Quantum propagation of guided matter waves: Anderson localization and atom laser

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Ultracold atoms and condensed matter

**Ultracold atoms**

*Bose-Einstein condensate*
Anderson *et al.*
Science 1995

**Condensed matter**

*Superfluid*  
*Supraconductors*

**Geometries:** 1D, 2D, 3D, lattices…

« clean » potentials without absorption

**Observation tools:**
wavefunction, momentum distribution…

**Tunable interactions:**
Feshbach resonances, BEC dilution

**Conductivity measurement**

Presence of phonons

Coulomb interactions between electrons

**Matterwaves**

$\lambda_{dB} \sim d$
Ultracold atoms and condensed matter

Ultracold atoms = « simple » and well controlled systems
⇒ condensed matter simulators

Mott transition (Superfluid – Insulator)

M. Greiner et al. Nature 2002

Bose -Einstein condensate
Anderson et al.
Science 1995

Matterwaves
\[ \lambda_{dB} \sim d \]

Superfluid
Supraconductors
Quantum transport phenomena

Mainly studied in Condensed Matter → Fundamental concept in physics (conduction of electrons)

Single particle effect (no interactions) : linear propagation

- Tunneling effect / quantum reflection
- Fabry-Perot cavity effect
- Bloch oscillations in periodic potential
- Anderson localization through disorder

Many body effect (interactions) : non linear propagation

- Superfluidity
- Atomic blockade (analog to Coulomb blockade)
- Solitonic propagation (Bright/ Dark)
Quantum transport with Bose-Einstein Condensates

Linear propagation:

Quantum reflection on surfaces:

- T. Pasquini et al. PRL 2006
- E. Cornell group Jila, Boulder 2005

Bloch oscillations:

- M. Ben Dahan et al. PRL 1996
- G. Roati et al. PRL 2004

Non linear: bright or dark solitons / shock waves

- L. Khaykovich et al. Science 2002
- E. Cornell group Jila, Boulder 2005
- I. Carusotto et al. PRL 2006
- P. Engels and C. Atherton PRL 2007
Optical waveguide (YAG@1064nm)

Magnetic trap (Ferromagnetic)

BEC + horizontal guide = guided matterwaves

\[ \lambda_{dB} = \frac{h}{mv} \approx \mu m \]

\[ \delta = \omega_{\text{laser}} - \omega_0 \]

\( D_2 (780nm) \)

Quantum propagation through optical potentials: size \( \sim \mu m \)
Quantum propagation through optical potentials: size \( \approx \mu m \)

\[ \lambda_{dB} = \frac{h}{mv} \approx \mu m \]

\[ \Rightarrow \]

Quantum propagation through optical potentials: size \( \approx \mu m \)

1. Anderson localisation of an expanding BEC in presence of disorder

2. Developpement of a new atomic source: guided atom laser
1. Anderson localization

*Predicted for condensed matter* (electrons):


⇒ Metal-Insulator transition induced by disorder (*no interactions*)

Anderson Localization with expanding BEC in disorder

First experiments in 2005

General context: *disorder + interactions*

New quantum phase (Bose Glass...)

B. Damski et al. PRL 2003
2. Guided Atom Laser

Well defined energy $E$
Controlled flux $\Rightarrow$ interactions

$\lambda_{dB} \simeq \mu$m
2. Guided Atom Laser

Spectral linewidth measurement:

Well defined energy $E$
Controlled flux $\Rightarrow$ interactions

$\lambda_{dB} \simeq \mu m$

$\sigma_E = 380 +/- 60$ Hz rms
Outline

1. Anderson Localization (AL)
   • Introduction and motivations
   • Scheme of 1D Anderson localization in laser speckle
   • Experimental realization and results
   • Conclusion

2. Perspectives
Outline

1. Anderson Localization (AL)
   • Introduction and motivations

P.W. Anderson
Phys. Rev. 1958
Localization of waves

Weak disorder: weak localization

\[ \ell^* \]

Interferences on closed loops

\[ \lambda \ll \ell^* \]

