A Multi-Scale Spatial General Equilibrium Model Applied to the USA and France

Laurent Faucheux

École doctorale n° 528 : École doctorale Ville, Transports et Territoires

THÈSE

pour obtenir le grade de docteur délivré par

Université Paris Est

Spécialité doctorale "Sciences Économiques"

présentée et soutenue publiquement par

Laurent FAUCHEUX

le 20 décembre 2018

A Multi-Scale Spatial General Equilibrium Model
Applied to the USA and France

Directeur de thèse : **Jean-Charles HOURCADE**

Jury

Jacques-François Thisse,	Professeur Emérite	Président
Stéphane De Cara,	Directeur de recherche	Rapporteur
Arnaud Diemer,	Professeur assistant	Rapporteur
Aurélie Méjean,	Chargée de recherche	Examinateur

Site du Jardin tropical
45 bis av. de la belle Gabrielle
94736 Nogent sur Marne Cedex
UMR 8568 - CIRED

Thèse effectuée au sein du Centre International de Recherche sur l'Environnement et le Développement

Site du Jardin tropical,
45 bis avenue de la Belle Gabrielle,
94736 Nogent sur Marne Cedex (FRANCE)

Aknowledgments

Je remercie Jean-Charles Hourcade de m'avoir intégré à l'équipe du CIRED, ainsi que pour son aide lors de la production finale de ce document. Je remercie Franck Lecocq de m'avoir soutenu dans ma volonté de conduire un doctorat en sciences économiques et de pouvoir le faire dans des conditions sereines. Je remercie Henri Waisman pour l'élan formidable qu'il m'a donné lors de mon arrivée au CIRED, élan grâce auquel j'ai pu traverser ces années de thèse avec un objectif précis et sans temps mort. Je remercie Franck Nadaud pour les centaines d'heures de discussion et de transmission sur les thèmes de l'économie et des statistiques. Je remercie Cédric Allio pour la remarquable dynamique d'équipe avec laquelle j'ai pu commencer mes premières années au CIRED. Aussi, le modèle de simulation formalisé dans ce document n'existerait tout bonnement pas sans lui. Je remercie ma partenaire de vie, Marion Dupoux, pour son aide permanente durant ces années de thèse, entre autre et très concrètement quant à la production de ce document. Enfin, je remercie l'équipe du CIRED pour la bonne ambiance dont j'ai pu bénéficier durant ces années.

Executive summary

The creation of the C40 Cities Climate Leadership group (C40) in 2005 is a noteworthy example that the urban scale is considered as a major leeway to mitigate CO_2 emissions. Nevertheless, the adequacy between this recent awareness and the number of modeling tools capable of quantifying this leeway in a spatially explicit integrated way is still missing. This thesis aims at bridging this gap. The outcome consists of a model that incorporates general equilibrium theory with an explicit representation of space at multiple scales. The model is designed as an autonomous numerical entity connectable to any pre-existing modeling architecture. This thesis hinges around three chapters, *i.e.* the presentation of the model, the calibration of the model and its application to France and the USA.

In the first chapter, we describe our so-called GEMSE model whose aim is to investigate the interplays between aggregate and local dimensions of economic activity while quantifying GHG emissions associated to mobility. The model is based on Urban Economics and the New Economic Geography to model on multiple spatial scales the economic development of urban areas in interaction.

In the second chapter, we describe the data and calibrate the model by using, for some parameters, spatial econometric techniques. Notably, we propose a new method to specify the spatial weight matrix, operationalized by using a numerical tool developed on purpose, namely PyOKNN, independent of GEMSE. Applied to Greater Paris, the tool identifies in a tangible way some key elements of its spatial structure, and yields values for the parameters under study that are similar to those of the literature.

In the third chapter, we run simulations of our model for both France and the USA. We analyze the baseline case and the impacts of two transport policies on several relevant dimensions for the long-term development of urban areas. The first measure – the decrease in private vehicle speed limitation – stimulates economic activity in a pro-environmental fashion by contracting GDP in a first phase but then allowing it to reach higher levels, resulting in a positive sum game. The second measure is the implementation of a CO_2 tax to private vehicles whose collected revenues are used to finance an increase in public transport speeds. The main policy insight is that setting a price of 100€ per tonne of CO_2eq represents virtually nothing once converted per commuter-kilometer and deters only marginally the use of cars. These two measures, the change in speed limitation or the recycling of the tax, encourage the use of cheaper and less polluting modes of transport, which induces a low-carbon growth.

Overall, these conclusions call for policy designs that internalize distortive effects, *e.g.* changes in mobility habits, the reorientation of demands, unbalances in labor markets via people's relocations and firms' improvements in terms of economies of scale. The results can rarely be generalized in terms of magnitude from one region to another, which shows the necessity to consider local specificities as well as the framework within which they interact.

Résumé en français

La création du groupe C40 Cities Climate Leadership (C40) en 2005 illustre bien le fait que l'échelle urbaine est dorénavant considérée comme comportant des leviers d'action importants afin d'atténuer les émissions de CO_2. Il n'y a cependant toujours pas adéquation entre cette prise de conscience et le nombre d'outils de modélisation capables de quantifier cette marge de manœuvre de manière spatialement explicite et intégrée. Cette thèse vise à combler cette lacune. L'objet produit consiste en un modèle d'équilibre général spatialisé et multi-échelle, pensé de sorte à être relié à toute architecture de modélisation préexistante. Cette thèse s'articule autour de trois chapitres, *i.e.* la présentation du modèle, sa calibration et son application à la France et aux USA.

Dans le premier chapitre, nous décrivons le modèle, baptisé GEMSE, dont l'objectif est d'étudier les interactions entre les dimensions agrégées et locales de l'activité économique tout en quantifiant les émissions de GES associées à la mobilité. Le modèle s'appuie sur l'Economie Urbaine et la Nouvelle Economie Géographique en vue de modéliser sur plusieurs échelles spatiales le développement économique de régions urbanisées en interaction.

Dans le deuxième chapitre, nous décrivons les données et calibrons le modèle en utilisant, pour certains paramètres, des techniques d'économétrie spatiale. Nous proposons notamment une méthode pour spécifier la matrice de poids spatial, laquelle méthode est opérationnalisée en utilisant un outil numérique développé à ces fins, PyOKNN, indépendant de GEMSE. Appliqué au Grand Paris, l'outil identifie de façon tangible des éléments clés de sa structure spatiale, et génère pour les paramètres étudiés des valeurs similaires à celles de la littérature.

Dans le troisième chapitre, nous appliquons le modèle à la France et aux Etats-Unis. Nous y analysons le scénario de référence, puis les impacts de deux politiques de transport. La première mesure – la baisse des limitation de vitesse des véhicules privés – stimule l'activité économique de manière pro-environnementale en réduisant le PIB dans un premier temps, mais en lui permettant ensuite d'atteindre des niveaux plus élevés, aboutissant à un jeu à somme positive. L'autre mesure simule la mise en place d'une taxe CO_2 pour les véhicules privés dont les recettes servent à financer l'augmentation des vitesses des transports publics. Il en résulte qu'un prix de 100€ par tonne de CO_2eq n'a qu'un faible effet incitatif car ne représente presque rien par kilomètre-voyageur. Ces deux mesures, le changement de norme ou le recyclage de la taxe, poussent à l'utilisation de modes de transport moins couteux et moins polluants qui stimule une croissance à plus faible intensité carbone.

Dans l'ensemble, ces conclusions plaident en faveur de politiques qui internalisent les effets distorsifs, *e.g.* les changements dans les habitudes de mobilité, la réorientation des demandes, les déséquilibres du marché du travail via les délocalisations des personnes et des changements induits en matière d'économies d'échelle externes. Les résultats ne sont dans leur ampleur que peu généralisables dans l'espace et montrent la nécessité de considérer les spécificités locales et le cadre dans lequel elles s'insèrent en terme d'interactions.

Contents

Contents **vi**

Figures **viii**

Tables **xi**

Introduction **1**

An attempt to follow Thisse's programm . . . 3

Objectives of the thesis . . . 8

Structure of the thesis . . . 9

1 Model description **14**

1.1 Short-run equilibrium . . . 16

1.2 Long-run modeling framework . . . 34

Appendix 1.A Determination of demands . . . 45

Appendix 1.B Transport modes' specified utility terms . . . 47

Appendix 1.C Website's variables descriptions . . . 49

2 Data and Calibration **66**

2.1 Data . . . 67

2.2 Calibration methods . . . 71

2.3 Econometric validation of the budget shares . . . 78

Appendix 2.A Models selected for estimation . . . 104

Appendix 2.B Models development . . . 111

3 Simulations **118**

3.1 Agglomerations and the economy: the baseline case . . . 119

3.2 Agglomerations and the economy under alternative policy scenarios . . . 137

3.3 Main findings . . . 150

Conclusions **153**

Methodological contributions . . . 154

Substantive contributions 155
Limits and future research 157
An attempt to overcome the 'blackbox' syndrome 159

Figures

1.1 Schematic representation of quasi dynamic models 16
1.2 A country as a set of $N + 1$ areas 16
1.3 A urban area j as a set of N_j places 17
1.4 Nested-structure of C 18
1.5 Example of $\mathbf{\Pi}_j$, given an agglomeration j network structure 21
1.6 Schematic representation of the GEMSE articulation 37

2.1 Iceberg melting cost symmetric matrix of the USA and France 68
2.2 The 22 American agglomerations' 8785 places (merged census block groups) considered by GEMSE 69
2.3 Illustration on New-York (private vehicle speed deciles km/min) of the census block groups merging process 70
2.4 The 12 French agglomerations' 2253 places (communes) considered by GEMSE . 70
2.5 Columbus polygon 25's (a) normalized weights and (b) mapped distances from it 81
2.6 The 428 places considered in the simulation (blue) *versus* the 300 ones considered in the econometric study. 92
2.7 Polynomial Times of n 93
2.8 Housing rents, R_i, and Road travel times, t_i 94
2.9 Residual incomes, $Y - P_i^a$, and Accessibility, T/t_i 95
2.10 SARIMA$(\varnothing, \varnothing, \varnothing)$-residual's mapped biciles, terciles, quartiles and deciles. . . . 97
2.11 SARIMA$(\varnothing, \varnothing, \varnothing)$ residual's plot and (full) autocorrelogram 98
2.12 SARIMA$(\varnothing, \varnothing, \varnothing)$ residual's full autocorrelogram against orders (left) and kilometers (right) 99
2.13 SARIMA$(\varnothing, \varnothing, \varnothing)$ residual's partial autocorrelogram against orders (left) and kilometers (right) 99
2.14 SARIMA$(\varnothing, \{7, 14, 37\}, \{1, 4, 12\})$ residual's full (left), partial autocorrelograms and residual plot 101
2.15 Bootstrapped budget shares' density plots 103
2.16 SARIMA$(\varnothing, \{7, 14, 34, 37\}, \{1, 4, 12\})$'s (P)ACFs, residual plot and bootstrap-based density plots of spatial parameters (1st rank) 105

2.17 SARIMA($\varnothing, \{7, 14, 37\}, \{1, 4, 12\}$)'s (P)ACFs, residual plot and bootstrap-based density plots of spatial parameters (2nd rank) . . . 107
2.18 SARIMA($\{2\}, \{7, 14, 37\}, \{1, 4, 12\}$)'s (P)ACFs, residual plot and bootstrap-based density plots of spatial parameters (3rd rank) . . . 109

3.1 US ($) and French (€) domestic prices per liter of fuel . . . 120
3.2 Levels of amenity in American and French areas . . . 120
3.3 Population attractiveness ranks of each nation largest urban space, the top three and rural . . . 121
3.4 Firms attractiveness ranks of each nation largest urban space and the top three 121
3.5 ULR dividers in the USA and France . . . 122
3.6 Variation in ULR dividers in the USA and France (%) . . . 123
3.7 Firms-number related HHI of the USA and France . . . 123
3.8 Population related HHI of the USA and France . . . 125
3.9 Welfares in the USA and France . . . 125
3.10 Variation in average urban costs in the USA and France (%) . . . 126
3.11 Variation in densities in the USA and France (%) . . . 126
3.12 Employment rates of each nation largest urban space and rural . . . 126
3.13 Average transport budget in the USA and France . . . 127
3.14 Modal share of car of each nation largest urban space and first ranked . . . 128
3.15 Transport budgets of each nation largest urban space and first ranked . . . 128
3.16 Density of each nation largest urban space and first ranked (hab/km²) . . . 128
3.17 Distribution of $\Delta^{P^{a,PV}}$-resilient modal shifts in 2030 . . . 132
3.18 Boxplots of all urban areas distributions of $\Delta^{P^{a,PV}}$-resilient modal shifts in the USA and France in 2030 . . . 134
3.19 Weighted average (over places by population) transport modes indifference levels in the USA and France . . . 136
3.20 Annualized variations in workers in the USA and France . . . 137
3.21 Places concerned (yellow) in 2017 by the Tr^{Std} 40% decrease in free private vehicle speeds limitation in NewYork and Paris . . . 140
3.22 Consequences of Tr^{Std} on housing rents in terms of variations in Baltimore and Grenoble . . . 141
3.23 Consequences of Tr^{Std} on housing rents in terms of variations in SanJosé and Marseille . . . 141
3.24 Consequences of Tr^{Std} on population shares in terms of variations in the USA and France . . . 142
3.25 Consequences of Tr^{Std} on GDP in terms of differences in the USA ($) and France (€) . . . 143

3.26 Consequences of Tr^{Std} on GHG emissions in terms of differences in the USA and France (kg/year) . . . 143
3.27 NPV of one kilometer of road, ξ_{km}, in the USA and France . . . 146
3.28 NPV of one kilometer per minute, $\xi_{\mathrm{km/min}}$, in the USA and France . . . 146
3.29 Revenues of the tax in the USA and France . . . 147
3.30 Translation of 100 \$ or € per ton of CO_2eq into its car-kilometer-based counterpart in the USA (\$) and France (€) . . . 147
3.31 Tr^{Tx}-financed speed upgrades in SanDiego and Marseille in 2018 . . . 148
3.32 Consequences of Tr^{Tx} on cars modal shares in terms of differences in SanDiego and Marseille . . . 148
3.33 Consequences of Tr^{Tx} in terms of differences on population welfare in the USA and France . . . 149
3.34 Consequences of Tr^{Tx} on GDP in terms of variations in the USA and France . . . 149
3.35 Consequences of Tr^{Tx} on GHG emissions in terms of differences in the USA and France (kg/year) . . . 150

Tables

1.1 Summary table of the sliced versions of $\mathbf{\Pi}_j$ . . . 32
1.2 Global scale related variables . . . 49
1.3 Inter-areas scale related variables . . . 56

2.1 Summary statistics . . . 95
2.2 SARIMA $(\varnothing, \varnothing, \varnothing)$ . . . 96
2.3 SARIMA$(\varnothing, \{7, 14, 37\}, \{1, 4, 12\})$-grounded eligible models . . . 102
2.4 SARIMA $(\varnothing, \{2, 7, 14, 34, 37\}, \{1, 4, 12\})$ . . . 106
2.5 SARIMA $(\varnothing, \{7, 14, 37\}, \{1, 4, 12\})$ . . . 108
2.6 SARIMA $(\{2\}, \{7, 14, 37\}, \{1, 4, 12\})$ . . . 110

3.1 Shares in each nation largest urban space (%) . . . 122

Introduction

Contents

An attempt to follow Thisse's programm **3**

Incorporating CGE theory 5

Involving geographical realism 5

Involving multiple spatial scales 7

Objectives of the thesis **8**

Structure of the thesis **9**

Over the past decade, the role of urban areas in climate change mitigation strategies has increasingly been considered. Typical of this new awareness was the creation of the C40 Cities Climate Leadership Group (C40), a network of cities that share their experiences on ways to reduce greenhouse gas (GHG) emissions.

Nevertheless, a gap still persists between this diagnosis and the capacity of the modeling tools used in the Integrated Assessment community to integrate the core characteristics of the urban issue, that is, its spatial dimension. The main reason for this gap is historical in nature. The first modeling community involved in the climate affair was this of the energy economists that built upon their experience in long term prospective tools in the context of the debate about resources exhaustion and controversies about the nuclear energy. Their leadership was established by the end of the 90th and, in 1993, Hourcade launched an alert about the necessity of a dialogue between energy modelers and the specialists of transportation systems (Hourcade, 1993). But only a few efforts were conducted is this direction and a real dialogue started only very recently, each "modeling tribe" preferring to pursue its initial track (Crassous, 2008).

A typical and recent example of the persistence of this gap is the New Policies Scenario (NPS) of IEA (2015) produced by the International Energy Agency. This scenario projects global energy demands by 2040 on the basis of assumed levels of urbanization rates amongst other indicators of on going trends.[1] Even though this modeling approach provides quantitative insights about the responsibility of urban areas in climate change, it *de facto* resorts to inter-scale correlations between macro trends (such as densities, GDP) and urbanization dynamics (captured at an aggregated level). It thus ignores the existence of the complex[2] role played by space in the dynamics of economic development. Going beyond the use of such correlations requires the explicit representation of the internal structures and the shapes of urban areas, as well as their interactions within regional systems and with a world economy under a globalization process (Schafer, 2012; Waisman et al., 2013).

Models have been developed so as to overcome the limits of uncontrolled global correlations but they do so by mobilizing the various sub-fields[3] of regional economics in a non-integrated way. This led to partial policy recommendations with no certainty about their overall consistency, whereas at a theoretical level, each of these subfields relate to considerations that complement each other. As stated by Thisse (2010), integrating space in economic analyses relates to *(i)* the use of a computable general equibrium (CGE) framework to endogenize the formation of incomes and relative prices, *(ii)* the realism of the modeled geographical space, the necessity of being multidimensional, asymmetric and empirically grounded and *(iii)* the number of encompassed spatial scales. Otherwise, it is difficult to figure out how spatial forces shape

[1] That are levels of growth in energy prices, population, economic development and industrialization.

[2] Which implies relying on computing architectures that can lead to surprising or counter-intuitive conclusions given the high number of antagonistic effects put at stake.

