Instabilité des prix agricoles et politiques optimales de stabilisation
Christophe Gouel

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Agricultural price instability and optimal stabilisation policies

Instabilité des prix agricoles et politiques optimales de stabilisation

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$^1$ http://www.agfoodtrade.eu
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CHAPTER 1

INTRODUCTION

1.1 Motivation

This research, which began in 2008, was motivated by the observation that successive Common Agricultural Policy reforms were resulting in European farmers being exposed to more and more price instability, a problem that they had been protected from by previous trade and public storage policies. Price instability is thus a new problem for European farmers, and was also not high on the agendas of European agricultural economists, who felt no compulsion to study an issue that was not affecting the European market.

Based on my experience in applied agricultural and trade policy analysis, the situation is the same in these fields in so far as policy analyses extensively employ applied general equilibrium models in which static economic behaviour can be represented in great detail (Hertel, 2002), but which usually neglect dynamics and risk, the traditional motivations for agricultural policies. This neglect is not new and is not caused by the recent development of large-scale applied models. Twenty years ago, Tyers (1991) noted that the models used to analyse the Uruguay Round focused on aspects amenable to comparative static analysis, but ignored questions about dynamics and risk.

Analyses of the Doha Round show that nothing has changed. There are better data and more complex models, but the instability of agricultural markets is not considered in policy models despite its importance in negotiations. A demonstration of its importance is the failure to agree on the design of a Special Safeguard Mechanism, which led to stalemate in the July 2008 World Trade Organization negotiations.¹

In this thesis I eventually do not work on European agricultural markets or applied trade policy analysis. My work is motivated by an acknowledgement of the neglect of dynamics and risk in these areas, but the 2007–08 food crisis refocused my attention on food price instability in developing countries, which emerged as a more pressing issue. Also, given our understanding

¹ Some attempts have been made to include stochastic issues in the computable general equilibrium model GTAP (Hertel et al., 2005, Valenzuela et al., 2007, Hertel et al., 2010). However, it is difficult to represent stochastic environments with consistent expectations in a large-scale general equilibrium model.
of the nature of agricultural markets, agricultural economists rarely see public policies aimed at stabilising agricultural prices in developed countries as welfare enhancing (Gardner, 1996b, Myers et al., 2010, Sumner et al., 2010). In many situations a degree of instability is needed for the efficient allocation of resources. The action of private storers provides some stability, and the additional stability provided by costly public intervention would be unlikely to increase welfare. The situation is different in developing countries: food represents a much larger share of consumers’ budgets, making them very vulnerable to price risk; financial markets are less developed and do not allow farmers and middlemen to cover their risk efficiently; and governments are more prone to adopting erratic food and trade policies in the pursuit of short-run political gains.

Price instability in developing countries has received a lot of attention in the food policy literature. This literature is mostly qualitative with excellent accounts of world food crises (Timmer, 2010), and existing and past policies in southern Africa (Jayne and Jones, 1997, Poulton et al., 2006, Tschirley and Jayne, 2010), Indonesia (Timmer, 1997, Dorosh, 2008), and many other countries. Quantitative work (e.g., Jha and Srinivasan, 1999, Srinivasan and Jha, 2001, Brennan, 2003) is not explicit about the market imperfections that would justify intervention and so does not design optimal stabilisation policies. This thesis tries to provide a theoretical framework for analysing food price stabilisation policies in poor countries.

1.2 Approach

The approach in this thesis draws its inspiration from the modern macroeconomics literature. The new synthesis in macroeconomics is built around the neoclassical stochastic growth model (Woodford, 2009). It recognises the need to study agents’ interactions based on their microeconomic behaviour, it acknowledges the importance of consistent intertemporal decisions and the endogeneity of expectations. In this setting, public policies, especially monetary policies, have been shown to be effective in improving welfare since macroeconomic dynamics is affected by several market imperfections, such as prices or wages rigidity.

The canonical model for the study of commodity price behaviour and stabilisation policies, the rational expectations storage model, shares several of these features (Wright, 2001). It represents price dynamics as stemming from the behaviour of optimising agents endowed with rational expectations. Productivity shocks are the primary source of dynamics, and these shocks are smoothed by private storage, which follows an intertemporal arbitrage equation. This simple setting is sufficient to explain most characteristics of price dynamics (Caﬁero et al., 2011). However, it is not directly applicable to characterise stabilisation policies in developing countries where numerous market imperfections are likely to make price dynamics sub-optimal.

The lack of ability to cope with risks is probably the most important source of market imperfections when treating price volatility. For example, many poor countries do not have futures markets, which limits the abilities of farmers to cover their risks. Even without risk markets, profit fluctuations can be managed through saving and borrowing, but poor farmers may be unable to access the
formal financial market and lack the good collateral needed to back borrowing. This applies also to consumers, who may not be able to cover their price risk and may suffer from the volatility. Among the many imperfections that could justify public intervention, I focus on the inability of consumers to insure against food price risk. Formally, this imperfection is represented by a combination of consumer risk aversion and market incompleteness. Introducing this hypothesis in an otherwise optimal storage model may provide justification for some public intervention to increase welfare.

The variety of policies in developing countries shows that interventions to achieve food security take many forms such as conditional cash transfers, public distribution programmes, food-for-work schemes, public storage and trade policy. In this work, I focus on policies that try to stabilise market prices. The instruments considered are counter-cyclical supply subsidy, public storage, trade policy and private storage subsidy. These policies are second-best policies, since a first-best policy would be to provide households with state-contingent cash transfers similar to what they would get from futures markets. Price stabilisation policies, however, are used frequently in developing countries and their capacity to help to alleviate episodes of food scarcity should be evaluated.

Because these price policies are designed to affect the whole market, they are not amenable to assessment by randomised trials as is the case for policies targeting households (Miller et al., 2011). These price policies affect the whole economy and each policy experiment could be very costly. In this situation, stabilisation policies can be tested and analysed without entailing any human and fiscal costs by using simulated economies that represent most of the important facts related to commodity price behaviour. This task is reminiscent of what Lucas (1980, p. 709–710) sees as the work-programme of macroeconomists working on business cycles issues:

“Our task as I see it [...] is to write a FORTRAN program that will accept specific economic policy rules as ‘input’ and will generate as ‘output’ statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies.”

1.2.1 Previous approaches

The determination of welfare-enhancing storage rules historically has been the object of numerous research and approaches (see, for an early survey, Labys, 1980, and Wright, 2001, for a recent one). These various approaches are presented succinctly (see more detail in the succeeding chapters of the thesis).

Dynamic programming Modern studies of storage rules begins with Gustafson (1958a), who proposed a dynamic programming solution to the rational expectations storage model. He uses dynamic programming to maximise the intertemporal sum of social surplus, thus the solution to this problem is an optimal storage rule. Gustafson (1958a,b) recognises, however, that, although optimal, this storage rule is the same as would be achieved by private storers maximising their expected profit, a result stemming from the absence of market imperfection.
Gustafson’s approach, up to the early 1980s, was followed by various studies that derived optimal storage policies through dynamic programming (Gardner, 1979a,b, Kennedy, 1979, Knapp, 1982). Gardner (1979a) extends Gustafson’s method to account for elastic supply, and proposes an optimal storage rule in situations where high prices entail external costs. Knapp (1982) considers optimal storage when trade is possible. Yet the dynamic programming approach has some limitations: it is impossible to solve for the decentralised equilibrium of a model with market imperfections, since this situation is not amenable to a maximisation problem; and when introducing imperfections, as in Gardner (1979b), the optimal policy is the solution to a benevolent planner problem, not to a policy maker maximising social welfare while accounting for the behaviour of private agents.

Exogenous storage rules With the development of new numerical methods, the analysis shifted in the 1980s from the study of optimal storage rules to the study of a decentralised system in which speculative storers play a central role (Wright and Williams, 1982a, 1984). Finding optimal policies subject to this equilibrium behaviour is difficult (it is the object of chapters 4–6), and except in Wright and Williams (1982b) and Williams and Wright (1991), has not been tackled. Most works introduce exogenous storage rules and assess their incidence (Miranda and Helmberger, 1988, Wright and Williams, 1988a, Gardner and López, 1996, Srinivasan and Jha, 2001). Because these works do not introduce market imperfection, the optimal situation is always the situation without intervention. The analysis of exogenous storage rules in this setting allows for the identification of their effects on the market, but says little about what would be optimal were public intervention useful.

Marshallian analysis Before the new developments of the rational expectations storage model in the 1980s, most work on the welfare effects of stabilisation used the Waugh-Oi-Massell framework, a surplus analysis in a simple market model. This work was focused more on the benefits to be expected from complete price stabilisation than on the precise rules for achieving it. Drawing on the work of Waugh (1944) and Oi (1961), Massell (1969) shows that price stabilisation is likely to increase total welfare. This approach has been extended in various directions. For example, Turnovsky (1974a) shows that higher welfare gains can be expected if producers have to plan their production before realisation of uncertainty, since stabilisation will allow more efficient allocation of resources. Turnovsky (1976) considers a situation with multiplicative rather than additive disturbances and shows that this assumption can reverse the results. Edwards and Hallwood (1980) introduce the cost of buffer stock schemes and determine the optimal degree of price stabilisation. Nevertheless, none of these improvements is able to correct for the principal problem in this approach: the absence of private storage. Assessing the effect of public storage policies while neglecting private storage is equivalent to saying that government has at its disposal a technology (storage) that is not available to private agents. It is not surprising, therefore, that public stabilisation of markets is shown to improve welfare since the benchmark does not account for the already stabilising role of private actions.
ECONOMETRIC POLICY EVALUATION This literature builds on the work on macroeconometric modelling. It estimates structural econometric models and uses the results to simulate stabilisation policies. The simplest models are supply-demand models, which consist of four equations: demand, supply, price and market equilibrium (see Labys, 2006, Ch. 1, for a survey). Supply is determined as a function of lagged prices, and price is a function of inventories. This simple framework has been extended to represent many commodities and many countries in one model.

One of the first applications of this method is Desai (1966), who estimates a model of the world tin market and uses it to test alternative stockpiling policies. Ghosh et al. (1982, 1984, 1985, 1987) and Hughes Hallett (1984, 1986), analysing the copper market, propose determination of optimal stabilisation policies by minimising a quadratic objective function, measuring the squared deviation of endogenous variables from a target trend, subject to the equations estimated in the econometric model. These studies compare the optimal solution to simple rules of stabilisation. This literature led to optimistic conclusions with regard to the possibility of stabilising commodity markets. However, this approach is subject to the same criticisms as the macroeconometrics literature that inspired it (see Miranda and Helmberger, 1988, Salant, 1983, for criticisms of this approach), particularly the Lucas (1976) critique. Policy recommendations based on relationships estimated on historical data can be misleading because the model parameters are not structural and would change with the policy. In addition, without any microfoundations, the objective of the maximisation problem cannot be grounded on welfare theory, which makes the interpretation of optimal policies difficult.

1.2.2 A BENCHMARK, NOT A POLICY PRESCRIPTION

In proposing optimal stabilisation policies, I focus on the risk-bearing problem of consumers and abstract from many aspects. For example, consumers are assumed not to be able to accumulate savings that would help them spread the risk over several periods, even in the absence of other insurance mechanisms. Any problems regarding the risk management ability of producers are also ignored, so producers are assumed to be risk neutral, not liquidity constrained and to have rational expectations. I have chosen to simplify the framework of analysis in order to be able to interpret in detail all the results.

This choice was also motivated by the need to keep the dimensionality of the model as low as possible. The solutions have no closed-form representation and we always have to solve for them numerically. The numerical methods available for these kinds of models have the drawback of being highly sensitive to the number of state variables, since numerical complexity increases exponentially with the number of state variables. Here, the models are chosen so as never to exceed two state variables, which allows graphical representation of decision rules and reasonable time to solve the
problem. The results should not be considered, therefore, as policy prescriptions, but rather as providing insights about the behaviour of stabilisation rules.

The main objective of this thesis is to feed the discussion on food price stabilisation by providing insights using the modern economics toolbox and not to propose realistic policies. The methods used to derive optimal policies could be applied to more realistic situations, although the numerical resolution would start to be challenging and the number of state variables would be too high to allow for a graphical representation of decision rules making the results more difficult to interpret. So the extension to more realistic settings is left for future work.

1.2.3 Small welfare gains and relation to the equity premium puzzle

Since this work adopts the expected utility approach, the welfare gains obtained are numerically very weak: they never exceed 1% of the budget share devoted to the stabilised commodity. Despite these small gains, the optimal interventions imply large public involvement with price volatility decreasing by one-third under optimal policies. This problem is not specific to my work and applies to most studies of macroeconomic policies using expected utility. This recalls the famous Lucas’s (1987) remark that people do not want to sacrifice more than one-tenth of a percent of their consumption to live in a world without macroeconomic volatility.

How should we interpret these small welfare gains? Following Lucas (1987, 2003), we could conclude that because people care so little for volatility, it is not good to devote large resources to stabilisation policies, and efforts should be concentrated on policies that increase long-term growth rate. In our case, this would preclude expensive storage policies, and divert attention instead to policies enhancing agricultural productivity and development opportunities for poor populations. Another interpretation, proposed by Grant and Quiggin (2006, 2007), makes the link with the equity premium puzzle, which states that the large difference in returns between stocks and government bonds cannot be explained by traditional models of behaviour under risk. This puzzle can be interpreted as demonstrating that systematic risk is not assigned a sufficiently high price in the expected utility framework. If risk is much more costly than our models indicate, then, depending on the reasons for this discrepancy—still under discussion—large stabilisation policies may be justified.

The welfare effects from risk aversion are second-order and tend to be dominated even by small mean price changes. My focus on risk aversion is justified by two considerations: consistency with the literature on the cost of food price volatility, and simplicity since it allows me to define a very simple welfare objective. Other modelling might have made the food price risk numerically more important, but have not been sufficiently studied to be incorporated easily in an optimal policy model. For instance, there is the problem of the influence of price risk on the investment decision. In the presence of investments that are only partly reversible, risk could deter investment (Pindyck, 1988) and harm long-run growth (Barlevy, 2004). Another aspect that could make welfare gains of first-order importance is the peculiar role of food, which cannot be substituted by other consumption goods. This lack of substitution makes a decrease in food consumption particularly costly. This
effect, however, is difficult to account for within a traditional welfare framework. Although all these
imperfections could make the case for stronger price stabilisation, risk aversion is probably sufficient
to obtain some insights into the features of optimal food price stabilisation policies and to provide a
tentative benchmark.

1.3 ORGANISATION OF THE THESIS

This thesis is organised in five main chapters. The first two survey respectively the literature on
agricultural price instability, and the numerical methods for solving the competitive storage model.
The last three analyse various optimal food price stabilisation policy schemes. The appendix includes
a paper addressed to a non-academic audience that explains the causes of the 2007–08 food price
spike.

Chapter 2: Survey of agricultural price instability

This chapter, published in the *Journal of Economic Surveys*, is organised as a confrontation between two explanations of agricultural
price dynamics. One follows cobweb logic, models fluctuations driven by expectation errors, and
emphasises that these expectations create complex dynamics and possibly chaos. The other stems
from the rational expectations tradition of dynamics driven by real shocks. Following the rational ex-
pectations revolution, we might have expected explanations based on backward-looking expectations
to be driven out of the scientific scene. But new justifications for these expectations schemes emerged
in the 1990s and even without these new arguments this idea shows somewhat puzzling persistence,
at least in Europe, in public debate, which induces me to consider them seriously. While not
conclusive, the empirical evidence tends to support the rational expectations explanation, through
the competitive storage model. In its simplest form, the storage model generates an optimal dynamic
path where no improvement can be expected from public intervention. However, if we account for
all potential market failures in agricultural markets, and especially in developing countries, this
conclusion might require some qualification, although an appropriate policy design for stability has
still to be achieved.

Chapter 3: Numerical methods for solving the storage model

The rational expectations storage model has no closed-form solution and, since storage implies an inequality constraint, the
model solution is highly nonlinear and the numerical methods require careful consideration. This
chapter compares different numerical methods for solving the competitive storage model. The model
is solved using value function iteration, and several projection approaches, including parameterised
expectations and decision rules approximation. Using a penalty function approach to smooth
the inequality constraint, perturbation methods are also applied. Parameterised expectations
approximation proves the most accurate method, while perturbation techniques are shown to be
inadequate for solving this highly nonlinear model. The endogenous grid method allows rapid
solution if supply is assumed to be inelastic. The storage models developed in the next chapters are
complex, and I have used the results of this comparison of numerical methods to develop a solver able to solve them in an automatic way. It is based on the framework developed by Miranda and Fackler (2002) and Fackler (2005) and uses the mixed complementarity problem solver PATH (Dirkse and Ferris, 1995).

Chapter 4: Optimal food price stabilisation policies in a closed economy This chapter proposes a framework for designing optimal food price stabilisation policies in self-sufficient developing countries. This framework is applied in the succeeding chapters to different settings, but the theoretical implications of optimal policies are mostly developed in this chapter. The approach combines three literatures: the literature on the storage model provides a perfect tool to represent agricultural market dynamics and discuss the effects of policies (Williams and Wright, 1991); the economics of risk provides abundant discussion of the need for public intervention which may arise from market incompleteness combined with risk aversion (Newbery and Stiglitz, 1981); and the modern literature on optimal dynamic policies provides tools to derive optimal food price stabilisation policies (Kydland and Prescott, 1980, Marcet and Marimon, 1999). The model is a rational expectations storage model with risk-averse consumers and incomplete markets. Government stabilises food prices by carrying public stocks and by applying state-contingent subsidies/taxes to production. The policy rules are designed to maximise intertemporal social welfare. The optimal policy under commitment crowds out all private stockholding activity by removing the profit opportunity from speculation. It increases both consumer welfare and short-run producer profits. The countercyclical subsidy to production contributes little to welfare gains, most of which come from stabilisation achieved through public storage.

Chapter 5: Rules versus discretion in food storage policies Chapter 4 analysed a benchmark policy; the optimal policy requires commitment and is highly nonlinear, which makes it doubtful that it could ever be implemented. Governments, especially in developing countries, lack mechanisms of commitment and they are more likely to adopt discretionary policies. Another criticism of optimal state-contingent policies is their possible lack of robustness to alternative environments. These criticisms lead me to consider more practical policies in chapter 5: a discretionary policy and policies following optimal simple rules. I consider two state-contingent policies, under commitment and under discretion, and two optimal simple rules of storage, a constant private storage subsidy and a price-band with capacity constraint on public storage. The four policies are designed to be welfare maximising. Commitment allows government to manipulate producers’ expectations and induce them to stabilise prices to complement the effect of public storage. Simple rules of stabilisation, more easily implementable than state-contingent policies, when designed optimally can achieve more than four-fifths of the maximum gain. The price-band maximising social welfare is a price-peg scheme: the floor and ceiling prices are the same, and the capacity constraint represents 18% of the steady-state production level.
Chapter 6: Optimal food price stabilisation in a small open economy  This chapter, written in collaboration with Sébastien Jean, extends the previous work to an open-economy setting and analyses food price stabilisation policies in a small, open, developing country, self-sufficient for average shocks. Without public intervention, price dynamics are driven by domestic productive shocks and international prices. Trade and/or storage policies are optimally designed to increase welfare, in a context where consumers are risk averse and markets are incomplete. An optimal storage policy alone fails to protect consumers, since most additional storage is actually used to serve the world market. In contrast, an optimal combination of storage and trade policies results in a powerful stabilisation of domestic food prices. This policy mix includes export restrictions that are harmful to the country’s partners. However, refraining from using export restrictions is costly and entails substantial transfers from consumers to producers. Total welfare gains are small in comparison to the transfer effects created among agents. Thus, to pursue small welfare gains, government has to create significant distributive changes.

Appendix A: How to explain the food price spike?  This appendix consists of a paper written in French in summer 2008, which explains the food price spike. It was aimed at a non-academic audience and was published as a chapter in the CEPII yearbook, which describes the state of the world economy.

Boom and bust events tend to characterise agricultural markets. In the food crises of 1973–74 and 2007–08, the prices of several agricultural commodities more than doubled in the course of a few months, before an even more rapid decline to lower levels (see figure 2.1). Such episodes inevitably trigger concerns about the peculiarities of agriculture and the need of public intervention in such essential markets. Public involvement in stabilisation of food prices has been commonplace for a very long time. According to the Bible, Joseph stored grain during seven years of abundance to face seven years of famine (Genesis 41–47). In early modern Europe, grain market regulation was widespread and deregulation started only in the nineteenth century with the greater integration of national and international markets (Persson, 1999). Until recently, international agreements have been aimed at reducing the price volatility of several commodities (Gilbert, 1996).

With rising incomes, the share of staple foods in families’ budgets has become so low that sheltering consumers in developed countries from price instability has become of less concern than protecting producers. Many developed country agricultural policies are now aimed explicitly at stabilisation of producer prices and incomes.

This chapter offers a review of some of the current explanations of agricultural price dynamics, and discusses the way that governments should intervene in markets. Since this chapter makes the link between the dynamics and public intervention, it focuses on theoretical justifications for price dynamics: it does not examine empirical explanations, such as those provided by time-series models (surveyed in Labys, 2006). It concentrates on issues of annual fluctuations.

Historically, two explanations have been proposed for the price dynamics in the agricultural sector. First, that prices are driven by real shocks, an explanation that fits perfectly with the rational expectations framework. Second, that price dynamics stem from forecasting errors, which
is based on the coordination issue created by price instability. These two explanations lead to opposite policy conclusions. If real shocks affect supply, price adjustment is a natural correction process. Policy makers could want to mitigate their adverse effects on fragile populations, but they should not alter the overall dynamics. If price volatility is caused by a failure to forecast next period market conditions, future scarcity is not driving resource allocation and the state of the economy could be improved through public intervention. Of course, these two explanations are not mutually exclusive. The literature has, however, rarely mixed them. So, in order to clarify the properties of each theory, the opposition is maintained in the chapter.

This chapter focuses on models related to annual crops not perennial crops or breeding. Volatility is not less an issue for these sectors. The explanations of their instability also present the opposition between endogenously and exogenously driven dynamics. However, the dynamically complex production processes involved in perennial crops and breeding would introduce additional complexities to the models. In a sense, the production process of annual crops could also be made complex by considering models featuring technology relying on capital investments with costly adjustments. For the sake of simplicity, these issues of investments are not treated here.

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1 As real events show (Headey and Fan, 2008), a complete characterisation of agricultural prices should also include the effects of macroeconomic shocks or oil price fluctuations. This chapter does not discuss these factors, but models focused on production, price and storage adjustments, such as those examined below, can be extended to include these drivers.

2.1 THE LINEAR COWBWEB AND ITS CRITIQUES

Ezekiel (1938) proposes one of the first formalisations of agricultural price dynamics with his famous cobweb theorem.\(^3\) This model describes a salient feature of agricultural markets: that the productive decision is made before its realisation, leading to a short-run inelastic supply. When this is accompanied by a low elastic demand the implication is that any market disturbance will have sizeable effects on price. Since adjustment can come only from a very rigid demand, prices need to change considerably to induce significant change in demand. This model is the basis for all current approaches to this problem and is presented in full in table 2.1. Demand (2.1) and supply (2.2) curves are linear. Supply is subject to additive white noise disturbance \(\epsilon_t\). At each period, producers plan future output on the basis of the current price (2.4).

\[
\begin{align*}
q_D^t &= a - bp_t & \text{Demand} & \quad (2.1) \\
q_S^t &= c + d\hat{p}_t + \epsilon_t & \text{Supply} & \quad (2.2) \\
q_D^t &= q_S^t & \text{Market equilibrium} & \quad (2.3) \\
\hat{p}_t &= p_{t-1} & \text{Naive expectations} & \quad (2.4)
\end{align*}
\]

The deterministic part of the dynamics depends on the demand and supply slopes. Starting from an equilibrium that is different from the deterministic steady state, the sequence of equilibrium prices can display three different behaviours, depending on the relative demand and supply slopes. When \(d/b < 1\), prices converge with dampened oscillations, to the steady-state price \(p^* = (a - c) / (b + d)\). Oscillations are explosive if \(d/b > 1\), and steady if \(d/b = 1\).

This model suffers from serious internal contradictions, which are set out in Buchanan (1939). Diverging and oscillatory regimes with systematic forecasting errors involve greater losses than profits for producers. An economically consistent model would require explicit assumptions about the entry of more producers willing to waste money on an unprofitable business.

The most common critiques concern explosive dynamics, which implies diverging and negative prices. That the supply curve is more elastic than demand seems natural in agricultural markets, which would make the explosive regime the norm. Consequently, some authors tried to extend the stability range. Hooton (1950) and Akerman (1957) propose that supply is probably not so inelastic in the short term. In times of affluence, speculators stock grain surpluses, which they can sell later at higher prices. Stockpiling behaviour will tend to make demand (supply) more elastic for low (high) prices, thus stabilising the market. Akerman (1957) notes that farmers will probably not follow immediately a sudden change in prices, and will adjust their production schedules but with a delay. Nerlove (1958) formalises this idea by proposing adaptive expectations; producers revise their

\(^3\) Waugh (1964) discusses earlier contributions to this model.
expectations depending on their last period forecasting errors,

\[ \hat{p}_t = (1 - w) \hat{p}_{t-1} + wp_{t-1}. \]  

This scheme encompasses the naive case for \( w = 1 \). Because producers react less to price change, this scheme extends the stability range. Prices converge to equilibrium when \( d/b < 2/w - 1 \).

Even after extending the stability of the cobweb model, it remains that the main dynamic behaviour derives from systematic forecasting errors occurring in a rather simple model. Muth (1961) uses the cobweb model as an illustration of the interests of rational expectations. These systematic errors imply that producers are wasting scarce information. He advocates that expectations should be consistent with economic theory, which means that agents should base their decisions on the information available at the time:

\[ \hat{p}_t = E_{t-1}(p_t). \]  

Under rational expectations, the dynamics of the previous system collapses to random fluctuations around the steady state \( p^e \),

\[ p_t = p^e - \frac{\epsilon_t}{b}. \]  

The rational expectations critique has not completely eliminated the cobweb model from the economist’s toolbox. It took some time for rational expectations to become a standard economic assumption, and use of the linear cobweb model, at least in its Nerlovian setting, has remained popular in applied works especially for identifying short and long-run price elasticities (see Sadoulet and de Janvry, 1995, Ch. 4).

### 2.2 Nonlinear dynamics models

Twenty years ago, the article “cobweb theorem” from the *New Palgrave Dictionary of Economics* noted a decreasing interest in cobweb models (Pashigian, 1987). However, the situation has reverted. The simplicity of the cobweb model makes it a useful framework for studying market stability and the role of expectations; it is widely used for modelling demand for education, in the learning literature and the nonlinear dynamics literature. The cobweb model widely spreads out from agricultural economics to become an alternative matrix to rational expectations, for time lag decisions. One of the developments relevant to agricultural markets is the extension of the linear cobweb model to nonlinear dynamics.

The dynamics in a simple linear cobweb are limited to explosive, convergence to steady state and two-period cycles. However, the introduction of nonlinear features, such as nonlinear curves, heterogeneous agent or risk aversion, generates complex dynamics and can lead to chaos. These additions respond in part to the criticisms of the traditional cobweb models.

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4 There are an infinity of two-period cycles. When demand slope equal supply slope, every couple of prices that are symmetric with respect to the steady-state price constitutes a cycle.
Nonlinear dynamics is about systems characterised by nonlinear time evolution equations. It has important consequences for dynamic behaviour. In linear models, small changes in parameters lead to small changes in behaviour. For instance, we can consider how the dynamics changes in the linear cobweb when the parameter $b$, which governs demand elasticity, is changed. Starting from the diverging oscillations regime, when demand elasticity increases, the oscillations diverge more and more slowly until demand elasticity is sufficient to achieve a two-period regime. An additional increase in demand elasticity changes the behaviour to converging oscillations, beginning with a very slow convergence, close to regular oscillation. In nonlinear systems, however, a change in parameters can induce dramatic behavioural changes. Qualitative change to model behaviour caused by a parameter variation is referred to as bifurcation. Nonlinear models can produce chaotic dynamics. Extreme sensitivity to initial conditions is characteristic of chaos, which implies that we cannot forecast what will happen in the future based on the current conditions, because of our inability to observe without error these conditions. We know only that the time path is bounded (Zhang, 2006).

2.2.1 Nonlinear cobweb models

In a cobweb model, with naive expectations, if the supply and demand curves are monotonic, the behaviour is qualitatively the same as in the linear case. Only three types of behaviour will occur: convergence to a fixed point; two-period cycles; and divergence. Artstein (1983), and Jensen and Urban (1984) represent a departure from this case with the introduction of non-monotonic curves. This simple change introduces the possibility of chaotic dynamics. The assumption of non-monotonicity in demand or supply curves, however, is quite strong and is not adopted in other studies. For example, Chiarella (1988) and Hommes (1991, 1994) show that simple S-shaped monotonic supply curves and adaptive expectations are sufficient to generate nonlinear deterministic dynamics. Table 2.2 provides a representation of their model, which is illustrated in figure 2.2. This is similar to Nerlove’s model apart from its S-shaped supply curve (2.2′). It also abstracts from stochastic shocks. In trying to emphasise the endogenous nature of fluctuations, these studies focus only on deterministic dynamics: no disturbance to supply is included.

**Table 2.2. Cobweb model with monotonic S-shaped supply curve**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_D^t = a - bp_t$</td>
<td>Demand (2.1)</td>
</tr>
<tr>
<td>$q_S^t = \arctan(\lambda p_t)$</td>
<td>Supply (2.2′)</td>
</tr>
<tr>
<td>$q_D^t = q_S^t$</td>
<td>Market equilibrium (2.3)</td>
</tr>
<tr>
<td>$\hat{p}<em>t = (1 - w) \hat{p}</em>{t-1} + wp_{t-1}$</td>
<td>Adaptive expectations (2.4′)</td>
</tr>
</tbody>
</table>

What is interesting about this model, and contrasts with the linear case, is the unstable case. The range of stability is still defined for $q_S^t(p^e)/q_D^t(p^e) < 2/w - 1$ (value close to 0.29 in figure 2.2(a)). Beyond this, prices are bounded and follow either stable or chaotic oscillations. As Nerlove (1958) shows, adaptive expectations tend to increase the range of stability, but in this setting the effects are
more complex. As expected, the amplitude of the fluctuations decreases when the weighting factor for expectations, \( w \), decreases from 1 (naive expectations) to 0, but the nature of the dynamics changes, shifting from two-period oscillations to a convergence to the steady state, after passing through chaos. Thus, the range of fluctuations decreases, but paradoxically prices become unpredictable. Figure 2.2(b) illustrates this extreme sensitivity to initial conditions created by chaos. Starting from two very close price expectations, trajectories are distinct after a few periods.

Other nonlinear features are explored in the literature. Boussard (1996) introduces risk aversion in the linear cobweb model using a mean-variance framework. The variance introduces a quadratic term on prices potentially leading to chaos. Onozaki et al. (2000) generate chaotic dynamics by introducing adjustment costs in a cobweb model.

Nonlinear cobweb models respond to several of the criticisms of the simple linear cobweb model. They introduce a more realistic dynamics than the three regimes in the linear case. However, Buchanan’s (1939) critique that such models often risk of representing sectors that are likely to go bankrupt is not addressed seriously. This is confirmed by Commendatore and Currie (2008), who demonstrate that neglecting borrowing constraints in a nonlinear cobweb model results in financial crisis. Introducing borrowing constraints limits too risky commitments. As most nonlinear cobweb

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Figure 2.2. Behaviour of the nonlinear cobweb with S-shaped supply curve. Parameters: \( a = 1 \), \( b = 0.25 \), \( \lambda = 4 \). In the bifurcation diagram (a), for \( w \) close to 0 there is a stable equilibrium; increasing \( w \) to 0.29 produces a bifurcation to stable two-period oscillations; after several more bifurcations there are chaotic oscillations, then further increasing \( w \) close to 1 produces a return to stable oscillations. For the two chaotic trajectories \( (w = 0.7) \), initial points are very close, with the initial expected prices equal to 0.3 (dotted line) and 0.31 (solid line), but, after a few iterations, the trajectories are completely different.
models lack this financial aspect, they are subject to the same limitation and are not internally consistent.

2.2.2 Heterogenous agent models

With bounded rationality à la Baumol and Quandt (1964), rule-of-thumb expectations can be rationalised as a trade-off between costly rational expectations and cheap backward-looking expectations. This logic is formalised in a seminal paper by Brock and Hommes (1997). If building good expectations is costly, and agents make rational choices between different expectation schemes based on their past performance, a complex dynamics emerges. In times of limited instability, rational expectations are too costly to be used widely and agents switch to naive expectations, which tends to destabilise the market. Stability returns when a sufficient share of agents switches back to rational expectations. This scheme, developed by Brock and Hommes (1997), justifies backward-looking expectations as a rational trade-off between good but costly, and bad but cheap expectations.

This model (table 2.3) is built from simple elements of the linear cobweb with linear demand (2.1) and supply (2.6) functions. Supply is the sum of the contributions of agents with naive and rational expectations (2.7). The share of each agent in the population, \( n_{ht} \), evolves through a discrete choice model (2.11) based on last period profits (2.10), with \( h \) indexing the type of agent.\(^5\) \( \beta \) is intensity of choice, with naive expectations supposed to be freely available \((C_2 = 0)\), and rational expectations costly \((C_1 = C)\). The interaction between the two types of agents leads to doldrums followed by booms and busts (see figure 2.3(b) top panel). When prices are stable, rational expectations are not worth their cost, thus, most agents switch to naive expectations, which are destabilising. In periods of market exuberance, naive expectations give rise to costly forecasting errors, and the share of rational expectations agents rises rapidly until the market stabilises (figure 2.3(b) bottom panel). This dynamic behaviour is extremely sensitive to the intensity of choice (figure 2.3(a)). For small values, agents tend to stay with their expectations schemes and prices converge to a steady state. Complex behaviours emerge when the intensity of choice increases.

This literature responds to Buchanan’s (1939) critique of survival producers with irrational expectations, who should be driven out of the market by producers with rational expectations. Backward-looking expectations become a viable strategy when expectations are costly and dynamics complex.

There are various extensions of this model.\(^6\) Brock and Hommes (1998) apply the framework to an asset pricing model. Goeree and Hommes (2000) and Lasselle et al. (2005) generalise Brock and Hommes’s (1997) model by introducing nonlinear demand and supply curves, and adaptive expectations.

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\(^5\) This model is a simplified version of Brock and Hommes model. It considers that the fitness measure collapses to last period profits without taking into account more distant profits. It does not change the main conclusions of the model.

\(^6\) See Hommes (2006) for a survey of the heterogeneous agent models literature.
### Table 2.3. Cobweb model with heterogeneous expectations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_t^D = a - bp_t$</td>
<td>Demand (2.1)</td>
</tr>
<tr>
<td>$q_{ht}^S = d\hat{p}_ht$</td>
<td>Supply by agent type (2.6)</td>
</tr>
<tr>
<td>$q_t^S = \sum_{h=1}^{2} n_{ht}q_{ht}^S$</td>
<td>Total supply (2.7)</td>
</tr>
<tr>
<td>$q_t^D = q_t^S$</td>
<td>Market equilibrium (2.3)</td>
</tr>
<tr>
<td>$\hat{p}_t = p_t$</td>
<td>Rational expectations agents (2.8)</td>
</tr>
<tr>
<td>$\hat{p}<em>t = p</em>{t-1}$</td>
<td>Naive expectations agents (2.9)</td>
</tr>
<tr>
<td>$\Pi_{ht} = q_{ht}^S p_{ht} - (q_{ht}^S)^2 / 2d - C_h$</td>
<td>Profit (2.10)</td>
</tr>
<tr>
<td>$n_{ht} = \frac{\exp(\beta\Pi_{ht,t-1})}{\sum_{j=1}^{2} \exp(\beta\Pi_{jt,t-1})}$</td>
<td>Discrete choice (2.11)</td>
</tr>
</tbody>
</table>

In Muth (1961), backward-looking expectations are criticised because they generate systematic errors with a strong cyclic pattern. Under chaotic dynamics, we would expect rule-of-thumb expectations to have better properties than with linear dynamics: we would expect that it would be more difficult to identify a pattern in forecasting errors. Hommes (1998) investigates this using the nonlinear cobweb models in Hommes (1991) and Brock and Hommes (1997). Hommes (1998) shows that forecasting errors all have a strong negative autocorrelation at the first lag, so, despite chaotic dynamics, nonlinear cobweb models with backward-looking expectations show little consistency between expectations and realisations.

### 2.3 THE COMPETITIVE STORAGE MODEL

In the simple linear model with a production lag, Muth’s (1961) introduction of rational expectations restricts the dynamics to exogenous shocks around the steady state. Producers always plan to produce the same amounts and actual production is just a perturbation around this steady-state level. In the same article, Muth models the effect of inventory speculation on price dynamics. He shows that the introduction of storage creates positive first-order serial correlation in prices. Speculation smooths shocks over several periods, so the effect of one shock is spread across subsequent periods, causing positive serial correlation. On the other hand, the simple rational expectations and cobweb models generate respectively zero and negative correlations. Both are inconsistent with the observations (Deaton and Laroque, 1992), which show evidence of high positive autocorrelation. Storage contributes to explaining one of the main features of commodity price series and reintroduces in the production lag model dynamic features other than exogenous shocks.

A technically correct treatment of storage is difficult, because it involves a regime-switching behaviour, since storage can be either positive or null. Muth gets around this problem by allowing negative storage. The same issue of inventory speculation under rational expectations was
Figure 2.3. Behaviour of the nonlinear cobweb with heterogeneous expectations. Parameters: \(a = 0, b = 0.5, d = 1.35, C = 1\). In the bifurcation diagram (a), for small intensity of choice, \(\beta\), the steady state is stable. Beyond a critical value, there is a two-period cycle. After infinite period doubling bifurcations, chaos occurs. The top panel of the trajectory (b) shows a chaotic path of price and the bottom panel the corresponding share of rational agents for \(\beta = 5\).

solved earlier by Gustafson (1958a) with the non-negativity constraint but without supply reaction. Gustafson’s work was really path-breaking. Not only he built a rational expectations storage model three years prior to Muth’s publication, but he proposed as well pioneering numerical methods to solve dynamic models with binding constraints, methods that had to wait thirty years to be applied to the similar problem of optimal consumption with stochastic income (Zeldes, 1989). Gustafson anticipates much of what we now know about the rational expectations storage model. In particular, he recognises that in the absence of distortions an optimal governmental storage programme coincides with the behaviour of private storers in a free market.

The competitive storage model differs only slightly from the simple linear model under rational expectations, as can be seen from table 2.4. Storers carry-over positive stocks (2.12) when they expect the next period price to cover purchasing costs, marginal stocking costs (function \(\phi\) of amount stocked \(X_t\)), opportunity costs (at the interest rate \(r\)) and depreciation (at the constant

\[\text{Modern literature on price stabilisation or storage points out that a multiplicative disturbance would be more realistic than the additive disturbance assumed in (2.2). Productivity shocks (e.g., weather) are more likely to affect output in proportion to planned production levels than independently. This is important, because, under multiplicative disturbance, a rational producer does not react to the expected price but takes account of the inverse correlation between shocks and price. It also greatly affects the assessment of price stabilisation policies (Turnovské, 1976, Wright, 1979). For the sake of simplicity, the usual assumption of additive disturbance is retained.}\]
rate $\delta$). When the expected marginal profit is negative, there are no stocks.\(^8\) The market clearing condition (2.3\textsuperscript{′}) takes account of storage. Total supply consists of current production and stocks carried over from the previous period after depreciation, while demand includes consumption and carry over stocks. Introducing a non-negativity constraint makes the model analytically intractable. The rational expectations equation (2.4\textsuperscript{′′}) is not a traditional algebraic expression. It captures the internal consistency of the model: that expectations must be consistent with all the information known about the model. Simple linear models with rational expectations can be solved using the undetermined coefficients method: the rational expectations condition is guessed at and its unknown coefficients are defined by solving the remaining equations. This strategy does not work for this model because of the complementarity condition (2.12). When storage is allowed to become negative, as in Muth (1961), an analytical solution exists. Finding a non-negative storage rule under rational expectations requires the resolution of a functional equation problem, which in turn requires numerical computations.\(^9\)

Table 2.4. The competitive storage model

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
<th>(Number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_t^D = a - b p_t$</td>
<td>Consumption</td>
<td>(2.1)</td>
</tr>
<tr>
<td>$q_t^S = c + d \hat{p}_t + \epsilon_t$</td>
<td>Production</td>
<td>(2.2)</td>
</tr>
<tr>
<td>$q_t^D = q_t^S + (1 - \delta) X_{t-1} - X_t$</td>
<td>Market equilibrium</td>
<td>(2.3\textsuperscript{′})</td>
</tr>
<tr>
<td>$\hat{p}<em>t = E</em>{t-1} (p_t)$</td>
<td>Rational expectations</td>
<td>(2.4\textsuperscript{′′})</td>
</tr>
<tr>
<td>$p_t \geq \hat{p}_{t+1} (1 - \delta) / (1 + r) - \phi' (X_t)$ $\perp X_t \geq 0$</td>
<td>Storage arbitrage</td>
<td>(2.12)</td>
</tr>
</tbody>
</table>

The implications of storage for market behaviour were studied by Gardner (1979a) and Wright and Williams (1982a), and summarised in Williams and Wright (1991). The market for a storable commodity presents specific characteristics. It includes two different regimes. When stocks are exhausted (because the price is too high for storers to expect profits from carrying over grains), behaviour is similar to the simple rational expectations model: producers plan the same production whatever the availability and tomorrow’s supply is independent of today’s. With positive stocks, shocks are smoothed over several periods and prices are positively correlated. Stocks decrease expectations of high prices in the next period, so producers decrease their planned production according to the levels of stocks. These two regimes are depicted in figure 2.4(b). Below an availability close to the steady-state level, market demand comes only from consumption because

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\(^8\) Complementarity conditions in equation (2.12) and in what follows are written using the “perp” notation ($\perp$). This means that the expressions on either side of the sign are orthogonal. If one holds with strict inequality, the other must have an equality.

\(^9\) A complete presentation of the corresponding numerical methods is beyond the scope of this chapter. Briefly, the problem boils down to choosing a parametrisation for the rational expectations condition, usually a polynomial or a finite element interpolation. Parameters are found by iterative solving of the model on a grid of possible availabilities. At each iteration, parameters are updated to minimise the discrepancy with the rational expectations condition. Iterations can be generated by a simple successive approximation algorithm or a Newton-based solver. Detailed presentation of the algorithms used for storage problems can be found in Williams and Wright (1991), Miranda and Glauber (1995), Miranda (1997), Miranda and Fackler (2002), and chapter 3 of this thesis.
the high price deters storage. For higher total supply, demand for storage is added to consumption and makes market demand more reactive to price.

Figure 2.4. Characterisation of the competitive storage model behaviour. Parameters: \( a = 1.2, b = 0.2, c = 0.8, d = 0.2, \delta = 1\%, r = 3\%, \phi'(X) = 0.02, \epsilon \sim \mathcal{N}(0, 0.05^2) \). These parameters imply mean production and price at 1 when there is no shock, and supply and demand elasticities of 0.2 and −0.2. In the demand curves (b), the dashed line is consumption demand, the solid line includes consumption demand and demand for carryover.

The possibility of storage affects the price distribution (figure 2.4(a)). Prices are less volatile and their distribution is positively skewed. Storers take advantage of low prices to stockpile and, by doing so, reduce the probability of low prices. Compared to the situation without storage, the incidence of high prices diminishes, but not as much as for low prices, because stockouts happen when prices are high. The variance of the next period price increases with the current period price (Deaton and Laroque, 1992, p. 8), because the higher the current price, the lower the stocks and the less their damping effects. Beyond a threshold price there is no inventory to make a link between the current and the next period, so the price variance is constant. Storage creates price paths along which stable periods are interrupted by price spikes, corresponding to periods of stockout.

The competitive storage model under rational expectations has become the workhorse of neo-classical studies on price volatility. It has been extended in various directions. Its simplest version describes only inter-harvest fluctuations. But agricultural production is highly seasonal and must be stored at time of harvest to permit year round consumption. This creates predictable patterns in intra-annual prices. Prices should rise between harvests to cover storage costs (Samuelson, 1957). Between harvests prices do not just evolve deterministically, intra-seasonal periods are also periods
of incoming information about future harvest conditions. Spot price and storage react to incoming news, because the storage arbitrage condition links the spot price to the expected price, even when current market conditions are unchanged. The storage model was extended by Lowry et al. (1987), Williams and Wright (1991, Ch. 8), Chambers and Bailey (1996), Ng and Ruge-Murcia (2000) and Osborne (2004) to account for intra-seasonal shocks on demand or future supply.

Despite widespread protectionist policies, agricultural markets are very integrated and the international market often has a strong influence on domestic price volatility. The interaction between international trade and storage is studied in Williams and Wright (1991, Ch. 9) and Miranda and Glauber (1995). Makki et al. (1996) use a storage-trade model for studying export subsidies. Economically, trade and storage obey the same laws (Samuelson, 1957). In markets separated by space or time, the prices of a good may differ, but their difference must stay below the costs of shipping them to a more advantageous location or period. The two situations differ in that commodities can be stored only for future transactions, and arbitraging between periods is risky, while spatial arbitraging is less so. Trade generates the same kind of complementarity conditions as in equation (2.12), commodities are traded only when the trade cost is covered by the difference in international prices.\(^{10}\)

Because goods flow from low price locations to higher price locations, trade, like storage, contributes to stabilising the market. Williams and Wright (1991) note that costless storage is more effective than costless trade for stabilising prices. This stabilising effect depends strongly on market features, storage and trade costs. If storage and trade are costly, an importing country does not carry-over stocks from one period to the next. Stocking imported goods incurs interest charges and depreciation on imported commodities. It is cheaper to wait for the next period harvest and, if necessary, decide then to import. However, this conclusion holds only if trade is instantaneous. If trade takes time as in Coleman (2009a), it can be rational to store imported commodities.

Two recent theoretical advances in the storage model are worthy of a mention. First, Bobenrieth et al. (2002) propose a storage model that differs slightly from the standard model in the tradition of Gustafson (1958a). Its specific assumptions are that zero harvest has positive probability, and marginal utility at zero consumption is infinite. The implication is that stockout never occurs—as we observe in reality. There is no need to explicitly consider the non-negativity constraint on storage, since storers always find profitable to keep stocks to be able to profit from a zero harvest. Second, Nishimura and Stachurski (2009) describe the dynamics of a multisector model of commodity markets with storage. They prove the stationarity of the state process and the equivalence between the competitive equilibrium and the planner’s problem. Such work could be valuable in the future given the important comovements between commodity prices.

