Essais sur la Taxation et la Stabilisation Macroéconomique

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ESSAIS SUR LA TAXATION ET LA STABILISATION MACROÉCONOMIQUE

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Résumé

Cette thèse a pour thème central l'influence stabilisatrice de politiques de taxation appropriées sur la dynamique économique. La stabilisation est ici entendue comme la possibilité d'atténuer voire d'éliminer, par de telles politiques, la probabilité d'apparition de fluctuations endogènes dans l'économie. Le premier chapitre examine, dans le cadre d'une économie réelle, les propriétés stabilisatrices de taxes linéairement progressives, proches des codes d'imposition par tranches prévalant dans de nombreux pays développés. Le deuxième chapitre met en évidence que l'influence stabilisatrice (contracyclique) de la taxation progressive diminue lorsqu'on introduit une disposition fiscale réaliste permettant aux entreprises de déduire de leur profit avant impôt les dépenses variables de maintenance. Le troisième chapitre évalue l'impact de la taxation des revenus dans une économie monétaire avec agents hétérogènes. Cette étude confirme que la progressivité fiscale réduit la probabilité d'apparition de fluctuations endogènes dans l'économie. Cependant on montre dans ce cadre monétaire, à la différence des économies réelles étudiées dans les deux premiers chapitres, que même un niveau élevé de progressivité ne peut éliminer complètement la possibilité de fluctuations endogènes. Enfin, le dernier chapitre analyse comment la politique fiscale peut affecter la volatilité agrégée et la croissance dans des économies sujettes à des contraintes de crédit. Dans un modèle de croissance endogène combinant des frictions sur le marché du capital et une inégalité d'accès aux opportunités d'investissement entre les individus, il est montré qu'une politique fiscale bien choisie, même linéaire, taxant les revenus du travail et organisant des transferts vers l'investissement innovant, peut remédier au défaut de financement de ces investissements lors des crises, stabiliser la dynamique et placer l'économie sur un sentier de croissance permanente et soutenue.

Mots clés : cycles économiques ; fluctuations endogènes ; stabilisation ; politique fiscale ; imposition proportionnelle ; imposition progressive ; exemptions fiscales ; maintenance du capital ; utilisation du capital ; encaisses préalables ; agents hétérogènes ; contraintes de crédit ; accès à l'investissement ; indétermination, tâches solaires.

Abstract

The general topic of this work is the stabilizing influence of appropriate fiscal policies on the economy's dynamics. In this thesis, stabilization policies are understood as a means to lower or rule out the likelihood of endogenous fluctuations in the economy. The first chapter examines, in a real economy, the stabilizing properties of linearly progressive income taxes, resembling the tax codes with brackets that prevail in many developed countries. The second chapter highlights the fact that the stabilizing (counter-cyclical) influence of income tax progressivity is weakened with the introduction of a realistic fiscal scheme allowing firms to deduct maintenance and repair expenditures when calculating pre-tax profits. The third chapter studies the impact of income taxation in a monetary economy with heterogeneous agents. The analysis confirms the idea that fiscal progressivity lowers the probability of endogenous fluctuations in the economy. However, in such a monetary frame, in contrast with the real economies pictured in the first two chapters, it is shown that even high levels of tax progressivity can not completely rule out the occurrence of endogenous fluctuations. Eventually, the last chapter investigates how fiscal policy can affect aggregate volatility and growth in credit constrained economies. In an endogenous growth model combining capital market frictions with unequal access to investment opportunities across individuals, it is shown that an appropriate fiscal policy, even linear, consisting of taxing labor income and organizing transfers towards innovating investment, is able to remedy the shortage of funding for these investments during slumps, stabilize the economy's dynamics and place it on a sustained permanent growth path.

Keywords : business cycles ; endogenous fluctuations ; stabilization ; fiscal policy ; linear taxation ; progressive taxation ; fiscal exemptions ; capital maintenance ; capital utilization ; cash-in-advance ; heterogeneous agents ; credit constraints ; access to investment ; indeterminacy, sunspots.

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Nicolas Dromel

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Introduction générale

Cette Thèse s'insère dans le champs de recherche étudiant comment certaines décisions fiscales affectent les grands agrégats économiques, la croissance et les fluctuations, en utilisant les outils et concepts de l'analyse macroéconomique moderne. Plus particulièrement, on s'intéresse à la manière dont certaines règles en matière fiscale affectent les fluctuations conjoncturelles, à l'aide de modèles dynamiques présentant des cycles d'affaires endogènes.

Depuis de nombreuses années, l'analyse des *stabilisateurs automatiques* a préoccupé grand nombre d'économistes, et engendré une littérature volumineuse. Ces mécanismes, intégrés au code des impôts et à la législation sociale, sont censés permettre des ajustements budgétaires rapides et symétriques contre les fluctuations, réduisant la pression sur la demande en périodes de forte croissance, et soutenant l'économie lors de ralentissements, sans requérir d'autre intervention de la part des autorités publiques. Le développement récent des modèles d'équilibre général dynamique a suscité un regain d'intérêt sur cette thématique, et de nouvelles questions concernant l'impact des décisions fiscales sur les agrégats.

Dans son ouvrage de 1959, *The Theory of Public Finance*, Richard Musgrave [117] fournit une typologie des objectifs de la politique économique, pouvant être résumés à partir des trois fonctions de l'Etat suivantes :

– La fonction d'allocation, concernant les politiques structurelles. La production de services collectifs, les nationalisations ou privatisations, l'aménagement du territoire, la planification, la politique industrielle... sont les actions de l'État qui infléchissent l'allocation des ressources productives par rapport à l'allocation qui résulterait du libre jeu du marché. L'objectif est en général de créer des conditions plus favorables à la croissance et au développement économiques.

- La fonction de redistribution concerne toutes les actions relatives à la protection sociale, au versement des revenus de transfert. Ces interventions se justifient par le fait que la répartition (primaire) des revenus qui résulte du marché est jugée inéquitable. Elles se justifient aussi dans la mesure où la répartition des revenus primaires et la répartition du patrimoine ne permettent pas à certaines catégories de la population de faire face à certains risques ; l'intervention de l'État relève alors de son rôle tutélaire. L'action de l'État sur la répartition des revenus peut se justifier aussi par l'existence d'une fonction d'utilité collective dont l'une des variables est l'altruisme des individus. Enfin, la redistribution des revenus peut se justifier par le fait que la répartition qui résulte du marché n'est pas optimale au sens de Pareto ; l'État recherche alors un optimum de second rang.
- la fonction de stabilisation concerne la régulation conjoncturelle de l'activité économique.

On comprend aisément que ces trois fonctions sont interdépendantes. Les essais proposés dans cette thèse, sur la stabilisation macroéconomique par la taxation, correspondent directement à au moins deux des trois fonctions de Musgrave (répartition et stabilisation). De plus, indirectement, la politique fiscale et la stabilisation jouent de manière évidente sur l'allocation des ressources.

C'est dans ce sillage que se situent mes recherches doctorales, en tentant de contribuer à l'analyse des effets macroéconomique de la taxation sur les fluctuations conjoncturelles. Cette thèse est constituée de quatre articles, dont j'expose un résumé dans les sections suivantes.

Taxation Linéairement Progressive et Stabilisation

*Ce travail, réalisé en collaboration avec Patrick Pintus, a été publié dans **Research in Economics**, volume **61**, numéro 1, Mars 2007.*

La littérature récente a montré comment la taxation progressive sur le revenu peut réduire la probabilité d'émergence d'états stationnaires dits "indéterminés"[1] et même conduire à une convergence de type point-selle[2] dans des modèles à rendements d'échelle croissants (cf. les travaux de Christiano et Harrison [45], Guo et Lansing [88], Guo [84]). Cependant, les contributions dans ce domaine reposent souvent sur l'hypothèse d'un taux marginal d'imposition continûment croissant avec le revenu, ce qui n'est pas vraiment une caractéristique partagée par la plupart des systèmes fiscaux actuels. (cf. par exemple "The Statistical Abstracts of the United States", ou "Tax and the Economy : A Comparative Assessment of OECD Countries", OECD Tax Policy Studies n.6, 2002).

Dans ce chapitre, nous incorporons une formulation alternative de taxation progressive, présentant une particularité assez proche de la réalité, dans le modèle de Benhabib et Farmer [14] : il est supposé qu'un taux d'imposition constant est appliqué au revenu seulement lorsque ce dernier est supérieur à un niveau d'exemption fiscale donné. Cette formulation semble en effet plus proche des codes d'imposition par tranches prévalant dans la plupart des pays de l'OCDE. Ces derniers se caractérisent généralement par un taux d'imposition constant à l'intérieur de chaque tranche de revenu, ainsi qu'un saut discret du taux marginal lorsque le revenu atteint la tranche supérieure. Nous montrons qu'il existe un seuil critique d'exemption fiscale au-delà duquel l'indétermination locale est éliminée et la stabilité de type point-selle assurée. Le mécanisme expliquant ce résultat est le suivant. Supposons que l'économie en situation de *laissez-faire* présente un état-stationnaire

[1] Un état stationnaire est dit localement indéterminé dès lors qu'il existe un nombre infini de trajectoires d'équilibre avec prévision parfaite convergeant vers cet état stationnaire.

[2] De manière alternative, un état stationnaire de type point-selle est localement déterminé, car il existe alors une unique trajectoire d'équilibre avec prévision parfaite convergeant vers l'état-stationnaire.

indéterminé, et introduisons des taxes linéairement progressives. L'accroissement du seuil d'exemption conduit à une diminution du taux moyen d'imposition, ce qui implique une progressivité plus élevée étant donnée la constance du taux marginal. De fait, un niveau suffisamment élevé d'exemption engendre la stabilité de type point-selle, en imposant un niveau de progressivité suffisamment élevé pour éliminer au moyen des taxes les conséquences des anticipations auto-réalisatrices des agents. Ceci rappelle le résultat selon lequel la détermination locale de l'état stationnaire est assurée lorsque la progressivité fiscale est suffisamment forte. Toutefois, notre analyse va plus loin et montre que cet effet stabilisant ne repose pas sur l'hypothèse d'un taux marginal continûment croissant lorsque la taxation linéairement progressive est considérée, ce qui complète les résultats existants. Notre résultat s'additionne aussi aux récentes conclusions soulignant l'inefficacité de la taxation linéaire (sans exemption) comme mécanisme visant à prévenir l'émergence de fluctuations entretenues par les croyances des agents.(e.g. Dromel and Pintus [51]).

En résumé, notre résultat peut être interprété de la manière suivante. Notre modèle se situe entre celui de Guo et Lansing [88] et une version élargie de Guo et Harrison [86] avec rendements d'échelle croissants. Dans une telle économie, où la politique fiscale introduite présente un aspect important des codes d'impositions actuels, nous montrons que des taxes sur le revenu linéairement progressives peuvent éliminer les équilibres dits à "tâches solaires", ce qui suggère que les seuils d'imposition, que l'on peut observer dans les codes fiscaux en vigueur dans de nombreux pays, peuvent contribuer à la stabilité agrégée de l'économie.

Politique Fiscale, Déduction des Dépenses de Maintenance et Cycles Endogènes

Sur la période récente, une littérature abondante a été consacrée à l'existence d'équilibres multiples à anticipations auto-réalisatrices dans les modèles d'équilibre général dynamique. Par exemple, Benhabib et Farmer [14] et Farmer et Guo [62] ont montré qu'un modèle de cycles d'affaires réels (RBC ci-après) à un secteur peut, lorsqu'il est sujet à des rendements d'échelles suffisamment élevés au niveau agrégé, présenter un état stationnaire indéterminé (c'est à dire un "puits"), à partir duquel peuvent être engendrés des cycles d'affaires conduits par les *esprits animaux* des agents. [3] En présentant les anticipations comme sources de chocs indépendantes, ces modèles dits à "tâches solaires" peuvent justifier le recours à des politiques de stabilisation, ayant pour but de réduire l'amplitude des cycles d'affaires. En suivant cette idée, Guo et Lansing [88] ont montré qu'une taxation progressive du revenu peut assurer la stabilité de type point-selle [4] dans le modèle de Benhabib-Farmer et Guo, et de fait protéger l'économie des conséquences des anticipations autoréalisatrices. [5] Toutefois, cette littérature fait l'hypothèse que l'économie en *situation de laissez-faire* est sujette à des degrés de rendements d'échelle agrégés très élevés et peu plausibles, au regard notamment des évaluations empiriques (cf. Burnside [36] ; Basu et Fernald [9]). Comme l'ont mis en évidence Christiano et Harrison ([45] p.20), l'incitation à stabiliser l'économie contre les fluctuations conduites par les croyances dépend du rapport entre deux facteurs ayant des effets opposés. Premièrement, toutes choses étant égales par ailleurs, la concavité d'une fonction d'utilité implique qu'un équilibre à "tâches so-

[3] Nous utiliserons ici les expressions "esprits animaux", "tâches solaires" et "anticipations auto-réalisatrices" de manière interchangeable. Chacune désigne une évolution aléatoire de l'économie, n'étant pas reliée à une incertitude concernant les variables *fondamentales* de l'économie que sont la technologie, les préférences et les dotations.

[4] Ici, nous adoptons l'idée souvent avancée qu'une politique est stabilisatrice lorsqu'elle conduit à la stabilité de type point-selle, et donc à la détermination de l'état stationnaire.

[5] Cf. Benhabib et Farmer [16] pour une revue de littérature des développement récents dans ce champs.

laires" sera Pareto-dominé par un équilibre déterministe et constant. Nous appellerons ceci l'*effet de concavité*. Toutefois, la présence de rendements d'échelle croissants signifie que la consommation peut être augmentée en moyenne sans augmenter le niveau moyen de l'emploi. Nous appellerons ceci l'*effet de bunching*. En conséquence, lorsque les rendements croissants sont suffisamment élevés, l'effet de bunching peut dominer l'effet de concavité, de sorte que les sentiers d'équilibres volatils peuvent améliorer le bien-être, par rapport aux allocations stationnaires. Dans cette situation, on peut dès lors douter du côté souhaitable de ce type de politiques de stabilisation.

Cependant, bien que les premières versions de ces modèles reposaient sur des valeurs de paramètres difficilement plausibles, les travaux plus récents sont basés sur des fondations dont le réalisme est croissant. Plusieurs auteurs ont montré que des modèles RBC à secteurs de production multiples (cf. Benhabib et Farmer [15] ; Perli [127] ; Weder [144]) ; Harrison [95]) ou présentant une utilisation endogène du capital (Wen [146]) peuvent engendrer de l'indétermination locale pour des valeurs beaucoup plus faibles de rendements croissants.[6] Weder [145] introduit une nouvelle formulation de l'utilisation endogène du capital, dans laquelle le coût d'utilisation apparaît sous la forme de dépenses variables de maintenance, et montre que l'indétermination de l'état stationnaire peut émerger pour des rendements d'échelles agrégés quasi-constants, défiant ainsi le point de vue selon lequel l'indétermination est un phénomène n'étant pas plausible du point de vue empirique. Dans un article récent, Guo et Lansing [90] explorent les effets liés à l'introduction de dépenses de maintenance et de réparation effectuées par les entreprises dans le modèle à utilisation variable du capital de Wen, et montrent aussi que l'indétermination apparaît pour de (très) faibles degrés de rendements croissants.

De toute évidence, les développements récents de ces modèles permettent d'étudier

[6]A l'exception notable de Benhabib et Nishimura [20] ; Benhabib, Meng et Nishimura [21], et Nishimura, Shimomura et Wang [120], entre autres, la plupart des études dans cette littérature postulent des rendements constants au niveau privé. Nous maintenons cette hypothèse à travers l'analyse.

l'indétermination et les tâches solaires pour des rendements d'échelle quasi-constants, c'est à dire lorsque l'effet de "bunching" est (très) faible. On présume alors que la volatilité conduite par les croyances engendre des pertes de bien-être (par l'effet de concavité, ou d'aversion au risque), constituant une justification claire pour une politique de stabilisation. Il nous paraît donc utile de ré-examiner les propriétés stabilisatrices de la progressivité fiscale dans le cas de rendements d'échelle quasi-constants, où la stabilisation semble *a priori* souhaitable du point de vue du Bien-Être.

Dans ce chapitre, nous étudions dans quelle mesure le pouvoir stabilisant de la progressivité fiscale, initialement mis en évidence dans cette littérature par Christiano et Harrison [45] et Guo et Lansing [88], est affecté lorsque les entreprises sont autorisées à déduire leurs dépenses de maintenance lors du calcul de leur profit avant-impôt (comme tel est le cas pour de nombreux codes d'imposition). [7]

Dans une version en temps continu du modèle de Guo et Lansing [90] avec dépenses de maintenance, nous trouvons que l'introduction d'une autorisation de déduction desdites dépenses affaiblit les propriétés stabilisatrices de la progressivité fiscale. Bien qu'une taxe progressive soit toujours à même de rendre la trajectoire d'équilibre vers l'état stationnaire unique, nous montrons que le niveau de progressivité requis afin de protéger l'économie des fluctuations conduites par les croyances est fonction croissante de la part des dépenses de maintenance dans le PIB. En d'autres termes, la possibilité pour les entreprises de déduire leurs dépenses de maintenance et réparation de leur profit avant-impôt accroît la probabilité d'indétermination locale de l'état stationnaire et favorise la volatilité excessive liée aux esprits animaux des agents. Nous trouvons ici aussi confirmation de l'inefficacité des taxes linéaires en tant que mécanisme stabilisateur contre les fluctuations engendrées par les croyances.

[7] En guise d'illustration, aux États-Unis "it has been held that expenses for small parts of a large machine, made in order to keep the machine in efficient working condition, were deductible expenses and not capital expenditures even though they may have a life of two or three years" (Commerce Clearing House, Chicago, Standard Federal Tax Reports, 1999, p.22, 182 [46])

Au delà de l'approche en temps continu et de l'introduction de la taxation, notre papier se distingue aussi de Guo and Lansing [90] dans la mesure où nous proposons des conditions formelles claires pour l'analyse de stabilité, sans se baser exclusivement sur des simulations numériques.

Il a été montré par Mc Grattan et Schmitz [115]), que les dépenses de maintenance sont "trop importantes pour ne pas être prises en compte", fortement pro-cycliques et d'importants substituts potentiels aux investissements. Cette propriété de substituabilité peut être utilisée pour fournir une discussion intuitive du mécanisme expliquant notre résultat. Supposons que les agents ont des anticipations optimistes concernant, par exemple, un rendement prochain du capital plus élevé. Les entreprises voudraient naturellement investir davantage en capital. Mais, du fait de la progressivité du système fiscal, elles savent qu'elles seraient alors confrontées à un taux d'imposition plus élevé. Ainsi, au lieu d'investir en nouveau capital physique (à travers de nouvelles machines ou de nouvelles structures), les entreprises préfèreront substituer la maintenance à l'investissement. La réduction conséquente de la base fiscale implique qu'un niveau plus élevé de progressivité fiscale sera nécessaire pour stabiliser l'économie contre les cycles engendrés par les croyances.

Progressivité des Taxes en Économie Monétaire avec Agents Hétérogènes

Ce travail, réalisé en collaboration avec Patrick Pintus, est à paraître dans le ***Journal of Public Economic Theory*** *(2008).*

Les taxes et transferts dépendant du revenu ont été proposés comme stabilisateurs automatiques efficaces depuis, au moins, les travaux de Musgrave et Miller [1948], Vickrey [1947, 1949], Slitor [1948] et Friedman [1948]. Ces dernières années, le développement des modèles d'équilibre général dynamique a montré l'utilité d'étudier d'une manière plus précise comment, en particulier, les politiques fiscales progressives peuvent stabiliser les variables agrégées de l'économie. En particulier, Christiano et Harrison [1999], Guo et Lansing [1998], Guo [1999], Guo et Harrison [2001], et Sim [2005] ont montré que la progressivité des taxes sur le revenu peut éliminer l'indétermination locale - engendrée par la présence de rendements d'échelle croissants - et restaurer la convergence de type point-selle.

Toutefois, comme nous l'avons mentionné plus haut, cette littérature fait l'hypothèse que l'économie en *situation de laissez-faire* est sujette à des degrés de rendements d'échelle agrégés très élevés et peu plausibles, au regard notamment des évaluations empiriques.

Dans ce papier, nous étudions l'impact d'une taxation progressive dans une économie monétaire présentant des rendements d'échelle constants, en prolongeant les résultats de Woodford [147] et Grandmont *et al.* [82]. Le "bunching effect" étant absent, on suppose que la volatilité entretenue par les croyances conduira de manière non-ambiguë à des pertes de bien-être (du fait de l'effet de concavité ou d'aversion devant le risque), justifiant le recours a une politique de stabilisation. Comme il a été montré par Woodford [147] et Grandmont *et al.* [82], la présence de monnaie comme actif dominé est cruciale pour engendrer l'indétermination locale dans l'économie de laissez-faire sans taxes. Nous montrons que l'indétermination locale (et donc les tâches solaires et cycles endogènes) sont *robustes* à l'introduction de la progressivité fiscale, lorsque cette dernière est fixée à des niveaux plausibles (i.e., faibles). Plus précisément, bien que l'on montre que les taxes progressives sur le revenu du travail réduisent, dans l'espace des paramètres, la probabilité d'émergence d'indétermination locale, nous mettons en évidence que de faibles et plausibles niveaux de progressivité fiscale n'empêchent pas la possibilité de tâches solaires et de cycles (de type Hopf ou flip). De plus, des taxes régressives sur le revenu élargiraient l'ensemble des valeurs de paramètres associées à l'indétermination locale.

En conséquence, on peut interpréter ce résultat comme une remise en question de l'idée selon laquelle la taxation progressive sur le revenu est un stabilisateur automatique efficace. Ceci rappelle quelques débats plus anciens, concernant l'importance pratique de la stabilisation automatique (e.g. Musgrave et Miller [118] (cf. aussi Vickrey [142, 143], Slitor [139]).[8] Le fait que nous portions notre attention sur de faibles valeurs concernant la progressivité fiscale provient de l'observation des estimations disponibles. Notre analyse fait abstraction d'une taxe progressive sur le revenu du capital. Ceci est cohérent avec de nombreux codes d'imposition actuels, dont la progressivité des taxes sur le revenu du travail est bien supérieure à celle des taxes sur le revenu du capital (cf. Hall et Rabushka [93]). Enfin, nous suivons Feldstein [65], Kanbur [103], Persson [128] en considérant des systèmes fiscaux présentant une progressivité du revenu résiduel constante.

Le mécanisme expliquant notre résultat est le suivant. Le travail est offert de façon élastique par les détenteurs de monnaie, et dépend du salaire réel courant et du taux d'inflation anticipé. Quand les travailleurs ont des anticipations optimistes (par exemple, concernant une baisse de l'inflation), ils voudront augmenter leur consommation et, de fait, consacrer une part plus importante de leur "dotation temporelle" au travail pour accroître leur revenu, ce qui engendre une expansion. Du fait de la présence de taxes progressives sur le revenu du travail, l'offre de travail est de moins en moins élastique aux variations du salaire réel et de l'inflation anticipée. Il existe toutefois une différence majeure dans la façon avec laquelle l'offre de travail réagit à ces deux variables : finalement, lorsque la progressivité atteint des niveaux maximum, l'inflation anticipée continue d'exercer un impact négatif sur l'offre de travail, alors que l'effet du salaire réel (avant impôt) tend vers 0. En d'autres termes, *la progressivité fiscale ne neutralise pas les effets de l'inflation anticipée sur l'offre de travail courante*, ce qui laisse une possibilité d'émergence de cycles d'affaires entretenus par les croyances.

Typiquement, cet effet ne peut être observé dans la littérature de type Ramsey avec rendements croissants et sans monnaie. La réaction de l'offre de travail aux anticipations

[8]Notre conclusion n'est pas incohérente avec certains résultats obtenus dans des modèles bi-sectoriels par Guo and Harrison [85] et Sim [138], dans lesquels les mécanismes conduisant à l'indétermination sont quelque peu différents, puisqu'ils reposent sur des rendements d'échelle croissants..

d'inflation, canal de transmission important des esprits animaux des agents, n'est effectivement pas prise en compte dans les deux premiers chapitres. Notons aussi que la politique fiscale progressive étudiée dans le chapitre 3 porte sur le salaire réel (ce qui est une hypothèse plutôt réaliste). On imagine donc aisément que si le salaire nominal était plutôt choisi comme base fiscale, ce canal d'anticipation d'inflation serait encore plus important, et pourrait engendrer encore plus de volatilité excessive.

Au-delà de l'hypothèse de rendements constants, et de la présence d'encaisses monétaires, la structure que nous considérons diffère de celles de Christiano et Harrison [45] et Guo et Lansing [88] au sens où nous faisons abstraction de la taxation du revenu du capital. Pour faciliter les comparaisons avec ces précédents papiers, nous introduisons de faibles rendements d'échelle croissants et vérifions la robustesse des résultats que nous obtenons avec des rendements constants. Notons que dans un papier récent Seegmuller [137, section 5.2.2] étudie, dans un exemple, les effets de taxes non-linéaires dans le même modèle, mais restreint son analyse à celle de la taxation régressive. Nous devons aussi préciser que la définition de progressivité que nous avons retenu est la suivante : le système fiscal auquel font face les agents est tel que le taux marginal est supérieur au taux moyen d'imposition.

Subventions aux Investissements et Stabilisation en Présence d'Imperfections sur le Marché du Capital

Le but de ce chapitre est d'analyser comment des subventions aux investissements innovateurs, financées par une taxation du revenu du travail, affectent la volatilité agrégée et la croissance dans une économie sujette à des imperfections sur le marché du crédit. Le modèle est basé sur un article d'Aghion, Banerjee et Piketty [3], dans lequel la présence de frictions sur le marché du capital combinée à une inégalité d'accès aux opportunités d'investissement entre individus peut engendrer des fluctuations endogènes et permanentes du PIB, de l'investissement et des taux d'intérêts. Dans cette structure, les épargnants et investisseurs sont "séparés" selon deux dimensions. La première, purement physique, est justifiée par le fait qu'un grand nombre d'épargnants n'ont pas la possibilité d'investir directement en capital physique (par manque d'idées, de talent, ou du fait de certaines distances sociales ou géographiques). La seconde dimension de séparation réside dans une contrainte pesant sur le montant que les investisseurs peuvent emprunter auprès des épargnants (du fait d'asymétries d'informations dans l'économie). Aghion et al. [3] montrent que lorsque le développement du marché du crédit est suffisamment élevé, et que le degré de séparation entre épargnants et investisseurs est suffisamment bas, l'économie peut se trouver en situation dite d'*expansion permanente*. A contrario, un fort degré de séparation entre épargnants et investisseurs, conduira l'économie à fluctuer autour de son sentier stationnaire. Plus précisément, il est montré qu'en combinant à la fois un fort degré de séparation entre épargnants et investisseurs et un très faible développement du marché du crédit, l'économie convergera toujours vers un cycle autour du son sentier stationnaire, à moins que les frictions sur le marché du capital soient si fortes que l'économie bascule en situation dite de *crise permanente*. Des économies présentant des marchés financiers peu développés et une forte séparation entre épargnants et investisseurs tendront vers une plus grande volatilité et de plus faibles taux de croissance. Pour un certain nombres de raisons évidentes, ces deux dimensions sont susceptibles d'être importantes dans les économies émergentes. Toutefois, certaines régularités empiriques montrent que ce mécanisme basé sur le fonctionnement du marché du crédit est aussi pertinent pour la compréhension des propriétés cycliques d'économies plus avancées. Par exemple, ce type d'analyse peut expliquer le cas d'économies de marché avancées telles que la Finlande, où le développement financier est néanmoins toujours un peu en retard, et ayant fait l'expérience d'une forte volatilité macroéconomique sur la dernière décennie (cf. Honjapohja et Koskela [99]). De plus, même pour une économie très financièrement développée telles que celle des

États-Unis, l'analyse reste pertinente pour le cas des petits investisseurs, dont le montant d'investissement est très souvent corrélé avec leurs cash-flows courants.

La contribution de ce chapitre est de montrer qu'une politique fiscale appropriée peut éliminer l'apparition de crises, et protéger l'économie contre des fluctuations endogènes permanentes du produit national, du taux d'intérêt et de l'investissement. Pour des niveaux de développement du crédit donnés dans cette économie, nous proposons des valeurs particulières de paramètres fiscaux pouvant éliminer la probabilité d'occurrence de crises, renforcer la croissance à long-terme et placer l'économie sur un chemin d'expansion permanente.

Le principal mécanisme expliquant ce résultat est le suivant. Le type de politique fiscale que nous analysons, à savoir introduire une taxe sur le revenu du travail des épargnants et redistribuer les montants prélevés vers les investisseurs productifs, semble équivalent à une augmentation de la fraction de la force de travail ayant un accès direct aux opportunités d'investissement.

Nous analysons comment les paramètres de la politique fiscale stabilisatrice sont affectés lorsque le niveau des frictions dans l'économie varie. Enfin, nous étudions comment le système fiscal modifie la réponse de l'économie aux différents chocs de productivité pouvant survenir.

Notre résultat est un complément aux résultats d'Aghion et al. En effet, nous n'utilisons pas les mêmes instruments de politique économique. Dans leur papier, en dernière section, il suggèrent la possibilité pour le gouvernement d'absorber l'épargne oisive pendant les périodes de crise par une émission de dette publique, afin de pallier à l'insuffisance de capacité d'investissement des emprunteurs. Cependant, le recours à la dette est un instrument contraint, dans de nombreux pays. La taxation du revenu du travail est, elle, quasi généralisée dans les économies modernes. Il est donc intéressant d'observer qu'une politique fiscale toute simple peut aussi aider à la stabilisation dans des économies sujettes à des frictions sur le marché du capital. Notre travail se distingue aussi d'Aghion et al [3] en proposant une caractérisation analytique complète des possibilités de régimes dynamiques, en fonction des taux de taxes et du niveau des frictions dans l'économie.

Bien que les fluctuations endogènes étudiées dans ce dernier chapitre ne soient pas de la même nature que celles étudiées dans les trois premiers, ce modèle a pour avantage de pouvoir explorer très simplement des dimensions plus complexes à analyser dans les modèles d'équilibre général dynamique (EGD) indéterminés. Il est aussi l'occasion de montrer que bien qu'elles n'aient pas ou peu d'effet stabilisant sur la dynamique des modèles EGD traités dans les trois premiers chapitres, les taxes linéaires ont bel et bien un effet stabilisant dans un modèle AK avec imperfections sur le marché du capital.

Chapter 1

Linearly Progressive Income Taxes and Stabilization[1]

It has been shown that progressive income taxes may lead to saddle-point convergence when the marginal tax rate is assumed to be a continuously increasing function of income. This paper shows that *linearly progressive* taxes may also immunize the economy against indeterminacy and sunspot equilibria. Therefore, our analysis suggests that exemption thresholds, as featured by prevailing tax codes, may help to stabilize the economy.

Key Words: Business Cycles; Progressive Income Taxes; Sunspots.

JEL Class.: D33; D58; E32; E62; H24; H30.

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1.1 Introduction

The recent literature has shown how progressive income taxes may reduce the likelihood of indeterminate equilibria and even lead to saddle-point stability (Christiano and Harrison [45], Guo and Lansing [88], Guo [84]). Contributions in this area rely on the assumption of a *continuously* increasing marginal tax rate, which is not a feature of most actual tax schedules, as casual observation suggests.

In this paper, we incorporate an alternative formulation of progressive taxation into the Benhabib and Farmer [14] model, which captures an ubiquitous real-world aspect: it is assumed that a constant tax rate is applied to income only when the latter exceeds an exemption threshold. This seems to get closer to the tax codes with brackets that are prevailing, for instance, in most OECD countries. We prove that there exists a critical exemption threshold above which local indeterminacy is ruled out and saddle-point stability ensured. The basic mechanism behind this result seems straightforward. Assume that the laissez-faire economy without taxes exhibits indeterminacy and then introduce linearly progressive taxes. As shown below, increasing the exemption threshold leads to a lower average tax rate, which implies higher tax progressivity given the constant marginal tax rate. Therefore, a large enough exemption restores saddle-point stability by imposing a level of progressivity that is high enough to tax away the benefits of self-fulfilling expectations. This is reminiscent of the result that local determinacy is ensured when tax progressivity is sufficiently large (Guo and Lansing [88], Guo [84]). However, our analysis further shows that this stabilizing effect does not need to rely on a continuously increasing marginal tax rate when linear progressivity is considered, which complements the existing conclusions. Our result also complements some recent conclusions underlining that flat-rate taxation without exemption may not promote macroeconomic stability (e.g. Dromel and Pintus [51]).

In summary, one may interpret our results in the following way. Our model is in between Guo and Lansing's [88] and an extended version of Guo and Harrison's [86] with increasing returns to scale. In such an economy with a fiscal policy that captures an important aspect of actual tax codes, we show that linearly progressive income taxes may rule out sunspot equilibria, which suggests that exemption thresholds, as featured by tax codes prevailing in many countries, may help to stabilize the economy.

The rest of the paper is organized as follows. The next section presents the economy with linearly progressive income taxes while section 3 studies the local dynamics and shows how progressivity may lead to saddle-point stability. Some concluding remarks are gathered in section 4.

1.2 The Economy

This paper introduces linearly progressive income taxes into a benchmark growth model. To ease comparison with results by Guo and Lansing [88], Christiano and Harrison [45] and Guo [84], we focus on the one-sector version with constant capital utilization studied by Benhabib and Farmer [14]. However, our analysis could be easily adapted to richer assumptions and is expected to yield similar results, for instance in a slightly different formulation with variable capital utilization and smaller externalities (by building on Wen's [146] insight).

1.2.1 Firms, Households and Government

Following Benhabib and Farmer [14], we assume that a unique final good y is produced by using capital k and labor l, according to the following (aggregate) technology:

$$y = k^{\alpha} l^{\beta}, \tag{1.2.1}$$

where $\alpha, \beta \geq 0$ and $\alpha + \beta > 1$. For simplicity, we assume increasing returns due to the presence of externalities. It would be straightforward to modify the analysis to cover the case with imperfect competition.