[3] Some of these subfields are Spatial Competition theory (Hotelling, 1929), Economic Geography, Urban Economics (Alonso, 1964), New Economic Geography (Krugman, 1991).

the so-called Space-economy[4] (Isard, 1949) and, *a fortiori*, what policy or what institutional change can influence them if they are judged to go in an undesired direction.

An attempt to follow Thisse's programm

To bridge this gap, we developed a spatial GE model, baptized *GE model of the Space-economy* (*GEMSE*). GEMSE relies on Thisse's recommendations and operationalizes the unification of New Economic Geography (NEG)(Krugman, 1991) and Urban Economics (UE)(Alonso, 1964; Mills, 1967; Muth, 1969).[5] GEMSE generates simulation data from the two UE- and NEG-related scales by explicitly accounting for the *geographical structure of local economies*. In parallel, the two-scale spatial system is influenced by an additional set of exogenous trends representing the nation's context – be it macroeconomic, demographic or technological –, constitutive of its third spatial scale, *i.e.* the global sphere.

GEMSE aims at providing a substitute to the correlation coefficients that are used to figure out how exogenous signals coming from the global sphere (economy, demography, prices) distribute among the determinants of the inferior scales. This modeling objective is at the source of one GEMSE's distinctive feature: among trends that are representative of the nation's context, some actually constrain the aggregated results of local scale dynamics. Put differently and using a neologism, GEMSE is said to *microcast* baseline-like macro-conditions. This neologism is defined by analogy with backcasting, in which the past is used in combination with long-term visions to calculate the drivers of the trajectories linking the present and future states of the economy. In a microcasting exercise, the local scale stands for the past and the aggregated results of local scale dynamics stands for the targeted future. This modeling approach allows us to address questions like: in a given space for all points in time, what are the baseline underliers linking the evolution of the fuel price, GDP and employment rate? What does it imply in terms of housing rents, mobility patterns, transport costs, residual income, densities and GHG emissions? Does the disaggregation process bring to light disparities that are hidden by using only average values?

The underlying assumption behind this approach is that the spatialization of robust equations – with socio-economical data differentiated in function of the geographical areas – will yield a modeling architecture more apt to deliver insights useful for policy debates and scientific discussions because its degree of realism is upgraded by the degree of consistency of the three scales of the projected system. Talking about long-term projections, the notion of degree of realism does not suggest degree of probability. It relates to the fact that the material for economic analysis is made of technically and economically consistent and plausible futures.

[4]In the following decades, it was decided not to use this expression to avoid confusions with emerging space technologies, which have led to the second best choice, that is regional economics. See Nijkamp (2013) for an holistic perspective of the concepts related to regional economics.

[5]To be precise, citing Mills (1967) and Muth (1969) in addition to Alonso (1964) rather relates to monocentric Land-Use Theory than to UE.

Once the "baseline world" is microcasted over the prospective horizon, the model runs in dual perspective by taking (baseline) microcasted parameters as exogenously-shockable and releasing macro-conditions into aggregates that are responsive to these shocks. In other words, in this second modeling stage, GEMSE outputs *ascending* inter-scales responses with respect to any (arbitrarily chosen) change in baseline underliers. This second modeling stage allows to answer for all points in time to questions such as: for a given counterfactual baseline world, what would be the consequences on GDP, employment rate, avoided cost of imported fossil fuels and/or income spatial redistribution of, *e.g.* new investments in transport infrastructure, changes in legally assigned road speed limits, the implementation of CO_2 based motor vehicle taxes?

In this thesis, this two-stage modeling approach is applied to both USA and France. For the USA, the model is calibrated over multiple 2010 spatial databases that collect data at the local scale for 22 major urban agglomerations.[6] This is done as well for France over 2008 spatial databases that collect data at the local scale for its 12 major urban agglomerations.[7] This wealth of information allows for capturing for most of the interaction mechanisms that take place between the areas of the two countries. Note that other areas' characteristics of each nation, not explicitly represented in space, are deduced to comply with national aggregates. The model also captures the impacts that are expected at a more aggregate scale of the economy, *e.g.* areas' economic performance, welfare, density, public and private transport sectors emissions.

The GHGs that are considered by GEMSE are CO_2, CH_4 and N_2O. These gases are converted into CO_2-equivalent (CO_2eq) following the IPCC (2007)'s computing method, *i.e.* accounting for their respective global warming potential (GWP).[8] The model computes Levasseur et al. (2010)'s dynamic GWPs to better consider the temporal profile of emissions.[9] Indeed, the model also aims at providing an assessment of mitigation potentials that are offered by policies with different implementation timings.

[6]The US agglomerations are Atlanta, Baltimore, Boston, Chicago, Dallas, Denver, Detroit, Houston, Losangeles, Miami, Minneapolis, Newyork, Philadelphia, Phoenix, Pittsburgh, Portland, Sandiego, Sanfrancisco, Sanjose, Seattle, Stlouis and Washington. Note that american urban areas' maps are available online according to the syntax `https://gemse.alwaysdata.net/usa/<area-name>`, *e.g.* `https://gemse.alwaysdata.net/usa/newyork`.

[7]The French agglomerations are Bordeaux, Grenoble, Lille, Lyon, Marseille, Montpellier, Nantes, Nice, Paris, Rennes, Strasbourg and Toulouse. Note that french urban areas' maps are available online according to the syntax `https://gemse.alwaysdata.net/france/<area-name>`, *e.g.* `https://gemse.alwaysdata.net/france/paris`.

[8]Where *GWP* is sometime prefixed with the term *relative*, which outlines the computing method, in which a horizon-dependent comparison of each gas radiative forcing (absolute GWP) is carried out relatively to CO_2.

[9]The Python library developed and used for the calculation of the GWP is available at `https://github.com/lfaucheux/PyGWP`.

Incorporating CGE theory

While the question of *what* to do to influence the space-economy is linked to the direct effects of policy implementation, the question of *how* to act to do so cannot be disconnected from their indirect effects, namely the impact of policies on industry, income distribution and employment. Addressing the question through the prism of economics mainly means being able to capture most general equilibrium effects, be they purely distributional or related to the expansion or the contraction of the production frontier (PF).[10] The modeling architecture to develop in order to spatialize a GE is implicitly specified by Thisse (2010), who, instead of soft-linking compact descriptions of the economy and spatial formalisms, insist on the necessity of their full entanglement. This intellectual movement is the same as the movement toward hybrid models approach to couple GE models and engineering-based energy models (Hourcade et al., 2006). In the case of spatial economics, this hybridization passes through the use of New Economic Geography (NEG), which intrinsically implies GE theory. Indeed, GE concerns are what makes Economic Geography *New* (Krugman, 1998).

As discussed by Simmonds et al. (2013, p.1058), *composite modeling*[11] – which stands for a programming design in which the processing order of submodels is not result-neutral – creates problems of consistency or of plausibility, worsened in the case of circularly dependent variables. Computable GE models are canonically representative of simultaneously interacting processes, *e.g.* the circularity of income, which is trivially emphasized when prices must balance markets. The convergence toward the equilibrium implies the use of modeling wrinkles such that well-shaped production functions, which comes to assume that over the long-run frictions of the real world have vanished. Space, and even more in a multi-scale framework, adds new layers of friction for the convergence of supplies and demands for all goods and services. Yet the implications of using one or other submodel sequence are rarely discussed in technical reports,[12] if ever. Failing that, the *computation-intensive* approach – implemented in this thesis – that consists of ensuring the parallel processing of submodels until their convergence (on some key indicators) is a reliable solution to avoid any disagreement on that matter.

Involving geographical realism

A few monoscale modeling frameworks incorporate GE effects but the realistic representation of the geographical space is not the norm.

At the scale (implicitly) involved in Urban Economics (UE), *i.e.* the urban scale, Anas and Xu (1999) describe a local economy whose differentiation *à la* Dixit and Stiglitz (1977) of the product space directly stems from a stylized geographical space. In this specification, in which the sole asymmetries relate to firms' and households' antagonist propensity to ag-

[10]The expansion of the PF results in different national aggregates while distributional ones do not.

[11]Composite modeling is similar to "hybrid modeling" in the Integrated Assessment Community.

[12]For an extensive description of this lack of discussion in the field of energy/climate economy models, see Lefevre (2016).

glomerate,[13] the GE resolution is equivalent to the search for an optimal spatial organization of economic activities, without any account being taken of distributional effects.[14] Nitzsche and Tscharaktschiew (2013) simulate transport speeds as an endogenizing key determinant of the spatial distribution of activities and population, and assess the corresponding impacts in environmental, safety and economic terms. While in the latter study, speeds are space-differentiated, the internal structure of the metropolitan area remains stylized. Choi et al. (2015) simulate the distributional effects on housing market fundamentals of a public income-neutral fiscal change. There is however no realistic considerations of space in the study given the linear patterns of the daily trips, completed at a constant speed. Remarkably, Anas and Liu (2007)'s RELU-TRAN model dynamically describes in a unified theoretical manner a metropolitan LUTI-based[15] space economy, in which the convergence of the system towards a unique equilibrium has been tested for a realistic set of calibration parameters, be they geographic or economic.

If we collapse the internal structure of a metropolitan area into a single point,[16] we conventionally jump to the field and the scale of analysis of New Economic Geography (NEG). Because at its early stages the analytic tractability of the spatial processes governing the interactions of areas was a key objective, one NEG's notable feature is its lack of realism with respect to the geographical distribution of areas, *e.g.* distributed in a mono-dimensional world as in Krugman (1993)'s Racetrack economy[17] or in a part of it as in Fujita et al. (1999b)'s Line economy, as well as in a bi-dimensional world as in Puga (1999)'s side-by-side Equidistant Economy (EE). Since then, adding more real spaces components to NEG has been thought of as the main step for future development (Behrens and Thisse, 2007; Fujita and Krugman, 2003). In this light, Behrens et al. (2007) develop a realistic geographical structure that can be seen as a bi-dimensional Tree economy, featuring transit and junction points, sole transit points, network edge points, and cycles, while still preserving analytical solvability. Bosker et al. (2007) numerically test NEG's theoretical predictions[18] against the spatial distribution of European metropolitan areas, by progressively complexifying Puga (1999)'s EE along the canonical axes of both geographic and economic mass symmetries. Starting from a bi-dimensional and asymmetric version of Krugman (1993)'s model, Stelder (2005) takes advantages of the complexity of the geographical shape of Europe to endogenize locations of the metropolitan areas. He de-

[13]This propensity is centrifugal for firms via agglomeration economies, while it is centripetal for households via congestion.

[14]Indeed, GE agents must be differentiated in one way or another to allow for the study of distributional effects.

[15]Recalling that LUTI means Land Use Transport interaction, LUTI-based space here stands for a spatial structure whose explicitation derives from those of the sectors of housing and transport.

[16]A point, whose inhabitants' maximization behaviors are aggregated into a GE agent in interaction with homologous versions of herself standing in other points of the spatial system.

[17]Not explicitly named as such in 1993. In the following years, this points setting is called "equidistant circle" by Brakman et al. (1996), finally ending in Fujita et al. (1999a)'s terminology, *i.e.* Racetrack economy.

[18]NEG's theoretical predictions are related to impacts changes in trade costs and in the mobility of production factors.

termines unequivocally the values of the parameters of the underlying mechanisms that explain its current structure.[19] Brakman et al. (2006) also start from an hyper-version of Krugman (1993)'s model and statistically test NEG predictions related to the "migration-driving power" of the wage equation across the NUTS2 European regions. Of greater complexity with the use of non-linear numerical techniques coming from different fields, Sheng et al. (2016) simulate four Canadian's regions. They infer each region's internal dynamic of inner-cities size via Markov chain whose temporal transition[20] matrix is estimated by maximum likelihood. At the top of the empirical complexity pyramid,[21] Mercenier et al. (2016)'s RHOMOLO model simulates the NUTS2 European regions in a fully asymmetric geographical space. Highly configurable, RHOMOLO's solvability can include more than one million equations depending on chosen options.

Involving multiple spatial scales

Both UE- or NEG-related models rely on exogenous signals coming from upper geographical scales, be them national, regional or global.[22] But it is harder to find modeling approaches in which these links are explicitly involved.

To bridge this gap, Waisman et al. (2013) develop a multi-scale multi-sectoral multi-regional modeling framework that operationalizes the unification of UE and NEG over the 74 major OECD cities. This GE-system of cities in interactions is then coupled with the integrated assessment model (IAM) IMACLIM-R (Waisman et al., 2012), which endogenizes macroeconomic and physical determinants. Given that this IAM also computes a (deeply detailed) GE, its consistency with the spatial one is ensured via the numerical correspondence of aggregates[23] that are computed on both sides. Allio (2016) also unifies UE and NEG, and presents a two-scales monosectoral[24] two-region model. This model innovates with realistic descriptive elements on both NEG and UE sides. On the NEG side, the two production factors, both mobile *à la* Tabuchi-Helpman (Murata and Thisse, 2005), formalize a labor market in each region, whose equilibrium is reached via wages. On the UE side, urban costs sum up over Land-Use Transport Interaction (LUTI) concerns, whose transport components are grounded on an explicit consideration of fully endogenous congestion-based transport speeds. Following Waisman et al. (2013), Allio (2015) operationalizes the exercise on France,[25] coupling it with

[19] USA are also modeled, but considered by the author as potentially less illustrative of the explanatory power of the model because of its too simple shape, which is indeed not so far from being represented by a rectangle.

[20] Transition of a city between relative size based groups.

[21] The numerical complexity of RHOMOLO is justified by its strong operational scope. Indeed, it is the dynamic spatial general equilibrium model of the European Commission, developed to undertake the ex-ante impact assessment of EU policies and structural reforms.

[22] Although contextual to the region we consider, examples of *(i)* national signals that can be population growth, labor force growth, investment growth and employment rate, *(ii)* regional signals, thinking from the European Union viewpoint, can be regulatory and *(iii)* global signals can be energy prices or technical progress.

[23] Namely the total production value, the totally available labor force, the total salary mass and the population.

[24] Indeed, there is no rural area as a venue of the homogeneous sector.

[25] He incorporates most of France's urban spatial structure through its twelve greater metropolitan areas.

IMACLIM-R.

Objectives of the thesis

The primary objective of this thesis is methodological in nature. As described in Crassous (2008), the history of energy modeling seems to be the confrontation or juxtaposition of a very few "modeling tribes" since the seventies. The modeling industry implies indeed high investment in human capital, in formation and in technical apparatus that cannot be changed overnight. This is why, to maximize chances for new models to inform policy debates, they must be graftable as easily as possible to any preexisting modeling architecture,[26] making them be ready-to-use in prospective exercises such as those that are usually produced by big institutions. Practically, this means that such models must be designed in a *nexum* fashion, *i.e.* they must have as element of their exogenous plug-in interface the most common economic drivers on which institutional organizations (such as OECD, World Bank, European Commission) make projections, *e.g.* GDP, factors productivity, investment growth, technical change, fuel price, population growth. Once these projections are spatialized, these models should be switchable to a responsive mode, both to explore the sensitivity of their baseline to other sets of parameters and the sensitivity of the system to various policy packages.

This is why we developed a multi-scale architecture in which the "for-all-scale" characteristics of the baseline are derived from the macro projections that are assumed by any other macroeconomic models. We consider explicitly three elements of representation. First, we consider the spatial structure of local economies so as to *e.g.* endogenize urban costs and delineate GHG emissions from urbanization rates. Second, we consider a unified multi-scale space economy (Scott and Storper, 2015; Thisse, 2010) to account for the reorganization of the geography of production and the interactions that take place within and between areas. Third, we consider the intertwined influences of signals descending from the global sphere to the local scale and *vice-versa*, so as to account for *e.g.* the influence of the fossil fuels cost on the spatial organization of urban areas and on their inner transport-housing trade-offs (Lampin et al., 2013; Waymire and Waymire, 1980).

An important effort has been made to calibrate the model on empirical data to make it applicable to *indigenous* policy-relevant questions and demonstrate how GEMSE could represent some progress with respect to WEO reports. From a programming standpoint, we also integrate as far as possible the critic of Simmonds et al. (2013) regarding the issues raised by composite (or sequential) modeling in terms "of consistency or of plausibility". To do so, the sets of variables that are connected by two-way feedback links[27] are modeled in a circular

[26] Modeling architecture whose zero-level version is a set of arbitrarily forcasted trends to which the model would be connected.

[27] In computer programming, when one piece of code requires the result from another, but that code needs the result from the first, this is often called *circular reference*.

fashion until convergence.

To summarize, the research objectives are about *(i)* creating a modeling tool capable of explicitly consider the geo-economical characteristics of national, regional and local economies, and endogenizing their interplay, *(ii)* creating a spatial modeling tool whose architecture will maximize its chance to be disseminated within the spheres of policy deliberations,[28] *(iii)* resorting to heavy-handed methods when dealing with assessable concerns of plausibility, *(iv)* providing a first set of results that show what the model can provide and *(v)* studying the consequences of some pro-environmental measures currently discussed in the political sphere.

Structure of the thesis

Chapter 1 presents the static model (or short-run model) and how its initial and terminal conditions relate to each other to constitute the recursive structure of the long-run modeling framework. We show how the model is switched from a configuration that determines baseline trends to a responsive mode that allows for analyzing the consequences of any exogenous shock performed on any model's parameter.

Chapter 2 presents the American and French data processed by the model and the calibration equations. In this chapter, a subsection is devoted to the (spatial) econometric validation of the budget shares related to housing and transport services in workers expenditures. In parallel, it also contributes to the theoretical spatial-econometric debate on the appropriate choice of the spatial weight matrix and makes a new proposal for it.

Chapter 3 operationalizes the model, using it in a first step to microcast and analyze projected trends of the USA and France metropolitan areas in their respective baseline world.[29] In a second phase, the model is used against baseline trends to carry out analyses that investigate the GHG abatement potential and the consequences on economic activities of designing transport policies. The choice of the policies is made so as to echo with the current policy questions, about the consequences of speed reductions in urban areas' center, and the introduction of a CO_2 price-based tax to finance public transport infrastructure.

