Prevention and insurance of natural disasters

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Prévention et assurance des catastrophes naturelles

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Résumé

Ces trente dernières années, l'ensemble des pertes économiques engendrées par les catastrophes naturelles a crû à un rythme plus soutenu que la richesse mondiale. Cette évolution s'explique par l'accroissement de la population dans les régions exposées aux catastrophes naturelles, ainsi que par la faiblesse des actions de prévention mises en place dans ces régions. Le changement climatique laisse présager une accélération de cette évolution avec des risques accrûs de tempêtes, d'inondations et de sécheresses entre autres. Par ailleurs, les catastrophes naturelles sont faiblement assurées puisque seulement un tiers environ des pertes mondiales sont aujourd'hui couvertes. Bien que la part assurée ait augmenté au cours des dernières décennies, le niveau moyen de couverture reste encore trop bas pour limiter la variabilité de la richesse des populations concernées. C'est pourquoi la réduction des pertes engendrées par les catastrophes naturelles et l'accroissement des couvertures d'assurance sont des enjeux majeurs.

Les thèmes de cette thèse sont la prévention et l'assurance des catastrophes naturelles dont les faibles niveaux actuels sont dus aux nombreuses imperfections de marché mais aussi aux déficiences des politiques publiques, comme l'explique le chapitre introductif de ce travail. En s'appuyant sur la modélisation des comportements individuels, des marchés et des politiques publiques, cette thèse a pour objectif d'étudier quels sont les actions de prévention et les mécanismes d'assurance qui permettraient de diminuer efficacement les pertes et aussi la variabilité de la richesse pour les agents averses au risque.

Le chapitre 1 porte sur les choix de prévention dans le contexte du développement des villes. Il résulte du modèle d'économie urbaine développé dans ce chapitre que les zones risquées sont plus développées près du centre-ville que loin du centre-ville et que l'investissement dans la résilience des bâtiments mène à des villes plus compactes. Ce modèle met aussi en évidence que subventionner les assurances entraîne une exposition excessive aux risques en augmentant la densité dans les zones les plus risquées et en abaissant les efforts de prévention. Cette analyse illustre les effets pervers des subventions en ce domaine et le rôle que doivent jouer les politiques publiques urbaines telles que les restrictions de densité ou les codes de construction pour restreindre ces effets pervers.

Les chapitres suivants abordent la problématique des mécanismes d'assurance lorsque les risques individuels ne sont pas indépendants. En effet, une des caractéristiques majeures des catastrophes naturelles est qu'elles affectent simultanément de nombreux agents économiques.

Le chapitre 2 s'attache à développer un modèle d'équilibre général où les agents économiques sont averses au risque et sont exposés à des risques individuels potentiellement dépendants. Il y est établi que, sans imperfection de marché, une allocation Pareto-optimale des risques est atteinte en présence d'un marché compétitif de compagnies d'assurance et d'un nombre restreint d'actifs financiers. Ce résultat est valide sous réserve que la responsabilité des agents économiques dans leurs engagements soit illimitée. En pratique, leur responsabilité est limitée et les politiques publiques requièrent que des réserves financières soient constituées pour limiter les défauts de paiement dans les états catastrophiques. C'est pourquoi les chapitres 3 et 4 abordent la question du coût des réserves financières et de leur impact sur la demande de couverture et sur la forme optimale des contrats qui en découle.

Le chapitre 3 étudie l'impact du coût des réserves financières sur le prix de l'assurance et le taux de couverture demandé par des agents averses au risque. Si le prix de l'assurance est composé d'une prime correspondant au risque individuel et d'une surprime correspondant au coût des réserves financières (qui permettent à l'assureur de faire face au risque agrégé des assurés), il apparaît qu'à risque collectif donné, le poids de la surprime est d'autant plus fort dans le prix de l'assurance que la probabilité individuelle d'être sinistré est faible. Dans ces conditions, il est établi qu'à risque collectif donné, le taux de couverture demandé par les agents averses au risque décroît quand la probabilité individuelle décroît.

Le chapitre 4 analyse la forme optimale des contrats d'assurance pour une communauté d'agents qui sont averses au risque et qui sont exposés à des risques individuels corrélés. S'il n'est pas coûteux pour la communauté de constituer les réserves nécessaires, le contrat optimal pour un risque individuel donné consiste en une couverture totale, quelques soient les pertes collectives, à laquelle s'ajoute un dividende qui permet de redistribuer le cas échéant les réserves non utilisées. Dans le cas contraire, le contrat optimal pour un risque individuel donné consiste en une couverture seulement partielle quand les pertes collectives sont élevées.

Contents

Introduction

People and assets are exposed to natural risks which have weather, geological, biological or spatial origin. For instance, weather[1] processes can lead to storms, floods, droughts and extreme temperatures; geological mechanisms can generate volcanic eruptions and earthquakes with potential tsunamis; while biological risks involve epidemics and spatial risks include asteroids. These natural phenomena are called natural disasters when they have significant human or economic consequences within affected regions.[2] Depending on the socio-economic context, natural disasters may affect population from a few people to hundred millions of people and generate a variety of economic damage, ranging from thousands to over hundred billions of dollars. The worst natural disasters in terms of affected people are weather-related in densely inhabited regions of Asia, in particular large droughts in India and large floods in China. The worst natural disasters in terms of economic losses have both geological and weather origins and are more widespread in the world. The costliest event is the 2011 earthquake which generated the tsunami hitting the Fukushima nuclear power plant in Japan (210 billion dollars of losses). The second costliest is hurricane Katrina which stroke the region of New Orleans in United States in 2005 (125 billion dollars of losses). Figure 1 illustrates the 15 worst natural disasters in terms of economic losses[3], in the world up to now.

[1] "Weather" is used here in a wide sense which includes meteorological, hydrological and climatological phenonema.

[2] Information and data on natural disasters in the world can be found on the EM-DAT website of the International Disaster Database (http://www.emdat.be/).

[3] The economic losses shown on the different graphs include only direct losses, such as the destruction of assets. Indirect losses, such as the loss of production due to the destruction of productive assets, are not included.

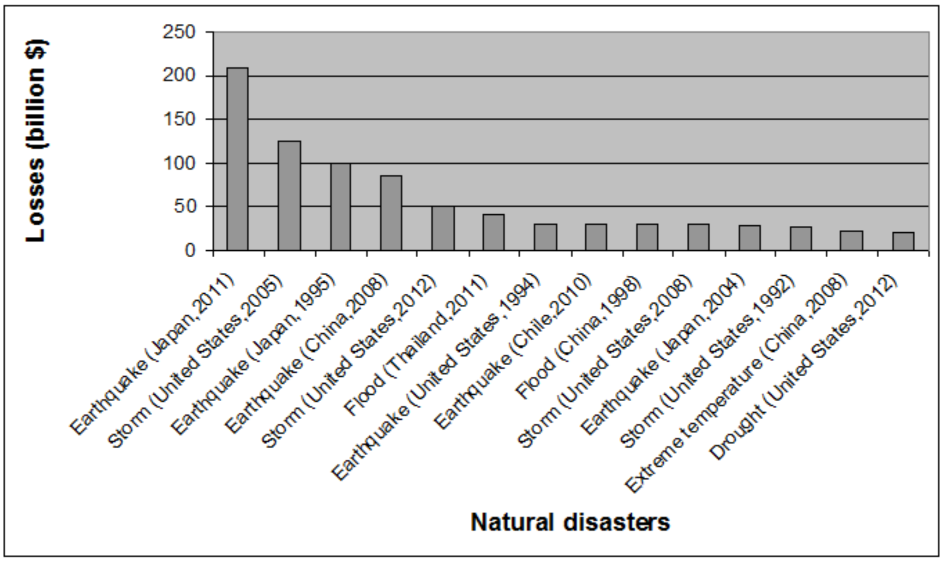


Figure 1: Worst natural disasters in terms of economic losses (Source: EM-DAT database).

World economic losses due to natural disasters have increased exponentially in the last three decades and a major part of these losses are weather-related, as shown by figure 2. For weather-related natural disasters, the real annual growth rate of losses has been of 4.1% in average since 1980, which is 1.1% above the world GDP growth rate (Aon Benfield, 2014). The fast increase of losses can be explained by increasing population in risky areas and low prevention measures in those same regions. In particular, nearly half of the world population lives nowadays within 150km of an ocean coastline and can be subject to floods, as it happened in 2005 with hurricane Katrina in New Orleans or more recently in 2012 with hurricane Sandy in New York. Beyond relocation in safer areas, prevention actions can consist for example for flooding risks in elevating or waterproofing structures or in building dams or levees. With rising sea levels, more severe rainfall patterns and higher temperature due to climate change, the trend of losses due to weather-related natural disasters is even expected to worsen in the coming years (IPCC, 2014). In this perspective, reducing natural disaster losses through prevention actions has become a main challenge for our societies.

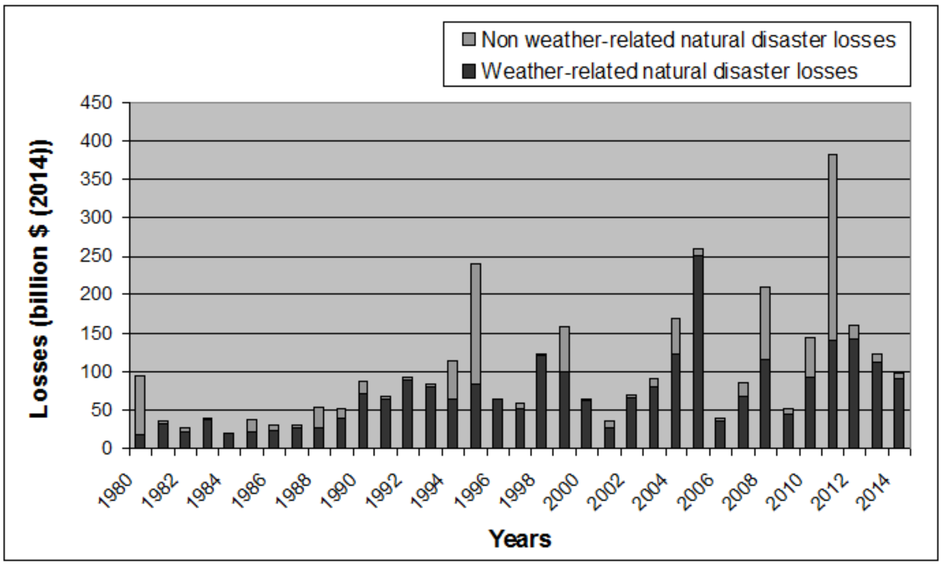


Figure 2: World total losses due to natural disasters (weather-related or not) (Source: EM-DAT database)

Moreover, losses due to natural disasters are weakly insured in the world, as illustrated by figure 3 for weather-related natural disasters. Even though insured losses have increased faster than total losses with an average annual rate of 7.7% since 1980, insured losses still represent only a third of total losses (Aon Benfield, 2014). In a highly developed country such as the USA, only 45% of households currently have insurance for weather-related risks. The low penetration of insurance can generate undesirable wealth fluctuation for affected population. In this context, increasing insurance coverage for natural disaster risks is another main challenge.

The present thesis addresses prevention and insurance issues relative to natural disaster risks. Prevention and insurance are valuable when their costs of implementation are lower than their benefits obtained through the decrease of losses and wealth variability for risk-averse agents. In an ideal decentralized world, the optimal level of prevention would be set by agents (such as households and firms) and the optimal level of insurance would be purchased by them through efficient

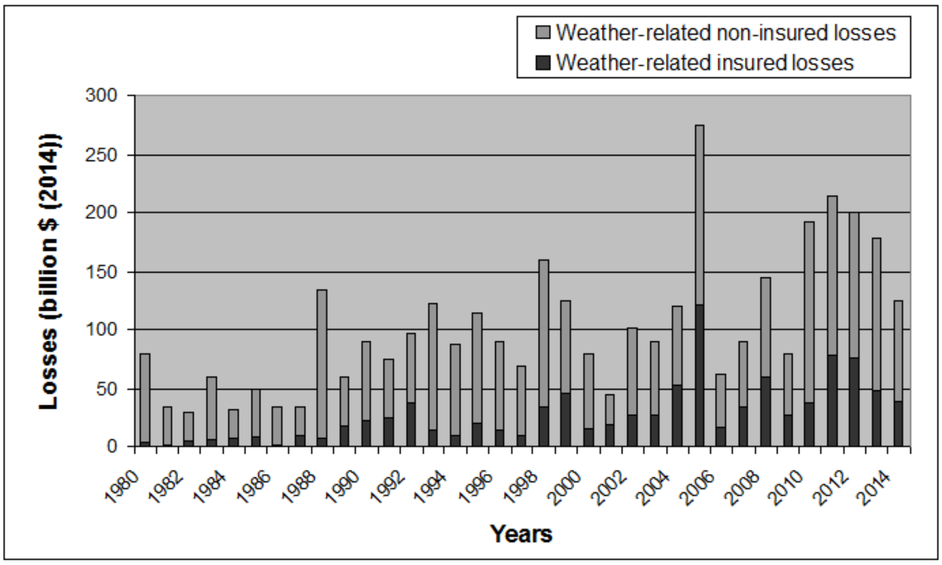


Figure 3: World total and insured losses due to weather-related natural disasters (Source: AON Benfield (2014)).

insurance markets, without public intervention.[4] However, there are market imperfections for natural disaster risks and public intervention has a role to play in this perspective (Gollier, 2005; Kunreuther & Michel-Kerjan, 2009). For instance, some preventive actions can be done at an individual level, such as elevating houses or making them waterproof, but others have to be done at a collective level, such as building dams or levees. The public good nature of these prevention actions or the externalities generated by them justify public intervention. There are also information issues related to natural disaster risks. Agents can be poorly informed about risks and public intervention can play a role to acquire and transfer information to them in order to improve behaviors both in terms of prevention and insurance. Agents can also lack information on the capacity of insurers to pay claims and public intervention can play a role to constraint insurers to secure capital and pay claims in catastrophic states. Beyond efficiency issues, public intervention is concerned with redistribution issues. The lack of insurance coverage is

[4]The optimal level of prevention corresponds to the marginal cost-benefit tradeoff for a prevention measure and the optimal level of insurance corresponds to the full elimination of individual risks and the share of collective risks.

often partly compensated by insurance subsidization and public relief. In the end, low current levels of prevention measures and insurance coverage can be explained by numerous market imperfections and poorly-designed public policies. Through modeling of individual behaviors, markets and public policies, this thesis aims at characterizing prevention actions and insurance mechanisms that could mitigate efficiently losses and wealth variability for risk averse agents. Because risk prevention choices and risk sharing mechanisms interact, it is essential to analyze them together (Dionne, 2013). Indeed, insurance prices affect the preventive behavior of risk exposed agents and prevention measures affect the level of required capital for insurance coverage.

The first part of my thesis, corresponding to chapter 1, deals with the impact of risk sharing policies on preventive behaviors. The underestimation of risks and the expectancy of public relief lead risk exposed agents to underinsure and underinvest in prevention (Kunreuther, 1984, 1996; Raschky & Weck-Hannemann, 2007). In this perspective, many governments have implemented public policies to increase risk mutualisation and decrease distress after natural disasters. For instance, public reliefs have been implemented in Canada and Germany, while public insurance systems with subsidization have been implemented in the USA (National Flood Insurance Program) and France (CatNat program) (Grislain-Letrémy & Peinturier, 2010; Michel-Kerjan, 2010). Even though these policies can partly improve risk sharing, they lead to even less prevention measures because of the lack of incentives (Courbage et al., 2013). This has been observed for example in the USA with the NFIP (Bagstad et al., 2007; Browne & Hoyt, 2000). To increase insurance purchase and prevention measures, more constraining policies can also be implemented. For instance, the purchase of insurance can be made mandatory to have access to credit and prevention measures can be enforced through urban policies such as zoning restrictions and building codes. However, in the USA, insurance contracts required for access to credit are withdrawn after a few years and constraining urban policies are still weakly implemented (Michel-Kerjan et al., 2012; Kunreuther & Michel-Kerjan, 2013), which confirms that insurance and prevention policies still have to be improved.

Chapter 1 of my thesis, entitled *Risk prevention in cities prone to natural hazards*, analyzes how insurance policies affect preventive choices in terms of location and resilience in the context of urban development in risk exposed regions. Cities located in regions prone to natural hazards such as flooding are not uniformly exposed to risks because of sub-city local characteristics. Spatial heterogeneity thus raises the issue of how these cities have spread and should continue to develop. Chapter 1 investigates these questions by featuring an urban model in which each location is characterized by a transport cost to the city center and a risk exposure. At market equilibrium, riskier areas are developed nearer to the city center than further away and investment in building resilience leads to more compact cities. At a given distance to the city center, riskier areas have lower land prices and get lower household density and higher building resilience. Actuarially fair insurance generates optimal density and resilience, while an increase of insurance subsidization leads to an increase of density in the riskiest areas and a general decrease of resilience. This analysis highlights the limits of insurance subsidization in the development of cities and addresses the role of public policy in terms of urban development, such as density restrictions or building codes, to limit risk over-exposure.

The second part of my thesis, which includes chapters 2, 3 and 4, deals with risk correlation issues and the ability of insurers to pay claims. Because natural disasters often have wide spatial impacts with tremendous losses, insurers are themselves exposed to risks. To limit the default of insurers in paying claims in catastrophic states, insurers are required to secure high levels of costly financial reserve. For natural disaster risks, the main issue for the supply of insurance thus rests on reinsurance and capital market imperfections (Froot, 2001; Jaffee & Russell, 1997). Because insurers have to secure costly capital to face uncertain catastrophic losses, insurance prices can be high and subject to fluctuations through time (Bernard, 2013; Weiss, 2007; Winter, 1994). Following years with high losses, private insurers increase their premiums to lower their liabilities and rebuild their reserves because purchasing high level of external capital through reinsurance or capital markets would be costlier. The premium increase was particularly strong at the beginning of the nineties because of hurricane Andrew and Northridge earthquake respectively hitting Florida in 1992 and California in 1994 (Kousky, 2010). In this context, market-based solutions have emerged through insurance-linked securities such as Cat-bonds which give a better access to capital to insurers and reinsurers (Cummins, 2006; Cummins & Barrieu, 2013). Also, public intervention can play a role to improve weak private insurance supply (Charpentier & Le Maux, 2014). For instance, after hurricane Andrew and Northridge earthquake, Florida and California implemented public reinsurance through the Florida Hurricane Catastrophe Fund and the California Earthquake Authority.

Chapter 2 of my thesis, entitled *The role of insurance companies in a risky economy*, analyzes the role of insurance companies when individual risks are potentially correlated. In a classic Arrow-Debreu economy, complete financial markets consist in one financial asset per state of nature, which allows a Pareto optimal allocation. Yet this setting requires a prohibitive number of financial assets. This limit finds its solution in the emergence of insurance companies. Chapter 2 analyzes their role in a model with multiple commodities and agents having different preferences and distributions of endowments, with potential risk dependence across agents. In this chapter and in the following chapters, it is assumed that individuals are risk-averse, rational and well-informed about risks. Pareto optimality is reached with stock insurance companies in competition plus one financial asset for every state of nature corresponding to the same aggregate endowments. In this case, agents can fully cover their endowment risks thanks to fair multi-risk contracts supplied by insurance companies. For a given endowment risk, the higher the correlation with the aggregate risk, the higher the premium. Also, agents can choose their share of the aggregate risk through the insurance stock shares and the financial assets. The latter allow agents to hedge commodity price risks due to the aggregate risk. In this setting without market imperfections, individual risks are fully covered even though there might be risk correlation. However, because of liability issues, insurers are required in practice to secure capital. In this case, if capital is costly to secure, correlated individual risks are more difficult to insure, which is analyzed in the following chapters.

Chapter 3 of my thesis, entitled *Insurability of low-probability catastrophic risks* and co-authored with Alexis Louaas, analyzes how the probability of a risk affects the purchase of insurance by risk-exposed individuals. With standard insurance costs and competitive pricing, agents are more inclined to insure for low-probability risks than for high-probability risks. Yet, these observations are at odds with the low insurance take-up rates for low-probability catastrophic risks. The explanation is that the risks for which underinsurance is most prevalent display substantial aggregate uncertainty. This uncertainty generates an additional fixed cost for insurers due to required financial reserves, which increases the insurance loading factor when the loss probability decreases and eventually discourages people from purchasing coverage. The analysis thus explains why some low-probability risks such as damages from lightnings are efficiently handled by the insurance sector whereas others, such as earthquakes or floods, are not.

Chapter 4 of my thesis, entitled *Pooling natural disaster risks in a community* and co-authored with Alexis Louaas, examines the optimal design of insurance contracts when individual risks are correlated across risk-averse agents in a community. The community is equipped with a public insurer which supplies insurance contracts to its members and has access to costly reinsurance outside the community. Without transaction costs inside the community, risk-averse agents fully insure against their individual risk and share collective risk by getting some dividend in normal states. With premiums raised ex-ante and generating an opportunity cost, they only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and potentially get some dividend in normal states. We illustrate the emergence of the latter contracts for the community of the Caribbean countries exposed to natural disaster risks.

Chapter 1

Risk prevention in cities prone to natural hazards

Abstract: Cities located in regions prone to natural hazards such as flooding are not uniformly exposed to risks because of sub-city local characteristics (e.g. topography). Spatial heterogeneity thus raises the issue of how these cities have spread and should continue to develop. The current paper investigates these questions by using an urban model in which each location is characterized by a transport cost to the city center and a risk exposure. Riskier areas are developed nearer to the city center than further away. Investment in building resilience leads to more compact cities. At a given distance to the city center, riskier areas have lower land prices and get lower household density and higher building resilience. Actuarially fair insurance generates optimal density and resilience. An increase of insurance subsidization leads to an increase of density in the riskiest areas and a general decrease of resilience. In this case density restrictions and building codes have to be enforced to limit risk over-exposure.

Keywords: natural disaster risks, city development, insurance, prevention, urban density, building resilience.

JEL classification: Q54, O18, G22, R52, H23.

1.1 Introduction

In October 2012, hurricane Sandy hit the East Coast of the USA, killing 54 people and generating more than 50 billion dollars of losses.[1] The damage was tremendous in Greater New York: 17% of the city was flooded and 150,000 homes were damaged (The Economist, 2012, 2013). Insurance indemnities were paid to affected households that were insured, and relief had to be organized for those that were not covered. People whose houses were destroyed wondered if they should abandon or rebuild them, and if so, how high they should elevate their new homes. Governments wondered if they should authorize development in risky areas like Oakwood Beach on Staten Island, and if so, according to which building codes. Sandy is only one example of extreme meteorological events that have caused large flooding damages in the world in the recent years. Among those, Xynthia superstorm affected the European coast in February 2010, hurricane Katrina struck the New Orleans region in the USA in August 2005 and Maharashtra heavy rains flooded the area of Mumbai in India in July 2005. Each time, these events and their devastating losses have raised the same questions about the necessity of better managing urban development in areas prone to natural hazards.

Most risk-prone regions were initially urbanized because of the many advantages they offered to communities. In particular, many cities are located near seas and/or rivers, as they can provide natural resources and transport facilities. Nowadays, many industries and services rely on these specificities, and agglomeration forces continue to drive urbanization at these locations (Fujita & Thisse, 2002). However, these locations are often double-edged because of exposure to flooding in the case of extreme meteorological events. Natural hazards coupled with expanding urbanization have already increased losses in the last few decades, and these are expected to escalate with the rising sea level and more severe rainfall patterns due to climate change (IPCC, 2014). At a sub-urban scale in risk-prone cities, locations are differentiated not only by their distance to valuable amenities such as the city center but also by exposure to risk due to local characteristics

[1] http://www.emdat.be/

(e.g. topography for flooding risks). The sub-city spatial heterogeneity raises the essential question of how these cities have spread until now and how they should continue to develop in the future.

The paper investigates these issues by using an urban model in which each location is characterized by a transport cost to the city center (or to other valuable amenities) and a risk of natural hazard (such as flooding). It focuses on the impacts of risk spatial variation and insurance subsidization on city development.[2] My results are the following. Riskier areas are developed nearer to the city center than further away. Investment in building resilience leads to more compact cities. At a given distance to the city center, riskier areas have lower land prices and get lower household density and higher building resilience. Actuarially fair insurance promotes the optimal development of the city in terms of risk prevention, with optimal household density and optimal building resilience. I analyze how an increase of insurance subsidization affects the city development. If the subsidy is financed by households in the city, it leads to an increase/decrease of density in the riskiest/safest areas. If the subsidy is financed by households in the country, it leads to a general increase of density in the city because it attracts households from other cities. Moreover, in any case, an increase of insurance subsidization leads to a general decrease of building resilience in the city. These results show that density and zoning restrictions as well as building codes have to be enforced in the city to limit risk over-exposure when insurance is subsidized.[3]

Academics in insurance economics have shown much interest in natural disasters, in particular because of the numerous imperfections in natural disaster insurance markets (Kunreuther, 1984; Kunreuther & Michel-Kerjan, 2009). On

[2] In the present framework, households deliberately purchase full insurance because they are risk-averse and insurance is supplied at or below actuarially fair prices. The model does not consider charity hazard or risk perception bias. Note however that the expectancy of assistance or the under-estimation of risk should have effects similar to insurance subsidization on the city development in terms of risk over-exposure.

[3] Density restriction at one location consists in limiting urban density while zoning restriction at one location consists in completely forbidding urban development. Building codes consist in imposing a minimal level of building resilience.

the supply side (Charpentier & Le Maux, 2014; Jaffee & Russell, 1997), diversification issues lead private insurers to supply contracts at prices largely above actuarially fair rates. On the demand side (Botzen et al., 2015; Kunreuther et al., 2007; Raschky & Weck-Hannemann, 2007), households under-insure even if insurance is fair, in particular because they under-estimate the risk or they expect free assistance (charity hazard). In this context, policy makers have implemented natural disaster public policies such as the National Flood Insurance Program (NFIP) in the USA and various programs in Europe like the CatNat in France (Bouwer et al., 2007; Kunreuther & Michel-Kerjan, 2009). To deal with diversification issues, public insurance/relief can complement the weak private insurance supply (e.g. in the USA) or public reinsurance can help private insurance to supply contracts at lower prices (e.g. in France). However, these policies cannot solve the weak insurance demand issues without subsidizing insurance or/and making it mandatory. For instance, the NFIP in the USA subsidizes contracts in risky areas thanks to taxpayers, and insurance is requested for access to loans. Meanwhile, the CatNat Program in France subsidizes contracts in risky areas with the other contracts and insurance is mandatory to avoid adverse selection. If the advantage of subsidization is to improve insurance demand and risk sharing (Browne & Hoyt, 2000; Grace et al., 2004), the disadvantage is to lead to risk over-exposure because it does not provide the right incentives for individual risk prevention (Bagstad et al., 2007; Courbage et al., 2013; Picard, 2008).

Academics in urban economics have focused on natural disaster issues in the context of city development. As modeled first by Alonso (1964), households spread out in the space surrounding the city center to commute there for consumption or work, and those settled further away incurring higher transport costs are compensated by lower land rent, which explains the increasing housing lot sizes and the decreasing density with distance to the city center. Polinsky & Shavell (1976) and Scawthorn et al. (1982) add in their model the existence of a negative amenity such as exposure to natural hazard. These models show that, at a given distance from the city center, the land price decreases when the loss exposure increases. Many empirical studies have confirmed this effect for natural disaster risks, as summarized in the meta-analysis by Daniel et al. (2009). Because households do not want to incur too much transport cost or natural disaster cost, Frame (1998) demonstrates that riskier areas are developed nearer to the city center than further away, and some risky areas inside the city outer boundary may stay undeveloped. The tradeoff between transport cost and natural disaster cost has been observed empirically for instance by Smith (1993) and Atreya & Czajkowski (2014). Frame (1998) also points out that insurance subsidization decreases the land price difference between risky areas and safe areas, as confirmed empirically by Shilling et al. (1989). Furthermore, Frame (2001) shows theoretically that risk aversion can lead households to under-develop risky areas. However, many empirical studies, such as Browne & Hoyt (2000), Harrison et al. (2001) and Michel-Kerjan et al. (2012), suggest that households are more inclined to risk over-exposure because of insurance subsidization, risk under-estimation or charity hazard, than to risk under-exposure because of risk aversion.[4] In this case, urban regulation should be enforced to limit over-exposure, in particular in terms of zoning/density restrictions and building codes (Bagstad et al., 2007; Kunreuther, 1996; Kunreuther & Michel-Kerjan, 2013). In an urban theoretical model with risk exposure but no transport costs, Grislain-Letrémy & Villeneuve (2014) show that zoning restrictions can be Pareto improving in the case of full insurance subsidization. In empirical analysis, Czajkowski & Simmons (2014) and McKenzie & Levendis (2010) respectively observe that investments in building resilience reduce natural disaster losses and increase housing values.

