Insurability of catastrophic risks

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Thèse de doctorat

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Résumé

D'origine naturelle ou industrielle, les grandes catastrophes attirent périodiquement l'attention des médias. La conjonction entre un nombre important de victimes et des conséquences dramatiques pour chacune d'entre elles, rend particulièrement difficile la gestion de ces crises majeures. Si la prévention et la protection des populations peuvent en limiter l'occurrence et/ou les conséquences, il est rarement possible d'éliminer tout risque.

Partant de ce constat, cette thèse étudie les mécanismes d'assurance qui peuvent être mis en place pour couvrir les risques catastrophiques. Ces mécanismes ont pour objectif d'apporter à chaque victime assurée, une indemnité financière qui compense, au moins partiellement, les pertes matérielles, sanitaires ou financières subies lors de la catastrophe.

L'assurance repose sur le principe de la mutualisation des risques entre assurés. Chaque assuré paie une prime ouvrant droit à une indemnisation de montant supérieur à la prime, et versée uniquement en cas de perte. Afin que le montant total des primes couvre l'intégralité des indemnités, il est nécessaire que les primes soient prélevées *avant* que les assurés ne puissent savoir s'ils feront l'objet d'une indemnisation.[1]

Il est donc crucial de pouvoir anticiper le nombre de victimes, ainsi que la perte moyenne par victime, afin de calculer le montant de primes qui permettra de financer l'intégralité des indemnités. Or, si certains risques, tels que le risque de vol de voiture ou de mortalité naturelle, se prêtent bien à ce type de prévision, les

[1] Dans le cas contraire, les personnes non-victimes n'auraient aucun intérêt à s'assurer. Seules les victimes voudraient s'assurer mais il serait alors impossible de leur verser une indemnité supérieure au montant de leur prime.

risques catastrophiques sont, par nature, beaucoup moins prévisibles. Ce problème est une menace très sérieuse pour les systèmes d'assurance. Prélever un montant de prime trop faible conduit à l'incapacité de payer une partie des indemnités, pourtant vitales pour les assurés. D'un autre côté, prélever un montant de prime trop important conduit à décourager la souscription d'assurance.

L'imprévisibilité est donc un défi important pour l'assurabilité des risques catastrophiques. Néanmoins, l'aide financière apportée aux victimes, se trouvant parfois dans des situations de grande difficulté matérielle et financière, est loin d'être négligeable. Il convient donc d'étudier à la fois les limites précises du mécanisme d'assurance traditionnel, ainsi que les mécanismes alternatifs qui peuvent lui être adjoints pour en améliorer l'efficacité et la viabilité.

Le premier chapitre de cette thèse montre que la demande d'assurance pour des risques de très faibles probabilités peut rester importante lorsque les assurés font face à une perte potentielle suffisamment grande et si la prime d'assurance est proportionnelle à la probabilité de la catastrophe. Le rôle d'instruments financiers hybrides, mi-financiers, mi-assurantiels, pour limiter le coût de l'assurance est mis en avant, notamment à travers l'exemple des obligations catastrophes (ou *cat-bonds*). Ces contrats permettent d'emprunter des capitaux importants dont le remboursement n'est pas exigé en cas de catastrophe (mais dont le taux d'intérêt est supérieur à un emprunt traditionnel). Notre application au cas du nucléaire en France révèle que, malgré des prix plus élevés pour les risques de faibles probabilités, il est possible, et vraisemblablement souhaitable, d'étendre la couverture d'assurance au-delà de ce que prévoit la loi en vigueur en France.

Le second chapitre étudie les limites de l'assurance traditionnelle dans deux cas distincts. Dans une première partie, nous montrons que les risques non catastrophiques sont plus faciles à assurer lorsque leur probabilité d'occurrence est faible. Ce résultat théorique est en accord avec des expériences contrôlées menées en laboratoire, au cours desquelles des personnes étaient interrogées sur leur disposition à acheter de l'assurance pour des risques non catastrophiques de différentes probabilités. Dans une deuxième partie, nous prouvons que ce résultat est inversé pour les risques catastrophiques, qui sont particulièrement difficiles à assurer

lorsqu'ils sont associés à de faibles niveaux de probabilités. Ce résultat est conforme avec les observations réalisées sur les taux de souscriptions d'assurance contre les risques d'inondations catastrophiques aux Etats-Unis notamment.

Dans le troisième chapitre, trois alternatives sont étudiées pour améliorer l'efficacité de l'assurance traditionnelle face aux risques catastrophiques. Lorsqu'une communauté fait face à un risque catastrophique, difficilement prévisible, il lui est parfois possible de transférer ce risque en contrepartie d'un paiement. Ce type de transfert, qui peut prendre la forme d'un *cat-bond* par exemple, est très efficace en terme de gestion du risque. Nous montrons qu'en l'absence de coûts de transaction, le transfert permet à tous les agents de la communauté de s'assurer complètement. En pratique, le transfert peut cependant s'avérer coûteux. Une seconde alternative consiste à ce que l'assureur s'engage à rembourser le trop-perçu de prime, le cas échéant. Ce type de contrat, appelé contrat mutualiste, est très efficace lorsque le transfert est impossible ou trop coûteux. Enfin, l'ajustement à la baisse de l'indemnité en cas de catastrophe, apporte une flexibilité qui n'est utilisée en complément du transfert et des contrats mutualistes que lorsque ces deux solutions sont coûteuses.

Enfin, le quatrième chapitre étudie l'utilisation d'obligations catastrophes pour assurer le risque de variations du prix des matières premières agricoles consécutives à des aléas climatiques extrêmes. En émettant une obligation catastrophe, l'entreprise qui s'approvisionne en matières premières emprunte un capital qu'elle peut conserver en cas de catastrophe, lorsque ses coûts d'approvisionnement sont élevés. Sur le plan théorique, elle présente deux avantages par rapport à la solution traditionnelle, qui consiste à assurer les variations de prix par des achats sur les marchés à terme.[2] D'une part, elle permet d'acheter une couverture globale pour plusieurs matières premières, réduisant ainsi la facture d'assurance par effet de diversification. D'autre part, elle permet d'ajuster la couverture aux contraintes logistiques et stratégiques particulières de l'entreprise.

[2]Ces marchés permettent de sécuriser à l'avance un prix pour une livraison d'une matière première à une date future.

Contents

Introduction

On 11 March 2011 a 9.0-magnitude earthquake shook the waters of the pacific ocean off the eastern coast of Honshu, the main Japanese island. The tremendous amounts of energy released under water raised a tsunami that unfurled over ten kilometers inland and hit the nuclear power plant of Fukushima-Daichii. The presence of a nuclear installation on the course of the wave combined with technological, human and organizational flaws converted the natural catastrophe into one of the worst industrial disaster of all times. Fears of radiological contamination lead the authorities to evacuate 150 000 people within a perimeter of 20 kilometers around the plant. Among the drastic measures dictated by the emergency of the situation, the Japanese authorities decided the construction of 30 meters deep ice wall, hence literally freezing the grounds around the power plant, to limit contaminated water leakages into the Pacific ocean.

It is impossible to know the extent of total damage yet. Solely for the purpose of decontamination, indemnification and decommissioning, the Japanese government expects a cost of 177 billion euros. Obtaining an idea of the total cost necessitates to add many indirect costs such as loss of land value (Kawaguchi & Yukutake (2017)), re-adaptation of the energy mix, loss in terms of image for the industrial sector. Including these indirect costs, the Institute for Radiological Protection and Nuclear Safety (IRSN) estimates the median cost of a major accident such as Fukushima in France would be 430 billion euros, that is more than 20% of the country's annual gross domestic product.

Despite these striking figures, the Fukushima-Daiichi accident is not an isolated event in history. Barro (2006) has revived Rietz (1988)'s idea that the many low probability - high severity events such as armed conflicts, economic depressions, or natural hazards, that have plagued the XXth century, have had significant macroeconomic consequences. This is because they do not only affect individual lives, threatening until the very survival of the most exposed, but also have far-reaching consequences that affect many people at the same time.

On the very long-run, the ever increasing interconnectedness between people plays an important role in the rise of large-scale catastrophes. Geographical proximity facilitates the spread epidemics of diseases and worsens the consequences of floods in exposed areas. Globalization makes national economies more vulnerable to aggregate shocks and the reliance on the internet generates new forms of highly dependent risks.

Yet and despite those risks, the secular trend is an increasing concentration of people around the major centers of activity and an increasing degree of dependence between people's interests. Concentration, indeed has many advantages. Innovation is facilitated by the large number of exchanges that large cities allow. Globalization, despite its flaws and biases, has enabled many countries to grow out of poverty and the internet now offers possibilities that were unimaginable a few years ago. The secular trend of increasing concentration is therefore likely to persist, with its benefits and its risks.

In addition to the increasing dependence between individual risks, the global increase in wealth levels has lead mechanically to an increase in wealth exposed to hazards.

Natural disasters are an important source of risk for individuals and collectivities, that sometimes interacts with technological risks, as for the Fukushimai nuclear accident. While Figure 1 highlights the global aspect of natural catastrophe risks, Figure 2 shows the drastic increase in losses due to natural or weather related events (the year 2011 includes the Fukushima-Daiichi accident) over the past decades. Both insured and non-insured losses have followed a positive trend. It is not clear however, whether the share of insured losses increases over time. In fact, the question of why such a large proportion of the total losses remains uninsured, despite the development of global, integrated financial markets, remains a

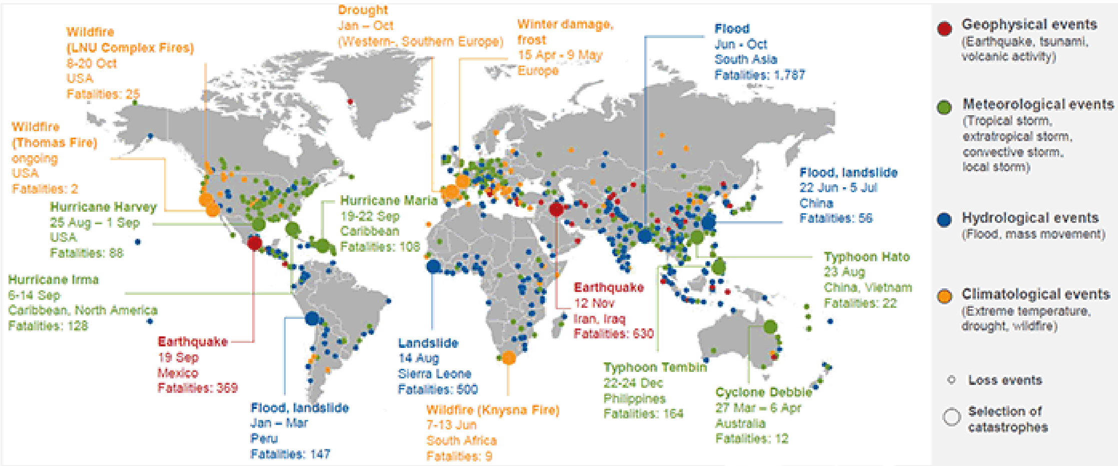


Figure 1: Map of natural catastrophes 2017 (2018 Munich Re, Geo Risks Research, NatCatSERVICE. As of January 2018)

puzzle. This question is at the heart of this thesis. The four papers presented in this work investigate the extent to which insurance can play a role in mitigating the consequences of catastrophic risks.[3]

Traditional insurance is the most simple form of mutualization. It requires all individuals in a group to contribute a fixed and known-in-advance amount of resources used to indemnify those who are unfortunate enough to experience a personal loss.[4] An important condition for insurance to work well, is that people face risks that are not caused by a common source. In this case, the Law of Large Numbers guarantees that it is possible to use past information on the number of claims to predict, with a high accuracy, the number of claims to come.[5] As the

[3]Other mitigation strategies such as prevention, ex-post disaster management are also available. They may be substitute of complement to insurance depending on the situation (Ehrlich & Becker (1972)). Other works, such as Goussebaile (2016) investigate the relationship between insurance and these other mitigation strategies.

[4]The fact that some people receive a net indemnity while others pay a net contribution often leads to the widespread misconception that mutualization mechanisms rely on altruism. In fact, the solidarity that the mechanism described above creates *de facto* between people does not rely on altruism. The cost that the net contributors pay should rather be interpreted as the price of the risk reduction they benefit from themselves, independently of whether their own risk materialized or not.

[5]Technically, the Law of Large Numbers guarantees that that the average loss actually incurred by each policyholder becomes arbitrarily close to the theoretical expected loss as the number of people in the insurance pool increases. The information that insurance providers can gather through past experiences about the theoretical distribution of losses is therefore useful to

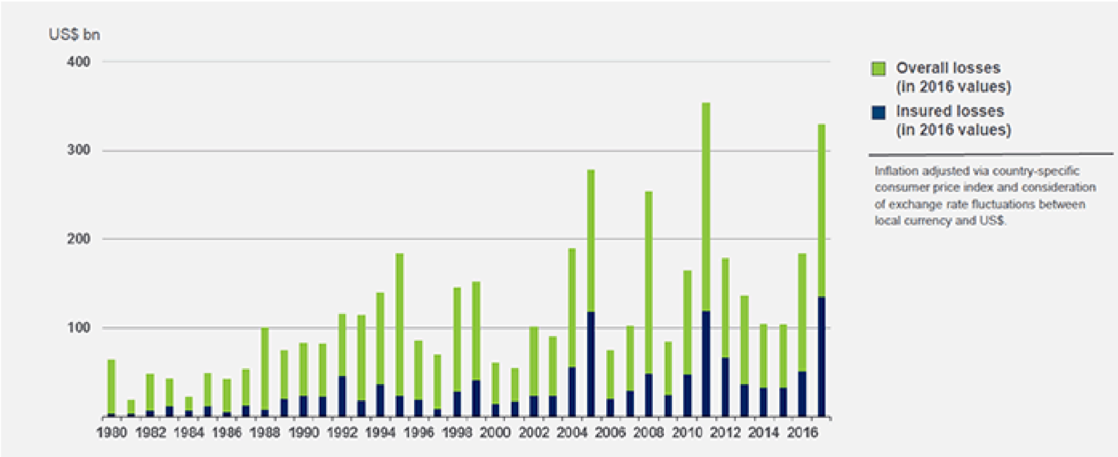


Figure 2: Weather related catastrophe losses 1990-2017 (2018 Munich Re, Geo Risks Research, NatCatSERVICE. As of January 2018)

number of people in the insurance pool increases, it therefore becomes possible to ask every person to pay an indemnity equal to the expected loss of his own risk, which any rational, risk averse, expected utility maximizing agent would accept. The strength of insurance lies in the fact that if losses are independent, sharing the risk enables each agent to find a pool of other agents like him, in which the average *ex-post* loss turns out to be very close from his own *ex-ante* expected loss. In this case, paying the premium allows him to reduce his risk without lowering his expected wealth.

This ideal mechanism has many limitations. Asymmetries of information, lack of competition or/and of information are examples of market failures that increase the cost of insurance compared to what it would be in the ideal world described above. Yet, the mechanism of risk pooling remains interesting in many situations. Matters complicate when the losses experienced by different policyholders are generated by a common source. The extreme case of perfect dependence between individual losses, in which either everybody is affected by a loss or nobody is, illustrates well the issue since it then becomes impossible to use the premium of the many unaffected policyholders to pay the claims of the few unfortunate victims. In this extreme perfect dependence example, the only premium that guarantees

predict actual losses.

that all claims are paid is a premium as high as the loss itself. In other words, insurance becomes completely inefficient.

In practice, most risks lie in between this extreme and the ideal world of independent losses. When the dependence between losses is weak, the Law of Large Numbers remains a good approximation and insurance continues to function well. In the case of collective catastrophes however, a common event (meteorological, industrial, epidemiological, etc) causes the losses to affect many people simultaneously. Without reaching the perfect dependence case, risks that are linked by a common causal relation are unpredictable, in the sens that the Law of Large Numbers does not apply to them. It then becomes impossible to predict the average loss as with independent risks. Figure 3 exhibits natural disaster losses in the Carribean countries region from 1965 to 2014. Collective losses are widely variable from one year to another because natural disasters in the region can have large spatial impacts. Year 2010 corresponds to the highest losses, with over 8 billion dollars in damages due to a dramatic earthquake hitting Haiti in January and a highly active hurricane season in the Caribbean during the second part of the year. Year 2004 corresponds to the second-highest losses, with nearly 7 billion dollars in damages also due to a highly active hurricane season which affected many countries such as the Bahamas, the Cayman Islands, Grenada and Jamaica.

Figure 3 illustrates clearly the issue of risk correlation. If Caribbean countries want to set-up an insurance scheme and require each country to pay its expected loss, then the insurance scheme will come short of money to pay the claims during the years of high losses. Increasing premium may be unacceptable for individual countries, whose risk aversion (if any) only guarantees their willingness to purchase insurance if the price is close to their expected loss (the price is said to be actuarially fair). When the price is above the expected loss, individuals typically lower their demand for insurance. Eventually, as the price becomes too high they stop purchasing insurance, leaving them with the full risk of loss. Traditional insurance then becomes useless.

In a nutshell, risk correlation makes aggregate losses unpredictable. And if traditional insurance allows to handle very efficiently individual risks, whose ag-

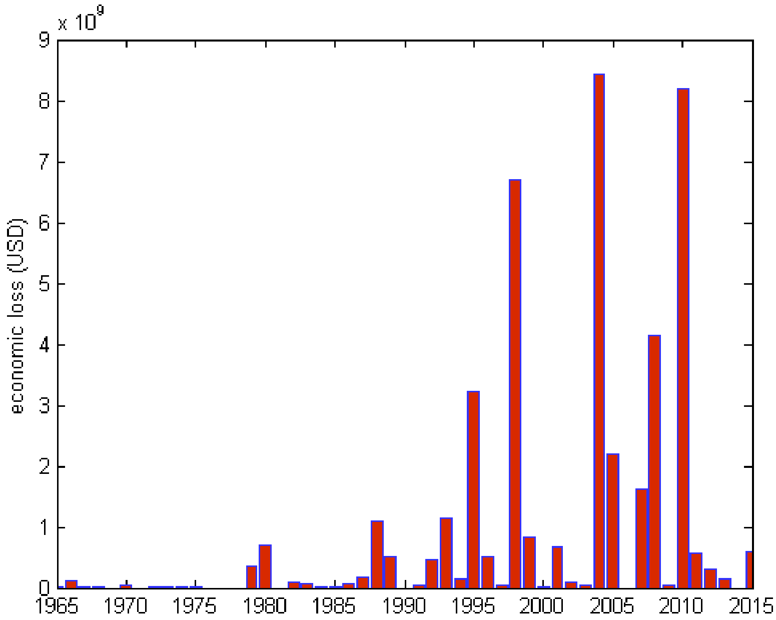


Figure 3: Natural disaster losses in the Caribbean countries (www.emdat.be)

gregate losses can be predicted, it is inoperative against unpredictable collective risks.

The theory of risk-sharing provides interesting insights about how risks can be optimally shared among people. In the absence of transaction costs, Borch (1962)'s mutuality principle states that the idiosyncratic component of individual risks, that is not correlated with the average wealth of the economy, should be fully insured. In order to achieve this in a free market economy, Arrow (1951) and Debreu (1959) showed that complete financial markets should exist in order to allow people to trade their risk exposures. The existence of sufficiently many financial instruments and sufficiently many investors allows each individual to tailor his demand/supply to his exact need, and the equilibrium prices are such that purely idiosyncratic risks are traded at their expected value,[6] hence allowing risk averse agents to be fully insured against their idiosyncratic risk.

[6]The complete market hypothesis guarantees that idiosyncratic risks can be split in small fraction, each borne by one of the many investors. The risk per investor therefore becomes very small when the number of investors is sufficiently large.

Complete markets however, is a very stringent requirement since there must exist as many securities as there are possible states of the world. This would require a number of securities much larger than the number of people in the economy, which in practice in unthinkable.[7] Malinvaud (1974) latter showed that insurance contracts could significantly reduce the number of securities needed to reach Borch's optimality principle. In fact, one insurance contract per agent is sufficient to reach optimality almost surely when the individual risks are independent and the number of people is sufficiently large. Simple insurance contracts however, are not sufficient when individual risks are inter-dependent because they do not allow individuals to trade their exposures to the aggregate risk.

From an insurance perspective, two main solutions can be considered. The first one is to transfer the aggregate risk to a set of agents outside of the economy, who are therefore unaffected by the aggregate risk. If such a transfer can be done without transaction cost, it is always optimal to do it, which settles the issue of correlated risks. Another alternative is to allow the indemnity or the premium to depend on the average loss. This types of mutual insurance contracts with the possibility of default expand the span of the traded securities by allowing people to exchange their exposures to aggregate risk. By bringing the economy closer to completeness, it may improve overall welfare.

The theory of risk-sharing however, ignores the many transaction costs, such as administrative and auditing costs that insurance arrangements necessarily involve. Lack of competition may also result in prices higher than the expected loss. In addition, the theory of risk-sharing does not distinguish mutualization (within the economy) from transfer.[8]

This thesis builds on the theory of risk-sharing to investigate the effect of loss correlation on the insurability of catastrophe risks in the presence of transaction

[7]The existence of a fixed cost for setting a contract could explain why only a limited number of contracts exist in practice.

[8]In the theory of risk-sharing, it is possible to consider that two sets of agents face different risks, which could capture the geographical exposure disparity between two countries or two sets of agents, but the cost of a financial contract is always the same between two agents, be they from the same area or not. This is an important practical limitation of the theory since international risk transfer contracts are typically more expensive than within-country risk pooling contracts.

costs. The four articles presented in this thesis illustrate how insurability is affected by the characteristics of the risk considered, people's behavior toward risk, and by the types of contracts that can be signed in a given economy.

In a first paper, co-authored with Pierre Picard, we study how innovative risk transfer contracts can be used to reduce the social cost of catastrophes. Alternative risk transfer markets indeed allow countries to purchase insurance against that fraction of catastrophe risk that cannot be diversified across national agents due to loss correlation. We therefore analyze how disaster risks within a given country can be transferred through international financial markets. The necessity to combine risk pooling with risk transfer naturally leads us to consider hybrid forms of insurance and reinsurance contracts traded on the alternative risk transfer markets (Cummins & Barrieu (2013)) such as cat-bonds. Cat-bonds are bonds with an embedded default option that can be triggered upon specific and pre-defined condition (the catastrophe).

Our analysis focuses on low probability - high severity accidents such as nuclear catastrophes. The historical frequency of nuclear accidents is 0.07% per year and per reactor (Rangel & Lévêque (2014))[9] while safety experts provide theoretical estimates between 0.002% and 0.0000001% per year and per reactor.[10] [11] Since 1952 and the beginnings of civil nuclear programs, only a handful of severe nuclear accidents have been registered. Fukushima was ranked at the highest severity level on the International Nuclear Events Scale (INES).[12] Reaching low probabilities of occurrence for catastrophes can sometimes be achieved through investment in prevention, such as mitigating technologies, safety standards and procedures, etc.

[9]In the terminology of safety specialists, an accident occurs when there is partial meltdown of the core of a reactor.

[10]The length of this interval can be partly explained by an heterogeneity among reactors. More recent reactors are built with higher safety standards, and some reactors are more exposed to natural hazard than others.

[11]The disparity between the empirical frequency and the theoretical probability estimates provided by experts is the subject of an on-going debate. See Rangel & Lévêque (2014) for more on this subject.

[12]This scale, engineered by the International Atomic Energy Agency, ranges from 1 for mere anomalies to 7 for major accidents. As of today, only Chernobyl (1986) and Fukushima (2011) were ranked at the highest level 7. The accident of Kyshtym (1957), in Russia was ranked at level 6 and Windscale Pile (1957) in Canada, Chalk River (1952) in Great-Britain, Three-Mile Island (1979) in the US have been ranked at level 5.

No matter how much we invest in prevention though, a small risk is bound to remain. How should societies handle these low probability but high severity risks? One intuition might be that risks below a certain probability level are unworthy of consideration. We show that this intuition is wrong and that hybrid risk pooling - risk transfer mechanisms can be very effective at handling low probability - high severity risks. Finally, the theory developed in the paper is applied to the French case. This application reveals that risk protection could be significantly strengthened thanks to adequate combination of risk pooling and risk transfer.

In a second article, co-authored with Arnaud Goussebaïle, we study how insurance demand changes with the loss probability. In a first section, we show that uncorrelated risks can be priced with a traditional rule linear in the loss probability. In this case, we find that insurance demand is actually decreasing in the loss probability, meaning that low probability risks tend to be better insured than higher probability risks. This finding is coherent with controlled experiments, in which people are found to purchase more insurance for low probability events. In a second section, we show that the presence of correlation between losses modifies the pricing rule that an insurance provider can apply. Loss correlation indeed, creates variability in the aggregate loss which is priced by the market if this aggregate risk is systemic, which is most likely the case of catastrophe risk. As a consequence, the insurance premium for a given risk must include a risk premium that depends on the correlation between this risk and other risks. Because this premium diminishes at a lower speed than the willigness to pay for insurance, insurance take-up can diminish when the probability becomes small enough. This explains why low probability systemic risks are, unlike low probability unsystemic risks, difficult to insure.

In a third article, co-authored with Arnaud Goussebaïle, we analyze how mutual contracts can be combined with default, transfer contracts and traditional insurance to handle correlated risks. Mutual contracts condition the premium payments to the aggregate loss. Under a mutual contract, policyholders pay higher premiums when the aggregate loss is high. The possibility of default also allows the insurance provider to lower the indemnity payment when the catastrophe occurs. This mutual and default contracts allow to share the collective risk when it cannot be transferred. It can also be used to reduce the cost of transfer. We obtain Borch's mutuality principle as a corner solution, when mutual contracts entail no transaction costs and transfers outside the economy are prohibitively costly. In this case, all idiosyncratic risks are fully insured and the collective risk is shared between the agents of the economy thanks to mutual contracts. The opposite situation arises when transfers outside the economy are possible with no transaction costs. In this case, it is optimal to transfer all the risks without using mutual contracts, such that all agents end-up perfectly insured. We also show that contracts with default are never used when mutual contracts are costless, that is when it is not problematic to ask policyholder to pay a high premium on which they obtain a dividend if the catastrophe does not materialized. In fact, default is only used to reduce the cost of the mutual contracts. Finally, when both transfer and mutual contracts have transaction costs, we show how mutual contracts can be used to lower the transfer bill.

Finally, a fourth paper, co-authored with Pierre Picard, investigates how a firm can take advantage of the positive correlation between several risk lines to gain a better protection at an advantageous price. We propose an innovative way of insuring procurement risk for companies that purchase commodities in large quantities throughout the world. The proposed strategy consists in issuing a catastrophe bond whose payoff depends on an index of spot prices and meteorological data. This proposed strategy is contrasted with the traditional future purchase strategy for hedging commodity price risks. First, we show that if market investors demand a risk premium when they sell future contracts, the firm may obtain coverage at a lower price by purchasing insurance contracts on its average loss, whose volatility is reduced through home-made diversification, rather than buying separate contracts for each line of risk. Second, we argue that a well calibrated cat bond strategy could lower basis risk. Indeed, the traditional future-based hedging strategy provides an indemnity that is based on the spot prices of the underlying commodities and not on the exact loss incurred by the company. In contrast, a cat-bond approach could use firm specific information to improve the adequacy between the firms's risk and the strategy's payoff.

Chapter 1

Optimal insurance for catastrophic risk

Theory and application to nuclear corporate liability

This chapter is co-authored with Pierre Picard.

Abstract: We analyze the socially optimal insurance for low probability - high severity accidents, both from theoretical and applied standpoints. Our main objective is to characterize the determinants of nuclear corporate liability insurance. We identify individual preferences under which the willingness to pay to avoid risk remains significant, even when the probability of a catastrophe is very small, and we characterize the corresponding optimal asymptotic insurance coverage. We calibrate a model of nuclear risk insurance with French data, in a setting where the risk is transferred to financial markets through catastrophe bonds. We conclude that the current liability limit is probably inferior to the socially optimal level.

Keywords: nuclear accident, liability insurance, catastrophic risk, risk aversion.
JEL classification: D81, D86, G22, G28, Q48.

1.1 Introduction

How should low probability - high severity disaster risks be covered by insurance contracts? The present paper approaches this question from theoretical and applied perspectives, with the objective of analyzing the optimal insurance of nuclear corporate liability.

In Europe, nuclear corporate liability is governed by the international conventions of Paris (1960) and Brussels (1963) that set a lower bound on the liability that governments impose on the nuclear operators within their territories. The 2004 protocol to amend the Paris Convention raised this lower bound to 700 million euros per accident. In France, the protocol was ratified and the operator's liability was set at the new 700 million euros lower bound. In contrast, Germany has defined a 2.5 billion euros liability cap for each accident on its territory[1] and in the United-States, the Price-Anderson Act provides for an overall limit higher than 10 billion dollars. Highlighting the determinants of an optimal nuclear corporate liability is the aim pursued in what follows.

This requires a preliminary step, in which we investigate how low probability-high severity risks can be viewed through the lens of insurance microeconomics. To do so, we first extend the Arrow (1963a) and Pratt (1964) approximation of the risk premium to account for the large deviations from the mean, and we show that a high absolute risk aversion (or, equivalently, a low risk tolerance) in the accident state may entail a significant willingness to pay to avoid risk, even if the accident probability is very low. As an illustration, we investigate the optimal insurance coverage of an individual who faces the risk of an accident with a very low probability. We take the canonical models of the optimal insurance design literature (Mossin (1968) and Raviv (1979)) to the limit case, and we analyze the convergence of the optimal insurance coverage when the accident probability goes to zero. Finally, we complete these theoretical foundations by considering the risk of a large scale industrial accident, such as a nuclear catastrophe, that

[1] On top of the operator's liability, the Paris convention also specifies tranches of liability born by governments, so that total coverage available for indemnifying the victims are at least 1.5 billion euros.

may affect the entire population of a country. Should an accident occur, the firm has to indemnify the victims according to liability law, and it purchases insurance to prevent any insolvency. When the accident probability goes to zero, insurance coverage converges toward a straight deductible indemnity schedule, common to all agents and capped by an upper limit. This extends Arrow (1963b)'s result about the optimality of a deductible to the case of a socially optimal disaster insurance scheme.

We subsequently study nuclear corporate liability insurance by drawing on these theoretical foundations. There are three main building blocks in this analysis: firstly, the assessment of damages that may be caused by a nuclear accident, secondly, the modelling of risk preferences, and finally, the cost of the capital needed to sustain the insurance coverage of a non-diversifiable risk such as the nuclear accident risk.

With regard to the two first points, our methodology builds on a paper by Eeckhoudt et al. (2000), who tried to evaluate the social cost of nuclear risk. Postulating a Constant Relative Risk Aversion (CRRA) utility function, they conclude that the cost estimate is strongly sensitive to the level of risk aversion. This lack of robustness of the CRRA specification for cost-benefit analysis in the presence of catastrophe risk is also present in Weitzman (2009)'s dismal theorem. For our part, we will use the more general Harmonic Absolute Risk Aversion (HARA) functions, that have been shown to be more robust to tail risk (Millner (2013), Ikefuji et al. (2015)).

Measuring the cost of capital leads naturally to consider the Alternative Risk Transfer instruments, surveyed in Cummins & Barrieu (2013). We elaborate on Lane (2000), Major & Kreps (2002), Lane et al. (2008), and Braun (2016) to build a model of catastrophe bond pricing.[2] Our model differs from existing ones along two dimensions: it allows for realistic price estimates for low probability risks and for a non constant marginal cost of capital. Our estimates are consistent with previous studies, and our model's performance compares favorably to existing

[2]Carayannopoulos & Perez (2015), have shown that cat bond returns feature little correlation with other asset prices. This suggests that cat bonds could be used to secure capital at reasonable prices, hence allowing for higher levels of coverage against catastrophes.

models on the probability interval for which observations are available, with much more reasonable price predictions for low probability events.

Using these three building blocks together, and considering the case of France, allows us to evaluate the socially optimal liability insurance scheme for nuclear risk. Our simulations suggest that the French nuclear liability law should be more ambitious than it currently is, even after the 2004 revision of the Paris Convention.

Others before us have identified diverse consequences of low probability disasters, including on asset prices (Rietz (1988), Barro (2009), Gabaix (2012) and Farhi & Gabaix (2015)), business cycles (Barro (2006) and Gourio (2012)), mitigation strategies (Martin & Pindyck (2015)) and welfare (Weitzman (2009)). To our knowledge, our paper is the first to build a model that assesses the welfare gain of a socially optimal insurance scheme for low probability - high severity risks. This assessment highlights the importance of two parameters: unsurprisingly the individuals' risk aversion, but also the cost of capital that sustains the risk transfer mechanism. We thus share the views of Jaffee & Russell (1997b), Froot (2001a), Niehaus (2002) and Zanjani (2002) about capital markets imperfections as significant impediments to the insurability of catastrophic risks. Our paper extends the scope of this literature by modeling simultaneously the demand and supply sides of the insurance market for low probability-high severity risks.

Two other papers can be related to ours. Concerning nuclear liability, Schneider & Zweifel (2004) use a survey approach to evaluate the willingness to pay for risk reduction and they infer the welfare gain that would result from an increase in corporate nuclear liability in Switzerland. Although their methodology is deeply different from ours, they obtain comparable estimates of the optimal level of coverage. On the use of cat-bonds in catastrophe insurance schemes, Borensztein et al. (2017) study the welfare gain that can be reached when cat bonds allow governments to smooth the potential cost of natural catastrophes. However they focus on the case of a representative agent, while we will contemplate a setting where damages affects a population and an indemnification rule has to be defined for each inhabitant.

The paper is organized as follows. Section 2 analyzes the risk premium and the insurance demand for a low probability - high severity accident from the perspective of a risk averse individual. Section 3 characterizes the optimal corporate liability insurance when a large scale industrial accident may affect the whole population of a country. On these theoretical grounds, Section 4 builds a calibrated model of nuclear catastrophe coverage where insurance risk is transferred to financial markets through catastrophe bonds. Section 5 concludes, Section 6 is an appendix that contains proofs and tables.

1.2 Risk premium and insurance demand for catastrophic risks

1.2.1 The risk premium of low-probability and high-severity risks

The Arrow (1963a) and Pratt (1964) approximation of the risk premium notoriously characterizes individual's willingness to pay to avoid exposure to small risks. It states that the risk premium can be approximated by half the absolute risk aversion multiplied by the variance of the risk. This however, only holds for risks that display small deviations around their mean. As a preliminary analysis of our study of optimal insurance against catastrophic risks, this section characterizes the willingness to pay to avoid low-probability high-severity risks.

Consider an expected utility risk-averse individual with a von Neumann-Morgenstern utility function $u(x)$ such that $u' > 0$ and $u'' < 0$, where x is the individual's wealth. Let $A(x) = -u''(x)/u'(x)$ and $T(x) = 1/A(x)$ be her indices of absolute risk aversion and of risk tolerance, respectively. He holds an initial wealth w, and he is facing the risk of a loss $L < w$ with probability p. Thus $m(p, L) = pL$ and $\sigma^2(p, L) = p(1-p)L^2$ are the expected loss and the variance of the loss, respectively. The certainty equivalent $C(p, L)$ of this lottery is defined by

$$u(w - C) = (1 - p)u(w) + pu(w - L).$$

We also denote

$$\theta(p,L) \equiv \frac{C(p,L) - m(p,L)}{\sigma^2(p,L)},$$

the normalized risk premium, that is the risk premium per unit of variance of the risk. This will provide a metric to assess the relevance of insuring low probability risks as we will show in Section 2.2. In particular, we will show that a necessary and sufficient condition for insurance to remain relevant for low probability risks is that $\theta(p,L)$ remains sufficiently large when p goes to 0. Straightforward calculations give

$$\begin{aligned} C'_p(p,L) &= \frac{u(w) - u(w-L)}{u'(w-C)} > 0, \\ C''_{p^2}(p,L) &= -C'_p(p,L)^2 A(w-C) < 0. \end{aligned}$$

Thus, $C(p,L)$ is increasing and concave with respect to p, and of course we have $C(0,L) = 0$.

Put informally, the risk (p,L) may be considered catastrophic for the individual if $C(p,L)$ is non-negligible although p is very small. Risk aversion implies that $C(p,L) > pL$. L'Hôpital's rule allows us to write the limit ratio of certainty equivalent to expected loss as

$$\lim_{p\to 0} \frac{C(p,L)}{pL} = \frac{C'_p(0,L)}{L},$$

which is proportional to $C'_p(0,L)$ for L given. Using l'Hôpital's rule again gives

$$\theta(0,L) \equiv \lim_{p\longrightarrow 0} \theta(p,L) = \frac{C'_p(0,L) - L}{L^2}. \tag{1.1}$$

Thus, analyzing the determinants of $\theta(0,L)$ is an intermediate step to understanding why $C'_p(0,L)$ may be large and thus why $C(p,L)$ may be significant although p is very small.

We know from the Arrow-Pratt approximation that the risk premium of low-severity risks per unit of variance is proportional to the index of absolute risk aversion. Indeed, we have

$$\lim_{L\longrightarrow 0}\theta(p,L)=\frac{A(w)}{2}\text{ for all }p\in(0,1),$$

which of course also holds when p goes to 0, that is

$$\lim_{L\longrightarrow 0}\theta(0,L)=\frac{A(w)}{2}.$$

When L is large, it is intuitive that the size of the risk premium depends on function $A(x)$ not only in the neighborhood of $x = w$, but over the whole interval $[w-L, L]$. Proposition 1 and its corollaries confirm this intuition. Proposition 1 provides an exact formula for $\theta(0, L)$ which is a weighted average of $A(x)\exp\{\int_x^w A(t)dt\}/2$ when x is in $[w - L, w]$. Corollary 1.1 directly deduces a lower bound for $\theta(0, L)$, and Corollary 1.2 considers the case where $L = w$ and the index of relative risk aversion $R(x)$ is larger or equal to one.[3] In this case, the lower bound of $\theta(0, L)$ is the (non-weighted) average of $A(x)$ when $x \in [0, w]$.

Proposition 1 *For all $L > 0$, we have*

$$\theta(0,L)=\frac{1}{2}\int_{w-L}^{w}[k(x)A(x)\exp\{\int_x^w A(t)dt\}]dx$$

where $k(x) = 2[x-(w-L)]/L^2$ and

$$\int_{w-L}^{w}k(x)dx=1.$$

Corollary 1.1 *For all $L > 0$, we have*

$$\theta(0,L)>\frac{1}{2}\int_{w-L}^{w}k(x)A(x)dx.$$

[3] Most empirical studies usually lead to values of $R(x)$ that are larger (and sometimes much larger) than one, and thus the assumption made in Corollary 1.2 does not seem to be, in practice, very restrictive.

Corollary 1.2 *If $L = w$, $R(x) \equiv xA(x) \geq 1$ for all x and $u(0) \in \mathbb{R}$ then*

$$\theta(0, L) > \frac{1}{2w} \int_0^w A(x)dx.$$

With the DARA case in mind, Proposition 1 and its corollaries suggest that $\theta(0, L)$ may be large if $A(x)$ is large when x goes to $w - L$.

Symmetrically, Proposition 2 shows that, under non-increasing absolute risk aversion, the normalized risk premium $\theta(p, L)$ may be large when p is close to zero only if $A(w - L)$ is very large, that is, only when the individual's risk tolerance is very small in the accident state.

Proposition 2 *Assume $R(x) \equiv xA(x) \leq \overline{\gamma}$ for all $x \in [w - L, w]$. Then, under non-increasing absolute risk aversion, we have*

$$\theta(0, L) < \frac{(\overline{\gamma} + 1)A(w - L)}{2},$$

and

$$C(p, L) < pL \left[1 + \frac{(\overline{\gamma} + 1)A(w - L)}{2} L\right].$$

Proposition 2 provides upper bounds for the normalized risk premium $\theta(0, L)$ and for the certainty equivalent $C(p, L)$ when the individual displays non-increasing risk aversion. $\overline{\gamma}$ is an upper bound for the index of relative risk aversion $R(x)$ when x is in the interval $[w - L, w]$. The upper bound of $\theta(0, L)$ is proportional to $A(w - L)$, which is the index of absolute risk aversion in the loss state. Consequently, $C(p, L)$ may be non-negligible when p is very small, say as a proportion of loss L, only if $A(w - L)$ is large. On the contrary, assume $A(w - L) = A(w)$, i.e., the index of absolute risk aversion remains constant in $[w - L, w]$. In that case, we would have $R(x) < R(w)$ for all $x < w$, and thus $\overline{\gamma} = R(w)$, which implies

$$C(p, L) < pL \left[1 + \frac{R(w)}{2} + \frac{R(w)^2}{2}\right].$$

Assuming $R(w) = 2$ or 3 would give $C(p, L) < 4pL$ or $C(p, L) < 7pL$, respectively. Thus, if p is very small, then $C(p, L)/L$ is very small.[4]

Thus, under non-increasing absolute risk aversion, we may conclude that the risk premium of low-probability high-severity accidents may be non-negligible (and thus that the coverage of such a risk is a relevant issue) if and only if the risk tolerance is very low in such catastrophic cases.

$CRRA$ preferences are an instance of such a case with $T(x) = x/\gamma$, where γ is the index of relative risk aversion. We then have $T(x) \longrightarrow 0$ and $A(x) \longrightarrow \infty$ when $x \longrightarrow 0$. However, $CRRA$ preferences are not very satisfactory from a theoretical standpoint, since the utility is not defined when wealth is nil. This corresponds to discontinuous preferences in which any lottery with zero probability for the zero wealth state is preferred to any lottery with a positive probability for this state. If preferences are of the $HARA$ type, then risk tolerance is a linear function of wealth, and we may write $T(x) = \eta + x/\gamma$, with $0 < \eta < 1$ and $\gamma > 0$. In such a case, we have $A'(x) < 0, A(0) = \eta$ and $R(x) > 1$. In particular, the individual's absolute risk aversion index is decreasing but upper bounded. A straightforward calculation then gives

$$\frac{1}{2w}\int_0^w A(x)dx = \frac{\gamma}{2w}\ln\left(1 + \frac{w}{\gamma\eta}\right),$$

and thus, Corollary 1.2 shows that for all $M > 0$, we have $\theta(0, L) > M$ if

$$\eta < \frac{w}{\gamma[\exp(2wM/\gamma) - 1]}.$$

The right-hand side of the previous inequality is positive, decreasing in M and increasing in γ. Thus, $\theta(0, L)$ is arbitrarily large if $\eta = T(0)$ is small enough and/or if $1/\gamma = T'(x)$ is small enough.

[4]For the sake of numerical illustration, consider the case of a large scale nuclear disaster that may occur with probability $p = 10^{-5}$, with total losses of \$100b evenly spread among 1 million inhabitants (think of people living in the neighborhood of the nuclear plant). In the case of an accident, each inhabitant would suffer a loss $L = \$100,000$, with expected loss pL equal to \$1, and risk premium equal to \$4 or \$7, which would be negligible, say as a proportion of their annual electricity expenses. Postulating larger but still realistic values of the index of relative risk aversion would not substantially affect this conclusion.

1.2.2 Insurance demand for catastrophic risks

We now assume that the individual can purchase insurance for a low-probability high-severity risk (p, L). Insurance contracts specify the indemnity I in the case of an accident, i.e., when the individual suffers a loss L, and the premium P to be paid to the insurer, proportional to the expected indemnity $P = (1+\lambda)pI$, where $\lambda > 0$ is the loading factor. Assuming $p(1+\lambda) < 1$ rules out the trivial cases in which the policyholder chooses no coverage.

