t-2}(\Delta p_{t-2}), E_{t-2}(\Delta p_{t-2})</td>
<td>DLC and EHP</td>
<td>AR(2)</td>
<td>SMUC, TESIRC, i.i.d.</td>
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<tr>
<td>Clarida, Galí and Gertler (1998)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>TESIRC, ORQLF, i.i.d.</td>
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<td>DLC and EHP</td>
<td>AR(2)</td>
<td>SMUC, TESIRC, i.i.d.</td>
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<tr>
<td>Demery and Duck (2002)</td>
<td>$E_t(\Delta p_{t-1})$, $y_t$</td>
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<td>Doménech, Ledo and Taguas (2001)</td>
<td>$E_t{\Delta p_{t+1}}, \Delta p_{t-1}, y_{t-1}$</td>
<td>SBLPS</td>
<td>i.i.d.</td>
<td>-</td>
<td>$E_t{y_{t+1}}, y_{t+1}, y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}, r_{t-2} - E_{t-2}{\Delta p_{t-1}}$</td>
<td>EHP</td>
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<tr>
<td>Estella and Fuhrer (1999)</td>
<td>$E_t{\Delta p_{t+1}}, y_t$</td>
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<td>i.i.d. or AR(2)</td>
<td>-</td>
<td>$E_t{y_{t+1}}, r_t - E_t{\Delta p_{t+1}}$</td>
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<td>Fuhrer (1997)</td>
<td>$E_t{\Delta p_{t+1}}, E_t{\Delta p_{t+2}}, E_t{\Delta p_{t+1}}, \Delta p_{t-2}, y_t$, and possibly $\Delta p_{t-3}$</td>
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<td>i.i.d.</td>
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<tr>
<td>Gali and Gertler (1999)</td>
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<td>Gali, Gertler and López-Salido (2001, 2002)</td>
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<td>SBLPS</td>
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<td>-</td>
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<tr>
<td>Kara and Nelson (2002)</td>
<td>$E_t{\Delta p_{t+1}}, y_t$ and open-economy variables</td>
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<td>Leith and Malley (2001)</td>
<td>$E_t{\Delta p_{t+1}}, \Delta p_{t-1}, y_t$</td>
<td>SBLPS</td>
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<td>-</td>
<td>$E_t{y_{t+2}}, E_t{y_{t+1}}, y_{t+1}, r_t - E_t{\Delta p_{t+1}}$</td>
<td>EHP</td>
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<td>Leith and Malley (2002)</td>
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<td>Specifications</td>
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<td>AR(1)</td>
<td>GSS</td>
<td>i.i.d. or AR(1)</td>
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<td>Lindé (2002)</td>
<td>(E_t(\Delta p_{t+1}), \Delta p_{t-1}, y_t)</td>
<td>AESPS, DPUS, RWC, SBLPS, SPUS</td>
<td>i.i.d. or AR(1)</td>
<td>-</td>
<td>E_t(y_{t+1}), y_{t-1}, (r_t - E_t(\Delta p_{t+1})) and possibly (y_{t-2}, y_{t-3}, y_{t-4})</td>
<td>EHP</td>
</tr>
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<td>Lubik and Schorfheide (2002)</td>
<td>(E_t(\Delta p_{t+1}), y_t)</td>
<td>-</td>
<td>AR(1)</td>
<td>SPO</td>
<td>E_t(y_{t+1}), (r_t - E_t(\Delta p_{t+1}))</td>
<td>-</td>
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<td>McCallum and Nelson (1999b)</td>
<td>(E_t(\Delta p_{t+1}), y_t \ast p_{t-1}, E_{t-1}(p_t))</td>
<td>PBM</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>E_t(y_{t+1}), (r_t - E_t(\Delta p_{t+1}))</td>
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<td>Mehra (1999)</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Neiss and Nelson (2002)</td>
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<td>-</td>
<td>i.i.d.</td>
<td>CPS</td>
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<td>Roberts (1995)</td>
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<td>i.i.d. or AR(1)</td>
<td>PE, UDW</td>
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<td>-</td>
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<tr>
<td>Roberts (1997)</td>
<td>(E_t(\Delta p_{t+1}), y_t, y_{t-1} \text{ and possibly } \Delta p_{t-1})</td>
<td>RWC, TSCM</td>
<td>MA(1)</td>
<td>RWC, TSCM</td>
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<tr>
<td>Roberts (2001)</td>
<td>(E_t(\Delta p_{t+1}), y_t, \text{and possibly } \Delta p_{t-1})</td>
<td>EQAC, RWC, AESPS</td>
<td>i.i.d. or AR(1)</td>
<td>IDP</td>
<td>-</td>
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<tr>
<td>Source/Year</td>
<td>Equation Structure</td>
<td>Methodology 1</td>
<td>Methodology 2</td>
<td>Methodology 3</td>
<td>Methodology 4</td>
<td>Methodology 5</td>
</tr>
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<td>---------------</td>
<td>---------------</td>
<td>---------------</td>
<td>---------------</td>
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<td>Rotemberg and Woodford (1997)</td>
<td>$E_{t+1}{\Delta p_{t+1}}, E_{t+2}{\Delta p_{t+2}}, E_{t+1}{y_{t+1}}, E_{t+1}{\Delta y_{t+1}}, E_{t}{\Delta y_{t}}, E_{t-1}{\Delta y_{t-1}}$</td>
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<td>SPO</td>
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<td>DLC</td>
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<td>Rudebusch (2002)</td>
<td>$E_{t}{\Delta^{4}p_{t+4}}, \Delta p_{t+1}, \Delta p_{t+2}, \Delta p_{t+3}, \Delta p_{t+4}, y_{t+1}$</td>
<td>IRPAL</td>
<td>i.i.d.</td>
<td>-</td>
<td>$y_{t+1}, y_{t+2}, E_{t+1}{\Delta^{4}p_{t+4}}$</td>
<td>i.i.d.</td>
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<td>Sbordone (2002)</td>
<td>$E_{t}{\Delta p_{t+1}}, y_{t}$</td>
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<td>-</td>
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<tr>
<td>Smets and Wouters (2002)</td>
<td>$E_{t}{\Delta p_{t+1}}, \Delta p_{t+1}, y_{t}$</td>
<td>DPUS</td>
<td>AR(1)</td>
<td>CPS, PS</td>
<td>$E_{t}{y_{t+1}}, y_{t+1}, r_{t}-E_{t}{\Delta p_{t+1}}$</td>
<td>EHP</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>$E_{t}{\Delta p_{t+1}}, \Delta p_{t+1}, y_{t}$ and open-economy variables</td>
<td>SBLPS</td>
<td>AR(1)</td>
<td>PS</td>
<td>$E_{t}{y_{t+1}}, r_{t}-E_{t}{\Delta p_{t+1}}$, open-economy variables and possibly $y_{t+1}$</td>
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</tbody>
</table>
Table 2.9: data and estimations in the literature.

<table>
<thead>
<tr>
<th>Study</th>
<th>Countries²</th>
<th>Data frequency³</th>
<th>Dates</th>
<th>Measure(s) of P</th>
<th>Measure(s) of y</th>
<th>Measure(s) of r</th>
<th>RF and/or S⁴</th>
<th>M and/or U⁵</th>
<th>Estimation method⁶</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adam and Padula (2003)</td>
<td>US</td>
<td>Q</td>
<td>1968:4 to 2000:1</td>
<td>• GDP deflator</td>
<td>• HP detrended log of real GDP • log of the labour income share</td>
<td>-</td>
<td>RF and S</td>
<td>U</td>
<td>OLS</td>
</tr>
<tr>
<td>Balakrishnan and López-Salido (2002)</td>
<td>UK</td>
<td>Q</td>
<td>1970:1 to 1999:3 or 1975:1 to 1999:3</td>
<td>• GDP deflator</td>
<td>• log of labour income share • log of adjusted labour income share • quadratically detrended log of real GDP</td>
<td>-</td>
<td>RF</td>
<td>U</td>
<td>GMM</td>
</tr>
</tbody>
</table>

² AUS: Australia; EUR: Eurozone; FRA: France; GER: Germany; ITA: Italy; JAP: Japan; NOR: Norway; UK: United Kingdom; US: United states.
³ A: annual; S: semi-annual; M: monthly; Q: quarterly.
⁴ RF: Reduced-Form parameters estimation; S: Structural parameters estimation.
⁵ M: Multivariate estimation; U: Univariate estimation.
⁶ BEM: Bayesian Estimation Method; FIML: Full Information Maximum Likelihood; GLS: Generalized Least Squares; GMM: Generalized Method of Moments (usually amounting to two-stage least squares); MDE: Minimum Distance Estimation; OLS: Ordinary Least Squares.
<table>
<thead>
<tr>
<th>Study</th>
<th>Country</th>
<th>Frequency</th>
<th>Time Period</th>
<th>Variables</th>
<th>Sample</th>
<th>Method</th>
<th>Estimator</th>
</tr>
</thead>
</table>
• CPI  
• quadratically detrended log of real GDP  
• HP detrended log of real GDP  
• log of the labour income share | -      | RF       | U    | FIML, GMM |
• high-pass frequency filtered log of real GDP  
• linearly detrended log of real GDP  
• quadratically detrended log of real GDP | • federal funds rate | S       | M    | MDE       |
• CPI  
• Congressional Budget Office measure of detrended GDP  
• linearly detrended real GDP  
• quadratically detrended real GDP | • federal funds rate  
• 3 month Treasury bill rate | RF      | M    | FIML     |
• real GDP | • federal funds rate  
• 3 month Treasury bill rate | S       | M    | MDE       |
• quadratically detrended log of industrial production | • interbank lending rate | RF      | U    | GMM       |
<table>
<thead>
<tr>
<th>Year Period</th>
<th>Country</th>
<th>Deflator</th>
<th>Data Source and Methodology</th>
</tr>
</thead>
</table>

- **Deflated**: Log of detrended real GDP, detrended log of GDP, log of GDP.
- **Methodology**: By a spline method, HP filter, and mean.
- **Unit Labour Costs**: log of per capita GDP, detrended log of GDP, linearly detrended log of GDP.
- **3 month Treasury bill rate**: Federal funds rate.
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<td>Country</td>
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Part II

A New Open Economy Macroeconomics perspective on two open economies
Chapter 3

Coordination, cooperation, contagion and currency crises

Abstract

Chapter 3, entitled “Coordination, cooperation, contagion and currency crises”, presents a micro-founded model where governments have an incentive to devalue to increase the national market share in a monopolistically competitive sector. Currency crises generated by self-fulfilling expectations are possible because workers demand high wages when they expect a devaluation. This decreases the competitiveness and profits of national firms and induces the government to devalue. We show that the more important trade competition, the more likely self-fulfilling speculative crises are and the larger the set of multiple equilibria. Coordination decreases the possibility of simultaneous self-fulfilling speculative crises in the region and reduces the set of multiple equilibria. However, regional coordination, even though welfare improving, makes countries more dependent on other countries’ fundamentals so that it may induce more contagion.

Abstract in French

Le chapitre 3, intitulé “Coordination, coopération, contagion et crises de change”, présente un modèle micro-fondé dans lequel les gouvernements sont incités à dévaluer pour augmenter la part de marché de leur pays dans un secteur à concurrence monopolistique. Les crises de change dues à des anticipations auto-réalisatrices sont possibles du fait que les travailleurs exi-
3.1 Introduction

Recent currency crises, such as the 1992-1993 EMS crisis, the crash of the Mexican Peso in 1994 and its Tequila effect on other countries as well as the Asian crisis, have involved several countries in the same geographical region. Most recently, the Asian crisis broke out in Thailand in May 1997, but spread rapidly to Malaysia, the Philippines, Indonesia and South Korea. In the European case, contagion was also important as the crisis hit five countries (Finland, the UK, Italy, Sweden and Norway) in its first year. By 1993 all countries except Netherlands had to widen the band of fluctuation with the DM. The attack on the Peso was itself followed by attacks on several Latin American countries.

Despite the fact that currency crises typically involve several countries that fix their currencies either to the dollar or to the DM, existing models of currency crisis look at the problem in a two-country framework where the actions of the country that pegs its currency are key\(^1\). This is the case of models of the "first generation" type à la Krugman (1979), where the crisis comes with a run on the Central Bank’s reserves, because speculators understand that monetary authorities conduct a policy inconsistent with the fixed parity. This is also the case with "second generation" models à la Obstfeld (1997), which consider devaluation as an intentional decision of a government that weights advantages and disadvantages: the cost of opting out of the fixed exchange rate system is primarily considered as a political cost; as for the cost of staying in, it can be

modeled as high interest rates\textsuperscript{2} or as unemployment\textsuperscript{3}.

As argued by Glick and Rose (1998), "from the perspective of most speculative attack models, it is hard to understand why currency crises tend to be regional". They argue that trade linkages should be first among the suspects for explaining regional contagion of currency crises, and give strong empirical support to this channel using five different crises. Eichengreen, Rose and Wyplosz (1996), in an empirical study using thirty years of panel data from twenty industrialized countries, also conclude in favor of a stronger explanatory power of international trade linkages than of macroeconomic similarities. Even though not modeled explicitly by these authors, the role of trade linkages is that in the presence of price rigidities a devaluation brings a short term competitive advantage to the country that devalues and therefore increases the cost for trade partners not to devalue.

The existence of these spillovers raises the issue of international cooperation: if governments take into account the negative externalities of devaluation on other countries, it might be easier to stop the snowball, perhaps even before it starts. Actually, some steps had been taken before the outbreak of the Asian crisis towards increased monetary cooperation between East Asian countries\textsuperscript{4}. As prophesied by Bayoumi and Eichengreen (1996), these unassuming measures however proved "insufficient to repel an all-out attack on an Asian currency comparable to the Mexican or ERM crises".

In this chapter we analyze the role of trade linkages in currency crises. To do this, we present a three-country model that builds on three separate literatures: 1) the literature on international monetary cooperation, especially Canzoneri and Henderson's (1991) theoretical approach, 2) the literature on currency crises, or more precisely on the "escape clause" approach of fixed exchange rate systems, 3) the micro-founded New Open Economy Macroeconomics framework initiated by Obstfeld and Rogoff (1995) and reviewed by Lane (1998).

Our model corresponds to a three-player, sequential game. The players involved are private agents and the two governments that unilaterally peg their currency to a third one. The two governments have an incentive to devalue

\textsuperscript{2}See for example Obstfeld (1994) or Ozkan and Sutherland (1994).

\textsuperscript{3}See for example Bessler and Jeanne (1994) or Drazen and Mason (1994). For empirical evidence on the role of self-fulfilling speculation in currency crises, see Jeanne (1997).

\textsuperscript{4}We are referring to: i) November 1995 repurchase agreements between the Hong Kong Monetary Authority and the central banks of Australia, Malaysia, Indonesia and Thailand, which enabled these central banks to intervene more heavily at short notice by allowing them to borrow dollars from one another; ii) February 1996 support of the dollar/yen rate by Hong Kong and Singapore; iii) March 1996 decision of the Bank of Japan, after that of Singapore and the Philippines' central banks, to sign the repurchase agreements. Besides, Fraser (1995) may be read as a suggestion to go further, up to a common peg.
because countries compete on a monopolistically competitive good that they both export. Currency crises that are generated by self-fulfilling expectations are possible because private agents rationally demand high wages when they expect a devaluation. This decreases the competitiveness of national firms and induces the government to devalue. To our knowledge, this framework is the first attempt to introduce micro-foundations in models of devaluations with self-fulfilling expectations.

We show that even when in equilibrium devaluations do not give any short term competitive advantage, strong trade competition increases the likelihood of currency crises that are induced by self-fulfilling expectations and magnify regional instability by increasing the number of possible multiple equilibria. We also show that countries that export goods in monopolistic sectors are more prone to devaluation induced by self-fulfilling expectations than countries specialized in competitive sectors.

We analyze the role of international coordination and cooperation in this context. Coordination is defined as in Canzoneri and Henderson (1991): governments coordinate on the best Nash equilibrium so that it does not require any commitment technology. Policymakers do not give up sovereignty in this case and all that is required is that they meet and coordinate on a good non-cooperative solution, for example in a regional forum. Because multiple equilibria due to self-fulfilling expectations are a natural outcome in this type of setup, the question of the feasibility of coordination on a specific equilibrium is a natural and important one. Cooperation is more demanding as it implies that governments maximise a joint welfare function. It therefore requires a commitment technology in the form of a supra-national institution that enforces the agreement. Both coordination and cooperation decrease but do not eliminate the possibility of simultaneous self-fulfilling speculative crises and reduce instability by limiting the set of multiple equilibria. However, regional coordination, even though welfare improving, makes countries more dependent on other countries' fundamentals so that it may induce more contagion: if one country is more likely to devalue because of a worsening of its fundamentals, this increases the possibility of a currency crisis in both countries because it reduces the credibility of coordination between the two countries.

This chapter is related to Buitcr, Corsetti and Pesenti (1995, 1998) who analyze the beneficial role of cooperation in the context of exchange rate crises in Europe. It differs in several dimensions. First, our model is based on micro-foundations. Second, expectations of the private sector play no role in their analysis of international cooperation so that the currency crisis they obtain are
not due to self-fulfilling expectations and multiple equilibria do not arise. Finally, because they do not allow for the possibility of multiple equilibria induced by self-fulfilling expectations, they do not analyze coordination but only look at the more demanding form of cooperation where governments minimize a joint loss function.

Another related paper is Corsetti, Pesenti, Roubini and Tille (2000), who present a micro-based model of competitive devaluations. Our model is different in that it analyzes the strategic interactions that lead to possible currency crises, whereas their paper studies the different welfare consequences of a devaluation. In contrast to their model, self-fulfilling expectations of private agents play a crucial role in our analysis. Finally, because we assume the two countries only trade with the country to which they fix their exchange rate, we restrict ourselves to the case when the spillovers of a devaluation are negative. This is in contrast to the two papers cited above as well as to Obstfeld and Rogoff (1995) who show that in presence of trade, the spillovers from a devaluation can be positive via the terms of trade effects.

We present a simple micro-founded model of competitive devaluations in section 3.2. We then solve this model and analyze the different possible equilibria in section 3.3.

3.2 The basic framework

The model corresponds to a three-country world: country A, country B and country Z. Country Z represents a large country to which countries A and B have pegged their currencies. To fix ideas, we will call the currency of country Z the dollar. We will assume that country Z is large compared to A and B, is not affected by their policy and therefore does not act strategically. Country Z plays two roles in our model: 1) it issues the numeraire currency, 2) it enables us to differentiate competition between countries that fix their exchange rate to a third one (A and B) and competition between a fixing country (A or B) and the country to which it pegs its currency (Z).

As in Obstfeld and Rogoff (1995, 1996), we introduce a monopolistically competitive sector. Countries A and B are both fully specialized in different varieties of this sector. In contrast to Obstfeld and Rogoff, these varieties are only exported to country Z. There are $n_A$, $n_B$, and $n_Z$ firms (respectively in countries A, B and Z) which each produce a different variety. Because of the small country assumption for A and B, the world aggregate demand for a composite intermediate good made of the different varieties is assumed to be
exogenous and given by

$$Y = \left[ \sum_{j=1}^{J_S} y_{A_j}^{1-\frac{1}{\delta}} + \sum_{j=1}^{J_B} y_{B_j}^{1-\frac{1}{\delta}} + \sum_{j=1}^{J_Z} y_{Z_j}^{1-\frac{1}{\delta}} \right]^{\frac{1}{1-\frac{1}{\delta}}} , \quad (3.2.1)$$

where $y_{A_j}$ represents the production of firm $j$ in country $A$ and $\sigma > 1$ is the elasticity of substitution between the different varieties. We note $N = n_A + n_B + n_Z$. As the world demand for the composite good increases, world trade increases for a given number of firms in the world. Hence, we focus on an environment where countries $A$ and $B$ compete on the same third markets. This is consistent with the measure of trade linkage used by Glick and Rose (1998) who focus on competition on third markets rather than on direct bilateral trade. Eichengreen, Rose and Wyplosz (1996) also note that "Finland's devaluation in August 1992 was widely regarded as having had negative repercussions for Sweden, not so much because of direct trade between the two countries but because their exporters competed on the same third markets".