The present paper aims to further analyze the role of natural hazard exposure and insurance subsidization in the development of risk-prone cities with transport costs. Relative to the previously cited theoretical papers on urban economics, the present paper adds building resilience modeling and analyzes how densities and

[4]Browne & Hoyt (2000) observe that households do not usually buy insurance at fair prices, Harrison et al. (2001) notice that the housing rent difference between risky and safe areas is below the expected loss difference and Michel-Kerjan et al. (2012) point out that insured households let their insurance contract lapse after a few years even when those are below fair prices. All this would not be possible if risk aversion was the dominating factor.

resiliences are affected by natural hazard exposure and insurance subsidization. This analysis is essential from the perspective of implementing efficient urban regulation, in terms of zoning restrictions, density restrictions and building codes, for cities with transport costs and natural disaster risks. The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 provides an analysis of city development. Section 4 provides an analysis of the impact of a change in insurance subsidization. Section 5 concludes.

1.2 Risk-prone city model

I consider a static model of a city with commuting transport costs and natural hazard exposure, in the spirit of Frame (1998, 2001), Polinsky & Shavell (1976) and Scawthorn et al. (1982). The city is inhabited by N identical households.[5] The sub-city scale grid is modeled by a two-dimensional continuous space with the coordinate system $x = (x_1, x_2)$. Because of spatial heterogeneity due to transport costs and natural hazards, all variables potentially depend on location x. Moreover, each variable has a unique value at each location x because I consider identical households and identical housing developers. The city has a pre-established center located at $x = (0, 0)$, also called the central business district where work and consumption activities are concentrated.

Households compete to spread out in the space around the city center and commute between their housing location and the city center. They choose their housing location x and the quantity of goods purchased in the city center, aggregated in a composite good denoted $z(x)$. Besides composite good consumption, households value their housing good consumption, characterized by lot size, measured in land area unit and denoted $s(x)$.[6] The utility function of each household, denoted $v(.)$, depends on $z(x)$ and $s(x)$ and is classically supposed to be twice con-

[5] I consider identical households in order to analyze the average development of the city. Inequality or asymmetric information issues are not the purpose the analysis.

[6] The lot size for one household is the land area for this household. For example, for a building occupying $400m^2$ of land and inhabited by 10 households, the lot size of one household is $40m^2$.

tinuously differentiable, strictly increasing in each argument (with $\partial_z v(0, s) = \infty$ and $\partial_s v(z, 0) = \infty$) and globally concave. The composite good supplied in the city center is considered as the numéraire (i.e. price equal to 1 for one unit of good) and the housing good supplied by housing developers at location x has a housing unit price denoted $p_h(x)$ (i.e. price for one land area unit with housing). The composite good expenses and the housing rent for one household located at x are thus respectively $z(x)$ and $p_h(x)s(x)$.

Besides composite good expenses and housing rent, households incur commuting transport costs and expenses related to natural hazards. One household settling at location x incurs the given transport cost $t(x)$ because of commuting between its housing location and the city center (or potentially other valuable amenities). For example, a city located next to an estuary is depicted in figure 1.1.[7] On the land, the darkness of the square units characterizes the commuting transport cost $t(x)$ for each household located at x. Darker areas represent locations further from the city center with higher transport costs. In stylized models, transport costs are often considered to be proportional to the distance to the city center. However, real transport costs are more complex than this stylized form, in particular because of transport system complexity. Moreover, other potential amenities (e.g. the positive amenities of being near the water-front) should be taken into account in the transport costs. Note also that transport costs should include different costs, in particular the direct transport cost but also the time opportunity cost.

One household settling at location x is also exposed to natural hazards (such as flooding), with the given probability of impact $\pi(x)$. The level of the loss in case of impact, denoted $l(.)$, depends on the housing lot size $s(x)$ and on the building resilience, denoted $b(x)$. The loss function $l(.)$ is assumed to be twice continuously differentiable. It is decreasing with b at a decreasing rate because the most efficient resilience investments are made first. Besides, if it is reasonably assumed that more households on a land unit leads to more total losses on this land unit (for a given

[7]In figures 1.1 and 1.2, the space is represented by a discrete grid even if the model is continuous.

building resilience level), the loss function is such that $\frac{l(s,b)}{s} \geq \frac{\partial l}{\partial s}(s,b)$ for any s and b.[8] Note that losses should include direct and indirect losses. The city depicted in figure 1.1 is also represented in figure 1.2 for natural hazard exposure. On the land, the darkness of the square units characterizes the probability $\pi(x)$ of being affected by a natural hazard for each household located at x. The higher the risk, the darker the location. For flooding risks, locations at lower altitude are usually more subject to flooding and should be darker. The probability of being affected by a natural hazard can correspond for example to the probability that the water level reaches a threshold level that induces significant losses for households. Besides, I consider that insurance is supplied to households at or below fair prices because I do not consider any insurance transaction cost and I consider potential insurance subsidy. As households are risk-averse (i.e. their utility function is concave), they deliberately purchase full insurance coverage and bear a certain cost related to natural disaster risks, which is the insurance premium. With insurance subsidy corresponding to a fraction $\lambda \in [0,1]$ of expected losses, the premium paid by a household located at x is $(1-\lambda)\pi(x)l(s(x),b(x))$. The higher λ, the higher the subsidy. The insurance subsidy can be financed either by the city through a lump-sum tax on household wealth or by another party outside the city. In the former case, the tax borne by each household in the city is $\overline{\tau} = \frac{\lambda}{N}\iint \pi(x)l\Big(s(x),b(x)\Big)n(x)dx_1dx_2$ (in which $n(x)$ is the household density at location x). In the latter case, the tax borne by each household in the city is $\overline{\tau} = 0$.[9] For one household located at x, transport cost, insurance premium and tax are thus respectively $t(x)$, $(1-\lambda)\pi(x)l(s(x),b(x))$ and $\overline{\tau}$.

[8]With $\frac{1}{s}$ households on a developed land unit, each one having a lot size s, the total loss on the land unit is $\frac{1}{s}l(s,b)$. If more households on a land unit leads to more total losses on this land unit, the loss function is such that $\frac{l(s_1,b)}{s_1} \leq \frac{l(s_2,b)}{s_2}$ for any $\frac{1}{s_1} \leq \frac{1}{s_2}$ and b. In this case, for any s and $ds \geq 0$, $\frac{l(s+ds,b)}{s+ds} \leq \frac{l(s,b)}{s}$, which leads to $s\big(l(s,b) + \frac{\partial l}{\partial s}(s,b)ds\big) \leq (s+ds)l(s,b)$ and then $\frac{\partial l}{\partial s}(s,b) \leq \frac{l(s,b)}{s}$ with a first order development.

[9]The latter case is representative of an insurance subsidized by the country which is large relative to the city. Besides, note that a natural disaster like flooding usually strikes many locations of a city at the same time and thus has an aggregate risk component at the city level. However, an insurance system organized at the country level enables to better diversify risk.

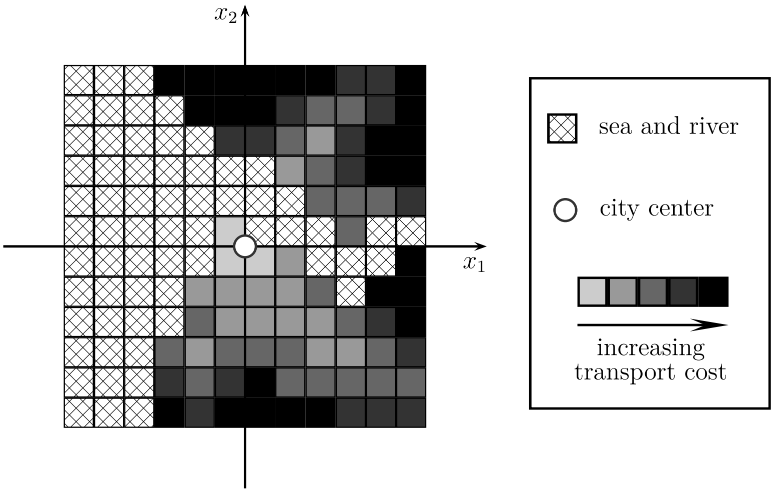


Figure 1.1: Commuting transport costs in the city.

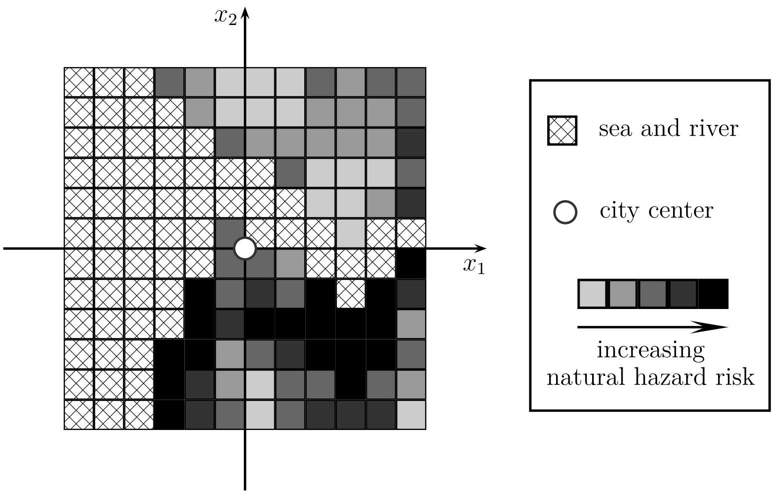


Figure 1.2: Natural hazard risks in the city.

As households are identical in terms of preferences and wealth, denoted $\overline{y}$, they reach the same utility level $\overline{v}$ at equilibrium. Otherwise, households with lower utility levels at location x would have settled at location x' where other households reach a higher utility level, which would have decreased at x and increased at x' housing unit price until the equilibrium with spatially uniform utility level had been reached. Following Alonso (1964) and Fujita & Thisse (2002), competition between households over where to settle leads housing prices to be the solutions of bid price problems: at each location x, the housing unit price $p_h(x)$ corresponds to the highest price that can be afforded by households. The wealth minus expenses except housing rent, divided by the lot size, is the maximal amount that can be paid by one household for one land unit with housing. As households are free to choose their composite good consumption and reach the utility level $\overline{v}$, the housing unit bid price problem at location x can then be expressed as follows:

$$\begin{aligned} p_h(x) = \max_{z(x)} \; & \frac{\overline{y} - \overline{\tau} - z(x) - t(x) - (1-\lambda)\pi(x) l\Big(s(x), b(x)\Big)}{s(x)} \\ s.t. \; & v\Big(z(x), s(x)\Big) = \overline{v}. \end{aligned} \tag{1.1}$$

Housing goods are supplied by identical housing developers in competition. Housing developers observe the housing unit price (1.1) resulting from the competition between households. They compete to acquire land from absentee land owners at the land unit price denoted $p_l(x)$ at each location x (i.e. price for one land area unit without housing). They choose the housing lot size $s(x)$ and the building resilience $b(x)$ for urban development at each location x. Besides the cost of land, they incur the cost of housing lot development, denoted $c(s(x), b(x))$ for lot size $s(x)$ and resilience $b(x)$ for one household. The cost function $c(.)$ is assumed to be twice continuously differentiable. It is increasing with b at an increasing rate because the less costly resilience investments are made first. Besides, if it is reasonably assumed that more households on a land unit leads to more total housing development costs on this land unit (for a given building resilience level), the cost function is such that $\frac{c(s,b)}{s} \geq \frac{\partial c}{\partial s}(s, b)$ for any s and b.[10] At each location, housing developers are constrained by the availability of one land area unit. Similarly to the housing unit price, the land unit price $p_l(x)$ is determined at each location x by the highest price that can be afforded by housing developers because of competition. The housing rent per household minus the development cost, multiplied by the household density denoted $n(x)$, is the maximal amount that can be paid by one housing developer for one land area unit. As housing

[10] The proof is similar to the one in the footnote 8 for the natural disaster loss function.

developers are free to choose housing lot sizes and building resiliences, and they observe the housing unit price (1.1) and face land constraints, the land unit bid price problem at location x can then be expressed as follows:

$$\begin{aligned} p_l(x) = \max_{s(x),b(x)} \Big(p_h(x)s(x) - c\big(s(x), b(x)\big)\Big)n(x) \\ s.t.\ (1.1) \ \text{ and } \ n(x)s(x) \leq 1. \end{aligned} \tag{1.2}$$

The boundaries of the city correspond to the locations where the land unit price $p_l(x)$ is equal to the land opportunity rent denoted $\overline{p_a}$ (e.g agricultural rent).

Finally, the city is characterized by its number of households:

$$N = \iint n(x)dx_1dx_2. \tag{1.3}$$

With a given number of households (i.e. N given), (1.3) indirectly determines the welfare level $\overline{v}$ in the city. This characterizes in particular a "closed city" in terms of population. With a given welfare level (i.e. $\overline{v}$ given), (1.3) determines the number of households N in the city. This characterizes in particular an "open city" in which the welfare level depends on the welfare level outside the city.

1.3 Risk-prone city development

Outside the boundaries of the city, housing development is not profitable ($n(x) = 0$ and $p_l(x) = \overline{p_a}$). On the boundaries, land may be partly developed ($0 \leq s(x)n(x) \leq 1$) because housing development is equally profitable to agriculture ($p_l(x) = \overline{p_a}$). Inside the boundaries, land is fully developed because housing development is more profitable than agriculture and thus the household density is:

$$n(x) = \frac{1}{s(x)}. \tag{1.4}$$

As explained in the previous section, households settled in the city reach the same welfare level $\overline{v}$ at equilibrium. As the utility function $v(.)$ is strictly increasing in z, $\tilde{z}(s, v)$ can be defined such that $v(\tilde{z}(s, v), s) = v$ and $\tilde{z}(.)$ is decreasing with s at a decreasing rate because $v(.)$ is concave (proof in appendix 1.6.1). Thus, the composite good consumption $z(x)$ purchased by one household settled at location x can be expressed as a function of the housing lot size $s(x)$ and the uniform welfare level $\overline{v}$:

$$z(x) = \tilde{z}\Big(s(x), \overline{v}\Big). \tag{1.5}$$

With (1.5), the housing unit bid price problem (1.1) boils down to the housing unit price:

$$p_h(x) = \frac{\overline{y} - \overline{\tau} - \tilde{z}\Big(s(x), \overline{v}\Big) - t(x) - (1-\lambda)\pi(x) l\Big(s(x), b(x)\Big)}{s(x)}. \tag{1.6}$$

With the housing unit price (1.6) and the household density (1.4), the land unit bid price problem (1.2) boils down to:

$$p_l(x) = \max_{s(x),b(x)} \frac{\overline{y} - \overline{\tau} - \tilde{z}\Big(s(x), \overline{v}\Big) - t(x) - (1-\lambda)\pi(x) l\Big(s(x), b(x)\Big) - c\Big(s(x), b(x)\Big)}{s(x)}. \tag{1.7}$$

The housing lot size $s(x)$ and the building resilience $b(x)$ chosen by housing developers at location x inside the boundaries of the city are the solutions of the first order conditions of (1.7) (proof in appendix 1.6.1):

$$\frac{\partial_s v}{\partial_z v}\Big(s(x), \tilde{z}(s(x), \overline{v})\Big) = (1-\lambda)\pi(x)\frac{\partial l}{\partial s}\Big(s(x), b(x)\Big) + \frac{\partial c}{\partial s}\Big(s(x), b(x)\Big) + p_l(x), \tag{1.8}$$

$$-(1-\lambda)\pi(x)\frac{\partial l}{\partial b}\Big(s(x), b(x)\Big) = \frac{\partial c}{\partial b}\Big(s(x), b(x)\Big). \tag{1.9}$$

(1.8) states that the housing lot size $s(x)$ for one household at location x is chosen such that it equalizes the marginal rate of substitution to the marginal housing unit cost (over the composite good price, i.e. the numéraire). The marginal rate of substitution characterizes the marginal benefit of increasing the housing lot size for the household, which decreases from $+\infty$ to 0 when $s(x)$ increases from 0 to $+\infty$. The marginal housing unit cost is composed of the marginal insurance premium borne by the household, the marginal housing development cost and the land unit price. (1.9) relates that the building resilience $b(x)$ for housing at location x is chosen such that it equalizes the marginal benefit of decreasing insurance premium for the household to the marginal cost of increasing building resilience for the housing developer. Note that if $-(1-\lambda)\pi(x)\partial_b l(s(x),0) \leq \partial_b c(s(x),0)$, $b(x)$ is binding in 0. With $s(x)$ and $b(x)$ being determined by (1.8) and (1.9), (1.7) then indirectly gives the land unit price:

$$p_l(x) = \frac{\overline{y} - \overline{\tau} - \tilde{z}\Big(s(x),\overline{v}\Big) - t(x) - (1-\lambda)\pi(x)l\Big(s(x),b(x)\Big) - c\Big(s(x),b(x)\Big)}{s(x)}. \tag{1.10}$$

Proposition 1 *With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the housing lot size $s(x)$ and the building resilience $b(x)$ vary in space as follows:*[11]

$$A_1(x)\frac{ds}{d\overrightarrow{x}} = \frac{1}{s(x)}\frac{dt}{d\overrightarrow{x}} + (1-\lambda)\left(\frac{l\Big(s(x),b(x)\Big)}{s(x)} - \frac{\partial l}{\partial s}\Big(s(x),b(x)\Big)\right)\frac{d\pi}{d\overrightarrow{x}}, \tag{1.11}$$

$$A_2(x)\frac{db}{d\overrightarrow{x}} = -(1-\lambda)\frac{\partial l}{\partial b}\Big(s(x),b(x)\Big)\frac{d\pi}{d\overrightarrow{x}}, \tag{1.12}$$

in which $A_1(x)$ and $A_2(x)$ are positive.

Proposition 1 is proved in appendix 1.6.1. (1.11) tells that, at a given risk of natural hazard, the housing lot size $s(x)$ increases while translating further away from the city center. Thus, the household density $n(x)$ (i.e. the inverse of the housing lot size $s(x)$) decreases while translating further away from the city center, as first explained by Alonso (1964). With the reasonable assumption $\frac{l(s,b)}{s} \geq \frac{\partial l}{\partial s}(s,b)$ for any s and b, the coefficient in front of $\frac{d\pi}{d\overrightarrow{x}}$ in (1.11) is positive and (1.11) says that, at a given distance to the city center, the housing lot size $s(x)$ increases while translating towards riskier areas if insurance is not fully subsidized ($\lambda < 1$). Thus, the household density $n(x)$ decreases while translating towards riskier areas in this case. As $l(.)$ is decreasing with b, (1.12) points out that the building resilience increases while translating towards riskier areas if insurance is not fully subsidized ($\lambda < 1$).

Proposition 2 *The housing unit price $p_h(x)$ and the land unit price $p_l(x)$ vary in space as follows:*

$$\frac{dp_h}{d\overrightarrow{x}} = \frac{dp_l}{d\overrightarrow{x}} + \frac{\frac{\partial c}{\partial s}(s(x),b(x)) - \frac{c(s(x),b(x))}{s(x)}}{s(x)}\frac{ds}{d\overrightarrow{x}} + \frac{\frac{\partial c}{\partial b}(s(x),b(x))}{s(x)}\frac{db}{d\overrightarrow{x}}, \tag{1.13}$$

[11] $d\overrightarrow{x}$ corresponds to any small move in space: $d\overrightarrow{x} = (dx_1, dx_2)$.

$$\frac{dp_l}{d\vec{x}} = -\frac{1}{s(x)}\frac{dt}{d\vec{x}} - \frac{(1-\lambda)l(s(x),b(x))}{s(x)}\frac{d\pi}{d\vec{x}}. \tag{1.14}$$

In proposition 2, (1.14) is obtained by spatial derivation of (1.7) with the envelop theorem, while (1.13) is obtained by spatial derivation of the combination of (1.6) and (1.10). (1.14) relates firstly that, at a given risk of natural hazard, the land unit price $p_l(x)$ decreases while translating further away from the city center, as first explained by Alonso (1964). (1.14) tells secondly that, at a given distance to the city center, the land unit price $p_l(x)$ decreases while translating towards riskier areas if insurance is not fully subsidized ($\lambda < 1$). Moreover, the higher the insurance subsidization (λ), the lower the land price difference between risky areas and safe areas. These observations confirm the results of Frame (1998), Polinsky & Shavell (1976) and Scawthorn et al. (1982) in a context including building resilience. (1.13) indicates that the housing unit price $p_h(x)$ is modified through three channels while moving in the city: the land unit price $p_l(x)$, the housing lot size $s(x)$ and the building resilience $b(x)$. At a given risk of natural hazard, the housing unit price $p_h(x)$ decreases while translating further away from the city center because firstly the land unit price decreases, secondly the effect through the housing lot size is negative (because $ds/d\vec{x} \geq 0$ and with the reasonable assumption $\frac{c(s,b)}{s} \geq \frac{\partial c}{\partial s}(s,b)$) and thirdly the effect through the building resilience is null. At a given distance to the city center, translating towards riskier areas leads to the decrease of housing unit price $p_h(x)$ through the decrease of land unit price $p_l(x)$ and the increase of housing lot size $s(x)$ (as far as $\frac{c(s,b)}{s} \geq \frac{\partial c}{\partial s}(s,b)$), while on the other hand it leads to the increase of housing unit price $p_h(x)$ through the increase of building resilience $b(x)$. This differentiates slightly the spatial variation of land unit price $p_l(x)$ and housing unit price $p_h(x)$, contrary to the previously cited papers which do not consider building resilience. Moreover, it is coherent with the empirical observation by McKenzie & Levendis (2010) that investments in building resilience increase housing prices.

Proposition 3 *If the probability of natural hazard is denoted $\pi^*(t)$ on the city*

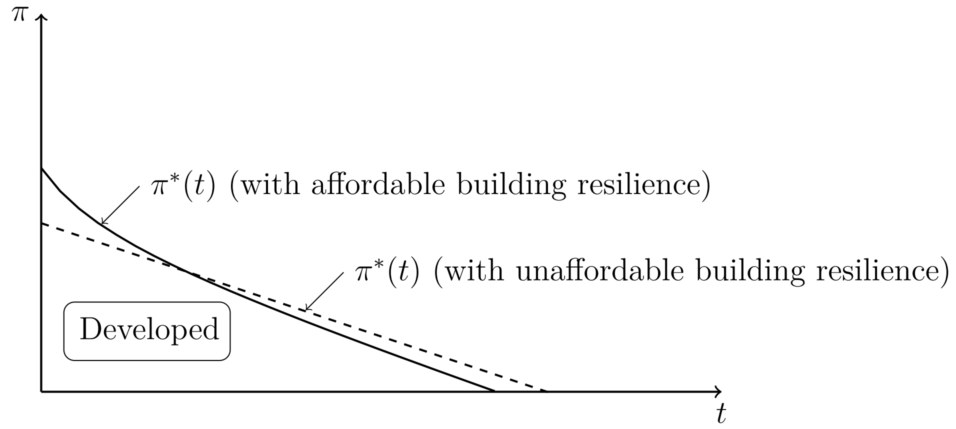


Figure 1.3: City boundaries and developed areas, as a function of transport cost t and probability π of natural hazard.

boundaries, $\pi^*(t)$ is such that:

$$\frac{d\pi^*}{dt} = -\frac{1}{(1-\lambda)l(s(x),b(x))}. \tag{1.15}$$

Proposition 3 is directly deduced from (1.14) because the land unit price $p_l(x)$ is constant and equal to the opportunity rent $\overline{p_a}$ on the city boundaries. (1.15) expresses that riskier locations are developed near the city center because of lower transport cost, which confirms the result of Frame (1998) in a context including building resilience. A location x at a distance t from the city center is developed if $\pi(x) \leq \pi^*(t)$. The outer boundary of the city corresponds to the developed area the furthest away from the city center. The inner boundaries of the city correspond to the riskiest developed area for each distance to the city center. Figure 1.3 illustrates on a graph, with transport and risk as coordinates, the city boundaries and the developed areas. The slope of π^* relative to t is steeper when building resilience is implemented. Thus, more households are located near the city center (i.e. the city is more compact) when building resilience is more affordable.

1.4 The impact of insurance subsidization

With actuarially fair insurance ($\lambda = 0$), the allocation of resources is Pareto optimal (proof in appendix 1.6.2). Actuarially fair insurance policy leads to the Pareto optimal allocation of resources because it gives the right incentives to households and housing developers in terms of density development and building resilience. In practice, actuarially fair insurance is hardly ever implemented, and policy makers usually implement insurance subsidy.

Proposition 4 *With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the increase of insurance subsidization has the following impact on urban development at each location x of the city:*

$$s(x)A_1(x)\frac{ds(x)}{d\lambda} = \pi(x)s(x)\left(\frac{\partial l}{\partial s}(s(x),b(x)) - \frac{l(s(x),b(x))}{s(x)}\right) + \alpha(x)\frac{d\overline{v}}{d\lambda} + \frac{d\overline{\tau}}{d\lambda}, \quad (1.16)$$

$$A_2(x)\frac{db(x)}{d\lambda} = \pi(x)\frac{\partial l}{\partial b}(s(x),b(x)), \quad (1.17)$$

in which $\alpha(x) = \frac{\partial \tilde{z}}{\partial v}(s(x),\overline{v}) - s(x)\frac{\partial^2 \tilde{z}}{\partial v \partial s}(s(x),\overline{v})$ and $A_1(x)$ and $A_2(x)$ are positive.

Proposition 4 is proved in appendix 1.6.2. (1.16) characterizes how the increase of insurance subsidy (λ) affects the housing lot size $s(x)$ and thus the household density $n(x)$ at each location in the city. The direct impact corresponds to the first term on the right-hand side of (1.16), which is negative with the reasonable assumption $\frac{l(s,b)}{s} \geq \frac{\partial l}{\partial s}(s,b)$ for any s and b. A given increase of λ gives, through this direct effect, a density increase which is proportional to the probability $\pi(x)$. The indirect impact through the levels of welfare $\overline{v}$ and tax $\overline{\tau}$ corresponds to the second and third terms. If the number N of households is fixed (which characterizes a "closed city" with $\overline{v}$ endogenously determined), the density cannot increase everywhere in the city[12] and the increase of insurance subsidy reallocates households from safer areas to riskier areas because of the direct effect. If the welfare level $\overline{v}$

[12] With N fixed, the population constraint (1.3) gives $0 = \iint \frac{1}{s(x)^2}\frac{ds(x)}{d\lambda}dx_1dx_2$, which means that $\frac{ds(x)}{d\lambda}$ cannot be negative at all locations. Thus, $\alpha(x)\frac{d\overline{v}}{d\lambda} + \frac{d\overline{\tau}}{d\lambda}$ in (1.16) cannot be negative for all the location in the city.

is fixed (which characterizes an "open city" with N endogenously determined), $\overline{v}$ is not affected by an increase of λ while the impact on $\overline{\tau}$ depends on who bears the cost of insurance subsidization. If the households in the city bear this cost through the lump-sum tax $\overline{\tau} = \frac{\lambda}{N} \iint \pi(x) l\Big(s(x), b(x)\Big) n(x) dx_1 dx_2$, an increase of $\overline{\tau}$ due to an increase of λ makes the city less attractive, which explains why it increases lot sizes and decreases densities. Thus, in this case, the increase of insurance subsidy leads to a density increase in strongly risky areas and a density decrease in weakly risky areas. If the households in the city do not bear the cost of insurance subsidization (i.e. $\overline{\tau} = 0$), an increase of λ does not have this negative effect on the city attractiveness. Thus, in this case, the increase of insurance subsidy leads to a general density increase in the city. These results explain in which direction density policies should be enforced in risk-prone cities when insurance subsidy is implemented. Besides, (1.17) points out how the increase of insurance subsidy (λ) modifies the building resilience $b(x)$ at each location in the city. The impact is negative and proportional to the local probability $\pi(x)$. A given increase of λ leads to a higher building resilience decrease in risky areas than in safe areas. As a consequence, whether with a closed city or an open city and whoever subsidizes insurance, the increase of insurance subsidy leads to a general decrease in building resilience in the city. Note that this decrease is null if the building resilience is already binding in zero (which is the case at a risk-free location). These results show that resilience policies should be enforced when insurance subsidy is implemented.