The policyholder therefore faces the lottery (w_1, w_2), with corresponding probabilities $1-p$ and p, where w_1 and w_2 denote respectively the wealth in the no-loss and loss states, with $w_1 = w - P$ and $w_2 = w - P - L + I$. The full coverage lottery $(w-P, w-P)$ is preferred to the no coverage lottery $(w, w-L)$ if and only if the willingness to pay $C(p, L)$ is higher than the price of full coverage $P = (1+\lambda)pL$, that is

$$C(p, L) \geq (1+\lambda)pL.$$

Rearranging the terms of the inequality and applying l'Hôpital rule with $p \to 0$ gives

$$\frac{C_p'(0, L) - L}{L} \equiv \theta(0, L)L \geq \lambda.$$

Hence the following Lemma.

Lemma 1 *$\theta(0, L)L \geq \lambda$ is a necessary and sufficient condition for the agent to prefer full insurance to no insurance when the loss probability tends to zero.*

Lemma 1 illustrates the importance of the normalized risk premium $\theta(0, L)$ analyzed in the previous section. For insurance to remain attractive despite the vanishingly low probability of accident, the normalized risk premium has to be larger than the loading λ divided by the loss. A direct consequence of Lemma 1 is that $\theta(0, L)L \geq \lambda$ is a sufficient condition for the optimal insurance cover to remain positive as p goes to zero.[5]

[5] Indeed, if the individual prefers full coverage to no coverage, extending his opportunity set does not make him switch to zero coverage. It is easy to check that the optimal limit cover (denoted I^* below) is positive when $\lambda < [u'(w-L) - u'(w)]/u'(w)$ and that this condition is implied by $\theta(0, L)L \geq \lambda$.

Combining Corollary 1.1 and Lemma 1 provides a statistic that can be used to assess whether insurance remains valuable for low probability events.

Corollary 2.1 *Assume $R(x)$ is non decreasing. If $\lim_{x\to 0} R(x) \geq \lambda$, then for L large enough, the individual prefers full insurance to no insurance (and therefore the optimal cover is positive) when p goes to zero.*

We now derive the optimal insurance coverage for a low probability accident. In the (w_1, w_2) plan represented in Figure 2.2, the set of feasible lotteries is delimited by the straight line that represents the equation

$$[1 - p(1+\lambda)]w_1 + (1+\lambda)pw_2 = w - (1+\lambda)pL, \tag{1.2}$$

and

$$w_2 - w_1 + L \geq 0, \tag{1.3}$$

represents the sign condition $I \geq 0$. The optimal lottery maximizes the individual's expected utility

$$(1-p)u(w_1) + pu(w_2),$$

in the set of feasible lotteries. It is such that the marginal rate of substitution $-dw_2/dw_{1|Eu=ct.} = (1-p)u'(w_1)/pu'(w_2)$ is equal to the slope (in absolute value) of the feasible lotteries lines, that is

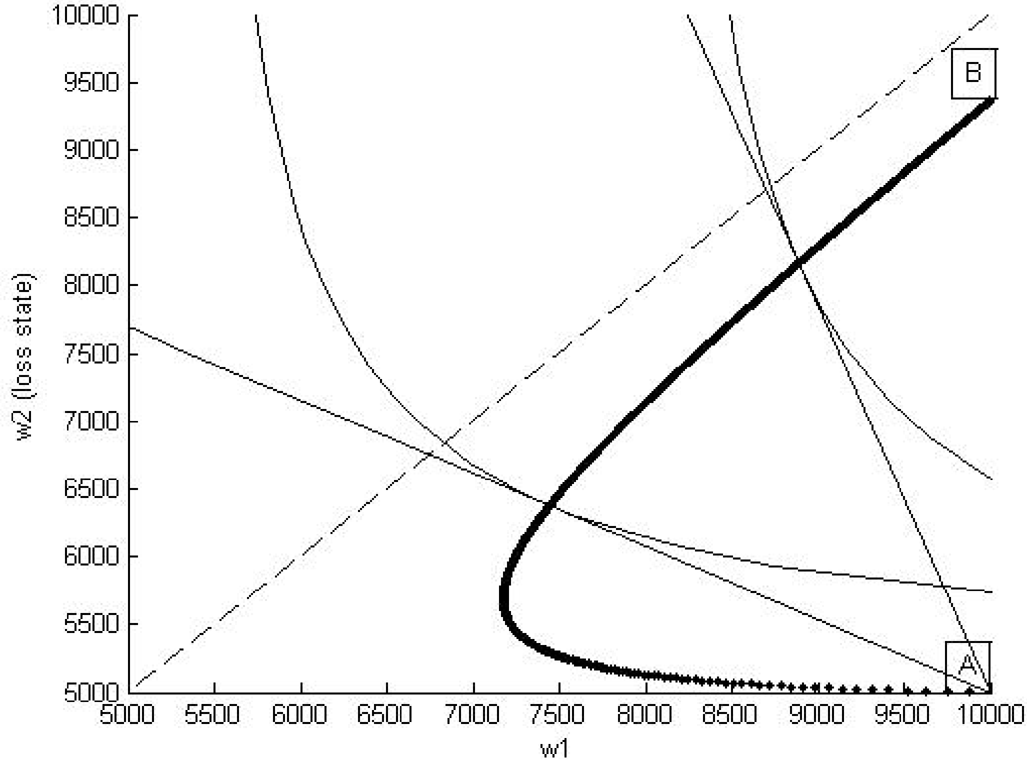
$$(1-p)(1+\lambda)u'(w_1) = [1-(1+\lambda)p]u'(w_2). \tag{1.4}$$

Figure 2.2 shows the locus of optimal lotteries in the (w_1, w_2) plane when p changes. Point A represents the situation with no insurance, and point B represents the optimal lottery when p goes to zero.

Let $w_1(p, L)$, $w_2(p, L)$ denote the optimal state-contingent wealth levels when $I > 0$, that is, when λ is not too large. Let us also denote

$$\begin{aligned} w_1^*(L) &\equiv \lim_{p\longrightarrow 0} w_1(p, L) = w, \\ w_2^*(L) &\equiv \lim_{p\longrightarrow 0} w_2(p, L), \end{aligned}$$

Figure 1.1: Comparative statics in the space of lotteries



Each thick black point represents an optimal lottery for a given probability, from p high in point A to p close to zero in point B. The optimal lottery moves closer to the 45 degree line, that represents full insurance, as the probability p becomes smaller. The calibration is $w = 10000$, $L = 5000$, $u(x) = -\frac{x^{-3}}{3}$.

with

$$u'(w_2^*(L)) = (1 + \lambda)u'(w), \tag{1.5}$$

which implies $w_2^*(L) < w = w_1^*(L)$. Thus, when p goes to 0, the optimal insurance contract (P, I) goes to a limit (P^*, I^*), with $P^* = 0$ and $I^* = w_2^*(L) + L - w_1^*(L) < L$. When p is positive but close to 0, we still have $I < L$ and $P = (1 + \lambda)pI \simeq (1 + \lambda)pI^*$. Since $w_2^*(L) = w - L + I^*$, (1.5) gives

$$u'(w - L + I^*) = (1 + \lambda)u'(w),$$

or

$$I^* = u'^{-1}((1+\lambda)u'(w)) - w + L,$$

and thus I^* is decreasing with λ.

We may characterize the effect of a change in L and/or w on optimal insurance coverage. An increase $dL > 0$ for w given induces an equivalent increase $dI^* = dL$. A simultaneous increase $dw = dL > 0$ induces an increase $dI^* > 0$ in coverage, while an increase in wealth with unchanged loss $dw > 0, dL = 0$ entails a decrease in optimal coverage $dI^* < 0$ under $DARA$ references, i.e. when $A' < 0$. Of course, there is nothing astonishing here. These are standard comparative statics results, which are extended to the asymptotic characterization of catastrophic risk optimal insurance. They are summarized in Proposition 3.

Proposition 3 *When p goes to 0, the optimal insurance coverage I goes to a limit I^*, and when p is close to 0, coverage I and premium P are close to I^* and $(1+\lambda)pI^*$, respectively. I^* is lower than L, and is decreasing with λ. A simultaneous uniform increase in L and w induces an increase in I and P. Under $DARA$, an increase in w with L unchanged induces a decrease in I and P.*

1.3 Optimal catastrophic risk coverage for a population

1.3.1 Catastrophic risk with corporate liability insurance

With the case of nuclear accident risk in mind, we now consider a population of individuals who face the risk of a catastrophic event (called "the accident") caused by a firm. Such an accident may affect the individuals differently, according to their risk exposure and also to their good or bad luck. The population is represented by a continuum of individuals with unit mass. It is composed of n groups or types indexed by $i = 1, ..., n$, and a proportion α_i of the population belongs to group i, with $\alpha_1 + \alpha_2 + ... + \alpha_n = 1$. In the case of a nuclear accident caused by a given reactor, the groups correspond to various locations that may be more or less distant from the nuclear power plant. The accident occurs with probability π. In the case of an accident, a proportion $q_i \in [0,1]$ of type i individuals suffers damage, with financial damage $\widetilde{x}_i$ for each individual in this subgroup of victims. $\widetilde{x}_i$ is a random variable, whose realization is denoted x_i, and which is distributed over the interval $[0, \overline{x}_i]$ with c.d.f. $F_i(x_i)$ and density $f_i(x_i) = F_i'(x_i)$. The random variables $\widetilde{x}_i$ are independently distributed among type i individuals. Thus, we assume that in group i the victims are randomly drawn with probability q_i, and the law of large numbers guarantees that the proportion of affected individuals is equal to q_i, while victim's losses are independently distributed. The total cost of an accident is equal to

$$\sum_{i=1}^{n} \alpha_i q_i \left[\int_0^{\overline{x}_i} x_i f_i(x_i) dx_i \right] = \sum_{i=1}^{n} \alpha_i q_i E\widetilde{x}_i.$$

Under our assumptions, this total cost is given, but the distribution of loss between members of each group is random. This provides a simple correlation structure of losses. There is one single accident risk, which is thus non-diversifiable. In the case of an accident, the losses per individual are equal to $q_i E\widetilde{x}_i$ in each group $i = 1, ..., n$, and thus aggregate losses per individuals $\widetilde{L}$ are equal to $\sum_{i=1}^{n} \alpha_i q_i E\widetilde{x}_i$ with probability π, and $\widetilde{L} = 0$ with probability $1 - \pi$. Hence, $\widetilde{L}$ has expected value $E(\widetilde{L}) = \pi \sum_{i=1}^{n} \alpha_i q_i E\widetilde{x}_i$ and standard deviation $\sigma(\widetilde{L}) = \sqrt{\pi(1-\pi)} \sum_{i=1}^{n} \alpha_i q_i E\widetilde{x}_i$, and its coefficient of variation is $CV(\widetilde{L}) = \sigma(\widetilde{L})/E(\widetilde{L}) = \sqrt{(1-\pi)/\pi}$. $CV(\widetilde{L})$ goes to infinity when π goes to zero, which reflects the high volatility of the accident risk when its probability is small.

Each type i individual is covered by an insurance contract that specifies an indemnity $I_i(x_i) \geq 0$ for all x_i in $[0, \overline{x}_i]$. This insurance coverage is taken out by the firm at price P. With the nuclear liability law in mind, we assume that the firm has to indemnify the victims according to the legal rule $I_i(x_i)$ and also - in order to prevent any bankruptcy risk - that it has to purchase insurance to cover its liability. Thus, $I_i(x_i)$ is at the same time the payment by the firm to type i individuals and the transfer from the insurer to the firm. The firm pays a premium P per individual, and this premium is passed on to the prices of the firm's product (say, on to the consumers' electricity bills). We assume that all consumers purchase the same quantity of the firm's products, and thus it is as if the insurance premium were paid by the individuals themselves.

Assume that the insurer allocates an amount of capital per individual K in order to pay indemnities, should an accident occur. The usual mutualization mechanism cannot be effective in the case of a low probability - high severity risk, and some alternative risk transfer is required. A simple approach (at least from a conceptual standpoint) consists in the insurer issuing a cat bond with par value K. The cat bond will pay some return (a spread above the risk-free rate of return), and will be reimbursed to investors only if no accident occurs. Otherwise, the cat bond will default, and its proceeds will be used to cover the claims for victims' compensation.[6]

We know from the law of large numbers that the average indemnity paid to type i victims in the case of an accident is

$$\int_0^{\overline{x}_i} I_i(x_i) f_i(x_i) dx_i,$$

and thus the total indemnity payment can be financed if

$$K = (1+\lambda) \sum_{i=1}^{n} \alpha_i q_i \int_0^{\overline{x}_i} I_i(x_i) f_i(x_i) dx_i,$$

where λ is a loading factor that represents the claim handling costs that the insurer faces beyond the indemnification costs. This cost of capital is covered by the premiums raised by the insurer, so we have

$$P = c(\pi, K)$$

[6]In practice, a Special Purpose Vehicle (SPV) is created by the sponsor (here, the firm) as a legal entity able to host the cat bond. This SPV acts as an insurer or reinsurer with respect to the sponsor. It issues the bond, delivered to the investors in exchange for the principal payment, which entitles them to a regular coupon. Upon the occurrence of a contractually defined event, called the trigger, the bond defaults and the sponsor gets to keep the principal. Cat bonds are used by insurers and reinsurers to hedge against large losses among their portfolios of insured people, and by large corporations to cover catastrophic events.

with capital cost $c(\pi, K)$ twice continuously differentiable, $c'_K > 0$, $c \to c(0,K) \geq 0$ and $c'_K \to 0$ when $\pi \to 0$, $c'_\pi > 0, c''_{K^2} \geq 0$ and $c''_{\pi K} \geq 1$.[7]

Let w_1 and $w_{2i}(x_i)$ be the wealth of a type i individual if he is not affected by an accident (which occurs with probability $1 - \pi q_i$), and if he is affected with loss x_i (which occurs with probability πq_i and conditional loss density $f_i(x_i)$), respectively. We have

$$\begin{aligned} w_1 &= w - P, \\ w_{2i}(x_i) &= w - P - x_i + I_i(x_i). \end{aligned}$$

All individuals have the same initial wealth w and the same risk preferences represented by utility function u, with $u' > 0, u'' < 0$.

Let C_i be the certainty equivalent loss of type i individuals. The set of feasible allocations $\{w_1, w_{21}(x_1), ..., w_{2n}(x_n), C_1, ..., C_n, K\}$ is defined by

$$u(w - C_i) = (1 - \pi q_i)u(w_1) + \pi q_i \int_0^{\bar{x}_i} u(w_{2i}(x_i))f_i(x_i)dx_i, \tag{1.6}$$

$$w_{2i}(x_i) - w_1 + x_i \geq 0 \text{ for all } i = 1, ..., n, \tag{1.7}$$

$$K = (1 + \lambda)\sum_{i=1}^{n} \alpha_i q_i \int_0^{\overline{x}_i} I_i(x_i)f(x_i)dx_i, \tag{1.8}$$

$$w_1 = w - c(\pi, K). \tag{1.9}$$

Equation (1.6) defines C_i and equation (1.7) is a sign constraint for the insurance coverage. (1.8) defines the capital required to pay indemnities, and (1.9) follows from $w_1 = w - P$ and $P = c(\pi, K)$.

[7] If capital is levied through a cat bond, then $c(\pi, K)/K$ is the spread over LIBOR, i.e. the compensation per euro required by investors for running the risk of losing their capital with probability π. Under a zero risk-free interest rate, a risk neutral investor would require $c(\pi, K) = \pi K$ to accept this risk. Note that we may have $c(0, K) > 0$ if levying capital K induces fixed costs. See Section 4 for further developments.

1.3.2 Optimal contract

We consider a utilitarian regulator that designs the risk coverage mechanism in order to minimize the social cost of an accident, which is the weighted sum of certainty equivalent to individuals' losses. The corresponding optimization program is also a way of characterizing the Pareto optimal allocations when ex-ante transfers between groups are possible.[8] This may be written as minimizing

$$\sum_{i=1}^{n} \alpha_i C_i,$$

with respect to $\{w_1, w_{21}(x_1), ..., w_{2n}(x_n); C_1, C_2, ..., C_n, K\}$, subject to conditions (1.6),(1.7), (1.8) and (1.9). Proposition 4 characterizes the optimal solution of this problem when π goes to 0 and $K > 0$.

Proposition 4 *When π goes to zero with $K > 0$, all the optimal indemnity schedules $I_i(x_i)$ converge toward a common straight deductible indemnity schedule $I^*(x_i) = \max(x_i - d^*, 0)$ and K converges toward K^* defined by*

$$u'(w - d^*) = (1 + \lambda)u'(w - c_0^*)c''_{\pi K}(0, K^*),$$

$$K^* = (1 + \lambda) \sum_{i=1}^{n} \alpha_i q_i \left[\int_{d^*}^{\overline{x}_i} (x_i - d^*) f_i(x_i) dx_i \right],$$

where $c_0^* = c(0, K^*)$.

Proposition 4 shows that the optimal indemnity schedule for small π involves full coverage of the victims above a straight deductible d^* (the same for all individuals whatever their type). This amounts to saying that the victims should be ranked in order of priority on the basis of their losses: the victims with loss x_i should receive an indemnity only if the victims with loss x'_i larger than x_i receive at least $x'_i - x_i$. This simple characterization of optimal indemnification will be used in the simulation conducted in Section 1.4. As in the simple model of Section 2.1, we may derive comparative statics properties for the asymptotic deductible d^*.

[8]See Proposition 5 in the appendix for details.

In particular, it is increasing in λ and, under $DARA$ preferences, it is increasing in wealth.

More importantly, Proposition 4 shows how d^* and K^* are affected by the cost of capital. If the investors were risk neutral, we would have $c(\pi, K) = \pi K$, i.e. the cost of capital would just be equal to the risk premium that compensates for the expected loss due to the default. We would have $c''_{\pi K}(\pi, K) = 1$ and, in such a case, the cost of capital would not affect the optimal indemnity schedule.

However, as we will see in more detail in Section 1.4 with the example of the cat bond market for low-probability triggers, because of the aversion of investors towards risk, or for other reasons, it is much more realistic to keep the cost of capital in a more general form $c(\pi, K)$. In that case the cost of capital does affect the optimal indemnity schedule as highlighted in Proposition 4.[9]

The optimality of straight deductible contracts was first established by Arrow (1963b)[10] in a different perspective. While Arrow studied individual insurance decisions, we are concerned with the design of a socially optimal insurance scheme, where an entire population is exposed to a common source of risk and the cost of insurance is uniformly spread among inhabitants. This implies that there is cross-subsidization from the less exposed to the more exposed individuals. In Arrow (1963b), both the optimal price and deductible depend on the risk profile of a particular agent. In contrast, Proposition 4 indicates that all indemnity schedules converge toward a single coverage rule, characterized by d^*, K^*, and the associated premium $P^* = c_0^*$. The fact that the deductible does not depend on type i is true

[9] Note that $c''_{\pi,K}(0, K^*) = \lim_{\pi \to 0} (1-\pi)c'_K(\pi, K^*)/\pi$ from L'hôpital's rule. Then, Proposition 4 yields, for π small enough

$$\frac{\pi u'(w - d^*)}{(1-\pi)u'(w - c_0^*)} \approx (1+\lambda)c'_K(\pi, K^*).$$

The left-hand side of this equality is the individual's marginal rate of substitution between the states where he receives an indemnity after an accident and where no accident occurs, respectively. The right-hand side is the marginal cost of capital needed to sustain the insurance coverage, inflated by the loading factor λ. Hence, the first condition in Proposition 4 may be interpreted as the equality between marginal willingness to pay and marginal cost of coverage. The second equation is just a rewriting of equation (1.8) for the indemnity schedule $I^*(x_i)$.

[10] This result has been generalized in many directions. Gollier & Schlesinger (1996) for example, demonstrate that a deductible second-degree stochastically dominates any other feasible insurance policy. For more on the robustness of Arrow's optimality result, see Gollier (2013).

only asymptotically when $\pi \longrightarrow 0$. Otherwise, the optimal indemnity schedule would involve type-dependent deductibles d_i, with $I_i(x_i) = \max\{x_i - d_i, 0\}$. This is because lower deductibles would allow the regulator to transfer wealth from more to less risky types (say from the groups with q_i high to the groups with q_i low if the conditional distribution of losses $F_i(x_i)$ is the same for all groups). For low probability risks, this compensatory effect vanishes as π goes to 0.

1.4 The nuclear corporate liability case

1.4.1 The cost of capital

Financial innovations have been developed during the two last decades in order to transfer large scale catastrophic risks to financial markets.[11] Focusing attention on the cat bond market, we may write $c(\pi, K) = s(\pi, K)K$, where $s(\pi, K)$ denotes the spread over LIBOR for a cat bond.

The empirical literature has developed a number of cat bond pricing models, of which we present four examples in Appendix 1.6.7. However, these models suffer from a lack of theoretical foundations and they predict unrealistically high spreads for cat bonds with very low probability triggers.[12] We therefore develop in Appendix 1.6.2 a simple one factor cat bond pricing model with the following features. The representative investor is assumed to be risk averse. In addition to the compensation for his expected loss, he therefore demands a premium for the systemic component of the risk that is correlated with his own wealth. He also requires a compensation for the underwriting and verification costs induced by the cat bond transaction. Our predictions for low probability cat bonds will therefore lie between two extremes. Spreads will be lower than those predicted by the existing models, presented in Appendix 1.6.7, but higher than those predicted in a model with risk neutral investors and no fixed cost. Our pricing equation is

[11] See Cummins & Barrieu (2013).

[12] In these models, either $c_0^* = c(0, K)$ is prohibitively large or $c''_{\pi K}(0, K) = +\infty$, which makes risk coverage unattractive when π is very small.

as follows

$$s = \pi(1+\mu)\mathbb{E}(\tilde{x}) + \eta\kappa(1+\mu)\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K + \frac{D}{K}, \tag{1.10}$$

where $\tilde{x}$ is the fraction of the cat bond's capital lost by investors when the cat bond defaults, and η and κ respectively reflect the representative investor's degree of risk aversion and the exposure of his own wealth to the catastrophe. Finally, μ is a loading that covers the verification costs that the investor incurs when the cat bond defaults. While the first term of equation (1.10) is the spread that would be required by a risk neutral investor, the second term reflects a risk premium. Finally, D is a fixed underwriting cost independent of the size K or probability π of a capital loss.

Based on this model, we estimate the following regression

$$s_i = \beta_0\pi_i\mathbb{E}(\tilde{x_i}) + \beta_1\pi_i[\mathbb{E}(\tilde{x_i}^2) - \pi(\mathbb{E}\tilde{x_i})^2]K_i + \frac{\beta_2(1+\sum_i \gamma_i X_i)}{K_i} + \varepsilon_i, \tag{1.11}$$

by using information from the Artemis database on cat bond transactions.[13] s_i denotes the spread over LIBOR of cat bond $i = 1, ..., n$. If m€K_i is issued through cat bond i, the corresponding cost of capital incurred by the issuer is $c_i = s_i K_i$. The spread of cat bonds is explained by the expected loss per €, π_i, conditional expected loss $\mathbb{E}(\tilde{x_i})$, conditional expected loss squared,[14] capital issued K_i, and a vector of observable controls X_i, such as year of issuance and zone of peril covered that may affect the fixed underwriting cost.

The Artemis database contains more than two-hundred issues, some of which are divided into several tranches, characterized by different levels of risk, and therefore by different spreads. We restrict our analysis to 185 of the most recent tranches, spanning an interval of six years (2011-2017), for which we have complete information, including the nature of perils, types of trigger, probability of a

[13]http://www.artemis.bm/

[14]We only possess information on the expected value of the random variable $\tilde{x}$. We therefore compute $\mathbb{E}(\tilde{x_i}^2)$ by making the assumption that $\tilde{x}_i$ is uniformly distributed over an interval $[a_i, 1]$. We then calibrate a_i to match the expected value of the uniform distribution with its empirical counterpart $\mathbb{E}(\tilde{x_i})$.

capital loss, expected loss,[15] spreads, and identity of sponsors. Relevant controls also include the year of issuance, the area of the peril covered, and the type of trigger. Appendix 1.6.5 and 1.6.7 show that the cat bonds in our data set have characteristics similar to those used in a recent study by Braun (2015).

Table 1.1 gives the main OLS estimates of this regression.[16] All parameters

Table 1.1: OLS estimates for model 1.11

	β_0	β_1	β_2
Estimates	1.4693***	0.0027**	0.5129
t-statistics	(10.5472)	(2.1438)	(0.9366)
R^2	0.7794		

are positive and consistent with theory. The first parameter β_0 is estimated to be 1.4693, which indicates the presence of a loading around forty-seven percent. The second parameter β_1, that identifies the representative investor's risk aversion, is statistically significant at a 5% level. The second term of the regression will play an important role, due to the large values taken by K, the cat bond's capital. Finally, the third parameter β_2, that captures the cat bond's fixed cost D, is estimated at 0.5129, which implies a fixed cost of €512,900. For a 100 million euros cat bond, this corresponds to a spread of 0.51% due to the fixed underwriting costs, a much more reasonable estimate than the 2.64% predicted by the standard linear model of Lane et al. (2008).[17]

Compared with alternative models, ours features three main differences. First, unlike competing models, we allow for a cost of capital $c = s(\pi, K)K$ which is non-linear in K, giving rise to increasing marginal cost of capital. Secondly, when multiplied by K the positive intercept of our regression is a fixed cost (i.e., a component of the capital cost independent from π and K) that can be interpreted as an underwriting cost, absent from other models. The third difference lies in that our model satisfies condition $c'_K \to 0$ when $\pi \to 0$. Violation of this condition in most competing models comes from the positive intercept in the spread equation

[15] The probability of a capital loss and the distribution of losses are evaluated by modeling companies independent from the sponsor and the investor.

[16] The full table, along with alternative specifications is reported in Appendix 1.6.6.

[17] See Appendix 1.6.8.

(or from a very concave relationship between the spread and the loss probability) which has no theoretical foundation. Appendix (1.6.8) compares our model with competing models in the literature. Overall, goodness of fit is comparable to that of competing models.

In the model of section 1.3, we have assumed $\tilde{x} = 1$, which, for our cat bond,[18] gives

$$c(\pi, K) = \beta_0 \pi K + \beta_1 \pi (1 - \pi) K^2 + \beta_2, \tag{1.12}$$

and in particular

$$c''_{K\pi}(0, K) = \beta_0 + 2\beta_1 K,$$

which is an ingredient of the formula provided in Proposition 4.

1.4.2 Individual lotteries

As in Eeckhoudt et al. (2000), we make use of the aggregate information on costs and probabilities drawn from Probabilistic Safety Assessment (PSA) studies[19] to construct individual lotteries. We consider the risk associated with one major accident on the French territory.[20] The 58 French nuclear reactors are gathered into 19 power plants. Based on Eeckhoudt et al. (2000), we assume that 2 million people live around each power plant. Therefore 38 million people are located near a power plant (less than 100km) and 28 million people live further away. We index these two groups by $i = 1, 2$, with shares in the population $\alpha_1 = 38/66$ and

[18]For simplicity, we have designed a simple cat bond that defaults entirely in case of a catastrophe. In addition, the cat bond we are interested in belongs by design to the reference group of our econometrics specification, which is why the dummy controls do not appear in equation (1.12).

[19]The Probabilistic Safety Assessment (PSA) studies assess the odds and the stakes of a major accident along several dimensions: sanitary, environmental, economic, etc. They deliver probability and cost estimates for various accident scenarios, presented in Dreicer et al. (1995) and Markandya (1994). Additional studies from international agencies, such as the French Institute for Radioprotection and Nuclear Safety (IRSN (2013)) and the Nuclear Energy Agency (NEA (2000)), also develop the methodology for estimating the costs associated with the various accident scenarios predicted by PSA studies.

[20]We use ST21 as a benchmark for the number of direct victims in our baseline scenario. The PSA studies referenced in the previous footnote provide the technical background on which ST21 relies.

$\alpha_2 = 28/66$, respectively. We let π denote the probability that a major nuclear accident affects the territory. Most PSA studies provide very low estimates ranging from 10^{-4} to 10^{-9} per year and per reactor. We will use in our computation $\pi = 58 * 10^{-5}$,[21] but since we approximate the optimal level of capital by its limit value, this calibration does not affect our results about the optimal coverage and deductible K^*, d^*, but it does affect the premium P.

For any individual, the potential direct consequences of a nuclear accident may include financial losses, severe disease and death, and it is these losses that may be subject to compensation under corporate liability law. Other losses are supposed to be evenly spread over the whole population. When an accident occurs, an individual of group 1 has a probability 1/19 of living nearby the damaged power plant ($<$ 100 km),[22] in which case he can die, or suffer a severe disease, or a financial loss if he lives in the plume of radioactivity. With probability 18/19, he lives away from the damaged power plant ($\geq$ 100 km), similar to a person from group 2, and can die or suffer a severe disease. The direct financial losses are incurred only by people in group 1, and may result from the impossibility to stay in a contaminated area.

We use figures similar to Eeckhoudt et al. (2000) to calibrate our baseline scenario. The number of direct victims in the baseline scenario (scenario 1) is summarized in Table 1.2.

Table 1.2: Repartition of losses in scenario 1

Distance	Population	Financial loss	Death	Severe disease
$<$ 100 km	2 million	10,000	500	1,000
$\geq$ 100 km	64 million	0	3,000	6,000

We assume that each person in the most exposed group (i.e., individuals from group 1, living within 100 km of a power plant) can potentially be in 6 distinct states (3 health states $\times$ 2 financial states) $s_1 = 1, ..., 6$. Other individuals never incur the direct financial loss, so they can only be in three different health states

[21] We neglect the possibility that accidents may occur simultaneously in several power plants.

[22] For simplicity, we assume that the 19 power plants have the same number of reactors. This approximation has very little impact on our results.

$s_2 = 1, ..., 3$. The lotteries associated with the baseline scenario are summarized in Tables 1.3 and 1.4. The initial wealth w is calibrated in euros, as the sum of the asset value currently held, plus the expected discounted future wealth of the average French citizen, which yields $w = 875,310$ euros.[23]

People from group 1 die in states $s_1 = 1$ and 2. They also suffer a financial loss in state $s_1 = 1$ (and not in state $s_1 = 2$). The worst possible case is represented by the loss of a fraction $1 - \omega$ of total wealth, where ω can be interpreted as a bequest parameter. We choose the parameter ω so as to match the value of a statistical life (VSL) recommended for cost-benefit analysis with a HARA utility function.[24] In particular, our baseline calibration with $\omega = 10\%$ implies Values of a Statistical Life between 3 and 4 million euros, consistent with the estimates provided in Viscusi & Aldy (2003) meta-analysis and with Quinet et al. (2014), which sets the standard for cost-benefit analysis in France.[25]

People in state $s_1 = 3$ do not die but they face the combined consequences of a severe disease and financial losses. In states $s_1 = 4$ and $s_1 = 5$, they suffer either the severe disease or the financial shock, respectively, while in state $s_1 = 6$ they do not incur direct losses. Table 1.3 presents these loss levels and the corresponding probability conditional on the occurrence of a nuclear accident.[26]

Concerning group 2, individuals die in state $s_2 = 1$, suffer a severe disease in state $s_2 = 2$ and face no direct loss in state $s_2 = 3$.

To these direct consequences, subject to compensation under corporate liability law, one must add more diffuse economic costs that are qualified as indirect costs in Schneider (1998) and subsequent studies. They are difficult to quantify and attribute to a given individual. Examples of such costs are: the loss of attractiveness of an impacted territory, loss in terms of image for the industrial sector, etc.[27] For

[23]The details of this calibration are presented in Appendix 1.6.4.

[24]The HARA utility function does not display a divergent index of absolute risk aversion when ω goes to zero, except in the limit CRRA case. See equation 2.7 below.

[25]Appendix 1.6.9 shows the robustness of our analysis to a change in the parameter ω.

[26]The state probabilities in Tables 1.3 and 1.4 are also conditional on belonging to group 1 and 2, respectively.

[27]Here we do not discuss the effect of the catastrophe on growth, as the literature has not reached a consensus on the growth effect of disasters. For example, Gignoux & Menéndez (2016) find a positive effect for the case of an earthquake in India, while Strobl (2012) finds a negative

Table 1.3: Lotteries for type $i = 1$

State	Description of direct losses	Direct loss	Total loss	Probability
$s_1 = 1$	Death + financial loss	787,780	787,780	7.8947e-08
$s_1 = 2$	Death	717,780	719,220	5.7513e-05
$s_1 = 3$	Disease + financial loss	330,000	331,440	1.3158e-07
$s_1 = 4$	Disease	260,000	261,440	1.1500e-04
$s_1 = 5$	Financial loss	70,000	71,440	2.6297e-04
$s_1 = 6$	No direct loss	0	1,440	9.996e-01

simplicity, we assume that these costs are evenly shared by all individuals in the economy[28] and we keep the total cost of the accident fixed at 100 billion euros. In group $i = 1$, agents in state $s_1 = 6$ only face the indirect loss from the accident. Total losses are obtained by adding direct and indirect losses.

Alternative scenarios (scenario 2,3,4 and 5) are generated by multiplying the number of direct victims considered in Table 1.2 by 2,3,4 and 5, respectively, while reducing the value of indirect losses so as to keep the total cost fixed at 100 billion euros. Total direct losses range from 5 billion euros in scenario 1 to approximately 25 billion euros in scenario 5. Total indirect losses therefore vary between 75 and 95 billion euros. In tune with the more recent studies on nuclear risk (Rabl & Rabl (2013)), we consider scenario 3 as the central scenario and baseline scenario 1 as a lower bound on the consequences of a large-scale accident. Because we assume that indirect losses are mutualized, they only marginally affect the optimal coverage level. Hence, as far as corporate liability is concerned, the assumption that total cost is 100 billion euros is innocuous.[29]

effect for the case of hurricanes in the Caribbean.

[28]We could also treat these indirect costs as uninsurable background risks. Under the risk vulnerability assumption, these background risks would increase the degree of risk aversion to insurable risks.

[29]In particular, assuming a total cost of 50 or 200 billion euros would not significantly modify our results.

Table 1.4: Lotteries for type $i = 2$

State	Description of direct losses	Direct loss	Total loss	Probability
$s_2 = 1$	Death	717,780	719,220	4.6875e-05
$s_2 = 2$	Disease	260,000	261,440	9.3750e-05
$s_2 = 3$	No direct loss	0	1,440	9.999e-01

1.4.3 Optimal coverage

We postulate a harmonic absolute risk aversion (HARA) utility function

$$u(x) = \zeta\Big(\eta + \frac{x}{\gamma}\Big)^{1-\gamma},$$

whose domain is such that $\eta + (x/\gamma) > 0$, and with the condition $\zeta(1-\gamma)/\gamma > 0$, that guarantees that $u(x)$ is increasing and concave. With affine risk tolerance $T(x) = 1/A(x) = \eta + x/\gamma$, the coefficient of relative risk aversion is

$$R(x) = x\Big(\eta + \frac{x}{\gamma}\Big)^{-1}. \tag{1.13}$$

The HARA class nests the constant relative risk aversion (CRRA) case when $\eta = 0$, and the constant absolute risk aversion (CARA) case when $\gamma \to +\infty$. Except for the CARA and CRRA limit cases, HARA functions satisfy decreasing absolute risk aversion and increasing relative risk aversion. Studies on individual data, such as Levy (1994) and Szpiro (1986), have isolated a plausible range between 1 and 5 for the index of relative risk aversion. We therefore perform simulations over this plausible range of values.

The optimal values of the deductible and capital are deduced from Proposition 4 and Section (1.4.1). They are reported in Table 1.5 for a level of relative risk aversion $\overline{R} := R(w) = 2$, which is our baseline assumption.[30] Since the relative risk aversion has two degrees of freedom in the HARA case, we let $\underline{R} := R(w - L(s_1))$, where $L(s_1)$ is the loss incurred in state s_1 by group 1 individuals, vary across columns.[31] The scenario considered varies across lines.

[30] A wider set of assumptions, with an index of relative risk aversion $\overline{R}$ varying from 1 to 5, is considered in Appendix 1.6.9.

[31] In other words, $\overline{R}$ and $\underline{R}$ denote the index of relative risk aversion, in the no accident state

Table 1.5: Optimal cover (in €billion), Welfare gain, Annual premium (in €millions), Deductible (in €hundreds of thousands), $\overline{R} = 2$

$\underline{R}$	1		2	
Scenario	Cover	Welfare	Cover	Welfare
1	0.6982	0.0562	0.7636	0.0791
2	0.9829	0.0825	1.1204	0.1213
3	1.1693	0.0972	1.3740	0.1472
4	1.3060	0.1056	1.5726	0.1637
5	1.4125	0.1030	1.7360	0.1742
$\underline{R}$	1		2	
Scenario	Premium	Deductible	Premium	Deductible
1	1.8759	5.6588	2.0825	5.5150
2	2.8731	6.1122	3.4459	5.9612
3	3.6640	6.3355	4.6588	6.1855
4	4.3138	6.4742	5.7502	6.3278
5	4.8604	6.5708	6.7409	6.4286

Optimal levels of coverage (in billion euros) and their associated welfare gains are read from the top panel of Table 1.5. Annual premiums (in millions of euros) and deductibles (in hundreds of thousands of euros) are read from the bottom panel. If we consider a central set of assumptions with scenario 3, $\underline{R} = 2$ and $\overline{R} = 2$ (i.e. the CRRA case), we find an optimal level of coverage K^* equal to €1.3740 billion, an associated welfare gain of 14.72%, a deductible of €$618,550$ per inhabitant, and an annual premium of €4.6588 million (just below 7 cents per person). This yields a spread $s = 4.6588/1374.0 = 0.39\%$ that is one order of magnitude above the spread that a risk neutral investor would require in the absence of underwriting costs. In principle, these fixed underwriting costs can be an issue for the insurability of low probability events, but in our setting they are divided among a large number of agents and therefore have a small impact on each agent.

Table 1.5 highlights the dependence of the coverage and annual premium on the catastrophe scenario. When $\underline{R} = 1$ and $\overline{R} = 2$, multiplying the number of people in each category of loss by 5 (i.e. comparing scenario 1 and 5) induces an

and in the worst case state, respectively.

increase in cover by a factor 2.02 and 2.27, respectively. The fact that coverage increases at a slower pace than direct losses is an intuitive result that is due to the increasing marginal cost of capital.

The deductible varies between €$551,500$ and €$657,080$ in Table 1.5. This represents more than half of the individual's wealth, which implies that only people in the worst states ($s_1 = 1, 2$ for group 1 and $s_2 = 1$ for group 2) are indemnified. Table 1.5 also confirms the intuition that deductibles should decrease with risk aversion, but the effect is quantitatively limited. Finally, the deductible increases with the severity of the loss scenario, which reflects our previous remark on the effect of increasing marginal cost of capital on optimal coverage. As more capital is needed to compensate the victims with the largest losses, it is optimal to increase the deductible in order to avoid a sharp increase in premiums.

The welfare gain is computed as the reduction in the loss certainty equivalent induced by the cover in comparison with the case without any compensation.[32] The welfare gain is therefore estimated at least at 14.72% under scenario 3 with $\overline{R} = 2$ and $\underline{R} = 2$. This means that the average monetary equivalent cost of the nuclear risk is lowered by 14.72% thanks to the indemnity schedule when $K^* =$ €1.3740 billion. Of course, welfare gains for group 2, taken separately, would be higher. Higher values for the coefficients of relative risk aversion, or a more pessimistic loss scenario would lead to much higher values of K^* and substantially higher welfare gains.

Note finally that in scenario 3, K^* is substantially higher than the lower bound of nuclear operator's liability adopted in 2004 through the revision of the Paris convention, which is €700 million for each accident. Only scenario 1 under the assumption $\underline{R} = 1$ and $\overline{R} = 2$ yields an optimal liability slightly lower than €700 million, while all other cases considered here deliver higher values. The fact that several other European countries[33] have set nuclear corporate liability at higher

[32]Since group 1 and group 2 do not face the same risk exposure, this reduction differs from one group to the other. The figure presented in Table 1.5 is an average of these two gains weighted by group size.

[33]Countries have their own legislation, in line with international conventions. For instance, in Germany, the nuclear corporate liability is set at €2.5 billion for each accident. This could be rationalized in our model with scenarios more severe than our scenario 5, or with higher levels

levels is coherent with such a conclusion.

1.5 Conclusion

The purpose of this paper was to analyze the insurability of low-probability high-severity events, simultaneously from the individual's standpoint and from a public policy perspective.

We have shown that the risk premium of such catastrophic events can remain large when the accident probability is close to zero, if the index of absolute risk aversion is sufficiently large (or equivalently if risk tolerance is sufficiently low) in the accident state. In addition, the optimal indemnity converges to a positive limit that reflects both the individual's attitude toward risk and the cost of insurance. In the case of an industrial catastrophe that may affect the whole population of a country, the asymptotic indemnity schedule is characterized by a straight deductible, common to all individuals.

Based on these results, we have analyzed the features of an optimal insurance scheme that covers the nuclear coporate liability, in which the risk is transferred to financial markets through cat bonds. Using recent cat bond data and safety studies on nuclear reactors allows us to compute the optimal level of coverage. Our results, calibrated with French data, suggest that the nuclear liability law could be more ambitious than it currently is, unlike in other countries, such as Germany, where this liability has been extended far beyond the requirements of international conventions.

The ratification of the 2004 Protocol in France was made difficult by the large induced capital costs, but also by the insurers' reluctance to extend the validity of health-related claims to thirty years. Added to the conservative position of the nuclear industry, this explains why the corporate nuclear liability has not been extended further.

Our analysis presents a certain number of limits that we shall now discuss.

of risk aversion, such as the ones considered in Appendix 1.6.9. Note that the Paris convention also specifies tranches of liability born by governments, so that total liability toward the victims are at least €1.5 billion.

First, we implicitly assume that market insurance is the only tool available to deal with catastrophic risk. In practice, individuals and societies have other means at their disposal. The effect of self-insurance -a reduction in the size of the loss- and self-protection -a reduction in the loss probability- were studied in a seminal paper by Ehrlich & Becker (1972). Because market insurance and self-insurance are perfect complements, our theoretical results would apply directly to self-insurance. On the other hand, Ehrlich and Becker have shown that self-protection and market insurance, can be complements. The complex analysis of the interaction between self-insurance, self protection and market insurance for catastrophic risk is not addressed in our model and is left for further research.

Our conclusions should also be put in the broader context of how law - be it in the common law or civil law traditions - define corporate liability, and how the liability regime affects the role of insurance.[34] Based on the international conventions on nuclear tort law, we have assumed that liability is strict. As opposed to a negligence rule, strict liability dictates that a nuclear operator must indemnify victims in case of accident, whether or not he was negligent. This strict liability avoids time-consuming litigation about who the liable entity is, hence facilitating a prompt indemnification of victims. Another approach would consist in adopting the negligence rule, according to which the operator of the nuclear installation is held liable for accident losses only if it exercised a level of care lower than a level specified by courts, hence without liability cost for a firm that complied with safety regulation. The main drawbacks of the negligence rule when applied to corporate nuclear liability are related to the difficulties for courts to establish that the nuclear operator did not exercise due care levels. This weakens the incentives for risk prevention and, furthermore, it opens the door to legal actions that create obstacles to the rapid compensation of victims. If, after an accident, the court considers that the operator has not been negligent and should be exonerated from liability, then the negligence rule shifts the compensation of victims from corporate liability insurance onto the government's budget. The cost of public fund would then be a key ingredient of the optimal liability level.

[34]For more on the impact of tort law on incentives, see Shavell (2009) and Cooter & Ulen (2008).

The upper limit that some countries grant to nuclear operators can be seen as a compensation for the heavy burden that strict liability imposes on the nuclear operators. In any case, it generates the necessity for states to provide coverage on the upper layers of risk. The 2004 revision of the Paris Convention indeed requires governments to provide for at least an additional €800 million in coverage but in practice, the damages not covered by liability law are also incurred by governments and individuals.