In countries $A$ and $B$, agents (whose numbers are respectively $n_A$ and $n_B$) share the same preferences: they derive utility from consumption of good $x$ imported from country $Z^6$ and disutility from effort in labor measured by $l$. The utility function of a representative agent is linear and given by

$$U_i = x_i - \gamma (l_i + \delta_i C_i) \quad (3.2.2)$$

for $i \in \{A, B\}$, where $\gamma$ measures the relative disutility from effort. We have also added a fixed cost $C_i$ for $i \in \{A, B\}$, in the case the government devalues: $\delta = 0$ if the government decides not to devalue and 1 if the government devalues. We interpret this cost as arising from the disruption of the economic activity in case of devaluation. In models of speculative attacks like those of Obstfeld (1990) and Jeanne (1997), this cost is borne by the government that loses credibility in the case of a devaluation. In our framework, we choose to model only a specific disruption cost$^6$: the devaluation, because it implies to change prices of imported goods in local currency, requires from the agents to spend a fixed effort in changing menus and enters the utility accordingly.

The budget constraint of a representative agent in country $i \in \{A, B\}$ is given by the following equation:

$^6$Our small country assumption on $A$ and $B$ implies that we disregard any impact of the demand of these countries for $x$ on its price. This greatly facilitates the analysis.

$^6$We do not need this fixed cost to get devaluations induced by self-fulfilling expectations and multiple equilibria. However, due to perfect foresight, this cost is necessary for the equilibrium without devaluation to be preferred to the equilibrium with devaluation.
\[ w_i i_i + t_i = e_A x_i. \] (3.2.3)

This budget constraint is given in terms of the local currency. A representative agent derives income from labor paid at wage \( w_A \) in country \( A \) and from a government transfer \( t_A \) which will be detailed later. The imported good \( x \) is paid at the nominal exchange rate \( e_A \) between country \( A \) and \( Z \). We normalize and fix the price of good \( x \) in \( Z \) to one and we assume zero inflation in that country during the period. For a consumer in \( A \), the cost of the imported good in local currency will therefore be either \( e_A = 1 \) if the fixed exchange rate is maintained or \( e_A = d \) if the government in \( A \) decides to devalue at the exogenous rate \( d \).

The timing of decisions by the different players is displayed in figure 3.1. First, wages paid in local currency are set. They can not be changed afterwards. This is the source of rigidity in the model which, together with the assumption that wages can not be paid in dollars, explains why devaluations may be optimally chosen. Second, each government decides whether to devalue or not. Third, monopolistic firms set their prices to maximize profits. Demands for exported goods are then realized, as well as demands for imported goods. Finally, production and consumption take place.

There are two sources of inefficiency in this set-up. First, the good market is monopolistic so that production will be too low. Second, in an application of double marginalization, the labor market is also monopolistic because there is a single union per firm. This will push the wage rate too high.

The price elasticity for a single producer of the export good is \(-\sigma\), so that we get the usual price rule (expressed in local currency) for a monopolistic producer who maximizes profits: \( P = \frac{w d \gamma}{\sigma - 1} \) where \( \beta \) is the labor requirement (common to all three countries) in the export sector.

We assume that there is a single union per firm. The monopolistic union in each firm knows the price rule above and the labor demand function when it decides on the wage. It therefore takes into account that the wage elasticity of production and labor is \(-\sigma\). Its objective is to maximize the expected utility of the representative worker in the union subject to the budget constraint. This means that the union has to form expectations on the exchange rate policy. Each union is small (there are as many unions as firms) so that it does not internalize the fact that it influences the exchange rate policy nor the transfer

\[^2\text{In Loisel and Martin (2001a), we analyze the case of an endogenous devaluation rate in a simpler model.}\]
(see below) when it chooses nominal wages. Appendix 5.1 shows that the wage rate is the mark-up over the marginal disutility from effort in labor:

$$w_i = \frac{\sigma}{\sigma - 1} \gamma e^*_i$$  \hspace{1cm} (3.2.4)

for $i \in \{A, B\}$, where $e_i$ is the expected exchange rate between country $i$ and country $Z$. We want the real wage in all situations, even out of equilibrium, to be higher than the marginal disutility of work. This imposes a parameter restriction such that the elasticity of substitution is small enough: $\frac{\sigma}{\sigma - 1} > \delta^0$. In country $Z$, the wage rate is determined in the same manner but because we fix the price of $x$ in that country to 1, the wage rate is simply $\frac{2\sigma}{\sigma - 1}$. Depending on whether a devaluation is expected or not in the countries that fix their exchange rate to the dollar, the nominal wages will be high or low. Hence, the expectation on the exchange rate will influence the marginal cost of monopolistic firms and therefore their price decision: $p_i = \beta \gamma \left( \frac{\sigma}{\sigma - 1} \right)^2 e^*_i$ in local currency and $\beta \gamma \left( \frac{\sigma}{\sigma - 1} \right)^2 \frac{e^*_i}{e^*_a}$ in dollars for $i \in \{A, B\}$. An unexpected devaluation therefore decreases this price.

Using the monopolistic price rule, equation (3.2.4), and the fact that all firms in a specific country are symmetric and face the same exchange rate, we get that the demand and the production levels of representative monopolistic firms in each country that pegs its currency to the dollar are (see appendix 5.2 for a more detailed derivation):

$$y_A = \frac{1}{\gamma} \left[ n_A + n_B \left( \frac{e^*_A}{e_A} \frac{e^*_B}{e_B} \right)^{\sigma - 1} + n_Z \left( \frac{e^*_A}{e_A} \right)^{\sigma - 1} \right] \frac{\sigma}{\sigma - 1},$$

$$y_B = \frac{1}{\gamma} \left[ n_B + n_A \left( \frac{e^*_B}{e_B} \frac{e^*_A}{e_A} \right)^{\sigma - 1} + n_Z \left( \frac{e^*_B}{e_B} \right)^{\sigma - 1} \right] \frac{\sigma}{\sigma - 1}. \hspace{1cm} (3.2.5)$$

Under our perfect foresight assumption ($e^*_i = e_i$ for $i \in \{A, B\}$), we can see already that fully expected devaluations will have no real impact as in this case, the devaluation is exactly compensated by a higher wage and higher marginal cost. However, suppose that private agents in country $A$ and $B$ expect no devaluation ($e^*_A = e^*_B = 1$), so that nominal wages are low, then a surprise devaluation ($e_A = d > 1$) will increase production of monopolistic firms in $A$ and lower production in monopolistic firms in $B$. Hence, from that point of view, a surprise devaluation is a "beggar-thy-neighbor" policy in this model. These equations illustrate the spillovers at play in the model. The dependence of production in $A$ on the exchange rate policy of $B$ can increase in two ways:

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6This inequality corresponds to $\frac{\sigma}{\sigma - 1} > \gamma$ in the case where $e^*_i = 1$ and $e_i = d$. 
first, if world trade $\mathcal{Y}$ expands and second if $n_B$, the number of firms in $B$ competing on third markets, increases.

Note also, that when the private sector in country $A$ expects a devaluation ($e_A^d = d > 1$), then a policy of no devaluation ($e_A = 1$) is costly as national production decreases. This explains why self-fulfilling expectations will occur in this setup.

The monopolistic profits of firms are taxed and fully redistributed to the private agents in the form of a lump-sum transfer. Given that the number of firms and the number of agents are identical, the transfer to the representative agent is the profit of the representative firm:

$$ t_i = \beta \gamma e_i^s y_i \frac{\sigma}{(\sigma - 1)^2} $$

(3.2.6)

for $i \in \{A, B\}$, with $y_i$ given by equation (3.2.5). It is easy to check that profits of firms and therefore the per-capita transfer decrease with exchange rate expectations and increase with the exchange rate of the country. Because each small union does not internalize the impact of its wage decision on the aggregate transfer, nominal wages will be too high.

Taking into account this transfer in the consumer budget constraint, the equilibrium utility of a representative agent is given by:

$$ U_i = \beta \gamma \left[ \frac{\sigma^2}{\sigma - 1} \left( \frac{\sigma}{\sigma - 1} \right)^2 - 1 \right] y_i - \delta_i \gamma C_i $$

(3.2.7)

for $i \in \{A, B\}$, where $y_i$ is given by equation (3.2.5) and the term in bracket is always positive due to the parameter restriction: $\frac{\sigma}{\sigma - 1} > d$. On top of the standard devaluation cost $C_i$, this equation shows that an unexpected devaluation has two welfare consequences. On the one hand it reduces the purchasing power of consumers by increasing the price of imported goods; the term $\frac{\sigma^2}{\sigma - 1}$ in the bracket takes the value $\frac{1}{2}$ in the case of an unexpected devaluation. On the other hand, as shown in equations (3.2.5) and (3.2.6), it increases domestic production, profits and income. The problem of each government will be to set the exchange rate to $1$ (no devaluation) or $d$ (devaluation), taking exchange rate expectations and the exchange rate policy of the other country as given, in order to maximize the utility of the representative consumer.

The absence of a non-traded local good implies that the consumption price index is the exchange rate and this simplifies greatly the analysis. If such a good were introduced, the wage rate would depend on the expected price index that would comprise both local and imported goods. Hence, an expected devaluation
would lead to an increase in nominal wages of lower magnitude. From that point of view, self-fulfilling expectations would be less important in generating currency crises. On the other hand, because the purchasing power cost of a devaluation is lower in the presence of local goods, this should lead to a stronger incentive to devalue.

3.3 Rationally expected Nash equilibria

Because we assume that country \( Z \) does not play strategically \( \textit{vis-à-vis} \) countries \( A \) and \( B \), the relevant players are the following: unions, the government of country \( A \) and the government of country \( B \). There are two games: the first one takes place between the private sector on the one hand and the governments on the other hand, and the second one between the two governments. Because unions, when they form expectations on a devaluation, base them on the expected result of the game between the governments, we solve the game by backward induction. We will first analyze the equilibrium of the game between the governments before looking at the private sector's expectations. The equilibrium in this game depends on the institutional setting that governs the relations between the governments of \( A \) and \( B \), which we assume is known by the private sector when it sets its expectations.

In the "no coordination" case, governments play Nash. A Nash equilibrium is such that no government has a unilateral incentive to deviate. If several equilibria are possible in the policy game, governments choose the one expected by private agents.

In the "coordination" case, governments coordinate on the Nash equilibrium that Pareto-dominates all Nash equilibria. We adopt here the terminology of Canzoneri and Henderson (1991, p. 4): coordination refers to the way policymakers settle on one solution out of several in a non-cooperative game. It corresponds to a relatively weak requirement: when the governments coordinate with each other, they do not need to trust each other, they just need to consult with each other to simultaneously move to a specific Nash equilibrium. Once they coordinate on an outcome, none of them has any incentive neither to deviate unilaterally from it nor to deviate in a coordinated manner from it. This stability makes coordination credible to private agents.

In the "cooperation" case, the governments choose the outcome that maximizes the sum of the utility levels of representative consumers. Cooperation imposes solidarity between both governments. Since it may lead to outcomes which prove not unilaterally stable, because they are not Nash equilibria, coop-
eration may not appear credible to private agents. We will therefore consider cooperative equilibria as benchmark equilibria. We will need to assume their credibility, through a commitment technology that may involve some institutional structure for instance.

We assume that unions know the nature of the game governments play: no coordination, coordination or cooperation. We assume moreover that unions and governments have full information on all parameter values. Since no uncertainty exists, rational expectations equilibria correspond to perfect foresight equilibria.

3.3.1 No coordination

Suppose that private agents expect none of the governments to devalue: \( e_A^* = e_B^* = 1 \). Governments then play this specific game. If parameters are such that neither \( A \) nor \( B \) has an incentive to deviate, then \( e_A^* = e_B^* = e_A = e_B = 1 \) is a rationally expected Nash equilibrium. This will be the case when

\[
\begin{cases} 
U_A (e_A^* = e_B^* = e_A = e_B = 1) \geq U_A (e_A^* = e_B^* = e_B = 1, e_A = d) \\
U_B (e_A^* = e_B^* = e_A = e_B = 1) \geq U_B (e_A^* = e_B^* = e_A = 1, e_B = d)
\end{cases}
\]

Rational private agents may indeed expect \( e_A^* = e_B^* = 1 \) when these two inequalities are satisfied: they correspond to \( C_A \geq \overline{C}_A \) and \( C_B \geq \overline{C}_B \). These threshold values for the costs of devaluation are given in appendix 5.3. In figure 3.2, the equilibria have been illustrated in relation to the value of parameters.

Now, suppose that private agents expect both governments to devalue: \( e_A^* = e_B^* = d \). The governments, in the policy stage, then play a different game. If parameters are such that:

\[
\begin{cases} 
U_A (e_A^* = e_B^* = e_A = e_B = d) \geq U_A (e_A^* = e_B^* = e_B = d, e_A = 1) \\
U_B (e_A^* = e_B^* = e_A = e_B = d) \geq U_B (e_A^* = e_B^* = e_A = d, e_B = 1)
\end{cases}
\]

then \( e_A = e_B = d \) is a rationally expected Nash equilibrium. Accordingly, in figure 3.2, this equilibrium exists for parameters such that \( C_A \leq \overline{C}_A \) and \( C_B \leq \overline{C}_B \). Hence, the possibility of multiple equilibria arises in the middle rectangle of figure 3.2. The existence of asymmetric equilibria, \((e_A^*, e_B^*) = (1, d)\) and \((e_A, e_B) = (d, 1)\), is checked through a similar strategy.

We can show analytically that \( \overline{C}_A^0 > C_A \) and \( \overline{C}_B^0 > C_B \) when \( C_A^1 > 0 \), \( C_B^1 > 0 \) and \( d \) is sufficiently small (see appendix 5.4). Numerical simulations did not lead to any example where this would not be the case for high values of \( d \), as long as \( \overline{C}_A \) and \( \overline{C}_B \) are positive.
The area of maximum instability is given in the middle square where all equilibria are possible due to self-fulfilling expectations. The intuition for the possibility of self-fulfilling expectations and devaluation when governments do not coordinate should be clear by now. If agents in country $A$ expect a devaluation, then they ask for high nominal wages. In this case, if the government does not devalue, monopolistic firms of the country lose competitiveness and income decreases. If the fixed cost of devaluation is not too high (below the threshold $\overline{C}_i^d$ for $i \in \{A, B\}$), then devaluation is an equilibrium. When the cost of devaluation is very low in both countries (below the threshold $\overline{C}_i^d$ for $i \in \{A, B\}$), then the only equilibrium is that both countries devalue simultaneously. This is because in this case, the temptation to engage in competitive devaluation (even if private agents did not expect such a devaluation) is very high. This equilibrium would be eliminated if the labor market was perfectly competitive.

We can analyse the effect of the competitive structure of the industry by looking at the impact of an increase in the world number of firms $N$ in the industry on the threshold values in figure 3.2. To simplify the exercise we look at the effect of an increase of $N$ on the threshold values in the case where $n_A = n_B = n = \frac{N}{2}$ so as to keep the relative level of competition between countries symmetric. It is easy to check then that $\frac{\partial \overline{C}_i^d}{\partial N} < 0$ and $\frac{\partial \overline{C}_i^d}{\partial N} < 0$. Hence, countries specialized in more competitive industries (higher $N$ implies more competition and less profits), are less subject to speculative devaluations. It is also easy to check that the area of maximum instability (all four possible equilibria) decreases with $N$, our measure of the degree of competition in the industry. The reason for these results is that as $N$ rises, the potential profit gain of an unexpected devaluation decreases. This is recognized by private agents who adjust their expectations accordingly. In the same vein, when the elasticity of substitution tends to infinity, the goods market becomes perfectly competitive, and $\overline{C}_i^d$ for $i \in \{A, B\}$ tends to zero so that governments never devalue. This fits quite well with the observation that the attacks have been mostly concentrated on new industrialized countries. Non industrialized countries such as African countries of the CFA zone that export mostly raw products have had little experience with attacks on their fixed exchange rate. Other important reasons exist however: for example, countries exporting mostly raw materials also typically have less open capital markets.

Note that trade spillovers have an important impact on the possibility of devaluations and of self-fulfilling expectations. An increase in the world demand and trade of the composite good $\overline{Y}$ increases both threshold values for both
countries. This implies that the set of parameters for which both countries devalue simultaneously expands, and that the set of parameters for which no country devalues narrows. Note also that as $\bar{Y}$ increases, the difference between $\bar{C}^2_i$ and $C^2_i$ for $i \in \{A, B\}$ increases so that the middle square of maximum instability with self-fulfilling expectations expands. Even though in equilibrium devaluations have no real impact because they are fully expected given our perfect foresight assumption, their potential impact out of equilibrium explains the increased incentive to devalue (and therefore the increased possibility of self-fulfilling expected devaluations) when trade increases. With more trade, the potential gain of an unexpected unilateral devaluation is higher as utility increases with profits that themselves increase with production and exports\(^9\). Because this potential gain is fully expected by private agents, trade magnifies the possibility of devaluations induced by self-fulfilling expectations.

Evaluated at $n_A = n_B = n_2 = \frac{Q}{S}$ and holding $N$ and $n_2$ constant, we can also show that $\frac{\partial C^1_A}{\partial n_2} > 0$ and $\frac{\partial C^2_A}{\partial n_2} > 0$, with symmetric results for the threshold levels in country $B$. A higher number of competitors in $B$ increases the incentive of country $A$ to devalue and expands the set of multiple equilibria with all four equilibria. The intuition is that a higher number of competitors in $B$ increases the importance of trade spillovers for $A$.

One can also check that when $n_B$ and $n_2$ are sufficiently small compared to $n_A$, $C^A$ becomes negative so that $A$ does not devalue whatever $B$ does. In this case, trade spillovers are small in the sense that country $A$ does not compete much with country $B$ on third markets. This also says that large countries are less prone to currency crises than small ones.

### 3.3.2 Coordination

Because devaluations in equilibrium have no real effect on output as they are fully expected and because of the fixed cost of devaluation, the equilibrium where no government devalues always Pareto-dominates the equilibrium where one or both governments devalue. Hence, in the case of multiple equilibria, coordination on the Nash equilibrium with no devaluation can improve welfare. We want here to determine under which circumstances such a coordination is feasible.

Suppose that private agents expect both governments to devalue: $e^*_A = e^*_B = \frac{Q}{S}$.

\[^9\]In addition to this effect, a higher level of world trade induces an increase in national income. Our assumption of linear utility in consumption implies that this increase in income does not diminish the incentive to devalue because the marginal benefit of a devaluation is the same in good and bad times.
d. If parameters are such that \( C_A \leq \overline{C}_A^2 \) and \( C_B \leq \overline{C}_B^2 \) then the equilibrium where both governments devalue is a Nash equilibrium. However, if

\[
\begin{align*}
U_A (e_A^* = e_B^* = d, e_A = e_B = 1) & \geq U_A (e_A^* = e_B^* = d, e_B = 1, e_A = d) \\
U_B (e_A^* = e_B^* = d, e_A = e_B = 1) & \geq U_B (e_A^* = e_B^* = d, e_A = 1, e_B = d)
\end{align*}
\]

then even if agents expected both countries to devalue, governments could, for this set of parameters, coordinate on the Nash equilibrium where they jointly decide not to devalue. In this case, governments will choose to coordinate on this equilibrium because it can be checked that it Pareto dominates the one with simultaneous devaluation. None of the governments has any incentive to do so unilaterally though and this is the reason why coordination is required. The coordinated equilibrium with no devaluation is credible in the sense that no country has any incentive to devalue, neither unilaterally nor jointly.

The two above inequalities correspond to \( C_A \geq \overline{C}_A^{\text{coord}} \) and \( C_B \geq \overline{C}_B^{\text{coord}} \), with \( \overline{C}_A^{\text{coord}} \) and \( \overline{C}_B^{\text{coord}} \) defined in appendix 5.3. Therefore, when \( \overline{C}_A^{\text{coord}} \leq C_A \leq \overline{C}_A^2 \) and \( \overline{C}_B^{\text{coord}} \leq C_B \leq \overline{C}_B^2 \), the governments coordinate on changing their strategies and thus play \( e_A = e_B = 1 \) rather than \( e_A = e_B = d \). Agents accordingly change their expectations. Situations in which agents do not expect simultaneous devaluations lead to results similar to the no coordination case. All possible equilibria are illustrated on figure 3.3.11

As shown in figure 3.3, coordination reduces but does not eliminate the set of parameters where self-fulfilling expectations of devaluations in both countries are equilibria. However, it does not reduce the set of parameters for which a devaluation in one country only is an equilibrium so that its impact is only in situations when private agents expect both countries to devalue. Trade competition between the two countries explains the possible difference between coordination and no coordination. If for example country A does not compete with country B (\( n_B = 0 \)), then it is easy to check that \( \overline{C}_A = \overline{C}_A^{\text{coord}} \), so that coordination with B is useless.