The Ramsey households have preferences represented by:

$$\int_0^\infty e^{-\rho t}\{\log[c(t)] - A\frac{[l(t)]^{1+\gamma}}{1+\gamma}\}dt, \tag{1.2.2}$$

where, at each date $t \geq 0$, $c > 0$ is consumption, $l > 0$ is labor supply, while $A > 0$ is a scaling parameter, $1/\gamma \geq 0$ is the labor supply elasticity to the real wage, and $\rho > 0$ is the discount rate. The representative consumer owns the inputs and rents them to firms through competitive markets. Therefore, we can write down, for sake of brevity, the consolidated budget constraint as:[2]

$$\dot{k} = y - \tau(y - E) - \delta k - c, \tag{1.2.3}$$

where $1 > \tau \geq 0$ is the tax rate that is imposed on that part of income which exceeds the exemption threshold $E \geq 0$. On the other hand, $\tau = 0$ when $E \geq y$. Both τ and E are assumed to be constant through time. In the sequel, we will assume that parameter values are such that $y > E$ in steady state so that taxes are strictly positive.

As is usual, government is assumed to finance public expenditures g that do not affect private decisions by taxing output in a progressive way, when the latter exceeds the exemption threshold, i.e. $g = \tau(y - E)$ when $y \geq E$.

1.2.2 Intertemporal Equilibria

Households' decisions follow from maximizing (1.2.2) subject to the budget constraint (1.2.3), given the initial stock $k(0) \geq 0$. Straightforward computations yield the following first-order conditions:

$$\begin{aligned} \dot{c}/c &= a(1-\tau)y/k - \rho - \delta, \\ Acl^\gamma &= b(1-\tau)y/l, \end{aligned} \tag{1.2.4}$$

[2] From now on, we omit the time dependence of all variables to save on notation.

where $1 \geq a, b \geq 0$ denote, respectively, the share of capital income and the share of labor income in total output, with $a + b = 1$. In contrast with the first-order conditions emerging in a laissez-faire economy (Benhabib and Farmer [14]), equations (1.2.4) depend on the after-tax marginal returns to capital and labor.

We may rewrite the budget constraint, from (1.2.3), as:

$$\dot{k}/k = (1-\tau)y/k + \tau E/k - \delta - c/k. \tag{1.2.5}$$

Equations (1.2.1) and (1.2.4)-(1.2.5) characterize the dynamics of intertemporal equilibria with perfect foresight, given $k(0)$. It is easily checked that the transversality constraint is met in the following analysis, as we consider orbits that converge towards an interior steady state.

Our setting nests both Benhabib and Farmer's [14] model, when $\tau = E = 0$, and a benchmark case of Guo and Harrison's [86], when $1 - \alpha - \beta = E = 0$ (with a uniform tax rate on capital and labor incomes).

1.2.3 Linearized Dynamics

We derive and linearize, around the steady state, the dynamical system describing intertemporal equilibria which consists of equations (1.2.4)-(1.2.5), together with equations (1.2.1). The first step is to rewrite, from the static condition in (1.2.4), the following equation:

$$[\gamma + 1 - \beta]\hat{l} = \log\left(b(1-\tau)/A\right) + \alpha\hat{k} - \hat{c}, \tag{1.2.6}$$

where the variables with a 'hat' are logs of the original variables (so that, for instance, $\hat{l} = \log(l)$), using the fact that $\hat{y} = \alpha\hat{k} + \beta\hat{l}$ from taking logs in equations (1.2.1). This yields, by using equation (1.2.6):

$$\hat{y} - \hat{k} = \lambda_0 + \lambda_1\hat{k} + \lambda_2\hat{c}, \tag{1.2.7}$$

where

$$\begin{aligned}\lambda_0 &= \beta \log\left(b(1-\tau)/A\right)/(\gamma+1-\beta),\\ \lambda_1 &= [\beta+(\alpha-1)(\gamma+1)]/(\gamma+1-\beta),\\ \lambda_2 &= -\beta/(\gamma+1-\beta).\end{aligned} \tag{1.2.8}$$

By rewriting equations (1.2.4)-(1.2.5) in logs and using equations (1.2.8), it is easy to get:

$$\begin{aligned}\dot{\hat{c}} &= a(1-\tau)\exp\left(\lambda_0+\lambda_1\hat{k}+\lambda_2\hat{c}\right)-\rho-\delta,\\ \dot{\hat{k}} &= (1-\tau)\exp\left(\lambda_0+\lambda_1\hat{k}+\lambda_2\hat{c}\right)+\tau E\exp\left(-\hat{k}\right)-\delta-\exp\left(\hat{c}-\hat{k}\right).\end{aligned} \tag{1.2.9}$$

It is straightforward to show that, under our assumptions, the differential equations (1.2.9) possess a steady state $\hat{c}^*, \hat{k}^*$. More precisely, $\dot{\hat{c}} = 0$ yields, from the first equation of system (1.2.9):

$$\exp\left(\lambda_0+\lambda_1\hat{k}^*+\lambda_2\hat{c}^*\right) = (\rho+\delta)/[a(1-\tau)], \tag{1.2.10}$$

On the other hand, $\dot{\hat{k}} = 0$ then yields, from the second equation of system (1.2.9):

$$\exp\left(\hat{c}^*-\hat{k}^*\right)-\tau E\exp\left(-\hat{k}^*\right) = \theta. \tag{1.2.11}$$

with $\theta \equiv [\rho+\delta(1-a)]/a \geq 0$.

One can then easily establish that the two latter equations have solutions $\hat{c}^*$ and $\hat{k}^*$, provided that the scaling parameter A is appropriately chosen. More precisely, one can set, without loosing generality, $\hat{k}^* = 0$ (that is, $k^* = 1$) by fixing $A = [b(1-\tau)/(\theta + \tau E)][(\rho+\delta)/(a(1-\tau))]^{(\beta-\gamma-1)/\beta}$. Moreover, one then has $\exp(\hat{c}^*) = c^* = \theta + \tau E$. Such a procedure aims at proving the existence of at least one steady state (which may not be necessarily unique), and at making sure that it "persists" after any arbitrarily small perturbation of the original two-dimensional map. For brevity, multiplicity of steady states is not studied. However, due to our normalization procedure, one can discuss the possibility to analyze a single steady-state, satisfying the stationary state condition $y^* > E$.

In stationary state, the euler equation (1.2.4) gives $y^*/k^* = (\rho+\delta)/(a(1-\tau))$. One can choose a A to get $k^* = 1$, and $l^{*\beta} = \frac{\rho+\delta}{a(1-\tau)}$. Moreover, $y^* > E \Leftrightarrow (k^*)^\alpha (l^*)^\beta > E$. Therefore, $l^{*\beta} > E \Leftrightarrow \tau E > E - \frac{\rho+\delta}{a}$. One can choose an A such that E is small enough and ensures $y^* > E$.

1.3 Analysis of the Dynamics

We linearized equations (1.2.1) and (1.2.4)-(1.2.5) around an interior steady state. Straightforward computations yield the following expressions for trace T and determinant D of the Jacobian matrix of the dynamical system derived from equations (1.2.1) and (1.2.4)-(1.2.5) (see equations (1.2.9)):

Proposition 1.3.1 (Linearized Dynamics around a Steady State)

Linearized dynamics for deviations $\hat{c} - \hat{c}^*$ and $\hat{k} - \hat{k}^*$ are determined by linear map such that, in steady-state:

$$\begin{aligned} T &= \rho + \tfrac{(\rho+\delta)(\alpha-a)(\gamma+1)}{a(\gamma+1-\beta)}, \\ D &= \tfrac{\rho+\delta}{\gamma+1-\beta}\{\theta(\alpha-1)(\gamma+1) + \tau E[\beta + (\alpha-1)(\gamma+1)]\}, \end{aligned} \tag{1.3.12}$$

with $\theta \equiv [\rho + \delta(1-a)]/a \geq 0$.

1.3.1 Local Determinacy with Progressive Taxes

Indeterminacy is defined as follows.

Definition 1.3.1 (Indeterminacy of the Steady State)

The equilibrium is indeterminate if there exists an infinite number of perfect foresight equilibrium sequences.

The variable k is predetermined since k_0 is given by the initial conditions of the economy while c_0 is free to be determined by the behavior of the agents in the economy. Suppose that the steady-state $\{k^*, c^*\}$ is completely stable in the sense that all equilibrium trajectories which begin in the neighborhood of $\{k^*, c^*\}$ converge back to the steady state. In this case, there will be a continuum of equilibrium path $\{k(t), c(t)\}$, indexed by c_0, since any path that converges to $\{k^*, c^*\}$ necessarily satisfies the transversality condition. Completely stable steady states giving rise to a continuum of equilibria are termed indeterminate and in this case the stable manifold has dimension 2. Indeterminacy requires that both eigenvalues of the Jacobian matrix have negative real parts (the steady-state is a sink).

— Figure 2.1 about here —

Alternatively, if there is a one-dimensional manifold in $\{k, c\}$ space with the property that trajectories that begin on this manifold converge to the steady-state but all other trajectories diverge then the equilibrium will be locally unique in the neighborhood of the steady-state. In this case, for every k_0 in the neighborhood of k^* there will exist a unique c_0 in the neighborhood of c^* that generates a trajectory converging to $\{k^*, c^*\}$. This c_0 is the one that places the economy on the stable branch of the saddle point $\{k^*, c^*\}$.

— Figure 2.2 about here —

Since the Trace of the Jacobian measures the sum of the roots and the Determinant measures the product we can use information on the sign of the Trace and the Determinant to check the dimension of the stable manifold of the steady-state $\{k^*, c^*\}$. Indeterminacy can be restated as $T < 0 < D$. Similarly the steady-state is saddle-path stable if $D < 0$, and is unstable (a source) if $T > 0$ and $D > 0$. Since the eventual fate of trajectories that diverge from the steady state cannot be determined from the properties of the Jacobian evaluated at the steady state, we will not further elaborate on the source case [3].

[3]Trajectories may eventually violate non-negativity constraints or may settle down to a limit cycle or to some more complicated attracting set

— Figure 2.3 about here —

Assume $a < \alpha < 1$, as in Benhabib and Farmer [14]. Direct inspection of equations (1.3.12) shows that $T < 0$, a necessary condition for local indeterminacy, implies that $\beta > \gamma + 1$ in our economy with linearly progressive taxes, as in the model without taxes of Benhabib and Farmer [14] where the latter inequality is also sufficient. We assume that the latter condition is met, so that local indeterminacy would prevail in the laissez-faire economy, and show that it does not suffice for local indeterminacy in the economy with taxes. Our main task is to underline the conditions such that linearly progressive taxes lead to saddle-point stability.

On the other hand, equations (1.3.12) show that, under the assumption that $\beta > \gamma + 1$, the sign of D is given by the sign of $\{\theta(1-\alpha)(\gamma+1) - \tau E[\beta + (\alpha-1)(\gamma+1)]\}$. When $\alpha < 1$, which implies that endogenous growth is ruled out, one therefore ensures saddle-point convergence, that is, $D < 0$, if and only if τE is large enough. More precisely, the following holds.

Proposition 1.3.2 (Local Determinacy Through Linearly Progressive Income Taxes)

Assume $a < \alpha < 1$ and $\beta > \gamma + 1$. Then the following holds:

(i) in the economy without taxes or without exemption (that is, when $\tau = 0$ or $E = 0$), the local dynamics of consumption c and capital k given by equations (1.2.1) and (1.2.4)-(1.2.5) around the positive steady state (c^*, k^*) exhibit local indeterminacy (that is, $T < 0 < D$) if β is not too large, as in Benhabib and Farmer [14].

(ii) however, in the economy with linearly progressive income taxes (that is, when $\tau, E > 0$), the local dynamics of c and k given by (1.2.1) and (1.2.4)-(1.2.5) exhibit local determinacy (that is, $D < 0$) if and only if $\tau E > [\theta(1-\alpha)(\gamma+1)]/[\beta + (\alpha-1)(\gamma+1)]$, with $\theta \equiv [\rho + \delta(1-a)]/a \geq 0$.

Proof: See the appendix 1.5.

The basic mechanism behind part (*ii*) of Proposition 1.3.2 is straightforward. Assume that the laissez-faire economy without taxes (such that $\tau = E = 0$) exhibits indeterminacy. Then introduce linearly progressive taxes, with a constant the marginal tax rate $1 > \tau > 0$ taken as given. As easily shown, increasing the exemption threshold leads to a lower average tax rate, which implies *higher tax progressivity* given the constant marginal tax rate. To see this, define tax progressivity, following Musgrave and Thin [119], as $\pi = (\tau_m - \tau_a)/(1 - \tau_a)$, where τ_m (resp. τ_a) defines the marginal (resp. average) tax rate. In our formulation, $\tau_m = \tau$ while $\tau_a = \tau(1 - E/y)$ so that $\pi = \tau E/[(1 - \tau)y + \tau E]$. This implies that progressivity π increases with E. Therefore, a large enough exemption E restores saddle-point stability by imposing a level of progressivity that is sufficiently large to tax away the benefits of self-fulfilling expectations. This is reminiscent of the result that local determinacy is ensured when tax progressivity is sufficiently large (Guo and Lansing [88], Guo [84]). However, our analysis further shows that this stabilizing effect does not need to rely on a continuously increasing marginal tax rate when linear progressivity is considered, which complements the existing conclusions. Note also that similar a conclusion is drawn if, instead, we would increase τ from zero, keeping $E > 0$ constant: the condition for determinacy appearing in part (*ii*) of Proposition 1.3.2 can equivalently be interpreted in terms of the marginal tax rate τ, as progressivity π is also shown to be an increasing function of τ.

By ignoring variable capacity, we admittedly focus on possibly large levels of increasing returns to scale. However, it is clear from the above analysis that incorporating variable utilization (as in Wen [146]) is expected to deliver qualitatively similar results with much smaller externalities. Moreover, it is expected that our analysis carries through, with similar results, if progressive taxes apply, maybe more realistically, to labor income only (with linear taxes on capital income). Finally, although our discussion is restricted for simplicity to a two-bracket schedule, it can easily be extended to the more plausible configuration with increasing tax rates along with multiple brackets. Our analysis turns out to remain essentially unchanged, with τ as the tax rate associated with the relevant bracket (to which belongs the income of the representative household) and the exemption level E as an average of all lower exemption thresholds weighted by the corresponding tax increments.

1.4 Conclusion

The recent literature has shown that progressive income taxation reduces, in parameter space, the likelihood of indeterminate equilibria and leads to saddle-point stability when the marginal tax rate is assumed to be a continuously increasing function of income. Using a formulation that arguably captures an ubiquitous real-world aspect, this paper has proved that setting the exemption threshold at a sufficiently large level ensures local determinacy, by imposing a strong enough tax progressivity. Therefore, our analysis suggests that linearly progressive tax codes prevailing in most OECD countries may help to immunize the economy against sunspot-driven business cycles.

Appendix. Linearly Progressive Income Taxes and Stabilization

1.5 Proof of Proposition 1.3.2

Straightforward computations lead to the following expressions of T and D, respectively the trace and determinant of the Jacobian matrix associated with equations (1.2.9), evaluated at the steady state $\hat{c}^*, \hat{k}^*$ that has been normalized following the procedure at the end of the previous appendix:

$$\begin{aligned} T &= \rho + \tfrac{(\rho+\delta)(\alpha-a)(\gamma+1)}{a(\gamma+1-\beta)}, \\ D &= \tfrac{\rho+\delta}{\gamma+1-\beta}\{\theta(\alpha-1)(\gamma+1)+\tau E[\beta+(\alpha-1)(\gamma+1)]\}, \end{aligned} \tag{1.5.13}$$

with $\theta \equiv [\rho + \delta(1-a)]/a \geq 0$.

Direct inspection of equations (1.5.13) shows that $T < 0$, which is a necessary condition for local indeterminacy, only if $\beta > \gamma + 1$, just as in the model without taxes of Benhabib and Farmer [14], regardless of both the tax rate τ and the exemption level E.

To show (i), set $\tau = E = 0$ and assume $\alpha < 1$. Then one has that $D = \theta(\rho + \delta)(\alpha - 1)(\gamma + 1)/(\gamma + 1 - \beta)$. Therefore, $T < 0 < D$ if and only if $\beta > \gamma + 1$ with β not too large (to ensure $T < 0$).

To show (ii), assume $\tau, E > 0$ and $\alpha < 1$. Then equations (1.5.13) show that, when $\beta > \gamma + 1$, the sign of D is given by the sign of $\{\theta(1-\alpha)(\gamma+1) - \tau E[\beta + (\alpha - 1)(\gamma + 1)]\}$. One therefore ensures saddle-point stability, that is, $D < 0$, if and only if the exemption level E is large enough, that is, if and only if $\tau E > [\theta(1-\alpha)(\gamma+1)]/[\beta + (\alpha-1)(\gamma+1)]$. $\square$

Figure 1.1: Local Indeterminacy: the Steady-State is a Sink

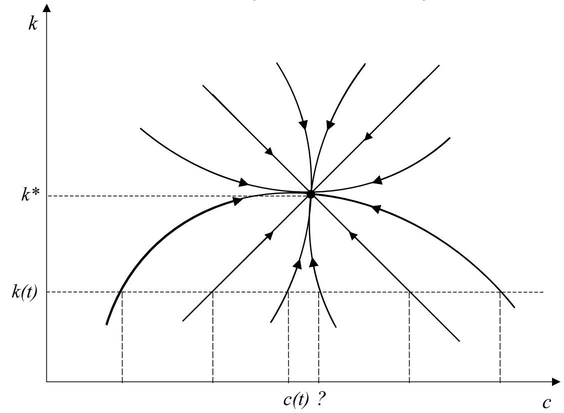


Figure 1.2: Saddle-Path Stability: the Steady-State is Locally Determinate

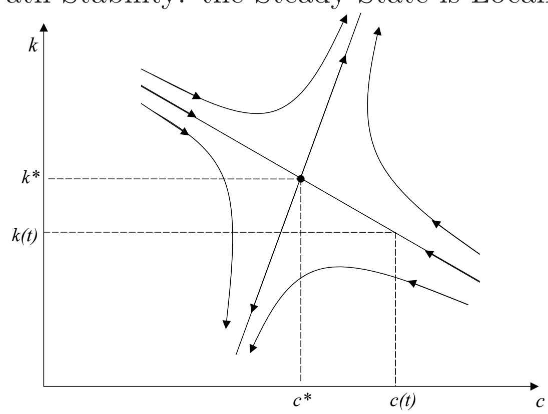
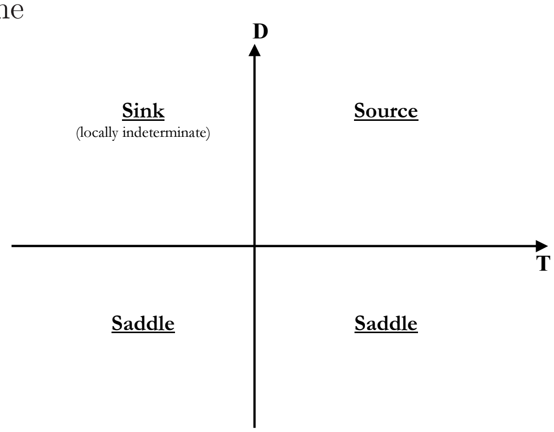


Figure 1.3: Jacobian's Trace-Determinant Diagram: Stability Regimes of Steady State in Continuous Time



Chapter 2

Fiscal Policy, Maintenance Allowances and Expectation-Driven Business Cycles[1]

Maintenance and repair activity appears to be a quantitatively significant feature of modern industrial economies. Within a real business cycle model with arguably small aggregate increasing returns, this paper assesses the stabilizing effects of fiscal policies with a maintenance expenditure allowance. In this setup, firms are authorized to deduct their maintenance and repair expenditures from revenues in calculating pre-tax profits, as in many prevailing tax codes. While flat rate taxation does not prove useful to insulate the economy from self-fulfilling beliefs, a progressive tax can render the equilibrium unique. However, we show that the required progressivity to protect the economy against sunspot-driven fluctuations is increasing in the maintenance-to-GDP ratio. Taking into account the

[1] I thank, without implicating, Jess Benhabib, Roger Farmer, Monique Florenzano, Jean-Michel Grandmont, Jang-Ting Guo, Kevin Lansing, Etienne Lehmann, Guy Laroque, Patrick Pintus, Thomas Seegmuller, Bertrand Wigniolle; seminar participants at Université Paris 1 Panthéon-Sorbonne, CREST; conference participants at the 8th Meeting of the Society for the Advancement of Economic Theory (SAET 2007, Kos) and the 8th Meeting of the Association for Public Economic Theory (PET07, Vanderbilt University) for helpful discussions on this work.

maintenance and repair activity of firms, and the tax deductability of the related expenditures, would then weaken the expected stabilizing properties of progressive fiscal schedules.

Key Words: Business Cycles; Maintenance and Repair Allowances; Capital Utilization; Progressive Income Taxes; Indeterminacy and Sunspots.

JEL Class.: D33; D58; E30; E32; E62; H20; H30.

2.1 Introduction

In recent years, there has been an extensive literature that examines the existence of multiple, self-fulfilling rational expectations equilibria in dynamic general equilibrium models. For example, Benhabib and Farmer [14] and Farmer and Guo [62] have shown that a one-sector real business cycle (RBC) model with sufficient aggregate increasing returns-to-scale may exhibit an indeterminate steady state (i.e. a sink) that can be exploited to generate business cycles driven by animal spirits.[2] By emphasizing expectations as an independent source of shocks, these so-called "sunspot" models create an opportunity for stabilization policies that are designed to mitigate belief-driven cycles. Following this idea, Guo and Lansing [88] have shown that a progressive income tax policy can ensure saddle path stability in the Benhabib-Farmer-Guo model, and thereby stabilize[3] the economy against "self-fulfilling beliefs".[4] However, this literature assumes that the laissez-faire economy is subject to large and implausibly high increasing returns to scale (Burnside [36]; Basu and Fernald [9]). As pointed out by Christiano and Harrison ([45] p.20), the desirability of stabilizing the economy against sunspot fluctuations is determined by the relative magnitude of two opposing factors. First, ceteris paribus, a concave utility function implies that a sunspot equilibrium is welfare-inferior to a constant, deterministic equilibrium (*concavity* or risk-aversion effect). However, other things are not the same. The increasing returns means that by bunching hard work, consumption can be increased on average without raising the average level of employment (*bunching* effect). As a consequence, when increasing returns are strong enough, the bunching effect may dominate the concavity effect, so that volatile paths may indeed improve welfare, in comparison with stationary allocations. In that situation, one may question the desirability of any stabilization policy.

Although initial versions of these models appear to rely on empirically implausible

[2] we use the terms "animal spirits", "sunspots" and self-fulfilling beliefs" interchangeably. All refer to any randomness in the economy that is not related to uncertainties about economic fundamentals such as technology, preferences and endowments.

[3] Here, we adopt the common view that a policy is stabilizing when it leads to saddle-point stability, hence to determinacy.

[4] See Benhabib and Farmer [16] for a survey of recent developments in this area

parameter values, recent vintages are based on increasingly realistic foundations. Many authors have shown that RBC models with multiple sectors of production (Benhabib and Farmer [15]; Perli [127]; Weder [144]); Harrison [95]) or endogenous capital utilization (Wen [146]) can generate local indeterminacy with much lower degrees of increasing returns.[5] Weder [145] introduces a new formulation of the endogenous capital utilization, in which the utilization costs appear in the form of variable maintenance expenses, and shows that indeterminacy can arise at approximately constant returns to scale, challenging the viewpoint that indeterminacy is empirically implausible. In a recent paper, Guo and Lansing [90] explore the effects of introducing maintenance and repair expenditures in Wen's variable capacity utilization model, and also show that indeterminacy can occur with a mild degree of increasing returns.

As a matter of fact, the latest developments of these models allow to study indeterminacy and sunspots for close-to-constant returns, that is when the "bunching effect" is (very) weak. One suspects then expectation-driven volatility to unambiguously lead to welfare losses (by the concavity, or risk-aversion, effect) that would call for stabilization. This proves useful to re-investigate the stabilizing properties of fiscal progressivity in the close-to-constant returns to scale case, where stabilization is *a priori* more desirable from a welfare standpoint.

In this paper, we investigate how the stabilizing power of fiscal progressivity, initially pushed forward in this literature by Christiano and Harrison [45] and Guo and Lansing [88], is affected when firms are authorized to deduct their maintenance expenditures from revenues in calculating pre-tax profits (as in many prevailing tax codes [6]). Because of tax ramifications of categorizing an expenditure as either maintenance and repair or in-

[5]With the noted exceptions of Benhabib and Nishimura [20]; Benhabib, Meng and Nishimura [21], and Nishimura, Shimomura and Wang [120], among others, most studies in this literature postulate constant returns-to-scale at the individual firm level. We maintain this assumption throughout the analysis.

[6]As an illustration, in the United States "it has been held that expenses for small parts of a large machine, made in order to keep the machine in efficient working condition, were deductible expenses and not capital expenditures even though they may have a life of two or three years" (Commerce Clearing House, Chicago, Standard Federal Tax Reports, 1999, p.22, 182 [46])

vestment, there are standard definitions used in the accounting literature. *Maintenance and repair* expenditures are made for the purpose of keeping the stock of fixed assets or productive capacity in good working order during the life originally intended. These include costs incurred to forestall breakdowns of equipment and structures (*maintenance*) and costs induced to restore fixed assets to a state of good working condition after malfunctioning (*repair*). Capital expenditures, or *investment spending*, are costs of all new plants, machinery and equipment which normally have a life of more than a year; these expenditures include purchases of new assets as well as major improvements or alterations to existing assets.

In a continuous-time version of the Guo and Lansing [90] maintenance expenditures model, we find that introducing maintenance allowances weakens the expected stabilizing properties of tax progressivity. Although a progressive tax can still render the equilibrium unique, we show that the required degree of progressivity to protect the economy against sunspot-driven fluctuations is increasing in the maintenance-to-GDP ratio. Put differently, the possibility for firms to deduct maintenance and repair expenditures from their pre-tax profits increases the likelihood of local indeterminacy and excess volatility due to animal spirits. Moreover, a flat tax schedule does not prove to be a useful and effective stabilizer.

Aside from dealing with continuous time and introducing taxation, our paper departs from Guo and Lansing [90] in that we provide clear necessary and sufficient conditions for the stability analysis and do not rely on numerical simulations.

It has been argued and documented (see for instance Mc Grattan and Schmitz [115]), that maintenance expenditures are "too big to ignore", strongly procyclical and important potential substitutes for investment. This substitutability feature can be used to provide an intuitive discussion of the basic mechanism driving our result. Let us suppose agents have optimistic expectations about, say, a higher return on capital in the next period. Firms will naturally want to invest more in the form of capital. But, due to the fiscal scheme progressivity, they know they will have to face in that case a higher tax rate. Thus, instead of investing in new physical capital (equipments or structures), firms prefer to substitute maintenance to investment. The consequent reduction in the tax base implies that a higher level of fiscal progressivity will be needed to stabilize the economy against belief-driven cycles.

Our result can be linked to a parallel strand of the literature, investigating the stabilizing properties of non-linear tax schedules in constant returns to scale, segmented asset markets economies (see for instance Lloyd-Braga, Modesto and Seegmuller [112]). In a monetary economy with constant returns to scale, Dromel and Pintus [51] show that tax progressivity reduces, in parameter space, the likelihood of local indeterminacy, sunspots and cycles. However, considering plausibly low levels of tax progressivity does not ensure saddle-point stability and preserves as robust the occurrence of sunspot equilibria and endogenous cycles. Exploiting a different mechanism, our paper gives also support to the view that low levels of tax progressivity may not be able to ensure the determinacy of equilibria.

The remainder of this paper proceeds as follows. The next section presents the model, while section 3 analyses dynamics and (in)determinacy conditions, showing how fiscal progressivity may lead to saddle-path stability. Some concluding remarks are gathered in section 4.

2.2 The Economy

This paper introduces fiscal policy, depreciation allowance and maintenance expenditures deductions into a continuous-time version of the Guo and Lansing [90] model. The decentralized economy consists of an infinite lived representative household that supplies labor, taking the real wage as given. The household owns a representative firm, acting in his best interest while making decisions about production, investment, maintenance and capital utilization.

2.2.1 Firms

There is a continuum of identical competitive firms, with the total normalized to unity, acting so as to maximize a discounted stream of profits. The representative firm i is endowed with k_0 units of capital and produces an homogeneous final good y_i using the following Cobb-Douglas technology

$$y_i = \bar{e}(u_i k_i)^\alpha n_i^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{2.2.1}$$

where k_i and n_i are firm i's usage of physical capital (equipment and structures) and labor hours, respectively[7]. The variable $u_i \in (0,1)$ designates the capital utilization rate. Although each firm is competitive, we assume that the economy as a whole is affected by organizational synergies that cause the output of the ith firm to be higher if all other firms in the economy are producing more. These productive external effects, denoted by $\bar{e}$, are outside of the scope of the market, and cannot be traded. Taken as given by each firm, they are specified as

$$\bar{e} = (\bar{u}\bar{k})^{\alpha\eta} \bar{n}^{(1-\alpha)\eta}, \qquad \eta \geq 0 \tag{2.2.2}$$

where $\bar{u}\bar{k}$ and $\bar{n}$ are economy-wide average levels of utilized capital and production labor inputs, respectively. We look at a symmetric equilibrium, in which all firms would take the same actions such that $u_i = \bar{u} = u$, $k_i = \bar{k} = k$ and $n_i = \bar{n} = n$, for all t. As a result, equation (2.2.2) can be substituted into equation (2.2.1) to obtain the following aggregate production technology, that may display increasing returns-to-scale:

$$y = \left[(uk)^\alpha n^{1-\alpha}\right]^{1+\eta} \tag{2.2.3}$$

where $1 + \eta$ characterizes the degree of aggregate increasing returns. When $\eta = 0$, the model boils down to the standard Ramsey formulation with constant returns-to-scale at both private and social levels.

[7] To save on notation, time dependence of all variables will be dropped in the sequel.

We assume an endogenous capital depreciation rate, $\delta \in (0,1)$, such that

$$\delta = \chi \frac{u^{\theta}}{(\frac{m}{k})^{\phi}}, \qquad \chi > 0, \quad \theta > 1, \quad \phi \geq 0 \tag{2.2.4}$$

where m represents maintenance and repair expenditures. The ratio m/k denotes the magnitude of the maintenance and repair per unit of capital. When the depreciation elasticity to maintenance (ϕ) is positive, a rise in maintenance activity will lower capital depreciation. On the other hand, an increase in the capital utilization rate u_t will speed up the depreciation. If $\phi = 0$, the model resembles the one analyzed in Wen [146], while if $\theta \to \infty$, it reduces to an economy with constant utilization like in the standard Benhabib-Farmer-Guo setup.

2.2.2 Households

The economy is populated by a large number of identical Ramsey households, each endowed with one unit of time, choosing their consumption c_t and labor supply n_t so as to maximize:

$$\int_0^{\infty} e^{-\rho t} \left\{ \log[c] - A \frac{n^{1+\gamma}}{1+\gamma} \right\} dt, \tag{2.2.5}$$

where $\rho > 0$ is the discount rate, A is a scaling parameter, $\gamma \geq 0$ is the inverse of the intertemporal elasticity of substitution in labor supply. The representative consumer owns the inputs and rents them to firms through competitive markets. We can write down the consolidated budget constraint as:

$$\dot{k} = (1-\tau)x - c, \quad \text{with} \quad x = y - \delta k - m \tag{2.2.6}$$

where $1 > \tau \geq 0$ is the tax rate imposed on the income net of capital depreciation and maintenance expenditures. We assume the capital stock is predetermined $k(0) = k_0$, and both consumption and capital are non-negative $k \geq 0$, $c \geq 0$.

2.2.3 Government

The government chooses tax policy τ and balances the public budget at each point in time. Hence, the instantaneous government budget constraint is $g = \tau x$, where g represents government spending on goods and services that are assumed not to contribute to either production or household utility. The aggregate resource constraint of the economy is given by:

$$c + \dot{k} + \delta k + g + m = y$$

The government is assumed to set τ according to the following tax schedule:

$$\tau = 1 - \nu \left(\frac{\bar{x}}{x} \right)^{\psi}, \qquad \nu \in (0,1); \quad \psi \in (0,1) \tag{2.2.7}$$

where $\bar{x}$ denotes a base level of income, net of depreciation and maintenance, that is taken as given. Here, $\bar{x}$ is set to the steady-state level of that income. The parameters ν and ψ govern the level and slope of the tax schedule, respectively. When $\psi > 0$, the tax rate τ increases with the household's taxable income, that is, households with taxable income above $\bar{x}$ face a higher tax rate than those with income below $\bar{x}$. When $\psi = 0$, all households face the same tax rate $1 - \nu$ regardless of their taxable income. For sake of simplicity, we only consider here flat and progressive taxation. [8] We clearly see at this point that when $\tau = 0$ (i.e. $\psi = 0$ and $\nu = 1$), even though the model is in continuous time, it is identical to Guo and Lansing [90].

In making decisions about how much to consume, work, invest in new capital, and spend on maintenance of existing capital over their lifetimes, households take into account the way in which the tax schedule affects their earnings. To understand the progressivity feature of the above tax schedule, it is useful to distinguish between the average and marginal tax rates. The average tax rate τ, given by (2.2.7), is equal to the total taxes

[8] However, the present analysis could easily be adapted to the case of weak regressivity (i.e. when $\psi < 0$), as in Guo and Lansing [88]. For instance, our results still hold if $\psi \in ((\alpha_k - 1)/\alpha_k, 1)$, with $0 < ((\alpha_k - 1)/\alpha_k) < 1$ and α_k being the elasticity of aggregate output with respect to capital, to be precisely defined later on in the text.

paid by each household divided by its taxable income x. The marginal tax rate τ_m is defined as the change in taxes paid divided by the change in taxable income. The expression for τ_m is

$$\tau_m = \frac{\partial(\tau x)}{\partial x} = 1 - (1-\psi)\nu\left(\frac{\bar{x}}{x}\right)^{\psi} \tag{2.2.8}$$

We require $\tau < 1$ to prevent government from confiscating all productive resources, and $\tau_m < 1$ so that households have an incentive to supply labor and capital services to firms. From (2.2.7) and (2.2.8), we notice that $\tau_m = \tau + \nu\psi(\bar{x}/x)^{\psi}$. Therefore, the marginal tax rate will be above the average tax rate when $\psi > 0$. In this case, the tax schedule is said to be "progressive". When $\psi = 0$, the average and marginal tax rates coincide at the value $(1-\nu)$ and the tax schedule is said to be "flat".