\(^{10}\) The trade aspect of storage-trade models is modelled in the spirit of spatial models in the manner of Takayama and Judge (1971).
2.4 ENDogenous OR exogenous FLUCTUATIONS?

There are two modern, and opposing theories on agricultural price fluctuations. One rests on cobweb logic and views fluctuations as chaotic and as originating in forecasting errors. The other follows the rational expectations hypothesis stating that volatility results from real shocks, with price dynamics determined by the optimal reactions of agents (competitive storers, farmers) to these shocks. This literature raises two empirical questions. First, do the data require the introduction of nonlinearities in the models? Second, are fluctuations driven endogenously by forecasting errors or exogenously by real shocks?

For convenience, most of the empirical literature on expectations (Irwin and Thraen, 1994) uses linear models. The introduction of nonlinearities through storage, nonlinear supply curves or heterogeneous expectations brings important complexities to the estimation process (discussed below), and so must be justified. Several studies find that price dynamics exhibit significant nonlinearities, which leads to the linear framework being rejected. Both types of nonlinearity are found in the literature: nonlinearity in mean (Ng, 1996, Westerhoff and Reitz, 2005) and nonlinearity in variance (Yang and Brossen, 1992, Shively, 1996, Beck, 2001, Chatrath et al., 2002, Adrangi and Chatrath, 2003). These findings confirm the need to introduce nonlinear features in the models.

What differentiates these two theories is the type of expectations: simple backward-looking, or rational. Two main strategies have been developed to identify how agents form their expectations (Irwin and Thraen, 1994, Nerlove and Bessler, 2001). In the first, when direct measures of expectations are unavailable, a structural model of supply and/or market equilibrium is required. This framework allows to confront several different expectations schemes. Orazem and Miranowski (1986) build such a model for three crops on the US market. Their estimations do not allow to conclude which expectations scheme is used by agents. In estimating models with private storage, Miranda and Glauber (1993a) and Frechette (1999) find evidence of a better fit from assuming agents to be endowed with rational expectations. But, in a survey of several of these studies, Irwin and Thraen (1994) are of the opinion that it is difficult to draw any robust conclusions about the formation of expectations from these estimations. For the same market, one study might opt for adaptive expectations, another for rational expectations and yet another might favour naive expectations. This lack of robustness might be explained by the linearity of all the models used in the works surveyed, the inability of the linear model to account for price dynamics being inherent.

The second strategy is to search directly for the expectations scheme in the expectations data—whether survey or experimental. In a survey of this literature, Irwin and Thraen (1994) highlight a lack of consensus regarding the rationality of expectations. Nerlove and Bessler (2001), however, are more positive. They find that agents try to adapt their forecasts according to the underlying stochastic process, but not in an optimal manner.

Given the lack of agreement about the formation of expectations, the rest of the chapter adopts Prescott’s (1977, p. 30) view that: “Like utility, expectations are not observed, and surveys cannot be used to test the rational expectations hypothesis. One can only test if some theory, whether it
incorporates rational expectations or, for the matter, irrational expectations, is or is not consistent with observations”. Thus, in what follows we assess the empirical performance of the two types of models.

2.4.1 Estimations of competitive storage models

The first estimation of a competitive storage model can be found in Deaton and Laroque (1992).\textsuperscript{11} Exploiting a model without supply reaction, from the storage arbitrage condition (2.12) without storage cost Deaton and Laroque deduce that there is a cutoff price $p^*$ above which the next period price is no longer linked to the current price. When the current price is so high that storage is not profitable, the next period expected price is constant and matches expectations of a simple rational expectations model without storage. The expected price function is given by

$$E_t(p_{t+1}) = \min(p^*, p_t) \frac{1 + r}{1 - \delta}. \tag{2.13}$$

Deaton and Laroque estimate this equation using a generalised method of moments technique. They suppose that only prices are observable. This is very convenient because long price series are available for commodities, which is not true for other data (such as stocks, harvested areas). This approach was adopted in subsequent works. Deaton and Laroque use yearly prices for 13 primary commodities for the period 1900–1987. In equation (2.13), they estimate $p^*$ and $\gamma = (1 + r) / (1 - \delta)$.

This estimation leads to mixed results. Predicted conditional means and variances conform to actual commodity prices. The model also explains price dynamics better than a simple random walk, but the estimates of $r + \delta$ are implausibly small and the cutoff price is often not well-determined, which would be consistent with infrequent stockouts.

This work paved the way to all further estimations. Chambers and Bailey (1996) use the same method to estimate a model using monthly data with periodic disturbances. Their conclusions are also mixed. Ng (1996) and Beck (2001) do not estimate the model, but they do test some of its empirical consequences, based on the findings in Deaton and Laroque (1992). Ng (1996) examines the existence of two regimes and the absence of serial correlation in the stockout regime. Using a threshold autoregressive model, she confirms an infrequent stockout regime but in many cases rejects the absence of serial correlation in this regime.\textsuperscript{12} Beck (2001) tests the autoregressive conditional heteroscedasticity (ARCH) implied by inventory carryover. She confirms that storable commodities follow an ARCH process, while this is not the case for non-storable ones.

Deaton and Laroque (1995, 1996) propose another estimation method using pseudo-likelihood techniques. They reach the same conclusions as in their first paper, that speculative storage model cannot account for high levels of serial correlation. This claim, however, must be qualified. Michaeilides and Ng (2000), using Monte Carlo simulations, compare several estimation methods for

\textsuperscript{11} For a recent survey of empirical assessments of the storage model see Cafiero and Wright (2006).
\textsuperscript{12} Being based on a very limited number of observations, this latter finding is subject to caution.
the storage model and show that the pseudo-maximum likelihood estimation tends to bias results. Cafiero and Wright (2006) cite other potential problems that influence the estimation (e.g., the inclusion of storage costs, the non-stationarity of price series, constant interest rate).

Under alternative assumptions, the model fits the data very well. For example, Miranda and Rui (1999) consider a different modelling of storage costs. They neglect any depreciation during storage, but introduce storage costs following the classical “supply of storage” theory (Kaldor, 1939, Working, 1949), which posits that costs are increasing with stock levels and negative for low stocks (i.e., convenience yield hypothesis). With a maximum likelihood estimation, they find that the storage model can explain the high autocorrelation of commodity prices very well. Cafiero et al. (2011) confirm the importance of storage costs modelling by applying Deaton and Laroque’s (1995) estimation method to a constant marginal storage cost model. This specification improves the model ability to yield high price autocorrelation. They also show that Deaton and Laroque’s (1995, 1996) estimations bias autocorrelation downward, because they approximate the equilibrium price function with an insufficient precision.

Estimations of agricultural price dynamics could be criticised on the grounds that, in most countries, public policies strongly affect prices. Miranda and Glauber (1993a) take account of the effects of public policies in estimating a rational expectations model for the US soybean market with private and government stockpiling. They use a log-linear approximation of the storage rule rather than the optimal rule (it is the optimal storage rule that creates the estimation problem solved by Deaton and Laroque). Miranda and Glauber show that the model behaves better under rational expectations than with autoregressive expectations. They apply the same method for estimating intra-seasonal stockpiling behaviour in the potato market (Miranda and Glauber, 1993b).

In the case of the Ethiopian grain market, Osborne (2004) uses the speculative storage model to assess the effect of news about future production on market price. She builds an intra-seasonal model including the arrival of news (rainfall) related to the next harvest and estimates it using nonlinear least squares. Incorporating news improves the model, particularly its ability to represent the highly autocorrelated prices. However, the model fails to explain all seasonal variability.

Although the model is able qualitatively to represent the main features of agricultural prices (Peterson and Tomek, 2005), Deaton and Laroque (1992, 1996), who were the first to provide econometric estimates, were disappointed by the limited fit between the model and the data. Subsequent contributions are much more positive (Miranda and Rui, 1999, Cafiero et al., 2011), and given that the competitive storage models estimated were quite simple (they can often be reduced to one nonlinear equation) a better fit could be expected from more elaborate models.

2.4.2 Empirical relevance of endogenous dynamics models

Nonlinear cobweb models emerge not as a result of ex ante empirical observations, but as a response to the theoretical critiques of the linear cobweb model, and to demonstrate the possible existence of complex dynamics. Their empirical validation is very difficult due to the lack of mathematical
tools to structurally estimate chaotic models (Barnett and He, 2001). We therefore considered other strategies including comparison with stylised facts, estimation of a related time-series model and testing for the presence of chaos in the time-series.

Do the predictions of nonlinear cobweb models agree with the stylised facts on agricultural prices? Studies such as Deaton and Laroque (1992), Yang and Brorsen (1992), Deaton (1999), Cashin et al. (2002) and Chatrath et al. (2002) identify some of these stylised facts. They show that agricultural prices are not normally distributed, present high positive first-order autocorrelation, are positively skewed and display positive excess kurtosis. They show evidence of volatility clustering (generalised ARCH or GARCH) and price cycles that are asymmetric, with slumps longer than upturns. The first generation of nonlinear cobweb models (Artstein, 1983, Jensen and Urban, 1984, Chiarella, 1988, Hommes, 1991, 1994) does not satisfy these properties. Figure 2.2(b) shows that their backward-looking expectations create negative serial correlation in prices, because years of abundance and low prices are predicted to be followed by more abundant years, which limits planned production and leads eventually to high prices.

The only features common to models that produce nonlinear deterministic dynamics are a decision lag and backward-looking expectations. The empirical failure of some models does not preclude the empirical success of other models belonging to the same family. Heterogeneous agent models appear empirically more promising than simple nonlinear cobweb models: they provide a better fit with the stylised facts. The interaction between rational and backward-looking producers in Brock and Hommes (1997) creates the commonly observed succession of doldrums, booms and bust periods (figure 2.3(b) top panel).

Voituriez (2001) studies the evolution of the palm-oil market from the early nineteenth century. Without carrying out a formal estimation, he shows that a nonlinear dynamics model, using agents with different expectations horizons, can be used for a qualitative representation of the successive periods of volatility experienced by this market when demand shifted from Europe to Asia, and supply shifted from Africa to Indonesia and Malaysia. To our knowledge, Westerhoff and Reitz (2005) is the only claim of an econometric test of an heterogeneous agent model. They analyse a US commodity market, corn, driven not by supply and demand for the physical good, but by the interaction between technical and fundamental traders (as theorised in Brock and Hommes, 1998). They emphasise the financial speculative nature of such markets. They test their model by considering that the interaction between different traders can be captured with a smooth transition autoregressive GARCH (STAR-GARCH) model. Deviations from long-run equilibrium value attract more technical traders, justifying the smooth transition. The strategies of these traders produce time-varying effects justifying the GARCH components. The monthly data confirm the STAR-GARCH model.13 The precise theoretical link between a deterministic, chaotic, heterogeneous agent model and a STAR-GARCH model, however, is unclear in Westerhoff and Reitz’s work.

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13 A STAR-GARCH model could be also compatible with the competitive storage model since its properties have been tested with an ARCH model and a self exciting threshold autoregressive (SETAR) model by Beck (2001) and Ng (1996).
Since the direct estimation of nonlinear deterministic models is so difficult, we can test time-series data for the presence of chaos, characteristic of nonlinear cobweb models. Chatrath et al. (2002) and Adrangi and Chatrath (2003) adopt this strategy and confirm the existence of nonlinearity in price data, but not that this nonlinearity is caused by chaos.\textsuperscript{14}

2.4.3 Which model to explain price dynamics?

In deciding among competing explanations we need to look at the empirical relevance of their counterfactual conclusions. Each type of model predicts a specific effect of trade on volatility. The rational expectations models imply a stabilising role of a larger market, while backward-looking models with no random shocks, predict a price dynamics not related to market size. In the context of early modern Europe, Persson (1999) shows how price volatility has declined with falling trade costs and reduced administrative barriers to trade. Jacks et al. (forthcoming) confirm these findings. They show that world market integration brings less commodity volatility, and that periods of isolation due to wars or autarkic policies present more volatility. Sarris (2000) finds no evidence of increased volatility in the international cereal market, after the late 1960s. The tendency towards more open agricultural markets would be expected to lead to more stability, but there are other factors that might influence the markets (use of new high-yielding varieties, general decrease in public stocks), and prevent definitive conclusions. The evidence regarding the link between trade and volatility tends to confirm the hypothesis of a dynamics driven by external shocks that would be smoothed by the presence of a larger market. This does not confirm the rational expectations hypothesis, but it clearly rejects the pure endogenous dynamics model. A significant part of the dynamics is driven by external shocks.

Given this argument’s longevity, the difficulty involved in empirically resolving the issue of endogenous and exogenous fluctuations is unsurprising. First generation agricultural price models, namely linear cobweb models for endogenous dynamics and rational expectations models for exogenous dynamics, have been proved to be observationally equivalent (Eckstein, 1984, 1985). Eckstein constructed a quadratic approximation of the rational expectations model, which embeds adjustment costs, and shows that it can be observationally equivalent to Nerlovian or simple cobweb models. We cannot prove similar equivalence for the second generation models, because they are not analytically tractable. But, since a chaotic trajectory can be confounded by random draws (a feature used by random number generators), it would not be surprising that a deterministic, nonlinear model could produce time series that are similar to the competitive storage model.

The remaining uncertainty should not outweigh the importance of previous findings: the significance of nonlinearity, the good fit of the rational expectations storage model, and the identification of a set of stylised facts. As shown above, most nonlinear models of endogenous dynamics do not comply with the stylised facts and must be rejected as relevant explanations. Also, evidence on

\textsuperscript{14} Given the lack of robustness of the tests for chaos for small sample sizes, finding evidence of chaos is difficult (Barnett et al., 1995, 1997).
the effects of trade on volatility shows that real shocks are important drivers of fluctuations. A tentative conclusion would be to assume support for the competitive storage model under rational expectations. The ability to decide which theory is best suited to representing agricultural markets becomes all the more critical when the competing theories lead to completely opposite conclusions regarding public intervention: one model represents Pareto-optimal dynamics; the other provides justification for public intervention.

2.5 Public Intervention in Volatile Markets

Since the appropriateness of the public intervention depends on the cause of the fluctuations, the structural explanations of agricultural price dynamics proposed above should be essential for this issue. The distinction between endogenous and exogenous fluctuations, however, has framed the debate for positive rather than for normative questions. Most normative models assume real shocks and rational expectations.

2.5.1 Marshallian Analysis

Modern welfare analysis of price stabilisation began with Waugh (1944), whose approach framed the debate for the next several decades. He analyse surpluses to show that consumers are better off under conditions of price instability than if prices are stabilised at no cost, at their arithmetic mean. This result is based on the flexibility provided to consumers by price variability. When the demand curve is downward sloping, users consume more at the lower prices and avoid the welfare losses of high prices by consuming less. Oi (1961) and Massell (1969), among others, use the same methodology.

This strand of the literature shows that it is possible to determine a priori the general welfare effects of a policy for producers and consumers. However, this finding is of limited relevance because an ideal stabilisation of prices at their arithmetic means is not feasible, and the costs of a stabilising policy are not included in the welfare evaluation. The last set of contributions to this methodology are by Wright (1979), Turnovsky et al. (1980) and Wright and Williams (1988b). Wright and Williams (1988b) derive a second order approximation of the equivalent variation for consumption stabilisation at the arithmetic mean, \( \bar{q} \),

\[
\frac{\bar{q} P(\bar{q}, Y)}{2 \eta^D} \left[ \frac{\theta}{\eta^D} (\eta^Y - \rho) + C - 1 \right] \Delta \sigma^2, \tag{2.14}
\]

where \( P(\bar{q}, Y) \) is the inverse demand function for income \( Y \), \( \eta^D \) and \( \eta^Y \) are the price and income elasticities, \( \theta \) is the budget share, \( \rho \) is the coefficient of relative risk aversion, \( C \) is the relative

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15 This analytical literature is surveyed in Wright (2001).
16 Turnovsky et al. (1980) obtain a similar approximation for the stabilisation of prices at their arithmetic mean.
curvature of demand \( (C = -qP_{qq}(\bar{q}, Y)/P_q(\bar{q}, Y)) \) and \( \Delta\sigma_q^2 \) is the reduction in the variance of consumption.

The budget share of a single agricultural commodity in the developing countries can be above 10%. This high budget share, combined with risk aversion, can induce important gains from consumption stabilisation in poor countries. In developed countries, the share is low, which means we can ignore the first term in the brackets. For these consumers, the curvature of demand drives the welfare effects. With a linear demand curve, \( C = 0 \), consumers lose from quantity stabilisation. This result can be overturned for a different demand curvature. For constant demand elasticity, consumers gain if demand is inelastic. Producer welfare effects are also analysed in Wright (1979).

2.5.2 A MODERN APPROACH: INTRODUCING THE ROLE OF EXPECTATIONS AND STABILISATION COSTS

The previous analyses lack applicability, because ignoring the role of stabilisation costs can lead to misleading recommendations. Policy conclusions should be based on analyses that combine the reasons for price fluctuations and the costs of stabilisation.

Marshallian analyses describe consumer and producer behaviours as being derived from complete, but changing information. Turnovsky (1974a) remarks that this is not appropriate for producers who have to plan production before knowing the selling price. Production lag implies a strong role for price expectations and alters the welfare results. Net gains from price stabilisation are higher under expected than actual prices, because stabilisation avoids resources being wasted when expectations are wrong. Turnovsky shows that the results of the Marshallian analyses hold when introducing a decision lag with rational expectations; this is not the case for adaptive expectations, which introduce errors in resources allocation.

The fact that stabilisation can bring net gains should not be surprising, because it is provided by a costless technology within this framework (a “self-liquidating” buffer stock in Turnovsky, 1974a). Introducing price stabilisation costs changes this. A costless technology does not exist, nor is it possible to perfectly stabilise markets. Storage does not allow perfect stabilisation, because a finite buffer stock cannot prevent stockouts.

The introduction of expectations and stabilisation costs makes the link to the previous sections. The two kinds of explanation for price volatility, exogenous and endogenous dynamics, give rise to different narratives concerning public intervention. In the first, within an exogenous dynamics model, the scarcity of agricultural products changes yearly with changes in yield, acreage and/or demand; price changes are normal economic adjustments to these new market conditions. Prices help to allocate a scarce resource to a more productive use. In traditional rational expectations storage models, agents are risk neutral, there is no market imperfection. Supply varies with weather shocks, optimal stock carryover and optimal reactions from farmers. Under these conditions, aggregate welfare cannot be improved by intervention. This optimality of competitive storage was emphasised in Gustafson (1958a) and Gardner (1979a), and formally proved by Scheinkman and Schechtman.
In the absence of market imperfections, a dynamic optimisation of the surplus can be recast as a set of decisions taken by individuals in a market context. So, in the framework of the competitive storage model, there is no rationale for public intervention.

This result does not hold for models following the cobweb logic. Cobweb models explain a significant part of the fluctuations as arising from systematic forecasting errors. The resulting price instability effectively reflects a change in scarcity, but this change is not the result of optimal producer reaction. Welfare optimality of the competitive equilibrium does not hold in this framework. The price path in cobweb-type models is socially suboptimal because farmers take decisions that are grounded not on the true expected variables but on backward-looking information, and production is not allocated in relation to expected scarcity.

As in the case of optimality of a decentralised equilibrium in general equilibrium theory, the social optimality of a market under rational expectations must not be taken as a positive statement which rules out any public intervention. This theory helps us to understand under which circumstances the interactions of private agents lead to socially optimal results when markets are volatile. Models with backward-looking expectations can be seen as a possible deviation from this benchmark. Other deviations, such as market incompleteness or externalities, are analysed later.

Starting from the rational expectations benchmark, several authors have examined various public interventions. Given that the competitive storage model is Pareto optimal without the introduction of market failures, any public policy that would stabilise prices in addition to private storage would lower welfare. Policy maker can infer some general conclusions about the effect of the available instruments. Miranda and Helmberger (1988) analyse in detail the introduction of a price band policy. Wright and Williams (1988a) compare the effects of floor price schemes, deficiency payments and extra-market disposal. Both papers find that short and long-run effects differ widely and may be of opposite signs. This result questions the reliability of comparative statics results obtained by highly stylised models in the tradition of Newbery and Stiglitz (1981). The short and long-run effects are different because stocks must be built-up before they can become operational. The additional demand for building the public stocks raises prices. On the asymptotic distribution, on the contrary, the mean price is lower under a public stockpiling policy than without it. Because future losses are discounted, the short-run gains dominate and producers are expected to gains from the implementation of price-stabilisation schemes. The result is opposite if one looks only at long-run effects. The dynamic analysis of producers welfare must also consider the issue of capitalisation of their benefits in land price. Glauber et al. (1989) compare different approaches to market stabilisation. They point out that market stabilisation can be achieved efficiently through subsidised private storage. If the main concern is about producer price stabilisation, they show that deficiency payments are the best choice. Gardner and López (1996) show that interest-rate subsidies are not appropriate for stimulating private stockpiling and that subsidising direct storage is a better

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17 This equivalence is the result of the application of the Second Welfare Theorem in a dynamic setting (Stokey and Lucas, 1989).

18 More discussion on dynamic issues in agriculture can be found in Wright (1993).
option. In the case of Indian foodgrain policy, Jha and Srinivasan (2001) demonstrate that market stabilisation is achieved more easily via trade under variable levies/subsidies than under autarky and price band policy.

Practical policy making cannot be based on these works. Even though many agricultural policies are known to be inefficient, they have often emerged initially to correct for some market imperfection. A fair policy assessment must take account of these imperfections, especially since this might completely reverse the more traditional conclusions. A few papers study the interaction of market imperfections and public intervention for price stabilisation; however they do not necessarily fit with the competitive storage framework. For convenience, they often use highly stylised rational expectations models.

The following sections discuss the justifications for public intervention based on four aspects: sensitivity of the results to the modelling assumptions; interventions in developing countries; interventions in developed countries; and the limited knowledge on the importance of market imperfections in relation to price dynamics.

2.5.3 Sensitivity to modelling assumptions

Some deviations from the canonical rational expectations model (with or without storage) have such strong effects that they may justify public intervention that is in complete opposition to conventional analyses. This is the case for risk aversion under incomplete markets or backward-looking expectations. These situations (Newbery and Stiglitz, 1984, Innes, 1990, Boussard et al., 2006) correspond to second best situations (Lipsey and Lancaster, 1956), where attempts to satisfy some of the Paretian conditions (e.g., liberalising trade or suppressing agricultural policies) do not improve welfare. Policy recommendations when prices are volatile are very sensitive to the modelling context.

Innes (1990) makes this point, showing that, in an incomplete market setting with risk aversion, a public policy such as deficiency payments, usually thought of as welfare decreasing, can be Pareto improving. A similar story is told by Newbery and Stiglitz (1984), who analyse a general equilibrium model involving two goods and two countries. The production of one good is subject to random shocks. Autarky plays an income insurance role for risk-averse producers because domestic prices vary inversely with domestic production. In contrast, trade smooths prices by averaging production shocks, but destroys the insurance effects since prices are linked to international rather than domestic conditions. This illustrates that price fluctuation is not the first concern of producers, who worry more about income fluctuation. The increased income risk deters them from producing a risky commodity and raises its price. Ultimately, welfare is lower under free trade than under autarky, because the reaction of risk-averse producers is stronger than the price stabilisation gains. Introducing risk aversion in an incomplete market can completely change the welfare effect of a policy.
The stabilising property of international trade is also sensitive to the modelling assumptions. If price fluctuations come mainly from forecasting errors, trade will not help to smooth idiosyncratic shocks, but, on the contrary, could increase instability. The study by Boussard et al. (2006) demonstrates this. They build a multi-country computable general equilibrium model with supply lagged one period, risk-averse producers and Nerlovian expectations. They show that the movement in agricultural prices is amplified by market liberalisation, and welfare results go in opposite way compare to traditional expectations. Voituriez (2001) reaches a similar conclusion with his palm-oil chaotic market model. He shows that the increase in market size increases the instability of this market. The absence of exogenous shocks in both of these models explains these strong conclusions. Such a hypothesis is understandable for the examination of the theoretical effects of backward looking expectations on dynamics, as discussed in section 2.1. But, in normative analysis, the focus on forecast errors may lead to spurious conclusions, which can be reversed by considering exogenous shocks. For example, by introducing both Nerlovian expectations and random shocks in a partial equilibrium model, Tyers and Anderson (1992) find a stabilising effect of increased market integration, because a wider market allows the shocks to be averaged out, despite the forecasting errors.

2.5.4 RATIONALE FOR PUBLIC INTERVENTION IN DEVELOPING COUNTRIES

Distinctions can be made in terms of the policy recommendations directed to developed and developing countries. Risk aversion has a significant effect for economically poorer consumers, who are required to allocate a large share of their budget to the purchase of staple food (see equation (2.14)). Newbery (1989) provides an analytical framework for analysing public intervention when consumers are risk averse and are required to spend a large share of their income on staple foods. Newbery determines the conditions that make it beneficial to stockpile above the level achieved in a competitive market, and shows that consumer protection can also be achieved through the sale of rations at low prices.

In poor societies, low prices for staple foods are essential for social order. Evidence from early modern England shows that the number of thefts rose during dearth periods (Walter and Wrightson, 1976). The food crisis in poor countries in the first part of 2008 culminated in hunger riots in Cameroon, Côte d’Ivoire and Haiti. Price stability generates gains that are not exclusive to grain storers (Persson, 1996). These externalities are a traditional motivation for public intervention. Private storage on its own cannot provide a socially optimal level of stability in the presence of externalities.

Externalities created by positive deviations from mean prices are discussed in Gardner (1979a). He proposes a method for finding an optimal storage rule, which could be implemented by a public agency, in response to this type of externality. This would crowd out all private speculative carryover because the optimal stockpiling would increase the carryover to a point where expected returns would be negative (optimal storage is higher where there are external benefits to price stability). The traditional practice of public storage, however, does not follow an optimal storage rule, but a buffer
stock with a price band rule, which leaves room for speculative stockpiling. Another way of correcting this externality would be to subsidise private storage, but neither of these two interventions, price band or subsidy, even if carefully designed, can substitute for an optimal storage rule. In a real context, the externality and numerous model parameters would not be measurable, which would call for robust storage policies that were not over sensitive to ill-known aspects. Gardner (1979a) shows that a small buffer stock is the best solution.\(^{19}\)

Even in the absence of externalities, social disorder presents government with a commitment problem. If government cannot commit to not imposing a policy that would prevent stock holders from benefiting from high prices (e.g., a price ceiling, an export tax or a seizure of existing stocks), stock holders can expect limited profits from their activities, and private storage will be reduced below the optimal level.\(^{20}\) When grain stock holders are blamed for hoarding and making money out of people’s hunger, and when political stability rests critically on the availability of staples, it is difficult for governments not to react against private stockpiling in times of grain shortages. In this case, public storage would be required to reach a social optimum. Wright and Williams (1982b) provide a formal treatment of such a situation in the case of disruption to oil supplies. They show that public stocks can alleviate the adverse effects of lack of government commitment to not imposing a price ceiling.

In addition to the above theoretical rationales for intervention, we should consider a practical issue: the most important staple for developing countries is also the most subject to extreme behaviours. Rice is thinly traded on international markets (7% of total production). And at the first signs of price tensions, the exporters impose export controls to ease their domestic markets. In the 1973–74 crisis, such export bans made the world rice market disappeared for nine months (Timmer, 2010). The 2007–08 crisis saw the same behaviour with most exporters closing successively their markets, fuelling in this way the panic in the rice market (Slayton, 2009). Given this lack of commitment of exporters to being reliable suppliers, importing countries should consider the case for national strategic reserves (Wright, forthcoming). Following the 1970s oil crisis, the related issue of managing strategic petroleum reserves in case of supply disruption have been analysed in several papers (e.g., Wright and Williams, 1982b, Murphy et al., 1987).

### 2.5.5 Intervention in Developed Countries?

In rich countries, agricultural price instability is of more limited concern to consumers, who devote only a limited share of their budgets to agricultural commodities. The rationale for public intervention in these countries will be based more on imperfections in the storage and production markets. Market imperfections in agricultural markets are numerous, but are rarely introduced in models that account for both the origins of price fluctuations and the public policies designed to mitigate them. Analyses

\(^{19}\) Concerns about the robustness of policies to model mis-specification have been the object of recent work in macroeconomic dynamics using new tools from robust control theory (Hansen and Sargent, 2007). Gardner (1979a) is a crude test of robustness that does not rely on these tools.

\(^{20}\) During the 2008 food crisis, numerous developing countries adopted such policies, especially for the rice market.
of farm programmes addressing instability concentrate on economic costs by comparing policies to the competitive storage benchmark.\textsuperscript{21} Hence, the discussion below highlights only the possible effects of market imperfections in developed countries.

In the storage model, both producers and storers may be concerned by risk aversion. Storers will be seen as risk neutral if they can hedge their position on futures markets (see Williams and Wright, 1991, p. 28). Holthausen (1979) and Feder et al. (1980) demonstrate the following separation property: when futures markets are available, a risk-averse firm under price uncertainty behaves as if it were risk neutral. Its risk aversion only affects its position in the futures market. This separation between productive choice and hedging does not hold for farmers who face both price and output uncertainty (McKinnon, 1967).\textsuperscript{22} Hence, welfare analyses of agricultural policies should take account of farmers’ risk aversion. Lence (2009) is the only example of a storage model that includes risk-averse producers. He demonstrates the counterintuitive effect of the introduction of a futures market for risk-averse producers. While such a policy is often thought of as helpful to producers, Lence finds that producers lose from the creation of a futures market while consumers gain. Producers who hedge part of their production are more responsive to changes in market conditions. Their output is higher, which lowers prices and profits. As a result, their welfare is lower.

The agricultural production sector is often regarded as a good example of a perfectly competitive market. This is not true of the upstream and downstream sectors. The storage and marketing of grains cannot always be seen as competitive markets. Some studies address the issue of market structure. Newbery (1984) analyses the case of producers’ market power. Williams and Wright (1991, Ch. 11) treat the case of market power over storage and show that storage is lower under monopoly than in a competitive market. McLaren (1999) extends the analysis to an oligopoly with restricted entry. McLaren confirms Williams and Wright’s results of a level of storage increasing with the level of competition, and thus decreasing price volatility. These results contradict the popular tenet that market power over storage explains the price spikes caused by excessive hoarding by non-competitive storers.

Leathers and Chavas (1986) examine the effects of price uncertainty on farm default. Because of market incompleteness and price shocks, indebted farmers may default on their loans. Such defaults are costly to society because of the immobility of capital assets. Public intervention could improve market outcomes, but the design of such policies would be challenging, because of the need that adverse economic incentives were not being created.

Thirty years ago, Gardner (1979a, p. 150) noted that “the current state of knowledge does not permit the specification of [a socially optimal stockpiling] regime”. The position is not better today. Since his work, several justifications for public intervention have been proposed. But robust

\textsuperscript{21} As noted by Leathers and Chavas (1986), this issue is not specific to price volatility analysis. Analyses of agricultural policies focus on the economic costs of policies without introducing the underlying market imperfections that were their justifications.

\textsuperscript{22} Moreover, futures cannot completely remove price uncertainty because of basis risk, i.e., the spread between the futures price and the producer price.
conclusions, including both the reasons for price instability and the public interventions relevant for the market imperfections, should only be based on empirically relevant models. These two aspects are still being studied separately. We can say tentatively that imperfections in agricultural markets justify public intervention, at least in developing countries, to mitigate price instabilities. In developed countries, since public policies are studied in an optimal world that does not justify any intervention, we do not know what would be the best way to stabilise imperfect markets. Most of the applied studies of public intervention to achieve price stability (e.g., Makki et al., 1996, 2001, Jha and Srinivasan, 1999, 2001, Srinivasan and Jha, 2001, Lence and Hayes, 2002, Brennan, 2003) do not consider market imperfections, with the exception of Brennan (2003), who examines the effect of an imperfect credit market for private storers.

2.5.6 THE MISSING CONNECTION BETWEEN PRICE DYNAMICS AND MARKET IMPERFECTIONS

The agricultural sector is not isolated from the rest of the economy and its price dynamics can be affected by non-agricultural factors, such as the business cycle. For instance, Andrews and Rausser (1986) explain how the macroeconomic effects on the agricultural sector were an important justification for the agricultural policies introduced in the first half of the twentieth century.

A possible link between macroeconomic conditions and commodity prices is described in the overshooting theory proposed by Frankel (1986). He introduces a model that is similar to Dornbusch’s (1976) model. It assumes that agricultural prices are flexible, while other prices are sticky. An unexpected rise in the money supply, which should lead to a nominal price increase, results in a decrease in real interest rates due to price stickiness. As a result of the storage arbitrage condition (2.12), if the next period prices change according to the new inflation expectations, the fall in interest rates will result in an overshooting of the current-period price. This conclusion has been criticised on theoretical grounds (Obstfeld, 1986, Lai et al., 2005), but Frankel’s work provides a framework for studying monetary impacts on commodity behaviour, whether over or under shooting. Several empirical studies confirm the link between these markets and monetary variables (Chambers and Just, 1982, Chambers, 1984, Frankel and Hardouvelis, 1985, Orden and Fackler, 1989, Robertson and Orden, 1990).

The existence of monetary effects implies that agricultural prices do not always match market scarcity. Because of imperfections in non-agricultural markets, the agricultural sector price path is not optimal. The link between market imperfections and observed price dynamics is rarely made, except in the case of overshooting caused by monetary effects. But if market imperfections really affect agents’ behaviour to the point of justifying public intervention, they should be reflected in price dynamics, which should differ from those in efficient markets. For other aspects than monetary shocks, the possibility that market imperfections may explain commodity dynamics and lead to a better fit remains an open question.
2.6 Summary and Perspectives

Economic understanding of agricultural price fluctuations has improved greatly since the beginning of the 1980s. One of the newer explanations for these fluctuations adopts the cobweb logic of endogenous dynamics, showing that, in the context of chaotic dynamics, relying on rule-of-thumb expectations is not such a bad choice. Another explanation follows the path of rational expectations and emphasises the importance of storage for explaining price dynamics. The theory has improved, but the question of the origin of volatility has not been definitively settled.

Empirical tests of the storage model under rational expectations confirm its relevance for qualitatively explaining many of the stylised facts in this area. And contrary to first econometric results, which led to mixed conclusions, recent estimates find that the storage model provides a very good fit. It is more difficult to test the alternative explanation of endogenous dynamics, because it gives rise to chaotic models that cannot be estimated. But most endogenous dynamics explanations do not generate price series that are consistent with the stylised facts, namely positive serial correlation and positive skewness, a failure that also applies to the linear cobweb model. We can tentatively conclude that the rational expectations scheme is better supported.

If agricultural price volatility is driven by real shocks in an economy populated by rational and risk-neutral agents, there is no public intervention that can improve the welfare. Such a strong conclusion requires to be tempered. Since this conclusion is based on there being a complete absence of market failure, the rational expectations model might be seen better as a normative benchmark than an accurate description of reality. In poor countries, consumer risk aversion and the social externalities of high food prices make a strong case for public intervention in grain markets. In developed countries, the case for intervention is less clear. Introducing producer risk aversion or liquidity constraints in the storage model would provide a rationale for public intervention, but this is not explored in the literature.

To define some next research steps, we can draw some parallels with the recent history of macro-dynamics literature. Macro-dynamics presents some similarities with agricultural price dynamics. Both fields aim at explaining the origins of economic fluctuations and designing policies to mitigate their effects.

The real business cycle model of Kydland and Prescott (1982) has become a workhorse for the study of macroeconomic fluctuations since the early 1980s. This canonical model was extended in various ways in order to try to deal with some of its empirical limitations such as lack of persistence in the time series, or the overemphasis on total factor productivity shocks, which implies that recessions are times of technological regress. The standard models now include nominal (prices and wages) and real rigidities (e.g., adjustment costs and habit formation), and monetary shocks (Rebelo, 2005, Woodford, 2009). However, there is no similar path in agricultural economics. The competitive storage model has not become the corner stone for series of works that include other features that can be tested for their effects on dynamics. Instead, most externalities have been analysed within a static framework and their dynamic effects are unknown.
To follow the business cycle programme in agricultural economics, the models should be extended, and econometrically validated, to include externalities and interactions between sectors. Recent papers (Lence and Hayes, 2002, Osborne, 2004, Park, 2006, Lence, 2009) have made some efforts to extend the storage model to more empirically relevant situations. However, these developments are more challenging in the context of the storage model than when applied to macroeconomic models. Macroeconomic models are usually linearised, or approximated at small orders, around the steady state, which makes them extendable to medium/large scale problems. The storage model is strongly nonlinear, which rules out such approximations. The available numerical methods suffer from the “curse of dimensionality”, namely the exponential rise in computing time when problem dimensions increase. New developments in computational methods, such as sparse grid interpolation (Krueger and Kubler, 2004), might help to overcome this problem.

Given the on-going debate over backward-looking versus rational expectations, another improvement would be to examine more deeply the question of expectations building and information. The opposition between backward-looking and rational expectations models is surely too simple. Rational agents can devote a limited attention to all incoming information and so will take decisions that are non-optimal with respect to a full information benchmark (Sims, 2006). Even fully informed agents will have to achieve coordination on the rational expectations equilibrium, which might prove difficult. For example, following the eductive stability concept of Guesnerie (1992), Guesnerie and Rochet (1993) and Calvo-Pardo (2009) studied expectations coordination in traditional agricultural market settings. Both studies show that international trade or speculation on futures market may be destabilising when there is no Walrasian auctioneer and agents must coordinate by forecasting the forecasts of others.
CHAPTER 3

COMPARING NUMERICAL METHODS FOR SOLVING THE COMPETITIVE STORAGE MODEL

The competitive storage model is the workhorse of neoclassical studies on the price behaviour of storable commodities (Williams and Wright, 1991, Wright, 2001). Although its empirical properties continue to be debated (Cafiero and Wright, 2006), this model provides justification for the main properties of commodity prices: positive serial correlation, skewness, successions of long periods of doldrums and short periods of high prices. It also serves as a normative benchmark for analysing public intervention in commodity markets (Miranda and Helmberger, 1988). Like most dynamic stochastic problems, it cannot be solved analytically. For this reason, understanding the properties of available numerical solutions is important for securing precise econometric estimates or reliable policy conclusions. One example of the importance of good numerical solutions is provided by Cafiero et al. (2011), who show that Deaton and Laroque’s (1995, 1996) econometric estimates are not reliable due to the imprecise methods used to approximate the model. Another aspect that requires good knowledge of numerical solutions is its extension to higher dimensional problems. Many important questions on commodity prices (e.g., the comovement of commodity prices, the relationship between trade and prices, the consequences of price stabilisation programmes, and the effect of monetary policies on prices) imply problems with several state and decision variables that become very challenging to solve numerically.

This chapter compares different approaches to solving the competitive storage model. It implements three methods not tested previously on this model: perturbation, endogenous grid methods and storage rule approximation. As perturbation methods only work with smooth problems, two alternative models are considered: one with a non-negativity constraint on storage and the other with a precautionary motive for storage that rules out zero stock situations and smoothes the model. Comparing solution methods for this smooth model is the object of Miranda’s (1997) study. Our approach differs from Miranda’s in considering the model with a non-negativity constraint—the more traditional representation of storage problems—and in its different focus. Miranda (1997) studies mainly the effect of varying approximation methods for a given projection algorithm which parameterises current price. The present chapter tests various algorithms, but limits variations
between approaches by holding the set of tools constant: functions are approximated by cubic splines; convergence to the rational expectations equilibrium is achieved by successive approximations; value function iteration and projection problems are resolved by collocation; and the same degree of approximation is used for each method. The precision of the solutions is assessed using a measure of the Euler equation error that is derived to account for the switching-regime behaviour.

There is a large body of work on the comparison of numerical methods in economics. Most concerns the stochastic growth model (see, e.g., Aruoba et al. (2006) and Heer and Maußner (2008) for two recent studies). This literature covers only some of the methods used for the storage model. The specificities of the storage model come from the non-negativity constraint on stocks. For these kinds of constraints, Christiano and Fisher (2000) study numerical methods in a stochastic growth model with irreversible investment. This chapter uses some of the same methods as Christiano and Fisher, but proposes also additional ones.

The study of numerical methods for the storage model presents interests beyond the study of commodities behaviour. The storage model, in effect, is formally very close to the consumption/saving problem under income uncertainty (Deaton, 1991, Carroll, 2001). Their problematics are identical: how much an agent should consume and save today when future resources are volatile. In the optimal consumption problem, a consumer’s borrowing capacity is limited or he applies a limit as a precaution. At each period, the consumer chooses how much to save to protect his future consumption against adverse income shocks. In the storage model, there are three agents: a consumer, a storer acting competitively, and a producer using a stochastic production function. Without distortion, this problem can be stated as a planner problem. The planner must choose how much grain to carry over to the next period. There is one main difference between the two models: storage is costly and grain may get spoilt during storage, while in a consumption/saving problem savings are remunerated. But the impatience of consumers makes saving costly, which renders the two model almost identical. This similarity was first noted by Deaton, who worked on both fields (Deaton, 1991, Deaton and Laroque, 1992). The longstanding ignorance about this resemblance had some consequences: a numerical method for the storage model was proposed by Gustafson (1958a), while the consumption/saving problem under income uncertainty was solved using the misleading certainty equivalence or log-linearisation until the work by Zeldes (Barsky et al., 1986, Zeldes, 1989).

This chapter provides several notable results. First, the parameterised expectations approach stands out as the most precise method. It has well known good properties for solving models with occasionally binding constraints (Christiano and Fisher, 2000), but in our case its good performances extend also to the smooth version of the model. Indeed, unlike other methods that approximate highly nonlinear functions, this method approximates a function that is almost linear. Second, perturbation methods are shown to be inadequate for this model. Given the nonlinearity of storage behaviour, low-order perturbations are imprecise and lead to negative storage, whereas high-order perturbations present diverging behaviour. Third, when supply is assumed to be inelastic, as is the case in most econometric studies, the endogenous grid method is by far the fastest, which makes it
3.1 THE COMPETITIVE STORAGE MODEL

3.1.1 Model equations

The storage model analysed in Wright and Williams (1982a) is used throughout the chapter. It features a market for a storable commodity with a competitive storer, a producer whose output is subject to multiplicative shocks and a final demand.

The activity of the competitive risk-neutral storer is to transfer the commodity from one period to the next. Storing the quantity $S_t$ from period $t$ to period $t+1$ entails a physical cost $\Phi(S_t)$, a purchase cost $P_t S_t$, with $P_t$ the market price, and an opportunity cost. A share $\delta$ of the commodity deteriorates during storage. The benefits valued in period $t$ are $P_{t+1} S_t \cdot (1 - \delta) / (1 + r)$, with $r$ the interest rate. The profit expected by the storer is

$$E_t \left( \Pi^S_{t+1} \right) = \left[ \frac{1 - \delta}{1 + r} E_t (P_{t+1}) - P_t \right] S_t - \Phi(S_t),$$

(3.1)

where $E_t$ is the expectation operator conditional on period $t$ information. Taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), expected profit maximisation yields the following complementary condition

$$S_t \geq 0 \quad \perp \quad \frac{1 - \delta}{1 + r} E_t (P_{t+1}) - P_t - \Phi'(S_t) \leq 0,$$

(3.2)

which means that inventories are null when the marginal cost of storage, including purchase cost, is not covered by the expected marginal benefits; for positive inventories, the arbitrage equation holds with equality.

A representative producer makes his productive choice one period before bringing the output to market. He plans in period $t$ a production level $H_t$ for period $t+1$, but a disturbance affects final production (e.g., weather disturbances). The expected profit can be written as

$$E_t \left( \Pi^H_{t+1} \right) = \frac{E_t (P_{t+1} H_t \epsilon_{t+1})}{1 + r} - \Psi(H_t),$$

(3.3)

where $\Psi(H_t)$ is the cost of planning the production $H_t$, and $H_t \epsilon_{t+1}$ is the realised production. $\epsilon_{t+1}$ is the realisation of a stochastic process, supposed i.i.d. and distributed following a normal centred on 1 with standard error $\sigma$. The planned production derived from expected profit maximisation satisfies

$$E_t (P_{t+1} \epsilon_{t+1}) = (1 + r) \Psi'(H_t).$$

(3.4)
Final demand is given by the inverse demand function \( P(D) \), where \( D \) is the quantity consumed. The shocks being i.i.d., the state of the model is defined by total availability

\[
A_t = (1 - \delta) S_{t-1} + H_{t-1} \epsilon_t. \tag{3.5}
\]

Accounting for carry over storage, market equilibrium can be written as

\[
A_t = D_t + S_t. \tag{3.6}
\]

Using equation (3.6), the current price can be substituted from the storage arbitrage condition (3.2) by a function of availability and storage:

\[
S_t \geq 0 \quad \perp \quad \frac{1 - \delta}{1 + r} E_t (P_{t+1}) - P (A_t - S_t) - \Phi'(S_t) \leq 0. \tag{3.7}
\]

Thanks to this substitution, the storage model is defined by two equilibrium equations, the first-order conditions (3.4) and (3.7), by the transition equation (3.5), and by the rational expectations hypothesis. A restriction of this model to inelastic supply is the standard tool for econometric works. The equilibrium reduces to equation (3.7), with \( H_t \) taken as constant. Although this simplified model is not the object of this work, we will discuss some of its numerical specificities.

### 3.1.2 Model Parameterisation

Physical storage costs are usually assumed to be proportional to the amount stored (Wright and Williams, 1982a). In our first parameterisation, we take \( \Phi'(S) = 0.01 \) (i.e., 1% of the deterministic steady-state price assumed to be 1). One alternative is to include in the storage cost a convenience yield, which would tend to take high negative values for low stocks, preventing the occurrence of stockouts. Miranda (1997) uses a logarithmic function to achieve this. We follow his approach by taking \( \Phi'(S) = 0.3 + 0.1 \log(S) \), which conveniently removes the complementarity condition from equation (3.7). As the logarithm tends to minus infinity for low stocks, stocks always remain positive. The two models show similar, but distinctive, behaviours (see figure 3.1), the model with convenience yield being smoother than the model with the non-negativity constraint. Negative marginal storage costs can be justified on the grounds that they account for both physical storage and convenience yield. To reconcile the fact that low stocks exist for apparently negative returns to storage, some authors, such as Kaldor (1939) and Working (1948), conclude that storers can expect a convenience yield from holding stock despite an apparent negative yield. This convenience yield comes from the possibility of being able to use stock at any moment. With a convenience yield, storage can be seen as responding to different motives: a speculative motive that allows stockouts and a precautionary motive that precludes them (Carter and Revoredo Giha, 2007).