Finally, an important component of this thesis, in a pro-transparency stance, is a website that makes dynamically[30] available all data used and generated by GEMSE for each country considered, each scenario, each variable, each year, each urban area as well as for each of its

[28] The development of a website that compiles and dynamically displays the results of the model fits into this strategy.

[29] Where energy prices, technical progresses, national investments in capital, average national labor force productivities and gross domestic products (GDP) all jointly redefine the context of their respective urban system through the prospective horizon.

[30] The term "dynamically" means that all charts and maps are clickable, configurable and browsable on users' demands.

constitutive places.[31] Moreover an alpha version documentation of GEMSE as modeling tool is available online.[32]

[31] The website can also be HATEOAS (Hypermedia As Engine of Application State) driven, see *e.g.* https://gemse.alwaysdata.net/france/?wSce=baseline, whereas the term HATEOS simply involves rendering website-like content with no superfluous interface since it is addressed to remote programs.

[32] Available at https://gemse.alwaysdata.net/static/gemse/GEMSE_doc_alpha.txt.

References

Allio, C. (2015). "Local Policies, Urban Dynamics and Climate Change: Development of a Multiscale Modeling Framework". PhD thesis, AllioT2015.

Allio, C. (2016). "Interurban population distribution and commute modes". *The Annals of Regional Science* 1991.

Alonso, W. (1964). *Location and land use: toward a general theory of land rent.* Harvard University Press, p. 204.

Anas, A. and Y Liu (2007). "A regional economy, land use, and transportation model (relutran((c))): Formulation, algorithm design, and testing". *Journal of Regional Science* 47.3, pp. 415–455.

Anas, A. and R. Xu (1999). "Congestion, Land Use, and Job Dispersion: A General Equilibrium Model". *Journal of Urban Economics* 45, pp. 451–473.

Behrens, K. and J. F. Thisse (2007). "Regional economics: A new economic geography perspective". *Regional Science and Urban Economics* 37.4, pp. 457–465.

Behrens, K., A. R. Lamorgese, G. I. P. Ottaviano and T. Tabuchi (2007). "Changes in transport and non-transport costs: Local vs global impacts in a spatial network". *Regional Science and Urban Economics* 37.6, pp. 625–648.

Bosker, M, S Brakman, H Garretsen and M Schramm (2007). "Adding geography to the new economic geography". *Cesifo Working Paper* 2038.

Brakman, S., H. Garretsen, R. Gigengack, C. van Marrewijk and R. Wagenvoort (1996). *Negative Feedbacks in the Economy and Industrial Location*.*

Brakman, S., H. Garretsen and M. Schramm (2006). "Putting new economic geography to the test: Free-ness of trade and agglomeration in the EU regions". *Regional Science and Urban Economics* 36.5, pp. 613–635.

Choi, K.-w., D. L. Sjoquist, C. Ki-Whan and D. L. Sjoquist (2015). "Economic and Spatial Effects of Land Value Taxation in an Urban Area: An Urban Computable General Equilibrium Approach". *Land Economics* 91.August, pp. 536–555.

Crassous, R. (2008). "Modéliser le long terme dans un monde de second rang : Application aux politiques climatiques", p. 345.

Dixit, A. K. and J. E. Stiglitz (1977). "Monopolistic Competition and Optimum Product Diversity". *The American Economic Review* 67.3, pp. 297–308.

Fujita, M. and P. Krugman (2003). "The new economic geography: Past, present and the future". *Papers in Regional Science* 83.1, pp. 139–164.

Fujita, M., P. R. Krugman and A. Venables (1999b). *The Spatial Economy: Cities, Regions, and International Trade.* MIT Press, p. 367.

Fujita, M., P. Krugman and A. J. Venables (1999a). "The Spatial Economy: Cities, Regions, and International Trade". *Southern Economic Journal* 67.2, p. 491.

Hotelling, H. (1929). *Stability in competition.*

Hourcade, J. C. (1993). “Modelling long-run scenarios. Methodology lessons from a prospective study on a low CO2 intensive country”. *Energy Policy* 21.3, pp. 309–326.

Hourcade, J.-C., M. Jaccard, C. Bataille and F. Ghersi (2006). “Hybrid Modeling: New Answers to Old Challenges”. *The Energy Journal*.

IEA (2015). *World Energy Outlook 2015*. World Energy Outlook. OECD Publishing.

IPCC (2007). *Climate Change 2007: The Physical Science Basis*. Ed. by S Solomon, D Qin, M Manning, Z Chen, M Marquis, K. B. Averyt, M Tignor and H. L. Miller. Cambridge University Press, p. 996.

Isard, W. (1949). “The General Theory of Location and Space-Economy”. *The Quarterly Journal of Economics* 63.476.

Krugman, P. (1991). “Increasing Returns and Economic Geography”. *Journal of Political Economy* 99.3, pp. 483–499.

Krugman, P. (1993). “On the number and location of cities”. *European Economic Review* 37.2-3, pp. 293–298.

Krugman, P. (1998). “What ’ S New About the New Economic Geography ?” *Oxford Review of Economic Policy* 14.2, pp. 7–17.

Lampin, L. B., F. Nadaud, F. Grazi and J.-C. Hourcade (2013). “Long-term fuel demand: Not only a matter of fuel price”. *Energy Policy* 62, pp. 780–787.

Lefevre, J. (2016). “Hybridization challenges in energy-economy integrated models and representation of the low carbon transition. An application to the Brazilian case”. PhD thesis. PARIS-SACLAY, p. 398.

Levasseur, A., M. Margni and L. Desche (2010). “Considering Time in LCA : Dynamic LCA and Its Application to Global Warming Impact Assessments”. 44.8, pp. 3169–3174.

Mercenier, J., F. Di Comite, O. Diukanova, D. Kancs, P. Lecca and M. Lopez Cobo (2016). *RHOMOLO-v2 Model Description*. Tech. rep. Institute for Prospective Technological Studies.

Mills, E. S. (1967). “An Aggregative Model of Resource Allocation in a Metropolitan Area”. *The American Economic Review* 57.2, pp. 197–210.

Murata, Y. and J.-F. Thisse (2005). “A simple model of economic geography à la Helpman–Tabuchi”. *Journal of Urban Economics* 58.1, pp. 137–155.

Muth, R. F. (1969). *Cities and Housing: The Spatial Pattern of Urban Residential Land Use*. University of Chicago Press, p. 355.

Nijkamp, P. (2013). “The Spatial Economy – A Holistic Perspective Research Memorandum 2013-37 Peter Nijkamp Waldemar Ratajczak The Spatial Economy – A Holistic Perspective”.

Nitzsche, E. and S. Tscharaktschiew (2013). “Efficiency of speed limits in cities: A spatial computable general equilibrium assessment”. *Transportation Research Part A: Policy and Practice* 56, pp. 23–48.

Puga, D. (1999). “The rise and fall of regional inequalities”. *European Economic Review* 43.2, pp. 303–334.

Schafer, A. (2012). “Introducing behavioral change in transportation into energy/economy/environment models”. *World Bank Policy Research Working Paper*.

Scott, A. J. and M. Storper (2015). “The nature of cities: The scope and limits of urban theory”. *International Journal of Urban and Regional Research* 39.1, pp. 1–15.

Sheng, K., J. Fan, W. Sun and H. Ma (2016). “From sequential to parallel growth of cities: Theory and evidence from Canada”. *Chinese Geographical Science* 26.3, pp. 377–388.

Simmonds, D., P. Waddell and M. Wegener (2013). “Equilibrium versus Dynamics in Urban Modelling”. *Environment and Planning B: Planning and Design* 40.6, pp. 1051–1070.

Stelder, D. (2005). “Where do cities form? A geographical agglomeration model for Europe”. *Journal of Regional Science* 45.4, pp. 657–679.

Thisse, J.-F. (2010). “Toward a Unified Theory of Economic Geography and Urban Economics”. *Journal of Regional Science* 50.1, pp. 281–296.

Waisman, H., J. Rozenberg, J Sassi and J.-C. Hourcade (2012). “Peak Oil Profiles Through the Lens of a General Equilibrium Assessment”. *Energy Policy* 48, pp. 744–753.

Waisman, H.-D., C. Guivarch and F. Lecocq (2013). “The transportation sector and low-carbon growth pathways: modelling urban, infrastructure, and spatial determinants of mobility”. en. *Climate Policy* 13.sup01, pp. 106–129.

Waymire, B. and E. Waymire (1980). “Effects of rising energy prices on urban space”. *Regional Science and Urban Economics* 10.3, pp. 407–422.

Chapter 1

Model description

Contents

1.1 Short-run equilibrium **16**

1.1.1 The Urban Spaces 17

1.1.2 The Rural Space 24

1.1.3 Regionwide equilibria 26

1.1.4 Urbanwide equilibria 28

1.2 Long-run modeling framework **34**

1.2.1 Step 1 – Exogenous dynamics 38

1.2.2 Step 2 – Endogenous dynamics 42

Appendix 1.A Determination of demands **45**

Appendix 1.B Transport modes' specified utility terms **47**

Appendix 1.C Website's variables descriptions **49**

GEMSE investigates interplays between aggregate and local dimensions of economic activity, which generates populations and firms migration, transportation, transport energy use and associated GHG emissions (N_2O, CO_2, CH_4). It is based on UE monocentric Land-Use Theory (MLU) (Alonso, 1964; Mills, 1967; Muth, 1969) and NEG (Fujita et al., 1999b; Krugman, 1991) whose economic determinants underpin at each of these two spatial scales the recursive disaggregation of "aspatial" trends (energy prices, technical progress, investments, the average national labor productivity, the national GDP and the national employment rate).

To describe the spatial dimension of urban development pathways, the NEG approach is applied to a set of conurbations (areas) to describe trade among them, as well as population migration[1] and capital migration.[2] As mentioned in the introductory Chapter, NEG models originally fail to explicitly capture the determinants of intra-area location. This restrains their empirical validation and limit the policy-relevance of their results. To overcome these limitations, we introduce two innovative features of crucial importance.

On the one hand, we introduce some empirically adapted[3] elements of UE to describe the internal structure of urban areas, notably the introduction of a specific Central Business District (CBD) where activity is concentrated and around which workers locate in function of a housing versus transport costs trade-off. On the other hand, we move beyond traditional two-areas/two-sectors NEG models to tackle inter-urban localization behaviors in a multi-areas/multi-sectors context, each of them represented by their main local economic development drivers (demographic growth, firms and population localization, transport demand, housing rents) and their underlying long term evolution mechanisms (agglomerations size and density, transport infrastructures type, individual preferences in terms of transport and housing).

GEMSE is a recursive dynamic model.[4] As shown in Figure 1.1, the model initiates the solving process by *(i)* taking the terminal states of the previous period, *(ii)* introducing endo- and/or exogenous shocks on these states and *(iii)* using these states as a set of conditions to be used to initiate the next recursion. In other words, this modeling approach consists of articulated short-run models that iteratively generate the trajectory of the long-run process.

Sections 1.1 and 1.2 that follow describe the wherefores falling respectively under these short- and long-run timescales.

[1]Called "mobile Labor model" in Krugman (1991)'s.
[2]Called "footloose capital model" in Martin and Rogers (1995)'s.
[3]Each agglomeration space is conceived as monocentric, but not as axisymmetric nor one-dimensional.
[4]Type of model that Wilson (1970) refers to as quasi dynamic model.

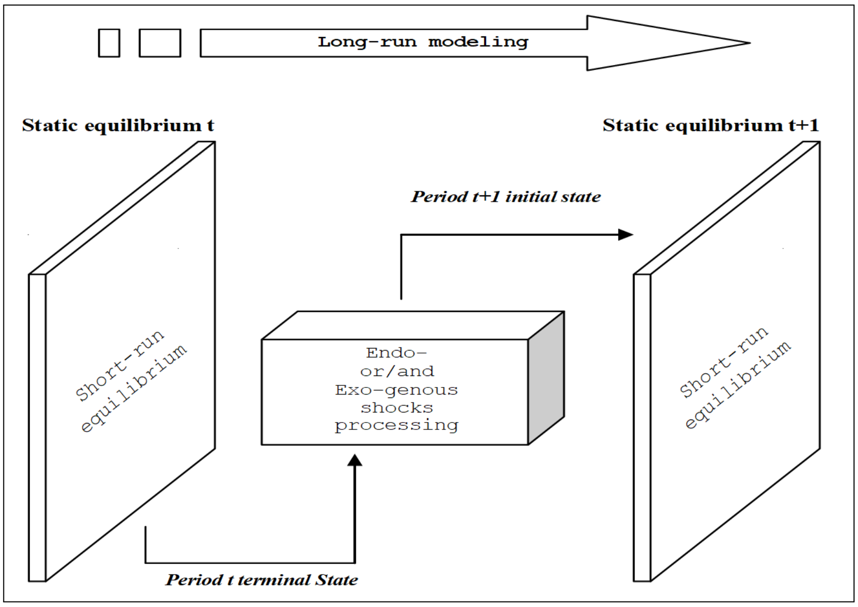


Figure 1.1: Schematic representation of quasi dynamic models

1.1 Short-run equilibrium

As illustrated on France in Figure 1.2, a country is envisaged as a set of $N+1$ areas, with N ag-

Figure 1.2: A country as a set of $N+1$ areas

glomerations ( ) and a rural area, z ( ). In the former, land is conceived as a heterogeneous space for workers and firms produce a number of varieties of a differentiated (manufactured) good M under increasing returns to scale. In the rural area, land is conceived as a homogeneous space, the rural workers are strictly identical and production is made of a homogeneous good F under constant returns to scale. Whether in rural or urban areas, note that labor force, $\mathcal{P}$, means workers, L, within an employment factor.[5]

[5] As explained further, the distinction between $\mathcal{P}$ and L is anything but innocuous since the dynamics of migration *between* the $N+1$ areas imply $\mathcal{P}$, while the dynamics of relocation *within* each of those areas imply L.

1.1.1 The Urban Spaces

In each urban agglomeration j, there is an explicit consideration of space. As illustrated in Figure 1.3 on NewYork, USA, an agglomeration j is envisaged as a monocentric set of

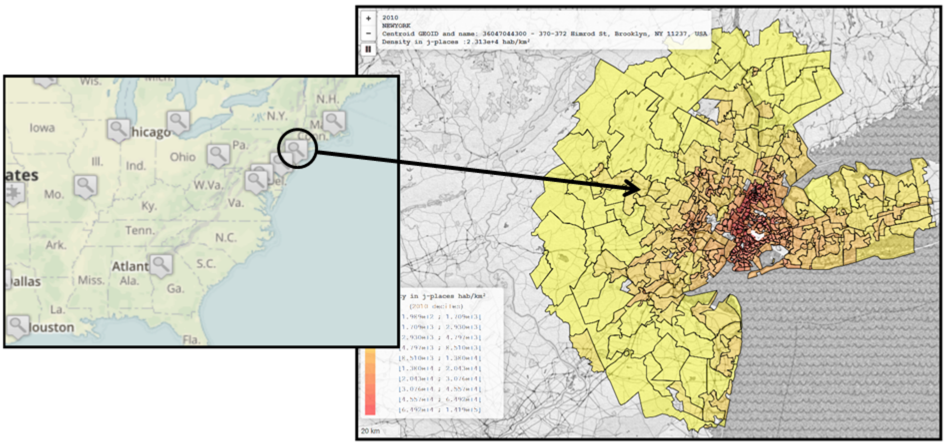


Figure 1.3: A urban area j as a set of N_j places

N_j atomistic spatial units – hereafter referred to indifferently as "places" – each located in a discrete two-dimensional space. Places in this two-dimensional space are indexed by $i \in [\![1; N_j]\!]$, scattered around the central business district (CBD), taken as the agglomeration center. In each agglomeration, three types of agents are operating: n_j firms, $\mathcal{P}_j$ active employees, and housing developers. All economic activities take place in the j-CBD, *i.e.* firms are assumed to locate in the CBD of the agglomeration.

1.1.1.1 Firms

Manufacturing production uses capital and labor as spatially mobile input factors. Production costs differ across agglomerations because of heterogeneous unitary labor requirements, $l_j \neq l_k$, $\forall \{j, k\} \in [\![1; N]\!]$, whereas these requirements are identical for all firms of a given agglomeration j. Employed labor force L_j is a variable factor of production and is subject to external economies of scale. The unitary labor costs c_j^L are lower in a larger market, as follows:[6]

$$c_j^L = \frac{l_j}{n_j^\eta} w_j \tag{1.1}$$

n_j is the number of firms in j. $\eta > 0$ is the elasticity of labor costs to the size of the market, which is measured by the number of active firms in area j. Equation (1.1) captures the improvement of effective productivity permitted by the agglomeration of production through

[6] As can be noted, we neglected to represent the phase during which the arrival of new workers in a given firm make the production function exhibiting decreasing returns to scale for a given stock of capital.

facilitated technology spillover. w_j are the wages paid per employee in agglomeration j. Capital is the fixed factor of production, and, with fixed input requirement $\varkappa$, the amount K_j of productive capital in agglomeration j is proportional to the number of domestic firms, n_j:

$$K_j = \varkappa n_j \tag{1.2}$$

Let r_j be the unitary return of capital K_j. The total cost c_j^q of producing q_j for a firm settled in agglomeration j is expressed as:

$$c_j^q = \varkappa r_j + c_j^L q_j \tag{1.3}$$

1.1.1.2 Workers

Utility of workers is differentiated in function of their position in the two-dimensional space of urban area j. In each place i, workers' utility $u_{j,i}$ derives from:

(i) the consumption of composite goods, $C_{j,i}$ – hereafter referred to indifferently as "Cobb-Douglas good"[7] –, resulting from a two-level decision tree as shown in Figure 1.4.

$$\overbrace{M^{\beta} F^{1-\beta}}^{C}$$
$$\overbrace{\left[\sum_{k=1}^{N} n_k (m_k)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}}$$

Figure 1.4: Nested-structure of C

where M is a composite index – of the consumption of manufactured goods m_k – defined by a constant-elasticity-of-substitution (CES) function, F represents the consumption of the traditional good coming from rural area, and β is a constant representing the share of M in workers' expenditures of Cobb-Douglas goods. As just outlined, M is actually a subutility CES function defined over a discrete and countable set of varieties of manufactured goods, m_k, whose (j,i)-demand, $m_{kj,i}$, reads as the demand of workers living in place i of urban area j for a variety produced in urban area k. Finally, $\varepsilon > 1$ is the elasticity of substitution among the varieties of manufactured goods.