2.2.4 Intertemporal Equilibria

Households' decisions follow from maximizing (2.2.5) subject to the budget constraint (2.2.6), given the initial capital stock $k(0) \geq 0$. Straightforward computations yield the following first-order conditions:

$$n \quad : \quad Acn^{\gamma} = (1-\tau_m)(1-\alpha)\frac{y}{n} \tag{2.2.9}$$

$$u \quad : \quad \frac{\alpha}{\theta}\frac{y}{k} = \delta \tag{2.2.10}$$

$$m \quad : \quad 1 = \phi\frac{\delta k}{m} \tag{2.2.11}$$

$$k \quad : \quad \frac{\dot{c}}{c} = (1-\tau_m)\frac{\alpha\big[\theta-(1+\phi)\big]}{\theta}\frac{y}{k} - \rho \tag{2.2.12}$$

where (2.2.9) equates the slope of the representative household's indifference curve (utility trade-off between leisure and consumption) to the after-tax real wage. Equation (2.2.12) is the consumption Euler equation. Equation (2.2.10) shows that the firm utilizes capital to the point where the marginal benefit of more output is equal to the marginal cost of faster depreciation. Equation (2.2.11) shows that the firm undertakes maintenance activity to the point where one unit of goods devoted to maintenance is equal to the marginal reduction in the firm's depreciation expense. Notice that the household's decisions regarding labor supply and capital investment are governed by the marginal tax rate τ_m.

We may rewrite the budget constraint, from (2.2.6), as:

$$\dot{k}/k = (1-\tau)x/k - c/k. \tag{2.2.13}$$

Equations (2.2.3), (2.2.9)-(2.2.12) and (2.2.13) characterize the dynamics of intertemporal equilibria with perfect foresight, given $k(0)$.

The transversality condition writes as:

$$\lim_{t\to+\infty} e^{-\rho t}\frac{k}{c} = 0 \tag{2.2.14}$$

It is easily checked that the transversality constraint is met in the following analysis, as we consider orbits that converge towards an interior steady state. From equation (2.2.10), we get $\delta k = (\alpha/\theta)y$ which gives, when plugged into (2.2.11),

$$m = \frac{\phi\alpha}{\theta}y$$

The equilibrium maintenance-to-GDP ratio is constant, and maintenance expenditures perfectly correlated with output. This reminds the procyclicality of maintenance documented by McGrattan and Schmitz [115].

As we want to characterize the reduced-form social technology as a function of k and n, we use equations (2.2.7) and (2.2.10) to solve [9] for u.

$$u = \left[\left(\frac{\phi}{\theta}\right)^{\phi}\left(\frac{\alpha y}{k}\right)^{1+\phi}\right]^{\frac{1}{\theta}}$$

[9] Since the parameter χ has no independent influence on the model's steady-state and dynamics around the steady-state, we simply set $\chi = 1/\theta$. Note that if we set ϕ to zero (when capital depreciation is inelastic to the maintenance activity), we recover the same optimal capacity utilization as in Wen [146].

Then we substitute this optimal rate of capacity utilization into (2.2.3) to finally get

$$y = Bk^{\alpha_k} n^{\alpha_n}$$

where the B, α_k and α_n write as:

$$\begin{aligned} B &= \left[\left(\frac{\phi}{\theta}\right)^{\phi} \alpha^{(1+\phi)}\right]^{\frac{\alpha(1+\eta)}{\theta-\alpha(1-\eta)(1+\phi)}} \\ \alpha_k &= \frac{\alpha(1+\eta)(\theta-1-\phi)}{\theta-\alpha(1+\eta)(1+\phi)} \quad (2.2.15) \\ \alpha_n &= \frac{(1-\alpha)(1+\eta)\theta}{\theta-\alpha(1+\eta)(1+\phi)} \quad (2.2.16) \end{aligned}$$

We restrict our attention to the case where $\alpha_k < 1 \Leftrightarrow 1 > \alpha(1+\eta)$, so that the externality on capital is not strong enough to generate sustained endogenous growth. We further assume that $\theta - 1 - \phi > 0$ to guarantee $\alpha_k > 0$. Equations (2.2.15) and (2.2.16) together imply $\partial(\alpha_k + \alpha_n)/\partial\phi > 0$ whenever $\eta > 0$. Hence, a higher degree of aggregate increasing returns can be achieved through a rise in ϕ.

2.2.5 Linearized Dynamics

The dynamics of intertemporal equilibria with perfect foresight, given $k(0)$, are characterized by

$$\begin{aligned} \tfrac{\dot{c}}{c} &= (1-\tau_m)\alpha \tfrac{\left[\theta-(1+\phi)\right]}{\theta} \tfrac{y}{k} - \rho \\ \tfrac{\dot{k}}{k} &= (1-\tau) \tfrac{(y-\delta k-m)}{k} - \tfrac{c}{k} \end{aligned}$$

To facilitate our analysis, we make the following logarithmic transformation of variables: $\hat{c} = \log(c)$, $\hat{k} = \log(k)$ and $\hat{y} = \log(y)$. With this transformation, the equilibrium conditions (2.2.9)-(2.2.14)can be rewritten as

$$\begin{aligned} \dot{\hat{c}} &= \alpha\nu(1-\psi)\bar{x}^{\psi} \tfrac{\left[\theta-(1+\phi)\right]}{\theta} \tfrac{\left[\theta-\alpha(1+\phi)\right]}{\theta}^{-\psi} e^{(1-\psi)\hat{y}-\hat{k}} - \rho \\ \dot{\hat{k}} &= \nu\bar{x}^{\psi} \tfrac{\left[\theta-\alpha(1+\phi)\right]}{\theta}^{1-\psi} e^{(1-\psi)\hat{y}-\hat{k}} - e^{\hat{c}-\hat{k}} \end{aligned}$$

where $\hat{c} = \log(c)$, $\hat{k} = \log(k)$ and $\hat{y} = \log(y)$.

We can obtain, from the static condition in (2.2.12), the following equation:

$$[\gamma + 1 - (1-\psi)\alpha_n]\hat{n} = \log\frac{\Gamma}{A} + (1-\psi)\log B + (1-\psi)\alpha_k\hat{k} - \hat{c}, \tag{2.2.17}$$

where $\Gamma = (1-\alpha)\nu(1-\psi)\bar{x}^{\psi}\left[\frac{\theta-\alpha(1+\phi)}{\theta}\right]^{-\psi}$. This yields, by using equation (2.2.17):

$$(1-\psi)\hat{y} - \hat{k} = \xi_0 + \xi_1\hat{k} + \xi_2\hat{c},$$

where

$$\begin{aligned} \xi_0 &= (1-\psi)Z & (2.2.18) \\ \xi_1 &= \frac{(1-\psi)\alpha_n + (\gamma+1)[\alpha_k(1-\psi)-1]}{\gamma+1-(1-\psi)\alpha_n} & (2.2.19) \\ \xi_2 &= -\frac{\alpha_n(1-\psi)}{\gamma+1-(1-\psi)\alpha_n} & (2.2.20) \end{aligned}$$

where $Z = \log B + \frac{\alpha_n}{\gamma+1-(1-\psi)\alpha_n}\left[\log\frac{\Gamma}{A} + (1-\psi)\log B\right]$. By rewriting equations (2.2.12)-(2.2.13) in logs and using equations (2.2.18)-(2.2.20), it is easy to get:

$$\begin{aligned} \dot{\hat{c}} &= \alpha\nu(1-\psi)(\bar{x})^{\psi}\frac{\theta-(1+\phi)}{\theta}\left[\frac{\theta-\alpha(1+\phi)}{\theta}\right]^{-\psi} e^{\xi_0+\xi_1\hat{k}+\xi_2\hat{c}} - \rho, \\ \dot{\hat{k}} &= \nu(\bar{x})^{\psi}\left[\frac{\theta-\alpha(1+\phi)}{\theta}\right]^{1-\psi} e^{\xi_0+\xi_1\hat{k}+\xi_2\hat{c}} - e^{\hat{c}-\hat{k}} \end{aligned} \tag{2.2.21}$$

It is straightforward to show that, under our assumptions, the differential equations (2.2.21) possess a steady state $\hat{c}^*, \hat{k}^*$. More precisely, $\dot{\hat{c}} = 0$ yields, from the first equation of system (2.2.21):

$$\exp(\xi_0 + \xi_1\hat{k}^* + \xi_2\hat{c}^*) = \frac{\rho}{\alpha\nu(1-\psi)(\bar{x})^{\psi}\frac{\theta-(1+\phi)}{\theta}\left[\frac{\theta-\alpha(1+\phi)}{\theta}\right]^{-\psi}} = \Omega$$

On the other hand, $\dot{\hat{k}} = 0$ then yields, from the second equation of system (2.2.21):

$$\exp\left(\hat{c}^* - \hat{k}^*\right) = \frac{[\theta - \alpha(1+\phi)]\rho}{\alpha(1-\psi)[\theta - (1+\phi)]} = \Upsilon$$

One can then easily establish that the two latter equations have solutions $\hat{c}^*$ and $\hat{k}^*$, provided that the scaling parameter A is appropriately chosen. More precisely, one can set, without loosing generality, $\hat{k}^* = 0$ (that is, $k^* = 1$) by fixing $A = \Upsilon^{-1}\Omega^{\frac{(1-\psi)\alpha_n-(\gamma+1)}{\alpha_n(1-\psi)}} B^{\frac{\gamma+1}{\alpha_n}}\Gamma$.

Such a procedure aims at proving the existence of at least one steady state, and at making sure that it "persists" after any arbitrarily small perturbation of the original two-dimensional map. For a given A, the steady-state is unique. On can easily compute the stationary labor supply $n^* = \left[\frac{(1-\alpha)\theta(1-\psi)}{A[\theta-\alpha(1+\phi)]}\right]^{\frac{1}{\gamma+1}}$, and deduce stationary capital stock $k^* = \left[\frac{B\alpha(1-\psi)\nu[\theta-(1-\phi)]}{\rho\theta}(n^*)^{\alpha_n}\right]^{\frac{1}{1-\alpha_k}}$ along with stationary consumption $c^* = \frac{\rho[\theta-\alpha(1+\phi)]}{(1-\psi)\alpha[\theta-(1+\phi)]]}\left[\frac{B\alpha(1-\psi)\nu[\theta-(1-\phi)]}{\rho\theta}(n^*)^{\alpha_n}\right]^{\frac{1}{1-\alpha_k}}$

2.3 Analysis of the Dynamics

We linearized equations (2.2.3) and (2.2.12)-(2.2.13) around an interior steady state. Straightforward computations yield the expressions for the trace T and the determinant D of the Jacobian matrix of the dynamical system.

Proposition 2.3.1 (Linearized Dynamics around a Steady State)

Linearized dynamics for deviations $\hat{c} - \hat{c}^*$ and $\hat{k} - \hat{k}^*$ are determined by linear map such that, in steady-state:

$$\begin{aligned} T &= \frac{\rho(1+\eta)\left\{(\gamma+1)\left[\theta-\alpha(1+\phi)\right]-(1-\alpha)\theta\right\}}{\left[\theta-(1+\phi)\alpha(1+\eta)\right]\left[\gamma+1-(1-\psi)\alpha_n\right]} \\ D &= \frac{(\gamma+1)\left[\alpha_k(1-\psi)-1\right]}{\gamma+1-(1-\psi)\alpha_n}\frac{\rho^2\left[\theta-\alpha(1+\phi)\right]}{\alpha(1-\psi)\left[\theta-(1+\phi)\right]}, \end{aligned} \tag{2.3.22}$$

2.3.1 Local Determinacy with Progressive Taxes

Indeterminacy is defined as follows.

Definition 2.3.1 (Indeterminacy of the Steady State)

The equilibrium is indeterminate if there exists an infinite number of perfect foresight equilibrium sequences.

The variable k is predetermined since k_0 is given by the initial conditions of the economy while c_0 is free to be determined by the behavior of the agents in the economy. Suppose that the steady-state $\{k^*, c^*\}$ is completely stable in the sense that all equilibrium trajectories which begin in the neighborhood of $\{k^*, c^*\}$ converge back to the steady state. In this case, there will be a continuum of equilibrium path $\{k(t), c(t)\}$, indexed by c_0, since any path that converges to $\{k^*, c^*\}$ necessarily satisfies the transversality condition. Completely stable steady states giving rise to a continuum of equilibria are termed indeterminate and in this case the stable manifold has dimension 2. Indeterminacy requires that both eigenvalues of the Jacobian matrix have negative real parts (the steady-state is a sink).

— Figure 2.1 about here —

Alternatively, if there is a one-dimensional manifold in $\{k, c\}$ space with the property that trajectories that begin on this manifold converge to the steady-state but all other trajectories diverge then the equilibrium will be locally unique in the neighborhood of the steady-state. In this case, for every k_0 in the neighborhood of k^* there will exist a unique c_0 in the neighborhood of c^* that generates a trajectory converging to $\{k^*, c^*\}$. This c_0 is the one that places the economy on the stable branch of the saddle point $\{k^*, c^*\}$.

— Figure 2.2 about here —

Since the Trace of the Jacobian measures the sum of the roots and the Determinant measures the product we can use information on the sign of the Trace and the Determinant to check the dimension of the stable manifold of the steady-state $\{k^*, c^*\}$. Indeterminacy can be restated as $T < 0 < D$. Similarly the steady-state is saddle-path stable if $D < 0$, and is unstable (a source) if $T > 0$ and $D > 0$. Since the eventual fate of trajectories that diverge from the steady state cannot be determined from the properties of the Jacobian evaluated at the steady state, we will not further elaborate on the source case [10].

— Figure 2.3 about here —

Let us notice that ν, the parameter characterizing the level of the fiscal schedule, does not appear neither in the Trace, nor in the Determinant (although it affects the steady state, cf. A1). Put differently, ν only affects the level of the steady-state, but not the dynamics around it. Hence, when the tax progressivity parameter is set to zero, the flat-rate fiscal structure does not seem to have any effect on the dynamics, in the neighborhood of a stationary state. This result complements some recent conclusions underlining that flat-rate taxation does *not* promote macroeconomic stability (see e.g., among others, Dromel and Pintus [51], Dromel and Pintus [50]).

Our main task is now to underline the conditions such that the steady-state is locally determinate or indeterminate. Direct inspection of equations (2.3.22) gives the following Proposition:

Proposition 2.3.2 (Local Stability of the Steady-State)

Assume $0 < \alpha_k < 1$. *Then the following holds:* $(1-\psi)\alpha_n > \gamma + 1$ *is a necessary and sufficient condition for the occurrence of local indeterminacy.* ***Proof.*** *C.f. Appendix 2.6.*

Corollary 2.3.1 (Local Indeterminacy and Sunspots)

[10] Trajectories may eventually violate non-negativity constraints or may settle down to a limit cycle or to some more complicated attracting set

Assume $0 < \alpha_k < 1$. *Then, the following holds:*
in the economy without taxes or with linear taxes, (that is, when $\psi = 0$*), the local dynamics of consumption* c *and capital* k *given by Eqs. (2.2.3) and (2.2.12)-(2.2.13) around the positive steady-state* (c^*, k^*) *exhibit local indeterminacy (that is* $T < 0 < D$*) if an only if* $\alpha_n > \gamma + 1$.

Corollary 2.3.2 (Saddle-Path Stability through Progressive Taxation)

Assume $0 < \alpha_k < 1$. *Then, the following holds:*
in the economy with progressive income taxes (that is, when $\psi \in (0, 1)$*, the local dynamics of* c *and* k *given by Eqs. (2.2.3) and (2.2.12)-(2.2.13) exhibit saddle-path stability (that is,* $D < 0$*) if and only if*
$\psi > \psi_{\min} = \frac{(1-\alpha)(1+\eta)\theta-(\gamma+1)\left[\theta-\alpha(1+\eta)(1+\phi)\right]}{(1-\alpha)(1+\eta)\theta}$.

A particular threshold of fiscal progressivity is thus able to immunize the economy from local indeterminacy. However, it is straightforward to show that $\psi_{\min}$ is decreasing in θ while increasing in ϕ (cf. Appendix 2.7). When the tax code displays some capital depreciation allowance and maintenance/repair deductions (as in the US tax code) the required degree of fiscal progressivity to protect the economy against sunspot-driven fluctuations is increasing in the equilibrium maintenance-to-GDP ratio. Consequently, the possibility for firms to deduct maintenance and repair expenditures from their pre-tax profit tends to weaken the stabilizing power of progressive fiscal schemes established by Guo and Lansing [88] and Guo [84] among others.

As shown in Fig. 2.4 and Fig. 2.5, the geometrical locus depicting the sensitivity of $\psi_{\min}$ to η is upward sloping and concave. An increase in the equilibrium maintenance ratio (achieved though an increase in ϕ or a decrease in θ) translates this locus upwards (its slope remains exactly the same, regardless of the level of the maintenance-to-GDP ratio) (cf. Appendix 2.8). Hence, for a given level of externalities in the economy, the tax deduction on maintenance and repair expenditures makes local indeterminacy more likely, since a higher level of fiscal progressivity is required to insulate the economy from belief-driven fluctuations.

— Figure 2.4 and 2.5 about here —

The recent empirical literature has shown that maintenance and repair activity is "too big to ignore" (Mc Grattan and Schmitz [115]): for instance, in Canada, expenditures devoted to maintenance and repair of existing equipment and structures averaged 6.1 percent of GDP from 1961 to 1993. Accordingly, in our theoretical setup, one could possibly expect an arguably high tax progressivity threshold necessary to eliminate indeterminacy.

2.3.2 Intuition

To gain insight into the mechanism that drives our result, it is useful to analyse how maintenance activity affects the equilibrium elasticity of social output with respect to labor α_n (cf. Appendix 2.9). It is easy to check that α_n is decreasing in θ, and increasing in ϕ, thus increasing in the equilibrium maintenance-to-GDP ratio.

The sufficient condition for saddle-path stability in this model is

$$(1-\psi)\alpha_n - 1 < \gamma$$

We clearly see that the higher ψ, the lower the left-hand-side, and the lower the likelihood of indeterminacy. Given the fact that the elasticity of output with respect to labor is higher when firms undertake maintenance activity, and a fortiori even more when the maintenance-to-GDP ratio is increased, ψ_{min} (the level of fiscal progressivity needed to render the equilibrium unique) is also higher.

The procyclicality of maintenance expenditures, assumed in the model, explains intuitively the excess of volatility added to the economy. On the empirical ground, these procyclical properties have been well established and documented by Mc Grattan and

Schmitz [115]. Using some unique survey data for Canada, these authors find that detrended maintenance and repair expenditures in Canada are strongly procyclical, exhibiting a correlation coefficient with GDP of 0.89. The Canadian survey also suggests that the activities of maintenance and repair and investment are to some degree close *substitutes* for each other. For example, during slumps, maintenance and repair spending falls in a lower extent than investment spending does. Similarly, during booms, maintenance and repair expenditures increase less than investment does. The standard deviation of maintenance and repair expenditures only represents 60 percent of the investment spending standard deviation, the difference being even sharper in the manufacturing industry. This tends to push forward the idea that during crises, new capital acquisitions are postponed, and existing equipment/structures are maintained and repaired to a larger extent. In other words, there would be a good deal of substitutability over the business cycle between the activities of maintenance and of investment.

This substitutability property can be used to provide an intuitive discussion of the basic mechanism driving our result. Let us suppose agents have optimistic expectations about, say, a higher return on capital in the next period. Firms will naturally want to invest more in the form of capital. But, due to the fiscal scheme progressivity, they know they will have to face in that case a higher tax rate. Thus, instead of investing in new physical capital (equipments or structures), firms will prefer to substitute maintenance to investment. The consequent reduction in the tax base implies that a higher level of fiscal progressivity will be needed to stabilize the economy against belief-driven cycles.

2.4 Conclusion

Maintenance and repair activity appears to be a quantitatively significant feature of modern industrial economies. Within a real business cycle model with arguably small aggregate increasing returns, this paper assesses the stabilizing effects of fiscal policies with a maintenance expenditure allowance. In this setup, firms are authorized to deduct their maintenance and repair expenditures from revenues in calculating pre-tax profits, as in many prevailing tax codes. While flat rate taxation does not prove useful to insulate the economy from self-fulfilling beliefs, a progressive tax can render the equilibrium unique. However, we show that the required progressivity to protect the economy against sunspot-driven fluctuations is increasing in the maintenance-to-GDP ratio. Taking into account the maintenance and repair activity of firms, and the tax deductability of the related expenditures, would then weaken the expected stabilizing properties of progressive fiscal schedules.

Some directions for further research seem natural. A calibration and simulation exercise would be useful in assessing the stabilizing level of fiscal progressivity in this economy and its plausibility. Indeed it remains to be seen if, for plausible values of increasing returns and realistic progressivity features, self-fulfilling beliefs can be a reasonable explanation for the excess of aggregate volatility. In addition, it seems relevant to introduce in this setup, following Guo [84], different progressivity features for labor and capital income, consistent with many OECD countries tax codes. Also, this paper (as many of the contributions in the area) consider fiscal progressivity with a continuously increasing marginal tax rate, which is not a shared feature by most actual tax schedules, as casual observation suggests. Considering linearly progressive taxation instead (as in Dromel and Pintus [50]) could be of interest, in order to get closer to the tax codes with brackets prevailing in most developed economies.

Appendix. Maintenance Allowances and Expectation-Driven Business Cycles

2.5 The Jacobian Matrix

The steady-state Jacobian matrix is derived from:

$$\begin{bmatrix} \dot{\hat{k}} \\ \\ \dot{\hat{c}} \end{bmatrix} = \begin{bmatrix} j_{11} & j_{12} \\ \\ j_{21} & j_{22} \end{bmatrix} \begin{bmatrix} \hat{k} - \hat{k}^* \\ \\ \hat{c} - \hat{c}^* \end{bmatrix}$$

where

$$j_{11} = \frac{\rho\big[\theta - \alpha(1+\phi)\big]}{\alpha(1-\psi)\big[\theta - (1+\phi)\big]}(\xi_1 + 1)$$

$$j_{12} = \frac{\rho\big[\theta - \alpha(1+\phi)\big]}{\alpha(1-\psi)\big[\theta - (1+\phi)\big]}(\xi_2 - 1)$$

$$j_{21} = \xi_1.\rho$$

$$j_{22} = \xi_2.\rho$$

2.6 Proof of Proposition 2.3.2

Assume $0 < \alpha_k < 1$. We can analyse the sign of the steady-state Jacobian's Trace as follows:

$$T = \frac{\overbrace{\rho(1+\eta)}^{>0}\big\{(\gamma+1)\big[\theta - \alpha(1+\phi)\big] - (1-\alpha)\theta\big\}}{\underbrace{\big[\theta - (1+\phi)\alpha(1+\eta)\big]}_{>0}\big[\gamma + 1 - (1-\psi)\alpha_n\big]}$$

$(\gamma+1)\big[\theta - \alpha(1+\phi)\big] - (1-\alpha)\theta$ can be re-written as $\underbrace{\gamma\big[\theta - \alpha(1+\phi)\big]}_{>0} + \underbrace{\alpha(\theta - 1 - \phi)}_{>0} > 0$. We get $T < 0$, which is a necessary condition for local indeterminacy, whenever $\gamma + 1 <$

$(1-\psi)\alpha_n$. It is easy to check with the steady-state Jacobian's determinant that this necessary condition is also sufficient:

$$D = \frac{\overbrace{(\gamma+1)}^{>0}\overbrace{\left[\alpha_k(1-\psi)-1\right]}^{<0}}{\gamma+1-(1-\psi)\alpha_n}\,\frac{\overbrace{\rho^2\left[\theta-\alpha(1+\phi)\right]}^{>0}}{\underbrace{\alpha(1-\psi)}_{>0}\underbrace{\left[\theta-(1+\phi)\right]}_{>0}}$$

The other necessary condition for indeterminacy, namely $D>0$, is obtained whenever

$$\gamma+1<(1-\psi)\alpha_n$$

. Hence, $\gamma+1<(1-\psi)\alpha_n$ is a Necessary and Sufficient Condition for the occurrence of local indeterminacy.

□

2.7 Sensitivity of $\psi_{\min}$ to ϕ and θ

Since $\frac{\partial\psi_{\min}}{\partial\theta} = -\frac{\alpha(\gamma+1)(1+\phi)}{(1-\alpha)\theta^2} < 0$ and $\frac{\partial^2\psi_{\min}}{\partial\theta^2} = \frac{2\alpha(\gamma+1)(1+\phi)\theta}{(1-\alpha)\theta^3} > 0$, $\psi_{\min}$ is convexly decreasing in θ. Moreover, as $\frac{\partial\psi_{\min}}{\partial\phi} = \frac{\alpha(\gamma+1)}{(1-\alpha)\theta} > 0$ and $\frac{\partial^2\psi_{\min}}{\partial\phi^2} = 0$, $\psi_{\min}$ is linearly increasing in ϕ.

2.8 Sensitivity of $\psi_{\min}$ to η

When the equilibrium maintenance-to-GDP ratio is set to zero ($\phi = 0$ and or $\theta \to \infty$), the level of fiscal progressivity $\bar{\psi}_{\min}$ needed to ensure saddle-path stability is $\bar{\psi}_{\min} = \frac{(1-\alpha)(1+\eta)-(\gamma+1)}{(1-\alpha)(1+\eta)}$. Since $\frac{\partial\bar{\psi}_{\min}}{\partial\eta} = \frac{\gamma+1}{(1-\alpha)(1+\eta)^2} > 0$ and $\frac{\partial^2\bar{\psi}_{\min}}{\partial^2\eta} = -\frac{2(\gamma+1)}{(1-\alpha)(1+\eta)^3} < 0$, $\bar{\psi}_{\min}$ is concavely increasing. If $\bar{\psi}_{\min} = 0$, then $\eta_{\min}\,|_{\psi_{\min}=0} = \frac{\gamma+\alpha}{1-\alpha}$.

As mentioned earlier in the text, when firms do undertake maintenance activity, and deduct the related expenditures from their pre-tax profit, the fiscal progressivity level required to ensure saddle-path stability is $\psi_{\min} = \frac{(1-\alpha)(1+\eta)\theta-(\gamma+1)\left[\theta-\alpha(1+\eta)(1+\phi)\right]}{(1-\alpha)(1+\eta)\theta}$. Since $\frac{\partial\psi_{\min}}{\partial\eta} = \frac{\gamma+1}{(1-\alpha)(1+\eta)^2} = \frac{\partial\bar{\psi}_{\min}}{\partial\eta} > 0$ and $\frac{\partial^2\psi_{\min}}{\partial^2\eta} = -\frac{2(\gamma+1)}{(1-\alpha)(1+\eta)^3} < 0 = \frac{\partial^2\bar{\psi}_{\min}}{\partial^2\eta}$, we notice that $\psi_{\min}$ as a function of (η) exhibits the same slope, whether or not maintenance and repair activity is effective.

If $\psi_{\min} = 0$, then $\eta_{\min} \mid_{\psi_{\min}=0}= \frac{(\gamma+1)[\theta-\alpha(1+\phi)]-\theta(1-\alpha)}{(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)}$. It is easily checked that since $\frac{\partial\eta|_{\psi_{\min}=0}}{\partial\theta} = \frac{(\gamma+1)^2\alpha(1+\phi)}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^2} > 0$ and $\frac{\partial^2\eta|_{\psi_{\min}=0}}{\partial\theta^2} = -\frac{(\gamma+1)^2\alpha(1+\phi)2(1-\alpha)}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^3} < 0$, $\eta \mid_{\psi_{\min}}= 0$ is concavely increasing in θ. Moreover, since $\frac{\partial\eta|_{\psi_{\min}=0}}{\partial\phi} = -\frac{(\gamma+1)^2\alpha\theta}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^2} < 0$ and $\frac{\partial^2\eta|_{\psi_{\min}=0}}{\partial\phi^2} = \frac{(\gamma+1)^3\alpha^2\theta 2}{[(1-\alpha)\theta+(\gamma+1)\alpha(1+\phi)]^3} > 0$, $\eta \mid_{\psi_{\min}}= 0$ is convexly decreasing in ϕ.

Given the equilibrium maintenance ratio writes as $\frac{m}{y} = \frac{\phi\alpha}{\theta}$, we know that an increase in this indicator can be achieved though an increase in ϕ or a decrease in θ. Consequently, when the equilibrium maintenance-to-GDP ratio rises, $\eta \mid_{\psi_{\min}}= 0$ falls.

2.9 Sensitivity of α_n to ϕ and θ

Since $\frac{\partial\alpha_n}{\partial\theta} = -\frac{\alpha(1+\eta)(1-\phi)}{[\theta-\alpha(1-\eta)(1+\phi)]^2} < 0$ and $\frac{\partial^2\alpha_n}{\partial\theta^2} = \frac{2\alpha(1+\eta)(1+\phi)[\theta-\alpha(1-\eta)(1+\phi)]}{[\theta-\alpha(1-\eta)(1+\phi)]^4} > 0$, α_n is convexly decreasing in θ. Moreover, as $\frac{\partial\alpha_n}{\partial\phi} = \frac{\theta\alpha(1-\alpha)(1+\eta)^2}{[\theta-\alpha(1-\eta)(1+\phi)]^2} > 0$ and $\frac{\partial^2\alpha_n}{\partial\phi^2} = \frac{2\theta\alpha(1-\alpha)\alpha^2(1+\eta)^3[\theta-\alpha(1-\eta)(1+\phi)]}{[\theta-\alpha(1-\eta)(1+\phi)]^4} > 0$, α_n is convexly increasing in ϕ.

Figure 2.1: Local Indeterminacy: the Steady-State is a Sink

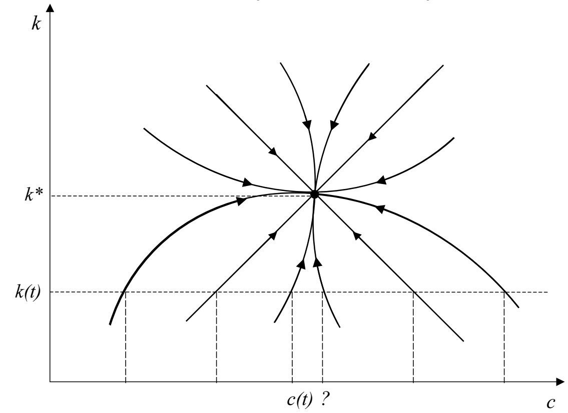


Figure 2.2: Saddle-Path Stability: the Steady-State is Locally Determinate

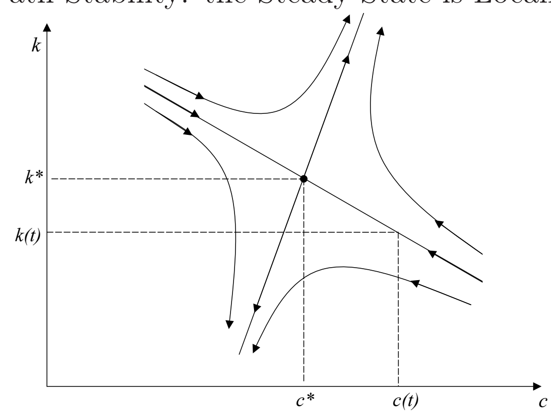


Figure 2.3: Jacobian's Trace-Determinant Diagram: Stability Regimes of Steady State in Continuous Time

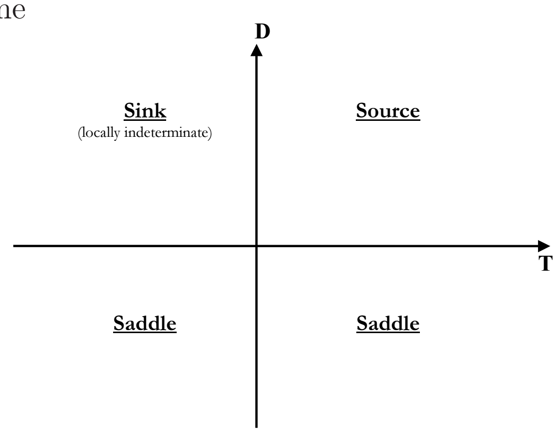


Figure 2.4: Sensitivity of $\psi_{\min}$ with respect to η when the equilibrium maintenance-to-GDP ratio is set to zero

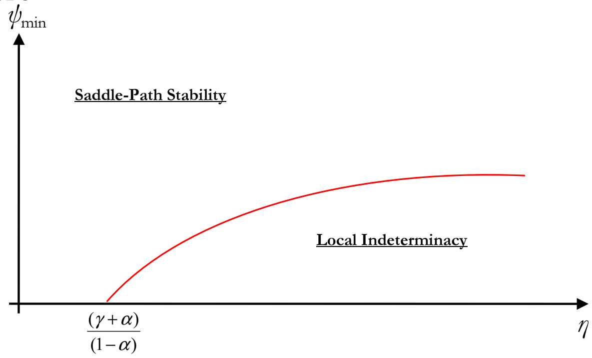
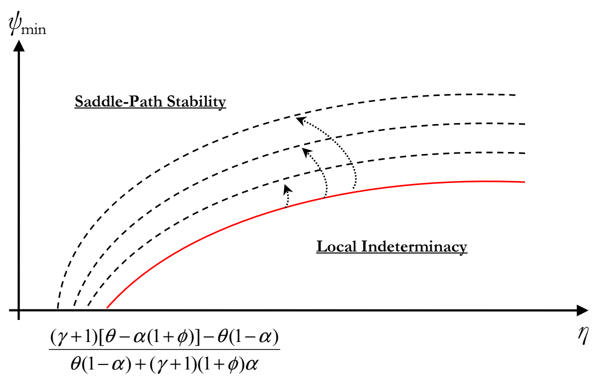


Figure 2.5: Sensitivity of $\psi_{\min}$ with respect to η when the equilibrium maintenance-to-GDP ratio rises



Chapter 3

Are Progressive Income Taxes Stabilizing?[1]

We assess the stabilizing effect of progressive income taxes in a monetary economy with constant returns to scale. It is shown that tax progressivity reduces, in parameter space, the likelihood of local indeterminacy, sunspots and cycles. However, considering plausibly low levels of tax progressivity does not ensure saddle-point stability and preserves as robust the occurrence of sunspot equilibria and endogenous cycles. It turns out that increasing

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progressivity, through its impact on after-tax income, makes labor supply more inelastic. However, even when large, tax progressivity does not neutralize the effects of expected inflation on current labor supply which may lead to expectation-driven business fluctuations.