In a dynamic model with uninsurable risk, prudent agents save to constitute a buffer stock, used in case of loss. If market insurance is sold at actuarially fair prices, expected utility maximizing agents should purchase full insurance and make no precautionary savings. However, positive insurance loadings may lower the demand for market insurance, substituted with precautionary savings. Gollier (2003) showed, with a calibrated example, that the demand for market insurance may become quite low whenever assets enable agents to transfer wealth across periods. However, his example only discusses the case of small losses. The strategy of substituting market insurance with precautionary savings would not be feasible at the level of individual agents who risk up to their lives. It could more realistically be set up at the level of a state who would face a choice between a funded and a pay-as-you-go system, in which it simply borrows when a catastrophe occurs. Both strategies would entail benefits comparable to the insurance scheme proposed in our paper but would involve a cost of public fund. In this regard, Borensztein et al. (2017) show that cat bonds may yield substantial gains to governments, for they lower their risk of default in case of catastrophe. This suggests that funding the coverage of catastrophic risks through cat bonds, and transferring losses to future periods through credit markets should not be seen as substitutes: they are complements.

1.6 Appendix

1.6.1 Complement to section 1.3.1

Let us assume that the government can redistribute wealth between groups through ex ante lump sum transfers. We denote t_i the net transfer paid to each individual of group i, the government budget constraint being written as

$$\sum_{i=1}^{n} \alpha_i t_i = 0.$$

Now we have

$$\begin{aligned} w_1 &= w - P + t_i, \\ w_{2i}(x_i) &= w - P - x_i + I_i(x_i) + t_i. \end{aligned}$$

and the certainty equivalent loss incurred by type i individuals is still denoted by C_i, with

$$\begin{aligned} u(w - C_i + t_i) &= (1 - \pi q_i) u(w_1 + t_i) \\ &+ \pi q_i \int_0^{\overline{x}_i} u(w_{2i}(x_i) + t_i) f(x_i) dx_i. \end{aligned} \tag{1.14}$$

An allocation is written as $\mathcal{A} = \{w_1, w_{21}(x_1), ..., w_{2n}(x_n), C_1, ..., C_n, t_1, ..., t_n, K\}$, and $\mathcal{A}$ is feasible if (1.7), (1.8),(1.9) and (1.14) are satisfied.

Definition 1 *$\mathcal{A}$ is Pareto-optimal if it is feasible and if there does not exist another feasible allocation $\widehat{\mathcal{A}} = \{\widehat{w}_1, \widehat{w}_{21}(x_1), ..., \widehat{w}_{2n}(x_n), \widehat{C}_1, ..., \widehat{C}_n, \widehat{t}_1, ..., \widehat{t}_n, \widehat{K}\}$ such that $\widehat{C}_i - \widehat{t}_i \leq C_i - t_i$ for all $i = 1, ..., n$, with $\widehat{C}_{i_0} - \widehat{t}_{i_0} < C_{i_0} - t_{i_0}$ for at least one group i_0.*

Proposition 5 *$\mathcal{A} = \{w_1, w_{21}(x_1), ..., w_{2n}(x_n), C_1, ..., C_n, t_1, ..., t_n, K\}$ is a Pareto-optimal allocation if and only if it minimizes $\sum_{i=1}^{n} \alpha_i C_i$ in the set of feasible allocations.*

1.6.2 A cat bond pricing model

This section presents the cat bond pricing model. The cat bond is issued at $t = 0$. Part of its capital is used at time $t = 0$ to pay the underwriting costs and the remainder constitutes the principal. At time $t = 1$ the principal K is returned to the investor if the accident did not occur. In the opposite case, the cat bond defaults and the sponsor uses a fraction $\tilde{x}$ of the capital to indemnify the victims. The remaining portion of capital is returned to the investors. From the standpoint of the investor, the cat bond 's payoff is therefore

$$\tilde{q} = \begin{cases} (1+R)K - (1+\mu)\tilde{x}K & \text{with probability} \quad \pi, \\ (1+R)K & \text{with probability} \quad 1-\pi. \end{cases}$$

In compensation for the option to default on the principal, the investors require a coupon of rate $R = r + s$, where r denotes the risk free rate and s denotes the spread. We let $D/(1 + r)$ be the value of the underwriting costs (i.e. D is the corresponding value at time $t = 1$), and μ is a loading that covers the verification costs.

Let CE be the certainty equivalent of the cat bond payoff $\tilde{q}$ to investors at time $t = 1$. Following the Consumption Capital Asset Pricing Model, we write

$$CE = \mathbb{E}\tilde{q} - \eta cov(\tilde{z}, \tilde{q}),$$

where $\tilde{z}$ denotes the wealth of the representative investor at $t = 1$, and η reflects his risk aversion. There are two states: with probability π, the accident occurs, the cat bond defaults and investors suffer a loss $(1 + \mu)\tilde{x}K$; with probability $1 - \pi$, the accident does not occur and the principal is returned to the investor. In both cases, the coupon RK is paid to the investor.[35] We assume that the representative

[35] Hence we assume that default affects the repayment of the capital to the investor first. The coupon payment is affected only when the loss $\tilde{x}$ is very large and $1-(1+\mu)\tilde{x}$ becomes negative. This assumption is made for simplicity, but of course other definitions of cat bonds are possible.

investor bears a fraction κ of the underlying loss.[36] We therefore write

$$\tilde{z} = \begin{cases} w - \kappa K \tilde{x} & \text{with probability} \quad \pi, \\ w & \text{with probability} \quad 1 - \pi. \end{cases}$$

Thus

$$\begin{aligned} \mathbb{E}\widetilde{q} &= [R + 1 - \pi(1+\mu)\mathbb{E}(\tilde{x})]K, \\ cov(\tilde{z}, \widetilde{q}) &= (1+\mu)\kappa\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K^2. \end{aligned}$$

and

$$CE = [R + 1 - \pi(1+\mu)\mathbb{E}(\tilde{x})]K - \eta(1+\mu)\kappa\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K^2.$$

Purchasing the cat bond is analogous to making an investment K with additional cost $D/(1+r)$ at $t = 0$ and random payoff $\widetilde{q}$, with certainty equivalent CE, at $t = 1$. Thus, in the absence of arbitrage, we have

$$K + \frac{D}{1+r} = \frac{CE}{1+r},$$

which may be rewritten as

$$K(1+r) = [R + 1 - \pi(1+\mu)\mathbb{E}(\tilde{x})]K - \eta(1+\mu)\kappa\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K^2 - D.$$

Let $s = R - r$ be the spread over the risk-free rate. We obtain

$$s = \pi(1+\mu)\mathbb{E}(\tilde{x}) + \eta\kappa(1+\mu)\pi[\mathbb{E}(\tilde{x}^2) - \pi(\mathbb{E}\tilde{x})^2]K + \frac{D}{K}. \qquad (1.15)$$

In order to estimate this equation on our data set, we assume each $\tilde{x}_i$ is uniformly distributed in an interval $[a_i, 1]$. This enables us to find $\mathbb{E}(\tilde{x}_i{}^2)$ which, in turn, leads to the regression performed in section 1.4.1.[37] We only have one loss scenario in

[36] We do not restrict κ and will estimate it from the data. From a theoretical perspective, the precise value of κ depends on the identity of the representative investor. If the representative investor is not exposed to the underlying risk transferred by the cat bond, we should have $\kappa \equiv 0$.

[37] The Artemis data base provides π_i and $\mathbb{E}\tilde{x}_i$ for each cat bond i in the sample. We deduce

our numerical analysis. Hence, the cat bond must completely default in case of accident, which implies that $\mathbb{E}(\tilde{x}) = 1$ for our cat bond. The cost of capital $c(\pi, K) \equiv s(\pi, K)K$ is therefore

$$c(\pi, K) = \pi(1+\mu)K + \eta\kappa(1+\mu)\pi(1-\pi)K^2 + D,$$

which is coherent with the assumptions used to derive Proposition 4.

1.6.3 Proofs

Proof of Proposition 1

From equation (1.2.1), we have

$$C_p'(0, L) = \frac{u(w) - u(w-L)}{u'(w)} = \int_{w-L}^{w} \frac{u'(x)}{u'(w)} dx.$$

Since

$$u'(x) = u'(w) - \int_x^w u''(t)dt,$$

for all $x \in [w-L, w]$, we may write

$$\begin{aligned} C_p'(0, L) &= L - \int_{w-L}^{w} \left[\int_x^w \frac{u''(t)}{u'(w)} dt \right] dx \\ &= L + \int_{w-L}^{w} \left[\int_x^w A(t) \frac{u'(t)}{u'(w)} dt \right] dx, \end{aligned}$$

and thus

$$\theta(0, L) = \frac{1}{L^2} \int_{w-L}^{w} \left[\int_x^w A(t) \frac{u'(t)}{u'(w)} dt \right] dx.$$

Integrating by parts gives

$$\theta(0, L) = \frac{1}{2} \int_{w-L}^{w} k(x) A(x) \frac{u'(x)}{u'(w)} dx, \tag{1.16}$$

$\mathbb{E}(\tilde{x}_i)^2$.

where $k(x) = 2[x - (w - L)]/L^2$, with

$$\int_{w-L}^{w} k(x)dx = 1.$$

In addition, we have

$$u'(x) = u'(w)\exp\{\int_{x}^{w} A(x)dx\},$$

which completes the proof.

Proof of Corollary 1.2

When $L = w$, we have

$$\theta(0, L) > \frac{1}{w}\int_{0}^{w} \frac{xu'(x)}{wu'(w)} A(x)dx,$$

from Proposition 1. Furthermore, we have

$$\begin{aligned} \frac{d[xu'(x)]}{dx} &= xu''(x) + u'(x) \\ &= -u'(x)[R(x) - 1], \end{aligned}$$

and thus

$$\frac{d[xu'(x)]}{dx} \leq 0 \text{ if } R(x) \geq 1.$$

We deduce

$$\theta(0, L) > \frac{1}{w}\int_{0}^{w} A(x)dx \text{ if } R(x) \geq 1.$$

Proof of Proposition 2

Using $A' \leq 0$ in equation (1.16) allows us to write

$$\theta(0, L) \leq \frac{A(w - L)}{L^2 u'(w)} \int_{w-L}^{w} [x - (w - L)]u'(x)dx$$

Using $R(x) \leq \overline{\gamma}$ and $u''(x) < 0$ yields

$$\begin{aligned}
\frac{d}{dx}[(x-(w-L))u'(x)] &= u'(x)\Big[1 - R(x) - \frac{u''(x)}{u'(x)}(w-L)\Big] \\
&\geq u'(x)[1-R(x)] \\
&\geq u'(x)(1-\overline{\gamma}) \\
&\geq u'(w)(1-\overline{\gamma}),
\end{aligned}$$

for all $x \in [w-L, w]$. Hence, we have

$$\begin{aligned}
\Big[x-(w-L)\Big]u'(x) + (w-x)u'(w)(1-\overline{\gamma}) &\leq [w-(w-L)]u'(w) \\
\Big[x-(w-L)\Big]u'(x) &\leq Lu'(w) + (w-x)u'(w)(\overline{\gamma}-1) \\
&= u'(w)[L+(w-x)(\overline{\gamma}-1)],
\end{aligned}$$

for all $x \in [w-L, w]$. Consequently,

$$\begin{aligned}
\theta(0,L) &\leq \frac{A(w-L)}{L^2u'(w)} \int_{w-L}^{w} \{u'(w)[L+(w-x)(\overline{\gamma}-1)]\}\, dx \\
&= \frac{A(w-L)}{L^2}\left[\frac{L^2(\overline{\gamma}+1)}{2}\right] \\
&= \frac{A(w-L)(\overline{\gamma}+1)}{2}.
\end{aligned}$$

Using $C_p'' < 0$ and $C(0,L) = 0$ allows us to write

$$\begin{aligned}
C(p,L) &< C'(0,L)p \\
&= pL + \theta(0,L)pL^2 \\
&\leq pL\left[1 + \frac{A(w-L)(\overline{\gamma}+1)L}{2}\right].
\end{aligned}$$

Proof of Corollary 2.1

Combining Lemma (1) with Corollary (1), shows that

$$\lambda \;\leq\; \frac{L}{2}\int_{w-L}^{w}\frac{k(x)}{x}R(x)dx$$

is a sufficient condition insurance take-up to be positive.

If $R(x)$ is non decreasing, then

$$\begin{aligned}
\frac{L}{2}\int_{w-L}^{w}\frac{k(x)}{x}R(x)dx &\geq \frac{LR(w-L)}{2}\int_{w-L}^{w}\frac{k(x)}{x}dx \\
&= \frac{R(w-L)}{L}\int_{w-L}^{w}\frac{x-(w-L)}{x}dx \\
&= R(w-L)[1-(\frac{w-L}{L})\ln\frac{w}{w-L}] \\
&\equiv \Psi(L) \quad L\in[0,w].
\end{aligned}$$

Noticing that $\lim_{L\to w}\psi(L) = \lim_{x\to 0}R(x)$ provides the result.

Proof of Proposition 4

The planner's program is to minimize $\sum_i \alpha_i C_i$ under constraints (1.6), (1.7), (1.8) and (1.9). The Kuhn-Tucker multipliers associated with each set of constraints are respectively γ_i, $\phi_i(x_i)$, η and ρ. The optimality conditions are

$$\alpha_i - \gamma_i u'(w-C_i) = 0 \tag{1.17}$$

$$\gamma_i\pi q_i u'(w_{2i}(x_i))f_i(x_i) - \eta(1+\lambda)\alpha_i q_i f_i(x_i) + \phi_i(x_i) = 0, \tag{1.18}$$

$$u'(w_1)\sum_{i=1}^{n}(1-\pi q_i)\gamma_i - \sum_{i=1}^{n}\int_0^{\bar{x_i}}\phi_i(x_i)dx_i - \rho + \eta(1+\lambda)\sum_{i=1}^{n}\alpha_i q_i = 0, \tag{1.19}$$

$$-\eta + \rho c'_K(\pi, K) = 0, \tag{1.20}$$

$$\phi_i(x_i)\geq 0 \qquad \text{and} \qquad \phi_i(x_i) = 0 \quad \text{if} \quad w_{2i}(x_i) - w_1 + x_i > 0 \qquad \forall i. \tag{1.21}$$

Let x_i be such that $w_{2i}(x_i) - w_1 + x_i > 0$. Thus, we have $\phi_i(x_i) = 0$ from (1.21), and (1.18) gives

$$\pi\gamma_i u'(w_{2i}(x_i)) = \eta(1+\lambda)\alpha_i. \tag{1.22}$$

(1.17) and (1.22) yield

$$u'(w_{2i}(x_i)) = \frac{\eta}{\pi}(1+\lambda)u'(w - C_i). \tag{1.23}$$

Hence, if there exist $x_i^0, x_i^1 \in [0, \bar{x}_i]$ such that $w_{2i}(x_i^0) - w_1 + x_i^0 > 0$ and $w_{2i}(x_i^1) - w_1 + x_i^1 > 0$, then we must have

$$u'(w_{2i}(x_i^0)) = u'(w_{2i}(x_i^1)),$$

which implies

$$w_{2i}(x_i^0) = w_{2i}(x_i^1).$$

Consequently, $w_{2i}(x_i)$ is constant over the set of x_i for which $w_{2i}(x_i) - w_1 + x_i > 0$, and we can write

$$w_{2i}(x_i) = w_1 - d_i,$$

with $d_i < x_i$ for all x_i in this set, and from 1.23 we have

$$u'(w_1 - d_i) = \frac{\eta}{\pi}(1+\lambda)u'(w - C_i). \tag{1.24}$$

Now let x_i be such that $w_{2i}(x_i) - w_1 + x_i = 0$. Using (1.17), (1.18) and (1.21) allows us to write

$$u'(w_{2i}(x_i)) = u'(w_1 - x_i) \leq \frac{\eta}{\pi}(1+\lambda)u'(w - C_i).$$

Using (1.23), and $u'' < 0$ we deduce $x_i \leq d_i$. Thus, we have established that there exists d_i such that

$$\begin{aligned} w_{2i}(x_i) &= w_1 - d_i \quad \text{if} \quad x_i > d_i, & (1.25)\\ w_{2i}(x_i) &= w_1 - x_i \quad \text{if} \quad x_i \leq d_i. & (1.26)\end{aligned}$$

Let $K \to K^*$ when $\pi \to 0$ and $c_0^* \equiv \lim_{\pi \to 0} c(\pi, K^*)$. When $\pi \to 0$, we have $w_1 \longrightarrow w - c_0^*$ and $C_i \longrightarrow c_0^*$ from (1.9) and (1.6) respectively. (1.24) then gives

$d_i \longrightarrow d^* \quad \forall i$ with

$$u'(w-d^*) = (1+\lambda)u'(w-c_0^*)\lim_{\pi\to 0}\frac{\eta}{\pi}. \tag{1.27}$$

Using (1.17), (1.19), (1.20) and $\sum_{i=1}^{n}\alpha_i = 1$ imply

$$\lim_{\pi\to 0}\Big[1 - \frac{\eta}{c'_K(\pi,K^*)} + \eta(1+\lambda)\sum_{i=1}^{n}\alpha_i q_i - \sum_{i=1}^{n}\int_0^{\bar{x_i}}\phi_i(x_i)dx_i\Big] = 0. \tag{1.28}$$

Suppose that η does not go to zero when π does. In such a case, we would have $\eta/c'_K(\pi,K^*) \longrightarrow +\infty$ when $\pi \longrightarrow 0$ since $c'_K(\pi,K^*) \longrightarrow 0$, and thus

$$\lim_{\pi\to 0}[\eta[\frac{1}{c'_K(\pi,K^*)} - (1+\lambda)\sum_{i=1}^{n}\alpha_i q_i]] = +\infty.$$

Since $\phi_i(x_i) \geq 0 \quad \forall i$, this is in contradiction with (1.28). Thus, we have

$$\lim_{\pi\longrightarrow 0}\left[1 - \frac{\eta}{c'_K(\pi,K^*)} - \sum_{i=1}^{n}\int_0^{\bar{x_i}}\phi_i(x_i)dx_i\right] = 0. \tag{1.29}$$

If $d_i \leq 0$, we have $w_{2i}(x_i) - w_1 + x_i > 0$ and $\phi_i(x_i) = 0 \quad \forall x_i > 0$. Hence

$$\int_0^{\bar{x_i}}\phi_i(x_i) = 0.$$

If $d_i > 0$, we have $\phi_i(x_i) = 0$ for $x_i > d_i$, and thus (1.17), (1.18) and (1.26) give

$$\begin{aligned}\int_0^{\bar{x_i}}\phi_i(x_i)dx_i &= \int_0^{d_i}\phi_i(x_i)dx_i & (1.30)\\ &= -\pi\alpha_i q_i\int_0^{d_i}[\frac{u'(w-x_i)}{u'(w-C_i)} - \frac{\eta}{\pi}(1+\lambda)]f_i(x_i)dx_i. & (1.31)\end{aligned}$$

Using the fact that $\eta \longrightarrow 0$ when $\pi \longrightarrow 0$ gives

$$\lim_{\pi\to 0}\int_0^{\bar{x_i}}\phi_i(x_i)dx_i = 0,$$

and from (1.29) we derive

$$\lim_{\pi \to 0} \frac{\eta}{c_K'(\pi, K^*)} = 1.$$

Using (1.27) together with L'hôpital's rule, we finally deduce

$$\begin{aligned} u'(w - d^*) &= (1+\lambda) u'(w - c_0^*) c_{\pi K}''(0, K^*) \\ &> u'(w), \end{aligned}$$

where the last inequality derives from $\lambda > 0$ and $c_{\pi K}''(0, K^*) \geq 1$. Using $u'' < 0$ gives $d^* > 0$. Since $I_i(x_i) = w_{2i}(x_i) + x_i - w_1$, we deduce that $I_i(x_i) \longrightarrow I^*(x_i) = \max(x_i - d^*, 0)$ when $\pi \longrightarrow 0$.

Proof of Proposition 5

Assume that $\mathcal{A}$ minimizes $\sum_{i=1}^n \alpha_i C_i$ in the set of feasible allocations, and suppose that it is not Pareto-optimal, then there exists a feasible allocation $\widehat{\mathcal{A}}$ and a group i_0 such that $\widehat{C}_i - \widehat{t}_i \leq C_i - t_i$ for all i and $\widehat{C}_{i_0} - \widehat{t}_{i_0} < C_{i_0} - t_{i_0}$. Consequently,

$$\sum_{i=1}^n \alpha_i (\widehat{C}_i - \widehat{t}_i) < \sum_{i=1}^n \alpha_i (C_i - t_i). \tag{1.32}$$

Since $\mathcal{A}$ and $\widehat{\mathcal{A}}$ are feasible, we have

$$\sum_{i=1}^n \alpha_i t_i = \sum_{i=1}^n \alpha_i \widehat{t}_i = 0, \tag{1.33}$$

and thus (1.32) and (1.33) give

$$\sum_{i=1}^n \alpha_i \widehat{C}_i < \sum_{i=1}^n \alpha_i C_i,$$

which contradicts the fact that $\mathcal{A}$ minimizes $\sum_{i=1}^n \alpha_i C_i$ in the set of feasible allocations.

Conversely, assume that $\mathcal{A}$ is a Pareto-optimal allocation, and suppose that it does not minimize $\sum_{i=1}^n \alpha_i C_i$ in the set of feasible allocations. Thus there exists a feasible allocation $\widehat{\mathcal{A}}$ such that $\sum_{i=1}^{n}\alpha_i\widehat{C}_i < \sum_{i=1}^{n}\alpha_i C_i$, and thus

$$\sum_{i=1}^{n}\alpha_i(\widehat{C}_i-\widehat{t}_i) < \sum_{i=1}^{n}\alpha_i(C_i-t_i). \tag{1.34}$$

Let us choose $\widehat{t}_i$ such that

$$\widehat{t}_i=\widehat{C}_i+t_i-C_i$$

for all $i\neq i_{i_0}$, which does not contradict the feasibility of $\widehat{\mathcal{A}}$ if we choose

$$\widehat{t}_{i_0}=-\sum\nolimits_{i\neq i_0}\widehat{t}_i. \tag{1.35}$$

We have

$$\widehat{C}_i-\widehat{t}_i=C_i-t_i \ \text{ for all } i\neq i_0. \tag{1.36}$$

Furthermore, (1.34),(1.35) and (1.36) give

$$\widehat{C}_{i_0}-\widehat{t}_{i_0}<C_{i_0}-t_{i_0}. \tag{1.37}$$

(1.36) and (1.37) contradict the fact that $\mathcal{A}$ is Pareto-optimal.

1.6.4 Calibration of initial wealth and losses

INSEE, the French national statistical agency, provides an average estimated Gross National Product per capita of 32,227 euros[38] and an average age of 39.2 year old[39]. The French National Institute on Demographics (INED) provides an estimated life expectancy of 73.2 for the average 39.2[40] years old citizen. Lifetime wealth is obtained as the annual GDP per capita discounted at a 2% rate on a 34 year horizon. This yields an expected discounted future wealth of 805,310 euros. INSEE also provides an estimated average of 70,000 euros of current assets, which will be the financial loss that victims may incur. We therefore consider that initial wealth is

[38] http://www.bdm.insee.fr
[39] http://www.insee.fr/
[40] http://www.ined.fr/

875,310 euros.

Group 1

The worst case scenario is a fatal outcome that occurs in states $s_1 = 1$ and 2. As in Eeckhoudt et al. (2000) we assume that when this worst state materializes, the individual (in practice, her heir) is only able to retain a fraction, equal to $\omega = 10\%$ of her initial wealth, that can be interpreted as a bequest parameter. In state $s_1 = 2$, the agent dies but does not suffer the financial loss. Direct losses in these catastrophic states are therefore equal to $875,310(1-\omega) = 787,780$ in state $s_1 = 1$ and $875,310(1-\omega) - 70,000 = 717,780$ in state $s_1 = 2$. In state $s_1 = 3$, the agent suffers a severe health loss due to exposure to radioactivity, as well as a direct financial loss of all her financial assets. The cost of health treatment and the health induced reduction in future income is estimated in Eeckhoudt et al. (2000) at 260,000 euros. The direct loss in this state is therefore equal to 330,000 euros. In state $s_1 = 4$, the agent faces the $260,000$ euros health loss and in state $s_1 = 5$, he faces the 70,000 euros financial loss.

Total losses are obtained by adding to the direct losses the indirect cost of the accident, assumed to be mutualized between all the agents who did not die. In the baseline scenario, the indirect loss is 1,440 euros per inhabitant.

Group 2

Agents in group 2 die in state $s_2 = 1$, face a severe disease in state $s_2 = 2$ and a financial loss in state $s_2 = 3$. Their direct losses are therefore calibrated at 717,780 and 260,000 euros.

1.6.5 Descriptive statistics

Table 1.6 provides the summary statistics for the main variables. At 6.38%, the average spread is lower than in Braun (2016) who finds an average of 8.18% for the period 1997-2012. Average expected loss is very close to Braun (2016) (2.35%

Table 1.6: Descriptive statistics for the 185 cat bonds

Variable	Mean	Median	S.D.	Max	Min
Spread	0.0638	0.0525	0.0392	0.2000	0.0175
Expected loss	0.0235	0.0160	0.0232	0.1306	0.0001
Size (€million)	134.86	108.8984	113.6927	1128.8	17.9453

versus 2.08%) and the average value of capital issued (size) is higher in our data set (134.86 €million versus 97.34 €million), perhaps due to our inability to observe small private transactions.

1.6.6 OLS Estimates

Table 1.7 provides the estimates of regression (1.11) for our fully specified model, by excluding the fixed cost and/or the risk premium among the explanatory variables. Our preferred specification, used to compute the cost of capital in the main text, is given in the three first columns. Expected loss, Risk premium and Size, respectively represent the terms $\pi_i \mathbb{E}(\tilde{x})_i$, $\pi_i[\mathbb{E}(\tilde{x_i}^2) - \pi(\mathbb{E}\tilde{x_i})^2]K_i$, and K_i^{-1}. 2017, Europe and Indemnity are the reference groups for the times dummies, the geographical area covered, and the trigger types, respectively. The coefficient estimates of Expected loss and Risk premium are positive and significant across the four specifications. Concerning the control variables, 2012 was a period of high prices, followed by a decline from 2013 to 2016. The geographical dummies point at the fact that cat bonds covering perils in the US are more expensive than in other countries. This is in accordance with Braun (2015). Table 1.7 also shows that parametric triggers have a lower spread than indemnity triggers, which may be explained by the lower moral hazard entailed by parametric triggers. Finally, the variables RMS, $EQECAT$ and $MILL$ (Milliman) represent three of the four risk modelers that were in charge of the deal. The reference group was taken to be the risk modeler AIR.

The four regressions highlight the important role played by the risk premium term. We report, in the penultimate line of each table, the optimal level of coverage under scenario 3 and assumption $\underline{R} = \overline{R} = 2$. Without the risk premium term, the marginal cost of capital would be constant, hence the higher levels of coverage reporter at the bottom of columns 8 and 11. On the other hand, the fixed cost term does not play a quantitatively important role. It is indeed divided among a large number of people, and therefore represents only a few cents per person. The last lines of each table report the premium paid under the same set of assumptions.

Table 1.7: Pricing equation estimation

	coeff	t stat	sign	coeff	t stat	sign	coeff	t stat	sign	coeff	t stat	sign
Expected loss	1.4693	10.5472	* * *	1.4784	10.8979	* * *	1.7833	15.3571	* * *	1.7829	15.5918	* * *
Risk premium	0.0027	2.1438	**	0.0027	2.1556	**						
Size	0.5129	0.9366					−0.0302	−0.0493				
Size×2011	3.2608	2.9438	* * *	3.5171	3.2699	* * *	3.5163	2.9987	* * *	3.5012	3.1328	* * *
Size×2012	3.0592	7.2809	* * *	3.3496	8.9532	* * *	3.3310	6.9637	* * *	3.3139	8.6398	* * *
Size×2013	1.0007	2.2483	**	1.2867	3.4372	* * *	1.2562	2.4620	**	1.2393	3.0708	* * *
Size×2014	−0.1566	−0.3759		0.1659	0.5986		−0.0484	−0.0949		−0.0676	−0.2069	
Size×2015	−0.0875	−0.2112		0.1817	0.4825		0.1210	0.2471		0.1051	0.2270	
Size×2016	−0.5392	−1.4499		−0.2473	−0.7781		−0.5823	−1.2491		−0.5997	−1.6023	
Size×$Japan$	0.4795	1.1553		0.6737	2.0989	**	0.9291	2.1376	**	0.9178	2.4865	**
Size×US	0.6534	2.2530	**	0.8425	4.3147	* * *	0.8244	2.7429	* * *	0.8132	3.7267	* * *
Size×$Other$	−0.0799	−0.1697		0.1430	0.3863		0.0107	0.0207		−0.0026	−0.0062	
Size×$Index$	−0.1103	−0.2788		−0.0503	−0.1284		−0.1159	−0.2492		−0.1195	−0.2555	
Size×$Param$	−0.9960	−2.9505	* * *	−0.9671	−2.9170	* * *	−1.1100	−3.0231	* * *	−1.1117	−3.0788	* * *
RMS	1.5653	4.7824	* * *	1.6095	5.1344	* * *	1.8956	5.4912	* * *	1.8932	5.5791	* * *
EQECAT	−0.2593	−0.3561		−0.1625	−0.2272		0.6917	1.8531	*	0.6865	1.8602	*
MILL	0.3158	1.0118		0.4695	1.9151	*	0.5640	1.5902		0.5550	1.8602	*
R^2	0.7985			0.7980			0.7405			0.7405		
adjustedR^2	0.7794			0.7801			0.7175			0.7191		
$s^2(\times 10^{-04})$	3.3951			3.3836			4.3477			4.3221		
AIC$(\times 10^{-04})$	3.7051			3.6747			4.7216			4.6709		
BIC$(\times 10^{-04})$	4.9811			4.8549			6.2381			6.0646		
K(billions)	1.3740			1.8576			6.6142			6.6150		
P(millions)	4.6588			6.9791			6.8108			6.8404		

1.6.7 Comparison with alternative data set

This section compares our data set with Braun (2016)'s. In order to do so, we compare the four cat bond pricing models estimated in Braun (2016), on a sample of 466 cat bond tranches covering a period from 1997 to 2012 (Table 1.8), with the same models estimated on our data set (Table 1.9). The first model specifies spreads as a linear function of expected loss

$$s_i = \hat{\alpha} + \hat{\beta}\pi_i\mathbb{E}(\tilde{x})_i. \tag{1.38}$$

The second model has spread as a polynomial of the natural logarithm of the expected loss

$$s_i = \hat{\alpha} + \hat{\beta}\ln\pi_i\mathbb{E}(\tilde{x})_i + \hat{\gamma}[\ln\pi_i\mathbb{E}(\tilde{x})_i]^2. \tag{1.39}$$

The third model is from Lane (2000) and specifies

$$s_i = \pi_i\mathbb{E}(\tilde{x})_i + \hat{\alpha}\pi_i^{\hat{\beta}}\mathbb{E}(\tilde{x})_i^{\hat{\gamma}}. \tag{1.40}$$

Finally, Major & Kreps (2002) model posits

$$s_i = \hat{\alpha}(\pi_i\mathbb{E}(\tilde{x})_i)^{\hat{\beta}}. \tag{1.41}$$

For comparison purposes, spreads are converted into basis points and expected losses are expressed in percentage points. Tables 1.8 and 1.9 display very similar estimates. All variables are significant, except $\hat{\gamma}$ estimated in Lane (2000) model, both with our own and Braun (2016) data sets.

1.6.8 Comparison with alternative models

This section compares our model to alternative models reviewed inBraun (2016). These models mostly aim at reflecting the practice of commercial reinsurers concerned by the comparative costs of various risk transfer instruments. In particular, Major & Kreps (2002) consider simultaneously the pricing of reinsurance and cat bond tranches. Models (1.38), (1.39), (1.40) and (1.41) are estimated on our data base (Table 1.11) and compared to our preferred specification (Table 1.7). To allow for a fair comparison, we augment models (1.38), (1.39), (1.40) and (1.41) with the same controls as in our preferred specification.

Table 1.10 summarizes our preferred specification to the alternative models in terms of adjusted R^2, sum of the squared residuals (s^2), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The four measures yield the same ordering. Our preferred specification performs slightly below models (1.38), (1.39) but significantly higher than models (1.40) and (1.41).

Table 1.8: Artemis database

	Linear in EL		Polynomial in ln(EL)		Lane (2000)		Major/Kreps (2002)	
	coeff	sign	coeff	sign	coeff	sign	coeff	sign
$\hat{\alpha}$	307.6273	$***$	423.5275	$***$	411.8256	$***$	470.2197	$***$
$\hat{\beta}$	140.4200	$***$	307.2772	$***$	0.4130	$***$	0.3610	$***$
$\hat{\gamma}$			62.2991	$***$	-0.0068			
adjustedR^2	0.6899		0.6817		0.5880		0.5564	

Table 1.9: Braun (2015) estimates

	Linear in EL		Polynomial in ln(EL)		Lane (2000)		Major/Kreps (2002)	
	coeff	sign	coeff	sign	coeff	sign	coeff	sign
$\hat{\alpha}$	312.70	$***$	594.58	$***$	415.74	$***$	588.54	$***$
$\hat{\beta}$	232.02	$***$	399.49	$***$	0.37	$***$	0.43	$***$
$\hat{\gamma}$			73.32	$***$	0.08			
adjustedR^2	0.80		0.73		0.54		0.71	

Table 1.10: Model comparisons

Measure	Preferred	L. in EL	P. in Ln(EL)	Lane	Major/Kreps
AdjustedR^2	0.7794	0.8391	0.8146	0.5613	0.7244
$s^2(\times 10^{-04})$	3.3951	2.4764	2.8531	1188	927
AIC$(\times 10^{-04})$	3.7051	2.6894	3.1137	1296	1007
BIC$(\times 10^{-04})$	4.9811	3.5532	4.1860	1743	1330

1.6.9 Robustness analysis

Finally, Tables 1.12 to 1.21 summarize the robustness of our numerical results of section 1.4. Each table presents either optimal coverage or welfare gain for a given set of hypotheses. The cost of handling claims is set to $\lambda = 0.3$, which is viewed as a reasonable estimate in the literature. However, changes in this parameter have a very limited impact on the simulation results. The scenarios that are considered vary across lines. All results are expressed in euros. Within each table, we fix $\overline{R}$ and let $\underline{R}$ vary through the columns. From left to right, we therefore increase the agent's risk aversion. For each level of $\overline{R}$ we provide two tables. The first delivers our estimates for the optimal level of coverage and the second computes the welfare gain relative to the no-coverage situation.

The most sensitive parameter is usually the subsistence level ω. Our results indicate that, while the optimal coverage is robust to changes in ω, the estimated welfare gains are quite sensitive. As expected, optimal coverage increases with the severity of the scenario under consideration and with the degree of risk aversion.

Table 1.11: Pricing equation estimation

	model 1.38			model 1.39			model 1.40			model 1.41		
	coeff	t stat	sign	coeff	t stat	sign	coeff	t stat	sign	coeff	t stat	sign
Intercept	0.0264	11.2126	$***$	0.3232	19.1976	$***$	−2.4922	−16.5827	$***$	−1.5722	−8.2625	$***$
Expected loss	1.5274	19.5593	$***$									
lnExpected loss				0.0914	16.5073	$***$				0.3445	8.1731	$***$
lnExpected loss^2				0.0064	14.6497	$***$						
$\ln(\pi)$							0.2362	7.0940	$***$			
$\ln(\mathbb{E}(\tilde{x}))$							0.0412	0.4215				
Size×2011	3.2886	3.9351	$***$	2.0058	2.2380	$**$	43.5799	2.5619	$**$	16.5116	1.1434	
Size×2012	2.7696	7.0018	$***$	2.0080	5.7786	$***$	40.8487	5.0611	$***$	5.9194	2.9963	$***$
Size×2013	1.1241	3.2866	$***$	0.3304	0.8614		12.4222	1.2237		−1.3320	−0.1361	
Size×2014	0.0086	0.0345		−0.5318	−2.0018	$**$	−2.7942	−0.3396		−6.2024	−0.6809	
Size×2015	−0.2819	−1.0513		−1.0972	−3.3311	$***$	−14.3656	−1.6679	$*$	−12.1281	−1.3163	
Size×2016	−0.4109	−1.6012		−0.8335	−2.9396	$***$	−12.0038	−1.5695		−7.1473	−0.8272	
Size×$Japan$	−0.8727	−2.0804	$**$	0.5331	1.4899		−25.7024	−1.8015	$**$	−22.3950	−1.5465	
Size×US	0.0678	0.2993		0.9068	3.9390	$***$	12.9411	1.7528	$**$	16.3709	2.1157	$**$
Size×$Other$	−0.4308	−1.1733		−0.2868	−0.6126		−5.0268	−0.5402		−1.7345	−0.2084	
Size×$Index$	−0.5966	−1.4803		−0.3699	−0.6417		−1.0433	−0.1084		7.3993	0.9304	
Size×$Param$	−1.0386	−3.6465	$***$	−1.0153	−3.3000	$***$	−17.1171	−2.6368	$***$	−6.8371	−1.2982	
RMS	−0.1327	−0.4718		0.3554	1.1571		−8.9933	−1.0699		−11.5646	−1.6202	
EQECAT	−2.3256	−5.5687	$***$	−2.2378	−2.8404	$***$	−75.4859	−7.3274	$***$	−49.4767	−6.3498	$***$
MILL	0.0120	0.0577		0.5667	2.3743	$**$	4.8421	0.6245		9.2355	1.0622	
R^2	0.8522			0.8307			0.5994			0.7468		
adjustedR^2	0.8391			0.8146			0.5613			0.7244		
$s^2(\times 10^{-04})$	2.4764			2.8531			1188			927		
AIC$(\times 10^{-04})$	2.6894			3.1137			1296			1007		
BIC$(\times 10^{-04})$	3.5532			4.1860			1743			1330		

Optimal coverage and welfare gains with $\omega = 0.90$

Table 1.12: Coverage, $\overline{R} = 1$

R	1	
Scenario	Cover	Welfare
1	0.2865	0.0102
2	0.3538	0.0131
3	0.3883	0.0144
4	0.4097	0.0151
5	0.4244	0.0154

Table 1.13: Coverage, $\overline{R} = 2$

R	1		2	
Scenario	Cover	Welfare	Cover	Welfare
1	0.6982	0.0562	0.7636	0.0791
2	0.9829	0.0825	1.1204	0.1213
3	1.1693	0.0972	1.3740	0.1472
4	1.3060	0.1056	1.5726	0.1637
5	1.4125	0.1103	1.7360	0.1742

Table 1.14: Coverage, $\overline{R} = 3$

R	1		2		3	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.0323	0.1481	1.1132	0.2319	1.1407	0.2822
2	1.5700	0.2213	1.7583	0.3419	1.8239	0.4077
3	1.9657	0.2615	2.2658	0.4022	2.3724	0.4744
4	2.2824	0.2842	2.6944	0.4373	2.8434	0.5130
5	2.5472	0.2967	3.0697	0.4583	3.2615	0.5364

Table 1.15: Coverage, $\overline{R} = 4$

$\underline{R}$	1		2		3		4	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.2993	0.2834	1.3861	0.4608	1.4151	0.5610	1.4296	0.6228
2	2.0780	0.4013	2.2919	0.6009	2.3645	0.6949	2.4010	0.7474
3	2.6911	0.4592	3.0447	0.6628	3.1662	0.7500	3.2275	0.7967
4	3.2073	0.4894	3.7066	0.6944	3.8802	0.7776	3.9680	0.8210
5	3.6572	0.5049	4.3053	0.7115	4.5331	0.7925	4.6486	0.8341

Table 1.16: Coverage, $\overline{R} = 5$

$\underline{R}$	1		2		3		4		5	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.5137	0.4409	1.6016	0.6837	1.6306	0.7908	1.6450	0.8452	1.6536	0.8768
2	2.5071	0.5759	2.7315	0.7929	2.8063	0.8709	2.8436	0.9073	2.8659	0.9275
3	3.3239	0.6336	3.7033	0.8330	3.8308	0.8984	3.8945	0.9280	3.9328	0.9441
4	4.0343	0.6611	4.5792	0.8516	4.7638	0.9110	4.8562	0.9373	4.9117	0.9516
5	4.6700	0.6743	5.3875	0.8608	5.6321	0.9173	5.7549	0.9420	5.8287	0.9555

Optimal coverage and welfare gains with $\omega = 0.975$

Table 1.17: Coverage, $\overline{R} = 1$

$\underline{\text{R}}$	1	
Scenario	Cover	Welfare
1	0.4132	0.0204
2	0.5504	0.0288
3	0.6342	0.0336
4	0.6930	0.0366
5	0.7192	0.0384

Table 1.18: Coverage, $\overline{R} = 2$

$\underline{\text{R}}$	1		2	
Scenario	Cover	Welfare	Cover	Welfare
1	0.9278	0.1436	0.9469	0.1661
2	1.4136	0.2203	1.4569	0.2545
3	1.7824	0.2668	1.8509	0.3081
4	2.0868	0.2962	2.1807	0.3424
5	2.3484	0.3153	2.4677	0.3651

Table 1.19: Coverage, $\overline{R} = 3$

$\underline{\text{R}}$	1		2		3	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.3251	0.4429	1.3471	0.5303	1.3544	0.5669
2	2.1618	0.5841	2.2156	0.6691	2.2337	0.7024
3	2.8528	0.6484	2.9418	0.7284	2.9718	0.7594
4	3.4589	0.6823	3.5852	0.7594	3.6277	0.7891
5	4.0068	0.7014	4.1716	0.7769	4.2272	0.8062

Table 1.20: Coverage, $\overline{R} = 4$

$\underline{\text{R}}$	1		2		3		4	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.6273	0.7644	1.6499	0.8553	1.6574	0.8858	1.6612	0.9019
2	2.7660	0.8524	2.8234	0.9141	2.8426	0.9340	2.8521	0.9446
3	3.7489	0.8832	3.8460	0.9337	3.8784	0.9500	3.8946	0.9589
4	4.6385	0.8973	4.7785	0.9427	4.8253	0.9575	4.8487	0.9658
5	5.4629	0.9044	5.6479	0.9474	5.7099	0.9615	5.7408	0.9696

Table 1.21: Coverage, $\overline{R} = 5$

$\underline{\text{R}}$	1		2		3		4		5	
Scenario	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare	Cover	Welfare
1	1.8614	0.9244	1.8836	0.9676	1.8909	0.9787	1.8946	0.9840	1.8968	0.9874
2	3.2514	0.9559	3.3090	0.9817	3.3282	0.9883	3.3377	0.9916	3.3434	0.9937
3	4.4844	0.9657	4.5833	0.9861	4.6162	0.9913	4.6326	0.9940	4.6424	0.9957
4	5.6219	0.9699	5.7659	0.9880	5.8137	0.9927	5.8376	0.9952	5.8519	0.9967
5	6.6919	0.9719	6.8836	0.9890	6.9474e	0.9935	6.9793	0.9958	6.9984	0.9973

Chapter 2

On the insurability of low-probability risks

This chapter is co-authored with Arnaud Goussebaïle.

Abstract: Why are catastrophic events so difficult to insure? Their extreme severity advocates for a wide-spread use of insurance, yet they are typically excluded from US homeowner policies. It is also puzzling to notice that some low-probability risks such as damages from lightnings are covered under all US homeowner policies whereas others, such as earthquakes and floods are not. Our model explains this puzzle by showing that the effect of aggregate uncertainty, a well-known threat to insurability, is amplified when the individual loss probability is small. Even in a simple expected utility framework, low-probability risks with aggregate uncertainty, such as earthquakes and floods, therefore display lower insurance take-up rates than low-probability risks without aggregate uncertainty such as lightning strikes.

Keywords: low-probability risks, catastrophic risks, insurance, capital cost.
JEL classification: D86, G22, G28, Q54.