(ii) housing services proxied by the consumption of $h_{j,i}$ squared meters:

$$h_{j,i} = \frac{H_{j,i}}{L_{j,i}} \tag{1.4}$$

[7] Admittedly, qualifying a good of being "Cobb-douglas" is abusive since, strictly speaking, what can be so-qualified are the consumers' preferences/tastes or the producers' technologies relative to the goods that are aggregated in such way. However, this linguistic liberty is relatively common, even more when the good is aggregated using a CES specification, see *e.g.* Bosi et al. (2010, p.13), Behrens and Robert-Nicoud (2011, p.218), Hungerland (2017, p.33), Ottaviano and Martin (2001, p.951) or Gaspar (2017, p.13).

$H_{j,i}$ refers to the overall consumption/supply of squared meters in place i of agglomeration j.

and *(iii)* transport services proxied by the consumption of $a_{j,i}$ round trips to the center business district (CBD), measuring the accessibility in i:

$$a_{j,i} = \frac{T}{d_{j,i}} v_{j,i} \tag{1.5}$$

For a given transport mode, $v_{j,i}$ is the observed speed to reach the CBD. $d_{j,i}$ is the corresponding traveling distance. T is the stable daily travel time budget of Zahavi and Talvitie (1980), assumed to be identical throughout the country for the sake of simplicity.

This leads to the utility function $u_{j,i}$:

$$u_{j,i} = u_j^0 C_{j,i}^{\delta^c} h_{j,i}^{\delta^H} a_{j,i}^{\delta^a} \tag{1.6}$$

with $\delta^c = 1 - \delta^a - \delta^H$, parameters δ^a and δ^H respectively referring to the elasticities of utility with respect to $a_{j,i}$ and $h_{j,i}$ quantities. u_j^0 captures all the amenities associated with residing in agglomeration j and is not differentiated across places.

By introducing the disposable income of the workers living in urban area j, Y_j – not differentiated across places either –, their consumption have to satisfy their budget constraint:

$$Y_j = p_z F_{j,i} + \sum_{k=1}^{N} n_k p_{kj} m_{kj,i} + h_{j,i} R_{j,i} + P_{j,i}^a \tag{1.7}$$

where p_z is the price of the homogeneous good produced in rural area, p_{kj} is the price of a variety of the manufactured good M coming from agglomeration k. At the level of agglomeration j's place i, $R_{j,i}$ is the average unitary land rent, $h_{j,i} R_{j,i}$ is the housing cost and $P_{j,i}^a$ the (transport mode-specific) commuting cost. Note that $h_{j,i} R_{j,i} + P_{j,i}^a$ is the "urban cost" incurred by a worker living at $d_{j,i}$ kilometers from the j-CBD.

Under the double constraint of time T and income Y_j, the demands for accessibility and land are:

$$\begin{cases} a_{j,i} & = \frac{T}{d_{j,i}} v_{j,i} \\ h_{j,i} & = \frac{\delta^H}{1-\delta^a} \frac{Y_j - P_{j,i}^a}{R_{j,i}} \end{cases} \tag{1.8}$$

For the traditional good and a given variety of the manufactured good, the maximization of utility under budget constraint leads respectively to the following demands:

$$\begin{cases} F_{j,i} & = \frac{(1-\beta)\delta^c}{1-\delta^a} \frac{Y_j - P_{j,i}^a}{p_z} \\ m_{kj,i} & = \frac{\beta \delta^c}{1-\delta^a} \left(\frac{P_j}{p_{kj}} \right)^{\varepsilon} \frac{Y_j - P_{j,i}^a}{P_j} \ \forall k \in [\![1; N]\!] \end{cases} \tag{1.9}$$

where P_j is the price of the CES good in agglomeration j,

$$P_j = \left[\sum_{k=1}^{N} n_k p_{kj}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{1.10}$$

1.1.1.3 Income formation

In each urban agglomeration j, the total disposable income, Y_j, results from four different sources, namely: wages paid to workers, w_j; dividends from capital invested in the Cobb-Douglas production, Y^K; incomes from real estate development, Y_j^H; and incomes from transport consumption, Y_j^a, as follows:

$$Y_j = w_j + Y^K + Y_j^H + Y_j^a \tag{1.11}$$

Wages derive from a wage-unemployment curve (Blanchflower and Oswald, 1995) and tensions between firms demand L_j and supply $\mathcal{P}_j$ of labor force are formalized by:

$$w_j = w_j^{min} \left(1 - \frac{L_j}{\mathcal{P}_j}\right)^{\sigma} \tag{1.12}$$

where w_j^{min} is the minimum wage in agglomeration j. $\sigma < 0$ is the elasticity of wages to the unemployment rate. We reiterate that $\mathcal{P}_j$ is the labor force supply in agglomeration j while L_j is the labor force *effectively* employed.

Nationwide total incomes from capital $\widetilde{Y^K}$ available in the Cobb-Douglas sector are given by:

$$\widetilde{Y^K} = k_z r_z q_z + \sum_{j=1}^{N} K_j r_j = k_z r_z q_z + \varepsilon^{-1} \sum_{j} p_j n_j q_j \tag{1.13}$$

where r_j (r_z) is the return on capital K_j ($k_z q_z$) invested in each agglomeration j (in the rural area z). For the sake of simplicity, we assume that productive capital is equally possessed by workers over the whole nation, so that dividends are uniformly redistributed among them. Thus, each of the L_j (L_z) workers living in agglomeration j (rural area) receives an "aspatial" income Y^K given by:

$$Y^K = \frac{\widetilde{Y^K}}{L_z + \sum_{k=1}^{N} L_k} \tag{1.14}$$

Recalling that $h_{j,i} R_{j,i}$ is the housing cost incurred by a worker located in place i, the total income generated in the housing sector of agglomeration j is:

$$\widetilde{Y_j^H} = \sum_{i=1}^{N_j} L_{j,i} h_{j,i} R_{j,i} \tag{1.15}$$

For the sake of simplicity, we assume that these incomes are redistributed among local workers, as follows:

$$Y_j^H = \frac{\widetilde{Y_j^H}}{L_j} \tag{1.16}$$

With the same redistribution rule as in eq.(1.16), the representative dividend generated by the transport sector is:

$$Y_j^a = \frac{\widetilde{Y_j^a}}{L_j} = \frac{\sum_{i=1}^{N_j} L_{j,i} P_{j,i}^a}{L_j} \tag{1.17}$$

1.1.1.4 Transport sector

In each place i of urban agglomeration j, workers have the choice à la Allio (2016) between two transport modes: public transport (PT) or private vehicle (PV). The PT average speed, $v_{j,i}^{PT}$, is constant regardless of the number of commuting workers, keeping in mind that congestion in public transport mode does not alter speed (subways, trams, less true for buses). In contrast, the PV average speed, $v_{j,i}^{PV}$, depends on the number of workers commuting on the urban road network, which therefore must be explicitly represented.

First, with occupancy rates of cars that are set to 1, the number of commuting cars is equal to the number of commuting workers. In closed-form, the $N_j \times 1$ vector of the numbers of cars in agglomeration j, $\boldsymbol{\Lambda}_j$, is:[8]

$$\boldsymbol{\Lambda}_j = [\Lambda_{j,i}] = [L_{j,i}\alpha_{j,i}] \tag{1.18}$$

where $\alpha_{j,i}$ is the modal share of cars in place i of urban area j.

Second, to model how cars interact, *i.e.* how they reciprocally influence their speed and ultimately, how they jointly create traffic jam, we must explicitly model the structure of the road network. Figure 1.5 provides an example of how a given network structure is considered numerically in the case of a symbolic agglomeration composed of 3 places A, B and C. The structure is transcribed by a square matrix, $\boldsymbol{\Pi}_j$, entrywise specified à la Allio with a percentage of route travel time in common between a given place and any other place while reaching the CBD of agglomeration j.

$$\boldsymbol{\Pi}_j = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 1 & 0.6 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

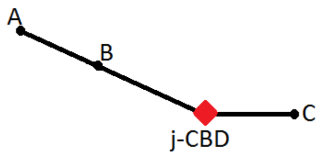


Figure 1.5: Example of $\boldsymbol{\Pi}_j$, given an agglomeration j network structure

In Figure 1.5, we see that: cars commuting from place A share 60% of their travel time with

[8] Note that $\boldsymbol{\Lambda}_j$ could have been named $\boldsymbol{L}_j^{PV}$, so as to be notation-consistent with the parallel use of $\boldsymbol{L}_j^{PT}$. However, as will be seen later in this section, this would have resulted in a lack of readability when superscripting $\boldsymbol{L}_j^{PV}$ by something else, *e.g.* $\boldsymbol{L}_j^{{PV}^{PV}}$.

those commuting from place B; cars commuting from place B share 100% of their travel time with those commuting from place A and; cars commuting from place C share no travel time with place A or B and thus have no network effect on those and reciprocally. For the sake of simplicity, $\mathbf{\Pi}_j$ is assumed to be the same whatever the commuting sense.

With such a representation of the road network, it is straightforward to compute the agglomeration's volume of private vehicles, $\mathrm{V}_{j,i}$, which as i-users takes the same route to commute to the j-CBD, given by:

$$\mathrm{V}_{j,i} = \sum_{o=1}^{N_j} \Lambda_{j,o} \Pi_{j,io} \quad \forall i \in [\![1; N_j]\!] \tag{1.19}$$

Private vehicle speeds observed on average along the route from place i to reach the j-CBD are computed using Geroliminis and Daganzo (2008)'s *macroscopic fundamental diagram of traffic flow* that establishes a relation between the number of cars over a given period of time and the number of cars over a given distance:

$$v_{j,i}^{PV} = \frac{v_{j,i}^{0,PV}}{1 + \left(\frac{\mathrm{V}_{j,i}}{\mathcal{K}_{j,i}}\right)^4} \tag{1.20}$$

$v_{j,i}^{0,PV}$ is the PV average free speed – with no congestion – and $\mathcal{K}_{j,i}$ is the road network capacity at a medium congestion state between place i and its j-CBD.

The consideration of the transport mode that is chosen in place i of urban area j leads to derive a specific accessibility component in the utility, $a_{j,i}^{PV}$ and $a_{j,i}^{PT}$, as follows:

$$a_{j,i}^{PV} = \frac{T}{d_{j,i}} v_{j,i}^{PV} \quad ; \quad a_{j,i}^{PT} = \frac{T}{d_{j,i}} v_{j,i}^{PT} \tag{1.21}$$

$a_{j,i}^{PV}$ and $a_{j,i}^{TC}$ are the numbers of round trips that a worker can perform given the speed of her transportation mode, her chosen place i to live in agglomeration j and the time T – not space-differentiated – she allocates to her mobility. Which leads to formalize $u_{j,i}^{PV}$ and $u_{j,i}^{PT}$:

$$u_{j,i}^{PV} = u_j^0 C_{j,i}^{\delta^c} h_{j,i}^{\delta^H} a_{j,i}^{PV^{\delta^a}} \quad ; \quad u_{j,i}^{PT} = u_{j,i}^{0,PT} u_j^0 C_{j,i}^{\delta^c} h_{j,i}^{\delta^H} a_{j,i}^{PT^{\delta^a}} \tag{1.22}$$

where $u_{j,i}^{0,PT}$ is an amenity factor that explains why workers use public transport in places where the private vehicle's speed unit is cheaper than that of public transport. This amenity factor consists of:

$$u_{j,i}^{0,PT} = \left(\frac{\nu_{j,i}^{PV}}{\nu_{j,i}^{PT}}\right)^{\delta^a} \left(\frac{Y_j - P_{j,i}^{a,PV}}{Y_j - P_{j,i}^{a,PT}}\right)^{1-\delta^a} \tag{1.23}$$

where $\nu_{j,i}^{PV}$ and $\nu_{j,i}^{PT}$ – instead of their v-like counterpart – stand for the reference speeds whose ratio internalizes the relative difference between the two modes of transport in terms of speed. $Y_j - P_{j,i}^{a,PV}$ and $Y_j - P_{j,i}^{a,PT}$ are the residual incomes stemming from the use of each of the two transport modes, private vehicles and public transport respectively, and whose ratio internalizes the relative difference between them on the monetary plan.

The PT commuting costs, $P^{a,PT}_{j,i}$, are location-dependent (charging zone) and constant regardless of the annual browsed pendulum distance, such that

$$P^{a,PT}_{j,i} = \overline{P^{a,PT}_{j,i}} \tag{1.24}$$

The private vehicle commuting costs is divided into two parts, one being distance-dependent, $\widehat{P^{a,PV}}$, the other distance-constant, $\overline{P^{a,PV}}$, as follows:

$$P^{a,PV} = \overline{P^{a,PV}} + \widehat{P^{a,PV}} = \overline{P^{a,PV}} + c_a p_a D \tag{1.25}$$

where c_a is the average liter consumption per kilometer, p_a the average price per liter and D the annual distance commuted by a PV-only user.

Thus, the transport budget of a (i-representative) worker living in place i, $P^a_{j,i}$, is:

$$P^a_{j,i} = \alpha_{j,i} P^{a,PV} + (1 - \alpha_{j,i}) P^{a,PT}_{j,i} \tag{1.26}$$

Recalling that $\alpha_{j,i}$ is the private vehicle modal share.

1.1.1.5 Housing sector

Housing developers have an instantaneous i-specific production function of *residential* square meters $H_{j,i}$, as follows:

$$H_{j,i} = \frac{S^{1-\gamma_{j,i}}_{j,i} \chi^{\gamma_{j,i}}_{j,i}}{\tau^H} \tag{1.27}$$

where $\gamma_{j,i}$ is the return to scale of installed capital specific to place i, which captures all omitted i-specificities in terms of regulation and of ease of construction. τ^H is the number of years that it takes to developers to build $H_{j,i}$ square meters.[9] Following Muth (1961), the only variable input is the stock of capital installed in place i, $\chi_{j,i}$, while $S_{j,i}$, the buildable surface, is fixed and does not influence the profit maximization program of developers. Also, the effective number of residential floors in place i of agglomeration j, $f_{j,i}$, consists of a ratio-relationship between the building land and the stock of square meters available locally, as follows:

$$f_{j,i} = \frac{H_{j,i}}{S_{j,i}} \tag{1.28}$$

[9] Which is likely to represent many years, given that $H_{j,i}$ is a quantity related to a whole place, say, a block of houses, with potentially thousands of people.

1.1.2 The Rural Space

In rural areas, there is no internal space considerations and land is taken as homogeneous and adimensional.

1.1.2.1 Firms

Rural firms, whose total number is not explicit, produce the traditional and homogeneous good under constant returns to scale. Both labor and capital are variable factors of production but only labor is spatially mobile. Letting r_z be the unitary return of capital K_z, the total cost c_z^q of producing q_z for a firm settled in rural areas is:

$$c_z^q = (l_z w_z + k_z r_z) q_z \quad ; \quad K_z = q_z k_z \tag{1.29}$$

1.1.2.2 Workers

The utility of rural workers, u_z, derives *(i)* from synthetic services h_z, representative of rural land-transportation services and *(ii)* (as for urban workers) from the consumption of "Cobb-Douglas goods", C, nesting a manufactured good imported from the urban space M and a traditional good produced locally, F [see Figure 1.4]. Except for land-transportation services, u_z is formalized as its urban counterpart, as follows:

$$u_z = u_z^0 C_z^{\delta^c} h_z^{\delta^a + \delta^H} \tag{1.30}$$

Since $1 - \delta^c = \delta^a + \delta^H$, note that income share attributed to rural land-transportation services, *i.e.* to h_z, equals the sum of the urban shares of income attributed to transport and housing in urban areas, δ^a and δ^H respectively. The idea is to keep the preferences of workers constant through the entire nation space. As in the urban space, u_z^0 captures all the amenities associated with residing in rural area.

Denoting Y_z the income of a worker living in the rural area, the consumer has to satisfy the following budget constraint:

$$Y_z = p_z F_z + \sum_{k=1}^{N} n_k p_{kz} m_{kz} + h_z R_z \tag{1.31}$$

where R_z is the land-transportation good price. The L_z workers pay for a synthetic good that aggregates both transport and housing services. Their demand for this good is given by:

$$h_z = (\delta^a + \delta^H) \frac{Y_z}{R_z} \tag{1.32}$$

The maximization of utility under budget constraint leads to the conditions:

$$\begin{cases} F_z & = (1-\beta)\delta^c \frac{Y_z}{p_z} \\ m_{kz} & = \beta\delta^c (\frac{P_z}{p_k})^\varepsilon \frac{Y_z}{P_z} \ \forall k \in [\![1; N]\!] \end{cases} \tag{1.33}$$

where P_z is the cost of differentiated good index in the rural area z:

$$P_z = \left[\sum_{k=1}^{N} n_k p_{kz}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{1.34}$$

1.1.2.3 Income formation

The total disposable income of a worker living in the rural area results from three different sources, namely: wages paid to workers, w_z; as in agglomerations, dividends from capital invested in the Cobb-Douglas goods production, Y^K; and dividends from land-transportation services Y_z^H:

$$Y_z = w_z + Y^K + Y_z^H \tag{1.35}$$

As in the urban space, wages calculations are based on a wage-unemployment curve, so that tensions between rural demand and supply of labor forces are formalized (Blanchflower and Oswald, 1995) via

$$w_z = w_z^{min} \left(1 - \frac{L_z}{\mathcal{P}_z}\right)^\sigma \tag{1.36}$$

where w_z^{min} is the minimum wage in the rural area. $\mathcal{P}_z$ is the active population in the rural area z, *i.e.* the labor supply, and L_z the rural labor force currently employed.