Key Words: Progressive Income Taxes; Business Cycles; Sunspots; Stabilization.

JEL Class.: D33; D58; E32; E62; H24; H30.

3.1 Introduction

Income-dependent taxes and transfers have been proposed as efficient automatic stabilizers since, at least, Musgrave and Miller [118] (see also Vickrey [142, 143], Slitor [139], Friedman [66]). In recent years, the development of dynamic general equilibrium models has proved useful to study how progressive fiscal policy may stabilize the economy's aggregate variables. This strand of literature specifically allows to evaluate the level of social insurance provided by given fiscal schemes in the presence of various shocks. In particular, Christiano and Harrison [45], Guo and Lansing [88] have shown that progressive income taxes can rule out local indeterminacy and restore saddle-path convergence (see also Guo [84], Dromel and Pintus [51]). However, this literature assumes that the laissez-faire economy is subject to (large) increasing returns to scale, so that volatile paths may indeed improve welfare, in comparison with stationary allocations. In fact, Christiano and Harrison [45, p. 20] give some examples such that, in their terminology, the bunching effect dominates the concavity effect).

In the present paper, we assess the impact of progressive taxation in a monetary economy with *constant* returns to scale, by extending results due to Woodford [147] and Grandmont *et al.* [82]. Absent the bunching effect, one suspects expectation-driven volatility to unambiguously lead to welfare losses (by the concavity, or risk-aversion, effect) that call for stabilization[2] As shown by Woodford [147] and Grandmont *et al.* [82], the presence of money as a dominated asset, is critical to generate local indeterminacy in the laissez-faire economy without taxes. We show that local indeterminacy (hence sunspots and endogenous cycles) is *robust* with respect to the introduction of tax progressivity when it is set at plausibly low values. More specifically, although progressive taxes on labor income are shown to reduce, in parameter space, the likelihood of local indeterminacy (see Propositions 3.2.3 and 3.2.4), considering plausibly low values of tax progressivity still leaves room for sunspots and (Hopf or flip) cycles. As a corollary, we also conclude that regressive in-

[2]Here, we adopt the common view that fiscal policy is stabilizing when it leads to saddle-point stability, hence to determinacy.

come taxes would, on the contrary, enlarge the set of parameter values that are associated with local indeterminacy.

In consequence, one may view our results as casting some doubt on the idea that progressive income taxes are useful automatic stabilizers. This seems reminiscent of earlier debates about the practical importance of "built-in flexibility" (e.g. Musgrave and Miller [118] (see also Vickrey [142, 143], Slitor [139]).[3] Our focus on low progressivity is dictated by the available evidence (e.g. our own computations that we present on page 15, or Bénabou [13], Cassou and Lansing [39]). Our analysis abstracts from progressive taxation of capital income. This is consistent with the US tax code, which sets tax rates that are less progressive on income from capital than on wage income (see Hall and Rabushka [93]). Finally, we follow Feldstein [65], Kanbur [103], Persson [128] by considering tax schedules that exhibit constant residual income progression.

The mechanism at the heart of our main result is the following. Labor is elastically supplied by money holders and it depends on both the current real wage and the expected inflation rate. When workers have optimistic expectations (say, about falling inflation), they want to raise their consumption today and, accordingly, they devote a higher fraction of their time endowment to work so as to increase their income, which originates an expansion. With increasingly progressive taxes on wage income, labor supply becomes less and less responsive to the real wage and to expected inflation. There is, however, a major difference with respect to how labor reacts to both variables: eventually, as progressivity tends to its maximal level, expected inflation keeps having a negative impact on labor, whereas the effect of (before-tax) real wage tends to zero. In other words, *tax progressivity does not neutralize the effects of expected inflation on current labor supply* which leaves room for expectation-driven business cycles.

Typically, this effect could not be observed in the related literature dealing with increasing returns in a non-monetary Ramsey model, where the main effect of progressive tax

[3] Our main conclusion is not inconsistent with some recent results obtained in two-sector models by Guo and Harrison [85] and Sim [138], in which the mechanisms leading to indeterminacy are somewhat different, as they rely on increasing returns.

rates is described as "taxing away the higher returns from belief-driven labor or investment spurts" (Guo and Lansing [13, p. 482]). In such models, the mechanisms leading to indeterminacy are different from those at work in our setup. Particularly, as we postulate constant returns, labor demand has a conventional negative slope as a function of real wage.

Aside from assuming constant returns and money holdings, the setting with taxes that we consider differs from those of Christiano and Harrison [45], Guo and Lansing [88] in that we abstract from capital income taxation. To ease comparison with the latter papers, we introduce small increasing returns to scale and we check the robustness of our main findings obtained under constant returns. Finally, note that a recent paper by Seegmuller [137, section 5.2.2] studies, in an example, the effects of nonlinear tax rates in the same model, but he restricts the analysis to regressive taxation. We should also stress at the outset that the property of progressivity we focus on is the following: the income tax schedule that agents face is such that the marginal tax rate is higher than the average tax rate (see Assumption 3.2.2).

The rest of the paper is organized as follows. Section 3.2 presents the monetary economy with constant returns and discusses how progressive taxes and transfers make expectation-driven fluctuations less likely. Section 3.3 checks the robustness of our results with respect to the introduction of small increasing returns to scale. Finally, some concluding remarks and directions of future research are gathered in Section 3.4, while two appendices present proofs.

3.2 Progressive Income Taxes in a Monetary Economy with Constant Returns

In this section, we sketch the benchmark model, following the lines set out in Woodford [147] and Grandmont *et al.* [82], to which we add progressive income taxes. The economy consists of two types of competitive agents (workers and capitalists) who consume and have perfect foresight during their infinite lifetime. Identical agents called *workers* consume and work during each period. They supply a variable quantity of labor hours and may save a fraction of their income by holding two assets: productive capital and nominal outside money. A financial constraint is imposed on workers: their expenditures must be financed out of their initial money balances or out of the returns earned on productive capital. On the other side, capitalists consume and save an income composed of money balances and returns on capital. Most importantly, it is assumed that capitalists discount future utility less than workers. Therefore, capitalists end up holding the whole capital stock and the resulting nonautarkic steady state is characterized by the modified golden rule, i.e. the stationary real rental rate on productive capital (net of capital depreciation) equals the discount rate of capitalists. Therefore, at the steady state (and nearby), the real return on capital is positive and larger than that of money balances, which is assumed to be zero, so that capitalists choose not to hold outside money.

To summarize, the steady state (and nearby) savings structure is the following: capitalists own the whole capital stock and workers hold the entire nominal money stock. Finally, the financial constraint faced by workers becomes a liquidity constraint which is obviously binding at the steady state. In that framework, Woodford [147] showed that although workers have an infinite lifetime, they behave like a two period living agent: they choose optimally their labor supply for today and consequently their next period consumption demand. Equivalently, workers know the current nominal wage and the next period price for the consumption good (along an intertemporal equilibrium with perfect foresight) and choose the (unique under usual assumptions) optimal bundle of labor today and consumption tomorrow on their offer curve. Therefore, the liquidity constraint allows one to interpret the length of the period as, say, a month and eventual endogenous fluctuations occur at business cycle frequency.

3.2.1 Fiscal Policy and Intertemporal Equilibria

A unique good is produced in the economy by combining labor $l_t \geq 0$ and the capital stock $k_{t-1} \geq 0$ resulting from the previous period. Production exhibits constant returns to scale, so that output is given by:

$$F(k,l) \equiv Alf(a), \tag{3.2.1}$$

where $A \geq 0$ is a scaling parameter and the latter equality defines the standard production function defined upon the capital labor ratio $a = k/l$. On technology, we shall assume the following.

Assumption 3.2.1

The production function $f(a)$ is continuous for $a = k/l \geq 0$, C^r for $a > 0$ and r large enough, with $f'(a) > 0$ and $f''(a) < 0$.

Competitive firms take real rental prices of capital and labor as given and determine their input demands by equating the private marginal productivity of each input to its real price. Accordingly, the real competitive equilibrium wage is:

$$\omega = \omega(a) \equiv A[f(a) - af'(a)], \tag{3.2.2}$$

while the real competitive gross return on capital is:

$$R = \rho(a) + 1 - \delta \equiv Af'(a) + 1 - \delta, \tag{3.2.3}$$

where $1 \geq \delta \geq 0$ is the constant depreciation rate for capital.

Fiscal policy is supposed to map labor income x into disposable labor income $\phi(x)$. Disposable labor income is obtained from labor market income by adding transfers and subtracting taxes. For simplicity, we assume that $x \geq \phi(x)$, so that taxes net of transfers (taxes from now on) are positive. In this formulation, there are two benchmark cases.

When $\phi(x)$ is proportional to x, then ϕ has unitary elasticity and the tax rate is flat. Decreasing the elasticity of $\phi(x)$ from one (when the net tax rate is constant) to zero may be interpreted as *increasing fiscal progressivity.* More precisely, one can postulate the following (see Musgrave and Thin [119] for an early definition, and, for example, Lambert [109, chap. 7-8]).

Assumption 3.2.2

Disposable labor income $\phi(x)$ is a continuous, positive function of labor income $x \geq 0$, with $x \geq \phi(x)$, $\phi'(x) > 0$ and $0 \geq \phi''(x)$, for $x > 0$. The income tax-and-transfer scheme exhibits progressivity*, that is, $\phi(x)/x$ is non-increasing for $x > 0$ or, equivalently, $1 \geq \psi(x) \equiv x\phi'(x)/\phi(x)$.*

Then $\pi(x) \equiv 1 - \psi(x)$ is a measure of income tax progressivity. In particular, the fiscal schedule is linear when $\pi(x) = 0$, or $\psi(x) = 1$, for $x > 0$, and the higher $\pi(x)$, the more progressive the fiscal schedule.

One can reinterpret the condition $1 \geq \psi(x)$ as the property that the *marginal* tax rate $\tau_m \equiv \partial(x - \phi(x))/\partial x$ is larger than the average tax rate $\tau \equiv (x - \phi(x))/x$: it is easily shown that $\tau_m - \tau = \phi(x)/x - \phi'(x)$ so that $\tau_m \geq \tau$ when $1 \geq \psi(x)$ or $\pi(x) \geq 0$ for all positive x. One can easily check that the latter condition also implies that the average tax rate is an increasing function of pre-tax income. Finally, note that fiscal progressivity is naturally measured, for some x, by $\pi \equiv 1 - \psi$ when one notes that $\pi = (\tau_m - \tau)/(1 - \tau)$. To put it differently, $\psi = 1 - \pi$ measures (local) residual income progression (as defined by Musgrave and Thin [119, p. 507]). In the next subsection, we simplify the analysis of the local dynamics by assuming that π is constant.

As in most papers in the literature (e.g. Guo and Lansing [88]), we assume that the proceeds of taxes, net of transfers, are used to produce a flow of public goods g, with $g = x - \phi(x)$. Therefore, the government budget is balanced.

To complete the description of the model, we now characterize the behavior of both classes of agents, following Woodford [147] and Grandmont *et al.* [82]. A representative worker solves the following utility optimization problem, as derived in Appendix 3.5:

$$\text{maximize } \{V_2(c^w_{t+1}/B) - V_1(l_t)\} \text{ such that } p_{t+1}c^w_{t+1} = p_t\phi(\omega_t l_t),\ c^w_{t+1} \geq 0,\ l_t \geq 0, \quad (3.2.4)$$

where $B > 0$ is a scaling factor, c^w_{t+1} is next period consumption, l_t is labor supply, $p_{t+1} > 0$ is next period price of output (assumed to be perfectly foreseen), $\omega_t > 0$ is real wage, and $\phi(\omega_t l_t)$ is disposable wage income, as described in Assumption 3.2.2. To keep things simple, we assume in this section that progressive taxes and transfers are applied to labor income only. Capital income taxes are studied in Dromel and Pintus [50], where we show that similar results hold. In particular, we show that flat-rate taxation of capital income has no impact on parameter values that are compatible with local indeterminacy and bifurcations. We consider the case such that leisure and consumption are gross substitutes and assume therefore the following:

Assumption 3.2.3

The utility functions $V_1(l)$ and $V_2(c)$ are continuous for $l^* \geq l \geq 0$ and $c \geq 0$, where $l^* > 0$ is the (maybe infinite) workers' endowment of labor. They are C^r for, respectively, $0 < l < l^*$ and $c > 0$, and r large enough, with $V_1'(l) > 0$, $V_1''(l) > 0$, $\lim_{l \to l^*} V_1'(l) = +\infty$, and $V_2'(c) > 0$, $V_2''(c) < 0$, $-cV_2''(c) < V_2'(c)$ (that is, consumption and leisure are gross substitutes).

The first-order condition of the above program (3.2.4) gives the optimal labor supply $l_t > 0$ and the next period consumption $c^w_{t+1} > 0$, which can be stated as follows.

$$v_1(l_t) = \psi(\omega_t l_t) v_2(c^w_{t+1}) \text{ and } p_{t+1}c^w_{t+1} = p_t\phi(\omega_t l_t), \quad (3.2.5)$$

where $v_1(l) \equiv lV_1'(l)$ and $v_2(c) \equiv cV_2'(c/B)/B$. Assumption 3.2.3 implies that v_1 and v_2 are increasing while v_1 is onto $\mathbf{R}_+$. Therefore, Assumption 3.2.3 allows one to define, from Eqs. (3.2.5), $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$ (whose graph is the offer curve), which is a monotonous, increasing function only if the elasticity of ψ is either negative or not too large when positive.

Capitalists maximize the discounted sum of utilities derived from each period consumption. They consume $c_t^c \geq 0$ and save $k_t \geq 0$ from their income, which comes exclusively from real gross returns on capital and is not affected by fiscal rules. We assume, following Woodford [147], that capitalists' instantaneous utility function is logarithmic. As easily shown (for instance by applying dynamic programming techniques), their optimal choices are then given by a constant savings rate:

$$c_t^c = (1-\beta)R_t k_{t-1},\ k_t = \beta R_t k_{t-1}, \tag{3.2.6}$$

where $0 < \beta < 1$ is the capitalists' discount factor and $R_t > 0$ is the real gross rate of return on capital.

As usual, equilibrium on capital and labor markets is ensured through Eqs. (3.2.2) and (3.2.3). Since workers save their wage income in the form of money, the equilibrium money market condition is:

$$c_t^w = \phi(\omega(a_t)l_t) = M/p_t, \tag{3.2.7}$$

where $M \geq 0$ is money supply, assumed to be constant in the sequel, and p_t is current nominal price of output. Finally, Walras' law accounts for the equilibrium in the good market, that is, $c_t^w + c_t^c + k_t - (1-\delta)k_{t-1} + g_t = F(k_{t-1}, l_t)$. From the equilibrium conditions in Eqs. (3.2.2), (3.2.3), (3.2.5), (3.2.6), (3.2.7), one easily deduces that the variables c_{t+1}^w, c_t^w, l_t, p_{t+1}, p_t, c_t^c and k_t are known once (a_t, k_{t-1}) are given. This implies that intertemporal equilibria may be summarized by the dynamic behavior of both a and k.

Definition 3.2.1

An intertemporal perfectly competitive equilibrium with perfect foresight is a sequence

(a_t, k_{t-1}) *of* $\mathbf{R}^2_{++}$, $t = 1, 2, \ldots,$ *such that, given some* $k_0 \geq 0$,

$$\begin{cases} v_2(\phi(\omega(a_{t+1})k_t/a_{t+1})) &= v_1(k_{t-1}/a_t)/\psi(\omega(a_t)k_{t-1}/a_t), \\ k_t &= \beta R(a_t)k_{t-1}. \end{cases} \tag{3.2.8}$$

For simplicity, we assume that ϕ has a *constant elasticity* $\psi = 1 - \pi$ with $1 > \pi \geq 0$. Convenience aside, it appears that economic theory does not place strong restrictions on how the *elasticity* $\psi(x)$ of after-tax income varies with pre-tax income x (see e.g. Lambert [109]). Therefore, we choose to be parsimonious and introduce fiscal progressivity through a *single* constant parameter, that is, $\pi = 1-\psi$. In other words, we assume *constant residual income progression*, as in Feldstein [65], Kanbur [103], Persson [128] (in static models), and Guo and Lansing [88], Bénabou [13] (in growth models). Let us notice that in Guo and Lansing [88], $\phi(x) = \nu\bar{x}^{\pi}x^{(1-\pi)}$, we do obtain $\psi(x) = 1 - \pi$. All results of this chapter could be derived with this functional form.

In view of Eqs. (3.2.8) and recalling that $a = k/l$, the nonautarkic steady states are the solutions $(\overline{a}, \bar{l})$ in $\mathbf{R}^2_{++}$ of $v_2(\phi(\omega(\overline{a})\bar{l})) = v_1(\bar{l})/\psi(\omega(\overline{a})\bar{l})$ and $\beta R(\overline{a}) = 1$. Equivalently, in view of Eq. (3.2.3), the steady states are given by:

$$\begin{cases} v_2(\phi(\omega(\overline{a})\bar{l})) &= v_1(\bar{l})/\psi(\omega(\overline{a})\bar{l}), \\ \rho(\overline{a}) + 1 - \delta &= 1/\beta. \end{cases} \tag{3.2.9}$$

Proposition 3.2.1 (Existence of a Normalized Steady State)

Under Assumptions 3.2.1, 3.2.2 and 3.2.3, $\lim_{c\to 0} cV_2'(c) < V_1'(1)/\psi(\omega(1)) < \lim_{c\to+\infty} cV_2'(c)$, $(\overline{a}, \overline{k}) = (1, 1)$ *is a steady state of the dynamical system in Eqs. (3.2.8) if and only if* $A = (1/\beta - 1 + \delta)/f'(1)$ *and* B *is the unique solution of* $\psi(\omega(1))\phi(\omega(1))V_2'(\phi(\omega(1))/B)/B = V_1'(1)$.

Proof.

In view of Eqs. (3.2.8) and recalling that $a = k/l$, the nonautarkic steady states are the solutions $(\overline{a},\bar{l})$ in $\mathbf{R}^2_{++}$ of $v_2(\phi(\omega(\overline{a})\bar{l})) = v_1(\bar{l})/\psi(\omega(\overline{a})\bar{l})$ and $\beta R(\overline{a}) = 1$. We shall solve the existence issue by setting appropriately the scaling parameters A and B, so as to ensure that one stationary solution coincides with, for instance, $(\overline{a},\bar{l}) = (1,1)$. The second equality of Eqs. (3.2.9) is achieved by scaling the parameter A, while the first is achieved by scaling the parameter B. That is, we set $A = (1/\beta - 1 + \delta)/f'(1)$ to ensure that $\overline{a} = 1$. On the other hand, $\psi(\omega(\overline{a})\bar{l})v_2(\overline{c}) = v_1(\bar{l})$ is then equivalent to

$$\psi(\omega(1))\frac{\phi(\omega(1))}{B}V_2'(\frac{\phi(\omega(1))}{B}) = V_1'(1). \tag{3.2.10}$$

From Assumption 3.2.3, v_2 is decreasing in B so the latter condition is satisfied for some unique B if and only if:

$$\lim_{c\to 0} cV_2'(c) < V_1'(1)/\psi(\omega(1)) < \lim_{c\to +\infty} cV_2'(c). \tag{3.2.11}$$

Such a normalization procedure aims at proving the existence of at least one steady state, and at making sure that it "persists" after any arbitrarily small perturbation of the original two-dimensional map. Let us now discuss the uniqueness of the stationary state. The second equation of (3.2.9) yields $\rho(\overline{a}) = (1-\beta)/\beta + \delta$. Since $\rho(a)$ is decreasing, the technology determines uniquely the steady-state capital-labor ratio if and only if $\lim_{a\to 0}\rho(a) > (1-\beta)/\beta + \delta > \lim_{a\to\infty}\rho(a)$. To find $\overline{k}$, or the stationary labor supply $\bar{l} = \overline{k}/\overline{a}$, one looks next at the first equation in (3.2.9) which reads in fact $v_1(\bar{l}) = \psi v_2(\overline{c})$, where $\overline{c} = \phi(\omega(\overline{a})\bar{l})$ is the steady-state consumption, or equivalently $V_1'(\bar{l}) = \psi(\omega(\overline{a})\bar{l})\frac{\phi(\omega(\overline{a})\bar{l})}{\bar{l}}V_2'(\phi(\omega(\overline{a})\bar{l}))$. From assumption 3.2.3, the left-hand side of this equation, considered as a function of $\bar{l}$, is increasing and tends to $+\infty$ when $\bar{l}$ goes to l^*. As regards the right-hand side, since ψ, the elasticity of ϕ, is lower than unity for all l, $\psi(\omega(\overline{a})\bar{l})\frac{\phi(\omega(\overline{a})\bar{l})}{\bar{l}}$ is decreasing in $\bar{l}$ given $\overline{a}$. Moreover, from the same assumption 3.2.3, $V_2'(.)$ is also decreasing in $\bar{l}$ given $\overline{a}$. Consequently, when a stationary solution exists, it is unique.

3.2.2 Sunspots and Cycles under Progressive Income Taxes

We now study the dynamics of Eqs. (3.2.8) around $(\overline{a}, \overline{k})$. These equations define locally a dynamical system of the form $(a_{t+1}, k_t) = G(a_t, k_{t-1})$ if the derivative of $\omega(a)/a$ with respect to a does not vanish at the steady state, or equivalently if $\varepsilon_\omega(\overline{a}) - 1 \neq 0$, where the notation ε_ω stands for the elasticity of $\omega(a)$ evaluated at the steady state under study.

Straightforward computations yield the following proposition.

Proposition 3.2.2 (Linearized Dynamics around the Steady State)

Under the assumptions of Proposition 3.2.1, suppose that ϕ has constant elasticity at the steady state $(\overline{a}, \overline{k})$ of the dynamical system in Eqs. (3.2.8), i.e. $\psi(x) = 1-\pi$, with $0 < \pi < 1$ measuring labor tax progressivity. Let ε_R, ε_ω, ε_γ be the elasticities of the functions $R(a)$, $\omega(a)$, $\gamma(l)$, respectively, evaluated at the steady state $(\overline{a}, \overline{k})$ and assume that $\varepsilon_\omega \neq 1$. The linearized dynamics for the deviations $da = a - \overline{a}$, $dk = k - \overline{k}$ are determined by the linear map:

$$\left\{ \begin{array}{rcccc} da_{t+1} & = & -\frac{\varepsilon_\gamma/(1-\pi)+\varepsilon_R}{\varepsilon_\omega - 1} da_t & + & \frac{\overline{a}}{\overline{k}} \frac{\varepsilon_\gamma/(1-\pi)-1}{\varepsilon_\omega - 1} dk_{t-1}, \\ dk_t & = & \frac{\overline{k}}{\overline{a}} \varepsilon_R da_t & + & dk_{t-1}. \end{array} \right. \tag{3.2.12}$$

The associated Jacobian matrix evaluated at the steady state under study has trace T and determinant D, where

$$T = T_1 - \frac{\varepsilon_\gamma - 1}{(1-\pi)(\varepsilon_\omega - 1)}, \quad \textit{with} \quad T_1 = 1 + \frac{|\varepsilon_R| - 1/(1-\pi)}{\varepsilon_\omega - 1},$$
$$D = \varepsilon_\gamma D_1, \quad \textit{with} \quad D_1 = \frac{|\varepsilon_R| - 1}{(1-\pi)(\varepsilon_\omega - 1)}.$$

Moreover, one has $T_1 = 1 + D_1 + \Lambda$, where $\Lambda \equiv -\pi|\varepsilon_R|/[(1-\pi)(\varepsilon_\omega - 1)]$.

Our main goal now is to show that sunspots and cycles are robust to the introduction of tax progressivity, when the latter is set at plausibly low levels. Direct inspection of Eqs. (3.3.18) shows that the case of *flat-rate* taxes ($\pi = 0$ or $\psi = 1$) is equivalent to the case without government taxes, as studied by Grandmont *et al.* [82]. In addition, when $1 > \pi > 0$, ε_γ is replaced, in the model with progressive taxes, by $\varepsilon_\gamma/(1-\pi) > \varepsilon_\gamma$. Therefore, *one expects the picture that is obtained when π is not too large to be similar to the configuration occurring in the model without taxes* (or, for that matter, with flat-rate taxes), up to the change of parameter $\varepsilon_\gamma \to \varepsilon_\gamma/(1-\pi)$. This is what we now show.

We assume, without loss of generality, that the steady state has been normalized at $(\overline{a}, \overline{k}) = (1, 1)$ (see Proposition 3.2.1). Then fix the technology (i.e. ε_R and ε_ω), at the steady state, and vary the parameter representing workers' preferences $\varepsilon_\gamma > 1$. In other words, consider the parameterized curve $(T(\varepsilon_\gamma), D(\varepsilon_\gamma))$ when ε_γ describes $(1, +\infty)$. Direct inspection of the expressions of T and D in Proposition 3.2.2 shows that this locus is a half-line Δ that starts close to (T_1, D_1) when ε_γ is close to 1, and whose slope is $1-|\varepsilon_R|$, as shown in Fig. 3.1. The value of $\Lambda = T_1 - 1 - D_1$, on the other hand, represents the deviation of the generic point (T_1, D_1) from the line (AC) of equation $D = T - 1$, in the (T, D) plane.

— Figure 3.1 about here —

The task we now face is locating the half-line Δ in the plane (T, D), i.e. its origin (T_1, D_1)

and its slope $1 - |\varepsilon_R|$, as a function of the parameters of the system. The parameters we shall focus on are the depreciation rate for the capital stock $1 \geq \delta \geq 0$, the capitalist's discount factor $0 < \beta < 1$, the share of capital in total income $0 < s = \overline{a}\rho(\overline{a})/f(\overline{a}) < 1$, the elasticity of input substitution $\sigma = \sigma(\overline{a}) > 0$, and fiscal progressivity $1 > \pi = \pi(\omega(\overline{a})\overline{l}) \geq 0$, all evaluated at the steady state $(\overline{a}, \overline{k})$ under study. In fact, it is not difficult to get the following expressions.

$$\begin{aligned} &D_1 = (\theta(1-s) - \sigma)/[(1-\pi)(s-\sigma)], \quad && \Lambda = -\pi\theta(1-s)/[(1-\pi)(s-\sigma)], \\ &T_1 = 1 + D_1 + \Lambda, && slope_\Delta = 1 - \theta(1-s)/\sigma, \end{aligned} \tag{3.2.13}$$

where $\theta \equiv 1 - \beta(1-\delta) > 0$ and all these expressions are evaluated at the steady state under study.

Our aim now is locating the half-line Δ, i.e. its origin (T_1, D_1) and its slope in the (T, D) plane when the capitalists' discount rate β, as well as the technological parameters δ, s, and the level of fiscal progressivity π at the steady state are fixed, whereas the elasticity of factor substitution σ is made to vary. We get confirmation that the benchmark economy with constant tax rate $\pi = 0$ is equivalent to the no-tax case presented in Grandmont *et al.* [82]: the origin (T_1, D_1) of Δ is located on the line (AC), i.e. $\Lambda \equiv 0$ (see Fig. 3.1). The immediate implication of the resulting geometrical representation is that local indeterminacy and endogenous fluctuations emerge only for low values of σ (for σ lower than s, the share of capital in output) while, on the contrary, local determinacy is bound to prevail for larger values of σ. One corollary of this is *that flat tax rates on labor income do not affect the range of parameter values that are associated with local indeterminacy and bifurcations* (see also Guo and Harrison [86]for a related discussion).

The key implication of increasing progressivity π from zero can be seen, starting with the benchmark case with linear taxes, by focusing on how the following intersection points vary with π (see Fig. 3.1). First, direct inspection of Eqs. (3.3.19) shows that Λ (the deviation of (T_1, D_1) from (AC)) is negative when σ is small enough (that is, $T_1 < 1 + D_1$ when $\sigma < s$). In fact, the locus of (T_1, D_1) generated when σ increases from zero describes a line Δ_1 which intersects (AC) at point I when $\sigma = +\infty$ (i.e. $\Lambda = 0$). From Eqs. (3.3.19), one immediately sees that $D_1(\sigma = +\infty)$ increases with π, so that point I goes north-east when π increases from zero. Second, Eqs. (3.3.19) imply that Δ_1 intersects the T-axis of equation $D = 0$ when $\sigma = \theta(1 - s)$ (that is, $D_1(\theta(1 - s)) = 0$), and that $\Lambda(\theta(1 - s))$ decreases, from zero, with π. An equivalent way of summarizing these two observations is that, when π increases from zero, point I (where Δ_1 intersects (AC)) goes north-east, along (AC), whereas the *slope of* Δ_1 decreases from one, so that several configurations occur in the (T, D) plane. We concentrate, in the next proposition, on the most plausible case such that progressivity is not too large. All other cases, associated with larger progressivity, can be derived through the very same analysis and are presented in Dromel and Pintus [50].

Proposition 3.2.3 (Local Stability and Bifurcations of the Steady State)

Consider the steady state that is assumed to be set at $(\overline{a}, \overline{k}) = (1, 1)$ *through the procedure in Proposition 3.2.1. If, moreover,* $\theta(1-s) < s$ *and* $0 < \pi < 1 - \theta(1-s)/s$ *(that is, fiscal progressivity is not too large), the following generically holds (see Fig. 3.1).*[4]

1. $0 < \sigma < \sigma_F$*: the steady state is a sink for* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*, where* $\varepsilon_{\gamma H}$ *is the value of* ε_γ *for which* Δ *crosses* $[BC]$*. Then the steady state undergoes a Hopf bifurcation (the complex characteristic roots cross the unit circle) at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$*, and is a source when* $\varepsilon_\gamma > \varepsilon_{\gamma H}$*.*

2. $\sigma_F < \sigma < \sigma_H$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma H}$*. Then the steady state undergoes a Hopf bifurcation at* $\varepsilon_\gamma = \varepsilon_{\gamma H}$ *and is a source when* $\varepsilon_{\gamma H} < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs (one characteristic root goes through* -1*) at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle when* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

3. $\sigma_H < \sigma < \sigma_I$*: the steady state is a sink when* $1 < \varepsilon_\gamma < \varepsilon_{\gamma F}$*. A flip bifurcation occurs at* $\varepsilon_\gamma = \varepsilon_{\gamma F}$ *and the steady state is a saddle if* $\varepsilon_\gamma > \varepsilon_{\gamma F}$*.*

4. $\sigma_I < \sigma < s$ *and* $s < \sigma$*: the steady state is a saddle when* $\varepsilon_\gamma > 1$*.*

Proof: See Appendix 3.6

Proposition 3.2.3 reveals an important implication. The upper bound that is imposed on π appears to be large for plausible parameter values. In fact, $\theta = 1-\beta(1-\delta)$ is bound to be close to zero when the period is commensurate with business-cycle length, as $\beta \approx 1$ and $\delta \approx 0$ when the period is, say, a month. Therefore, the condition that $\pi < 1 - \theta(1-s)/s$ is not restrictive. In other words, *for sensible parameter values, tax progressivity does not rule out local indeterminacy by ensuring saddle-point convergence, even when it is quite large.* The next statement shows how critical values involved in Proposition 3.2.3 (see also Fig. 3.1) move with π.

Proposition 3.2.4 (Income Tax Progressivity and Local Indeterminacy)

[4] The expressions of σ_F, σ_H, σ_I, σ_J, $\varepsilon_{\gamma H}$ and $\varepsilon_{\gamma F}$ are given in Proposition 3.2.4.

The critical values involved in Proposition 3.2.3 (see Fig. 3.1) are such that $\sigma_F = \theta(1-s)/2$ *and* $\sigma_H = s[1+\theta(1-s)/s+\sqrt{1-\theta(1-s)/s}]/2$ *are independent of* π*, while* $\varepsilon_{\gamma F} = (1-\pi)(2s+\theta(1-s)-2\sigma)/[2\sigma-\theta(1-s)]$*,* $\varepsilon_{\gamma H} = (1-\pi)(s-\sigma)/[\theta(1-s)-\sigma]$ *and* $\sigma_I = \theta(1-s)/2+s(1-\pi)/(2-\pi)$ *are decreasing functions of* π*. In consequence, income tax progressivity reduces the set of parameter values that are associated with local indeterminacy.*

Suppose now that we extend our results to consider negative values of π. In view of Assumption 3.2.2, $\pi < 0$ implies that income taxes are *regressive*.[5] Then one corollary of Proposition 3.2.4 is that decreasing π from zero would enlarge the set of parameter values that are associated with local indeterminacy, as in Guo and Lansing [88] or Seegmuller [137].

Besides the qualitative statements contained in Proposition 3.2.4, it may be informative to assign numerical values to parameters and then ask how sensitive the critical values are with respect to tax progressivity. The first thing to notice is that the assumption of a finance constraint imposed on workers suggests to interpret the period as, say, a month. Then setting $\beta = 0.997$ and $\delta = 0.008$ is the monthly counterpart of the values adopted in the literature and based on annual data (that is $\beta = 0.96$ and $\delta = 0.1$). With $s = 1/3$, on then has $\theta \approx 0.01$, which implies that the upper bound appearing in Proposition 3.2.3 is $\pi < 1-\theta(1-s)/s \approx 0.98$. This corroborates our previous conclusion that the case pictured in Fig. 3.1 is the most relevant. Moreover, based on US marginal tax rates that are provided by Stephenson [140], our own computations deliver that income tax progressivity has ranged in $[4\%-11\%]$ over 1940-93, with an average around 6% (in accord with Bénabou [13] or Cassou and Lansing [39]). In view of Proposition 3.2.4, one expects σ_F and sih to be close to zero when θ is close to zero, as is the case in our numerical example. Therefore, we focus on σ_I which is equal to $[\theta(1-s)+s]/2 \approx 0.17$ when $\pi = 0$. With tax progressivity, one gets that $\sigma_I = \theta(1-s)/2 + s(1-\pi)/(2-\pi) \approx 0.16$ when $\pi = 0.11$, that is, when the tax progressivity is set at the modal value observed over the period $1940-93$. Therefore,

[5] In that case, one has to modify Assumption 3.2.2 so as to impose that the absolute value of π is not too large to ensure concavity of the worker's decision problem.

plausibly low values of income tax progressivity leaves the range $(0, \sigma_I)$ of parameter values associated with local indeterminacy virtually unchanged. in contrast, reducing σ_I by 50% requires $\pi \approx 0.67$, which seems improbable in view of available evidence.