Note that coordination, whose role is to eliminate the worse possible equilibrium, is feasible and useful especially when countries are sufficiently similar and when the devaluation cost in both countries is sufficiently high relative to the potential devaluation gain. Otherwise, the equilibrium where no country devalues cannot be a Nash equilibrium when agents expect both countries to

---

10We focus on the case where these inequalities are fulfilled. The conditions are given in appendix 5.4.

11Under the same conditions as in figure 3.2, we have \( \overline{C}_A^{\text{coord}} > \overline{C}_A^1 \) and similarly for country B.
devalue as one country at least will have an incentive to deviate and devalue. In this case, coordination on the no devaluation equilibrium is not sustainable. In other words, an announcement by both governments that they will not devalue when nominal wages are high is no longer credible.

This also illustrates a channel of contagion of currency crises different of those usually identified. In figure 3.3, it can be checked that if \( C_A \) becomes less than \( C_{A}^{\text{coord}} \) (either because \( C_A \) decreases or because \( C_{A}^{\text{coord}} \) increases), then the possibility of a self-fulfilling currency crisis increases not only in country \( A \) but also in country \( B \): when \( C_{A}^{\text{coord}} \) increases, the area of equilibria (11-1d-d1) is reduced at the expense of the area of equilibria (dd-11-1d-d1). This change in the economic situation of country \( A \) makes coordination between the two countries more difficult to sustain and an announcement that governments coordinate on the no-devaluation equilibrium will not be credible in this sense. If private agents now expect both countries to devalue, the coordinated equilibrium where both countries decide not to devalue is no longer a Nash equilibrium. Country \( A \) will be induced to deviate and devalue. This itself induces country \( B \) to devalue. This channel of contagion is different from the classic one. Here, contagion of the currency crisis comes from the fact that regional coordination becomes less credible in the eyes of private agents when fundamentals in one country are such that this country is more likely to suffer a currency crisis.

It illustrates the ambiguous effect that coordination has on the issue of regional contagion. Compared to no coordination, it reduces the possibility of simultaneous speculative crises in a region with important trade spillovers and in this sense is welfare improving. However, because the credibility of coordination itself is dependent on parameters of both countries, it introduces a new channel of contagion.

### 3.3.3 Cooperation

Cooperation between two countries of different size is difficult to analyze so we choose to restrict our attention to the case where the two countries are identical: \( n_A = n_B \) and \( C_A = C_B^{12} \). Like Buitier, Corsetti and Pesenti (1995, 1998), we assume a national horizontal equity constraint, according to which no international agreement is enforceable unless countries that are identical \( \text{ex ante} \) end up having an identical level of welfare \( \text{ex post} \). This excludes the possibility of a cooperative equilibrium where one country devalues and the other does not. Cooperation then implies that governments choose jointly the same exchange

\[12\text{Loisel and Martin (2001a) fully analyze the case of cooperation. The conclusions are not very different from those presented there.}\]
rate policy that maximizes the utility of a common representative consumer. In effect, cooperation is identical to the case where the two countries form a monetary union.

Suppose that private agents expect the governments to cooperate on the no-devaluation equilibrium: \( e_A^* = e_B^* = 1 \). If parameters are such that

\[
U_A (e_A^* = e_B^* = e_A = e_B = 1) = U_B (e_A^* = e_B^* = e_A = e_B = 1) \geq U_A (e_A^* = e_B^* = 1, \ e_A = e_B = d) = U_B (e_A^* = e_B^* = 1, \ e_A = e_B = d),
\]

then \( e_A^* = e_B^* = e_A = e_B = 1 \) is a rationally expected cooperative equilibrium. Suppose on the contrary that private agents expect governments to cooperate on the devaluation equilibrium. If parameters are such that

\[
U_A (e_A^* = e_B^* = e_A = e_B = d) = U_B (e_A^* = e_B^* = e_A = e_B = d) \geq U_A (e_A^* = e_B^* = d, \ e_A = e_B = 1) = U_B (e_A^* = e_B^* = d, \ e_A = e_B = 1),
\]

then \( e_A^* = e_B^* = e_A = e_B = d \) is a rationally expected cooperative equilibrium. The first inequality corresponds to \( C_A = C_B \geq \overline{C}^{\text{coop}}_1 \). The second inequality corresponds to \( C_A = C_B \leq \overline{C}^{\text{coop}}_2 \). The values of \( \overline{C}^{\text{coop}}_1 \) and \( \overline{C}^{\text{coop}}_2 \) are given in appendix 5.3.

Figure 3.4 shows that even though a devaluation has no real impact in equilibrium, cooperation between A and B is not enough to eliminate the possibility of self-fulfilling currency crises. The reason is that, even though governments can commit to each other they still can not commit to private agents. Out of equilibrium, they can cooperate and surprise private agents through a devaluation and gain competitiveness relative to country Z firms. The equilibrium with devaluation is fully eliminated only when \( n_Z = 0 \). In this case, \( \overline{C}^{\text{coop}}_2 \) is negative because governments in A and B have no incentive to devalue.

It can be checked that for \( n_A = n_B, \overline{C}^{\text{coop}}_A = \overline{C}^{\text{coop}}_B > \overline{C}^{\text{coop}}_1 \) and \( \overline{C}^{\text{coop}}_2 = \overline{C}^{\text{coop}}_B > \overline{C}^{\text{coop}}_2 \). This implies that cooperation narrows the set of parameters for which both countries devalue and expands the set of parameters for which both countries decide not to devalue. Because \( \overline{C}^{\text{coop}}_2 \) is lower than \( \overline{C}^{\text{coord}}_A = \overline{C}^{\text{coord}}_B \) (with \( n_A = n_B \)), cooperation reduces the set of parameters for which both countries may devalue and improves welfare relative to the coordination equilibrium. It is easy to see that when trade spillovers are
eliminated for $A$ ($n_B$ tends to zero), then $C_1^{coop} = C_A^1$ and $C_2^{coop} = C_A^2 = C_A^{coord}$ so that cooperation between $A$ and $B$ is useless.

Contrary to coordination, cooperation requires a strong commitment technology to be sustainable so that we view it mainly as a benchmark case. However, if we interpret the European integration process and the political institution building that goes with it as such a commitment device, we can analyze the impact of European monetary cooperation on contagion in this specific context.

### 3.4 Conclusion

In this chapter, we have asked two sets of questions. First, how do regional trade structures influence the fragility of a fixed exchange rate regime and the probability that it collapses simultaneously in the region? The answer to this question is that the more trade competition between countries in a monopolistic sector, the more fragile fixed exchange rate regimes are. We have shown that this is the case even in a model where there is no real gain to a devaluation in equilibrium because the devaluation is perfectly expected by agents. The result is likely to be even stronger if we allow for the possibility of unexpected shocks that would lead to unexpected devaluations. Second, do coordination and cooperation reduce instability and contagion? We have shown that the answer to this second question is more ambiguous. Neither coordination nor cooperation at the regional basis, eliminate the possibility of crises induced by self-fulfilling expectations. However, coordination and cooperation reduce the set of fundamental parameters for which simultaneous devaluations is an equilibrium and are therefore welfare improving. Both are stabilizing in the sense that they reduce the number of equilibria. Because multiple equilibria due to self-fulfilling expectations are a natural outcome of this type of model, the role of coordination on a specific equilibrium is important. In contrast to cooperation, coordination does not require a commitment technology, and should therefore be relatively easy. We have shown however that coordination reduces the possibility of simultaneous devaluations, at the cost of making each country's more dependent on the other's fundamentals.

### 3.5 Appendix

#### 3.5.1 Wage determination

The representative monopolistic union in country $i \in \{A, B\}$ chooses the wage rate $w_i$ to maximize expected utility:
taking into account the following perceived elasticities: \( \frac{\partial u_i}{\partial u_i} = \frac{\partial u_i}{\partial u_i} = 0 \) and \( \frac{\partial u_i}{\partial v_i} = -\sigma \). Equation (3.2.4) in the text follows.

### 3.5.2 Demand determination

We solve a standard problem of cost minimization under the constraint of total demand. This constraint is given by equation (3.2.1), which can be written in terms of the production levels of representative firms:

\[
Y = \left[ n_A Y_A^{1-\frac{1}{\beta}} + n_B Y_B^{1-\frac{1}{\beta}} + n_Z Y_Z^{1-\frac{1}{\beta}} \right]^{\frac{1}{1-\frac{1}{\beta}}}.
\]

Denominated in dollar, the total cost is:

\[
\frac{P_A}{e_A} n_A Y_A + \frac{P_B}{e_B} n_B Y_B + p_Z n_Z Y_Z.
\]

This leads to the following demand level of a representative variety produced in country A:

\[
y_A = Y \left[ n_A + n_B \left( \frac{e_A}{P_A} \right)^{\sigma-1} + n_Z \left( \frac{1}{e_A p_Z} \right)^{\sigma-1} \right]^{\frac{\sigma}{\beta}}.
\]

Using the price formula as a function of expectations of the exchange rate, we get equation (3.2.5) in the text. The demand levels for varieties produced in B are derived similarly.

### 3.5.3 Threshold levels for the cost of devaluation

\[
\bar{C}_A = \beta Y \left\{ \left[ \frac{\sigma}{\sigma - 1} \right] \left[ \frac{1}{d} - 1 \right] \left[ n_A + n_B d^{1-\sigma} + n_Z d^{1-\sigma} \right] \right\}^{\frac{\sigma}{\beta}}
\]

\[
- \left[ \frac{\sigma}{\sigma - 1} \right] \left[ \frac{1}{d} - 1 \right] n \right\}^{\frac{\sigma}{\beta}}
\]

\[
\bar{C}_B = \beta Y \left\{ \left[ \frac{\sigma}{\sigma - 1} \right] \left[ \frac{1}{d} - 1 \right] \left[ n_B + n_A d^{1-\sigma} + n_Z d^{1-\sigma} \right] \right\}^{\frac{\sigma}{\beta}}
\]

\[
- \left[ \frac{\sigma}{\sigma - 1} \right] \left[ \frac{1}{d} - 1 \right] n \right\}^{\frac{\sigma}{\beta}}
\]
\[ C_A^2 = \beta \gamma \left\{ \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 - 1 \right] \eta \frac{\epsilon}{\epsilon} \right. \\
- \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 d - 1 \right] [n_A + n_B d^{\sigma - 1} + n_Z d^{\sigma - 1}] \frac{\epsilon}{\epsilon} \left\}, \right. \]

\[ \bar{C}_B^2 = \beta \gamma \left\{ \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 - 1 \right] \eta \frac{\epsilon}{\epsilon} \right. \\
- \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 d - 1 \right] [n_B + n_A d^{\sigma - 1} + n_Z d^{\sigma - 1}] \frac{\epsilon}{\epsilon} \right\}, \]

\[ \bar{C}^{coord}_A = \beta \gamma \left\{ \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 - 1 \right] [n_A + n_B d^{1 - \sigma} + n_Z] \frac{\epsilon}{\epsilon} \right. \\
- \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 d - 1 \right] [n_A + n_B + n_Z d^{\sigma - 1}] \frac{\epsilon}{\epsilon} \left\}, \right. \]

\[ \bar{C}^{coord}_B = \beta \gamma \left\{ \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 - 1 \right] [n_B + n_A d^{1 - \sigma} + n_Z] \frac{\epsilon}{\epsilon} \right. \\
- \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 d - 1 \right] [n_A + n_B + n_Z d^{\sigma - 1}] \frac{\epsilon}{\epsilon} \left\}, \right. \]

\[ \bar{C}^{coop}_1 = \beta \gamma \left\{ \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 \frac{1}{d} - 1 \right] [n_A + n_B + n_Z d^{1 - \sigma}] \frac{\epsilon}{\epsilon} \right. \\
- \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 \frac{1}{d} - 1 \right] N \frac{\epsilon}{\epsilon} \left\}, \right. \]

\[ \bar{C}^{coop}_2 = \beta \gamma \left\{ \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 - 1 \right] \eta \frac{\epsilon}{\epsilon} \right. \\
- \left[ \left( \frac{\sigma}{\sigma - 1} \right)^2 d - 1 \right] [n_A + n_B + n_Z d^{\sigma - 1}] \frac{\epsilon}{\epsilon} \right\}. \]
3.5.4 Derivation of conditions for \( 0 \leq \overline{C}_A^{\ell} \leq \overline{C}_A^{\text{coord}} \leq \overline{C}_A^2 \)

Consider \( \overline{C}_A^{\ell}, \overline{C}_A^{\text{coord}} \) and \( \overline{C}_A^2 \) as functions of parameter \( d \geq 1 \): \( \overline{C}_A^{\ell} (d), \overline{C}_A^{\text{coord}} (d) \) and \( \overline{C}_A^2 (d) \). It can be checked that \( \overline{C}_A^{\ell} (1) = \overline{C}_A^{\text{coord}} (1) = \overline{C}_A^2 (1) = 0 \) and

\[
\overline{C}_A^{\ell} (1) = \overline{C}_A^{\text{coord}} (1) = \overline{C}_A^2 (1) = \frac{2 \beta \sigma Y N \tau \bar{x}_2}{(\sigma - 1)^2} \left[ -\sigma n_A + (\sigma - 1)(n_B + n_Z) \right].
\]

These equations imply that \( \overline{C}_A^{\ell} (d) \) is positive in the neighborhood of \( d = 1 \) if and only if \( \frac{n_A + n_Z}{n_A} \geq \frac{\sigma}{\sigma - 1} \). It can be also checked that:

\[
\overline{C}_A^{\text{coord}}' (1) - \overline{C}_A^{\ell} (1) = \frac{2 \beta \sigma Y N \tau \bar{x}_2}{(\sigma - 1)^2} \left[ -\sigma n_A^2 + \sigma (\sigma - 1) n_B^2 + \sigma (\sigma - 2) n_A n_B 
+ (\sigma - 1)(4\sigma - 1) n_A n_Z + \sigma (\sigma - 1) n_B n_Z \right],
\]

\[
\overline{C}_A^{\text{coord}}'' (1) - \overline{C}_A^{\ell} (1) = \frac{2 \beta \sigma Y n_B N \tau \bar{x}_2}{(\sigma - 1)^2} \left\{ \left[ \sigma^2 - (\sigma - 1)^2 \right] n_A 
- \sigma (\sigma - 1)(n_B + n_Z) \right\}
\]

The first expression proves positive when \( \frac{n_A + n_Z}{n_A} \geq \frac{\sigma}{\sigma - 1} \), that is, when \( \overline{C}_A^{\ell} (d) \) is positive in the neighborhood of \( d = 1 \). The second expression proves positive if and only if \( \frac{n_A + n_Z}{n_A} \leq \frac{\sigma^2 - (\sigma - 1)^2}{\sigma (\sigma - 1)} \). We know that \( \frac{\sigma}{\sigma - 1} \leq \frac{\sigma^2 - (\sigma - 1)^2}{\sigma (\sigma - 1)} \) as \( \sigma \geq 1 \). Hence, if \( \frac{\sigma}{\sigma - 1} \leq \frac{n_A + n_Z}{n_A} \leq \frac{\sigma^2 - (\sigma - 1)^2}{\sigma (\sigma - 1)} \), then \( \overline{C}_A^{\ell} (1) \leq \overline{C}_A^{\text{coord}}'' (1) \leq \overline{C}_A^{\text{coord}}' (1) \).

This implies that for \( \frac{\sigma}{\sigma - 1} \leq \frac{n_A + n_Z}{n_A} \leq \frac{\sigma^2 - (\sigma - 1)^2}{\sigma (\sigma - 1)} \), and for \( d \) sufficiently close to one, we have \( 0 \leq \overline{C}_A^{\ell} \leq \overline{C}_A^{\text{coord}} \leq \overline{C}_A^2 \). Conditions under which \( 0 \leq \overline{C}_B^{\ell} \leq \overline{C}_B^{\text{coord}} \leq \overline{C}_B^2 \) are symmetric.

Figure 3.3 on the effects of coordination corresponds to \( 0 \leq \overline{C}_A^{\ell} \leq \overline{C}_A^{\text{coord}} \leq \overline{C}_A^2 \) and \( 0 \leq \overline{C}_B^{\ell} \leq \overline{C}_B^{\text{coord}} \leq \overline{C}_B^2 \). It is therefore valid for \( \frac{\sigma}{\sigma - 1} \leq \frac{n_A + n_Z}{n_A} \leq \frac{\sigma^2 - (\sigma - 1)^2}{\sigma (\sigma - 1)} \) and \( \frac{\sigma}{\sigma - 1} \leq \frac{n_B + n_Z}{n_Z} \leq \frac{\sigma^2 - (\sigma - 1)^2}{\sigma (\sigma - 1)} \).
Figure 3.1: timing.

- Wages are fixed
- Government decides on devaluation
- Export prices are set
- Work, production and consumption take place
Figure 3.2: equilibria in the "no coordination" case\textsuperscript{13}.

\begin{figure}
\centering
\begin{tikzpicture}
\begin{axis}[
    xlabel={$C_A$},
    ylabel={$C_B$},
    xmin=0, xmax=20,
    ymin=0, ymax=20,
    xtick={0,10,20},
    ytick={0,10,20},
    xticklabels={$\overline{C}_A^1$, $\overline{C}_A^2$},
    yticklabels={$\overline{C}_B^1$, $\overline{C}_B^2$},
]
\addplot [only marks] table {
    1  1  d1
    1  2  11-d1
    2  1  11
    2  2  11-1d
    1  2  dd-d1
    2  1  11-dd-1d-d1
    1  1  dd-1d
    2  2  1d
}
\end{axis}
\end{tikzpicture}
\end{figure}

\textsuperscript{13} To save space, we note 11 the equilibrium where both countries stay in the fixed exchange rate, d1 the equilibrium where country A devalues and B does not, etc.
Figure 3.3: equilibria in the "coordination" case.
Figure 3.4: equilibria in the "cooperation" case\textsuperscript{14}.

\textsuperscript{14} Figure 3.4 can be interpreted as a collapsed version of figure 3.2 in which the countries are symmetric, the 1d and d1 equilibria are excluded by cooperation, and all cooperative equilibria lie along a 45° line through the origin. Under the same conditions as in figure 3.2 (with \(n_A = n_B\)), we have \(C_1^{coop} < C_2^{coop}\).
Chapter 4

Endogenously asymmetric demand shocks under alternative exchange rate regimes

Abstract

Chapter 4, entitled "Endogenously asymmetric demand shocks under alternative exchange rate regimes", presents a two-country, two-differentiated-good, one-period monetary model, with intermediate inputs and transport costs, which builds a bridge between the New Economic Geography and the New Open Economy Macroeconomics literatures. Endogenously asymmetric shocks arise in this model when the exchange rate regime in force favours the concentration of each industry in one country, thus turning industry-specific shocks into country-specific shocks. In the presence of nominal wage rigidity, the occurrence of endogenously asymmetric demand shocks (EADS) affects the location incentives of firms, even though the latter are risk-neutral, because of the conjunction of substitution and income effects. As the smoothing of these shocks depends on the exchange rate regime, so do the location incentives of firms and hence eventually the occurrence of such shocks. Both the emergence and the sustainability of EADS are found more favoured by a fixed exchange rate regime or a monetary union than by a floating exchange rate regime.
Abstract in French

Le chapitre 4, intitulé "Chocs de demande endogènément asymétriques sous différents régimes de change", présente un modèle monétaire à deux pays, deux industries et une période, avec biens intermédiaires et coûts de transport, qui fait le lien entre les littératures de New Economic Geography et de New Open Economy Macroeconomics. Les chocs endogènément asymétriques apparaissent dans ce modèle lorsque le régime de change en vigueur favorise la concentration de chaque industrie dans un seul pays, c'est-à-dire la spécialisation de chaque pays, de telle sorte que les chocs spécifiques à une industrie donnée ont alors des effets asymétriques entre les pays. En présence de rigidité des salaires nominaux, l'existence de chocs de demande endogènément asymétriques affecte les décisions de localisation des entreprises, bien que ces dernières soient neutres au risque, du fait de la conjonction d'effets de substitution et de richesse. Comme l'effet de ces chocs dépend du régime de change, les décisions de localisation des entreprises et donc finalement l'existence même de ces chocs dépendent aussi du régime de change. Nous trouvons que le régime de change fixe et l'union monétaire favorisent l'émergence et la soutenabilité des chocs de demande endogènément asymétriques davantage que le régime de change flexible.

4.1 Introduction

This chapter investigates the emergence and the sustainability of endogenously asymmetric shocks under various exchange rate regimes, focusing on demand shocks. Endogenously asymmetric shocks arise when the exchange rate regime in force promotes greater national specialization, thus turning industry-specific shocks into country-specific shocks.