The inverse demand function is taken to be isoelastic: \( P(D) = D^{1/e} \). The production cost function also follows an isoelastic form: \( \Psi(H) = H^{\alpha+1}/[(1 + r)(\alpha + 1)] \). The production cost is
normalised by $1 + r$ to ensure that the deterministic steady-state production is at 1. Table 3.1 presents the parameters used to calibrate the model. They are assumed to be such that the deterministic steady state of the model with non-negativity constraint is 1 for availability, price and production. Convenience yield implies that there is always some stock, even when price is constant and there is no speculative opportunity, so the deterministic steady state differs slightly with stock, availability, price and production respectively at 0.0338, 1.0336, 0.9998 and 1.0001. Supply elasticity is 0.2 and demand elasticity −0.3.

Table 3.1. Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\delta$</th>
<th>$r$</th>
<th>$\epsilon$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.01</td>
<td>0.03</td>
<td>−0.3</td>
<td>5</td>
<td>0.10</td>
</tr>
</tbody>
</table>

For this calibration, the storage model reproduces some stylised facts about agricultural commodities markets (table 3.2). Prices are positively serially correlated. Their distribution is asymmetric with a higher frequency of high than low prices, because storage alleviates low prices by stockpiling, but cannot alleviate all episodes of high prices as stocks cannot be negative. The occurrence of zero stock periods should be small to preserve the serial correlation (it is storage that creates autocorrelation in prices) and to limit episodes of high prices. Because market availability is smoothed by the storage and supply reaction, final demand oscillates little (5% and 6% of coefficient of variation). In opposition to prices, final consumption is positively skewed, because high consumption is prevented by storage, and periods of low consumption occur because of stockouts.

Table 3.2. Storage model behaviour

<table>
<thead>
<tr>
<th></th>
<th>With non-negativity constraint</th>
<th>With convenience yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occurrence of stockouts (%)</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>Price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.27</td>
<td>0.23</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.20</td>
<td>0.24</td>
</tr>
<tr>
<td>Skewness</td>
<td>3.72</td>
<td>2.71</td>
</tr>
<tr>
<td>Consumption demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Skewness</td>
<td>−2.04</td>
<td>−0.79</td>
</tr>
</tbody>
</table>

Figure 3.1 illustrates the behaviour of the storage model. It shows the numerical difficulties we encounter when deciding about the different approximation schemes. The difficulties arise from the non-negativity constraint of storage. Below a threshold availability, the expected marginal profit from stockpiling grains is negative, as the expected price does not cover storage costs and the high current price. For a negative expected marginal profit, there is no stock carried over to the next period. So the storage rule encompasses two regimes: a no-stock regime below the threshold, and a stock increasing with availability above the threshold. This behaviour affects all the variables in the model. Below the threshold, the market price is set by consumption demand, but above it,
the demand for storage adds to consumption demand. So, the market demand function presents a kink at the availability threshold. The behaviour of producers depends on the expected price in the next period. This expected price is constant for low availability because there is no stock to connect successive periods. Therefore, planned production is constant until the threshold availability is reached and then decreases with the increase in stocks. For a more complete description of the properties of the storage model see Wright (2001).

The model parameterised with convenience yield has no threshold and is smooth. However, its behaviour is similar to the model with inequality constraint: it displays highly nonlinear curves that feature a transition between a regime with stocks driven not by speculation but by convenience yield to a speculative regime. Planned production or expected price curves are strongly nonlinear when represented with respect to availability, but much more linear when stock is used in $x$-axis (figures 3.1(c) and 3.1(d)).

3.2 Solution methods

3.2.1 General framework

Before detailing the methods used to solve the storage model, we present an overview of how rational expectations, stochastic, discrete time, continuous state models can be approximated. We adopt the framework used in Fackler (2005), and Winschel and Krätzig (2010). For brevity, we ignore models defined by complementarity equations, but the extension is straightforward. A rational expectations model is composed of four kinds of variables: $s$ the state variables, $x$ the response variables, $z$ the expectations and $e$ the shocks. The model is defined by the following three groups of equations

$$s_{t+1} = g(s_t, x_t, e_{t+1})$$  \hspace{1cm} \text{state transition,} \hspace{1cm} (3.8)

$$0 = f(s_t, x_t, z_t)$$  \hspace{1cm} \text{equilibrium,} \hspace{1cm} (3.9)

$$z_t = E_t[h(s_t, x_t, e_{t+1}, s_{t+1}, x_{t+1})]$$  \hspace{1cm} \text{expectations.} \hspace{1cm} (3.10)

For the storage model, the state transition equation is (3.5), and the equilibrium equations are (3.4) and (3.7). The expectations equations are already included in the equilibrium equations, but the expectations variable can be defined as the vector $z_t = [E_t(P_{t+1}e_{t+1}) \ E_t(P_{t+1})]$.

The difficulty of solving this type of models arises from the consistency between expectations and outcomes. Without expectations in equation (3.9) or with exogenous expectations, for example with a backward-looking structure, the method would be to solve (3.9) using a nonlinear solver and to iterate for next period with (3.8). In a stochastic, infinite-horizon model, it is not possible to solve for all possible future situations in order to build consistent expectations. They are several approaches to tackle this problem. A perturbation method considers equations (3.8)–(3.10) as a system and derives a Taylor approximation of the system around its steady state. Projection methods define (3.8)–(3.10) as a functional equation problem and find an approximation of a function
Figure 3.1. Characterisation of the storage model behaviour. Black curves correspond to the model with non-negativity constraint, grey curves to the smoother model with convenience yield. For the price curves (b), the dashed line corresponds to demand for final consumption; total market demand (solid lines) includes demand for storage. For the production rule (c) and the expected price function (d), the plot includes another version of the same functions, but drawn against the decision variable, stock, rather than the state variable, availability. These additional curves show the important differences in nonlinearity depending on the explanatory variable.

allowing to define next-period conditions. Various functions can be used to approximate future conditions. They, however, differs in their degree of nonlinearity and their algorithmic performance,
as we show below with the storage model.

For projection, four functions are approximated in the literature (we note below an approximating function as $\lambda(\cdot)$). (i) The response variables with respect to the state variables: $x_t = \lambda(s_t)$ (implemented, for example, in Fackler, 2005, Winschel and Krätzig, 2010). (ii) The expectations with respect to the state: $z_t = \lambda(s_t)$ (den Haan and Marcet, 1990). (iii) The expectations with respect to the response: $z_t = \lambda(x_t)$ (Wright and Williams, 1984). (iv) When the expectations function, $h$, does not depend on time $t$ variables and on shocks, it can be approximated as a function of next-period state: $h(\cdot, \cdot, s_{t+1}, x_{t+1}) = \lambda(s_{t+1})$ (Miranda and Glauber, 1995). We compare below for the storage model approximations (i), (iii) and (iv).

In addition, when the recursive equilibrium problem can be derived from an optimisation problem, such as a central planner problem, it can be solved using dynamic programming. In this case, equations (3.8)–(3.10) can be seen as the first-order conditions of

$$V(s_t) = \max_{x_t} \{b(s_t, x_t) + \beta E_t[V(s_{t+1})]\}$$

subject to (3.8).

### 3.2.2 Existing approaches to solving the storage model

The seminal paper by Gustafson (1958a) proposes several solution methods: iterating the value function, iterating the marginal value function, and approximating the storage rule by a piecewise linear function, but he carries his work with value function iteration alone. Gardner (1979a) extends Gustafson’s value function iteration method to a case with elastic supply (for an early review of the dynamic programming approach to the storage problem, see Plato and Gordon, 1983). Value function iteration is known to be a reliable method. It has convergence properties based on dynamic programming theory, but it is slow and, since it solves a central planner problem, it does not enable finding a suboptimal competitive equilibrium.

Two more recent methods are generally used to solve the storage model. Wright and Williams (1982a) were the first to propose a solution based on Euler equations. They use low-order polynomials to fit expected price to current stock. They believe that low-order polynomials provide sufficient precision because the expected price function is smooth and not too nonlinear (the small plot in figure 3.1(d)). They apply their method to various settings, such as a storage-trade model, a model featuring the interaction of public buffer-stock and private stock, and a storage model with news arrival (Williams and Wright, 1991). This numerical strategy is applied in several works on the storage model, such as Miranda and Helmberger (1988), Lence and Hayes (2002), and Park (2006).

Another method, which has two alternative forms, was proposed by Deaton and Laroque (1992) (a method similar to that applied to optimal consumption problems under liquidity constraints by Deaton, 1991) and Miranda and Glauber (1995). Both forms approximate the current price function.
The first solves the system using a fixed-point algorithm; the second applies a collocation method. Miranda (1997) compares various approximation schemes for this latter method.

The methods described above do not exhaust the possibilities, however. For a model with convenience yield, Judd (1998) suggests direct approximation of the storage rule, but does not apply it. Approximating decision rules is a usual method for solving the stochastic growth model (Aruoba et al., 2006) and other dynamic models. The method is often used because it allows direct simulations once decision rules are identified. It is generally not used for the storage model because the storage rule is kinked, which makes it difficult to approximate. However, its limited accuracy could be compensated by higher speed; therefore, in the present chapter we implement decision rules approximation.

Newbery and Stiglitz (1982a) propose to expand the storage rule as a Taylor series at the kink point. We adopt a similar method in this chapter. The model is approximated at low orders around its steady state. The use of a Taylor expansion is restricted to smooth functions, so we apply it only to the case with convenience yield. This method, the perturbation method, is commonly used with the stochastic growth model.

With the exception of the perturbation methods, all the methods applied in this chapter share the same set of numerical issues: how to pass from an integral over $\epsilon$ to a finite dimensional problem, i.e., how to discretise the shocks; how to solve the first-order conditions (3.4) and (3.7), which present specific problems caused by the complementarity condition; and which method to use to approximate the unknown functions. Following the treatment of these common issues, we present the different methods.

### 3.2.3 Shocks discretisation

Productivity shocks $\epsilon$ are discretised and the integrals over the shocks are calculated using a 7-node Gauss-Hermite quadrature, allowing the exact integration of degree 15 polynomials weighted by a normal distribution. The Gaussian formula transforms an expectation term into a sum. For example, next-period expected price expressed as a function of next-period availability is approximated as

$$E[P(A)] = E[P((1 - \delta)S + He)] \approx \sum_{l=1}^{L} w_lP((1 - \delta)S + He_l),$$

with $\epsilon_l$ and $w_l$ the nodes and weights of the quadrature.

### 3.2.4 Solving the equations

The equilibrium equations of competitive storage model reduce to two equations: the storage arbitrage condition (3.7) and the producer incentive equation (3.4). The storage arbitrage condition takes a complementarity form when marginal storage costs are constant. This is not a traditional equation that can be solved with a regular nonlinear solver. For positive stocks levels, it behaves as
does a traditional equation with stock level adjusting to ensure the nullity of marginal profit. But when stock hits the zero lower bound, stock level does not adjust any more and marginal profit is allowed to become negative. We solve this complementarity problem by using the \texttt{ncpsolve} solver (Miranda and Fackler, 2002).\footnote{For more complex complementarity problems, the use of more robust solvers can be contemplated. \textsc{Path} (Dirkse and Ferris, 1995) and \textsc{LMCP} (Kanzow and Petra, 2004, 2007) are both good candidates for this task.} When the inequality constraint is smoothed by the convenience yield hypothesis, the problem can be solved using any of the traditional nonlinear solvers.\footnote{Even if smoother this problem is not simple to solve. Indeed, the logarithm does not tolerate negative storage level and the solver must be prevented from exploring this region.} In our case, we use also the \texttt{ncpsolve} solver, which can also solve simple nonlinear problems. Equations are solved with a precision of $10^{-9}$.

Given the simplicity of the problem, we could avoid having to use a complementarity solver. The two equations can be solved by any nonlinear solver and, once a solution is found, the positivity of storage can be checked. For negative storage, we would force it at zero and solve for the planned production. This is the approach taken by Williams and Wright (1991, p. 83). However, this is not advisable for a problem with more state variables, such as a multi-commodity model or a storage-trade model. A complementarity solver introduces only limited overheads compared to a traditional nonlinear solver, so we prefer this more direct solution.

With inelastic supply, it is necessary to solve only the storage arbitrage condition. The problem can be written as a fixed-point problem and solved through simple operations without the need for solvers (Deaton and Laroque, 1992, Judd, 1998). Deaton and Laroque (1992) propose this solution and it has been applied in most econometric work on the storage model because of its speed (Chambers and Bailey, 1996, Michaelides and Ng, 2000, Osborne, 2004).

### 3.2.5 Approximation methods

The first numerical methods for the storage model involve approximating the value function by space discretisation (Gustafson, 1958a, Gardner, 1979a). Later, Wright and Williams (1984) use low-order polynomials to approximate conditional expectations. The polynomials are fitted to grid values by ordinary least-squares. Miranda and Glauber (1995) solve the storage model using Chebychev collocation methods.

The virtues of the various approximation schemes used to study dynamic economic problems have been discussed by various authors (see, e.g., Judd, 1992, 1998, Christiano and Fisher, 2000). The choice simply reduces to splines versus Chebychev polynomials. In this context, the storage model presents a specific difficulty. Because of the non-negativity constraint on stocks, most functions of the model present a kink. They are continuous, but not differentiable (see figure 3.1), which creates problems for both interpolation schemes and requires a high number of free parameters to achieve a satisfactory level of precision. The difficulty is greater with polynomials, since the discontinuity affects the entire interpolation interval (Miranda and Fackler, 2002), which means that, in this case, splines should behave better. For the storage model with convenience yield, which has no
kink, Miranda (1997) compares various approximation schemes. He finds that approximation by cubic splines is preferable to the other approaches. We follow his result and use only cubic splines interpolation.

The interpolation is done with the Matlab CompEcon toolbox, developed to accompany Miranda and Fackler (2002). Functions defined over storage are approximated over the interval \([0, 0.5]\) \(([2.2 \times 10^{-16}, 0.5]\) with convenience yield). Functions defined over availability are approximated over the interval \([\min(\epsilon_l), 1.7]\). These intervals were chosen to include the values we are most likely to find during a simulation. They are determined by tâtonnement until the smallest intervals including most points inside the empirical distribution were found. The bounds play a crucial role in the precision achieved by an algorithm. Since most of the distribution is included in these intervals, enlarging them can only decrease precision.

Values outside intervals may be encountered during the resolution of the functional equations problem, and, in these cases, there is no extrapolation: values are taken as approximations of the last points of the interval. For all methods except perturbation where we rely on a separate software, convergence to the true approximating function is assured by a successive approximation algorithm.

Algorithm performance and convergence depend greatly on the initial guess about the approximated function. Our initial guess is based on the corresponding functions at steady-state values. When price or expected price are approximated, we correct the first guess for the expected effects of storage. Storage will occur in periods of low prices and, thus, will prevent situations of extremely low prices. We, therefore, restrict the first guess to values above 70% of the steady state.

Approximation methods make it possible to transform an infinite-dimension problem (finding a function satisfying some conditions over a continuum) into a finite dimension problem (finding the values satisfying the conditions at some nodes and approximate between the nodes). The function \(f(\cdot)\)—to be defined later for each method—is approximated by \(f_n(\cdot)\): \(f(x) \approx f_n(x) = \sum_{j=1}^{J} \theta_j^n B_j(x)\). \(\theta_j^n\) are basis coefficients to be determined and \(B_j(\cdot)\) are the basis functions of a cubic spline (Judd, 1998). At each iteration of \(n\), the basis coefficients are updated in a fitting step. Except for endogenous grid method where the coefficients are updated through a least-squares regression, the fitting step consists of solving the linear system of \(J\) equations in the unknown \(\theta_j^n\):

\[
\sum_{j=1}^{J} \theta_j^n B_j(x_i) = f(x_i) \quad \text{for } i = 1, \ldots, J, \tag{3.13}
\]

with \(\{x_1, x_2, \ldots, x_J\}\), the collocation nodes.
3.2.6 Value function iteration

The model can be re-cast as a planner problem (Scheinkman and Schechtman, 1983). Its recursive dynamic formulation is

\[
V(A_t) = \max_{S_t \geq 0, H_t} \int_D^{A_t - S_t} P(D) \, dD - \Phi(S_t) - \Psi(H_t) + \frac{1}{1+r} E_t V( (1 - \delta) S_t + H_t \epsilon_{t+1}),
\]

(3.14)

where \( V(\cdot) \) is the value function of the problem. When \( \bar{D} \) is a positive number, \( \int_{A_t - S_t}^{A_t} \bar{D} P(D) \, dD \) is gross consumer benefit. Without any loss of generality, we make \( \bar{D} \) sufficiently high such that \( \int_{A_t - S_t}^{A_t} \bar{D} P(D) \, dD = (A_t - S_t)^{1+1/e} / (1 + 1/e) \).

To solve by value function iteration (VFI), we define a grid on availability, \( \{A_1, A_2, \ldots, A_J\} \). For a given value function, it is necessary to determine optimal storage and planned production by solving the two arbitrage conditions. The value function approximation can be updated using these optimal decisions. We stop the iterations when changes in the value function decrease below a threshold. The algorithm runs as follows:

1. Initialise by taking the first guess: \( V_0(A) = A^{(1+1/e)} / (1 + 1/e) - \Psi(H^{SS}) \), with \( V_0(\cdot) \) a \( J \)-breakpoint spline.

2. For each \( A_i \) in \( \{A_1, A_2, \ldots, A_J\} \), find \( S_i \) and \( H_i \) that solve the first-order conditions

\[
S_i \geq 0 \quad \downarrow \quad -P(A_i - S_i) - \Phi'(S_i) + \frac{1 - \delta}{1+r} \sum_{l=1}^L w_l V'_n((1 - \delta) S_i + H_i \epsilon_l) \leq 0, \quad (3.15)
\]

\[
- (1 + r) \Psi'(H_i) + \sum_{l=1}^L w_l \epsilon_l V'_n((1 - \delta) S_i + H_i \epsilon_l) = 0. \quad (3.16)
\]

3. Update the \( J \)-breakpoint spline \( V_{n+1}(\cdot) \) using the system of \( J \) equations:

\[
V_{n+1}(A_i) = \frac{(A_i - S_i)^{1+1/e}}{1 + 1/e} - \Phi(S_i) - \Psi(H_i) + \frac{1}{1+r} \sum_{l=1}^L w_l V_n((1 - \delta) S_i + H_i \epsilon_l)
\]

for \( i = 1, \ldots, J \).

\[
V_n((1 - \delta) S_i + H_i \epsilon_l) \text{ and its derivative are computed by interpolation.}
\]

4. If \( \|V_{n+1}(A) - V_n(A)\|_2 \geq 10^{-7} \) then increment \( n \) to \( n + 1 \) and go to step 2.

Comparing this crude dynamic programming algorithm with the projection methods that follow is unfair, since there are several procedures that could accelerate a value function iteration algorithm (Rust, 1996). But, given VFI is limited to models amenable to a planner problem, a deeper analysis of this algorithm does not seem relevant.

\[\text{Variables with the superscript } SS \text{ are the deterministic steady-state values.}\]
3.2.7 Projection methods

Projection methods are the most standard methods applied to the storage model (Wright and Williams, 1982a, Miranda and Helmberger, 1988). Its use was formalised in economics by Judd (1992) and consists of defining an approximation scheme and using it to minimise a residual function that defines the rational expectations equilibrium. Storage model is a perfect test-case for projections methods, since they can be applied in various ways depending on the functions being approximated. The present chapter demonstrates that choosing the smoothest function for approximation can result in significant precision gains.

Projection methods can differ in terms of which functions they approximate, but the residual is always defined by the two Euler equations (3.4) and (3.7). As it is not possible to make the residual function zero for all points, several possibilities exist for defining objectives. They differ in their definition of the norm over the residual. The Galerkin method forces the inner product of the residual with the basis functions to be zero. The collocation method requires the interpolant to make the residual function equal to zero for all nodes. Collocation only obliges that the model be solved for the same number of points as the number of free parameters in the interpolant, and has proved slightly less precise than Galerkin for the resolution of significantly fewer points (Judd, 1992); thus in this chapter we use collocation for all projection methods, except endogenous grid.

We follow the usual practice in the literature on storage model in implementing the simplest method to converge to the true rational expectations equilibrium, i.e., time iteration. At each iteration step, the new approximation is defined by the application of the current approximation to the next-period problem. The algorithm stops when the function used for two successive periods are almost identical. The collocation conditions could also be approached as a nonlinear problem and solved using a Newton solver.

We present three versions of the projection method, which differ only in the functions they approximate: current price approximation; parameterisation of the expected price; storage rule approximation.

Parameterised current price algorithms

Approximating the relation between price and availability is equivalent to approximating the expectations function, the projection method (iv) in 3.2.1, since the expectations only depend on price and productive shock. The literature proposes two approaches to solving the storage model by parameterising the price function. Deaton and Laroque (1992) propose a fixed-point algorithm, which is very fast because it does not require any rootfinding step, but is applicable only to cases without supply reaction. So it cannot be used here. The other approach is suggested by Miranda and Glauber (1995), who use a projection algorithm with Chebychev collocation.

In this chapter, we implement two algorithms to solve the model with a parameterised price function. First, Miranda and Glauber’s approach, which we call time iteration approach. Second, a fixed-point algorithm, based on the endogenous grid method (EGM) proposed by Carroll (2006),
which is faster than Deaton and Laroque’s algorithm and is applicable to all the situations of interest in this chapter.

THE TIME ITERATION APPROACH  Miranda and Glauber’s algorithm and the value function iteration are almost identical. The former iterates the marginal of the value function—i.e., the price function—rather than the value function. More precise results can be expected because VFI requires a good approximation of both the value function and its derivative.

1. Initialise $g_0(A) = \max \left[ P(A), 0.7P^{SS} \right]$ with $g_0(\cdot)$ a $J$-breakpoint spline.

2. For each $A_i$ in $\{A_1, A_2, \ldots, A_J\}$, find $S_i$ and $H_i$ that solve the first-order conditions

\begin{align*}
S_i & \geq 0 \quad \perp -P(A_i - S_i) - \Phi'(S_i) + \frac{1-\delta}{1+r} \sum_{l=1}^{L} w_l g_n \left( (1-\delta) S_i + H_i \epsilon_l \right) \leq 0, \quad (3.18) \\
- (1 + r) \Psi'(H_i) + \sum_{l=1}^{L} w_l \epsilon_l g_n \left( (1-\delta) S_i + H_i \epsilon_l \right) & = 0. \quad (3.19)
\end{align*}

3. Determine the $J$-breakpoint spline $g_{n+1}(\cdot)$ that solves the system:

$$g_{n+1}(A_i) = P(A_i - S_i) \quad \text{for} \quad i = 1, \ldots, J. \quad (3.20)$$

4. If $\|g_{n+1}(A) - g_n(A)\|_2 \geq 10^{-7}$ then increment $n$ to $n + 1$ and go to step 2.

THE ENDOGENOUS GRID METHOD  Carroll (2006) proposes a very efficient method for solving small-scale dynamic stochastic optimisation problems. The idea is to solve the Euler equation by iterations of the end-of-period decision variable rather than the state variable. This suppresses the need to solve a non-linear equation and replaces the rootfinding step by arithmetic operations. Applied to the model with convenience yield and inelastic supply, it would simply consist of iterating the function $k_n(A)$ on a grid of storage points, $\{S_1, S_2, \ldots, S_M\}$. $k_n(A)$ approximates the current price function, with

$$A_i = S_i + P^{-1} \left( \frac{1-\delta}{1+r} E \left[ k_n \left( (1-\delta) S_i + H_i \epsilon_l \right) \right] - \Phi'(S_i) \right), \quad (3.21)$$

and $k_{n+1}(A_i) = P(A_i - S_i)$. The grid $\{A_1, A_2, \ldots, A_M\}$ on which the function is approximated changes at each iteration, hence the name endogenous grid method. Since there is no rootfinding step, we do not need the derivative of the function $k_n$. It can, thus, be approximated by linear interpolation instead of spline, which helps further speed the method.
Carroll (2006) explains also how the method can be applied to problems with inequality constraints. One needs only to change the iteration step in $k_n$ and make sure that the grid on $S$ includes zero:

$$k_{n+1}(A) \text{ approximates } \begin{cases} P(A_i - S_i) & \text{for all grid points,} \\ P(A) & \text{for } A \leq \min(A_i). \end{cases}$$

As soon as there is more than one decision variable, the methods becomes trickier. Barillas and Fernández-Villaverde (2007) extend the method to a case with two decision variables. As these decision variables must be mutually consistent, it is not possible to define a grid on both variables. The idea is to define the grid on one decision variable and to find the other by solving the corresponding first-order equation. In our case, we define a grid on storage, solve equation (3.4) for planned production, and compute availability and price with (3.21). We lose one of the interests of EGM, i.e., avoiding rootfinding step, but in this case rootfinding is quite simple and we can accelerate the algorithm by applying it every two or three iterations. The algorithm runs as follows:

1. Initialise $k_0(A) = \max\{P(A), 0.7P^{SS}\}$ with $k_0(\cdot)$ a $J$-breakpoint spline.

2. For each storage level $S_i$ in $\{S_1, S_2, \ldots, S_M\}$ find $H_i$ that solve the Euler equation for production

\[-(1 + r) \Psi'(H_i) + \sum_{l=1}^{L} w_l \epsilon_l k_n((1 - \delta) S_i + H_i \epsilon_l) = 0. \tag{3.22}\]

To accelerate the algorithm, this step can be applied every two or three iterations.

3. For each combination $\{S_i, H_i\}$ calculate $A_i$ with

$$A_i = S_i + P^{-1}\left(\frac{1 - \delta}{1 + r} \sum_{l=1}^{L} w_l k_n((1 - \delta) S_i + H_i \epsilon_l) - \Phi'(S_i)\right). \tag{3.23}\]

4. Determine the $J$-breakpoint spline $k_{n+1}(\cdot)$ that best fits (in a least-squares meaning) the points $\{A_i, P(A_i - S_i)\}$ and the function $P(A)$ for $A < \min(A_i)$.

5. If $\|k_{n+1}(A) - k_n(A)\|_2 \geq 10^{-7}$ then increment $n$ to $n + 1$ and go to step 2.

Unlike the other methods, we take more grid points for the EGM than the number of breakpoints in the approximating function. Consequently, the approximation is updated by a least-squares step. This is required by the fact that the definition of the spline requires at least one observation between two breakpoints, which cannot be guaranteed if we take as many observations as breakpoints because breakpoints and grid points are chosen independently.

Since both the simple time iteration approach and the endogenous grid method approximate the relation between price and availability, they lead to close precision performances, except for the small differences introduced by the fitting step (collocation or least squares). In the following, thus, they are only distinguished with respect to their computing time.
**Parameterised expectations algorithm**

The parameterised expectations algorithm (PEA) approximates the expectations that appear in the Euler equations. Popularised in the macroeconomics literature by den Haan and Marcet (1990), this algorithm was originally developed by Wright and Williams (1982a) for the storage model.\(^4\) Christiano and Fisher (2000) note that the two methods are slightly different. Den Haan and Marcet approximate conditional expectations with respect to the state variables when Wright and Williams approximate conditional expectations with respect to the current-period decision.\(^5\)

In the case of the storage model, the den Haan and Marcet PEA would approximate
\[ E(P_{t+1}|A_t) \]
whereas Williams and Wright parameterise
\[ E(P_{t+1}|S_t) \].

The latter function should be smoother than the former, because expected price with respect to availability shows two regimes with a kink, and is quite nonlinear. For low availabilities, the expected price is constant. It is the stockout case. After a threshold, expected price decreases with current availability. The relation between expected price and storage is smoother and more linear, as it does not include the first stockout regime (compare the two plots in figure 3.1(d)). This difference was already appreciated in the case of the stochastic growth model with irreversible investment in Christiano and Fisher (2000). In what follows, we implement only Wright and Williams’s approach.

The algorithm below is adapted from Wright and Williams (1984). This method requires approximation of two functions: expected price with respect to storage, \( f_S^n(S) \approx E(P|S) \), and producer incentive price with respect to storage, \( f_H^n(S) \approx E(P\epsilon|S) \).\(^6\)

1. Initialise
\[ f_H^0(S) = E\{\max\left[ P\left( (1-\delta)S + H^{SS}\epsilon - S^{SS} \right), 0.7P^{SS} \right] \epsilon \} \]
and
\[ f_S^0(S) = E\{\max\left[ P\left( (1-\delta)S + H^{SS}\epsilon - S^{SS} \right), 0.7P^{SS} \right] \}, \]
both are \( J \)-breakpoint splines.

2. For each storage level \( S_i \) in \( \{S_1,S_2,\ldots,S_J\} \) and shocks \( \epsilon_l \) define the availability
\[ A_{i,l} = (1-\delta)S_i + \left( \Psi' \right)^{-1} \left( f_H^n(S_i) / (1+r) \right) \epsilon_l. \] (3.24)

3. For each element \( A_{i,l} \) solve the storage Euler equation to find \( S_{i,l} \)
\[ S_{i,l} \geq 0 \quad \perp \quad \frac{1-\delta}{1+r} f_S^n(S_{i,l}) - P(A_{i,l} - S_{i,l}) - \Phi'(S_{i,l}) \leq 0. \] (3.25)

---

\(^4\) They describe the algorithm in Wright and Williams (1984).

\(^5\) The two versions differ also in the method used to compute the approximation. Wright and Williams use a projection method, and den Haan and Marcet implement a stochastic simulation approach, dubbed by Judd et al. (2009) as simulation-based PEA. Stochastic PEA may be more intuitive than its projection counterpart, but it is less accurate and its convergence is not warranted, so it is not implemented here.

\(^6\) Wright and Williams (1984) approximate directly the value of planned production instead of the producer incentive price. The two schemes are equivalent, but to follow the logic of PEA we parameterise the expectation term.
4. Determine the $J$-breakpoint splines $f_{n+1}^S(\cdot)$ and $f_{n+1}^H(\cdot)$ that solve the systems

\[f_{n+1}^S(S_i) = \sum_{l=1}^{L} w_l P (A_{i,l} - S_{i,l}) \quad \text{for } i = 1, \ldots, J, \quad (3.26)\]

\[f_{n+1}^H(S_i) = \sum_{l=1}^{L} w_l \epsilon_l P (A_{i,l} - S_{i,l}) \quad \text{for } i = 1, \ldots, J. \quad (3.27)\]

5. If $\|f_{n+1}^S(S) - f_n^S(S)\|_2 \geq 10^{-7}$ or $\|f_{n+1}^H(S) - f_n^H(S)\|_2 \geq 10^{-7}$ then increment $n$ to $n + 1$ and go to step 2.

We expect this method to be more precise than the others, since the conditional expectations are smoother than most of the other functions to be approximated in the model. However, this method requires two conditional expectations to be approximated, each corresponding to a different decision variable. The parameterised current price approach or VFI, on the other hand, are more parsimonious, since they require only one function approximation.

**Storage rule approximation**

Although widely used with other dynamic models, decision rules approximation has not been applied to solving the storage model. The method was suggested in Judd (1998) for the case with convenience yield, but here we apply it to both cases: convenience yield and non-negativity constraint. In a time iteration approach, the approximated storage rule is used inside the expectation terms to define next-period storage. Storage undertaken at the current period, found by solving the first-order conditions over a grid of availability points, allows the approximation to be updated. The production rule is not necessary to define the rational expectations equilibrium.

1. Initialise $s_0(A) = 0$ with $s_0(\cdot)$ a $J$-breakpoint spline.

2. For each $A_i$ in \{$A_1, A_2, \ldots, A_J$\}, find $S_i$ and $H_i$ that solve the first-order conditions

\[S_i \geq 0 \quad \downarrow \quad -P (A_i - S_i) - \Psi'(S_i)\]

\[+ \frac{1 - \delta}{1 + r} \sum_{l=1}^{L} w_l P ((1 - \delta) S_i + H_i \epsilon_l - s_n ((1 - \delta) S_i + H_i \epsilon_l)) \leq 0, \quad (3.28)\]

\[- (1 + r) \Psi'(H_i) + \sum_{l=1}^{L} w_l \epsilon_l P ((1 - \delta) S_i + H_i \epsilon_l - s_n ((1 - \delta) S_i + H_i \epsilon_l)) = 0. \quad (3.29)\]

3. Update the approximation by finding the $J$-breakpoint spline $s_{n+1}(\cdot)$ that solves the system

\[s_{n+1}(A_i) = S_i \quad \text{for } i = 1, \ldots, J. \quad (3.30)\]
4. If $\|s_{n+1}(A) - s_n(A)\|_2 \geq 10^{-7}$ then increment $n$ to $n + 1$ and go to step 2.

The storage rule approximation method can also be solved by fixed-point iteration, as suggested in Judd (1998) or by EGM.

3.2.8 Perturbation methods

For the storage model, perturbation methods have been barely considered. Miranda (1997) tries linearisation of the decision rules, but not higher-order approximations. In other settings, particularly dynamic stochastic general equilibrium modelling, perturbation methods are widely used because they allow one to work with a large number of state variables and are easy to implement through user-friendly packages such as *Dynare*. The method consists of approximating the dynamic problem around the steady state by its Taylor development, which requires a smooth problem.

The storage problem is normally not amenable to such methods because of the non-negativity constraint. The introduction of convenience yield smooths the model and allows the use of perturbation. This transformation to a smooth problem from a problem with inequality constraint is not uncommon in numerical methods, where it is called the penalty function or barrier method (Judd, 1998, p. 123–125). It is used in the macroeconomic context by Preston and Roca (2007) and Kim et al. (2010) to smooth the borrowing constraint in a consumption/savings decision.

The storage model with convenience yield is approximated around its steady state from the first to the fifth order by using the software *Dynare++*, included in *Dynare* v. 4.1.3.\(^7\)

3.3 Numerical results

A benchmark solution is used to generate the true distribution of the state variable. The benchmark solution is the PEA with 5,000 nodes. The benchmark simulation is defined over 10,000 periods. The same shocks are used for both versions of the model.

3.3.1 Euler equation error

Since the storage model has no analytical solution, we cannot compare the approximate solutions obtained from the various methods with an exact benchmark. For this case, Judd (1992) designed a bounded rationality measure, the Euler equation error, to measure by how much solutions violate the optimising conditions. This measure defines the resources wasted by using approximated decision rules instead of true ones. In what follows, we derive the definitions of the two measures of the Euler equation error for the storage model. This differs from the literature on numerical methods in that the measure has to account for the existence of two regimes in the Euler equation.

\(^7\) http://www.dynare.org
We can define the Euler equation error function with convenience yield as

\[ S(A_t) \geq 0 \perp \frac{1 - \delta}{1 + r} E_t \left[ P \left( \left( 1 - \delta \right) S(A_t) + H(A_t) \epsilon_{t+1} - S \left( \left( 1 - \delta \right) S(A_t) + H(A_t) \epsilon_{t+1} \right) \right) \right] - P(A_t - S(A_t)) - \Phi'(S(A_t)) \leq 0. \]  \tag{3.31}

To simplify the notations in the following equations, we use \( A_{t+1} = (1 - \delta) S(A_t) + H(A_t) \epsilon_{t+1} \). We need to distinguish two situations. First, when storage costs include a convenience yield, the Euler equation holds exactly without a complementarity condition. This leads to the following Euler equation

\[ P(A_t - S(A_t)) = \frac{1 - \delta}{1 + r} E_t \left[ P(A_{t+1} - S(A_{t+1})) \right] - \Phi'(S(A_t)). \]  \tag{3.32}

As the decision rules are approximations, this equation will not hold exactly for all availabilities. We can define the Euler equation error function with convenience yield as

\[ EE_S(A_t) = 1 - \frac{P^{-1} \left( \frac{1 - \delta}{1 + r} E_t \left[ P(A_{t+1} - S(A_{t+1})) \right] - \Phi'(S(A_t)) \right)}{A_t - S(A_t)}. \]  \tag{3.33}

Second, for the case without convenience yield, we can rewrite the Euler equation as

\[ P(A_t - S(A_t)) = \max \left\{ P(A_t), \frac{1 - \delta}{1 + r} E_t \left[ P(A_{t+1} - S(A_{t+1})) \right] - \Phi'(S(A_t)) \right\}. \]  \tag{3.34}

This formulation displays the two regimes. For low next-period expected price, there is no storage, and the current price is given simply by the inverse demand function applied to current availabilities. For positive storage, current price is linked to next-period price. As the decision rules are approximations, this equation does not hold exactly for all availabilities. We can define the Euler equation error as

\[ EE_S(A_t) = 1 - \frac{P^{-1} \left( \max \left\{ P(A_t), \frac{1 - \delta}{1 + r} E_t \left[ P(A_{t+1} - S(A_{t+1})) \right] - \Phi'(S(A_t)) \right\} \right)}{A_t - S(A_t)}. \]  \tag{3.35}

As \( D_t = A_t - S(A_t) \), this error can be interpreted as a unit-free representation of the amount of commodity consumption that should adjust to make the Euler equation hold exactly.

The error for the production Euler equation is derived in the same way:

\[ EE_H(A_t) = 1 - \frac{\left( \Phi' \right)^{-1} \left( \frac{1}{1 + r} E_t \left[ P(A_{t+1} - S(A_{t+1})) \right] \right)}{H(A_t)}. \]  \tag{3.36}

It is equivalent to the level of production that should be adjusted to make the Euler equation hold.

We consider two measures of the Euler equation error, its maximum and its average. Its maximum is taken over an interval of availability defined by the first and the last percentiles of the benchmark simulation. The maximum error is useful to identify the algorithm limits. Average error is the error arising from the use of decision rules for a sequence of decisions. We calculate it by averaging
absolute Euler equation errors over the benchmark simulation. To facilitate their reading, the errors are presented in a base-10 logarithm. A result of $-3$ for $\log_{10}|EE_S|$ should be read as a $\$1$ error in every $\$1,000$ of commodity consumption; $-6$, $\$1$ error for $\$1,000,000$ of consumption.

### 3.3.2 Limitations of perturbation methods

Because perturbation methods perform significantly worse than other methods, we give them special treatment in order to understand why they are of limited interest in this case whereas it is the method of choice in several other settings. Figure 3.2 presents the storage rules obtained by perturbation methods, at various orders, and compares them with the true one, obtained using the benchmark method. A figure for the production rule would be similar, so is not displayed here.

![Figure 3.2. Storage rules obtained with perturbation methods up to the 5th order.](image) Density of availability from the benchmark model above the plot. The “True” storage rule is generated by the benchmark method. Deterministic steady state is the state reached in the absence of shocks and ignoring future shocks. Risky steady state is the fixed point of the stochastic problem in the absence of shocks, i.e., the state reached in the absence of shocks but accounting for the likelihood of future shocks (Coeurdacier et al., 2011).

---

8 They are also treated apart because the storage rules found by perturbation are negative for some availability. The Euler equation error, used to assess the precision of the other methods, is not defined for negative storage, because of the logarithm in $\Phi'(\cdot)$. 
Given the nonlinearity of decision rules, a first-order perturbation could not be expected to yield an appropriate approximation. For low availability, the first-order storage rule proposes negative storage and presents a marginal propensity to store current availability much lower than the true rule. These are the classical properties of the certainty equivalent solution to the consumption/saving model under uncertainty (Zeldes, 1989). The certainty equivalent means that the variance of future shocks does not matter. The first-order storage rule passes by the deterministic steady state, which is not part of the true storage rule, because, under certainty, there is no speculative motive for storing, only a convenience yield. Beyond first-order perturbation, the approximated decision rules follow the true one quite closely for availability between 0.94 and 1.16, which corresponds to 69% of the distribution. Below or beyond these bounds, approximations are very poor. Odd-order approximations present negative storage for high availability. And, despite the fact that Blanchard-Kahn conditions are satisfied, 4th and 5th order solutions are unstable. They are locally stable around the steady state, but lose stability farther away from it. The best precision is achieved with second-order approximation. This degradation of precision with perturbation order was identified by Feigenbaum (2005) for the consumption problem and by Aruoba et al. (2006) for one parameterisation of the stochastic growth model.

There are two problems related to perturbation methods applied to the competitive storage model. Even smoothed by a penalty function, the storage model remains highly non-linear, which calls for high-order perturbation. As perturbation methods are in essence local methods, they should behave correctly around the steady state, which is what we observe. But the typical shocks in agricultural production drive the behaviour in regions distant from the steady state. This work confirms some recent evidence that higher-order perturbation may not be well-behaved. Feigenbaum (2005) and den Haan and de Wind (2009) note that a drawback of perturbation methods is the lack of control over the radius of convergence. In this case, we see that the radius of convergence covers only a small part of the availability distribution. The use of a logarithm as the barrier function may have increased the lack of convergence of the Taylor development, because its radius of convergence is limited. But den Haan and de Wind (2009) show that this problem arises also with a well behaved barrier, such as an exponential.

### 3.3.3 Comparison of methods precision

Before comparing methods, it is useful to assess the relation between storage and production Euler equation errors, and the precision achievable from the most precise method. Table 3.3 presents detailed results for the PEA—proved below to be the most accurate algorithm—when the number of breakpoints in the spline approximation varies.

The production Euler equation error is systematically lower than the storage Euler equation error. However, they are close and vary in the same way. Augmenting the number of nodes improves the precision slowly for the model with non-negativity constraint. The general shape is well approximated at a low-order (for information, the papers of Williams and Wright that have derived
most of theoretical properties of the storage model used an order-5 polynomial). With convenience yield, the expected price approximation improves faster with the number of breakpoints. Based on these first results, we decide to restrict presentation of the subsequent results to the storage Euler equation error and to a 20-breakpoint approximation.

Table 3.4 presents the precision achieved by the various methods. The PEA is consistently more precise than other methods. Current price approximation and storage rule approximation achieve very close precision. This is confirmed at various orders in figure 3.3. Value function iteration performs similarly to storage or price approximation. For all methods, the logarithm of the Euler equation error decreases linearly with the logarithm of the approximation order. The mean slope is $-2$, which means a 10-fold increase in the number of breakpoints leads to an error decrease of 2. If at 20 breakpoints, the average error is $1$ in every $1,000$; at 200 breakpoints it will be $1$ in every $100,000$. Whatever the approximation level, PEA is the most precise method, with always one order of precision below the other methods.

### 3.3.4 Simulating the Model

We now address an issue often neglected in computational economics, which is how the model should be simulated. They are two principal approaches. The first, and most commonly used, consists in approximating the decision rules and simulating the model by applying them recursively. The second uses the function being approximated (the decisions rules, the conditional expectations or the expectation function as shown in section 3.2.1) to approximate next-period expectations and

<table>
<thead>
<tr>
<th>Order</th>
<th>Storage Euler equation</th>
<th>Production Euler equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max error</td>
<td>Average error</td>
</tr>
<tr>
<td>With non-negativity constraint</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$-2.88$</td>
<td>$-3.20$</td>
</tr>
<tr>
<td>10</td>
<td>$-2.80$</td>
<td>$-3.57$</td>
</tr>
<tr>
<td>20</td>
<td>$-3.20$</td>
<td>$-4.20$</td>
</tr>
<tr>
<td>50</td>
<td>$-3.91$</td>
<td>$-5.14$</td>
</tr>
<tr>
<td>100</td>
<td>$-4.25$</td>
<td>$-5.76$</td>
</tr>
<tr>
<td>200</td>
<td>$-4.23$</td>
<td>$-6.27$</td>
</tr>
<tr>
<td>1000</td>
<td>$-5.26$</td>
<td>$-7.79$</td>
</tr>
<tr>
<td>With convenience yield</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$-3.54$</td>
<td>$-3.94$</td>
</tr>
<tr>
<td>10</td>
<td>$-4.68$</td>
<td>$-5.12$</td>
</tr>
<tr>
<td>20</td>
<td>$-6.26$</td>
<td>$-6.87$</td>
</tr>
<tr>
<td>50</td>
<td>$-8.17$</td>
<td>$-8.74$</td>
</tr>
<tr>
<td>200</td>
<td>$-10.28$</td>
<td>$-10.87$</td>
</tr>
<tr>
<td>1000</td>
<td>$-12.30$</td>
<td>$-12.56$</td>
</tr>
</tbody>
</table>
Table 3.4. Storage Euler equation error ($\log_{10} |EE_S|$) for various methods with a 20-breakpoint spline approximation

<table>
<thead>
<tr>
<th>Method</th>
<th>With non-negativity constraint</th>
<th>With convenience yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max error</td>
<td>Average error</td>
</tr>
<tr>
<td>Value function iteration</td>
<td>−3.06</td>
<td>−3.86</td>
</tr>
<tr>
<td>Parameterised expectations</td>
<td>−3.20</td>
<td>−4.20</td>
</tr>
<tr>
<td>Current price approximation</td>
<td>−2.66</td>
<td>−3.24</td>
</tr>
<tr>
<td>Storage rule approximation</td>
<td>−2.66</td>
<td>−3.24</td>
</tr>
</tbody>
</table>

solve the equilibrium equations to find the current decisions (in a time-iteration approach). For example, if applied to the approximation of the conditional expectations ($z_t = \lambda(x_t)$), this method would solve for $x_t$ the following equation

$$0 = f(s_t, x_t, \lambda(x_t)). \quad (3.37)$$

The two approaches differ in speed and in precision. Simulating a model through the recursive application of approximated decision rules is very fast since it does not require any nonlinear solve. The second approach is at least 10 times slower in the case of the storage model since each period requires solving a system of complementarity equations. However, its precision is much higher, because the approximation is used only for the expectations of next-period conditions. The precision gain is all the more important when decision rules have kinks, which makes them difficult to approximate. Wright and Williams (1984), who first proposed the PEA, suggest using the second
method for simulating the storage model.

Above results were produced by the second, more precise method. In contrast, table 3.5 presents the Euler equation errors achieved when approximating the true decision rules at various orders and using this approximation for simulation. The decision rules approximated by a 20-breakpoint spline generate maximum and average storage Euler equation errors of $-2.04$ and $-2.96$, a level of precision lower than what was achieved previously by explicitly solving the first-order conditions (see table 3.4 for comparison). This applies especially to the maximum error, which decreases very slowly with the number of breakpoints because of the difficulty involved in approximating the kinks.