3.2.3 Explaining the Limits of Progressive Taxes as Automatic Stabilizers

Our last step is to provide some intuitive explanation of the mechanisms at work. In the related literature dealing with increasing returns in a non-monetary Ramsey model, the main effect of progressive tax rates is described as "taxing away the higher returns from belief-driven labor or investment spurts" (Guo and Lansing [88, p. 482]).In such models, the mechanisms leading to indeterminacy are different.We postulate instead constant returns so that, in particular, labor demand is (as a function of real wage) downward sloping. What we now illustrate intuitively is that expectation-driven business cycles occur because of an expected inflation effect that is absent from the related literature. Most importantly, we would like to understand why even *large* income tax progressivity fails to ensure saddle-point stability. As we now illustrate, key to the results is the fact that the more progressive taxes on labor income, the more stable disposable wage income and, therefore, the less responsive workers' labor supply. However, this effect is not strong enough to neutralize the impact of *expected inflation* on labor supply, which can lead to "self-fulfilling beliefs".

It is helpful to start with the benchmark case of a constant tax rate (which also covers the case with zero taxes and transfers) on labor income. In that case, workers' decisions are summarized by Eqs. (3.2.5) that may be written as follows, as ϕ reduces to the identity function and $\psi = 1$:

$$v_1(l_t) = v_2(p_t \omega_t l_t / p_{t+1}). \tag{3.2.14}$$

which defines implicitly labor supply $l(p_t \omega_t / p_{t+1})$. The latter first-order condition shows that when workers expect, in period t, that the price of goods p_{t+1} will go, say, down tomorrow, they wish to increase their consumption at $t + 1$ and, therefore, to work more today (remember that gross substitutability is assumed) so as to save more in the form of money balances to be consumed tomorrow. Moreover, the dynamical system in Eqs. (3.2.8) may be written as follows:

$$\begin{cases} v_2(\omega_{t+1}l_{t+1}) &= v_1(l_t), \\ k_t &= \beta R(k_{t-1}/l_t)k_{t-1}. \end{cases} \tag{3.2.15}$$

A higher labor supply l_t will lead to greater output, larger consumption and a smaller capital-labor ratio k_{t-1}/l_t and, therefore, to a higher return on capital R_t, so that, from Eqs. (3.2.15), capital demand k_t and investment will increase. Moreover, a larger capital stock k_t tomorrow will tend to increase tomorrow's real wage ω_{t+1} which will trigger an increase in tomorrow's labor supply. However, a higher capital stock will also tend to increase the ratio of capital/labor and, eventually, the effect on capitalists' savings will turn negative: a higher capital-labor ratio leads to a lower rate of return on capital and, therefore, to lower capital demand and investment. This will lead to lower wage, lower labor supply, etc: the economy will experience a reversal of the cycle. Note that this intuitive description relies on the presumption that both wage and interest rate are elastic enough to the capital-labor ratio: the elasticity of input substitution σ must be small enough.

Now, we would like to shed some light on why although progressive income taxation makes the occurrence of self-fulfilling fluctuations less likely, it does not rule them out. Assume again, for simplicity, that ϕ has constant elasticity at steady state. In that case, Eqs. (3.2.5) reduce to:

$$v_1(l_t) = \psi v_2(p_t\phi(\omega_t l_t)/p_{t+1}). \tag{3.2.16}$$

When π increases from zero to one, the volatility of wage income decreases to zero: eventually, a highly progressive tax rate on labor income (that is, π close to one) leads to an almost constant wage bill, which in turn leads to a more stable consumption and, thereby, to a smaller reaction of labor supply in comparison to the case of flat-rate taxes.

More specifically, Eq. (3.2.16) shows that a large progressivity π decreases the elasticity of labor supply. To see this, differentiate Eqs. (3.2.16) to get:

$$(\varepsilon_\gamma - 1 + \pi)\frac{dl}{l} = -\frac{d\iota^e}{\iota^e} + (1-\pi)\frac{d\omega}{\omega}, \tag{3.2.17}$$

where $\gamma \equiv v_2^{-1} \circ [v_1/\psi]$, ι^e denotes expected inflation, that is, $\iota^e_{t+1} \equiv p_{t+1}/p_t$. Eq. (3.2.17) clearly shows how the higher fiscal progressivity π, the less responsive labor supply to the real wage: this elasticity tends to zero when π tends to one. *However, large progressivity does not completely neutralize the impact of expected inflation on labor supply*: the corresponding elasticity does not vanish when $\pi = 1$. Consequently, optimistic expectations (say, a reduction in p_{t+1}) still lead to an increase of consumption and labor when fiscal policy is highly progressive so that expectation-driven business-cycles occur.

3.3 Extending the Analysis: the Case of Small Externalities

The purpose of this section is to ask whether our results are robust with respect to the introduction of small increasing returns to scale. The presence of either externalities or internal increasing returns (as in Guo and Lansing [88]) has been shown to enlarge the range of capital-labor substitution elasticities compatible with local indeterminacy (Cazzavillan et al. [41]). More precisely, local indeterminacy and bifurcations occur when σ belongs to some interval, provided that ε_γ is small enough (Cazzavillan et al. [41, Prop. 4.2]. We now show that adding progressive labor taxes tends to restrict such an interval of values for σ. In other words, extending the analysis to introduce small externalities does not change our main conclusion: *increasing the level of tax progressivity π reduces the set of parameter values such that local indeterminacy and bifurcations occur.*

So as to focus on the relevant cases, we assume that externalities come from labor only. In the notation of (Cazzavillan et al. [41, Ass. 2.1]), we set $\epsilon_\psi = 0$ and let $\nu >$

0 be the level of externalities. This is consistent with the existing literature, which has stressed that labor externalities are necessary for local indeterminacy to arise, but that capital externalities are not. In addition, we focus on small external effects by assuming that ν is smaller than a (large) threshold. This assumption accords with most empirical studies, that have shown how the assumption of constant returns to scale is consistent with the available data. Exactly as in the previous section, we use the fact that, here again, adding progressive labor taxes in Cazzavillan et al. [41] amounts to the parameter change $\varepsilon_\gamma \to \varepsilon_\gamma/(1-\pi)$. Then it is not difficult to the derive the following statement, which is the analog of Proposition 3.2.2.

Proposition 3.3.5 (Linearized Dynamics around the Steady State with Small Externalities)

The linearized dynamics for the deviations $da = a - \overline{a}$, $dk = k - \overline{k}$ are determined by the linear map:

$$\left\{\begin{array}{rcl} da_{t+1} & = & -\frac{\varepsilon_\gamma/(1-\pi)+\varepsilon_{R,a}(1+\varepsilon_{\Omega,k})}{\varepsilon_{\Omega,a}-1} da_t + \frac{\overline{a}}{\overline{k}} \frac{\varepsilon_\gamma/(1-\pi)-(1+\varepsilon_{\Omega,k})(1+\varepsilon_{R,k})}{\varepsilon_{\Omega,a}-1} dk_{t-1}, \\ dk_t & = & \frac{\overline{k}}{\overline{a}} \varepsilon_{R,a} da_t + (1+\varepsilon_{R,k}) dk_{t-1}. \end{array}\right. \tag{3.3.18}$$

The associated Jacobian matrix evaluated at the steady state under study has trace T and determinant D, where

$$T = T_1 - \frac{\varepsilon_\gamma - 1}{(1-\pi)(\varepsilon_{\Omega,a}-1)}, \quad \textit{with} \quad T_1 = 1 + \frac{|\varepsilon_{R,a}| - 1/(1-\pi) - \varepsilon_{R,k} + \varepsilon_{R,k}\varepsilon_{\Omega,a} + \varepsilon_{\Omega,k}|\varepsilon_{R,a}|}{\varepsilon_{\Omega,a}-1},$$

$$D = \varepsilon_\gamma D_1, \quad \textit{with} \quad D_1 = \frac{|\varepsilon_{R,a}| - 1 - \varepsilon_{R,k}}{(1-\pi)(\varepsilon_{\Omega,a}-1)}.$$

Moreover, one has $T_1 = 1 + D_1 + \Lambda$, where $\Lambda \equiv \frac{(1-\pi)[\varepsilon_{R,k}\varepsilon_{\Omega,a}+\varepsilon_{\Omega,k}|\varepsilon_{R,a}|]+\pi[|\varepsilon_{R,k}|-\varepsilon_{R,a}]}{[(1-\pi)(\varepsilon_{\Omega_a}-1)]}$.

Furthermore, we directly borrow from Cazzavillan et al. [41, p.81] the expressions of the elasticities of Ω and R that are related to technology, to get the following expressions.

$$\begin{aligned} &D_1 = (\theta(1-s)-\sigma)/[(1-\pi)(s-\sigma(\nu+1))], && \Lambda = \theta[\nu-\pi(1-s+\nu)]/[(1-\pi)(s-\sigma(\nu+1))], \\ &T_1 = 1 + D_1 + \Lambda, && slope_\Delta = 1-\theta(1-s)/\sigma, \end{aligned} \tag{3.3.19}$$

where $\theta \equiv 1-\beta(1-\delta) > 0$ and all these expressions are evaluated at the steady state under study.

Assume that $\nu < (s-\theta(1-s))/(\theta(1-s))$, which is easily identified if we focus on small externalities and remember that θ is likely to be close to zero. Then two cases have to be considered. If $\pi < \nu/(1+\nu)$, the geometrical picture arising when $\pi > 0$ is similar to Cazzavillan et al. [3, Fig. 7], so that local indeterminacy and bifurcations occur when σ belongs to $(0, \sigma_{F2}) \cup (\sigma_{H2}, +\infty)$, provided that ε_γ is small enough. If, on the contrary, $\pi > \nu/(1+\nu)$, then local indeterminacy occurs only if σ belongs to $(0, \sigma_{F2})$. In that configuration, increasing returns are so small that they are dominated by the opposite impact of tax progressivity. As a consequence, indeterminacy occurs only for low values of σ, as in the case without externalities. In summary, if $\pi > \nu/(1+\nu)$ then income tax progressivity eliminates local indeterminacy when $\sigma > \sigma_{H2}$ but preserves it when $\sigma < \sigma_{F2}$. On the other hand, the change of parameter $\varepsilon_\gamma \to \varepsilon_\gamma/(1-\pi)$ ensures that all the critical values of ε_γ, below which local indeterminacy prevails, are decreasing functions of π. This allows us to state the following.

Proposition 3.3.6 (Tax Progressivity and Local Indeterminacy under Small Externalities)

Assume that labor externalities are small enough, that is $\nu < (s-\theta(1-s))/(\theta(1-s))$. *Then the range of values of* σ *that is compatible with local indeterminacy and bifurcations is either* $(0, \sigma_{F2}) \cup (\sigma_{H2}, +\infty)$ *when* $\pi < \nu/(1+\nu)$, *or* $(0, \sigma_{F2})$, *when* $\pi > \nu/(1+\nu)$. *The critical values are such that* $\sigma_{F2} = [(1-\pi)(2s+\theta(1-s+\nu))+\theta(1-s)]/[2+2(1-\pi)(\nu+1)]$ *is a decreasing function of* π. *Therefore, income tax progressivity reduces the set of para-*

meter values that are associated with local indeterminacy.

3.4 Conclusion

We have shown, in a monetary economy model with constant returns, that sunspots and endogenous cycles are robust to the introduction of tax progressivity, when it is set at realistic levels. Although fiscal progressivity reduces, in parameter space, the likelihood of sunspot equilibria and endogenous cycles, considering plausibly low values of tax progressivity still leaves room for sunspots and (Hopf or flip) cycles. In an expanded version, Dromel and Pintus [50], we show that similar results hold with capital income taxes, or in an OLG economy with consumption in old age.

Some directions for future research naturally follow. It would be useful to generalize the analysis to the realistic case whereby progressivity is increasing with income (that is, when $\pi'(x) > 0$). In the OLG setting, it seems relevant to introduce consumption/savings choices in the first period of life. In that context, one expects that a smaller level of progressivity on *both* capital and labor incomes could rule out local indeterminacy and bifurcations by stabilizing both young and old agents' consumptions. It remains to be seen if this is more in line with actual levels of fiscal progressivity. It is also expected that progressive taxes and transfers are inefficient to rule out endogenous fluctuations when consumption is financed, even partially, by the returns from financial assets that would remain untaxed. Moreover, although we have studied the stabilizing power of income taxation, similar results are expected in other frameworks with business taxes, e.g. in models with credit-constrained firms and collateral requirements related to cash-flows.

Appendix. Are Progressive Income Taxes Stabilizing?

3.5 Progressive Labor Income Taxes and Workers' Choices

In this section, we show how workers' decisions can be reduced to a two-period problem. Workers solve the following problem:

$$\max \sum_{t=1}^{+\infty} \beta_w^{t-1} V_2(c_t^w/B) - \beta_w^t V_1(l_t), \tag{3.5.20}$$

subject to

$$M_{t-1}^w + (r_t + (1-\delta)p_t)k_{t-1}^w + p_t\phi(\omega_t l_t) \geq p_t c_t^w + p_t k_t^w + M_t^w, \tag{3.5.21}$$

$$M_{t-1}^w + (r_t + (1-\delta)p_t)k_{t-1}^w \geq p_t c_t^w + p_t k_t^w, \tag{3.5.22}$$

where $B > 0$ is a scaling parameter, $0 < \beta_w < 1$ is the discount factor, $c_t^w \geq 0$ is consumption, $l_t \geq 0$ is labor supply. On the other hand, $M_{t-1}^w \geq 0$ and $k_{t-1}^w \geq 0$ are respectively money demand and capital holdings at the beginning of period t, $p_t > 0$ is the price of the consumption good, $w_t > 0$ is nominal wage, $r_t > 0$ is nominal return on capital and $1 \geq \delta \geq 0$ is capital depreciation, while $p_t\phi(\omega_t l_t)$ is disposable wage income (see Section 3.2) and $\omega = w/p$ defines real wage.

Define $\lambda_t \geq 0$ and $\epsilon_t \geq 0$ as the Lagrange multipliers associated, respectively, to (3.5.21) and (3.5.22) at date t. Necessary conditions are then the following.

$$\begin{aligned} 0 &\geq \beta_w^{t-1} V_2'(c_t^w/B)/B - (\lambda_t + \epsilon_t)p_t, && = 0 \text{ if } c_t^w > 0, \\ 0 &\geq -(\lambda_t + \epsilon_t)p_t + (\lambda_{t+1} + \epsilon_{t+1})(r_{t+1} + (1-\delta)p_{t+1}), && = 0 \text{ if } k_t^w > 0, \\ 0 &\geq -\lambda_t + \lambda_{t+1} + \epsilon_{t+1}, && = 0 \text{ if } M_t^w > 0, \\ 0 &\geq -\beta_w^t V_1'(l_t) + \lambda_t p_t \omega_t \phi'(\omega_t l_t), && = 0 \text{ if } l_t > 0. \end{aligned} \tag{3.5.23}$$

Therefore, capital holdings are zero at all dates ($k_t^w = 0$) if the second inequality of (3.5.23) is not binding, that is, if:

$$V_2'(c_t^w/B) > \beta_w(r_{t+1}/p_{t+1} + 1 - \delta)V_2'(c_{t+1}^w/B), \tag{3.5.24}$$

if one assumes that $c_t^w > 0$ for all t (we will show that this is the case around the steady state). Condition (3.5.24) implies that workers choose not to hold capital, and it depends on workers' preferences because of the financial constraint (3.5.22).

Moreover, the financial constraint (3.5.22) is binding if $\epsilon_t > 0$, that is, if:

$$\omega_t \phi'(\omega_t l_t) V_2'(c_t^w/B)/B > \beta_w V_1'(l_t), \tag{3.5.25}$$

if one assumes that $l_t > 0$ (again, we will show that this is the case around the steady state). Condition (3.5.25) therefore implies that (3.5.22) is binding.

Under conditions (3.5.24) and (3.5.25), workers spend their money holdings , i.e. $p_t c_t^w = M_{t-1}^w$, and save their wage income in the form of money, i.e. $M_t^w = p_t \phi(\omega_t l_t)$, so as to consume it tomorrow, i.e. $p_{t+1} c_{t+1}^w = M_t^w$. Therefore, workers choose $l_t \geq 0$ and $c_{t+1}^w \geq 0$ as solutions to:

$$\max \, \{V_2(c_{t+1}^w/B) - V_1(l_t)\} \text{ s.t. } p_{t+1} c_{t+1}^w = p_t \phi(\omega_t l_t). \tag{3.5.26}$$

The solutions to (3.5.26) are unique under Assumption 3.2.2 and 3.2.3 and characterized by the following first-order condition, which is identical to (3.2.5) in the main text:

$$v_1(l_t) = \psi(\omega_t l_t) v_2(c^w_{t+1}),\ p_{t+1} c^w_{t+1} = p_t \phi(\omega_t l_t), \tag{3.5.27}$$

where $v_2(c) \equiv cV_2'(c/B)/B$, $v_1(l) \equiv lV_1'(l)$.

Finally, it is straightforward to show that, under the assumptions that capitalists discount future less heavily than workers (that is, $\beta_w < \beta$) and that $\beta_w < 1$, conditions (3.5.24) and (3.5.25) are met at the steady state under study defined in Proposition 3.2.1. □

3.6 Proof of Propositions 3.2.3

To prove formally the proposition, our first task is to show that the point $(T_1(\sigma), D_1(\sigma))$, as a function of σ, indeed describes part of a line Δ_1. From the fact that $T_1(\sigma) = 1 + D_1(\sigma) + \Lambda(\sigma)$ and $D_1(\sigma)$ are fractions of first degree polynomials in σ with the same denominator (see Eq. (3.3.19)), we conclude that the ratio of their derivatives $D_1'(\sigma)/T_1'(\sigma)$, or $D_1'(\sigma)/(D_1'(\sigma) + \Lambda'(\sigma))$, is independent of σ. Straightforward computations show that the slope of Δ_1 is:

$$slope_{\Delta_1} = \frac{D_1'(\sigma)}{T_1'(\sigma)} = \frac{s - \theta(1-s)}{s - \theta(1-s) + \pi\theta(1-s)}. \tag{3.6.28}$$

From Eq. (3.3.19), we conclude that $\Lambda(\sigma)$ vanishes when σ goes to infinity. It follows that Δ_1 intersects the line (AC) at a point I of coordinates $(T_1(+\infty), D_1(+\infty))$, where $D_1(+\infty) = 1/(1-\pi) > 0$ (see Figs. 1-3). We shall focus throughout on the configuration presented in Figs. 1-3, where $D_1(+\infty) \geq 1$ and the slope of Δ_1 is smaller than 1 (that is, $\pi \geq 0$). We shall ensure the latter condition by imposing, as in the case of linear (or of no) taxes, that $\theta(1-s) < s$ (that is, the share of capital is large enough). This condition is not very restrictive when $\theta = 1 - \beta(1-\delta)$ is small, which is bound to be the case when the period is short since β is then close to one and δ is close to zero. Note that the geometrical method can be applied as well when these conditions are not met.

Then it follows that both $\Lambda(\sigma)$ and $D_1(\sigma)$ are decreasing functions (see Eq. (3.3.19)), so that $T_1(\sigma)$ is also a decreasing function, i.e. $T_1'(\sigma) = D_1'(\sigma) + \Lambda'(\sigma) < 0$. Accordingly, the slope of Δ_1 is smaller than one. From the above assumptions, one also gets all the necessary information to appraise the variations of $(T_1(\sigma), D_1(\sigma))$ as well as of the slope of Δ, when σ moves from 0 to $+\infty$. In particular, $T_1(0)$ and $D_1(0) = \theta(1-s)/[s(1-\pi)]$ are positive and the corresponding point is below I on the line Δ_1 when $\pi > 0$ (see Fig.1). As σ increases from 0, $T_1(\sigma)$ and $D_1(\sigma)$ are decreasing and tend to $-\infty$ when σ tends to s from below. When $\sigma = s$, the function $\omega(a)/a$ has a critical point, i.e. its derivative with respect to a vanishes, and the dynamical system derived from Eqs. (3.2.8) is not defined. When σ increases from s to $+\infty$, $T_1(\sigma)$ and $D_1(\sigma)$ are still both decreasing, from $+\infty$ to $(T_1(+\infty), D_1(+\infty))$, which is represented by the point I in Fig. 1. In addition, the slope of Δ as a function of σ increases monotonically from $-\infty$ to 1 as σ moves from 0 to $+\infty$, and vanishes when $D_1(\sigma) = 0$. Moreover, the half-line Δ is above Δ_1 when $\sigma < s$, and below it when $\sigma > s$.

Therefore, several configurations arise when π increases from zero. When $\pi < 1 - \theta(1-s)/s$ (Proposition 3.2.3) then $D_1(0) < 1$: the geometric picture is as in Fig. 1 and it is not qualitatively different from the case of linear (or no) taxes. Second, $D_1(0) > 1$ when $\pi > 1 - \theta(1-s)/s$, and two cases arise depending on whether π is smaller or larger than $[s - \theta(1-s)]/[s - \theta(1-s)/2]$. The latter cases are presented in Dromel and Pintus [50].

We now derive all bifurcation values as functions of the structural parameters. We define $\theta \stackrel{\text{def}}{=} 1 - \beta(1-\delta)$, and $s_\Delta(\sigma) \stackrel{\text{def}}{=} 1 - \theta(1-s)/\sigma$ as the slope of the half-line Δ.

An eigenvalue of -1: the flip bifurcation.

The equality $s_\Delta(\sigma) = -1$ allows one to derive $\sigma_F = \theta(1-s)/2$, so that $s_\Delta(\sigma) < -1$ when $\sigma < \sigma_F$.

Equation $1 + T(\varepsilon_\gamma) + D(\varepsilon_\gamma) = 0$ yields $\varepsilon_{\gamma F} = (1-\pi)(2s + \theta(1-s) - 2\sigma)/[2\sigma - \theta(1-s)]$. The condition that $1 + T_1(\sigma) + D_1(\sigma) = 0$ or, equivalently $\varepsilon_{\gamma F} = 1$, gives the last flip bifurcation value $\sigma_I = [\theta(1-s)(2-\pi)+2s(1-\pi)]/[2(2-\pi)]$ so that $\varepsilon_{\gamma F} > 1$ when $\sigma < \sigma_I$.

A pair of eigenvalues of modulus 1: the Hopf bifurcation.

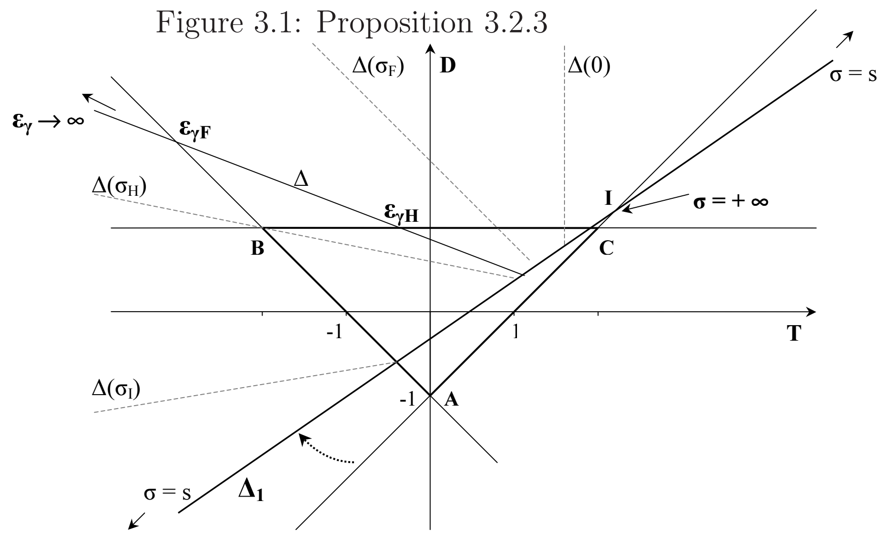
The condition that $T(\varepsilon_{\gamma H}) = -2$ when $D = 1$, i.e. when $\varepsilon_\gamma = \varepsilon_{\gamma H} = 1/D_1$, is rewritten as $Q_H(\sigma) \stackrel{\text{def}}{=} a\sigma^2 + b\sigma + c$, the roots of which contain the bifurcation value σ_H. The coefficients of $Q_H(\sigma)$ are:

$$\begin{aligned} a &= 4, \\ b &= -4[s + \theta(1-s)], \\ c &= \theta(1-s)[\theta(1-s) + 3s]. \end{aligned}$$

It is easily shown that there must exist two distinct real roots, and that $\sigma_H = s[1+\theta(1-s)/s - \sqrt{1-\theta(1-s)/s}]/2$ is the lowest.

The condition $D_1(\sigma) = 1$ yields, in view of Eqs. (3.3.19), $\sigma_J = [\theta(1-s) - s(1-\pi)]/\pi$. Moreover, the bifurcation value $\varepsilon_{\gamma H} = (1-\pi)(s-\sigma)/[\theta(1-s) - \sigma)]$ follows from $D = \varepsilon_\gamma D_1 = 1$, i.e. $\varepsilon_{\gamma H} = 1/D_1$.

Figure 3.1: Proposition 3.2.3



Chapter 4

Investment Subsidies and Stabilization in Credit Constrained Economies[1]

We analyse how fiscal policy can affect aggregate volatility and growth in economies subject to capital market imperfections. The model is based upon Aghion, Banerjee and Piketty [3], in which the combination of frictions on the capital market and unequal access to investment opportunities among individuals can generate endogenous and permanent fluctuations. We show that appropriate fiscal policy parameters are able to rule out the occurrence of slump regimes, and immunize the economy against endogenous fluctuations in GDP, investment and interest rates. For given levels of the credit market development in the economy, we provide specific fiscal parameters able to insulate the economy from crises, fuel its long-run growth rate and place it on a *permanent boom* dynamic path. We analyse how the conditions on the stabilizing fiscal parameters are modified when frictions

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in the economy evolve. Eventually, we study how the tax system impacts the answer of the economy to temporary and permanent productivity shocks.

Key Words: Endogenous Business Cycles; Capital Market Imperfections; Access to Productive Investment; Fiscal Policy; Macroeconomic Stabilization.

JEL Class.: E22; E32; E62; H20; H30.

4.1 Introduction

The growth and amplification effects of capital market frictions have been the subject of a large literature. At the aggregate level, a number of studies have highlighted the importance of credit constraints in explaining fluctuations in activity (see, in particular, Bernanke [22], Eckstein and Sinai [58], and Friedman [67]). Bernanke and Gertler [23] develop a simple neoclassical model of the business cycle in which the condition of borrowers' balance sheets is a source of output dynamics. Higher borrower net worth reduces the agency costs of financing real capital investments. Business upturns improve net worth, lower agency costs, and increase investment, which amplifies the upturn; vice versa, for downturns. Shocks that affect net worth (as in a debt-deflation) can initiate fluctuations. Kiyotaki and Moore [107] construct a model of a dynamic economy in which lenders cannot force borrowers to repay their debts unless the debts are secured. The dynamic interaction between credit limits and asset prices turns out to be a powerful transmission mechanism by which the effects of shocks persist, amplify, and spill over to other sectors. They show that small, temporary shocks to technology or income distribution can generate large, persistent fluctuations in output and asset prices. Holmstrom and Tirole [98] study an incentive model of financial intermediation in which firms as well as intermediaries are capital constrained. They show that all forms of capital tightening (a credit crunch, a collateral squeeze, or a savings squeeze) hit poorly capitalized firms the hardest, but that interest rate effects and the intensity of monitoring will depend on relative changes in the various components of capital. Bernanke, Gertler and Gilchrist [25] develop a dynamic general equilibirum model intended to help clarify the role of credit market frictions in business fluctuations, from both a qualitative and a quantitative standpoint. The framework exhibits a "financial accelerator", i.e. endogenous developments in credit markets work to amplify and propagate shocks to the macroeconomy. Their use of money and price stickyness allows them to study how credit market frictions may influence the transmission of the monetary policy. In addition, they allow for lags in investment which enables the model to generate both hump-shaped output dynamics and a lead-lag relation between asset prices and investment, as is consistent with the data. Finally, they allow for heterogeneity among firms to capture the fact that borrowers have differential access to capital markets. Under reasonable parametrizations of the model, the financial accelerator has a significant influence on business cycle dynamics.

The purpose of this paper is to analyse how investment subsidies, financed through labor-income taxation, can affect aggregate volatility and growth in economies subject to capital market imperfections. The model is based upon Aghion, Banerjee and Piketty [3], in which the combination of frictions on the capital market and unequal access to investment opportunities among individuals can generate endogenous and permanent fluctuations in aggregate GDP, investment and interest rates. In this setup, savers and investors are separated along two dimensions: first, a pure simple physical separation, since many people who save are in no position to invest directly in physical capital (as distinct from financial capital); and a more market based separation embodied in the constraints on the amounts investors can borrow from savers. Aghion et al. [3] show that when the credit market development is high enough, and the separation between savers and investors is low enough, the economy can stay in a *permanent boom regime*. In contrast, a high degree of such separation leads the economy to fluctuate around its steady-state growth path. More specifically, it is shown that under a relatively high degree of physical separation of savers and investors and a poorly functioning capital market, the economy will always converge to a cycle around its trend growth path, unless the capital market frictions are so high that the economy falls into a permanent slump regime. Economies with less developed financial markets and a sharper physical separation between savers and investors will then tend to be more volatile and grow more slowly. For a number of obvious reasons, both of these dimensions of separation are likely to be greater in emerging market economies. However, there is at least some evidence that this kind of mechanism based on the functioning of the credit market is also relevant for understanding the business cycle properties of more developed economies. For instance, this type of analysis may shed some light on the case of advanced market economies such as Finland, where financial development is still lagging behind and which has experienced high macroeconomic volatility over the past decade (see Honkapohja and Koskela [99]). Moreover, even in financially developed economies like the United States, the analysis remains relevant for the case of small investors whose investments turn out to be significantly correlated with current cash flows.

The contribution of the paper is to show that appropriate fiscal policy parameters are able to rule out the occurrence of slump regimes, and immunize the economy against endogenous and permanent fluctuations in GDP, investment and interest rates. For given levels of the credit market development in the economy, we provide specific fiscal parameters able to insulate the economy from crises, fuel its long-run growth rate and place it on a *permanent boom* dynamic path.

The main mechanism driving our result is the following. The fiscal policy we analyse, introducing a tax on savers' labor income and transferring the proceeds into investors' wealth, is tantamount to an increase in the fraction of the labor force having direct access to capital investment opportunities (and therefore to a decrease in the fraction of agents unable to invest directly in the production process). More precisely, a structural policy that would remove institutional obstacles and rigidities separating savers and investors to promote growth, stability and equity at the same time, would presumably have a similar effect on the economy's dynamics.

We analyse how the conditions on the stabilizing fiscal parameters are modified when frictions in the economy evolve. Eventually, we study how the tax system impacts the response of the economy to temporary and permanent productivity shocks. Typically, aside from its direct growth-enhancing effects, it is shown that this type of fiscal policy moderates the wealth distribution effects following a productivity shock in a slump episode.

Our findings complement the conclusions of Aghion et al. [3]. In the last part of their paper, they suggest a government could absorb idle savings in the economy by public debt issuance, and finance investment subsidies so as to counteract the limited debt capacity of investors in slumps. However, in many actual economies, the public debt option is a constrained policy tool. In contrast, labor income taxation is a feature shared by the quasi totality of modern economies. It is then interesting to see in our setup that basic income taxation can also have stabilizing properties in economies where capital markets are subject to frictions. Another dimension in which our paper departs from Aghion et al. [3] is that we provide an exhaustive analytic characterization of dynamic regimes possibilities, depending on the tax rate level and the friction parameters values.

Our results can be linked to a parallel strand of the macroeconomic literature, investigating the stabilizing properties of fiscal policies on local indeterminacy and belief-driven endogenous fluctuations. Many papers in that field (e.g., among others, Dromel and Pintus [50]) show that flat rate taxation is not efficient in stabilizing business fluctuations. In contrast here, a standard linear tax does have an effect on dynamics, and can effectively isolate the economy from (another type of) endogenous cycles (than those analyzed in the sunspot literature).

The remainder of this paper proceeds as follows. The next section lays out the model, while section 3 analyses dynamics and regime change conditions, showing how fiscal policy may help getting out of slumps, cycles, and fuel the growth rate. Section 4 discusses the comparative statics properties of the suggested stabilizing and growth-enhancing fiscal policy. Section 5 investigates how this policy changes the response of the economy to productivity shocks. Section 6 briefly discusses the question of Welfare in this setup, and announces directions for further research. Some concluding remarks are gathered in section 7.

4.2 The Economy

This paper introduces fiscal policy, through labor-income taxation and lump-sum transfers, into the positive long-run growth AK model with capital market imperfections studied in Aghion et al. [3]. For ease of comparison with this benchmark model, we keep the same notations and dynamic analysis methods.

4.2.1 Production Technology

An homogeneous good is produced and serves both as capital and as a consumption commodity. In each period $t \in \mathbb{N}$, agents are endowed with one unit of time.

The good is produced according to the technology: $F(K, L) = AK^{\beta}L^{1-\beta} = Y$. We assume the growth rate of the workforce to be at least equal to the one of the capital stock. Then, all agents are willing to work at a wage greater than or equal to one, so that the equilibrium labor price can be set to unity.

Assumption 4.2.1

$\frac{\partial F}{\partial L} = 1 \Rightarrow L = \big((1-\beta)A\big)^{1/\beta} K \Rightarrow Y = \sigma K$ *with* $\sigma = A\big((1-\beta)A\big)^{(1-\beta)/\beta}$

Positive long-run growth can be generated from this AK type setting. The parameter β stands for the capital share in final output, whereas $(1-\beta)$ denotes the labor share.

4.2.2 Dualism

The economy is physically split into two categories of agents. Only a fraction $0 \leq \mu \leq 1$ of the workforce (called the *productive investors*) can directly invest in physical capital. The other individuals (called the *savers*), can either lend their savings to the productive investors at current interest rate r, or invest in a low-yield asset with a return $\sigma_2 < \sigma_1 = \beta\sigma$. When μ increases from 0 to 1, the separation between savers and investors becomes thinner.