The concept of endogenously asymmetric shocks goes back to the early 90's. At that time, the New Economic Geography theory emerges to show that a fall in the cost of transport of goods and services across countries can affect the specialization patterns of these countries, for instance lead to more national specialization by providing an incentive to reap scale economies and agglomeration benefits. The European Monetary Union (EMU) could then have the same effect, as it said, if the launch of the euro, as a further step on the road to economic integration, can be interpreted at first sight as a fall in the transport cost across member countries, through the complete elimination of exchange rate risk.

In other words, the Optimal Currency Area (OCA) criterion identified by
Kenen (1969), i.e. the degree of national economic diversification in the presence of industry-specific shocks, should be considered as endogenous\footnote{Beine (1999), the OECD (1999, pp. 106-109) and The Economist ("Birds of a feather", May 27th, 1999) provide good discussions on that topic.}. Insofar as its susceptibility to country-specific shocks is concerned, EMU might be less justifiable \textit{ex post} than \textit{ex ante}, or vice versa\footnote{Carré, Levasseur and Portier (2000) also stress that a rigorous assessment of whether the Eurozone satisfies this OCA criterion is hardly possible as long as trade liberalization within the zone has not been achieved, because the specialization patterns of member countries depend on the degree of trade liberalization.}.

Pointing to the fact that the United States experience a higher degree of local specialization than Europe, Krugman (1991, 1993) thus argues that EMU will tend to develop inter-industry rather than intra-industry international trade, and hence favour the emergence of endogenously asymmetric shocks\footnote{"Theory and the experience of the US suggest that EC regions will become increasingly specialized, and that as they become more specialized they will become more vulnerable to region-specific shocks. Regions will, of course, be unable to respond with counter-cyclical monetary or exchange rate policy" (Krugman, 1993, p. 260).}. On the contrary, empirical works by Fontagné and Freudenberg (1999), Fontagné (1999, 2000) reach the conclusion that EMU will not make member countries more specialized, simply because previous economic integration has not.

On the theoretical ground, very little has been done to modelize explicitly endogenously asymmetric shocks within a monetary union and thus illustrate the endogeneity of Kenen's OCA criterion. Such a modelization would be welcome however, if only because the relevance of the argument exposed above, which consists in interpreting the elimination of exchange rate risk as a fall in the transport cost across countries, implicitly rests on two disputable assumptions: exchange rate variations are exogenous, and firms behave as risk-adverse agents.

This naturally raises the question of what would happen, were these two assumptions to be relaxed. One may wish to relax the assumption of risk-adverse firms by sheer curiosity, as it seems no more relevant than the alternative assumption of risk-neutral firms. And one may wish to relax the assumption of exogenous exchange rate variations simply because it stands at odds with the core assumption of the OCA literature, namely the assumption that country-specific shocks are smoothed by endogenous exchange rate variations under a flexible exchange rate regime\footnote{If exchange rate variations are exogenous, then Krugman's point may apply, making country-specific shocks bigger or more frequent under a fixed exchange rate regime than under a flexible exchange rate regime. But then a flexible exchange rate regime may not be preferable to a fixed exchange rate regime, because it does not smooth country-specific shocks anyway: on the contrary, it brings additional noise into the system through these exogenous exchange rate variations.}.\footnote{Beine (1999), the OECD (1999, pp. 106-109) and The Economist ("Birds of a feather", May 27th, 1999) provide good discussions on that topic.}
This chapter aims at filling in this gap in the literature, as it presents a model of endogenously asymmetric shocks where exchange rate variations are endogenous and firms behave as risk-neutral agents. Its main virtue may be pedagogical however, as the economic mechanisms unveiled seem likely to be of second-order importance compared to those predicted (though yet to be modeled) by Krugman (1991, 1993) and followers.

To our best knowledge actually, Ricci's (1997, 1998) models are so far the only ones to tackle the issue of endogenously asymmetric shocks under fixed and flexible exchange rate regimes. These models do not belong *stricto sensu* to the New Economic Geography literature, as they do not modelize transport costs and hence do not feature forward and backward linkages. Nonetheless, they show explicitly that countries' specialization patterns depend on the exchange rate regime when firms behave as risk-adverse agents.

Ricci's (1997) is a one-period, two-country, two-industry monetary model, where the exchange rate (when flexible) adjusts endogenously to shocks so as to balance trade. The point is the following: under a floating exchange rate regime, firms of a given industry have an incentive to locate in the country relatively specialized in this industry, because industry-specific shocks are smoothed there by endogenous exchange rate variations. Countries tend therefore to be more specialized under flexible than under fixed exchange rate regimes.\(^5\)

Ricci's (1998) is a one-period, n-country monetary model, where the exchange rate (when flexible) is by contrast an exogenous random variable. The point is the following: under a floating exchange rate regime, firms have an incentive to locate in the same country so as to enjoy a large market share there and hence benefit from a low variability of sales. Floating exchange rate regimes tend therefore to promote the agglomeration of economic activity.

In Ricci's models, firms' risk-adverse behaviour stems from the assumptions of decreasing returns to scale and price rigidities, which make the profit function concave in output, so that firms dislike variability of sales. Absent price rigidity, the profit function becomes linear in output, and one has to assume the existence of firing, inventory or bankruptcy costs (which are specified in an *ad hoc* fashion) for the results to hold.

We examine the same issue as Ricci, namely exchange rate regimes and location, but start from the more commonly adopted assumption that firms behave as risk-neutral agents. Our model thus features flexible prices and increasing returns to scale. Firms have no shareholders in this simple framework, but

\(^5\)Ricci (1997) also considers the case where the nominal exchange rate (when flexible) moves exogenously. This case leads to the same conclusion: countries tend to be more specialized under flexible than under fixed exchange rate regimes.
we shortly discuss in conclusion the case where the rationale for firms' risk-neutrality is that their shareholders welcome risky shares which enable them to smooth their labour income.

Like Ricci's (1997), the model presented here is a two-country, two-industry, one-period monetary model, where the exchange rate (when flexible) adjusts endogenously to shocks so as to balance trade. Similarities stop here however. As far as assumptions are concerned, as highlighted in table 4.1, the main differences to be mentioned (in the form: his model vs. ours) are risk-adverse vs. risk-neutral firms, rigid vs. flexible prices, decreasing vs. increasing returns to scale, the absence vs. the presence of intermediate goods and transport costs. Moreover, Ricci considers many different kinds of shocks, whereas our model only rests on the occurrence of industry-specific demand shocks\(^6\).

Our model builds a bridge between the New Economic Geography and the New Open Economy Macroeconomics literatures. These frameworks both hinge on monopolistic competition in the goods market and get along quite well under a few simplifying assumptions. The New Economic Geography features of our model deal with the long term, while its New Open Economy Macroeconomics features deal with the short term. From the New Economic Geography literature, we borrow the industrial clustering model of Fujita, Krugman and Venables (1999, chapter 16), which features intermediate inputs and transport costs. We reformulate this model in a stochastic environment, where exchange rate variations have real effects in the presence of short-run nominal wage rigidity.

The traditional dispersion force (based on the local competition effect) and concentration forces (based on backward and forward linkages), familiar to the New Economic Geography literature, are present in our model. But new forces enter the stage when industry-specific demand shocks are considered. These new forces explain why, despite firms behaving as risk-neutral agents in our framework, countries' specialization patterns still depend on the exchange rate regime\(^7\). The argument goes as follows.

Consider first the case where the two countries form a monetary union. In that case, asymmetric shocks are not smoothed. Let us focus on a firm of a given industry located in the country relatively specialized in this industry. In the case of a positive demand shock on that industry, this firm benefits from

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\(^6\)A more minor difference between his paper and our chapter is that the number of firms is exogenous in Ricci's (1997) main specification and endogenous in our model, as we allow for free entry and specify a fixed cost of production (expressed in terms of labour).

\(^7\)Our model thus contrasts with Krugman's (1991, 1993) argument and Ricci's (1997) model, which would both predict that the exchange rate regime has no impact on countries' specialization patterns under the risk-neutrality assumption.
a substitution effect (households spend relatively more on the goods produced by that industry) as well as from an income effect (the firm's local market gets larger as the country gets wealthier, and this matters in the presence of transport costs). The conjunction of this substitution effect and this income effect turns the firm's profit into a convex function of the stochastic parameter representing the industry-specific shock, so that the firm's average profit is increased. This gives firms of the same industry the incentive to locate in the same country. Competition between them raises wages (until average profits go back to zero) and therefore attracts local workers into their industry: a new concentration force arises.

Now consider the case of a floating exchange rate regime. Asymmetric shocks are then smoothed by endogenous exchange rate variations, so that the income effect mentioned above disappears, and so does the newly identified concentration force. Instead, another mechanism is at work, opposite to Ricci's (1997). Indeed firms of a given industry have an incentive to locate in the country relatively specialized in the other industry, because they then benefit from the conjunction of two substitution effects: a positive demand shock on their industry makes households spend relatively more on the goods produced by that industry, and makes the currency of their country depreciate. As previously, the induced competition between these firms raises wages (until average profits go back to zero) and therefore attracts local workers into their industry: a new dispersion force arises.

Our model thus predicts that countries tend to be more specialized within a monetary union, or under a fixed exchange rate regime, than under a flexible exchange rate regime. These results differ significantly from Ricci's (1997). They differ quantitatively: in Ricci's main specification, every specialization pattern is an equilibrium under a fixed exchange rate regime, and the only equilibrium under a flexible exchange rate regime is full specialization; outcomes are not so extreme in our model. And more importantly, as shown in table 4.2, they differ qualitatively: Ricci finds that countries tend to be more specialized under flexible than under fixed exchange rate regimes; we find quite the opposite.

Related literatures are seeking to assess the effects of exchange rate regimes on foreign direct investment, or the effects of exchange rate variability on trade, but they take countries' specialization patterns as given. Another related literature is examining the endogeneity of the OCA criteria. Frankel and Rose (1996, 1997, 1998) thus provide some empirical evidence supporting the idea

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8 Closer to our analysis is Bayoumi's (1994) model of optimum currency areas, which features rigid wages, transaction costs and national specializations, but considers countries' specialization patterns as exogenous.
that the adoption of a common currency, by developing international trade, no matter whether intra- or inter-industry, tends to increase the correlation between national business cycles. Their findings are however challenged by Imbs (1998).

In the remaining of the chapter, we first present our theoretical framework, and then examine the stability properties of two particular cases: the concentration of each industry in one country, and the dispersion of each industry across countries. A few concluding remarks follow.

4.2 Theoretical framework

4.2.1 Overview

This section presents our one-period, two-country, two-industry monetary model. We focus on a single period as we assume away any possibility for intertemporal trade, so that trade is constantly balanced. The two countries considered, labelled home and foreign, are identical in the sense that they share the same structural parameters, including size. Taken as a whole, they form a closed economy, which is a good approximation for the Eurozone.

The two industries considered produce tradable goods. The choice of only two industries has been made for simplicity. This assumption seems natural in the case of small countries (think for example of the importance of the telecommunications industry in Finland), but it may also be relevant for bigger countries like Germany and France if industries are re-interpreted as sectors, such as manufacturing or tradable services. What actually matters here is that there are few industries, so that each of them can represent a big share of the productive structure in a country, and thus industry-specific shocks can have non-negligible (income) effects on national economies. Similarly, the absence of non-tradable goods in our model is primarily a matter of practical convenience. The presence of a non-tradable goods sector would not alter the main results of the chapter, provided each tradable goods industry is big enough for industry-specific shocks to carry national income effects.

Many general assumptions in our model aim at replicating the structural characteristics of the European economy\(^9\). Member countries of EMU are commonly thought to lack of the adjustment mechanisms which could in theory compensate for the loss of national monetary sovereignty. Indeed, labour is rather immobile across European countries, at least compared to across US

\(^9\)Brüllhart and Torstensen (1996) provide some empirical evidence on how well the New Economic Geography theory fits the European economy.
states, and some among the main obstacles to cross-country labour mobility, such as linguistic barriers, are likely to persist for some time to come. Wages and prices may lack of flexibility. Fiscal redistribution across countries is almost non-existent, in sharp contrast with the American situation, and the prospect of a Eurozone-wide budget, which would institutionalize interregional transfers, looks far distant in the future. Finally, national fiscal policies are currently constrained by the Stability and Growth Pact, so that there is limited scope (left by the Ricardian equivalence) for countercyclical budgets to smooth disposal income.

Thus, insofar as the adaptability to country-specific shocks is concerned, the Eurozone is no OCA, and such shocks, should they occur, would then prove costly under EMU in terms of macroeconomic volatility. The assumptions chosen in our model ensure that such is the case indeed. The labour force is mobile between industries within the same country, but immobile between countries. Wages are sticky in the short run, though prices are flexible. There is neither fiscal redistribution across countries, nor national fiscal policies, nor individual intertemporal borrowing. The (industry-specific) shocks considered are temporary demand shocks; when specialization patterns make these shocks country-specific, they are (at least partially) smoothed only under a flexible exchange rate regime, through the expenditure-switching role of the exchange rate. Though temporary demand shocks can typically be accommodated by monetary policy, we choose for simplicity to keep national monetary policies exogenously passive in our framework.

Risk-sharing across countries is one more possible adjustment mechanism within a monetary union which we choose not to consider in the main body of our chapter (the implications of such a risk-sharing are shortly drawn in conclusion). It is a well established empirical regularity that there is little risk-sharing across countries in general, and across European countries in particular (Atkeson and Bayoumi, 1993; Sørensen and Yoshia, 1998). This risk-sharing could in theory be achieved by cross-ownership of productive assets via the capital market, or by intertemporal trade via the credit market. But none of these channels of risk diversification is much developed currently, thus providing a very limited amount of insurance against fluctuations in national income. As a consequence, fluctuations in national consumption are much more correlated with fluctuations in national production than with fluctuations in the European production as a whole.

It may be argued however that the creation of a common currency may in itself eventually enhance capital market integration, through the unification
of European equity markets for instance, thus lifting the home bias\textsuperscript{10}. Mélitz and Zumer's (1999) results suggest that EMU will promote the smoothing of shocks \textit{via} market channels, in particular by encouraging cross-ownership of productive assets. This seems to be the only shock-absorber mechanism in sight which would emerge in the foreseeable future to speed up adjustment and limit the potential cost of foregoing national monetary policy independence. This development should not be taken for granted however, as informational barriers to cross-ownership of productive assets, as well as governmental restrictions on the proportion of shares which can be owned by non-residents, may be slow to disappear.

The three agents involved, firms, households and monetary authorities, are introduced in turn in the next three subsections. To keep things simple, notations are (mainly) borrowed from Fujita, Krugman and Venables (1999, ch. 16), and can be shortly described as follows. Superscripts usually refer to households and firms, indexed by integers $i$ and $j$ respectively. Subscripts usually refer to the industry considered (1 or 2). Variables specific to the foreign country are denoted with an asterisk ($\ast$). All nominal variables are expressed in local currency. Our presentation focuses on the home country, but equations for the foreign country are defined or derived in a similar way.

4.2.2 Firms

We assume monopolistic competition in the goods market, so that each industry is composed of a large number of differentiated tradable goods, called varieties. We note $\theta$ (with $\theta > 1$) the constant elasticity of substitution (CES) between these varieties, the same for both industries. We define a firm as a producer of a single variety\textsuperscript{11}.

We assume that firms face a fixed cost of production, expressed in real terms. As is the rule in the monopolistic competition literature, we modelize this fixed cost as a fixed labour input requirement, noted $F$, the same for both industries and both countries. Considering a fixed cost of production enables us to endog-

\textsuperscript{10}[...\textit{over time investors' behaviour will change, creating something that looks more like a single European equity market. At present, most European fund managers are barred from putting more than 20% of their funds into foreign-currency investments. Others choose not to invest heavily abroad because of the costs of hedging their currency risks. The euro will lift both constraints. Fund managers will start to invest more of their money across European borders so as to build investment portfolios that reflect Europe's economy rather than their own domestic one. This growing band of investors will divide up Europe's investment opportunities by industry sector, not by nationality, just as in America.} (The Economist, “Euro neurosis”, May 7th, 1998.)

\textsuperscript{11}Being risk-neutral, firms have no incentives to diversify their production in our framework, neither across varieties within the same industry, nor across industries.
enize the number of firms, because with free entry, there are new entrants until expected benefits equal the fixed cost of production (so that expected profits are nil), thus ensuring a finite number of firms in equilibrium.

We note \( n_h \) and \( n^*_h \) the number of firms in industry \( k \) respectively in the home and the foreign country. Because industries are modeled in a symmetric way from both the demand and the supply sides (as will be seen below), there is the same number \( N \) of firms, or equivalently of varieties, in each industry: \( N = n_1 + n^*_1 = n_2 + n^*_2 \). Moreover, since our two countries share the same structural parameters, including size, they have a mirror-image production structure: \( n_1 = n^*_2 \) and \( n_2 = n^*_1 \).

Every firm produces a variety of good which is consumed by all households (final consumption) and used by all firms as a production input (intermediate consumption) in both countries. The production functions are the same for firms within the same industry, but differ between industries. They are written

\[
Y^j_l = A \left( L^j_1 \right)^{\beta} \left( X^j_{11} \right)^{\alpha} \left( X^j_{12} \right)^{\gamma} \\
Y^j_f = A \left( L^j_2 \right)^{\beta} \left( X^j_{21} \right)^{\gamma} \left( X^j_{22} \right)^{\alpha}
\]

where, for firm \( j \) in industry \( k \), \( Y^j_k \) is the production level, \( L^j_k \) the amount of labour employed (excluding the fixed labour input requirement \( F \) ), \( X^j_{kk'} \) the quantity index of intermediate goods bought from industry \( kk' \), and where \( \alpha, \beta \) and \( \gamma \) are strictly positive parameters such that \( \alpha + \beta + \gamma = 1 \). We choose to impose the condition \( \alpha > \gamma \), which says that linkages within industries are stronger than those between industries and seems a natural assumption to be made.

There is monopolistic competition in the goods market with the same constant elasticity of substitution (CES) between goods, noted \( \theta \) (with \( \theta > 1 \)), for both final and intermediate consumptions, so that \( X^j_{kk'} \) in particular is defined as

\[
X^j_{kk'} = \left[ \sum_{j'=1}^{N} \left( X^j_{kk'} \right)^{\frac{\theta}{\theta - 1}} \right]^{\frac{\theta - 1}{\theta}} \quad (k, k') \in \{1, 2\}^2,
\]

where \( X^j_{kk'} \) represents the amount of goods sold by firm \( j' \) in industry \( kk' \) to firm \( j \) in industry \( k \). Although firms buy themselves intermediate goods, we will assume they use no money.

The timing of events is the following: 1) firms choose their location, 2) wages are set, 3) a shock occurs, and 4) prices are set, production and consumption take place. Wages are thus assumed not to react to shocks, contrary to prices,
and this is the source of nominal rigidity in our model. Note that this nominal rigidity applies only in the short term. In the long term indeed, wages are flexible and depend on the pattern of location of the firms. In other words, steps 1 and 2 belong to the long term, while steps 3 and 4 belong to the short term.

At step 1, firms choose whether to produce their single variety in the home country or in the foreign country. Each firm chooses the production location which maximizes its expected profit. Firms thus behave as risk-neutral agents. We assume in the following that firms have no shareholders and redistribute their profits to their own workers, which is compatible with their risk-neutral behaviour. In the conclusion is discussed an alternative specification, based on the assumption that firms have risk-adverse shareholders who can perfectly diversify their portfolios.

At step 4, prices are set by firms, since we have monopolistic competition in the goods market. Following Obstfeld and Rogoff (2000), who empirically motivate their choice, we rule out local-currency-pricing (LCP). In other words, the price of a given good is assumed to be denominated in the producer's currency, as opposed to the buyer's. This ensures that the exchange rate can play its traditional Keynesian expenditure-switching role, because the exchange-rate pass-through to import prices is equal to one.

We adopt moreover the pricing-to-market (PTM) approach, that is to say that by assumption producers can charge a price for the home market and another for the foreign market. However, even though they can discriminate between countries, they choose not to. Indeed, as usual in monopolistic competition frameworks, the price charged by a given producer is equal to her marginal cost times a mark-up which depends only on the elasticity of substitution between goods. Now, because each producer faces by assumption the same elasticity $\theta$ in the home and the foreign markets, she chooses to charge the same price in both countries.