Proposition 5 *The increase of insurance subsidization has the following impact on housing and land prices at each location x in the city:*

$$\frac{dp_h(x)}{d\lambda} = \frac{dp_l(x)}{d\lambda} + \frac{\frac{\partial c}{\partial s}(s(x), b(x)) - \frac{c(s(x),b(x))}{s(x)}}{s(x)} \frac{ds(x)}{d\lambda} + \frac{\frac{\partial c}{\partial b}(s(x), b(x))}{s(x)} \frac{db(x)}{d\lambda}, \tag{1.18}$$

$$\frac{dp_l(x)}{d\lambda} = \frac{1}{s(x)} \left(\pi(x) l(s(x), b(x)) - \beta(x) \frac{d\overline{v}}{d\lambda} - \frac{d\overline{\tau}}{d\lambda} \right), \tag{1.19}$$

in which $\beta(x) = \frac{\partial \tilde{z}}{\partial v}\Big(s(x), \overline{v}\Big)$ is positive.

In proposition 5, (1.19) is obtained at a given location by derivation of (1.7) relative to λ with the envelop theorem, while (1.18) is obtained at a given location by derivation of the combination of (1.6) and (1.10) relative to λ. (1.19) relates how the increase of insurance subsidy (λ) affects the land price $p_l(x)$ at each location in the city. The direct impact corresponds to the first term on the right-hand side which is positive and proportional to the local probability $\pi(x)$. For a given increase of λ, the riskier the location, the higher the land price increase through this direct effect. The indirect impact through the levels of welfare $\overline{v}$ and tax $\overline{\tau}$ corresponds to the second and third terms. If the welfare level $\overline{v}$ is fixed (which characterizes an "open city" with N endogenously determined), $\overline{v}$ is not affected by an increase of λ while the impact on $\overline{\tau}$ depends on who bears the cost of insurance subsidization. If the households in the city bear this cost through the lump-sum tax $\overline{\tau} = \frac{\lambda}{N} \iint \pi(x) l\Big(s(x), b(x)\Big) n(x) dx_1 dx_2$, an increase of $\overline{\tau}$ due to an increase of λ decreases their wealth and thus land prices. Thus, in this case, the increase of insurance subsidy leads to a land price increase in strongly risky areas and a land price decrease in weakly risky areas. If the households in the city do not bear the cost of insurance subsidization (i.e. $\overline{\tau} = 0$), an increase of λ does not have this negative effect on land prices. Thus, in this case, the increase of insurance subsidy leads to a general land price increase in the city because the increase of attractiveness of the city is not lowered by a tax on households in the city. Figure 1.4 illustrates the impact of a subsidy increase on the city boundaries for an "open city" in the case where the subsidy is borne by households in the city and in the case where the subsidy is not borne by households in the city. Besides, (1.18) states that the housing price $p_h(x)$ is modified through three channels while increasing insurance subsidy (λ): the land unit price $p_l(x)$, the housing lot size $s(x)$ and the building resilience $b(x)$. The direction of the impacts through the land unit price and the housing lot size depends on the location, similarly to these two variables. The impact through the building resilience decreases the housing unit price because the increase of insurance subsidy decreases the building resilience and thus its cost.

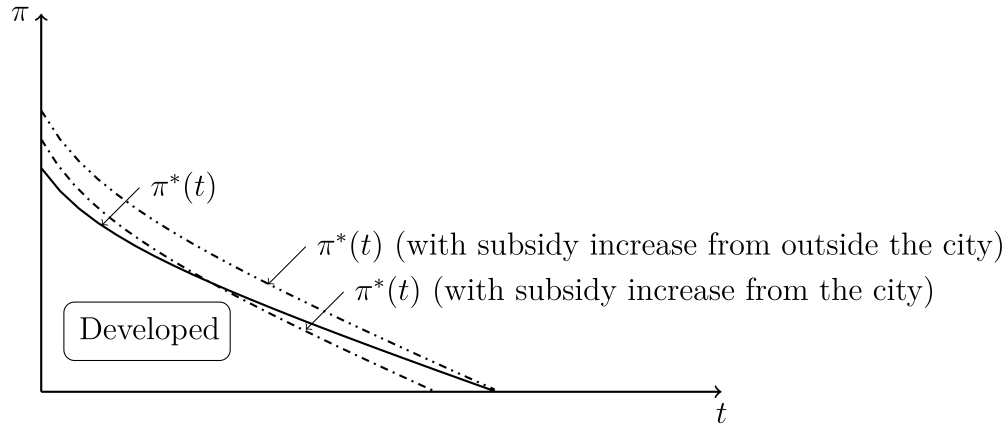


Figure 1.4: City boundaries and developed areas, as a function of transport cost t and probability π of natural hazard.

1.5 Conclusion

The paper has analyzed urban development choices in a city prone to natural disasters. It complements previous studies, in particular by including building resilience choices. Riskier areas are developed nearer to the city center than further away. Investment in building resilience leads to more compact cities. At a given distance to the city center, riskier areas have lower land prices and get lower household density and higher building resilience if insurance is not fully subsidized. Actuarially fair insurance leads households to optimally settle in space in terms of density and resilience. An increase of insurance subsidization leads to an increase of density in the riskiest areas of the city, in particular displacing inner boundaries towards riskier areas near the city center. Moreover an increase of insurance subsidization leads to a general decrease of building resilience in the city. To avoid excessive exposure to risk in the case of insurance subsidization, policy makers have to complement their policies by enforcing density and zoning restrictions as well as building codes. In this perspective, the present paper tells that, in the case of insurance subsidization, density and zoning restrictions have to be enforced at least in the riskiest areas of the city, in particular near the city center where land is attractive because of low transport costs. It also tells that, in the case of insurance subsidization, building codes should be generally enforced in the city for Pareto improvement.

1.6 Appendix

1.6.1 Risk-prone city development

Characteristics of $\tilde{z}(.)$

The derivation of $v(\tilde{z}(s,v),s) = v$ relative to s gives:

$$\frac{\partial v}{\partial z}\frac{\partial \tilde{z}}{\partial s} + \frac{\partial v}{\partial s} = 0, \tag{1.20}$$

which can be rewritten:

$$\frac{\partial \tilde{z}}{\partial s} = -\frac{\partial_s v}{\partial_z v}. \tag{1.21}$$

Because $v(.)$ is increasing with z and s, $\tilde{z}(.)$ is decreasing with s. Besides, the derivation of (1.20) relative to s gives:

$$\frac{\partial^2 v}{\partial z^2}\left(\frac{\partial \tilde{z}}{\partial s}\right)^2 + \frac{\partial v}{\partial z}\frac{\partial^2 \tilde{z}}{\partial s^2} + 2\frac{\partial^2 v}{\partial z \partial s}\frac{\partial \tilde{z}}{\partial s} + \frac{\partial^2 v}{\partial s^2} = 0, \tag{1.22}$$

which can be rewritten with (1.21):

$$\frac{\partial v}{\partial z}\frac{\partial^2 \tilde{z}}{\partial s^2} = -\frac{\partial^2 v}{\partial z^2}\left(\frac{\partial_s v}{\partial_z v}\right)^2 + 2\frac{\partial^2 v}{\partial z \partial s}\frac{\partial_s v}{\partial_z v} - \frac{\partial^2 v}{\partial s^2}. \tag{1.23}$$

The term on the right-hand side of (1.23) is positive because $v(.)$ is concave and the determinant of the Hessian matrix of $v(.)$ is positive. Thus, $\frac{\partial^2 \tilde{z}}{\partial s^2}$ is positive and $\tilde{z}(.)$ is decreasing with s at a decreasing rate.

Derivation of (1.8) and (1.9)

With (1.21) and the expression (1.7) of $p_l(x)$, the first order conditions of (1.7) relative to $s(x)$ and $b(x)$ are respectively:

$$\frac{\frac{\partial_s v}{\partial_z v}\Big(s(x), \tilde{z}(s(x), \overline{v})\Big) - (1-\lambda)\pi(x)\frac{\partial l}{\partial s}\Big(s(x), b(x)\Big) - \frac{\partial c}{\partial s}\Big(s(x), b(x)\Big)}{s(x)} - \frac{p_l(x)}{s(x)} = 0, \tag{1.24}$$

$$-(1-\lambda)\pi(x)\frac{\partial l}{\partial b}\Big(s(x),b(x)\Big)-\frac{\partial c}{\partial b}\Big(s(x),b(x)\Big)=0, \quad (1.25)$$

which respectively give (1.8) and (1.9).

Proof of proposition 1

As the first order conditions of (1.7) correspond to a maximum, the second order conditions of (1.7) are negative at the solutions $s(x)$ and $b(x)$. Thus, the following expressions which are called $A_1(x)$ and $A_2(x)$ are positive:

$$A_1(x)=\frac{\partial^2\tilde{z}}{\partial s^2}\Big(s(x),\overline{v}\Big)+(1-\lambda)\pi(x)\frac{\partial^2 l}{\partial s^2}\Big(s(x),b(x)\Big)+\frac{\partial^2 c}{\partial s^2}\Big(s(x),b(x)\Big)\geq 0, \quad (1.26)$$

$$A_2(x)=(1-\lambda)\pi(x)\frac{\partial^2 l}{\partial b^2}\Big(s(x),b(x)\Big)+\frac{\partial^2 c}{\partial b^2}\Big(s(x),b(x)\Big)\geq 0. \quad (1.27)$$

With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the spatial derivation of (1.8) and (1.9) respectively gives:

$$A_1(x)\frac{ds}{d\vec{x}}=\frac{1}{s(x)}\frac{dt}{d\vec{x}}+(1-\lambda)\left(\frac{l\Big(s(x),b(x)\Big)}{s(x)}-\frac{\partial l}{\partial s}\Big(s(x),b(x)\Big)\right)\frac{d\pi}{d\vec{x}}, \quad (1.28)$$

$$A_2(x)\frac{db}{d\vec{x}}=-(1-\lambda)\frac{\partial l}{\partial b}\Big(s(x),b(x)\Big)\frac{d\pi}{d\vec{x}}, \quad (1.29)$$

which gives proposition 1.

1.6.2 The impact of insurance subsidization

Optimal allocation

The first welfare theorem predicts the Pareto optimality with $\lambda=0$ because efficient insurance markets would lead to actuarially fair insurance. For the formal proof, the optimal allocation with uniform welfare level $\overline{v}$ is obtained by minimizing the total expenditure of the city with N households:

$$\begin{aligned}\min_{s(.),b(.),n(.)} & \iint \Big(\tilde{z}(s(x),\overline{v})+t(x)+\pi(x)l\big(s(x),b(x)\big)+c\big(s(x),b(x)\big)\Big)n(x)dx_1dx_2 \\ \text{s.t. } & n(x)s(x)\leq 1,\forall x \\ & N=\iint n(x)dx_1dx_2.\end{aligned} \quad (1.30)$$

The first order conditions give similar equations to the decentralized economy with $\lambda = 0$. Thus, the actuarially fair insurance policy (i.e. with $\lambda = 0$) implements the Pareto optimal allocation of resources.

Proof of proposition 4

The proof of proposition 4 is similar to the proof of proposition 1. The positive $A_1(x)$ and $A_2(x)$ are defined by (1.26) and (1.27). Contrary to proposition 1, the derivation of (1.8) and (1.9) relative to λ at a given location x do not have terms with derivatives of $t(x)$ and $\pi(x)$ but have terms with derivatives of $\overline{v}$ and $\overline{\tau}$. With null cross derivation for $l(.)$ and $c(.)$ relative to their two arguments, the derivation of (1.8) and (1.9) relative to λ at a given location x respectively gives:

$$s(x)A_1(x)\frac{ds(x)}{d\lambda} = \pi(x)s(x)\left(\frac{\partial l}{\partial s}(s(x),b(x)) - \frac{l(s(x),b(x))}{s(x)}\right) + \alpha(x)\frac{d\overline{v}}{d\lambda} + \frac{d\overline{\tau}}{d\lambda}, \quad (1.31)$$

$$A_2(x)\frac{db(x)}{d\lambda} = \pi(x)\frac{\partial l}{\partial b}(s(x),b(x)), \quad (1.32)$$

in which $\alpha(x) = \frac{\partial \tilde{z}}{\partial v}(s(x),\overline{v}) - s(x)\frac{\partial^2 \tilde{z}}{\partial v \partial s}(s(x),\overline{v})$. This gives proposition 4.

Chapter 2

The role of insurance companies in a risky economy

Abstract: I consider an exchange economy with multiple commodities and agents having different preferences and distributions of endowments. In an Arrow-Debreu setting, complete financial markets consist in one financial asset per state of nature and allow to reach a Pareto optimal allocation. Yet, this setting requires a prohibitive number of financial assets. With a competitive insurance market, it is sufficient to have only one financial asset for all the states of nature corresponding to the same aggregate endowments. In this case, agents can fully cover their endowment risks thanks to fair multi-risk contracts supplied by stock insurance companies. For a given endowment risk, the higher the correlation with the aggregate risk, the higher the premium. Besides, financial assets allow agents to hedge commodity price risks due to the aggregate risk and to choose a share of the aggregate risk.

Keywords: individual risk, aggregate risk, complete markets, general equilibrium, securities, insurance.

JEL classification: D53, D86, G10.

2.1 Introduction

The combination of risk exposure and risk aversion creates the demand for risk sharing and the emergence of risk sharing mechanisms in a decentralized economy. In this context, complete financial markets can lead to a Pareto optimal allocation of risks, as firstly shown by Arrow (1953) and Debreu (1959). In a classic Arrow-Debreu economy, complete financial markets consist in having one financial asset (also called security) per state of nature, in which a state of nature is characterized by a full specification of the individual endowments obtained by all the agents in the economy. Yet, in the real world, this setting requires a tremendous amount of assets and necessitates to make public the realized individual state of each agent to know the realized state of nature. These limits find their solution in the emergence of insurance companies. The present paper shows that markets are complete for risk sharing with only one financial asset for all the states of nature corresponding to the same aggregate endowments, when there are in addition competitive insurance companies supplying standard insurance contracts and owned through stock markets.

I consider a static exchange economy with multiple commodities and multiple heterogeneous agents facing heterogeneous risks, in the sense that agents have different preferences and different distributions of endowments. I define an "Arrow-Debreu" state of nature as a full specification of the individual endowments in this state obtained by all the agents in the economy. With a financial asset associated to each Arrow-Debreu state and a spot market for each commodity, the economy reaches a Pareto optimal allocation. The collection of all the "Arrow-Debreu" states displaying the same aggregate endowments in the economy defines a "fundamental" state of nature. I consider that the agents have only access to a financial asset for each fundamental state and to a spot market for each commodity. Besides, there are M insurance companies in competition. They supply fair multi-risk insurance contracts to agents, with an indemnity compensating for the individual commodity losses in exchange for a premium corresponding to the expected indemnity. Insurance contracts are standard in the sense that they do not include participation on the insurer profits.[1] The insurance companies are owned by agents in the economy through stock markets. With frictionless markets, I show that this economy reaches a Pareto optimal allocation.

In the general case, agents are exposed to commodity price risks in addition to individual endowment risks because individual endowment risks generate aggregate endowment risks. Insurance contracts with insurance companies enable agents to eliminate their individual endowment risks and security markets enable agents to eliminate their exposure to price risks. Besides, agents purchase a share of aggregate risks through insurance stock markets and security markets. Thanks to insurance companies, the number of financial assets can be tremendously reduced relative to the classic Arrow-Debreu economy. For illustration, we can consider a simple setting with 1 commodity, 3 identical agents, 10 individual states, with individual endowment from 1 to 10 commodity units and probability 1/10 for each state. In this case, the number of Arrow-Debreu states is 10^3 and the number of fundamental states is 28 (total endowment reaching potentially each integer between 3 and 30). While Pareto optimality is reached in the Arrow-Debreu economy with 1000 financial assets, it is also reached in an economy with 28 financial assets and 2 insurance companies which compete to supply to each of the 3 agents an insurance contract for individual endowment risks. Besides, while the Arrow-Debreu economy requires to make public the realized individual endowments of each agent to determine the Arrow-Debreu state and allow transactions, the reduced number of financial assets requires to make public only the realized aggregate endowments in the economy (which are indirectly known trough the spot prices because one vector of spot prices correspond to one vector of aggregate endowments).

Insurance companies serve as intermediaries that pool individual endowment risks. Because of competition, they sell fair contracts to be able to catch policyholders on one side and shareholders on the other side. A fair premium for an insured risk is equal to the expected indemnity, in which the indemnity in each

[1] Standard contracts are from stock companies by opposition to mutual contracts from mutual companies, in which there is a participation on the insurer profits.

state of nature depends on the commodity losses and on the commodity prices. For a given endowment risk, the higher the correlation with the aggregate risk (i.e. with the commodity price risk), the higher the expected indemnity and thus the premium. That is why a fair contract includes an aggregate risk loading factor in the premium.[2] The loading factor is potentially different from one agent to another for the same individual risk and can be positive or negative. As insurance contracts are fairly priced relative to individual risks and their correlation with the aggregate risk, agents purchase full coverage for their individual risks. Shareholders of an insurance company are exposed to the risk resulting from the pool of individual risks insured by the company. The higher the correlation between these risks and the aggregate risk, the higher the raised premiums, which compensates shareholders for bearing a less diversifiable risk.

The economics literature on risk has already addressed the role played by insurance companies for risk sharing. In an exchange economy with one commodity and agents having different preferences and distributions of endowments, Kihlstrom & Pauly (1971) and Ellickson & Penalva-Zuasti (1997) show that the economy can reach a Pareto optimal allocation with one contract per agent or per risk and without any financial assets, but they cannot explain who supplies these contracts. To go beyond this supply issue, Marshall (1974a,b) explains that agents have to sign mutual contracts which enable to eliminate individual risks and share aggregate risks. Malinvaud (1973) and Cass et al. (1996) consider an exchange economy with one commodity and groups of agents having identical preferences and facing independently and identically distributed risks. They show that Pareto optimality is reached with mutual insurance contracts within each group of agents and a limited number of financial assets (i.e. all the Arrow-Debreu states corresponding to rearrangements between identical agents necessitates only one financial asset). Besides, Doherty & Dionne (1993) and Doherty & Schlesinger (2002) explain that mutual insurance contracts can be replaced by "homemade mutualization",

[2]Contrary to an insurance contract based on a financial loss, an insurance contract based on commodity losses naturally includes an aggregate risk loading factor through the product between commodity losses and commodity prices for the value of the expected indemnity.

in which agents purchase standard contracts from stock insurance companies to eliminate individual risks and invest in insurance companies through stock markets to share aggregate risks. Penalva-Zuasti (2001, 2008), as well as the present paper, relaxes the condition of mutual contracts thanks to insurance companies shared on the stock markets. Penalva-Zuasti (2008) considers an exchange economy with one commodity and agents having different preferences and different initial wealth but facing independently and identically distributed risks. He shows that Pareto optimality is reached with standard insurance contracts supplied by stock insurance companies and a limited number of financial assets (i.e. all the Arrow-Debreu states corresponding to the same aggregate endowment necessitates only one financial asset).[3] Relative to Penalva-Zuasti (2008), the present paper relaxes the hypothesis of independently and identically distributed risks.[4]

The main contribution of the present paper is to extend the analysis of the role played by insurance companies to an economy with multiple commodities and agents having different preferences and different risks with potential risk dependence across agents (i.e. the present model is not restricted to independently and identically distributed risks). The analysis highlights in particular that individual

[3]Penalva-Zuasti (2008) considers actually a dynamic economy with uncertainty resolving nicely through time and continuous trading on financial markets. In this context, it is even sufficient to have only two financial assets, a risk-free one and a risky one relative to the aggregate risk. Note that the dynamic approach does not change the role of insurance because insurance contracts are not traded continuously, that is why I remain in a static model in the present paper.

[4]Another part of the economics literature has focused on the role of financial intermediaries when market frictions are considered. Gorton & Winton (2003) reviews the literature on the role of financial intermediaries when there are transaction costs or asymmetric information between agents. This topic has been particularly developed for bank-like intermediaries which play an intermediary role between borrowers and lenders (Freixas & Rochet (1997)). With transaction costs due to asymmetric information, contracts through financial intermediaries can Pareto dominate direct contracts if financial intermediaries are able to diversify. Indeed, even though financial intermediaries increase the number of transactions, diversification decreases the uncertainty and thus the asymmetric information issue for the less informed agents. As a consequence, with market imperfections, risk correlation represents a limit for financial intermediaries.

risks do not have to be identical or independent to be insured by insurance companies (in a context without market frictions, similarly to the quoted papers). Even insurance companies weakly diversified are able to sell contracts at fair prices, because the shareholders diversify their risk thanks to other markets. Besides, because risks are not independently and identically distributed, the insurance premium for an individual risk is specific to the individual risk and includes an aggregate risk loading factor characterizing the correlation between the individual risk and the aggregate risk in the economy. For a given risk, the higher the risk correlation with the aggregate risk, the higher the premium. The paper is organized as follows. Section 2 sets up the model. Section 3 reminds the classic results of Borch, Arrow and Debreu. Section 4 analyzes the decentralized equilibrium with insurance companies. The last section concludes.

2.2 The model of the risky economy

I consider a static pure exchange economy with N agents $(i = 1,..,N)$ and C commodities $(c = 1,..,C)$. Agents have uncertain endowment, are risk-averse and have objective probabilities over the risks. I denote $\overline{\boldsymbol{e_i}}$[5] the vector with C components characterizing the highest possible endowment of each commodity for agent i. Agent i can face S_i individual states $(s_i = 1,..,S_i)$. The endowment loss in state s_i relative to $\overline{\boldsymbol{e_i}}$ is denoted $\boldsymbol{l_i}(s_i)$. The endowment $\boldsymbol{e_i}$ is thus a random variable with $\boldsymbol{e_i}(s_i) = \overline{\boldsymbol{e_i}} - \boldsymbol{l_i}(s_i)$ in state s_i. The preferences satisfy the von Neumann-Morgenstern axioms with $v_i(.) : \mathbb{R}_+^C \to \mathbb{R}$ the corresponding utility function which is strictly increasing in each argument, globally concave and twice continuously differentiable.

Definition 1 *An "Arrow-Debreu" state is a full specification of the individual endowments obtained by all the agents in the economy.*

The number of Arrow-Debreu states in the economy depends on the number of agents, the number of individual states for each agent and on the risk

[5]The vectors are denoted in bold letters in the paper. In mathematic formula, the product of two vectors correspond to the scalar product.

dependences between agents. In the absence of full dependence between individual risks, the number of Arrow-Debreu states in the economy is: $Z = \prod_i S_i$ (i.e. $Z = S^N$ with identical S_i), because each combination of individual states has a positive probability to occur. I denote by $z = 1, .., Z$ the Arrow-Debreu states and $\pi(z) \geq 0$ the probability of obtaining the Arrow-Debreu state z (with $\pi(z) > 0$ and $\sum_z \pi(z) = 1$). With objective probabilities over the risks, all the agents consider this probability $\pi(z)$ for the Arrow-Debreu state z to occur. For any agent i, I denote by $s_i(.) : [1, Z] \rightarrow [1, S_i]$ the function which gives the individual state of agent i associated to each Arrow-Debreu state. I denote by $\boldsymbol{x_i}(z)$ the commodity consumption plan of agent i in the Arrow-Debreu state z. Thus, $\boldsymbol{x_i} = (\boldsymbol{x_i}(1), .., \boldsymbol{x_i}(Z))$ is the commodity consumption plan of agent i in all the Arrow-Debreu states, $\boldsymbol{x}(z) = (\boldsymbol{x_1}(z), .., \boldsymbol{x_N}(z))$ is the commodity consumption plan of all the agents in the Arrow-Debreu state z and $\boldsymbol{x} = ((\boldsymbol{x_1}(1), .., \boldsymbol{x_1}(Z)), .., (\boldsymbol{x_N}(1), .., \boldsymbol{x_N}(Z)))$ is the commodity consumption plan of all the agents in all the Arrow-Debreu states.

Definition 2 *A "fundamental" state is a full specification of the aggregate endowments in the economy.*

All the Arrow-Debreu states, which have the same aggregate endowments for the C commodities, correspond to the same fundamental state. I denote by $t = 1, .., T$ the fundamental states in the economy (with T the number of fundamental states) and $\boldsymbol{\mathcal{E}}(t)$ the total endowment vector in the fundamental state t. The set of Arrow-Debreu states included in the fundamental state t is denoted F_t (with $\sum_t \#F_t = Z$ and $\forall z \in F_t, \sum_i \boldsymbol{e_i}(s_i(z)) = \boldsymbol{\mathcal{E}}(t)$). I denote by $t(.) : [1, Z] \rightarrow [1, T]$ the function which gives the fundamental state associated to each Arrow-Debreu state. The probability of obtaining the fundamental state t is denoted $\tilde{\pi}(t)$ (we have $\tilde{\pi}(t) = \sum_{z \in F_t} \pi(z)$ and $\sum_t \tilde{\pi}(t) = 1$). The probability of obtaining the Arrow-Debreu state z conditioned on being in the fundamental state t is denoted $\pi(z|t)$ (we have $\pi(z) = \pi(z|t)\tilde{\pi}(t)$ and $\sum_{z \in F_t} \pi(z|t) = 1$).

2.3 Pareto optimality and Arrow-Debreu economy

This section aims at reminding the classic results of Borch, Arrow and Debreu. Firstly, in the spirit of Borch, the characteristics of the Pareto optimal allocations are analyzed. Secondly, in the spirit of Arrow and Debreu, it is shown that the decentralized economy with a complete set of financial assets allows to reach a Pareto optimal allocation. This section is useful to understand in the following section the functioning of a decentralized economy with insurance companies and a reduced number of financial assets.

2.3.1 Borch Mutuality Principle

The Pareto optimal allocations in an economy are by definition the ones that maximize a weighted sum of the welfare of all the agents under the constraints of the economy. In the present economy, the only feasibility constraints that an allocation must satisfy are the aggregate endowment constraints for all the Arrow-Debreu states:

$$\sum_i \boldsymbol{x_i}(z) \leq \boldsymbol{\mathcal{E}}(t(z)), \quad \forall\ z. \tag{2.1}$$

Definition 3 *A feasible allocation x is Pareto optimal if there exists a positive μ_i for each agent i such that the allocation is the solution of the following concave programming problem with Z inequality constraints:*

$$\begin{aligned} \max_{\boldsymbol{x}} \ & \sum_i \mu_i \sum_z \pi(z) v_i(\boldsymbol{x_i}(z)) \\ s.t. \ & \sum_i \boldsymbol{x_i}(z) \leq \boldsymbol{\mathcal{E}}(t(z)), \quad \forall\ z. \end{aligned} \tag{2.2}$$

Proposition 6 *A feasible allocation $\boldsymbol{x}$ is Pareto optimal if and only if there exist a positive μ_i for each agent i and a positive vector $\boldsymbol{\lambda}(z)$ of dimension C for each Arrow-Debreu state z such that:*

$$\mu_i \boldsymbol{\nabla} v_i(\boldsymbol{x_i}(z)) = \boldsymbol{\lambda}(z), \quad \forall\ i, z, \tag{2.3}$$

$$\sum_i \boldsymbol{x_i}(z) = \boldsymbol{\mathcal{E}}(t(z)), \quad \forall\ z, \tag{2.4}$$

in which $\boldsymbol{\nabla} v_i(\boldsymbol{x_i}(z))$ is the vector of the C derivatives of $v_i(\boldsymbol{x_i}(z))$ relative to each commodity.

Proposition 6 is the classic equivalence for the characterization of Pareto optimal allocations, obtained with the first order conditions of (2.2) (with $\pi(z)\boldsymbol{\lambda}(z)$ the positive Lagrangian multiplier of constraint z) and the binding constraints of (2.2) (because v_i is strictly increasing in each argument). With this notation, $\boldsymbol{\lambda}(z)$ and $\pi(z)\boldsymbol{\lambda}(z)$ can also be called the vector of shadow spot prices and the vector of shadow contingent prices. (2.3) tells that, in a Pareto optimal allocation, the marginal rate of substitution between any two Arrow-Debreu states for one commodity is equal for each agent to the ratio of Lagrangian multipliers of the two Arrow-Debreu states for this commodity, which means that it is identical across agents.