Due to asymmetric information issues (moral hazard), capital market is subject to a borrowing constraint. There is a constant $0 \leq \nu \leq 1$ such that anyone who wants to invest an amount I must have assets of at least νI. In other words, $1/\nu$ is nothing else than a credit multiplier. Indeed, when ν decreases from 1 to 0, credit market development improves.

4.2.3 Interest rate setting

The AK type technology used in this framework implies that the equilibrium interest rate will take two possible values. When investment exceeds savings (i.e., demand for savings is (very) large), the gross interest rate will take its "high" value $\sigma_1 = \beta\sigma$. In contrast, when savings are in excess supply, the gross equilibrium interest rate will drop to $\sigma_2 < \sigma_1$.

4.2.4 Government

The government chooses tax policy and balances the budget at each point in time. The public authority is assumed to care about the level of productive investment in the economy. To this end, linear taxes are applied on savers' labor income, to finance a transfer than can be thought of as an investment subsidy.[2]

4.2.5 Timing of the model

In the beginning of a period (say, in the morning), the respective amounts of planned investment and available savings in the economy are compared. Depending on their relative magnitude, the interest rate is set. If investment runs ahead of savings, the higher interest rate prevails, and the non-investors are willing to lend all their savings to the productive investors. Thus, during these *boom* episodes, all available savings in the economy will be invested in the high-yield activity. In contrast, if investment plans are not large enough to absorb all available savings, the interest rate will be set at its low value σ_2. Then savers will be indifferent between issuing low-return loans, or investing in the low-yield asset. At the end of the day, returns to investment are realized, borrowers pay back their debt to lenders, wages are paid. Taxes are also levied and income transfers occur. Consumption finally takes place, from the net resources of the day. For sake of simplicity, and to ease

[2] Even though we choose to present the model with an exceedingly pared-down fiscal structure, the following results are robust to the introduction of a richer tax structure, as long as the fiscal scheme is tantamount to a net transfer to investors.

comparisons with the Aghion et al. [3] benchmark model, a linear savings rate is assumed, as in the case of a standard logarithmic utility function. The non-consumed part of the day's net resources constitutes the amount of available savings in the next morning.

4.2.6 Agents' Wealth Accumulation

Let W_B^t and W_L^t respectively represent the wealth levels of the borrowers (productive investors) and of the lenders (savers) in the morning of period $(t+1)$. We denote by S_t the total amount of savings: $S_t = W_B^t + W_L^t$, and $I_{t+1}^d = W_B^t/\nu$ the total planned investment in the morning of period $(t+1)$.

In a boom, the investment capacity of investors is higher than the available amount of savings $((I_{t+1}^d \geq S_t)$. The prevailing interest rate is $\sigma_1 = \beta\sigma$, such that all aggregate savings $(W_B^t + W_L^t)$ are invested in the high-yield activity. The wealth accumulation of borrowers and lenders can be summarized as follows:

BOOM

$$W_B^{t+1} = (1-\alpha)\big[(1-\tau)\mu(1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma(W_B^t + W_L^t) - \beta\sigma W_L^t + T^t\big] \quad (4.2.1)$$

$$W_L^{t+1} = (1-\alpha)\big[(1-\tau)(1-\mu)(1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma W_L^t\big] \quad (4.2.2)$$

The total return of the high-yield activity $\sigma(W_B^t + W_L^t)$, is shared between labor income with a fraction $(1-\beta)$, and capital income. Productive investors (resp. savers) represent a fraction μ (resp. $1-\mu$) of the total labor share in output. Only borrowers take advantage of the return $\beta\sigma$ on physical capital investment, but have to refund and pay the high level interest rate on the amount they borrowed (the whole W_L^t, since at the high interest rate $\sigma_1 = \beta\sigma$, investing in the low-yield asset is a dominated strategy for lenders). To support productive investment, the government operates a transfer of resources by taxing labor income at rate $0 < \tau < 1$ and reallocating the proceeds T^t in the form of an investment subsidy.

As public budget is balanced in each period:

$$T^t = [\tau\mu + \tau(1-\mu)](1-\beta)\sigma(W_B^t + W_L^t) \tag{4.2.3}$$

Hence, we can re-write the borrowers' wealth motion equation in a boom as:

$$W_B^{t+1} = (1-\alpha)\big\{[\mu + \tau(1-\mu)](1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma W_B^t\big\} \tag{4.2.4}$$

In a slump, the investment capacity of productive investors is lower than the level of aggregate savings ($I_{t+1}^d < S_t$). The prevailing interest rate is then $\sigma_2 < \beta\sigma$, such that only $\frac{W_B^t}{\nu}$ can be invested in the high-yield activity, generating a total revenue equal to $\sigma\frac{W_B^t}{\nu}$.

SLUMP

$$W_B^{t+1} = (1-\alpha)\big[(1-\tau)\mu(1-\beta)\sigma\frac{1}{\nu}W_B^t + \beta\sigma\frac{1}{\nu}W_B^t - \sigma_2(\frac{1}{\nu}-1)W_B^t + T^t\big] \tag{4.2.5}$$

$$W_L^{t+1} = (1-\alpha)\big[(1-\tau)(1-\mu)(1-\beta)\sigma\frac{1}{\nu}W_B^t + \sigma_2(\frac{1}{\nu}-1)W_B^t + \sigma_2(W_L^t - (\frac{1}{\nu}-1)W_B^t)\big] \tag{4.2.6}$$

The total revenue $\sigma\frac{W_B^t}{\nu}$ remunerates labor up to a fraction $(1-\beta)$, with borrowers (resp. lenders) getting a share μ (resp. $1-\mu$) of that wage income. Only the borrowers get the fraction β of the total revenue, remunerating physical capital investment. As productive investors have actually borrowed $W_B^t/\nu - W_B^t$, they repay this amount to the lenders with the interest σ_2 prevailing in a slump period. Aside from this repayment, lenders get also the return σ_2 from investing the rest of their savings in the low-yield activity. Once again, the government operates a transfer of resources by taxing labor income and reallocating the proceeds in the form of an investment subsidy.

As the public budget is balanced, we can re-write the borrowers' wealth motion equation in a slump as:

$$W_B^{t+1} = (1-\alpha)\big\{[\mu + \tau(1-\mu)](1-\beta)\sigma\frac{1}{\nu}W_B^t + \beta\sigma\frac{1}{\nu}W_B^t - \sigma_2(\frac{1}{\nu}-1)W_B^t\big\} \tag{4.2.7}$$

Let us notice that, both during booms or slumps, borrower's *physical* capital income could also be taxed (say, at rate τ_k). As a matter of fact, there would be no difference in eq. (4.2.4) or (4.2.7) because of the balanced budget assumption (cf. Appendix 4.8). However, taxing lender's *financial* capital income (say, at rate τ_f) would make little sense since it would completely discourage loans to productive investors. As mentioned earlier, during slumps, the interest rate is at its low value σ_2. After-tax rates of return on loans would then be equal to $(1-\tau_f)\sigma_2 < \sigma_2$, and lending to borrowers would be a dominated strategy with respect to investment in the low-yield asset.

Besides, an interesting feature is that a proportional investment subsidy would have the same effect on dynamics as the lump-sum transfer to the borrowers we consider here (cf. Appendix 4.8).

4.3 Analysis of the Dynamics

Defining $q^t = \frac{S_t}{I^d_{t+1}} = \frac{W^t_B + W^t_L}{W^t_B}\nu$ as the ratio of aggregate savings over investment plans in the high-yield activity in the morning of period $(t+1)$, we can obtain from the previous wealth motion laws the two following difference equations, allowing the global dynamics analysis of this economy.

When at the beginning of period $t+1$ planned investment runs ahead of savings $(q_t \leq 1)$, the economy is in a boom:

$$\frac{1}{q^{t+1}} = \frac{[\mu + \tau(1-\mu)](1-\beta)}{\nu} + \frac{\beta}{q^t} \tag{BB}$$

In contrast, when $q_t > 1$ the economy is experiencing a slump:

$$q^{t+1} = \frac{\left[(\sigma - \sigma_2) + \sigma_2 q^t\right]}{[\mu + \tau(1-\mu)](1-\beta)(\sigma/\nu) + \beta\sigma/\nu - \big((1/\nu) - 1)\sigma_2} \tag{SS}$$

It is worth noticing that if we set τ to zero, we recover the benchmark model of Aghion et al. [3].

These two difference equations behavior can be studied graphically in the (q^t, q^{t+1}) plane. It is straightforward to show (cf. Appendix 4.9) that (BB) is monotonic, increasing and concave while (SS) is linearly increasing. As shown by Aghion et al. [3], there are only three dynamic regimes the economy can actually experience, corresponding to the three possible rankings between 1, b and s, where s and b are steady-state values of the savings to planned investment ratio, respectively determined by the intersections between (SS) and (BB) with the 45^0 line. When $q^t \leq 1$ (i.e., when the planned investment volume is higher than the aggregate savings amount), only the (BB) curve is relevant, while if $q^t > 1$, only the (SS) locus prevails.

The steady-state savings to planned investment ratio in a boom writes as $b = \frac{\nu}{\mu+\tau(1-\mu)}$. As soon as $b \leq 1$, the economy is in a permanent boom.

— FIGURE 4.1 ABOUT HERE —

From any initial $q^t < 1$, the economy will converge to b, and the long-run growth rate is nothing else than the Harrod-Domar one, that is the product of the savings rate by the average productivity of capital $g^* = (1-\alpha)\sigma$ (cf. Appendix 4.10). The condition for a permanent boom can also be written in terms of the fiscal parameter τ: the economy will experience a permanent boom regime if and only if

$$\tau \geq \frac{\nu - \mu}{1 - \mu} = \tau_b$$

Increasing τ lowers the level of (BB) in the plane and the savings to planned investment steady-state level b. The ordinate to origin of (BB) $q^{t+1}|_{q^t=0} = 0$ remains equal to zero, whatever the tax rate.

The steady-state savings to planned investment ratio in a slump writes as $s = \frac{(\sigma-\sigma_2)\nu}{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2}$. As soon as $s > 1$, the economy will go through a permanent slump.

— Figure 4.2 about here —

The condition for the permanent slump can be also stated as function of the fiscal parameter τ:

$$\tau < \frac{\nu(\sigma - \sigma_2) + \sigma_2 - \beta\sigma - \mu(1-\beta)\sigma}{(1-\beta)\sigma(1-\mu)} = \tau_s$$

Increasing τ lowers the level of (SS) in the plane and the steady-state savings to planned investment ratio s. The ordinate to origin of (SS) drops when the tax rate is increased since $\frac{\partial}{\partial \tau}(q^{t+1}|_{q^t=0}) < 0$. The long-run growth rate in a slump can be written as $g_s = \frac{(1-\alpha)}{\nu}\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)\} = (1-\alpha)\{\frac{\sigma}{s}+(1-\frac{1}{s})\sigma_2\}$, and clearly depends positively on the fiscal parameter τ. Since, $b - s = \frac{\nu(\beta\sigma-\sigma_2)(1-\tau)(1-\mu)}{[\mu+\tau(1-\mu)]\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2\}} > 0$, we know that b is always greater than s. However, the distance between the two steady-state savings to planned investment ratios shrinks as soon as τ increases from 0 to 1.

The remaining case corresponds to the intermediary situation where $\tau_s < \tau < \tau_b$. In that situation, a cyclical regime prevails.

— Figure 4.3 about here —

The economy will then keep back and forth between episodes of booms and periods of slumps, and will eventually converge to a limit cycle, which periodicity depends upon some deep parameters of the model. The logic behind the cycles is the following. Periods of slow growth are periods when savings are plentiful relative to the limited debt capacity of potential investors, which implies a low demand for savings and therefore low equilibrium interest rates. This in turn implies that the investors can retain a high proportion of their profits (since the interest rate and hence the debt burden on investors is low), which allows them to rebuild their reserves and debt capacity and expand their investment. This in turn, generates more profits and more investment until, eventually, planned investment runs ahead of savings forcing the interest rates to rise. Then, the debt burden on the investors will to be higher, retained earnings will be lower, and investment will collapse, taking the economy back to a period of slower growth.

We can summarize the different dynamic regimes by the following proposition:

Proposition 4.3.1 (Stabilizing and Growth-Enhancing Fiscal Policy)

1. When $\nu < \mu$, the economy is in a permanent boom whatever τ. This case is covered in Aghion et al. [3]

2. When $\mu < \nu$, the economy

1. *is in a permanent boom if $\tau > \frac{\nu-\mu}{1-\mu} = \tau_b \Leftrightarrow b < 1$*
2. *is in a permanent slump if $\tau < \frac{\nu(\sigma-\sigma_2)+\sigma_2-\beta\sigma-\mu(1-\beta)\sigma}{(1-\beta)\sigma(1-\mu)} = \tau_s \Leftrightarrow s > 1$.*
3. *cycles if $\tau_s < \tau < \tau_b \Leftrightarrow s < 1 < b$.*

Let $\nu = \bar{\nu}(\mu) = \frac{\mu(1-\beta)\sigma+\beta\sigma-\sigma_2}{\sigma-\sigma_2} \Leftrightarrow \tau_s = 0$. We will have $\tau_b > 0 \Leftrightarrow \nu > \mu$ and $\tau_s > 0 \Leftrightarrow \nu > \bar{\nu}(\mu)$.

- *For any $\mu < \nu < \bar{\nu}(\mu)$: if $0 < \tau < \tau_b$ then the economy experiences a cyclical regime, alternating between phases of expansion and downturns; if $\tau_b < \tau$, the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.*
- *For any $\bar{\nu}(\nu) < \nu < 1$: if $0 < \tau < \tau_s$, the economy is trapped into a permanent slump regime; if $\tau_s < \tau < \tau_b$ then the economy experiences a cyclical regime alternating between phases of expansion and downturns; if $\tau_b < \tau$, the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.*

Similarly, one can get related results by analyzing the dynamics regimes for fixed values of μ (cf. Appendix 4.11)

If the tax rate initially set to $\tau < \tau_s$ is raised to a value $\tau_s < \tau' < \tau_b$, the dynamic regime goes from a permanent slump to a cyclical motion (cf. Fig. 4.4). Starting from the previous permanent slump steady-state level of the savings to planned investment ratio, the economy hits the new (SS') locus, and enters a cycle, alternating between temporary booms and slumps.

— Figure 4.4 about here —

If the tax rate initially set to $\tau_s < \tau < \tau_b$ is raised to a value $\tau' = \tau_b$, the dynamic regime goes from a cycle to a permanent boom (cf. Fig. 4.5). In that case, b is set equal to 1, so that the economy can not go back to the slump zone, since when $q \leq 1$, only (BB) is relevant.

— Figure 4.5 about here —

Eventually, if the tax rate initially set to $\tau < \tau_s$ is raised to a value $\tau' = \tau_b$, the dynamic regime goes from a permanent slump to a permanent boom (cf. Fig. 4.6). In that case, b is set equal to 1, so that the economy can not go back to the slump zone, since when $q \leq 1$, only (BB) is relevant.

— Figure 4.6 about here —

4.3.1 Intuition

To gain insight into the mechanism that drives our result, it is useful to analyse how the introduction of this fiscal scheme affects the fractions of the labor share going respectively to investors and to savers. Actually, introducing a tax on savers' labor income and transferring the proceeds into the investors' wealth is tantamount to an increase in the fraction of the labor force having direct access to capital investment opportunities. As a matter of fact, the original setup studied in Aghion et al. [3] is modified up to the following parameter changes : μ becomes $\mu+(1-\mu)\tau$, which is increasing in τ, and $(1-\mu)$ becomes $(1-\mu)(1-\tau)$, decreasing in τ.

In other words fiscal policy, usually mobilized as a conventional countercyclical tool, affects here the economy in the same way as a *structural reform* would do. More precisely, a structural policy that would remove institutional obstacles and rigidities separating savers and investors to promote growth, stability and equity at the same time, would presumably have a similar effect on the economy dynamics. In general, such structural policies may be difficult to implement(especially in the short-run), and are in some cases just not feasible: governments cannot simply decide that access to credit and investment opportunities should be extended. Interestingly, with a very basic setup, the fiscal policy we feature can impact the dynamics as if a structural policy had managed to improve the access to productive investment opportunities.

4.4 Comparative Statics

In the following section, we assess how τ_b and τ_s behave when one of the friction parameters ν and μ is made to vary (cf. Appendix 4.11 for detailed expressions).

4.4.1 Comparative Statics Properties of τ_b and τ_s with respect to ν

Let $\tau_{b,\nu}$ and $\tau_{s,\nu}$ be the geometrical loci respectively depicting the sensitivity of τ_b and τ_s with respect to ν, *caeteris paribus.*

— Figure 4.7 about here —

$\tau_{b,\nu}$ and $\tau_{s,\nu}$ are linear. Since $0 < \frac{\partial \tau_b}{\partial \nu} < \frac{\partial \tau_s}{\partial \nu}$, both are upward sloping, but $\tau_{s,\nu}$ is steeper than $\tau_{b,\nu}$. It can be easily shown that $0 < \mu = \nu|_{\tau_b=0} < \nu|_{\tau_s=0} = \bar{\nu}(\mu) < 1$, so that $\tau_{s,\nu}$ hits the abscissa axis for a higher value of ν than $\tau_{b,\nu}$ does. As $\tau_b - \tau_s > 0$, $\tau_{b,\nu}$

is always "higher" than $\tau_{s,\nu}$ in the plane, for any value of $\nu > \mu$. The intersection of $\tau_{b,\nu}$ and $\tau_{s,\nu}$ eventually occurs at $\nu = 1$.

Let us now turn to the effect of a variation in μ (i.e., a change in the access to productive investment opportunities) on the respective properties of $\tau_{b,\nu}$ and $\tau_{s,\nu}$. We suppose μ goes from μ_1 to $\mu_2 > \mu_1$, i.e. the separation between savers and productive investors is smaller.

— FIGURE 4.8 ABOUT HERE —

Since $0 < \frac{\partial^2 \tau_b}{\partial\mu\partial\nu} < \frac{\partial^2 \tau_s}{\partial\mu\partial\nu}$, when μ increases, both $\tau_{b,\nu}$ and $\tau_{s,\nu}$ become steeper, but the steepness rise is higher for $\tau_{s,\nu}$. Moreover, as $0 < \frac{\partial}{\partial\mu}\bar{\nu}(\mu) < \frac{\partial}{\partial\mu}(\nu|_{\tau_b=0}) = 1$, both abscissa to origin values of $\tau_{b,\nu}$ and $\tau_{s,\nu}$ increase following a rise in μ, but the abscissa to origin value of $\tau_{b,\nu}$ reacts more. Hence, when the degree of separation between savers and investors decreases (μ goes up from μ_1 to μ_2), the permanent boom likelihood is increased for any $\nu \geq \mu_1$ (permanent boom can be achieved with a lower τ) and the permanent slump likelihood reduces for any $\nu \geq \bar{\nu}(\mu_1)$ (we can get out of slumps with a lower τ). The cycles likelihood reduces for any $\mu_1 < \nu < \bar{\nu}(\mu_1)$ and expands for any $\nu > \bar{\nu}(\mu_1)$. We also notice that $\frac{\partial}{\partial\mu}(\tau_b - \tau_s) > 0$. Very intuitively, improving the access to investment opportunities facilitates the conditions needed to reach a permanent boom, or to get out from a permanent slump trap.

Besides, $\tau_{b,\nu}$ and $\tau_{s,\nu}$ can also be affected by a productivity shock (namely, a rise in σ, from σ_1 to $\sigma_2 > \sigma_1$). Since $0 = \frac{\partial^2 \tau_b}{\partial\sigma\partial\nu} < \frac{\partial^2 \tau_s}{\partial\sigma\partial\nu}$ and $0 = \frac{\partial}{\partial\sigma}(\nu|_{\tau_b=0}) < \frac{\partial}{\partial\sigma}\bar{\nu}(\mu)$, both the slope and the abscissa to origin value of $\tau_{s,\nu}$ will go up, whereas $\tau_{b,\nu}$ will remain unchanged. Hence, if a productivity shock occurs, the likelihood of permanent booms will remain unchanged for any $0 < \nu < 1$, while the permanent slump likelihood will reduce and the cycles likelihood will increase for any $\nu > \bar{\nu}(\mu)|_{\sigma=\sigma_1}$.

4.4.2 Comparative Statics Properties of τ_b and τ_s with respect to μ

Let $\tau_{b,\mu}$ and $\tau_{s,\mu}$ be the geometrical loci respectively depicting the sensitivity of τ_b and τ_s with respect to μ, *caeteris paribus.*

— Figure 4.9 about here —

Since $\frac{\partial \tau_s}{\partial \mu} < \frac{\partial \tau_b}{\partial \mu} < 0$ and $\frac{\partial^2 \tau_s}{\partial \mu^2} < \frac{\partial^2 \tau_b}{\partial \mu^2} < 0$, both $\tau_{b,\mu}$ and $\tau_{s,\mu}$ are decreasing and concave, but $\tau_{s,\mu}$ is more concave. The ordinate and abscissa to origin values of $\tau_{b,\mu}$ are the same. The locus $\tau_{s,\mu}$ shares the same property, such that: $0 < \tau_s|_{\mu=0} = \mu|_{\tau_s=0} = \bar{\mu}(\nu) < \mu|_{\tau_b=0} = \tau_b|_{\mu=0} = \nu < 1$.

Let us now turn to the effect of a variation in ν, (i.e. a change in the credit market development), on the properties of $\tau_{b,\mu}$ and $\tau_{s,\mu}$. We suppose ν goes from ν_1 to $\nu_2 > \nu_1$, i.e. the credit market development gets poorer.

— Figure 4.10 about here —

Since $0 < \frac{\partial^2 \tau_b}{\partial \nu \partial \mu} < \frac{\partial^2 \tau_s}{\partial \nu \partial \mu}$, and $0 < \frac{\partial}{\partial \nu}(\mu|_{\tau_b=0}) = \frac{\partial}{\partial \nu}(\tau_b|_{\mu=0}) < \frac{\partial}{\partial \nu}(\tau_s|_{\mu=0}) = \frac{\partial}{\partial \nu}\bar{\mu}(\nu)$, the upward shift and the concavity reduction of $\tau_{s,\mu}$ following a rise in ν is stronger than the reaction of $\tau_{b,\mu}$. Hence, when conditions on the credit market deteriorate (ν increases from ν_1 to ν_2), the permanent slump likelihood increases for any $0 < \mu < \bar{\mu}(\nu_2)$ (we need a higher τ to get out from the permanent slump) and the permanent boom likelihood decreases for any $0 < \mu < \nu_2$. The cycles likelihood reduces for any $0 < \mu < \bar{\mu}(\nu_2)$, but increases for any $\bar{\mu}(\nu_2) < \mu < \nu_2$. We also notice that $\frac{\partial}{\partial \nu}(\tau_b - \tau_s) < 0$. Very intuitively, a deterioration in the credit market development makes stronger the conditions needed to reach a permanent boom, or to get out from a permanent slump.

Besides, $\tau_{b,\mu}$ and $\tau_{s,\mu}$ can also be affected following a productivity shock (namely, a rise in σ from σ_1 to $\sigma_2 > \sigma_1$). Since $\frac{\partial^2 \tau_s}{\partial\sigma\partial\mu} < \frac{\partial^2 \tau_b}{\partial\sigma\partial\mu} = 0$ and $\frac{\partial}{\partial\sigma}\bar{\mu}(\nu) = \frac{\partial}{\partial\sigma}(\tau_s|_{\mu=0}) < \frac{\partial}{\partial\sigma}(\mu|_{\tau_b=0}) = \frac{\partial}{\partial\sigma}(\tau_b|_{\mu=0}) = 0$, we know that both the slope, the abscissa to origin and the ordinate to origin values of $\tau_{s,\mu}$ will decrease following a productivity shock, whereas $\tau_{b,\mu}$ will remain unchanged. Hence, for any $\mu < \bar{\mu}(\nu)|_{\sigma=\sigma_1}$ the likelihood of permanent slumps will reduce and the likelihood of cycles will increase. However, the likelihood of permanent boom will remain unchanged for any $0 < \mu < 1$.

4.5 Response to Shocks

Is the fiscal structure featured in this economy able to affect its reaction to productivity shocks (such as shocks on σ)? As in the benchmark case of Aghion et al. [3], σ does not appear in the expression of (BB). It can be easily shown in a boom that, following a shock on σ (may it be permanent or temporary), loans repayments and investment returns vary in the exact same proportion, so that the distribution of wealth between savers and investors remains unchanged (indeed, q provides a direct measure of any evolution in this repartition). A productivity shock during a boom does affect the Harrod-Domar growth rate g^*. But as q is not affected by any variation in the productivity level, all the shock will be registered instantly in g^* (there will be no indirect effects due to a change in wealth distribution). Put differently, the tax schedule has no effect whatsoever on the way g^* reacts to σ.

In contrast, during slumps, (SS) does react to any variation in σ (Cf. Appendix 4.10 and Appendix 4.12). Since $\frac{\partial^2 q^{t+1}}{\partial\sigma\partial q^t} < 0$ a positive productivity shock on σ decreases the slope of the (SS) curve. We can see from $\frac{\partial^3 q^{t+1}}{\partial\tau\partial\sigma\partial q^t} > 0$ that increasing τ makes $\frac{\partial^2 q^{t+1}}{\partial\sigma\partial q^t}$ less negative. In other words, the reduction in the slope of (SS) due to a productivity shock is lower when the tax rate is high. Moreover, following a productivity shock, the ordinate to origin of (SS) goes up since, $\frac{\partial}{\partial\sigma}(q^{t+1}|_{q^t=0}) > 0$. Increasing τ moderates this increase, as $\frac{\partial}{\partial\tau}[\frac{\partial}{\partial\sigma}(q^{t+1}|_{q^t=0})] < 0$.

— Figure 4.11 about here —

Following a permanent productivity shock (cf. Fig. 4.11), the steady-state savings to planned investment ratio in a slump s decreases, since $\frac{\partial s}{\partial \sigma} < 0$. Hence, aside from the direct effect of σ on g_s, $\frac{\partial g_s}{\partial \sigma} > 0$, the drop in s adds an indirect effect on growth, via the shift in wealth distribution in favor of the productive investors as q goes down. However, when the tax rate is increased, s still decreases following a productivity shock, but to a lower extent, since: $\frac{\partial^2 s}{\partial \tau \partial \sigma} > 0$. Hence, although $\frac{\partial^2 g_s}{\partial \tau \partial \sigma} > 0$ which basically says that τ reinforces the direct positive effect on g_s of a positive shock on σ, the indirect growth effects linked to the convergence to the new steady-state savings to planned investment ratio will be smaller when taxes are increased.

When the productivity shock is only temporary (cf. Fig. 4.12), the steady-state savings to planned investment ratio does not change. When τ is increased, both the short-run and the long-run convergence path will be shorter, also meaning smaller indirect wealth distribution effects.

— Figure 4.12 about here —

4.6 Welfare Analysis: an Avenue for Further Research

A crucial dimension that we plan to seriously investigate in the soon future is the issue of Welfare. Although this model is not particularly adequate for a rigorous, well-founded Welfare analysis, we can provide however some intuitions based upon the evolution of agents' consumption through time, whether the fiscal policy we analysed is implemented or not. Let us compare the situations when τ is set to τ_b (ensuring permament boom), and when τ is zero.

It is obvious that the borrowers (cf. Fig. 4.13) will favor the type of fiscal policy featured in this paper. Their wealth and consumption is instantaneously increased at soon as the tax schedule is implemented. Moreover, the indirect positive effects on labor and capital income of such a growth-enhancing policy will make them even more well-off.

— Figure 4.13 about here —

In contrast, although they will undoubtedly benefit from future increases in labor and interest income of this growth-promoting policy, lenders have to incur today the whole cost of this transfer mechanism(cf. Fig. 4.14). The extent to which they accept such a fiscal scheme depends on two crucial parameters: their degree of impatience, and the speed of convergence.

— Figure 4.14 about here —

In a one period environment, the degree of patience is directly linked to the notion of altruism. The parameter α captures these notions in the mobilized setup. The shift from $\tau = 0$ to τ_b can be Pareto improving if and only if lenders are patient or altruistic enough (that is, if α is low enough). In other words, τ_b will be effectively implemented as long as there exists a level of altruism for lenders such that τ_b is preferred to $\tau = 0$, despite the loss in consumption they incur at the date of the fiscal change.

A more precise characterization of preferences, notably about the intertemporal elasticity of substitution in consumption, and an endogenous savings behavior would certainly provide new answers to the questions about welfare emerging from this setup. This is an issue we are currently working on.

4.7 Conclusion

The purpose of this paper is to analyse how investment subsidies, financed through labor-income taxation, can affect aggregate volatility and growth in economies subject to capital market imperfections. Within a model featuring both frictions on the capital market and unequal access to investment opportunities among individuals, we have shown that appropriate fiscal policy parameters are able to rule out the occurrence of slump regimes, and immunize the economy against endogenous fluctuations in GDP, investment and interest rates. For given levels of the credit market development, we provide specific fiscal parameters able to insulate the economy from crises, fuel its long-run growth rate and place it on a *permanent boom* dynamic path.

The main mechanism driving our result is the following. The fiscal policy we analyse, introducing a tax on savers' labor income and transferring the proceeds into the investors' wealth, is tantamount to an increase in the fraction of the labor force having direct access to capital investment opportunities (and therefore to a decrease in the fraction of agents unable to invest directly in the production process). We analyse how conditions on the stabilizing fiscal parameters are modified when frictions in the economy evolve. Eventually, we study how the tax system impacts the economy's response to temporary and permanent productivity shocks. Typically, aside from its direct growth-enhancing effects, it is shown that this type of fiscal policy moderates the wealth distribution effects following a productivity shock in a slump episode.

These findings complement the conclusions of Aghion et al. [3]. Abstracting from the utilization of public debt issuance, which can be a constrained instrument in many modern economies, we show that labor income taxation (which is a widely available instrument) has also some stabilizing properties in economies where capital markets are subject to frictions.

Some directions for further research naturally follow. We have seen in this economy that the shift from the situation without taxes to the tax rate ensuring a permanent boom can be Pareto-improving if and only if lenders are patient or altruistic enough. Augmenting the micro-foundations of the model, through a more precise characterization of preferences, notably about the intertemporal elasticity of substitution in consumption, and an endogenous savings behavior would certainly provide new answers to the questions about welfare emerging from this setup. Besides, an analysis of "optimal fiscal rules", aimed at achieving both stabilization and inequality reduction in this economy would certainly be of interest.

Appendix. Stabilizing Fiscal Policy with Capital Market Imperfections

4.8 Alternative Fiscal Structures

4.8.1 Adding capital income taxation for borrowers

If borrowers were also to pay capital income taxes, we would rigorously obtain the same reduced-form wealth accumulation equations as in the case with labor income taxes only, due to the balanced public budget assumption.

For instance in a boom:

$$W_B^{t+1} = (1-\alpha)\big[(1-\tau)\mu(1-\beta)\sigma(W_B^t+W_L^t)+(1-\tau_k)\beta\sigma(W_B^t+W_L^t)-\beta\sigma W_L^t+T^t\big] \quad (4.8.8)$$

$$W_L^{t+1} = (1-\alpha)\big[(1-\tau)(1-\mu)(1-\beta)\sigma(W_B^t+W_L^t)+\beta\sigma W_L^t\big] \quad (4.8.9)$$

As public budget is balanced in each period:

$$T^t = \{[\tau\mu+\tau(1-\mu)](1-\beta)+\tau_k\beta\}\sigma(W_B^t+W_L^t) \quad (4.8.10)$$

Hence, we can re-write the borrowers' wealth motion equation in a boom as:

$$W_B^{t+1} = (1-\alpha)\big\{[\mu + \tau(1-\mu)](1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma W_B^t\big\} \tag{4.8.11}$$

which is identical to eq. (4.2.4).

On the other hand, during a slump:

$$W_B^{t+1} = (1-\alpha)\big[(1-\tau)\mu(1-\beta)\sigma\frac{1}{\nu}W_B^t + (1-\tau_k)\beta\sigma\frac{1}{\nu}W_B^t - \sigma_2(\frac{1}{\nu}-1)W_B^t + T^t\big] \tag{4.8.12}$$

$$W_L^{t+1} = (1-\alpha)\big[(1-\tau)(1-\mu)(1-\beta)\sigma\frac{1}{\nu}W_B^t + \sigma_2(\frac{1}{\nu}-1)W_B^t + \sigma_2(W_L^t - (\frac{1}{\nu}-1)W_B^t)\big] \tag{4.8.13}$$

As public budget is balanced, we can re-write the borrowers' wealth motion equation in a slump as:

$$W_B^{t+1} = (1-\alpha)\big\{[\mu + \tau(1-\mu)](1-\beta)\sigma\frac{1}{\nu}W_B^t + \beta\sigma\frac{1}{\nu}W_B^t - \sigma_2(\frac{1}{\nu}-1)W_B^t\big\} \tag{4.8.14}$$

which is identical to eq. (4.2.7).

4.8.2 Proportional rather than lump-sum investment subsidy

If the investment subsidy was granted in a proportional rather than a lump-sum way, we would also obtain the same reduced-form wealth accumulation equations.

For instance in a boom:

$$W_B^{t+1} = (1-\alpha)\big[(1-\tau)\mu(1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma(W_B^t + W_L^t) - \beta\sigma W_L^t\big](1+\gamma) \tag{4.8.15}$$

$$W_L^{t+1} = (1-\alpha)\big[(1-\tau)(1-\mu)(1-\beta)\sigma(W_B^t + W_L^t) + \beta\sigma W_L^t\big] \tag{4.8.16}$$

As public budget is balanced in each period:

$$(1-\alpha)\gamma\big[(1-\tau)\mu(1-\beta)\sigma(W_B^t+W_L^t)+\beta\sigma(W_B^t+W_L^t)-\beta\sigma W_L^t\big] = (1-\alpha)[\tau\mu+\tau(1-\mu)](1-\beta)\sigma(W_B^t+W_L^t) \tag{4.8.17}$$

Hence, we can re-write the borrowers' wealth motion equation in a boom as:

$$W_B^{t+1} = (1-\alpha)\big[(1-\tau)\mu(1-\beta)\sigma(W_B^t+W_L^t)+\beta\sigma(W_B^t+W_L^t)-\beta\sigma W_L^t\big]+(1-\alpha)[\tau\mu+\tau(1-\mu)](1-\beta)\sigma(W_B^t+W_L^t) \tag{4.8.18}$$

$$W_B^{t+1} = (1-\alpha)\big\{[\mu+\tau(1-\mu)](1-\beta)\sigma(W_B^t+W_L^t)+\beta\sigma W_B^t\big\} \tag{4.8.19}$$

which is identical to eq. (4.2.4).