As urban workers, each rural worker holds an identical part of the Cobb-Douglas sector and receives the same capital-related income Y^K given by:

$$Y^K = \frac{\widetilde{Y^K}}{L_Z + \sum_{k=1}^{N} L_k} \tag{1.37}$$

We assume that dividends generated by the land-transportation sector, Y_z^H, are exclusively distributed between rural workers:

$$Y_z^H = (\delta^a + \delta^H) Y_z \tag{1.38}$$

1.1.2.4 Land-Transport sector

As already explained, given the non-explicit spatial representation of rural area, housing and transport services are aggregated into one unique service referred to as land-transportation service, h. This service is auto-generated by rural workers and its price is indexed on the gross rural product so that its consumption is kept constant in quantity.

1.1.3 Regionwide equilibria

First, note that trade is allowed across urban areas, as well as between urban areas and the rural area. We use the 'iceberg' form of transport costs associated with trade of the composite goods (Samuelson, 1952). If one variety of differentiated good is shipped from agglomeration j to agglomeration k (to the rural area, z), only a fraction τ_{jk} – where *a priori* $\tau_{jk} \neq \tau_{kj}$, $\forall\ j, k \in [\![1; N]\!]$ – will reach the destination, the remainder melting during the shipment.

Any unit produced in agglomeration j provides the same revenue independently from the location where it is sold, so that a variety sold at price p_j in its production location j is charged in consumption location k (the rural area z) at price p_{jk} (p_{jz}), as follows:

$$p_{jk} = \tau_{jk} p_j \quad ; \quad p_{jz} = \tau_{jz} p_j \tag{1.39}$$

We assume that the homogeneous good is freely traded across areas, so that:

$$p_{zj} = p_z \ \forall\ j \tag{1.40}$$

1.1.3.1 Heterogeneous good market equilibrium

The production q_j of a firm located in agglomeration j equals the sum of local consumptions and exports. The market clearance condition then imposes

$$\begin{aligned} q_j &= \beta\delta^c \left(\tau_{jz} L_z \frac{Y_z}{P_z} \left(\frac{P_z}{p_{jz}}\right)^{\varepsilon} + \frac{1}{1-\delta^a} \sum_{k=1}^{N} \tau_{jk} \sum_{i=1}^{N_k} L_{k,i} \frac{Y_k - P^a_{k,i}}{P_k} \left(\frac{P_k}{p_{jk}}\right)^{\varepsilon} \right) \\ &= \tau_{jz} L_z\ m_{jz} + \sum_{k=1}^{N} \tau_{jk} \sum_{i=1}^{N_k} L_{k,i}\ m_{jk,i} \end{aligned} \tag{1.41}$$

where $m_{jk,i}$ is the demand for the variety produced in urban area j of a worker who lives in place i of urban area k [see eq.(1.9)], m_{jz} is the demand for the variety produced in urban area j of a worker who lives in rural area [see eq.(1.33)]. Note that each demand is inflated by an iceberg-melting factor, $\tau_{j.}$, which shows that the producers of urban area j overproduce in order to offset the quantities of goods that melt during shipments.

Under Dixit-Stiglitz monopolistic market, firms set their price by assuming a constant elasticity of substitution (CES), $\varepsilon > 1$, and profit maximization leads to a constant mark-up on variable cost

$$p_j = \frac{\varepsilon}{\varepsilon - 1} c^L_j \tag{1.42}$$

As a consequence of the profit maximization behavior, the j-number of firms is such that profits are zero, as an equilibrium condition of monopolistic competition. Thus by setting zero profit, the return to capital r_j at the equilibrium is

$$r_j = \frac{q_j}{\varkappa}(p_j - c_j^L) \tag{1.43}$$

1.1.3.2 Homogeneous good market equilibrium

Market clearing imposes that

$$\begin{aligned} q_z &= (1-\beta)\delta^c \left(L_z \frac{Y_z}{p_z} + \frac{1}{1-\delta^a} \sum_{k=1}^{N} \left[\sum_{i=1}^{N_k} L_{k,i} \frac{Y_k - P_{k,i}^a}{p_z} \right] \right) \\ &= L_z F_{zz} + \sum_{k=1}^{N} \sum_{i=1}^{N_k} L_{k,i} F_{zk,i} \end{aligned} \tag{1.44}$$

where the first term on the right-hand side is the consumption of goods produced locally and the second one the total consumption from workers living in urban areas. Perfect competition implies marginal cost pricing, so that

$$p_z = l_z w_z + k_z r_z \tag{1.45}$$

1.1.3.3 Location choice equilibrium

A la Martin and Rogers (1995), urban firms relocate between urban areas. Firms/capitals base their migration choice on rate-of-return differentials across all urban areas, *i.e.* they have incentive to relocate as long as higher rates of return persist elsewhere, *i.e.* as long as $\{r_j > \hat{r} | j \in [\![1; N]\!]\} \neq \varnothing$. Put differently, they stop to relocate once they observe that

$$r_j = \hat{r} \quad \forall\, j \in [\![1; N]\!] \tag{1.46}$$

where r_j is the rate of return of firms settled in urban area j.

A la Krugman (1991), active populations (hereafter referred to indifferently as "population", "people" or labor force supply) base their migration choice on welfare differentials across all areas, *i.e.* they have incentive to relocate as long as a higher welfare level exists, *i.e.* as long as $\{W_r > \hat{W} | r \in [\![1; N+1]\!]\} \neq \varnothing$. In other words, populations have no incentive to relocate once they observe that

$$W_r = \hat{W} \quad \forall\, r \in [\![1; N+1]\!] \tag{1.47}$$

where W_r is the level of welfare of the r^{th} urban or rural area.

1.1.3.4 Labor market equilibrium

In each urban area, $\mathcal{P}_j$ denotes the quantity of labor force supplied by the population, *i.e.* the active population. In parallel, the quantity of goods that area j's firms produce implies a given level of labor requirement, $l_j n_j^{1-\eta} q_j$, which defines the firms' demand of labor force at the labor-market equilibrium, L_j, *i.e.* the quantity of labor force that is *effectively employed* by firms, such that

$$L_j = l_j n_j^{1-\eta} q_j \text{ for } L_j \in [0; \mathcal{P}_j],\ \forall j \in [\![1; N]\!] \tag{1.48}$$

In the rural area, total labor requirements for production of the homogeneous good is given by $l_z q_z$, and, as in the urban space, from this demand derives the quantity of labor force that is *effectively employed*, L_z, as follows

$$L_z = l_z q_z \text{ for } L_z \in [0; \mathcal{P}_z] \tag{1.49}$$

At the interface between the regional and the local scales, each area's labor market links the welfare of (active) population to the utility level of workers by an employment factor, E_r, as follows

$$\frac{L_r}{\mathcal{P}_r} u_r = E_r u_r = W_r \quad \forall r \in [\![1; N+1]\!] \tag{1.50}$$

Thus, W_r can be seen as an augmented version of u_r, related to the people's concern of losing and finding a job when/where they move.

1.1.4 Urbanwide equilibria

For any endowment of transport mode in any place i, all must comply with the criterion of equalization of all utility levels within urban area j, given by:

$$u_{j,i}^{PV} = u_{j,i}^{PT} = u_j^* \ \forall i \in [\![1; N_j]\!] \ \wedge \sum_{i=1}^{N_j} L_{j,i} = L_j \wedge \ u_j^* \propto u_j = W_j \frac{\mathcal{P}_j}{L_j} \ \forall j \in [\![1; N]\!] \tag{1.51}$$

Since welfare level (at the NEG-related scale) means utility level (at the UE-related scale) within a local employment rate factor [see eq.(1.50)], $u_j^* \propto u_j$ stands for the fact that u_j^* varies from u_j – given when initiating urban solving – so as to get a value reachable by all the places of the urban area j. Thus, any difference between u_j^* and u_j defines the impacts at the urban scale of exogenous and/or endogenous changes in the long-run of GEMSE [see Section 1.2].

Equation (1.51) is ensured $\forall i \in [\![1; N_j]\!]$ via a balanced trade-off based on an iterative and *circular* meeting between the consumptions of goods, $C_{j,i}^*$, of square meters (housing services), $h_{j,i}^*$, and of round trips (accessibility), $a_{j,i}^*$. The term *circular* is of critical importance to describe the way each GEMSE's urban system achieves its equilibrium. For example, in a given place i, changes in the number of residents may affect local and neighboring speeds in a differentiated manner and thus not to allow for the homogeneously-driven realization of eq.(1.51). Equivalently, i-changes in density or in accessibility may translate (via network effects) here and there into too low or too high residual incomes, which could also prevent the realization of eq.(1.51) on the two plans of housing and goods consumptions. In parallel/addition, the sustainability of eq.(1.51) would be undermined instantaneously if an incentive to modal shift persists in only one location of the urban system. This would translate into changes in private vehicle speeds on the one hand and of residual (of transport cost) income on the other hand, thereby leading to changes in demands and prices of goods and services, finally reorienting the entire urban system toward a new equilibrium level of utility.

These examples illustrate how critical the spatial distribution of workers, $L_{j,i} \forall i \in [\![1; N_j]\!]$, and of mobility habits, $\alpha_{j,i} \forall i \in [\![1; N_j]\!]$, are in the agglomeration for achieving the urban equilibrium. Put differently, $\boldsymbol{L}_j$'s and $\boldsymbol{\alpha}_j$'s components are the degrees of freedom of the equilibrium and must be those that, when translated into residual incomes and densities – density that is actually the object at the housing/transport interface – lead to comply with eq.(1.51), such that

$$\{C^*_{j,i}(L^*_{j,i}, \alpha^*_{j,i}), h^*_{j,i}(L^*_{j,i}, \alpha^*_{j,i}), a^*_{j,i}(\boldsymbol{L}^*_j, \boldsymbol{\alpha}^*_j)\} = \underset{C_{j,i},\ h_{j,i},\ a_{j,i}}{\operatorname{argmax}} \ \mathscr{L}^u_{j,i}(C_{j,i},\ h_{j,i},\ a_{j,i},\ \mu^u_{j,i}) \text{ s.t.(1.51)} \tag{1.52}$$

where $\mathscr{L}^u_{j,i}$ stands for the Lagrangian $u_{j,i}$-surfunction that incorporates the income constraint of workers living in place i of agglomeration j and $\mu^u_{j,i}$ is the i-specific infinitesimal change in the utility level arising from an infinitesimal change in the income constraint. While only the local number of workers (directly) matters for the demand related to housing and goods, $h^*_{j,i}(L^*_{j,i}, \alpha^*_{j,i})$ and $C^*_{j,i}(L^*_{j,i}, \alpha^*_{j,i})$, the existence of (road) network effects [see from eq.(1.18) to eq.(1.22)] involves to express each local accessibility demand as a function of the two entire spatial distributions of workers and mobility habits, hence the term of accessibility that is expressed as a function of vectors, *i.e.* expressed as $a^*_{j,i}(\boldsymbol{L}^*_j, \boldsymbol{\alpha}^*_j)$.

We subsequently expose the numerical system of equations to build so as to provide a urban area j with a geography that puts it at its equilibrium state, *i.e.* to provide a urban area j with $\boldsymbol{L}^*_j$ and $\boldsymbol{\alpha}^*_j$.

1.1.4.1 Housing/Transport equilibria

To comply with eq.(1.52), we adapt a computing method that is well-known in urban transportation planning: the four-step (4S) model.[10] The four steps that sequence the transportation-modeling into four basic building blocks are, namely, the steps of: *(i) trip generation* that consists of determining the number of commuters from each place i of agglomeration j, *(ii) trip distribution* that is about determining each commuter's destination, *(iii) modal choice* and *(iv) trip assignment* that consists of determining the route taken by commuters to reach their destination. Steps *(ii)* and *(iv)* are asserted structurally by the monocentric and simply-connected[11] nature of urban spaces that are modeled in GEMSE. Thus only the notions (and

[10] For details see *e.g.* McNally (2008).

[11] A terminology that comes from Topology, used abusively to say that the shortest path to reach the CBD of agglomeration j from any place i is unique.

their underlying variables) that are implied in Steps *(i)* and *(iii)* are invoked to comply with eq.(1.52).

In GEMSE's discrete representation of space, complying with eq.(1.52) leads to the use of discrete optimization techniques. Indeed – as outlined by the coexistence of two modal-choice-dependent utility functions [see eq.(1.22)] – $\boldsymbol{L}_j$ and $\boldsymbol{\alpha}_j$ are characterized over a quantifiable number of situations from which it is question to start so as to put the overall urban area at equilibrium. This means breaking the problem stated via eq.(1.52) into three sub-problems according to which precise condition [see eq.(1.51)] the utility of workers that live in place i is subject to. In some places, this condition reduces to $u_{j,i}^{PV} = u_j^*$ – *e.g.* places not served by public transports –, while in others it is rather $u_{j,i}^{PT} = u_j^*$, whereas in some other places eq.(1.51) is representative without restriction.

To formalize these three sub-problems, define $\mathscr{I}_j^W$, $\mathscr{I}_j^{PV}$ and $\mathscr{I}_j^{PT}$: three sets such that $\mathscr{I}_j^W \cup \mathscr{I}_j^{PV} \cup \mathscr{I}_j^{PT} = [\![1; N_j]\!]$. These three sets contain the indexes related to places where workers use, respectively, the two transport modes, only private vehicles and only public transports. Nothing prevents at most two of these three sets from being empty.

Depending on the transport mode that is chosen by workers, their utility function are specified differently [see eq.(1.22)]. However, in some places, their indirect specification leads to deal with sub-determined systems of equations while trying to comply with eq.(1.52). Let's first see this on the housing side by seeing how each i-local housing market clearance is achieved. The clearance of the housing market is achieved via the variation of housing rents per square meter, *i.e.* getting $R_{j,i} = R_{j,i}^*$, which depends on the sum of all local residual incomes (net of transport costs), as follows

$$R_{j,i}^* = \begin{cases} \frac{\delta^H}{1-\delta^a} \frac{\Lambda_{j,i}^*\left(Y_j - P_{j,i}^{a,PV}\right)}{H_{j,i}} & \text{if } i \in \mathscr{I}_j^{PV} \\ \frac{\delta^H}{1-\delta^a} \frac{\Lambda_{j,i}^*\left(Y_j - P_{j,i}^{a,PV}\right) + L_{j,i}^{PT*}\left(Y_j - P_{j,i}^{a,PT}\right)}{H_{j,i}} & \text{if } i \in \mathscr{I}_j^{W} \\ \frac{\delta^H}{1-\delta^a} \frac{L_{j,i}^{PT*}\left(Y_j - P_{j,i}^{a,PT}\right)}{H_{j,i}} & \text{otherwise (if } i \in \mathscr{I}_j^{PT}) \end{cases} \qquad \forall i \in [\![1; N_j]\!] \tag{1.53}$$

We recall that $H_{j,i}$ is a quantity of supplied square meters coming from the maximization program of housing developers [see eq.(1.27)]. Considered together, equations (1.102) and (1.53) show that all workers' indirect utility levels (are directly functions of these equilibrium housing rents and) incorporate the mobility habits of other workers living in the same place as them. At this stage and considering only the *housing side*, it therefore seems more judicious to figure u_j^* out either through $\mathscr{I}_j^{PV}$-places or through $\mathscr{I}_j^{PT}$-places, *i.e.* where the determination of u_j^* does not rely on the simultaneous knowledge of $\Lambda_{j,i}^*$ and $L_{j,i}^{PT*}$.

However, it is on the *transport side* that the choice of finding u_j^* (through $\mathscr{I}_j^{PV}$-places or through $\mathscr{I}_j^{PT}$-places) is constrained. As explained previously, the flows of cars that commute from $\mathscr{I}_j^{PV}$-places have an influence via (road) network effects on their own level of utility, as well as on the levels of utility in $\mathscr{I}_j^{W}$-places, *i.e.* the places that are endowed with a valuable *PV versus PT* trade-off. In $\mathscr{I}_j^{W}$-places, complying with eq.(1.51) translates into the use of the first principle of Wardrop (1952), whose underlying is an user-optimized Nash equilibrium reached when no user may lower her transportation cost through unilateral change in mobility habit. By equalizing indirect workers' utility derived from public transport with the one derived from private vehicle [see from eq.(1.101) to eq.(1.107)], we get the following transport-modes-indifference condition:

$$\frac{\upsilon_{j,i}^{PT}}{\nu_{j,i}^{PT}} = \frac{\upsilon_{j,i}^{PV}}{\nu_{j,i}^{PV}} \ \forall i \in \mathscr{I}_j^W \subseteq [\![1; N_j]\!] \tag{1.54}$$

where – as already introduced in eq.(1.23) – *(i)* $\nu_{j,i}^{PT}$ stands for the reference speed in public transport, which may be different from $\upsilon_{j,i}^{PT}$, *e.g.* a transport policy may have been implemented and *(ii)* $\nu_{j,i}^{PV}$ is the reference speed in private vehicle that may also be different from $\upsilon_{j,i}^{PV}$ because of changes in congestion on the road network and/or because of a modification in speed limitation, $\nu_{0j,i}^{PV}$, on which it depends [see eq.(1.20)].