Once the maximization is done, we can drop all indices $j$, as in equilibrium all firms, conditionally to the industry they belong to, end up charging the same price

$$ P_1 = \frac{\theta}{2} (W_1)^{\theta} (G_1)^{\alpha} (G_2)^{\gamma} $$
$$ P_2 = \frac{\theta}{2} (W_2)^{\theta} (G_1)^{\gamma} (G_2)^{\alpha} $$

(4.2.1)

with the normalization $A = (\alpha \beta \gamma)^{-1}$, where $W_k$ and $G_k$ are respectively the nominal wage and the price index in industry $k$ (detailed below).
4.2.3 Households

Each country is composed of a given number $\eta$ of households, who supply labour and consume final goods. Households are assumed to be mobile across industries and immobile across countries, which is more or less the case in the Eurozone. The utility function of household $i$ is written

$$U^i = (C^i)^{\lambda} \left( \frac{M^i}{P} \right)^{1-\lambda},$$

where $C^i$ is her consumption level, $M^i$ her nominal money balance, $P$ the price index, and where $\lambda$ is a constant parameter ($0 < \lambda < 1$).\(^{12}\)

Note that labour $A^i \equiv L^i + F^i$ supplied by household $i$, where $L^i$ represents the productive part and $F^i$ the non-productive part (used for the fixed labour input requirement), does not enter the utility function above. If no further assumption were made, this would imply that households are indifferent *ceteris paribus* between working a lot for a low wage and working little for a high wage (such that labour income is identical in both alternatives). To rule out such a case, we impose the average amount of labour supplied by every household to be exogenously fixed at a given level, noted $\Lambda$:

$$\forall i, \quad E(A^i) = \Lambda. \quad (4.2.2)$$

Thus, in our framework, the only way labour affects the utility level of household $i$ is through fluctuations in labour income and therefore in consumption $C^i$, linked to labour income through the budget constraint. We assume that labour is demand-determined, workers supplying all the labour demanded by firms following the shock.

We also impose each household $i$ to work in only one firm, for a wage $W^i = W_k$ where $k$ is the industry the firm belongs to. In other words, households cannot have a diversified job portfolio. The budget constraint faced by household $i$ is thus the following:

$$M^i + PC^i = \overline{M}^i + PT^i + W^i A^i + \Pi^i, \quad (4.2.3)$$

where $\overline{M}^i$ and $M^i$ denote respectively her initial and final stocks of domestic currency, $T^i$ the lump-sum transfer she receives from the central bank (as defined below) and $\Pi^i$ the share of firms’ profits she receives.\(^{12}\)

\(^{12}\)The indirect utility function, derived from the initial utility function by using the first-order condition of households’ optimisation problem, will be linear in consumption. Households are therefore risk-neutral in our framework.
We adopt a "super home bias" assumption, under which every firm redistributes its profits equally among its own workers, so that $\Pi^i$ represents a certain share of the profit made by the firm which household $i$ works in. (An alternative way of modeling $\Pi^i$, shortly discussed in conclusion, consists in considering a possibly nationally and internationally diversified share portfolio owned by household $i$.)

Finally, the consumption index of household $i$ is written

$$C^i = (C_i^1)^\mu (C_i^2)^{1-\mu}$$

with

$$C_k^i = \left[ \sum_{j=1}^{N} \left( C_k^{ij} \right)^{\frac{1}{1-\mu}} \right]^\frac{1}{\mu} \quad k \in \{1, 2\},$$

where $C_k^i$ is household $i$'s consumption of goods from industry $k$, and $C_k^{ij}$ her consumption of goods of the variety produced by firm $j$ from industry $k$. Parameter $\mu$ is stochastic ($0 < \mu < 1$), drawn at each period, with mean $E(\mu) = 1/2$ and variance $V(\mu)$. We assume that its probability distribution is symmetric around 1/2, so that $\mu$ and $1-\mu$ have the same distribution.

Parameter $\mu$ corresponds to a temporary shock on the relative preference for goods from industries 1 and 2, and represents the only source of exogenous perturbations in the model. This shock is mentioned as an industry-specific shock in the following, though it is actually modeled for mere reasons of practical convenience as an industry-antisymmetric shock, in the sense that any increase in the demand faced by one industry is accompanied by a decrease of the same amount in the demand faced by the other industry.

As a consequence, the general consumer price index is

$$P = \frac{(G_1)^\mu (G_2)^{1-\mu}}{\mu^\mu (1-\mu)^{1-\mu}}.$$

### 4.2.4 Monetary policy

Household $i$ chooses $C^i$ so as to maximize $U^i$ subject to her budget constraint. The first-order condition of this optimization problem is

$$C^i = \frac{\lambda}{1-\lambda} M^i.$$  \hspace{1cm} (4.2.4)

Suppose that $P$ does not react to variations in the money stock. Then the monetary policy transmission mechanism works as follows: in case of an unexpected increase in money supply, each consumer is willing to exchange money
for goods (to restore the first order condition of her optimisation problem), and this results in an increase in the aggregate demand for goods. Then the supply of goods adjusts, raising consumption, until the demand for money equals the money supply.

Now, there are actually some cases where $P$ does react to variations in the money stock. Indeed, under a floating exchange rate regime, money supply affects the nominal exchange rate and hence the price of imported intermediate goods. This in turn results in a change in marginal costs, and prices are adjusted accordingly.

Under a fixed exchange rate regime however, or when countries form a monetary union, firms face constant marginal costs, because then neither the price of imported intermediate goods (due to the fixity of the exchange rate) nor nominal wages (rigid by assumption) vary. As firms also face a constant elasticity of substitution between goods, they have no incentive to change their prices. In that case, though perfectly flexible, prices do not react to the shock nor to monetary policy, so that they could be set at any time after step 2.

This in turn implies that holds then a well-known property of models with monopolistic competition in the goods market, with rigid nominal wages and without intermediate goods, namely that output is demand-determined. Indeed, prices do not react to a demand shock and since they are strictly higher than marginal costs, firms have an incentive to adjust supply so as to meet demand.

Monetary policy is conducted at the national level in case of a flexible or fixed exchange rate regime (one central bank per country) and at the supranational level in case of a monetary union (one central bank for both countries). The monetary authorities are assumed to rebate all lump-sum transfers in the form of money:

$$ PT^i = M^i - \bar{M}^i. $$

(4.2.5)

Though our framework enables us to consider optimal monetary policies, we choose not to and expose below the reasons why. Monetary policy is optimal when the monetary authorities aim at implementing the egalitarian social optimum, which amounts to maximize the sum of households' utility levels. This optimisation problem is constrained in the sense that monetary policy rules are being considered. In other words, there is no scope for monetary surprises to offset monopolistic distortions, and monetary policy aims only at stabilizing the economy.

Considering utility-maximizing monetary policies would enable us to address the issue of (the lack of) cooperation between central banks. We could then
compute the Nash equilibrium under a floating exchange rate regime, and the Stackelberg equilibrium under a fixed exchange rate regime, with one central bank acting as a leader—call it Buba—and the other central bank as a follower, as may have been the case under the European Monetary System (EMS).

Instead, we choose not to consider optimal monetary policies, so that in most of the exchange rate regimes examined in the following, money supply is exogenous and, as we do not introduce monetary shocks, kept constant. In other words, monetary authorities are passive. We consider three distinct exchange rate regimes: a monetary union, a floating exchange rate regime, and an (asymmetrically) fixed exchange rate regime. Only in the latter case is money supply endogenous (though not utility-maximizing), in the country which unilaterally pegs its currency to the other country’s.

The main reason why we choose not to consider optimal monetary policies is that in our framework these policies turn out to be pro-cyclical under a floating exchange rate regime when central banks do not cooperate and when countries are specialized. This result can be easily obtained by first expressing $U$ as a function of $M$, $M^*$ and $\mu$, then using the lagrangian method to solve the home country central bank optimization problem, that is to say to choose a monetary policy rule, the function $\mu \mapsto M(\mu)$, so as to maximize $E[U(\mu,M(\mu),M^*(\mu))]$ subject to $E[M(\mu)] = \bar{M}$ and considering the function $\mu \mapsto M^*(\mu)$ as given.

To see why this leads to pro-cyclical monetary policies, consider the case where countries are fully specialized, with all of industry 1 concentrated in the home country. Then the elasticity of the consumer price index $P$ with respect to the nominal exchange rate is decreasing in $\mu$, because less foreign goods are consumed when there is a positive demand shock on the home country’s products. Thus the nominal currency depreciation following an expansionary monetary policy fuels domestic inflation all the less than $\mu$ is large. As equation (4.2.4) enables us to express the (indirect) utility level $U$ as a function of $\frac{M}{P}$ only, monetary policy efficiency increases with $\mu$, so that monetary authorities have an incentive to increase $M$ following a positive shock on $\mu$.

Utility-maximizing monetary policies can therefore be pro-cyclical in our framework, so that we choose not to consider them. (As will become clear later on, taking them into account would actually strengthen our results at the expense of greater mathematical complexity.) Under a floating exchange rate regime, central banks are therefore assumed not to react to whatever shock on

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18Such an approach is standard in the New Open Economy Macroeconomics literature, where monetary policy is often considered as a source of shocks rather than as a stabilization tool.
\( \mu \) occurs and to leave their money stocks unchanged. The nominal exchange rate is then assumed to adjust so as to balance international trade\(^{14}\).

### 4.2.5 International trade

We assume that shipping a good from one country to the other is costly. This transport cost, noted \( T \), is modelized as an iceberg cost (\( T \geq 1 \)). Despite the full implementation of the Single Market Process and despite technological innovations, there remain some obstacles to international trade within Europe, which \( T \) can account for\(^{15}\).

We note \( e \) the nominal exchange rate, defined as the price of the foreign country currency in terms of the home country currency. Thus the (consumer) price index in industry \( k \) is written:

\[
G_k = \left[ n_k (P_k)^{1-\theta} + n_k^* (P_k^* T e)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad k \in \{1,2\}. \tag{4.2.6}
\]

To close the model, we only need now to specify goods market clearing conditions, which ensure that international trade is balanced. To that aim, we first have to determine total expenditures in each country on each industry. Consider a firm in a given industry employing a labour force \( l \) (excluding the fixed labour input requirement) paid at wage \( w \). Profit-maximization, or equivalently price-setting, then gives a profit equal to \( \frac{wl}{\beta(\theta - 1)} - wF \). In the specification without shareholding, profits are redistributed within firms, so that households' revenue from working in that firm and receiving its profit is \( w(1 + F) + \frac{wl}{\beta(\theta - 1)} - wF = H(1,0)wl \), where function \( H \) is defined as:

\[
H(z_1, z_2) = \frac{1 + \beta(\theta - 1)}{\beta(\theta - 1)} z_1 + \frac{z_2}{\beta}.
\]

All households working in a given firm supply the same \( L_i \) and \( F_i \), so that the revenue per household from working in that firm and receiving its profit is \( H(1,0)W^iL^i \). Thus, using (4.2.5), the budget constraint (4.2.3) simplifies to:

\[
PC_i = H(1,0)W^iL^i. \tag{4.2.7}
\]

\(^{14}\)An alternative modelling strategy would consist in removing money from the representative household's utility function and assuming that the central bank conducts an exchange rate policy - quite a reasonable assumption in our opinion, given the absence of non-traded goods in our model. Indeed, if it perfectly controls the nominal exchange rate (when flexible), the central bank will choose its value such that international trade is balanced, because this value is precisely the one which offsets the impact of the shocks on real variables, as will be seen later on.

\(^{15}\)Should this transport cost not exist in our model (\( T = 1 \)), the location of economic activity would then be indeterminate under any of the exchange rate regimes considered here, as will be seen below.
Moreover, costs minimization leads this firm to spend \( \frac{w_l}{K} \) on intermediate goods from the same industry and \( \frac{w_l}{K} \) on intermediate goods from the other industry, so that summing intermediate and final consumptions, we obtain the following expressions for \( E_k \), total expenditure in the home country on industry \( k \):

\[
E_1 = H (\mu, \alpha) n_1 W_1 L_1 + H (\mu, \gamma) n_2 W_2 L_2,
\]

\[
E_2 = H (1 - \mu, \gamma) n_1 W_1 L_1 + H (1 - \mu, \alpha) n_2 W_2 L_2,
\]

where \( L_k \) is the amount of labour employed (excluding the fixed labour input requirement) by each firm in industry \( k \).

Now consider a firm in industry \( k \), charging price \( p \) and employing a labour force \( l \) (excluding the fixed labour input requirement) paid at wage \( w \). The non-fixed part of its production costs being \( \frac{w_l}{K} \), the nominal value of its production is \( \frac{w_l}{\beta} \). Total expenditure on that variety, expressed in the home country currency, is \( p^{1-\theta} (G_k)^{\theta -1} E_k \) in the home country and \( (p_F^*)^{1-\theta} (G_k^*)^{\theta -1} e E_k^* \) in the foreign country. Goods market clearing implies for that variety that total expenditure should be equal to the nominal value of production, so that we eventually obtain:

\[
\frac{\theta}{(\theta - 1)} \frac{W_k L_k}{\beta} = (P_k)^{1-\theta} [G_k^{\theta -1} E_k + G_k^{\theta -1} T^{1-\theta} e^{\theta} E_k^*] \quad k \in \{1, 2\}. \tag{4.2.9}
\]

The presentation of the model is now complete. Solving this model analytically in the general case proves both mathematically complex and of little interest, in that it does not help to understand the mechanisms at work. We therefore choose to focus in the next sections on two degenerated cases: concentration (defined as each industry entirely located in one country) and dispersion (defined as half of each industry in each country).

Note that our framework is thus very close to that of the father of the OCA theory: Mundell (1961) considers first a shift in demand from the goods produced in a country to the goods produced in another (this corresponds to our concentration case), then a shift in demand from goods like lumber products, produced in the Western parts of both Canada and the United States, to goods like cars, produced in the Eastern parts of both countries (our dispersion case).

### 4.3 The sustainability of EADS

This section focuses on the effect of various exchange rate regimes on the sustainability of endogenously asymmetric demand shocks (EADS). In other words, for
each exchange rate regime we derive here the conditions for which concentration (defined as each industry entirely located in one country) becomes sustainable.

4.3.1 Concentration

Concentration corresponds here to all of industry 1 in the home country and all of industry 2 in the foreign country. Thus \( N = n_1 = n_2^* \) and \( n_1^* = n_2 = 0 \), so that consumer price equations (4.2.6) become

\[
G_1 = N^{1+\gamma} P_1, \quad G_2 = N^{1+\gamma} P_2 T \varepsilon, \\
G_1^* = N^{1+\gamma} P_1^* T, \quad G_2^* = N^{1+\gamma} P_2^*.
\]

Total expenditures (4.2.8), in each country and for each industry, are now written

\[
E_1 = NW_1 L_1 H (\mu, \alpha), \quad E_2 = NW_1 L_1 H (1 - \mu, \gamma), \\
E_1^* = NW_2^* L_2^* H (\mu, \gamma), \quad E_2^* = NW_2^* L_2^* H (1 - \mu, \alpha),
\]

and using (4.3.10), the goods market clearing conditions (4.2.9) simplify to

\[
\frac{\theta}{\theta - 1} \frac{NW_1 L_1}{\beta} = E_1 + \varepsilon E_1^*, \quad \frac{\theta}{\theta - 1} \frac{NW_2^* L_2^*}{\beta} = \frac{1}{\varepsilon} E_2 + E_2^*,
\]

so that we obtain the following balanced trade condition:

\[
\frac{W_1 L_1}{W_2^* L_2^*} = \frac{H (\mu, \gamma)}{H (1 - \mu, \gamma)}. \quad (4.3.12)
\]

Moreover, we know that \( \Pi_1 = \frac{W_1 L_1}{\beta (\theta - 1)} - FW_1 \) and that free entry implies \( E (\Pi_1) = 0 \), with similar equations for the foreign country, so that we get:

\[
E (L_1) = E (L_2^*) = F \beta (\theta - 1). \quad (4.3.13)
\]

In order to derive the general conditions for the sustainability of concentration, we need to consider the case of one firm from industry 2 settling in the home country. Using (4.2.1), (4.3.10) and (4.3.11), the corresponding goods market clearing condition in (4.2.9) is written:

\[
\frac{\theta}{(\theta - 1)} \frac{W_2 L_2}{\beta} = \left( \frac{W_2^*}{W_2} \right)^{\beta (\theta - 1)} T^{-(\theta - 1)(\alpha - \gamma)} \varepsilon^\beta (\theta - 1) \left[ T^{\theta - 1} W_1 L_1 H (1 - \mu, \gamma) + T^{-(\theta - 1)} \varepsilon W_2^* L_2^* H (1 - \mu, \alpha) \right].
\]
This firm makes a profit equal to $\Pi_2 = \frac{W_2 L_2}{\beta (\theta - 1)} - FW_2$, and free entry implies $E [\Pi_2] = 0$, so that

$$E (L_2) = F \beta (\theta - 1).$$

(4.3.14)

Thus we eventually get the following wage ratio equation:

$$\left( \frac{W_2}{W_1} \right)^{\beta (\theta - 1)+1} = \frac{T^{-(\theta - 1)(\alpha - \gamma)}}{\theta F} \left( \frac{W_2^*}{W_1^*} \right)^{\beta (\theta - 1)} E \left[ e^{\beta (\theta - 1)} \left( T_{\theta - 1} L_1 \right. \right. \\
H (1 - \mu, \gamma) + T^{-(\theta - 1)} \frac{W_2^*}{W_1^*} e^{\gamma L_2^*} \left(1 - \mu, \alpha \right) \left] \right].$$

(4.3.15)

Now consider household $i$ in the home country. She can be employed either in one firm from industry 1, supplying the amount $\Lambda_1^i = L_1^i + F_1^i$ of labour for a wage $W_1$, or in that firm from industry 2, working $\Lambda_2^i = L_2^i + F_2^i$ for $W_2$. She will choose the first alternative if and only if it provides her a higher expected utility level than the second alternative. Given equation (4.2.7), the necessary and sufficient condition for concentration to be sustainable is thus $E [W_2 L_2^i] \leq E [W_1 L_1^i]$.

Equations (4.3.13) and (4.3.14) imply that the firm considered (from industry 2) employs the same average amount of productive labour as firms from industry 1: $E (L_2) = E (L_1)$. Since they also face the same fixed labour input requirement $F$, they employ the same average total amount of labour, and therefore the same number of workers, due to equation (4.2.2). This in turn implies $E (L_2) = E (L_1)$, and the condition for the sustainability of concentration can therefore be rewritten: $W_2 \leq W_1$.

Equation (4.3.15) is therefore central to our analysis. Concentration is sustainable if and only if the right-hand side of this wage ratio equation is lower than one. In the next subsections, we examine how this wage ratio equation can be rewritten depending on the exchange rate regime considered.