⇒ Enhanced (x2) return probability

⇒ Decrease of diffusion constant
Localization of waves

Weak disorder: weak localization

Interferences on closed loops

⇒ Enhanced (x2) return probability

⇒ Decrease of diffusion constant

Strong disorder: localized – extended states transition

⇒ Wavefunction are exponentially localized

⇒ Dimensionality

\[
1D, 2D : all \text{ states localized} \\
3D : \text{transition (mobility edge)}
\]
Strong localization with classical waves

Problematic:
Discrimination absorption / localization

Classical waves:
Ultrasound, μ-waves, light

Geometries:
- quasi-1D, 2D, 3D
- Photonic crystals

Signatures:
- Transmission (static / time resolved)
- Fluctuations
- Wavefunction imaging

D. Wiersma et al. Nature 97
C.M. Aegerter et al. EPL 06
T. Schwartz et al. Nature 07
D. Laurent et al. PRL 08
H. Hu et al. Nature Physics 08
Anderson localization: still an active field!

Remaining challenges

Behavior of the transition (3D)? (critical exponents)

C.M. Aegerter et al. EPL 2006

Effects of non-linearities? (interaction)

1D: Y. Lahini et al. PRL 2008
Anderson localization: still an active field!

Remaining challenges

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Experiments with matterwaves?

- *Since 80’s: indirect observations (conductivity) with electrons*
Anderson localization: still an active field!

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Dynamical localisation with cold atoms

J. Chabé et al. PRL 2009
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Dynamical localisation with cold atoms

J. Chabé et al. PRL 2009

Since 2005: experimental activity with cold atoms

D. Clément et al. PRL 2005
C. Fort et al. PRL 2005
T. Schulte et al. PRL 2005
Anderson localization with cold atoms?

→ Cold atoms = controlled systems
→ Direct observation of localized wavefunctions
→ Critical exponents in 3D?

Controlled random optical potentials:

- Speckle pattern
  - Horak et al. PRA 1998

- Bi-chromatic lattice
  - Damski et al. PRL 2003

+ Impurities in optical lattice
  - Gavish & Castin PRL 2005

- Atomic chips
  - Esteve et al. PRA 2004
A controlled disorder:

\[ \Rightarrow \text{Disorder strength} = \text{laser intensity} \]
\[ \sigma_Y = \langle V \rangle \equiv V_R \]

\[ \Rightarrow \text{Correlation length} = \text{numerical aperture} \]
\[ \Delta z = \frac{\lambda}{2(\text{N.A.})} \]

Laser speckle: diffraction from a rough plate

Blue detuned (atomic transition @ 780 nm) = repulsive potential

\[ V_{\text{random}} \propto \frac{I(r)}{\delta} \]
1. Anderson Localization (AL)

- *Scheme of 1D Anderson localization in laser speckle*
1D Anderson localization of matterwave

Propagation in weak disorder

\[ E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \gg V_R \]

No classical trapping

Classical: atoms « fly » above disorder
1D Anderson localization of matterwave

**Diagram:**
- Energy axis labeled as $E$.
- Potential $V_R$ axis.
- Exponential decay indicated.
- Single particle effect noted.
- Classical: atoms « fly » above disorder.
- Quantum: destructive multiples interferences.
  - single particule effect (no interaction)

**Equations:**
- Lyapunov exponent:
  $$\frac{1}{L_{loc}(k)} = \gamma(k) = -\lim_{|z| \to +\infty} \frac{\log\{r(z)\}}{|z|}$$
  $$p = \hbar k$$

**Propagator in weak disorder:**
$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \gg V_R$$
No classical trapping.
**Weak disorder regime**

1st order calculation (Born approximation):

\[ \gamma(k_{\text{atom}}) \propto C(2k_{\text{atom}}) \quad (p = \hbar k) \]

**Bragg condition**

(momentum conservation)

**Frequency distribution of disorder**

**Localization at 1st order:**

Disorder contains the spatial component \(2k_{\text{atom}}\)

**Energy**

\[ E \]

\[ V_R \]

\[ z \]

Exponential decay

\[ \Delta z \]
2 important distributions

Atomic momentum distribution:\ $D(k_{atom})$

(S↔ expansion from a BEC)

Spatial frequency distribution:\ $C(2k_{atom})$

(S↔ speckle characteristics)