2.1 Introduction

Why are catastrophic events so difficult to insure? Natural and man-made catastrophes such as earthquakes, floods, nuclear accidents and terrorists attacks have severe consequences at the individual level, affect large numbers of people at the same time, but have small individual probabilities of occurrence. The extreme severity of these risks advocates for a wide-spread use of prevention and insurance. Yet, they are typically excluded from US homeowner policies.[1] Private insurance markets are often non-existant and, even when specific contracts, benefiting from public subsidies, are available, relatively few people purchase them.[2]

It is also puzzling to notice that some low-probability risks such as damages from lightnings are efficiently handled by the insurance sector and covered under standard US homeowner policies whereas others, such as earthquakes and floods are not, despite comparable probability levels (Table 2.1). Our model explains this puzzle by showing how the low probability and systemic aspects of these risk combine to deter insurance take-up.

Earthquake	Flood	Hurricane
0.001-0.01[3]	0.01[4]	0.04[5]
Lightning strike	Death at 45 yo	Airplane crash
0.005[6]	0.003244[7]	10^{-6}

Table 2.1: Systemic low probability risks such as earthquakes, floods and hurricanes tend not to be insured while non-systemic low probability risks such as lightning strikes, death at 45 years old or airplanes crashes feature higher take-up rates.

We begin by examining the case without aggregate uncertainty. In this framework, we show that low-probability risks are actually easier to insure than high-probability risks. It is known since Mossin (1968) that expected utility maximizing agents optimally purchase partial coverage when insurance is sold above actuarially fair price. In this case, a decrease in the loss probability has two effects on

[1]http://www.iii.org/article/which-disasters-are-covered-by-homeowners-insurance

[2]Kunreuther (1973), Kousky & Kunreuther (2014) and Grislain-Letrémy (2015) document low take-up rates for catastrophe insurance while Cole et al. (2014) and Mobarak & Rosenzweig (2013) try to explain low rates in the micro-insurance industry.

the demand for coverage. On the one hand, the cost of providing insurance diminishes, which translates into lower premiums for policyholders. On the other hand, the likelihood of receiving the indemnity declines as well. Our contribution in this framework is to show that the cost-reduction effect dominates the likelihood-reduction effect under risk aversion. As the loss probability declines, the ratio of willingness to pay to cost of coverage always increases. In a population heterogeneous in wealth or preferences, take-up rates are therefore predicted to increase as the loss probability decreases.[8] In addition, we show that given the opportunity to do so, people optimally choose higher levels of coverage for low-probability risks if their index of absolute risk aversion does not decrease too fast with wealth.

These results are in line with Laury et al. (2009), who experiment in the laboratory the effect of a change in the loss probability. A previous experiment by McClelland et al. (1993) also found a decreasing mean ratio of willingness to pay to expected indemnity, indicating that people tend to be willing to pay higher loadings for low-probability risks than for high-probability risks.

These observations, however, are at odds with the low take-up rates for disaster risks. Our explanation is that the risks for which underinsurance is most prevalent display substantial aggregate uncertainty. Natural and man-made catastrophes feature geographically correlated individual losses which translates into aggregate loss uncertainty, even within very large pools of policyholders. In order to remain solvent, the insurance provider must either raise prohibitively high levels of premiums or more realistically, it must have access to capital to fill the gap between the premiums raised and the amount of claims due in case of catastrophe. The cost of allocating this capital to a specific line of business[9] depends on a risk premium that emerges only for systemic risks. Because this premium decreases at a lower pace than willigness to pay for insurance, it results in low take-up rates for low probability risks.

Several literatures have attempted to explain the low take-up rates for disaster coverage. Kunreuther & Slovic (1978), Hertwig et al. (2004) and Kunreuther et al.

[8]With the exception of the pathological case of Giffen behaviors which, to our knowledge have never been observed in an insurance market.
[9]See Zanjani (2002) and Froot (2001a).

(2001) rely on departures from the expected utility paradigm, arguing that low-probabilities are more difficult to process than high probabilities. In the expected utility framework, Kunreuther et al. (2001) show that search costs may generate a probability threshold below which coverage is not purchased while Coate (1995) shows how government relief can crowd-out the demand for insurance. Raschky et al. (2013), Kousky et al. (2013) and Grislain-Letrémy (2015) find empirical support for this hypothesis in various countries. Finally, in the context of micro-insurance Cole et al. (2014) suggests that learning is an important determinant of the demand for insurance.

Overall, much less attention has been devoted to the supply side of the market. Kunreuther et al. (1995) explain that ambiguity may provide a rationale for high disaster insurance premiums. In the context of micro-insurance markets, Mobarak & Rosenzweig (2013) argue that basis risk can explain low take-up rates.

Our contribution to the literature is twofold. First, we provide a simple, yet general framework, that nests an alternative explanation to the observed low take-up rates for disaster risks.[10] Second, we explain this apparently puzzling observation that some low-probability events are well insured while others are not. It is in fact the combination of aggregate loss uncertainty with low-probability that makes earthquakes, floods and terrorism risks difficult to insure.

In addition, our model for systemic risk predicts that for a given systemic insurance line, say flooding, people with lower probabilities of loss purchase less insurance than people with higher loss probability, even-though a risk-based pricing approach is used. This prediction is common with that of the adverse selection literature (Akerlof (1970)) but its underpinning is of a very different nature.

2.2 Insuring non systemic low-probability risks

In this section, we show that in the baseline insurance framework with independent losses, low-probability events are in fact more likely to be insured than higher

[10]Jaffee & Russell (1997b) discuss the problem of capital allocation cost but provide no formal model while Kousky & Cooke (2012) have a simulated model, calibrated to analyze flood insurance coverage in Broward County, Florida.

probability events. The intuition behind this result is that individual's willingness to pay is concave in the probability, while the cheapest feasible contract is linear. Therefore, if a risk is insurable somewhere over the parameter space, there is a probability threshold below which it is insurable and above which it is not.

We consider risk-averse agents with a twice continuously differentiable and concave utility function $u(x)$ and initial wealth w. $A(x) = -\frac{u''(x)}{u'(x)}$ denotes the Arrow-Pratt index of absolute risk aversion.

The insurance provider can be private or public. It is represented by a risk-neutral agents that sells an amount of coverage $\tau \in [0, L]$ at a premium α. Following Raviv (1979) we assume that, in addition to the payment of the indemnity τ, the insurance provider faces a cost $c(\tau)$ with

$$c(0) = 0, \quad c'(0) = b > 0, \quad c'(\tau) > 0 \quad \text{and} \quad c''(\tau) \geq 0,$$

which represents the various expenses associated with the payment of an indemnity τ to all affected agents. It can be interpreted as an administrative cost, as a cost of expertise, or more broadly as a dead-weight loss, resulting either from an asymmetry of information between the insurance company and the policy holder, or by imperfect competition. The actuarial and insurance literatures often make the simplifying assumption that the marginal cost $c'(\tau)$ is equal to a constant λ called the loading factor. In this case, the dead-weight cost is simply a fraction of the indemnity. We call loading the ratio $\alpha/p\tau$ premium to expected indemnity. If $c'(\tau) = \lambda$, the loading is $1 + \lambda$ and we have $c''(\tau) = 0$, which is indeed a particular case of our model.

A necessary condition for the insurance to provide such a contract is that its expected profit is positive. This motivates the following definition

Definition 2 *A contract is called feasible if and only if the insurance provider can realize at least a zero expected profit.*

In the absence of aggregate loss uncertainty, a contract is therefore feasible if and only if

$$\alpha \geq p\tau + pc(\tau).$$

The first notion of insurability that we develop is the following

Definition 3 *A risk is strongly insurable at a level τ if and only if individuals are willing to purchase the proposed level of coverage τ of some feasible contract.*

In order to know whether a risk is strongly insurable or not, it is sufficient to verify that individuals are willing to purchase the zero expected profit contract, for which the insurer breaks even. If they reject the zero expected profit feasible contract, agents will also reject all the other more expansive contracts, and if they accept the zero expected profit feasible contract, then the risk is insurable. In the remaining of the paper, we use the word contract to mean zero expected profit contract.

Independently of the supply side constraints, the highest price $C(p,\tau)$ that an individual would pay for a level of coverage τ is given by

$$pu(w-L+\tau-C)+(1-p)u(w-C)=pu(w-L)+(1-p)u(w). \tag{2.1}$$

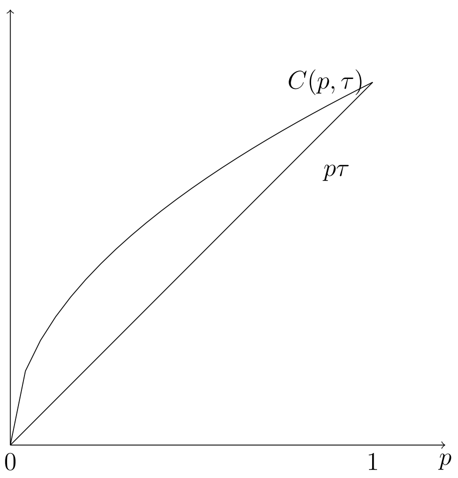
First notice that $C(0,\tau)=0$ and $C(1,\tau)=\tau$. The willingness to pay for the coverage of a a zero probability event is zero and the willingness to pay for the coverage of a sure event is just the coverage itself. Total differentiation of (2.1) gives

$$C'_p(p,\tau)=\frac{u(w)-u(w-L)-[u(w-C)-u(w-L+\tau-C)]}{pu'(w-L+\tau-C)+(1-p)u'(w-C)}\geq 0 \quad \forall\tau\leq L,$$

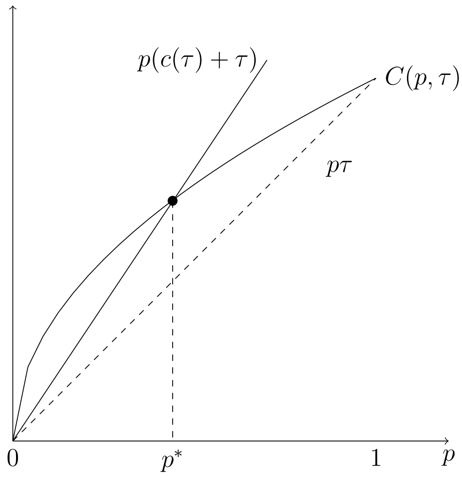
and

$$\begin{aligned} C''_{pp}(p,\tau)= & -\quad 2C'_p\frac{u'(w-L+\tau-C)-u'(w-C)}{pu'(w-L+\tau-C)+(1-p)u'(w-C)} \\ & +\quad (C'_p)^2\frac{pu''(w-L+\tau-C)+(1-p)u''(w-C)}{pu'(w-L+\tau-C)+(1-p)u'(w-C)}\leq 0 \quad \forall\tau\leq L, \end{aligned}$$

where C'_p and C''_p represent the first and second order partial derivative with respect to p. The agent's willingness to pay C is therefore increasing and concave in the probability of loss p. Figure 2.1a represents the agent's willingness to pay and the cost of coverage as a function of the loss probability p for the case $c(\tau)=0$. When



(a) Actuarially fair coverage



(b) Non-actuarially fair coverage

coverage is sold at an actuarially fair price, the agent is always willing to purchase it. The surplus she derives from the transaction however, may vary with the loss probability.

Figure 2.1b represents the case where coverage is available at a cost higher than the actuarially fair price. For values of p lower than a threshold p^*, willingness to pay is above the price of coverage. An insurance market should therefore emerge in this case. For values of p higher than p^*, willingness to pay is below the price of the zero expected profit contract, resulting in the absence of any market.

Proposition 6 *In the absence of aggregate loss uncertainty, a risk is strongly insurable at level τ if and only if the individual probability of loss p is below a threshold p^*, where p^* is such that*

$$C(p^*,\tau) = p^*\tau + p^*c(\tau).$$

For a given level of coverage, low-probability events are therefore more likely to be covered than high-probability events.

Laury et al. (2009) investigate in an experiment how insurance purchase decisions evolve with p. They observe that the fraction of their sample that purchases full-coverage decreases with the probability of loss p, given a constant expected loss and loading. Proposition (6) confirms that this should indeed be the case but delivers an even stronger prediction: the fraction of a population that purchases coverage should increase as p decreases even when the loss L is fixed.

To see this, assume that the population is heterogeneous in wealth and preferences. An agent i endowed with wealth w^i and utility function u^i purchases a feasible contract providing coverage τ if and only if

$$C_i(p, \tau) \geq p\tau + pc(\tau) \tag{2.2}$$

Each individual therefore has a probability threshold p_i^* above which she stops purchasing insurance. The distribution of wealth and preferences generates a distribution G over the thresholds p_i^*. The fraction of the population that purchases insurance is $\mathbb{P}(p_i^* \geq p) = 1 - G(p_i^*)$, which is decreasing in p.

An insurance contract providing a level of coverage τ is therefore more likely to be purchased when the probability of loss p is smaller. In this sense, low-probability events are more insurable than higher-probability events. However, the exogeneity of the level of coverage τ may appear as a limit to this analysis. If people have control over the level of coverage, the decision is not a take-it-or-leave-it problem anymore.

With this idea in mind, we propose the following notion of insurability:

Definition 4 *A risk is weakly insurable if and only if individuals are willing to purchase a positive amount of coverage of some feasible contract.*

This notion of insurability is weaker in the sense that a strongly insurable risk is necessarily weakly insurable. If a person agrees to purchase a level of coverage τ rather than no insurance when the loss probability is p, she would never optimally choose $\tau = 0$. However, she may select a level of coverage lower than τ to reduce the cost of insurance.

With endogenous coverage, the agent solves

$$\begin{aligned} \max pu(w - L + \tau - \alpha) + (1-p)u(w - \alpha) \\ \text{s.t.} \quad \alpha = p\tau + pc(\tau) \\ \tau \geq 0. \end{aligned} \tag{2.3}$$

The first order condition of this problem is

$$[1-(1+c'(\tau))p)]u'(w_1) = (1-p)(1+c'(\tau))u'(w_2), \tag{2.4}$$

in which $w_1 = w - L + \tau - p\tau - pc(\tau)$ and $w_2 = w - p\tau - pc(\tau)$ are the levels of wealth in the loss and no-loss states. It is easy to check that $c'(\tau) > 0$ implies that the agent chooses partial coverage, so that $\tau < L$ at any interior solution.

Individuals purchase a positive amount of coverage if and only if

$$(1-p)(1+b)u'(w) < [1-(1+b)p]u'(w-L). \tag{2.5}$$

Re-arranging the terms of (2.5) yields the following Proposition.

Proposition 7 *In the absence of aggregate loss uncertainty, a risk is weakly insurable if and only if its probability p is such that*

$$p < \frac{1}{1+b} - \frac{b}{1+b}\frac{u'(w)}{u'(w-L)-u'(w)}. \tag{2.6}$$

For a given p, an event is uninsurable when the marginal cost of coverage $b = c'(0)$ is too high or the size of the loss L is not sufficiently large to generate a significant difference between marginal utility in the loss state $u'(w-L)$ and marginal utility in the no-loss state $u'(w)$. Proposition (7) stresses once more that, in the absence of aggregate loss uncertainty, low-probability events are easier to insure than high-probability events.

In addition, Proposition 8 and its corollary give conditions under which a strictly positive optimal coverage increases when the probability p decreases.

When the loss probability diminishes, two effects interact. On the one hand, the risk of experiencing the loss L diminishes, lowering the incentive to pay the dead-weight cost $c(\tau)$. When p diminishes, a higher level of risk aversion is therefore required to justify the purchase of coverage. On the other hand, the dead-weight marginal cost $pc'(\tau)$ diminishes, making insurance more attractive at the margin. The total effect on optimal coverage cannot be signed for any utility function, but the following proposition and corollary enables to identify some interesting and realistic cases.

Proposition 8 *In the absence of aggregate loss uncertainty, the optimal coverage τ of a weakly insurable risk is strictly decreasing in p if and only if*

$$A(w_1) - A(w_2) < \frac{c'(\tau)}{[\tau + c(\tau)](1-p)[1-(1+c'(\tau))p]}$$

at the optimum.

Proof The proof is given in Appendix 2.5.1

The left-hand side of the inequality is positive when $A(w_1) \geq A(w_2)$, while the right-hand side is positive when $1-(1+c'(\tau))p > 0$, which is a necessary condition for an interior solution. Since $w_1 < w_2$, the condition is trivially satisfied for any increasing or constant absolute risk aversion functions (IARA or CARA). For the class of decreasing absolute risk aversion (DARA), the condition puts an upper bound on the variation of risk aversion between the loss and the no-loss state.

The empirical literature most often fails to reject DARA as a realistic hypothesis such as in Guiso & Paiella (2008) and Levy (1994). In addition, Rabin (2000)'s calibration theorem, showing that aversion to small-stake gambles implies rejection of gambles that people take in their lives, can be interpreted as a rejection of the CARA (and even more of the IARA) hypothesis. We would therefore like to know whether classical utility functions, satisfying the DARA property, also feature an optimal coverage decreasing in the probability p. This is the purpose of Corollary 8.1 that deals with the case of Harmonic Absolute Risk Aversion (HARA) functions. We define a HARA utility function as in Gollier (2004)

$$u(x) = \zeta\Big(\eta + \frac{x}{\gamma}\Big)^{1-\gamma},$$

whose domain is such that $\eta + \frac{x}{\gamma} > 0$ and the condition $\zeta\frac{1-\gamma}{\gamma} > 0$ guarantees that the function is indeed increasing and concave. The coefficient of absolute risk aversion is

$$A(x) = \Big(\eta + \frac{x}{\gamma}\Big)^{-1}. \tag{2.7}$$

Except for the limit case $\gamma \to +\infty$, the HARA functions satisfy the DARA property when $\gamma > 0$, which makes them appealing with respect to the literature discussed previously.

Corollary 8.1 *If a risk-averse agent has preferences represented by a HARA utility function with $\gamma \geq 1$, then the optimal coverage τ of a weakly insurable risk is strictly decreasing in p in the absence of aggregate loss uncertainty.*

Proof The proof is given in Appendix 2.5.2

The HARA class nests two of the most widely used classes of utility functions. The Constant Absolute Risk Aversion (CARA) is obtained when $\gamma \to +\infty$. Solving the differential equation (2.7) yields the specification of this function

$$u(x) = -\eta \exp(-\frac{1}{\eta}x).$$

The second class of functions within the HARA class is the set of Constant Relative Risk Aversion (CRRA) which is obtained for $\eta = 0$. Solving (2.7) in this case yields

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}. \tag{2.8}$$

If people's preferences can be represented by a CARA or by a CRRA utility function with $\gamma \geq 1$, then their optimal coverage is always a decreasing function of p. For the CRRA case, Szpiro (1986) and Barsky et al. (1997) find values of relative risk aversion respectively between 1.2 and 1.8 for the first and 4.17 for the second. According to Gollier (2004) (p.69), the "range of acceptable values of relative risk aversion [is] $[1,4]$". A complete survey of the literature on risk preferences elicitation would reveal some estimates below one, such as Chetty (2006), but overall our (sufficient but not necessary) condition $\gamma \geq 1$ seems a very plausible assumption.

Finally, Louaas & Picard (2014) have shown the following proposition

Proposition 9 *In the absence of aggregate loss uncertainty and if $u'(w-L) > (1+b)u'(w)$, the optimal coverage converges toward a positive limit τ defined as*

$$u'(w-L+\tau) = (1+c'(\tau))u'(w).$$

It may sound surprising that the agent is willing to pay a positive loading factor for a risk whose probability tends to zero, but the pricing rule $\alpha = p\tau + pc(\tau)$ implies that the premium tends to zero with the probability. The agent therefore receives an indemnity with an infinitesimal probability whose price also becomes infinitesimal.

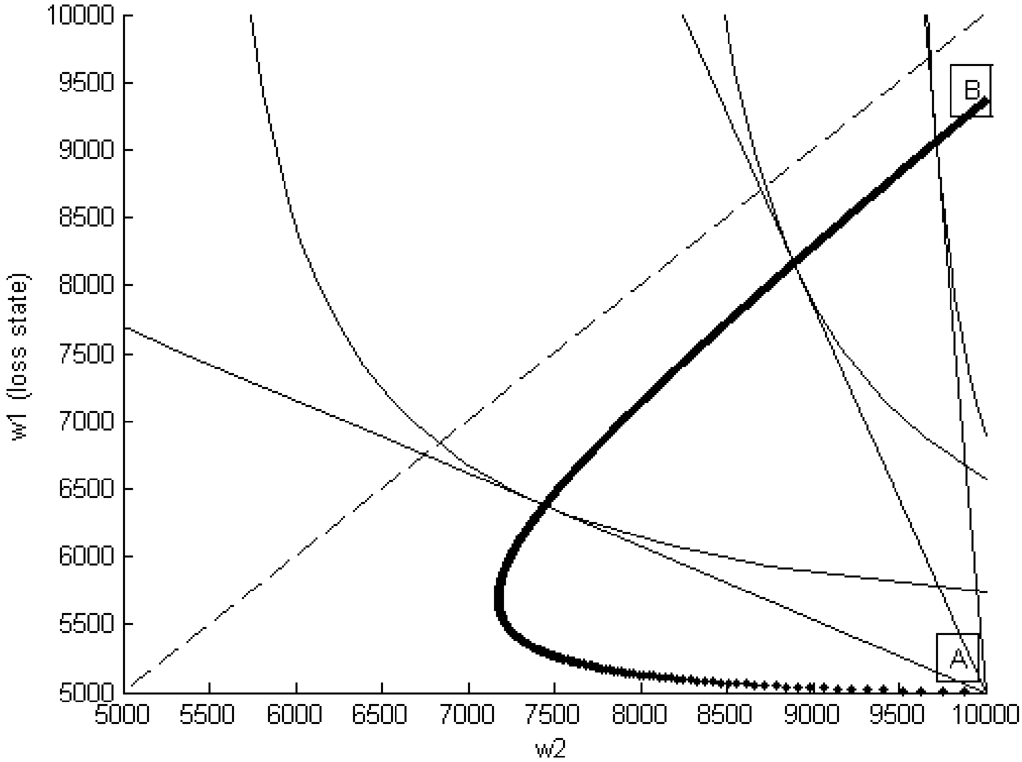


Figure 2.2: $w = 10000, L = 5000, u(x) = \frac{x^{-3}}{3}$

Figure 2.2 gives an illustration of the previous results in the (w_1, w_2) space. Point A represents the optimal lottery at the highest probability p for which the insured chooses a positive amount of coverage. The thick curve represents the locus of optimal lotteries as p diminishes and Point B is the limit optimal lottery when $p \to 0$. In agreement with Proposition 8, the optimal lottery gets closer from the 45 degree line as p decreases, until it (almost) reaches point B when $p \to 0$.

Our theoretical results indicate that under reasonable assumption on individual's preferences, low-probability events should be more insured than higher probability events. Gollier (1997) shows that the deductible chosen by a risk averse agent is decreasing in p when p is sufficiently small and the agent's utility function features IARA, which is equivalent[11] to purchase more insurance when the probability is very small. Our results are more general since they apply everywhere on the domain of p where the agent purchases a positive amount of coverage. In addition, we are able to accommodate some of the more realistic DARA utility functions.

Low-probability and high-stake risks are characterized by a large potential loss L. In order to compare insurance choice for these risks to choices concerning more traditional risks, it may be useful to fix the expected loss, as in Laury et al. (2009), so that the loss increases as its probability diminishes. In this case and independently of any assumption on individual's preferences, agents purchase more coverage as the probability of loss decreases.

Proposition 10 *In the absence of aggregate loss uncertainty, and if the expected loss pL remains constant, the optimal coverage is a decreasing function of the probability p for any risk averse individual.*

Proof The proof is given in Appendix 2.5.3

The requirement that expected loss be fixed makes it easier for the condition of proposition (8) to hold. Indeed as p diminishes, the loss becomes larger, providing agents with additional incentives to purchase insurance. This in turn reduces the gap between wealth in the non loss state w_1 and wealth in the loss state w_2.

This section has shown that in the absence of aggregate loss uncertainty, low-probability risks are easier to insure than high-probability risks. This may explain why typical homeowners insurance cover perils as unlikely as lightening strikes. In contrast, low-probability correlated risks such as earthquakes, flooding, windstorms, nuclear hazards and acts of wars are typically excluded from homeowner policies. The next section provides an explanation as to why low-probability correlated risks are also difficult to insure.

[11] In a model where agents only face two states: loss or no loss, deductible and partial coverage are strictly equivalent.

2.3 Insuring systemic low-probability risks

We now consider different risks, potentially correlated with each other. Each policy covers the risk of loss $x_i = L_i X_i$, for all $i = 1, ..., n$, where X_i is a Bernoulli random variable taking value 1 with probability p_i and 0 with probability $1 - p_i$. L_i is a parameter that reflects the intensity of the loss.

As an example, assume that the insurance provider is a firm which insures n people against a risk of flood. The n policyholders face a risk that has a common source and which is systemic if the flood is important. But the probabilities p_i and intensities L_i can be different, depending on how far they live from the river bed.

We call $\sigma_i^2 = p_i(1-p_i)$, the variance of the Bernoulli variable X_i and σ_{ij} the covariance between risks X_i and X_j. The dependence between these individual risks is captured by the pairwise coefficients of correlation $\rho_{ij} = \text{cov}(X_i, X_j)/(\sigma_i \sigma_j)$ for all $j \neq i$. Each policyholder can purchase a coverage τ_i, provided by the insurance when $x_i = L_i$. In addition to the indemnity, the insurance provider faces a deadweight cost $c(\tau_i)$ per policy. As a consequence, the cost per policy faced by the insurance provider is

$$I_i(x_i) = \begin{cases} \tau_i + c(\tau_i) & \text{with probability} \quad p_i \\ 0 & \text{with probability} \quad 1 - p_i \end{cases}$$

Because insurability depends on the indemnity costs $I_i(x_i)$ faced by the insurer, it is useful to write the moments of these costs given the exogeneously given moments of the risks i.

Lemma 2 *Let μ_i and χ_i^2 be the expected value and variance of the indemnity cost $I_i(x_i)$, and χ_{ij} and δ_{ij} be the covariances and correlations between $I_i(x_i)$ and $I_j(x_j)$, where $j \neq i$. Then*

$$\mu_i = p_i(\tau_i + c(\tau_i)),$$

$$\chi_i^2 = p_i(1-p_i)(\tau_i + c(\tau_i))^2,$$

$$\chi_{ij} = \rho_{ij}(\tau_i + c(\tau_i))(\tau_j + c(\tau_j))\sqrt{p_i(1-p_i)}\sqrt{p_j(1-p_j)},$$

$$\delta_{ij} = \rho_{ij}.$$

Proof The proof is provided in Appendix 2.5.4

The presence of correlation between the individual loss Bernoulli variables X_i translates into a positive correlation between the indemnity costs $I_i(x_i)$ faced by the insurer.

To cover the total indemnity costs, the insurance provider purchases n contracts that each deliver a payoff

$$y = \frac{1}{n}\sum_{i=1}^{n} I_i(x_i),$$

equal to the average loss per policy. These n contracts can be secured under the form of equity contracts (the owners of the insurance provider are directly liable for the payment of y), by the purchase of reinsurance or alternative risk transfer contracts.[12] The expected value of each of these contracts is the population average expected indemnity cost

$$\mathbb{E}(y) = \frac{1}{n}\sum_{i=1}^{n} \mu_i,$$

and the variance is

$$\mathrm{V}(y) = \frac{1}{n^2}[\sum_{i=1}^{n} \chi_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} \chi_{ij}]. \tag{2.9}$$

Equation (2.9) can also be written

$$\mathrm{V}(y) = \frac{1}{n}\bar{\chi}^2 + \frac{n-1}{n}\bar{\chi}^c, \tag{2.10}$$

where $\bar{\chi}^2 = 1/n \sum_{i=1}^{n} \chi^2$ is the population average variance and

$$\bar{\chi}^c = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{n-1}\sum_{j\neq i}^{n} \chi_{ij},$$

is the population average covariance between losses. If we consider $\bar{\chi}$ and $\bar{\chi}^c$ to be

[12]This paper abstracts from the possibility of default, examined in Charpentier & Le Maux (2014a) for example. Since default is not allowed from the counter-parties of the insurance providers, the solvency requirement do not lead to an immobilization of capital that would add an extra cost. In practice, regulatory agencies often require insurance companies to immobilize capital to secure their liability toward policyholders. For example, they may require the firms to hold a level of capital k such that the probability of default remains below a probability η. This requirement imposes an opportunity cost on the capital $k = F_y^{-1}(1-\eta) - \alpha$ that is immobilized.

fixed, equation (2.10) shows that the average loss y's variance tends to 0 when the number of policyholders n tends to $+\infty$ if and only if $\bar{\chi}^c = 0$, that is if and only if the average correlation between two losses is zero.

Assume that the market value of the random payoff y can be assessed with the one factor model

$$P_y = \mathbb{E}y - A\mathrm{cov}(y, w_I) \tag{2.11}$$

where w_I and A are the representative investor's wealth and index of absolute risk aversion.[13] Market completeness pre-supposes that all investor's idiosyncratic risks have been eliminated from the economy thanks to adequate contracting on the financial markets. As a consequence only systemic risks give rise to a risk premium. The covariance term of equation (2.11) captures the idea that an asset whose payoff is negatively correlated with the investor's wealth offers consumption insurance, for which investors are willing to pay a risk premium.

We assume that the representative investor's[14] wealth is

$$w_0 - \frac{1}{n}\sum_{i=1}^{n} a_i x_i, \tag{2.12}$$

where the positive weights a_i capture the dependance between the representative investor wealth and policyholder i's loss.[15] Positive a_i may reflect uninsurable spillovers from the source of aggregate risk to the investor's assets. Such spillovers, particularly likely to occur in the case of catastrophes, include the many indirect costs such as the loss of attractiveness of the affected territory. $a_i = 0$ for all i means no spillovers, while $a_i = a$ for all i means that the representative investor

[13]This expression characterizes the market equilibrium if markets are complete (see Gollier (2004)). It is an exact expression if investors have quadratic utility functions, or CARA utility functions when returns are normally distributed. With any other specification, it remains a good approximation if the risk y is sufficiently small compared to the representative investor's wealth.

[14]Since Mankiw & Zeldes (1991), an entire strand of literature has documented and attempted to explain the limited financial market participation puzzle. According to Grinblatt et al. (2011), only 50 % of U.S. households own some stocks. Guiso et al. (2008) shows that this shares falls to 31.5 % in U.K., 26 % in France, and only 8 % in Italy. In order to account for this important fact, we consider here that the investors and policyholders are potentially different agents.

[15]This formulation can be obtained by assuming that each of the m investors have a wealth $w_j = w_{0j} - 1/n\sum_{i=1}^{n} a_{ij}x_i$. In this case, the average wealth is $1/m\sum_{j=1}^{m} w_j = w_0 + 1/n\sum_{i=1}^{n} a_i x_i$, where $a_i = 1/m\sum_{j=1}^{m} a_{ij}$ is the average dependence parameter of policyholder i.

is himself exposed to the average loss.

Equation (2.11) then re-writes

$$P_y = \frac{1}{n}\sum_{i=1}^{n}\mu_i + \frac{A}{n^2}(\sum_{i=1}^{n}\sum_{j=1}^{n}a_j\mathrm{cov}(I_i(x_i), x_j)).$$

Using the expressions of the expected value and variance of the indemnity cost (2.9), we obtain a total cost

$$nP_y = \sum_{i=1}^{n}\mu_i + \frac{A}{n}(\sum_{i=1}^{n}\sum_{j=1}^{n}a_j(\tau_i + c(\tau_i))L_j\rho_{ij}\sigma_i\sigma_j).$$

Since the "purchase" of n securities paying each the payoff y perfectly matches the total indemnity cost, the no-profit condition

$$\alpha - nP_y = 0$$

defines the smallest total premiums raised α, that makes insurance sustainable. This total cost is split among policyholders in such a way that each policy pays for its own marginal contribution to the total cost of the insurance scheme, which in our case corresponds to

$$\alpha_i = \mu_i + \frac{A}{n}(\tau_i + c(\tau_i))(a_iL_i\sigma_i^2 + \sum_{j\neq i}a_jL_j\rho_{ij}\sigma_i\sigma_j). \tag{2.13}$$

Importantly, our assumption reflects the actuarial practice of *risk based* pricing, in which no cross-subsidization takes place. The premium paid by agent i contains a risk premium only if it is correlated with the representative investor's wealth, either directly ($a_i > 0$), or indirectly through its correlation with other systemic risks ($\rho_{ij} > 0$ and $a_j > 0$). The risk premium comprises a term related to the variance of the individual loss σ_i^2 and a sum of terms related to the covariance between risk i and the other risks. In a sufficiently large pool, the impact of risk i's own variance on the premium can be neglected and only the covariance terms matter. In a large pool, the insurance for risk i therefore features a risk premium if it is correlated with other systemic risks. Indeed, risks uncorrelated with the investor's wealth do not give rise to a risk premium in equilibrium. In addition, a pool of risks correlated with the investor's wealth but uncorrelated with each other can be diversified within the pool, hence eliminating the volatility of the average loss that is transferred to the market investors.

Lemma 3 *For given risk exposure of the insurance provider* $(p_i, L_i, \rho_{ij}, a_i)_{ij}$*, the loading factor* $\lambda(p_i) = \alpha_i/p_i\tau_i - 1$ *of a risk* i *is a decreasing function of the loss probability* p_i *and*

$$\lim_{p_i \to 0} \lambda(p_i) = +\infty$$

iff $\sum_{j \neq i} a_j L_j \rho_{ij} \sqrt{pj(1-p_j)} > 0$.

Proof The proof is given in Appendix 2.5.5.

Taking the risk exposure of the insurance provider as given, low probability risks have a higher loading $\alpha_i/p_i\tau_i$ and the very low probability risks have arbitrarily high loadings, despite the fact that prices are set to reflect individual exposures. This is due to the fact that the covariance, reflected in the insurance premium, decreases at a lower pace than the probability p_i. As a consequence

Proposition 11 *Assume that the insurance provider has a risk exposure* $(p_i, L_i, \rho_{ij}, a_i)_{ij}$*, then for all lines* i*, there is a threshold* p_i^* *below which risk* i *becomes uninsurable.*

Corollary 11.1 *Assume that the insurance provider has a risk exposure* $(p_i, L_i, \rho_{ij}, a_i)_{ij} = (p_i, L, \rho, a)$ *for all* i, j*, then there exists a threshold* p^* *such that the lines with probability* $p_j < p^*$ *are not insurable.*

Proof The proof is given in Appendix 2.5.6

Proposition 11 shows that if we fix the exposure of the insurance provider, then there exist a level of probability below which an agent i stops purchasing insurance. This threshold p_i^* depends on how systemic the risk considered is (a_i), how correlated with other policyholder's risk it is (ρ_{ij}), and how severe the loss L_i is. Assuming that all other characteristics but the loss probabilities p_i are identical across policyholders, yields a common threshold p^*, below which agents do not purchase insurance. Due to the high loading that they face, policyholders who have a low probability of loss are less likely to purchase insurance.

In the example where a single firm insures n policyholders heterogeneously exposed to a risk of flooding, this means that take-up should decrease as one considers areas away from the river bed, which is the source of the common risk.

Propositions (11) and its corollary (11.1) show how correlation is a threat to the insurability of the lower probability risks, despite the fact that policies are priced to reflect individual exposure. Perhaps surprisingly, it is not the fact that a risk is systemic that makes it uninsurable. Instead, the fact of being correlated with other systemic risks generates a risk premium that remains high even when the loss probability becomes very small. As a consequence, the loading becomes very high and deters potential buyers from purchasing insurance.

2.4 Conclusion

In this paper, we show that the low insurance take-up rates for low-probability risks can be explained by the presence of aggregate uncertainty. Low probability risks without aggregate uncertainty, are easy to insure because people's willingness to pay, expressed in terms of loading, decreases with the loss-probability. On the contrary, low-probability risks with aggregate uncertainty can be very difficult to insure as the cost of providing coverage, expressed in terms of loading, explodes as the loss probability becomes small. This may explain why risks such as lightnings are covered under US homeowner policies, while earthquakes, floods and nuclear accidents are typically excluded.

Indeed most insurance companies propose coverage on several lines of risk. As we have showed, the risks with the lowest probabilities also have the highest loading factors if they are systemic and correlated with each other. Under these circumstances, they are also more difficult to insure.

This fact may explain how entities such as the National Flood Insurance Program or the Californian Earthquake Authority can mitigate this issue by insuring a single line of risk. However, some heterogeneity of exposure is bound to remain since not all policyholders live at an equal distance from the source of risk. As a consequence, we should observe the take-up rates diminish as we consider areas further away from the source of the risk considered, even with these mono-line insurance providers. Testing this empirical prediction is an interesting venue for further research.

2.5 Proofs

2.5.1 Proposition 8

We consider the case of an interior solution. From the first order condition (2.4), we obtain

$$\frac{d\tau}{dp} = \frac{(1+c'(\tau))[u'(w_1)-u'(w_2)]}{(1-p)p(1+c'(\tau))^2u''(w_2)+[1-(1+c'(\tau))p]^2u''(w_1)-c''(\tau)[pu'(w_1)+(1-p)u'(w_2)]}$$

$$+ \frac{[\tau+c(\tau)]\Big([1-(1+c'(\tau))p]u''(w_1)-(1-p)(1+c'(\tau))u''(w_2)\Big)}{(1-p)p(1+c'(\tau))^2u''(w_2)+[1-(1+c'(\tau))p]^2u''(w_1)-c''(\tau)[pu'(w_1)+(1-p)u'(w_2)]}.$$

Since u is concave and c convex we have

$$\begin{gathered}\tfrac{d\tau}{dp} < 0 \\ \Leftrightarrow \\ u'(w_1)+\tfrac{\tau+c(\tau)}{1+c'(\tau)}(1-(1+c'(\tau))p)u''(w_1) > u'(w_2)+\tfrac{\tau+c(\tau)}{1+c'(\tau)}(1-p)(1+c'(\tau))u''(w_2) \\ \Leftrightarrow \\ u'(w_1)[1-\tfrac{\tau+c(\tau)}{1+c'(\tau)}(1-(1+c'(\tau))p)A(w_1)] > u'(w_2)[1-\tfrac{\tau+c(\tau)}{1+c'(\tau)}(1-p)(1+c'(\tau))A(w_2)].\end{gathered} \tag{2.14}$$

Using the first order condition (2.4), this last inequality holds if and only if

$$1-\frac{\tau+c(\tau)}{1+c'(\tau)}(1-(1+c'(\tau))p)A(w_1) > \frac{1-(1+c'(\tau))p}{(1-p)(1+c'(\tau))}[1-\frac{\tau+c(\tau)}{1+c'(\tau)}(1-p)(1+c'(\tau))A(w_2)].$$

which can be written as

$$\frac{c'(\tau)}{(1-p)(1+c'(\tau))} > \frac{\tau+c(\tau)}{1+c'(\tau)}[A(w_1)-A(w_2)][1-(1+c'(\tau))p].$$

2.5.2 Corollary 8.1

Since $u()$ is a concave function and $\tau < L$ we know that $u'(w_1) > u'(w_2)$. A sufficient condition for inequality (2.14) to be satisfied is

$$1 - [1 - (1 + c'(\tau))p]\frac{\tau + c(\tau)}{1 + c'(\tau)}A(w_1) > 1 - (1 - p)(1 + c'(\tau))\frac{\tau + c(\tau)}{1 + c'(\tau)}A(w_2)$$

This can also be written as

$$(1 - p)(1 + c'(\tau))A(w_2) > [1 - (1 + c'(\tau))p]A(w_1)$$

Using the expression 2.7 of the coefficient of relative risk aversion for a HARA function, we obtain

$$\frac{(1 - p)(1 + c'(\tau))}{1 - (1 + c'(\tau))p} > \frac{\eta + \frac{w_2}{\gamma}}{\eta + \frac{w_1}{\gamma}} \tag{2.15}$$

At an interior solution, the first order condition (2.4) yields

$$\frac{(1 - p)(1 + c'(\tau))}{1 - (1 + c'(\tau))p} = \Big(\frac{\eta + \frac{w_2}{\gamma}}{\eta + \frac{w_1}{\gamma}}\Big)^{\gamma}$$

So for any $\gamma > 1$, this implies that (2.15) is verified and optimal coverage is indeed decreasing in p.

2.5.3 Proposition 10

Assume that L is now a function of p such that a change in p is compensated by a change in L that maintain pL constant, i.e.

$$L'(p) = -\frac{L}{p} \tag{2.16}$$

Let τ^*, w_1^* and w_2^* be the optimal values of coverage, loss state wealth and no-loss state wealth. The first order condition now writes

$$[1-(1+c'(\tau^*))p]u'(w+\tau^*-p\tau^*-pc(\tau^*)-L(p)) = (1-p)(1+c'(\tau^*))u'(w-p\tau^*-pc(\tau^*)) \tag{2.17}$$

Proceeding as in the proof of Proposition 8, we find the necessary and sufficient condition for $\frac{d\tau^*}{dp} < 0$

$$\frac{c'(\tau^*)}{(1-p)[1-(1+c'(\tau^*))p]} > [\tau^* + c(\tau^*)][A(w_1^*) - A(w_2^*)] + A(w_1^*)L'(p) \tag{2.18}$$

Using equation 2.16

$$\begin{aligned}
[\tau^* + c(\tau^*)][A(w_1^*) \quad - \quad & A(w_2^*)] + A(w_1^*)L'(p) = [\tau^* + c(\tau^*)][A(w_1^*) - A(w_2^*)] - A(w_1^*)\frac{L}{p} \\
= \quad & [\tau^* + c(\tau^*)]\Big(A(w_1^*) - A(w_2^*) - A(w_1^*)\frac{L}{[\tau^* + c(\tau^*)]p}\Big) \\
= \quad & [\tau^* + c(\tau^*)]\frac{A(w_1^*)[\tau^* + c(\tau^*)]p - A(w_2^*)[\tau^* + c(\tau^*)]p - A(w_1^*)L}{[\tau^* + c(\tau^*)]p} \\
= \quad & [\tau^* + c(\tau^*)]\frac{A(w_1^*)\{[\tau^* + c(\tau^*)]p - L\} - A(w_2^*)}{[\tau^* + c(\tau^*)]p}. \qquad (2.19)
\end{aligned}$$

At any interior solution, the left-hand side of 2.17 must be positive. Therefore, it must be the case that

$$p(1 + c'(\tau^*)) < 1.$$

By convexity of $c(\tau)$, we have

$$p(1 + c'(\tau)) < p(1 + c'(\tau^*)) < 1 \quad \forall \tau < \tau^*.$$

Hence

$$\int_0^{\tau^*} p(1 + c'(\tau))d\tau \;\; < \;\; \int_0^{\tau^*} d\tau.$$

Or equivalently, using $c(0) = 0$

$$p(\tau^* + c(\tau^*)) < \tau^* < L.$$

Therefore

$$A(w_1^*)\Big([\tau^* + c(\tau^*)]p - L\Big) < 0,$$

which, from 2.19 implies

$$[\tau^* + c(\tau^*)][A(w_1^*) - A(w_2^*)] + A(w_1^*)L'(p) < 0.$$

The necessary and sufficient condition 2.18 is therefore always satisfied.