Besides, one can note that the optimal allocation obtained from problem (2.2) consists actually in separated optimal allocation problems for the different Arrow-Debreu states. It is thus separable in Z programming problems which are for $z = 1, .., Z$:

$$\begin{aligned} \max_{\boldsymbol{x}(z)} \ & \sum_i \mu_i v_i(\boldsymbol{x_i}(z)) \\ \text{s.t.} \ & \sum_i \boldsymbol{x_i}(z) = \boldsymbol{\mathcal{E}}(t(z)). \end{aligned} \tag{2.5}$$

Proposition 7 *(Borch (1960, 1962))*[6] *In a Pareto optimal allocation, the consumption plans $\boldsymbol{x}(z)$ and the shadow spot prices $\boldsymbol{\lambda}(z)$ are identical across Arrow-Debreu states z corresponding to the same fundamental state t.*

[6] Borch (1960, 1962) demonstrates in a model with one commodity that individual risks are eliminated and the aggregate risk is shared in a Pareto optimal allocation. Arrow (1996) confirms in the case of many commodities that "the allocation of consumption in any given state (i.e. in any Arrow-Debreu state) depends only on the total in that state (i.e. on the fundamental state)". Gollier (2004) has some more insight on this mutuality principle in Chapter 21 of his book.

Proposition 7 is due to the fact that the programming problem (2.5) is identical for all the Arrow-Debreu states z corresponding to the same fundamental state t because they have the same aggregate endowments $\boldsymbol{\mathcal{E}}(t(z))$. That is why the consumption plans $\boldsymbol{x}(z)$ are identical for all the Arrow-Debreu states z corresponding to the same fundamental state t and can be denoted $\tilde{\boldsymbol{x}}(t)$. Moreover, thanks to (2.3), the vectors $\boldsymbol{\lambda}(z)$ are identical for all the Arrow-Debreu states z corresponding to the same fundamental state t and can be denoted $\tilde{\boldsymbol{\lambda}}(t)$. Proposition 7 states that the individual risks are fully eliminated in each fundamental state (i.e. one agent gets the same quantity of commodity in two Arrow-Debreu states corresponding to the same fundamental state). This result firstly shown by (Borch (1960, 1962)) is usually called the Borch mutuality principle. Besides, (2.3) tells that the increase of a component of $\tilde{\boldsymbol{\lambda}}(t)$ leads to the decrease of consumption of the corresponding commodity by all the agents because $v_i(.)$ is concave. This means that, if two fundamental states have identical $\tilde{\boldsymbol{\lambda}}(t)$ except for one commodity, the fundamental state that has a higher shadow spot price for this commodity necessarily has a lower aggregate endowment of this commodity. Thus, aggregate endowment and shadow spot price for each commodity are inversely related and aggregate endowment uncertainty is translated into shadow spot price uncertainty.

2.3.2 Decentralized equilibrium à la Arrow-Debreu

I now consider a decentralized economy equipped with a contingent market for each commodity and each Arrow-Debreu state (i.e. a financial asset for each commodity in each Arrow-Debreu state). A contingent market for one commodity and one Arrow-Debreu state corresponds to the market for this commodity contingent on this Arrow-Debreu state before the Arrow-Debreu state has been revealed. Markets are considered frictionless. In this economy, all the deals are made before the Arrow-Debreu state has been revealed. The price vector for commodities contingent on the Arrow-Debreu state z is denoted $\pi(z)\boldsymbol{p}(z)$. At equilibrium, each agent i maximizes her utility level under her (binding) budget constraint and the market conditions clear:

$$\begin{aligned} \max_{\boldsymbol{x_i}} \ & \sum_z \pi(z) v_i(\boldsymbol{x_i}(z)) \\ \text{s.t.} \ & \sum_z \pi(z) \boldsymbol{p}(z) \boldsymbol{x_i}(z) = \sum_z \pi(z) \boldsymbol{p}(z) \boldsymbol{e_i}(s_i(z)), \end{aligned} \tag{2.6}$$

$$\sum_i \boldsymbol{x_i}(z) = \boldsymbol{\mathcal{E}}(t(z)), \quad \forall\, z. \tag{2.7}$$

Proposition 8 *(Arrow (1953, 1964) and Debreu (1959))*[7] *In the decentralized economy with a contingent market for each commodity and each Arrow-Debreu state ($C * Z$ contingent markets), the allocation is Pareto optimal at equilibrium.*

Proposition 8 is obtained with proposition 6. By denoting $\frac{1}{\nu_i}$ the positive Lagrangian multiplier of the constraint in problem (2.6), the consumption plan $\boldsymbol{x_i}$ is such that the first order conditions of (2.6) are verified:

$$\nu_i \boldsymbol{\nabla} v_i(\boldsymbol{x_i}(z)) = \boldsymbol{p}(z), \quad \forall\, z. \tag{2.8}$$

Thus, at equilibrium, there exist a positive μ_i for each i (i.e. ν_i) and a positive vector $\boldsymbol{\lambda}(z)$ for each z (i.e. $\boldsymbol{p}(z)$) such that (2.3) and (2.4) are verified, which gives proposition 8. Proposition 8 is an application of the first welfare theorem, firstly shown by Arrow and Debreu. They have also shown that Pareto optimality is reached with a spot market for each commodity and a security market for each Arrow-Debreu state (i.e. a financial asset for each Arrow-Debreu state) ($C + Z$ markets), instead $C * Z$ contingent markets.[8] Thanks to proposition 7, proposition (8) tells that the consumption plans $\boldsymbol{x}(z)$ and the spot prices $\boldsymbol{p}(z)$ are identical across Arrow-Debreu states corresponding to the same fundamental state. Either with $C * Z$ contingent markets or with C spot markets and Z security markets,

[7] Arrow (1953, 1964) and Debreu (1959) have firstly shown this result. Gollier (2004) has some more insight on the decentralized equilibrium with these types of markets in Chapter 22 of his book.

[8] A spot market for one commodity corresponds to the commodity market once the Arrow-Debreu state has been revealed and in this case the spot market prices correspond to $\boldsymbol{p}(z)$. A security associated to one Arrow-Debreu state enables to get 1 money unit if this Arrow-Debreu state is revealed in exchange for $\pi(z)$ money unit ex-ante.

the Arrow-Debreu economy requires at least one financial asset for each state of nature, characterized by a full specification of the individual endowments obtained by all the agents in the economy, which is an unrealistic setting. The first limit is that it supposes an incommensurable number of financial assets ($C * Z$ or $C + Z$) in the real world to hedge all the individual risks. Indeed, in an economy with $N = 7{*}10^9$ people, each one facing $S = 10$ individual states, the required number of financial assets is prohibitive with $Z = 10^{7*10^9}$. The second limit is that it requires to make public the realized individual endowments of each agent in the economy. Indeed, exchanges made on these markets aim at securing transfers ex-post which are contingent on the realized Arrow-Debreu state and thus necessitate to make public the realized individual state of each agent in the economy. Besides, as explained by Arrow (1996), agents buy risk sharing contracts to insure against the uncertainty of their individual endowments and not against the uncertainty of other agent endowments. This corresponds to more usual insurance contracts between two agents, in which one exposed agent buys coverage against her individual risk from another agent playing the role of an insurance company, as introduced in the following section.

2.4 Decentralized equilibrium with insurance companies

2.4.1 The setting

I now consider a decentralized economy equipped with a spot market for each commodity and a security market for each fundamental state (i.e. a financial asset for each fundamental state). Moreover, there are M insurance companies in competition, which supply standard insurance contracts and are shared through stock markets. All markets are frictionless. Spot markets correspond to the commodity markets once the Arrow-Debreu state has been revealed. In the Arrow-Debreu state z, the price vector for the C commodities is denoted $\boldsymbol{p}(z)$. With these spot markets, agents cannot transfer wealth from one Arrow-Debreu state to another because the deals are made after the Arrow-Debreu state has been revealed. Sells and purchases of securities, as well as purchases of insurance contracts and insurer shares, enable these transfers. I consider a security market for each of the T fundamental states. The security associated to the fundamental state t gives 1 money unit in all the Arrow-Debreu states z corresponding to t, in exchange for $\sum_{z \in F_t} \pi(z) = \tilde{\pi}(t)$ money unit before the state of nature z is revealed. The quantity of securities "purchased" by agent i is denoted $a_i = (a_i(1), .., a_i(T))$.[9] I consider M insurance companies in competition. They supply fair multi-risk standard insurance contracts. The insurance contract supplied to agent i consists in compensating her in any state s_i for her loss $\boldsymbol{l_i}(s_i)$ (i.e. an indemnity $\tau_i(\boldsymbol{p}(z), s_i(z)) = \boldsymbol{p}(z)\boldsymbol{l_i}(s_i(z))$ in any state $z \in Z$), in exchange for a premium α_i paid before the state of nature z has been revealed. The quantity of insurance purchased by agent i is denoted n_i.[10] This is an insurance contract in the sense that the contract for one agent does not depend on the individual states of other agents (i.e. on the Arrow-Debreu state) but on her individual state s_i and on aggregate data (i.e. commodity price vector $\boldsymbol{p}(z)$). Thanks to competition and no transaction costs, the insurance contract is fair in the sense that the premium denoted α_i is equal to the expected indemnity ($\alpha_i = \sum_{z'} \pi(z')\boldsymbol{p}(z')\boldsymbol{l_i}(s_i(z'))$). The insurance contract is multi-risk in the sense that it covers for the different individual risks to which the agent is exposed.[11] The insurance contract is standard and not mutual in the sense that the premium is paid ex-ante and there is no participation on the insurer profit in the contract. Each insurance company $k \in M$ invests the premiums raised $\sum_{j \in N_k} \alpha_j n_j$ in financial assets, in which N_k is the group of agents purchasing a contract from the insurance company k. The quantity of securities purchased by insurer k is denoted $b_k = (b_k(1), .., b_k(T))$ and thus $\sum_t \tilde{\pi}(t) b_k(t) = \sum_{j \in N_k} \alpha_j n_j$. Insurance companies are owned through stock markets. The profit of insurer $k \in M$

[9] $a_i(t)$ can be either positive or negative. A positive $a_i(t)$ corresponds to a security purchase. A negative $a_i(t)$ corresponds to a security sell.

[10] n_i can take any positive value. For instance, $n_i = 0$ corresponds to zero coverage, $n_i = 1$ corresponds to full coverage and a value strictly between 0 and 1 corresponds to partial coverage.

[11] Different contracts for different risks could also be offered to each agent, but one multi-risk contract per agent is sufficient.

in state z for the shareholders is $r_k(z) = b_k(t(z)) - \sum_{j \in N_k} \tau_j(\boldsymbol{p}(z), s_j(z))n_j$. The share of the insurance company k purchased by agent i is denoted m_{ki}.[12]

2.4.2 The equilibrium

At equilibrium, each agent i maximizes her utility level under her (binding) budget constraints (before and after the state of nature is revealed), plus the clearing market conditions (for spot markets, security markets and stock markets):

$$\begin{aligned} \max_{\boldsymbol{x_i}, n_i, m_{ki}, a_i} \quad & \sum_z \pi(z) v_i(x_i(z)) \\ \text{s.t. } & \boldsymbol{p}(z)\boldsymbol{x_i}(z) = \boldsymbol{p}(z)\boldsymbol{e_i}(s_i(z)) + \tau_i(\boldsymbol{p}(z), s_i(z))n_i + \sum_k r_k(z)m_{ki} + a_i(t(z)), \quad \forall\ z \\ & \alpha_i n_i + \sum_t \tilde{\pi}(t)a_i(t) = 0 \end{aligned} \tag{2.9}$$

$$\sum_i \boldsymbol{x_i}(z) = \boldsymbol{\mathcal{E}}(t(z)), \quad \forall\ z \in Z \tag{2.10}$$

$$\sum_i a_i(t) + \sum_k b_k(t) = 0, \quad \forall\ t \in T \tag{2.11}$$

$$\sum_i m_{ki} = 1, \quad \forall\ k \in M \tag{2.12}$$

Proposition 9 *In the decentralized economy with C spot markets for commodities, T financial assets associated to fundamental states and stock insurance companies in competition, the allocation is Pareto optimal at equilibrium.*

Proof We show the equivalence in the consumption plan $\boldsymbol{x}$ between the Arrow-Debreu economy as given by (2.6) and (2.7) and the economy with insurance companies:

[12] I do not model the managers of insurance companies because, with multiple insurance companies in competition and frictionless markets, they cannot extract any rent. Besides, note that m_{ki} cannot be negative.

(i) Suppose first that $\boldsymbol{x}$ and $\boldsymbol{p}$ are the solutions of (2.6) and (2.7). Thus, (2.10) is verified. By choosing $n_i = 1$, $m_{ki} = \frac{1}{N}$, $b_k(t) = \sum_{j \in N_k} \alpha_j n_j$ and $a_i(t(z)) = \boldsymbol{p}(z)(\boldsymbol{x_i}(z) - \overline{\boldsymbol{e_i}}) - \frac{1}{N} \sum_k r_k(z)$ (which is possible for $a_i(t(z))$ because $\boldsymbol{p}(z)$, $\boldsymbol{x_i}(z)$ and $\sum_k r_k(z)$ are identical across z corresponding to the same t in this case), the first constraint in (2.9) is verified and (2.12) is verified. Then, the first constraint in (2.9) with the constraint in (2.6) implies that the second constraint in (2.9) is verified. Finally, the first constraint in (2.9) with (2.10) and (2.12) implies that (2.11) is verified.

(ii) Suppose now that $\boldsymbol{x}$ and $\boldsymbol{p}$ are the solutions of (2.9), (2.10), (2.11) and (2.12). Thus, (2.7) is verified and the constraint in (2.6) is verified thanks to the constraints in (2.9).

The equivalence between the two economies finally tells us that the allocation in the economy with insurance companies is Pareto optimal at equilibrium, just as in the Arrow-Debreu economy.

2.4.3 The role of insurance companies and financial assets

Proposition 9 tells us that the decentralized economy with insurance companies in competition allows to reach a Pareto optimal allocation. The proof of proposition 9 shows that the consumption plan $\boldsymbol{x_i}$ chosen by each agent i is the same as the one that is chosen in the Arrow-Debreu economy. It also tells that one way to reach this consumption plan for each agent i is to purchase full insurance coverage ($n_i = 1$), an ownership share of insurance companies ($m_{ki} = \frac{1}{N}$) and some amount of securities ($a_i(t) = \boldsymbol{p}(z)(\boldsymbol{x_i}(z) - \overline{\boldsymbol{e_i}}) - \frac{1}{N} \sum_k r_k(z)$ which may be positive (i.e. purchase) or negative (i.e. sell)), if the insurers invest safely the premiums raised ($b_k(t) = \sum_{j \in N_k} \alpha_j n_j$ independent from t). The purchase of full insurance coverage ($n_i = 1$) enables agent i to get in the Arrow-Debreu state z the wealth $\boldsymbol{p}(z)\overline{\boldsymbol{e_i}} - \sum_{z'} \pi(z')\boldsymbol{p}(z')\boldsymbol{l_i}(s_i(z'))$ instead of the endowment value $\boldsymbol{p}(z)(\overline{\boldsymbol{e_i}} - \boldsymbol{l_i}(s_i(z)))$. Contrary to the endowment value, the wealth with full insurance coverage depends only on the fundamental state (because $\boldsymbol{p}(z)$ is identical across z corresponding to the same fundamental state t). Thus, insurance allows each agent to fully eliminate individual risks within each fundamental state. Besides, individual endowment risks generate aggregate endowment risk characterized by the multiplicity of fundamental states. The aggregate risk materializes for agents through commodity price risk, which affects the agent wealth with full insurance coverage ($\boldsymbol{p}(z)\overline{\boldsymbol{e_i}} - \sum_{z'} \pi(z')\boldsymbol{p}(z')\boldsymbol{l_i}(s_i(z'))$). For instance, the wealth of an agent, who is endowed with only one type of commodity, is very dependent on the price of this commodity. This agent may want to protect against price risk to be able to consume other commodities when the commodity price and thus her wealth are low. By buying or selling some securities ($\boldsymbol{p}(z)(\boldsymbol{x_i}(z) - \overline{\boldsymbol{e_i}})$ in $a_i(t)$), agent i is protected against the price risk in order to reach the targeted consumption plan $\boldsymbol{x_i}$. Thus, securities allow each agent to hedge the price risk.[13] In her targeted consumption plan, each agent chooses a fraction of the aggregate risk thanks to shares in stock insurance companies and sells/purchases of securities. The share of the aggregate risk targeted by agent i depend on her risk aversion relative to others: the higher the risk aversion relative to others, the lower the purchased share of the aggregate risk.

2.4.4 Insurance prices

Insurance companies have to sell fair contracts to catch policyholders on one side and shareholders on the other side because of competition. On the one hand, competition between insurance companies lead them to decrease their prices at least until fair prices to catch some policyholders. On the other hand, they cannot decrease them below fair prices, otherwise they would not have enough shareholders for the insurance stock market to be at equilibrium. As insurance for individual risks is fairly priced, agents have incentives to purchase full coverage for their individual endowment losses ($n_i = 1$). For agent i, a full coverage insurance contract gives an indemnity corresponding to the individual loss in any state $z \in Z$, in exchange for a premium equal to the expected indemnity before the state of nature has been revealed. The insurance premium $\alpha_i = \sum_{z'} \pi(z')\boldsymbol{p}(z')\boldsymbol{l_i}(s_i(z'))$ for agent

[13]The price risk could also be hedged with other security derivative financial products such as options.

i can also be written:

$$\alpha_i = (1+\gamma_i)\overline{\boldsymbol{p}}\sum_{z'}\pi(z')\boldsymbol{l_i}(s_i(z')), \tag{2.13}$$

in which $\overline{\boldsymbol{p}} = \sum_{z'}\pi(z')\boldsymbol{p}(z')$ is the average commodity price vector and γ_i is an aggregate risk loading factor such that:

$$\gamma_i = \frac{1}{\overline{\boldsymbol{p}}\sum_{z'}\pi(z')\boldsymbol{l_i}(s_i(z'))}\sum_{z'}\pi(z')(\boldsymbol{p}(z')-\overline{\boldsymbol{p}})\boldsymbol{l_i}(s_i(z')). \tag{2.14}$$

(2.13) gives an expression for the insurance premium, in which the loading factor γ_i expressed in (2.14) captures the correlation between the individual risk (expressed in the uncertainty of $\boldsymbol{l_i}(s_i(z'))$) and the aggregate risk (expressed in the uncertainty of $\boldsymbol{p}(z')-\overline{\boldsymbol{p}}$). For a given loading factor, the higher the individual risk, the higher the premium. For a given individual risk, the higher the correlation with the aggregate risk, the higher the correlation with the commodity price risk and the higher the premium.[14] Contrary to the cases with independently and identically distributed risks, the premium and the loading factor are not uniform across agents. Bad fundamental states with high aggregate losses are characterized by higher prices than good fundamental states with low aggregate losses. The loading factor is positive (respectively negative) if the agent individual risk is positively (respectively negatively) correlated with the price risk. With some aggregate risk, the factor is positive for the agents facing higher losses in bad fundamental states than in good fundamental states, whereas it is negative for the agents facing the opposite. Without any aggregate risk, the factor is null for any agent because $\boldsymbol{p}(z') = \overline{\boldsymbol{p}}$ for any z'. Shareholders of an insurance company are exposed to the risk resulting from the pool of individual risks insured by the company. The higher the correlation between these risks and the aggregate risk, the higher the raised premiums. Shareholders raise higher premiums in this case at equilibrium because they cannot diversify a higher part of the company risk. Lastly, as the aggregate

[14] If agents were able to reduce their risk through prevention expenses, they would have incentives through insurance premiums to reduce both their individual risks and their correlation with the aggregate risk. In other words, insurance premiums would give individual incentives to reduce both the aggregate endowment losses and the aggregate endowment uncertainty.

risk results from the aggregation of individual risks, there are globally more individual endowment risks positively correlated with the aggregate risk, which means that the average loading factor in the economy is positive.

2.5 Conclusion

I extend the analysis of the role played by insurance companies to an economy with multiple commodities and agents having heterogeneous preferences and facing heterogeneous risks. With competitive insurance companies that supply fair multi-risk insurance contracts and are owned through stock markets, Pareto optimality is reached with a reduced number of financial assets relative to the Arrow-Debreu setting. Instead of one financial asset per Arrow-Debreu state (characterized by a full specification of the individual endowments of all the agents in the economy), it is sufficient to have only one financial asset per fundamental state (characterized by the aggregate endowments in the economy). The higher the risk faced by one agent or the higher the correlation between her risk and the aggregate risk, the higher her premium. Correlation between individual risks does not prevent individual risks to be fully insured, even though it can generate aggregate risk which is not insurable. Even insurance companies weakly diversified are able to sell contracts at fair prices, because the shareholders diversify their risk with other insurance stocks and securities. However, this result is valid only without market failures. In particular, agents have to be liable for insurance company shares they purchase and securities they sell, which means that they have to bear insurance company deficits if some occur and to pay amounts of securities they owe in any state of nature. In the case of asymmetric information on the capacity of agents to honor these transactions, public policies require agents to build financial reserve in order to limit default. Yet financial reserves are costly to build for instance because of the opportunity cost. In this case, risk correlation becomes an issue for insurance companies because it leads to more variability on total claims and thus requires more financial reserves, as it appears for natural disaster risks.

Chapter 3

Insurability of low-probability catastrophic risks

This chapter is co-authored with Alexis Louaas.

Abstract: We analyze how the risk probability affects the insurance purchase of rational and well-informed risk-averse individuals. With standard insurance costs and competitive pricing, we show that agents are more inclined to insure for low-probability risks than for high-probability risks. Yet, these observations are at odds with the low insurance take-up rates for low-probability catastrophic risks (e.g. earthquakes and floods). Our explanation is that the risks for which underinsurance is most prevalent display substantial aggregate uncertainty. This uncertainty generates an additional fixed cost for insurers, which increases the insurance loading factor when the loss probability decreases and eventually discourages people from purchasing coverage.

Keywords: low-probability risks, catastrophic risks, insurance, reserve.

JEL classification: D86, G22, G28, Q54.

3.1 Introduction

Why are catastrophic events so difficult to insure? Natural and man-made catastrophes such as earthquakes, floods, nuclear accidents and terrorists attacks have severe consequences at the individual level, affect large numbers of people at the same time, but have small individual probabilities of occurrence. The extreme severity of these risks advocates for a wide-spread use of prevention and insurance. Yet, they are typically excluded from US homeowner policies.[1] Private insurance markets are often non-existant and even when specific contracts benefiting from public subsidies are available, relatively few people purchase them.[2] It is also puzzling to notice that some low-probability risks such as damages from lightnings are efficiently handled by the insurance sector and covered under the US standard homeowner policies whereas others, such as earthquakes or floods, are not. Our model explains this puzzle by showing that the effect of aggregate uncertainty, a well-known threat to insurability, is amplified when the individual loss probability is low. Low-probability risks characterized by aggregate uncertainty, such as earthquakes and floods, are therefore predicted to feature lower take-up rates than low-probability risks without aggregate uncertainty.

We begin by examining the case without aggregate uncertainty. In this framework, we show that low-probability risks are actually easier to insure than high-probability risks. It is known since Mossin (1968) that expected utility maximizing agents optimally purchase partial coverage when insurance is sold above actuarially fair price. In this case, a decrease in the loss probability has two effects on the demand for coverage. On the one hand, the cost of providing insurance diminishes, which translates into lower premiums for policyholders. On the other hand, the likelihood of receiving the indemnity declines as well. Our contribution in this framework is to show that the cost-reduction effect dominates the likelihood-reduction effect under risk aversion. As the loss probability declines, the ratio of

[1] http://www.iii.org/article/which-disasters-are-covered-by-homeowners-insurance

[2] Kousky & Kunreuther (2014) document low take-up rates for catastrophe insurance while Cole et al. (2014) try to explain low rates in the micro-insurance industry.

willingness to pay to cost of coverage always increases. In a population heterogeneous in wealth or preferences, take-up rates are therefore predicted to increase as the loss probability decreases.[3] In addition, we show that given the opportunity to do so, people optimally choose higher levels of coverage for low-probability risks if their index of absolute risk aversion does not decrease too fast with wealth. These results are in line with Laury et al. (2009), who experiment in the laboratory the effect of a change in the loss probability. A previous experiment by McClelland et al. (1993) also found a decreasing mean ratio of willingness to pay to expected indemnity, indicating that people tend to be willing to pay higher loadings for low-probability risks than for high-probability risks.

These observations, however, are at odds with the low insurance take-up rates for low-probability catastrophic risks. Our explanation is that the risks for which underinsurance is most prevalent display substantial aggregate uncertainty. Natural and man-made catastrophes feature geographically correlated individual losses which translates into aggregate loss uncertainty, even within very large pools of policyholders. In order to remain solvent, the insurance provider must either raise prohibitively high levels of premiums or more realistically, it must have access to capital to fill the gap between the premiums raised and the amount of claims due in case of catastrophe. The cost of allocating this capital to a specific line of business[4] depends on the size of the potential worst case aggregate loss. This fixed cost causes the loading to increase when the loss probability decreases and eventually discourages people from purchasing coverage. In a world where agents are heterogeneous in wealth or preferences, a decrease in probability may therefore induce lower take-up rates. This prediction is much more in line with the observations of the take-up rates in various settings.

Several literatures have attempted to explain the low insurance take-up rates for low-probability catastrophic risks. Kunreuther & Slovic (1978) and Kunreuther et al. (2001) rely on departures from the expected utility paradigm, arguing that

[3]With the exception of the pathological case of Giffen behaviors which, to our knowledge have never been observed in an insurance market.

[4]See Zanjani (2002) and Froot (2001).

low-probabilities are more difficult to process than high probabilities. In the expected utility framework, Kunreuther et al. (2001) show that search costs may generate a probability threshold below which coverage is not purchased while Coate (1995) shows how government relief can crowd-out the demand for insurance. Raschky et al. (2013) and Kousky et al. (2013) find empirical support for this hypothesis in various countries. In the context of micro-insurance markets, Cole et al. (2014) suggests that learning is an important determinant of the demand for insurance. Overall, much less attention has been devoted to the supply side of the market. Kunreuther et al. (1995) explain that ambiguity may provide a rationale for high disaster insurance premiums. In the context of micro-insurance markets, Mobarak & Rosenzweig (2013) argue that basis risk can explain low take-up rates. Finally, Jaffee & Russell (1997) and Kousky & Cooke (2012) suggest that the low take-up rates are due to the high premiums required by insurers that have to secure costly capital for catastrophic events.

Our contribution to the literature is twofold. First, we provide a simple, yet general framework, that provides an alternative explanation to the observed low take-up rates for disaster risks.[5] Second, we explain this apparently puzzling observation that some low-probability events are well insured while others are not. It is in fact the combination of aggregate loss uncertainty with low-probability that makes earthquakes, floods and terrorism risks difficult to insure.

3.2 A model with aggregate loss uncertainty

This section lays out the framework for the analysis of the insurability of low-probability events. Our model permits the analysis of correlated risks and encompasses the standard independent losses model as a particular case.

We consider a population of unit mass, in which each individual may or may

[5]Jaffee & Russell (1997) discuss the problem of capital allocation cost but provide no formal model while Kousky & Cooke (2012) have a simulated model, calibrated to analyze flood insurance coverage in Broward County, Florida.

not be affected by a loss of size L. Similarly to Charpentier & Le Maux (2014), the share of the population affected by a loss is a random variable $\tilde{q}$. It is distributed with cumulative distribution function $F(.)$ defined and increasing on $[0,1]$, with expected value p and variance σ^2. From the insurer's standpoint, there is therefore a continuum of states of the world, that we call aggregate states, characterized by a fraction q of the population being affected by a loss. Notice that almost all lines in the insurance business feature some kind of aggregate loss uncertainty. Finding the distribution of the number of claims and therefore of the fraction of the policyholders who will file a claim, constitutes an important task of actuaries.

Conditionally on the occurrence of a particular aggregate state of the world, losses are independently distributed across people. Calling $\tilde{x}_i$ the random loss of agent i, the probability that an individual faces a loss is therefore p.[6] The individual loss is therefore a Bernouilli random variable with parameter p equal to the expected value of $\tilde{q}$. Individual losses are in general not independent random variables. We show in Appendix (3.6.1) that the coefficient of correlation between two individual risks can be written $\delta = \frac{\sigma^2}{p(1-p)} \leq 1$. Given a fixed expected value p, increasing the variance of the distribution F always increases the correlation coefficient. For a given σ^2, a decrease in p results in an increase in δ if $p < 1/2$.

This model of loss exposure nests the usual independent losses model as a specific case in which the fraction q is known. In this case, the probability cumulative distribution function $F(.)$ is such that $F(x) = 0$ for $x \leq q$ and $F(x) = 1$ for $x \geq q$. Individual losses are usual Bernoulli random variables with parameter $p = q$ and since $\sigma^2 = 0$, the coefficient of correlation δ is null.

Finally, we consider that agents are risk-averse with a twice continuously differentiable and concave utility function $u(x)$ and initial wealth w. $A(x) = -\frac{u''(x)}{u'(x)}$ denotes the Arrow-Pratt index of absolute risk aversion.