On the other hand, during a slump:

$$W_B^{t+1} = (1-\alpha)\big[(1-\tau)\mu(1-\beta)\sigma\frac{1}{\nu}W_B^t+\beta\sigma\frac{1}{\nu}W_B^t-\sigma_2(\frac{1}{\nu}-1)W_B^t\big](1+\gamma) \tag{4.8.20}$$

$$W_L^{t+1} = (1-\alpha)\big[(1-\tau)(1-\mu)(1-\beta)\sigma\frac{1}{\nu}W_B^t+\sigma_2(\frac{1}{\nu}-1)W_B^t+\sigma_2(W_L^t-(\frac{1}{\nu}-1)W_B^t)\big] \tag{4.8.21}$$

As public budget is balanced, we can re-write the borrowers' wealth motion equation in a slump as:

$$W_B^{t+1} = (1-\alpha)\big\{[\mu+\tau(1-\mu)](1-\beta)\sigma\frac{1}{\nu}W_B^t+\beta\sigma\frac{1}{\nu}W_B^t-\sigma_2(\frac{1}{\nu}-1)W_B^t\big\} \tag{4.8.22}$$

which is identical to eq. (4.2.7).

4.9 Properties of the BB and SS loci

4.9.1 The BB locus

Since $\frac{\partial q^{t+1}}{\partial q^t} = \frac{\beta}{[\mu+\tau(1-\mu)](1-\beta)\frac{q^t}{\nu}+\beta} > 0$, and $\frac{\partial^2 q^{t+1}}{\partial (q^t)^2} = -\frac{2\beta(1-\beta)[\mu+\tau(1-\mu)]}{\nu\{[\mu+\tau(1-\mu)](1-\beta)\frac{q^t}{\nu}+\beta\}^3} < 0$, (BB) is positively sloped and concave.

Moreover when τ is increased, (BB) moves downwards: $\frac{\partial^2 q^{t+1}}{\partial\tau\partial q^t} = -\frac{2\beta(1-\beta)(1-\mu)q^t}{\nu\{[\mu+\tau(1-\mu)](1-\beta)\frac{q^t}{\nu}+\beta\}^3} < 0$.

The steady-state level in a boom writes as: $b = \frac{\nu}{\mu+\tau(1-\mu)}$.

Since $\frac{\partial b}{\partial\tau} = -\frac{\nu(1-\mu)}{[\mu+\tau(1-\mu)]^2} < 0$, the steady-state level b decreases when τ increases.

The ordinate to origin $q^{t+1}|_{q^t=0} = 0$ remains zero, whatever the tax rate.

4.9.2 The SS locus

Since $\frac{\partial q^{t+1}}{\partial q^t} = \frac{\sigma_2\nu}{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)} > 0$, (SS) is linear and positively sloped.

The ordinate to origin of (SS) writes as $q^{t+1}|_{q^t=0} = \frac{\sigma-\sigma_2}{[\mu+\tau(1-\mu)](1-\beta)(\sigma/\nu)+\beta\sigma/\nu-\left(1/\nu-1\right)\sigma_2}$.

Since $\frac{\partial}{\partial\tau}q^{t+1}|_{q^t=0} = -\frac{(\sigma-\sigma_2)(1-\mu)(1-\beta)\sigma}{\nu\{[\mu+\tau(1-\mu)](1-\beta)(\sigma/\nu)+\beta\sigma/\nu-\left(1/\nu-1\right)\sigma_2\}^2} < 0$, increasing τ lowers the ordinate to origin.

Moreover, since $\frac{\partial^2 q^{t+1}}{\partial\tau\partial q^t} = -\frac{\sigma_2(1-\mu)(1-\beta)\sigma\nu}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)\}^2} < 0$, increasing τ decreases the slope of (SS).

The steady-state level in a slump writes as: $q^{t+1} = q^t = s = \frac{(\sigma-\sigma_2)\nu}{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2}$.

Since $\frac{\partial s}{\partial\tau} = -\frac{\nu(\sigma-\sigma_2)(1-\mu)(1-\beta)\sigma}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2\}^2} < 0$ the steady-state level s is lower when τ increases.

Nota: the (BB) curve always lies above the (SS) locus at $q^t = 1$:
$q^{t+1}(q^t = 1, r_{t+1} = \sigma_1) = \left\{\frac{[\mu+\tau(1-\mu)](1-\beta)}{\nu} + \beta\right\}^{-1} > q^{t+1}(q^t = 1, r_{t+1} = \sigma_2) = \left\{\left[\mu + \tau\right](1-\beta)/\nu\right] + (\beta/\nu) - \left[(1/\nu-1)\right](\sigma_2/\sigma)\right\}^{-1}$

4.9.3 Steady state levels

Since $b-s=\frac{\nu(\beta\sigma-\sigma_2)(1-\tau)(1-\mu)}{[\mu+\tau(1-\mu)]\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2\}}>0$, we know that b is always greater than s.

Since $\frac{\partial}{\partial\tau}(b-s)=\frac{-\nu(\beta\sigma-\sigma_2)(1-\mu)\langle[\mu+\tau(1-\mu)](1-\beta)\sigma\{2-[\tau(1-\mu)+\mu]\}+\beta\sigma-\sigma_2\rangle}{[\mu+\tau(1-\mu)]^2\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2\}^2]}<0$, we know that increasing τ reduces the gap between b and s, from $(b-s)|_{\tau=0}=\frac{\nu(\beta\sigma-\sigma_2)(1-\mu)}{\mu[\mu(1-\beta)\sigma+\beta\sigma-\sigma_2]}>0$ to $(b-s)|_{\tau=1}=0$.

4.10 Growth rates

In a boom $I^d_{t+1}\geq S_t$, the actual investment in the high-yield activity is $\min(S_t,I^d_{t+1})=S_t$.

The boom growth rate can then be measured by: $\frac{Y_{t+2}}{Y_{t+1}}=\frac{\sigma K_{t+2}}{\sigma K_{t+1}}=\frac{\sigma S_{t+1}}{\sigma S_t}=(1-\alpha)\sigma=g^*$.

In a slump, $I^d_{t+1}<S_t$, the actual investment in the high-yield activity is $\min(S_t,I^d_{t+1})=I^d_{t+1}$.

The slump growth rate will be therefore written as: $\frac{K_{t+2}}{K_{t+1}}=\frac{W_B^{t+1}/\nu}{W_B^t/\nu}=\frac{(1-\alpha)}{\nu}\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)\}=g_s=(1-\alpha)\{\frac{\sigma}{s}+(1-\frac{1}{s})\sigma_2\}$.

Since $\frac{\partial g_s}{\partial\tau}=\frac{(1-\alpha)}{\nu}(1-\mu)(1-\beta)\sigma>0$, raising τ increases long-run growth during slumps.

Indeed $g_s|_{\tau=0}=\frac{(1-\alpha)}{\nu}[\mu(1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu))]<g_s|_{\tau>0}$.

Very intuitively, a positive productivity shock on σ affects positively the growth rate, both during booms and during slumps: $\frac{\partial g^*}{\partial\sigma}=(1-\alpha)>0$ and $\frac{\partial g_s}{\partial\sigma}=\frac{(1-\alpha)}{\nu}\{[\mu+\tau(1-\mu)](1-\beta)+\beta\}>0$. Since $\frac{\partial^2 g_s}{\partial\tau\partial\sigma}=\frac{(1-\alpha)}{\nu}(1-\mu)(1-\beta)>0$, we know that increasing τ reinforces the positive effect on g_s of a positive shock on σ.

4.11 Proposition 4.3.1 Continued

One can get related results by analyzing the dynamics regimes for fixed values of μ:

$\mu = \bar{\mu}(\nu) = \frac{\nu(\sigma-\sigma_2)+\sigma_2-\beta\sigma}{(1-\beta)\sigma} \Leftrightarrow \tau_s = 0$. We will have $\tau_b > 0 \Leftrightarrow \nu > \mu$ and $\tau_s > 0 \Leftrightarrow \mu < \bar{\mu}(\nu)$.

- For any $\bar{\mu}(\nu) < \mu < \nu$: if $0 < \tau < \tau_b$ then the economy experiences a cyclical regime, alternating between phases of expansion and downturns; if $\tau_b < \tau$, the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.

- For any $0 < \mu < \bar{\mu}(\nu)$: if $0 < \tau < \tau_s$, the economy is trapped into a permanent slump regime; if $\tau_s < \tau < \tau_b$ then the economy experiences a cyclical regime alternating between phases of expansion and downturns; if $\tau_b < \tau$, the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.

Nota : $\bar{\mu}(\nu) = \frac{[\bar{\nu}(\mu)+\nu](\sigma-\sigma_2)-\mu(1-\beta)\sigma-2(\beta\sigma-\sigma_2)}{(1-\beta)\sigma}$.

Appendix 4.E. Comparative Statics (Detailed Expressions)

4.11.1 Comparative Statics Properties of τ_b and τ_s with respect to ν

Let $\tau_{b,\nu}$ and $\tau_{s,\nu}$ be the geometrical loci respectively depicting the sensitivity of τ_b and τ_s with respect to ν, *caeteris paribus.*

$\tau_{b,\nu}$ and $\tau_{s,\nu}$ are linear. Since $0 < \frac{1}{(1-\mu)} = \frac{\partial \tau_b}{\partial \nu} < \frac{\sigma-\sigma_2}{(1-\beta)\sigma}\frac{\partial \tau_b}{\partial \nu} = \frac{\partial \tau_s}{\partial \nu}$, both are upward sloping, but $\tau_{s,\nu}$ is steeper than $\tau_{b,\nu}$. Since $0 < \mu = \nu|_{\tau_b=0} < \nu|_{\tau_s=0} = \bar{\nu}(\mu) = \frac{\mu(1-\beta)\sigma+\beta\sigma-\sigma_2}{\sigma-\sigma_2} < 1$, $\tau_{s,\nu}$ hits the abscissa axis for a higher value of ν than $\tau_{b,\nu}$ does. Since, $\tau_b - \tau_s = \frac{(\beta\sigma-\sigma_2)(1-\nu)}{(1-\beta)\sigma(1-\mu)} > 0$, $\tau_{b,\nu}$ is always "higher" than $\tau_{s,\nu}$ in the plane, for any value of $\nu >= \nu|_{\tau_b=0} = \mu$. The intersection of $\tau_{b,\nu}$ and $\tau_{s,\nu}$ occurs at $\nu = 1$.

Let us now turn to the effect of a variation in μ (i.e. a change in the access to productive investment opportunities) on the properties of $\tau_{b,\nu}$ and $\tau_{s,\nu}$.We suppose μ goes from μ_1 to $\mu_2 > \mu_1$, i.e. the separation between savers and productive investors is smaller.

Since $0 < \frac{1}{(1-\mu)^2} = \frac{\partial^2 \tau_b}{\partial\mu\partial\nu} < \frac{\sigma-\sigma_2}{(1-\beta)\sigma}\frac{\partial^2 \tau_b}{\partial\mu\partial\nu} = \frac{\partial^2 \tau_s}{\partial\mu\partial\nu}$, when μ increases, $\tau_{b,\nu}$ and $\tau_{s,\nu}$ become steeper, but the steepness rise is higher for $\tau_{s,\nu}$. Moreover as $0 < \frac{\partial}{\partial\mu}\bar{\nu}(\mu) = \frac{\partial}{\partial\mu}(\nu|_{\tau_s=0}) = \frac{(1-\beta)\sigma}{\sigma-\sigma_2} < \frac{\partial}{\partial\mu}(\nu|_{\tau_b=0}) = 1$, both abscissa to origin values of $\tau_{b,\nu}$ and $\tau_{s,\nu}$ increase following a rise in μ, but abscissa to origin value of $\tau_{b,\nu}$ reacts more. Hence, when the degree of separation between savers and investors decreases (μ goes up from μ_1 to μ_2), the permanent boom likelihood is increased for any $\nu \geq \mu_1$ (permanent boom can be achieved with a lower τ) and the permanent slump likelihood reduces for any $\nu \geq \bar{\nu}(\mu_1)$ (we can get out of slumps with a lower τ). The cycles likelihood reduces for any $\mu_1 < \nu < \bar{\nu}(\mu_1)$ and expands for any $\nu > \bar{\nu}(\mu_1)$. We also notice that $\frac{\partial}{\partial\mu}(\tau_b - \tau_s) = \frac{(\beta\sigma-\sigma_2)(1-\nu)}{(1-\beta)\sigma(1-\mu)^2} > 0$. Very intuitively, improving the access to investment opportunities facilitates the conditions needed to reach a permanent boom, or to get out from a permanent slump trap.

Besides, $\tau_{b,\nu}$ and $\tau_{s,\nu}$ can also be affected by a productivity shock (namely, a rise in σ, from σ_1 to $\sigma_2 > \sigma_1$). Since $0 = \frac{\partial^2 \tau_b}{\partial\sigma\partial\nu} < \frac{\partial^2 \tau_s}{\partial\sigma\partial\nu} = \frac{\sigma_2}{(1-\beta)\sigma^2(1-\mu)}$ and $0 = \frac{\partial}{\partial\sigma}(\nu|_{\tau_b=0}) < \frac{\partial}{\partial\sigma}(\nu|_{\tau_s=0}) = \frac{\partial}{\partial\sigma}\bar{\nu}(\mu) = \frac{\sigma_2(1-\beta)(1-\mu)}{(\sigma-\sigma_2)^2}$, both the slope and the abscissa to origin value of $\tau_{s,\nu}$ will go up, whereas $\tau_{b,\nu}$ will remain unchanged. Hence, if a productivity shock occurs, the likelihood of permanent booms will remain unchanged for any $0 < \nu < 1$, while the permanent slump likelihood reduces and the cycles likelihood increases for any $\nu > \bar{\nu}(\mu)|_{\sigma=\sigma_1}$.

4.11.2 Comparative Statics Properties of τ_b and τ_s with respect to μ

Let $\tau_{b,\mu}$ and $\tau_{s,\mu}$ be the geometrical loci respectively depicting the sensitivity of τ_b and τ_s with respect to μ, *caeteris paribus*.

Since $\frac{\partial \tau_s}{\partial\mu} = \frac{\sigma-\sigma_2}{(1-\beta)\sigma}\frac{\partial \tau_b}{\partial\mu} < \frac{\partial \tau_b}{\partial\mu} = -\frac{1-\nu}{(1-\mu)^2} < 0$ and $\frac{\partial^2 \tau_s}{\partial\mu^2} = \frac{\sigma-\sigma_2}{(1-\beta)\sigma}\frac{\partial^2 \tau_b}{\partial\mu^2} < \frac{\partial^2 \tau_b}{\partial\mu^2} = -\frac{2(1-\nu)}{(1-\mu)^3} < 0$, both $\tau_{b,\mu}$ and $\tau_{s,\mu}$ are decreasing and concave, but $\tau_{s,\mu}$ is more concave. The ordinate and abscissa to origin values of $\tau_{b,\mu}$ are the same. The locus $\tau_{s,\mu}$ shares the same property, such that: $0 < \frac{\nu(\sigma-\sigma_2)+\sigma_2-\beta\sigma}{(1-\beta)\sigma} = \tau_s|_{\mu=0} = \mu|_{\tau_s=0} = \bar{\mu}(\nu) < \mu|_{\tau_b=0} = \tau_b|_{\mu=0} = \nu < 1$.

Let us now turn to the effect of a variation in ν, (i.e. a change in the credit market development), on the properties of $\tau_{b,\mu}$ and $\tau_{s,\mu}$. We suppose ν goes from ν_1 to $\nu_2 > \nu_1$, i.e. the credit market development gets poorer.

Since $0 < \frac{1}{(1-\mu)^2} = \frac{\partial^2\tau_b}{\partial\nu\partial\mu} < \frac{\sigma-\sigma_2}{(1-\beta)\sigma}\frac{\partial^2\tau_b}{\partial\nu\partial\mu} = \frac{\partial^2\tau_s}{\partial\nu\partial\mu}$, and $0 < 1 = \frac{\partial}{\partial\nu}(\tau_b|_{\mu=0}) = \frac{\partial}{\partial\nu}(\mu|_{\tau_b=0}) < \frac{\partial}{\partial\nu}(\tau_s|_{\mu=0}) = \frac{\partial}{\partial\nu}(\mu|_{\tau_s=0}) = \frac{\partial}{\partial\nu}\bar{\mu}(\nu) = \frac{\sigma-\sigma_2}{(1-\beta)\sigma}$, the upward shift and the concavity reduction of $\tau_{s,\mu}$ following a rise in ν is stronger than the reaction of $\tau_{b,\mu}$. Hence, when conditions on the credit market deteriorate (ν increases from ν_1 to ν_2), the permanent slump likelihood increases for any $0 < \mu < \bar{\mu}(\nu_2)$ (we need a higher τ to get out from the permanent slump) and the permanent boom likelihood decreases for any $0 < \mu < \nu_2$). The cycles likelihood reduces for any $0 < \mu < \bar{\mu}(\nu_2)$, but increases for any $\bar{\mu}(\nu_2) < \mu < \nu_2$. We also notice that $\frac{\partial}{\partial\nu}(\tau_b - \tau_s) = -\frac{(\beta\sigma-\sigma_2)}{(1-\beta)\sigma(1-\mu)} < 0$. Very intuitively, a deterioration in the credit market development makes stronger the conditions needed to reach a permanent boom, or get out from a permanent slump.

Besides, $\tau_{b,\mu}$ and $\tau_{s,\mu}$ can also be affected following a productivity shock (namely, a rise in σ from σ_1 to $\sigma_2 > \sigma_1$). Since $-\frac{\sigma_2(1-\nu)}{(1-\beta)\sigma^2(1-\mu)^2} = \frac{\partial^2\tau_s}{\partial\sigma\partial\mu} < \frac{\partial^2\tau_b}{\partial\sigma\partial\mu} = 0$ and $-\frac{\sigma_2(1-\nu)}{(1-\beta)\sigma^2} = \frac{\partial}{\partial\sigma}(\mu|_{\tau_s=0}) = \frac{\partial}{\partial\sigma}\bar{\mu}(\nu) = \frac{\partial}{\partial\sigma}(\tau_s|_{\mu=0}) < \frac{\partial}{\partial\sigma}(\mu|_{\tau_b=0}) = \frac{\partial}{\partial\sigma}(\tau_b|_{\mu=0}) = 0$, both the slope, the abscissa to origin and the ordinate to origin values of $\tau_{s,\mu}$ will decrease following a productivity shock, whereas $\tau_{b,\mu}$ will remain unchanged. Hence, for any $\mu < \bar{\mu}(\nu)|_{\sigma=\sigma_1}$ the likelihood of permanent slumps will reduce and the likelihood of cycles will increase. However, the likelihood of permanent boom will remain unchanged for any $0 < \mu < 1$.

4.12 Response to Productivity Shocks during Slumps

Since $\frac{\partial^2 q^{t+1}}{\partial\sigma\partial q^t} = \frac{-\nu\sigma_2\{[\mu+\tau(1-\mu)](1-\beta)+\beta\}}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)\}^2} < 0$ a positive productivity shock on σ decreases the slope of the (SS) curve. We can see from $\frac{\partial^3 q^{t+1}}{\partial\tau\partial\sigma\partial q^t} = \frac{\nu\sigma_2(1-\mu)(1-\beta)\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma+\sigma_2(1-\nu)\}}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)\}^3} > 0$ that increasing τ makes $\frac{\partial^2 q^{t+1}}{\partial\sigma\partial q^t}$ less negative. In other words, the reduction in the slope of (SS) due to a productivity shock is lower when the tax rate is high.

Moreover, following a productivity shock, the ordinate to origin of (SS) goes up since: $\frac{\partial}{\partial\sigma}(q^{t+1}|_{q^t=0}) = \frac{\nu\sigma_2\{[\mu+\tau(1-\mu)](1-\beta)+\beta-(1-\nu)\}}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)\}^2} > 0$. Increasing τ moderates this increase in the ordinate to origin, since:
$\frac{\partial}{\partial\tau}[\frac{\partial}{\partial\sigma}(q^{t+1}|_{q^t=0})] = \frac{\nu\sigma_2(1-\mu)(1-\beta)(\sigma_2-2\sigma)\{[\mu+\tau(1-\mu)](1-\beta)+\beta-(1-\nu)\}}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2(1-\nu)\}^3} < 0$.

Following a productivity shock, the steady-state level in a slump s decreases, since $\frac{\partial s}{\partial\sigma} = \frac{-\nu\sigma(1-\beta)(1-\mu)(1-\tau)}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2\}^2} < 0$. However, when the tax is increased, s still decreases following a productivity shock, but in a lower extent, since: $\frac{\partial^2 s}{\partial\tau\partial\sigma} = \frac{-\nu\sigma_2(1-\beta)(1-\mu)\{(1-\beta)\sigma[\mu+\tau(1-\mu)-1]+\sigma_2-\sigma\}}{\{[\mu+\tau(1-\mu)](1-\beta)\sigma+\beta\sigma-\sigma_2\}^3} > 0$.

Figure 4.1: The Permanent Boom

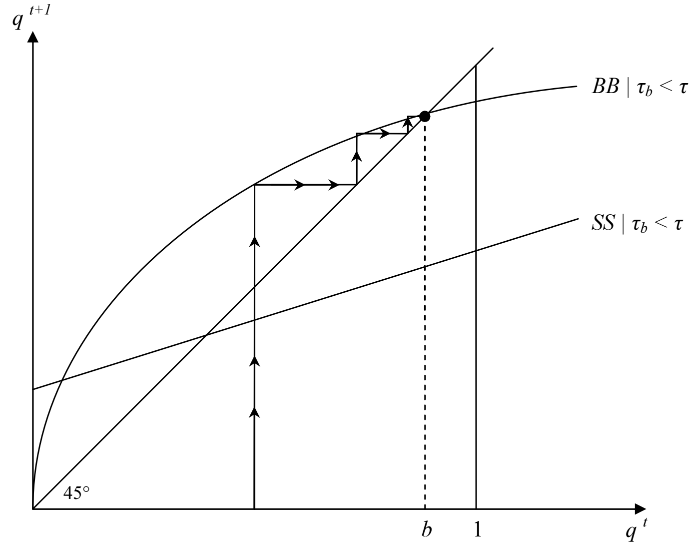


Figure 4.2: The Permanent Slump

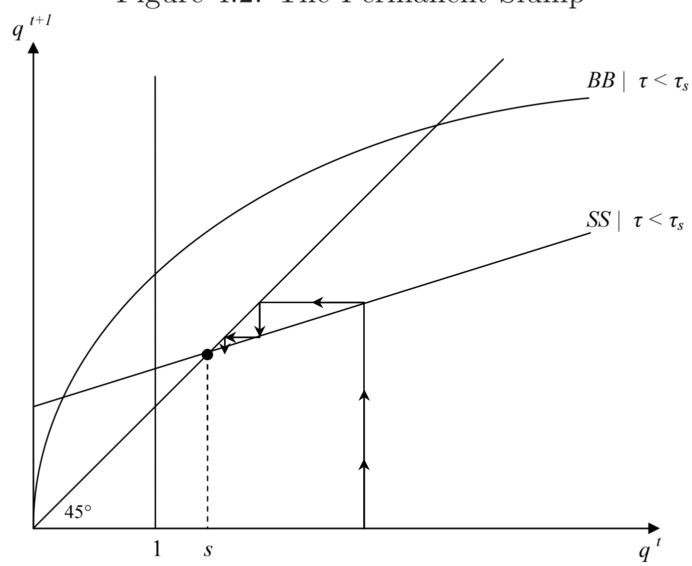


Figure 4.3: The Cyclical Regime

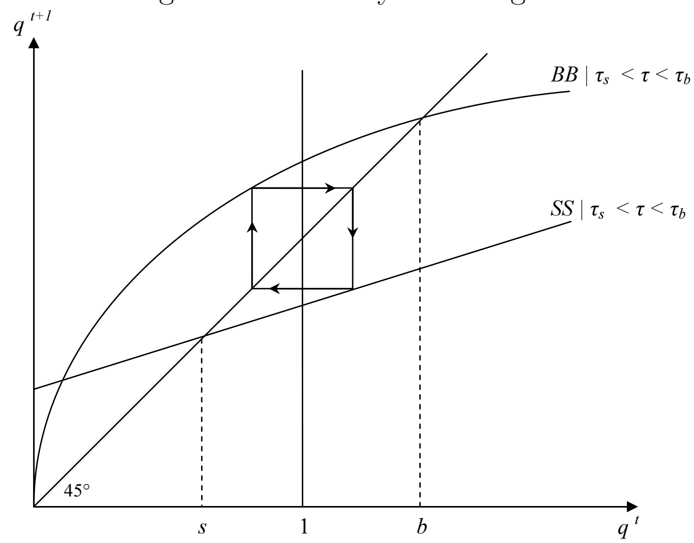


Figure 4.4: From a Permanent Slump to a Cycle

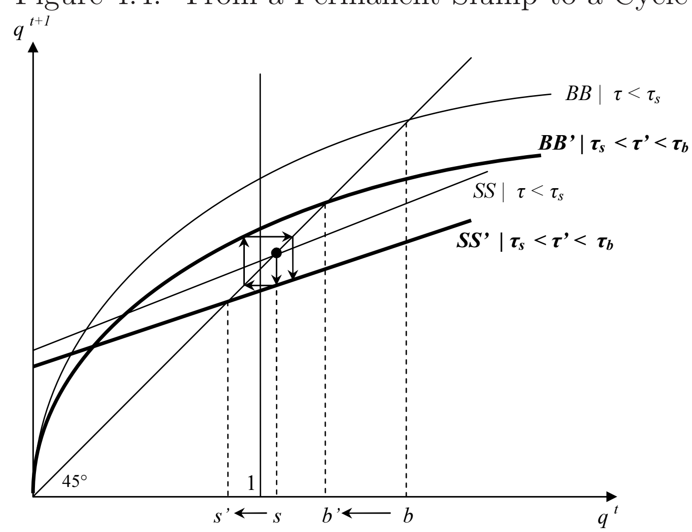


Figure 4.5: From a Cycle to a Permanent Boom

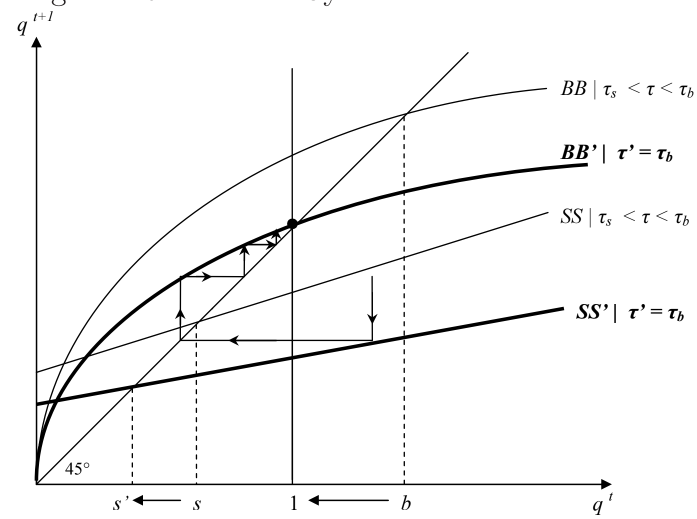


Figure 4.6: From a Permanent Slump to a Permanent Boom

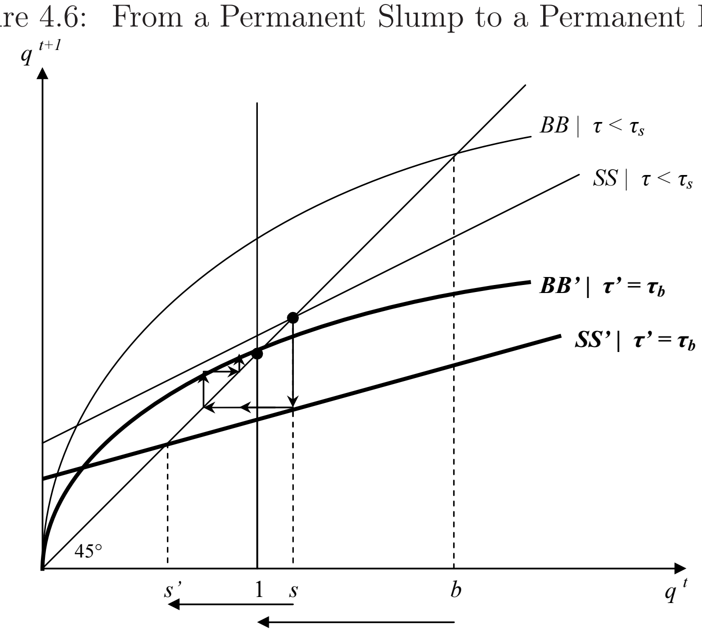


Figure 4.7: Comparative Statics Properties of τ_b and τ_s with respect to ν

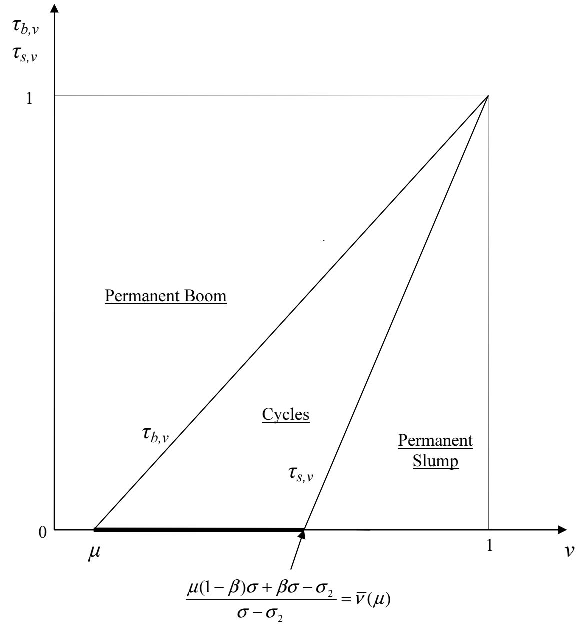


Figure 4.8: Comparative Statics Properties of τ_b and τ_s with respect to ν, when μ rises

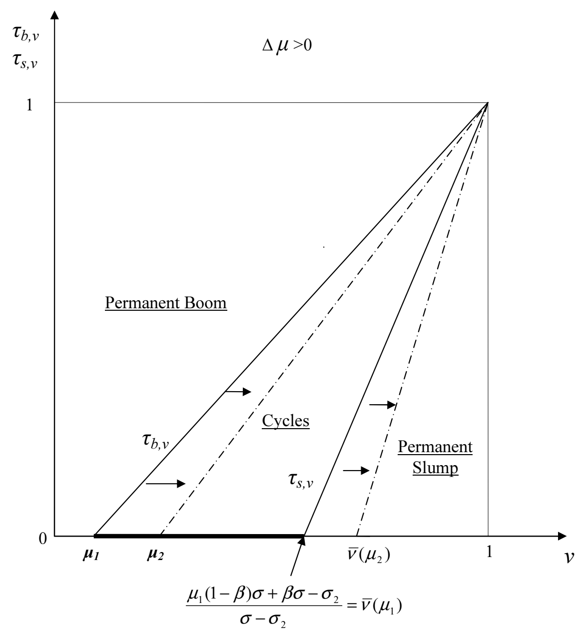


Figure 4.9: Comparative Statics Properties of τ_b and τ_s with respect to μ

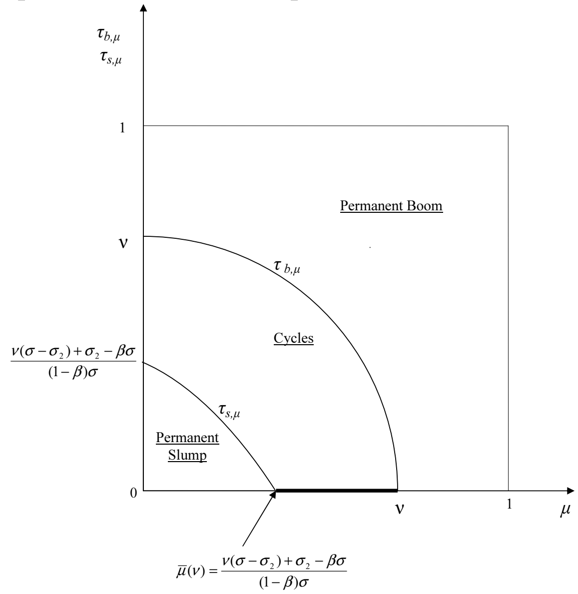


Figure 4.10: Comparative Statics Properties of τ_b and τ_s with respect to μ, when ν rises

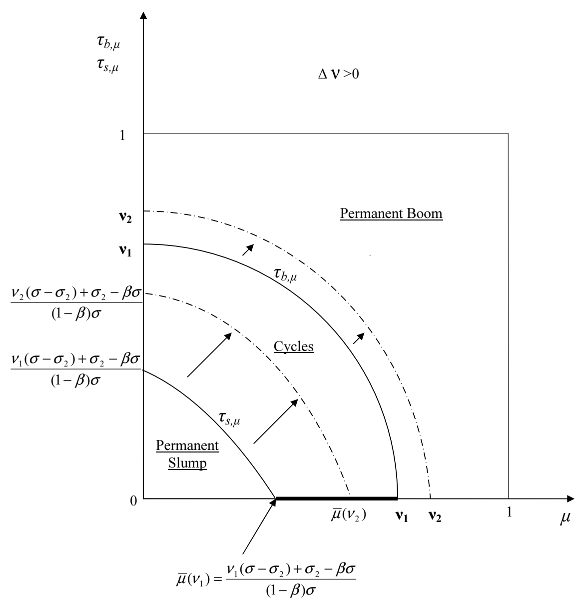


Figure 4.11: Effect of τ on the Response to a Permanent Productivity Shock in a Slump