Table 3.5. The effect of approximation order on precision ($\log_{10} |EE|$) for a spline approximation of the true decision rules, for the model with non-negativity constraint.

<table>
<thead>
<tr>
<th>Order</th>
<th>Storage Euler equation</th>
<th>Production Euler equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max error</td>
<td>Average error</td>
</tr>
<tr>
<td>5</td>
<td>$-1.35$</td>
<td>$-1.82$</td>
</tr>
<tr>
<td>10</td>
<td>$-1.77$</td>
<td>$-2.50$</td>
</tr>
<tr>
<td>20</td>
<td>$-2.04$</td>
<td>$-2.96$</td>
</tr>
<tr>
<td>50</td>
<td>$-2.50$</td>
<td>$-3.88$</td>
</tr>
<tr>
<td>100</td>
<td>$-3.08$</td>
<td>$-4.47$</td>
</tr>
<tr>
<td>200</td>
<td>$-3.14$</td>
<td>$-5.00$</td>
</tr>
<tr>
<td>1000</td>
<td>$-4.05$</td>
<td>$-6.47$</td>
</tr>
</tbody>
</table>

3.3.5 Implementation and computing time

This subsection focuses on the time needed to solve the model. As all the existing econometric work uses the model without supply reaction, we include a comparison of computing time over this more restricted model. Comparison of its Euler equation errors would be less interesting since they are very close to those for the model with supply reaction.

The results reported are for a 2.6 GHz Intel PC running Windows Vista and Matlab R2010b. We would expect a faster solve for a model with inelastic supply, because the model is simpler. This is not the case for all methods (see table 3.6). While the model has one less equation, the function being approximated is farther away from the first guess and so requires more iterations. The kink point of the storage rule appears at a lower availability level when supply elasticity decreases. For a low availability level, it is better to increase future supply by increasing production rather than by increasing stock, because building stocks are expensive. And the lower the kink, the higher will be the number of iterations needed. This higher number of iterations is not compensated for by the fact that iterations are simpler except for VFI and EGM. In the case of EGM, solving the model without supply reaction requires only simple calculations and there is no nonlinear problem to solve. It is very fast. It is the case where this method shows all its interest.

The value function iteration, as expected, is the most time-consuming method. It converges very slowly (several hundred iterations) to rational expectations. EGM is the fastest method. The
Table 3.6. Time (seconds) to solve the rational expectations equilibrium with a 20-breakpoint spline approximation

<table>
<thead>
<tr>
<th>Method</th>
<th>With non-negativity constraint</th>
<th>With convenience yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inelastic supply</td>
<td>Elastic supply</td>
</tr>
<tr>
<td>Value function iteration</td>
<td>3.45</td>
<td>3.62</td>
</tr>
<tr>
<td>Parameterised expectations</td>
<td>0.78</td>
<td>0.44</td>
</tr>
<tr>
<td>Current price approximation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time iteration</td>
<td>0.42</td>
<td>0.36</td>
</tr>
<tr>
<td>Endogenous grid method</td>
<td>0.02</td>
<td>0.26</td>
</tr>
<tr>
<td>Storage rule approximation</td>
<td>0.55</td>
<td>0.47</td>
</tr>
</tbody>
</table>

* For inelastic supply, equation (3.4) is excluded of the model and planned production is fixed at 1, its deterministic steady state.

most precise method, parameterised expectations algorithm, is the slowest for a given approximation order (20 breakpoints). However, this is not to imply that it is slower than other methods for a given precision.

In terms of speed, EGM outperforms other methods by a factor of at least 20 in the case of inelastic supply. The speed of this method makes it a particularly suitable alternative to Deaton and Laroque’s fixed point algorithm, for econometric work. When extended to a situation with elastic supply, EGM loses part of its interest. It requires a nonlinear solve and so loses its very high speed, but still outperforms other methods.

The model with convenience yield is smooth. Its functions are simpler to approximate than those of the model with non-negativity constraint. Hence, it takes less time to solve it; convergence is achieved with fewer iterations.

The time needed to find the rational expectations equilibrium and to simulate the model is only a small part of the time devoted to the model. The time needed to program the solution method is likely the largest part of the whole. In this regard, EGM is very simple to implement in the model with inelastic supply, because it boils down to iterations with arithmetic operations. With elastic supply, it requires a nonlinear solve, so it is comparable to other methods. The various projection methods present the same difficulty of implementation, PEA being less intuitive because of its two nested loops, the first on storage and the second on availability.

3.4 Conclusion

This chapter compared several solution methods for the competitive storage model. Three methods are popular in the related literature: value function iteration, parameterised expectations algorithm, and current price approximation. We introduced three other methods, borrowed from the literature on numerical analysis: perturbation, endogenous grid, and decision rules approximation. Perturbation methods proved to be of limited use in this context, because the storage model, smoothed by the convenience yield hypothesis, is too nonlinear, and the disturbances drive approximation out of its
radius of convergence. The endogenous grid method is a good alternative to Deaton and Laroque’s (1992) algorithm for a fast solve of the model for estimation purpose. One method stands out: the parameterised expectations algorithm, which is very precise, even at small approximation orders, because it approximates smooth and close to linear functions. These results could be generalised to the consumption/saving problem under uncertainty, where the equations are close to the storage model.

Because of modern computational power, any method, except perturbation, would provide a satisfying solution to the competitive storage model, and in reasonable time. Numerical challenges arise when the model is extended to include several state variables, such as in storage-trade interaction or the multiple commodities model. In these settings, the advantages of the parameterised expectations algorithm over other methods may be even greater, because it is more precise, even for a small number of nodes.
Most developing countries pursue some form of food price stabilisation policy. For example, China and India operate very large stabilisation programmes (Dorosh, 2008). In China, government controls most international grain trade and carries large stocks (in the 2000s Chinese stocks of cereals—corn, rice, wheat—represented 40% of world stocks). India operates the world’s largest public food distribution system, covering about 600 million people: government builds stocks from procurements at minimum support prices and sells subsidised rations to the poorest people. The Egyptian government subsidises cooking oil, sugar, bread and flour (Löfgren and El-Said, 2001). Bread is also subsidised in Iran, involving an outlay of 6.5% of government expenses in 2001 (Amid, 2007). These policies have been largely analysed in the food policy literature with general lessons drawn from these experiences. However, the findings are generally qualitative, and few quantitative analyses of food policies have been conducted.

This chapter defines a framework for studying food stabilisation policies in self-sufficient developing countries. We build a model representing the main features of food prices. The model includes a market imperfection justifying public intervention: market incompleteness associated with consumer risk aversion. The model features the behaviours of a competitive storer, a producer exposed to productivity shocks, and a risk-averse consumer. Prices are stabilised through an optimal policy under commitment using as instruments a countercyclical subsidy/tax on planned production and a public stock. This work builds on two existing approaches to commodity price stabilisation: the literature on competitive storage and the works of Newbery and Stiglitz.

Much attention has been paid to the study of commodity price stabilisation schemes (for a survey, see Wright, 2001), with the purpose mainly of analysing agricultural policies, such as deficiency payments or floor-price programmes (Miranda and Helmberger, 1988, Wright and Williams, 1988a, Glauber et al., 1989, Gardner and López, 1996). These works impose exogenous policy rules in the rational expectations storage model. Although very stylised, this model reproduces the most

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1 Its traditional form is presented and analysed in depth by Wright and Williams (1982a).
important features of commodity markets (Cafiero et al., 2011). A setting common to most studies based on this model is perfect market equilibrium, which implies that there is no rationale for public intervention, and stabilising markets beyond what is done by private agents is a waste of public money.

One exception is the work of Gardner (1979b) who considers that periods of high food prices generate external costs that justify public intervention. To increase welfare, he designs a discretionary optimal stockpiling policy. He also discusses price-band policies and analyses the uncertainties that could hinder the implementation of such policies. Unlike Gardner (1979b), the present chapter is more explicit about the source of market failure in considering market incompleteness, which is in line with the abundant literature on the cost of commodity price instability against which consumers are assumed to be unable to insure (Waugh, 1944, Turnovsky et al., 1980, Helms, 1985b, Wright and Williams, 1988b). It also makes a methodological contribution since, in addition to solving the model numerically, it follows the modern literature on optimal dynamic policies (Marcet and Marimon, 1999) in allowing the derivation of first-order conditions for an optimal dynamic policy, which delivers interpretable conditions.

Newbery and Stiglitz’s approach departs from the framework of the infinite-horizon storage model. Newbery (1989) shows that market incompleteness and consumer risk aversion justify government intervention in the form of public storage or distribution of food rations. This study enlarges on his previous work with Stiglitz, which considers the role of public intervention in incomplete commodity markets (Newbery and Stiglitz, 1981, 1982b, 1984). Their work exploits two-period models, which cannot account for the stylised facts of food prices. For example, commodity prices are positively serially correlated (Deaton and Laroque, 1992), so storage management must account for the fact that high prices may be followed by high prices, making it suboptimal to dump all the stock in one period, as would be recommended by Newbery’s model. More generally, two-period models allow a single optimal stockpile size to be identified, whereas any applications based on a realistic framework require an optimal storage rule that stipulates how stock should be accumulated or sold given the prevailing market conditions. Such a storage strategy is the result of an infinite-horizon problem, as in the present chapter.

Our analysis provides three main findings. First, on the transitional dynamics following policy implementation, provided that the budget share of food and risk aversion are sufficiently high, there is an optimal food price stabilisation policy that improves both consumers’ and producers’ welfare. The optimal policy consists of a public storage rule and a subsidy/tax on planned production, which works like a set-aside programme. The management of public stock is such that it removes any profit opportunity from speculation and crowds out all private storage. Second, producers enjoy short-run gains because the launch of the policy is accompanied by a phase of transitional stock build-up which pushes up prices. In the long-run, producers may lose because stabilisation entails lower prices on average. Third, the countercyclical subsidisation of production contributes little to welfare gains, most of which come from stabilisation through public storage.
Section 4.1 defines the stylised facts on food prices and evaluates the potential gains to be expected from a stabilisation policy by calculating the welfare gains from perfect stabilisation. Section 4.2 describes the storage model without public policy. Section 4.3 presents the model under optimal policy. We define a social welfare function that serves as policy objective, and then derive and analyse the first-order conditions of the optimal policy problem. In section 4.4, we calibrate the model and analyse the numerical results, discussing the various decision rules, then separately analysing transitional and asymptotic behaviours. Section 4.5 concludes.

### 4.1 Welfare gains from an ideal stabilisation

Before we consider the effect of a stabilisation policy, this section evaluates the consumer’s gain from perfect stabilisation, i.e., costless stabilisation at a given price. It allows us to quantify the potential welfare gains. This section draws on the works of Turnovsky et al. (1980), Helms (1985a), and Wright and Williams (1988b).

Because stabilisation gains depend heavily on the initial price distribution, some facts about staple prices are presented in table 4.1. The first moments of the price distributions of maize, rice and wheat show similar properties. Detrended cereal prices show a high and positive one-year autocorrelation, between 0.4 and 0.61, meaning that periods of high or low prices tend to come in a row. The coefficient of variation is around 20%. Prices are positively skewed, so the tails of the price distribution are longer for high prices than low prices, and most of the values are below the mean price—a pattern often related to storage. Visually, agricultural prices display long periods of doldrums followed by booms and busts. The model proposed in the next section tries to account for these facts.

<table>
<thead>
<tr>
<th></th>
<th>Maize</th>
<th>Rice</th>
<th>Wheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-year autocorrelation</td>
<td>0.40</td>
<td>0.61</td>
<td>0.57</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.20</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.68</td>
<td>0.98</td>
<td>0.98</td>
</tr>
</tbody>
</table>

*Note:* Statistics obtained after HP-filtering (smoothing parameter of 400) of the logarithm of price indexes deflated by the manufactures unit value index.

*Source:* Underlying data are annual commodity price indexes from Pfaffenzeller et al. (2007).

Modern analysis of the welfare effect of price stabilisation started with Waugh (1944). Using a Marshallian surplus analysis, Waugh demonstrated that price stabilisation makes consumers worse off. This counterintuitive proposition stems from the negative slope in the consumer demand curve, which allows consumers to benefit from low prices by consuming more, and to limit the nuisance of high prices by consuming less. Marshallian consumer surplus analysis, however, rests on the hypothesis of an income elasticity equal to the coefficient of relative risk aversion (Turnovsky et al., 1980). In the absence of this hypothesis, the welfare change from stabilisation should be evaluated...
by the equivalent variation, $EV$, implicitly defined by

$$E[v(P, Y + EV)] = E[v(\bar{P}, Y)], \quad (4.1)$$

with $v(\cdot, \cdot)$, an indirect utility function; $P$, the price before stabilisation; $\bar{P}$, the stabilised price; $Y$, the income; and $E(\cdot)$, the expectation operator. The price after stabilisation follows a probability distribution with lower variance than the price before stabilisation. Using subscripts to denote partial derivatives, the equivalent variation can be approximated (see the appendix for the derivation) at the second order by

$$EV \approx [v_P(\bar{P}, Y) \Delta \bar{P} + v_{PP}(\bar{P}, Y) \Delta \sigma^2_P/2] / v_Y(\bar{P}, Y), \quad (4.2)$$

$$\approx D(\bar{P}, Y) \cdot \{-\Delta \bar{P} + [\gamma (\eta - \rho) - \alpha] \cdot \Delta \sigma^2_P / (2 \bar{P})\}, \quad (4.3)$$

where $\alpha < 0$ and $\eta$ stand for the price and income elasticities of demand; $D(\bar{P}, Y)$ is demand at the mean price, $\bar{P}$; $\Delta \bar{P}$ and $\Delta \sigma^2_P$ are the changes in mean price and variance; and $\gamma$ and $\rho$ are the commodity budget share and the relative risk aversion parameter.

Using Roy’s identity, the second term on the right-hand side of equation (4.2) can be reformulated as $(-v_{PP}/v_P) \cdot (D \Delta \sigma^2_P/2)$, which isolates a term similar to a coefficient of absolute risk aversion, but defined here with respect to price risk. This coefficient is shown in (4.3) to depend on relative risk aversion, commodity budget share, and price and income elasticity.

Waugh’s result is apparent in equation (4.3). For stabilisation at the mean price (i.e., $\Delta \bar{P} = 0$) of a risk-neutral consumer with income inelastic demand ($\rho = \eta = 0$), the equivalent variation is equal to $-\alpha \Delta \sigma^2_P \cdot D(\bar{P}, Y) / (2 \bar{P})$, which is always negative. Risk aversion can compensate for this effect and make stabilisation beneficial but only if the budget share is sufficiently high. It implies that stabilisation at the mean price would be detrimental to consumers from developed countries, since they devote a low share of their budget to staple food.

To extend these qualitative statements, we provide a numerical illustration. The equivalent variation defined in equation (4.3) is calculated in table 4.2 for complete stabilisation of a price distribution with mean 1 and standard deviation 0.2. Demand at mean prices is assumed to be equal to $\gamma Y$. By assuming income to be equal to 100, the equivalent variation can be interpreted as a percentage change in income. Two stabilisation policies are considered: stabilisation at the mean price and stabilisation at 2% below the mean price. The latter situation accounts for the possible depressing effect of storage policies on prices (Wright and Williams, 1982a).

Stabilisation at mean prices is always detrimental to consumers spending 1% of their budget on the volatile commodity, but the welfare loss is negligible given the low budget share for the consumer. It is also detrimental to a risk-neutral consumer. The importance of the loss depends on the budget share and the elasticities, especially the price elasticity. Income elasticity has a limited effect on all the results. In the above approximation, it is supposed to have the same effect as the risk aversion
parameter, but, given its limited range of possible variations,\(^2\) it has little effect on welfare. If we consider a relative risk aversion of 9 and a budget share of 30%, the welfare gains can reach 1.56% of income.

When prices are stabilised at 2% below their mean, the mean price change is responsible for a large share of overall gains. In particular, consumers with low risk aversion or low budget share earn most of their gains from the change in the mean price.

This analysis motivates the fact that in the following we consider a model calibrated to the situation of a developing country where consumers are likely to suffer from price instability of food because of the high budget share they devote to it.

### 4.2 The Model without Public Intervention

Analysis of food price stabilisation policy should take account of the origin of food price volatility and reproduce its main features, as characterised in table 4.1. The model used in most of the work on commodity price stabilisation schemes, such as Miranda and Helmberger (1988) and Wright and Williams (1988a), is the single-country competitive storage model. Cafiero et al. (2011) recently proved that this model performs well for explaining international commodity price behaviour.

Our framework extends this model to account for consumer risk aversion, market incompleteness, and government stabilisation policy. Time is discrete. This partial equilibrium model features an annual market for a storable commodity with a competitive storer, a producer whose output is subject to multiplicative shocks, and a final demand.

\(^2\) In an estimation of food consumption elasticities, Seale and Regmi (2006) find income elasticities for breads and cereals between 0.04 and 0.62 for 114 countries.
Despite the fact that international prices are an important driver of the domestic price in many
countries, our focus is on a closed economy and we assume that domestic productivity shocks are
the only source of dynamics. Agricultural markets are the object of many protectionist policies
(Anderson, 2010), including self-sufficiency (e.g., cereals in China and India), making the autarkic
case a not uncommon situation.

4.2.1 **Consumers**

The economy is populated with risk-averse consumers whose final demand for food has an isoelastic
specification: \( D(P_t, Y) = d P_t^\alpha Y^{\eta} \), where \( d \) is a normalisation parameter. Assuming there are only
two goods and the second good is the numeraire, the integration of this demand function gives the
following instantaneous indirect utility function (Hausman, 1981)

\[
\hat{v}(P_t, Y) = \frac{Y^{1-\eta}}{1-\eta} - \frac{d P_t^{1+\alpha}}{1+\alpha}. \tag{4.4}
\]

This utility function has a relative risk aversion equal to the income elasticity of demand. To
distinguish income elasticity from risk aversion, we follow Helms (1985a) and apply a monotone
transformation to the indirect utility function,

\[
v(P_t, Y) = \frac{\hat{v}(P_t, Y)^{1+\theta}}{1+\theta}. \tag{4.5}
\]

This specification is still consistent with the isoelastic demand function, but its coefficient of relative
risk aversion is

\[
\rho(P_t, Y) = \eta - \theta \frac{Y^{1-\eta}}{\hat{v}(P_t, Y)}, \tag{4.6}
\]

with \( \theta \) indexing the degree of risk aversion.\(^3\)

The representative consumer is assumed to adopt hand-to-mouth behaviour. He consumes current
income and does not save to smooth out fluctuations. This assumption simplifies the dynamics
of the problem, since consumer’s “cash on hand” does not have to be included as a state variable;
however, it overestimates the effect of public policy by neglecting the possibility of self-insurance
added by saving.\(^4\)

Given the absence of saving, the consumer does not solve an intertemporal problem. At each
period, the consumer is concerned only with current-period demand, which is not affected by the
degree of risk aversion. So speculators face the same profit opportunities if the economy is populated

---

\(^3\) Following the same method, other demand curves could have been contemplated. Few, however, have an indirect
utility function with a known form. In such case, the numerical algorithm of Vartia (1983) can be used to find a
numerical approximation of the indirect utility function.

\(^4\) Introducing saving would create important difficulties, such as the need to consider borrowing constraint, which
creates strong nonlinearities (Deaton, 1991), whereas the model proposed below is already strongly nonlinear.
by risk-averse or risk-neutral consumers. This absence of effect of risk aversion on demand creates the need for public intervention, since private storers do not account for it.

4.2.2 Storers

There is a single representative speculative storer, which is risk neutral and acts competitively. Its activity is to transfer a commodity from one period to the next. Storing the quantity $S_t$ from period $t$ to period $t+1$ entails a purchasing cost, $P_t S_t$, and a storage cost, $k S_t$, with $k$ the unit physical cost of storage. The benefits in period $t$ are the proceeds from the sale of previous stocks: $P_t S_{t-1}$.

The storer follows a storage rule that maximises its expected profit as stated by

$$\max_{\{S_{t+i} \geq 0\}, i=0}^\infty \{ \sum_{i=0}^{\infty} \beta^i [P_{t+i} S_{t+i-1} - (P_{t+i} + k) S_{t+i}] \}$$

with $\beta = 1/(1+r)$, the discount factor. The storer’s problem can be expressed in a recursive form using the following Bellman equation:

$$V^S (S_{t-1}, P_t) = \max_{S_t \geq 0} \{ P_t S_{t-1} - (P_t + k) S_t + \beta E_t \{ V^S (S_t, P_{t+1}) \} \}.$$  

This equation has two state variables: the price, whose dynamics is considered by the storer to be exogenous, and the stock carried over from the previous year. Using the first-order condition on $S_t$ and the envelope theorem, and taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), this problem yields the following complementary condition

$$S_t \geq 0 \perp \beta E_t (P_{t+1}) - P_t - k \leq 0,$$

which means that inventories are null when the marginal cost of storage is not covered by expected marginal benefits; for positive inventories, the arbitrage equation holds with equality. So this is a situation of a stabilising speculation, the storer buys when prices are low and when he rationally expects that they will be higher later.

4.2.3 Producers

A representative producer makes his productive choice one period before bringing output to market. He puts in production in period $t$ a level $H_t$ for period $t+1$, but a multiplicative disturbance affects final production (e.g., a weather disturbance). The producer chooses the production level by solving the following maximisation of expected profit:

$$\max_{\{H_{t+i}\}} \{ \sum_{i=0}^{\infty} \beta^i [P_{t+i} \xi_{t+i} H_{t+i-1} - \Psi (H_{t+i})] \}.$$
where \( \Psi (H_t) \) is the cost of planning the production \( H_t \) and \( H_t \epsilon_{t+1} \) is the realised level. \( \epsilon_{t+1} \) is the realisation of an i.i.d. stochastic process of mean 1 exogenous to the producer, which follows a translated beta distribution. Like the storer’s problem, this problem can be reformulated into a recursive form, and its solution gives the following Euler equation:

\[
\beta E_t (P_{t+1} \epsilon_{t+1}) = \Psi' (H_t).
\]

(4.11)

This equation has a straightforward interpretation: it is the equality between the marginal cost of production and the expected discounted marginal benefit of one unit of planned production. The production cost function is assumed to be convex, as increasing production requires increasing the use of less fertile lands, and to follow an isoelastic form

\[
\Psi (H) = h H^{1+\mu} \frac{1}{1 + \mu},
\]

(4.12)

where \( h \) is a scale parameter and \( \mu \) is the inverse of supply elasticity.

4.2.4 Recursive Equilibrium

At the beginning of each period, three predetermined variables define the state of the model: \( S^o_{t-1} \), \( H_{t-1} \) and \( \epsilon_t \). They can be combined in one state variable, availability, the sum of production and private carry-over:

\[
A_t = S^o_t + H_{t-1} \epsilon_t.
\]

(4.13)

Market equilibrium can be written as

\[
A_t = D(P_t) + S^o_t.
\]

(4.14)

From the above, we can define the recursive equilibrium of the problem without public policies:

**Definition 1.** In the absence of stabilisation policy, a recursive equilibrium is a set of functions, \( S^o (A) \), \( H (A) \) and \( P (A) \), defining storage, production and price over the state \( A \) and the transition equation (4.13) such that (i) storer solves (4.7), (ii) producer solves (4.10), and (iii) the market clears.

4.3 Optimal Policy Approach

We assume that government is able to commit at the first period to following a policy rule that maximises expected intertemporal welfare (the so-called Ramsey policy). We design the optimal policy by following the modern literature on optimal policy in dynamic economies (Marcet and Marimon, 1999). As the policy is conceived to correct for market incompleteness, i.e., the inability of

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5 As income is assumed to be constant, in what follows demand function is only expressed as a function of price.
consumers to insure against food price volatility, the first-best policy would be to mimic the role of the futures market by providing state-contingent cash payments to consumers. In the real world, few policies work in this way, but some can be related to this logic. Food and cash-for-work schemes, for example, are labour-intensive work programmes offering in kind or cash payments (Barrett, 2002). They are designed to combat food insecurity and to target those suffering adverse shocks. Other types of policies more frequent in the developing world are food price stabilisation policies. Because of their practical importance, we focus on these second-best policies, which target price behaviour. Here, the stabilisation instruments are subsidy/tax on planned production and public stocks, where government is assumed to be able to manage storage facilities at the same costs as private storers.

The policy starts at period 0 and is unanticipated by the agents. The parameters of the stabilisation policy are determined by maximising a social welfare function that aggregates consumer utility, other agents’ surpluses and fiscal cost. Initial availability is taken as being at its deterministic steady-state level and initial private stocks are assumed to be null.

4.3.1 Social Welfare Function

Working with a partial equilibrium model imposes some constraints. For example, it is not possible to take the expected sum of discounted consumer’s utility as the objective of the policy authority as is often done in optimal policy problems. Instead, all effects on the welfare of other agents must be included in the objective. In a general equilibrium model, there is no need for this since the consumers’ income includes all earnings. The approach adopted here also differs from most partial equilibrium analyses due to its consideration of risk aversion. In partial equilibrium models, policy design is based on maximising the sum of all agents’ surpluses. This does not take account of risk aversion, though, since the expected consumer surplus is not a measure of the welfare of risk-averse agents (Helms, 1985b, Stennek, 1999).

Instead of using the sum of agents’ surpluses, we have to define the preferences of the government. We assume that government tries to maximise a social welfare function that weights linearly the welfare of each agent. We want to leave aside all distributional considerations. For this purpose, the determination of weights has to ensure that transfers between agents do not affect welfare. The weights should remove the transfers and leave in the social welfare function only efficiency gains from stabilisation and the cost of the policy. At period \( t \), social welfare is given by:

\[
W_t = v(P_t, Y) + w_H [P_t H_{t-1} e_t + \zeta_t H_t - \Psi (H_t)] + w_S [P_t S_t^g - (P_t + k) S_t^r] - w_C Cost_t, \tag{4.15}
\]

where \( w_H, w_S \) and \( w_C \) are the weights given in social welfare to producers’ income, storers’ realised profit and fiscal cost; \( \zeta_t \) is a state-contingent subsidy/tax on planned production, received by producers at planting time; and \( Cost_t \) is the fiscal cost of the policy. Since producers and storers are both assumed to be risk neutral, if government has no distributional bias, there is no reason to weight their profits differently, so their weights can be considered equal, \( w_S = w_H \). Defined in
this way, the social welfare function shows that the optimal policy will consist in a risk-sharing arrangement between risk-averse consumers and risk-neutral agents, government in particular.

To rule out transfers between agents, we need to know how the consumer values a monetary transfer in utility terms. For a given price $P$, a small monetary transfer $\tau$ to the consumer increases utility by approximately (at the first order) $v_Y(P, Y) \tau$. Thus, we could use the marginal utility with respect to income as the weight. But, given that the price is stochastic, we have to consider the marginal utility of income over all the price distribution and not just for one price. The computation is based on the ergodic distribution of price without public policies:

$$w_H = E[v_Y(P, Y)].$$

This is the unconditional expectation of marginal utility over income.\(^6\)

Although also a monetary value, fiscal cost does not necessarily have the same weight as agents’ profit. Unless raised by a lump-sum tax, income raised for public expenses entails distortionary costs due to revenue collection. We ignore these costs and assume $w_C = w_H$. This assumption can be removed, but turns out to be convenient as described below.

The cost of the policy is the cost of carrying public stock plus the cost of subsidising planned production. The cost of public storage is similar to the profit from private storage. It is equal to the difference between the purchasing plus the storage costs of new stock and the revenue derived from selling previous stock, $S_{t-1}^{cs}$:

$$Cost_t = (P_t + k) S_t^{cs} - P_t S_{t-1}^{cs} + \zeta_t H_t.$$  

(4.17)

Because of public stock, availability is now defined by

$$A_t = S_t^{cs} - H_{t-1} + S_{t-1}^{cs} + \epsilon_t.$$  

(4.18)

Using $w$ as the unique weight of monetary terms in social welfare, we can write social welfare as

$$W_t = v(P_t, Y) + w [P_t A_t - \Psi(H_t) - (P_t + k) (S_t^{cs} + S_{t-1}^{cs})].$$  

(4.19)

Here the common money measure is used to sum the predetermined terms, $H_{t-1} \epsilon_t$, $S_{t-1}^{cs}$ and $S_{t-1}^{cs}$, to obtain current availability, $A_t$. This means that it does not matter for the welfare evaluation who holds the availability; whether it is in the producers’ hands, in private stock or in public stock is a matter of indifference. As a result, production, private and public stock need not be considered as

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\(^6\) This weighting scheme takes as reference the situation without intervention, so it is defined in an equivalent variation approach. In a compensating variation logic, the weight would be different and would be equal to the unconditional expectation of marginal values of utility over income on the ergodic distribution of price after stabilisation. Numerically, they do not differ much, but the compensating variation approach poses the problem that the welfare weight is endogenous to the optimal policy determination.
separate state variables. They are combined in one state variable, which is very convenient for the numerical solution since complexity increases exponentially with the number of state variables.\footnote{Other social welfare functions could have been contemplated (McLaren, 1997, Slesnick, 1998, Im, 2001). Agricultural and food policies are often designed to serve particular interests such as those of the urban poor in developing countries or of farmers in developed countries, alternative weighting schemes could reflect such distributional biases. The weighting scheme chosen here, however, has the advantage to focus on efficiency effects; it obeys the compensation principle and leads to easily interpretable equations.}

4.3.2 OPTIMAL POLICY PROBLEM

The government maximises the expected sum of the discounted instantaneous social welfare function

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ v(P_t, Y) + w [P_t A_t - \Psi(H_t) - (P_t + k)(S_t^s + S_t^g)] \},$$

subject to the constraints imposed by private agents’ behaviour and market equilibrium. In the previous equation, it is assumed that the consumer’s rate of time preference is equal to the discount parameter, $\beta$, which corresponds to a discount by the real interest rate. This equality does not generally occur, but this assumption avoids the notational clutter of having to use two parameters to discount utility and monetary terms separately.

Among the constraints imposed by private agents’ behaviour, the first-order condition for the storers’ behaviour (4.9) is a complementarity equation and cannot enter directly as constraint in a maximisation problem. To restate this complementarity equation, we introduce a slack variable, $\phi$, with its associated complementarity slackness conditions

$$\phi_t = P_t + k - \beta E_t (P_{t+1}) ,$$

where $S_t^s \geq 0$ and $\phi_t \geq 0$. $\phi$ refers to the marginal loss from storage. If positive, arbitraging by storage entails a loss in expectation, and so there is no private storage. If it is null, prices are arbitraging between the two successive periods.

The first-order condition for producers now includes the subsidy/tax and is

$$\beta E_t (P_{t+1 \epsilon_{t+1}}) + \zeta_t = \Psi' (H_t) .$$

The market equilibrium equation includes the public stock:

$$A_t = D (P_t) + S_t^s + S_t^g .$$

By using equations (4.18) and (4.21)–(4.24) as constraints to the maximisation problem, introducing the corresponding present-time Lagrange multipliers, and applying the law of iterated
expectations, the dynamic optimisation problem with its Lagrangian can be defined as follows:

\[
\min_{\{\Phi_t\}_{t=0}} \max_{\{\Omega_t\}_{t=0}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ v(P_t, Y) + w[P_t A_t - \Psi(H_t) - (P_t + k) (S_t^s + S_t^G)] \right] + \lambda_t (\beta P_{t+1} + \phi_t - P_t - k) + \delta_t S_t^g \phi_t + \nu_t \left[ \Psi'(H_t) - \zeta_t - \beta P_{t+1} \epsilon_{t+1} \right] + \chi_t \left[ A_t - D(P_t) - S_t^e - S_t^G \right] + \gamma_t \left( A_{t+1} - S_t^e - S_t^G - H_t \epsilon_{t+1} \right),
\]

(4.25)

where \( \Phi_t = \{\lambda_t, \delta_t, \nu_t, \chi_t, \gamma_t\} \), \( \Omega_t = \{S_t^e \geq 0, H_t, P_t, S_t^G \geq 0, \phi_t \geq 0, \zeta_t, A_{t+1}\} \), and \( A_0 \) is given. Since government surprises other agents in starting the stabilisation policy, it is not constraint by past private expectations, hence \( \lambda_{-1} = \nu_{-1} = 0 \).

From the first-order conditions, and after some manipulation we get the following system of complementarity conditions, which define the dynamics of the optimal policy under commitment

\[
S_t^e : S_t^G \geq 0 \perp -w P_t - \chi_t - w k + \beta E_t (w P_{t+1} + \chi_{t+1}) + \delta_t \phi_t \leq 0,
\]

(4.26)

\[
H_t : \beta E_t (\chi_{t+1} \epsilon_{t+1}) - w \zeta_t - \nu_t \Psi''(H_t) = 0,
\]

(4.27)

\[
P_t : v P_t (P_t, Y) + w D(P_t) - \lambda_t + X_t - \chi_t D'(P_t) = 0,
\]

(4.28)

\[
S_t^e : S_t^G \geq 0 \perp -w P_t - \chi_t - w k + \beta E_t (w P_{t+1} + \chi_{t+1}) \leq 0,
\]

(4.29)

\[
\phi_t : \phi_t \geq 0 \perp \lambda_t + \delta_t S_t^G \leq 0,
\]

(4.30)

\[
\lambda_t : \phi_t = P_t + k - \beta E_t (P_{t+1}),
\]

(4.31)

\[
\delta_t : S_t^G \phi_t = 0,
\]

(4.32)

\[
\zeta_t : \nu_t = 0,
\]

(4.33)

\[
\nu_t : \beta E_t (P_{t+1} \epsilon_{t+1}) + \zeta_t = \Psi'(H_t),
\]

(4.34)

\[
\chi_t : A_t = D(P_t) + S_t^e + S_t^G,
\]

(4.35)

with transition equations (4.18) and

\[
X_t = \lambda_{t-1} + \nu_{t-1} \epsilon_t.
\]

(4.36)

Corresponding to the two forward-looking equations (4.21) and (4.23), two Lagrange multipliers, \( \lambda \) and \( \nu \), appear as lagged variables in the first-order conditions. As expressed in (4.36), they are summed together in the state variable \( X \). The introduction of these lagged variables as new state variables gives the problem its recursive form (Kydland and Prescott, 1980, Marcet and Marimon, 1999). Government, in its decisions, has to account for private agents’ forecasts of its own future actions. Indeed in a rational expectations equilibrium, government confirms the earlier expectations.
of private agents. The need to satisfy private expectations makes the problem history-dependent. Government choice depends on both the natural state variable, $A$, and the history of the state. The role of the lagged multipliers is to summarise this trajectory by measuring the social cost of confirming past expectations in current behaviour.

4.3.3 Discussion of first-order conditions

The first-order conditions can be interpreted as follows. The first-order condition on public storage (4.29) is an arbitrage equation in the utility metric. While similar to the arbitrage equation of private storage without policy (4.9), it differs in one important aspect: public storage arbitrages the total marginal values of the commodity across two consecutive periods, while private storage arbitrages only on the private marginal values. The total marginal value of the commodity is the sum of its social marginal value, summarised by the Lagrange multiplier on market clearing equation, $\chi$, and its private marginal value (i.e., price) converted in the utility metric, $wP$.

Equation (4.33) states that the Lagrange multiplier on the producers Euler equation is zero. It means two things, which are related. First, the condition on producers’ behaviour does not constrain welfare maximisation and, second, the forward-looking behaviour of producers does not imply any time-inconsistencies for the optimal policy. More specifically, a technical distinction between a policy with a credible precommitment and a discretionary policy would be that in the former case the policy maker can manipulate expectations, but not in the latter. Here, the policy maker can use an instrument equivalent to manipulating the producers’ expectations since he can choose the level of the subsidy, which, in terms of the producers’ behaviour, has the same incentive effect as changing expectations.

Using $\nu_t = 0$, the first-order condition on production (4.27) can be written as

$$\zeta_t = \beta E_t (\chi_{t+1} \epsilon_{t+1}) / w,$$

which has a straightforward interpretation. The subsidy is equal to the discounted expected marginal social value in the money metric of one unit of planned production. This implies that production is no longer planned with respect to prices only, as in equation (4.11), but is based on total marginal values.

Private storers’ behaviour is hard to identify in the first-order conditions. So we rely on intuitions confirmed in the numerical analysis carried in the next section. Private and public stocks have perfectly symmetric roles in the social welfare function, market equilibrium and definition of availability, since, in these equations, they are always summed. As policy instruments they have the same effects on price dynamics, even though private storage imposes an additional constraint: private storers are profit-seeking and behave according to equations (4.21)–(4.22). Since the two instruments are equivalent, but private storage imposes constraints, the maximisation should privilege public
storage, and as long as public storage leads to higher stock level than private storage it should crowd out this latter.\footnote{There is one situation where there is no crowding out: the situation of a risk-neutral consumer. In this case, there is no rationale for public intervention and the optimal policy is to stockpile exactly as would be done by the private storer alone. Here, equations (4.21)–(4.22) do not impose constraints since the optimal stock level is the profit maximising one (Scheinkman and Schechtman, 1983).}

In the absence of private storage, price behaviour can be characterised through equation (4.29), by reformulating it as

\[ P_t = \max \left[D^{-1}(A_t) \cdot \beta E_t \left( P_{t+1}/w \right) - \chi_t/w - k \right]. \quad (4.38) \]

This equation defines a two-regime behaviour for the price function: for low availability there is no storage, price is defined by the inverse of the demand function at current availability (first term). For higher availability, public storage links current price to the future marginal value of the commodity.

The issue of commitment comes from the expectations of private agents. Producers’ expectations are indirectly under the control of the subsidy/tax on planned production. And since, under an optimal policy, speculative storers do not stock anything, their expectations are irrelevant for public decision. Hence, the problem can be simplified. Without private stock, equation (4.21) does not impose any constraint and \( \lambda = X = 0 \). The state variable \( X \) is introduced to account for the effect of lagged multipliers on current decision. These multipliers being null, the problem has one state variable: availability. The crowding out of private storage implies that there is no commitment problem. The public decision would be identical under commitment and under discretion.

This absence of commitment is specific to the combination of the instruments implemented. The government takes control of all intertemporal decisions, so it does not face any commitment problem. If only one instead of two instruments of stabilisation is considered, the distinction between commitment and discretion re-emerges. Stabilisation through public storage without production-related policy would crowd out private storage, but producers’ expectations of future public actions would create a time-inconsistency problem. The same applies to a policy using only the subsidy/tax on planned production: storers’ expectations affect the time-consistency of the policy. Since in analysing numerical results it is important to identify the contribution of each policy instrument to stabilisation, in the following section, we simulate the model with each instrument separately.

From the previous system of equations, an optimal public storage policy is obtained by removing equation (4.33) and all appearances of \( \zeta \). An optimal policy of a subsidy/tax to production is the solution of the previous system without equation (4.29) and terms in \( S^g \).

Equation (4.28) gives some insights into the behaviour of the social marginal value of the commodity. Using \( \lambda = X = 0 \) and Roy’s identity, this equation gives

\[ \chi_t = \frac{P_t}{\alpha} \left[ w - v_Y(P_t, Y) \right]. \quad (4.39) \]
The sign of $\chi_t$ can be discussed by comparing $w$ and $v_Y$. $w$ is the average value of marginal utility over income for the price distribution without stabilisation. If the stabilisation is not too important (and the stabilisation policy cost will prevent it), the marginal utility over income should still fluctuate around $w$ and the sign of $\chi_t$ should alternate. For a relative risk aversion superior to income elasticity, $v_Y$ increases with price.\(^9\) So the social marginal value of the commodity is positive for low prices, and correspondingly high availability, and negative for high prices, and correspondingly low availability. This implies that the optimal policy should try to move the commodity from periods of high availability, when the social marginal value is negative, to periods of low availability, when it is positive. This discussion sheds some light on the behaviour of the subsidy/tax on production as defined by (4.37). When the next-period availability is expected to be high, which implies that a large stock is carried over, producers are taxed in order to limit supply in a period with negative social marginal value. On the contrary, for low stock levels, the next-period availability is expected to be low and production is subsidised.

4.4 Numerical results

The model without stabilisation policy, and a fortiori the model with optimal policy, cannot be solved analytically. Decision rules must be approximated numerically. The problems are solved under Matlab using a projection method with a collocation approach. The solver is similar to Fackler (2005), except that the equilibrium equations are solved by the mixed complementarity problems solver PATH (Dirkse and Ferris, 1995). A grid on the state variables is chosen, on which decision rules are approximated by splines and the number of grid nodes is chosen such that the use of these decision rules entails on average less than a $1 error every $1,000 of decision (measured by the Euler equation error).

4.4.1 Calibration

Parameterisation is required in order to quantify the welfare effects of introducing a food stabilisation policy. Table 4.3 presents the parameters used to calibrate the model. The parameters are set such that, at the model’s deterministic steady state, price, production, consumption and availability are equal to 1. Since steady-state consumption and price are equal to 1, income, which is assumed to be constant, is equal to the inverse of the commodity budget share, $1/\gamma$. An annual interest rate of 5% is used for discounting.

Seale and Regmi (2006) estimate elasticities for food consumption across 144 countries. From their research, we choose cereal elasticities typical of low-income countries: $-0.4$ for price elasticity and $0.5$ for income elasticity. For poor households, the budget share devoted to one commodity can be important. For example, rice expenditures represent roughly 20–35% of total expenditure for the

\(^9\) By differentiating Roy’s identity with respect to income and by using the definition of income elasticity and relative risk aversion, we have $v_{Y,P} = v_{Y,Y} D (\eta/\rho - 1)$.\n
Table 4.3. Parameterisation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Economic interpretation</th>
<th>Assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Annual discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Income elasticity</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Own-price demand elasticity</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>Commodity budget share</td>
<td>0.15</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Inverse of supply elasticity</td>
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</tr>
<tr>
<td>$Y$</td>
<td>Income</td>
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</tr>
<tr>
<td>$d$</td>
<td>Normalisation parameter of demand function</td>
<td>0.39</td>
</tr>
<tr>
<td>$h$</td>
<td>Normalisation parameter of production cost function</td>
<td>0.95</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Parameter defining risk aversion</td>
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</tr>
<tr>
<td>$k$</td>
<td>Physical storage cost</td>
<td>0.02</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Probability distribution of yield</td>
<td>$B(2,2) \cdot 0.5 + 0.75$</td>
</tr>
</tbody>
</table>

bottom quintile in Bangladesh, India, Indonesia and Philippines (for a share of food expenditures around 62–70%, Asian Development Bank, 2008). For the top quintile, it is about 5–10% of total expenditure. Since the main focus of a food security policy is the poorest populations, the representative consumer is assumed to spend, at the steady state, 15% of its income on the staple.

There is less evidence on which to base the level of risk aversion since the literature on risk aversion parameters does not converge on a consensus.\(^{10}\) In our case, the parameter has to be sufficiently high to justify a stabilisation policy. From equation (4.3), this implies that we need $\rho$ such that $\gamma (\eta - \rho) - \alpha < 0$, that is, with our parameters, $\rho > 3.17$. We assume at the steady state a relative risk aversion parameter of 4, implying $\theta = -6.13$. For this calibration, the third derivative of indirect utility with respect to price, $v_{PPP}$, is negative. Given that utility decreases with price, $v_{PPP} < 0$ implies an aversion to downside price risk (Eeckhoudt and Schlesinger, 2006). For a given mean and standard deviation, the consumer prefers a price distribution with higher third moment, that is, shifting the volatility toward low rather than high prices. It matters for the design of stabilisation policies, because storage tends to skew prices upwards.

We follow Lence and Hayes (2002) and assume a per-unit storage cost of 2% of the steady-state price (i.e., $k = 0.02$). Supply elasticity is set at 0.1, so $\mu = 10$. This value is confirmed by Roberts and Schlenker (2010) who analyse a supply and demand model of world commodities, and estimate global supply elasticity for the caloric equivalent of corn, rice, soybean and wheat between 0.08 and 0.13. FAPRI’s elasticities database\(^{11}\) confirms this order of magnitude. For example, for developing countries they propose supply elasticity for wheat between 0.07 and 0.43. The productive shocks, $\epsilon$,

---

\(^{10}\) For example, Pålsson (1996) analyses risk taking by Swedish households and finds a relative risk aversion of the order 10 to 15 for all real assets, and 2 to 4 when restricted to financial assets. Estimating a DSGE model van Binsbergen et al. (2010) find a value of 79. Chetty (2006) shows that to be consistent with observed labour supply, relative risk aversion must be low, with an upper bound at 2 and a central estimate at 1. Part of this variability may be explained by a large heterogeneity in individual risk aversion (Guiso and Paiella, 2008), which can be related to changes in the environment (e.g., imperfections in financial markets may discourage risk-taking behaviours).

are assumed to follow a beta distribution of shape parameters, $2$ and $2$, which makes it unimodal at 0.5 and symmetric. The distribution is recentred and rescaled to vary between 0.75 and 1.25, which implies a coefficient of variation of 11.2%.

4.4.2 Decision rules

Storage rules

Storage rules are presented in figure 4.1 (left panel). Without public policy, private storers (solid line) do not stock anything for low availability (the threshold is close to the steady-state availability level, 1), and increase the level of stock with market availability above the threshold. When normal consumption is satisfied, any additional quantity in the market tends to lower prices. The speculator takes opportunity of these lower prices to accumulate stock that can be sold in periods of lower availability. The middle panel shows the behaviour of the marginal loss from storage, $\phi$. It is positive for low availability, which explains the absence of storage: profit is negative. As soon as stock is positive, marginal loss is null. Private storers operate in expectations at zero profit.

Stabilisation through an optimal public storage rule (dashed line) results in levels of public stock that are always superior to the levels reached by private storage without policy. In consequence, public stocks discourage any speculative storage, and the marginal loss from storage is always positive. The crowding-out of private storage means that the degree of involvement of government in storage has to go beyond the difference between the private storage rule without intervention and the optimal rule. Because public storage removes profit opportunity, public intervention means government is responsible for both speculative storage and additional storage motivated by consumer risk aversion. This complete crowding-out is absent in the optimal storage rules in Wright and Williams (1982b) and Williams and Wright (1991, Ch. 15), which analyse the management of strategic petroleum reserves. Two features explain the coexistence of both public and private stocks in these works: in the first study, private storers are assumed to receive a convenience yield from the holding of stocks, implying that they hold stock even if the apparent return is negative; in the
second study, they suppose that public stock is not held at the same location as private stock, which
gives justification for the existence of private storers that are closer to the market. For these reasons,
and because private storers hold stocks to smooth the natural seasonality of agriculture production,
it is reasonable to think an optimal public storage policy would not in practice completely crowd out
private storage. But there will be very little scope for private storage to obey a speculative motive
in the presence of welfare-maximising public storage.

