Étude asymptotique des algorithmes stochastiques et calcul du prix des options parisiennes

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Résumé

Cette thèse traite de deux sujets indépendants. La première partie est consacrée à l'étude des algorithmes stochastiques. Dans un premier chapitre introductif, je présente l'algorithme de [55] dans un parallèle avec l'algorithme de Newton pour l'optimisation déterministe. Ces quelques rappels permettent alors d'introduire les algorithmes stochastiques aléatoirement tronqués de [21] qui sont au cœur de cette thèse. La première étude de cet algorithme concerne sa convergence presque sûre qui est parfois établie sous des hypothèses assez changeantes. Ce premier chapitre est l'occasion de clarifier les hypothèses de la convergence presque sûre et d'en présenter une preuve simplifiée. Dans le second chapitre, nous poursuivons l'étude de cet algorithme en nous intéressant cette fois à sa vitesse de convergence. Plus exactement, nous considérons une version moyenne mobile de cet algorithme et démontrons un théorème centrale limite pour cette variante. Le troisième chapitre est consacré à deux applications de ces algorithmes à la finance : le premier exemple présente une méthode de calibration de la corrélation pour les modèles de marchés multidimensionnels alors que le second exemple poursuit les travaux de [7] en améliorant ses résultats.

La seconde partie de cette thèse s'intéresse à l'évaluation des options parisiennes en s'appuyant sur les travaux de Chesney, Jeanblanc-Picqué, and Yor [23]. La méthode d'évaluation se base sur l'obtention de formules fermées pour les transformées de Laplace des prix par rapport à la maturité. Nous établissons ces formules pour les options parisiennes simple et double barrières. Nous étudions ensuite une méthode d'inversion numérique de ces transformées. Nous établissons un résultat sur la précision de cette méthode numérique tout à fait performante. A cette occasion, nous démontrons également des résultats liés à la régularité des prix et l'existence d'une densité par rapport à la mesure de Lebesgues pour les temps parisiens.

Abstract

This thesis is split into two parts. The first one deals with the study of stochastic algorithms. In an introductory chapter, we present the [55] algorithm while making a parallel with the Newton algorithm commonly used in deterministic optimisation problems. These reminders naturally lead to the presentation of randomly truncated stochastic algorithms as first introduced by [21]. The first study of these randomly truncated stochastic algorithms is concerned with their almost sure convergence which has already been established under varying hypotheses. The first chapter gives us the opportunity to try to clarify the assumptions a little and to present a simplified proof of the almost sure convergence. The second chapter is devoted to the study of the convergence rate. More precisely, we consider a moving window version of the algorithm and establish a central limit theorem. The last chapter of this first part presents two applications of stochastic algorithms to finance. The first one deals with the calibration of the correlation in a multidimensional market model, while the second one is based on the work of [7]. Meanwhile, we improve the results Arouna had obtained.

The second part of the thesis is concerned with the pricing of Parisian options. The valuation technique is based on computing closed form formula for the Laplace transforms of the prices following the seminar work of Chesney, Jeanblanc-Picqué, and Yor [23] on the topic. First, we determine these formulae for the single barrier Parisian options following closely [23], second we do the same for double barrier Parisian options. Then, we study the numerical inversion of these Laplace transforms based on a contour integral technique. We establish the accuracy of the method we use. To do so, we prove the regularity of the Parisian option prices and establish the existence of a density with respect to the Lebesgue measure for the "Parisian time".

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