[6] The probability that an individual faces a loss is $\int_0^1 \mathbb{P}(\tilde{x}_i = L|\tilde{q} = q)dF(q) = \int_0^1 q dF(q) = p$.

3.3 Insurability without aggregate uncertainty

In this section, we show that in the baseline insurance framework with independent losses, low-probability events are in fact more likely to be insured than higher probability events. The intuition behind this result is that individual's willingness to pay is concave in the probability, while the cheapest feasible contract is linear. Therefore, if a risk is insurable somewhere over the parameter space, there is a probability threshold below which it is insurable and above which it is not.

The insurance provider can be private or public. It is represented by a risk-neutral agents that sells an amount of coverage $\tau \in [0, L]$ at a premium α. Following Raviv (1979) we assume that, in addition to the payment of the indemnity τ, the insurance provider faces a cost $c(\tau)$ with:

$$c(0) = 0, \quad c'(0) = b > 0, \quad c'(\tau) > 0 \quad \text{and} \quad c''(\tau) \geq 0,$$

which represents the various expenses associated with the payment of an indemnity τ to all affected agents. It can be interpreted as an administrative cost, as a cost of expertise, or more broadly as a dead-weight loss, resulting either from an asymmetry of information between the insurance company and the policy holder, or by imperfect competition. The actuarial and insurance literatures often make the simplifying assumption that the marginal cost $c'(\tau)$ is equal to a constant λ called the loading factor. In this case, the dead-weight cost is simply a fraction of the indemnity. We call loading the ratio $\alpha/p\tau$ premium to expected indemnity. If $c'(\tau) = \lambda$, the loading is $1 + \lambda$ and we have $c''(\tau) = 0$, which is indeed a particular case of our model. A necessary condition for the insurance to provide such a contract is that its expected profit is positive. This motivates the following definition.

Definition 4 *A contract is called feasible if and only if the insurance provider can realize at least a zero expected profit.*

In the absence of aggregate loss uncertainty, a contract is therefore feasible if and only if:

$$\alpha \geq p\tau + pc(\tau).$$

3.3.1 Strong insurability

The first notion of insurability that we develop is the following.

Definition 5 *A risk is strongly insurable at a level τ if and only if individuals are willing to purchase the proposed level of coverage τ of some feasible contract.*

In order to know whether a risk is strongly insurable or not, it is sufficient to verify that individuals are willing to purchase the zero expected profit contract, for which the insurer breaks even. If they reject the zero expected profit feasible contract, agents will also reject all the other more expansive contracts, and if they accept the zero expected profit feasible contract, then the risk is insurable. In the remaining of the paper, we use the word contract to mean zero expected profit contract.

Independently of the supply side constraints, the highest price $C(p,\tau)$ that an individual would pay for a level of coverage τ is given by:

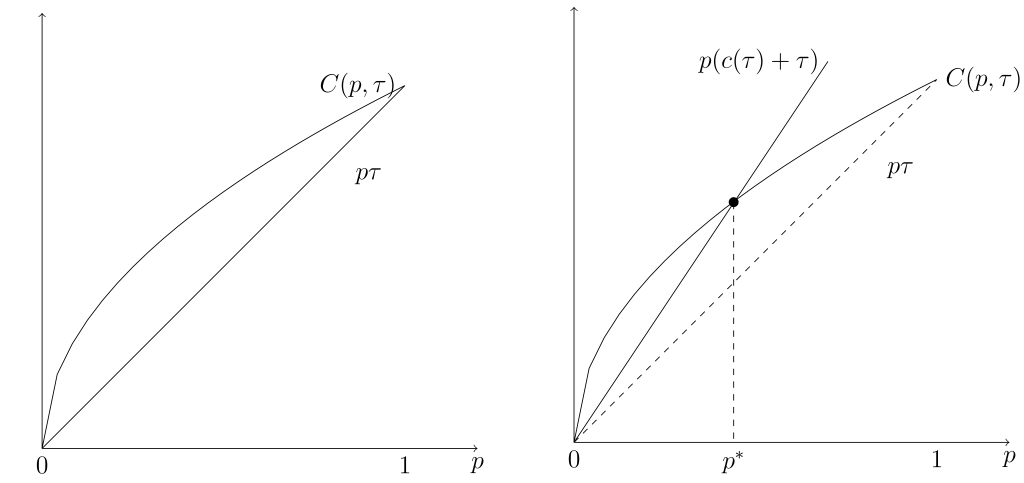
$$pu(w-L+\tau-C)+(1-p)u(w-C)=pu(w-L)+(1-p)u(w). \tag{3.1}$$

First notice that $C(0,\tau)=0$ and $C(1,\tau)=\tau$. The willingness to pay for the coverage of a a zero probability event is zero and the willingness to pay for the coverage of a sure event is just the coverage itself. Total differentiation of (3.1) gives:

$$C'_p(p,\tau)=\frac{u(w)-u(w-L)-[u(w-C)-u(w-L+\tau-C)]}{pu'(w-L+\tau-C)+(1-p)u'(w-C)}\geq 0 \quad \forall\tau\leq L,$$

$$\begin{aligned}C''_{pp}(p,\tau)= & - \quad 2C'_p\frac{u'(w-L+\tau-C)-u'(w-C)}{pu'(w-L+\tau-C)+(1-p)u'(w-C)} \\ & + \quad (C'_p)^2\frac{pu''(w-L+\tau-C)+(1-p)u''(w-C)}{pu'(w-L+\tau-C)+(1-p)u'(w-C)}\leq 0 \quad \forall\tau\leq L,\end{aligned}$$

where C'_p and C''_{pp} represent the first and second order partial derivatives with respect to p. The agent's willingness to pay C is therefore increasing and concave in the probability of loss p.



(a) Actuarially fair coverage (b) Non-actuarially fair coverage

Figure 3.1: Coverage without aggregate loss uncertainty

Figure 3.1a represents the agent's willingness to pay and the cost of coverage as a function of the loss probability p for the case $c(\tau) = 0$. When coverage is sold at an actuarially fair price, the agent is always willing to purchase it. The surplus she derives from the transaction however, may vary with the loss probability. Figure 3.1b represents the case where coverage is available at a cost higher than the actuarially fair price. For values of p lower than a threshold p^*, willingness to pay is above the price of coverage. An insurance market should therefore emerge in this case. For values of p higher than p^*, willingness to pay is below the price of the zero expected profit contract, resulting in the absence of any market.

Proposition 10 *In the absence of aggregate loss uncertainty, a risk is strongly insurable at level τ if and only if the individual probability of loss p is below a threshold p^*, where p^* is such that:*

$$C(p^*, \tau) = p^*\tau + p^*c(\tau).$$

For a given level of coverage, low-probability events are therefore more likely to be covered than high-probability events. Laury et al. (2009) investigate in an experiment how insurance purchase decisions evolve with p. They observe that the fraction of their sample that purchases full-coverage decreases with the probability of loss p, given a constant expected loss and loading. Proposition 10 confirms that this should indeed be the case but delivers an even stronger prediction: the fraction of a population that purchases coverage should increase as p decreases even when the loss L is fixed. To see this, assume that the population is heterogeneous in wealth and preferences. An agent i endowed with wealth w^i and utility function u^i purchases a feasible contract providing coverage τ if and only if $C_i(p,\tau) \geq p\tau + pc(\tau)$. Each individual therefore has a probability threshold p_i^* above which she stops purchasing insurance. The distribution of wealth and preferences generates a distribution G over the thresholds p_i^*. The fraction of the population that purchases insurance is $\mathbb{P}(p_i^* \geq p) = 1 - G(p_i^*)$, which is decreasing in p.

An insurance contract providing a level of coverage τ is therefore more likely to be purchased when the probability of loss p is smaller. In this sense, low-probability events are more insurable than higher-probability events. However, the exogeneity of the level of coverage τ may appear as a limit to this analysis. If people have control over the level of coverage, the decision is not a take-it-or-leave-it problem anymore. With this idea in mind, we propose the following notion of insurability.

3.3.2 Weak insurability

The second notion of insurability that we develop is the following.

Definition 6 *A risk is weakly insurable if and only if individuals are willing to purchase a positive amount of coverage of some feasible contract.*

This notion of insurability is weaker in the sense that a strongly insurable risk is necessarily weakly insurable. If a person agrees to purchase a level of coverage τ rather than no insurance when the loss probability is p, she would never optimally choose $\tau = 0$. However, she may select a level of coverage lower than τ to reduce the cost of insurance.

With endogenous coverage, the agent solves:

$$\max_{\tau} \quad pu(w - L + \tau - \alpha) + (1-p)u(w-\alpha)$$
$$\text{s.t.} \quad \alpha = p\tau + pc(\tau), \quad \tau \geq 0. \tag{3.2}$$

The first order condition of this problem is:

$$[1 - (1 + c'(\tau))p)]u'(w_1) = (1-p)(1 + c'(\tau))u'(w_2), \tag{3.3}$$

in which $w_1 = w - L + \tau - p\tau - pc(\tau)$ and $w_2 = w - p\tau - pc(\tau)$ are the levels of wealth in the loss and no-loss states. It is easy to check that $c'(\tau) > 0$ implies that the agent chooses partial coverage, so that $\tau < L$ at any interior solution. Individuals purchase a positive amount of coverage if and only if:

$$(1-p)(1+b)u'(w) < [1 - (1+b)p]u'(w - L). \tag{3.4}$$

Re-arranging the terms of (3.4) yields the following Proposition.

Proposition 11 *In the absence of aggregate loss uncertainty, a risk is weakly insurable if and only if its probability p is such that:*

$$p < \frac{1}{1+b} - \frac{b}{1+b}\frac{u'(w)}{u'(w-L) - u'(w)}. \tag{3.5}$$

For a given p, an event is uninsurable when the marginal cost of coverage $b = c'(0)$ is too high or the size of the loss L is not sufficiently large to generate a significant difference between marginal utility in the loss state $u'(w - L)$ and marginal utility in the no-loss state $u'(w)$. Proposition 11 stresses once more that, in the absence of aggregate loss uncertainty, low-probability events are easier to insure than high-probability events. In addition, proposition 12 and its corollary give conditions under which a strictly positive optimal coverage increases when the probability p decreases. When the loss probability diminishes, two effects interact. On the one hand, the risk of experiencing the loss L diminishes, lowering the incentive to pay the dead-weight cost. On the other hand, the dead-weight marginal cost $pc'(\tau)$ diminishes, making insurance more attractive at the margin. The total effect on optimal coverage cannot be signed for any utility function, but the following proposition and corollary enables to identify some interesting and realistic cases.

Proposition 12 *In the absence of aggregate loss uncertainty, the optimal coverage τ of a weakly insurable risk is strictly decreasing in p if and only if:*

$$A(w_1) - A(w_2) < \frac{c'(\tau)}{[\tau + c(\tau)](1-p)[1-(1+c'(\tau))p]}.$$

The proof of proposition 12 is given in Appendix 3.6.2. Note that the right-hand side of the inequality in proposition 12 is positive when $1-(1+c'(\tau))p > 0$, which is necessary to have an interior solution. Since $w_1 < w_2$, the condition in proposition 12 is trivially satisfied for any increasing or constant absolute risk aversion functions (IARA or CARA). For the class of decreasing absolute risk aversion (DARA), the condition puts an upper bound on the variation of risk aversion between the loss and the no-loss state. The empirical literature most often fails to reject DARA as a realistic hypothesis such as in Guiso & Paiella (2008) and Levy (1994). We would therefore like to know whether classical utility functions, satisfying the DARA property, also feature an optimal coverage decreasing in the probability p. This is the purpose of Corollary 12.1 that deals with the case of Harmonic Absolute Risk Aversion (HARA) functions. We define a HARA utility function as in Gollier (2004):

$$u(x) = \zeta\Big(\eta + \frac{x}{\gamma}\Big)^{1-\gamma},$$

whose domain is such that $\eta + \frac{x}{\gamma} > 0$ and the condition $\zeta\frac{1-\gamma}{\gamma} > 0$ guarantees that the function is indeed increasing and concave. The coefficient of absolute risk aversion is:

$$A(x) = \Big(\eta + \frac{x}{\gamma}\Big)^{-1}. \tag{3.6}$$

Except for the limit case $\gamma \to +\infty$, the HARA functions satisfy the DARA property when $\gamma > 0$, which makes them appealing with respect to the literature discussed previously.

Corollary 12.1 *If a risk-averse agent has preferences represented by a HARA utility function with $\gamma \geq 1$, then the optimal coverage τ of a weakly insurable risk is strictly decreasing in p in the absence of aggregate loss uncertainty.*

The proof of corollary 12.1 is given in Appendix 3.6.3. The HARA class nests two of the most widely used classes of utility functions. The Constant Absolute Risk Aversion (CARA) is obtained when $\gamma \to +\infty$. Solving the differential equation (3.6) yields the specification of this function:

$$u(x) = -\eta \exp\left(-\frac{1}{\eta}x\right). \tag{3.7}$$

The second class of functions within the HARA class is the set of Constant Relative Risk Aversion (CRRA) which is obtained for $\eta = 0$. Solving (3.6) in this case yields:

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}. \tag{3.8}$$

If people's preferences can be represented by a CARA or by a CRRA utility function with $\gamma \geq 1$, then their optimal coverage is always a decreasing function of p. For the CRRA case, Szpiro (1986) and Barsky et al. (1997) find values of relative risk aversion respectively between 1.2 and 1.8 for the first and 4.17 for the second. According to Gollier (2004) (p.69), the "range of acceptable values of relative risk aversion [is] $[1, 4]$". A complete survey of the literature on risk preferences elicitation would reveal some estimates below one, such as Chetty (2006), but overall our (sufficient but not necessary) condition $\gamma \geq 1$ seems a very plausible assumption.

Besides, Louaas & Picard (2014) have shown the following proposition.

Proposition 13 *In the absence of aggregate loss uncertainty and if* $u'(w - L) > (1+b)u'(w)$*, the optimal coverage converges, as* p *goes to* 0*, toward a positive limit* τ *such that:*

$$u'(w - L + \tau) = (1 + c'(\tau))u'(w).$$

It may sound surprising that the agent is willing to pay a positive loading factor for a risk whose probability tends to zero, but the pricing rule $\alpha = p\tau + pc(\tau)$ implies that the premium tends to zero with the probability. The agent therefore receives an indemnity with an infinitesimal probability whose price also becomes infinitesimal.

Finally, figure 3.2 gives an illustration of the previous results in the (w_1, w_2) space. Point A represents the optimal lottery at the highest probability p for which the insured chooses a positive amount of coverage. The thick curve represents the locus of optimal lotteries as p diminishes and Point B is the limit optimal lottery when $p \to 0$. In agreement with proposition 12, the optimal lottery gets closer from the 45 degree line as p decreases, until it (almost) reaches point B when $p \to 0$. Our theoretical results indicate that under reasonable assumption on individual's preferences, low-probability events should be more insured than higher probability events. Gollier (1997) shows that the deductible chosen by a risk averse agent is increasing in p when the agent's utility function features IARA and p is sufficiently small, which is equivalent[7] to purchase more insurance when the probability decreases. Our results are more general since they apply everywhere on the domain of p where the agent purchases a positive amount of coverage. In addition, we are able to accommodate some of the more realistic DARA utility functions.

Note that low-probability and high-stake risks are usually characterized by a large potential loss L. In order to compare insurance choice for these risks to choices concerning more traditional risks, it may be useful to fix the expected loss, as in Laury et al. (2009), so that the loss increases as its probability diminishes. In this case and independently of any assumption on individual's preferences, agents purchase more coverage as the probability of loss decreases.

Proposition 14 *In the absence of aggregate loss uncertainty, and if the expected loss pL remains constant, the optimal coverage is a decreasing function of the probability p for any risk averse individual.*

The proof of proposition 14 is given in Appendix 3.6.4. The requirement that expected loss be fixed makes it easier for the condition of proposition 12 to hold. Indeed as p diminishes, the loss becomes larger, providing agents with additional incentives to purchase insurance. This in turn reduces the gap between wealth in the non loss state w_1 and wealth in the loss state w_2.

[7] In a model where agents only face two states: loss or no loss, deductible and partial coverage are strictly equivalent.

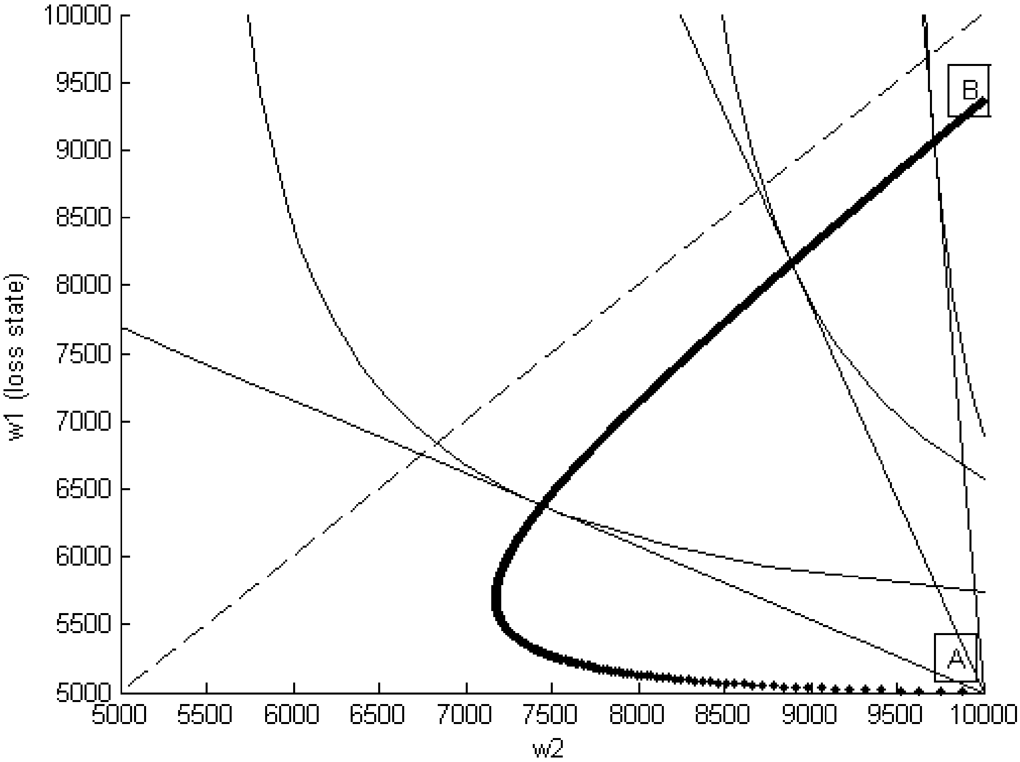


Figure 3.2: $w = 10000, L = 5000, u(x) = \frac{x^{-3}}{3}$

This section has shown that in the absence of aggregate loss uncertainty, low-probability risks are easier to insure than high-probability risks. This may explain why typical homeowners insurance cover perils as unlikely as lightening strikes. In contrast, low-probability correlated risks such as earthquakes, flooding, wind-storms, nuclear hazards and acts of wars are typically excluded from homeowner policies. The next section provides an explanation as to why low-probability correlated risks are also difficult to insure.

3.4 Insurability with aggregate uncertainty

This section considers the case of dependent risks. We relax the assumption that the fraction q of the population affected by the loss is known. Instead, $\tilde{q}$ is a random variable with cumulative distribution function $F(q)$, expected value p and variance σ^2. Because the insurance company raises the premiums ex-ante, the total amount of premium raised is unlikely to match the amount required to pay-off all claims. When it raises a premium α, it must therefore have access to an amount of capital per policy k such that:

$$k = \bar{q}[\tau + c(\tau)] - \alpha, \tag{3.9}$$

where $\bar{q}$ is the highest fraction of affected policyholders with a non-zero measure. This capital can be financed with an internal reserve of liquidities, with some form of reinsurance contract[8] or by borrowing.[9] In any case, the availability of this capital comes at a cost per monetary unit called r. When k is a reserve, r represents the foregone investment opportunities associated with having to keep the reserve. If the insurance company is private, r can be measured as the internal rate of return of the insurance company. If the insurance provider is public, r can be seen as the cost of public funds. When k is a reinsurance or an alternative risk transfer contract, r is simply the rate of return paid to the seller of the contract. The zero expected profit feasible contract is therefore such that:

$$\alpha - (\tau + c(\tau)) \int_0^1 q dF(q) - kr = 0.$$

Replacing k by its expression (3.9) yields the price of the contract:

$$\alpha = \frac{\tau + c(\tau)}{1 + r}(p + r\bar{q}).$$

In the absence of aggregate loss uncertainty, $\bar{q} = p$ and we find a lower premium $\alpha = p\tau + pc(\tau)$ which corresponds to the premium derived in the previous section. Aggregate loss uncertainty therefore generates an additional cost determined by the cost of capital allocation and by the tail of the distribution.[10]

[8]Insurance providers traditionally rely on reinsurance companies to transfer the layers of risk they do not wish to retain. The past two decades have also seen the emergence of Insurance Linked Security (ILS) markets, enabling insurance and reinsurance providers to transfer layers of risk to the financial markets.

[9]This is particularly true for public insurance providers, who have greater access to capital through fiscal policy.

[10]We assume here that the insurance company is not allowed to default. In practice, insurance companies are subject to regulatory solvency constraints that impose a probability of default

3.4.1 Strong insurability

Figure 3.3b displays the highest price that an agent is willing to pay $C(p,\tau)$ and the price of the zero expected profit feasible contract in the presence of aggregate loss uncertainty. The two curves intersect at $\underline{p}$ and $\overline{p}$, hence defining three zones on the $[0,1]$ interval. Coverage is purchased on the domain $[\underline{p},\overline{p}]$. Below $\underline{p}$, willingness to pay falls below the cost of insurance, which explains the failure to insure low-probability correlated risks.

Proposition 15 *If a risk is strongly insurable at some probability level, then in the presence of aggregate loss uncertainty, there exist a probability threshold $\underline{p}$, below which the risk becomes uninsurable (in the strong sense), with $\underline{p}$ such that:*

$$\underline{p} = \min\{p | C(p,\tau) = \frac{\tau + c(\tau)}{1+r}(p + r\bar{q})\}.$$

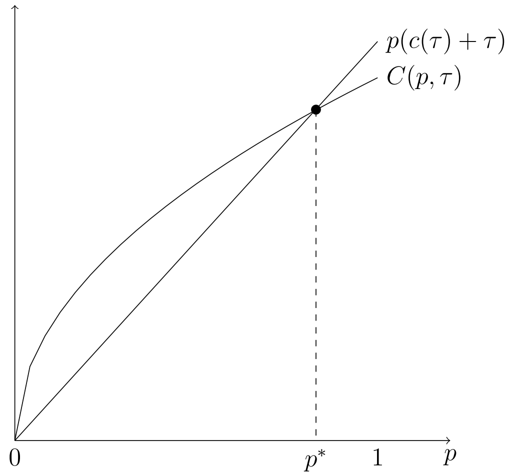
This result underlies the fact that in the presence of aggregate loss uncertainty, low-probability risks are indeed more difficult to insure because of the cost of the reserve.

3.4.2 Weak insurability

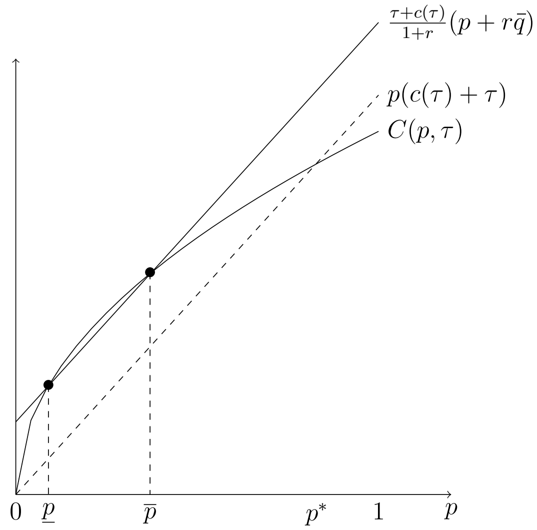
As in the section without aggregate uncertainty, we now proceed to endogenize the level of coverage τ. People are now able to lower the coverage as the probability decreases, hence mitigating the increase in the loading $\alpha/p\tau$. The agent's program is:

$$\begin{aligned} \max_{\tau} \quad & pu(w+\tau-\alpha-L) + (1-p)u(w-\alpha) \\ \text{s.t.} \quad & \alpha = \tfrac{\tau+c(\tau)}{1+r}(p+r\bar{q}), \quad \tau \geq 0, \end{aligned}$$

lower than an exogenously defined level η. On the one hand, it would require the insurance provider to have access to a lower level of capital $k = F^{-1}(1-\eta)[\tau + c(\tau)] - \alpha$, which would translate in a lower premium $\alpha = \frac{\tau+c(\tau)}{1+r}(\int_0^{F^{-1}(1-\eta)} q dF(q) + rF^{-1}(1-\eta))$. On the other hand, policyholders would have weaker coverage because of the default risk and default could generate additional costs. Thus, the optimal default risk depends on the tradeoff between benefit and costs but is not the focus of the present paper.



(a) Non-actuarially fair coverage with no aggregate loss uncertainty



(b) Non-actuarially fair coverage with aggregate loss uncertainty

Figure 3.3: Coverage without and with aggregate loss uncertainty

and the associated first order condition is:

$$p[1-\frac{1+c'(\tau)}{1+r}(p+r\bar{q})]u'(w_1)=(1-p)\frac{1+c'(\tau)}{1+r}(p+r\bar{q})u'(w_2).$$

The problem has an interior solution if and only if:[11] $Q(p) = -p^2[u'(w-L) - u'(w)]+p[\frac{1+b}{1+r}u'(w-L)+r\bar{q}(u'(w)-u'(w-L))-u'(w)]-r\bar{q}u'(w) > 0$. Remark that the left-hand side of this last inequality is quadratic in p with a negative intercept. Figure 3.4 represents an example of this quadratic form. There may or may not exist an interval of p where the inequality is satisfied (where the agent purchases a positive amount of coverage), depending on the relative values of the parameters, but when such an interval exists, it is delimited downward by a lower bound p^{**}.

Proposition 16 *If a risk is weakly insurable at some probability level, then in the presence of aggregate loss uncertainty, there exists a probability threshold* p^{**}*,*

[11]From the first order condition, we obtain $p[1-\frac{1+b}{1+r}(p+r\bar{q})]u'(w-L)-(1-p)\frac{1+b}{1+r}(p+r\bar{q})u'(w) > 0$.

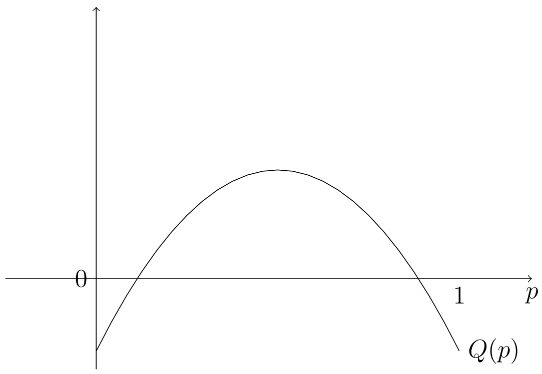


Figure 3.4: $Q(p)$

below which the risk becomes uninsurable (in the weak sense), with p^{**} *such that:*

$$p^{**} = \min\{p|Q(p) = 0\}.$$

This results emphasizes that even when the agent is free to choose the level of coverage τ, there is a probability threshold p^{**} below which she will stop purchasing insurance. This also implies the following corollary.

Corollary 16.1 *In the presence of aggregate loss uncertainty, the optimal coverage tends to zero as the probability* p *goes to* 0*.*

This contrasts with proposition 13 of the previous section. In the presence of aggregate loss uncertainty, the premium α has a component that does not vary with p. As p becomes very small the cost-reduction effect of a decrease in p vanishes but the likelihood-reduction effect remains and eventually dominates completely, leading the agent to stop purchasing insurance.

Finally, figure 3.5 shows a calibrated example in the simple case where $\tilde{q}$ takes value $q = 0.5$ with probability π and 0 with probability $1 - \pi$. The individual probability of loss is $p = \pi q$. The locus of optimal lotteries as the probability π varies is represented by the set of dark dots. When $\pi = 1$, $p = 0.5$ is too large for the agent to purchase any coverage. The optimal lottery is therefore $(w, w - L)$, which is located at the extreme south-east part of the graph. As π diminishes, the cost-reduction effect dominates, driving the optimal lottery closer from the 45 degree line. At a certain point however, the cost-reduction effect becomes weaker and the likelihood-reduction effect dominates, leading coverage down to zero in the limit. The optimal lottery returns to its original $(w, w-L)$ position.