### 4.3.2 Monetary union

When the countries form a monetary union, the exchange rate is fixed and normalized to one: $\epsilon = 1$. Besides, there is one (common) central bank, which sets $M + M^* = \overline{M} + \overline{M}^*$. In each country, the money stock is linked to consumption through the first-order condition (4.2.4) of the households’ optimization problem. Consumption in turn is related to income through the budget constraint (4.2.7) faced by households, so that in the end we can write:
\[ W_1 L_1 + W_2^* L_2^* = \frac{\lambda (\overline{M} + \overline{M}^*)}{N (1 - \lambda) H (1, 0)}. \]

Together with equation (4.3.12), this leads to
\[
\begin{align*}
W_1 L_1 &= \frac{\lambda (\overline{M} + \overline{M}^*)}{2N (1 - \lambda) H (1, 0) H (\frac{1}{2}, \gamma)} H (\mu, \gamma), \\
W_2^* L_2^* &= \frac{\lambda (\overline{M} + \overline{M}^*)}{2N (1 - \lambda) H (1, 0) H (\frac{1}{2}, \gamma)} H (1 - \mu, \gamma).
\end{align*}
\] (4.3.16)

The profit made by each firm in the home country is equal to \( \Pi_1 = \frac{W_1 L_1}{\beta(\theta - 1)} - FW_1 \), with free entry implying \( E[\Pi_1] = 0 \), and similarly in the foreign country, so that using (4.3.16), we obtain:

\[ W_1 = W_2^*, \]

\[
L_1 = F \beta (\theta - 1) \frac{H (\mu, \gamma)}{H (\frac{1}{2}, \gamma)}, \quad L_2^* = F \beta (\theta - 1) \frac{H (1 - \mu, \gamma)}{H (\frac{1}{2}, \gamma)}. \] (4.3.17)

The wage ratio equation (4.3.15) thus becomes:

\[ \left( \frac{W_2}{W_1} \right)^{\theta (\theta - 1) + 1} = \frac{\beta (\theta - 1) T - (\theta - 1)(\alpha - \gamma)}{H (\frac{1}{2}, \gamma)} E \left[ H (1 - \mu, \gamma) \right] \left[ T^{\theta - 1} H (\mu, \gamma) + T - (\theta - 1) H (1 - \mu, \alpha) \right]. \]

### 4.3.3 Floating exchange rate regime

National central banks remain passive: \( M = \overline{M} \) and \( M^* = \overline{M}^* \). To simplify, we choose to normalise \( \overline{M} = \overline{M}^* \) without any loss of generality. Using (4.2.4) and (4.2.7), we get

\[
\begin{align*}
W_1 L_1 &= \frac{\lambda \overline{M}}{N (1 - \lambda) H (1, 0)}, \\
W_2^* L_2^* &= \frac{\lambda \overline{M}}{N (1 - \lambda) H (1, 0)},
\end{align*}
\] (4.3.18)

so that zero-profit conditions (holding on average) now lead to:

\[ W_1 = W_2^*, \]

\[ L_1 = L_2^* = F \beta (\theta - 1). \]
Together, (4.3.18) and (4.3.12) imply that the nominal exchange rate is 
\[ \varepsilon = \frac{H(1 - \mu, \gamma)}{H(\mu, \gamma)}. \]

The wage ratio equation (4.3.15) therefore becomes:
\[
\left( \frac{W_2}{W_1} \right)^{\delta(\theta-1)+1} = \frac{\beta(\theta - 1)}{\theta} T^{-(\theta-1)(\alpha-\gamma)} E \left[ \frac{H(1 - \mu, \gamma)}{H(\mu, \gamma)} \right]^{\delta(\theta-1)+1}
\left[ T^{\theta-1} H(\mu, \gamma) + T^{-(\theta-1)} H(1 - \mu, \alpha) \right].
\]

### 4.3.4 Fixed exchange rate regime

We consider the case where the foreign country unilaterally pegs its currency to the home country's. Thus the home country central bank remains passive \((M = \overline{M})\), while the foreign country's adjusts its money supply so as to maintain the parity between the two currencies. We note \(\overline{M^*} = E(\overline{M^*})\). The fixed exchange rate is normalized to one: \(\varepsilon = 1\).

Following the same steps as previously with the floating exchange rate regime, we easily reach:

\[ \frac{W_1}{\overline{M}} = \frac{W_2}{\overline{M^*}}, \]

\[ L_1 = F \beta(\theta - 1), \quad L_2 = F \beta(\theta - 1) \frac{\overline{M^*}}{\overline{M}}, \]

\[ \overline{M^*} = \overline{M} \frac{H(1 - \mu, \gamma)}{H(\mu, \gamma)}, \quad \overline{M^*} = \overline{M} E \left[ \frac{H(1 - \mu, \gamma)}{H(\mu, \gamma)} \right]. \]

The wage ratio equation is therefore:
\[
\left( \frac{W_2}{W_1} \right)^{\delta(\theta-1)+1} = \frac{\beta(\theta - 1)}{\theta} T^{-(\theta-1)(\alpha-\gamma)} \left( E \left[ \frac{H(1 - \mu, \gamma)}{H(\mu, \gamma)} \right] \right)^{\delta(\theta-1)}
E \left[ \frac{H(1 - \mu, \gamma)}{H(\mu, \gamma)} \right] \left[ T^{\theta-1} H(\mu, \gamma) + T^{-(\theta-1)} H(1 - \mu, \alpha) \right].
\]

Had the home country been the one unilaterally pegging its currency to the foreign country's, we would have obtained instead as a wage ratio equation:
\[
\left( \frac{W_2}{W_1} \right)^{\beta(\theta-1)+1} = \frac{\beta(\theta - 1)}{\theta} T^{-((\theta-1)(\alpha - \gamma))} \left( E \left[ \frac{H(\mu, \gamma)}{H(1 - \mu, \gamma)} \right] \right)^{-\beta(\theta-1)-1} \\
T^{\theta-1} \left( \frac{1}{2}, \gamma \right) + T^{-((\theta-1))} H \left( \frac{1}{2}, \alpha \right) .
\]

### 4.3.5 Benchmark case

In addition to the various exchange rate regimes examined above, we consider the deterministic case, corresponding to \( V(\mu) = 0 \), as a benchmark case. In effect, this amounts more or less to replicate Fujita, Krugman and Venables' (1999, ch. 16) results\(^{16}\). The wage ratio equation in this benchmark case is obtained by replacing \( \mu \) by \( \frac{1}{2} \) in any of the wage ratio equations above:

\[
\left( \frac{W_2}{W_1} \right)^{\beta(\theta-1)+1} = \frac{\beta(\theta - 1)}{\theta} T^{-((\theta-1)(\alpha - \gamma))} \left[ T^{\theta-1} \left( \frac{1}{2}, \gamma \right) + T^{-((\theta-1))} H \left( \frac{1}{2}, \alpha \right) \right] .
\]

### 4.3.6 Sustain points

This subsection summarizes, interprets and discusses the results obtained above, from a positive point of view. In the following, superscripts refer either to the exchange rate regime considered: \( mu \) (monetary union), \( fg \) (floating exchange rate regime), \( fd \) (fixed exchange rate regime); or to the benchmark case, noted \( bk \). As is clear from the wage ratio equations, there exists in each of these cases a threshold value \( T(S) \) such that concentration is sustainable for all \( T \leq T(S) \) and unsustainable for all \( T \geq T(S) \)\(^{17}\). We call this threshold value the sustain point and we note it \( T^{mu}(S) \), \( T^{fg}(S) \), \( T^{fd}(S) \) or \( T^{bk}(S) \), depending on the case considered (\( mu, fg, fd \) or \( bk \))\(^{18}\).

The existence of such a sustain point is familiar to the New Economic Geography literature. In the usual deterministic framework, corresponding here

\(^{16}\)More or less do we say, because even then our framework still differs from theirs on one point: they choose to express the fixed cost of production in terms of the composite input (including labour and intermediate goods), whereas we choose to express it in terms of labour only.

\(^{17}\)This says in particular that in all the four cases considered (\( mu, fg, fd \) and \( bk \)), concentration is not sustainable for sufficiently high transport costs. In other words, there is no need for imposing a no-black-hole condition on the parameters, as is done in Fujita, Krugman and Venables (1998), to rule out the possibility for concentration to be sustainable for all \( Ts \). Once again, the (rather minor) differences between their results and ours in the \( bk \) case are due to different assumptions on the nature of the fixed cost of production.

\(^{18}\)In the (asymmetric) \( fd \) case, where one country unilaterally pegs its currency to the other, concentration is no longer sustainable as soon as it is no longer sustainable in either country, so that \( T^{fd}(S) \) represents the lowest of the two sustain points defined by the two wage ratio equations.
to the benchmark case \((bk)\), two concentration forces can be identified. They correspond to the incentive for a firm to settle in the neighbourhood of other firms of the same industry in order to benefit there from cheaper intermediate goods (forward linkage) and from a larger market (backward linkage). Both these concentration forces exist here because of our assumption that linkages within industries are stronger than those between industries \((\alpha > \gamma)\).

Opposite to these concentration forces is a dispersion force, which consists in a local competition effect: firms of a given industry have an incentive to settle in the country not specialized in this industry, in order to supply this market at a lower price than its competitors from the other country. What happens then is that the concentration forces are stronger than the dispersion force at low values of \(T\), and that inversely the dispersion force dominates the concentration forces at high values of \(T\); hence the existence of a sustain point.

Things change when industry-specific demand shocks are considered. Traditional concentration and dispersion forces still apply obviously, but new forces enter the stage too. Let us first focus on the \(mu\) case. As can be seen above, as far as employment is concerned, a monetary union does not smooth asymmetric shocks. When a positive demand shock on industry 1 occurs \((\mu > \frac{1}{2})\), a firm of industry 1 located in the home country benefits from a substitution effect (demand shifts from goods of industry 2 to goods of industry 1) as well as from an income effect (the firm's local market gets larger as the home country, specialized in industry 1, gets wealthier, and this matters in the presence of transport costs).

The conjunction of this substitution effect and this income effect makes the term \(\mu^2\) enter the profit function of this firm, turning this profit function into a function convex in \(\mu\), so that the average profit is increased. This in turn gives firms of the same industry the incentive to settle in the same country, and the induced competition between them raises \(W_1\) relatively to \(W_2\) until their average profit goes back to zero. Households in the home country are therefore attracted into industry 1: a new concentration force arises.

Now consider the \(fg\) case. As can be seen above, as far as employment is concerned, shocks are perfectly smoothed in both countries under a floating exchange rate regime. As (say) a positive demand shock occurs in the home country \((\mu > \frac{1}{2})\), the substitution effect still holds, but the income effect mentioned above disappears, as employment is kept constant thanks to an appreciation of the home country currency (a decrease in \(e\)). This implies that the concentration force identified above in the \(mu\) case does not apply here in the \(fg\) case.
Instead, another mechanism is at work, and a new dispersion force arises, which stems from the conjunction of two substitution effects. Consider indeed a firm of industry 1 located in the foreign country, specialized in industry 2. A shock $\mu > \frac{1}{2}$ makes households spend relatively more on the goods of industry 1, and makes also the foreign country currency depreciate. The conjunction of these two substitution effects turns the profit of our firm into a convex function of $\mu$ and therefore increases its average profit. Firms of industry 1 face therefore an incentive to locate in the foreign country, and competition between them raises $W_1^*$ relatively to $W_2^*$ until the average profit goes back to zero. Households in the foreign country are therefore attracted into industry 1: a new dispersion force arises.

To summarize, the occurrence of industry-specific demand shocks within a monetary union adds one concentration force to the benchmark case, and the occurrence of industry-specific demand shocks under a floating exchange rate regime adds one dispersion force to the benchmark case. One should therefore expect concentration to be more sustainable within a monetary union than under a floating exchange rate regime, the benchmark case being intermediate.

And such is the case indeed. From the wage ratio equations above, it can be shown that $T^{fe}(S) \leq T^{fd}(S) \leq T^{bk}(S) \leq T^{mu}(S)$. This result, displayed in figure 4.1, says that the range of parameter $T$ for which concentration is sustainable is the largest in the $mu$ case, then in the $bk$ case, then in the $fd$ case and finally in the $fg$ case. In other words, a monetary union tends to strengthen the sustainability of EADS, compared to a fixed exchange rate regime and (even more) to a floating exchange rate regime.

The case of a fixed exchange rate regime happens to be intermediate between the $fg$ and $mu$ cases. This is because the dispersion force present in the $fg$ case disappears here, as the exchange rate is fixed, and the concentration force present in the $mu$ case is weakened, as shocks are not smoothed in the country which unilaterally pegs its currency, but are in the other country.

4.3.7 Welfare analysis

This subsection draws some normative implications of the results obtained above. Using the first-order condition (4.2.4) of the households' optimization problem, we can express the utility level $U$ as a linear function of consumption $C$, itself a function of $C_1$ and $C_2$. The latter are derived by using $G_1C_1 = H(\mu, 0) n_1W_1L_1$, $G_2C_2 = H(1 - \mu, 0) n_1W_1L_1$, and getting $G_1$ and $G_2$ out of equations (4.3.10) and (4.2.1). Noting that $N = \frac{\eta^A}{\eta^A + \eta^{(1-N)}}$, we eventually obtain the utility level $U$ as a function of the exogenous parameters. We
reach in the benchmark case:

\[ U_{conc}^{bk} = \left( \frac{1 - \lambda}{\lambda} \right)^{1-\lambda} (\eta \Lambda)^{H(1,0)} [F[1 + \beta (\theta - 1)]]^{\mu(\frac{1}{2} - \tau)} \left( \frac{\theta}{\theta - 1} \right)^{-\frac{\tau}{\theta}} \frac{T^{-(\frac{1}{1+\frac{\tau}{\theta}})}}{2}, \]

and in the monetary union case:

\[ U_{conc}^{mu} = U_{conc}^{bk} 2\mu (1 - \mu) \left( \frac{1 - \mu}{\frac{1}{2}} \right) T^{\mu - \frac{1}{2}} \frac{H(\mu, \gamma)}{H(\frac{1}{2}, \gamma)}. \] (4.3.19)

From these equations, one can show that \( U_{conc}^{bk} < E[U_{conc}^{mu}] \). When countries are fully specialized, the occurrence of industry-specific demand shocks within a monetary union is welfare-improving. There are two reasons which explain this result. One is that the differentiated good which matters less in the utility function (by definition of an industry-specific demand shock) is less consumed. This consumption basket effect corresponds to the factor \( 2\mu (1 - \mu) \) in equation (4.3.19), which is larger than one whatever \( \mu \) in-between zero and one.

The other reason why \( U_{conc}^{bk} < E[U_{conc}^{mu}] \) stems from the conjunction of the substitution effect and the income effect identified in the previous subsection. When \( \mu > \frac{1}{2} \), the substitution effect lowers the consumer price index in the home country, as producer prices remain the same and as goods of industry 2, more expensive than goods of industry 1 because produced abroad, matter less; this corresponds to the factor \( T^{\mu - \frac{1}{2}} \) in equation (4.3.19). And when \( \mu > \frac{1}{2} \), the income effect consists in an increase in labour income in the home country, as shown by equation (4.3.17); this corresponds to the factor \( \frac{H(\mu, \gamma)}{H(\frac{1}{2}, \gamma)} \) in equation (4.3.19). The product of these two factors is a convex function of \( \mu \) and has therefore a strictly positive average impact on \( U_{conc}^{mu} \).

In the floating exchange rate regime case, computations (carried out in the same way as above) lead to the following result:

\[ U_{conc}^{fs} = U_{conc}^{bk} 2\mu (1 - \mu) \left( \frac{1 - \mu}{\frac{1}{2}} \right) T^{\mu - \frac{1}{2}} \frac{H(\mu, \gamma)}{H(1 - \mu, \gamma)}. \] (4.3.20)

The comparison between \( E[U_{conc}^{fs}] \) on the one hand and \( U_{conc}^{bk} \) or \( E[U_{conc}^{mu}] \) on the other hand proves mathematically complex. One can show however that when \((\beta - 2\gamma) H(1, 0) + 4\gamma \geq 0\), we have \( \forall \mu, U_{conc}^{fs} < U_{conc}^{mu} \), so that \( E(U_{conc}^{fs}) < E(U_{conc}^{mu}) \); on the contrary, when \((\beta - 2\gamma) H(1, 0) + 4\gamma \leq -\frac{H(1, 0)^2}{(\theta - 1)H(1, \gamma)} \), we have \( \forall \mu, U_{conc} < U_{conc}^{fs} \), so that \( E(U_{conc}^{mu}) < E(U_{conc}^{fs}) \).
In equation (4.3.20) can be identified the same consumption basket effect (factor $2\mu^{(1 - \mu)}$) as in equation (4.3.19), as well as the same substitution effect (factor $T^{\mu - \frac{1}{2}}$), even though producer prices now vary with the nominal exchange rate. The last factor in equation (4.3.20) corresponds to the contribution of the nominal exchange rate, as $\varepsilon = \frac{H(1 - \mu, \gamma)}{H(\mu, \gamma)}$. When $\mu > \frac{1}{2}$, the home country currency appreciates ($\varepsilon < 1$), so that households can afford more foreign goods. However, when $\mu > \frac{1}{2}$ households are also less willing to consume foreign goods, so that in the end the exchange rate effect is ambiguous at first sight.

4.4 The emergence of EADS

This section focuses on the effect of various exchange rate regimes on the emergence of EADS. In other words, for each exchange rate regime we derive here the conditions for which dispersion (defined as half of each industry in each country) becomes unstable.

4.4.1 Dispersion

Dispersion corresponds to half of each industry established in each country. We note $n = n_1 = n_2^* = n_1^* = n_2 = \frac{N}{2}$. Thus both countries are perfectly identical to each other. This implies that even when flexible, the exchange rate keeps equal to one, whatever the shock on $\mu$ (provided the normalisation $\overline{M} = \overline{M}^*$ is done). Moreover, in this section, we choose for the moment not to consider the case of an asymmetrically fixed exchange rate regime, focusing only on the case of a monetary union (noted $mu$), that of a floating exchange rate regime (noted $fg$), and the benchmark case (noted $bk$). We therefore have by symmetry: $W_1 = W_2 = W_1^* = W_2^*$. Together with equations (4.2.1) and (4.2.6), this implies that whatever the case considered (be it $bk$, $mu$ or $fg$), producer prices and consumer price indexes are equal across industries and across countries. We note $P = P_1 = P_2 = P_1^* = P_2^*$ and $G = G_1 = G_2 = G_1^* = G_2^*$.

Labour employed is derived in the same way as when each industry is concentrated in only one country, by using equations (4.2.9), (4.3.13), (4.2.4), (4.2.7), and noticing that by symmetry $L_k = L_k^*$ for $k \in \{1, 2\}$. We thus reach, whatever the case considered (be it $bk$, $mu$ or $fg$):

$$L_1 = L_1^* = F \beta (\theta - 1) \frac{H(\mu, \gamma)}{H(\frac{1}{2}, \gamma)}.$$
\[ L_2 = L^*_2 = F \beta (\theta - 1) \frac{H (1 - \mu, \gamma)}{H \left( \frac{1}{2}, \gamma \right)}. \]

In order to examine the stability of dispersion, we consider a change in \( n_1 \), noted \( dn_1 \), leading to a change in \( W_1 \), noted \( dW \). Because we do not consider the case of an asymmetrically fixed exchange rate regime in this section, we have, by symmetry, \( dn_1 = -dn_2 = -dn_1^* = dn_2^* \) and \( dW_1 = -dW_2 = -dW_1^* = dW_2^* \). Dispersion will be an equilibrium, i.e. it will be stable, if and only if \( \frac{dW}{dn} \leq 0 \).

### 4.4.2 Benchmark case

This case corresponds to \( V(\mu) = 0 \). By symmetry we have \( \frac{dL_1}{p} = -\frac{dL_2}{p} = -\frac{dL_1^*}{p} = -\frac{dL_2^*}{p} \) and \( \frac{dL_1}{\nu} = -\frac{dL_2}{\nu} = -\frac{dL_1^*}{\nu} = -\frac{dL_2^*}{\nu} \). We note \( \frac{dL}{p} = \frac{dL^*}{p} \) and \( \frac{dL}{\nu} \equiv \frac{dL^*}{\nu} \). Using the producer price equations (4.2.1) and the consumer price index equations (4.2.6), we easily get \( \frac{dL}{p} \) and \( \frac{dL^*}{\nu} \) as functions of \( \frac{dn}{n} \) and \( \frac{dW}{W} \) only (since \( \frac{dL}{\nu} = 0 \)).

We then differentiate one of the goods market clearing conditions (4.2.9), noticing that \( dE_k = dE_k^* = 0 \) for \( k \in \{1, 2\} \), because \( \mu = \frac{1}{2} \) and because \( dL_k = dL_k^* = 0 \) for \( k \in \{1, 2\} \) (due to the zero profit condition (4.3.13)). This differentiated goods market clearing condition corresponds to a relationship between \( \frac{dL}{p}, \frac{dL^*}{\nu} \) and \( \frac{dW}{W} \), so that using the expressions of \( \frac{dL}{p} \) and \( \frac{dL^*}{\nu} \) previously obtained (as functions of \( \frac{dn}{n} \) and \( \frac{dW}{W} \) we finally reach:

\[ Q^{bk} (Z) \frac{dW}{W} = R^{bk} (Z) \frac{dn}{n} \]

where

\[ Z \equiv \frac{1 - T^{1 - \theta}}{1 + T^{1 - \theta}}, \]

\[ Q^{bk} (Z) \equiv 1 + \beta (\theta - 1) - (\alpha - \gamma) \left( \frac{2\theta - 1}{\theta} \right) Z + (\theta - 1) \left[ \left( \frac{\alpha - \gamma}{\theta} \right)^2 - \beta \right] Z^2, \]

\[ R^{bk} (Z) \equiv (\alpha - \gamma) \left( \frac{2\theta - 1}{\theta} \right) Z - \left[ 1 + (\alpha - \gamma)^2 \left( \frac{\theta - 1}{\theta} \right) \right] Z^2. \]

### 4.4.3 Monetary union

Since \( \frac{dx}{c} = 0 \), we get \( \frac{dL}{p} = \frac{dL^*}{p} = -\frac{dL}{p} = -\frac{dL^*}{p} = \frac{dL}{\nu} \) and \( \frac{dL}{\nu} \equiv \frac{dL^*}{\nu} \) as functions of \( \frac{dn}{n} \) and \( \frac{dW}{W} \) exactly in the same way as in the benchmark case.
We then differentiate one of the goods market clearing conditions (4.2.9). This time however, \( dE_k \neq 0 \) and \( dE^*_k \neq 0 \) for \( k \in \{1, 2\} \), because \( dL_1 + dL_2 = -(dL^*_1 + dL^*_2) = k(2\mu - 1) \), where \( k \) depends on \( \frac{dn}{n}, \frac{dW}{W}, \alpha, \gamma, \theta \) and \( T \), as can be shown with the help of the other goods market clearing conditions.