2.5.4 Lemma 2

$I_i(x_i) = X_i(\tau_i + c(\tau_i))$ implies

$$\mathrm{cov}(I_i(x_i), I_j(x_j)) = (\tau_i + c(\tau_i))(\tau_j + c(\tau_j))\mathrm{cov}(X_i, X_j), \tag{2.20}$$

and, by definition, $\rho_{ij} = \mathrm{cov}(X_i, X_j)/(\sqrt{p_i(1-p_i)}\sqrt{p_j(1-p_j)})$. As a consequence, the covariance between two indemnity costs $I_i(x_i)$ and $I_j(x_j)$ is

$$\mathrm{cov}(I_i(x_i), I_j(x_j)) = \rho_{ij}(\tau_i + c(\tau_i))(\tau_j + c(\tau_j))\sqrt{p_i(1-p_i)}\sqrt{p_j(1-p_j)}. \tag{2.21}$$

The coefficient of correlation between two indemnity costs can therefore be written

$$\delta_{ij} = \rho_{ij}.$$

2.5.5 Lemma 3

Equation (2.13) with $\sigma_i = \sqrt{p_i(1-p_i)}$ yields

$$\alpha_i = \mu_i + \frac{A}{n}a_i(\tau_i + c(\tau_i))[L_i\sigma_i^2 + \sum_{j\neq i} L_j\rho_{ij}\sqrt{p_i(1-p_i)}\sqrt{p_i(1-p_i)}].$$

Dividing by $p_i\tau_i$ gives

$$\alpha_i/p_i\tau_i = 1 + \frac{c(\tau_i)}{\tau_i} + \frac{A}{n}a_i(\tau_i + c(\tau_i))[\frac{L_i(1-p_i)}{\tau_i} + \sqrt{\frac{1-p_i}{p_i}}\sum_{j\neq i}\rho_{ij}L_j\sqrt{p_j(1-p_j)}].$$

2.5.6 Proposition 11.1

Line i is insurable (in the strong sense) if and only if

$$[1-(1+b)p_i(1+\lambda(p_i))]u'(w-L_i) > (1-p_i)(1+b)(1+\lambda(p_i))u'(w),$$

Notice that

$$\lim_{p_i\to 0} p_i(1+\lambda(p_i)) = 0,$$

which implies

$$\lim_{p_i\to 0}[1-(1+b)p_i(1+\lambda(p_i))]u'(w-L_i) = u'(w-L_i).$$

Also

$$\lim_{p_i\to 0}\lambda(p_i) = +\infty$$

implies that

$$\lim_{p_i\to 0}(1-p_i)(1+b)(1+\lambda(p_i)) = +\infty.$$

Therefore, there exist a threshold p_i^* such that $p_i < p_i^*$ implies

$$[1-(1+b)p_i(1+\lambda(p_i))]u'(w-L_i) < (1-p_i)(1+b)(1+\lambda(p_i))u'(w),$$

and $\tau = 0$ at optimum. p_i^* is defined by

$$[1-(1+b)p_i(1+\lambda(p_i))]u'(w-L_j) = (1-p_i)(1+b)(1+\lambda(p_i))u'(w).$$

Chapter 3

Pooling natural disaster risks in a community

This chapter is co-authored with Arnaud Goussebaïle.

Abstract: We analyze the design of contracts when individual risks are correlated across risk-averse agents in a community. The community is equipped with a public insurer which supplies insurance contracts to its members and has access to costly reinsurance outside the community. Without transaction costs inside the community, risk-averse agents fully insure against their individual risk and share collective risk by getting some dividend in normal states. With premiums raised ex-ante and generating an opportunity cost, they only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and potentially get some dividend in normal states. We illustrate the emergence of the latter contracts for the community of the Caribbean countries exposed to natural disaster risks.

Keywords: individual risk, collective risk, insurance, mutual insurance.
JEL classification: D86, G22, G28, Q54.

3.1 Introduction

The Caribbean countries are located in a region of the world widely exposed to large natural disaster risks, such as earthquakes, hurricanes and flooding events. Even though aggregate damages are usually lower than 1 billion dollars per year in this region, the hurricane season in 2004 affected many countries with more than 6 billion dollars of aggregate losses and a large earthquake in January 2010 caused in Haiti more than 8 billion dollars of damages.[1] In this context, the non-for-profit Caribbean Catastrophe Risk Insurance Facility (CCRIF) is designed to supply insurance contracts to the Caribbean countries (CCRIF SPC (2014)). To deal with the high collective risks due to the spatial correlation of losses, the CCRIF purchases reinsurance outside the community. As reinsurance companies or other investors on financial markets supply reinsurance contracts[2] above fair prices (Jaffee & Russell (1997a), Cummins (2006) and Froot (2001b)), the CCRIF only partially reinsures the collective risks and supplies to its members insurance contracts which are mutual in the sense that they depend on collective losses. The indemnities for given individual losses are lower when collective losses are high than when collective losses are low. Moreover, dividends are given to the insureds when collective losses are low.[3] The CCRIF, created in 2007, is one example of such facilities that have emerged in different regions of the world exposed to natural disasters in the last twenty years. The Florida Hurricane Catastrophe Fund (FHCF) and the California Earthquake Authority (CEA) respectively created in 1993 and 1996 are other examples (Kousky (2010) and Kunreuther & Michel-Kerjan (2009)).

The present paper analyzes the optimal design of insurance contracts by a pooling insurance facility when the collective risks are not negligible and reinsur-

[1]Information on natural disaster losses in the Caribbean countries can be found on the EM-DAT International Disaster Database (http://www.emdat.be/).

[2]Investors on financial markets supply insurance-linked securities such as cat-bonds, which are similar for insurers to standard reinsurance contracts supplied by reinsurance companies. However, insurance-linked securities have emerged in the nineties because financial markets have larger financial capacities to supply contracts for very large risks.

[3]Dividends are given through premium discounts after a year with low collective losses.

ance is above fair prices. We consider a community of identical risk-averse agents (representing for instance the Caribbean countries). Each agent faces two individual states: she can either suffer a loss or not. At the collective level, there are two states of nature, the normal one and the catastrophic one, respectively characterized by low and high fraction of the agents affected.[4] We consider a non-for-profit pooling insurance facility for the community (representing for instance the CCRIF for the Caribbean countries). The insurance facility supplies mutual insurance contracts to the agents in the community. For one contract, it charges a premium and pays an indemnity to the insured if affected. The indemnity level in the normal state may differ from the indemnity level in the catastrophic state. The insurance contract may also include a dividend if the normal state occurs. Besides, the insurance facility has access to reinsurance outside the community. We analyze the characteristics of the optimal insurance and reinsurance contracts for the community, when reinsurance is above fair prices.

Without any transaction costs inside the community, the optimal insurance contract consists in full coverage for individual losses in both the normal state and the catastrophic state. Moreover, it includes a strictly positive dividend in the normal state because reinsurance is above fair prices. The higher the cost of reinsurance, the higher the premium and the dividend because the insurer substitutes reinsurance by a higher reserve from the agents to pay the high total indemnities of the catastrophic state. However, requiring high amount of premiums ex-ante[5] can generate an opportunity cost for the agents in the community. Indeed, this capital cannot be used for other purpose (i.e consumption or investment) which thus may require the agents to borrow more costly external capital. In this case, it is Pareto improving to implement a contract with a lower indemnity for individual loss in the catastrophic state than in the normal state. Moreover, the optimal contract still has full coverage and dividend in the normal state if and only if the

[4]We consider only two individual states to keep the model tractable. At the collective level, we consider two and only two states of nature respectively to model collective risks and to keep the model tractable.

[5]Premiums are required ex-ante to pay reinsurance premiums and to secure a reserve which avoids participation default. Moreover, raising the premiums ex-ante enables to transfer indemnities quickly to affected agents.

marginal opportunity cost is low enough relative to the marginal reinsurance cost.

The economics literature has already addressed the question of optimal insurance contract when there are collective risks in a community. Doherty & Schlesinger (1990), Hau (1999), Cummins & Mahul (2004) and Mahul & Wright (2004, 2007) consider the case where the indemnity level in normal states increases with the premium level but the indemnity level in catastrophic states is null whatever the premium level. In this case, the optimal contract consists in partial coverage for individual losses in normal states in order to preserve their welfare level in catastrophic states. However, these papers do not address the issue of the insurer financial capacity which would explain why indemnities cannot be paid in catastrophic states.[6] Charpentier & Le Maux (2014b) focuses on the issue by considering an insurer with an exogenously given amount of reserve besides premiums. In this case, the indemnity level increases with the premium level in normal states and in catastrophic states because raised premiums increase the financial capacity. However, the optimal insurance contract consists in full coverage for individual losses in normal states but not in catastrophic states because of insurer limited reserve. Relative to Charpentier & Le Maux (2014b), we relax the assumption of partially exogenous financial capacity by introducing reinsurance outside the community. Moreover, we allow a better participation of the insureds in the reserve by introducing dividends in the contracts, which gives more flexibility in the design of contracts for risk sharing. Indeed, as explained by Borch (1962) and Marshall (1974), in a community where agents are exposed to individual risks with collective components, it is Pareto optimal to eliminate individual risks and to share collective risks (mutuality principle). Malinvaud (1973) and Cass et al. (1996) show that a mutual contract with dividend supplied by the insurance company enables to reach the mutuality principle. Penalva-Zuasti (2001) and Penalva-Zuasti (2008) show that it is also reached with agents purchasing a standard contract from the insurance company and investing in the insurance company through stock market.

[6] This contingency can be seen as contractual or as a "default risk" with right perception by insureds and no cost of default. In the quoted theoretical papers, "default risk" is used in this sense.

When reinsurance outside a community is costly, Doherty & Dionne (1993) and Doherty & Schlesinger (2002) show that the optimal contract consists in the full elimination of individual risks in each state of nature, plus partial coverage of the collective risks. Relative to Doherty & Dionne (1993) and Doherty & Schlesinger (2002), we analyze how the cost of reinsurance and the correlation between individual risks affect the optimal contract. Moreover, we introduce and analyze the impact of the opportunity cost of capital potentially generated by raising premiums ex-ante.

The first contribution of the present paper is to develop a simple and tractable model to analyze the optimal design of insurance contracts by a pooling insurance facility to manage individual and collective risks. The second contribution is to study the impact of reinsurance costs and risk correlation on the optimal insurance contract further than previous works. The third contribution is to consider the opportunity cost of capital potentially generated by raising premiums ex-ante. The paper is organized as follows. Section 2 presents the example of the Caribbean countries and their insurance facility. Section 3 sets up the model of a community with individual and collective risks and the insurance and reinsurance contracts. Section 4 provides an analysis of the optimal insurance and reinsurance contracts. Section 5 concludes.

3.2 Caribbean countries and natural disasters insurance

The Caribbean countries are located in a region of the world exposed to important natural disaster risks. Figure 3.1[7] exhibits natural disaster losses in this region in the last fifty years. Collective losses are widely variable from one year to another because natural disasters in the region can have large spatial impacts. Year 2010 corresponds to the highest losses with a large earthquake affecting Haiti in January with more than 8 billion dollars of damages. Year 2004 corresponds to the

[7]EMDAT International Disaster Database (http://www.emdat.be/)

second highest losses with a particularly dramatic hurricane season affecting many countries such as the Bahamas, the Cayman Islands, Grenada and Jamaica.

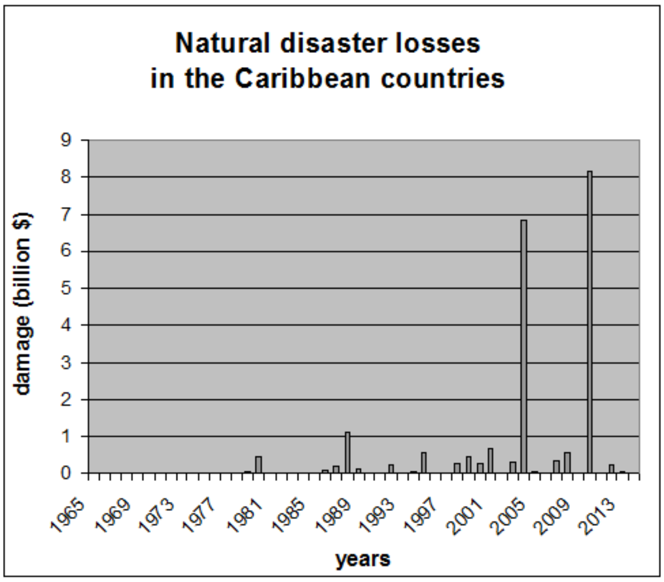


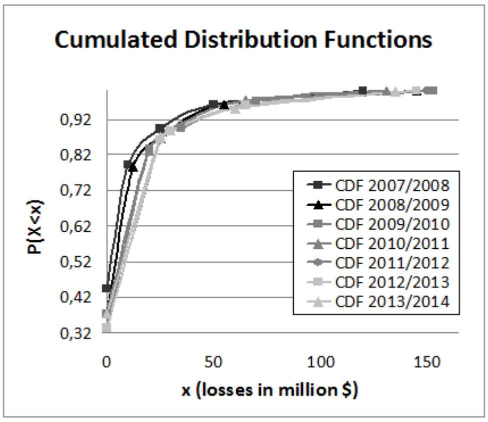
Figure 3.1: Natural disaster losses in the Caribbean countries.

In this context, the Caribbean Catastrophe Risk Insurance Facility (CCRIF) is a non-for-profit multi-country insurance pool. Created in 2007, it currently offers disaster-relief insurance policies covering public losses of sixteen Caribbean countries, protecting them against earthquake, hurricane and excess rainfall losses. Its effectiveness during the five first years of existence has conducted the program to be extended to Central American countries, starting from 2016. The facility aims at pooling the risks faced by its members and reduce the cost the members would individually face if they directly insured on the reinsurance market. The annual reports of the CCRIF are publicly available[8] and provide useful information about the catastrophe insurance contracts proposed to the sixteen members. The CCRIF reports a stable number of 29 or 30 sold policies each year since its inception.[9] The collective risk faced by the CCRIF has remained rather stable as well. Figure 3.2a displays the cumulative density function of the aggregation of the risks covered by

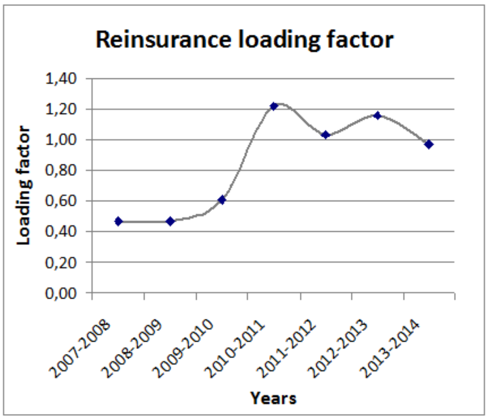
[8]http://www.ccrif.org/content/publications/reports/annual

[9]The CCRIF can sell more than one policy per country per year because insurance policies for the different types of natural disasters are separated.

the CCRIF and reported in its annual reports since year 2007-2008. The darker lines represent the cumulative distribution of this aggregate risk faced by the pool in the earlier periods of its existence.[10] Using the cumulative distribution functions with the information about the structure of the reinsurance scheme bought by the CCRIF, we can compute an estimated loading factor paid by the organization as : $\lambda^R = \frac{\alpha^R}{\mathbb{E}(L)} - 1$, where α^R is the premium paid by the CCRIF to reinsurers and $\mathbb{E}(L)$ is the expected loss reinsured. Figure 3.2b displays its evolution through the years and shows that the CCRIF faces a significant loading factor on reinsurance, which clearly explains why it only partially reinsures the collective risk. The figure shows that reinsurers increased their prices a lot in 2010, following the large earthquake affecting Haiti.



(a) Aggregate risks faced by the CCRIF



(b) Reinsurance loading factor

Figure 3.2: Aggregate risks resulting from risk pooling by the CCRIF and estimated reinsurance loading factor faced by the CCRIF.

As the CCRIF only partially reinsures the collective risk, it supplies to its members insurance contracts which are mutual in the sense that they depend on collective losses. In addition to the regular insurance premiums, the facility requires its members to pay an up-front participation fee. Audited financial state-

[10]Insured losses in figure 3.2a are much lower than total losses due to natural disasters in figure 3.1. This is due to the fact that the CCRIF covers only public losses which represent only a small fraction of the total losses incurred in a country when a natural disaster occurs. It is also due to the fact that the Caribbean countries purchase from the CCRIF only partial insurance for natural disaster risks.

ments report that "it is Management's intent that participation fee deposits are available to fund losses in the event that funds from retained earnings, reinsurers and the Donor Trust are insufficient. If deposits are used to fund losses, it is also Management's intent that any subsequent earnings generated by the Group will be used to reinstate the deposits to their original carrying value". Figure 3.3 shows that the total amount of premiums was effectively much higher the first year than the following years. It has not been necessary to raise high premiums the following years because no extremely large claims had to be paid during these years. In terms of claims to be paid, the worth year is 2010, during which the CRIFF had to transfer a bit less than 8 million dollars to Haiti for the large earthquake affecting the country. The yearly insurance contract is similar to a contract with a high premium requested at the beginning of the year in exchange for an indemnity if the insured is affected during the year and a dividend at the end of the year if collective losses are not too catastrophic. In the present case, the dividend is given through a premium discount at the beginning of the following year. Besides, the CCRIF acknowledges the possibility of lowered indemnities in catastrophic states: "The CCRIF can currently survive a series of loss events with a less than 1 in 10,000 chance of occurring in any given year. Due to planned premium reductions, the safety level drops somewhat through the course of our 10-year forward modeling. However, the lowest projected survivability for the CCRIF in the 10-year modeled period is about 1 in 3000 chance of claims exceeding capacity in any one year." In other words, the CCRIF acknowledges to supply contracts such that the indemnity for one individual loss level is lower in highly catastrophic states than in the other states of nature.

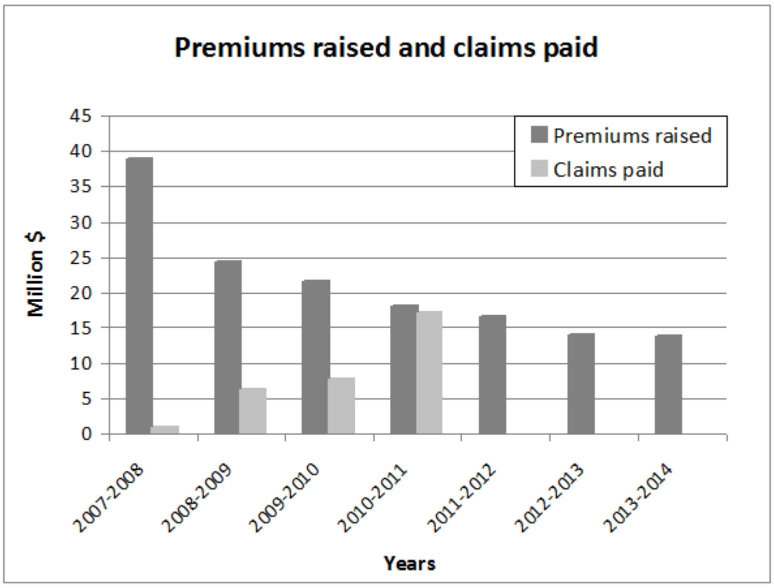


Figure 3.3: Premiums raised and claims paid by the CCRIF.

3.3 The model

3.3.1 The community of agents

We consider a community of N agents identical in terms of preferences, initial wealth and exposure to risk.[11] The preferences of the representative agent satisfy the von Neumann-Morgenstern axioms, with $u(.)$ the corresponding utility function which is strictly increasing, globally concave and twice continuously differentiable. The representative agent has an initial wealth w and is exposed to a potential loss l. The individual risks can generate a significant collective risk either because N is not large enough or because individual risks are correlated. To model the collective risks, we consider two states of nature, one catastrophic and one normal. Ex-ante, the representative agent knows that with a probability p (such that $0 < p < 1$), a catastrophe occurs and the fraction of agents enduring a loss of size l is q_c. In the normal state, the fraction of agents enduring the same loss l is $q_n < q_c$.[12] In this

[11]Heterogeneity of individuals raises questions related to asymmetric information that are out of the scope of our analysis.

[12]As pointed out by Malinvaud (1973) and Cass et al. (1996), considering two different individual loss levels in the normal and catastrophic states could be considered as two different

template, the individual probability of enduring a loss l is q_c in the catastrophic state and q_n in the normal state, and the unconditional individual probability of enduring a loss l is: $\overline{q} = (1-p)q_n + pq_c$. The individual random wealth without risk sharing scheme is characterized in figure 3.4. Besides, the collective random wealth of the N agents is characterized in figure 3.5. With N large, the coefficient δ of correlation between individual risks is well approximated by: $\delta = \frac{p(1-p)}{\overline{q}(1-\overline{q})}(q_c - q_n)^2$ (proof in appendix 3.6.1). The higher the difference between the fraction q_c of affected agents in the catastrophic state and the fraction q_n of affected agents in the normal state, the higher the risk correlation between agents.[13] Finally, q_n and q_c can be expressed as functions of the individual probability $\overline{q}$ of being affected, the correlation δ between individual risks and the probability p of catastrophe: $q_n = \overline{q} - p(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ and $q_c = \overline{q} + (1-p)(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$.

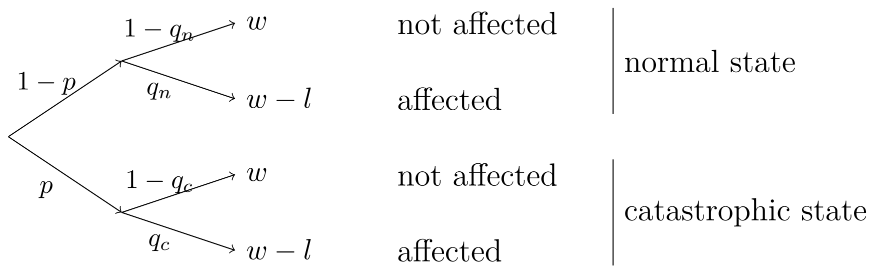


Figure 3.4: individual random wealth of the representative agent

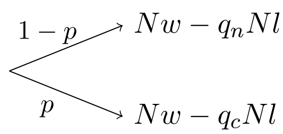


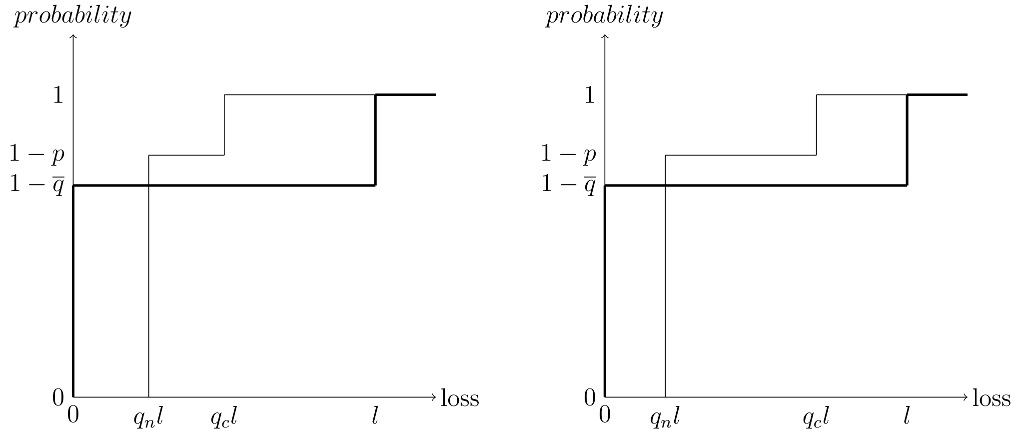
Figure 3.5: collective random wealth of the N agents

In this template, average individual loss depends on the state of nature, its value is q_nl in the normal state and q_cl in the catastrophic state. Thus, the expected value

risks.

[13]The fully correlated case ($\delta = 1$) is characterized by $q_n = 0$, $q_c = 1$ and $0 < p < 1$, in which everyone endures a loss or no one. The no-correlated case ($\delta = 0$) would correspond to $q_n = q_c$, $p = 1$ or $p = 0$, in which there is only one collective state.

of the average individual loss is $\overline{q}l$ and its variance is $\overline{q}(1-\overline{q})\delta l^2$ (proof in appendix 3.6.1). The higher the individual probability $\overline{q}$ of being affected, the higher the expected average loss. The more correlated the individual risks, the more volatile the average loss.[14] Figure 3.6 illustrates for two different sets of parameters the cumulative distribution functions for the individual loss (thick bars) and for the average individual loss (thin bars). The spread between q_n and q_c is smaller in 3.6a than in 3.6b, while in both cases $p = 0.2$ and $\overline{q} = 0.3$. The individual probability of being affected $\overline{q}$ is similar for the two sets of parameters, whereas the correlation across individual risks δ is smaller in 3.6a than in 3.6b, which makes a difference for risk sharing mechanism as detailed in the paper.



(a) $p = 0.20$, $q_n = 0.25$, $q_c = 0.50$, $\overline{q} = 0.3$, $\delta = 0.048$

(b) $p = 0.20$, $q_n = 0.20$, $q_c = 0.70$, $\overline{q} = 0.3$, $\delta = 0.190$

Figure 3.6: Cumulative distribution functions for individual loss (thick bars) and average individual loss (thin bars) for two different sets of parameters in 3.6a and 3.6b.

3.3.2 Insurance and reinsurance contracts

We consider that the community is equipped with a pooling insurance facility, also called the insurer. The insurer faces two states of nature, the normal one and the

[14] Cummins (2006) and Cummins & Trainar (2009) have more insights on the relation between the risk correlation and the average loss volatility.

catastrophic one, in which it respectively has a fraction q_n and a fraction q_c of its insureds that have to be indemnified. A standard-type insurance contract is a couple (α, τ). In this case, α is the premium paid by the agent and $\tau \geq 0$ is the indemnity received by the agent if the latter endures a loss l. A mutual-type insurance contract is a quadruple $(\alpha, \tau, \epsilon, \pi)$. In this case, α is the premium paid by the agent, $\tau \geq 0$ is the indemnity received by the agent in the normal state if the latter endures a loss l, $\tau - \epsilon \geq 0$ is the indemnity received by the agent in the catastrophic state if the latter endures a loss l and $\pi \geq 0$ is the dividend received by the agent in the normal state. This contract is called mutual-type contract because each agent shares a fraction of the collective risk of the community. Indeed, ϵ and π make the insurance contract directly depend on the collective losses, contrary to the standard contract. The standard contract is a specific case of the mutual contract with $\epsilon = 0$ and $\pi = 0$.[15] Besides, we consider that the contract can generate an opportunity cost for the insured. When premiums are raised ex-ante while indemnities and dividends are given ex-post, the secured capital cannot be used for other purpose (i.e. consumption or investment). Thus, the agents may have to raise more costly external capital instead of using this capital, which generates an opportunity cost for the agents. The higher the required premium α, the higher should be the marginal opportunity cost because the costlier should be the marginal external capital. We denote $\lambda^l(\alpha)$ the opportunity cost function which is increasing and convex relative to the premium α. The agent wealth profile with a mutual-type contract is represented in figure 3.7.

The insurer has to manage the collective risks generated by the aggregation of the insured individual risks. It can purchase reinsurance outside the community, to be able to pay the higher total claims of the catastrophic state. Purchasing a reinsurance contract, with an indemnity $\tau^R \geq 0$ in the catastrophic state occuring with a probability p, costs $(1 + \lambda^R)p\tau^R$, in which $\lambda^R \geq 0$ is the reinsurance loading

[15] The mutual contract defined here is in the spirit of the contracts supplied by the CCRIF to the Caribbean countries. The premium α corresponds to the regular premium plus the up-front participation fee in the contracts supplied by the CCRIF. The dividend π corresponds to the premium discount of the following year if losses are not too catastrophic. The indemnity gap ϵ between normal state and catastrophic state is also acknowledged by the CCRIF.

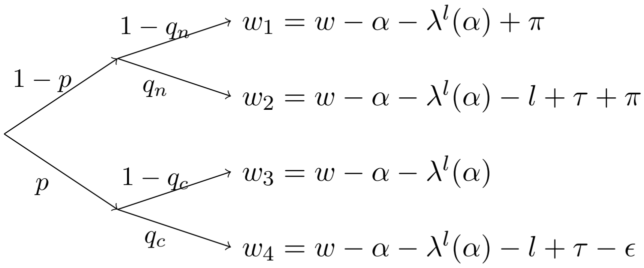


Figure 3.7: agent wealth profile with an insurance contract

factor.[16] For reinsurance to be relevant, we need to have $(1+\lambda^R)p < 1$.[17] With the insurance contracts supplied to the agents and the reinsurance contract purchased outside the community, the insurer wealth profile is detailed in figure 3.8.

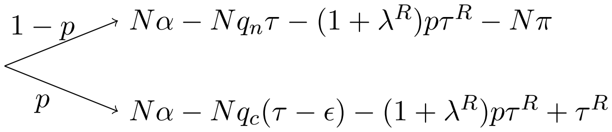


Figure 3.8: insurer profit profile

3.4 Optimal insurance and reinsurance

The optimal insurance and reinsurance contracts for the community consist in maximizing the expected utility of the representative agent under the budget constraints.

3.4.1 Budget constraints

The mutual insurance facility cannot pay claims unless it has secured the funds either through raised premiums or purchased reinsurance. With the budget constraints in both the normal state and the catastrophic state (budget expressions in

[16] λ corresponds to frictional costs with reinsurers or investors, as detailed in Froot (2001b).

[17] If $(1+\lambda^R)p \geq 1$, purchasing reinsurance would have no sense for the CCRIF because it would lose money in both the normal state and the catastrophic state with the reinsurance contract.

figure 3.8), the optimal insurance and reinsurance contracts are thus the solution of the following maximization problem:

$$\begin{aligned}
&\max_{\alpha,\tau,\epsilon,\pi,\tau^R} \mathbb{E}(u(\tilde{w})) \\
&\text{s.t. } N\alpha - Nq_n\tau - (1+\lambda^R)p\tau^R - N\pi \geq 0 \\
&\qquad N\alpha - Nq_c(\tau-\epsilon) - (1+\lambda^R)p\tau^R + \tau^R \geq 0 \\
&\qquad \tau \geq 0,\ \tau-\epsilon \geq 0,\ \pi \geq 0,\ \tau^R \geq 0.
\end{aligned} \tag{3.1}$$

Because utility is increasing with wealth, the budget constraints are binding in the two states of nature, the catastrophic one and the normal one. The subtraction of the two binding budget constraints gives the purchased reinsurance indemnity τ^R:

$$\tau^R = Nq_c(\tau-\epsilon) - Nq_n\tau - N\pi \geq 0. \tag{3.2}$$

The insurance facility has to purchase a reinsurance indemnity in order to cover the difference between the amount due in the catastrophic state ($Nq_c(\tau-\epsilon)$) and the amount due in the normal state ($Nq_n\tau + N\pi$). With (3.2), the binding budget constraints give the required premium:

$$\alpha = \left(1 + \frac{p(q_c-q_n)}{\bar{q}}\lambda^R\right)\bar{q}\tau + \left(1 - \frac{p}{1-p}\lambda^R\right)(1-p)\pi - \left(1+\lambda^R\right)pq_c\epsilon, \tag{3.3}$$

which simplifies, if reinsurance is binding ($\tau^R = 0$), to:

$$\alpha = q_c(\tau-\epsilon). \tag{3.4}$$

To be able to pay indemnities and dividends, the insurance facility requires the premium (3.3) (if $\tau^R > 0$) or (3.4) (if $\tau^R = 0$) for the contract $(\alpha,\tau,\epsilon,\pi)$. If it is not valuable to purchase reinsurance, the insurance facility has to raise premiums (3.4) in order to be able to pay all the indemnities in the catastrophic state. If it is valuable to purchase reinsurance, the insurance facility has to pass on the cost of reinsurance to insureds, which explains the loading factor $\frac{p(q_c-q_n)}{\bar{q}}\lambda^R$ in front of τ in (3.3). As shown by (3.2), allowing a dividend in the normal state ($\pi > 0$) or a lower indemnity in the catastrophic state ($\epsilon > 0$) enables to lower the purchase of reinsurance. With a dividend in the normal state ($\pi > 0$), the premium is affected in two opposite directions. The first channel is straightforward: a higher dividend implies a higher premium. The second channel is due to the fact that the insurer has to purchase less reinsurance thanks to the reserve from the insureds and appears through λ^R in the coefficient in front of π in (3.3). Note that the factor in front of π in (3.3) is globally positive because $(1+\lambda^R)p < 1$. With a lower indemnity in the catastrophic state ($\epsilon > 0$), the premium is reduced through two channels. The first channel is straightforward: a lower indemnity implies a lower premium. The second channel is due to the fact that the insurer has to purchase less reinsurance and appears through λ^R in the coefficient in front of ϵ in (3.3). If the agents in the community can have direct access to reinsurance with the same loading factor λ^R, it is valuable to insure through the insurance facility because: $1 + \frac{p(q_c - q_n)}{\bar{q}}\lambda^R < 1 + \lambda^R$, thanks to partial diversification done by the insurance facility. This is true with standard insurance contracts and thus also true with mutual contracts. The higher the cost of reinsurance, the more valuable the facility. The lower the correlation between participants, the more efficient the pooling and thus the more valuable the facility. This could explain why the Caribbean countries would like to extend their facility to South American Countries in 2016. However, we have assumed that there are no management costs for the insurance facility. If there are, the pooling insurance facility is valuable if the cost of implementing the facility generates a loading factor λ^i such that: $1 + \lambda^i + \frac{p(q_c - q_n)}{\bar{q}}\lambda^R < 1 + \lambda^R$. In the case of the Caribbean countries, extending the insurance facility to South American Countries will be valuable if it does not add too much management costs.

3.4.2 Insurance and reinsurance contracts

With the binding budget constraints, the maximization problem (3.1) for the optimal contracts boils down to:[18]

$$\begin{aligned}
&\max_{\tau,\epsilon,\pi} \mathbb{E}(u(\tilde{w}))\\
&\text{s.t. } \alpha = \left(1+\frac{p(q_c-q_n)}{\bar{q}}\lambda^R\right)\bar{q}\tau + \left(1-\frac{p}{1-p}\lambda^R\right)(1-p)\pi - \left(1+\lambda^R\right)pq_c\epsilon\\
&\qquad \tau \geq 0,\ \pi \geq 0,\ (q_c-q_n)\tau - \pi - q_c\epsilon \geq 0.
\end{aligned} \tag{3.5}$$

Note that, with standard insurance contracts ($\pi = 0$ and $\epsilon = 0$), the maximization problem (3.5) corresponds to the standard Mossin problem, in which the optimal coverage level is obtained by the marginal tradeoff between the aversion to risk and the cost due to reinsurance ($\frac{p(q_c-q_n)}{\bar{q}}\lambda^R$).

Without opportunity cost ($\lambda^l(\alpha) = 0$)

We first consider the case in which raising premiums ex-ante does not generate an opportunity cost for the insured ($\lambda^l(\alpha) = 0$). The first order conditions of (3.5) are derived in appendix 3.6.2. If it is valuable to purchase reinsurance (i.e. $\frac{\tau^R}{N} = (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0$ is not binding), the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w-\alpha-l+\tau+\pi)}{u'(w-\alpha+\pi)} = 1, \tag{3.6}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w-\alpha-l+\tau-\epsilon)}{u'(w-\alpha)} = 1, \tag{3.7}$$

$$\frac{u'(w_3)}{u'(w_1)} = \frac{u'(w-\alpha)}{u'(w-\alpha+\pi)} = \frac{1+\lambda^R}{1-\frac{p}{1-p}\lambda^R}. \tag{3.8}$$

[18]The last inequality constraint in (3.5) corresponds to $\tau^R \geq 0$. The inequality constraint $\tau - \epsilon \geq 0$ is not written because it is necessarily verified with the other inequality constraints. Indeed, we have at least as much money for indemnities in the catastrophic state as the amount of money for indemnities and dividends in the normal state.

If it is not valuable to purchase reinsurance (i.e. $\frac{\tau^R}{N} = (q_c - q_n)\tau - \pi - q_c\epsilon \geq 0$ is binding), the optimal contract has indemnities τ and $\tau - \epsilon$ plus dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w - \alpha - l + \tau + \pi)}{u'(w - \alpha + \pi)} = 1, \tag{3.9}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w - \alpha - l + \tau - \epsilon)}{u'(w - \alpha)} = 1, \tag{3.10}$$

$$\pi = (q_c - q_n)\tau - q_c\epsilon. \tag{3.11}$$

Whether reinsurance is purchased or not, the optimal insurance contract is such that: $w_1 = w_2$ and $w_3 = w_4$ (thanks to (3.6) and (3.7) or (3.9) and (3.10)), which means that $\tau = l$ and $\epsilon = 0$. Besides, when $\lambda^R = 0$, (3.8) tells that $\pi = 0$, (3.3) gives $\alpha = \overline{q}l$ and (3.2) gives $\tau^R = N(q_c - q_n)l$. When $0 < \lambda^R < \lambda^{R^*}$, (3.8) tells that $\pi > 0$, (3.3) gives $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$ and (3.2) gives $\tau^R = N(q_c - q_n)l - N\pi > 0$. λ^{R^*} is determined with (3.8) and the additional constraint $\frac{\tau^R}{N} = (q_c - q_n)l - \pi = 0$, which tells that $\pi = (q_c - q_n)l$ and $\alpha = q_c l$. When $\lambda^{R^*} \leq \lambda^R$, (3.11) tells that $\pi = (q_c - q_n)l$, (3.4) gives $\alpha = q_c l$ and reinsurance is not purchased $\tau^R = 0$.

Proposition 12 *The optimal insurance and reinsurance contracts are such that:*

(i) when $\lambda^R = 0$*:* $\tau = l$, $\epsilon = 0$, $\pi = 0$, $\alpha = \overline{q}l$, $\tau^R = N(q_c - q_n)l$*;*

(ii) when $0 < \lambda^R < \lambda^{R^*}$*:* $\tau = l$, $\epsilon = 0$, $\pi > 0$, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$, $\tau^R = N(q_c - q_n)l - N\pi > 0$*;*

(iii) when $\lambda^{R^*} \leq \lambda^R$*:* $\tau = l$, $\epsilon = 0$, $\pi = (q_c - q_n)l$, $\alpha = q_c l$, $\tau^R = 0$*;*

in which λ^{R^*} *is such that* $\frac{u'(w - q_c l)}{u'(w - q_n l)} = \frac{1 + \lambda^{R^*}}{1 - \frac{p}{1-p}\lambda^{R^*}}$.

Proposition 12 states that the optimal insurance contract has full coverage for a given individual loss in both normal and catastrophic states ($\tau = l$ and $\epsilon = 0$) whatever the cost of reinsurance (λ^R). The optimal contract eliminates individual risks, which is in line with Borch mutuality principle. Besides, proposition 12 states that the optimal contract has dividend ($\pi > 0$) in the normal state if and only if reinsurance is supplied above fair prices ($\lambda^R > 0$). If reinsurance is fair ($\lambda^R = 0$), the insurance facility fully reinsures the collective risk ($\tau^R = N(q_c - q_n)l$) and the optimal insurance contract is standard, i.e. without any dividend in the normal state ($\pi = 0$). If reinsurance is not fair ($\lambda^R > 0$), a mutual contract (i.e. with $\pi > 0$) is better than a standard contract because it enables the risk-averse agent to bear a part of the collective risk contrary to the standard contract, which is valuable because reinsurance is costly. If reinsurance is excessively above fair prices ($\lambda^{R*} \leq \lambda^R$), the insurance facility does not purchase reinsurance ($\tau^R = 0$) and the optimal insurance contract is with dividend in the normal state corresponding to the indemnity difference between the catastrophic state and the normal state ($\pi = (q_c - q_n)l$). If reinsurance is reasonably above fair prices ($0 < \lambda^R < \lambda^{R*}$), the insurance facility partially reinsures the collective risk ($\tau^R > 0$) and the optimal insurance contract is with dividend in the normal state ($\pi > 0$).

Proposition 13 *With $0 < \lambda^R < \lambda^{R*}$ (and a CARA utility function*[19]*), we have for the optimal insurance and reinsurance contracts:* $\frac{d\pi}{d\lambda^R} > 0$, $\frac{d\alpha}{d\lambda^R} > 0$, $\frac{d\tau^R}{d\lambda^R} < 0$.

Proposition 13 is proved in appendix 3.6.2. It states that the higher the reinsurance cost (i.e. λ^R), the lower the reinsurance purchase and the higher the premium and the dividend in the normal state. Indeed, to be able to cover individual losses in the catastrophic state when reinsurance purchase is decreased, the insurance facility has to increase the reserve financed by the insureds through higher premiums. Moreover, it has higher dividends to give to the insureds if the catastrophic state does not occur. In the extreme case where λ^R reaches λ^{R*}, reinsurance is

[19]The coefficient of absolute risk aversion of a utility function $u(.)$ is by definition $A(.) = -\frac{u''(.)}{u'(.)}$. We consider here a utility function with a constant absolute risk aversion A (also called CARA utility function). If the utility function is not CARA, there is an additional wealth effect. However, as long as this effect is of secondary order, it does not change the results. Note that if this effect was not of secondary order, it would have been observed that insurance can be a Giffen good (i.e. a higher premium leading to a higher purchase of insurance). To our knowledge, empirical analysis on the purchase of natural disaster insurance have not observed such behaviors. For instance, Browne & Hoyt (2000) and Grace et al. (2004) observe that when insurance price increases, the demand for insurance decreases.

not purchased ($\tau^R = 0$) and the premium and the dividend respectively reach the highest levels $\alpha = q_c l$ and $\pi = (q_c - q_n)l$.

Proposition 14 *We have for the optimal insurance and reinsurance contracts:*

(i) when $\lambda^R = 0$*:* $\frac{d\pi}{d\delta} = 0$, $\frac{d\alpha}{d\delta} = 0$, $\frac{d\tau^R}{d\delta} > 0$*;*

(ii) when $0 < \lambda^R < \lambda^{R^*}$ *(with a CARA utility function):* $\frac{d\pi}{d\delta} = 0$, $\frac{d\alpha}{d\delta} > 0$, $\frac{d\tau^R}{d\delta} > 0$*;*

(iii) when $\lambda^{R^*} \leq \lambda^R$*:* $\frac{d\pi}{d\delta} > 0$, $\frac{d\alpha}{d\delta} > 0$, $\frac{d\tau^R}{d\delta} = 0$.

Proposition 14 is obtained thanks to proposition 12, recalling that $q_n = \overline{q} - p(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ and $q_c = \overline{q} + (1-p)(\frac{\overline{q}(1-\overline{q})}{p(1-p)}\delta)^{0.5}$ ((i) and (iii) are obvious and (ii) is proved in appendix 3.6.2). Firstly, it states that the insurance contract is affected by a change of correlation δ if and only if reinsurance is not fair ($\lambda^R > 0$). If reinsurance is fair, only the average probability $\overline{q}$ and the loss l affects the insurance contract because the collective risk is fully reinsured without any cost. If reinsurance is not fair, the higher the correlation δ, the larger the collective risk and the more expensive its coverage. If reinsurance is not too costly ($0 < \lambda^R < \lambda^{R^*}$), an increase of δ is managed by an increase of reinsurance purchase to be able to cover the higher total indemnities in the catastrophic state and the insurance facility has to translate the cost of reinsurance to insureds through higher premiums. If reinsurance is too costly ($\lambda^{R^*} \leq \lambda^R$), an increase of δ is managed by an increase of the reserve through higher premiums and the insurance facility has higher dividends to distribute if the catastrophe does not occur. In both cases, higher correlation δ leads to higher premiums.

To sum up, the premium α increases from $\overline{q}l$ to $q_c l$ when λ^R increases from 0 to high values and it also increases when risk correlation δ increases. Thus, with costly reinsurance and significant risk correlation, the required premiums can reach high levels for insureds if the individual loss l is significant. In this case, which is relevant for natural disaster risks, raising such levels of premiums ex-ante can generate an opportunity cost for insureds, which are considered in the following section.