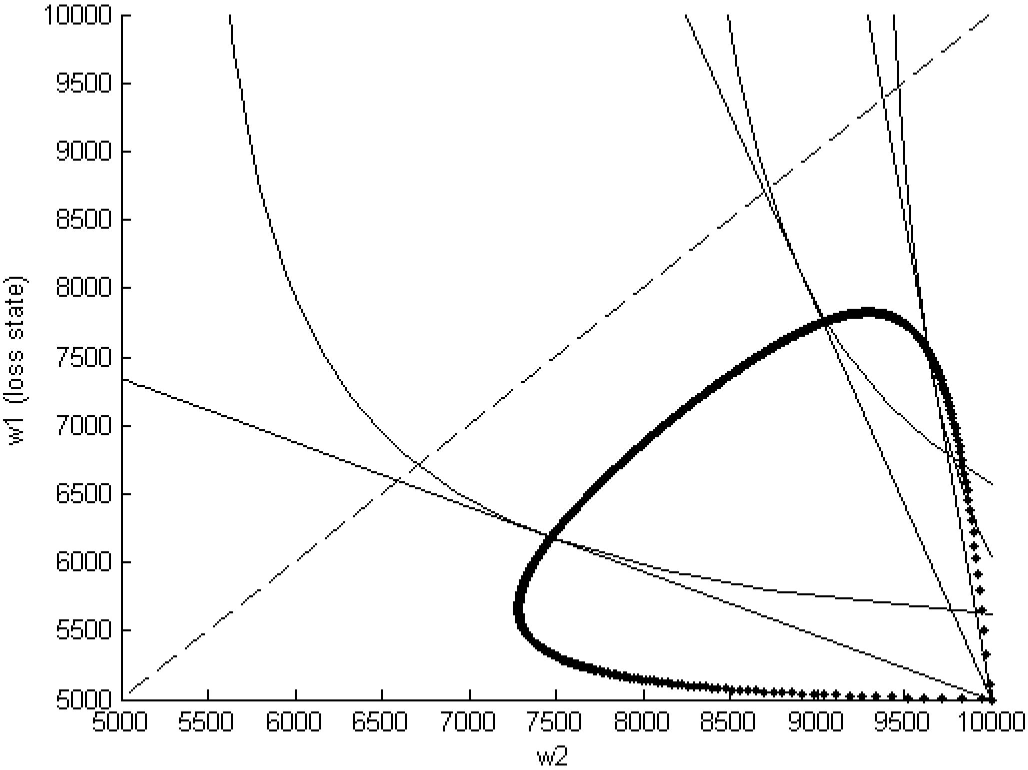


Figure 3.5: $w = 10000, L = 5000, u(x) = \frac{x^{-3}}{3}, r = 0.05, \lambda = 0.3, q = 0.5$

The actuarial and insurance literatures often use the loading $\alpha/p\tau$ as a measure of the cost of insurance. Apart from the case of Giffen behaviors, the higher the loading, the lower the demand for insurance coverage.[12]

Proposition 17 *In the presence of aggregate loss uncertainty, the loading* $\frac{\tau+c(\tau)}{\tau(1+r)}(1+\frac{r\bar{q}}{p})$ *is a decreasing function of* p *and diverges to* $+\infty$ *as* p *goes to* 0.

[12] Some DARA utility functions generate Giffen behaviors as shown by Briys et al. (1989). However, to our knowledge, empirical analysis on the purchase of natural disaster insurance have not observed such behaviors. For instance, Browne & Hoyt (2000) and Grace et al. (2004) observe that when insurance price increases, the demand for insurance decreases.

In the absence of aggregate loss uncertainty, or if the cost of capital allocation is null, the loading $1 + c(\tau)/\tau$ does not depend on the probability p. In this case, the simplifying assumption $c'(\tau) = \lambda$ gives an accurate and useful approximation of the price of insurance. However, it becomes less accurate when, holding other factors constant, the loss probability p becomes small. Empirical studies on the demand for insurance should therefore take this effect into account. Although we have little doubt that the traditional explanations of low take-up rates have a significant importance in the decision to purchase insurance, their effects would be overestimated should one omit to control for variations in the loading factor induced by changes in the loss probability.

3.5 Conclusion

We have shown that the low take-up rates for low-probability event coverage cannot be explained by rational behaviors in the absence of aggregate loss uncertainty. In the presence of aggregate loss uncertainty however, the cost of capital allocation may represent a significant obstacle for the insurability of low-probability events. The fact that uncorrelated low-probability events such as lightning strikes do not posit any insurability issue while correlated low-probability events such as hurricanes and nuclear accidents do, supports our theory. It is also interesting to remark that acts of terrorism were included in standard US homeowner policies until early 2002. One month after the 9/11 attacks, the Insurance Services Office filed a request to exclude acts of terrorism from standard US homeowner policies (Kunreuther & Michel-Kerjan (2005)). In a very similar way, the supply of coverage against losses due to earthquakes dramatically dropped after the 1994 Northridge earthquake that devastated California and generated more than $24 billions in insured losses. These facts suggest that supply side explanations account for a great share of the insurability problem for low-probability events. The explanation we propose is that the occurrence of large catastrophes changes the way actors of the insurance industry perceive the risks they insure. The 9/11 attacks and the Northridge earthquake made insurers realize that the worst case scenario was even worst than what they had imagined so far. The impact on the insurability of low-probability events is particularly salient because even small premium increase may generate important changes on the implied loading factor that people have to pay for coverage. Public-Private Partnership insurance structures, such as the Californian Earthquake Authority, were often created in the aftermath of large catastrophes to face the unavailability of insurance policies at reasonable prices. Their goal is precisely to reduce the cost of capital allocation by aggregating the risk at the highest possible level and by smoothing the shocks across time.

3.6 Appendix

3.6.1 Correlation

The coefficient of correlation δ between two individual losses writes for individuals i and j:

$$\delta = \frac{\mathbb{C}ov(\tilde{x}_i, \tilde{x}_j)}{(\mathbb{V}ar(\tilde{x}_i)\mathbb{V}ar(\tilde{x}_j))^{0.5}}.$$

Since we have an infinite number of agents:[13]

$$\tilde{x}_i\tilde{x}_j = \begin{cases} L^2 & \text{with probability} \quad \int_0^1 q^2 dF(q) \\ 0 & \text{with probability} \quad 1 - \int_0^1 q^2 dF(q), \end{cases}$$

the covariance between two individual losses can be written as:

$$\begin{aligned} \mathbb{C}ov(\tilde{x}_i, \tilde{x}_j) &= \mathbb{E}(\tilde{x}_i\tilde{x}_j) - \mathbb{E}(\tilde{x}_i)\mathbb{E}(\tilde{x}_j) \\ &= L^2 \int_0^1 q^2 dF(q) - (L \int_0^1 q dF(q))^2. \end{aligned}$$

Remark that:

$$\begin{aligned} L^2 \int_0^1 q^2 dF(q) &= L^2 \int_0^1 \Big((q-p)^2 - p^2 + 2pq\Big) dF(q) \\ &= L^2[\sigma^2 - p^2 + 2p^2] \\ &= L^2[\sigma^2 + p^2]. \end{aligned}$$

[13] An infinite number of agents enables us to find a simple expression for the coefficient of correlation since it implies that all the aggregate uncertainty comes from the dependence between the individual losses. But none of our results relies on this assumption.

Thus, we have:

$$\begin{aligned}\mathbb{C}ov(\tilde{x}_i, \tilde{x}_j) &= L^2[\sigma^2 + p^2] - p^2L^2 \\ &= L^2\sigma^2.\end{aligned}$$

Since $\tilde{x}_i$ and $\tilde{x}_j$ have the same variance equal to $p(1-p)L^2$, the coefficient of correlation between two individual losses writes:

$$\delta = \frac{\sigma^2}{p(1-p)}.$$

To show that $\delta \leq 1$, remark that:

$$\begin{aligned}\sigma^2 &= \int_0^1 (q-p)^2 dF(q) \\ &= \int_0^1 q^2 dF(q) - p^2 \\ &\leq p - p^2.\end{aligned}$$

3.6.2 Proposition 12

We consider the case of an interior solution. From the first order condition (3.3), we obtain:

$$\begin{aligned}\frac{d\tau}{dp} &= \frac{(1+c'(\tau))[u'(w_1) - u'(w_2)]}{(1-p)p(1+c'(\tau))^2u''(w_2) + [1-(1+c'(\tau))p]^2u''(w_1) - c''(\tau)[pu'(w_1) + (1-p)u'(w_2)]} \\ &+ \frac{[\tau + c(\tau)]\Big([1-(1+c'(\tau))p]u''(w_1) - (1-p)(1+c'(\tau))u''(w_2)\Big)}{(1-p)p(1+c'(\tau))^2u''(w_2) + [1-(1+c'(\tau))p]^2u''(w_1) - c''(\tau)[pu'(w_1) + (1-p)u'(w_2)]}.\end{aligned}$$

Since $u(.)$ is concave and $c(.)$ convex, we have:

$$\begin{gathered}\tfrac{d\tau}{dp} < 0 \\ \Leftrightarrow \\ u'(w_1) + \tfrac{\tau + c(\tau)}{1+c'(\tau)}(1-(1+c'(\tau))p)u''(w_1) > u'(w_2) + \tfrac{\tau+c(\tau)}{1+c'(\tau)}(1-p)(1+c'(\tau))u''(w_2) \\ \Leftrightarrow \\ u'(w_1)[1 - \tfrac{\tau+c(\tau)}{1+c'(\tau)}(1-(1+c'(\tau))p)A(w_1)] > u'(w_2)[1 - \tfrac{\tau+c(\tau)}{1+c'(\tau)}(1-p)(1+c'(\tau))A(w_2)].\end{gathered} \tag{3.10}$$

Using the first order condition (3.3), this last inequality holds if and only if:

$$1-\frac{\tau+c(\tau)}{1+c'(\tau)}(1-(1+c'(\tau))p)A(w_1) > \frac{1-(1+c'(\tau))p}{(1-p)(1+c'(\tau))}[1-\frac{\tau+c(\tau)}{1+c'(\tau)}(1-p)(1+c'(\tau))A(w_2)],$$

which can be written as:

$$\frac{c'(\tau)}{(1-p)(1+c'(\tau))} > \frac{\tau+c(\tau)}{1+c'(\tau)}[A(w_1)-A(w_2)][1-(1+c'(\tau))p].$$

3.6.3 Corollary 12.1

Since $u(.)$ is a concave function and $\tau < L$ we know that $u'(w_1) > u'(w_2)$. A sufficient condition for inequality (3.10) to be satisfied is:

$$1-[1-(1+c'(\tau))p]\frac{\tau+c(\tau)}{1+c'(\tau)}A(w_1) > 1-(1-p)(1+c'(\tau))\frac{\tau+c(\tau)}{1+c'(\tau)}A(w_2).$$

This can also be written as:

$$(1-p)(1+c'(\tau))A(w_2) > [1-(1+c'(\tau))p]A(w_1).$$

Using the expression (3.6) of the coefficient of relative risk aversion for a HARA function, we obtain:

$$\frac{(1-p)(1+c'(\tau))}{1-(1+c'(\tau))p} > \frac{\eta+\frac{w_2}{\gamma}}{\eta+\frac{w_1}{\gamma}}. \tag{3.11}$$

At an interior solution, the first order condition (3.3) yields:

$$\frac{(1-p)(1+c'(\tau))}{1-(1+c'(\tau))p} = \Big(\frac{\eta+\frac{w_2}{\gamma}}{\eta+\frac{w_1}{\gamma}}\Big)^{\gamma}.$$

So for any $\gamma > 1$, this implies that (3.11) is verified and optimal coverage is indeed decreasing in p.

3.6.4 Proposition 14

Assume that L is now a function of p such that a change in p is compensated by a change in L that maintains pL constant, i.e.:

$$L'(p) = -\frac{L(p)}{p}. \tag{3.12}$$

Let τ^*, w_1^* and w_2^* be the optimal values of coverage, loss state wealth and no-loss state wealth. The first order condition now writes:

$$[1-(1+c'(\tau^*))p]u'(w+\tau^*-p\tau^*-pc(\tau^*)-L(p)) = (1-p)(1+c'(\tau^*))u'(w-p\tau^*-pc(\tau^*)). \tag{3.13}$$

Proceeding as in the proof of proposition 12, we find the necessary and sufficient condition for $\frac{d\tau^*}{dp} < 0$:

$$\frac{c'(\tau^*)}{(1-p)[1-(1+c'(\tau^*))p]} > [\tau^* + c(\tau^*)][A(w_1^*) - A(w_2^*)] + A(w_1^*)L'(p). \tag{3.14}$$

Using equation (3.12) gives:

$$\begin{aligned}
[\tau^* + c(\tau^*)][A(w_1^*) &- A(w_2^*)] + A(w_1^*)L'(p) = [\tau^* + c(\tau^*)][A(w_1^*) - A(w_2^*)] - A(w_1^*)\frac{L}{p} \\
&= [\tau^* + c(\tau^*)]\Big(A(w_1^*) - A(w_2^*) - A(w_1^*)\frac{L}{[\tau^* + c(\tau^*)]p}\Big) \\
&= [\tau^* + c(\tau^*)]\frac{A(w_1^*)[\tau^* + c(\tau^*)]p - A(w_2^*)[\tau^* + c(\tau^*)]p - A(w_1^*)L}{[\tau^* + c(\tau^*)]p} \\
&= [\tau^* + c(\tau^*)]\frac{A(w_1^*)\{[\tau^* + c(\tau^*)]p - L\} - A(w_2^*)}{[\tau^* + c(\tau^*)]p}. \qquad (3.15)
\end{aligned}$$

At any interior solution, the left-hand side of (3.13) must be positive. Therefore, it must be the case that $p(1 + c'(\tau^*)) < 1$. By convexity of $c(\tau)$, we have:

$$p(1 + c'(\tau)) < p(1 + c'(\tau^*)) < 1 \quad \forall \tau < \tau^*.$$

Hence:

$$\int_0^{\tau^*} p(1 + c'(\tau))d\tau < \int_0^{\tau^*} d\tau.$$

Or equivalently, using $c(0) = 0$, we have:

$$p(\tau^* + c(\tau^*)) < \tau^* < L.$$

Therefore:

$$A(w_1^*)\Big([\tau^* + c(\tau^*)]p - L\Big) < 0,$$

which implies with (3.15):

$$[\tau^* + c(\tau^*)][A(w_1^*) - A(w_2^*)] + A(w_1^*)L'(p) < 0.$$

The necessary and sufficient condition (3.14) is therefore always satisfied.

Chapter 4

Pooling natural disaster risks in a community

This chapter is co-authored with Alexis Louaas.

Abstract: We analyze the design of contracts when individual risks are correlated across risk-averse agents in a community. The community is equipped with a public insurer which supplies insurance contracts to its members and has access to costly reinsurance outside the community. Without transaction costs inside the community, risk-averse agents fully insure against their individual risk and share collective risk by getting some dividend in normal states. With premiums raised ex-ante and generating an opportunity cost, they only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and potentially get some dividend in normal states. We illustrate the emergence of the latter contracts for the community of the Caribbean countries exposed to natural disaster risks.

Keywords: individual risk, collective risk, insurance contracts, mutual insurance.

JEL classification: D86, G22, G28, Q54.

4.1 Introduction

The Caribbean countries are located in a region of the world widely exposed to large natural disaster risks, such as earthquakes, hurricanes and flooding events. Even though aggregate damages are usually lower than 1 billion dollars per year in this region, the hurricane season in 2004 affected many countries with more than 6 billion dollars of aggregate losses and a large earthquake in January 2010 caused in Haiti more than 8 billion dollars of damages.[1] In this context, the non-for-profit Caribbean Catastrophe Risk Insurance Facility (CCRIF) is designed to supply insurance contracts to the Caribbean countries (CCRIF SPC (2014)). To deal with the high collective risks due to the spatial correlation of losses, the CCRIF purchases reinsurance outside the community. As reinsurance companies or other investors on financial markets supply reinsurance contracts[2] above fair prices (Jaffee & Russell (1997), Cummins (2006) and Froot (2001)), the CCRIF only partially reinsures the collective risks and supplies to its members insurance contracts which are mutual in the sense that they depend on collective losses. The indemnities for given individual losses are lower when collective losses are high than when collective losses are low. Moreover, dividends are given to the insureds when collective losses are low.[3] The CCRIF, created in 2007, is one example of such facilities that have emerged in different regions of the world exposed to natural disasters in the last twenty years. The Florida Hurricane Catastrophe Fund (FHCF) and the California Earthquake Authority (CEA) respectively created in 1993 and 1996 are other examples (Kousky (2010) and Kunreuther & Michel-Kerjan (2009)).

The present paper analyzes the optimal design of insurance contracts by a

[1]Information on natural disaster losses in the Caribbean countries can be found on the EM-DAT International Disaster Database (http://www.emdat.be/).

[2]Investors on financial markets supply insurance-linked securities such as cat-bonds, which are similar for insurers to standard reinsurance contracts supplied by reinsurance companies. However, insurance-linked securities have emerged in the nineties because financial markets have larger financial capacities to supply contracts for very large risks.

[3]Dividends are given through premium discounts after a year with low collective losses.

pooling insurance facility when the collective risks are not negligible and reinsurance is above fair prices. We consider a community of identical risk-averse agents (representing for instance the Caribbean countries). Each agent faces two individual states: she can either suffer a loss or not. At the collective level, there are two states of nature, the normal one and the catastrophic one, respectively characterized by low and high fraction of the agents affected.[4] We consider a non-for-profit pooling insurance facility for the community (representing for instance the CCRIF for the Caribbean countries). The insurance facility supplies mutual insurance contracts to the agents in the community. For one contract, it charges a premium and pays an indemnity to the insured if affected. The indemnity level in the normal state may differ from the indemnity level in the catastrophic state. The insurance contract may also include a dividend if the normal state occurs. Besides, the insurance facility has access to reinsurance outside the community. We analyze the characteristics of the optimal insurance and reinsurance contracts for the community, when reinsurance is above fair prices.

Without any transaction costs inside the community, the optimal insurance contract consists in full coverage for individual losses in both the normal state and the catastrophic state. Moreover, it includes a strictly positive dividend in the normal state because reinsurance is above fair prices. The higher the cost of reinsurance, the higher the premium and the dividend because the insurer substitutes reinsurance by a higher reserve from the agents to pay the high total indemnities of the catastrophic state. However, requiring high amount of premiums ex-ante[5] can generate an opportunity cost for the agents in the community. Indeed, this capital cannot be used for other purpose (i.e consumption or investment) which thus may require the agents to borrow more costly external capital. In this case, it is Pareto improving to implement a contract with a lower indemnity for individual

[4]We consider only two individual states to keep the model tractable. At the collective level, we consider two and only two states of nature respectively to model collective risks and to keep the model tractable.

[5]Premiums are required ex-ante to pay reinsurance premiums and to secure a reserve which avoids participation default. Moreover, raising the premiums ex-ante enables to transfer indemnities quickly to affected agents.

loss in the catastrophic state than in the normal state. Moreover, the optimal contract still has full coverage and dividend in the normal state if and only if the marginal opportunity cost is low enough relative to the marginal reinsurance cost.

The economics literature has already addressed the question of optimal insurance contract when there are collective risks in a community. Doherty & Schlesinger (1990), Hau (1999), Cummins & Mahul (2004) and Mahul & Wright (2004, 2007) consider the case where the indemnity level in normal states increases with the premium level but the indemnity level in catastrophic states is null whatever the premium level. In this case, the optimal contract consists in partial coverage for individual losses in normal states in order to preserve their welfare level in catastrophic states. However, these papers do not address the issue of the insurer financial capacity which would explain why indemnities cannot be paid in catastrophic states.[6] Charpentier & Le Maux (2014) focuses on the issue by considering an insurer with an exogenously given amount of reserve besides premiums. In this case, the indemnity level increases with the premium level in normal states and in catastrophic states because raised premiums increase the financial capacity. However, the optimal insurance contract consists in full coverage for individual losses in normal states but not in catastrophic states because of insurer limited reserve. Relative to Charpentier & Le Maux (2014), we relax the assumption of partially exogenous financial capacity by introducing reinsurance outside the community. Moreover, we allow a better participation of the insureds in the reserve by introducing dividends in the contracts, which gives more flexibility in the design of contracts for risk sharing. Indeed, as explained by Borch (1962) and (Marshall, 1974b), in a community where agents are exposed to individual risks with collective components, it is Pareto optimal to eliminate individual risks and to share collective risks (mutuality principle). Malinvaud (1973) and Cass et al. (1996) show that a mutual contract with dividend supplied by the insurance company enables to reach the mutuality principle. Penalva-Zuasti (2001)

[6] This contingency can be seen as contractual or as a "default risk" with right perception by insureds and no cost of default. In the quoted theoretical papers, "default risk" is used in this sense.

and Penalva-Zuasti (2008) show that it is also reached with agents purchasing a standard contract from the insurance company and investing in the insurance company through stock market. When reinsurance outside a community is costly, Doherty & Dionne (1993) and Doherty & Schlesinger (2002) show that the optimal contract consists in the full elimination of individual risks in each state of nature, plus partial coverage of the collective risks. Relative to Doherty & Dionne (1993) and Doherty & Schlesinger (2002), we analyze how the cost of reinsurance and the correlation between individual risks affect the optimal contract. Moreover, we introduce and analyze the impact of the opportunity cost of capital potentially generated by raising premiums ex-ante.

The first contribution of the present paper is to develop a simple and tractable model to analyze the optimal design of insurance contracts by a pooling insurance facility to manage individual and collective risks. The second contribution is to study the impact of reinsurance costs and risk correlation on the optimal insurance contract further than previous works. The third contribution is to consider the opportunity cost of capital potentially generated by raising premiums ex-ante. The paper is organized as follows. Section 2 presents the example of the Caribbean countries and their insurance facility. Section 3 sets up the model of a community with individual and collective risks and the insurance and reinsurance contracts. Section 4 provides an analysis of the optimal insurance and reinsurance contracts. Section 5 concludes.

4.2 Caribbean countries and natural disasters insurance

The Caribbean countries are located in a region of the world exposed to important natural disaster risks. Figure 4.1[7] exhibits natural disaster losses in this region in the last fifty years. Collective losses are widely variable from one year to another because natural disasters in the region can have large spatial impacts. Year 2010

[7]EMDAT International Disaster Database (http://www.emdat.be/)

corresponds to the highest losses with a large earthquake affecting Haiti in January with more than 8 billion dollars of damages. Year 2004 corresponds to the second highest losses with a particularly dramatic hurricane season affecting many countries such as the Bahamas, the Cayman Islands, Grenada and Jamaica.

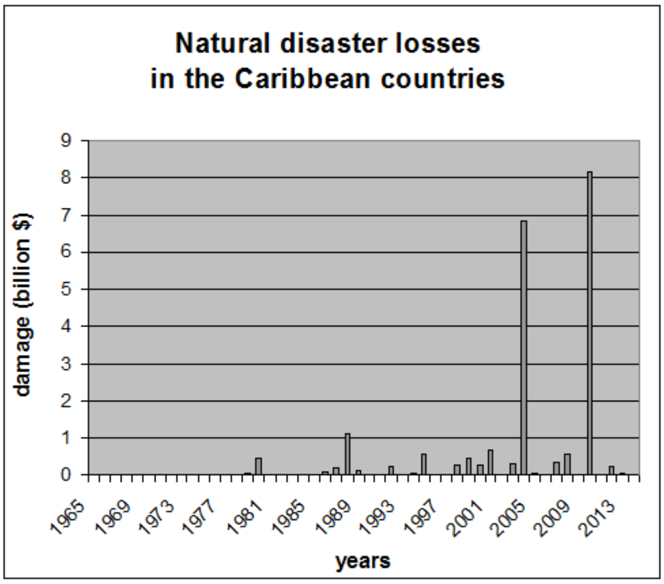


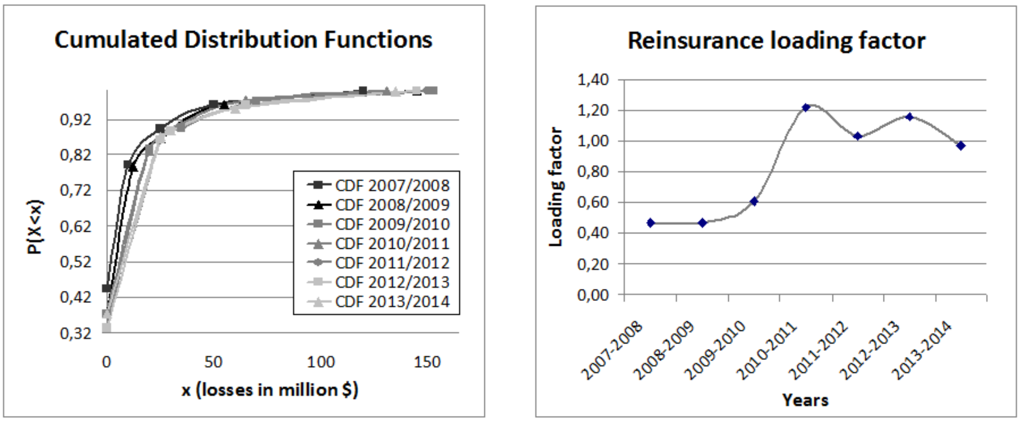
Figure 4.1: Natural disaster losses in the Caribbean countries.

In this context, the Caribbean Catastrophe Risk Insurance Facility (CCRIF) is a non-for-profit multi-country insurance pool. Created in 2007, it currently offers disaster-relief insurance policies covering public losses of sixteen Caribbean countries, protecting them against earthquake, hurricane and excess rainfall losses. Its effectiveness during the five first years of existence has conducted the program to be extended to Central American countries, starting from 2016. The facility aims at pooling the risks faced by its members and reduce the cost the members would individually face if they directly insured on the reinsurance market. The annual reports of the CCRIF are publicly available[8] and provide useful information about the catastrophe insurance contracts proposed to the sixteen members. The CCRIF reports a stable number of 29 or 30 sold policies each year since its inception.[9] The

[8]http://www.ccrif.org/content/publications/reports/annual

[9]The CCRIF can sell more than one policy per country per year because insurance policies

collective risk faced by the CCRIF has remained rather stable as well. Figure 4.2a displays the cumulative density function of the aggregation of the risks covered by the CCRIF and reported in its annual reports since year 2007-2008. The darker lines represent the cumulative distribution of this aggregate risk faced by the pool in the earlier periods of its existence.[10] Using the cumulative distribution functions with the information about the structure of the reinsurance scheme bought by the CCRIF, we can compute an estimated loading factor paid by the organization as : $\lambda^R = \frac{\alpha^R}{\mathbb{E}(L)} - 1$, where α^R is the premium paid by the CCRIF to reinsurers and $\mathbb{E}(L)$ is the expected loss reinsured. Figure 4.2b displays its evolution through the years and shows that the CCRIF faces a significant loading factor on reinsurance, which clearly explains why it only partially reinsures the collective risk. The figure shows that reinsurers increased their prices a lot in 2010, following the large earthquake affecting Haiti.



(a) Aggregate risks faced by the CCRIF (b) Reinsurance loading factor

Figure 4.2: Aggregate risks resulting from risk pooling by the CCRIF and estimated reinsurance loading factor faced by the CCRIF.

for the different types of natural disasters are separated.

[10]Insured losses in figure 4.2a are much lower than total losses due to natural disasters in figure 4.1. This is due to the fact that the CCRIF covers only public losses which represent only a small fraction of the total losses incurred in a country when a natural disaster occurs. It is also due to the fact that the Caribbean countries purchase from the CCRIF only partial insurance for natural disaster risks.