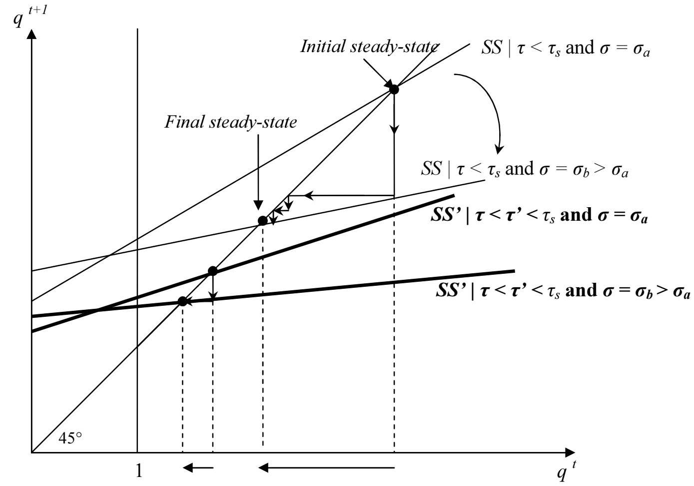


Figure 4.12: Effect of τ on the Response to a Temporary Productivity Shock in a Slump

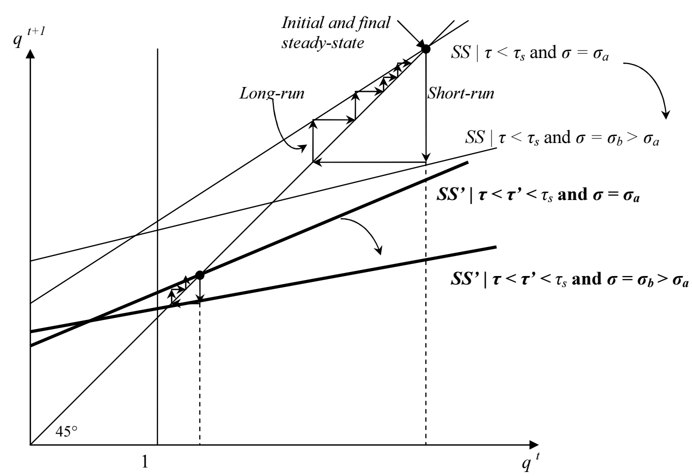


Figure 4.13: Effect of the $\tau = \tau_b$ Fiscal Policy on Borrowers' Consumption Growth

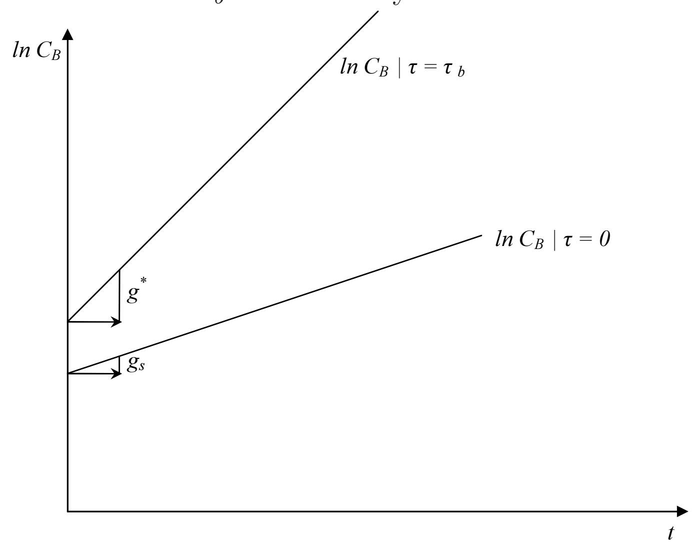
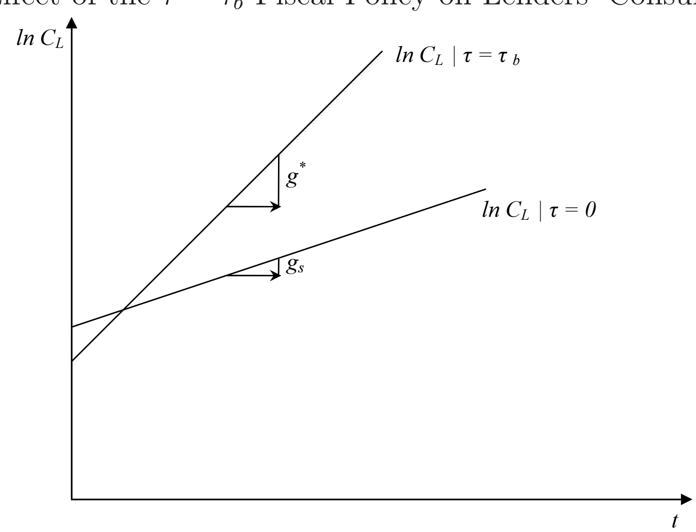


Figure 4.14: Effect of the $\tau = \tau_b$ Fiscal Policy on Lenders' Consumption Growth



Conclusion générale

Depuis de nombreuses années, l'analyse des *stabilisateurs automatiques* a engendré une littérature volumineuse. Ces mécanismes, intégrés au code des impôts et à la législation sociale, sont censés permettre des ajustements budgétaires rapides et symétriques contre les fluctuations, réduisant la pression sur la demande en périodes de forte croissance, et soutenant l'économie lors de ralentissements, sans requérir d'autre intervention de la part des autorités publiques. Le développement récent des modèles d'équilibre général dynamique a suscité un regain d'intérêt sur cette thématique, et de nouvelles questions concernant l'impact des décisions fiscales sur les agrégats.

Cette thèse propose d'étudier comment certaines décisions de politique fiscale affectent les fluctuations conjoncturelles, à l'aide de modèles macrodynamiques présentant des cycles d'affaires endogènes.

La littérature récente a montré comment la taxation progressive sur le revenu peut réduire la probabilité d'émergence d'états stationnaires dits "indéterminés", et même conduire à une convergence de type point-selle dans des modèles à rendements d'échelle croissants (cf. Christiano et Harrison [45], Guo et Lansing [88], Guo [84]). Cependant, les contributions dans ce domaine reposent souvent sur l'hypothèse d'un taux marginal d'imposition continûment croissant avec le revenu, ce qui n'est pas vraiment une caractéristique partagée par la plupart des systèmes fiscaux actuels. Le ***premier Chapitre*** tente de répondre à cette critique, en utilisant une formulation des taxes assez proche de la réalité. Il est supposé qu'un taux d'imposition constant est appliqué au revenu seulement lorsque ce dernier est supérieur à un niveau d'exemption fiscale donné. Cette formulation semble en effet peu éloignée des codes d'imposition par tranches prévalant dans la plupart des pays de l'OCDE. Nous montrons qu'il existe un seuil critique d'exemption fiscale au-delà duquel l'indétermination locale est éliminée et la stabilité de type point-selle assurée, le niveau de progressivité effective dans l'économie dépendant de manière positive du niveau d'exemption. Par conséquent, notre étude suggère que les codes d'imposition linéairement progressifs, en vigueur dans la plupart des économies modernes, peuvent aider à protéger l'économie contre les fluctuations entretenues par les croyances. Bien que l'analyse soit restreinte pour des raisons de simplicité à un système à deux tranches, elle peut très bien être étendue et généralisée au cas de tranches multiples. En ignorant l'utilisation variable du capital, nous nous concentrons sur des niveaux de rendements d'échelle agrégés pouvant être considérés comme trop élevés, au regard notamment des évaluations empiriques disponibles. Le chapitre suivant remédie à cet inconvénient, en tenant compte non seulement de l'utilisation variable du capital, mais aussi des dépenses de maintenance réalisées par les entreprises.

Le ***Chapitre 2*** étudie un autre type d'exemptions, dans un modèle de cycles d'affaires réels où l'indétermination locale émerge pour des valeurs (très) faibles des rendements d'échelle du fait de la prise en compte conjointe d'un degré variable d'utilisation du capital et de la présence de dépenses de maintenance dans l'économie. Bien qu'il existe un niveau de progressivité fiscale pouvant "déterminer" l'état stationnaire, on montre que l'exemption des dépenses de maintenance et du capital déprécié, déduites de la base fiscale des entreprises comme dans de nombreux codes fiscaux en vigueur (tels e.g. celui des États-Unis), affaiblit lesdites propriétés stabilisatrices de la progressivité. Nous proposons des conditions formelles claires pour l'analyse de stabilité, et montrons comment le seuil de progressivité stabilisatrice dépend positivement de l'activité de maintenance dans l'économie.

Après s'être intéressés aux modèles réels, indéterminés du fait de la présence de rendements d'échelle agrégés (faibles dans le chapitre 2), le ***Chapitre 3*** étudie l'impact d'une taxation progressive dans une économie monétaire présentant des rendements d'échelle constants. Les rendements croissants étant absents, on suppose que la volatilité entretenue par les croyances conduira de manière non-ambiguë à des pertes de bien-être (du fait de l'effet de concavité ou d'aversion devant le risque), justifiant le recours a une politique de stabilisation. Nous montrons que la progressivité fiscale réduit, dans l'espace des paramètres, la probabilité d'indétermination locale. Toutefois, si l'on considère des valeurs faibles et plausibles de progressivité, l'indétermination locale, les tâches solaires et cycles endogènes sont *robustes* à l'introduction de la progressivité fiscale. Il apparaît qu'accroître la progressivité rend l'offre de travail moins élastique, à travers son effet sur le revenu après impôt. Toutefois la progressivité fiscale ne neutralise pas les effets de l'inflation anticipée sur l'offre de travail courante, ce qui laisse une possibilité d'émergence de cycles d'affaires entretenus par les croyances.

Le ***Chapitre 4*** s'intéresse au pouvoir stabilisant des subventions aux investissements, financées par la taxation sur le revenu du travail, dans une économie sujette à des imperfections sur le marché du capital. Prenant pour référence la contribution d'Aghion, Banerjee et Piketty [3], on montre qu'une politique fiscale appropriée peut éliminer l'occurrence de crises, et protéger l'économie contre des fluctuations endogènes permanentes du produit national, du taux d'intérêt et de l'investissement. Pour des niveaux de développement du crédit donnés dans cette économie, nous proposons des valeurs particulières de paramètres fiscaux pouvant éliminer la probabilité d'occurrence de crises, renforcer la croissance à long-terme et placer l'économie sur un chemin d'expansion permanente. Le type de politique fiscale que nous analysons, à savoir introduire une taxe sur le revenu du travail des épargnants et redistribuer les montants prélevés vers les investisseurs productifs, semble équivalent à une augmentation de la fraction de la force de travail ayant un accès direct aux opportunités d'investissement. Nous analysons comment les paramètres de la politique fiscale stabilisatrice sont affectés lorsque le niveau des frictions dans l'économie varie. Proposant une caractérisation analytique complète des possibilités de régimes dynamiques, nous étudions comment le système fiscal modifie la réponse de l'économie aux différents chocs de productivité pouvant survenir. Notre résultat est un complément aux résultats d'Aghion et al. [3]. Dans leur papier, en dernière section, ils proposent un type d'intervention publique susceptible de pallier à l'insuffisance de capacité d'investissement des emprunteurs en période de crise. Ils suggèrent ainsi la possibilité pour le gouvernement d'absorber l'épargne oisive pendant les périodes de crise par une émission de dette publique, afin de financer des subventions ou réductions d'impôts pour les investisseurs. Cependant, le recours à la dette est un instrument contraint, dans de nombreux pays. Nous montrons que la taxation des revenu du travail, quasi généralisée dans les économies modernes et moins contrainte, peut aussi aider à la stabilisation des économies sujettes à des frictions sur le marché du capital. Bien que les fluctuations endogènes étudiées dans ce dernier chapitre ne soient pas de la même nature que celles étudiées dans les trois premiers, ce modèle a pour avantage de pouvoir explorer très simplement des dimensions plus complexes à analyser dans les modèles d'équilibre général dynamique (EGD) indéterminés . Il est aussi l'occasion de montrer que bien qu'elles n'aient pas ou peu d'effet stabilisant sur la dynamique des modèles EGD traités dans les trois premiers chapitres, les taxes linéaires ont bel et bien un effet stabilisant dans un modèle AK avec imperfections sur le marché du capital.

Bibliographie

[1] P. Aghion, P. Bacchetta, A. Banerjee, Financial development and the instability of open economies, J. Monet. Econ. **51** (2004) 1077-1106.

[2] P. Aghion, A. Banerjee, Volatility and Growth, Oxford University Press (2005).

[3] P. Aghion, A. Banerjee, T. Piketty, Dualism and macroeconomic volatility, Quart. J. Econ. **14** (1999) 1359-97.

[4] P. Aghion, P. Bolton, A trickle-down theory of growth and development, Rev. Econ. Stud. **64** (1997) 151-72.

[5] P. Aghion, E. Caroli, C. García - Peñalosa, Inequality and growth : the perspective of the new growth theories, J. Econ. Lit. **37** (1999), 1615-60.

[6] A. Alesina, D. Rodrik, Distributive politics and economic growth, Quart. J. Econ. **109** (1994), 465-90.

[7] C. Azariadis, Intertemporal macroeconomics, Blackwell, Oxford-Cambridge (1993).

[8] A. Banerjee, A. Newman, Occupational choice and the process of development, J. Pol. Econ. **101** (1993) 274-98.

[9] S. Basu, J.G. Fernald, Returns to scale in U.S. production : estimates and implications, J. Pol. Econ. **105** (1997) 249-283.

[10] R.A. Becker, On the long run steady state in a simple dynamic model of equilibrium with heterogeneous agents, Quart. J. Econ. **95** (1980) 375-82.

[11] R. Bénabou, Inequality and growth, in NBER Macroeconomics annual 1996 (Cambridge : MIT Press, 1996), 11-74.

[12] R. Bénabou, Unequal societies : income distribution and the social contract, Amer. Econ. Rev. **90** (2000), 96-129.

[13] R. Bénabou, Tax and education policy in a heterogeneous-agent economy : what levels of redistribution maximize growth and efficiency ?, Econometrica **70** (2002), 481-518.

[14] J. Benhabib, R.E.A. Farmer, Indeterminacy and increasing returns, J. Econ. Theory **63** (1994), 19-41.

[15] J. Benhabib, R.E.A. Farmer, Indeterminacy and sector-specific externalities, J. Monet. Econ. **37** (1996) 421-443.

[16] J. Benhabib, R.E.A. Farmer, Indeterminacy and sunspots in macroeconomics, in Handbook of Macroeconomics, 387-448. Eds. J.B. Taylor and M. Woodford. Amsterdam : North Holland (1999).

[17] J. Benhabib, G. Laroque, On competitive cycles in productive economies, J. Econ. Theory **35** (1988) 145-70.

[18] J. Benhabib, K. Nishimura, The Hopf bifurcation and the existence and stability of closed orbits in multisector models of optimal economic growth, J. Econ. Theory **21** (1979) 421-44.

[19] J. Benhabib, K. Nishimura, Competitive equilibrium cycles, J. Econ. Theory **35**(1985) 284-306.

[20] J. Benhabib, K. Nishimura, Indeterminacy and sunspots with constant returns, J. Econ. Theory **81** (1998) 58-96.

[21] J. Benhabib, Q. Meng, K. Nishimura, Indeterminacy under constant returns to scale in multisector economies, Econometrica **68** (2000) 1541-1548.

[22] B. Bernanke, Nonmonetary effects of the financial crisis in propagation of the great depression, Amer. Econ. Rev. **73** (1983) 257-76.

[23] B. Bernanke, M. Gertler, Agency costs, net worth and business fluctuations, in Business Cycle Theory, F. Kydland, ed. (Cheltenham, UK : Ashgate)(1995a) 197-214.

[24] B. Bernanke, M. Gertler, Inside the black box : the credit channel of monetary transmission, J. Econ. Perspect. **9** (1995b) 27-48.

[25] B. Bernanke, M. Gertler, S. Gilchrist, The Financial Accelerator in a Quantitative Business Cycle Framework, in Handbook of Macroeconomics, J. Taylor and M. Woodford, eds., Amsterdam : North Holland (1999).

[26] O.J. Blanchard, Debt, deficits and finite horizons, J. Pol. Econ. **93** (1985) 223-47.

[27] O.J. Blanchard, N. Kiyotaki, Monopolistic competition and the effects of aggregate demand, Amer. Econ. Rev. **77**(1987) 647-66.

[28] M. Boldrin, Paths of optimal accumulation in two-sector growth models, in W. Barnett, J. Geweke and K. Shell (eds.), Economic Complexity : Chaos, Sunspots, Bubbles and Nonlinearity, Columbia University Press, New York, (1989) 231-52.

[29] M. Boldrin, R. Deneckere, Sources of complex dynamics in two-sector growth models, J. Econ. Dynam. Control **14** (1990) 627-53.

[30] M. Boldrin, M. Horvath, Labor contracts and business cycles, J. Pol. Econ. **103** (1995), 972-1004.

[31] M. Boldrin, L. Montrucchio, On the indeterminacy of capital accumulation paths, J. Econ. Theory **40** (1986) 26-39.

[32] S. Bosi, F. Dufourt, F. Magris, Animal spirits in cash-in-advance economies, Rech. Econ. Louvain (forthcoming).

[33] S. Bosi, F. Magris, A. Venditti, Competitive equilibrium cycles with endogenous labor, J. Math. Econ. **41** (2005) 325-49.

[34] R. Breen, C. García - Peñalosa, Income inequality and macroeconomic volatility : an empirical investigation, Rev. Devel. Econ. **9** (2005) 380-98.

[35] W. Brock, Some results on the uniqueness of steady-states in multisector models of optimum growth when future utilities are discounted, Int. Econ. Rev. **14** (1973) 535-59.

[36] C. Burnside, Production function regression, returns to scale, and externalities, J. Monet. Econ. **37** (1996) 177-201.

[37] C. Burnside, M. Eichenbaum, S. Rebelo, Capital utilization and returns to scale, in NBER Macroeconomics Annual 1995, 67-100. Eds. B. Bernanke and J. Rotemberg. Cambridge : MIT Press.

[38] Canada. Statistics Canada. Various years. Capital and repair expenditures. Annual. Investment and Capital Stock Division, Statistics Canada. Available at http ://www.statcan.ca.

[39] S. Cassou, K.J. Lansing, Growth effects of shifting from a graduated-rate tax system to a flat tax, Econ. Inquiry **42** (2004), 194-213.

[40] E.M. Caucutt, S. Imrohoroglu, K.B. Kumar, Growth and welfare analysis of tax progressivity in a heterogenous-agent model, Rev. Econ. Dyn. **6** (2003), 546-77.

[41] G. Cazzavillan, T. Lloyd-Braga, PA Pintus, Multiple steady states and endogenous fluctuations with increasing returns to scale in production, J. Econ. Theory **80** (1998), 60-107.

[42] G. Cazzavillan, P.A. Pintus, Robustness of multiple equilibria in OLG economies, Rev. Econ. Dynam. **7** (2004) 456-75.

[43] Ch. Chamley, Optimal taxation of income in general-equilibrium with infinite lives, Econometrica **54** (1986) 607-622.

[44] L.J. Christiano, M. Eichenbaum, C. Evans, Nominal rigidities and the dynamic effects of a shock to monetary policy, J. Pol. Econ. **113** (2005) 1-46.

[45] L. Christiano, S. Harrison, Chaos, sunspots and automatic stabilizers, J. Monet. Econ. **44** (1999), 3-31.

[46] Commerce Clearing House (CCH), *Standard Federal Tax Reports* (7/29/99). Chicago.

[47] D. de la Croix, P. Michel, A Theory of economic growth : dynamics and policy in overlapping generations, Cambridge University Press (2002).

[48] R. Dos Santos Ferreira, F. Dufourt, Free entry and business cycles under the influence of animal spirits, J. Monet. Econ. **53** (2006), 311-28.

[49] R. Dos Santos Ferreira, F. Dufourt, Free entry equilibria with positive profits : a unified approach to quantity and price competition games, Int. J. Econ. Theory (forthcoming).

[50] N. Dromel, P. Pintus, Are progressive fiscal rules stabilizing ?, GREQAM working paper series n. 2006-04, 2006.

[51] N. Dromel, P. Pintus, Linearly progressive income taxes and stabilization, Res. Econ. **61** (2007), 25-29.

[52] N. Dromel, P. Pintus, Are progressive income taxes stabilizing ?, J. Pub. Econ. Theory (forthcoming).

[53] N. Dromel, Fiscal policy, maintenance allowances and expectation-driven business cycles, mimeo CREST, 2007.

[54] N. Dromel, Investment Subsidies and Stabilization in Credit Constrained Economies, mimeo CREST, 2007.

[55] F. Dufourt, Chocs de demande et fluctuations du taux de marge : une évaluation du modèle de collusion implicite, Rech. Econ. Louvain **67** (2001), 91-111.

[56] F. Dufourt, Demand and productivity components of business cycles : estimates and implications, J. Monet. Econ. **52** (2005), 1089-105.

[57] F. Dufourt, T. Lloyd-Braga, L. Modesto, Indeterminacy, bifurcations and unemployment fluctuations, Macroecon. Dynam. (forthcoming).

[58] O. Eckstein, A. Sinai, The mechanisms of the business cycle in the postwar era, in 'the American Business Cycle : Continuity and Change', edited by Robert J. Gordon. Chicago : Univ. Chicago Press (for NBER), (1986).

[59] T. Eicher, C. García - Peñalosa (Eds.), Institutions, development and economic growth, MIT Press (forthcoming).

[60] P. Englund, M. Persson, Housing prices and tenure choice with asymmetric taxes and progressivity, J. Pub. Econ. **19** (1982), 271-90.

[61] R.E.A Farmer, Deficits and cycles, J. Econ. Theory **40** (1986) 77-88.

[62] R.E.A. Farmer, J.-T. Guo, Real business cycles and the animal spirits hypothesis, J. Econ. Theory **63** (1994) 42-72.

[63] S. Fazzari, G. Hubbard, B. Petersen, Financial constraints and corporate investment, Brookings Pap. Econ. Act. **1** (1988) 141-95.

[64] S. Fazzari, B. Petersen, Working capital and fixed investment : new evidence on financial constraints, RAND J. Econ. **24** (1993) 328-42.

[65] M. Feldstein, The effects of taxation on risk taking, J. Pol. Econ. **77** (1969), 755-64.

[66] M. Friedman, A monetary and fiscal framework for economic stability, Amer. Econ. Rev. **38** (1948), 245-64.

[67] B.M. Friedman, Money, credit, and interest rates in the business cycles, 'the American Business Cycle : Continuity and Change', edited by Robert J. Gordon. Chicago : Univ. Chicago Press (for NBER), (1986).

[68] R.M. Goodwin, A growth cycle,in capitalism and growth, C. Feinstein, ed., Cambridge University Press.(1967)

[69] C. García - Peñalosa, Inequality and growth : a note on recent theories, Investigaciones economicas, (1994) 97-116.

[70] C. García - Peñalosa, S. Turnovsky, Growth and income inequality : a canonical model, Econ. Theory **28** (2006) 28-49.

[71] J.M. Grandmont, Money and Value, Econometric Society Monographs, Cambridge University Press, (1983). French version, Economica, 1986.

[72] J.M. Grandmont, On endogenous competitive business cycles, Econometrica **53** (1985) 995-1045.

[73] J.M. Grandmont, Stabilizing competitive business cycles, J. Econ. Theory **40** (1986) 57-76. Reprinted in Nonlinear Economic Dynamics, J. M. Grandmont (Ed.), Academic Press, (1987).

[74] J.M. Grandmont, Nonlinear Economic Dynamics (Ed.), Academic Press, (1987).

[75]

[76] J.M. Grandmont, Nonlinear difference equations, bifurcations and chaos : an introduction, CEPREMAP, June 1988, and Stanford Technical Reports, (1988).

[77] J.M. Grandmont, Temporary Equilibrium. Selected Readings (Ed.), Academic Press, (1988).

[78] J.M. Grandmont, Local bifurcations and stationary sunspots, in W.A. Barnett, J. Geweke and K. Shell (Eds.), Economic Complexity : Chaos, Sunspots, Bubbles and Nonlinearity, Cambridge University Press, (1989).

[79] J.M. Grandmont, Expectations Driven Business Cycles, Europ. Econ. Rev. **35** (1991) 293-99.

[80] J.M. Grandmont, G. Laroque, Stability of Cycles and Expectations, J. Econ. Theory **40** (1986) 138-51. Reprinted in Nonlinear Economic Dynamics, J. M. Grandmont (Ed.), Academic Press, (1987).

[81] J.M. Grandmont, G. Laroque, Stability, expectations and predetermined variables, in P. Champsaur et al. (Eds), Essays in Honour of E. Malinvaud, Vol. I, MIT Press, (1990).

[82] J.-M. Grandmont, P.A. Pintus, R.G. de Vilder, Capital-labor substitution and competitive nonlinear endogenous business cycles, J. Econ. Theory **80** (1998), 14-59.

[83] J. Greenwood, Z. Hercowitz, G.W. Huffman, Investment, capacity utilization, and the real business cycle, Amer. Econ. Rev. **78** (1988) 402-417.

[84] J.-T. Guo, Multiple equilibria and progressive taxation of labor income, Econ. Letters **65** (1999), 97-103.

[85] J.-T. Guo, S. Harrison, Tax policy and stability in a model with sector-specific externalities, Rev. Econ. Dyn. **4** (2001), 75-89.

[86] J.-T. Guo, S. Harrison, Indeterminacy with capital utilization and sector-specific externalities, Econ. Letters **72** (2001) 355-360.

[87] J.-T. Guo, S. Harrison, Balanced-budget rules and macroeconomic (in)stability, J. Econ. Theory **119** (2004), 357-63.

[88] J.-T. Guo, K.J. Lansing, Indeterminacy and stabilization policy, J. Econ. Theory **82** (1998), 481-90.

[89] J.-T. Guo, K.J. Lansing, Fiscal policy, increasing returns, and endogenous fluctuations, Macroecon. Dynam. **6** (2002) 633-64.

[90] J.-T. Guo, K.J. Lansing, Maintenance Expenditures and Indeterminacy under Increasing Returns to Scale, Int. J. Econ. Theory **3** (2007) 147-158.

[91] G. Haberler, Prosperity and depression, London : Allen and Unwin (1964).

[92] R. Hall, The relation between price and marginal cost in U.S. industry, J. Pol. Econ. **96** (1988) 921-48.

[93] R. Hall, A. Rabushka, "The flat tax", San Francisco CA : Hoover Institution Press, 1995.

[94] G. Hansen, Indivisible labor and the business cycle, J. Monet. Econ. **16** (1985) 309-328.

[95] S. Harrison, Indeterminacy in a model with sector-specific externalities, J. Econ. Dynam. Control **25** (2001) 747-764.

[96] S. Harrison, M. Weder, Tracing externalities as sources of indeterminacy, , J. Econ. Dynam. Control **26** (2002) 851-867.

[97] R.F. Harrod, An essay in dynamic theory, Econ. J. **49** (1939) 14-33.

[98] B. Holmstrom, J. Tirole, Private and public supply of liquidity, J. Pol. Econ. **106** (1998), 1-40.

[99] S. Honkapohja, E. Koskela, The economic crisis of the 1990s in finland, Econ. Pol. **29** (1999) 401-36.

[100] K.L. Judd, Redistributive taxation in a simple perfect foresight Model, J. Pub. Econ. **28** (1985), 59-83.

[101] K.L. Judd, Optimal taxation and spending in general competitive growth models, J. Pub. Econ. **71** (1999), 1-26.

[102] B. Jullien, Competitive business cycles in an overlapping generations economy with productive investment, J. Econ. Theory **46** (1988) 45-65.

[103] S. M. Kanbur, Of risk taking and the personal distribution of income, J. Pol. Econ. **87** (1979), 769-97.

[104] M.C. Kemp, N. Van Long, K. Shimomura, Cyclical and noncyclical redistributive taxation, Inter. Econ. Review **34** (1993), 415-29.

[105] J.M. Keynes, The general theory of employment, interest and money, New York : Harcourt and Brace (1936).

[106] R. King, C. Plosser, S. Rebelo, Production, growth and business cycles I : the basic neoclassical model, J. Monet. Econ. **21** (1988) 195-232.

[107] N. Kiyotaki, J. Moore, Credit cycles, J. Pol. Econ. **105** (1997) 211-48.

[108] J. Laitner, D. Stolyarov, Aggregate returns to scale and embodied technical change : theory and measurement using stock market data, J. Monet. Econ. **51** (2004) 191-233.

[109] P.J. Lambert, "The distribution and redistribution of income", Manchester : Manchester University Press, 2001.

[110] K.J. Lansing, Optimal redistributive capital taxation in a neoclassical growth model, J. Pub. Econ. **73** (1999), 423-53.

[111] O. Licandro, L.A. Puch, Capital utilization, maintenance costs, and the business cycle, Ann. Econ. Statist. **58** (2000) 143-164.

[112] T. Lloyd-Braga, L. Modesto, T. Seegmuller, Tax rate variability and public spending as sources of indeterminacy, CEPR Discussion Paper No. 5796 (2006).

[113] T. Lloyd-Braga, C. Nourry, A. Venditti, Indeterminacy in dynamic models : when Diamond Meets Ramsey, Int. J. Econ. Theory (forthcoming) (2006).

[114] T. Lloyd-Braga, C. Nourry, A. Venditti, Indeterminacy with small externalities : the role of non-separable preferences Int. J. Econ. Theory (forthcoming) (2006).

[115] E.R. McGrattan, J.A. Schmitz Jr., Maintenance and repair : too big to ignore, Fed. Reserve Bank Minneapolis Quart. Rev. **23** (1999) 2-13.

[116] C. Mulligan, X. Sala-i-Martin, Extensive margins and the demand for money at low interest rates, J. Pol. Econ. **108** (2000), 961-991.

[117] R.A. Musgrave, The theory of public finance, Mac-Graw Hill, New York (1959).

[118] R.A. Musgrave, M.H. Miller, Built-in flexibility, Amer. Econ. Rev. **38** (1948), 122-28.

[119] R.A. Musgrave, T. Thin, Income tax progression, 1928-49, J. Pol. Econ. **56** (1948), 498-514.

[120] K. Nishimura, K. Shimomura, P. Wang, Production externalities and local dynamics in discrete-time multi-sector growth models with general production technologies, J. Math. Econ. (forthcoming) (2006).

[121] K. Nishimura, A. Venditti, Indeterminacy in discrete-time infinite-horizon models, in Handbook od Economic Growth : Vol.1 : The discrete time horizon, R.A Dana, C. Le Van, K. Mitra and K. Nishimura (Eds.), Kluwer, 2006.

[122] K. Nishimura, A. Venditti, Indeterminacy in discrete-time infinite-horizon models with non linear utility and endogenous labor, J. Math. Econ. (forthcoming) (2006).

[123] C. Nourry, A. Venditti, Agents hétérogènes, croissance et détermination de l'équilibre, Ann. Econ. Statist. **59** (2000) 140-57.

[124] C. Nourry, A. Venditti, Determinacy of equilibria in an overlapping generations model with heterogeneous agents, J. Econ. Theory **96** (2001) 230-55.

[125] C. Nourry, A. Venditti, The OLG model with endogenous labor supply : a general formulation, J. Optimization Theory and Applications **128** (2006).

[126] J. Peck, K. Shell, Market uncertainty : correlated and sunspot equilibria in imperfectly competitive economies, Rev. Econ. Stud. **58** (1991) 1011-29.

[127] R. Perli, Indeterminacy, home production, and the business cycle : a calibrated analysis, J. Monet. Econ. **41** (1998) 105-125.

[128] M. Persson, The distribution of abilities and the progressive income tax, J. Pub. Econ. **22** (1983), 73-88.

[129] T. Piketty, The dynamics of the wealth distribution and interest rate with credit rationing, Rev. Econ. Stud. **64** (1997) 173-89.

[130] P. Reichlin, Equilibrium cycles in an overlapping generations economy with production, J. Econ. Theory **40** (1986) 89-103.

[131] P. Reichlin, Endogenous fluctuations in a two-sector overlapping generations economy, in J. Benhabib (ed.), Cycles and Chaos in Economic Equilibrium, Princeton University Press, Princeton (1992).

[132] P. Romer, Increasing returns and long-run growth, J. Pol. Econ. **94** (1986) 1002-37.

[133] E. Saez, Optimal progressive capital income taxes in the infinite horizon model, NBER working paper 9046, july 2002.

[134] P.-D. G. Sarte, Progressive taxation and income inequality in dynamic competitive equilibrium, J. Pub. Econ. **66** (1997), 145-71.

[135] S. Schmitt-Grohé, Comparing four models of aggregate fluctuations due to self-fulfilling expectations, J. Econ. Theory **72** (1997) 96-147.

[136] S. Schmitt-Grohe, M. Uribe, Balanced-budget rules, distortionary taxes, and aggregate instability, J. Pol. Econ. **105** (1997) 976-1000.

[137] T. Seegmuller, Market imperfections and endogenous fluctuations : a general approach, mimeo EUREQua, Paris, september 2004.

[138] N. Sim, Indeterminacy, stabilization policy and returns to scale : a re-investigation, Contr. to Macro. **5** (2005), 1-24.

[139] R.E. Slitor, The measurement of progressivity and built-in flexibility, Quart. J. Econ. **62** (1948), 309-13.

[140] E.F. Stephenson, Average marginal tax rates revisited, J. Monet. Econ. **41** (1998), 389-409.

[141] U.S. Department of Commerce. Various years. Expenditures for residential improvements and repairs. Current Construction Reports, C50. Quarterly. Bureau of the Census, U.S. Department of commerce. Available at http : www.census.gov.

[142] W. Vickrey, “Agenda for progressive taxation”, New-York : Ronald, 1947.

[143] W. Vickrey, Some limits to the income elasticity of income tax yields, Rev. Econ. Stat. **31** (1949), 140-44.

[144] M. Weder, Animal spirits, technology shocks and the business cycle, J. Econ. Dynam. Control **24** (2000) 279-295.

[145] M. Weder, Indeterminacy revisited : variable capital utilization and returns to scale, Finnish Econ. Pap. **18** (2005) 49-56.

[146] Y. Wen, Capacity utilization under increasing returns to scale, J. Econ. Theory **81** (1998), 7-36.

[147] M. Woodford, Stationary sunspot equilibria in a finance constrained economy, J. Econ. Theory **40** (1986), 128-137.