We finally reach:

\[
Q^{mu} (Z) \frac{dW}{W} = R^{mu} (Z) \frac{dn}{n}
\]

where

\[
Q^{mu} (Z) \equiv Q^{bk} (Z) + 2\beta \frac{(\theta - 1)^2}{\theta} Z (1 + Z) \frac{[H(1,0)]^2}{H \left( \frac{1}{2}, \gamma \right)} V(\mu),
\]

\[
R^{mu} (Z) \equiv R^{bk} (Z) + 2\beta \frac{(\theta - 1)}{\theta} Z (1 + Z) \frac{[H(1,0)]^2}{H \left( \frac{1}{2}, \gamma \right)} V(\mu).
\]

### 4.4.4 Floating exchange rate regime

In this case, computations are simplified by the normalisation \( \overline{M} = \overline{M}^* \). We first use the producer price equations (4.2.1) and the consumer price index equations (4.2.6) to get \( \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p}, \frac{dp}{p} \) and \( \frac{dp}{p} \) as functions of \( \frac{dn}{n}, \frac{dW}{W} \) and \( \frac{dW}{W} \). We then differentiate the goods market clearing conditions to get:

\[
\frac{dc}{c} = k' (1 - 2\mu),
\]

where \( k' \) depends on \( \frac{dn}{n}, \frac{dW}{W}, \alpha, \gamma, \theta \) and \( T \). Replacing \( \frac{dc}{c} \) by \( k' (1 - 2\mu) \) in one of the goods market clearing conditions, we finally reach:

\[
Q^{fs} (Z) \frac{dW}{W} = R^{fs} (Z) \frac{dn}{n},
\]

with

\[
Q^{fs} (Z) \equiv Q^{bk} (Z) - \beta (\theta - 1) (1 - Z^2) \left[ \frac{H(1,0)}{H \left( \frac{1}{2}, \gamma \right)} \right]^2 V(\mu),
\]

\[
R^{fs} (Z) \equiv R^{bk} (Z) - (1 - Z^2) \left[ \frac{H(1,0)}{H \left( \frac{1}{2}, \gamma \right)} \right]^2 V(\mu).
\]
4.4.5 Break points

This subsection summarizes, interprets and discusses the results obtained above, from a positive point of view. It can be shown that because $V(\mu) \leq \frac{1}{4}$, we have: $\forall Z \in [0,1]$, $Q^b_k(Z) > 0$, $Q^m_u(Z) > 0$ and $Q^f_s(Z) > 0$. The sign of $\frac{\delta W}{\delta m}$ is therefore that of $R^b_k(Z)$, $R^m_u(Z)$ or $R^f_s(Z)$, depending on the case considered. Dispersion will be sustainable if and only if this sign is negative. The values of $T$ strictly higher than one and such that $R^b_k(Z) = 0$, $R^m_u(Z) = 0$ or $R^f_s(Z) = 0$ are called break points.

There is one and only one $Z > 1$ solution of the equation $R^b_k(Z) = 0$, so that there is one and only one break point, which we note $T^b_k(B)$, in the benchmark case:

$$T^b_k(B) = \left[ \frac{(1 + \alpha - \gamma)[1 + (\alpha - \gamma)\left(\frac{\theta - 1}{\theta}\right)]}{(1 - \alpha + \gamma)[1 - (\alpha - \gamma)\left(\frac{\theta - 1}{\theta}\right)]} \right]^{\frac{1}{\gamma - 1}}.$$

Dispersion is sustainable for all $T \geq T^b_k(B)$ and unsustainable for all $T \leq T^b_k(B)$.

Under a floating exchange rate regime, we find that if $V(\mu)$ is larger than a certain threshold value, which we note $V_0$:

$$V_0 \equiv \frac{1}{2} \left[ \frac{H\left(\frac{1}{2}, \gamma\right)}{H(1,0)} \right]^2 \left[ 1 + (\alpha - \gamma)^2 \left(\frac{\theta - 1}{\theta}\right) \right] - \sqrt{\left[ 1 - (\alpha - \gamma)^2 \right] \left[ 1 - (\alpha - \gamma)^2 \left(\frac{\theta - 1}{\theta}\right)^2 \right]}.$$

then there is no break point, and dispersion is sustainable for all $T \geq 1$. If, on the contrary, $V(\mu) < V_0$, there are two break points, which we note $T^f_s(B')$ and $T^f_s(B)$, with $T^f_s(B') < T^f_s(B)$. Dispersion is then sustainable if and only if $T \leq T^f_s(B')$ or $T \geq T^f_s(B)$.

In the case of a monetary union, we find that if $V(\mu)$ is larger than a certain threshold value, which we note $V_b$:

$$V_b \equiv \frac{[1 - (\alpha - \gamma)][1 - (\alpha - \gamma)\left(\frac{\theta - 1}{\theta}\right)]}{4\beta\left(\frac{\theta - 1}{\theta}\right)\left[H(1,0)\right]^2\left[H\left(\frac{1}{2}, \gamma\right)\right]}.$$

then there is no break point, and dispersion is not sustainable, whatever the value of $T$. If, on the contrary, $V(\mu) < V_b$, there is one break point, which we note $T^m_u(B)$:
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\[ T^{mu}(B) = \left[ \frac{(1 + \alpha - \gamma) [1 + (\alpha - \gamma) \left( \frac{\theta - 1}{\delta} \right)]}{(1 - \alpha + \gamma) [1 - (\alpha - \gamma) \left( \frac{\theta - 1}{\delta} \right)] - 4\beta \left( \frac{\theta - 1}{\delta} \right) \frac{H(1, 0)2}{H(1, \gamma)} V(\mu)} \right]^{\frac{1}{2\alpha}}. \]

Moreover, one can show that

\[ T^{fs}(B') < T^{fs}(B) < T^{bk}(B) < T^{mu}(B). \]

All these results are displayed in figures 4.2, 4.3, 4.4 and 4.5. One can easily see that \( V_a > 0 \) and \( V_b > 0 \). Depending on the values of parameters \( \alpha, \beta, \gamma, \theta \) and \( T \), we can have \( V_a < V_b < \frac{1}{2} \), \( V_b < V_a < \frac{1}{2} \), \( V_a < \frac{1}{2} < V_b \), \( V_b < \frac{1}{2} < V_a \), \( \frac{1}{2} < V_a < V_b \), or \( \frac{1}{2} < V_b < V_a \). As \( 0 < V(\mu) < \frac{1}{2} \), we need therefore to distinguish between four different cases: \( 0 < V(\mu) < \min (V_a, V_b, \frac{1}{2}) \) (figure 4.2), \( V_a < V(\mu) < \min (V_b, \frac{1}{2}) \) (figure 4.3), \( V_b < V(\mu) < \min (V_a, \frac{1}{2}) \) (figure 4.4) and \( \max (V_a, V_b) < V(\mu) < \frac{1}{2} \) (figure 4.5).

These results say in substance that the range of parameter \( T \) for which dispersion is an equilibrium (i.e. for which dispersion is stable) is the largest in the \( fg \) case, then in the \( bk \) case and finally in the \( mu \) case. In other words, a monetary union tends to favour the emergence of EADS, compared to a floating exchange rate regime. This result is all the stronger than \( V(\mu) \) is large. When \( V(\mu) \) reaches the threshold \( V_a \), dispersion becomes stable whatever the value of \( T \) under a floating exchange rate regime. When \( V(\mu) \) reaches the threshold \( V_b \), dispersion becomes unstable whatever the value of \( T \) within a monetary union.

The mechanism which makes dispersion more unstable within a monetary union is the same as the one which makes concentration more sustainable within a monetary union. As countries start becoming specialized, industry-specific demand shocks start having a country-specific component which cannot be smoothed by an exchange rate adjustment. Firms of a given industry established in the country which starts specializing in this industry then benefit from the conjunction of a substitution effect and an income effect when industry-specific demand shocks occur. This gives birth to the concentration force mentioned above in subsection 4.3.6, and explains why \( T^{mu}(B) \) is increasing in \( V(\mu) \).

Under a floating exchange rate regime, exchange rate variations smooth asymmetric shocks as soon as countries start becoming specialized, so that this concentration force does not apply. Instead, the mechanism which makes dispersion more stable under a floating exchange rate regime is the same as the one which makes concentration less sustainable under a floating exchange rate regime. Firms of a given industry have an incentive to locate in the country
which starts specializing in the other industry, because they then benefit from the conjunction of two substitution effects: a positive demand shock on their industry makes households spend relatively more on the goods produced by that industry, and makes the currency of their country depreciate.

This gives birth to the dispersion force mentioned above in subsection 4.3.6, and explains why \( T^{fg} (B) \) is decreasing in \( V (\mu) \). It also explains why dispersion is stable for \( T \leq T^{fg} (B') \) and why \( T^{fg} (B') \) is increasing in \( V (\mu) \). For low values of \( T \) indeed, backward and forward linkages lose their strength, and so does the local competition effect, so that the traditional concentration and dispersion forces are dominated by our new dispersion force, whose existence does not rest on that of transport costs, and whose strength depends on \( V (\mu) \).

### 4.4.6 Welfare analysis

This subsection draws some normative implications of the results obtained above. Using the first-order condition (4.2.4) of the households’ optimization problem, we can express the utility level \( U \) as a linear function of consumption \( C \), itself a function of \( C_1 \) and \( C_2 \). The latter are derived by using \( G_1 C_1 = H (\mu, 0) [n_1 W_1 L_1 + n_2 W_2 L_2] \), \( G_2 C_2 = H (1 - \mu, 0) [n_1 W_1 L_1 + n_2 W_2 L_2] \), and getting \( G_1 \) and \( G_2 \) out of equations (4.2.6) and (4.2.1). Noting that \( N = \frac{m_b}{F [1 + B (\theta - 1)]} \), we eventually obtain the utility level \( U \) as a function of the exogenous parameters:

\[
U_{disp}^{\delta} = \left( \frac{1 - \lambda}{\lambda} \right)^{1 - \lambda} \left( \frac{\eta \Lambda}{2} \right)^{H (1, 0)} \left( \begin{array}{c} 
\theta \\
\theta - 1
\end{array} \right)^{-\frac{1}{2}} \left[ \frac{1 + T^{(\theta - 1)}}{F [1 + B (\theta - 1)]} \right]^{\gamma (\theta - 1)}.
\]

This implies that \( \forall \mu, U_{disp}^{\delta} < U_{disp}^{nu} = U_{disp}^{fg} \). Whatever the exchange rate regime, the presence of industry-specific demand shocks is welfare-improving. This results directly from the consumption basket effect, mentioned in subsection 4.3.7: the differentiated good which matters less in the utility function (by definition of an industry-specific demand shock) is less consumed. This effect is common to both exchange rate regimes considered (\( mu \) and \( fg \)) and is the only one at work here, so that \( U_{disp}^{nu} = U_{disp}^{fg} \).

When countries are perfectly diversified (half of each industry in each country), the exchange rate regime does not matter actually. Indeed, the nominal exchange rate keeps equal to one under a floating exchange rate regime, and national money stocks are constant within a monetary union, as countries, being
perfectly identical to each other, are affected by the industry-specific demand shock in exactly the same way.

Moreover, one can see that

\[ U_{\text{conc}}^{rh} > U_{\text{disp}}^{rh} \iff 2 \left[ T^{\theta-1} \right]^{\frac{1+\xi_1+\gamma_1}{2}} > 1 + T^{\theta-1}. \]

This says that in the absence of shocks, concentration is more desirable than dispersion for low values of \( T \), and less desirable for high values of \( T \).

### 4.5 Conclusion

In his classic contribution, Mundell (1961) examines in a two-country two-industry framework how exchange rate regimes should ideally be chosen given countries' specialization patterns. In this chapter, we inversely consider how exchange rate regimes influence the location of economic activity and hence countries' specialization patterns. To that aim, we introduce industry-specific demand shocks, in the presence of short-run nominal wage rigidity, into an otherwise canonical New Economic Geography model, that of Fujita, Krugman and Venables (1999, ch. 16).

Our results suggest that when firms behave as risk-neutral agents, the occurrence of industry-specific demand shocks makes the dispersion of each industry across countries less stable, and the concentration of each industry in one country more sustainable, within a monetary union than under a floating exchange rate regime. This is because when industry-specific demand shocks occur within a monetary union, a new concentration force arises, which stems from the conjunction of a substitution effect and an income effect. On the contrary, when industry-specific demand shocks occur under a floating exchange rate regime, a new dispersion force arises, which stems from the conjunction of two substitution effects. Thus, endogenous specialization patterns, and therefore EADS, are more likely to emerge and be sustained within a monetary union than under a floating exchange rate regime.

As shown in table 4.2, these results go in the same direction as Krugman's (1991, 1993) predictions in terms of endogenous specialization patterns, even though the mechanisms involved are very different from one framework to the other. Our model assumes that firms are risk-neutral and that a floating exchange rate regime does smooth asymmetric shocks, a flexible exchange rate moving endogenously so as to balance international trade\(^{19}\). On the contrary,
Krugman's point rests on risk-adverse firms and exogenous exchange rate variations. Finally, Ricci's (1997) model, based on risk-adverse firms and endogenous exchange rate variations, has opposite implications in terms of endogenous specialization patterns.

Compared to those predicted by Krugman (1991, 1993) or modelized by Ricci (1997), the economic mechanisms in force in our chapter seem likely however to be of second-order importance, so that our results should be taken with caution and the main virtue of our model may actually be pedagogical. It is worth noting moreover, and this represents one more reason to water them down, that our results rest only on the existence of industry-specific demand shocks, that is to say that other kinds of shocks would have no impact in our framework on the location incentives faced by firms.

The ultimate success of EMU may well depend upon two things: its exposure to country-specific shocks, and its ability to cope with them. This chapter argues that to a certain extent, these two points are not independent from each other, and that the very vulnerability of EMU to country-specific shocks may reinforce its susceptibility to such shocks, through a shift in industrial structure towards greater national specialization. We show however that such an outcome need not be a cause for concern, as it is not necessarily harmful and in some cases can actually be welfare-improving.

Our point may also apply to East Asia, where some countries specialized in a handful of industries now talk of forming some day a currency union, though East Asia as a whole cannot be considered as a closed economy. Moreover, our analysis has some implications for regions within a country too: as these regions form a currency union, what we say is that the mere fact that industry-specific shocks may occur is enough to pave the way for a new concentration force, which tends to make these regions more specialized. This concentration force is stronger, the smaller the size of the redistributive budget at the national level: indeed, a large redistribution between the regions weakens the income effect mentioned above, thus reducing the incentives for firms of a given industry to concentrate in the same region.

One extension to our framework would consist in relaxing the "super home

and Canzoneri, Valles and Vitais (1996) find that exchange rates do not really move to address international macroeconomic imbalances. As already mentioned in the introduction, we choose however to assume on the contrary that exchange rates do adjust to fundamentals so as to balance international trade, in order to keep as close as possible to the OCA literature.

Following a productivity shock for instance, maintaining the nominal wage rigidity assumption, prices would adjust so as to leave labour employed unchanged even though quantities produced are affected, and this would result in no income effect and no substitution effect.
bias" assumption, and considering II as the diversified share portfolio owned by household i. In that case, for firms to go on behaving as risk-neutral agents and maximize their expected profits, households must be willing to hold risky shares, and they are indeed when they can (at least partially) stabilize their total income by holding shares whose value is negatively correlated to their labour income\textsuperscript{21}. Since this hedging requires households to hold foreign shares when each country is entirely specialized in one industry, we then need to assume that there is no barrier to international equity trade.

This closer financial market integration can be expected to lead to less national specialization, as the income effect mentioned above would be smoothed by households' portfolio diversification, thus reducing the incentives for firms of a given industry to concentrate in the same country. These results would thus oppose those of Kalemli-Ozcan, Sørensen and Yoosha (1999), who argue that risk-sharing causes specialization. However, as already discussed in the presentation of the model, equity markets may remain segmented for some time to come within the euro zone, so that relaxing the "super home bias" assumption does not necessarily correspond to a more accurate and relevant modelization of European economies.

Another extension would be to allow for the presence of multinational firms in our framework, using Markusen and Venables' (1995, 1996) technical specification, which is compatible with our model and can be added without any difficulty. The existence of multinationals could thus easily be justified along the lines of the so-called OLI framework (for Ownership, Location and Internalization). The presence of multinationals can be expected to lead to less national specialization through two channels. One is direct and obvious, as a multinational is by definition a firm of a given industry settling in both countries. The other is indirect and based on our results: as a source of closer income linkage between countries, foreign direct investment should smooth (at least partially) the income effect mentioned above, and thus reduce the incentives for firms of a given industry to concentrate in the same country.

Another rationale for the emergence of multinationals would exist if firms did actually behave as risk-adverse agents, say because of firing, inventory or bankruptcy costs. Consider indeed the case of a monetary union. If the disappearance of national monetary policies and the fixity of exchange rates imply non-synchronized cycles for member countries facing asymmetric shocks, then firms may be more willing to have their activities dispersed in several coun-

\textsuperscript{21}This requires some risk-aversion from households. The utility function should therefore be modified accordingly.
tries\textsuperscript{22}. This effect should prevent national economies from becoming too much specialized within the monetary union.

Finally, we could test the predictions of our model if the data exist, that is to say examine empirically whether monetary unions tend to lead to more national specialization than floating exchange rate regimes. The evidence might be found in the data sets used by Glick and Rose (2001), Rose (2000), Rose and van Wincoop (2001).

\textsuperscript{22}This concept of "multinationality" as a risk-spreading device is explored at length in Rugman (1979).
Table 4.1: Ricci’s (1997) and chapter 4’s assumptions.

<table>
<thead>
<tr>
<th></th>
<th>Ricci (1997)</th>
<th>Chapter 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>firms</td>
<td>risk-adverse</td>
<td>risk-neutral</td>
</tr>
<tr>
<td>nominal rigidity</td>
<td>price rigidity (no wages)</td>
<td>wage rigidity (flexible prices)</td>
</tr>
<tr>
<td>returns to scale</td>
<td>decreasing</td>
<td>increasing</td>
</tr>
<tr>
<td>intermediate goods</td>
<td>absent</td>
<td>present</td>
</tr>
<tr>
<td>transport costs</td>
<td>absent</td>
<td>present</td>
</tr>
<tr>
<td>shocks</td>
<td>demand, supply, monetary, exchange rate shocks</td>
<td>demand shocks</td>
</tr>
</tbody>
</table>
Table 4.2: Assumptions and results of chapter 4 and other close studies.

<table>
<thead>
<tr>
<th>Risk-adverse firms</th>
<th>Endogenous exchange rate</th>
<th>Exogenous exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ricci’s (1997) case: the flexible exchange rate regime favours national specialization, the fixed exchange rate regime has no effect on national specialization.</td>
<td>Krugman’s (1991, 1993) case: the fixed exchange rate regime favours national specialization more than does the flexible exchange rate regime. Also Ricci’s (1997) case: the flexible exchange rate regime favours national specialization, the fixed exchange rate regime has no effect on national specialization.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Risk-neutral firms</th>
<th>Endogenous exchange rate</th>
<th>Exogenous exchange rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case of chapter 4: the fixed exchange rate regime and the monetary union favour national specialization more than does the flexible exchange rate regime.</td>
<td>Case a priori amounting to Fujita, Krugman and Venables’ (1999): the exchange rate regime (fixed or flexible) has no effect on national specialization.</td>
</tr>
</tbody>
</table>
Figure 4.1: Sustain points.

Floating exchange rate regime:
\[ 1 \quad T_{\text{fl}}(S) \rightarrow T \quad +\infty \]

Fixed exchange rate regime:
\[ 1 \quad T_{\text{f}}(S) \rightarrow T \quad +\infty \]

Benchmark case:
\[ 1 \quad T_{\text{bk}}(S) \rightarrow T \quad +\infty \]

Monetary union:
\[ 1 \quad T_{\text{mu}}(S) \rightarrow T \quad +\infty \]

: concentration sustainable.
Figure 4.2: Break points when $0 < V(\mu) < \min(V_u, V_u^\mu, 1/4)$.