Since there is a complete crowding out, the stabilisation brought by public storage could also
be decentralised through a state-contingent subsidy to private storage. The subsidy would consist
in covering the marginal loss from storage induced by the higher stock level to bring the expected
private storers’ profit to zero; hence the subsidy would follow $\phi$ as represented in figure 4.1.

**Production rules**

Like the storage rule, the production rule presents a kink at the stockout limit (figure 4.1, right
panel). When storage is null, whatever the current availability, the planned production level is the
same. For positive stock, the next-period expected price decreases with availability and so too does
planned production.

The effect of public policy is to make production more elastic to current availability. Production
is higher for low availability and lower for high availability. This is achieved through a subsidy to
planned production for low availability and a tax for high availability (see figure 4.2, left panel). For
low availability levels, the storage policy is ineffective, since there will be no stockpiling. But it is
possible to increase future availability by producing more. This is the point of subsidising planned
production. It has to be high (it can reach 15% of steady-state price) to compensate for the higher
costs (there are decreasing returns to scale) and the depressing effect of supplementary production
on price.

![Subsidy/tax to planned production, price rules, and social marginal value of the commodity](image-url)
A state-contingent subsidy policy makes sense only for a storable commodity. Without storage, the rational expectations solution to the model without public intervention is a constant planned production level (Muth, 1961). Storage creates variation in the expected price and therefore justifies the variation in production with growers producing more when availability is expected to be low. The subsidy/tax magnifies this effect beyond private behaviour. In essence, this instrument plays a complementary role to storage.

It may seem strange to tax producers in abundant years when they are suffering low prices, but taxation is the incentive part of the policy; it is possible to compensate producers for any loss through non-distortive transfer. It is possible also to design policy such that producers are not taxed but are paid to take land out of production. These types of policies were common in the US as part of supply management until the 1996 FAIR Act (Gardner, 2002, Ch. 7). The US Secretary of Agriculture decided on the percentage of land that farmers should set aside each year, in order to be eligible for the price support programme. This policy was seen as complementing storage policies, since it limited production when stocks were already high.

The supply control through subsidies and taxes to planned production may, however, be more difficult to implement in a developing country where a direct control of producers’ behaviour is much more difficult, given the administrative system required to verify acreages. An alternative instrument, which could partly mimic the role of a supply control policy, is a state-contingent fertiliser subsidy, since fertiliser subsidies have been common in many African countries (Morris et al., 2007).

Price and social marginal value of the commodity

The two regimes, without and with storage, are also apparent in the price rules (figure 4.2, middle panel). In a stockout situation, price is determined solely by current consumer demand and can increase a lot for small availability. For higher availability, demand for storage adds to consumer demand, which makes total demand more elastic and limits the decrease in the price with availability. Under an optimal policy, for positive stocks, the price is higher because demand for storage is higher.

The behaviour of the social marginal value, $\chi$, is represented in figure 4.2 (right panel). It is positive for low availability, meaning it is socially justifiable to try to transfer more resources to low availability periods, hence the higher level of stock. The social marginal value is negative for high availability, which explains taxation of production in these situations in order to prevent excess supply.

4.4.3 Dynamics under optimal stabilisation policy

When stabilisation policy is first introduced, the new asymptotic distribution is not reached immediately. Since the policy involves a higher mean stock level than without intervention, it begins with a phase of stock build-in. The transition is illustrated in figure 4.3 with the dynamics of mean total stock, mean government outlays and mean prices when the policy starts with the system on the asymptotic distribution without public intervention.
In the first years, the purchases made to accumulate stocks push the mean price above its long-run mean. After a few years, the mean price drops below its value without public intervention. Stabilisation affects the mean price through the curvature of the inverse demand function (Wright, 1979). Since we apply constant elasticity of demand, the inverse demand function is convex. Storage also makes the inverse of the total demand function convex, as shown in figure 4.2. Thus, the price at mean consumption is lower than the mean price. So stabilisation of the consumption results in a lower mean price.

Mean government outlays are very high in the first period, because stocks must be accumulated and there is no stock to sell. The first year cost reaches 12% of the steady-state commodity budget share and 1.7% of income. Public storage outlays decrease in later periods and, on the asymptotic distribution, the public stock is financially self-balanced: the proceeds from sales cover purchase and storage costs. The outlay from the subsidy/tax on production is always less than the amount from public stock and, after a few periods, it is negative. This means that production is more taxed than subsidised. Indeed, in the long run mean availability reaches 1.13 corresponding to a tax on planned production in figure 4.2.

The dynamic welfare effects of the policy are presented in table 4.4. In addition to the effects of an optimal policy, the welfare results from an optimal public storage policy on its own, and an optimal subsidy/tax to planned production are depicted. Consumer gains are \( \textit{ex ante} \) per-period equivalent variation calculated using

\[
\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ v(P_t, Y + EV) - v(\tilde{P}_t, Y) \right] \right\} = 0. \tag{4.40}
\]

Producers’ gains are the annualised changes in expected profit from equation (4.10). Government outlays are the annualised expected sum of the discounted costs from equation (4.17). Changes in storers’ surpluses are ignored since storers operate at zero profit on average and we have assumed...
no stock in the first period. The welfare results are all conditional expectations that depend on initial availability, which is assumed to be equal to its deterministic steady state level. The sensitivity to this starting point is assessed in section 4.4.6. To put the magnitude of the welfare results in perspective, they can be compared to income or to the steady-state commodity budget share. Total gains from the optimal policy using both instruments represent 0.05% of income and 0.32% of the commodity budget share. The low level of these gains is not surprising given that the gains resulting from a reduction of risk are of second order. The finding parallels Lucas’s (1987) insight that individuals would be unlikely to sacrifice more than 0.1% of their lifetime consumption to live in a world with no macroeconomic volatility.

Table 4.4. Welfare results on transitional dynamics (as a percentage of the steady-state commodity budget share.)

<table>
<thead>
<tr>
<th></th>
<th>Public storage</th>
<th>Subsidy/tax to production</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers gains</td>
<td>0.39</td>
<td>0.26</td>
<td>0.36</td>
</tr>
<tr>
<td>Producers gains</td>
<td>0.42</td>
<td>1.02</td>
<td>0.35</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0.53</td>
<td>1.20</td>
<td>0.39</td>
</tr>
<tr>
<td>Total gains</td>
<td>0.28</td>
<td>0.08</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Stabilisation policies result in transfers between agents (see Newbery and Stiglitz, 1981, Ch. 6 and 9, for a discussion of efficiency and transfer from price stabilisation). Consumers gains include two components: a transfer term corresponding to the change in mean expenditures, and an efficiency term corresponding to the risk reduction and the change in mean consumption. The mean expenditure change is a transfer with producers, and private and public storers. Public storage entails both storage costs and changes in mean price, i.e., a transfer with producers and consumers. The subsidy/tax on production is a transfer between government and producers. Overall, the transfers sum to zero and do not affect social welfare, which is composed only of efficiency gains from consumers, and changes in production and storage costs.

For a policy of public storage, stockpiling pushes prices up in the early periods and in later periods pushes them below their mean value without stabilisation. For producers, this implies long-term losses from lower prices. However, since later periods are discounted, overall producers gain from a stockpiling policy. These producers’ gains stem mainly from transfers from government through the effect of public storage on mean price.

Producers’ gains are higher under a pure subsidy/tax policy. This policy is a combination of subsidy in periods of low availability and tax in periods of abundance. But, without public storage, subsidies are more frequent than taxes, and producers benefit greatly from this instrument. Under this policy, situations of low availability are more frequent than when both instruments apply. It is

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12 These welfare results stem from expected infinite sums of discounted values (e.g., equation (4.10)). To calculate them, we follow the dynamic programming trick of transforming the infinite sums in recursive problems that can be solved easily by value function iteration.

13 This result does not depend on the value of the discount parameter. It holds for any reasonable value of $\beta$. 
precisely when availability is low that the producers are subsidised to produce more, which explains why producers gain more when output subsidy is the only policy.

Having accounted for all transfers, public storage contributes the most to overall welfare. A policy of planned production subsidy only complements public storage.

4.4.4 The long-run effects of stabilisation

Since it is not possible to borrow the commodity for the future (the non-negativity constraint on storage), increased stabilisation as a result of public storage requires an increase in mean stock levels, as stock dynamics show (figure 4.3). Following this transitory phase of stock building availability increases due to the higher stock levels as is apparent from the asymptotic distribution of availability (figure 4.4). Because public storage significantly shifts the distribution of availability toward higher values, it reduces the risk of shortfalls. Subsidising planned production acts differently on availability. It reduces the probability of low and high availability by making supply more elastic, although the effect appears marginal.

![Figure 4.4. Density of availability. The curves are obtained by kernel density estimation over 1,000,000 simulated points.](image_url)

Confirming previous results, table 4.5 shows that all stabilisation policies involving public storage decrease the mean price. This decrease brings long-run losses for producers, because stabilisation decreases mean profit (Turnovsky, 1976, shows that this is always true for a constant-elasticity demand where elasticity is below unity).

---

14 Long-run results are calculated over 1,000,000 sample observations from the asymptotic distribution.
Table 4.5. Descriptive statistics of price series on the asymptotic distribution

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Public storage</th>
<th>Subsidy/tax to production</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.021</td>
<td>1.012</td>
<td>1.020</td>
<td>1.015</td>
</tr>
<tr>
<td>One-year autocorrelation</td>
<td>0.283</td>
<td>0.475</td>
<td>0.227</td>
<td>0.401</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.194</td>
<td>0.141</td>
<td>0.178</td>
<td>0.130</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.709</td>
<td>2.409</td>
<td>2.245</td>
<td>1.790</td>
</tr>
<tr>
<td>Quantiles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>0.792</td>
<td>0.833</td>
<td>0.818</td>
<td>0.862</td>
</tr>
<tr>
<td>25%</td>
<td>0.891</td>
<td>0.924</td>
<td>0.902</td>
<td>0.935</td>
</tr>
<tr>
<td>50%</td>
<td>0.962</td>
<td>0.978</td>
<td>0.966</td>
<td>0.981</td>
</tr>
<tr>
<td>75%</td>
<td>1.071</td>
<td>1.054</td>
<td>1.056</td>
<td>1.050</td>
</tr>
<tr>
<td>99%</td>
<td>1.715</td>
<td>1.590</td>
<td>1.665</td>
<td>1.559</td>
</tr>
</tbody>
</table>

Public stockpiling increases price autocorrelation, since a higher level of stock allows a better smoothing of shocks over several periods. A production subsidy has the opposite effect: it makes supply more elastic to price expectations and tends to decrease price autocorrelation.

Most of the stabilisation is achieved through public storage, which on its own decreases the coefficient of variation from 0.19 to 0.14. Introducing in addition state-contingent subsidy decreases it to 0.13. Storage stabilises by moving the commodity from periods of low prices to periods of high prices. This is apparent in the fact that the probability of the occurrence of low and high prices decreases. Very low prices are perfectly alleviated by storage. The possibility of stockouts means that high prices are not all alleviated, and the skewness of prices increases with the level of storage.

The quantiles of prices show how storage policies operate. Because the public storage rule is more aggressive than the private storage rule, low price periods are less frequent since government uses these opportunities to build up stocks. More than half of the distribution ends up at higher prices, but the higher stock level allows the occurrence of high prices to reduce.

4.4.5 Nature of the risk-sharing arrangement

The stabilisation policies increase total welfare by transferring risk from risk-averse consumers to risk-neutral agents. Table 4.6 illustrates this risk sharing by presenting the standard deviation of each component of the social welfare function (4.15) on the asymptotic distribution: consumers’ utility, producers’ and storers’ profit and government outlays.

Table 4.6. Standard deviation of social welfare components

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Public storage</th>
<th>Subsidy/tax to production</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers utility</td>
<td>0.090</td>
<td>0.064</td>
<td>0.082</td>
<td>0.059</td>
</tr>
<tr>
<td>Producers profit</td>
<td>0.105</td>
<td>0.098</td>
<td>0.191</td>
<td>0.136</td>
</tr>
<tr>
<td>Storers profit</td>
<td>0.067</td>
<td>0</td>
<td>0.073</td>
<td>0</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0</td>
<td>0.087</td>
<td>0.130</td>
<td>0.093</td>
</tr>
</tbody>
</table>
As price volatility decreases with public intervention, the standard deviation of consumers’ utility decreases for all policies. The effects on producers’ profit are more contrasted. A public storage policy decreases the volatility of their profit, but the other policies increase it. The subsidy/tax has a procyclical behaviour with respect to producers’ profit. In periods of low availability, producers’ profit is high because the high prices more than compensate for the poor harvests; this is also the period when they receive subsidies to increase production. And conversely, they are taxed to limit production when their profit is the lowest, when availability is high. This increase in profit volatility creates no welfare loss in our model since producers are assumed to be risk neutral. This assumption is useful in limiting the number of state variables, but it is not realistic. Since a policy of subsidy/tax alone almost doubles the volatility of producers’ profit, this policy would not be followed as such. Even, in association with public storage, this instrument increases the volatility of profit by 30%. It does not imply, however, that a supply control policy is useless with risk-averse producers. Its precise form as stemming from the optimal policy design would just be different.

With regard to risk sharing, public storage appears the most balanced policy. It decreases the volatility perceived by both consumers and producers, and transfers the risk to government budget. The public storage policy found here would not be optimal with risk-averse producers, but it would probably increase social welfare, and the welfare of both consumers and producers.

4.4.6 Sensitivity to initial availability

Above we have considered the effects of stabilisation policies starting when availability is equal to its deterministic steady-state value. Given the importance of transitional dynamics effects, we analyse in this section the sensitivity to initial availability by considering the welfare gains when the policies starts with availabilities of 0.8, 1 (the benchmark) and 1.2. As before, private stocks are assumed to be null, which rules out transfers to private storers.

For all policies total gains are not much affected by initial availability; it is the distribution of gains that are more influenced by the starting point (see table 4.7). This means that the starting point influences more the transfers among agents than the efficiency gains. For a public storage policy, because storage entails an initial stock building phase, starting from higher availability reduces the costs associated with this phase by providing excess quantities to be stockpiled.

For a subsidy/tax to planned production, the distributive effects are important with changes in signs of welfare gains depending on the starting point. For an initial availability of 1.2, consumers lose from the policy. For this high availability, there are few efficiency gains to expect from the policy in the short-run, since private storers already provide stabilisation. Producers are initially taxed to induce them to produce less because the social marginal value of the commodity is negative for this availability level. This taxation entails higher prices, leading to welfare losses of consumers.

\[15\] With risk-averse producers, it would not have been possible to sum production and stocks into one state variable, availability. The increase in dimension would have made the problem more difficult to solve and the results more difficult to interpret.
Table 4.7. Sensitivity of welfare results to initial availability (welfare results expressed as percentages of the steady-state commodity budget share.)

<table>
<thead>
<tr>
<th></th>
<th>Public storage</th>
<th>Subsidy/tax to production</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_0$: 0.8</td>
<td>$A^a$: 1.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Consumers gains</td>
<td>0.42 0.39 0.39</td>
<td>0.35 0.26 −0.07</td>
<td>0.47 0.36 0.04</td>
</tr>
<tr>
<td>Producers gains</td>
<td>0.36 0.42 0.49</td>
<td>1.43 1.02 −0.24</td>
<td>0.78 0.39 −1.12</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0.52 0.53 0.58</td>
<td>1.70 1.20 −0.40</td>
<td>0.78 0.39 0.32</td>
</tr>
<tr>
<td>Total gains</td>
<td>0.26 0.28 0.29</td>
<td>0.08 0.08 0.08</td>
<td>0.30 0.32 0.33</td>
</tr>
</tbody>
</table>

Notes: Initial private stocks are assumed to be null, production is the only component of initial availability.

* Benchmark.

For low availability on the contrary, producers are subsidised in the short-run, which explains their high welfare gains. They increase their production, which decreases mean price and price volatility for consumers, explaining their welfare gains. This policy has a high fiscal cost, representing 1.7% of the commodity budget share, because of the high initial subsidies.

From the above, we can conclude that the instrument of subvention/tax to production leads to results which are dominated by transfer gains, efficiency gains being very limited in contrast.

4.5 Conclusion

In this chapter we proposed a framework for food price stabilisation policy design. This framework takes some inspiration from the method used in the modern literature on optimal monetary policy (Clarida et al., 1999): (i) we start with a simple model, the rational expectations storage model, that is able to account for the stylised facts of commodity prices; (ii) because markets are incomplete, consumers are unable to insure themselves, which could justify public intervention; (iii) corresponding to the market failure, a public policy maximising the welfare is designed.

This work brings together two approaches. That proposed by Newbery and Stiglitz, which uses very stylised models to propose stabilisation policies in imperfect markets, but does not result in implementable policies due to the restrictive framework, and the tradition of competitive storage modelling (e.g., Miranda and Helmberger, 1988, Wright and Williams, 1988a) in which public policies are discussed within a framework that mimics the main features of commodity markets, but neglects market imperfections. The resulting model, although still highly stylised, provides a theoretical benchmark against which food security propositions can be assessed.

We show that an optimal policy under commitment composed of state-contingent public storage and state-contingent subsidy/tax on planned production improves consumers’ and producers’ welfare and crowds out all private storage activity by removing any profit opportunity from speculation. Most of the welfare gains are from public storage. A countercyclical production policy, which provides incentives for increased production in times of scarcity and decreased production in times of abundance, makes little difference and can have undesirable consequences such as increasing the volatility of producers’ profit.
By choice, we kept the model as simple as possible. Future work could extend this study along a number of important dimensions. It could account for more realistic features, such as the effect of international trade or substitution among food staples (Bellemare et al., 2010), or, and perhaps more importantly, the welfare framework could be improved. The one proposed in this chapter is a first attempt to provide some justification for stabilisation policies. For example, our modelling of food demand is very simple and does not acknowledge the fact that food consumption cannot decrease too much without consequences for health or even death. Accounting for the specific role of food would reinforce the need to prevent the occurrence of high food prices. This could be done with the help of the survival function proposed by Ravallion (1987) or the microeconomic framework developed in Chavas (2000). Assigning to food a peculiar role as a consumption good should increase the welfare gains from stabilisation beyond the low values found here.

Also, the optimal policy with state-contingent public storage and subsidy to planned production implies heavy government involvement in the functioning of markets and the application of highly nonlinear policy rules. Historical experience of the involvement of states in grain markets shows mixed results and several instances of large inefficiencies (Dorosh, 2008). It would be useful to consider the design of simpler policies that would decentralise a part of the policy implementation to the private sector.

In this chapter, for simplicity, the analysis was restricted to a partial equilibrium framework. Because food price instability is a concern only for people who spend a large part of their budget on food, partial equilibrium reasoning might not account for important general equilibrium effects. For example, in developing countries, the poorest people are likely also to be subsistence farmers, and the high variability in food prices means that they devote limited acreage to cash crops and self-insure by producing food crops (Fafchamps, 1992). More stable food prices would reduce their need for insurance and allow them to specialise more.

Finally, we do not allow for heterogeneity among consumers. If people likely to suffer from high prices represent only a limited share of the total population, it may not be optimal to engage in large-scale public stock programmes. A better scheme might include policies specifically targeting people in need. They might take the form of targeted cash transfer programmes (Skoufias et al., 2001) or the distribution of subsidised rations. The public stock backing a targeted policy would not necessarily crowd out private storage since it would not be designed to affect the whole market.

\[ 4.A \quad \text{SECOND-ORDER APPROXIMATION OF WELFARE CHANGE} \]

A second-order approximation of the right-hand side of equation (4.1) around the mean price without stabilisation yields

\[
v (\hat{P}, Y) = v (\bar{P}, Y) + (\hat{P} - \bar{P}) v_P (\bar{P}, Y) + (\hat{P} - \bar{P})^2 v_{PP} (\bar{P}, Y) / 2 \quad (4.41)\]
And around $(\bar{P}, Y)$, the left-hand side yields

$$v(P, Y + EV) = v(\bar{P}, Y) + (P - \bar{P}) v_P (\bar{P}, Y) + EVv_Y (\bar{P}, Y) + \left[ (P - \bar{P})^2 v_{PP} (\bar{P}, Y) + EV^2 v_{YY} (\bar{P}, Y) + EV (P - \bar{P}) v_{PY} (\bar{P}, Y) \right] / 2.$$  \((4.42)\)

Taking the expectations of both sides, and for simplicity omitting the variables inside the partial derivatives of utility, we have

$$\Delta \bar{P} v_P + E \left[ (P - \bar{P})^2 \right] v_{PP} / 2 = EVv_Y + \left[ \sigma_P^2 v_{PP} + EV^2 v_{YY} \right] / 2,$$  \((4.43)\)

where $\sigma_P^2$ is price variance without stabilisation. Using

$$E \left[ (\bar{P} - \bar{P})^2 \right] = E \left[ (\bar{P} - \bar{P})^2 + (\bar{P} - \bar{P})^2 \right],$$  \((4.44)\)

where $\bar{P}$ is the mean price after stabilisation, we have

$$EVv_Y + EV^2 v_{YY} / 2 = \Delta \bar{P} v_P + \left[ (\Delta \bar{P})^2 + \sigma_P^2 - \sigma_P^2 \right] v_{PP} / 2.$$  \((4.45)\)

Terms $EV^2$ and $(\Delta \bar{P})^2$ are of order $\sigma_P^4$; they can be neglected for small $\sigma_P$, and equation (4.2) follows. Using Roy’s identity, we have

$$EV = -D \Delta \bar{P} + \Delta \sigma_P^2 \cdot v_{PP} / (2v_Y).$$  \((4.46)\)

Differentiating Roy’s identity gives the expression of $v_{PP}$,

$$v_{PP} = D^2 v_{YY} + D \frac{\partial D}{\partial Y} v_Y - \frac{\partial D}{\partial P} v_Y,$$  \((4.47)\)

from which equation (4.3) follows.
CHAPTER 5

RULES VERSUS DISCRETION IN FOOD STORAGE POLICIES

Should food prices in poor countries be stabilised? This question was prompted by the riots during the 2007–08 food crisis and, in its aftermath, most commentators agreed about the need for government intervention. The precise design of this intervention, however, is achieving much less consensus. Von Braun and Torero (2009) propose the establishment of a small global food reserve for emergency intervention, accompanied by a virtual reserve designed to reduce irrational speculation on futures markets. Wright (forthcoming) points to the lack of precise motivation for this type of financial scheme. This recalls a similar policy implemented by the US in the 1930s, which proved very expensive and was rapidly dismantled. Various storage schemes have been tried in the past in the bid to stabilise commodity prices, many of which were not motivated by food security. There have been international commodity agreements that relied on price-bands (Gilbert, 1996); in Latin America and the US, private storage subsidies, often in the form of interest-rate subsidies, were common (Gardner and López, 1996); India and the European Union introduced minimum support prices for cereals, but, because stock selling provisions were either unclear or ineffective, in both cases episodes of excessive stock levels occurred (Gardner, 1996a, Dorosh, 2009).

Here, we compare storage policies designed to stabilise food prices in a poor, self-sufficient country. Public storage policies are introduced into a rational expectations storage model, in order to stabilise markets beyond the efforts made by private agents. In traditional storage models, stabilisation policies cannot increase welfare since there is no market imperfection (Scheinkman and Schechtman, 1983). In our model, the additional stability brought by storage policies improves welfare because markets are assumed to be incomplete and consumers are risk averse. The use of a rational expectations storage model is motivated empirically by Cafiero et al. (2011), who show that this model performs well for explaining commodity price behaviour. As the present analysis adopts a theoretical approach, we do not try to reproduce the dynamics of a specific market, but rather calibrate the model to values typical of the cereals market in a developing country.

This chapter characterises fully optimal storage rules, which are nonlinear functions of state variables, and also optimal simple rules, which are simpler policies defined by a limited set of parameters. For simple rules, we consider a constant subsidy to private storage and a price-band
with capacity constraint on public stock levels. When government is able to commit to following an optimal state-contingent storage rule, it can induce producers to promote stabilisation by manipulating their expectations, which explains that a policy with commitment generates the highest welfare gains. The gains from a commitment policy, however, are small, and a discretionary policy achieves similar gains. If the parameters of simple rules of stabilisation are chosen so as to maximise welfare, such rules can achieve more than four-fifths of the maximum gains.

This work is related to studies of commodity price stabilisation schemes, for example, Miranda and Helmberger (1988) and Wright and Williams (1988a). The former analyse the behaviour of price-band programmes and the latter compare several stabilisation schemes applied frequently in agricultural policies. Both studies emphasise the importance of distinguishing between short-run and long-run effects. Glauber et al. (1989) analyse the cost-efficiency of various policies, and show that a programme of direct payments is the most efficient farm prices stabiliser. Gardner and López (1996) compare policies aimed at stimulating private stockpiling showing that interest-rate subsidies are inefficient and that it is less costly to increase stabilisation through direct storage subsidy. We draw on this literature although our approach differs in that in our work the storage policies, or the parameters that define them, are optimal, while previous studies generally impose exogenous policy rules. Previous studies also do not introduce rationales for public intervention, thus in their framework public interventions are bound to decrease welfare.

Some authors tried to combine stabilisation policies with market imperfections, using the latter as justification for the former. Brennan (2003) uses the limited development of financial markets, which implies prohibitively high private storage costs, to justify public intervention in the rice market in Bangladesh. She proposes stabilisation policies to increase welfare, but does not design them to be welfare maximising. A few works propose stabilisation policies resulting from government optimisation in reaction to market imperfections. Gardner (1979b) considers that periods of high prices generate external costs not accounted for by private storers. He proposes optimal storage policies that correct for these external costs. Wright and Williams (1982b) design optimal public storage rules where government is unable to commit to not using a price ceiling, which creates a disincentive for private storage.1 Although we build on these works, we extend the analysis of optimal policies by distinguishing between commitment and discretionary policies. We consider also the outcomes of suboptimal policies, such as storage subsidy or price-bands, which figure more frequently in policy discussion than fully optimal rules.

The problems created by discretionary intervention in food markets are referred to frequently in the food policy literature (e.g., Poulton et al., 2006, Tschirley and Jayne, 2010). In Africa, private agents cannot easily anticipate discretionary public interventions and so they limit their investments, which increases price instability. To identify the best way to intervene while accommodating to the expectations and behaviour of private agents, we use an approach that follows the literature on optimal dynamic policies. The first-order conditions of optimal state-contingent policies are

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1. See also McLaren (1997, 1998) for interesting discussions on optimal policies.
derived using the methods in Marcet and Marimon (1999) and Klein et al. (2008). The source of time-inconsistency in the optimal policy is identified and discussed (Kydland and Prescott, 1977). Comparison of fully optimal policies and optimal simple rules is influenced by related works in the theory of monetary and fiscal policies (Schmitt-Grohé and Uribe, 2007, Taylor and Williams, 2010), where simple rules are seen as a type of policy that is more easily implementable and more robust than a fully optimal policy.

Section 5.1 describes the rational expectations storage model without public intervention. Section 5.2 introduces a framework for designing optimal policies. In section 5.3, we describe the calibration of the model and discuss the numerical results. Section 5.4 concludes.

5.1 The model

Our analysis follows usual practice in studies of commodity price stabilisation schemes (Miranda and Helmberger, 1988, Wright and Williams, 1988a) and uses a rational expectations storage model. This is a partial equilibrium model featuring a market for a storable commodity with a competitive storer, a producer whose output is submitted to multiplicative shocks, and a final demand. We assume that consumers are risk averse and unable to insure because of market incompleteness. In our model, time is discrete, and one period is assumed to correspond to one year. Thus, we focus on inter-annual storage and ignore issues related to seasonal fluctuations.

5.1.1 Consumers

The economy is populated with risk-averse consumers whose final demand for food has an isoelastic specification: \( D(P_t, Y) = dP_t^\alpha Y^\eta \), where \( d \) is a normalisation parameter; \( P_t \) is the period \( t \) price; \( Y \) is income, which is assumed to be constant; and \( \alpha \) and \( \eta \) are the price and income elasticities. Assuming there are only two goods and the second good is the numeraire, the integration of this demand function gives the following instantaneous indirect utility function (Hausman, 1981)

\[
\hat{v}(P_t, Y) = \frac{Y^{1-\eta}}{1-\eta} - \frac{d P_t^{1+\alpha}}{1+\alpha}.
\]

This utility function has a relative risk aversion equal to the income elasticity of demand. To distinguish income elasticity from risk aversion, we follow Helms (1985a) and apply a monotone transformation to the indirect utility function,

\[
v(P_t, Y) = \frac{\hat{v}(P_t, Y)^{1+\theta}}{1+\theta}.
\]

This specification is still consistent with the isoelastic demand function, but its coefficient of relative risk aversion is

\[
r(P_t, Y) = \eta - \theta \frac{Y^{1-\eta}}{\hat{v}(P_t, Y)}.
\]
with \( \theta \) indexing the degree of risk aversion.

The representative consumer is assumed to adopt hand-to-mouth behaviour. He consumes current income and does not save to smooth out fluctuations. This assumption simplifies the dynamics of the problem, since consumer’s “cash on hand” does not have to be included as a state variable; however, it overestimates the effect of public policy by neglecting the possibility of self-insurance added by saving.

5.1.2 Storers

There is a single representative speculative storer, which is risk neutral and acts competitively. Its activity is to transfer a commodity from one period to the next. Storing the quantity \( S_t^S \) from period \( t \) to period \( t + 1 \) entails a purchasing cost, \( P_t S_t^S \), and a storage cost, \( k S_t^S \), with \( k \) the unit physical cost of storage. We introduce also a per-unit constant subsidy \( \zeta \), which is considered later as a possible stabilisation tool. The benefits in period \( t \) are the proceeds from the sale of previous stocks: \( P_t S_{t-1}^S \). The storer follows a storage rule that maximises its expected profit defined by

\[
V^S(S_{t-1}^S, P_t) = \max_{\{S_{t+1}, \geq 0\}_{t=0}^\infty} E_t \left\{ \sum_{i=0}^\infty \beta^i [P_{t+i} S_{t+i-1}^S - (P_{t+i} + k - \zeta) S_{t+i}^S] \right\},
\]

(5.4)

where \( E_t \) denotes the mathematical expectations operator conditional on information available at time \( t \), and \( \beta \) is the discount factor. The storer’s problem can be expressed in a recursive form using the following Bellman equation:

\[
V^S(S_{t-1}^S, P_t) = \max_{S_t^S \geq 0} \left\{ P_t S_{t-1}^S - (P_t + k - \zeta) S_t^S + \beta E_t \left[ V^S(S_{t+1}^S, P_{t+1}) \right] \right\}.
\]

(5.5)

This equation has two state variables: the price, whose dynamics is considered by the storer to be exogenous, and the stock carried over from the previous year. Using the first-order condition on \( S_t^S \) and the envelope theorem, and taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), this problem yields the following complementary condition

\[
S_t^S \geq 0 \perp \beta E_t (P_{t+1}) + \zeta - P_t - k \leq 0,
\]

(5.6)

which means that inventories are null when the marginal cost of storage is not covered by expected marginal benefits; for positive inventories, the arbitrage equation holds with equality. So this is a situation of a stabilising speculation, the storer buys when prices are low and when rationally it expects that they will become higher later.

5.1.3 Producers

A representative producer makes his productive choice one period before bringing output to market. He puts in production in period \( t \) a level \( H_t \) for period \( t + 1 \), but a multiplicative disturbance (e.g., a
weather disturbance) affects final production. The producer chooses the production level by solving the following maximisation of expected profit:

$$\max_{\{H_{t+1}\}_{t=0}^{\infty}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i [P_{t+i} \epsilon_{t+i} H_{t+i-1} - \Psi (H_{t+i})] \right\},$$

(5.7)

where $\Psi (H_t)$ is the cost of planning the production $H_t$ and $H_t \epsilon_{t+1}$ is the realised level. $\epsilon_{t+1}$ is the realisation of an i.i.d. stochastic process of mean 1 exogenous to the producer, which follows a translated beta distribution. Like the storer’s problem, this problem can be reformulated into a recursive form, and its solution gives the following Euler equation:

$$\beta E_t (P_{t+1} \epsilon_{t+1}) = \Psi' (H_t).$$

(5.8)

This equation has a straightforward interpretation: it is the equality between the marginal cost of production and the expected discounted marginal benefit of one unit of planned production. We assume decreasing returns to scale since a production increase requires the use of less fertile lands. The production cost function follows an isoelastic form

$$\Psi (H) = h H^{1+\mu} 1 + \mu,$$

(5.9)

where $h$ is a scale parameter and $\mu$ is the inverse of supply elasticity.

### 5.1.4 Recursive equilibrium

At the beginning of each period, three predetermined variables define the state of the model: $S^p_{t-1}$, $H_{t-1}$ and $\epsilon_t$. They can be combined in one state variable, private availability, the sum of production and private carry-over:

$$A^p_t = S^p_{t-1} + H_{t-1} \epsilon_t.$$  

(5.10)

When government pursues food price stabilisation through public storage, we can also define total availability as the sum of private availability and public carry-over noted $S^g$:

$$A^p_t = S^g_{t-1} + S^p_{t-1} + H_{t-1} \epsilon_t.$$  

(5.11)

Accounting for the presence of public stockholding, market equilibrium can be written as

$$A^p_t = A^p_t + S^g_{t-1} = D (P_t) + S^p_{t} + S^g_{t}.$$  

(5.12)

---

2 As income is assumed to be constant, in what follows the demand function is expressed only as a function of price.
From the above, we can define the recursive equilibrium of the problem without public intervention:

**Definition 2.** In the absence of a stabilisation policy (i.e., \( \zeta = S^i = 0 \)), a recursive equilibrium is a set of functions, \( S^s(A^p) \), \( H(A^p) \) and \( P(A^p) \), defining storage, production and price over the state \( A^p \) and a transition (5.10) such that (i) storer solves (5.4), (ii) producer solves (5.7), and (iii) the market clears.

### 5.2 Designing optimal stabilisation policies

The monetary policy literature distinguishes two types of policies (Clarida et al., 1999, Taylor and Williams, 2010). State-contingent policies, designed to be fully optimal in a given model, which serve as theoretical benchmarks, and simple rules or policies defined with few parameters, which are considered more practical since they are less model-dependent and are simpler to communicate and implement. We follow this logic and examine both optimal state-contingent storage policies under commitment and under discretion, and simple rules. State-contingent rules are designed using the methodology developed in Marcet and Marimon (1999) for commitment, and in Klein et al. (2008) for discretion.

Policies start at period 0 and are unanticipated by the agents. The parameters of stabilisation policies are determined by maximising a social welfare function that aggregates consumers’ utility, other agents’ surpluses and fiscal cost. Initial availability is taken as the deterministic steady-state level and initial private stocks are assumed to be null. Government is assumed to be able to manage storage facilities at the same costs as private storers.

#### 5.2.1 Social welfare function

In a partial equilibrium model, it is not possible to take the expected sum of discounted consumer utility as the policy authority objective as is usual in optimal policy problems. Instead, the welfare of all the agents must be included in the objective. The approach in this chapter also differs from most partial equilibrium analyses in considering risk aversion. In partial equilibrium models, policy design is based on maximising the sum of all agents’ surpluses. This takes no account of risk aversion, since the expected consumer surplus is not a measure of the welfare of risk-averse agents (Helms, 1985b). Instead of using the sum of agents’ surpluses, we combine the utility of consumers with the surplus of other agents.

Thus, we introduce a social welfare function, which weights linearly the welfare of each agent. At period \( t \), social welfare is given by

\[
W_t = v(P_t, Y) + w \left[ P_t H_{t-1} \epsilon_t - \Psi(H_t) + P_t S_{t-1}^s - (P_t + k - \zeta) S_t^s - Cost_t \right],
\]  

(5.13)
where $w$ is the weight in social welfare to monetary terms, and $\text{Cost}_t$ is the fiscal cost of the policy. The use of a unique weight for all monetary terms implies implicitly that any distortionary cost caused by tax revenue collection is neglected.

The weight $w$ is chosen such that it ensures that transfers (i.e., changes in average expenditures/profits/costs) between agents do not affect welfare. Storage policies affect price distributions, which enables the desired risk sharing. In addition to spreading risk among agents the changes in price distributions create monetary transfers. In order to focus on efficiency issues, we assume that the level of social welfare cannot be improved by these transfers and choose the weight accordingly. To rule out transfers, $w$ is defined as the value in utility terms of a unitary monetary transfer to consumers. The benchmark situation we consider to determine $w$ is the ergodic distribution of the model without public intervention. Given this definition, a small, permanent, monetary transfer, $\delta$, to consumers should not change welfare:

$$E \left[ v(P,Y + \delta) - w\delta \right] = E \left[ v(P,Y) \right]. \quad (5.14)$$

A first-order approximation of the left-hand side around $\delta = 0$ gives $w$ equal to unconditional expectation of the marginal utility over income:

$$w = E \left[ v_Y (P,Y) \right]. \quad (5.15)$$

The cost of the policy is either the amount of the subsidies given to private storers or the cost of carrying public stock, which is similar to the profit from private storage. It is equal to the difference between purchasing plus storage costs of new stock, and the revenue derived from selling previous period stock, $S_{t-1}^C$:

$$\text{Cost}_t = \zeta S_t^c + (P_t + k) S_t^c - P_t S_{t-1}^c. \quad (5.16)$$

Using equations (5.10) and (5.16), social welfare can be simplified as

$$W_t = v(P_t, Y) + w \left[ P_t \left( A_t^p + S_{t-1}^c \right) - \Psi(H_t) - (P_t + k) \left( S_t^a + S_t^c \right) \right]. \quad (5.17)$$

In the social welfare function, the subsidy is positive in storers’ profit and negative in public cost, so eventually it disappears.

### 5.2.2 State-contingent policies

State-contingent policies are public storage policies in which storage decisions are based on the state of the system. The state of the system is defined by total availability. We consider two state-contingent policies based on government’s ability to commit to a policy rule. In both cases, government maximises the expected intertemporal social welfare function. If it is able to commit to an intervention rule, its storage rule is defined at the first period of the policy implementation and applied for all the following periods, while without commitment, government chooses the optimal
storage level at each period. For conciseness, the dynamic programming problem and first-order conditions of the optimal policy problems are presented in the appendix (for a detailed interpretation of similar first-order conditions, see chapter 4).

**Policy under discretion**

We consider first a time-consistent Markovian policy (see, e.g., Klein et al., 2008, and Ambler and Pelgrin, 2010, for characterisation of the equilibrium concept). The government is, in each period, a Stackelberg leader, moving first while accounting for agents’ best response to the policy (i.e., their first-order conditions). The choice concerns current-period endogenous variables and depends only on the current state, total availability, and the value of future variables is taken as given based on rational expectations about them. So, at each period, the policy maker maximises the conditional expected sum of the discounted social welfare function subject to the equations defining the rational expectations equilibrium:

\[
\max_{S_t \geq 0, P_t, A_t, H_t} \sum_{i=0}^{\infty} \beta^i \left\{ v(P_{t+i}, Y) + w \left[ P_{t+i}A_{t+i}^r - \Psi(H_{t+i}) - (P_{t+i} + k) (S_{t+i}^g + S_{t+i}^s) \right] \right\} \tag{5.18}
\]

subject to equations (5.6), (5.8), (5.11) and (5.12), \(A_0\) given, and anticipating \(\{S^g_{t+i}, H_{t+i}, P_{t+i}, A^r_{t+i+1}, S^s_{t+i}\}\) for \(i \geq 1\).

**Policy under commitment**

The optimal policy rule under commitment is set at the initial period and maintained unchanged for all subsequent periods. It is defined to maximise intertemporal social welfare, and so solves the following problem

\[
\max_{\{S_t \geq 0, H_t, P_t, A_t\}} \sum_{t=0}^{\infty} \beta^t \left\{ v(P_t, Y) + w \left[ P_tA^r_t - \Psi(H_t) - (P_t + k) (S^g_t + S^s_t) \right] \right\} \tag{5.19}
\]

subject to equations (5.6), (5.8), (5.11) and (5.12), and \(A_0^r\) given.

One difference between commitment and discretion is that under the former, the storage rule is history-dependent: the storage level decided at one period depends on behaviour in previous periods. This dependence is created by the commitment. By committing to a rule government is able to manipulate the expectations of private agents. In a rational expectations equilibrium, government’s actions confirm private agents earlier expectations. So this manipulation of expectations comes at the cost of the need to respect past promises, which creates path-dependence (formally, these promises are represented in the first-order condition (5.30) in the appendix by lagged Lagrange multipliers, which are introduced as additional state variables).
5.2.3 Optimal simple rules

Simple rules are rules of public behaviour providing a simple feedback between observable variables and policy instruments. To be simple, these rules have to be defined by a small set of parameters, and not be infinite-dimension objects like the state-contingent policies defined above. We consider two simple rules commonly discussed in the literature: a constant subsidy for storage, which aims at stimulating private storers’ activity, and a price-band defended by a public stock. They are defined respectively by one and three parameters, which are chosen to maximise intertemporal social welfare.

Since these rules are not fully flexible with respect to the state of the system, in contrast to a state-contingent policy, they do not provide an optimal policy for each situation. Despite this limited flexibility, simple rules are discussed at length in the literature on monetary and fiscal policy, which recognises their good properties. Simple rules entail commitment. They also tend to be more robust than globally optimal policies. The need for robustness emerges from our limited knowledge of the way the economy works. The literature does not completely agree about the best explanations for agricultural price dynamics; and it might be better to find a simple rule that is robust across a variety of frameworks than to design optimal policies that are model-specific and whose good properties do not transfer well to another model (Levin et al., 2003). Gardner (1979b) analyses this issue of robustness in a situation where ignorance is related to the level of negative externality created by high commodity prices. He shows, in that context, that a price-band policy may be preferable to an optimal stockpiling policy. In the present study, we do not analyse the robustness of simple rules. We only consider, in a given framework, how simple rules compare with optimal state-contingent rules.

Unlike state-contingent rules, the parameters of optimal simple rules are not determined by solving the first-order conditions of a maximisation problem. The parameters that need to be determined are constant, and maximising over constant variables creates terms with infinite sums that are difficult to handle. To find these parameters, a method similar to the nested fixed-point approach in Rust (1987) is applied. It consists of two nested algorithms. The inner one solves the rational expectations problem for a given set of policy parameters. The outer one adjusts the parameters of the rule to maximise intertemporal social welfare by applying an optimisation solver.

Subsidy to storage

A constant subsidy to storage is a very simple policy, since it consists only of giving private storers a constant subsidy $\zeta$ for each stockpiled unit. The intervention does not vary, there is no feedback between observations and policy actions, the underlying idea being that private storers do a good job at reacting to the economic situation, but provide too little stabilisation and should be encouraged to do more. The subsidy makes physical speculation more profitable and increases stock levels, which stabilises prices more than in the absence of subsidy. Since there is no public storage, storage subsidy makes private availability equal to total availability.
The optimal subsidy is the value maximising the conditional expected sum of the discounted social welfare, defined by (5.17), subject to (5.6), (5.8), (5.10) and (5.12).

**Price-band**

A price-band programme is a controversial policy. On the one hand, it is probably the most commonly proposed stabilisation instrument in the policy arena. Its simplicity is appealing to policy makers. It consists of defining two prices: a floor and a ceiling price, which would be defended by a public stock. Such a scheme was proposed in 1942 by Keynes (1974) as a way to stabilise internationally traded raw materials. Subsequently, it was used in three international commodity agreements: cocoa, rubber and tin (Gilbert, 1996). BULOG, the Indonesian Food Logistics Agency, has defended, with some success in the rice market, a floor price to protect farmers and a ceiling price to protect consumers (Timmer, 1997).

On the other hand, theoretical analysis of price-bands has led to much criticism (Miranda and Helmberger, 1988, Williams and Wright, 1991, Ch. 13–14). One major critique of price-band programmes is that they tend to over-accumulate stock since nothing is sold until the ceiling price is reached. They may also lead to explosive stock levels in the case of poor combinations of floor and ceiling prices. Finally, policy makers expect these programmes to stabilise prices between the bounds, but the models predict that, on the contrary, prices tend to get stuck at the bounds, challenging them continually, but will rarely be lying between them.\(^3\)

In our implementation of a price-band, to avoid extreme over-accumulation, we place a capacity constraint, \(\bar{S}_g\), on public storage.\(^4\) Once public stock reaches its capacity constraint, public intervention ceases and the floor price is no longer defended until there is a decrease in the public stock. So this policy is governed by three parameters: a floor price, \(P_f\), a ceiling price, \(P_c\), and a capacity constraint. The parameters are determined optimally to maximise welfare, so if the capacity constraint does not improve welfare, it will be given a very high value thus making it redundant.

The behaviour of a price-band with capacity constraint obeys some simple principles. When the price is above the floor price, there is no accumulation of public stock:

\[
P_t > P_f \Rightarrow \Delta S_t^g \leq 0. \tag{5.20}
\]

Likewise, when the price is below the ceiling price, government does not sell stock:

\[
P_t < P_c \Rightarrow \Delta S_t^g \geq 0. \tag{5.21}
\]

\(^3\) For a non-technical summary of all criticisms, see Wright (forthcoming).

\(^4\) Alternatively, a maximum public spending could have been defined. However, a storage capacity constraint is simpler to express mathematically.
When the capacity constraint is reached, the floor price is not defended and the price can decrease below it:

\[ P_t < P^f \Rightarrow \Delta S^g_t = S^g_t - S^g_{t-1}. \]  
(5.22)

Finally, the ceiling price is not defended when the public stock is exhausted:

\[ P_t > P^c \Rightarrow \Delta S^g_t = -S^g_{t-1}. \]  
(5.23)

These four conditions are expressed in a concise mathematical formulation as two mixed complementarity equations in the appendix.

Under a price-band policy, the problem has two state variables: private availability, defined by (5.10), and previous period public stock. Public stock appears as a separate state variable, because it is not directly available for the market. It is used only to defend the price-band and so does not play the same role as private stock and production, which can be summed together.

5.3 RESULTS

The rational expectations storage model is known to lack a closed-form solution. It has to be approximated numerically. The numerical method used here is inspired by Miranda and Glauber (1995). It is a projection method with a collocation approach. A grid on the state variables is chosen. On this grid, the terms inside expectations are approximated by splines. Equilibrium equations are solved by the mixed complementarity problems solver PATH (Dirkse and Ferris, 1995). Once the rational expectations equilibrium is identified, the decision rules are approximated by splines and used to simulate the model. The number of grid nodes is chosen such that the use of these decision rules entails less than a $1 error on average for every $1,000 of decision (measured by the Euler equation error).