With opportunity cost ($\lambda^l(\alpha) \geq 0$)

We now consider the case in which raising premiums ex-ante generate an opportunity cost for the insureds ($\lambda^l(\alpha) \geq 0$). As explained in section 3.3.2, we assume that the opportunity cost function $\lambda^l(.)$ is increasing and convex relative to the premium α. We analyze how the marginal opportunity cost $\lambda^{l'}(\alpha)$ affects the optimal insurance and reinsurance contracts. We consider $\lambda^R \leq \lambda^{R^*}$, which means that purchasing some reinsurance is valuable. The first order conditions of (3.5) are derived in appendix 3.6.3. If it is valuable to have dividend in the normal state $\pi \geq 0$, the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w-\alpha-\lambda^l(\alpha)-l+\tau+\pi)}{u'(w-\alpha-\lambda^l(\alpha)+\pi)} = 1, \tag{3.12}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w-\alpha-\lambda^l(\alpha)-l+\tau-\epsilon)}{u'(w-\alpha-\lambda^l(\alpha))} = \frac{(1+\lambda^{l'}(\alpha))(1+\lambda^R)}{(1+\lambda^{l'}(\alpha))(1+\lambda^R)-\frac{\lambda^{l'}(\alpha)}{p(1-q_c)}}, \tag{3.13}$$

$$\frac{u'(w_3)}{u'(w_1)} = \frac{u'(w-\alpha-\lambda^l(\alpha))}{u'(w-\alpha-\lambda^l(\alpha)+\pi)} = \frac{(1+\lambda^{l'}(\alpha))(1+\lambda^R)-\frac{\lambda^{l'}(\alpha)}{p(1-q_c)}}{(1+\lambda^{l'}(\alpha))(1-\frac{p}{1-p}\lambda^R)}. \tag{3.14}$$

If it is not valuable to have dividend in the normal state (i.e. $\pi \geq 0$ is binding), the optimal contract has indemnities τ and $\tau - \epsilon$ and dividend π such that:

$$\pi = 0, \tag{3.15}$$

$$\frac{u'(w_2)}{u'(w_1)} = \frac{u'(w-\alpha-\lambda^l(\alpha)-l+\tau)}{u'(w-\alpha-\lambda^l(\alpha))} = \frac{(1+\lambda^{l'}(\alpha))(1-\frac{p}{1-p}\lambda^R)}{(1+\lambda^{l'}(\alpha))(1-\frac{p(q_c-q_n)}{1-\overline{q}}\lambda^R)-\frac{\lambda^{l'}(\alpha)}{1-\overline{q}}}, \tag{3.16}$$

$$\frac{u'(w_4)}{u'(w_3)} = \frac{u'(w-\alpha-\lambda^l(\alpha)-l+\tau-\epsilon)}{u'(w-\alpha-\lambda^l(\alpha))} = \frac{(1+\lambda^{l\prime}(\alpha))(1+\lambda^R)}{(1+\lambda^{l\prime}(\alpha))(1-\frac{p(q_c-q_n)}{1-\overline{q}}\lambda^R)-\frac{\lambda^{l\prime}(\alpha)}{1-\overline{q}}}. \tag{3.17}$$

Proposition 15 *The optimal insurance and reinsurance contracts are such that:*

(i) when $0 < \lambda^{l\prime}(\alpha) < \lambda^{l*}$*:* $\tau = l$, $\epsilon > 0$, $\pi > 0$, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi - (1+\lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$;

(ii) when $\lambda^{l\prime}(\alpha) = \lambda^{l*}$*:* $\tau = l$, $\epsilon > 0$, $\pi = 0$, $\alpha = (\overline{q}+p(q_c-q_n)\lambda^R)l-(1+\lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)l - Nq_c\epsilon > 0$;

(iii) when $\lambda^{l\prime}(\alpha) > \lambda^{l*}$*:* $\tau < l$, $\epsilon > 0$, $\pi = 0$, $\alpha = (\overline{q}+p(q_c-q_n)\lambda^R)\tau-(1+\lambda^R)pq_c\epsilon$, $\tau^R = N(q_c - q_n)\tau - Nq_c\epsilon > 0$;

in which $\lambda^{l*} = \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$.

Proposition 15 is derived from the first order conditions of (3.5) written above plus (3.2), (3.3) and (3.4).[20] Firstly, it states that the optimal insurance contract has lower coverage for a given individual loss in the catastrophic state than in the normal state ($\epsilon > 0$) when increasing the premium α generates an opportunity cost ($\lambda^{l\prime}(\alpha) > 0$). In this case, it is not valuable to cover fully individual losses in the catastrophic state, which means that the optimal contract does not fully eliminate individual risks and does not fulfill the Borch mutuality principle. This is a second-best insurance contract when insurance premiums have to be raised ex-ante and generate an opportunity cost. Besides, relative to the case without an opportunity cost, proposition 15 states that the optimal contract may not always have dividend in the normal state. If the marginal opportunity cost is too high relative to the reinsurance cost ($\lambda^{l\prime}(\alpha) \geq \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$), it is not valuable to have dividend in the normal state, which means that it is more valuable to spend all the premiums to reinsure rather than to keep some reserves which would be

[20] λ^{l*} is obtained with (3.14) equal to 1. Besides, $\lambda^{l\prime}(\alpha) > \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ tells that (3.16) is strictly greater than 1 and $\tau < l$.

given back through dividends in the normal state. In this case, it is even valuable to lower the indemnity in the normal state relative to full coverage ($\tau < l$) to increase reinsurance and the indemnity in the catastrophic state. We consider in the following a constant marginal opportunity cost $\lambda^{l\prime}(\alpha) = \lambda^l$.

Proposition 16 *With $0 < \lambda^l < \lambda^{l^*}$ (and a CARA utility function), we have for the optimal insurance and reinsurance contracts: $\frac{d\epsilon}{d\lambda^l} > 0$, $\frac{d\pi}{d\lambda^l} < 0$, $\frac{d\alpha}{d\lambda^l} < 0$, $\frac{d\tau^R}{d\lambda^l} > 0$.*

Proposition 16 is proved in appendix 3.6.3. It states that an increase of the marginal opportunity cost (λ^l) leads to a decrease of the premium to limit the opportunity cost for the insured. Thus, it leads to a decrease of the indemnity in the catastrophic state and a decrease of the dividend in the normal state. However, to limit the indemnity decrease in the catastrophic state, reinsurance purchase is increased in this case.

Proposition 17 *With $0 < \lambda^l < \lambda^{l^*}$ (and a CARA utility function), we have for the optimal insurance and reinsurance contracts: $\frac{d\epsilon}{d\delta} > 0$, $\frac{d\pi}{d\delta} < 0$, $\frac{d\alpha}{d\delta}$ ambiguous and $\frac{d\tau^R}{d\delta} > 0$.*

Proposition 17 is proved in appendix 3.6.3. It states that an increase of the correlation δ leads to an increase of reinsurance purchase because the collective risk increases with δ. On the one hand, the premium has to increase because reinsurance is costly. On the other hand, increasing the premium generates an additional opportunity cost. That is why the indemnity in the catastrophic state and the dividend in the normal state are lowered and finally the variation of the premium is ambiguous.

3.5 Conclusion

In the present paper, we have built a simple model to analyze the type of insurance contracts that emerge when risks are correlated across risk-averse agents in a community. For the sake of realism, we have considered that the community simultaneously chooses the type of contract sold to its members and the level of reinsurance it purchases, given that reinsurance is available at a cost higher than fair price. In this scheme, the insurer of the community supplies mutual contracts which are contingent on the state of nature. Without transaction costs in the community, risk-averse agents fully insure against their individual risk and share collective risk by getting some dividend in normal states of nature. Our model highlights the tradeoff between reinsurance and mutual types contracts. If reinsurance is costly, the promise of dividends in normal states enables the community to raise high premiums that are used as reserves to better indemnify in catastrophic states. With premiums raised ex-ante and generating an opportunity cost, risk-averse agents only partially insure against their individual risk, getting a lower indemnity in catastrophic states than in normal states, and get some dividend in normal states if the marginal cost of the reserve is low relative to the marginal cost of reinsurance. This analysis helps to understand the limits that risk correlation, costly reinsurance and costly reserve represent for risk sharing and how the contracts in a community can be improved through higher flexibility. Indeed, contracts with contingent indemnity and dividend enable to share better individual risks and collective risks. We have illustrated these mechanisms with the example of the Caribbean Catastrophe Risk Insurance Facility (CCRIF) that combines reinsurance and mutual contracts with indemnity and dividend contingent on the collective state.

3.6 Appendix

3.6.1 Risk correlation

With the loss represented by the random variable $\tilde{x}^i$ for individual i and the probability $\overline{q} = (1-p)q_n + pq_c$ of having a loss l, we have:

$$\tilde{x}^i = \begin{cases} -l & \text{with probability} \quad \overline{q} \\ 0 & \text{with probability} \quad 1-\overline{q} \end{cases}$$

In the normal state, the probability that individual i is affected is q_n. Besides, in the normal state, if individual i is affected, agent j is affected with a probability $\frac{q_n N-1}{N}$, which is well approximated by q_n when N is large. In the catastrophic state, this is similar with q_c instead of q_n. Thus, when N is large, we have with a good approximation:

$$\tilde{x}^i\tilde{x}^j = \begin{cases} l^2 & \text{with probability} \quad (1-p)q_n^2 + pq_c^2 \\ 0 & \text{with probability} \quad 1-(1-p)q_n^2 - pq_c^2 \end{cases}$$

The correlation between individual risks is:

$$\delta = \frac{COV(\tilde{x}^i, \tilde{x}^j)}{(VAR(\tilde{x}^i)VAR(\tilde{x}^j))^{0.5}}.$$

We have:

$$\begin{aligned} \mathrm{COV}(\tilde{x}^i, \tilde{x}^j) &= \mathbb{E}(\tilde{x}^i\tilde{x}^j) - \mathbb{E}(\tilde{x}^i)\mathbb{E}(\tilde{x}^j) \\ &= l^2((1-p)q_n^2 + pq_c^2) - (-l\overline{q})^2 \\ &= l^2((1-p)q_n^2 + pq_c^2 - \overline{q}^2), \end{aligned}$$

$$\begin{aligned} \mathrm{VAR}(\tilde{x}^i) &= \mathbb{E}((\tilde{x}^i)^2) - \mathbb{E}(\tilde{x}^i)^2 \\ &= l^2\overline{q} - (-l\overline{q})^2 \\ &= l^2\overline{q}(1-\overline{q}). \end{aligned}$$

Then, when N is large, the coefficient of correlation is with a good approximation:

$$\begin{aligned} \delta &= \frac{(1-p)q_n^2 + pq_c^2 - \overline{q}^2}{\overline{q}(1-\overline{q})} \\ &= \frac{p(1-p)}{\overline{q}(1-\overline{q})}(q_c - q_n)^2. \end{aligned}$$

With the average individual loss represented by the random variable $\tilde{X}$, we have:

$$\tilde{X} = \begin{cases} q_c l & \text{with probability} \quad p \\ q_n l & \text{with probability} \quad 1-p \end{cases}$$

This can also be written as $\tilde{X} = \tilde{q}l$ where:

$$\tilde{q} = \begin{cases} q_c & \text{with probability} \quad p \\ q_n & \text{with probability} \quad 1-p \end{cases}$$

Hence, the variance of the average individual loss is: $\mathrm{Var}(\tilde{X}) = \mathrm{Var}(\tilde{q})l^2$, with:

$$\tilde{q}^2 = \begin{cases} q_c^2 & \text{with probability} \quad p \\ q_n^2 & \text{with probability} \quad 1-p \end{cases}$$

$$\begin{aligned} \mathrm{Var}(\tilde{q}) &= \mathbb{E}(\tilde{q}^2) - (\mathbb{E}(\tilde{q}))^2 \\ &= (1-p)q_n^2 + pq_c^2 - \overline{q}^2. \end{aligned}$$

The variance of the average individual loss is then:

$$\mathrm{Var}(\tilde{X}) = \delta\overline{q}(1-\overline{q})l^2.$$

3.6.2 Without opportunity cost ($\lambda^l(\alpha) = 0$)

Derivation of the FOC of (3.5)

If the inequality constraints are not strictly binding in (3.5), the first order conditions of (3.5) relative to τ, ϵ and π are respectively:

$$-(1 + \frac{p(q_c - q_n)}{\overline{q}}\lambda^R)\overline{q}\mathbb{E}(u'(\tilde{w})) + (1-p)q_n u'(w_2) + pq_c u'(w_4) = 0, \tag{3.18}$$

$$(1+\lambda^R)pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0, \tag{3.19}$$

$$-(1-\frac{p}{1-p}\lambda^R)(1-p)\mathbb{E}(u'(\tilde{w}))+(1-p)(1-q_n)u'(w_1)+(1-p)q_n u'(w_2)=0. \quad (3.20)$$

Firstly, (3.19) gives:

$$u'(w_4)=(1+\lambda^R)\mathbb{E}(u'(\tilde{w})). \quad (3.21)$$

Secondly, with $\overline{q}=(1-p)q_n+pq_c$, the combination of (3.18) and (3.19) gives:

$$u'(w_2)=(1-\frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \quad (3.22)$$

Thirdly, (3.20) gives with the latter equation:

$$u'(w_1)=(1-\frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \quad (3.23)$$

Fourthly, with (3.21), (3.22), (3.23) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_3)=(1+\lambda^R)\mathbb{E}(u'(\tilde{w})). \quad (3.24)$$

If the inequality constraints are not strictly binding in (3.5) except $\boldsymbol{(q_c-q_n)\tau-\pi-q_c\epsilon\geq 0}$, we have then $\pi=(q_c-q_n)\tau-q_c\epsilon$ and (3.5) boils down to:

$$\begin{aligned}&\max_{\tau,\epsilon}\ \mathbb{E}(u(\tilde{w}))\\ &\text{s.t. } \alpha=q_c(\tau-\epsilon)\\ &\qquad \pi=(q_c-q_n)\tau-q_c\epsilon.\end{aligned} \quad (3.25)$$

The first order conditions of (3.25) relative to τ and ϵ are respectively:

$$-q_c\mathbb{E}(u'(\tilde{w}))+(q_c-q_n)(1-p)((1-q_n)u'(w_1)+q_n u'(w_2))+(1-p)q_n u'(w_2)+pq_c u'(w_4)=0, \quad (3.26)$$

$$q_c\mathbb{E}(u'(\tilde{w}))-q_c(1-p)((1-q_n)u'(w_1)+q_n u'(w_2))-pq_c u'(w_4)=0. \quad (3.27)$$

Firstly, the sum of (3.26) and (3.27) gives:

$$u'(w_2) = u'(w_1). \tag{3.28}$$

Secondly, (3.27) gives with the latter equation:

$$u'(w_4) = u'(w_3). \tag{3.29}$$

Comparative statics

We consider a CARA utility function $u(.)$, i.e. with $A = -\frac{u''(.)}{u'(.)} > 0$ constant.

With $0 < \lambda^R < \lambda^{R*}$, (3.8) gives:

$$(1 - \frac{p}{1-p}\lambda^R)u''(w_3)dw_3 - \frac{p}{1-p}u'(w_3)d\lambda^R = (1+\lambda^R)u''(w_1)dw_1 + u'(w_1)d\lambda^R. \tag{3.30}$$

With (3.8), (3.30) can be rewritten:

$$-A(dw_3 - dw_1) = (\frac{p}{1-p-p\lambda^R} + \frac{1}{1+\lambda^R})d\lambda^R, \tag{3.31}$$

which finally gives with $\pi = w_1 - w_3$:

$$\frac{d\pi}{d\lambda^R} = \frac{1}{A(1-p-p\lambda^R)(1+\lambda^R)}. \tag{3.32}$$

$\frac{d\pi}{d\lambda^R} > 0$ because $(1+\lambda^R)p < 1$. Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1-p-p\lambda^R)\pi$ and $\tau^R = N(q_c - q_n)l - N\pi > 0$ respectively give with (3.32):

$$\frac{d\alpha}{d\lambda^R} = p((q_c - q_n)l - \pi) + \frac{1}{A(1+\lambda^R)}, \tag{3.33}$$

$$\frac{d\tau^R}{d\lambda^R} = -\frac{N}{A(1-p-p\lambda^R)(1+\lambda^R)}. \tag{3.34}$$

$\frac{d\alpha}{d\lambda^R} > 0$ because $\frac{\tau^R}{N} = (q_c - q_n)l - \pi > 0$. $\frac{d\tau^R}{d\lambda^R} < 0$ because $(1+\lambda^R)p < 1$.

With $0 < \lambda^R < \lambda^{R*}$, (3.8) gives similarly:

$$\frac{d\pi}{d\delta} = 0. \tag{3.35}$$

Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi$ and $\tau^R = N(q_c - q_n)l - N\pi > 0$ respectively give with (3.35):

$$\frac{d\alpha}{d\delta} = p\frac{d(q_c - q_n)}{d\delta}\lambda^R l, \tag{3.36}$$

$$\frac{d\tau^R}{d\delta} = N\frac{d(q_c - q_n)}{d\delta}l. \tag{3.37}$$

Because $\frac{d(q_c - q_n)}{d\delta} > 0$, $\frac{d\alpha}{d\delta} > 0$ and $\frac{d\tau^R}{d\delta} > 0$.

3.6.3 With opportunity cost ($\lambda^l(\alpha) \geq 0$)

Derivation of the FOC of (3.5)

If the inequality constraints are not strictly binding in (3.5), the first order conditions of (3.5) relative to τ, ϵ and π are respectively:

$$-(1+\lambda^{l'}(\alpha))(1+\frac{p(q_c - q_n)}{\overline{q}}\lambda^R)\overline{q}\mathbb{E}(u'(\tilde{w}))+(1-p)q_n u'(w_2)+pq_c u'(w_4) = 0, \tag{3.38}$$

$$(1 + \lambda^{l'}(\alpha))(1 + \lambda^R)pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0, \tag{3.39}$$

$$-(1+\lambda^{l'}(\alpha))(1-\frac{p}{1-p}\lambda^R)(1-p)\mathbb{E}(u'(\tilde{w}))+(1-p)(1-q_n)u'(w_1)+(1-p)q_n u'(w_2) = 0. \tag{3.40}$$

Firstly, (3.39) gives:

$$u'(w_4) = (1 + \lambda^{l'}(\alpha))(1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{3.41}$$

Secondly, with $\overline{q} = (1-p)q_n + pq_c$, the combination of (3.38) and (3.39) gives:

$$u'(w_2) = (1 + \lambda^{l'}(\alpha))(1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{3.42}$$

Thirdly, (3.40) gives with the latter equation:

$$u'(w_1) = (1 + \lambda^{l'}(\alpha))(1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{3.43}$$

Fourthly, with (3.41), (3.42), (3.43) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_3) = \left((1 + \lambda^{l'}(\alpha))(1 + \lambda^R) - \frac{\lambda^{l'}(\alpha)}{p(1-q_c)}\right)\mathbb{E}(u'(\tilde{w})). \tag{3.44}$$

If the inequality constraints are not strictly binding in (3.5) except $\boldsymbol{\pi \geq 0}$, we have $\pi = 0$ (which states $u'(w_1) = u'(w_3)$) and the first order conditions of (3.5) relative to τ and ϵ are respectively:

$$-(1+\lambda^{l'}(\alpha))(1+\frac{p(q_c-q_n)}{\overline{q}}\lambda^R)\overline{q}\mathbb{E}(u'(\tilde{w}))+(1-p)q_n u'(w_2)+pq_c u'(w_4) = 0, \tag{3.45}$$

$$(1 + \lambda^{l'}(\alpha))(1 + \lambda^R)pq_c\mathbb{E}(u'(\tilde{w})) - pq_c u'(w_4) = 0. \tag{3.46}$$

Firstly, (3.46) gives:

$$u'(w_4) = (1 + \lambda^{l'}(\alpha))(1 + \lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{3.47}$$

Secondly, with $\overline{q} = (1-p)q_n + pq_c$, the combination of (3.45) and (3.46) gives:

$$u'(w_2) = (1 + \lambda^{l'}(\alpha))(1 - \frac{p}{1-p}\lambda^R)\mathbb{E}(u'(\tilde{w})). \tag{3.48}$$

Thirdly, with (3.47), (3.48) and the definition of $\mathbb{E}(u'(\tilde{w}))$, we get:

$$u'(w_1) = u'(w_3) = \left((1 + \lambda^{l'}(\alpha))(1 - \frac{p(q_c-q_n)}{1-\overline{q}}\lambda^R) - \frac{\lambda^{l'}(\alpha)}{1-\overline{q}}\right)\mathbb{E}(u'(\tilde{w})). \tag{3.49}$$

Comparative statics

We consider a CARA utility function function $u(.)$, i.e. with $A = -\frac{u''(.)}{u'(.)} > 0$ constant.

With $0 < \lambda^l < \lambda^{l*}$, (3.13) gives:

$$(1-\frac{\lambda^l}{p(1-q_c)(1+\lambda^R)(1+\lambda^l)})u''(w_4)dw_4 - \frac{\frac{1}{(1+\lambda^l)^2}}{p(1-q_c)(1+\lambda^R)}u'(w_4)d\lambda^l = u''(w_3)dw_3. \quad (3.50)$$

With (3.13), (3.50) can be rewritten:

$$-A(dw_4 - dw_3) = \frac{1}{p(1-q_c)(1+\lambda^R)(1+\lambda^l)^2 - \lambda^l(1+\lambda^l)}d\lambda^l, \quad (3.51)$$

which finally gives with $\epsilon = w_3 - w_4$:

$$\frac{d\epsilon}{d\lambda^l} = \frac{1}{A(p(1-q_c)(1+\lambda^R)(1+\lambda^l)^2 - \lambda^l(1+\lambda^l))}. \quad (3.52)$$

Similarly, (3.14) gives:

$$\frac{d\pi}{d\lambda^l} = -\frac{1}{A(p(1-q_c)(1+\lambda^R)(1+\lambda^l)^2 - \lambda^l(1+\lambda^l))}. \quad (3.53)$$

Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1 - p - p\lambda^R)\pi - (1+\lambda^R)pq_c\epsilon$ and $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$ respectively give with (3.52) and (3.53):

$$\frac{d\alpha}{d\lambda^l} = -(1 - p(1-q_c)(1+\lambda^R))\frac{d\epsilon}{d\lambda^l}, \quad (3.54)$$

$$\frac{d\tau^R}{d\lambda^l} = N(1-q_c)\frac{d\epsilon}{d\lambda^l}. \quad (3.55)$$

Note that $\lambda^l < \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ and $(1+\lambda^R)p < 1$ give: $p(1-q_c)(1+\lambda^R)(1+\lambda^l) - \lambda^l > 0$, which tells the sign of the four latter equations.

With $0 < \lambda^l < \lambda^{l*}$, (3.13) and (3.14) give similarly:

$$\frac{d\epsilon}{dq_c} = \frac{\lambda^l}{A(p(1-q_c)^2(1+\lambda^R)(1+\lambda^l) - \lambda^l(1-q_c))}, \tag{3.56}$$

$$\frac{d\pi}{dq_c} = -\frac{\lambda^l}{A(p(1-q_c)^2(1+\lambda^R)(1+\lambda^l) - \lambda^l(1-q_c))}. \tag{3.57}$$

Note that $\lambda^l < \frac{p(1-q_c)}{1-p-p(1-q_c)\lambda^R}\lambda^R$ and $(1+\lambda^R)p < 1$ give: $p(1-q_c)(1+\lambda^R)(1+\lambda^l) - \lambda^l > 0$. Thus, $\frac{d\epsilon}{dq_c} > 0$ and $\frac{d\pi}{dq_c} < 0$. Because $\frac{d\epsilon}{dq_n} = 0$ and $\frac{d\pi}{dq_n} = 0$, we thus have: $\frac{d\epsilon}{d\delta} > 0$ and $\frac{d\pi}{d\delta} < 0$. Besides, $\alpha = (\overline{q} + p(q_c - q_n)\lambda^R)l + (1-p-p\lambda^R)\pi - (1+\lambda^R)pq_c\epsilon$ and $\tau^R = N(q_c - q_n)l - N\pi - Nq_c\epsilon > 0$ respectively give with (3.56) and (3.57):

$$\frac{d\alpha}{d\delta} = p\frac{d(q_c - q_n)}{d\delta}\lambda^R l - \left((1 - p(1-q_c)(1+\lambda^R))\frac{d\epsilon}{dq_c} + (1+\lambda^R)p\epsilon\right)\frac{dq_c}{d\delta}, \tag{3.58}$$

$$\frac{d\tau^R}{d\delta} = \left(N(l-\epsilon) + N(1-q_c)\frac{d\epsilon}{dq_c}\right)\frac{dq_c}{d\delta} - Nl\frac{dq_n}{d\delta}. \tag{3.59}$$

Thus, $\frac{d\alpha}{d\delta}$ is ambiguous and $\frac{d\tau^R}{d\delta} > 0$.

Chapter 4

Securitizing the supply chain risk

This chapter is co-authored with Pierre Picard.

Abstract: This paper analyzes optimal insurance strategies of a firm facing several risks when the insurance tools at his disposal feature basis risk. Our aim is to characterize the optimal hedging strategy of a company who seeks to secure its supply chain procurement risk. In opposition with the current practice in risk management departments, we highlight the benefit of adopting an approach that bundles the various risk lines under a single insurance policy. We argue that such a policy could take the form of a catastrophe bond issued by the company. Compared to a traditional hedging strategy (via future contracts), catastrophe bonds combine the advantages of risk bundling and of lower basis risk.

Keywords: supply chain, catastrophic risks, insurance, catastrophe bonds.
JEL classification: G32, G22, G23, Q14.

4.1 Introduction

Supply chain events are a major source of risk for many large firms. When important volumes are exchanged among actors of the supply chain, even minor glitches or prices changes may have dramatic effects on production possibilities. In addition to the important contagion effects (Hertzel et al. (2008)), consequences for individual firms along the supply chain can be severe. Hendricks & Singhal (2005b) find that supply chain disruptions are associated with an abnormal return of almost -40% on a three year period starting one year before the disruption. In a second paper Hendricks & Singhal (2005a) also show that measures of operating performance, such as operating income, return on sales, and return on assets, are significantly and negatively affected by supply chain glitches.

Through their network of stores, large retail firms', such as Carrefour, Walmart, Sainsbury's,... core activity consists in purchasing and carrying goods where they are the most needed. Consequently, food procurement is an important component of their activity. When adverse events affect their suppliers' sale prices, these firms may either pay a higher price and maintain the relationship with the concerned suppliers or reorganize their supply chain. In both cases, substantial costs are to be expected and it is unlikely that they would be able to pass all the cost increase on to its customers. Procurement price risk therefore directly affects their profitability.

Traditional methods for handling price risk include financial hedging, through the purchase of financial instruments, such as futures and other options, and operational hedging (Cudahy et al. (2008), Johnson (2001)), that consists in purchasing "opportunities to delay and adjust investments and operating decisions over time in response to resolution of uncertainty" (Triantis (2000)). The complementarity between financial and operational hedging is an interesting subject studied in Dong & Tomlin (2012) but it is not what we discuss in the present paper. Instead, we focus on financial hedging.[1]

[1] In its 2015 annual report, Sainsbury's (2015) declares using forward contracts and options to hedge against currency risks (pp 117-123). Forward contracts also protect Sainsbury's own consumption of gas, electricity and fuel. No information is provided about procurement risk. Given the strategic role played by procurement, it is likely that Sainsbury's uses a combination

Purchases of future contracts are a conventional hedging strategy. In order to secure its procurement cost at date $t+h$, the firm buys at date t futures maturing at date $t + h$, hence securing the cost of its supplies. The aim of this paper is to show that this strategy is dominated by a bundled strategy without basis risk. Concretely, such a strategy could take the form of a catastrophe bond (henceforth cat-bond) issued by the firm to hedge the risk of large price surges.

Traded on Alternative Risk Transfer (ART) markets cat-bonds provide very flexible ways to insure risks and to tailor coverage to firm's special needs. They enable cedent companies to transfer a given risk to financial market investors by conditioning the repayment of the principal to the non-occurence of a pre-defined adverse event. In exchange, the cedent company offers a compensation that reflects the risk taken by investors. These market investors therefore act as the insurer of the cedent.

With more than 25 billion dollars of outstanding capital[2], the cat-bond market has been able to attract investors by offering spreads that feature little correlation with traditional asset returns, hence enabling portfolio managers to reach further diversification.

The hedging strategy based on the emission of a cat-bond may be more effective than the traditional future strategy currently followed by most firms for at least two reasons. Firstly, in-house diversification may lower the cost of insurance if the risk lines are sufficiently diversifiable among each other (i.e., they feature little correlations.) Secondly, hedging strategies based on future purchases are likely to feature significant amounts of basis risk. Indeed, future contracts are not available for all commodities purchased by a particular firm. Futures exchanged on the London market place for example, reflect the evolution of the average transaction price, without necessarily reflecting the specifics of the firm's logistic and strategic constraints. Futures should therefore be considered as indices, that imperfectly hedge the actual procurement risk. Cat-bonds, in contrast, allow for basis risk mitigation. While indemnity triggers directly condition the indemnity on the actual loss incurred by the cedent, parametric triggers condition the indemnity

of financial and operational hedging.

[2] *http://www.artemis.bm/*

on the level of an index, designed to replicate as closely as possible the actual loss of the firm. A minimal parametric trigger includes the market spot prices, that define the future strategy's payoff. Adding other variables to the index of a parametric trigger can therefore only reduce the basis risk of a cat-bond strategy compared to a future based strategy.

On the other hand, cat-bonds, and in particular cat-bonds with indemnity triggers (without basis risk) are costly. The theory of future pricing does not provide a clear indication as to whether future prices are above or below expected spot prices and two main theories compete to explain future prices : the theory of storage and the risk premium theory.

The theory of storage relies on an absence of arbitrage argument by which the net present values of holding a future must be equal to that of storing the commodity. This theory can only apply to the extent that the commodity is storable, which may be a reasonable assumption when considering a 12-months ahead hedging strategy for grains. It predicts that future prices may be slightly above (contango) or below (backwardation) the expected spot price, depending on the aggregate level of stored commodity. When aggregate inventory levels are high, there is little gain for a particular firm to store the product itself. In these periods, future prices tend to be high (contango). In contrast, when aggregate inventory levels are low, firms are willing to pay a premium (the convienience yield) to physically hold the commodity, which tends to lower the price of futures (backwardation). According to this theory, we may therefore expect high future prices in harvest seasons, when stocks of grains are replenished and lower future prices in-between.

The theory of the risk premium emphasizes the risk-return trade-off of futures contracts. As any other asset, futures should deliver a risk premium to the extent that their payoff is correlated with the market's payoff (positive betas). Nevertheless, empirical tests of the risk premium theory do not seem to reject the null hypothesis of a zero risk premium.

In any case, it is clear that the loading associated with future contract is likely to be lower than that associated with a cat-bond. As a consequence, the three benefits of our proposed hedging strategy (in-house diversification and lower basis risk) must be traded-off against the cost of issuing a cat-bond, which is the purpose of this paper.

The article is organized as follows. Section 4.2 highlights the gains of a bundled hedging (cat-bond) strategy over a line-by-line (future) strategy in the absence of basis risk. We show that the gain from a bundled strategy must be higher than that of a line-by-line strategy and we quantify this gain using four series of crop prices : maize, wheat, rice and soy, traded on the London stock market between 2001 and 2017. Our computations suggests that the cost of procurement risk could be lowered by a fraction comprised between 5 and 14 percent annually using a cat-bond rather than futures.

Section 4.3 then takes on the issue of basis risk. A discussion of the uni-variate case precedes the derivation of the optimal hedging behavior when the firm faces several risk lines. The gain from a hedging strategy with futures is compared with that of our proposed cat-bond strategy with an indemnity trigger (that is without basis risk). Taking basis risk into account therefore reduces the efficiency of the future based strategy. we find that our proposed cat-bond hedging strategy could lower the certainty equivalent by a figure comprised between 10 and 35 percent, compared to a future based approach.

4.2 Risk pooling in the absence of basis risk

We begin our investigation by ignoring basis risk to focus on the gain from bundling several risk lines together. Indeed, the risk that results from bundling all the product-specific price risks can be characterized as lower than that resulting from the non-aggregated sum of all price risks. This diversification effect exists as soon as the product-specific risks are not perfectly correlated, a technical condition that is highly likely to be met.

To see this argument, suppose a firm has n procurement lines. Its unit production cost, $\sum_{i=1}^{n} \alpha_i x_i$, is the sum of the input prices x_i weighted by quantities α_i, where $\sum_{i=1}^{n} \alpha_i = 1$. For simplicity, we assume that the firm faces a technology with complementary factors and constant return to scale, which is why the coefficients α_i remain fixed when the relative input prices change.

4.2.1 Line-by-line insurance

If the firm decides to insure all procurement lines separately, it receives n indemnities $I_i(x_i)$, each covering the price variation of a single unit of input in a single line. Typically, $I_i(x_i)$ can be a payment that increases linearly when the price is above a deductible, but the current analysis does not depend on the form of the indemnity schedule. The firm therefore receives a total indemnity

$$q \sum_{i=1}^{n} \alpha_i I_i(x_i),$$

where q is the exogeneously chosen quantity of output produced by the firm. Final wealth is

$$w_f^s = w(q) - q[\sum_{i=1}^{n} \alpha_i (x_i - I_i(x_i))] - P_s,$$

where $w(q)$ is an arbitrary production function and P_s is the price of the insurance contracts. A simple model used by most reinsurers to price contracts is

$$P_i = q(1+\lambda)\chi_i + \frac{c}{2} q^2 \xi_i^2, \tag{4.1}$$

where χ_i and ξ_i are the expected value and standard deviation of the line specific indemnity schedule $I_i(x_i)$.[3] λ is a loading that takes into account the various expenses associated with claim handling and payment. The parameter c reflects the degree of risk aversion of the insurer. Under this pricing assumption, the cost of covering separately the n procurement lines is therefore $P_s = P_1 + ... + P_n$, where P_i is the price of the line-specific indemnity i given by equation (4.1), hence

$$P_s = q(1+\lambda)\sum_{i=1}^{n}\alpha_i\chi_i + \frac{c}{2}q^2\sum_{i=1}^{n}\alpha_i\xi_i^2. \tag{4.2}$$

4.2.2 Bundled insurance

The firm can also choose to insure all procurement risks at once with one bundled policy $I(x_1, ...x_n)$ that pays an indemnity depending simultaneously on all its input prices. The firm's final wealth is in this case

$$w_f^b = w(q) - q[\sum_{i=1}^{n}\alpha_i x_i - I(x_1, ..., x_n)] - P_b,$$

In order to compare the price of the separate policies with that of the bundled policy, we set

$$I(x_1, ..., x_n) = \sum_{i=1}^{n}\alpha_i I_i(x_i), \tag{4.3}$$

so that final wealth under the bundled policy is identical to that under the line-by-line policy. The price of the bundled strategy is given by equations 4.1 and 4.3

$$P_b = q(1+\lambda)\sum_{i=1}^{n}\alpha_i\chi_i + \frac{c}{2}q^2[\sum_{i=1}^{n}\alpha_i^2\xi_i^2 + \sum_{i=1}^{n}\sum_{j\neq i}\alpha_i\alpha_j\rho_{ij}\xi_i\xi_j], \tag{4.4}$$

where ρ_{ij} is the coefficient of correlation between indemnities $I_i(x_i)$ and $I_j(x_j)$. The gain obtained by bundling the risks instead of purchasing separate policies for

[3] We keep the traditional notations μ_i and σ_i^2 for the expected value and variance of the underlying risks x_i's.

each procurement line is therefore

$$P_s - P_b = \frac{c}{2}q^2[\sum_{i=1}^{n}\alpha_i\xi_i^2 - \sum_{i=1}^{n}\alpha_i^2\xi_i^2 - \sum_{i=1}^{n}\sum_{j\neq i}\alpha_i\alpha_j\rho_{ij}\xi_i\xi_j], \tag{4.5}$$

which is always positive.

Proposition 18 *If the reinsurer is risk averse ($c > 0$) and not all prices are perfectly correlated ($\exists \quad \rho_{ij} < 1$), then for any indemnity schedules $I_i(x_i)_{i=1,...,n}$, the bundled policy is cheaper than the line-by-line policy.*

The gain between the two strategies increases with the degree of risk aversion c of the reinsurer and decreases with the pairwise correlation coefficient ρ_{ij}.

Proof The proof is given in Appendix 4.6.1.

This gain comes from the variance reduction that the firm is able to operate before transferring the risk. Provided with the form of a contract and with the share of each procurement line in a firm's business, it is possible to re-express equation (4.5) as a function of the characteristics of the underlying risks x_i, which is more customary but less general. If we consider, as an example, a case of full insurance ($I_i(x_i) = x_i$ for all $i = 1, ..., n$), we may write $\chi_i = \mu_i$ and $\xi_i = \sigma_i$, where μ_i and σ_i are the expected values and variances of the underlying risks x_i. Further assuming equal weights $\alpha_i = 1/n$, a symmetric variance covariance structure with $\sigma_i = \sigma$ for all procurement lines $i = 1, ...n$ and $\rho_{ij} = \rho$ for all pairs $i \neq j$ yields a simple expression for equation (4.5) to compute the gain from pooling risks as

$$P_s - P_b = \frac{c}{2}q^2\sigma^2[1 - \frac{1}{n}(1 + (n-1)\rho)].$$

When the coefficient of correlation $\rho = 1$ or when $n = 1$, there is no gain from pooling : $P_s - P_b = 0$. The gain from pooling decreases with ρ. As correlation diminishes, more diversification across lines takes place, hence inducing a decrease in the indemnity premium required by a risk averse insurer. The gain from pooling increases in the number of lines pooled n, in the variance σ^2 of the line specific risks, in the insurer's risk aversion c, and in the quantity q.

4.2.3 Optimal linear contract

We now turn to the question of the optimal contract. Assume that the firm can insure a quantity θ_i of each line of risk $i = 1, ..., n$. That is, it chooses the weights θ_i of the indemnification rule

$$I(x_1, ..., x_n) = \sum_{i=1}^{n} \alpha_i \theta_i x_i.$$

Let us denote $\psi_i = \theta_i \alpha_i$, or in vector form $\psi = \theta \cdot \alpha$, where ψ, θ and α are $n \times 1$ column vectors and $\cdot$ is the Hadamard element-wise product. Then, the indemnity can be written as $I(x) = \psi' x$ and the agent's final wealth is

$$w^f = w(q) + q(\psi - \alpha)' x - P.$$

With mean-variance preferences, the objective function is

$$V(\theta) = w(q) + q(\psi - \alpha)'\mu - P - \frac{\kappa}{2} q^2 (\psi - \alpha)' \Sigma (\psi - \alpha). \tag{4.6}$$

Full insurance is characterized by $\psi = \alpha$, μ is the $n \times 1$ mean vector and Σ is the matrix that collects the variances and covariances of the n risk lines. Finally the parameter κ captures the firm's degree of risk aversion.

Line-by-line pricing

We first consider the case where the price of the insurance policy is calculated on a line-by-line basis. This is the case if the firm uses futures to insure its various risk lines. We assume that future markets are well arbitraged and that the law of one price holds on each market. We further assume that the price of a future contract, delivering the payoff x at maturity is given by equation (4.1), where $\chi_i = \theta_i \mu_i$ is the expected payoff of the future contract, that is the expected spot price at maturity multiplied by the amount of future purchased and $\xi_i = \theta_i^2 \sigma_i^2$ is the strategy payoff's variance. Equation (4.2) be written as

$$P_s = q(1+\lambda)\sum_{i=1}^{n} \alpha_i \theta_i \mu_i + \frac{c}{2} q^2 \sum_{i=1}^{n} \alpha_i \theta_i^2 \sigma_i^2.$$

or, in matrix form

$$P_s = q(1+\lambda)\psi' \mu + \frac{c}{2} q^2 \psi' \hat{\Sigma}(\psi \cdot / \alpha), \tag{4.7}$$

where $\cdot/$ is the element-by-element division operator and $\hat{\Sigma}$ is a matrix of size $n \times n$ constructed by deleting all non-diagonal elements of the variance-covariance matrix Σ.

The case $\lambda = c = 0$ corresponds to a case where futures are sold exactly at the expected spot price on all markets, while $\lambda > 0$ and $c > 0$ reflect potential transaction costs and risk premiums, respectively. From the standpoint of the firm, insurance is sold above actuarially fair prices as soon as $\lambda > 0$ or $c > 0$. In this case, it is excluded that we obtain $\psi = \alpha$ as an optimum.

Differentiating the objective function (4.6) with $P = P_s$ yields the first order condition

$$-\lambda \mu - cq\hat{\Sigma}(\psi \cdot / \alpha) = -\kappa q \Sigma(\psi - \alpha).$$

Denoting $\alpha^* = (1/\alpha_1, ..., 1/\alpha_n)'$, we can write

$$\hat{\Sigma}(\psi \cdot / \alpha) = \hat{\Sigma}\alpha^* \psi,$$

to factor out the vector of co-insurance rates ψ. This yields the solution

$$\psi = \kappa(c\hat{\Sigma}\alpha^* + \kappa\Sigma)^{-1}\Sigma\alpha - \lambda(c\hat{\Sigma}\alpha^* + \kappa\Sigma)^{-1}\mu. \tag{4.8}$$

Now recalling that $\psi = \theta \cdot \alpha$ provides the formula for the optimal co-insurance rates

$$\theta = \kappa(c\hat{\Sigma}\alpha^* + \kappa\Sigma)^{-1}\Sigma\mathbb{1}_{n\times 1} - \lambda(c\hat{\Sigma}\alpha^* + \kappa\Sigma)^{-1}\mu \cdot / \alpha,$$

where $\mathbb{1}_{n\times 1}$ is a $n \times 1$ vector of ones. Remark that when $\lambda = c = 0$, we indeed obtain full insurance $\theta = \mathbb{1}_{n\times 1}$. Also remark that the optimal co-insurance formula (4.8) considerably simplifies when $c = 0$, in which case we obtain

$$\psi = \alpha - \frac{\lambda}{\kappa}\Sigma^{-1}\mu.$$

This section showed how futures can be used as a hedge against procurement price risk. The next section considers the case of a supply chain cat-bond.

Pooled insurance

One important difference between the future strategy discussed above and the cat-bond strategy proposed here is the in-house diversification that the latter allows. If the firm contracts on a set of risk lines rather than on a line-by-line basis, the volatility of the risk that it transfers to its counterpart is reduced, hence lowering the cost of insurance, as shown in Proposition 18. We now turn to the question of how to optimally design the insurance policy of a bundled insurance device such as our proposed cat-bond.

The matrix form of (4.4) gives the price of the bundled policy as

$$P = (1+\lambda)q\psi'\mu + \frac{c}{2}q^2\psi'\Sigma\psi \tag{4.9}$$

Differentiating (4.6) and (4.9) with respect to ψ yields a first order condition, equating marginal gain to marginal cost

$$\mu - q\kappa\Sigma(\psi - \alpha) = (1+\lambda)\mu + cq\Sigma\psi.$$

This gives

$$\psi = \frac{\kappa}{\kappa + c}\alpha - \frac{\lambda}{q(\kappa + c)}\Sigma^{-1}\mu. \tag{4.10}$$

Now recalling that $\psi = \theta \cdot \alpha$ provides the formula for the optimal co-insurance rates

$$\theta = \frac{\kappa}{\kappa + c}\mathbb{1}_{n\times 1} - \frac{\lambda}{q(\kappa + c)}\Sigma^{-1}\mu \cdot /\alpha. \tag{4.11}$$

Equation (4.11) characterizes the optimal co-insurance rates under the bundled policy. This formula comprises two terms. The first term $[\kappa/(\kappa+c)]\mathbb{1}_{n\times 1}$ is the solution when the loading factor λ is null. It is proportional to the relative risk aversion of the firm and of the insurer and it is simply the vector $\mathbb{1}_{n\times 1}$ when $c = 0$, i.e. the insurer has no risk aversion. The second term $\frac{\lambda}{q(\kappa+c)}\Sigma^{-1}\mu \cdot /\alpha$ is proportional to the loading cost $\lambda\mu \cdot /\alpha$. For a given line i, $\lambda_i\mu_i/\alpha_i$ reflects the loading cost adjusted for the relative importance of the line. The lower α_i, the less important line i is in the firm's business and therefore the lower the co-insurance rate θ_i. In addition, a correction is made for the variance of the risk through the inverse of the variance-covariance matrix Σ.