Floating exchange rate regime:

1  \( T^{G}(B') \)  \( T^{G}(B) \)  \( + \infty \)

Benchmark case:

1  \( T^{\text{bk}}(B) \)  \( + \infty \)

Monetary union:

1  \( T^{\text{mu}}(B) \)  \( + \infty \)

: dispersion sustainable.
Figure 4.3: Break points when $V_a < V(\mu) < \text{Min}(V_a, \frac{1}{4})$.

Floating exchange rate regime:

Benchmark case:

Monetary union:

: dispersion sustainable.
Figure 4.4: Break points when $V_b < V(\mu) < \text{Min}(V_a, 1/4)$.

Floating exchange rate regime:

```
1 \quad T^{\text{fr}}(B') \quad T^{\text{fr}}(B) \quad + \infty
```

Benchmark case:

```
1 \quad T^{\text{bk}}(B) \quad + \infty
```

Monetary union:

```
1 \quad + \infty
```

\[\square\] : dispersion sustainable.
Figure 4.5: Break points when $\max(V_a, V_b) < V(\mu) < \frac{1}{4}$.

Floating exchange rate regime:

Benchmark case:

Monetary union:

\[ \text{dispersion sustainable}. \]
General conclusion

What conclusions are to be drawn from our dissertation? On a chapter by chapter basis, table 1 briefly sums up our main findings and derives therefrom the straightforward implications for economic policy. We do not further comment on these results, however new and promising, as they have already been discussed at great length in the chapters. Such a kaleidoscopic view of our dissertation would moreover prove inopportune in this general conclusion. Instead, we focus on the general lessons to be drawn from our dissertation.

This dissertation has thrown a new light on the links between macroeconomic volatility, macroeconomic instability and the exchange rate regime, both from a positive point of view and from a normative point of view. It has notably clarified the form which macroeconomic volatility could take in a small open economy New Keynesian framework, both theoretically and empirically, under alternative exchange rate regimes. But perhaps most importantly of all, it has unveiled unknown sources of and proposed new remedies to macroeconomic instability under alternative exchange rate regimes - and here may lie our most significant original contribution to the existing literature.

These newly identified sources of and remedies to macroeconomic instability are presented in table 2. Although their nature may substantially differ from one chapter to the other, all these sources and remedies are inextricably linked to the exchange rate regime. Table 3 displays the resulting short-run macroeconomic instability, with or without remedy administered, and table 4 similarly displays the long-run macroeconomic instability, as well as the short-run macroeconomic instability under an irrevocably fixed exchange rate regime, both without remedy administered1. Note that there is usually no miracle remedy to macroeconomic instability. As indicated in table 3 indeed, the proposed remedies will often simply reduce the set of multiple equilibria, rather than

---

1The remedies to long-run macroeconomic instability, or to short-run macroeconomic instability under an irrevocable exchange rate regime, are not considered in table 4 simply because their implementation raises practical difficulties, as they correspond to structural policies.
completely eliminate the possibility of multiple equilibria.

Let us end our dissertation with a few considerations on the optimal exchange rate regime. Indeed, whether optimal or not, monetary policy has been considered throughout this dissertation as a policy conditional on the exchange rate regime in force. But we can naturally address the issue (situated upstream) of the optimal exchange rate regime. This optimal exchange rate regime is defined as the one which maximizes household welfare or minimizes the government’s loss function, and corresponds more or less to the one associated with the lowest ex post macroeconomic volatility and instability.

Table 5 ranks the exchange rate regimes according to ex post macroeconomic volatility, conditionally either on the absence of ex post macroeconomic instability, or on the comparability of the sets of multiple equilibria considered. Naturally, the flexible exchange rate regime with commitment comes out first in all but one case\(^2\), because the central bank has then full freedom to react to the fundamental shocks. The two fixed exchange rate regimes are usually placed equal, as they entail the same ex post macroeconomic volatility. It is worth noting moreover that they can be preferable to the flexible exchange rate regime without commitment. All these results displayed in table 5 are more or less in accordance with conventional wisdom.

Table 6 ranks the exchange rate regimes according to ex post macroeconomic instability. The novelty lies here both in the principle and in the results of this ranking. In the principle, because to our knowledge such a ranking has never been carried out in the literature. In the results, because this ranking brings back the irrevocably fixed exchange rate regime into favour. Indeed, this regime now comes out first in all cases but one, which ranks it ahead of the flexible exchange rate regime with commitment. The fixed but adjustable exchange rate regime comes out last in all cases, while the two flexible exchange rate regimes are placed equal in-between the two fixed exchange rate regimes.

The irrevocable exchange rate regime, whether a monetary union or a “dollarization”, is ranked first according to ex post macroeconomic instability thanks to its ability to anchor the private agents’ expectations. In the small open economy New Keynesian framework, this property ensures that divergent equilibria are ruled out a priori under an irrevocable exchange rate regime, because the private agents know that there will be no national central bank to react to these divergent equilibria. Under a flexible exchange rate regime, divergent equilibria may occur by contrast if the monetary policy rule followed does not rule them

\(^2\)The only exception concerns chapter 4, in the special case where macroeconomic volatility actually happens to be welfare-improving.
out, and annoyingly enough, monetary policy rules ruling out divergent equilibria may actually not exist under some specifications. The only case where the flexible exchange rate regime is preferable to the irrevocable exchange rate regime, as far as *ex post* macroeconomic instability is concerned, is the case where an adequate monetary policy rule can be found which rules out multiple equilibria under the flexible exchange rate regime, be they convergent or divergent, while the irrevocable exchange rate regime proves compatible with more than one convergent equilibrium.

Our final thoughts will be directed towards the European Monetary Union (EMU). Where do we stand altogether on the question of EMU’s desirability? Before everything, note that our dissertation mainly focuses on what is usually considered as the adjustment costs associated with the Euro. In other words, our operative field excludes what is usually considered as the structural benefits associated with the Euro. If anything, our dissertation will therefore tend to offer a biased euro sceptic point of view. That said, we actually prove rather in favour of EMU. Indeed, although we acknowledge the possibility that EMU might endogenously favour asymmetric shocks and thus raise macroeconomic volatility in the long term, we argue that such an outcome may not be necessarily welfare-decreasing. And as far as EMU’s short-term effects are concerned, we do certainly advocate EMU against the alternative of some kind of European Monetary System (EMS), which we view as fundamentally unstable. Against the alternative of a flexible exchange rate regime, no clear-cut (unconditionally settled) conclusion emerges though, and we simply recommend to take a closer look country by country on what would become not only of macroeconomic volatility, but also of macroeconomic instability under EMU-membership. The die is cast for twelve countries, but it is now Denmark, Sweden and the UK’s turn to decide whether to adopt the Euro or not. We but wish them a sound debate about the economics of EMU prior to their decision. In this respect, the recent release of eighteen high-quality so-called EMU studies by HM Treasury gives ground for optimism.
Table 1: main results and implications for economic policy.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Main results</th>
<th>Implications for economic policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>• whatever the exchange rate regime, the optimal monetary policy rules (i.e.</td>
<td>• whatever the exchange rate regime in force and the credibility of the central bank,</td>
</tr>
<tr>
<td></td>
<td>the rules ensuring the implementation of the optimal equilibrium, i.e. the</td>
<td>a forward-looking monetary policy rule should be adopted</td>
</tr>
<tr>
<td></td>
<td>rules ruling out macroeconomic instability and minimizing macroeconomic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>volatility) are necessarily forward-looking</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>• should the UK adopt the Euro now, it would not suffer from macroeconomic</td>
<td>• the UK should not adopt the Euro without first adapting its economy structurally</td>
</tr>
<tr>
<td></td>
<td>instability but would possibly experience a higher macroeconomic volatility</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>• a fixed but adjustable exchange rate regime is all the more vulnerable to</td>
<td>• when international cooperation is not credible for the private agents (for instance</td>
</tr>
<tr>
<td></td>
<td>currency crises than there is trade competition on a monopolistic sector</td>
<td>in the absence of any international institution enforcing the agreements),</td>
</tr>
<tr>
<td></td>
<td>between the countries considered • international cooperation is preferable to</td>
<td>the governments should coordinate each other to select the best non-cooperative</td>
</tr>
<tr>
<td></td>
<td>international coordination (itself preferable to neither cooperation nor</td>
<td>equilibrium</td>
</tr>
<tr>
<td></td>
<td>coordination) because it further reduces the risk of a currency crisis •</td>
<td></td>
</tr>
<tr>
<td></td>
<td>coordination introduces a new canal of transmission of currency crises</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>• in the presence of sectorial shocks, the fixed exchange rate regimes favour</td>
<td>• although the fixed exchange rate regimes entail a higher macroeconomic volatility</td>
</tr>
<tr>
<td></td>
<td>national specialization (and consequently shocks asymmetric across countries)</td>
<td>than the flexible exchange rate regime, there is no unconditionally preferable</td>
</tr>
<tr>
<td></td>
<td>more than does the flexible exchange rate regime</td>
<td>exchange rate regime, so that the choice of the exchange rate regime should be made</td>
</tr>
<tr>
<td></td>
<td></td>
<td>on a case by case basis</td>
</tr>
</tbody>
</table>
Table 2: sources of and remedies to macroeconomic instability.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Exchange rate regime concerned</th>
<th>Newly identified sources of macroeconomic instability</th>
<th>Newly identified remedies to macroeconomic instability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>flexible exchange rate regime (with or without commitment), fixed but adjustable exchange rate regime</td>
<td>impossibility to preclude all non-optimal convergent and divergent equilibria with a purely backward-looking monetary policy rule</td>
<td>adoption of an adequate forward-looking monetary policy rule</td>
</tr>
<tr>
<td>2</td>
<td>irrevocably fixed exchange rate regime (unilateral)</td>
<td>existence of multiple (convergent) equilibria in the absence of any monetary policy</td>
<td>structural reforms</td>
</tr>
<tr>
<td>3</td>
<td>fixed but adjustable exchange rate regime</td>
<td>existence of commercial spillovers, responsible for the contagion of currency crises</td>
<td>international coordination, international cooperation</td>
</tr>
<tr>
<td>4</td>
<td>flexible exchange rate regime, irrevocably fixed exchange rate regime (bilateral)</td>
<td>existence of macroeconomic volatility (due to industrial demand shocks) in the presence of transport costs and intermediate goods</td>
<td>risk-sharing across countries</td>
</tr>
<tr>
<td>Chapter</td>
<td>Meaning of &quot;ex ante&quot;</td>
<td>$Ex\ ante$ multiplicity $M$ of equilibria</td>
<td>Meaning of &quot;ex post&quot;</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------</td>
<td>------------------------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>1</td>
<td>flexible exchange rate regime (with or without commitment) with an arbitrary monetary policy rule</td>
<td>$M = \infty$</td>
<td>flexible exchange rate regime (with or without commitment) with an optimal monetary policy rule</td>
</tr>
<tr>
<td></td>
<td>fixed but adjustable exchange rate regime with an arbitrary monetary policy rule</td>
<td>$M = \infty$</td>
<td>fixed but adjustable exchange rate regime with an optimal monetary policy rule</td>
</tr>
<tr>
<td>2</td>
<td>flexible exchange rate regime (with or without commitment) with an arbitrary monetary policy rule</td>
<td>$M = \infty$</td>
<td>flexible exchange rate regime (with or without commitment) with an optimal monetary policy rule</td>
</tr>
<tr>
<td></td>
<td>fixed but adjustable exchange rate regime with an arbitrary monetary policy rule</td>
<td>$M = \infty$</td>
<td>fixed but adjustable exchange rate regime with an optimal monetary policy rule</td>
</tr>
<tr>
<td>3</td>
<td>fixed but adjustable exchange rate regime without international cooperation or international coordination</td>
<td>$M = 1$</td>
<td>fixed but adjustable exchange rate regime with international coordination</td>
</tr>
<tr>
<td></td>
<td>$M = 2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$M = 4$</td>
<td>fixed but adjustable exchange rate regime with international coordination (equilibria symmetric across countries)</td>
<td>$M' \in {3, 4}$</td>
</tr>
<tr>
<td></td>
<td>fixed but adjustable exchange rate regime without international cooperation or international coordination (equilibria symmetric across countries)</td>
<td>$M = 1$</td>
<td>fixed but adjustable exchange rate regime with international coordination (equilibria symmetric across countries)</td>
</tr>
<tr>
<td></td>
<td>$M = 2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4: short-run and long-run macroeconomic instability.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Exchange rate regime</th>
<th>Set S of possible equilibria multiplicities $^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>irreversibly fixed exchange rate regime (unilateral)</td>
<td>$S = {1}$</td>
</tr>
<tr>
<td>2</td>
<td>irreversibly fixed exchange rate regime (unilateral)</td>
<td>$S \in {{0}, {1}, {\infty}}$</td>
</tr>
<tr>
<td>4</td>
<td>benchmark case</td>
<td>$S \in {{0, 1}, {1, 2}}$</td>
</tr>
<tr>
<td></td>
<td>flexible exchange rate regime</td>
<td>$S \in {{1, 2}, {0, 1, 2}}$</td>
</tr>
<tr>
<td></td>
<td>irreversibly fixed exchange rate regime (bilateral)</td>
<td>$S \in {{0, 1}, {0, 1, 2}}$</td>
</tr>
</tbody>
</table>

$^3$ Because we limit our attention to two degenerated equilibria in chapter 4 (the complete national specialization equilibrium and the perfect industrial dispersion equilibrium), the multiplicity of equilibria cannot exceed two.
Table 5: ranking of the exchange rate regimes according to *ex post* macroeconomic volatility.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>flexible exchange rate regime with commitment</th>
<th>flexible exchange rate regime without commitment</th>
<th>fixed but adjustable exchange rate regime</th>
<th>irrevocably fixed exchange rate regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>joint 3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>joint 3&lt;sup&gt;rd&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>joint 2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>joint 2&lt;sup&gt;nd&lt;/sup&gt;</td>
</tr>
<tr>
<td>2</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>-</td>
<td>-</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
</tr>
<tr>
<td>4</td>
<td>joint 1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>joint 1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>-</td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>joint 2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>joint 2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>-</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

4 The ranking criterion is more precisely household welfare (chapters 1 and 4), the government's loss function (chapter 3) or the variance of inflation and output (chapter 2), under the optimal monetary policy assumption when relevant (chapter 1). The exchange rate regime graded 1<sup>st</sup> is the one associated with the highest household welfare, the lowest government's loss function or the lowest variance of inflation and output. In the absence of macroeconomic instability, the ranking of two given exchange rate regimes raises no difficulty, as one unique equilibrium is compared to another. In the presence of macroeconomic instability, the ranking of two given exchange rate regimes is carried out only in the case where all possible equilibria under one exchange rate regime are preferable to all possible equilibria under the other exchange rate regime. Note finally that one given chapter may provide several rankings, depending on the value of the parameters in the corresponding model.
Table 6: ranking of the exchange rate regimes according to *ex post* macroeconomic instability$^5$.

<table>
<thead>
<tr>
<th>Chapter</th>
<th>flexible exchange rate regime with commitment</th>
<th>flexible exchange rate regime without commitment</th>
<th>fixed but adjustable exchange rate regime</th>
<th>irrevocably fixed exchange rate regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>joint 1$^{st}$</td>
<td>joint 1$^{st}$</td>
<td>joint 1$^{st}$</td>
<td>joint 1$^{st}$</td>
</tr>
<tr>
<td>2</td>
<td>1$^{st}$</td>
<td>-</td>
<td>3$^{rd}$</td>
<td>2$^{nd}$</td>
</tr>
<tr>
<td></td>
<td>2$^{nd}$</td>
<td>-</td>
<td>3$^{rd}$</td>
<td>1$^{st}$</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>2$^{nd}$</td>
<td>1$^{st}$</td>
</tr>
<tr>
<td>4</td>
<td>joint 2$^{nd}$</td>
<td>joint 2$^{nd}$</td>
<td>-</td>
<td>1$^{st}$</td>
</tr>
</tbody>
</table>

$^5$ The ranking criterion is simply the multiplicity of equilibria. The exchange rate regime graded 1$^{st}$ is the one associated with the lowest multiplicity of equilibria. Note that one given chapter may provide several rankings, depending on the value of the parameters in the corresponding model.
Bibliography


BIBLIOGRAPHY


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Summary - The present PhD dissertation is made of FOUR ESSAYS shedding a new light on the links between macroeconomic volatility, macroeconomic instability and the exchange rate regime. Macroeconomic volatility and instability are defined as the variability of key macroeconomic aggregates due to the occurrence of fundamental and sunspot shocks respectively. In other words, macroeconomic volatility concerns a given equilibrium, while macroeconomic instability arises in the presence of multiple equilibria. Three main exchange rate regimes are considered alternatively: the flexible, the fixed but adjustable and the irrevocably fixed exchange rate regimes. The FIRST ESSAY characterizes analytically, within the canonical New Keynesian model of a small open economy under alternative exchange rate regimes, the set of monetary policy rules which ensure the implementation of the optimal equilibrium by eliminating macroeconomic instability and minimizing macroeconomic volatility. The SECOND ESSAY proposes a simulation of the UK business cycle under EMU-membership, based on the predictions of a New Keynesian model estimated on pre-EMU data. The results suggest that the euroized UK would escape macroeconomic instability but experience a higher macroeconomic volatility. The THIRD ESSAY builds a three-country second-generation currency crises model based on the New Open Economy Macroeconomics literature. International coordination and cooperation are found to reduce macroeconomic instability by eliminating the possibility of self-fulfilling expectations of currency crises. The FOURTH ESSAY presents a two-country model based on the New Open Economy Macroeconomics and the New Economic Geography literatures. The fixed exchange rate regimes are found to be more likely than the flexible exchange rate regime to raise macroeconomic volatility in the long term by giving rise to endogenously asymmetric demand shock. Altogether, this dissertation points to the flexible and to the irrevocably fixed exchange rate regimes as the ones associated with the lowest macroeconomic volatility and instability respectively. Keywords - Macroeconomic volatility, macroeconomic instability, exchange rate regime.

Titre - Quatre essais sur la volatilité et l'instabilité macroéconomiques sous différents régimes de change. Résumé – Cette thèse est composée de QUATRE ESSAIS éclairant d'un jour nouveau les relations entre volatilité macroéconomique, instabilité macroéconomique et régime de change. La volatilité et l'instabilité macroéconomiques sont définies comme la variabilité d'agrégats macroéconomiques clefs due à l'occurrence de chocs fondamentaux et de chocs sunspot respectivement. En d'autres termes, la volatilité macroéconomique concerne un équilibre donné, tandis que l'instabilité macroéconomique apparaît en présence d'équilibres multiples. Trois principaux régimes de change sont considérés alternativement: les régimes de change flexible, fixe mais ajustable et irrévocablement fixe. Le PREMIER ESSAI caractérise analytiquement, dans le cadre du modèle nouveau-keynésien canoïque d'une petite économie ouverte sous différents régimes de change, l'ensemble des règles de politique monétaire qui assurent l'implémentation de l'équilibre optimal en éliminant l'instabilité et en minimisant la volatilité macroéconomiques. Le DEUXIÈME ESSAI propose une simulation du cycle macroéconomique du Royaume-Uni en Eurozone, basée sur les prédictions d'un modèle nouveau-keynésien estimé sur données pré-Euro. Les résultats suggèrent que le Royaume-Uni euroisé échapperait à l'instabilité macroéconomique, mais ferait l'expérience d'une volatilité macroéconomique accrue. Le TROISIÈME ESSAI construit un modèle de crises de change de seconde génération à trois pays basé sur la littérature de New Open Economy Macroeconomics. La coordination et la coopération internationales y réduisent l'instabilité macroéconomique en éliminant la possibilité d'anticipations auto-réalisatrices de crises de change. Le QUATRIÈME ESSAI présente un modèle à deux pays basé sur les littératures de New Open Economy Macroeconomics et de New Economic Geography. Les régimes de change fixe y sont davantage susceptibles que le régime de change flexible d'augmenter la volatilité macroéconomique dans le long terme en donnant naissance à des chocs de demande endogènement asymétriques. Au final, cette thèse fait apparaître les régimes de change flexible et irrévocablement fixe comme ceux associés aux moindres volatilité et instabilité macroéconomiques respectivement. Mots-clés - Volatilité macroéconomique, instabilité macroéconomique, régime de change.