Weak disorder regime

1st order calculation (Born approximation):

$\gamma(k_{atom}) \propto C(2k_{atom}) \quad (p = \hbar k)$

Bragg condition (momentum conservation)

Frequency distribution of disorder

Localization at 1st order:

Disorder contains the spatial component $2k_{atom}$
Scheme for localization of an expanding BEC

L. Sanchez-Palencia et al., PRL 98, 210401, 2007

1D expansion of BEC in weak disorder ($V_R \ll \mu_{in}$)

Exponential decay in the wings?
**Scheme for localization of an expanding BEC**

L. Sanchez-Palencia et al., PRL 98, 210401, 2007

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**Matterwave k-distribution:**

Interaction energy (∼ \( \mu_{in} \)) converted into kinetic energy

\[
E_{kin}^{max} = \frac{\hbar^2 k_{max}^2}{2m} = 2\mu_{in}
\]

---

**Disorder k-distribution:**

High frequency cut-off \( k_c \) given by diffraction limit

\[
k_c = \frac{\pi}{\Delta z} = \frac{2\pi}{\lambda} (N.A.)
\]

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1D expansion of BEC in weak disorder (\( V_R << \mu_{in} \))

Exponential decay in the wings?
Scheme for localization of an expanding BEC

L. Sanchez-Palencia et al., PRL 98, 210401, 2007

1D expansion of BEC in weak disorder ($V_R << \mu_{in}$)

Exponential decay in the wings?

Exponential localization of matterwave:
All k-components localize

$\textit{Preliminary experiments:}$

- $k_{\text{max}} >> k_c$
- Interactions

D. Clément et al. PRL 2005
C. Fort et al. PRL 2005
T. Schulte et al. PRL 2005
Outline

1. Anderson Localization (AL)

• Experimental realization and results
Experimental « road map »

Condition: $k_{\text{max}} < k_c$
**Experimental « road map »**

**Condition:**

\[ k_{\text{max}} < k_c \]

1. large \( k_c \) \( \rightarrow \) Very thin speckle
Experimental « road map »

Condition: \[ k_{\text{max}} < k_c \]

1. large \( k_c \) → Very thin speckle \( k_c = 3.85 \mu m^{-1} \)

Blue light @ 514 nm
High N.A. = 0.3

\( \Delta z = 0.8 \mu m \)

\(^{87}\text{Rb BEC}\)

Magnetic trapping (longitudinal)

Optical guide 1064 nm

Speckle

Atoms

rough plate

Beam @ 514 nm

guide
Experimental « road map »

Condition: \[ k_{\text{max}} < k_c \]

1. large \( k_c \) → Very thin speckle \( k_c = 3.85 \mu m^{-1} \)
2. small \( k_{\text{max}} \) → Dilute BEC
Experimental « road map »

Condition: \[ k_{\text{max}} < k_c \]

1. large \( k_c \) → Very thin speckle \( k_c = 3.85 \mu m^{-1} \)
2. small \( k_{\text{max}} \) → Dilute BEC

Low number of atoms: \( 1.7 \times 10^4 \)
Weak trap frequencies
\[ \mu_{in} = 220 \text{ Hz} \]
Experimental « road map »

Condition: \( k_{\text{max}} < k_c \)

1. large \( k_c \) \( \rightarrow \) Very thin speckle \( k_c = 3.85 \mu m^{-1} \)

2. small \( k_{\text{max}} \) \( \rightarrow \) Dilute BEC

3. large \( L_{\text{loc}} \) \( \rightarrow \) Expansion over few millimeters

\[ k_{\text{max}} = \frac{mv_{\text{max}}}{\hbar} \]
**AL - experiment**

**Experimental « road map »**

- **Condition:** \( k_{\text{max}} < k_c \)

1. large \( k_c \) → Very thin speckle \( k_c = 3.85 \mu m^{-1} \)
2. small \( k_{\text{max}} \) → Dilute BEC
3. large \( L_{\text{loc}} \) → Expansion over few millimeters

\[ k_{\text{max}} = 2.47 \mu m^{-1} < k_c = 3.85 \mu m^{-1} \]