In order to see how this adjustment term plays, let us consider a situation with two goods. Inverting the variance-covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$$

yields the solution

$$\theta_i = \frac{\kappa}{\kappa+c} - \frac{\lambda}{q\alpha_i(\kappa+c)(1-\rho^2)\sigma_i^2}\Big(\mu_i - \rho\mu_j\frac{\sigma_i}{\sigma_j}\Big). \tag{4.12}$$

First notice that when $\rho = 0$, the two risk lines are optimally insured independently of each other, which is not surprising since in this case there is no substitutability across lines that the policyholder can leverage to lower his insurance bill. Otherwised, an increase in the expected loss μ_j or a decrease of the variance σ_j^2 leads to an increase in the co-insurance rate on line i θ_i as soon as $\rho > 0$, due to the substitution across lines.

In fact, Equation (4.12) shows two effects of an increase in the correlation coefficient ρ. In the denominator of the right-hand side term, ρ has an adverse effect on θ_i because correlation makes it more costly to achieve a given level of insurance. Realizing it, the agent lowers its demand for insurance, as he would in response of an increase in the loading factor λ. In the second term of (4.12), ρ has a positive impact on θ_i because the agent may exploit the correlation to substitute across lines. If $\rho > 0$ and μ_j is sufficiently high with respect to μ_i, the firm will substitute insurance on line j with insurance on line i. In fact, ρ can be seen as a measure of the cost of insurance as well as a measure of substitutability across lines. The polar case $\rho = 1$ indicates perfect substitutability but absence of gain from pooling.

4.2.4 The welfare effect of bundling : empirical investigation

This section uses four times series to evaluate the potential gains derived from pooling risks together instead of having a line-by-line hedging strategy. The time series used span 16 years of monthly information from 2001 to 2017 on the UK market for maize, wheat, rice and soy crops. All prices are converted in UK pound per tonne. For simplicity, spot prices are proxied with one-month future prices.

Figures (4.1a), (4.1b), (4.1c) and (4.1d) show the evolution of the crop prices over time. Prices seem highly correlated and the four time series feature two peaks, corresponding to the inflationary periods of 2008 and 2012-2013. Table (4.1) shows the historical price averages. Soybean is the most expensive crop, followed by maize, wheat and rice. Table (4.2) displays the variance covariance matrix of the crop returns. All crops have positive pairwise correlation coefficients and Soybean and Maize returns are the most correlated lines.

	Maize	Wheat	Rice	Soy
μ	450.64	192.33	23.85	1379.59
σ	218.28	43.26	7.24	453.10

Table 4.1: Input prices mean and std dev. Jan 2001 - Sept 2017

Maize	Wheat	Rice	Soy
1.0000	0.36740	0.47758	0.70626
	1.0000	0.31280	0.38215
		1.0000	0.46251
			1.0000

Table 4.2: Correlation Jan 2001 - Sept 2017

Assessing the welfare gains from a bundling strategy requires to estimate the joint distribution of future crop prices. In order to do so, we assume that log-returns are jointly normally distributed

$$r_{t,t+1} \sim \mathcal{N}(\mu_r, \Sigma_r),$$

where $r_{t,t+1} = \ln x_{t+1} - \ln x_t$ is a 4×1 vector that contains the crop returns. μ_r is the 4×1 expected value vector and Σ_r is the 4×4 variance-covariance matrix. From the normality of log-returns, it follows that the h period ahead price follows a log-normal distribution

$$x_{t,t+h} \sim \mathcal{LN}(\mu_{t,t+h}, \Sigma_{t,t+h}),$$

with mean

$$\mu_{t,t+h} = x_t \cdot \exp[h(\mu_r + diag(\Sigma_r)/2)],$$

and variance-covariance matrix

$$\Sigma_{t,t+h} = \mu_{t,t+h}\mu'_{t,t+h} \cdot (\exp(h\Sigma_r) - \mathbb{1}_{n\times n}).$$

The joint distribution of crop prices, enable to estimate the certainty equivalent of the firm's procurement risk

$$CE_{t,t+h}(\theta_t) = q[\alpha\cdot(\mathbb{1}_{n\times 1}-\theta_t)]'\mu_{t,t+h}+\frac{\kappa}{2}q^2[\alpha\cdot(\mathbb{1}_{n\times 1}-\theta_t)]'\Sigma_{t,t+h}[\alpha\cdot(1-\theta_t)], \quad (4.13)$$

and the net gain of a policy θ_t

$$G_{t,t+h} = CE_{t,t+h}(0) - CE_{t,t+h}(\theta_t) - P_{\theta_t},$$

for various insurance strategies θ_t, where P_{θ_t} is the price of the policy associated with the strategy θ_t. In particular, the optimal insurance strategy with bundling is estimated with the rule (4.11) replacing Σ and μ by $\mu_{t,t+h}$ and $\Sigma_{t,t+h}$. The welfare gain from bundling is then obtained by comparing the net gain under bundling to the net gain under a line-by-line strategy with rule (4.8).

The exercise requires a set of calibrating assumptions summarized in Table (4.3).

κ	0.0005	h	12
c	0.002	q	200
λ	0.3	α_i	0.25

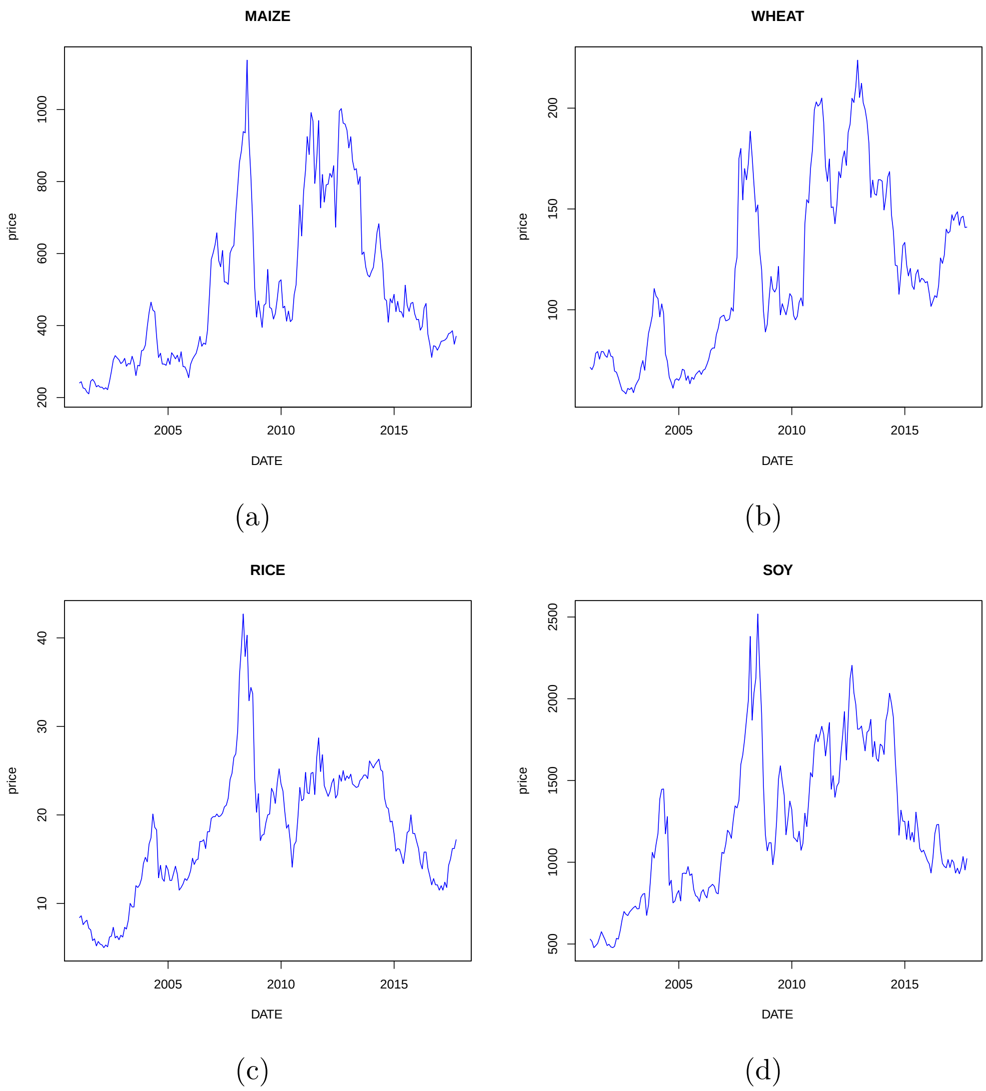
Table 4.3: Calibration

Figures (4.2a) and (4.2b) represent the certainty equivalent and the cost of full coverage (respectively) for a 12 month ahead risk at each time period (a month). On the left panel, the certainty equivalent follows closely the crop prices, for higher current prices produce higher expected prices and higher variances. On the right panel, the lower curve represents the cost of full coverage under a bundled strategy while the higher curve represents the cost of full coverage under a line-by-line strategy such as the future strategy. The lower cost of a bundled strategy is a direct consequence of Proposition 18.

Figures (4.3a), (4.3b), (4.3c) and (4.3d) contrast the optimal strategies under bundling (red) to the optimal strategies under a line-by-line strategy (black). The insurance demand on rice and wheat is much lower under the bundled scheme than under the line-by-line approach. These two crops are the ones with the smallest mean prices. As a consequence, their forecasted volatility is also lower than that of the two other crops. Since the line-by-line strategy puts a high price on volatility, the firm's demand for these highly volatile crops is substituted with demand on the less volatile crops. This substitution can take place because of the positive correlation between these two cheaper insurance lines and the more costly insurance on maize and soy.

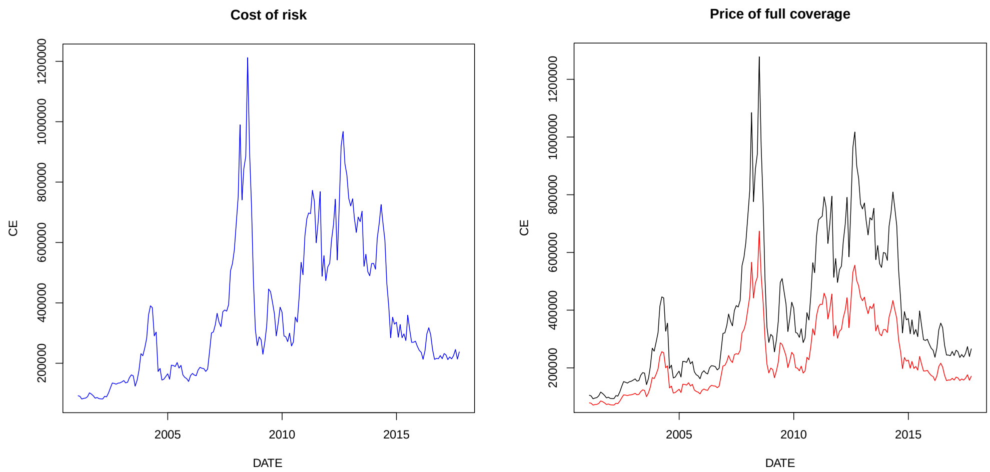
Under the line-by-line scheme, soy is very little insured due to its high volatility. The insurance on soy is substituted with insurance on maize, a crop highly correlated with soy. Demand for maize is important despite its high price due to this substitution effect.

Finally, Figure (4.4a) shows the reduction in the certainty equivalent that the bundled strategy allows, comprised between 25 and 55 percent. Figure (4.4b) shows the difference between the welfare gain from a bundled strategy and that from the line-by-line strategy, which can be interpreted as the gain from bundling the four risk lines. Due to our assumption on the stationary of returns, the gain from bundling is low when prices are low and increases as prices increase. According to our calculations, bundling could have yielded significant gains, up to a 14 percent reduction in the certainty equivalent during the 2008 price soar.



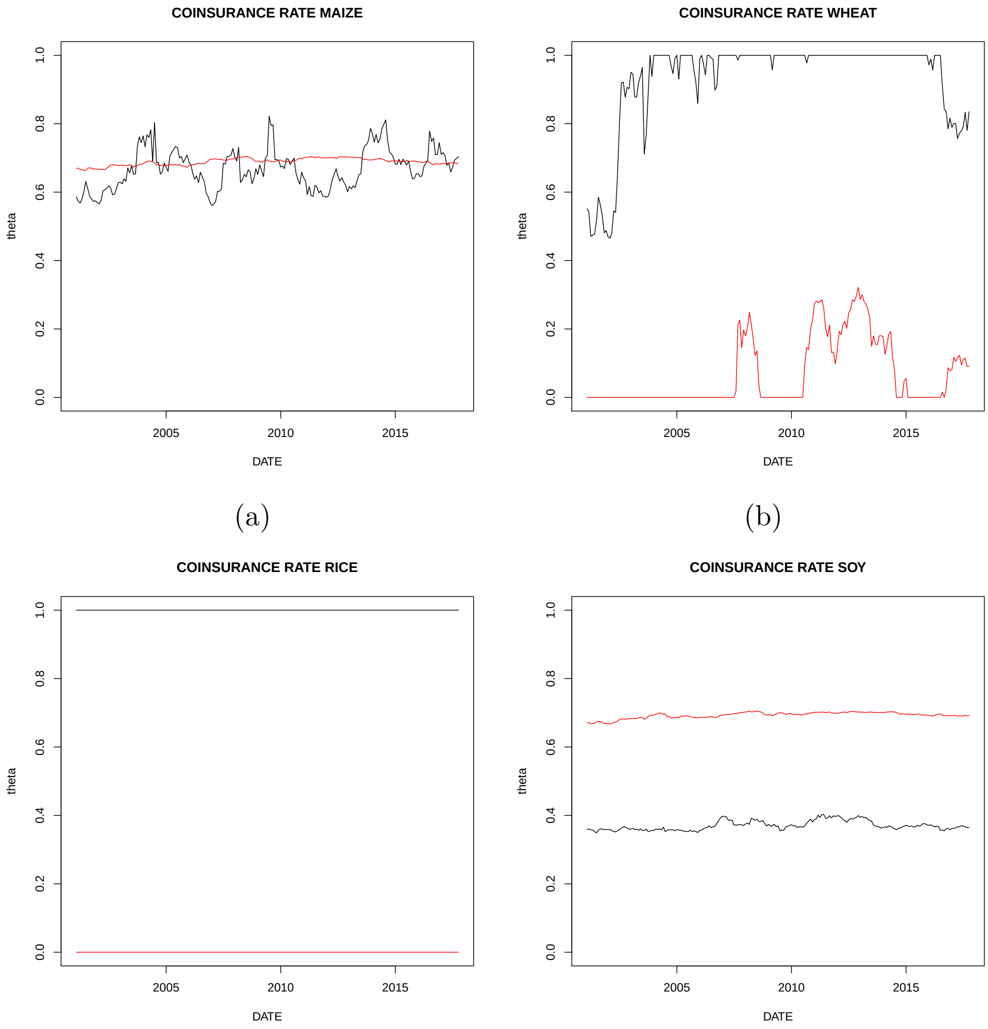
(a) (b) (c) (d)

Figure 4.1: Crop prices 2001-2017



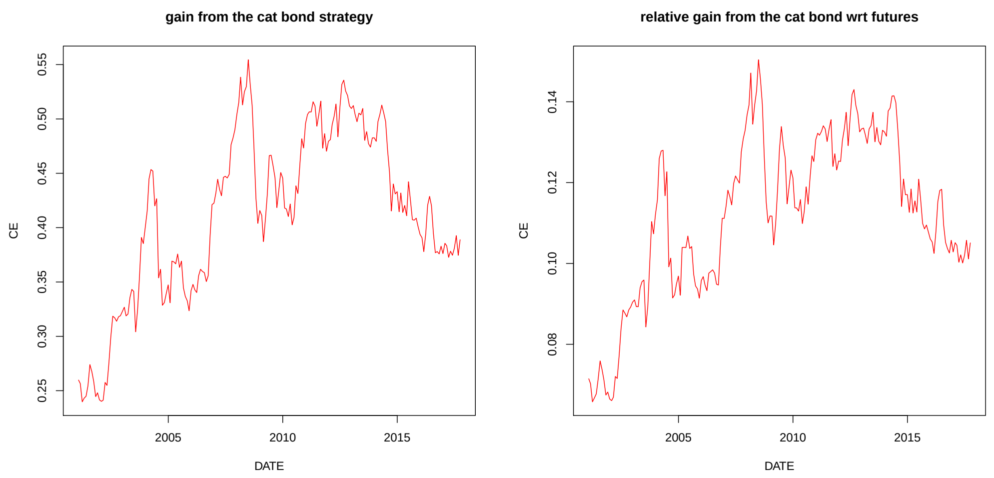
(a) (b)

Figure 4.2: Cost of risk and of full insurance



(a) (b) (c) (d)

Figure 4.3: Co-insurance rates line-by-line (blue) and bundled (red) strategy



(a) (b)

Figure 4.4: Gain from bundling

4.3 Insurance in the presence of basis risk

We have established so far that a bundled strategy is always superior to a line-by-line strategy. The next step toward our goal is therefore to compare a cat-bond strategy to a future strategy. We have highlighted one drawback of the future strategy compared to a cat-bond strategy : the future strategy cannot be bundled. Each future is typically sold by a given market investor. If futures are sold at actuarially fair prices (i.e. at the expected spot price of their corresponding maturity) then no cost reduction can be expected from bundling. However, in the presence of a risk premium, a future strategy would fail to exploit the gains from in-house diversification.

The theory of future pricing does not provide a clear indication as weather future prices are above or below expected spot prices. However, it is clear that the loading associated with future contracts is likely to be significantly lower than that associated with a cat-bonds. Nevertheless, cat-bonds allow for a better match between the indemnity and the actual loss of the company, that is for less basis risk. The next sections aim at analyzing the trade-off between cost and basis risk in order to clarify the relative merits of the cat-bond and future hedging strategies.

4.3.1 Accepting basis risk - The univariate case

We begin by considering a simple univariate insurance decision problem, where the firm wants to insure a risk x. In this univariate setting, the next sections compare the relative merits of a direct insurance scheme (such as an indemnity cat-bond), where the indemnity is based on the actual loss, hence eliminating basis risk, and those of an indirect insurance scheme (such as a purchase of future contracts), where the payoff is imperfectly correlated with the actual loss x.

Direct vs index insurance

A contract insuring directly the variable x entails a loading λ due for example to the high cost of setting up the contract (moral hazard,[4] monitoring costs, risk

[4]See Appendix 4.6.3.

modeling, placement, lack of competition among cat-bond investors, etc) On the other hand, the firm can purchase an index insurance, that delivers a payoff y, imperfectly correlated with the loss x. In our case, a future delivering the spot price at maturity can be regarded as an index insurance against procurement risk. For simplicity, we assume that the index insurance on y entails no loading (that is, the future is sold at the expected spot price.) We begin by discussing the case of direct insurance, where the index y is not available and the firm can only contract on x. The firm's final wealth in this case, is

$$w(q) - qx + q\theta_x x - P^d,$$

with $P^d = q(1+\lambda)\theta_x\mu_x$. Differentiating the firm's mean-variance utility function

$$V(\theta_x) = w(q) - q\mu_x + q\theta_x\mu_x - P^d - \frac{\kappa}{2}[q^2(\theta_x - \alpha)^2\sigma_x^2],$$

with respect to $\theta_x \geq 0$[5] gives the optimal insurance contract

$$\theta_x^* = 1 - \frac{\lambda\mu_x}{q\kappa\sigma_x^2}. \tag{4.14}$$

The firm chooses a level of coverage that is lower than its exposure due to the presence of a loading cost $\lambda\mu_x$. The higher the risk variance σ_x^2 or the level of risk-aversion κ, the closer from full coverage the optimal rule (4.14) is. The indirect utility function is

$$V(\theta_x^*) = w(q) - (1+\lambda)q\mu_x + \frac{\lambda^2\mu_x^2}{2\kappa\sigma_x^2}. \tag{4.15}$$

The first two terms represent the firm's wealth if it had purchased full direct insurance. The ability to choose partial insurance gives rise to the third term, which improves the firm's welfare.

We may now compare the direct insurance scheme to the future hedging strategy, where the firm pays a price P^i to receive the spot price y at the contract

[5]The constraint $\theta_x \geq 0$ prevents the firm from receiving a fixed payment $(1+\lambda)\theta_x\mu_x$ higher than the expected value of the variable indemnity $\theta_x x$ provided in exchange, which would be inconsistent with the interpretation of λ as a transaction cost.

maturity. The final wealth of the firm in this case is

$$w(q) - qx + q\theta_y y - P^i,$$

with $P^i = q\theta_y\mu_y$ is actuarially fair. The firm's mean-variance utility function is

$$\begin{aligned} V(\theta_y) = w(q) \quad &- \quad q\mu_x + q\theta_y\mu_y - P^i \\ &- \quad \frac{\kappa}{2}q^2[\theta_y^2\sigma_y^2 + \sigma_x^2 - 2\theta_y\mathrm{cov}(x,y)], \end{aligned}$$

and the optimal contract is

$$\theta_y^* = \frac{\mathrm{cov}(x,y)}{\sigma_y^2}. \tag{4.16}$$

The optimal insurance rate θ_y is equal to the slope coefficient's OLS estimate of a simple bivariate regression of x on y. When this coefficient is equal to one, the future contract y is a perfect substitute for direct insurance. Since we assumed that future are sold at actuarially fair prices, the firm's optimal strategy is full insurance $\theta_y^* = 1$ when $\mathrm{cov}(x,y)/\sigma_y^2 = 1$. The firm may want to buy a negative amount of insurance on the index y if it is negatively correlated with its loss x. For simplicity, we assume that the constraint $\theta_y \geq 0$ prevents such behavior.[6]

The indirect utility function with the future hedging strategy is

$$V(\theta_y^*) = w(q) - q\mu_x - \frac{\kappa}{2}q^2\sigma_x^2(1-\rho^2), \tag{4.17}$$

where $\rho = \mathrm{cov}(x,y)/\sigma_x\sigma_y$. When $\rho = 0$ or $\rho = 1$, the indirect utility is just equal to the utility without any insurance and with full insurance, respectively. The first two terms of equation (4.17) represent the firm's wealth if it takes full insurance at an actuarially fair price. The last term represents the cost of basis risk. When

[6]Buying a negative amount of insurance $\theta_y < 0$ would amount for the firm to paying an indemnity when the index is high (and therefore when the loss is low by equation 4.16) in exchange for a fixed payment P^i. This financial contract is rather atypical in an insurance-policyholder relationship, where the policyholder tends to pay a fixed premium and the insurer a variable indemnity. Since we have assumed that futures are available at an actuarially fair price, the possibility, evoked in footnote 5, for the firm to sell insurance at a price higher than the expected indemnity is absent here. However, we impose the constraints $\theta_x \geq 0$ and $\theta_y \geq 0$ for simplicity of exposition.

$\rho < 1$, even a situation of full index insurance $\theta_y^* = 1$ leaves some risk to the firm. Indeed, $\mathrm{cov}(x, y)/\sigma_y^2 = 1$ implies full index insurance, as shown in equation (4.16), and is equivalent to $\rho = \sigma_y/\sigma_x$. But the correlation coefficient can be smaller than one if the variance of the index is smaller than the variance of the actual loss. In this case, purchasing a quantity $\theta_y = 1$ of futures fails to deliver full insurance.

Therefore, it may be optimal for the firm to purchase more than full index insurance ($\theta_y > 1$), even if the index is perfectly correlated with the loss. For example, if $\rho = 1$ the optimal rate of insurance is $\theta_x = \sigma_x/\sigma_y > 1$ if $\sigma_x > \sigma_y$. Indeed, the variance of the index being smaller than the variance of the loss, a full index insurance $\theta_x = 1$ would not provide full insurance. In this case, the firm generates additional variance in its indemnity by purchasing more than full insurance.

Rearranging equations (4.15) and (4.17), the gain from using index insurance rather than direct insurance is

$$V(\theta_y^*) - V(\theta_x^*) = \frac{\kappa}{2} q^2 \sigma_x^2 [\rho^2 - (1 - \frac{\lambda \mu_x}{\kappa q \sigma_x^2})^2],$$

which is positive if and only if

$$\rho \geq 1 - \frac{\lambda \mu_x}{2 \kappa q \sigma_x^2}.$$

The firm prefers the index insurance if the index is sufficiently correlated with the loss, or if the cost of direct insurance $\lambda \mu_x$ is high relative to the cost $\kappa q \sigma_x^2$ of retaining the risk. Notice that if direct insurance entails no loading cost ($\lambda = 0$), only a perfectly correlated index would make the firm indifferent between index insurance and direct insurance.

Combining index and direct insurance

We now study the case where the firm has access to both the index insurance and to direct insurance. The firm's final wealth in this case writes

$$w - q[x - \theta_x x - \theta_y y] - P,$$

where θ_x and θ_y are the coinsurance rates on the variables x and y, respectively. The relevant loss for the firm is x but it can purchase insurance both on x and y. The price of the contract is

$$P = q[(1+\lambda)\theta_x\mu_x + \theta_y\mu_y],$$

and the firm's objective function writes

$$\begin{aligned} V(\theta_x,\theta_y) = w(q) &- q[\mu_x - \theta_x\mu_x - \theta_y\mu_y] - P \\ &- \frac{\kappa}{2}q^2[(1-\theta_x)^2\sigma_x^2 + \theta_y^2\sigma_y^2 - 2(1-\theta_x)\theta_y\mathrm{cov}(x,y)]. \end{aligned}$$

Optimizing with respect to the coinsurance rates $\theta_x \geq 0$ and $\theta_y \geq 0$ yields the first order conditions

$$-\lambda\mu_x - q\kappa[(1-\theta_x)\sigma_x^2 + \theta_y\mathrm{cov}(x,y)] \leq 0 \tag{4.18}$$

$$-\kappa\theta_y\sigma_y^2 - \kappa(1-\theta_x)\mathrm{cov}(x,y) \leq 0 \tag{4.19}$$

At an interior solution, the quantities of insurance purchased are

$$\theta_x^* = 1 - \frac{\lambda\mu_x}{q\kappa\sigma_x^2(1-\rho^2)} \tag{4.20}$$

$$\theta_y^* = \frac{\rho}{1-\rho^2}\frac{\lambda\mu_x}{q\kappa\sigma_x\sigma_y} \tag{4.21}$$

Compared to the solution without index insurance, described by equation (4.14), the quantity of direct insurance purchased given by equation (4.20), contains an additional factor $1/(1-\rho^2)$ that reflects the substitution of direct insurance with index insurance when the latter instrument is sufficiently correlated with the true loss. The corner solution $\theta_x^* = 0$ is obtained when

$$\rho^2 > 1 - \frac{\lambda\mu_x}{q\kappa\sigma_x^2}. \tag{4.22}$$

Inequality (4.22) gives the condition under which it is preferable to insure x only indirectly through the correlated variable y, when both direct and index insurance are available. It is always the case if $\rho = 1$, i.e. the two variables are perfectly correlated, because in this case y is a perfect proxy for x and does not entail a deadweight cost. The right hand side of (4.22) is increasing in $\lambda\mu_x$, which is the cost of insuring directly x, and decreasing in the firm's risk aversion κ, in the variance of the risk σ_x^2 and in the size q of the risk.

Equation (4.21) indicates that the firm always wants to purchase a positive amount of index insurance, as soon as the index is positively correlated with the loss. The corner solution $\theta_y^* = 0$ can therefore only be obtained when $\rho \leq 0$. θ_y^* is increasing in the correlation coefficient ρ, in the cost of direct insurance $\lambda\mu_x$ and decreasing in the index of risk aversion κ and standard deviations σ_x and σ_y.

Table 4.4 summarizes the results of Section 4.3.1. The parameter space can be divided into three disjoint subsets. When the correlation coefficient ρ between the index and the true loss is small, the firm prefers direct insurance to index insurance. If both instruments are available, it chooses to combine both instruments in its optimal mix. For intermediate values of ρ, the firm prefers the index instruments but would keep mixing if possible. Finally, when ρ is sufficiently large, the firm prefers the index and would choose not to purchase direct insurance, even when it is possible to combine both instruments.

	$0 \leq \rho < 1 - \frac{\lambda\mu}{q\kappa\sigma_x^2}$	$1 - \frac{\lambda\mu}{q\kappa\sigma_x^2} \leq \rho < [1 - \frac{\lambda\mu}{q\kappa\sigma_x^2}]^{(1/2)}$	$[1 - \frac{\lambda\mu}{q\kappa\sigma_x^2}]^{(1/2)} < \rho \leq 1$
direct vs index direct and index	direct $\succ$ index $\theta_x^* > 0, \quad \theta_y^* > 0$	index $\succ$ direct $\theta_x^* > 0, \quad \theta_y^* > 0$	index $\succ$ direct $\theta_x^* = 0, \quad \theta_y^* > 0$

Table 4.4: Summary

The bottom line is that when the cost of insuring directly a variable is too high the firm can be willing to accept some basis risk. A correlated variable, or more realistically a set of variables as we will see in the next section, can be defined as an index on which the indemnity payment is conditioned.

4.3.2 The case of multiple risk lines with basis risk

We now return to the case of multiple risk lines. The policyholder is the firm who seeks to secure its supply cost. It uses a quantity of input q composed of n

categories, each representing a fraction α_i of q. The input is transformed into an output $w(q)$. Procurement prices are random variables collected in a vector x, with mean μ_x and variance covariance matrix Σ_x. In order to secure its procurement cost, the firm can purchase a quantity θ of index insurance delivering a payoff $q(\alpha \cdot \theta)'y$ at price P, where y is a $n \times 1$ vector of indemnity payoffs, imperfectly correlated with the actual losses x. If the firm uses futures as a hedging strategy, the vector y represents the spot price of the commodity underlying the future contract. Because spot prices on a given market do not reflect perfectly the actual loss of the firm, such a strategy exposes the firm to basis risk. In contrast, an indemnity cat-bond's payoff depends directly on the actual loss incurred by the firm, hence eliminating basis risk. In our framework, this translate into the equality $x = y$ between the index and the loss.

The final wealth of the policyholder writes

$$w^f = w(q) + q(\psi' y - \alpha' x) - P,$$

with $\psi = \theta \cdot \alpha$. With mean-variance preferences, the objective function is

$$\begin{aligned} V(\psi) = w(q) \quad &+ \quad q[\psi'\mu_y - \alpha'\mu_x] - P && (4.23) \\ &- \quad q^2 \frac{\kappa}{2}\{\psi'\Sigma_y\psi + \alpha'\Sigma_x\alpha - 2\alpha'\mathrm{Cov}(x,y)\psi\}. && (4.24) \end{aligned}$$

where κ captures the firm's degree of risk aversion. Let us assume again that $P = (1+\lambda)q\psi'\mu_y + (c/2)q^2\psi'\Sigma_y\psi$. Differentiating with respect to ψ yields the optimal hedging strategy

$$\psi = \frac{\kappa}{\kappa + c}\Sigma_y^{-1}\mathrm{Cov}(x,y)\alpha - \frac{\lambda}{q(\kappa + c)}\Sigma_y^{-1}\mu_y. \tag{4.25}$$

If future prices are considered equal to expected spot prices, then $P = \mu_y$ and the optimal hedging strategy simplifies to

$$\psi = \Sigma_y^{-1}\mathrm{Cov}(x,y)\alpha.$$

4.3.3 The gains from a supply chain cat-bond : empirical investigation

This section compares the gain that the firm derives from using a future hedging strategy to that from using an indemnity cat-bond. The first strategy consists in purchasing at date t a quantity θ_t of futures, delivering the commodity at date $t+h$. At date $t+h$, the commodity is sold at spot price y_{t+h} while the firm must purchase its own input at price x_{t+h}. Assessing the effectiveness of the future hedging strategy requires to estimate the relationship between spot and procurement prices.

Co-integration often requires to estimate models on returns rather than prices. Also, let us assume that market price returns are normally distributed

$$r_{y,t} \sim \mathcal{N}(\mu_r, \Sigma_r),$$

with mean μ_r and variance Σ_r, and that, at each period the relationship between the price returns $r_{y,t} = (r_{y_1}, ..., r_{y_n})'_t$ and the procurement price returns $r_{x,t} = (r_{x_1}, ..., r_{x_n})'_t$ can be captured through the linear model

$$r_{x,t} - \mu_r = \beta_r(r_{y,t} - \mu_r) + \varepsilon_{x,t}, \tag{4.26}$$

where $\varepsilon_{x,t} \sim \mathcal{N}(0, \Sigma_\varepsilon)$. Consequently, $r_{x,t}$ is normally distributed with mean μ_r and variance $\beta_r'\Sigma_r\beta_r + \Sigma_\varepsilon$. Equation (4.26) can be interpreted as follows. The firm's specific risk, represented by the excess return $r_{x,t} - \mu_r$ is broken into a market price component, $r_{y,t} - \mu_r$ and an idiosyncratic component $\varepsilon_{x,t}$. The parameter β_r can be understood as the sensitivity of the firm to the market price volatility.

In our set-up, low values of β_r characterize a situation in which the firm's exposure to market risk is limited, for example because it has the operational capacity to restructure efficiently its supply chain in response to shocks on the market for commodities. In contrast, high values of β_r characterizes rigidities in the supply chain.

From this specification, we can derive the procurement and spot price characteristics as in Section 4.2.4. The mean and variance-covariance matrix of the spot prices are

$$\mu_{t+h}^{y} = y_t \cdot \exp[h(\mu_r + diag(\Sigma_r)/2)], \qquad \Sigma_{t+h}^{y} = \mu_{t+h}^{y}(\mu_{t+h}^{y})' \cdot (\exp(h\Sigma_r) - \mathbb{1}_{n\times n}).$$

The mean and variance-covariance matrix of the procurement prices are

$$\mu_{t+h}^{x} = x_t \cdot \exp[h(\mu_r + diag(\beta_r'\Sigma_r\beta + \Sigma_\varepsilon)/2)] \qquad \Sigma_{t+h}^{x} = \mu_{t+h}^{x}(\mu_{t+h}^{x})' \cdot (\exp(h(\beta_r'\Sigma_r\beta + \Sigma_\varepsilon)) - \mathbb{1}_{n\times n}).$$

Also remark that $\mathbb{cov}(r_{x,t}, r_{y,t}) = \beta_r\Sigma_r$, which enables to write the cross-covariance matrix

$$\mathbb{cov}_{t,t+h}(x_{t+h}, y_{t+h}) = \mu_{t+h}^{y}(\mu_{t+h}^{x})' \cdot (\exp(h(\beta_r\Sigma_r)) - \mathbb{1}_{n\times n}).$$

The optimal co-insurance rates in the presence of basis risk can then be assessed with equation (4.25), replacing μ_y, Σ_y and $\mathbb{cov}(x, y)$ by their empirical counterparts μ_{t+h}^{y}, Σ_{t+h}^{y} and $\mathbb{cov}_{t,t+h}(x_{t+h}, y_{t+h})$. As in Section 4.2.4, the welfare gain from a given policy is measured by the reduction in the certainty equivalent that it yields. In particular, we will compare the certainty equivalent when a future bundle is used (presence of basis risk but low cost) to the certainty equivalent when an indemnity cat-bond is used (no basis risk but high costs).

Because we do not possess the procurement price times series, we randomly generate a time series

$$\tilde{r}_{x,t} - \mu_r = \beta_r(r_{y,t} - \mu_r) + \varepsilon_{x,t}, \tag{4.27}$$

where $\varepsilon_{x,t}$ is drawn from a multivariate normal distribution with mean zero and variance covariance matrix

$$\hat{\Sigma}_\varepsilon \equiv \sigma_\varepsilon \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

We will let the coefficient σ_ε vary to represent the degree of basis risk, $\sigma_\varepsilon = 0$

indicating the absence of basis risk. Also, we assume that

$$\beta_r \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Table 4.5 provides the calibration that we use here. The only change compared to Section 4.2.4 is the presence of two additional costs $c_f < c$ and $\lambda_f < \lambda$, that define the pricing of the future strategies, and the parameter σ_ε that characterizes the amount of basis risk associated with the future strategy.

κ	0.0005	h	12
c	0.002	q	200
λ	0.3	α_i	0.25
c_f	0.002		
λ_f	0.05		

Table 4.5: Calibration

Figure 4.5 shows the net relative gain of using a cat-bond rather than a future strategy, computed as

$$\frac{CE_{t,t+h}^{cat-bond} - CE_{t,t+h}^{future}}{CE_{t,t+h}^{no\ insurance}}.$$

It is therefore positive (negative) when the cat-bond is more (less) advantageous than the future strategy. Unsurprisingly, when $\sigma_\varepsilon = 0$ (no basis risk), the gain from using a cat-bond strategy is inferior to that of using a future bundle strategy. In the absence of basis risk, the future strategy allows for an additional 3 to 8 percent welfare improvment due to its lower cost.

More interestingly, let us consider the case $\sigma_\varepsilon = 0.005$. For comparison purposes, table 4.6 reports the variance-covariance matrix Σ_r of the spot price time series. $\sigma_\varepsilon = 0.005$ is of the same order of magnitude than the variance of all four time series.

The series of procurement prices hence created are depicted in Figures 4.6a 4.6b 4.6c and 4.6d. The red lines represent the actual spot prices while the blue

Maize	Wheat	Rice	Soy
0.009463773	0.002629818	0.004219454	0.006346193
	0.005413991	0.002090242	0.002597261
		0.008248088	0.003879821
			0.008531711

Table 4.6: Variance-covariance matrix Σ_r

lines represent the simulated procurement prices. The two peaks of the spot price time series correspond to the 2008 and 2012-2013 world food price crisis.

Figures 4.7a, 4.7b, 4.7c and 4.7d report the optimal co-insurance rates under the future strategy (in black) and the cat-bond strategy (in red). Optimal insurance on rice and wheat is much lower under the future strategy than under the cat-bond strategy, as in Section 4.2.4. In addition, the demand for future insurance on the soy crop is much higher than it was in Section 4.2.4. This can be explained by the evolution of the soy procurement price series. Figure 4.6d shows that the procurement price for soy is above the spot price, with the exception of the period 2005-2008, which means that the firm faces more risk on the soy line than in Section 4.2.4, where the risk incurred by the firm was simply the spot price. Put differently, more future-based insurance is needed when procurement prices are high because the spot price becomes a less efficient hedge. The period 2005-2008, on the other hand, corresponds a lower demand for future-based insurance as the procurement price becomes lower than the spot price. Cat-bond insurance demand is unaffected by this dynamics of procurement prices since there is no basis risk.

Finally, Figures 4.8a and 4.8b display the gain from the cat-bond (expressed as the percentage decrease in the certainty equivalent) relative to no insurance and to the future-based strategy, respectively. The cat-bond yields a reduction in the certainty equivalent comprised between 25 and 55 percent, equal to the gain calculated in Section (4.2.4). However, the gain of a cat-bond relative to the future strategy is much higher and comprised between 8 and 35 percent. This suggests that the basis risk introduced in this section is sufficient to provide the cat-bond strategy with a quantitatively important advantage despite its higher price.