5.3.1 CALIBRATION

Table 5.1 presents the parameters used to calibrate the model. The parameters are set such that, at the model’s non-stochastic steady state, price, production, consumption and availability are equal to 1. An annual interest rate of 5% is used for discounting.

Seale and Regmi (2006) estimate elasticities for food consumption across 144 countries. From their research, we choose cereal elasticities typical of low-income countries: \(-0.4\) for price elasticity and \(0.5\) for income elasticity. We assume that consumers spend, at the steady state, \(\gamma = 15\%\) of their income on the staple (a value intermediate between what is observed for rice consumption in poor and affluent households in Asia, Asian Development Bank, 2008). Since steady-state consumption and price are equal to 1, income, which is assumed to be constant, is equal to the inverse of the commodity budget share, \(1/\gamma\).
Table 5.1. Parameterisation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Economic interpretation</th>
<th>Assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Annual discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Income elasticity</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Own-price demand elasticity</td>
<td>−0.4</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Commodity budget share</td>
<td>0.15</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Inverse of supply elasticity</td>
<td>2</td>
</tr>
<tr>
<td>$Y$</td>
<td>Income</td>
<td>6.67</td>
</tr>
<tr>
<td>$d$</td>
<td>Normalisation parameter of demand function</td>
<td>0.39</td>
</tr>
<tr>
<td>$h$</td>
<td>Normalisation parameter of production cost function</td>
<td>0.95</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Parameter defining risk aversion</td>
<td>−6.13</td>
</tr>
<tr>
<td>$k$</td>
<td>Physical storage cost</td>
<td>0.02</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Probability distribution of yield</td>
<td>$B(2, 2) \cdot 0.5 + 0.75$</td>
</tr>
</tbody>
</table>

Consumers enjoy welfare gains from a stabilisation at mean price if their indirect utility function is concave in price. Turnovsky et al. (1980) show that this is the case if $\gamma (\eta - \rho) - \alpha < 0$, that is, with our parameters, $\rho > 3.17$. We thus assume at the steady state a relative risk aversion parameter of 4, implying $\theta = −6.13$.

We follow Lence and Hayes (2002) and assume a per-unit storage cost of 2% of the steady-state price (i.e., $k = 0.02$). Estimation of the price elasticity of supply produces controversial results (Rao, 1989, Schiff and Montenegro, 1997). We choose a value of 0.5, often produced by related studies, and analyse later the sensitivity of results to this value. For this elasticity, we have $\mu = 2$. The productive shocks, $\epsilon$, are assumed to follow a beta distribution of shape parameters, 2 and 2, which makes it unimodal at 0.5 and symmetric. The distribution is centred and rescaled to vary between 0.75 and 1.25, which implies a coefficient of variation of 11.2%.

5.3.2 Storage behaviour

Without storage policy, private storers (solid line in figures 5.1–5.2) do not stock anything for low availability (the threshold is close to the steady-state availability level, 1), and increase the level of stock with market availability above the threshold. When normal consumption is satisfied, any additional quantity in the market tends to lower prices. The speculators take opportunity of these lower prices to accumulate stock that can be sold in periods of lower availability. If government intervenes, total stock is always higher for a given availability than without intervention.

State-contingent public storage

State-contingent public storage completely crowds out private storage. Public stockpiling starts at a lower total availability than would do private storers, and public storage rules present a higher marginal propensity to store than the private storage rule without intervention. The higher stock level results in prices that are more stable. This stabilisation is such that there is no profit opportunity left for speculation.
Under commitment, the storage rule is not defined only by total availability (see the grey dots in figure 5.1, these points are obtained by simulation of the optimal storage rule), which illustrates the history-dependence of the rule. As explained above, history-dependence arises as the need to respect past commitments in a rational expectations model. Since private storers are crowded out, government commitments concern only agents with intertemporal trade-offs: producers.

Path-dependence works as follows. For a given current availability $A_t$, the lower the previous availability, $A_{t-1}$, the higher will be the current public stock, $S_t^G$, and vice versa. When availability is low, it is difficult to stockpile without hurting consumers by raising prices too much. However, to ensure a future sufficient availability, government promises producers a high price at the next-period, thus creating incentives for high production levels. At the next-period, the only way for government to meet its commitments is to stockpile more in order to raise the price for producers. Conversely, when availability is high, government would like to decrease production more than producers would tend to do so. It thus promises a lower price at the next period and, in order to keep this promise, it has to stockpile less than it would have done without commitment in order to force the price down. Were government to renege on its promise, it would reset the lagged Lagrange multipliers to zero and choose another storage level. For most levels of availability, the level of storage under commitment may be below or above the level under discretion depending on the state’s history. However, for availabilities above 1.4, the storage rule under commitment is always below the rule under discretion. A situation with such high availability levels can only follow another period of high availability, hence it implies public engagement to limit stockpiling in order to decrease the expected price and limit production.

Public storage under discretion (short-dash line) is close to the level under commitment, one
main difference being that it is a function only of current availability and is not time-dependent. Under discretion, government cannot commit to a rule and so has no need to respect past promises.

**Subsidy to private storage**

A policy of subsidy to private storage shifts the storage rule to the left (black, long-dash curve in figure 5.2). It produces a storage rule that is close to the optimal, state-contingent, discretionary rule. The optimal level of subsidy is 0.041, which represents 58% of steady-state storage cost (physical cost plus opportunity cost for a price of 1).

![Figure 5.2. Storage rules for optimal simple rules policies](image)

We have assumed a constant subsidy. This is convenient for policy making since this policy is simple and implies commitment. The drawback is that it lacks the flexibility of a state-contingent public storage to adjust to a changing situation. Other schemes of subsidies could have been considered. For example, instead of being constant a subsidy to private storage could be state-contingent. A state-contingent subsidy allows decentralisation of an optimal public storage policy by compensating the speculator for its loss when following a welfare maximising storage-rule. Figure 5.3 represents such a state-contingent subsidy, which decentralises the discretionary optimal policy described above.

The profile of the state-contingent subsidy is determined by two opposite effects. Arbitraging the social marginal value between two successive periods would call for a decreasing subsidy, because it is socially beneficial to increase stockpiling more in low stock situations than in high stock situations. On the other hand, this subsidy has a feedback effect on producers’ behaviour. Since it increases storage, it reduces the incentive to produce. This effect would call for a low subsidy at availability.
close to steady state in order to limit the negative effect on production and a high subsidy at high availability in order to avoid adding more production to an expected glut. The combination of these two effects gives this subsidy profile in which the feedback on production dominates at low availability with an increasing curve and then the decreasing part coming from welfare arbitrage dominates.

**Price-band programme**

When the price-band policy is designed to maximise welfare, the optimal floor and ceiling prices collapse in a single price. This kind of policy, a floor-price with a capacity constraint on the public stockpile, is analysed in Salant (1983), Williams and Wright (1991, Ch. 14) and Brennan (2003), and dubbed a “price-peg”, the term we use in what follows. Government defends the target price by building stock, but when price is above target, public stock is sold. This behaviour is as is expected under a price-band except that the two bounds are confounded. This result contrasts with the intuition that to protect consumers, stocks should be accumulated at low prices and used to prevent prices from exceeding a limit. However, this is in line with the theory on price-band programmes. Miranda and Helmberger (1988, table 3) show that the deadweight loss from a price-band programme is the lowest for a zero bandwidth. Williams and Wright (1991, Ch. 14) also find that the price-peg scheme entails smaller losses than either a price-band or a simple floor-price. They also explain that a major flaw in price-band schemes is that they tend to over-accumulate. This is caused by the absence of public stock sales while the price is between the bounds. A price-band policy does not try to stabilise the price inside the bounds, but only to prevent the price from exceeding them. Under a price-band scheme, public stock is not considered as a potential source of supply, but only as a mean of defending the bounds, so the stock can reach very high levels if the bounds are set inappropriately. A price-peg scheme avoids this drawback by putting stock on the market as soon as it is accumulated. Excessive accumulation is also avoided by the existence of a capacity limit.
The optimal coincidence of floor and ceiling prices may be sensitive to one assumption rarely considered in the storage literature, but likely relevant: the transaction costs involved in adjusting stock levels (Chavas et al., 2000). For example, for storage in a silo, grain has to be cleaned and the temperature reduced to prevent pest infestation, which entails costs. These adjustment costs may justify a small wedge between the two prices to avoid too many stock level adjustments.

Under this price-peg programme, speculators still find profit opportunities (see the grey curves in figure 5.2). The optimal target price is 1.008, which is just above the deterministic steady-state price, 1, and above the price threshold under which other storage rules start to accumulate. Thus, for prices higher than the intervention price, there is some private storage. For lower prices, and so higher availability, the price would tend to go below the target and government would start to accumulate. At this point, there is no profit to be expected from holding stock so private speculators dump all their stock on the public stock. For still higher availability, government cannot defend the price-peg, because its stock bumps into its capacity constraint, optimally set at 0.18. At this point, the tendency of over-accumulation appears clearly. The storage rule under price-peg scheme is above all other rules and the marginal propensity to store is 1. When the capacity constraint is reached, no stock, public or private, is added to the existing stock. Private storers start to accumulate again for higher levels of availability.

These interactions between private speculation and public stock are illustrated in the simulation in figure 5.4. Public stock frequently reaches its capacity limit. This implies that the price-peg programme does not work as expected for a traditional price-band. The target price is not seriously defended since public stock is often at its limit. By its nature, a price-peg cannot be defended indefinitely if the initial public stock is finite. And, without the capacity constraint, the accumulation of public stock under this policy would be explosive given the high level of the intervention price. Actually, the price-peg is just a way to define a rule allowing the accumulation of public stocks to protect consumers from high prices and from fluctuations, it is not a target that government seriously tries to support.

5.3.3 Dynamics under food price stabilisation policies

In this section, we analyse the transitional behaviour following policy implementation. We do not analyse the dynamics of endogenous variables for specific simulations, only their expectations while they converge to their new asymptotic distributions. In any period, the realised values may be above or below these expectations. Since the policies involve higher mean stock levels than without intervention, they begin with a phase of stock build-in. The transition is illustrated in figure 5.5 with the dynamics of mean total stock, mean government outlays and mean prices. The average stock level almost doubles under an optimal policy. The lowest stock level increase occurs for the price-peg policy and the highest for the discretionary policy.

For public storage policies, mean government outlays are very high in the first period because stocks must be accumulated and there is no stock to sell. The first-year costs exceed 8% of the
Figure 5.4. Simulation of total availability, public stock and private stock under the optimal price-band programme

commodity budget share. In the long-run, once stocks have been accumulated, policies are financially self-balanced with proceeds from sales covering, in average, purchase and storage costs. A subsidy policy is cheaper in the short-run, but more expensive in the long-run, since government has to pay storers proportional to stock levels, which rise with time.

In the first years, the purchases made to accumulate stocks push mean prices above their long-run values. After a few years, mean prices drop below the value without public intervention. Because of the convexity of the demand curve, the additional storage decreases higher prices more than it increases low prices. Hence, stabilisation entails lower mean prices and the effects are almost identical for all policies.

The dynamic welfare effects of the policies are presented in table 5.2. Consumer gains are ex

Figure 5.5. Transitional dynamics. Before period 0, the system is on the asymptotic distribution of the model without public intervention. The unexpected policy is announced and starts in period 0. (Obtained by Monte Carlo simulation as the average over 100,000 simulated paths.)
ante per-period equivalent variation, EV, calculated using the following implicit definition

$$
E_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[ v(P_t, Y + EV) - v(\hat{P}_t, Y) \right] \right\} = 0,
$$

(5.24)

where $P$ and $\hat{P}$ are the prices before and after stabilisation. Producers gains are the annualised changes in expected profit from equation (5.7). Government outlays are the annualised expected sum of discounted costs defined by equation (5.16). Changes in storers’ surpluses are ignored since storers operate, in average, at zero profit and we have assumed no stock at the first period. Welfare increases for all policies. As expected, it is maximum for public storage under commitment.

Table 5.2. Welfare results on transitional dynamics (as a percentage of the steady-state commodity budget share).

<table>
<thead>
<tr>
<th></th>
<th>Commitment</th>
<th>Discretion</th>
<th>Subsidy</th>
<th>Price-band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers gains</td>
<td>0.506</td>
<td>0.465</td>
<td>0.441</td>
<td>0.451</td>
</tr>
<tr>
<td>Producers gains</td>
<td>0.347</td>
<td>0.339</td>
<td>0.293</td>
<td>0.210</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0.575</td>
<td>0.548</td>
<td>0.475</td>
<td>0.434</td>
</tr>
<tr>
<td>Total gains</td>
<td>0.278</td>
<td>0.256</td>
<td>0.258</td>
<td>0.227</td>
</tr>
</tbody>
</table>

Most welfare results differ little across policies. A first conclusion is that there are limited gains from commitment. This is explained by the source of time-inconsistency, which lies in producers’ expectations. The manipulation of producers’ expectations helps government achieve price stabilisation by giving it an additional instrument: a partial control on planned production, and changing planned production is an alternative to storage to affect next-period availability. However, because supply reaction is small (supply elasticity is 0.5), manipulating producers’ expectations entails limited gains (this issue is discussed in more detail in section 5.3.5).

An optimal constant subsidy for private storage achieves welfare results close to state-contingent policies. Comparison of the storage rules under discretion and under constant subsidy (figures 5.1–5.2) shows that they are fairly similar. In monetary policy theory, a rationale for using simple rules is to achieve most of the gains from commitment via a policy that is easier to implement than a globally optimal policy. The gains from commitment being small here, a constant subsidy does not achieve substantially higher welfare than the policy under discretion. A storage subsidy could still be defended on the grounds that it is a very simple policy to announce and decentralises the policy implementation to private agents.

The price-band, which in our case collapses to a price-peg mechanism, is the worst of the four policies. It manages, nevertheless, to reap more than four-fifths of the welfare gains from the policy under commitment. As shown in figure 5.2, one problem with the price-band policy is its tendency to over-accumulate. Because of its marginal propensity to store equal to 1 when the intervention price is defended, stocks accumulate much faster than what is prescribed by the optimal rules. Excessive accumulation is only prevented by the capacity constraint on storage.
The relative good performance of the price-band should be linked to the optimal choice of parameters. Table 5.3 illustrates what would be the social welfare changes from price-bands with different parameters. It considers three floor prices and three limits on public storage. Ceiling prices are chosen to illustrate price-pegs, symmetric price-bands and ceilings at the deterministic steady state. There is no monotonic relationship between welfare and any parameter. Increasing one parameter increases welfare for some combinations of the two other parameters and decreases it for other combinations. The capacity constraint is essential for reaching the highest gains; with no constraint there can be several situations with welfare losses. With a capacity constraint, welfare tends to increase with the floor price, because a higher floor increases the occurrence of interventions, but with no risk of over-accumulation since the size of public stocks is limited. Despite the intuitive appeal of defining a policy by a mean price with symmetric bounds, symmetric price-bands do not perform well, since the higher the upper bound, the longer the food is stored.

Table 5.3. Social welfare changes from non-optimal price-band policies (as a percentage of the steady-state commodity budget share).

<table>
<thead>
<tr>
<th>Floor price</th>
<th>Ceiling price</th>
<th>Capacity constraint on public stock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>0.85</td>
<td>0.85</td>
<td>−0.000</td>
</tr>
<tr>
<td>0.85</td>
<td>1.00</td>
<td>0.017</td>
</tr>
<tr>
<td>0.85</td>
<td>1.15</td>
<td>0.016</td>
</tr>
<tr>
<td>0.90</td>
<td>0.90</td>
<td>0.048</td>
</tr>
<tr>
<td>0.90</td>
<td>1.00</td>
<td>0.091</td>
</tr>
<tr>
<td>0.90</td>
<td>1.10</td>
<td>0.068</td>
</tr>
<tr>
<td>0.95</td>
<td>0.95</td>
<td>0.153</td>
</tr>
<tr>
<td>0.95</td>
<td>1.00</td>
<td>0.171</td>
</tr>
<tr>
<td>0.95</td>
<td>1.05</td>
<td>0.149</td>
</tr>
</tbody>
</table>

Speculative storers are essential for good functioning of the optimal price-band. They stock for prices below and above the target. Hence they provide some stabilisation in situations where the public storage rule is inactive. Without speculator activity, total welfare gains from the price-peg would be halved. This situation is all the more surprising since, in some situations, private storers make speculative attacks on public stocks. These interactions between speculators and the public stock are discussed in Salant (1983). He demonstrates the existence of speculative attacks on public stock when stocks fall below a threshold. This behaviour is depicted in figure 5.2. The public stock level does not decrease continuously to zero: at some availability, all public stock is bought by speculators, who expect that prices may increase in the next period. Given that speculation brings additional stability, the vulnerability of price-band schemes to speculative attack is not something that can be criticised: it is rather a feature of their limited behaviour. Under a price-band programme government intervenes only to defend the bounds and does not consider volatility inside and outside the bounds. Because there are profit opportunities beyond the bounds, speculators bring additional stabilisation.
5.3.4 The long-run effects of stabilisation policies

On the asymptotic distribution, stabilisation policies decrease the occurrence of low prices (see the shift in the distribution to the right in figure 5.6), because there is more storage in situations of high availability than without public intervention. This effect is less important for the price-band policy since public stockpiling starts at higher availability levels.

![Figure 5.6. Density of price under various intervention schemes.](image)

Price density without public intervention presents a long right tail, where consumers suffer from high prices. All stabilisation policies reduce the frequency of very high prices thanks to the additional storage made at high availability.

For the price-peg policy, the target price is located above the steady-state price. This means that, without public intervention, prices will often be below this target, implying that under a price-peg government intervention will be frequent. We observe that when prices are not on target, they are mostly below it. 44% of the time government defends the target by accumulating or selling stocks, but because the target is so high public stock often reaches its capacity constraint and government is forced to let the price fall below the target price. Thus, this policy often fails to defend the price-band. However, this should not be seen as a failure. The price-peg programme is designed to maximise the welfare of consumers that face incomplete risk markets. Hence failure to defend the price-peg is purposeful. The target price is a pretext for accumulating stocks to stabilise prices, and is not expected to be defended seriously. This behaviour shows that a price-band achieves reasonable welfare performance only by being designed in a way opposite to most expectations.

Optimal storage under discretion achieves the lowest coefficient of variation of price (see table 5.4), because of a mean stock level much higher than for other policies. In contrast, the policy under commitment achieves a similar level of stabilisation with much lower stocks. This is achieved by
manipulating producers’ expectations, which is confirmed by the fact that planned production is more variable under commitment than under discretion or any other policy. Stabilisation is more efficient under commitment because the policy maker, by manipulating expectations, can induce producers to stabilise in addition to public storage.

Table 5.4. Descriptive statistics on the asymptotic distribution

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Commitment</th>
<th>Discretion</th>
<th>Subsidy</th>
<th>Price-band</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of variation of price</td>
<td>0.175</td>
<td>0.117</td>
<td>0.115</td>
<td>0.121</td>
<td>0.128</td>
</tr>
<tr>
<td>Correlation between price and production</td>
<td>−0.728</td>
<td>−0.518</td>
<td>−0.572</td>
<td>−0.589</td>
<td>−0.602</td>
</tr>
<tr>
<td>Mean total stock</td>
<td>0.067</td>
<td>0.126</td>
<td>0.138</td>
<td>0.128</td>
<td>0.120</td>
</tr>
<tr>
<td>Coefficient of variation of planned production</td>
<td>0.026</td>
<td>0.030</td>
<td>0.025</td>
<td>0.025</td>
<td>0.027</td>
</tr>
<tr>
<td>Standard deviation of social welfare components</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumers’ utility</td>
<td>0.080</td>
<td>0.053</td>
<td>0.052</td>
<td>0.055</td>
<td>0.058</td>
</tr>
<tr>
<td>Producers’ profit</td>
<td>0.098</td>
<td>0.110</td>
<td>0.097</td>
<td>0.097</td>
<td>0.101</td>
</tr>
<tr>
<td>Storers’ profit</td>
<td>0.077</td>
<td>0</td>
<td>0</td>
<td>0.092</td>
<td>0.031</td>
</tr>
<tr>
<td>Government outlays</td>
<td>0</td>
<td>0.100</td>
<td>0.095</td>
<td>0.004</td>
<td>0.083</td>
</tr>
</tbody>
</table>

Note: statistics calculated over 1,000,000 sample observations from the asymptotic distribution.

Table 5.4 presents the standard deviations for each component of the social welfare function as defined in (5.13), which allows the analysis of risk-sharing among the agents. Risk is mostly transferred from consumers to government budget. The volatility of the producers’ profit is influenced by several endogenous components evolving in opposite ways: price volatility, volatility in planned production, and the correlation between price and production. Price volatility decreases for all policies. The volatility of planned production increases for the policy under commitment, since government tries to make supply more elastic by manipulating producers’ expectations. It increases also for the price-band policy. Price and production are negatively correlated, but this correlation decreases with a storage policy. The combined effect is an increase in the volatility of producers’ profit for the state-contingent policy under commitment, because of the increase in the volatility of planned production and the reduced price/production correlation. There is also an increase in producers’ profit volatility with a price-band policy.

5.3.5 The sensitivity of time-consistency to supply elasticity

Since the difference between commitment and discretion is caused by manipulation of producers’ expectations, we explore the sensitivity of this difference to supply elasticity, the most important parameter of producers’ behaviour. Supply elasticity affects both the importance of time-consistency and the volatility without public policy (table 5.5). A higher supply elasticity implies more stable prices and lower gains under a stabilisation policy since planned production reacts more to changes in expected prices. For a very low elasticity of 0.01, differences between commitment and discretion are negligible, since government cannot induce supply changes by manipulating expectations. The gains from commitment increase with the elasticity, but remain limited.
Table 5.5. Sensitivity to supply elasticity

<table>
<thead>
<tr>
<th>Supply elasticity ((1/\mu))</th>
<th>0.01</th>
<th>0.10</th>
<th>0.50</th>
<th>1.00</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total gains</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commitment</td>
<td>0.284</td>
<td>0.284</td>
<td>0.278</td>
<td>0.251</td>
<td>0.225</td>
</tr>
<tr>
<td>Discretion</td>
<td>0.284</td>
<td>0.280</td>
<td>0.256</td>
<td>0.222</td>
<td>0.197</td>
</tr>
<tr>
<td>Coefficient of variation of price</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without policy</td>
<td>0.202</td>
<td>0.194</td>
<td>0.175</td>
<td>0.162</td>
<td>0.150</td>
</tr>
<tr>
<td>Commitment</td>
<td>0.150</td>
<td>0.141</td>
<td>0.117</td>
<td>0.104</td>
<td>0.092</td>
</tr>
<tr>
<td>Discretion</td>
<td>0.149</td>
<td>0.139</td>
<td>0.115</td>
<td>0.103</td>
<td>0.092</td>
</tr>
</tbody>
</table>

Notes: Benchmark in bold. Welfare gains as a percentage of the steady-state commodity budget share.

5.4 Conclusion and perspectives

In this chapter, we evaluated the properties of different storage policies designed to maximise social welfare, in a model representing the cereals market in a self-sufficient developing country. Equilibrium without public intervention is non-optimal because markets are incomplete. We consider two state-contingent policies, with and without commitment, and two optimal simple rules of storage, a constant subsidy to private storage and a price-band defended by public stocks limited by a capacity constraint.

Among state-contingent storage rules, the policy under commitment achieves higher welfare results with lower average public stocks than the discretionary policy, because commitment induces producers to promote stabilisation. The gains numerically are small, however, since supply reaction is limited. Regarding simple rules of storage, the storage subsidy produces welfare results close to the discretionary policy, and the welfare gains under the price-band scheme are the lowest of all tested policies, but still represent 82% of the gains from the Ramsey-optimal policy.

A price-band policy is defined by three parameters: a floor price, a ceiling price and a capacity constraint on public storage. The parameters maximising welfare are a floor price equal to the ceiling price and close to the non-stochastic steady state, and a capacity constraint of 18% of steady-state production. This price-peg cannot be defended indefinitely since, at some point in time, the public stock will be exhausted or will reach its capacity constraint. The target price is set so high that the capacity constraint is most often binding and government is forced to let prices decrease below the intervention level. The target price is not meant to be seriously defended, instead it is a way to force the accumulation of public stocks above the private stock level without intervention.

The small welfare differences found between policies under commitment and discretion contrast with the accounts of food price policies in developing countries (Poulton et al., 2006; Tschirley and Jayne, 2010), where discretionary policies have proven very costly. Our idealised setting differs from real situations in at least one crucial aspect. In reality, the policy maker is not a benevolent planner. There are political cycles and people expect different governments to follow different policies (Alesina and Gatti, 1995). There are rent-seeking behaviours, which may create uncertainty about the group obtaining eventually the most important weight in the political process. These considerations,
traditionally absent in the design of optimal dynamic policies, weaken the case for discretionary policies and reinforce the need to study simple rules of stabilisation whose management can be decentralised to an independent agency.

The potential role of simple storage rules in stabilisation policies may be based also on considerations of model uncertainty. Volatility in agricultural prices is certainly caused by shocks other than just productive shocks. Demand shocks, monetary shocks and input price shocks are all candidates. Although a model featuring productivity shocks associated with private storage looks to be a good approximation for commodity price dynamics (Cañiero et al., 2011), there are still uncertainties about the true data generated process. There are so many uncertainties related to what is the appropriate model and the value of behavioural parameters that it might be better to rely on simple rules with good performance in a variety of settings, than on a fully optimal policy designed for a specific model, but which may behave poorly in other contexts.\(^5\)

\(^5\) This is one usual justification for focusing on simple rules in the conduct of monetary policies (Taylor and Williams, 2010).
5A First-order conditions for the optimal policy under commitment

To solve the optimal policy problems presented above, we need to reformulate the complementarity equation (5.6) since it cannot enter directly as a constraint in a maximisation problem. To restate this equation, we introduce a slack variable, \( \phi_t \), with its associated complementarity slackness conditions

\[
\phi_t = P_t + k - \beta E_t (P_{t+1}), \quad (5.25)
\]

\[
S_t^\phi \phi_t = 0, \quad (5.26)
\]

where \( S_t^\phi \geq 0 \) and \( \phi_t \geq 0 \). \( \zeta \) is taken to equal zero since there is no subsidy under a public storage policy.

We follow Marcet and Marimon (1999) and express the optimal policy problem under commitment as a saddle-point functional equation problem

\[
J (A_t^\tau, \lambda_{t-1} + \nu_{t-1} \epsilon_t) = \min_{\Phi_t} \max_{\Omega_t} \left\{ \nu (P_t, Y) + w [P_t A_t^\tau - \Psi (H_t) - (P_t + k) (S_t^s + S_t^g)] + \lambda_t (\phi_t - P_t - k) + \delta_t S_t^\phi \phi_t - \nu_t \Psi' (H_t) + (\lambda_{t-1} + \nu_{t-1} \epsilon_t) P_t + \chi_t [A_t^\tau - D (P_t) - S_t^s - S_t^g] + \beta E_t [J (S_t^s + S_t^g + H_t \epsilon_{t+1}, \lambda_t + \nu_t \epsilon_{t+1})] \right\}, \quad (5.27)
\]

where \( \Phi_t = \{ \lambda_t, \delta_t, \nu_t, \chi_t \} \) and \( \Omega_t = \{ S_t^s \geq 0, H_t, P_t, S_t^g \geq 0, \phi_t \geq 0 \} \). Lagrange multipliers corresponding to forward-looking terms (\( \lambda \) and \( \nu \)) are included as state variables to make the problem recursive. They represent the constraint for the policy maker to respect previous promises. Since both apply to price expectation, they can be summed together. Thus the system has two state variables, \( A_t^\tau \) and \( \lambda_{t-1} + \nu_{t-1} \epsilon_t \).

From the first-order conditions and the envelope theorem, and after some manipulations we get the following system of complementarity conditions that, in addition to the transition equation (5.11), defines the dynamics of an optimal policy under commitment

\[
S_t^s: S_t^s \geq 0 \quad \perp \quad -wP_t - \chi_t - wk + \beta E_t (w P_{t+1} + \chi_{t+1}) + \delta_t \phi_t \leq 0, \quad (5.28)
\]

\[
H_t: \beta E_t (\epsilon_{t+1} \chi_{t+1}) = \nu_t \Psi'' (H_t), \quad (5.29)
\]

\[
P_t: v_P (P_t, Y) + w D (P_t) - \lambda_t + \lambda_{t-1} + \nu_{t-1} \epsilon_t - \chi_t D' (P_t) = 0, \quad (5.30)
\]

\[
S_t^g: S_t^g \geq 0 \quad \perp \quad -wP_t - \chi_t - wk + \beta E_t (w P_{t+1} + \chi_{t+1}) \leq 0, \quad (5.31)
\]
\( \phi_t : \phi_t \geq 0 \quad \lambda_t + \delta_t S_t^s \leq 0, \) 
\( \lambda_t : \phi_t = P_t + k - \beta E_t (P_{t+1}), \) 
\( \delta_t : S_t^c \geq 0, \) 
\( \nu_t : \beta E_t (P_{t+1} \epsilon_{t+1}) = \Psi' (H_t), \) 
\( \chi_t : A_t^r = D (P_t) + S_t^s - S_t^c. \) 

The time-inconsistency of the policy under commitment shows up in equation (5.30) in the lagged Lagrange multipliers. If the policy maker were to re-optimise after date 0, this would reset the Lagrange multipliers to zero.

5.B CHARACTERISATION OF THE OPTIMAL DISCRETIONARY POLICY

For the discretionary problem, since we focus on a Markovian equilibrium, price can be characterised by a function of the state variable: 
\( P_t = \psi (A_t^r). \)
Using this function \( \psi \), the discretionary problem is formulated as a recursive problem similar to (5.27) except for the use of \( \psi \) to characterise price expectations and the absence of lagged Lagrange multipliers since there are no past promises to respect:

\[
J (A_t^r) = \min_{\phi_t} \max_{\Omega_t} \left\{ v (P_t, Y) + w [P_t A_t^r - \Psi (H_t) - (P_t + k) (S_t^a + S_t^c)] + \lambda_t \{ \beta E_t [\psi (S_t^a + S_t^c + H_t \epsilon_{t+1})] + \phi_t - P_t - k \} + \delta_t S_t^a \phi_t + \nu_t \{ \beta E_t [\psi (S_t^a + S_t^c + H_t \epsilon_{t+1}) \epsilon_{t+1}] - \Psi' (H_t) \} + \chi_t [A_t^r - D (P_t) - S_t^a - S_t^c] + \beta E_t [J (S_t^a + S_t^c + H_t \epsilon_{t+1})] \right\}. 
\] 

It is not possible to characterise the first-order conditions of this problem since we cannot assume \( \psi \) to be differentiable because of the occasionally binding constraints.\(^6\)

5.C EQUATIONS OF A PRICE-BAND PROGRAMME

For a rigorous mathematical characterisation of the behaviour of public stock under a price-band we need to introduce two variables: \( \Delta S_t^{a+} \) and \( \Delta S_t^{a-} \), which refer to increase and decrease in public stock. Both are positive and bounded from above. The increase in public stock is bounded from above by the remaining storage capacity, and the decrease in public stock by the level of existing stocks. To defend the price-band, public stocks are managed by the four conditions (5.20)–(5.23),

\(^6\) In practice, since \( \psi \) is approximated by a spline, which is differentiable everywhere, we numerically solve the dynamic programming problem by solving the corresponding first-order conditions.
which can be restated as two mixed complementarity equations:\footnote{Here the “perp” notation ($\perp$) is extended to situations with two complementarity constraints. The expression $a \leq X \leq b \perp F(X)$ is a compact formulation for $X = a \Rightarrow F(X) \geq 0, X \in ]a, b[ \Rightarrow F(X) = 0, X = b \Rightarrow F(X) \leq 0.$}

\begin{align}
0 & \leq \Delta S^g_t^+ \leq \bar{S}^g - S^g_{t-1} \perp P_t - P^p, \quad (5.38) \\
0 & \leq \Delta S^g_t^- \leq S^g_{t-1} \perp P^c - P_t. \quad (5.39)
\end{align}

Equation (5.38) means that public stocks increase to prevent the price from decreasing below floor price, $P^p$. The floor is defended until public stocks reach the limit $\bar{S}^g$. Equation (5.39) governs the decrease of public stocks. They decrease to prevent price from rising above the ceiling price, $P^c$.

The release of stocks is constrained by the existing level of stocks $S^g_{t-1}$.

Market equilibrium and public stock transition are defined by

\begin{align}
A^p_t &= D(P_t) + S^g_t + \Delta S^g_t^+ - \Delta S^g_t^-, \quad (5.40) \\
S^g_t &= S^g_{t-1} + \Delta S^g_t^+ - \Delta S^g_t^- . \quad (5.41)
\end{align}

The recursive equilibrium under a price-band programme is defined by the equilibrium equations (5.6), (5.8) and (5.38)–(5.40), and the transition equations (5.10) and (5.41).
In poor countries, the price of staple food products is a serious concern. These products frequently mobilise a significant share of poor households’ budget, who have limited insurance possibilities to face adverse price shocks. Price spikes may thus be very problematic for poor households that are not self-sufficient, jeopardising in some cases their capacity to feed themselves. Noting the strong correlation between domestic output and consumption of grains, Reutlinger and Knapp (1980) observed in their abstract that, in the 1970s, “neither stocks nor imports were apparently used aggressively enough to counter fluctuations in production”. Emphasising that “the effects of international trade, in general, and of trade policies, in particular, on domestic food grain supply and prices have not always been appreciated adequately”, Bigman and Reutlinger (1979, p. 657) also stressed the potential benefits that could be reaped from increased recourse to international trade. It is not clear whether such statements would still be valid, given that active policies aiming at counteracting food price instability are now common among developing countries. While a variety of policies exist, including targeted assistance to poor households and price support to farmers, storage and trade policies are the main two instruments used in practice to counter price instability. In most Asian countries, in particular, stabilising the domestic price of rice is a central objective, followed through both buffer stocks and trade restrictions (Timmer, 1989, Islam and Thomas, 1996, Dorosh, 2008). The question is probably less acute in Latin America, where most countries are net exporters of grains, but it is not irrelevant, as witnessed by the use by Chile of a price-band system on wheat and a few other food products (Bagwell and Sykes, 2004).

The relevance of trade for food security is not new. Suspending exports in periods of dearth was actually standard practice in pre-modern times, as emphasised by Outhwaite (1981) in the case of England. However, the link now seems pervasive, and the increasing importance of trade flows makes it more central. Based on large-scale database on agricultural price distortions, Anderson and

Chapter written with Sébastien Jean.
Nelgen (2010) show that countries tend to vary their nominal rate of assistance to agriculture so as to limit the transmission of world price variation to domestic prices.

As illustrated by the 2007–08 price spike in staple food products, the reliance on trade does not solve the problem of food security. To be sure, though, trade and trade policies are key ingredients worth accounting for when thinking about food security and price stabilisation in developing countries, and this raises numerous questions. Perhaps the most important one is how should storage and trade policies be combined by governments aiming at price stabilisation. Increased reliance on national buffer stocks is frequently pointed out as a possible remedy for developing countries faced with the significant volatility of world prices; is this a consistent policy per se, independently from trade policy interventions? Dorosh (2008) suggests that greater reliance on the world market allowed Bangladesh to achieve price stabilisation far more cost-effectively than India, which relied almost exclusively on massive public stocks, while severely restricting imports. Can it be taken for granted that greater trade openness, or more reactive trade policy, may reduce the amount of stocks needed to achieve a given stabilisation target, and to which extent?

Export restrictions raise a number of additional questions. Most analysts of the 2007–08 food crisis agree that trade policies have played a significant role in fuelling the international price spike (von Braun, 2008, Mitra and Josling, 2009, Headey, 2011). In particular, export bans enforced by several rice exporters during the 2007–08 food crisis seem to have heavily contributed to the astonishing price level reached (Slayton, 2009). Noting that the situation was similar in the 1973–74 crisis, Martin and Anderson (2011) emphasise the collective action problem created by export restrictions, since their use by some countries to shelter from price spikes aggravates the problem for others. The one-year restrictions imposed by Russia on its cereal exports following a drought in 2010 can only add to this concern. Yet a first step in coping with this problem is to understand better the motivations and consequences of export restrictions. Based on Marshallian surplus analysis, many authors conclude that such policies are harmful for the countries enacting them. Is it really so, or might export restrictions make economic sense for a small open economy? And in this case, is refraining from using export restrictions an important sacrifice for the country concerned? Should specific flanking policies be advised?

The way uncertainty alters trade theory results has already been widely studied, as discussed in the next section. However, a specificity of staple food products is to be storable, and this is not taken into account in these works. Storage and its consequences have actually been subject to a separate strand of literature, which includes analyses of its relationships with trade and trade policy. Although different cases were studied, these works did not allow optimal policies to be identified. Our chapter innovates by designing optimal stabilisation policies for a small open economy in a rational expectations storage model, using tools developed for the analysis of optimal dynamic policies. This is done with a focus on food security concerns in developing countries, assuming consumers to be risk averse without any insurance possibility and considering a country that is self-sufficient on average.
This objective is challenging because the combination of rational expectations with the non-negative storage and trade constraints render the model (which does not admit closed-form solutions) uneasy to solve, and even more to optimise along a dynamic path. Tractability thus requires simplifying the model as much as possible. This is why we focus on consumers’ risk aversion—most directly linked to food security concerns—but overlook producers’ risk aversion, despite the significant proportion of poor farmers in developing countries. We also disregard supply reaction, which, while being a potentially important mechanism, usually remains of limited quantitative importance. The need of simplicity is also the reason why we stick to a single-country model. This is frustrating given the coordination concerns surrounding export restrictions, but we shed some light on this debate by assessing the consequences for developing countries of refraining from using such restrictions.

6.1 Trade, uncertainty and storage: Related literature

Uncertainty is largely recognised as potentially affecting the main conclusions of trade theory. David Ricardo himself acknowledged that temporary tariffs on cereals may be justified to induce farmers to produce more in periods of sudden change in trade, such as wars (Ricardo, 1821, Ch. 19). The first formalisation of the issue is due to Brainard and Cooper (1968). Based on a portfolio approach, they show that diversification of a primary producing country decreases fluctuations in national income, thus increasing national welfare if the country is averse to risk. Based on a comparable framework, including risk aversion in a context where productive choices are made before uncertainty is resolved, several other papers have also challenged the idea of the optimality of free trade under uncertainty (Batra and Russell, 1974, Turnovsky, 1974b, Anderson and Riley, 1976).

Helpman and Razin (1978) point out that these results hinge crucially on the assumption of incomplete risk-sharing markets. They show that the main results of Ricardian and Heckscher-Ohlin theories of international trade, including the optimality of free-trade, carry over to uncertain environments when risk can be appropriately shared. In their model this is the case because the stock market allows households to diversify their capital, and cross-border trade in financial assets makes international risk-sharing arrangements available.

Helpman and Razin’s seminal contributions have clarified decisively the conditions underlying potential deviations from standard results and have paved the way for numerous insightful elaborations. Yet there is a variety of reasons why the conditions required to establish their results might not hold. This may be the case, for instance, because households need to invest their capital in a particular activity, without any possibility to diversify, to insure or to trade the corresponding risk. In this context, plausible in particular for rural households in developing countries, Eaton and Grossman (1985) show that the optimal trade policy for a small open economy differs from free trade. On average, the optimal policy entails an anti-trade bias. Similar conclusions can be reached when market incompleteness results from the lack of international trade in financial assets (Feenstra, 1987). In a specific-factor model with risk-averse factor owners, Cassing et al. (1986) also show that a state-contingent tariff policy can increase the expected utility of all agents. Newbery
and Stiglitz (1984) provide another illustration of this potential insurance role of trade restrictions, while extending the analysis to a two-country model. Without insurance market, they show that free-trade may be Pareto-inferior to no trade. Indeed, autarky links directly domestic prices to domestic output, thus providing, for a unitary price elasticity of demand, a perfect income insurance for farmers.

These different cases show that departure from free-trade can be motivated by risk-sharing objectives, when other arrangements are not available. When dealing with food security in developing countries, assuming incomplete insurance market seems obvious. Poor households have little opportunity to insure against the real income risk associated with variable food prices, and poor farmers (as well as many other poor workers) cannot diversify their income source, at least in the short run. As we focus on food security in developing countries, we adopt this assumption of market incompleteness and assume consumers to be risk averse, without any insurance schemes available.

Dealing with food security requires accounting also for the fact that staple food products are storable. This is especially important because storage is a central feature of these markets and can be thought of as an intertemporal risk-sharing arrangement. When dealing with food security, storage is thus a key feature, recognised as such for a long time. Early analyses of storage-trade interactions relied upon idealised or arbitrary storage technologies (Feder et al., 1977, Pelcovits, 1979, Bigman and Reutlinger, 1979, Reutlinger and Knapp, 1980). They tend to emphasise that trade is more cost-effective than storage at bringing price stability. While useful to ensure tractability, such simplified representations, not rooted in a consistent description of agent behaviours, do not reflect correctly the risk-sharing properties of storage. A consistent modelling of these properties requires accounting properly for agents’ expectations: the best way to do this is to work in a rational expectations, infinite-horizon framework.

Apart from specific analyses of oil-related problems, where world prices are the main source of uncertainty (Teisberg, 1981, Wright and Williams, 1982b), storage-trade interactions have first been studied in a rational expectations, infinite-horizon framework by Williams and Wright (1991, Ch. 9), where a small open market is studied as the extreme case of a two-country model. In a series of papers, Jha and Srinivasan (1999, 2001, Srinivasan and Jha, 2001) have used a model of Indian agricultural markets in relation with the world. They consider the rice market, in which India is a large country, and the wheat market, where India is a small country. In both markets, there are competitive private storers. World prices are randomly generated without accounting for serial correlation. They find international trade to be stabilising even when international price is more volatile than domestic price. Brennan (2003) considers the Bangladesh rice market. She shows that opening the market to trade stabilises as much as some public policies (such as subsidies to

Miranda and Glauber (1995) confirm Williams and Wright’s results and propose an improved numerical method. Makki et al. (1996, 2001) present a policy application of this model; based on a three-country model including the EU, the USA and the Rest of the World, they analyse the effects of removing current policy distortions such as export subsidies. Coleman (2009b) extends Williams and Wright’s work by considering that trade takes time. The time to ship then brings a new motive for stockpiling.
private storage or price ceiling) without any fiscal cost. In all these papers, welfare is assessed using surplus changes.

In contrast to the theoretical analyses of trade under uncertainty mentioned above, these storage-trade models all focus on the assessment of given exogenous policies. They give no hint about what the optimal policy would be. The present chapter extends to an intertemporal framework with storage under rational expectations the normative analyses of trade theory in an uncertain environment. Since designing optimal policies in a dynamic setting is already challenging, for tractability we consider a single-country model. We follow Williams and Wright’s insight by representing the world price as generated by a storage model and considered as exogenous by the country.

6.2 THE MODEL

We consider the market for a storable commodity in a small open economy. The country takes world price as given and faces a constant per-unit transport cost. Consumers are risk averse. Domestic food price volatility is driven by random output and stochastic world price.

6.2.1 CONSUMERS

The economy is populated with risk-averse consumers whose final demand for food has an isoelastic specification: \( D(P_t, Y) = dP_t^\alpha Y^\eta \), where \( d \) is a parameter of normalisation; \( P_t \) is period \( t \) price; \( Y \) is income, assumed to be constant over time; and \( \alpha \) and \( \eta \) are the price and income elasticities. Assuming there are only two goods and the second good is the numeraire, the integration of this demand function gives the following instantaneous indirect utility function (Hausman, 1981)

\[
\hat{v}(P_t, Y) = Y^{1-\eta} - dP_t^{1+\alpha} (1 + \alpha).
\]

This utility function has relative risk aversion equal to income elasticity of demand. To distinguish income elasticity from risk aversion, we follow Helms (1985a) and apply a monotone transformation to the indirect utility function,

\[
v(P_t, Y) = \frac{\hat{v}(P_t, Y)^{1+\theta}}{1 + \theta}.
\]

This specification is still consistent with the isoelastic demand function, but its coefficient of relative risk aversion is

\[
\rho(P_t, Y) = \eta - \theta \frac{Y^{1-\eta}}{\hat{v}(P_t, Y)}.
\]

with \( \theta \) indexing the degree of risk aversion.

The representative consumer is assumed to adopt a hand-to-mouth behaviour. He consumes current income and does not save to smooth out fluctuations. This assumption simplifies the dynamics, since consumer’s “cash on hand” does not have to be included as a state variable; however,
it overestimates the effect of public policy by neglecting the possibility of self-insurance added by saving.

Given the absence of saving, the consumer does not solve an intertemporal problem. At each period, he is only concerned with its current-period demand, which is not affected by the degree of risk aversion. Spatial and intertemporal arbitrages are thus independent from consumers’ risk aversion, which creates the need for public intervention.

6.2.2 Storers

The single representative speculative storer is assumed to be risk neutral and acts competitively. Its activity is to transfer a commodity from one period to the next. Storing the quantity $S_t$ from period $t$ to period $t+1$ entails a purchasing cost, $P_t S_t$, and a storage cost, $k S_t$, with $k$ the unit physical cost of storage. A (positive or negative) per-unit subsidy $\zeta_t$ to private storage is also considered. The benefits in period $t$ are the proceeds from the sale of previous stocks: $P_t S_{t-1}$. The storer maximises his expected profit as stated by

$$V^S(S_{t-1}, P_t, \zeta_t) = \max_{S_t \geq 0} \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ P_{t+i} S_{t+i-1} - (P_{t+i} + k - \zeta_{t+i}) S_{t+i} \right] \right\},$$

(6.4)

where $\mathbb{E}_t$ denotes the mathematical expectations operator conditional on information available at time $t$, and $\beta$ is the discount factor. The storer’s problem can be expressed in a recursive form using the following Bellman equation:

$$V^S(S_{t-1}, P_t, \zeta_t) = \max_{S_t \geq 0} \left\{ P_t S_{t-1} - (P_t + k - \zeta_t) S_t + \beta \mathbb{E}_t [V^S(S_t, P_{t+1}, \zeta_{t+1})] \right\}.$$  

(6.5)

This equation has three state variables: the price and the subsidy, whose dynamics are considered exogenous by the storer, and the stock carried over from the previous year. Using the first-order condition on $S_t$ and the envelope theorem, and taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), this problem yields the following complementary condition:

$$S_t \geq 0 \perp \beta \mathbb{E}_t (P_{t+1}) + \zeta_t - P_t - k \leq 0,$$

(6.6)

which means that inventories are null when the marginal cost of storage is not covered by expected marginal benefits; for positive inventories, the arbitrage equation holds with equality. The storer thus buys when present prices are low enough compared to their expected future level.