With this greater functional dependence of the urban equilibrium to what happens in $\mathscr{I}_j^{PV}$-places, the starting point is thus related to figuring out the equilibrium numbers of cars commuting from any place i of agglomeration j, $\Lambda_{j,i}^*$ [see eq.(1.18)], be them emitted by $\mathscr{I}_j^{PV}$-places or by $\mathscr{I}_j^W$-places. To do so, define $N_j^{PV} = \text{card}(\mathscr{I}_j^{PV})$ and $\mathbf{\Lambda}_j^{PV*}$, a $N_j^{PV} \times 1$ vector of the stock-numbers of cars that stem from $\mathscr{I}_j^{PV}$-places. Then, in combination with Geroliminis and Daganzo (2008)'s macroscopic fundamental diagram of traffic flow [see eq.(1.20)], turn eq.(1.54) into the corresponding indifference-preserving flow of indigenous cars,[12]

$$\tilde{\mathrm{V}}_{j,i} = \mathcal{K}_{j,i}\left(-1 + \frac{\upsilon_{j,i}^{0,PV}\nu_{j,i}^{PT}}{\upsilon_{j,i}^{PT}\nu_{j,i}^{PV}}\right)^{\frac{1}{4}} \ \forall i \in \mathscr{I}_j^W \tag{1.55}$$

Define $N_j^W = \text{card}(\mathscr{I}_j^W)$ and $\tilde{\mathbf{V}}_j^W$, a $N_j^W \times 1$ vector of the flow-numbers of cars between $\mathscr{I}_j^W$-places and the CBD of urban area j, whose closed-form content is given by eq.(1.55), *i.e.* $\tilde{\mathbf{V}}_j^W = [\tilde{\mathrm{V}}_{j,i}]$. The components of $\tilde{\mathbf{V}}_j^W$ then have to be adjusted to take into account the (non-indigenous-to-$\mathscr{I}_j^W$-places) cars that come from $\mathscr{I}_j^{PV}$-places in addition to its own. This involves carrying out a segmented treatment of impacts that are due to cars coming from the two types of places, which means dividing the road network of agglomeration j, $\mathbf{\Pi}_j$, into four sub-networks, *i.e.* one per type of impacted-impacting pairs of places. The characteristics of these matrices are listed in Table 1.1. With these sliced versions of $\mathbf{\Pi}_j$, it is straightforward to downward-adjust $\tilde{\mathbf{V}}_j^W$ by taking non-indigenous $\mathscr{I}_j^{PV}$-cars into account. Let's denote the downward adjusted version of $\tilde{\mathbf{V}}_j^W$ by $\mathbf{V}_j^{W*}$, calculated as follows

$$\mathbf{V}_j^{W*} = \tilde{\mathbf{V}}_j^W - \mathbf{V}_j^{PV*} = \tilde{\mathbf{V}}_j^W - \mathbf{\Pi}_j^{(ii)}\mathbf{\Lambda}_j^{PV*} \tag{1.56}$$

[12] I insist on the word used here: *"indigenous"*, not "endogenous", which in this context must be understood as "indigenous to place i of urban area j".

	Impacting users	
Places i with	Valuable PV/PT trade-off	No PT users
Impacted users: Valuable PV/PT trade-off	$\left(\mathbf{\Pi}_j^{(i)}\right)_{N_j^W \times N_j^W}$ [a]	$\left(\mathbf{\Pi}_j^{(ii)}\right)_{N_j^W \times N_j^{PV}}$ [b]
Impacted users: No PT users	$\left(\mathbf{\Pi}_j^{(iii)}\right)_{N_j^{PV} \times N_j^W}$ [c]	$\left(\mathbf{\Pi}_j^{(iv)}\right)_{N_j^{PV} \times N_j^{PV}}$ [d]

[a] $\mathbf{\Pi}_j^{(i)}$, square *a fortiori*, is designed to calculate impacts on the private vehicle speed of users commuting from $\mathscr{I}_j^W$-places due to users commuting from $\mathscr{I}_j^W$-places.

[b] $\mathbf{\Pi}_j^{(ii)}$, rectangular *a priori*, is designed to calculate impacts on the private vehicle speed of users commuting from $\mathscr{I}_j^W$-places, due to users commuting from $\mathscr{I}_j^{PV}$-places.

[c] $\mathbf{\Pi}_j^{(iii)}$, rectangular *a priori*, is designed to calculate impacts on the private vehicle speed of users commuting from $\mathscr{I}_j^{PV}$-places, due to users stemming from $\mathscr{I}_j^W$-places.

[d] $\mathbf{\Pi}_j^{(iv)}$, square *a fortiori*, is designed to calculate impacts on the private vehicle speed of users commuting from $\mathscr{I}_j^{PV}$-places due to users commuting from $\mathscr{I}_j^{PV}$-places.

Table 1.1: Summary table of the sliced versions of $\mathbf{\Pi}_j$

Box 1. Equation (1.56) must be carefully considered since one of its terms is the deepest fiber of the urban equilibrium, that is $\mathbf{\Lambda}_j^{PV*}$,[a] *whose number of components and components' values* must be determined *iteratively*. The iterative approach followed to determine $\mathbf{\Lambda}_j^{PV*}$, *i.e.* to get $\mathbf{\Lambda}_j^{PV} = \mathbf{\Lambda}_j^{PV*}$, is clarified in Box 2.

[a] As clarified shortly, $\mathbf{\Lambda}_j^{PV}$ is a common fiber of $\boldsymbol{L}_j$ and $\boldsymbol{\alpha}_j$.

With these (pro-equilibrium) flow-numbers of $\mathscr{I}_j^W$-cars in hand, $\mathbf{V}_j^{W*}$, still remains to transform them into punctual stocks distributed overall $\mathscr{I}_j^W$-places. Indeed, $\mathbf{V}_j^{W*}$ is representative of all the cars that are "currently" in circulation, regardless of which $\mathscr{I}_j^W$-place they come from. This transformation of circulating flows into residing stocks is done by pre-multiplying $\mathbf{V}_j^{W*}$ by the inverse of $\mathbf{\Pi}_j^{(iv)}$, as follows

$$\mathbf{\Lambda}_j^{W*} = \left(\mathbf{\Pi}_j^{(iv)}\right)^{-1} \mathbf{V}_j^{W*} \tag{1.57}$$

Define $\mathbf{\Lambda}_j^*$, a $N_j \times 1$ vector that is formed over the components of $\mathbf{\Lambda}_j^{PV*}$ and $\mathbf{\Lambda}_j^{W*}$, *i.e.* a vector whose components are the numbers of cars commuting from any place i of agglomeration j at equilibrium, such that

$$\Lambda_{j,i}^* = \begin{cases} \Lambda_{j,\hat{\imath}}^{PV*} & \text{if } i = \hat{\imath} \wedge i \in \mathscr{I}_j^{PV} \\ \Lambda_{j,\hat{\imath}}^{W*} & \text{if } i = \hat{\imath} \wedge i \in \mathscr{I}_j^{W} \\ 0 & \text{otherwise (if } i \in \mathscr{I}_j^{PT}) \end{cases} \quad \forall i \in [\![1; N_j]\!] \tag{1.58}$$

The knowledge of the spatial distribution of cars in urban area j, $\Lambda_{j,i}^* \forall i \in [\![1; N_j]\!]$, allows us to determine the spatial distribution of housing rents over the $\mathscr{I}_j^{PV}$-places [see eq.(1.53)], as well as that of their speeds, computed as follows

$$v_{j,i}^{PV*} = \frac{v_{j,i}^{0,PV}}{1 + \left(\frac{\left[\mathbf{\Pi}_j^{(iii)}\mathbf{\Lambda}_j^{W*}+\mathbf{\Pi}_j^{(iv)}\mathbf{\Lambda}_j^{PV*}\right]_i}{\mathcal{K}_{j,i}}\right)^4} \ \forall i \in \mathscr{I}_j^{PV} \tag{1.59}$$

From now on, we can determine the equilibrium utility level (of equalization), u_j^* [as formulated in eq.(1.102)], where we know all its terms, *i.e.* in $\mathscr{I}_j^{PV}$-places, which yields

$$u_{j,i} = u_{j,i}^0 u_j^2 \left(R_{j,i}^*\right)^{-\delta^H} \left(\frac{T}{d_{j,i}} v_{j,i}^{PV*}\right)^{\delta^a} \left(Y_j - P_{j,i}^{a,PV}\right)^{1-\delta^a} = u_j^* \ \forall i \in \mathscr{I}_j^{PV} \tag{1.60}$$

The equilibrium numbers of workers who take public transport in place i of urban area j, $L_{j,i}^{PT*}$, is determined by considering *(i)* eq.(1.58), *(ii)* $u_{j,i}^{PT} = u_j^*$ with $u_{j,i}^{PT}$ expressed as in eq.(1.105) and *(iii)* eq.(1.53) over non-$\mathscr{I}_j^{PV}$-places, which yields

$$L_{j,i}^{PT*} = \left(\frac{u_j^0 u_j^2 u_{j,i}^{0,PT}}{u_j^*}\right)^{\frac{1}{\delta^H}} \frac{1-\delta^a}{\delta^H} H_{j,i} \left(\frac{T}{d_{j,i}} v_{j,i}^{PT}\right)^{\frac{\delta^a}{\delta^H}} \left(Y_j - P_{j,i}^{a,PT}\right)^{\frac{\delta^c}{\delta^H}} - \Lambda_{j,i}^* \left(\frac{Y_j - P_{j,i}^{a,PV}}{Y_j - P_{j,i}^{a,PT}}\right) \tag{1.61}$$

Box 2. In relation with Box 1, the iterative approach followed until $\mathbf{\Lambda}_j^{PV} = \mathbf{\Lambda}_j^{PV*}$, is directly based on the components of $[\Lambda_{j,i}]$ [see eq.(1.58)] and of $[L_{j,i}^{PT}]$ [see eq.(1.61)]. This iterative approach requires to choose between *(i)* decreasing the size of $\mathbf{\Lambda}_j^{PV}$ by one element where $|\Lambda_{j,i}/L_{j,i}^{PT}|$ exists and exhibits the highest value, or *(ii)* increasing the size of $\mathbf{\Lambda}_j^{PV}$ by one element where $|L_{j,i}^{PT}/\Lambda_{j,i}|$ exists and exhibits the highest value. Proceed to *(i)* if $|\Lambda_{j,i'}/L_{j,i'}^{PT}|$ is the only one that exists or $|\Lambda_{j,i'}/L_{j,i'}^{PT}| > |L_{j,i"}^{PT}/\Lambda_{j,i"}|$, otherwise proceed to *(ii)*.

Finally – and relatively to the 4S model's steps of *(i)* trip generation and of *(iii)* modal choice –, $\boldsymbol{L}_j^*$ and $\boldsymbol{\alpha}_j^*$ consist of

$$\boldsymbol{L}_j^* = \mathbf{\Lambda}_j^* + \boldsymbol{L}_j^{PT*} \text{ and } \boldsymbol{\alpha}_j^* = \frac{\mathbf{\Lambda}_j^*}{\boldsymbol{L}_j^*} \tag{1.62}$$

This concludes the presentation of the short-run model.

1.2 Long-run modeling framework

Before explaining what the modeling framework of long-run consists of, let us summarize the short-run model. GEMSE disaggregates a national economy into a set of N urban areas (hereafter indifferently referred to as agglomerations or conurbations) plus one unique "aspatialized" area that is rural, z. Each agglomeration $j \in [\![1; N]\!]$ comprises local housing developers, n_j firms located in the Central Business District (CBD) and L_j workers empirically distributed within places around it. With (spillover-adjusted) unitary labor requirement l_j/n_j^{η} paid at wage w_j [see eq.(1.1)] and fixed capital costs with uniform amount per firm $\varkappa$ [see eq.(1.2)], each urban firm produces q_j units of a variety of composite good, M, under variable cost submitted to external economies of scale working through an elasticity of labor cost with respect to the size of the market.

In any place[13] $i \in [\![1; N_j]\!]$ of urban area j, workers both endure and generate urban costs resulting from services of housing and commuting. Housing costs depend on the aggregated demand of local workers for housing surface $h_{j,i}$ [see eq.(1.4)] and the level of equilibrium land rent $R_{j,i}$ [see eq.(1.53)] that derives from the supplied stock of square meters, $H_{j,i}$ [see eq.(1.27)]. Urban sprawl is endogenized via a ratio-relationship between the building land and the stock of square meters supplied locally [see eq.(1.28)] and is considered horizontal as long as the ratio is smaller than 1.[14] Transport costs $P^a_{j,i}$ [see eq.(1.26)] derive from the accessibility demand $a_{j,i}$, *i.e.* the number of round trips a worker can do given the speed of her transport mean, her chosen place i to live in agglomeration j and the daily travel time T she allocates to her mobility [see eq.(1.5)].

We refer to the stable daily travel time budget of Zahavi and Talvitie (1980), so that T is assumed to be identical for all urban workers. Utility maximization under the double constraint of time T and income Y_j gives the local demands for accessibility $a_{j,i}$, for housing surface $h_{j,i}$, for the manufactured good $M_{j,i}$ and the homogeneous z-specific good $F_{j,i}$ (imported from the rural area). At the urban scale, workers' income includes wages plus dividends from transport and housing sectors and, at the national scale, dividends from the production of goods [see eq.(1.11)].

In the rural area z, space is not explicitly represented and the L_z workers are strictly identical. Firms produce q_z units of the homogeneous good, F, under constant returns to scale with two input factors: the unitary capital and labor requirements, that are k_z and l_z respectively [see eq.(1.29)]. Rural workers pay for a good that aggregates both transport and housing services, h_z, whose rent, R_z, varies according to the two growths of local income Y_z and of population $\mathcal{P}_z$. Utility maximization under the simple constraint of income Y_z gives local demand for housing/transport h_z, for the manufactured good M_z imported from the urban space, and the homogeneous good F_z produced locally. As in the rest of the nation, the income

[13] Note that a 'place' is the most atomistic spatial unit of GEMSE.

[14] Note that empirically, this ratio is very often smaller than 1 in peripheral places, which consistently transcribes urban sprawl up to a certain level.

of rural workers includes wage plus dividends. These dividends come from the equitably owned production of goods and from the composite local housing/transport sector [see eq.(1.35)].

Trade occurs across agglomerations, as well as between agglomerations and the rural area. Trade costs related to manufactured goods delivery between areas are represented under an 'iceberg' formulation *à la* Samuelson (1952) [see eq.(1.39)], whereas homogeneous goods produced in the rural area are freely tradable [see eq.(1.40)].

The short-term equilibrium of GEMSE is defined by two sets of conditions, the first being related to the scale implied to represent interactions between areas, *i.e.* at the *inter*-regional scale, and the second to the *intra*-area one, *i.e.* the urban scale.

At the inter-regional scale, market equilibrium for the differentiated good under monopolistic competition *à la* Dixit and Stiglitz (1977) gives the equilibrium quantity q_j [eq.(1.41)], prices p_j [eq.(1.42)] and returns to capital r_j [eq.(1.43)]. For the homogeneous good under perfect competition, it gives the equilibrium quantity q_z [eq.(1.44)] and prices p_z [eq.(1.45)]. For each sector (differentiated in urban areas and homogeneous in rural area), equilibrium specificities, *i.e.* quantitities and prices, locally imply a specific firms' demand of labor forces, which gives L_j in urban areas [eq.(1.48)] and L_z in rural area [eq.(1.49)]. In each area, the availability of active populations – supplied labor force – and of workers – demanded labor force – then jointly determine *à la* Allio (2016) a local labor market whose equilibrium is reached via wages [eq.(1.12) (eq.(1.36))]. Note that population's welfare, only involved at the "NEG" scale (to compute migratory population flows), therefore means worker's utility at the urban scale, within a local employment rate factor [eq.(1.50)].

The second set of short term conditions, related to the urban scale, *i.e.* the "UE" scale, comprises an intra-agglomeration equalization of places' level of utility [eq.(1.51)], ensured via a balanced trade-off based on the (circularly dependent [see eq.(1.52)]) states of the bi-modal transport-network-use and of the housing market [from eq.(1.53) to eq.(1.62)]. Note that multiple equilibria exist at this scale and the model solves equilibrium equations from the shocked previous equilibrium to the closest one.[15]

Workers have the choice between two transport modes: public transport (PT) or private vehicle (PV). The PT average speed, $v_{j,i}^{PT}$, is constant regardless of the number of commuting workers, assuming that congestion in public transport does not alter speed. *A contrario*, the PV average speed, $v_{j,i}^{PV}$, depends on road congestion, which requires GEMSE explicitly accounting for the network structure of the conurbation.

The representation of agglomeration j's road network structure, denoted by $\mathbf{\Pi}_j$ [see Figure 1.5], is necessary to compute the whole volume of commuting cars, $\mathbf{V}_j$ [see eq.(1.19)], and all places' PV speeds, $v_{j,i}^{PV}$ [see eq.(1.20)]. As exposed analytically in Allio (2016), who uses the first principle of Wardrop (1952),[16] $v_{j,i}^{PV}$ must be such that it generates transport-mode indif-

[15]The term "closest" must be taken for what it is, *i.e.*, based on the Euclidean norm on $\mathbb{R}^d$, where d stands, in the local scope of this footnote only, for the number of degrees of freedom of the equilibrium system.

[16]Whose underlying is a user-optimized Nash equilibrium reached when no user lowers her transportation cost through unilateral action, *i.e.* through unilateral change in mobility habits.

ference [eq.(1.54)], supported in parallel by a housing market clearing that provides each place i with the exact density that preserves the transport-user-optimized Nash equilibrium on the one hand and the intra-agglomeration equalization of places utility levels on the other hand [see eq.(1.52)].

At equilibrium, this computing process gives the localization of workers among the places of urban area j, $L_{j,i}$, their modal share of cars, $\alpha_{j,i}$, their consumption of round trips $a_{j,i}$, their residual income and associated consumption of square meters $h_{j,i}$ and of "Cobb-Douglas" goods $M_{j,i}^{\beta}F_{j,i}^{1-\beta}$. All other meaningful variables that are entailed at this scale are also computed, such as densities, urban costs (housing rents plus transport budget over the two modes) and, ultimately, transport-related GHG flows and stocks.