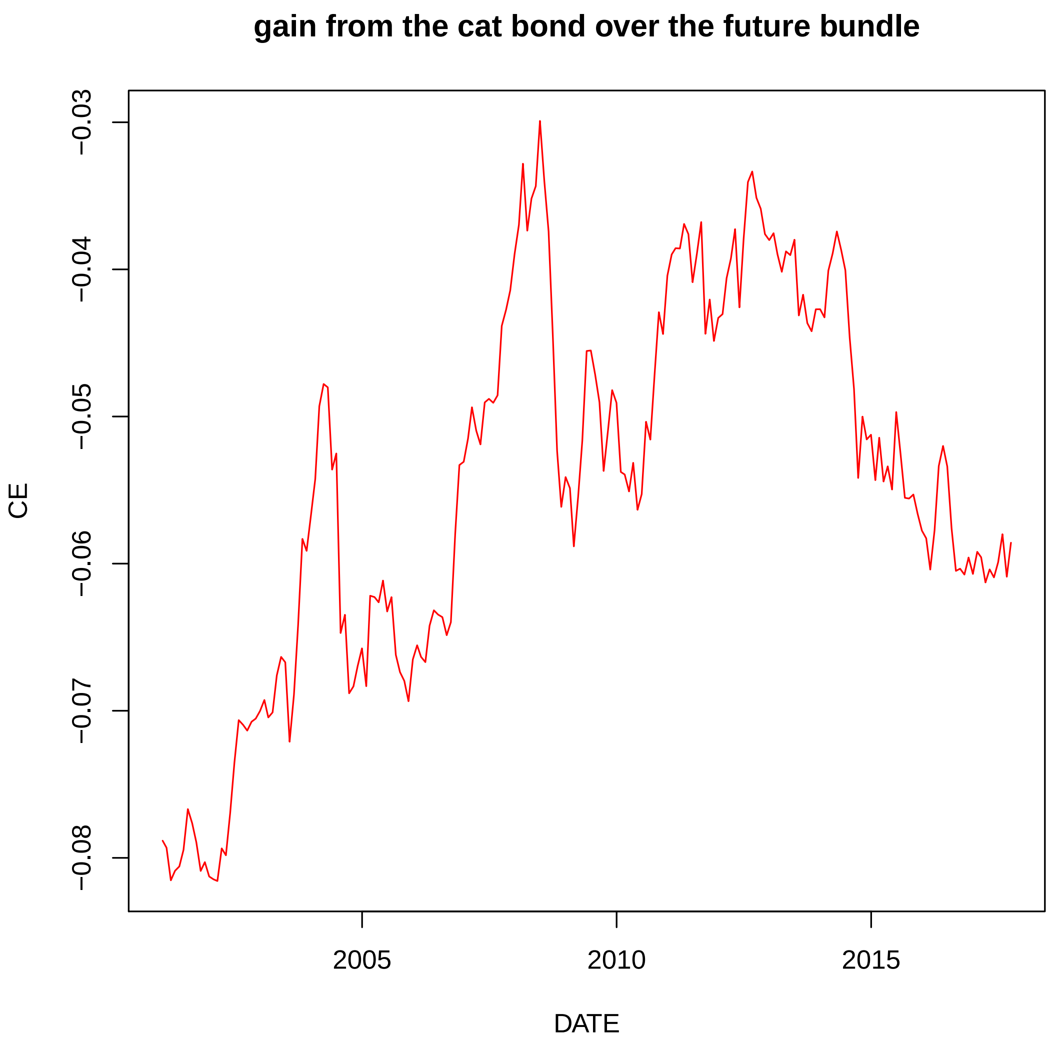

gain from the cat bond over the future bundle
CE
−0.03
−0.04
−0.05
−0.06
−0.07
−0.08
2005
2010
2015
DATE

Figure 4.5

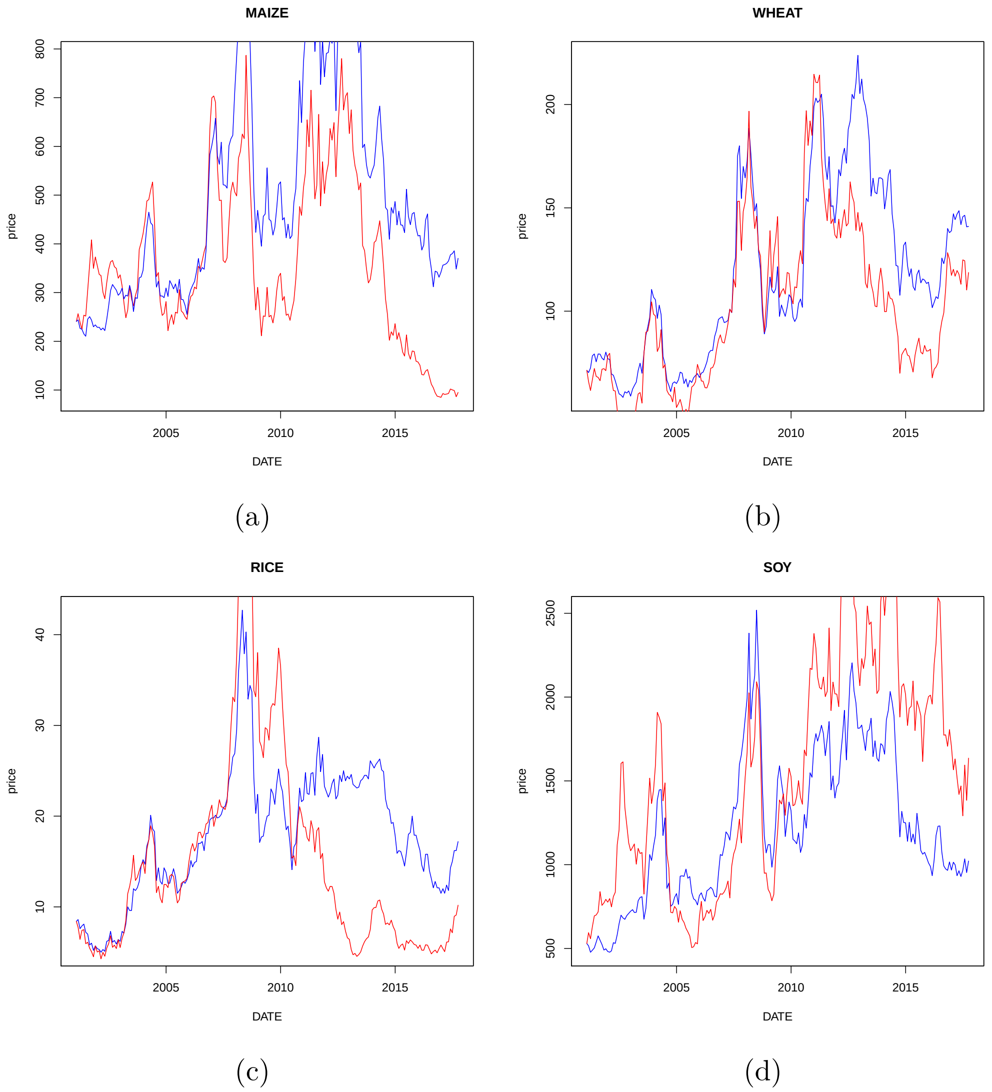
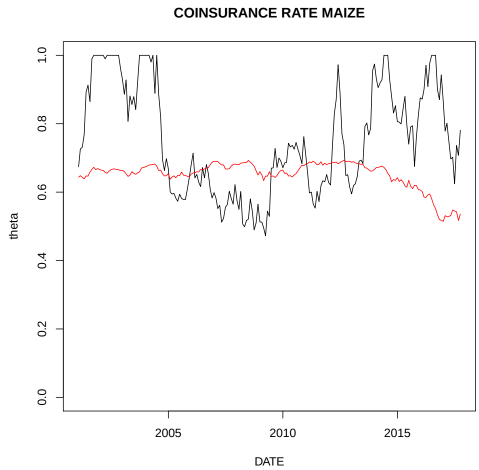
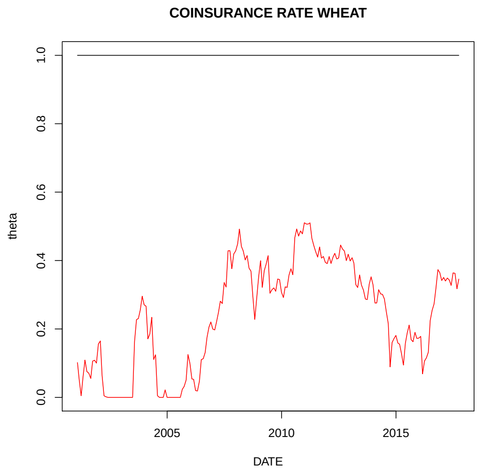


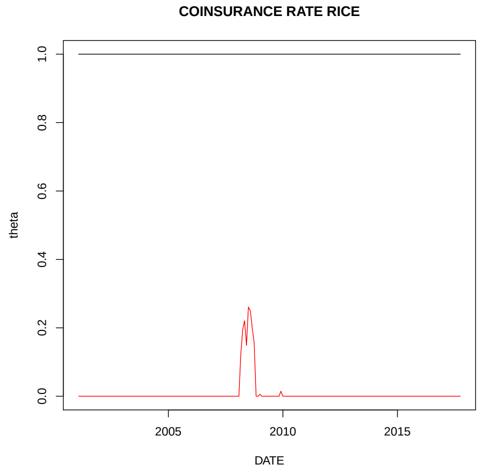
Figure 4.6: Crop spot (blue) and procurement (red) prices



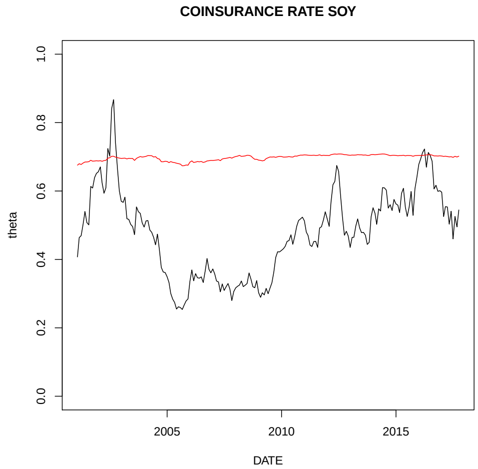
(a)



(b)

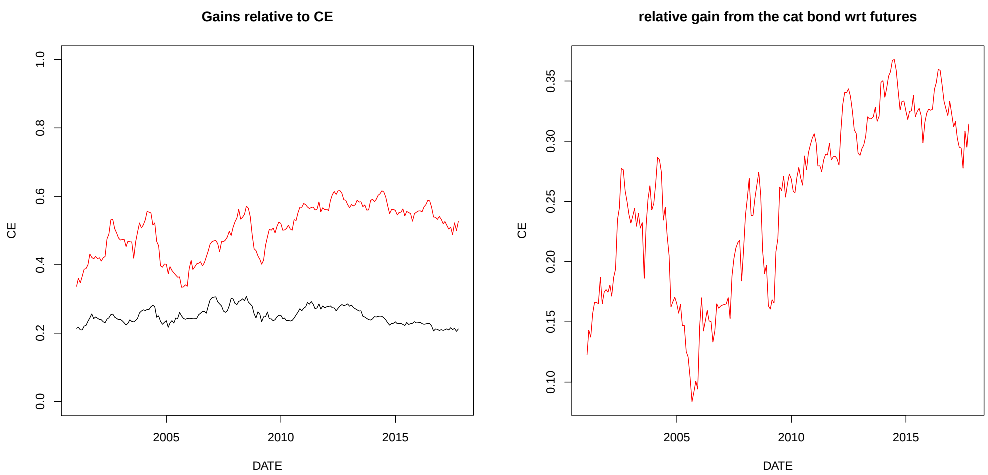


(c)



(d)

Figure 4.7: Co-insurance rates for cat-bond (red) and future (blue) strategies



(a) (b)

Figure 4.8: Gains from the cat-bond strategy

4.4 Reducing basis risk with climate indices

We have shown so far that cat-bonds with indemnity triggers could constitute a better hedging strategy than the purchase of future contracts even-though indemnity triggers are relatively expansive compared to futures. In fact, they are also more expansive than other types of triggers that cat-bonds can support. Considering alternative types of triggers could therefore lower the cost of our proposed cat-bond strategy. In this section, we consider the case of a parametric trigger. Parametric triggers rely on indices that aim at replicating the loss as closely as possible. The main difference with indemnity triggers is that parametric triggers do not depend on the actual loss of the policyholder. Because the index is observed by both the insured and the insurer, parametric triggers mitigate the cost of asymmetric information.

4.4.1 The univariate case

In order to illustrate the benefit of a parametric trigger, we begin by investigating the univariate case. As in earlier sections, the firms seeks to insure a risk x. In order to do so, it has access to futures, that provide a payoff y_1 at maturity. In addition, the indemnity can be made conditional on a climate index y_2. The indemnity has a form

$$I(y_1, y_2) = q(\theta_1 y_1 + \theta_2 y_2).$$

For simplicity, we assume here that parametrically triggered cat-bond are sold at actuarially fair prices and that y_1 and y_2 are independent random variables. In this case, the price of the insurance policy is $P = q(\theta_1 \mu_1 + \theta_2 \mu_2)$, where μ_1 and μ_2 are the expected values of y_1 and y_2. The firm's objective function is

$$\begin{aligned} V(\theta) = w(q) \quad &- \quad qx + q\theta_1 y_1 + q\theta_2 y_2 - P \\ &- \quad \frac{\kappa}{2} q^2 \{\sigma_x^2 + \theta_1 \sigma_{y_1}^2 + \theta_2 \sigma_{y_2}^2 - 2(\theta_1 \mathrm{Cov}(x, y_1) + \theta_2 \mathrm{Cov}(x, y_2))\}. \end{aligned}$$

And at an interior solution, the optimum is characterized by

$$\theta_1 = \frac{\mathrm{Cov}(x, y_1)}{\sigma_{y_1}},$$
$$\theta_2 = \frac{\mathrm{Cov}(x, y_2)}{\sigma_{y_2}}.$$

The optimal co-insurance rates are equal to the OLS estimates of the regressions of the loss variable x on the index variables y_1 (the spot price) and y_2 (the meteorological index). Also, the indirect objective function of the firm can now be written

$$V(\theta^*) = w(q) - q\mu_x - \frac{\kappa}{2}q^2\sigma_x^2[1 - \rho_1^2 - \rho_2^2],$$

where ρ_1 and ρ_2 are the correlation coefficients between y_1 and x, and y_2 and x, respectively. The addition of a climate index in the trigger of the cat-bond, in addition to the spot prices, therefore reduces basis risk. In practice, the extent to which the addition of a new variable in the index reduces basis risk depends on how it is correlated with the loss x, but also on how it is correlated with variables already present in the index. The more correlated with the actual loss a new variable is, the more it reduces basis risk. In contrast, the less correlated with other index variables a new variable is, the more information it brings about the actual loss, and therefore the lower the resulting level of basis risk.

4.4.2 The multivariate case

We now turn to the more realistic multivariate case. Let y be a $k \times 1$ vector of predictors, containing information about the $n \times 1$ vector x of procurement prices. The firm can choose the $k \times 1$ vector θ of co-insurance rates on each of these predictors. If only spot prices are considered as predictor variable, the strategy is equivalent to a purchase of futures. More generally, the vector θ can be interpreted as the vector of weights that define the trigger of the cat-bond. The price of the insurance scheme is $P = (1+\lambda)q\theta'\mu_y + c/2q^2\theta'\Sigma_y\theta$ and the firm's objective function writes

$$\begin{aligned} V(\theta) = w(q) &+ q\theta'\mu_y - q\alpha'\mu_x - P \\ &- \frac{\kappa}{2}q^2\{\theta'\Sigma_y\theta + \alpha'\Sigma_x\alpha - 2\theta'\text{Cov}(x,y)\alpha\}, \end{aligned}$$

where $\text{Cov}(x,y)$ is now a $k \times n$ matrix that contains the covariance terms between the price variables in x and the index variables in y. The first order condition

$$-\lambda\mu_y + \kappa q\text{Cov}(x,y)\alpha = q(c+\kappa)\Sigma_y\theta,$$

provides the optimal weights of the parametric trigger index

$$\theta^* = \frac{\kappa}{c+\kappa}\Sigma_y^{-1}\text{Cov}(x,y)\alpha - \frac{\lambda}{q(c+\kappa)}\Sigma_y^{-1}\mu_y.$$

This formula is almost identical to the optimal co-insurance rule in the case of a future hedging strategy, studied in Section 4.3. The only difference is that it allows for additional predictors to enter the indemnity rule, hence defining an index whose aim is to lower basis risk.

4.5 Conclusion

This paper constitutes a first attempt at quantifying the potential gains to be expected from a supply chain cat-bond strategy. Despite its high cost, a cat-bond would combine the advantages of lower basis risk (in particular with an indemnity trigger), and of bundling the risk, hence lowering the price and improving the efficiency of the hedging strategy. Our simulations suggest significant welfare gains, measured by a reduction in procurement risk certainty equivalent, between 8 and 55 percent. This estimates however, necessitate to be confronted to those obtained with real procurement data. In addition, other types of triggers could be implemented and tested in our model. If indemnity triggers feature the advantage of very low basis risk, they are expensive. Well designed parametric triggers could be used to lower the cat-bond emission costs without increasing too much the basis risk. Meteorological indices reflecting the weather conditions in the precise locations where the firm's suppliers grow their crops could help improve the prediction of loss, compared to a simple spot price measure.

4.6 Appendix

4.6.1 Proof of Proposition 18

Proof The gain from pooling is positive if

$$\sum_{i=1}^{n} \alpha_i \xi_i^2 \geq \sum_{i=1}^{n} \alpha_i^2 \xi_i^2 + \sum_{i=1}^{n} \sum_{j \neq i} \alpha_i \alpha_j \rho_{ij} \xi_i \xi_j .$$

Equivalently,

$$\sum_{i=1}^{n} \alpha_i \xi_i^2 (1 - \alpha_i) \geq \sum_{i=1}^{n} \sum_{j \neq i} \alpha_i \alpha_j \rho_{ij} \xi_i \xi_j .$$

Since $\rho_{ij} \leq 1 \quad \forall \quad i, j$, it is sufficient to show that

$$\sum_{i=1}^{n} \alpha_i \xi_i^2 (1 - \alpha_i) \geq \sum_{i=1}^{n} \sum_{j \neq i} \alpha_i \alpha_j \xi_i \xi_j . \tag{4.28}$$

First remark that

$$\sum_{i=1}^{n} \sum_{j \neq i} \alpha_i \alpha_j \xi_i \xi_j = 2 \sum_{i=1}^{n} \alpha_i \sum_{j > i} \alpha_j \xi_i \xi_j .$$

Also, $\sum_{i=1}^{n} \alpha_i = 1$ implies that

$$\sum_{i=1}^{n} \alpha_i \xi_i^2 (1 - \alpha_i) = \sum_{i=1}^{n} \alpha_i \xi_i^2 (\sum_{j \neq i} \alpha_j).$$

Inequality 4.28 therefore becomes

$$\sum_{i=1}^{n} \alpha_i \xi_i^2 (\sum_{j \neq i} \alpha_j) - 2 \sum_{i=1}^{n} \alpha_i \sum_{j > i} \alpha_j \xi_i \xi_j \geq 0.$$

Now, using the simple change in notations

$$\sum_{i=1}^{n} \alpha_i \xi_i^2 (\sum_{j \neq i} \alpha_j) = \sum_{i=1}^{n} \alpha_i \sum_{j > i} (\xi_i^2 + \xi_j^2),$$

gives

$$\sum_{i=1}^{n} \alpha_i \sum_{i=1}^{n} \alpha_i \sum_{j > i} (\xi_i^2 + \xi_j^2 - 2 \xi_i \xi_j) \geq 0,$$

or

$$\sum_{i=1}^{n} \alpha_i \sum_{j>i} \alpha_j (\xi_i - \xi_j)^2 \geq 0,$$

which is always true.

4.6.2 The case of a corner solution

We consider here the case of pooled insurance. Imposing the constraint $\theta \geq 0$ gives rise to the possibility of corner solutions. Since the co-insurance rates θ_i are (imperfect) substitutes, a corner solution on one line may impact the insurance take-up on the other lines. In order to investigate these effects, we denote γ the vector of Lagrange multipliers associated with the n constraints $\theta \geq 0$. The Lagrangian of the optimization program writes

$$\mathcal{L} = V(\psi) + \gamma' \psi.$$

The first order condition $V'(\theta) + \gamma = 0$ is

$$\gamma + q\mu - q^2 \kappa \Sigma(\psi - \alpha) = (1 + \lambda) q\mu + cq^2 \Sigma \psi,$$

which yields a solution for ψ

$$\psi = \frac{\kappa}{\kappa + c}\alpha - \frac{\lambda}{q(\kappa + c)}\Sigma^{-1}\mu + \frac{1}{q^2(c + \kappa)}\Sigma^{-1}\gamma. \tag{4.29}$$

The Karush-Kuhn-Tucker (KKT) condition implies that the last term is positive if and only if at least one of the positivity constraint is binding. In such case, all the co-insurance rates whose constraint is slack are affected. To make things concrete, take the two lines example discussed previously. The adjustment term $\Sigma^{-1}\mu$ that arise when at least one constraint binds writes

$$\Sigma^{-1}\gamma = (1 - \rho^2) \begin{pmatrix} \frac{\gamma_1}{\sigma_1^2} & -\frac{\rho\gamma_2}{\sigma_1\sigma_2} \\ -\frac{\rho\gamma_1}{\sigma_1\sigma_2} & \frac{\gamma_2}{\sigma_2^2} \end{pmatrix}.$$

Assume that $\psi_2 \geq 0$ binds while ψ_1 is an interior solution, then the KKT condition implies that $\gamma_1 = 0$ and $\gamma_2 > 0$, in which case the co-insurance rate ψ_1 has an adjustment term

$$-\frac{\rho\gamma_2}{\sigma_1\sigma_2}.$$

Adjustment is therefore downward when losses 1 and 2 are positively correlated. The intuition behind this result is that the binding constraint prevents the agent from selling insurance on the more expensive line to finance higher insurance purchases on the cheaper line. In the case of negatively correlated losses, the adjustment is upward. In the absence of a positivity constraint indeed, selling the most expensive line is actually used to purchased less insurance on the other line.

4.6.3 The example of moral hazard

This section provides micro foundations for the presence of a positive loading factor that we assumed in the direct insurance scheme of the previous section. A reason as to why such a cost may arise is moral hazard. In a nutshell, the problem of moral hazard can be summarized as follows. Agents who purchase insurance fail to internalize the positive externalities of their mitigating action, which leads to an under-provision of effort to mitigate.

We consider here a continuum of identical agents of mass 1, insured by a single company. Each agent i can change the distribution of the risk $dF(x, e_i)$ by applying an effort at cost $c(e_i)$, characterized by $c(0) = 0$, $c' > 0$ and $c'' > 0$. We call $\mu_x(e_i) = \int_0^{\bar{x}} x dF(x, e_i)$ the expected value of the loss conditional on effort level e being exercised. This effort is not observable, hence non contractible.

We study the set of contracts such that the insurance company breaks even in expected value, that is

$$P_i = \theta_i \int_0^1 \mu_x(e_i) di.$$

The price is adjusted depending on agent i's demand for insurance but it cannot be adjusted to the risk that agent i brings to the pool of policyholders because e_i is not observable. Instead, the price paid by agent i depends on the loss averaged across the pool of policyholders. Since each agent only represents an infinitesimal fraction of the population, the effect of its own effort on price is zero:

$$\frac{dP}{de_i} = 0.$$

Since all agents are similar, we obtain $\theta_i = \theta$ and $\mu_x(e_i) = \mu_x(e)\ \forall\ i$. We therefore drop the subscript i from now on and obtain

$$P = \theta\mu_x(e).$$

The agent's final wealth is

$$w_f = w - x + \theta x - P - c(e).$$

Using the mean-variance representation, the objective function writes

$$V^m = w - (1-\theta)\mu_x(e) - \theta\mu_x(e) - \frac{\kappa}{2}(1-\theta)^2\sigma_x^2$$

The first order conditions give

$$\begin{aligned} \theta^* &= 1 \\ (1-\theta^*)\mu'(e) - c'(e^*) &= 0, \end{aligned}$$

which implies $e^* = 0$. Conditionally on the effort level, insurance is actuarially fair, which induces the agents to purchase full insurance, a situation in which there is incentive to provide mitigating efforts. As a result, the price paid in equilibrium $P = \mu(0)$ is the highest price possible if effort has any effect on the expected loss. The indirect utility function is therefore

$$V^m = w - \mu(0). \tag{4.30}$$

The agent is perfectly insured but pays a high cost for its insurance. To circumvent this problem, the insurance company may try to induce a certain level of effort from agents either by providing partial insurance, or by spending resources to extract information about the level of effort.[7] An alternative route is to insure a risk that is correlated with the agent's actual loss and independant from the agents' actions, that is index insurance.

As in the previous sections, an index y is available with mean μ_y and variance σ_y^2. This time, the price at which insurance is available

$$P = \theta\mu_y$$

does not depend on the policyholder's effort. The objective function of the agent is now

$$V^i = w - \mu_x(e) + \theta\mu_y - P - \frac{\kappa}{2}[\sigma_x^2 + \theta^2\sigma_y^2 - 2\theta\rho\sigma_x\sigma_y]$$

The optimal break-even contracts are characterized by

$$\begin{aligned} \theta^* &= \rho\frac{\sigma_x}{\sigma_y} \\ -\mu'(e^*) = c'(e^*). \end{aligned}$$

The optimal rate of insurance depends on the correlation and variance ratio between the index and the actual loss and the optimal level of effort is here unaffected by the presence of insurance.[8] The agent fully internalizes the outcome of its effort which contributes to reducing its average loss, independently of the level of coverage provided by the index insurance contract. The optimal coverage under index insurance is therefore characterized by a first best level of effort. The quality of the index instrument on the other hand, depends on its correlation with the actual loss. The indirect utility under the index insurance scheme is

$$V^i = w - \mu(e^*) - c(e^*) - \frac{\kappa}{2}\sigma_x^2(1-\rho^2). \tag{4.31}$$

[7]It is not clear why the insurance would want to do that. In a competitive insurance sector, or if the insurance company provides a public service, the only relevant contracts are the break-even contracts.

[8]This separation between the insurance and effort decision may not be robust to changes of utility functions. Under prudence, the level of risk in the variable x, and therefore the level of effort exerted by the agent would affect the insurance decision. In particular, an increase in effort which translates into a risk reduction on x would command a lower optimal insurance rate.

ρ^2 captures the quality of the index instrument and only when $\rho = 1$ can full insurance be obtained.

We now compare direct insurance in the presence of moral hazard with index insurance. For the sake of simplicity, we assume that the function $\mu(e) = \mu(0) - ae$ is linear in e. We obtain that index insurance is preferred to direct insurance if $V^i > V^m$, that is from (4.30) and (4.31) if

$$ac'^{-1}(a) - c(c'^{-1}(a)) > \frac{\kappa}{2}\sigma_x^2(1-\rho^2),$$

that is if the gain from implementing the first best effort level is higher than the loss that results from the imperfect index instrument. Considering a power cost function $c(e) = e^{\phi}/\phi$ simplifies the condition further to

$$1 - \frac{1}{\phi} > \frac{\kappa}{2}\sigma_x^2(1-\rho^2).$$

Index insurance dominates direct insurance under moral hazard when the cost of effort is sufficiently high. When the cost of effort is low agents exert a high level of effort despite the fact that part of the benefit accrue to other agents. The problem of under-provision of effort is therefore mitigated compared to a situation with high costs where agents have little incentives to provide effort for themselves. Under this set of assumptions, it is also possible to characterize more precisely the optimal level of effort as

$$e^* = a^{\frac{1}{\phi-1}}.$$

The strength of index insurance is that it separates effort from insurance decisions, hence forcing agents to internalize fully the consequences of their effort choices.

Conclusion

This thesis addresses several aspects of the insurability of catastrophic risks. In a first chapter, we focus on very low probability events and we show how hybrid financial instruments can be used to extend the domain of insurable risks. Our application to the case of nuclear accidents using cat-bonds data in France shows that despite the higher price of reinsurance for low probability events, it is possible to insure more than is currently provided for by the French law. The second chapter takes on the issue of why insurance is more costly for low probability events. We show that because catastrophic risks have a systemic component, they give rise to a risk premium in equilibrium which decreases at a lower pace than the willigness to pay for insurance. We use this finding to explain why systemic low probability catastrophes are hard to insure. The third chapter investigates the role of mutual and participating contracts to improve insurability. Such contracts improve welfare by expanding the span of the financial market instruments available when individual losses are correlated. Finally, the fourth chapter investigates an innovative solution for firms that try to insure their procurement risk. The firm could issue a cat-bond with an index reflecting its losses. Such a solution would yield the advantage of risk-pooling, to lower the price of insurance. In addition, it could lower basis risk compared to more traditional hedging strategies such as future purchases, that imperfectly reflect the firm's exposure.

One central theme of this work is to show how financial innovations can be used to improve the insurability of catastrophic risks. An important limitation of our work is that we do not take into account the costs that this increasing financial complexity would yield. The global financial crisis of 2007 has indeed shown very clearly the risks associated with a lack of regulation of the financial industries. By failing to monitor the imperfectly internalized risks undertaken by systemic banks and insurance companies, our economies have actually endured a highly correlated and uninsured risk, that materialized with the failure of AIG, the willingness to save systemic institutions and the resulting European sovereign debt crisis in 2010. Without even considering the conflicting interests between the financial institutions that trade financial instruments and the population as a whole, tailoring the adequate incentive structures and/or monitoring the non internalized risks is in fact a costly activity, to which funds must be devoted. In practice therefore, the development of new financial instruments and markets should be accompanied by a parallel development in regulators' means and expertise if it is to truly reduce systemic risk.

Another related limitation is that we did not consider alternative protection mechanisms against catastrophes. If insurance and other mitigation strategies can sometimes be complements, some situations must lead to prefer prevention or protection to insurance. In particular, when it comes to physical or medical damages, it seems natural to think that, at a given price, avoiding the loss should be systematically preferred to an *ex-post* indemnity. In our first chapter on the insurance of nuclear accident, we have implicitly assumed that all adequate prevention and protection measures were already taken. In practice, the french nuclear safety authority (ASN) is in charge of defining and monitoring these measures. The insurance scheme we propose therefore aims at handling the residual risk. It should not be seen as a substitute for the preventive and protective actions that are necessary to reduce the probability and/or the consequences of the risk *ex-ante*.

Bibliography

Akerlof, G. A. (1970). The market for" lemons": Quality uncertainty and the market mechanism. The Quarterly Journal of Economics, 84(3), 488–500.

Arrow, K. J. (1951). An extension of the basic theorems of classical welfare economics. Technical report, STANFORD UNIVERSITY STANFORD United States.

Arrow, K. J. (1963a). Liquidity preference, lecture vi in" lecture notes for economics 285, the economics of uncertainty". undated, Stanford University, (pp. 33–53).

Arrow, K. J. (1963b). Uncertainty and the welfare economics of medical care. The American Economic Review, 53(5), 941–973.

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. The Quarterly Journal of Economics, 121(3), 823–866.

Barro, R. J. (2009). Rare disasters, asset prices, and welfare costs. American Economic Review, 99(1), 243–64.

Barsky, R. B., Juster, F. T., Kimball, M. S., & Shapiro, M. D. (1997). Preference parameters and behavioral heterogeneity: An experimental approach in the health and retirement study. The Quarterly Journal of Economics.

Borch, K. (1962). Equilibrium in a reinsurance market. Econometrica, 30(3), 424–444.

Borensztein, E., Cavallo, E., & Jeanne, O. (2017). The welfare gains from macro-insurance against natural disasters. Journal of Development Economics, 124, 142–156.

Braun, A. (2016). Pricing in the primary market for cat bonds: new empirical evidence. Journal of Risk and Insurance, 83(4), 811–847.

Browne, M. J. & Hoyt, R. E. (2000). The demand for flood insurance: Empirical evidence. Journal of Risk and Uncertainty, 20:3, 291–306.

Carayannopoulos, P. & Perez, M. F. (2015). Diversification through catastrophe bonds: Lessons from the subprime financial crisis. The Geneva Papers on Risk and Insurance-Issues and Practice, 40(1), 1–28.

Cass, D., Chichilnisky, G., & Wu, H.-M. (1996). Individual risk and mutual insurance. Econometrica, 64(2), 333–341.

CCRIF SPC (2014). Annual Report 2013-2014. Technical report, CCRIF SPC (Caribbean Catastrophe Reinsurance Insurance Facility).

Charpentier, A. & Le Maux, B. (2014a). Natural catastrophe insurance: How should the government intervene? Journal of Public Economics, 115, 1–17.

Charpentier, A. & Le Maux, B. (2014b). Natural catastrophe insurance: How should the government intervene? Journal of Public Economics, 115, 1–17.

Chetty, R. (2006). A new method of estimating risk aversion. The American Economic Review, (pp. 1821–1834).

Coate, S. (1995). Altruism, the samaritan's dilemma, and government transfer policy. The American Economic Review, (pp. 46–57).

Cole, S., Stein, D., & Tobacman, J. (2014). Dynamics of demand for index insurance: Evidence from a long-run field experiment. The American Economic Review, 104(5), 284–290.

Cooter, R. D. & Ulen, T. (2008). Law and economics international edition.

Cudahy, G. C., Mulani, N., & Cases, C. (2008). Mastering global operations in a multipolar world. Supply Chain Management Review, 12(2).

Cummins, J. D. (2006). Should the government provide insurance for catastrophes? Federal Reserve Bank of St. Louis Review, 88(4), 337–79.

Cummins, J. D. & Barrieu, P. (2013). Innovations in insurance markets: hybrid and securitized risk-transfer solutions. In Handbook of Insurance (pp. 547–602). Springer.

Cummins, J. D. & Mahul, O. (2004). The demand for insurance with an upper limit on coverage. The Journal of Risk and Insurance, 71(2), 253–264.

Cummins, J. D. & Trainar, P. (2009). Securitization, insurance, and reinsurance. Journal of Risk and Insurance, 76(3), 463–492.

Debreu, G. (1959). Theory of value: an axiomatic analysis of general equilibrium. Cowles Commission Monograph, 17.

Doherty, N. A. & Dionne, G. (1993). Insurance with undiversifiable risk: Contract structure and organizational form of insurance firms. Journal of Risk and Uncertainty, 6, 187–203.

Doherty, N. A. & Schlesinger, H. (1990). Rational insurance purchasing: Consideration of contract nonperformance. The Quarterly Journal of Economics, 105(1), 243–253.

Doherty, N. A. & Schlesinger, H. (2002). Insurance contracts and securization. The Journal of Risk and Insurance, 69(1), 45–62.

Dong, L. & Tomlin, B. (2012). Managing disruption risk: The interplay between operations and insurance. Management Science, 58(10), 1898–1915.

Dreicer, M., Tort, V., & Margerie, H. (1995). The external cost of the nuclear fuel cycle: Implementation in france. CEA Report, 238.

Eeckhoudt, L., Schieber, C., & Schneider, T. (2000). Risk aversion and the external cost of a nuclear accident. Journal of environmental Management, 58(2), 109–117.

Ehrlich, I. & Becker, G. S. (1972). Market insurance, self-insurance, and self-protection. Journal of political Economy, 80(4), 623–648.

Farhi, E. & Gabaix, X. (2015). Rare disasters and exchange rates. The Quarterly Journal of Economics, 131(1), 1–52.

Froot, K. A. (2001a). The market for catastrophe risk: a clinical examination. Journal of Financial Economics, 60(2), 529–571.

Froot, K. A. (2001b). The market for catastrophe risk: a clinical examination. Journal of Financial Economics, 60(2), 529–571.

Gabaix, X. (2012). Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. The Quarterly journal of economics, 127(2), 645–700.

Gignoux, J. & Menéndez, M. (2016). Benefit in the wake of disaster: Long-run effects of earthquakes on welfare in rural indonesia. Journal of Development Economics, 118, 26 – 44.

Gollier, C. (1997). About the insurability of catastrophic risks. Geneva Papers on Risk and Insurance. Issues and Practice, (pp. 177–186).

Gollier, C. (2003). To insure or not to insure?: an insurance puzzle. The Geneva Papers on Risk and Insurance Theory, 28(1), 5–24.

Gollier, C. (2004). The economics of risk and time. MIT press.

Gollier, C. (2013). The economics of optimal insurance design. In Handbook of Insurance (pp. 107–122). Springer.

Gollier, C. & Schlesinger, H. (1996). Arrow's theorem on the optimality of deductibles: a stochastic dominance approach. Economic Theory, 7(2), 359–363.

Gourio, F. (2012). Disaster risk and business cycles. American Economic Review, 102(6), 2734–66.

Goussebaile, A. (2016). Prevention and insurance of natural disasters. PhD thesis, Université Paris-Saclay.

Grace, M. F., Klein, R. W., & Kleindorfer, P. R. (2004). Homeowners insurance with bundled catastrophe coverage. Journal of Risk and Insurance, 71(3), 351–379.

Grinblatt, M., Keloharju, M., & Linnainmaa, J. (2011). Iq and stock market participation. The Journal of Finance, 66(6), 2121–2164.

Grislain-Letrémy, C. (2015). Natural disasters: exposure and underinsurance. Available at SSRN 2461822.

Guiso, L. & Paiella, M. (2008). Risk aversion, wealth, and background risk. Journal of the European Economic association, 6(6), 1109–1150.

Guiso, L., Sapienza, P., & Zingales, L. (2008). Trusting the stock market. the Journal of Finance, 63(6), 2557–2600.

Hau, A. (1999). A note on insurance coverage in incomplete markets. Southern economic journal, (pp. 433–441).

Hendricks, K. B. & Singhal, V. R. (2005a). Association between supply chain glitches and operating performance. Management science, 51(5), 695–711.

Hendricks, K. B. & Singhal, V. R. (2005b). An empirical analysis of the effect of supply chain disruptions on long-run stock price performance and equity risk of the firm. Production and Operations management, 14(1), 35–52.

Hertwig, R., Barron, G., Weber, E. U., & Erev, I. (2004). Decisions from experience and the effect of rare events in risky choice. Psychological science, 15(8), 534–539.

Hertzel, M. G., Li, Z., Officer, M. S., & Rodgers, K. J. (2008). Inter-firm linkages and the wealth effects of financial distress along the supply chain. Journal of Financial Economics, 87(2), 374–387.

Ikefuji, M., Laeven, R. J., Magnus, J. R., & Muris, C. (2015). Expected utility and catastrophic consumption risk. Insurance: Mathematics and Economics, 64, 306–312.

IRSN (2013). Methodology applied by the irsn to estimate the costs of a nuclear accident in france. Report PRP-CRI/SESUC/2013-00261.

Jaffee, D. & Russell, T. (1997a). Catastrophe insurance, capital markets, and uninsurable risks. The Journal of Risk and Insurance, 64(2), 205–230.

Jaffee, D. M. & Russell, T. (1997b). Catastrophe insurance, capital markets, and uninsurable risks. Journal of Risk and Insurance, (pp. 205–230).

Johnson, M. E. (2001). Learning from toys: Lessons in managing supply chain risk from the toy industry. California Management Review, 43(3), 106–124.

Kawaguchi, D. & Yukutake, N. (2017). Estimating the residential land damage of the fukushima nuclear accident. Journal of Urban Economics, 99, 148–160.

Kousky, C. (2010). Managing the risk of natural catastrophes: The role and functioning of state insurance programs. Technical report, Resources for the Future DP.

Kousky, C. & Cooke, R. (2012). Explaining the failure to insure catastrophic risks. The Geneva Papers on Risk and Insurance-Issues and Practice, 37(2), 206–227.

Kousky, C. & Kunreuther, H. (2014). Addressing affordability in the national flood insurance program. Journal of Extreme Events, 1(01), 1450001.

Kousky, C., Michel-Kerjan, E. O., & Raschky, P. (2013). Does federal disaster assistance crowd out private demand for insurance. Risk Management and Decision Processes Center The Wharton School, University of Pennsylvania, 10.

Kunreuther, H. (1973). Recovery From Natural Disasters: Insurance or Federal Aid? Washington.

Kunreuther, H., Meszaros, J., Hogarth, R. M., & Spranca, M. (1995). Ambiguity and underwriter decision processes. Journal of Economic Behavior & Organization, 26(3), 337–352.

Kunreuther, H. & Michel-Kerjan, E. (2009). At war with the weather. Massachusetts Institute of Technology.

Kunreuther, H., Novemsky, N., & Kahneman, D. (2001). Making low probabilities useful. Journal of Risk and Uncertainty, 23(2), 103–120.

Kunreuther, H. & Slovic, P. (1978). Economics, psychology, and protective behavior. The American Economic Review, 68(2), 64–69.

Lane, M., Mahul, O., et al. (2008). Catastrophe risk pricing: an empirical analysis. Technical report, The World Bank.

Lane, M. N. (2000). Pricing risk transfer transactions 1. ASTIN Bulletin: The Journal of the IAA, 30(2), 259–293.

Laury, S. K., McInnes, M. M., & Swarthout, J. T. (2009). Insurance decisions for low-probability losses. Journal of Risk and Uncertainty, 39(1), 17–44.

Levy, H. (1994). Absolute and relative risk aversion: An experimental study. Journal of Risk and Uncertainty, 8(3), 289–307.

Louaas, A. & Picard, P. (2014). Optimal insurance for catastrophic risk: Theory and application to nuclear corporate liability.

Mahul, O. & Wright, B. D. (2004). Implications of incomplete performance for optimal insurance. Economica, 71, 661–670.

Mahul, O. & Wright, B. D. (2007). Optimal coverage for incompletely reliable insurance. Economics Letters, 95(3), 456–461.

Major, J. A. & Kreps, R. E. (2002). Catastrophe risk pricing in the traditional market. Alternative Risk Strategies, (pp. 201–222).

Malinvaud, E. (1973). Markets for an exchange economy with individual risks. Econometrica, 41(3), 383–410.

Malinvaud, E. (1974). The allocation of individual risks in large markets. In Allocation under Uncertainty: Equilibrium and Optimality (pp. 110–125). Springer.

Mankiw, N. G. & Zeldes, S. P. (1991). The consumption of stockholders and nonstockholders. Journal of financial Economics, 29(1), 97–112.

Markandya, A. (1994). Externalities of fuel cycles' externe'project. economic valuation. economical valuation: An impact pathway approach.

Marshall, J. M. (1974). Insurance theory: Reserves versus mutuality. Economic Inquiry, 12(4), 476–492.

Martin, I. W. & Pindyck, R. S. (2015). Averting catastrophes: the strange economics of scylla and charybdis. American Economic Review, 105(10), 2947–85.

McClelland, G. H., Schulze, W. D., & Coursey, D. L. (1993). Insurance for low-probability hazards: A bimodal response to unlikely events. Journal of Risk and Uncertainty, 7(1), 95–116.

Millner, A. (2013). On welfare frameworks and catastrophic climate risks. Journal of Environmental Economics and Management, 65(2), 310–325.

Mobarak, A. M. & Rosenzweig, M. R. (2013). Informal risk sharing, index insurance, and risk taking in developing countries. The American Economic Review, 103(3), 375–380.

Mossin, J. (1968). Aspects of rational insurance purchasing. The Journal of Political Economy, (pp. 553–568).

NEA (2000). Methodologies for assessing the economic consequences of nuclear reactor accidents. OECD Nuclear Energy Agency, Paris.

Niehaus, G. (2002). The allocation of catastrophe risk. Journal of Banking & Finance, 26(2-3), 585–596.

Penalva-Zuasti, J. S. (2001). Insurance with frequency trading: A dynamic analysis of efficient insurance markets. Review of Economic Dynamics, 4, 790–822.

Penalva-Zuasti, J. S. (2008). A study of the interaction of insurance and financial markets: efficiency and full insurance coverage. Journal of Risk and Insurance, 75(2), 313–342.

Pratt, J. W. (1964). Risk aversion in the small and in the large. Econometrica: Journal of the Econometric Society, (pp. 122–136).

Quinet, E. et al. (2014). L'évaluation socioéconomique des investissements publics. Technical report, HAL.

Rabin, M. (2000). Risk-aversion for small stakes: A calibration theorem. Econometrica, 68, 1281–1292.

Rabl, A. & Rabl, V. A. (2013). External costs of nuclear: Greater or less than the alternatives? Energy Policy, 57, 575–584.

Rangel, L. E. & Lévêque, F. (2014). How fukushima dai-ichi core meltdown changed the probability of nuclear accidents? Safety Science, 64, 90–98.

Raschky, P. A., Schwarze, R., Schwindt, M., & Zahn, F. (2013). Uncertainty of governmental relief and the crowding out of flood insurance. Environmental and Resource Economics, 54(2), 179–200.

Raviv, A. (1979). The design of an optimal insurance policy. The American Economic Review, 69(1), 84–96.

Rietz, T. A. (1988). The equity risk premium a solution. Journal of monetary Economics, 22(1), 117–131.

Schneider, T. (1998). Integration of the indirect costs evaluation. Technical report, CEPN.

Schneider, Y. & Zweifel, P. (2004). How much internalization of nuclear risk through liability insurance? Journal of Risk and Uncertainty, 29(3), 219–240.

Shavell, S. (2009). Economic analysis of accident law. Harvard University Press.

Strobl, E. (2012). The economic growth impact of natural disasters in developing countries: Evidence from hurricane strikes in the central american and caribbean regions. Journal of Development economics, 97(1), 130–141.

Szpiro, G. G. (1986). Measuring risk aversion: an alternative approach. The Review of Economics and Statistics, (pp. 156–159).

Triantis, A. J. (2000). Real options and corporate risk management. Journal of Applied Corporate Finance, 13(2), 64–73.

Viscusi, W. K. & Aldy, J. E. (2003). The value of a statistical life: a critical review of market estimates throughout the world. Journal of risk and uncertainty, 27(1), 5–76.

Weitzman, M. L. (2009). On modeling and interpreting the economics of catastrophic climate change. The Review of Economics and Statistics, 91(1), 1–19.

Zanjani, G. (2002). Pricing and capital allocation in catastrophe insurance. Journal of Financial Economics, 65(2), 283–305.

université PARIS-SACLAY

ÉCOLE DOCTORALE
Sciences de l'homme et de la société (SHS)

Titre: Assurabilité des risques catastrophiques

Mots clés: Catastrophes, risques, assurance

Résumé: Cette thèse étudie l'assurabilité des risques catastrophiques. Le premier chapitre s'intéresse aux risques de très faibles probabilités. Nous montrons comment des instruments financiers hybrides, tels que les obligations catastrophes, peuvent être utilisés pour améliorer l'assurabilité des risques catastrophiques. Une application au cas du risque nucléaire français est développée. Le second chapitre s'attaque à la question du prix de la réassurance des risques de faibles probabilités. Les risques catastrophiques donnent lieu à une prime de risque dont le montant décroit avec la probabilité de la catastrophe moins rapidement que la disposition à payer d'un assuré typique. Cela explique pourquoi les risques systémiques de faibles probabilités sont difficiles à assurer. Le troisième chapitre étudie le rôle des contrats mutuels et participatifs. De tels contrats permettent aux assurés d'ajuster au mieux leur demande d'assurance, en prenant en compte la dimension systémique des risques auxquels ils sont exposés. Enfin, le quatrième chapitre étudie l'utilisation d'obligations catastrophes pour assurer le risque de variations extrêmes du prix des matières premières agricoles. En émettant une obligation catastrophe, l'entreprise emprunte un capital qu'elle peut conserver lorsque ses coûts d'approvisionnement sont élevés. Cette solution est évaluée qualitativement et quantitativement.

Title: Insurability of catastrophic risk

Keywords: Disasters, risks, insurance

Abstract: This thesis addresses several aspects of the insurability of catastrophic risks. In a first chapter, we focus on very low probability events and we show how hybrid financial instruments can be used to extend the domain of insurable risks. We develop an application to the case of nuclear risk in France. The second chapter takes on the issue of why reinsurance is costly for low probability events. We show that catastrophic risks give rise to a risk premium in equilibrium which decreases at a lower pace than the willingness to pay for insurance. We use this finding to explain why systemic low probability catastrophes are hard to insure. The third chapter investigates the role of mutual and participating contracts to improve insurability. Such contracts are necessary for people to adjust their demand for insurance when individual losses are correlated. Finally, the fourth chapter investigates the use of cat-bonds to hedge the risk of extreme agricultural supplies price variations. By issuing a cat-bond, the hedging firm borrows a capital that can be retained in case of catastrophic price surge. Such a solution would combine several advantages that are assessed both qualitatively and quantitatively.

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