⇒ **Condition for AL satisfied !**

**Expansion without disorder**

\[ k_{\text{max}} = \frac{mv_{\text{max}}}{\hbar} \]
Expansion in disorder

+ weak disorder $V_R/\mu_in = 0.12 \ll 1$

$k_{\text{max}} < k_c$

Fluorescence imaging (Camera EMCCD $\leftrightarrow 1\text{at}/\mu\text{m}$)

$\Rightarrow$ Matterwave is stopped by weak disorder (after around 1s of propagation)
BEC (t=0) \rightarrow \text{Exponential decay in the wings (no interactions)} \rightarrow \text{Stationary profiles (not shown)}

\Rightarrow \text{Exponential decay in the wings (no interactions)}
\Rightarrow \text{Stationary profiles (not shown)}

\text{Localization length } L_{\text{Loc}} = 530 \pm 80 \, \mu\text{m}
Localization length vs disorder amplitude

Born approximation:

\[ L_{\text{loc}} = \frac{2 \hbar^4 k_{\text{max}}^2 k_c}{\pi m^2 V_R^2 (1 - k_{\text{max}}/k_c)} \]

\[ k_{\text{max}} < k_c \]

⇒ Good agreement with no adjustable parameters
Beyond the effective mobility edge ($k_{\text{max}} > k_c$)

Same scheme with $N_{\text{at}}$ (x10):

$k_{\text{max}}$ increases $\Rightarrow k_{\text{max}} > k_c$

$N_{\text{at}} = 1.7 \times 10^5$

Theory: algebraic decay
LSP et al. PRL 2007

$n_{1D} \propto 1/z^2$
Beyond the effective mobility edge ($k_{\text{max}} > k_c$)

Same scheme with $N_{\text{at}}$ (x10):

$N_{\text{at}} = 1.7 \times 10^5$

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Theory: algebraic decay
LSP et al. PRL 2007

$D(k)$

$C(2k)$

Unlocalized atoms at 1st order

Observation of stationary localized profiles

$n_{1D} \propto 1/z^\beta$

with $\beta = 1.96 \pm 0.06$

AL - experiment
Conclusion

- 1D Anderson localization of matterwaves without interaction
- Speckle disorder: two localization regimes (exponentiel & algebraic)
- Good agreement theory / experiment
Conclusion

- 1D Anderson localization of matterwaves without interaction
- Speckle disorder: two localization regimes (exponential & algebraic)
- Good agreement theory / experiment

Related results in Florence (M. Inguscio group)


Disorder: bi-chromatic lattice
No interactions: Feshbach resonance $^{39}$K

Cold atoms = « good candidate » to study disordered systems
2. Perspectives
Anderson localization in higher dimensions

2D: critical dimensionality

3D: real transition (critical exponent, mobility edge position)

R.C. Kuhn et al. NJP 2007
S. Skipetrov et al. PRL 2008

On the experiment: 3D Anderson localisation

→ 3D expansion of matterwaves in laser speckle
Perspectives

Anderson localization in higher dimensions

2D: critical dimensionality
3D: real transition (critical exponent, mobility edge position)

R.C. Kuhn et al. NJP 2007
S. Skipetrov et al. PRL 2008

On the experiment: 3D Anderson localisation

→ 3D expansion of matterwaves in laser speckle

→ 3D speckle realisation: crossed speckles

→ 3D expansion of BEC:
magnetic levitation to compensate gravity
 Localization of the guided atom laser in presence of disorder:

**Transport experiment:**
linear and non-linear propagation

T. Paul et al. PRA 2009

→ Effects of interactions on AL: localization in « real systems »
Perspectives

1D Anderson localization with guided atom laser

Localization of the guided atom laser in presence of disorder:

Transport experiment:
linear and non-linear propagation
T. Paul et al. PRA 2009

→ Effects of interactions on AL: localization in « real systems »

Florence (1D)
Feshbach resonance $^{39}$K
B. Deissler et al. arxiv 2009
Quantum tunneling through a single thin optical barrier

Atomic Fabry-Pérot cavity: transport through a double barrier

- Frequency filtering
- Atom interactions $\rightarrow$ non classical atomic state

Towards blockade effect

I. Carusotto PRA 2001
Thanks…

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