As the CCRIF only partially reinsures the collective risk, it supplies to its members insurance contracts which are mutual in the sense that they depend on collective losses. In addition to the regular insurance premiums, the facility requires its members to pay an up-front participation fee. Audited financial statements report that "it is Managements intent that participation fee deposits are available to fund losses in the event that funds from retained earnings, reinsurers and the Donor Trust are insufficient. If deposits are used to fund losses, it is also Managements intent that any subsequent earnings generated by the Group will be used to reinstate the deposits to their original carrying value". Figure 4.3 shows that the total amount of premiums was effectively much higher the first year than the following years. It has not been necessary to raise high premiums the following years because no extremely large claims had to be paid during these years. In terms of claims to be paid, the worth year is 2010, during which the CRIFF had to transfer a bit less than 8 million dollars to Haiti for the large earthquake affecting the country. The yearly insurance contract is similar to a contract with a high premium requested at the beginning of the year in exchange for an indemnity if the insured is affected during the year and a dividend at the end of the year if collective losses are not too catastrophic. In the present case, the dividend is given through a premium discount at the beginning of the following year. Besides, the CCRIF acknowledges the possibility of lowered indemnities in catastrophic states: "The CCRIF can currently survive a series of loss events with a less than 1 in 10,000 chance of occurring in any given year. Due to planned premium reductions, the safety level drops somewhat through the course of our 10-year forward modeling. However, the lowest projected survivability for the CCRIF in the 10-year modeled period is about 1 in 3000 chance of claims exceeding capacity in any one year." In other words, the CCRIF acknowledges to supply contracts such that the indemnity for one individual loss level is lower in highly catastrophic states than in the other states of nature.

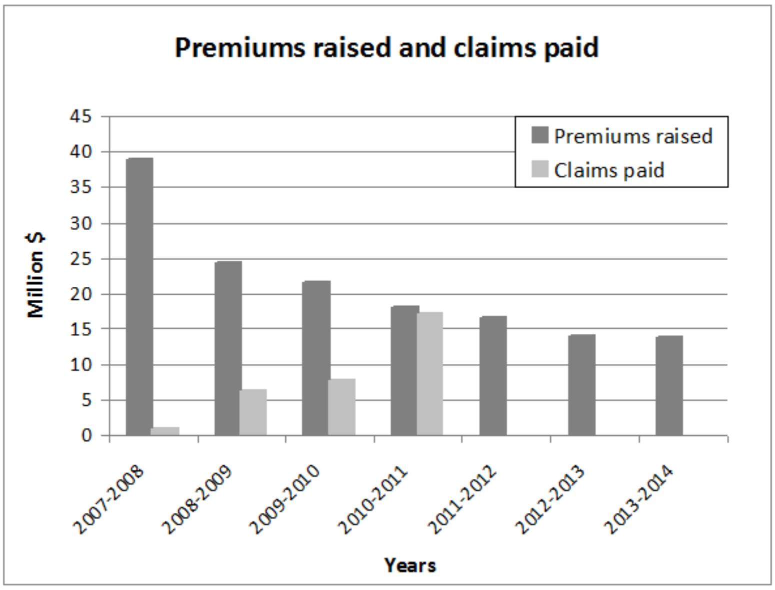


Figure 4.3: Premiums raised and claims paid by the CCRIF.

4.3 The model

4.3.1 The community of agents

We consider a community of N agents identical in terms of preferences, initial wealth and exposure to risk.[11] The preferences of the representative agent satisfy the von Neumann-Morgenstern axioms, with $u(.)$ the corresponding utility function which is strictly increasing, globally concave and twice continuously differentiable. The representative agent has an initial wealth w and is exposed to a potential loss l. The individual risks can generate a significant collective risk either because N is not large enough or because individual risks are correlated. To model the collective risks, we consider two states of nature, one catastrophic and one normal. Ex-ante, the representative agent knows that with a probability p (such that $0 < p < 1$), a catastrophe occurs and the fraction of agents enduring a loss of size l is q_c. In the

[11]Heterogeneity of individuals raises questions related to asymmetric information that are out of the scope of our analysis.

normal state, the fraction of agents enduring the same loss l is $q_n < q_c$.[12] In this template, the individual probability of enduring a loss l is q_c in the catastrophic state and q_n in the normal state, and the unconditional individual probability of enduring a loss l is: $\overline{q} = (1-p)q_n + pq_c$. The individual random wealth without risk sharing scheme is characterized in figure 4.4. Besides, the collective random wealth of the N agents is characterized in figure 4.5. With N large, the coefficient δ of correlation between individual risks is well approximated by: $\delta = \frac{p(1-p)}{\overline{q}(1-\overline{q})}(q_c - q_n)^2$ (proof in appendix 4.6.1). The higher the difference between the fraction q_c of affected agents in the catastrophic state and the fraction q_n of affected agents in the normal state, the higher the risk correlation between agents.[13] Finally, q_n and q_c can be expressed as functions of the individual probability $\overline{q}$ of being affected, the correlation δ between individual risks and the probability p of catastrophe: $q_n = \overline{q} - p(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ and $q_c = \overline{q} + (1-p)(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$.

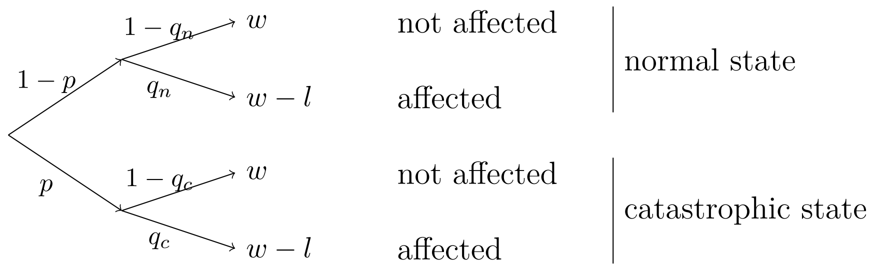


Figure 4.4: individual random wealth of the representative agent

In this template, average individual loss depends on the state of nature, its value is $q_n l$ in the normal state and $q_c l$ in the catastrophic state. Thus, the expected value of the average individual loss is $\overline{q}l$ and its variance is $\overline{q}(1-\overline{q})\delta l^2$ (proof in appendix 4.6.1). The higher the individual probability $\overline{q}$ of being affected, the higher the expected average loss. The more correlated the individual risks, the more volatile

[12]As pointed out by Malinvaud (1973) and Cass et al. (1996), considering two different individual loss levels in the normal and catastrophic states could be considered as two different risks.

[13]The fully correlated case ($\delta = 1$) is characterized by $q_n = 0$, $q_c = 1$ and $0 < p < 1$, in which everyone endures a loss or no one. The no-correlated case ($\delta = 0$) would correspond to $q_n = q_c$, $p = 1$ or $p = 0$, in which there is only one collective state.

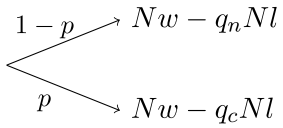
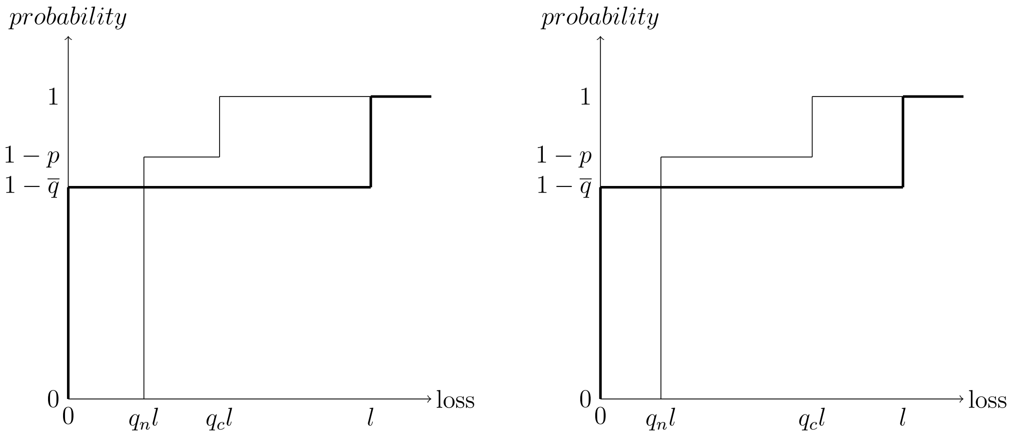


Figure 4.5: collective random wealth of the N agents

the average loss.[14] Figure 4.6 illustrates for two different sets of parameters the cumulative distribution functions for the individual loss (thick bars) and for the average individual loss (thin bars). The spread between q_n and q_c is smaller in 4.6a than in 4.6b, while in both cases $p = 0.2$ and $\overline{q} = 0.3$. The individual probability of being affected $\overline{q}$ is similar for the two sets of parameters, whereas the correlation across individual risks δ is smaller in 4.6a than in 4.6b, which makes a difference for risk sharing mechanism as detailed in the paper.



(a) $p = 0.20$, $q_n = 0.25$, $q_c = 0.50$, $\overline{q} = 0.3$, $\delta = 0.048$

(b) $p = 0.20$, $q_n = 0.20$, $q_c = 0.70$, $\overline{q} = 0.3$, $\delta = 0.190$

Figure 4.6: Cumulative distribution functions for individual loss (thick bars) and average individual loss (thin bars) for two different sets of parameters in 4.6a and 4.6b.

[14] Cummins (2006) and Cummins & Trainar (2009) have more insights on the relation between the risk correlation and the average loss volatility.

4.3.2 Insurance and reinsurance contracts

We consider that the community is equipped with a pooling insurance facility, also called the insurer. The insurer faces two states of nature, the normal one and the catastrophic one, in which it respectively has a fraction q_n and a fraction q_c of its insureds that have to be indemnified. A standard-type insurance contract is a couple (α, τ). In this case, α is the premium paid by the agent and $\tau \geq 0$ is the indemnity received by the agent if the latter endures a loss l. A mutual-type insurance contract is a quadruple $(\alpha, \tau, \epsilon, \pi)$. In this case, α is the premium paid by the agent, $\tau \geq 0$ is the indemnity received by the agent in the normal state if the latter endures a loss l, $\tau - \epsilon \geq 0$ is the indemnity received by the agent in the catastrophic state if the latter endures a loss l and $\pi \geq 0$ is the dividend received by the agent in the normal state. This contract is called mutual-type contract because each agent shares a fraction of the collective risk of the community. Indeed, ϵ and π make the insurance contract directly depend on the collective losses, contrary to the standard contract. The standard contract is a specific case of the mutual contract with $\epsilon = 0$ and $\pi = 0$.[15] Besides, we consider that the contract can generate an opportunity cost for the insured. When premiums are raised ex-ante while indemnities and dividends are given ex-post, the secured capital cannot be used for other purpose (i.e. consumption or investment). Thus, the agents may have to raise more costly external capital instead of using this capital, which generates an opportunity cost for the agents. The higher the required premium α, the higher should be the marginal opportunity cost because the costlier should be the marginal external capital. We denote $\lambda^l(\alpha)$ the opportunity cost function which is increasing and convex relative to the premium α. The agent wealth profile with a mutual-type contract is represented in figure 4.7.

The insurer has to manage the collective risks generated by the aggregation of

[15]The mutual contract defined here is in the spirit of the contracts supplied by the CCRIF to the Caribbean countries. The premium α corresponds to the regular premium plus the up-front participation fee in the contracts supplied by the CCRIF. The dividend π corresponds to the premium discount of the following year if losses are not too catastrophic. The indemnity gap ϵ between normal state and catastrophic state is also acknowledged by the CCRIF.

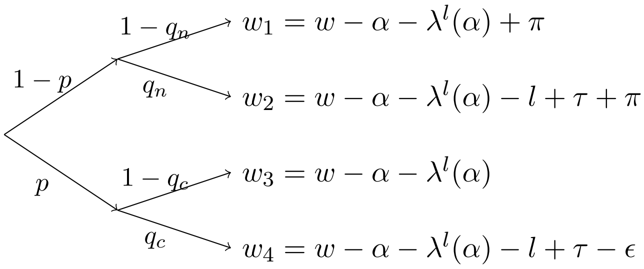


Figure 4.7: agent wealth profile with an insurance contract

the insured individual risks. It can purchase reinsurance outside the community, to be able to pay the higher total claims of the catastrophic state. Purchasing a reinsurance contract, with an indemnity $\tau^R \geq 0$ in the catastrophic state occuring with a probability p, costs $(1+\lambda^R)p\tau^R$, in which $\lambda^R \geq 0$ is the reinsurance loading factor.[16] For reinsurance to be relevant, we need to have $(1+\lambda^R)p < 1$.[17] With the insurance contracts supplied to the agents and the reinsurance contract purchased outside the community, the insurer wealth profile is detailed in figure 4.8.

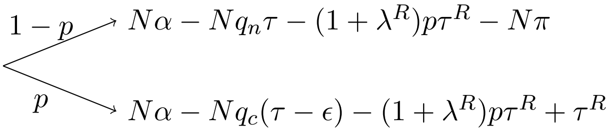


Figure 4.8: insurer profit profile

4.4 Optimal insurance and reinsurance

The optimal insurance and reinsurance contracts for the community consist in maximizing the expected utility of the representative agent under the budget constraints.

[16] λ corresponds to frictional costs with reinsurers or investors, as detailed in Froot (2001).

[17] If $(1+\lambda^R)p \geq 1$, purchasing reinsurance would have no sense for the CCRIF because it would lose money in both the normal state and the catastrophic state with the reinsurance contract.

4.4.1 Budget constraints

The mutual insurance facility cannot pay claims unless it has secured the funds either through raised premiums or purchased reinsurance. With the budget constraints in both the normal state and the catastrophic state (budget expressions in figure 4.8), the optimal insurance and reinsurance contracts are thus the solution of the following maximization problem:

$$\begin{aligned} \max_{\alpha,\tau,\epsilon,\pi,\tau^R} \ & \mathbb{E}(u(\tilde{w})) \\ \text{s.t. } & N\alpha - Nq_n\tau - (1+\lambda^R)p\tau^R - N\pi \geq 0 \\ & N\alpha - Nq_c(\tau-\epsilon) - (1+\lambda^R)p\tau^R + \tau^R \geq 0 \\ & \tau \geq 0,\ \tau-\epsilon \geq 0,\ \pi \geq 0,\ \tau^R \geq 0. \end{aligned} \tag{4.1}$$

Because utility is increasing with wealth, the budget constraints are binding in the two states of nature, the catastrophic one and the normal one. The subtraction of the two binding budget constraints gives the purchased reinsurance indemnity τ^R:

$$\tau^R = Nq_c(\tau-\epsilon) - Nq_n\tau - N\pi \geq 0. \tag{4.2}$$

The insurance facility has to purchase a reinsurance indemnity in order to cover the difference between the amount due in the catastrophic state ($Nq_c(\tau-\epsilon)$) and the amount due in the normal state ($Nq_n\tau + N\pi$). With (4.2), the binding budget constraints give the required premium:

$$\alpha = \left(1 + \frac{p(q_c-q_n)}{\bar{q}}\lambda^R\right)\bar{q}\tau + \left(1 - \frac{p}{1-p}\lambda^R\right)(1-p)\pi - \left(1+\lambda^R\right)pq_c\epsilon, \tag{4.3}$$

which simplifies, if reinsurance is binding ($\tau^R = 0$), to:

$$\alpha = q_c(\tau-\epsilon). \tag{4.4}$$

To be able to pay indemnities and dividends, the insurance facility requires the premium (4.3) (if $\tau^R > 0$) or (4.4) (if $\tau^R = 0$) for the contract $(\alpha,\tau,\epsilon,\pi)$. If it is not valuable to purchase reinsurance, the insurance facility has to raise premiums (4.4) in order to be able to pay all the indemnities in the catastrophic state. If it is valuable to purchase reinsurance, the insurance facility has to pass on the cost of reinsurance to insureds, which explains the loading factor $\frac{p(q_c-q_n)}{\bar{q}}\lambda^R$ in front of τ in (4.3). As shown by (4.2), allowing a dividend in the normal state ($\pi > 0$) or a lower indemnity in the catastrophic state ($\epsilon > 0$) enables to lower the purchase of reinsurance. With a dividend in the normal state ($\pi > 0$), the premium is affected in two opposite directions. The first channel is straightforward: a higher dividend implies a higher premium. The second channel is due to the fact that the insurer has to purchase less reinsurance thanks to the reserve from the insureds and appears through λ^R in the coefficient in front of π in (4.3). Note that the factor in front of π in (4.3) is globally positive because $(1 + \lambda^R)p < 1$. With a lower indemnity in the catastrophic state ($\epsilon > 0$), the premium is reduced through two channels. The first channel is straightforward: a lower indemnity implies a lower premium. The second channel is due to the fact that the insurer has to purchase less reinsurance and appears through λ^R in the coefficient in front of ϵ in (4.3). If the agents in the community can have direct access to reinsurance with the same loading factor λ^R, it is valuable to insure through the insurance facility because: $1 + \frac{p(q_c-q_n)}{\bar{q}}\lambda^R < 1 + \lambda^R$, thanks to partial diversification done by the insurance facility. This is true with standard insurance contracts and thus also true with mutual contracts. The higher the cost of reinsurance, the more valuable the facility. The lower the correlation between participants, the more efficient the pooling and thus the more valuable the facility. This could explain why the Caribbean countries would like to extend their facility to South American Countries in 2016. However, we have assumed that there are no management costs for the insurance facility. If there are, the pooling insurance facility is valuable if the cost of implementing the facility generates a loading factor λ^i such that: $1 + \lambda^i + \frac{p(q_c-q_n)}{\bar{q}}\lambda^R < 1 + \lambda^R$. In the case of the Caribbean countries, extending the insurance facility to South American Countries will be valuable if it does not add too much management costs.

4.4.2 Insurance and reinsurance contracts

With the binding budget constraints, the maximization problem (4.1) for the optimal contracts boils down to:[18]

$$\max_{\tau,\epsilon,\pi} \mathbb{E}(u(\tilde{w}))$$
$$\text{s.t. } \alpha = \left(1 + \frac{p(q_c - q_n)}{\bar{q}}\lambda^R\right)\bar{q}\tau + \left(1 - \frac{p}{1-p}\lambda^R\right)(1-p)\pi - \left(1+\lambda^R\right)pq_c\epsilon$$
$$\tau \geq 0,\ \pi \geq 0,\ (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0. \tag{4.5}$$

Note that, with standard insurance contracts ($\pi = 0$ and $\epsilon = 0$), the maximization problem (4.5) corresponds to the standard Mossin problem, in which the optimal coverage level is obtained by the marginal tradeoff between the aversion to risk and the cost due to reinsurance ($\frac{p(q_c-q_n)}{\bar{q}}\lambda^R$).

Without opportunity cost ($\lambda^l(\alpha) = 0$)

We first consider the case in which raising premiums ex-ante does not generate an opportunity cost for the insured ($\lambda^l(\alpha) = 0$). The first order conditions of (4.5) are derived in appendix 4.6.2. If it is valuable to purchase reinsurance (i.e. $\frac{\tau^R}{N} = (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0$ is not binding), the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - l + \tau + \pi)}{u'(w - \alpha + \pi)} = 1, \tag{4.6}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - l + \tau - \epsilon)}{u'(w - \alpha)} = 1, \tag{4.7}$$

$$\frac{u'(w_3)}{u'(w_1)} = \frac{u'(w - \alpha)}{u'(w - \alpha + \pi)} = \frac{1 + \lambda^R}{1 - \frac{p}{1-p}\lambda^R}. \tag{4.8}$$

[18]The last inequality constraint in (4.5) corresponds to $\tau^R \geq 0$. The inequality constraint $\tau - \epsilon \geq 0$ is not written because it is necessarily verified with the other inequality constraints. Indeed, we have at least as much money for indemnities in the catastrophic state as the amount of money for indemnities and dividends in the normal state.

If it is not valuable to purchase reinsurance (i.e. $\frac{\tau^R}{N} = (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0$ is binding), the optimal contract has indemnities τ and $\tau - \epsilon$ plus dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - l + \tau + \pi)}{u'(w - \alpha + \pi)} = 1, \quad (4.9)$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - l + \tau - \epsilon)}{u'(w - \alpha)} = 1, \quad (4.10)$$

$$\pi = (q_c - q_n)\tau - q_c\epsilon. \quad (4.11)$$

Whether reinsurance is purchased or not, the optimal insurance contract is such that: $w_1 = w_2$ and $w_3 = w_4$ (thanks to (4.6) and (4.7) or (4.9) and (4.10)), which means that $\tau = l$ and $\epsilon = 0$. Besides, when $\lambda^R = 0$, (4.8) tells that $\pi = 0$, (4.3) gives $\alpha = \overline{q}l$ and (4.2) gives $\tau^R = N(q_c - q_n)l$. When $0 < \lambda^R < \lambda^{R^*}$, (4.8) tells that $\pi > 0$, (4.3) gives $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$ and (4.2) gives $\tau^R = N(q_c - q_n)l - N\pi > 0$. λ^{R^*} is determined with (4.8) and the additional constraint $\frac{\tau^R}{N} = (q_c - q_n)l - \pi = 0$, which tells that $\pi = (q_c - q_n)l$ and $\alpha = q_c l$. When $\lambda^{R^*} \leq \lambda^R$, (4.11) tells that $\pi = (q_c - q_n)l$, (4.4) gives $\alpha = q_c l$ and reinsurance is not purchased $\tau^R = 0$.

Proposition 18 *The optimal insurance and reinsurance contracts are such that:*

(i) when $\lambda^R = 0$: $\tau = l$, $\epsilon = 0$, $\pi = 0$, $\alpha = \overline{q}l$, $\tau^R = N(q_c - q_n)l$;

(ii) when $0 < \lambda^R < \lambda^{R^*}$: $\tau = l$, $\epsilon = 0$, $\pi > 0$, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$, $\tau^R = N(q_c - q_n)l - N\pi > 0$;

(iii) when $\lambda^{R^*} \leq \lambda^R$: $\tau = l$, $\epsilon = 0$, $\pi = (q_c - q_n)l$, $\alpha = q_c l$, $\tau^R = 0$;

in which λ^{R^*} is such that $\frac{u'(w - q_c l)}{u'(w - q_n l)} = \frac{1 + \lambda^{R^*}}{1 - \frac{p}{1-p}\lambda^{R^*}}$.

Proposition 18 states that the optimal insurance contract has full coverage for a given individual loss in both normal and catastrophic states ($\tau = l$ and $\epsilon = 0$) whatever the cost of reinsurance (λ^R). The optimal contract eliminates individual risks, which is in line with Borch mutuality principle. Besides, proposition 18 states that the optimal contract has dividend ($\pi > 0$) in the normal state if and only if reinsurance is supplied above fair prices ($\lambda^R > 0$). If reinsurance is fair ($\lambda^R = 0$), the insurance facility fully reinsures the collective risk ($\tau^R = N(q_c - q_n)l$) and the optimal insurance contract is standard, i.e. without any dividend in the normal state ($\pi = 0$). If reinsurance is not fair ($\lambda^R > 0$), a mutual contract (i.e. with $\pi > 0$) is better than a standard contract because it enables the risk-averse agent to bear a part of the collective risk contrary to the standard contract, which is valuable because reinsurance is costly. If reinsurance is excessively above fair prices ($\lambda^{R*} \leq \lambda^R$), the insurance facility does not purchase reinsurance ($\tau^R = 0$) and the optimal insurance contract is with dividend in the normal state corresponding to the indemnity difference between the catastrophic state and the normal state ($\pi = (q_c - q_n)l$). If reinsurance is reasonably above fair prices ($0 < \lambda^R < \lambda^{R*}$), the insurance facility partially reinsures the collective risk ($\tau^R > 0$) and the optimal insurance contract is with dividend in the normal state ($\pi > 0$).

Proposition 19 *With $0 < \lambda^R < \lambda^{R*}$ (and a CARA utility function*[19]*), we have for the optimal insurance and reinsurance contracts: $\frac{d\pi}{d\lambda^R} > 0$, $\frac{d\alpha}{d\lambda^R} > 0$, $\frac{d\tau^R}{d\lambda^R} < 0$.*

Proposition 19 is proved in appendix 4.6.2. It states that the higher the reinsurance cost (i.e. λ^R), the lower the reinsurance purchase and the higher the premium and the dividend in the normal state. Indeed, to be able to cover individual losses in the catastrophic state when reinsurance purchase is decreased, the insurance facility has to increase the reserve financed by the insureds through higher premiums. Moreover, it has higher dividends to give to the insureds if the catastrophic

[19]The coefficient of absolute risk aversion of a utility function $u(.)$ is by definition $A(.) = -\frac{u''(.)}{u'(.)}$. We consider here a utility function with a constant absolute risk aversion A (also called CARA utility function). If the utility function is not CARA, there is an additional wealth effect. However, as long as this effect is of secondary order, it does not change the results. Note that if this effect was not of secondary order, it would have been observed that insurance can be a Giffen good (i.e. a higher premium leading to a higher purchase of insurance). To our knowledge, empirical analysis on the purchase of natural disaster insurance have not observed such behaviors. For instance, Browne & Hoyt (2000) and Grace et al. (2004) observe that when insurance price increases, the demand for insurance decreases.

state does not occur. In the extreme case where λ^R reaches λ^{R*}, reinsurance is not purchased ($\tau^R = 0$) and the premium and the dividend respectively reach the highest levels $\alpha = q_c l$ and $\pi = (q_c - q_n)l$.

Proposition 20 *We have for the optimal insurance and reinsurance contracts:*

(i) when $\lambda^R = 0$*:* $\frac{d\pi}{d\delta} = 0$, $\frac{d\alpha}{d\delta} = 0$, $\frac{d\tau^R}{d\delta} > 0$*;*

(ii) when $0 < \lambda^R < \lambda^{R*}$ *(with a CARA utility function):* $\frac{d\pi}{d\delta} = 0$, $\frac{d\alpha}{d\delta} > 0$, $\frac{d\tau^R}{d\delta} > 0$*;*

(iii) when $\lambda^{R*} \leq \lambda^R$*:* $\frac{d\pi}{d\delta} > 0$, $\frac{d\alpha}{d\delta} > 0$, $\frac{d\tau^R}{d\delta} = 0$*.*

Proposition 20 is obtained thanks to proposition 18, recalling that $q_n = \overline{q} - p(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ and $q_c = \overline{q} + (1-p)(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ ((i) and (iii) are obvious and (ii) is proved in appendix 4.6.2). Firstly, it states that the insurance contract is affected by a change of correlation δ if and only if reinsurance is not fair ($\lambda^R > 0$). If reinsurance is fair, only the average probability $\overline{q}$ and the loss l affects the insurance contract because the collective risk is fully reinsured without any cost. If reinsurance is not fair, the higher the correlation δ, the larger the collective risk and the more expensive its coverage. If reinsurance is not too costly ($0 < \lambda^R < \lambda^{R*}$), an increase of δ is managed by an increase of reinsurance purchase to be able to cover the higher total indemnities in the catastrophic state and the insurance facility has to translate the cost of reinsurance to insureds through higher premiums. If reinsurance is too costly ($\lambda^{R*} \leq \lambda^R$), an increase of δ is managed by an increase of the reserve through higher premiums and the insurance facility has higher dividends to distribute if the catastrophe does not occur. In both cases, higher correlation δ leads to higher premiums.

To sum up, the premium α increases from $\overline{q}l$ to $q_c l$ when λ^R increases from 0 to high values and it also increases when risk correlation δ increases. Thus, with costly reinsurance and significant risk correlation, the required premiums can reach high levels for insureds if the individual loss l is significant. In this case, which is relevant for natural disaster risks, raising such levels of premiums ex-ante can generate an opportunity cost for insureds, which are considered in the following section.