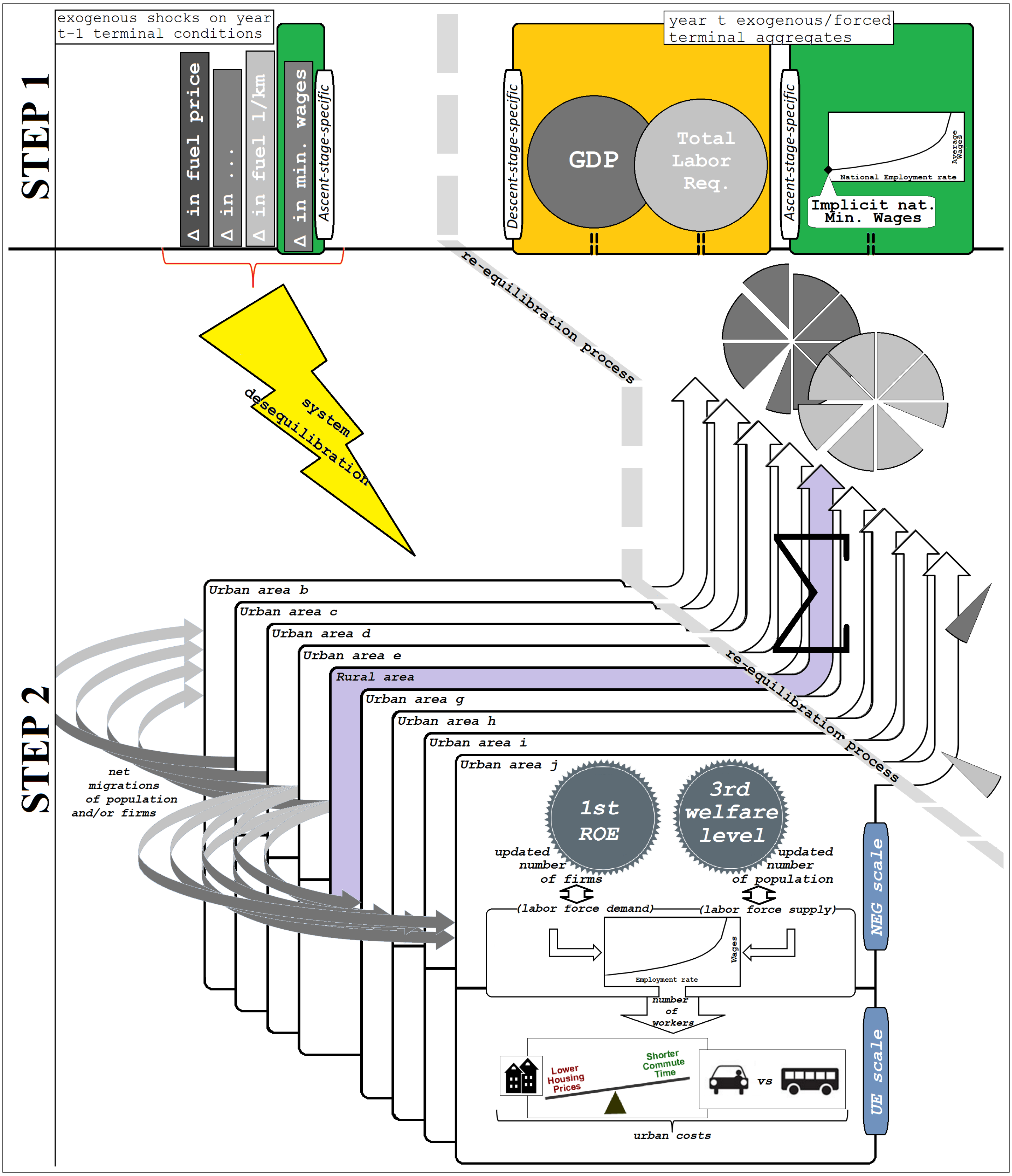
To conclude this summary, remind that the short-run model possesses a different set of boundary conditions whether it is involved in the (baseline driven) *microcasting*[17] stage or in the responding stage in which the model may diverge with respect to some (arbitrarily chosen) changes in *microcasted* baseline underliers. In what follows, the former stage is referred to as baseline microcasting or descension/disaggregation stage and the latter to as the microcasted baseline or ascent/reaggregation stage.

The long-run framework of GEMSE

The long-run modeling framework of GEMSE consists of a set of exogenous temporal trends, be them descending or ascending, used to inform the short-run model.

In the microcasting stage, trends are descending from the global sphere and are inputed to the short-run model that disaggregates them. Some of these descending trends simply change the parameters of the short-run model without involving the addition of degrees of freedom to its equilibrium system, *e.g.* energy price, direct investments. Some other descending trends constrain the values ascending from lower spatial scale – UE- and NEG-related –, thereby strictly characterizing the microcasting process. Technically, this is carried out at every recursion in two steps: *Step-1)* the two national aggregates of GDP, $\widetilde{\mathcal{Y}}_t$, and employment rate, $\widetilde{E}_t$, define a year-terminal set of conditions for the subsequent static equilibrium to comply with and; *Step-2)* GEMSE microcasts at regional and urban scales, the spatial dynamics that underpin and lead to these terminal aggregate conditions; all this ensures that the sum or the average value (over all areas) of each variable equals the value of the corresponding aggregate.

[17]Recall that by analogy, microcasting does with the upper and lower spatial scales what backcasting does respectively with future and past times: the lower scale representing the past, and the aggregation of this lower scale being seen as the desirable future.



[a] The yellow-backgrounded frame is specific to the descent/microcasting stage.
[b] The two green-backgrounded frames are specific to the ascent/microcasted/reaggregation stage.
[c] All other objects are common to the two stages.

Figure 1.6: Schematic representation of the GEMSE articulation

In the ascent stage,[18] the model runs from the dual perspective. Underliers that have been microcasted in the descent stage are taken as exogenous whereas macro-constraints are removed and henceforth become responsive to any change in UE- and/or NEG- related scales. Note that among aggregates that become free, some can only take values in a *functionally-constrained* admissible range. That is the case of the national employment rate and average wage, whose only possible values derive from the national *implicit* wage-curve parametrized during the descent stage.[19] The nationwide labor-market is said to be implicit since it has no implications on agents' income formation, which is processed at the scale of areas. It is only representative of a synthetic situation in which a labor market would prevail at the national scale. Technically, this is carried out in two steps: *Step-1)* the implicit nationwide wage-unemployment curve define a year-terminal *line* of conditions and; *Step-2)* GEMSE differentiates and reaggregates the regional and urban scales responses with respect to (arbitrarily chosen) changes in *microcasted* baseline underliers. Thus, in this stage of reaggregation, only the gdp, $\widetilde{\mathcal{Y}}_t$, is completely free while areas' employment rates and wages are functionally constrained at the level of their (implicit) national counterpart.

In Figure 1.6 each stage's respective steps *1)* and *2)* are illustrated. In this figure, note that *(i)* objects with a yellow background are specific to the descent stage, *(ii)* objects with a green background are specific to the ascent stage and *(iii)* all other objects are common to the two stages.

1.2.1 Step 1 – Exogenous dynamics

What follows holds $\forall t \in [\![0; \mathcal{T}]\!]$, where $\mathcal{T}$ stands for the last year of the prospective horizon.

1.2.1.1 Constraining framework

Descent stage

The sum of gross local products over all areas $r \in [\![1; N+1]\!]$ at year t, where r stands for all areas including rural, is given by $\sum_{r=1}^{N+1} Y_{r,t} L_{r,t}$. This sum must be such that it "microcasts" the baseline-scenarized national GDP, $\widetilde{\mathcal{Y}}_t$, as follows:

$$\sum_{r=1}^{N+1} Y_{r,t} L_{r,t} = \widetilde{\mathcal{Y}}_t \tag{1.63}$$

We then also ensure the equalization between the aggregate employment rate, $\widetilde{E}_t$, and its microcasted version at the regional scale, given by:

$$\frac{L_t}{\mathcal{P}_t} = \widetilde{E}_t \tag{1.64}$$

[18]We recall that the ascent stage can also be described as *microcasted* or *reaggregation* stage.

[19]We recall that the descent stage can also be described as *microcasting* or *disaggregation* stage.

where $L = \sum_{r=1}^{N+1} L_r$ and $\mathcal{P} = \sum_{r=1}^{N+1} \mathcal{P}_r$ are respectively the national number of workers – firms' demand of labor force [see eq.(1.48) and eq.(1.49)] – and the national active population – population's supply of labor force. The above condition obliges to add one more degree of freedom to the equilibrium system. This degree of freedom relates to the $(N+1)$ minimum wages, that we vary according to a non-space-differentiated factor, $1 + \underset{\sim}{\Delta}^w{}_t$, in such a way that

$$w_{r,t}^{min} = w_{r,t-1}^{min}(1 + \underset{\sim}{\Delta}^w{}_t) \ \forall r \in [\![1; N+1]\!] \tag{1.65}$$

Note that the addition of this degree of freedom to the equilibrium system – at the level of minimum wages – is a direct mathematical consequence of driving the system from the top using baseline-scenarized aggregates. As such, exogenous variations in aggregates $\widetilde{\mathcal{Y}}_t$ and $\widetilde{E}_t$ assert the expansion/contraction of the PF (production frontier), from which this (undifferentiated spatial) variation in minimum wages, $\underset{\sim}{\Delta}^w{}_t$, is the microcasting underlier.

Ascent stage

During the ascent stage, the constraints formulated in eq.(1.63) and eq.(1.64) are removed and $\widetilde{\mathcal{Y}}_t$ and $\widetilde{E}_t$ thus become endogenous and responsive to shocks that propagate (through the GE-system) to $Y_{r,t}$ and/or $L_{r,t}$. Note however that so as to uniquely determine the solution of the equilibrium system, eq.(1.65) must still hold. It serves to define the continuous set of national eligible labor market conditions, *i.e.* the national implicit labor market. To define the latter, take all areas minimum wages that have been microcasted at year t during the descent stage following eq.(1.65) and average them together to obtain the implicit minimum wage at the national scale, denoted by $\underset{\sim}{w}_t^{min}$, such that

$$\underset{\sim}{w}_t^{min} = \frac{\sum_{r=1}^{N+1} \underset{\sim}{w}_{r,t}^{min} \underset{\sim}{L}_{r,t}}{\underset{\sim}{L}_t} \tag{1.66}$$

where $\underset{\sim}{w}_r^{min}$ and $\underset{\sim}{L}_r$ are the minimum wages and the numbers of workers respectively, computed during the microcasting stage. Recall that the equilibria of areas' labor markets are reached via the deflation/inflation of their wages, w_r [see eq.(1.12) and eq.(1.36)]. To functionally constrain all areas' employment rates and wages at the level of their (implicit) national counterpart, we reverse the wage curve function and ensure the following condition

$$\underset{\sim}{w}_t^{min} = \frac{\sum_{r=1}^{N+1} w_{r,t} L_{r,t}}{L_t} \left(1 - \frac{L_t}{\mathcal{P}_t}\right)^{-\sigma} \tag{1.67}$$

Another element that is specific to the ascent stage, is the apparition by difference of what can be called a fiscal entity. Indeed, all the transport expenses that are microcasted (during the descent stage) must still be considered so that for all point in time the *inter-scenarios* accounting equation remains balanced. Note that only transportation expenses are considered this way since the sector is the only one to be autogenerated, *i.e.* with no explicitly modeled market, otherwise this would expose the model's income circuit to violate the conservation law. To comply with this law, any inter-scenario difference in transport expenses is computed at the national level and then redistributed to urban workers. Denote by $\mathrm{d}[\widetilde{Y^a}]$ the country's difference in total expenses incurred in transportation services between the ascent/microcasted stage and the descent/microcasting stage, such that

$$\mathrm{d}[\widetilde{Y^a}] = \widetilde{Y^a} - \underset{\sim}{Y^a} \tag{1.68}$$

where $\widetilde{Y^a}$ is the country's total expense incurred in transport services during the ascent stage and $\underset{\sim}{Y^a}$ is its (baseline) microcasted counterpart. We made the choice of injecting $\mathrm{d}[\widetilde{Y^a}]$ in the country's income circuit by directly interfering with urban areas' labor markets. We do so in a non-differentiated manner *across urban areas* by ensuring that wages, w_j, incorporate the (so-qualified) labor taxation change, as follows

$$w_j = w_j^{min}\left(1 - \frac{L_j}{\mathcal{P}_j}\right)^{\sigma} - \frac{\mathrm{d}[\widetilde{Y^a}]}{\sum_{j=1}^{N} L_j} \; \forall j \in [\![1; N]\!] \tag{1.69}$$

Note that rural area's wages are not concerned by the above equation, since rural workers do not incur urban-like transport costs. Also, this modeling choice is not neutral in terms of distributive effects within the urban space, since eq.(1.69) exhibits a redistribution rule that differs from that of transport sector's dividends, redistributed at the level of each urban area [see eq.(1.17)].

1.2.1.2 Contextualizing framework

As stated previously, GEMSE also possesses plug-in interfaces that allow for other top-down signals to change the context of the modeled country without involving adding degrees of freedom to its equilibrium system.

The annual variation of capital invested both in the manufactured and homogeneous productions at year t, Δ_t^K, uniformly controls the total stock of capital invested in all areas by imposing:

$$1 + \Delta_t^K = \frac{\sum_{r=1}^{N+1} K_{r,t}}{\sum_{r=1}^{N+1} K_{r,t-1}} \tag{1.70}$$

All area's unitary labor requirement annually change in an identical manner, according to Δ_t^l,

$$l_{r,t} = l_{r,t-1}(1 + \Delta_t^l) \forall r \in [\![1; N+1]\!] \tag{1.71}$$

Commuting-related monetary cost faced by a PV user living in place i of urban area j, $P_{j,i}^{a,PV}$ [see eq.(1.25)], also varies according to Δ_t^c, Δ_t^p and Δ_t^D: the variations of, respectively, the unitary fuel liter consumption per kilometer from vehicles, c^a, and the domestic price per liter of fuel, p^a, as follows:

$$c^a_t = (1 + \Delta^c_t)c^a_{t-1} \quad ; \quad p^a_t = (1 + \Delta^p_t)p^a_{t-1} \tag{1.72}$$

Relatively to the interregional trade, note that the matrix of transport costs *à la* Samuelson (1952) also varies within an elasticity factor, $\nabla^{\boldsymbol{\tau}}$, with respect to $\Delta^c_t\Delta^p_t$, as follows:

$$\boldsymbol{\tau}_t = \boldsymbol{\tau}_{t-1}(1 + \nabla^{\boldsymbol{\tau}}\Delta^c_t\Delta^p_t) \tag{1.73}$$

Public transport budgets of workers living in place i of urban area j, $P^{a,PT}_{j,i}$ [see eq.(1.24)], also follow an exogenous variation, $\Delta^{P^{a,PT}}_t$, such that:

$$P^{a,PT}_{j,i,t} = (1 + \Delta^{P^{a,PT}}_t)P^{a,PT}_{j,i,t-1} \quad \forall\{j,i\} \in [\![1;N]\!] \times [\![1;N_j]\!] \tag{1.74}$$

Commuting-related temporal cost of mobility, *i.e.* the stable daily travel time budget of Zahavi and Talvitie (1980), may change within a factor $(1 + \Delta^T_t)$, such that:

$$T_t = (1 + \Delta^T_t)T_{t-1} \tag{1.75}$$

To represent the repercussions of the reorientation of investments between the sectors of transport, of housing and of Cobb-Douglas goods,[20] parameters δ^a and δ^H [see eq.(1.6)],[21] vary exogenously,[22] according to $\Delta^{\delta^a}_t$ and $\Delta^{\delta^H}_t$:

$$\delta^a_t = (1 + \Delta^{\delta^a}_t)\delta^a_{t-1} \quad ; \quad \delta^H_t = (1 + \Delta^{\delta^H}_t)\delta^H_{t-1} \tag{1.76}$$

The share of manufactured goods, M, in workers expenditures of Cobb-Douglas goods, C, does so as well, following Δ^β_t:

$$\beta_t = (1 + \Delta^\beta_t)\beta_{t-1} \tag{1.77}$$

Dually, return on equity stated by the homogeneous sector also varies exogenously, according to $\Delta^{r_z}_t$:

$$r_{z,t} = (1 + \Delta^{r_z}_t)r_{z,t-1} \tag{1.78}$$

Finally, the labor force growth must reflect the growth of its national counterpart, $\Delta^{\mathcal{P}}_t$, such that:

$$\mathcal{P}_t = (1 + \Delta^{\mathcal{P}}_t)\mathcal{P}_{t-1} \tag{1.79}$$

[20]Recalling that the Cobb-Douglas good, C, is composed of one CES manufactured, M, and one traditional coming from rural areas, F [see Figure 1.4].

[21]Shares of investments allocated to the production on the one hand and consumers' budget shares on the other hand are strictly related given the shareholding structure set in GEMSE.

[22]Recalling that the share of income allocated to the consumption of Cobb-Douglas goods is deduced by negation of $\delta^a + \delta^H$.

1.2.2 Step 2 – Endogenous dynamics

Aggregates identified at step 1 are inequitably distributed according to the degree of heterogeneity that is empirically set across agglomerations at the scales of areas and inner places. This heterogeneity stems from four determinants: *(i)* the type of available labor force, whose productivity, l_j^{-1}, is differentiated across agglomerations via cost reducing spillover *à la* Brakman et al. (1996) and Østbye (2010); *(ii)* urban costs resulting from housing density and the quality of available modes of transport (*e.g.* speeds and road capacity); *(iii)* the local employment rate and; *(iv)* amenities of the area.

1.2.2.1 Inter-areas dynamics

Urban firms move towards the agglomerations markets that offer the most attractive rate of return r_j [see eq.(1.46)] (*ceteris paribus* due to a large home market and/or knowledge spillover, low employment rate and wages). Identically, populations move towards the agglomerations that offer the highest population's welfare W_r $\forall r \in [\![1; N+1]\!]$ [see eq.(1.47)] (high amenities, weak urban costs, great home market, high utility, high employment rate).

To capture the determinants of these migrations, two area-specific attractiveness indexes are built, A_j^n and $A_r^{\mathcal{P}}$, which respectively reflect firms' incentive to settle in the agglomeration j given its rate of return, and populations' incentive to migrate in the area r $\forall r \in [\![1; N+1]\!]$ given their welfare level. The inter-areas differences in these indexes of attractiveness drive the decisions of migrations by assuming that the relative variation of the number of firms (population) in a given urban area j (area r), is an increasing function of its attractiveness index A_j^n ($A_r^{\mathcal{P}}$).