6.2.3 International trade

Since the model describes a homogeneous product for a small open economy, international trade modelling collapses to two arbitrage conditions, between domestic prices on the one hand, and export
or import parity price on the other hand. Expressed in a complementarity form,

\begin{align*}
M_t &\geq 0 \perp P_t - \nu_t^M - (P^w_t + \tau) \leq 0, \\
X_t &\geq 0 \perp (P^w_t - \tau) - P_t - \nu_t^X \leq 0,
\end{align*}

where \( M_t \) and \( X_t \) are imports and exports; \( P^w_t \) is world price; and \( \tau \) represents per-unit import and export costs, assumed to be constant and identical. \( \nu_t^M \) and \( \nu_t^X \) denote (positive or negative) per-unit taxes on imports and exports. The complementarity equations for trade (6.7)–(6.8) imply that domestic price is restricted to evolve in a moving band defined by world price, trade costs, and trade taxes if any:

\[ P^w_t - \tau - \nu_t^X \leq P_t \leq P^w_t + \tau + \nu_t^M. \]

6.2.4 Recursive Equilibrium

Period \( t \) harvest is noted \( \epsilon^H_t \) and is an i.i.d. random variable. The model has three state variables: \( S_{t-1}, \epsilon^H_t \) and \( P^w_t \). The first two can be combined into one variable, availability \( (A_t) \), the sum of production and private carry-over:

\[ A_t = S_{t-1} + \epsilon^H_t. \]

The other state variable, the world price, follows a continuous Markov chain, defined next section and characterised by the following transition function:

\[ P^w_{t+1} = f \left( P^w_t, \epsilon^w_{t+1} \right), \]

where \( \epsilon^w \) is the random production in the world market, which is assumed to be uncorrelated from domestic production shock.

Market equilibrium can be written as:

\[ A_t + M_t = D(P_t) + S_t + X_t, \]

From the above, we can define the recursive equilibrium of the problem without public intervention:

**Definition 3.** In the absence of a stabilisation policy (i.e., \( \zeta_t = \nu_t^M = \nu_t^X = 0 \)), a recursive equilibrium is a set of functions, \( S(A, P^w), P(A, P^w), M(A, P^w) \) and \( X(A, P^w) \) defining private storage, price, import and export over the state \( \{A, P^w\} \) and transition equations (6.10)–(6.11) such that (i) storer solves (6.6), (ii) trade obeys the arbitrage equations (6.7)–(6.8), and (iii) market clears.

\[ \text{As income is assumed to be constant, in what follows the demand function is only expressed as a function of domestic price.} \]
6.2.5 World price

Modelling world price dynamics is a crucial issue here. Not only may the world price level directly influence domestic price through arbitrage with export and import parity prices. The world price also matters through expectations formed about its future level, which influence storage decisions and domestic price expectations, central to the issues considered here.

In single-country models, the world price is generally represented as a stochastic process following a standard distribution, including in some cases first-order autocorrelation (Srinivasan and Jha, 2001, Brennan, 2003). Such simplifications are not consistent with the stylised facts on agricultural prices (Deaton and Laroque, 1992), which can be correctly represented by a storage model (Cafiero et al., 2011). Fully taking into account this storage-based representation of world prices would thus require a two-country model, where the rest of the world would be modelled alongside the economy under study. The complexity cost of such an option would be high, and difficult to reconcile with our willingness to identify optimal policies. Following Williams and Wright (1991, Ch. 9), a way out of this dilemma is to consider the small-open economy model as a limit case of a two-country model as the size difference between the two countries increases. In this limit case, the small economy is negligible compared to the big one, meaning that the rest of the world can be modelled without paying attention to the small economy under study. In other words, overlooking the influence of the economy on world markets allows world prices to be modelled as a separate process, exogenous from the economy’s point of view. This assumption of a price-taker economy leads to disregard potential motivations linked to the influence of government decisions on world markets, and greatly simplifies the analysis.

World prices are thus assumed to result from a storage model with random inelastic production; they are set as a result of a system of three equations equivalent to (6.6), (6.10) and (6.12), without import and export variables, and without storage subsidy. This system has one state variable, availability. All variables and functions corresponding to the world market are indicated with the superscript \( w \). Given model’s structure, the observation of the price allows to define the state of the system; price dynamics can thus be defined as a continuous state Markov chain. Its expression can be derived by using the decision rules, \( P^w (A^w) \) and \( S^w (A^w) \). It gives

\[
P^w_{t+1} = P^w (A^w_{t+1})
\]

\[
= P^w (S^w (A^w_t) + \epsilon^w_{t+1})
\]

\[
= P^w (S^w ((P^w)^{-1} (P^w_t)) + \epsilon^w_{t+1}),
\]

from which equation (6.11) follows.

The calibration assumes the world market to have the same parameters’ values as the domestic country. This assumption means that international trade is not motivated by any structural difference, but only by the existence of country-specific production shocks uncorrelated to worldwide shocks. The symmetry between the rest of the world and the economy may seem arbitrary. For instance,
the diversification of risks across the many countries part of the rest of the world could be assumed to lead to a lesser variability of output. If the rest of world is richer on average than the country considered, it could also be argued that price elasticity should be assumed to be smaller in the rest of the world. This would make sense, but sticking to a perfectly symmetrical case allows focusing on economic mechanisms independent from structural differences between the economy and the rest of the world.

6.2.6 Calibration

The rational expectations storage model does not admit a closed-form solution. It has to be approximated numerically. The numerical method used here is a projection method with a collocation approach, inspired from Miranda and Glauber (1995). Once chosen a grid for state variables, terms inside expectations are approximated by splines. Equilibrium equations are then solved by the mixed complementarity problems solver PATH (Dirkse and Ferris, 1995). Once the rational expectations equilibrium is found, the decision rules are approximated by splines and used to simulate the model.

The parameters are set such that, at the non-stochastic steady-state equilibrium, price, production, consumption and availability are equal to 1, and import and export are equal to 0 (see table 6.1 for parameters’ values). As a result, the country is self-sufficient in the steady state and no trade takes place.

Table 6.1. Parameterisation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Economic interpretation</th>
<th>Assigned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Annual discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Income elasticity</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Own-price demand elasticity</td>
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</tr>
<tr>
<td>$\gamma$</td>
<td>Commodity budget share</td>
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</tr>
<tr>
<td>$Y$</td>
<td>Income</td>
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</tr>
<tr>
<td>$d$</td>
<td>Normalisation parameter of demand function</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>Parameter defining risk aversion</td>
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</tr>
<tr>
<td>$k$</td>
<td>Physical storage cost</td>
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</tr>
<tr>
<td>$\tau$</td>
<td>Trade cost</td>
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</tr>
<tr>
<td>$\epsilon_H, \epsilon_w$</td>
<td>Probability distribution of yield</td>
<td>$B(2, 2) \cdot 0.5 + 0.75$</td>
</tr>
</tbody>
</table>

An annual interest rate of 5% is used for discounting. Based inter alia on Korinek and Sourdin (2010), we set trade costs to 20%. This is more than the average cost cited in this study for agricultural products, reflecting our focus on grains for poor countries.\textsuperscript{3}

Seale and Regmi (2006) estimate elasticities for food consumption across 144 countries. From their research, we choose cereal elasticities typical of low-income countries: $-0.4$ for price elasticity and 0.5 for income elasticity. We assume that consumers spend, at the steady state, $\gamma = 15\%$ of their income on the staple (a value intermediate between what is observed for rice consumption in poor

\textsuperscript{3} Noteworthily, international trade also frequently entails beyond-average domestic transport costs.
and affluent households in Asia, Asian Development Bank, 2008). Since steady-state consumption and price are equal to 1, income, which is assumed to be constant, is equal to the inverse of the commodity budget share, $1/\gamma$.

Consumers enjoy welfare gains from a stabilisation at mean price if their indirect utility function is concave in price. Turnovsky et al. (1980) show that this is the case if $\gamma(\eta - \rho) - \alpha < 0$, that is, with our parameters, $\rho > 3.17$. We thus assume at the steady state a relative risk aversion parameter of 4, implying $\theta = -6.12$.

Following Lence and Hayes (2002), per-unit storage costs are assumed to be equal to 2% of the steady-state price (i.e., $k = 0.02$). The random production, $\epsilon_H$ and $\epsilon_W$, are both assumed to follow a beta distribution with shape parameters 2 and 2, which makes it unimodal at 0.5 and symmetric. The distribution is translated and rescaled to vary between 0.75 and 1.25, implying a coefficient of variation of 11.2%.

6.3 Dynamics without public policy

To understand the consequences of public policies, the situation without public intervention is a natural and useful benchmark. Since the model used here differs significantly from those discussed in the literature so far, we analyse this benchmark case in some detail.

6.3.1 Price, trade and storage behaviour

For a small open economy, absent any trade tax, domestic prices necessarily lie in a moving band defined by world prices plus or minus trade costs. Compared to a closed economy, this context modifies radically storage behaviour and its consequences. Abundant availability usually favours storage, but exporting is here another potential profitable outlet; when scarcity prevails, the stabilising effect of selling inventories may be partially redundant with the price ceiling imposed by import competition.

A first salient feature is that there is no storage when the country imports (see figure 6.1(a)). Indeed, in this case, the domestic price is exactly equal to the world price plus trade costs, which gives

$$\beta E_t(P_{t+1}) - P_t - k = \beta E_t(P_{t+1}) - P_t^w - \tau - k. \quad (6.16)$$

As the storage arbitrage condition (6.6) holds in the rest of the world (as assumed here given the way world prices are determined),

$$-P_t^w - k \leq -E_t(P^w_{t+1}), \quad (6.17)$$

which combined with (6.16) gives

$$\beta E_t(P_{t+1}) - P_t - k \leq \beta E_t(P_{t+1} - P^w_{t+1}) - \tau. \quad (6.18)$$
Given (6.9), domestic price is always inferior to import parity price, which also holds in expectations terms. As a result,

$$\beta E_t (P_{t+1}) - P_t - k \leq (\beta - 1) \tau < 0.$$  \hfill (6.19)

This implies that storage is not profitable domestically while importing, because the expected value of next year’s difference between domestic and world prices cannot exceed trade costs.

This feature, already emphasised *inter alia* by Williams and Wright (1991) in a two-country framework, reflects the fact that importing for the sake of storing never makes economic sense when intertemporal arbitrage is the same at home and abroad: differing until next year the decision as to whether importing will be necessary or not is always preferable.

Although possible, storage is not common either when the country exports, but both may coexist in cases where availability is relatively abundant. When the country exports, domestic price equals export parity price and the storage arbitrage equation becomes

$$S_t \geq 0 \quad \perp \quad \beta E_t (P_{t+1}) - P_t^w - k \leq 0.$$  \hfill (6.20)

*Figure 6.1. Stock, price and trade behaviour.* Negative trade values refer to imports.
For a high enough world price, expected domestic price cannot be so high as to make speculation profitable: exporting is more profitable than storing, and no storage takes place. The coexistence of storage and exports is only observed for intermediate world price levels: high enough compared to domestic prices so as to make exporting profitable, but not so large as to make the first unit of storage less profitable than exports. The interrelations between storage and exports are also illustrated by the storage rule, where exports are reflected in flat storage curves for relatively large availabilities. In the left panel of figure 6.1(a), this situation of non-zero storage is observed only with a world price equal to 1.2. As soon as the world price reaches 1.3, exporting is always more profitable than storing, so that the storage rule is flat throughout.

Noteworthily, current world price affects domestic storage even in the absence of trade. Indeed, world prices are positively autocorrelated since they are generated by a storage model. As a result, a higher current world price entails higher world price expectations for the next period. Accordingly, storage outside trade situations is slightly moved upward for higher world prices.

---

The returns to storage are declining due to its negative influence on expected future domestic prices.
Under the small economy assumption, exporting implies a complete disconnection of domestic prices from availability, as reflected in the flat segments of the price curves observed for large enough availabilities for world prices equal to 1.2 and above in the central panel of figure 6.1(a). Similarly, for limited availabilities, the domestic price is disconnected from availability when it reaches the import trigger price, equal to world price minus trade costs. In between these two cases, the price curve has a standard form in presence of storage, with a strongly downward sloping curve for availabilities under a given threshold, under which no storage takes place, and a smoother curve afterwards. As far as trade is concerned, assuming exogenous world prices implies that the net trade curve has a unitary slope whenever trade is not zero.

A sample simulation of world and domestic prices illustrates the link between world and domestic prices (figure 6.2). The domestic price tends to be set to the import parity price when the world price is low, and to the export parity price for high world prices. Most world price spikes are imported through trade to the domestic market.

![Figure 6.2. Simulated history of prices without public intervention](image)

### 6.3.2 Effects of trade and storage costs

While commodity trade is usually thought of as a way to smooth production shocks over different markets, trade does not necessarily decrease price volatility in the present case. Actually, price instability is the same when trade costs are zero and when they are prohibitive: in the first case, the domestic price is exactly equal to the world price, while in the second case the economy behaves as a closed one. Both situations are equivalent since we are assuming world prices to result from a closed economy behaviour, with the same parameters as those used for the economy studied.

When trade is costless, however, the strict link to world prices means that only storage in the world market matters, while storage in the small economy does not matter. Under prohibitive trade costs, in contrast, only domestic storage matters. Intermediate cases thus exhibit less volatility, with domestic and foreign storage combining to smooth price variability: domestic shortages can be
alleviated by imports while surplus can be exported to the world market, but the country remains partly shielded from world prices by trade costs. The result is a U-shaped relationship between trade cost and volatility (figure 6.3, left panel). The volatility is the lowest for trade costs close to 0.22.\(^5\)

**Figure 6.3. Dependence of price instability to trade and storage costs.** The value of storage cost is maintained equal to the benchmark when analysing the influence of the trade cost, and reciprocally.

This result contrasts with the influence of the domestic physical storage cost, which always increases instability (figure 6.3, right panel). For prohibitive storage costs, the instability is slightly lower than on the world market since trade costs partly insulate the country from worldwide instability. For zero physical storage cost, the instability is still significant for various reasons: storage is profit seeking and entails opportunity costs, so it will not provide a complete stabilisation; and there is some imported instability.

### 6.4 Optimal Stabilisation Policy

In modelling the relationships between storage and trade policies, we do not only want to analyse specific cases, we aim at assessing what the optimal use of these policies would be. To do this we assume that the government cannot commit to future policies and has to follow time-consistent policies. Dynamic programming is used to derive the Markov-perfect equilibrium associated with the discretionary policy. To design such optimal stabilisation policy, a meaningful objective is first spelt out. The optimisation problem of the discretionary optimal policy can then be stated.

#### 6.4.1 Social Welfare Function

In contrast to usual practice in optimal policy problems, the expected sum of discounted consumer’s utility is not a valid objective in the present context of a partial equilibrium. Instead, the welfare of all agents must be included in the objective. In partial equilibrium models, policy design is usually

---

\(^5\) Numerical results here and in the following sections are calculated over 100,000 sample observations from the asymptotic distribution.
based on maximising the sum of all agents’ surpluses. This standard practice is not valid either here, since expected surplus is not a suitable measure of risk-averse consumers’ surplus (Helms, 1985b).

The social welfare function used here thus combines the utility of consumers with the surplus of other agents. It is given by:

$$W_t = v(P_t, Y) + w[P_t A_t - (P_t + k - \zeta_t) S_t + (P_t - \nu_t M - P_w - \tau) M_t + (P_t - \tau - P_t - \nu_t X) X_t - Cost_t],$$

(6.21)

where $w$ is the weight given in social welfare to monetary terms and $Cost_t$ denotes period $t$ fiscal cost of public policies. The use of a unique weight for all monetary terms means that any distortionary cost caused by revenue collection is neglected.

In order to focus the analysis on efficiency issues, we assume that the conditional expectation of the social welfare cannot be improved by simple transfers. Thus, the value of the weight $w$ must be such that marginal transfers between agents do not affect total welfare (implying that $w$ is the value in utility terms of a unitary monetary transfer to consumers):

$$E_0 \sum_{t=0}^{\infty} \beta^t [v(P_t, Y + \delta) - w \delta] = E_0 \sum_{t=0}^{\infty} \beta^t v(P_t, Y),$$

(6.22)

for any small, permanent, monetary transfer $\delta$ to consumers. A first-order approximation of the left-hand side around $\delta = 0$ gives

$$w = (1 - \beta) E_0 \left[ \sum_{t=0}^{\infty} \beta^t v_Y(P_t, Y) \right].$$

(6.23)

Note that, were income elasticity and relative risk aversion to be equal to zero, this definition would make $w$ equal to 1, so that the social welfare function would actually be the classical sum of surpluses.

The cost of policy intervention is the cost of subsidising private storage plus the net tax cost of trade policy:

$$Cost_t = \zeta_t S_t - \nu_t^M M_t - \nu_t^X X_t.$$  

(6.24)

Using (6.10) and (6.24), the social welfare can be simplified to

$$W_t = v(P_t, Y) + w [P_t A_t - (P_t + k) S_t + (P_t - P_w - \tau) M_t + (P_t - \tau - P_t) X_t].$$

(6.25)

The per-unit storage subsidy enters positively in private agents’ profit and negatively in public cost, so it does not feature directly in the social welfare function. With opposite sign, the same holds for trade taxes.
6.4.2 Optimisation problem

The social welfare function is a natural objective for policy optimisation. In stating this problem, the policy is assumed to start at period 0 and to be unanticipated. Commitment is unlikely in most countries and especially in developing ones; the policy is thus assumed to be discretionary. Three state-contingent instruments may be used to stabilise prices: a tax or subsidy to private storage and a trade policy (import and/or export tax or subsidy). Initial state variables are taken as being at their non-stochastic steady-state level (i.e., \( A_0 = 1 \) and \( P_{w0} = 1 \)) and initial stocks are assumed to be null.

At each period, optimisation entails maximising the expected sum of the discounted social welfare function subject to the constraints imposed by private agents’ behaviour and market equilibrium. The optimisation is carried out over current endogenous and control variables, taking as given future variables:

\[
\max_{S_t \geq 0, P_t, M_t \geq 0, X_t \geq 0, A_{t+i}, \zeta_{t+i}, \nu^M_{t+i}, \nu^X_{t+i}} E_t \sum_{i=0}^{\infty} \beta^i \left\{ v\left( P_{t+i}, Y \right) + w\left[ P_{t+i} A_{t+i} - (P_{t+i} + k) S_{t+i} + \left( P_{t+i} - P_{w_{t+i}} - \tau \right) M_{t+i} + \left( P_{w_{t+i}} - \tau - P_{t+i} \right) X_{t+i} \right] \right\}
\]

subject to equations (6.6)–(6.8) and (6.10)–(6.12), \( A_t \) and \( P_t^w \) given, and anticipating \( \{ S_{t+i}, P_{t+i}, M_{t+i}, X_{t+i}, A_{t+i+1}, \zeta_{t+i}, \nu^M_{t+i}, \nu^X_{t+i}, P_{w_{t+i}} \} \) for \( i \geq 1 \).

The above problem defines an optimal stabilisation policy using both storage and trade taxes or subsidies. A policy using only one of the two instruments can be easily defined by removing from the objective and constraints all occurrences of \( \zeta \) to define a trade policy, or of \( \nu^M \) and \( \nu^X \) to define a storage policy. In each case, solving the problem requires reformulating the constraints defined as complementary equations and writing down explicitly the dynamic programming problem. See appendix for details.

6.5 Characterisation and consequences of optimal public interventions

The analytical framework laid out above allows optimal public interventions to be characterised. Three cases are considered here, corresponding to the optimal use of trade policy alone, of storage policy alone, and of both policies jointly. We first describe the nature of these policy interventions, and analyse what the consequences are for decision rules. We then assess the resulting consequences for welfare.

6.5.1 Optimal trade policy

Absent any storage policy, the optimal use of trade policy can be simply characterised as the composition of two types of interventions. The first one consists in subsidising imports when
availability is low. As illustrated in the central panel of figure 6.1(b), the corresponding subsidies can be substantial, with powerful trimming impact on the upper tail of the price distribution. For a small open economy, import subsidies are thus an efficient way to avoid very high prices: despite their cost, \textit{ad valorem} subsidies larger than 20\% prove to be optimal when domestic scarcity coincides with high world prices.

The second type of intervention consists in taxing exports when availability is abundant. Such interventions decrease the export parity price and avoid importing price spikes on the world market through exports. The corresponding tax levels remain moderate, typically under 10\% even for world prices as high as 1.4.

For the asymptotic distribution of prices, such optimal trade policy results in a decline in the mean (by 3.5\%) and a cut by more than one quarter in the standard deviation (table 6.2). This reduced variability is strongly asymmetrical, as illustrated by changes in quantiles: it mainly stems from a strong cut in high prices obtained thanks to import subsidies and export taxes (the top percentile of the price distribution is driven down from 1.58 without policy to 1.28, a cut in half in the deviation from the average price). The asymmetry typical of price distribution of storable commodities is thus strongly reduced, with a skewness divided by three compared to the benchmark.

\begin{table}[h]
\centering
\caption{Descriptive statistics of prices asymptotic distribution}
\begin{tabular}{lcccc}
\hline
 & Benchmark & Trade policy & Storage subsidy & Both instruments \\
\hline
Mean & 1.039 & 1.003 & 1.056 & 1.029 \\
One-year autocorrelation & 0.325 & 0.331 & 0.337 & 0.429 \\
Coefficient of variation & 0.150 & 0.109 & 0.127 & 0.076 \\
Skewness & 1.448 & 0.419 & 2.068 & 1.044 \\
Correlation coefficient & & & & \\
\quad with domestic shocks & -0.466 & -0.609 & -0.350 & -0.444 \\
\quad with world price & 0.765 & 0.653 & 0.798 & 0.760 \\
Quantiles & & & & \\
\quad 1\% & 0.820 & 0.813 & 0.902 & 0.902 \\
\quad 25\% & 0.930 & 0.911 & 0.974 & 0.973 \\
\quad 50\% & 1.004 & 0.996 & 1.015 & 1.016 \\
\quad 75\% & 1.109 & 1.080 & 1.089 & 1.067 \\
\quad 99\% & 1.583 & 1.283 & 1.577 & 1.275 \\
\hline
\end{tabular}
\end{table}

Not surprisingly, these trade policy interventions result in increased imports and decreased exports. Trimming the upper tail of the price distribution also reduces storage profitability. As a result, the storage rule is moved rightward, and the mean stock level is reduced by approximately one fourth on the asymptotic distribution (table 6.3). In cases where domestic and world prices would be consistent with the coexistence of exports and storage, were it not for public intervention, the export tax could counterbalance this effect by decreasing domestic price; even then, though, storage is reduced.
6.5.2 Optimal storage policy

In a closed economy, an optimal storage policy motivated by consumers’ risk aversion under incomplete insurance markets increases the level of storage with regards to the situation without intervention (chapter 4). In the present context of a small open economy without any trade policy intervention, this is only true when storage does not directly compete with exporting, i.e., when the domestic price remains above the export parity price, because availability is limited and/or the world price is not too high. In this case, the subsidy increases storage and domestic price, contributing to smooth down intertemporal price variability in the domestic market.

When availability is abundant and the world price is high enough, exporting is significantly more profitable than storage. This does not leave any room for a worthwhile storage policy.

The storage subsidy is positive when the current social marginal value of availability (i.e., the Lagrange multiplier on the market clearing equation (6.12), $\chi_t$ in appendix) is lower than its discounted, expected future value. For a world price equal to 1.2 and for intermediate availabilities (between 1 and 1.1), storage takes place while exports are not profitable (see figure 6.1(c)). In this situation, the small excess of availability with respect to steady-state consumption leads to a present social marginal value lower than its expected discounted value and so to a positive subsidy to storage. However, as soon as the domestic price reaches the export parity price, the present social marginal value of availability jumps to zero (when there is some trade, the market clearing equation is not binding, since foreign demand and supply are infinitely elastic). Since stock levels are already high (around 10% of the steady-state consumption), the expected future marginal value of availability is negative in this situation of glut, it then becomes optimal to tax storage. Hence the discontinuity observed in the storage rule and in the export curve for an availability around 1.2. Beyond this level, storage coexists with trade, and the optimal policy remains a storage tax (worth approximately 2%). This outcome is paradoxical: while public intervention is motivated by consumers’ risk aversion, it tends to discourage storage. Intuitively, this is a context of overly abundant availability, where dispensing with it through exports is socially preferable to retaining it through storage, even when the storer breaks even in doing so.

When storage and export coexist, there is not always a storage tax. For high world prices, and therefore relatively high domestic prices, storage profitability is quickly driven down to zero. However, the expected social marginal value of availability is positive in this situation, which justifies subsidising storage.

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Trade policy</th>
<th>Storage subsidy</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean stocks</td>
<td>0.060</td>
<td>0.044</td>
<td>0.105</td>
<td>0.098</td>
</tr>
<tr>
<td>Mean imports</td>
<td>0.014</td>
<td>0.021</td>
<td>0.011</td>
<td>0.015</td>
</tr>
<tr>
<td>Mean export</td>
<td>0.023</td>
<td>0.018</td>
<td>0.028</td>
<td>0.024</td>
</tr>
</tbody>
</table>
The impact of such optimal storage policy on the asymptotic price distribution is strongly constrained by foreign trade: as soon as the country exports or imports, its domestic price is determined by the world price, since no trade policy intervention is assumed to take place. The two main channels through which this optimal storage policy influences domestic prices are actually the following: storage subsidies increase domestic prices in situations of stock accumulation, when above-average availability is combined with low to intermediate world price, with the exception of the above-mentioned case of storage tax, when storage coexists with exports; ensuing higher stock levels decrease prices when stocks are sold, but this situation often coincides with situations of international trade, thus limiting the price fall.

As a result, an optimal public storage policy increases the mean price of the asymptotic distribution (see table 6.2). This seems paradoxical since the standard result obtained for a closed economy is that introducing private storage or increasing storage level by public intervention depresses average price (Wright and Williams, 1982a, Miranda and Helmberger, 1988). To interpret this puzzling result, it is useful to consider such a stabilisation policy through storage as a sequence of transfers of demand from one period \( t_1 \) to another \( t_2 \), where the price is higher \( (p_1 < p_2) \), as suggested by Newbery and Stiglitz (1981, p. 251, theorem 2). Consumers’ demand is reduced by any additional storage, and it is increased by the same amount (in the absence of spoilage) when the quantity stored is finally sold. To see how demand transfer influences prices, let us differentiate the market clearing equation (6.12) with regards to stocks:

\[
D'(P_t) \frac{\partial P_t}{\partial S_t} + 1 + \frac{\partial(X_t - M_t)}{\partial S_t} = 0.
\]

In a closed economy, the last term is pointless, and this equation collapses to \( \frac{\partial P_t}{\partial S_t} = -1/D'(P_t) \). As soon as the demand function is convex, this partial derivative is an increasing function of prices, meaning that it is larger in \( t_2 \) than in \( t_1 \). As a result, modifying storage to operate a small transfer of consumption from \( t_1 \) to \( t_2 \) would cut the price in \( t_2 \) by more than it would increase it in \( t_1 \). This explains why stabilisation through additional storage usually depresses mean prices in closed economy.

In an open economy, in contrast, the last term on the left hand side of (6.27) is not zero. In particular, it may be the case that the country exports when the consumption transfer takes place. In this case, a small enough additional storage will not drive exports down to zero, meaning that they will not change domestic price, which will remain equal to world price minus transport cost. Conversely, when there is trade, selling additional stocks does not depress domestic price. This latter effect dominates in practice, mainly because it means that the country cannot insulate its market from episodes of very high world prices. In this case, domestic stocks are sold on the world market (either directly or indirectly, when domestic consumption of domestic stocks displaces imports). This is profitable given the high price level, but it does not curb down domestic prices. Hence the limited efficiency of the storage policy in avoiding high-price episodes.

While this optimal storage policy reduces the standard deviation of prices by approximately 14%,
stabilisation only comes from the increase in low prices, a paradoxical result for a public intervention linked to consumers’ welfare. The upper quantiles of the asymptotic price distribution hardly change compared to the benchmark, while the first percentile is increased by 10% (table 6.2).

The impact on trade is not trivial. The significantly higher average stock reduces the frequency of scarce domestic availabilities and of the associated large imports. On average, imports are reduced by 20%. For the same reason, abundant availabilities are more frequent, and they increase exports. When storage coincides with exports, this export-enhancing effect is magnified by the storage tax mentioned above, which reduces the demand for storage, thus increasing the volumes of domestic output absorbed by the world market. On average, exports are increased by 20%. This increased importance of exports is also a strong driver of domestic price variability, as illustrated by the increased correlation between domestic and world prices. On the whole, and despite the absence of trade policy, this policy could be called opportunistic in trade terms, to the extent that the country tends to favour storage when world prices are low but to discourage it when they are high enough.

6.5.3 Optimal trade and storage policy

Combining optimally trade and storage policies allows a powerful stabilisation policy to be devised, with the standard deviation of domestic prices cut in half compared to the benchmark. This is not surprising given that trade policy is very efficient at preventing domestic prices to reach very high levels, while storage is a powerful tool to avoid excessively low prices. The basics of an optimal policy mix thus consists in using import subsidies to trim the upper tail of the distribution of domestic prices, and storage subsidies to trim its lower tail. The reduced variability of domestic prices comes from both ends of the distribution, which get substantially closer to the mean than in the benchmark (see quantiles in table 6.2). This outcome is comparable to the one obtained with storage policy for bottom quantiles, and with trade policy for upper quantiles.

Both instruments are not independent, though, and their interrelationships are especially strong when the country both exports and stores, as is the case for abundant availability under intermediate world prices (1.2 in figure 6.1(d)). As already mentioned, this is the case where trade policy alone leads to tax exports, while storage policy alone leads to tax storage. When both are combined, the optimal policy consists in slightly subsidising exports while significantly subsidising storage (for a world price of 1.2, the per-unit subsidy \( \zeta \) is 0.04, or 4% of the steady-state domestic price). Despite slightly lower stocks, the result is a domestic price slightly beyond the one that would prevail without intervention, i.e., closer to the distribution’s mean. This shift thus contributes to limiting price variability. For higher world price, however, private storage are still subsidised, but exports are taxed, which prevents domestic prices from reaching excessive levels.

The asymptotic distribution’s mean price is slightly lower than in the benchmark, an outcome intermediate between those found for trade and storage policies separately. It is also intermediate in terms of correlation of domestic prices with world prices and with domestic shocks.
6.5.4 Decomposition of welfare changes

By changing the distributions of prices, stabilisation policies transfer both risk and resources across agents. The weighting parameter used for welfare analysis, \( w \), has been defined so as to neutralise the aggregate welfare impact of small permanent monetary transfers across agents. Aggregate welfare changes should thus only reflect efficiency terms. To understand the distributive effects of these policies, welfare change can be decomposed for each agent in efficiency and transfer changes.

Consumer gains are \( \textit{ex ante} \) per-period equivalent variation, \( EV \), calculated using the following implicit definition

\[
E_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[ v \left( P_t, Y + EV \right) - v \left( \hat{P}_t, Y \right) \right] \right\} = 0,
\]

where \( P \) and \( \hat{P} \) are the prices before and after stabilisation. The equivalent variation includes two components: a transfer term corresponding to the change in mean expenditure, which is exactly matched by an opposite change in the average revenues of the other agents, and an efficiency term corresponding to risk benefits and mean consumption change. There is no closed-form solution for the efficiency term, which, following Newbery and Stiglitz (1981), is defined by difference between equivalent variation and change in mean expenditures:

\[
EV + (1 - \beta) E_0 \sum_{t=0}^{\infty} \beta^t \Delta \left( P_t D \left( P_t \right) \right),
\]

where the operator \( \Delta \) before an expression refers to the difference between its value after and before public stabilisation.

Changes in storers’ profit, defined by equation (6.4), contains an efficiency term representing changes in storage costs, \( E_0 \sum_{t=0}^{\infty} \beta^t k \Delta S_t \), and two transfer terms: \( E_0 \sum_{t=0}^{\infty} \beta^t \Delta \left[ P_t \left( S_{t-1} - S_t \right) \right] \), a transfer from other private agents, and \( E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t S_t \), a transfer from the government.

Producers are not represented explicitly since production is assumed to be inelastic but they are affected by policies through price changes. Their welfare change is a transfer term defined by their average benefits change: \( E_0 \sum_{t=0}^{\infty} \beta^t \Delta \left[ P_t \epsilon_H \right] \).

International trade can be decomposed into four effects, affected in table 6.4 to an agent labelled “shipper”. The mean change in imports and exports valued at domestic price, \( \tau E_0 \sum_{t=0}^{\infty} \beta^t \Delta \left[ P_t \left( M_t - X_t \right) \right] \), is a transfer from private domestic agents; the mean change in trade costs, \( \tau E_0 \sum_{t=0}^{\infty} \beta^t \Delta \left( M_t + X_t \right) \), is an efficiency term; the mean change of imports and exports valued at world price, \( \tau E_0 \sum_{t=0}^{\infty} \beta^t \Delta \left[ P^w_t \left( X_t - M_t \right) \right] \), which we call trade balance, is an efficiency term accounting for the possibility to buy foreign goods allowed by an export surplus in the food commodity; and there is a transfer from the government through the trade policy instrument.

---

6 See also Newbery and Stiglitz (1981, Ch. 6 and 9) for a discussion of efficiency and transfer gains from price stabilisation.
The government is only involved in transfers to private storers through the subsidy and to the shipper through the trade policy.

This decomposition is illustrated for the three optimal policies in table 6.4. Strikingly, total gains are small in comparison to transfers: to protect consumers from price fluctuations, public intervention induces comparatively large changes for other agents. These transfers mainly stem from two effects. The first one is the change in the mean price. A lower mean price, as results from optimal use of trade policy or both instruments, benefits consumers at the expense of producers. The reverse is true under storage policy alone. Changes in the covariance between prices and production shocks also originate transfers between producers and consumers but their importance is more limited. Changes in the foreign trade balance valued at domestic prices are the second main source of transfers, from shipper to consumers when the balance increases. Once trade costs and possible taxes are taken into account, changes in the trade balance actually affect the consumer, not the shipper (whose profit is assumed to remain zero throughout). The fiscal cost of policies is an additional source of transfer: trade policy intervention generates fiscal revenue, while storage subsidy entails costs. While the magnitude of these fiscal effects is limited compared to other effects, the cost of a storage policy is not negligible.

Table 6.4. Decomposition of welfare impacts of optimal policies on transitional dynamics (as percentage of the steady-state commodity budget share.)

<table>
<thead>
<tr>
<th></th>
<th>Trade policy</th>
<th>Storage subsidy</th>
<th>Both instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumers</td>
<td>3.64</td>
<td>-1.65</td>
<td>1.21</td>
</tr>
<tr>
<td>Efficiency</td>
<td>1.64</td>
<td>-0.57</td>
<td>0.74</td>
</tr>
<tr>
<td>Expenditures(a)</td>
<td>2.01</td>
<td>-1.09</td>
<td>0.46</td>
</tr>
<tr>
<td>Producers(a)</td>
<td>-3.47</td>
<td>1.99</td>
<td>-0.68</td>
</tr>
<tr>
<td>Storers</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Transfers(a)</td>
<td>-0.03</td>
<td>-0.17</td>
<td>-0.27</td>
</tr>
<tr>
<td>Storage costs</td>
<td>0.03</td>
<td>-0.08</td>
<td>-0.06</td>
</tr>
<tr>
<td>Storage subsidy(a)</td>
<td>-</td>
<td>0.24</td>
<td>0.34</td>
</tr>
<tr>
<td>Shipper</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Transfers(a)</td>
<td>1.50</td>
<td>-0.74</td>
<td>0.49</td>
</tr>
<tr>
<td>Trade costs</td>
<td>-0.03</td>
<td>-0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td>Trade balance</td>
<td>-1.44</td>
<td>0.75</td>
<td>-0.36</td>
</tr>
<tr>
<td>Trade policy(a)</td>
<td>-0.03</td>
<td>-</td>
<td>-0.11</td>
</tr>
<tr>
<td>Government</td>
<td>0.03</td>
<td>-0.24</td>
<td>-0.22</td>
</tr>
<tr>
<td>Storage subsidy(a)</td>
<td>-</td>
<td>-0.24</td>
<td>-0.34</td>
</tr>
<tr>
<td>Trade policy(a)</td>
<td>0.03</td>
<td>-</td>
<td>0.11</td>
</tr>
<tr>
<td>Total</td>
<td>0.20</td>
<td>0.09</td>
<td>0.30</td>
</tr>
</tbody>
</table>

\(a\) Transfer terms. They sum to zero, except for the numerical approximations, and do not contribute to total gains.

Efficiency gains eventually stem from four terms. Two of them, storage and trade costs, are comparatively small in all cases. Trade costs increase for all policies, because overall trade increases with all policies (see table 6.3). Changes in storage costs are positive for the trade policy, reflecting the decrease in storage resulting from trade policy intervention. For the other two policies, they are negative as storage increases. Most efficiency effects correspond to the remaining two effects,
namely consumers’ efficiency gains and trade balance changes. Consumers’ efficiency gains have themselves two components, not decomposed in table 6.4: the gains originating in the change in mean consumption, the traditional welfare triangle in a surplus analysis, and the reduction in the risk premium. The latter is necessarily positive since instability decreases with all policies, while the former may be positive or negative depending on the mean price change, which reveals the mean consumption change. With respect to the distribution of these various gains, the policies considered stand in stark contrast.

An optimal storage subsidy without trade policy intervention has counterintuitive impacts. While public intervention is motivated by consumers’ risk aversion, it actually results in efficiency losses for consumers: decreased price volatility is more than compensated by reduced mean consumption due to higher prices. The policy is still beneficial socially, because of the risk-premium decrease, but it does not increase consumers’ welfare. Absent any trade policy, a storage subsidy does not benefit consumers.

With an optimal trade policy, consumers enjoy significant efficiency gains, resulting from decreases in both mean price and price variability, in particular from less frequent price spikes; but this result is obtained at the cost of a significantly deteriorated trade balance. The order of magnitude of both effects is 1.5%, with a net gain worth approximately a eighth of this magnitude.

While intermediate between single-instrument policies, the impacts of an optimal combination of trade and storage policies is closer to those found for trade policy. Consumers enjoy efficiency gains, mainly reflecting the policy’s effectiveness in preventing price spikes, and the cost is a deterioration of the trade balance. However, public intervention is more effective in this case: while consumers’ efficiency gains do not reach 1%, total social gains exceed 0.3%. With less transfer, this policy thus achieves more gains. This finding illustrates the strong complementarity between trade and storage policies.

6.5.5 Consequences of a discipline on export restrictions

The use of export restrictions during the food crisis has generated a lot of criticisms and, to limit their future use, some have proposed that they should be the object of discipline, such as coming from a WTO agreement (Mitra and Josling, 2009). Our framework does not allow the impacts on the world market of such discipline to be assessed, but export restrictions no doubt exacerbate world price volatility, since they tend to limit supply when prices are high. Considering multilateral disciplines over export restrictions is thus meaningful, and our framework can shed light on the stakes of such disciplines, by assessing the consequences for a developing country of deviating from its optimal policy by committing not to using export restrictions. Given that existing WTO agreements also severely restrict export subsidies—and potential agreements would presumably strengthen further these disciplines—we deal with this issue by designing an optimal trade and storage policy excluding any export-related instrument (i.e., \( \nu^X = 0 \)).
Banning export taxes prevents the country to isolate its domestic market from the world market when world prices are high and domestic availability is large. Consequently, the optimal policy has a limited effect on high prices (the 1, 25, 50, .75 and 99 quantiles are respectively at 0.90, 0.97, 1.02, 1.07 and 1.55). Some high prices are alleviated by import subsidies or by the release of stocks, but most price spikes being caused by the world market, the occurrence of high prices is not decreased much. Since the optimal storage subsidy decreases the occurrence of low prices by increasing the stock level, the overall effect is a slight mean price increase to 1.046.

The decrease in volatility being limited (the coefficient of variation of price is 0.12), the welfare effects on consumers are dominated by the mean price increase and consumers lose from this policy, as was already the case with a storage policy alone. Export disciplines thus entail significant transfers from consumers toward producers. Overall the gains for the whole economy are divided by two in comparison to the optimal policy without constraint. Export taxes thus appear as key ingredients in the design of price-stabilising policies that benefit consumers.

These results should not be understood as a plea for export restrictions, the destabilising effects of which have already been emphasised. Rather, they may be useful in understanding why these restrictions are so often used, and in gauging the stakes of potential multilateral disciplines. For the poorest countries, in particular, banning export restrictions altogether may be politically difficult, at least if no substantial compensating measures are offered.

6.6 CONCLUSION

This chapter is to our knowledge the first attempt to design optimal dynamic food price stabilisation policies in an open economy setting. The model can only be solved numerically, and tractability requires simple specification. Our analysis focuses on the optimal use of trade and storage policies on the food market of a small, normally self-sufficient, developing economy, where public intervention is justified by the impossibility of risk-averse consumers to insure against price volatility. The framework developed here, however, could be applied to other cases, in terms of both parameterisation and specification.

Our results show that an optimal trade policy consists in subsidising imports and taxing exports. This policy truncates the upper half of the distribution and is not fiscally costly since proceeds from export taxation cover import subsidies’ fiscal cost. Import subsidies alleviate the traditional limit of food storage: its non-negativity. When stocks are zero, subsidising imports prevents price spikes.

When stabilisation is pursued through storage subsidies only, it does not improve consumers’ welfare. Additional storage increases low prices through additional demand for stockpiling, but it is not effective in preventing price spikes. In a small open economy, price spikes occur when the world price is high, in which case any additional stock is sold on the world market. While domestic prices are stabilised to some extent, the potential benefits for consumers are wiped out by the increase in the mean price. Such policy improves the country’s trade balance by giving it more resources to export when the world price is relatively high, but it does not benefit consumers. Since storage
policies are generally thought of as a way to help consumers, these results sound like a warning that storage policies designed without any flanking trade policy might be inconsistent: the limited isolation provided by trade costs—especially when they are relatively small—does not allow any independent food price policy to be pursued. In contrast, a well-designed combination of trade and storage taxes and subsidies can be a very cost-efficient price-stabilising policy.

These policies share one important limitation. They originate distributive welfare effects that are much larger than total gains. Reducing consumers’ risk bearing by manipulating prices in an open economy may thus face strong oppositions.

For the sake of tractability, the model overlooks supply reaction and producers’ risk aversion. However, our results show that an optimal combination of trade and storage policies trims both the lower and upper part of the distribution of domestic prices. If not necessarily the best one, the optimal policy identified here should thus remain welfare increasing even when producers are risk averse—they should value the trimming of low prices—and, when supply is reactive, since expected prices are not strongly modified, supply reaction should remain limited.

The important role played by import subsidies in our results may raise questions, since this instrument is not commonly used in practice. Faced with food prices deemed excessive, developing countries’ governments tend to favour instead consumption subsidies or direct price controls. These policies generally include public intervention on the food market, 
inter alia
through import resells at a loss. Such interventions, which are common practice, thus include import subsidies. Depending on how they are conducted exactly, they usually also entail output subsidies or taxes, a component that is not taken into account here.

Another striking finding is that export taxes are key ingredients in the design of price-stabilising policies that benefit consumers. This result should not be understood as a plea for export restrictions, the destabilising effects of which are well documented and are legitimately a serious source of concern at the multilateral level (Mitra and Josling, 2009). Nonetheless, this analysis may be useful in understanding better why export restrictions are used so often. They might also help gauging the stakes of potential multilateral disciplines. For the poorest countries, in particular, banning export restrictions altogether may be politically difficult, at least if no substantial compensating measures are offered. The collective action problem created by export restrictions certainly deserves closer scrutiny. In any case these constraints, disturbing as they are, are better acknowledged than ignored.

6.A FIRST-ORDER CONDITIONS OF THE OPTIMAL POLICY PROBLEM

To solve the optimal policy problem presented in section 6.4, we have to reformulate the complementarity equations (6.6)–(6.8) since they cannot enter directly as constraints of a maximisation problem. We restate these equations as a combination of inequalities and equations. For equation (6.6), we
introduce a slack variable, $\phi$, with its associated complementarity slackness conditions

$$\phi_t = P_t + k - \beta E_t (P_{t+1}) - \zeta_t$$  \hspace{1cm} (6.30)$$

$$S_t \phi_t = 0,$$  \hspace{1cm} (6.31)

where $S_t \geq 0$ and $\phi_t \geq 0$. To limit the number of equations and variables, the two trade policy instruments are merged into a single one with $\nu_t = \nu_t^X - \nu_t^M$, which is equivalent since each instrument is redundant when the other is active. The equations governing trade, (6.7)–(6.8), can be restated as

$$X_t [P_t^T - (P_t^w - \tau)] = 0,$$  \hspace{1cm} (6.32)$$

$$M_t [(P_t^w + \tau) - P_t^T] = 0,$$  \hspace{1cm} (6.33)$$

$$P_t^T = P_t + \nu_t,$$  \hspace{1cm} (6.34)

where $X_t \geq 0$, $M_t \geq 0$, and $P_t^w - \tau \leq P_t^T \leq P_t^w + \tau$.

Discretionary equilibriums are known to be difficult to characterise since they involve functional equations, often called generalised Euler equations (Klein et al., 2008, Ambler and Pelgrin, 2010). In a model setting with occasionally binding constraints, it is even more complex because first-order conditions cannot be derived (see appendix 6.C). For optimal policies with both instruments and with storage subsidy alone, this difficulty can be sidestepped by noting that the policy maker acts identically under commitment and under discretion. This peculiarity makes it possible to use the solution of the problem under commitment, more easily computed, as a solution to the problem under discretion.