With opportunity cost ($\lambda^l(\alpha) \geq 0$)

We now consider the case in which raising premiums ex-ante generate an opportunity cost for the insureds ($\lambda^l(\alpha) \geq 0$). As explained in section 4.3.2, we assume that the opportunity cost function $\lambda^l(.)$ is increasing and convex relative to the premium α. We analyze how the marginal opportunity cost $\lambda^{l'}(\alpha)$ affects the optimal insurance and reinsurance contracts. We consider $\lambda^R \leq \lambda^{R*}$, which means that purchasing some reinsurance is valuable. The first order conditions of (4.5) are derived in appendix 4.6.3. If it is valuable to have dividend in the normal state $\pi \geq 0$, the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - \lambda^l(\alpha) - l + \tau + \pi)}{u'(w - \alpha - \lambda^l(\alpha) + \pi)} = 1, \tag{4.12}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - \lambda^l(\alpha) - l + \tau - \epsilon)}{u'(w - \alpha - \lambda^l(\alpha))} = \frac{(1 + \lambda^{l'}(\alpha))(1 + \lambda^R)}{(1 + \lambda^{l'}(\alpha))(1 + \lambda^R) - \frac{\lambda^{l'}(\alpha)}{p(1-q_c)}}, \tag{4.13}$$

$$\frac{u'(w_3)}{u'(w_1)} = \frac{u'(w - \alpha - \lambda^l(\alpha))}{u'(w - \alpha - \lambda^l(\alpha) + \pi)} = \frac{(1 + \lambda^{l'}(\alpha))(1 + \lambda^R) - \frac{\lambda^{l'}(\alpha)}{p(1-q_c)}}{(1 + \lambda^{l'}(\alpha))(1 - \frac{p}{1-p}\lambda^R)}. \tag{4.14}$$

If it is not valuable to have dividend in the normal state (i.e. $\pi \geq 0$ is binding), the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\pi = 0, \tag{4.15}$$

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - \lambda^l(\alpha) - l + \tau)}{u'(w - \alpha - \lambda^l(\alpha))} = \frac{(1 + \lambda^{l'}(\alpha))(1 - \frac{p}{1-p}\lambda^R)}{(1 + \lambda^{l'}(\alpha))(1 - \frac{p(q_c - q_n)}{1-\bar{q}}\lambda^R) - \frac{\lambda^{l'}(\alpha)}{1-\bar{q}}}, \tag{4.16}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - \lambda^l(\alpha) - l + \tau - \epsilon)}{u'(w - \alpha - \lambda^l(\alpha))} = \frac{(1 + \lambda^{l'}(\alpha))(1 + \lambda^R)}{(1 + \lambda^{l'}(\alpha))(1 - \frac{p(q_c - q_n)}{1-\bar{q}}\lambda^R) - \frac{\lambda^{l'}(\alpha)}{1-\bar{q}}}. \tag{4.17}$$

Proposition 21 *The optimal insurance and reinsurance contracts are such that:*

(i) when $0 < \lambda^{l'}(\alpha) < \lambda^{l*}$*:* $\tau = l$, $\epsilon > 0$, $\pi > 0$, $\alpha = (\bar{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi - (1+\lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$*;*

(ii) when $\lambda^{l'}(\alpha) = \lambda^{l*}$*:* $\tau = l$, $\epsilon > 0$, $\pi = 0$, $\alpha = (\bar{q}+p(q_c-q_n)\lambda^R)l-(1+\lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)l - Nq_c\epsilon > 0$*;*

(iii) when $\lambda^{l'}(\alpha) > \lambda^{l*}$*:* $\tau < l$, $\epsilon > 0$, $\pi = 0$, $\alpha = (\bar{q}+p(q_c-q_n)\lambda^R)\tau-(1+\lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)\tau - Nq_c\epsilon > 0$*;*

in which $\lambda^{l*} = \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$.

Proposition 21 is derived from the first order conditions of (4.5) written above plus (4.2), (4.3) and (4.4).[20] Firstly, it states that the optimal insurance contract has lower coverage for a given individual loss in the catastrophic state than in the normal state ($\epsilon > 0$) when increasing the premium α generates an opportunity cost ($\lambda^{l'}(\alpha) > 0$). In this case, it is not valuable to cover fully individual losses in the catastrophic state, which means that the optimal contract does not fully eliminate individual risks and does not fulfill the Borch mutuality principle. This is a second-best insurance contract when insurance premiums have to be raised ex-ante and generate an opportunity cost. Besides, relative to the case without an opportunity cost, proposition 21 states that the optimal contract may not always have dividend in the normal state. If the marginal opportunity cost is too high relative to the reinsurance cost ($\lambda^{l'}(\alpha) \geq \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$), it is not valuable to have dividend in the normal state, which means that it is more valuable to spend all the premiums to reinsure rather than to keep some reserves which would be given back through dividends in the normal state. In this case, it is even valuable to lower the indemnity in the normal state relative to full coverage ($\tau < l$) to increase reinsurance and the indemnity in the catastrophic state. We consider in the following a constant marginal opportunity cost $\lambda^{l'}(\alpha) = \lambda^l$.

[20] λ^{l*} is obtained with (4.14) equal to 1. Besides, $\lambda^{l'}(\alpha) > \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ tells that (4.16) is strictly greater than 1 and $\tau < l$.

Proposition 22 *With* $0 < \lambda^l < \lambda^{l^*}$ *(and a CARA utility function), we have for the optimal insurance and reinsurance contracts:* $\frac{d\epsilon}{d\lambda^l} > 0$, $\frac{d\pi}{d\lambda^l} < 0$, $\frac{d\alpha}{d\lambda^l} < 0$, $\frac{d\tau^R}{d\lambda^l} > 0$.

Proposition 22 is proved in appendix 4.6.3. It states that an increase of the marginal opportunity cost (λ^l) leads to a decrease of the premium to limit the opportunity cost for the insured. Thus, it leads to a decrease of the indemnity in the catastrophic state and a decrease of the dividend in the normal state. However, to limit the indemnity decrease in the catastrophic state, reinsurance purchase is increased in this case.

Proposition 23 *With* $0 < \lambda^l < \lambda^{l^*}$ *(and a CARA utility function), we have for the optimal insurance and reinsurance contracts:* $\frac{d\epsilon}{d\delta} > 0$, $\frac{d\pi}{d\delta} < 0$, $\frac{d\alpha}{d\delta}$ *ambiguous and* $\frac{d\tau^R}{d\delta} > 0$.

Proposition 23 is proved in appendix 4.6.3. It states that an increase of the correlation δ leads to an increase of reinsurance purchase because the collective risk increases with δ. On the one hand, the premium has to increase because reinsurance is costly. On the other hand, increasing the premium generates an additional opportunity cost. That is why the indemnity in the catastrophic state and the dividend in the normal state are lowered and finally the variation of the premium is ambiguous.

4.5 Conclusion

In the present paper, we have built a simple model to analyze the type of insurance contracts that emerge when risks are correlated across risk-averse agents in a community. For the sake of realism, we have considered that the community simultaneously chooses the type of contract sold to its members and the level of reinsurance it purchases, given that reinsurance is available at a cost higher than fair price. In this scheme, the insurer of the community supplies mutual contracts which are contingent on the state of nature. Without transaction costs in the community, risk-averse agents fully insure against their individual risk and share collective risk by getting some dividend in normal states of nature. Our model highlights the tradeoff between reinsurance and mutual types contracts. If reinsurance is costly, the promise of dividends in normal states enables the community to raise high premiums that are used as reserves to better indemnify in catastrophic states. With premiums raised ex-ante and generating an opportunity cost, risk-averse agents only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and get some dividend in normal states if the marginal cost of the reserve is low relative to the marginal cost of reinsurance. This analysis helps to understand the limits that risk correlation, costly reinsurance and costly reserve represent for risk sharing and how the contracts in a community can be improved through higher flexibility. Indeed, contracts with contingent indemnity and dividend enable to share better individual risks and collective risks. We have illustrated these mechanisms with the example of the Caribbean Catastrophe Risk Insurance Facility (CCRIF) that combines reinsurance and mutual contracts with indemnity and dividend contingent on the collective state.

4.6 Appendix

4.6.1 Risk correlation

With the loss represented by the random variable $\tilde{x}^i$ for individual i and the probability $\overline{q} = (1-p)q_n + pq_c$ of having a loss l, we have:

$$\tilde{x}^i = \begin{cases} -l & \text{with probability} \quad \overline{q} \\ 0 & \text{with probability} \quad 1-\overline{q} \end{cases}$$

In the normal state, the probability that individual i is affected is q_n. Besides, in the normal state, if individual i is affected, agent j is affected with a probability $\frac{q_n N-1}{N}$, which is well approximated by q_n when N is large. In the catastrophic state, this is similar with q_c instead of q_n. Thus, when N is large, we have with a good approximation:

$$\tilde{x}^i\tilde{x}^j = \begin{cases} l^2 & \text{with probability} \quad (1-p)q_n^2 + pq_c^2 \\ 0 & \text{with probability} \quad 1-(1-p)q_n^2 - pq_c^2 \end{cases}$$

The correlation between individual risks is:

$$\delta = \frac{COV(\tilde{x}^i, \tilde{x}^j)}{(VAR(\tilde{x}^i)VAR(\tilde{x}^j))^{0.5}}.$$

We have:

$$\begin{aligned} \mathrm{COV}(\tilde{x}^i, \tilde{x}^j) &= \mathbb{E}(\tilde{x}^i\tilde{x}^j) - \mathbb{E}(\tilde{x}^i)\mathbb{E}(\tilde{x}^j) \\ &= l^2((1-p)q_n^2 + pq_c^2) - (-l\overline{q})^2 \\ &= l^2((1-p)q_n^2 + pq_c^2 - \overline{q}^2), \end{aligned}$$

$$\begin{aligned} \mathrm{VAR}(\tilde{x}^i) &= \mathbb{E}((\tilde{x}^i)^2) - \mathbb{E}(\tilde{x}^i)^2 \\ &= l^2\overline{q} - (-l\overline{q})^2 \\ &= l^2\overline{q}(1-\overline{q}). \end{aligned}$$

Then, when N is large, the coefficient of correlation is with a good approximation:

$$\begin{aligned} \delta &= \frac{(1-p)q_n^2 + pq_c^2 - \overline{q}^2}{\overline{q}(1-\overline{q})} \\ &= \frac{p(1-p)}{\overline{q}(1-\overline{q})}(q_c - q_n)^2. \end{aligned}$$

With the average individual loss represented by the random variable $\tilde{X}$, we have:

$$\tilde{X} = \begin{cases} q_c l & \text{with probability} \quad p \\ q_n l & \text{with probability} \quad 1-p \end{cases}$$

This can also be written as $\tilde{X} = \tilde{q}l$ where:

$$\tilde{q} = \begin{cases} q_c & \text{with probability} \quad p \\ q_n & \text{with probability} \quad 1-p \end{cases}$$

Hence, the variance of the average individual loss is: $\mathrm{Var}(\tilde{X}) = \mathrm{Var}(\tilde{q})l^2$, with:

$$\tilde{q}^2 = \begin{cases} q_c^2 & \text{with probability} \quad p \\ q_n^2 & \text{with probability} \quad 1-p \end{cases}$$

$$\begin{aligned} \mathrm{Var}(\tilde{q}) &= \mathbb{E}(\tilde{q}^2) - (\mathbb{E}(\tilde{q}))^2 \\ &= (1-p)q_n^2 + pq_c^2 - \overline{q}^2. \end{aligned}$$

The variance of the average individual loss is then:

$$\mathrm{Var}(\tilde{X}) = \delta\overline{q}(1-\overline{q})l^2.$$

4.6.2 Without opportunity cost ($\lambda^l(\alpha) = 0$)

Derivation of the FOC of (4.5)

If the inequality constraints are not strictly binding in (4.5), the first order conditions of (4.5) relative to τ, ϵ and π are respectively:

$$-(1 + \frac{p(q_c - q_n)}{\overline{q}}\lambda^R)\overline{q}\mathbb{E}(u'(\tilde{w})) + (1-p)q_n u'(w_2) + pq_c u'(w_4) = 0, \tag{4.18}$$

$$(1 + \lambda^R)pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0, \tag{4.19}$$

$$-(1 - \frac{p}{1-p}\lambda^R)(1-p)\mathbb{E}(u'(\tilde{w})) + (1-p)(1-q_n)u'(w_1) + (1-p)q_n u'(w_2) = 0. \tag{4.20}$$

Firstly, (4.19) gives:

$$u'(w_4) = (1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{4.21}$$

Secondly, with $\overline{q} = (1-p)q_n + pq_c$, the combination of (4.18) and (4.19) gives:

$$u'(w_2) = (1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{4.22}$$

Thirdly, (4.20) gives with the latter equation:

$$u'(w_1) = (1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{4.23}$$

Fourthly, with (4.21), (4.22), (4.23) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_3) = (1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{4.24}$$

If the inequality constraints are not strictly binding in (4.5) except $\boldsymbol{(q_c - q_n)\tau - \pi - q_c\epsilon \geq 0}$, we have then $\pi = (q_c - q_n)\tau - q_c\epsilon$ and (4.5) boils down to:

$$\begin{aligned} &\max_{\tau,\epsilon}\ \mathbb{E}(u(\tilde{w})) \\ &\text{s.t. } \alpha = q_c(\tau - \epsilon) \\ &\qquad \pi = (q_c - q_n)\tau - q_c\epsilon. \end{aligned} \tag{4.25}$$

The first order conditions of (4.25) relative to τ and ϵ are respectively:

$$-q_c\mathbb{E}(u'(\tilde{w})) + (q_c - q_n)(1-p)((1-q_n)u'(w_1) + q_n u'(w_2)) + (1-p)q_n u'(w_2) + pq_c u'(w_4) = 0, \tag{4.26}$$

$$q_c\mathbb{E}(u'(\tilde{w})) - q_c(1-p)((1-q_n)u'(w_1) + q_n u'(w_2)) - pq_c u'(w_4) = 0. \tag{4.27}$$

Firstly, the sum of (4.26) and (4.27) gives:

$$u'(w_2) = u'(w_1). \tag{4.28}$$

Secondly, (4.27) gives with the latter equation:

$$u'(w_4) = u'(w_3). \tag{4.29}$$

Comparative statics

We consider a CARA utility function $u(.)$, i.e. with $A = -\frac{u''(.)}{u'(.)} > 0$ constant.

With $0 < \lambda^R < \lambda^{R*}$, (4.8) gives:

$$(1 - \frac{p}{1-p}\lambda^R)u''(w_3)dw_3 - \frac{p}{1-p}u'(w_3)d\lambda^R = (1 + \lambda^R)u''(w_1)dw_1 + u'(w_1)d\lambda^R. \tag{4.30}$$

With (4.8), (4.30) can be rewritten:

$$-A(dw_3 - dw_1) = (\frac{p}{1 - p - p\lambda^R} + \frac{1}{1 + \lambda^R})d\lambda^R, \tag{4.31}$$

which finally gives with $\pi = w_1 - w_3$:

$$\frac{d\pi}{d\lambda^R} = \frac{1}{A(1-p-p\lambda^R)(1+\lambda^R)}. \tag{4.32}$$

$\frac{d\pi}{d\lambda^R} > 0$ because $(1+\lambda^R)p < 1$. Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1-p-p\lambda^R)\pi$ and $\tau^R = N(q_c - q_n)l - N\pi > 0$ respectively give with (4.32):

$$\frac{d\alpha}{d\lambda^R} = p((q_c - q_n)l - \pi) + \frac{1}{A(1+\lambda^R)}, \tag{4.33}$$

$$\frac{d\tau^R}{d\lambda^R} = -\frac{N}{A(1-p-p\lambda^R)(1+\lambda^R)}. \tag{4.34}$$

$\frac{d\alpha}{d\lambda^R} > 0$ because $\frac{\tau^R}{N} = (q_c - q_n)l - \pi > 0$. $\frac{d\tau^R}{d\lambda^R} < 0$ because $(1+\lambda^R)p < 1$.

With $0 < \lambda^R < \lambda^{R*}$, (4.8) gives similarly:

$$\frac{d\pi}{d\delta} = 0. \tag{4.35}$$

Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1-p-p\lambda^R)\pi$ and $\tau^R = N(q_c - q_n)l - N\pi > 0$ respectively give with (4.35):

$$\frac{d\alpha}{d\delta} = p\frac{d(q_c - q_n)}{d\delta}\lambda^R l, \tag{4.36}$$

$$\frac{d\tau^R}{d\delta} = N\frac{d(q_c - q_n)}{d\delta}l. \tag{4.37}$$

Because $\frac{d(q_c-q_n)}{d\delta} > 0$, $\frac{d\alpha}{d\delta} > 0$ and $\frac{d\tau^R}{d\delta} > 0$.

4.6.3 With opportunity cost ($\lambda^l(\alpha) \geq 0$)

Derivation of the FOC of (4.5)

If the inequality constraints are not strictly binding in (4.5), the first order conditions of (4.5) relative to τ, ϵ and π are respectively:

$$-(1+\lambda^{l'}(\alpha))(1+\frac{p(q_c - q_n)}{\overline{q}}\lambda^R)\overline{q}\mathbb{E}(u'(\tilde{w})) + (1-p)q_n u'(w_2) + pq_c u'(w_4) = 0, \tag{4.38}$$

$$(1+\lambda^{l'}(\alpha))(1+\lambda^{R})pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0, \tag{4.39}$$

$$-(1+\lambda^{l'}(\alpha))(1-\frac{p}{1-p}\lambda^{R})(1-p)\mathbb{E}(u'(\tilde{w}))+(1-p)(1-q_n)u'(w_1)+(1-p)q_n u'(w_2) = 0. \tag{4.40}$$

Firstly, (4.39) gives:

$$u'(w_4) = (1+\lambda^{l'}(\alpha))(1+\lambda^{R})\mathbb{E}(u'(\tilde{w})). \tag{4.41}$$

Secondly, with $\overline{q} = (1-p)q_n + pq_c$, the combination of (4.38) and (4.39) gives:

$$u'(w_2) = (1+\lambda^{l'}(\alpha))(1-\frac{p}{1-p}\lambda^{R})\mathbb{E}(u'(\tilde{w})). \tag{4.42}$$

Thirdly, (4.40) gives with the latter equation:

$$u'(w_1) = (1+\lambda^{l'}(\alpha))(1-\frac{p}{1-p}\lambda^{R})\mathbb{E}(u'(\tilde{w})). \tag{4.43}$$

Fourthly, with (4.41), (4.42), (4.43) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_3) = \left((1+\lambda^{l'}(\alpha))(1+\lambda^{R}) - \frac{\lambda^{l'}(\alpha)}{p(1-q_c)}\right)\mathbb{E}(u'(\tilde{w})). \tag{4.44}$$

If the inequality constraints are not strictly binding in (4.5) except $\boldsymbol{\pi \geq 0}$, we have $\pi = 0$ (which states $u'(w_1) = u'(w_3)$) and the first order conditions of (4.5) relative to τ and ϵ are respectively:

$$-(1+\lambda^{l'}(\alpha))(1+\frac{p(q_c-q_n)}{\overline{q}}\lambda^{R})\overline{q}\mathbb{E}(u'(\tilde{w}))+(1-p)q_n u'(w_2)+pq_c u'(w_4) = 0, \tag{4.45}$$

$$(1+\lambda^{l'}(\alpha))(1+\lambda^{R})pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0. \tag{4.46}$$

Firstly, (4.46) gives:

$$u'(w_4) = (1+\lambda^{l'}(\alpha))(1+\lambda^{R})\mathbb{E}(u'(\tilde{w})). \tag{4.47}$$

Secondly, with $\overline{q} = (1-p)q_n + pq_c$, the combination of (4.45) and (4.46) gives:

$$u'(w_2) = (1+\lambda^{l'}(\alpha))(1-\frac{p}{1-p}\lambda^{R})\mathbb{E}(u'(\tilde{w})). \tag{4.48}$$

Thirdly, with (4.47), (4.48) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_1) = u'(w_3) = \left((1+\lambda^{l\prime}(\alpha))(1-\frac{p(q_c-q_n)}{1-\overline{q}}\lambda^R) - \frac{\lambda^{l\prime}(\alpha)}{1-\overline{q}}\right)\mathbb{E}(u'(\tilde{w})). \tag{4.49}$$

Comparative statics

We consider a CARA utility function function $u(.)$, i.e. with $A = -\frac{u''(.)}{u'(.)} > 0$ constant.

With $0 < \lambda^l < \lambda^{l*}$, (4.13) gives:

$$(1-\frac{\lambda^l}{p(1-q_c)(1+\lambda^R)(1+\lambda^l)})u''(w_4)dw_4 - \frac{\frac{1}{(1+\lambda^l)^2}}{p(1-q_c)(1+\lambda^R)}u'(w_4)d\lambda^l = u''(w_3)dw_3. \tag{4.50}$$

With (4.13), (4.50) can be rewritten:

$$-A(dw_4 - dw_3) = \frac{1}{p(1-q_c)(1+\lambda^R)(1+\lambda^l)^2 - \lambda^l(1+\lambda^l)}d\lambda^l, \tag{4.51}$$

which finally gives with $\epsilon = w_3 - w_4$:

$$\frac{d\epsilon}{d\lambda^l} = \frac{1}{A(p(1-q_c)(1+\lambda^R)(1+\lambda^l)^2 - \lambda^l(1+\lambda^l))}. \tag{4.52}$$

Similarly, (4.14) gives:

$$\frac{d\pi}{d\lambda^l} = -\frac{1}{A(p(1-q_c)(1+\lambda^R)(1+\lambda^l)^2 - \lambda^l(1+\lambda^l))}. \tag{4.53}$$

Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1-p-p\lambda^R)\pi - (1+\lambda^R)pq_c\epsilon$ and $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$ respectively give with (4.52) and (4.53):

$$\frac{d\alpha}{d\lambda^l} = -(1-p(1-q_c)(1+\lambda^R))\frac{d\epsilon}{d\lambda^l}, \tag{4.54}$$

$$\frac{d\tau^R}{d\lambda^l} = N(1-q_c)\frac{d\epsilon}{d\lambda^l}. \tag{4.55}$$

Note that $\lambda^l < \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ and $(1+\lambda^R)p < 1$ give: $p(1-q_c)(1+\lambda^R)(1+\lambda^l) - \lambda^l > 0$, which tells the sign of the four latter equations.

With $0 < \lambda^l < \lambda^{l*}$, (4.13) and (4.14) give similarly:

$$\frac{d\epsilon}{dq_c} = \frac{\lambda^l}{A(p(1-q_c)^2(1+\lambda^R)(1+\lambda^l) - \lambda^l(1-q_c))}, \tag{4.56}$$

$$\frac{d\pi}{dq_c} = -\frac{\lambda^l}{A(p(1-q_c)^2(1+\lambda^R)(1+\lambda^l) - \lambda^l(1-q_c))}. \tag{4.57}$$

Note that $\lambda^l < \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ and $(1+\lambda^R)p < 1$ give: $p(1-q_c)(1+\lambda^R)(1+\lambda^l) - \lambda^l > 0$. Thus, $\frac{d\epsilon}{dq_c} > 0$ and $\frac{d\pi}{dq_c} < 0$. Because $\frac{d\epsilon}{dq_n} = 0$ and $\frac{d\pi}{dq_n} = 0$, we thus have: $\frac{d\epsilon}{d\delta} > 0$ and $\frac{d\pi}{d\delta} < 0$. Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1-p-p\lambda^R)\pi - (1+\lambda^R)pq_c\epsilon$ and $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$ respectively give with (4.56) and (4.57):

$$\frac{d\alpha}{d\delta} = p\frac{d(q_c - q_n)}{d\delta}\lambda^R l - \left((1 - p(1-q_c)(1+\lambda^R))\frac{d\epsilon}{dq_c} + (1+\lambda^R)p\epsilon\right)\frac{dq_c}{d\delta}, \tag{4.58}$$

$$\frac{d\tau^R}{d\delta} = \left(N(l-\epsilon) + N(1-q_c)\frac{d\epsilon}{dq_c}\right)\frac{dq_c}{d\delta} - Nl\frac{dq_n}{d\delta}. \tag{4.59}$$

Thus, $\frac{d\alpha}{d\delta}$ is ambiguous and $\frac{d\tau^R}{d\delta} > 0$.

Conclusion

This thesis addresses prevention and insurance issues relative to natural disaster risks. It tackles sub-efficient current levels of prevention measures and insurance coverage caused by market imperfections and poorly-designed public policies.

The first part of the thesis deals with the limits of public policies such as insurance subsidy or public relief. Even though they can improve risk sharing, these public policies do not give the right incentives in terms of risk exposure. Chapter 1 highlights that insurance subsidy leads to excessive population density and insufficient resilience investment in risky areas. To avoid risk over-exposure, this chapter recommends that public policies should focus on making agents aware of the risk and liable for their choice of risk exposure. Furthermore, if public policies have to manage inequality issues, they should not favor public aids based on risk exposure.

The second part of the thesis focuses on risk dependence and costs of financial reserves. Because insurers have to secure costly capital to face uncertain catastrophic claims, insurance prices can be high for natural disaster risks. Chapter 3 points out that the cost of securing capital limits the insurability of natural disaster risks, while chapter 4 shows that the design of insurance contracts can be improved with indemnities and dividends contingent on the collective states. To further improve the insurability of natural disaster risks, we advocate that public policies should promote international diversification with the development of competitive reinsurance markets and insurance-linked securities such as Cat-bonds.

Lastly, both the time and ambiguity dimensions of natural disaster risks should be further investigated. The time dimension is central in particular because natural disaster preventive actions consist usually in long-term investments and natural disaster insurance markets can be subject to fluctuations through time. The ambiguity dimension relates to the lack of information about risks which has proven to be a major obstacle for prevention and insurance of natural disaster risks. These two dimensions are all the more critical in the perspective of climate change which affects the patterns of natural hazards and our knowledge of them.

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Title: Prevention and insurance of natural disasters

Keywords: natural disasters, risks, insurance, prevention

This thesis deals with prevention and insurance of natural disasters. Low current levels of prevention measures and insurance coverage are explained by numerous market failures and poorly-designed public policies. Modeling individual behaviors, markets and public policies, this thesis aims at characterizing prevention actions and insurance mechanisms that could mitigate efficiently losses and wealth variability for risk averse agents. Chapter 1 investigates preventive behaviors in the context of city development. It shows that risky areas are more developed nearer to the city center than further away and that investment in building resilience leads to more compact cities. This chapter also highlights the perverse effects of insurance subsidies leading to risk over-exposure and the role that can be played by urban policies such as density restrictions and building codes. The following chapters focus on insurance mechanisms when individual risks are not independent, a main feature of natural disaster risks. Chapter 2 shows that, without market failures, Pareto optimal allocation of risks is reached thanks to stock insurance companies in competition and a reduced number of financial assets. In practice, agents have limited liability and public policies require agents to secure financial reserves to limit payment defaults in catastrophic states. That is why chapters 3 and 4 investigate the issue of the cost of financial reserves. Chapter 3 analyzes how the cost of financial reserves affects the insurance demand of agents exposed to correlated risks. If the financial reserves for the collective risk leads to an additional premium in the price of insurance for one agent, it appears that for a given collective risk, the purchased coverage rate decreases when the individual probability of being affected decreases. Chapter 4 examines the optimal design of insurance contracts when individual risks are correlated in a community. If it is not costly for the community to build reserves, the optimal contract for a given individual risk consists in full coverage, whatever the collective losses, plus a dividend if necessary to redistribute the remaining part of the reserves. Otherwise, the optimal contract for a given individual risk consists in partial coverage when collective losses are high.

Titre: Prévention et assurance des catastrophes naturelles

Mots-clés: catastrophes naturelles, risques, assurance, prévention

Cette thèse porte sur la prévention et l'assurance des catastrophes naturelles dont les faibles niveaux actuels sont dus aux nombreuses imperfections de marché mais aussi aux déficiences des politiques publiques. En s'appuyant sur la modélisation des comportements individuels, des marchés et des politiques publiques, cette thèse a pour objectif d'étudier quels sont les actions de prévention et les mécanismes d'assurance qui permettraient de diminuer efficacement les pertes et aussi la variabilité de la richesse pour les agents averses au risque. Le chapitre 1 porte sur les choix de prévention dans le contexte du développement des villes. Il montre que les zones risquées sont plus développées près du centre-ville que loin du centre-ville et que l'investissement dans la résilience des bâtiments mène à des villes plus compactes. Ce chapitre met aussi en évidence les effets pervers des subventions à l'assurance qui entrainent une surexposition au risque et le rôle que doivent jouer les politiques publiques urbaines telles que les restrictions de densité ou les codes de construction. Les chapitres suivants abordent la problématique des mécanismes d'assurance lorsque les risques individuels ne sont pas indépendants, ce qui est une des caractéristiques majeures des catastrophes naturelles. Dans le chapitre 2, il est établi que, sans imperfection de marché, une allocation Pareto-optimale des risques est atteinte en présence d'un marché compétitif de compagnies d'assurance et d'un nombre restreint d'actifs financiers. En pratique, la responsabilité des agents économiques est limitée et les politiques publiques requièrent que des réserves financières soient constituées pour limiter les défauts de paiement dans les états catastrophiques. C'est pourquoi les chapitres 3 et 4 abordent la question du coût des réserves financières. Le chapitre 3 étudie l'impact du coût des réserves financières sur le taux de couverture demandé par les agents exposés à des risques individuels corrélés. Si les réserves financières pour le risque collectif génèrent une surprime dans le prix de l'assurance individuelle, il apparaît qu'à risque collectif donné, le taux de couverture demandé décroît quand la probabilité individuelle d'être sinistré décroît. Le chapitre 4 analyse la forme optimale des contrats d'assurance pour une communauté d'agents qui sont exposés à des risques individuels corrélés. S'il n'est pas coûteux pour la communauté de constituer les réserves nécessaires, le contrat optimal pour un risque individuel donné consiste en une couverture totale, quelques soient les pertes collectives, à laquelle s'ajoute un dividende qui permet de redistribuer le cas échéant les réserves non utilisées. Dans le cas contraire, le contrat optimal pour un risque individuel donné consiste en une couverture seulement partielle quand les pertes collectives sont élevées.