The equivalence between commitment and discretion for these two policies arises because the storage subsidy allows fully controlling the only intertemporal trade-off, namely the storage decision. Formally, following Marcet and Marimon (1999), the optimal policy problem under commitment can be expressed as a saddle-point functional equation problem:

$$J (A_t, P_t^w, \lambda_{t-1}) = \min_{\Phi_t} \max_{\Omega_t} \left\{ v (P_t, Y) \right\}$$

$$+ w [P_t A_t - (P_t + k) S_t + (P_t - P_t^w - \tau) M_t + (P_t^w - \tau - P_t) X_t]$$

$$+ \chi_t [A_t + M_t - D (P_t) - S_t - X_t]$$

$$+ \lambda_t (\phi_t + \zeta_t - P_t - k)$$

$$+ \nu_t P_t$$

$$+ \delta_t S_t \phi_t$$

$$+ \delta_t^M M_t (P_t^w + \tau - P_t^T)$$

$$+ \delta_t^X X_t (P_t^T - P_t^w + \tau)$$

$$+ \kappa_t (P_t^T - P_t - \nu_t)$$

$$+ \beta E_t \left[ J \left( S_t + \epsilon_{t+1}, f \left( P_t^w, \epsilon_{t+1}^w \right), \lambda_t \right) \right],$$  \hspace{1cm} (6.35)$$
The transition from one period to the next is still governed by equations (6.10)–(6.11). Combining where 

must be considered in addition: equations (6.36), (6.41), (6.42), and (6.44)–(6.46) remain valid. The following first-order conditions

are no commitment problem. The equilibrium is, thus, recursive in its natural state variables and multipliers. Here, the multiplier is always null (equation (6.41)), which demonstrates that there is no commitment problem. The equilibrium is, thus, recursive in its natural state variables and time-consistent. It is identical under commitment and under discretion.

The transition from one period to the next is still governed by equations (6.10)–(6.11). Combining equations (6.38)–(6.39), (6.43) and (6.47)–(6.48) gives \( \nu_t = \chi_t / w \) for positive trade. It means that the optimal trade policy is to take trade decisions on the total marginal value of the commodity (the sum of private marginal value, domestic price, and social marginal value, \( \chi_t / w \)) instead of the price.

Usually, the time-inconsistency of policies under commitment shows up in the lagged Lagrange multipliers. Here, the multiplier \( \lambda \) is always null (equation (6.41)), which demonstrates that there is no commitment problem. The equilibrium is, thus, recursive in its natural state variables and time-consistent. It is identical under commitment and under discretion.

### 6.3 First-order conditions of the optimal storage subsidy

The first-order conditions of the optimal storage subsidy are obtained similarly, except that variables \( \nu \) and \( P_t \) have to be dropped from (6.35). From the first-order conditions with both instruments, equations (6.36), (6.41), (6.42), and (6.44)–(6.46) remain valid. The following first-order conditions must be considered in addition:

\[
S_t : S_t \geq 0 \quad \perp \quad -w [P_t + k - \beta E_t (P_{t+1})] - \chi_t + \beta E_t (\chi_{t+1}) + \delta^S_t \phi_t \leq 0,
\]

\[
P_t : v_P (t) + wD (P_t) - \chi_t D' (P_t) - \lambda_t + \lambda_{t-1} = 0,
\]

\[
M_t : M_t \geq 0 \quad \perp \quad w (P_t - P^w_t - \tau) + \chi_t + \delta^M_t (P^w_t + \tau - P^\tau_t) \leq 0,
\]

\[
X_t : X_t \geq 0 \quad \perp \quad w (P^w_t - \tau - P_t) - \chi_t + \delta^X_t (P^\tau_t - P^w_t + \tau) \leq 0,
\]

\[
\sigma_t : \lambda_t = 0,
\]

\[
\phi_t : \phi_t \geq 0 \quad \perp \quad \lambda_t + \delta^S_t S_t \leq 0,
\]

\[
\nu_t : P^\tau_t - P_t - \nu_t = 0,
\]

\[
\chi_t : A_t + M_t = D (P_t) + S_t + X_t,
\]

\[
\lambda_t : \phi_t + \sigma_t - P_t - k + \beta E_t (P_{t+1}) = 0,
\]

\[
\delta^S_t : S_t \phi_t = 0,
\]

\[
\delta^M_t : M_t (P^w_t + \tau - P^\tau_t) = 0,
\]

\[
\delta^X_t : X_t (P^\tau_t - P^w_t + \tau) = 0.
\]
\[ X_t : X_t \geq 0 \quad \perp -\chi_t + (\delta^X_t - w) (P_t - P_t^w + \tau) \leq 0, \quad (6.51) \]
\[ \delta^M_t : M_t (P_t^w + \tau - P_t) = 0, \quad (6.52) \]
\[ \delta^X_t : X_t (P_t - P_t^w + \tau) = 0. \quad (6.53) \]

6. C  CHARACTERISATION OF THE OPTIMAL TRADE POLICY

For the optimal trade policy, the solutions under commitment and under discretion being different, we have to characterise the discretionary solution directly. Since we focus on a Markovian equilibrium, price can be characterised by a function of the state variables: \( P_t = \psi(A, P_t^w) \). Using this function \( \psi \) to characterise price expectations, the value function is defined by the following Bellman equation

\[
J(A_t, P_t^w) = \min_{\Phi_t} \max_{\Omega_t} \left( v(P_t, Y) + w \left[ P_t A_t - (P_t + k) S_t + (P_t - P_t^w - \tau) M_t + (P_t^w - \tau - P_t) X_t \right] + \chi_t \left[ A_t + M_t - D (P_t) - S_t - X_t \right] + \lambda_t \left\{ \beta E_t \left[ \psi \left( S_t + \epsilon_{t+1}^H, f \left( P_t^w, \epsilon_{t+1}^w \right) \right) \right] + \phi_t - P_t - k \right\} + \delta^S_t S_t \phi_t \right) + \delta^M_t M_t (P_t^w + \tau - P_t^w) + \delta^X_t X_t (P_t^w - P_t^w + \tau) + \kappa_t (P_t^w - P_t - \nu_t) + \beta E_t \left[ J \left( S_t + \epsilon_{t+1}^H, f \left( P_t^w, \epsilon_{t+1}^w \right) \right) \right],
\]

In this setting with occasionally binding constraints, we cannot assume \( \psi \) to be differentiable everywhere. Thus, in theory, we cannot derive the first-order conditions of this problem since they would imply derivatives of \( \psi \). But since, in practice, \( \psi \) is approximated by a spline, which is differentiable everywhere, we numerically solve the dynamic programming problem by solving the corresponding first-order conditions.
“Having looked at monetary policy from both sides now, I can testify that central banking in practice is as much art as science. Nonetheless, while practicing this dark art, I always found the science quite useful. And I came to believe that the Federal Reserve and other central banks could profit from more disciplined and systematic thinking.”

—Alan S. Blinder (1997, p. 17)

7.1 General conclusion

If it is true, as the quote above from Alan S. Blinder would suggest, that the practice of monetary policy would benefit from more science, this probably applies also to the practice of food price stabilisation, which has been the object of many kinds of discretionary, and generally costly, policies. The objective of this thesis is to contribute to the science of food price stabilisation by providing a rationale for public intervention and discussing the optimal design of these policies.

The previous literature emphasises the stabilising role of private storers. Here, while acknowledging their role, we show that private storage alone is not sufficient to stabilise the economy when consumers suffer from price instability, because of incomplete markets. Government has two main ways to improve consumers’ welfare. It can provide them with countercyclical cash transfers or further stabilise prices in addition to the efforts of private agents. In this thesis, we study the latter types of policies.

Our first contribution is methodological. We show how to design optimal state-contingent food price stabilisation policies using tools commonly used in macroeconomics to design optimal monetary and fiscal policy. These methods allow policies to be distinguished between commitment and discretion. They allow the identification of problems created by the lack of commitment. We consider also simple stabilisation rules, such as a constant private storage subsidy or a price-band defended by public storage.

Our second contribution is to characterise optimal food price stabilisation policies. In a closed economy, an optimal storage policy consists of translation of the storage rule followed by speculative
storers to higher storage levels. A higher storage level implies that there is no longer a profit opportunity from speculation. There is no coexistence of private and public storage, public storage crowds out private storage. The optimal storage policy, however, could be decentralised through a state-contingent subsidy to private storage. The subsidy would compensate the storers for the losses suffered as a result of the optimal storage rule. Under government commitment, the optimal storage policy is history dependent. Government can manipulate the expectations of producers and induce them to be more responsive to expected price changes, which brings additional stabilisation. But since government has to keep its promises, the manipulation of expectations comes at the cost of adjusting the storage rule each period to respect past commitments made to producers.

In addition to storage, prices can be stabilised through a countercyclical supply policy, which tends to tax producers in situations of glut in order to decrease next-period stock levels, and to subsidise them when availability is limited. In summary, it makes supply more elastic. In this precise form, such a policy is unlikely since it would increase the volatility of farmers’ incomes, but it could be implemented through other mechanisms, which would respect incentives while compensating farmers for their losses.

In an open economy, some of the previous results may be reversed. An optimal storage policy decreases the long-run mean price in a closed economy because of the convexity of the demand function. In contrast, in an open economy an optimal storage policy increases the long-run mean price since when prices spike the economy is connected to the world market, so that selling stocks will not decrease prices because of international arbitrage. Additional storage is only profitable for consumers if complemented by trade policy. By insulating the domestic market from the world market, a trade policy allows a country to pursue a storage policy without its benefits leaking to the rest of the world. An optimal trade policy combines export restrictions to avoid import price spikes from the world market, and import subsidies to decrease domestic prices in periods of domestic scarcity.

This thesis is optimistic about the possibility of improving welfare through food price stabilisation policies, although we have shown that there are large distributive effects and small welfare gains from these policies. This optimism should be tempered somewhat by the potential practical difficulties. Even leaving aside the uncertainties that remain about models and parameter values, the implementation of stabilisation policies is dependent upon the political context. It may be that, even if an optimal policy would increase welfare over a situation without intervention, the policies actually implemented decrease welfare. This might be due to a regulatory capture by interest groups, to the impossibility to commit, or to the implementation of bad policies (recall the example of the price-bands in section 5.3.3 which decrease welfare when badly parameterised). Such a situation has been observed in southern Africa where bad policies have meant that domestic prices have exceeded import parity (Tschirley and Jayne, 2010), which should not happen in a free trade situation. We should not exclude the possibility that in many countries it could be better to decrease rather than increase public intervention in food markets.
7.2 Perspectives

While I hope this work helps to fill some of the gap that exists between the theory and the food policy applied, these results are not necessarily applicable in developing countries. The models were built to produce insights about public stabilisation policies, but, for tractability, many important aspects have been ignored. In order to enable a discussion of more realistic stabilisation policies, the framework adopted here should be extended to account for other important features.

Cash transfers or price stabilisation? While food price stabilisation policies, if optimally designed, can increase social welfare by spreading the risk from risk-averse consumers to risk-neutral government, they also create large distributive effects, particularly between producers and consumers, through the changes in the mean price. This is due to their second-best nature. They do not target the market imperfection precisely, as would conditional cash transfer policies, they only change the price distribution in a way that increases social welfare. This raises questions about the best way to intervene in developing countries: through targeted transfers or through stabilisation policies. In the simple environment we consider in this thesis, the answer is simple: targeted transfers allow the highest welfare since they are able to mimic futures markets. In reality, the opportunity cost of public funds, the administrative costs and corruption in the public aid system may drastically affect this conclusion; however, I am not aware of any study on this issue.

Consumers’ welfare cost of price volatility Another limitation of the proposed policies is the smallness of the welfare gains they produce. Public intervention to mitigate instability may be justified only if instability is costly for agents and if they do not have the ability to cope with it. However, as discussed in the introduction and subsequent chapters of this thesis, welfare losses stemming from consumers’ risk aversion are second order and the smallness of the potential gains could dissuade policy makers from trying to reach them. Risk aversion, however, is only one aspect of the cost of price volatility for consumers, allowing a straightforward design of the welfare objective. Consumers’ welfare is affected through other dimensions that we have ignored.

In our analysis, we do not take account of the dynamic effects of price instability on consumers, our evaluation accounts only for a static effect. There are at least two dynamic components: households savings, which are studied in Eaton (1980), Ni and Raymon (2004), and Nocetti and Smith (forthcoming), and dynamic effects which create irreversibilities in household welfare, such as removing children from school or decreasing food consumption to the extent that it has long-run health consequences. These latter effects would likely lead to much higher welfare losses than risk aversion since, in addition to decreasing welfare in periods of high prices, they also have an effect on all subsequent periods. There is much evidence of these dynamic consequences of high food prices. For instance, we know from de Janvry et al. (2006) that when facing adverse shocks poor parents may take their children out of school and send them out to work (see also Jensen, 2000, for other effects on children of agricultural volatility). Children growing up in times of famine or drought tend
to be shorter (Hoddinott and Kinsey, 2001, Gørgens et al., forthcoming). These effects may justify more aggressive stabilisation policies, but are delicate to integrate in a welfare framework and, thus, have been neglected in this thesis and in previous works on the welfare cost of food price instability.

**Producers’ welfare cost of price volatility** To economise on the number of state variables, producers’ risk aversion is not considered in this thesis. There is no methodological difficulty attached to doing this extension except for the added state variable. It would increase the concavity of the social welfare function, but would prevent any public policies that tend to destabilise producers’ revenues. The eventual welfare effect of stabilisation policies when accounting for producers’ risk aversion is linked to two dimensions: the effect of risk on producers’ welfare, which, as for consumers, is likely to be of second order, and the effect of risk on producers’ effort. This latter effect may be of importance (e.g., it drives the Pareto inferiority of free trade in Newbery and Stiglitz, 1984). A producer exposed to price risk and to multiplicative output risk and that exhibits decreasing absolute risk aversion reduces its effort when facing increased risk (Chavas, 2004, Ch. 8). Consequently, a stabilisation policy designed to increase producers’ welfare could also induce more production. This additional production could bring higher welfare benefits than those stemming from the risk-premium reduction.

**International risk-sharing** In chapter 6, we showed that for an open economy the most efficient way to stabilise food prices is to use non-cooperative trade policy, while a cooperative storage policy leads to welfare losses for consumers. This result, in addition to the important role of export restrictions in the 2007–08 and 2010–11 food price spikes, should motivate more study of international policy coordination. This issue has been barely addressed in the trade and uncertainty literature. Most papers study small open economies and do not discuss the potential international effects of the anti-trade results they find. One exception is Newbery and Stiglitz (1984). They conclude that a perfect international risk-sharing of yield shocks may not be optimal since it decreases the correlation between domestic shocks and domestic prices, which provides an insurance to risk-averse producers. Newbery and Stiglitz’s model is voluntarily simplistic and its conclusions do not hold in a more realistic environment (e.g., the inclusion of speculative storage would break the link between yield shocks and prices), but it has the virtue of opening debate on the optimality of free trade.

From our analysis, it is clear that without any commitment mechanism the optimal reaction of a trading country to high world prices is to impose export restrictions. The anticipation of this behaviour by the rest of the world leads to specialisation less pronounced than under a commitment to free trade. This situation could be analysed as a problem of two-sided lack of commitment (Ljungqvist and Sargent, 2004, Ch. 20). In a two-country world, since countries cannot commit to keeping their borders open in all situations, there may not be a perfect risk-sharing of agricultural risk and free trade may not be the equilibrium. Such an approach is taken, for instance, in the analysis of the optimal international risk-sharing of macroeconomic risk (Kehoe and Perri, 2002). The failure
of a country to commit to paying back all international debt means that the country will be able to borrow up to a limit where it will have no interest in defaulting and remaining subsequently in autarky. The resulting endogenous market incompleteness leads to limited international risk-sharing.

**Imperfect Information** We have assumed that state variables are perfectly observable. This is in line with the literature on the rational expectations storage model and much of the macroeconomics literature (for exceptions, see the survey by Mankiw and Reis, 2010). However, this is crucially important. In reality, prices are observable in real time, but quantity variables, in particular stocks and yields, are only observable with noise and with delay. Private and public decisions have to be taken on the basis of imperfect information, and decisions may have to be changed when new information comes in. It is not uncommon, for example, for governments to restrict imports based on optimistic output information only to see prices spike a few months later when production is shown to be mediocre (Jayne and Tschirley, 2009, Tschirley and Jayne, 2010). Understanding the behaviour of private agents in this setting and how government could intervene would be a fundamental step towards more appropriate policies.
APPENDIX A

COMMENT EXPLIQUER LA FLAMBÉE DES PRIX AGRICOLES?

This appendix consists of a paper written in French in summer 2008, which explains the food price spike. It was aimed at a non-academic audience and was published as a chapter in the CEPII yearbook, which describes the state of the world economy.¹

V / Comment expliquer la flambée des prix agricoles

Christophe Gouel*


De nombreux facteurs ont été avancés pour expliquer cette flambée des prix, depuis la croissance asiatique jusqu’aux biocarburants, en passant par la spéculation ou les événements climatiques. Toutefois, la responsabilité de chacun d’eux est délicate à distinguer du fait de la faible disponibilité de données récentes, du très grand nombre de paramètres à considérer et des difficultés à les intégrer dans un cadre d’analyse commun.

Ce chapitre tente de clarifier le rôle des principales forces ayant joué sur ces marchés en distinguant bien ce qui relève de deux temporalités souvent confondues dans les commentaires de la crise actuelle : les tendances de moyen terme, qui ont

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COMMENT EXPLIQUER LA FLAMBÉE DES PRIX AGRICOLES ?

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Graphique 1. Prix des matières premières agricoles depuis 1970
(indice trimestriel, 2000 = 100)

Source : FMI, Statistiques financières internationales.

Entraîné une augmentation régulière des prix agricoles depuis 2000 – principalement le renchérissement des autres matières premières et le déséquilibre croissant entre l’offre et la demande mondiales ; et les événements survenus ces deux dernières années, qui ont perturbé un équilibre déjà fragile. Nous commencerons par rappeler les principales caractéristiques des marchés agricoles.

Les caractéristiques des marchés agricoles

Des marchés mondialisés ?

Les principales denrées alimentaires sont des produits standardisés. Dès lors, leur origine géographique n’est pas une information pertinente. Les marchés agricoles peuvent donc être très intégrés au niveau mondial. Des cotations dans de grandes bourses (comme le Chicago Board of Trade pour le maïs ou le Minneapolis Grain Exchange pour le blé) servent de référence pour fixer les prix partout dans le monde. Tant que des barrières physiques ou commerciales n’isolent pas un marché du reste du monde, les prix domestiques suivent les prix mondiaux.

Mais, en même temps, le secteur agricole est bien connu pour être l’un des plus protégés au monde : les protections à la fron-
tière (contingents tarifaires et droits de douane) sont en moyenne trois fois plus élevées pour l’agriculture que pour les autres secteurs ; l’Union européenne consacre 42 % de son budget à sa politique agricole commune. On est donc en présence, selon les produits, de secteurs qui peuvent être complètement intégrés au marché mondial ou de secteurs très protégés, isolés des chocs extérieurs. Le marché des huiles végétales, par exemple, est très intégré (tableau I). À l’opposé, les céréales essentielles pour l’alimentation humaine (blé, riz et maïs) font l’objet de fortes protections limitant leur commerce. C’est particulièrement vrai du riz, très peu échangé au niveau mondial et dont la production est très soutenue dans toute l’Asie. Ces barrières peuvent aller jusqu’à déconnecter complètement le marché domestique du marché mondial : le prix du riz japonais est ainsi systématiquement au-dessus de 2 000 dollars la tonne, alors que le prix mondial n’a dépassé les 1 000 dollars qu’en 2008, après une augmentation vertigineuse.

Tableau I. Part des exportations dans la production mondiale
(en %)

<table>
<thead>
<tr>
<th>Années</th>
<th>Huile de palme</th>
<th>Soja</th>
<th>Blé</th>
<th>Maïs</th>
<th>Riz</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000-2005</td>
<td>72</td>
<td>30</td>
<td>18</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

Source : US Department of Agriculture.

Stockage et volatilité

Les prix agricoles sont très volatiles, car ils équilibrent des marchés où l’offre et la demande sont très rigides et sujettes à de nombreux chocs extérieurs. À court terme, la réaction de l’offre agricole est quasi nulle : ne peut être vendu que ce qui a été planté précédemment, aux variations de stocks prés. La demande est à peine plus flexible. L’alimentation répond à un besoin à la fois vital et peu extensible. La demande ne varie que très faiblement avec les prix, et cette rigidité augmente avec le revenu. Plus on est riche, plus la part de l’alimentation dans l’ensemble des dépenses est faible, et moins la consommation réagit aux prix alimentaires. L’inélasticité de la demande au prix et la rigidité de l’offre rendent les marchés agricoles très sensibles aux variations qui peuvent intervenir d’un côté ou de l’autre ; il faut alors que le prix s’ajuste fortement afin d’équilibrer le marché.
Le stockage, caractéristique fondamentale de ces marchés, vient limiter ces fluctuations. Les spéculateurs/stockeurs achètent lorsque les cours sont bas, pour revendre leurs stocks lorsque les cours sont élevés. Ce faisant, ils permettent de lisser les prix et limitent la dépendance des marchés aux conditions de l’année en cours. Lorsque les stocks sont élevés, les prix agricoles restent à un niveau faible, avec une instabilité limitée, la majeure partie des chocs pouvant être absorbée. Lorsque les stocks sont faibles, notamment parce qu’ils ont été utilisés pour amortir les chocs lors des saisons précédentes, les prix sont élevés et, surtout, très instables. Les marchés agricoles se caractérisent ainsi par de longues périodes de calme et de prix bas, suivies d’épisodes d’envolée des prix et d’instabilité.

Un contexte général haussier depuis 2000

Les autres matières premières : pétrole, fret et engrais

Depuis 2000, les prix de toutes les matières premières, et assimilés, ont augmenté, en particulier ceux du pétrole et du fret. Le pétrole est doublement au cœur de la tourmente actuelle. Tout d’abord, l’augmentation de son prix s’est ajoutée aux tensions géostratégiques pour servir de justification au lancement de programmes de soutien aux biocarburants. Ensuite, le pétrole a un impact important sur la production agricole, car en plus de sa forte utilisation dans le machinisme agricole, il est un intrant essentiel de la production d’engrais (potasse et phosphate sur le graphique 2). Le renchérissement du pétrole s’est donc traduit par une augmentation des coûts de production qui a limité la croissance de l’offre et conduit à la hausse des prix. Notons à ce propos que les agriculteurs ne sont pas, dans cette crise, aussi gagnants que les prix agricoles le laisseraient supposer. L’augmentation très forte des coûts de production a sensiblement amputé les gains provenant du renchérissement de leur production.

Tiré par le prix du pétrole, mais aussi par la demande chinoise et la congestion des capacités de transport, le prix du fret atteint lui aussi des sommets (graphique 2). Le transport d’une tonne de grain du golfe du Mexique vers le Japon coûtait le 25 avril 2008 119 dollars, contre 59 dollars quatre ans auparavant (USDA, Grain Transportation Report). Aux mêmes dates, le prix du blé était
Graphique 2. **Indices des prix alimentaires, du fret, des engrais et du pétrole, 1997-2007**

(Indices, 2000 = 100)

*Note :* le Baltic Dry Index est un indice représentant le fret maritime de matières premières sèches sur 26 routes maritimes.

*Sources :* FMI, Statistiques financières internationales pour les prix alimentaires et les engrais, Thomson financial pour le Baltic Dry Index et le pétrole.

respectivement de 292 et 160 dollars la tonne. Les pays éloignés des grands centres de production doivent donc payer aujourd’hui presque autant en transport que ce qu’ils payaient il y a quelques années pour la matière première elle-même.

**Une forte croissance de la demande**

La forte croissance des économies en développement au cours de ces dernières années a des conséquences importantes pour la demande alimentaire mondiale. Lorsque les revenus augmentent dans un pays riche, la consommation alimentaire n’augmente pas, le supplément de revenu est dépensé en produits manufacturés et services. Au contraire, dans les pays pauvres, l’enrichissement se traduit par une amélioration à la fois quantitativ et qualitative de l’alimentation. Le régime alimentaire chinois, par exemple, se rapproche de celui des pays riches : la consommation de riz et de blé diminue, au profit de celle de viande. En dix ans, la consommation chinoise a augmenté de 56 % pour le poulet, 23 % pour le porc et 71 % pour la viande bovine (tableau II), alors que la population chinoise n’augmentait que de 7 %.
Tableau II. Changement de consommation, variation 2007/1997
(volume, hors alimentation animale)

<table>
<thead>
<tr>
<th></th>
<th>Afrique subsaharienne</th>
<th>Chine</th>
<th>Inde</th>
<th>Monde</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blé</td>
<td>1,69</td>
<td>0,93</td>
<td>1,10</td>
<td>1,09</td>
</tr>
<tr>
<td>Maïs</td>
<td>1,34</td>
<td>1,59</td>
<td>1,42</td>
<td>1,59</td>
</tr>
<tr>
<td>Riz</td>
<td>1,53</td>
<td>0,96</td>
<td>1,18</td>
<td>1,12</td>
</tr>
<tr>
<td>Soja (graine et huile)</td>
<td>2,28</td>
<td>2,05</td>
<td>2,50</td>
<td>1,49</td>
</tr>
<tr>
<td>Viande bovine</td>
<td>1,30</td>
<td>1,71</td>
<td>1,63</td>
<td>1,16</td>
</tr>
<tr>
<td>Viande de porc</td>
<td>1,55</td>
<td>1,23</td>
<td>-</td>
<td>1,23</td>
</tr>
<tr>
<td>Poulet</td>
<td>1,97</td>
<td>1,56</td>
<td>3,86</td>
<td>1,51</td>
</tr>
</tbody>
</table>

Source : US Department of Agriculture.

Ce changement d’habitudes alimentaires, qui se produit dans l’ensemble des pays pauvres s’enrichissant, a d’importantes conséquences en termes d’usage des céréales, car pour produire un kilo de viande, il faut entre 3 et 7 kilos d’alimentation animale. Ainsi, pour répondre à l’augmentation de la consommation de produits carnés, la consommation animale de soja en Chine a été multipliée par 4,5 entre 1997 et 2007.

Les fortes demandes alimentaires chinoise et indienne conduisent souvent à faire de celles-ci les principales responsables de la crise actuelle. Après avoir fait baisser les prix des produits manufacturés, la Chine est aujourd’hui souffrante de faire flamber les cours des matières premières.

C’est vrai, la consommation de produits agricoles chinois a explosé. Cependant, la Chine a historiquement développé une politique d’autosuffisance alimentaire visant à assurer 95 % de ses besoins. C’est donc par une augmentation de sa production domestique qu’elle a répondu à cet accroissement de la demande. Seule sa production de soja n’a pas été en mesure d’y répondre : encore exportatrice au milieu des années 1990, la Chine importe désormais plus de la moitié du soja échangé mondialement.

L’Inde est dans une situation assez similaire, proche de l’autosuffisance. Sa position par rapport au reste du monde n’a pas vraiment changé ces dernières années. Bien que ses importations aient augmenté plus vite que ses exportations, elle est toujours exportatrice nette de produits alimentaires.

Si donc on considère à la fois l’offre et la demande agricoles des grands émergents asiatiques, ces derniers n’apparaissent pas
avoir joué un rôle central dans les tensions apparues sur les marchés. L’excès de demande actuel provient davantage de la demande en provenance du reste du monde en développement dans lequel les habitudes alimentaires se transforment aussi.

En revanche, il est vrai que la Chine a largement contribué à la diminution des stocks mondiaux de céréales. Les stocks de blé, comme des autres céréales, ont atteint en 2006 un niveau historiquement bas (graphique 3). La politique de commercialisation des grains, depuis le début des réformes en Chine, avait conduit à l’accumulation de stocks très importants chez les agriculteurs (Aubert, 2004). À la fin des années 1990, les stocks chinois de céréales atteignaient une année de production et représentaient la moitié des stocks mondiaux. La diminution du prix domestique qui s’est ensuite a entraîné un déstockage massif.

Graphique 3. Évolution des stocks de blé

En 2007, le ratio stock sur consommation est, pour toutes les céréales, à un niveau beaucoup plus faible que les années précédentes (tableau III). Pour 2008, les stocks de début de campagne en blé sont prévus à soixante-trois jours de consommation, alors qu’ils sont habituellement de près de trois mois. Ce niveau est inférieur à celui qui prévalait lors de la grande crise de 1973. C’est aussi le cas pour les produits laitiers et le sucre (mais non pour les oléagineux). Du fait de la faiblesse des stocks, la plupart
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Tableau III. Rapport stocks* sur consommation dans le monde, principaux produits agricoles (en %)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Céréales</td>
<td>15,9</td>
<td>26,1</td>
<td>Produits laitiers</td>
<td>3,6</td>
</tr>
<tr>
<td>Blé</td>
<td>20,1</td>
<td>30,3</td>
<td>Sucre</td>
<td>20,6</td>
</tr>
<tr>
<td>Maïs</td>
<td>13,7</td>
<td>24,9</td>
<td>Oléagineux</td>
<td>18,0</td>
</tr>
<tr>
<td>Riz</td>
<td>17,9</td>
<td>29,9</td>
<td>Soja</td>
<td>27,0</td>
</tr>
</tbody>
</table>

* Stocks de début de saison. Source : US Department of Agriculture.

des marchés agricoles sont dans une situation de dépendance particulièrement forte aux conditions de l’année en cours.

Un ensemble de chocs sur des marchés fragiles

C’est dans cette situation de grande vulnérabilité que les marchés agricoles ont dû, ces deux dernières années, faire face à plusieurs chocs : montée en puissance des politiques de soutien aux biocarburants, mauvaises récoltes, restrictions des exportations.

Le rôle des biocarburants, majeur sur le maïs et les oléagineux

Depuis 2003 aux États-Unis et 2005 en Europe, la production de biocarburants, largement encouragée par les politiques publiques, a fortement augmenté la demande de certains produits agricoles, réduisant leur disponibilité pour des usages alimentaires.

Il existe deux types de biocarburants, l’un correspondant à l’essence et l’autre au diesel. Le premier, l’éthanol, est produit principalement à partir de céréales et de sucre ; le biodiesel est obtenu à partir d’huiles végétales ou d’autres graisses.

Le développement des biocarburants contribue à l’augmentation des prix agricoles, directement en augmentant la demande de produits agricoles utilisés pour les fabriquer, mais aussi indirectement par un effet de concurrence pour l’usage des sols.

La consommation de céréales pour produire des biocarburants représentait 3 % de la production mondiale de céréales en 2007. Toutefois, seules certaines céréales sont concernées : le maïs aux États-Unis et en Chine, et le blé en Europe. Aux États-
Unis, 21 % de la production de maïs partait, en 2007, en production d'éthanol ; cette part devrait atteindre 30 % en 2008. Une telle augmentation de la demande doit se traduire, à production inchangée, par un très fort accroissement des prix. Anticipant cela, les producteurs américains ont augmenté les surfaces en maïs, permettant une récolte record en 2007, en hausse de 60 millions de tonnes (+ 24 %). Ils ont ainsi pu faire face à leur demande domestique de biocarburants et à leurs exportations, limitant la hausse du prix du maïs par rapport aux autres produits (12 % en 2007, contre 27 % pour l'ensemble des produits alimentaires, 80 % pour le blé et 74 % pour le soja).

Mais cette forte réaction de la production s'est faite au détriment des autres cultures, principalement celles de soja et de blé. Pour le soja, les surfaces plantées aux États-Unis ont diminué d'un sixième entre 2006 et 2007. Cette chute de production a entraîné avec elle les stocks propulsant rapidement le prix du soja vers des sommets.

Ce n'était que le début du jeu de domino des productions agricoles. Suite aux augmentations vertigineuses du prix du blé et du soja, les agriculteurs ont privilégié, dans la campagne de 2008, ces productions au détriment de celle de maïs. La chute de production qui devrait en résulter, combinée aux mauvaises conditions météorologiques outre-Atlantique ont conduit à une forte hausse des cours du maïs. Aujourd'hui, les stocks mondiaux de maïs sont, comme les stocks de blé, à un niveau historiquement faible.

Du côté des huiles végétales, en 2007 c'est 6 % de la production mondiale qui est transformée en biodiesel. Entre 2005 et 2007, plus de la moitié de l'augmentation de la demande d'huiles végétales est venue de l'industrie des biocarburants (OCDE/FAO, 2008). L'Europe convertit 40 % de ses huiles (principalement l'huile de colza) en biodiesel ; pour répondre à la consommation alimentaire, elle a dû importer massivement, alors même que la demande mondiale augmentait fortement.

Pour résumer, depuis deux ans, les biocarburants ont fortement affecté plusieurs secteurs, dont le maïs, le soja et les autres huiles végétales. M. W. Rosegrant (2008) estime ainsi qu'ils ont contribué à hauteur de 39 % à l'augmentation du prix du maïs. Les effets indirects sont difficiles à chiffrer, mais il est clair que les autres marchés céréaliers, notamment celui du blé, n'ont pas été déconnectés de ce qui se passait sur le marché du maïs. Les productions de blé et de maïs peuvent en effet se faire sur des
terres semblables, et leurs utilisations pour l'alimentation animale sont concurrentes.

Les événements climatiques ont profondément affecté le marché du blé

Les biocarburants ne sont qu'une des causes du prix élevé du blé, qui a surtout souffert ces dernières années de mauvaises récoltes dans un contexte déjà difficile où la production avait du mal à suivre la demande. La production mondiale de blé a baissé de 2000 à 2003 faisant plonger le niveau des stocks (graphique 3) et marquant le début de l'augmentation de la volatilité sur ce marché. Les bonnes récoltes des deux années suivantes ont permis une légère remontée des stocks, mais pas de baisse des prix alors que tous les marchés agricoles commençaient à afficher des signes de tension. En revanche, les deux récoltes suivantes ont été très mauvaises. L'Australie a perdu trois cinquièmes de sa production du fait d'une sécheresse exceptionnelle. L'Australie est certes loin derrière l'Europe, l'Inde, la Chine ou les États-Unis en termes de production de blé, mais, exportant la moitié de sa production, elle assure habituellement 10 % des exportations mondiales. L'année 2006 a aussi été assez mauvaise pour la production européenne, de sorte que la production mondiale de blé a chuté de 5 %. Les mauvaises récoltes se sont répétées en 2007.

L'augmentation des prix a alors été exacerbée, comme dans le cas du riz, par les mesures commerciales prises par les États : l'Ukraine et le Pakistan ont imposé des quotas d'exportation, l'Argentine et la Chine des taxes à l'exportation.

Le rôle des politiques commerciales : le cas du riz

Le cas du riz illustre bien l'étroitesse des marchés agricoles et leur dépendance à quelques grands acteurs. La production de riz n'a pas baissé depuis 2002 mais la demande n'a pu être satisfaite qu'au prix d'une baisse sensible des stocks (− 40 % entre 2000 et 2007).

Depuis le point bas du début 2000, le prix du riz a régulièrement augmenté jusqu'à atteindre 350 dollars la tonne fin 2007, le double de son prix sept ans auparavant. Face à cette augmentation, de nombreux pays exportateurs (qui sont aussi des pays assez pauvres) ont commencé à imposer des restrictions à leurs exportations pour maintenir l'approvisionnement local et
limiter la flambée des prix. Le Vietnam et l’Égypte ont donné le signal en novembre 2007, suivis par la plupart des pays exportateurs, à l’exception de la Thaïlande. Or le marché international du riz est très étroit. Les grands producteurs, la Chine et l’Inde, poursuivent des politiques d’autosuffisance ; elles n’interviennent sur les marchés mondiaux que pour y déverser leurs excédents. La Corée et le Japon protègent leurs marchés par des droits de douane supérieurs à 500 %. Au total, les échanges internationaux de riz ne représentent que 7 % de la production mondiale (tableau II). Les mesures de restriction des exportations adoptées fin 2007 ont donc renforcé l’étroitesse de ce marché à un moment où les stocks étaient déjà très faibles, poussant les prix vers des sommets. La spéculation a pu aussi jouer un rôle sur ce marché, mais mesurer son impact est extrêmement complexe.

La spéculation, un coupable idéal ?

Les produits agricoles s’échangent sur des marchés au comptant (marchés spot) pour une livraison immédiate et, pour des livraisons différées, sur des marchés à terme. Ces derniers permettent aux différents acteurs de la filière agricole de couvrir leurs risques de prix en achetant ou vendant aujourd’hui contre une promesse de livraison ultérieure. De nombreux acteurs financiers interviennent aussi sur ces marchés. Ils permettent d’accroître la liquidité ce qui, en principe, rend ces marchés plus stables que si n’y opéraient que les acteurs du monde agricole et agroalimentaire.

L’augmentation des prix des matières premières agricoles a suivi de peu la crise des subprimes. Cela a naturellement incité à soupçonner les fonds spéculatifs, qui ont reporté leurs liquidités sur les marchés de matières premières, d’être à l’origine de l’envolée actuelle des prix agricoles. Les doutes sur le rôle des spéculateurs ont aussi été renforcés par la volatilité accrue et les anomalies répétées enregistrées sur ces marchés.

Cependant, les premières conclusions de l’instance américaine de surveillance des marchés à termes, la Commodity Futures Trade Commission, dédouanent la spéculation des grands mouvements observés dont l’origine serait surtout liée aux fondamentaux économiques (Harris et Fenton, 2008). La meilleure illustration en serait que les marchés où les prix ont le plus augmenté sont justement ceux sur lesquels la spéculation financière a été la plus faible : blé dur ou riz.
Notons toutefois que la spéculation ne se limite pas aux opéra-
tions qui s’effectuent sur les marchés financiers. Ainsi, le fait
qu’il n’existe pas de marché financier significatif pour le riz
n’empêche aucunement qu’il puisse y avoir une spéculation sur
celui produit. Au Vietnam, face à l’envolée des cours du riz, toutes
les personnes capables de stocker ont parié sur une envolée
future des cours en achetant du riz qu’ils espéraient revendre à
meilleur prix plus tard. On a vu les producteurs de café ou de
poivre se lancer dans le négoce du riz. Les ménages, craignant
de nouvelles augmentations, ont acheté tout de suite, lorsqu’ils
en avaient les moyens, leur consommation des mois à venir.
Comme lors de toute panique de ce type, les anticipations ont
été autoréalisatrices : les achats de précaution massifs ont
entraîné mécaniquement la hausse des prix et le rationnement
redouté. Pour stopper le processus, le gouvernement vietna-
mien a interdit, fin avril 2008, le commerce de riz aux non-pro-
fessionnels de la filière.

En général, le niveau des stocks, ou du moins ce qu’on en
coune, semble toutefois confirmer le rôle limité joué par la
spéculation. Si la spéculation faisait monter le prix au-dessus du
prix équilibrant l’offre et la demande, l’offre excédentaire
devrait se retrouver dans les stocks ; pour d’autres matières
premières, il est possible de stocker en n’extrayant pas la ressource
du sous-sol, mais pour les produits agricoles il n’est pas possible
de laisser la plante sur pied. Or, il n’y a aucune évidence d’une
augmentation des stocks. Les stocks de céréales de début de sai-
son ont baissé de 5 % aux États-Unis et de 1 % dans le monde

S’il subsiste des incertitudes quant au rôle de la spéculation
dans la crise alimentaire actuelle, il existe au total peu d’élé-
ments permettant de penser que son rôle ait pu être détermi-
nant, alors que les fondamentaux économiques indiquent
vraiment une situation très tendue sur de nombreux marchés.

**Vers une sortie de crise**

De nombreux commentateurs estiment que nous sommes
duramente sortis d’une période de prix faibles des matières
premières. Pourtant, historiquement, les périodes de prix très
élevés ont souvent été rapidement suivies d’un retour à des prix
bas. Car, comme le rappellent l’OCDE et la FAO (2008), le pire
ennemi des prix élevés sont précisément les prix élevés. Ceux-ci inci-
tent en effet à mettre de nouvelles terres en culture et à aug-
menter les rendements ; l’augmentation de production qui en
résulte ramène les marchés dans une situation normale.

Les marchés du blé ou du riz, pour lesquels la conjonction de
chocs d’offre et de demande et de faibles stocks a été détermi-
nante, montrent déjà des signes importants de réaction. Les cours
du blé sont passés de 450 dollars la tonne à la mi-mars 2008 à
300 dollars en trois semaines, suite à des prévisions de récolte
exceptionnelle (la récolte mondiale passerait de 600 à 660 mil-
ions de tonnes). Une récolte exceptionnelle est aussi prévue pour
le riz. Toutefois, une détente sur les prix est peu probable tant que
les mesures de restriction des exportations n’auront pas été levées
et que les stocks n’auront pas été reconstitués.

Ces augmentations de production traduisent les marges de
mani?vre existantes dans les systèmes productifs agricoles. En
Europe, la Commission européenne a provisoirement levé la
jachère obligatoire afin de laisser les agriculteurs répondre aux
changements dans les prix mondiaux. Dans le monde, les sur-
faces en blé devraient augmenter en 2008 de 3,4 % et le ren-
dement à l’hectare de 4,7 %. Toutefois, tout retour des prix au
niveau du coût marginal de production se fera à un niveau plus
élevé que précédemment du fait de l’augmentation forte des
prix des intrants : pétrole ou fertilisants. De plus, sans changem-
ment de politique quant à la production de biocarburants,
aucune détente n’est à prévoir sur les marchés du maïs et des
huiles végétales, ce qui continuera à affecter durablement les
autres marchés agricoles.

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ABSTRACT  This thesis proposes a theoretical analysis of food price stabilisation policies in developing countries. It uses a rational expectations storage model, which is extended to situations justifying public intervention, and stabilisation policies are designed to correct for market imperfections. The use of the storage model as a workhorse is justified by a literature survey (Ch. 2), which presents theoretical explanations for the dynamics of agricultural commodity prices. There are two classical explanations for price dynamics: expectations errors (cobweb theory) and productivity shocks (rational expectations framework). The empirical evidence tends to support the latter, justifying the choice of the storage model. This model does not have a closed-form solution and is solved using numerical methods, which are analysed in chapter 3. The traditional formulation of the storage model does not include market imperfections. In order better to represent conditions in poor countries, consumers are assumed to be risk averse and unable to insure against price risk. This market incompleteness justifies public intervention and, in chapter 4, various stabilisation policies with government commitment are analysed using this framework. An optimal storage policy implies a level of storage higher than achievable without intervention. The additional storage crowds out private storers by removing the profit opportunity from speculation. The use of complex intervention rules, however, is unlikely in poor countries and governments may prefer to rely on simple rather than optimal rules that are strongly nonlinear. Chapter 5 compares state-contingent optimal food storage policies, with and without commitment, to simple storage rules such as a constant private storage subsidy or a price-band defended by public storage. The commitment of government brings additional welfare gains compared to a discretionary policy. These gains stem from the possibility of manipulating producers’ expectations and, thus, of inducing them to promote stabilisation. Simple stabilisation rules, if designed optimally, can achieve gains close to those achievable by state-contingent policies. Finally, the analysis of stabilisation policies is extended to an open economy framework in which the stabilisation instruments are trade and storage policies (Ch. 6). A storage policy alone does not benefit consumers, since most additional storage is used to serve the world market. In contrast, an optimal trade policy strongly decreases price volatility by exploiting the world market. Exports are taxed to avoid importing price spikes on the world market through exports and imports are subsidised to keep domestic prices low when local availability is insufficient. However, this type of policy is non-cooperative and, were it to be applied by many countries, could amplify world price volatility.

KEYWORDS  Cobweb, commitment, discretion, export restrictions, market dynamics, price expectations, food security, incomplete markets, market dynamics, price expectations, risk aversion, storage.
RéSUMÉ  Cette thèse propose une analyse théorique des politiques de stabilisation des prix alimentaires dans les pays en développement. Pour cela, le modèle de stockage à anticipations rationnelles est étendu à des situations justifiant une intervention publique, ce qui permet de proposer des politiques optimales de stabilisation pour corriger l’effet des imperfections de marché. L’usage du modèle de stockage comme cadre de référence est justifié par une revue de la littérature (Ch. 2) présentant les explications théoriques existantes de la dynamique des prix des matières premières agricoles. Deux explications sont traditionnellement proposées pour la dynamique des prix : les erreurs d’anticipation (théorie du cobweb) et les chocs productifs (modèles à anticipations rationnelles). La littérature empirique supporte plutôt la seconde explication, justifiant notre choix du modèle de stockage. Celui-ci n’a pas de solution analytique et est résolu à l’aide de méthodes numériques qui sont analysées dans le chapitre 3. Dans sa forme canonique, le modèle de stockage ne présente aucune imperfection de marché. Afin de mieux représenter la situation dans les pays pauvres, nous introduisons l’hypothèse que les consommateurs ne peuvent pas s’assurer contre le risque prix et qu’ils sont averse au risque. Cette incomplétude de marché justifie l’intervention publique et des politiques optimales de stabilisation avec engagement de l’État sont analysées (Ch. 4). Une politique optimale de stockage publique implique un niveau de stockage plus élevé que ce qui est fait par le privé sans intervention publique. Ce stockage additionnel entraîne l’évincement de tous les stockeurs privés en supprimant les opportunités de profit spéculatif. L’usage de règles complexes d’intervention publique est cependant peu probable dans des pays pauvres, et on peut imaginer que des gouvernements préféreraient suivre des règles simples plutôt que des règles optimales fortement non-linéaires. Le chapitre 5 compare ainsi des politiques optimales de stockage alimentaire, avec et sans engagement de l’État, à des règles simples comme une subvention au stockage privé ou une bande de prix définie par du stockage public. L’engagement du gouvernement entraîne des gains de bien-être par rapport à une politique discrétionnaire liés à la possibilité de manipuler les anticipations des producteurs et donc de les induire à stabiliser le marché en plus de ce qui est fait par le stockage public. Les règles simples de stabilisation permettent d’obtenir des gains proches de ceux obtenus avec des politiques optimales. Enfin, l’analyse des politiques de stabilisation est étendue à un cadre d’économie ouverte dans lequel les instruments de stabilisation sont la politique commerciale et le stockage (Ch. 6). Les résultats montrent qu’une politique de stockage non accompagnée d’une politique commerciale ne profite pas aux consommateurs, car les bénéfices de la stabilisation se dissipent sur le marché mondial. Au contraire, une politique commerciale optimale permet d’augmenter fortement la stabilisation en exploitant le marché mondial. Les exportations sont taxées pour éviter de trop faire augmenter les prix domestiques et les importations sont subventionnées pour faire baisser les prix lorsque la disponibilité intérieure est faible. Une telle politique est cependant particulièrement non-coopérative et, si utilisée par de nombreux pays, serait susceptible d’amplifier la volatilité des prix mondiaux.

MOTS-CLÉS  Anticipations de prix, aversion au risque, cobweb, discrétion, dynamique de marché, engagement, marchés incomplets, restrictions aux exportations, sécurité alimentaire, stockage.