In technical terms, to get the year-t-terminal number of firms in agglomeration j, $n_{j,t}$, we calculate $\Delta r_{k,t-1}$, that is each agglomeration k's rate of return deviation (from the average over all agglomerations, $\overline{r}_{t-1}$) that has closed the previous year:

$$\overline{r}_{t-1} = \frac{\sum_{k=1}^{N} r_{k,t-1} n_{k,t-1}}{\sum_{k=1}^{N} n_{k,t-1}} \quad ; \quad \Delta r_{k,t-1} = -1 + \frac{r_{k,t-1}}{\overline{r}_{t-1}} \quad \forall k \in [\![1; N]\!] \tag{1.80}$$

where $n_{k,t-1}$ is the year-$(t-1)$-terminal number of firms in agglomeration k. Based on these N differentials, the number of firms in agglomeration j at the end of year t is defined as follows:

$$n_{j,t} = n_{j,t-1}(1 + \nabla^n \Delta r_{j,t-1}) \quad \forall j \in [\![1; N]\!] \tag{1.81}$$

∇^n can either be seen as an inertia factor or as a migration probability, which relates to the migration speed of firms between agglomerations. The variation of the sum over the year-

t numbers of firms is then controlled to comply with the annual variation signal in capital invested both in manufactured and homogeneous productions, Δ_t^K [see eq.(1.70)].
In the same way, to get the year-t-terminal population (labor force supply) available in area r, $\mathcal{P}_{r,t}$ $\forall r \in [\![1; N+1]\!]$, we first calculate $\Delta W_{k,t-1}$, that is each area k's welfare level deviation (from the average over areas, $\overline{W}_{t-1}$) and then define $\mathcal{P}_{r,t}$ as follows:

$$\mathcal{P}_{r,t} = \mathcal{P}_{r,t-1}(1 + \nabla^{\mathcal{P}} \Delta W_{r,t-1}) \quad \forall r \in [\![1; N+1]\!] \tag{1.82}$$

where as for firms, ∇^W relates to the migration speed of populations between areas. The sum over populations of all areas is then also controlled so as to comply in variation with $\Delta_t^{\mathcal{P}}$, [see eq.(1.79)].

1.2.2.2 Intra-urban areas dynamics

Another endogenous dynamic that annually changes conditions of the statics of GEMSE, is the housing developers'. In imperfect competition, developers base their maximization program according to their anticipation of the local housing rents per square meter at year t, that is $\widetilde{\boldsymbol{R}}_{j,t}$. Inspired from the 'neighborhood effects' literature, $\widetilde{\boldsymbol{R}}_{j,t}$ is anticipated as resulting on average from itself in the past, $\boldsymbol{R}_{j,t-1}$, and from the past neighboring rents:

$$\widetilde{\boldsymbol{R}}_{j,t} = \hat{\boldsymbol{\Pi}}_j \boldsymbol{R}_{j,t-1} \tag{1.83}$$

where $\hat{\boldsymbol{\Pi}}_j$ stands for the row-stochastic[23] version of $\boldsymbol{\Pi}_j$ [see Figure 1.5] and thus implies the assumption that rents partially influence each other along the road network involved to reach the j-CBD. Anticipating imperfectly $\widetilde{R}_{j,i,t}$ and in pure competition, developers supply square meters given their Cobb-Douglas production function [see eq.(1.27)] in order to maximize their anticipated short-term profit perceived in place i at the end of year t, $\widetilde{\pi}^H_{j,i,t}$, which is given by

$$\widetilde{\pi}^H_{j,i,t} = \widetilde{R}_{j,i,t} H^*_{j,i,t} - \mathrm{d}\chi^*_{j,i,t-1} c^H (1 + r_H) \tag{1.84}$$

where c^H is the construction cost per installed unit of capital, which is capitalized at a r_H discount rate (the latter discount rate being specific to the housing sector) due to the one-year lag between the occurrence of installation costs and the perception of incomes. $\mathrm{d}\chi^*_{j,i,t-1}$ is the argument of maximization and consists of the flow of capital to install at year $t-1$ in place i to get the ideal corresponding stock of square meters, $H^*_{j,i,t}$, one year later. This flow results from the ideal $H^*_{j,i,t}$ minus all past years depreciated[24] and cumulative flows of square meters, such that

$$\mathrm{d}\chi^*_{j,i,t-1} = \chi^*_{j,i,t} - \left(\frac{H_{j,i,t-1}}{S_{j,i}^{1-\gamma_{j,i}}}\right)^{\frac{1}{\gamma_{j,i}}} (1 - \delta_H) = \chi^*_{j,i,t} - \chi_{j,i,t-1}(1 - \delta_H) \tag{1.85}$$

[23]The term 'row-stochastic' refers to a matrix with each row summing to 1.
[24]Note that depreciation relates to the stock of capital units, not directly to the stock of square meters.

where δ_H is the depreciation rate of installed capital. Deconstruction not being allowed, note that $\mathrm{d}\chi^*_{j,i,t-1}$ belongs to $[0;\infty[$ and thus equals 0 when any developer's maximization program yields negative flows. Consequently, the newly installed stock of square meters at year t, $H_{j,i,t}$, is such that

$$\frac{H_{j,i,t}}{S_{j,i}^{1-\gamma_{j,i}}} = (\chi_{j,i,t-1}(1-\delta_H))^{\gamma_{j,i}} + \frac{(\mathrm{d}\chi^*_{j,i,t-1})^{\gamma_{j,i}}}{\tau_H} \tag{1.86}$$

The short-term profit maximization program possesses an analytic solution, here given by unit of building land surface,

$$\frac{\chi^*_{j,i,t}}{S_{j,i}} = \left(\frac{c^H(1+r_H)}{\widetilde{R}_{j,i}\gamma_{j,i}(1-\delta_H)^{\gamma_{j,i}}}\right)^{\frac{1}{-1+\gamma_{j,i}}} \tag{1.87}$$

This concludes the presentation of the long-run modeling framework.

1.A Determination of demands

All what follows stands $\forall \{j,i\} \in (⟦1;N⟧ \cup \{z\}) \times ⟦1;N_j⟧$, where $⟦1;N⟧ \cup \{z\}$ stands for the set of indexes related to urban areas plus the rural area z. In order to preserve a general notation as much as possible, the particularities of the rural area are declined in the course of the model development. Given that the rural area is an "aspatial" region, the first of these particularities is that the number of places i therein, N_z, can be thought of as being equal to 1, and can thus be omitted, *i.e.* $u_{z,i=1} = u_z$.

As a first stage of resolution, we have

$$\max_{C_{j,i},h_{j,i}} \left(u_{j,i}(C_{j,i}, h_{j,i}, a_{j,i}) = u^0_{j,i} C^{\delta^c}_{j,i} h^{\delta^H}_{j,i} a^{\delta^a}_{j,i} \text{ s.t. } \begin{cases} Y_j & \geq p_C C_{j,i} + P^a_{j,i}(a_{j,i}) + R_{j,i} h_{j,i} \\ a_{j,i} & \leq \frac{T}{d_{j,i}} v_{j,i} \end{cases} \right) \tag{1.88}$$

which is equivalent to the maximization of the following utility sur-function

$$\mathscr{L}^u_{j,i}(C_{j,i}, h_{j,i}, \mu^u_{j,i}) = u^0_{j,i} C^{\delta^c}_{j,i} h^{\delta^H}_{j,i} a^{\delta^a}_{j,i} + \mu^u_{j,i} \left(Y_j - p_C C_{j,i} - P^a_{j,i}(a_{j,i}) - R_{j,i} h_{j,i} \right) \tag{1.89}$$

with $a_{j,i} = \frac{T}{d_{j,i}} v_{j,i}$. It follows that $\max\limits_{C_{j,i},h_{j,i},\mu^u_{j,i}} \mathscr{L}^u_{j,i}$ turns into

$$\begin{cases} \frac{\partial \mathscr{L}^u_{j,i}}{\partial C_{j,i}} & = 0 \\ \frac{\partial \mathscr{L}^u_{j,i}}{\partial h_{j,i}} & = 0 \\ \mu^u_{j,i} & \neq 0 \end{cases} \Rightarrow p_C C_{j,i} = \frac{\delta^c}{\delta^H} R_{j,i} h_{j,i} = Y_j - P^a_{j,i}(a_{j,i}) - R_{j,i} h_{j,i} \Leftrightarrow \begin{cases} C^*_{j,i} & = \frac{\delta^c}{1-\delta^a} \frac{Y_j - P^a_{j,i}(a_{j,i})}{p_C} \\ h^*_{j,i} & = \frac{\delta^H}{1-\delta^a} \frac{Y_j - P^a_{j,i}(a_{j,i})}{R_{j,i}} \end{cases} \tag{1.90}$$

Via setting $a_z = h_z$ and $P^a_z = 0$ for individuals located in rural areas, we get the following analogous-to-(1.89) Lagrangian

$$\mathscr{L}^u_z(C_z, h_z, \mu^u_z) = u^0_z C^{\delta^c}_z h^{\delta^H + \delta^a}_z + \mu^u_z \left(Y_z - p_C C_z - R_z h_z \right) \tag{1.91}$$

which is maximized at

$$\begin{cases} \frac{\partial \mathscr{L}^u_z}{\partial C_z} & = 0 \\ \frac{\partial \mathscr{L}^u_z}{\partial h_z} & = 0 \\ \mu^u_z & \neq 0 \end{cases} \Rightarrow p_C C_z = \frac{\delta^c}{\delta^H} R_z h_z = Y_z - R_z h_z \Leftrightarrow \begin{cases} C^*_z & = \delta^c \frac{Y_z}{p_C} \\ h^*_z & = \left(\delta^H + \delta^a \right) \frac{Y_z}{R_z} \end{cases} \tag{1.92}$$

Then, since $C_{j,i}$ is a quantity that Cobb-Douglas-aggregates the consumptions (in each place i of area j) of a manufactured good, $M_{j,i}$, and a homogeneous good, $F_{j,i}$, a second resolution stage turns to be as follows

$$\max_{M_{j,i},F_{j,i}} \left(M^{\beta}_{j,i} F^{1-\beta}_{j,i} \text{ s.t. } p_C C^*_{j,i} \geq M_{j,i} P_j + F_{j,i} p_z \right) \tag{1.93}$$

Or put differently,

$$\max_{M_{j,i},F_{j,i},\mu^C_{j,i}} \left(\mathscr{L}^C_{j,i}(M_{j,i},F_{j,i},\mu^C_{j,i}) = M^{\beta}_{j,i}F^{1-\beta}_{j,i} + \mu^C_{j,i}\left(p_C C^*_{j,i} - M_{j,i}P_j - F_{j,i}p_z\right)\right) \tag{1.94}$$

It follows that $\max\limits_{M_{j,i},F_{j,i},\mu^C_{j,i}} \mathscr{L}^C_{j,i}$ turns into

$$\begin{aligned}
&\begin{cases} \frac{\partial \mathscr{L}^C_{j,i}}{\partial M_{j,i}} & = 0 \\ \frac{\partial \mathscr{L}^C_{j,i}}{\partial F_{j,i}} & = 0 \Rightarrow P_j M_{j,i} = \dfrac{\beta}{1-\beta}F_{j,i}p_z = p_C C^*_{j,i} - F_{j,i}p_z \\ \mu^C_{j,i} & \neq 0 \end{cases} \\
&\Leftrightarrow \begin{cases} M^*_{j,i} & = \begin{cases} \beta\delta^c \frac{Y_j}{P_j} & \text{if } j = z \\ \beta\frac{\delta^c}{1-\delta^a}\frac{Y_j - P^a_{j,i}(a_{j,i})}{P_j} & \text{if } j \neq z \end{cases} \\ F^*_{j,i} & = \begin{cases} (1-\beta)\delta^c\frac{Y_j}{p_z} & \text{if } j = z \\ (1-\beta)\frac{\delta^c}{1-\delta^a}\frac{Y_j - P^a_{j,i}(a_{j,i})}{p_z} & \text{if } j \neq z \end{cases} \end{cases}
\end{aligned} \tag{1.95}$$

Finally, since $M_{j,i}$ is a quantity that CES-aggregates the consumption (in each place i of area j) of the varieties of the manufactured goods, $m_{kj,i}$, a third and last resolution stage consists of choosing the quantity of $m_{kj,i}$ which minimizes the cost of attaining $M^*_{j,i}$ regardless of its value, such that:

$$\min_{m_{.j,i}} \left(M^*_{j,i}P_j = \sum_{k=1}^{N} n_k p_{kj} m_{kj,i} \text{ s.t. } M^*_{j,i} = \left[\sum_{k=1}^{N} n_k (m_{kj,i})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \right) \tag{1.96}$$

Or put differently,

$$\min_{m_{.j,i},\mu^M_{j,i}} \left(\mathscr{L}^M_{j,i}(m_{.j,i},\mu^M_{j,i}) = \sum_{k=1}^{N} n_k p_{kj} m_{kj,i} + \mu^M_{j,i}\left(M^*_{j,i} - \left[\sum_{k=1}^{N} n_k (m_{kj,i})^{\gamma}\right]^{\frac{1}{\gamma}} \right)\right) \tag{1.97}$$

with $\gamma = (\varepsilon-1)/\varepsilon$ for the sake of clarity. From the first order conditions involved by $\min\limits_{m_{.j,i},\mu^M_{j,i}} \mathscr{L}^M_{j,i}$, it follows that

$$\frac{\partial \mathscr{L}^M_{j,i}}{\partial m_{.j,i}} = 0 \Rightarrow m_{lj,i} = M^*_{j,i} p_{lj}^{\frac{1}{\gamma-1}} \left[\sum_{k=1}^{N} n_k p_{kj}^{\frac{\gamma}{\gamma-1}}\right]^{-\frac{1}{\gamma}} \tag{1.98}$$

from which we can identify the j-price index of CES goods, P_j:

$$M^*_{j,i}P_j = M^*_{j,i}\left(\sum_{l=1}^{N} n_l p_{lj}^{\frac{\gamma}{\gamma-1}}\right)\left(\sum_{k=1}^{N} n_k p_{kj}^{\frac{\gamma}{\gamma-1}}\right)^{-\frac{1}{\gamma}} = M^*_{j,i}\underbrace{\left(\sum_{k=1}^{N} n_k p_{kj}^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}}}_{\boxed{\text{regional price index of CES goods}}} \tag{1.99}$$

which leads to clarify eq.(1.98) as

$$m^*_{kj,i} = \left(\frac{P_j}{p_{kj}}\right)^{\frac{1}{1-\gamma}} M^*_{j,i} = \begin{cases} \beta\delta^c \left(\frac{P_j}{p_{kj}}\right)^{\varepsilon} \frac{Y_j}{P_j} & \text{if } j = z \\ \frac{\beta\delta^c}{1-\delta^a} \left(\frac{P_j}{p_{kj}}\right)^{\varepsilon} \frac{Y_j - P^a_{j,i}}{P_j} & \text{if } j \neq z \end{cases}, \ \forall k \in [\![1;N]\!] \tag{1.100}$$

Evaluating the urban utility function at its demand solutions, and rearranging, yields:

$$u_{j,i} = u^0_{j,i} \left(\frac{\beta\delta^c}{P_j}\right)^{\beta\delta^c} \left(\frac{(1-\beta)\delta^c}{p_z}\right)^{(1-\beta)\delta^c} \left(\frac{\delta^H}{R_{j,i}}\right)^{\delta^H} \left(\frac{T}{d_{j,i}} v_{j,i}\right)^{\delta^a} \left(\frac{Y_j - P^a_{j,i}(a_{j,i})}{1-\delta^a}\right)^{1-\delta^a} \tag{1.101}$$

A more concise expression of eq.(1.101) is used in the chapter, rewritten as

$$u_{j,i} = u^0_{j,i} u^2_j R^{-\delta^H}_{j,i} \left(\frac{T}{d_{j,i}} v_{j,i}\right)^{\delta^a} \left(Y_j - P^a_{j,i}(a_{j,i})\right)^{1-\delta^a} \tag{1.102}$$

with $u^2_j = \left(\frac{\beta\delta^c}{P_j}\right)^{\beta\delta^c} \left(\frac{(1-\beta)\delta^c}{p_z}\right)^{(1-\beta)\delta^c} \left(\delta^H\right)^{\delta^H} \left(\frac{1}{1-\delta^a}\right)^{1-\delta^a}$
Another form is also used, which consists of rewriting eq.(1.102) as

$$u_{j,i} = u^0_{j,i} u^2_j \left(\frac{H^*_{j,i}}{L_{j,i}} \frac{1-\delta^a}{\delta^H}\right)^{\delta^H} \left(\frac{T}{d_{j,i}} v_{j,i}\right)^{\delta^a} \left(Y_j - P^a_{j,i}(a_{j,i})\right)^{1-\delta^a-\delta^H} \tag{1.103}$$

where $\frac{H^*_{j,i}}{L_{j,i}} = h^*_{j,i}$ internalizes in place i of agglomeration j the housing market equilibrium within the utility function.

1.B Transport modes' specified utility terms

The utility function is declined into two different versions according to which transport mode utility-maximizers choose. These two versions are obtained simply by substituting the terms related to transport cost, $P^a_{j,i}(a_{j,i})$, speed, $v_{j,i}$, and amenity term, $u^0_{j,i}$, for the corresponding ones. For PV-users, these substitutions, *e.g.* in eq.(1.103), lead to

$$u^{PV}_{j,i} = u^{0,PV}_{j,i} u^2_j \left(\frac{H^*_{j,i}}{L_{j,i}} \frac{1-\delta^a}{\delta^H}\right)^{\delta^H} \left(\frac{T}{d_{j,i}} v^{PV}_{j,i}\right)^{\delta^a} \left(Y_j - P^{a,PV}_{j,i}\right)^{1-\delta^a-\delta^H} \tag{1.104}$$

whereas for PT-users those lead to

$$u^{PT}_{j,i} = u^{0,PT}_{j,i} u^2_j \left(\frac{H^*_{j,i}}{L_{j,i}} \frac{1-\delta^a}{\delta^H}\right)^{\delta^H} \left(\frac{T}{d_{j,i}} v^{PT}_{j,i}\right)^{\delta^a} \left(Y_j - P^{a,PT}_{j,i}\right)^{1-\delta^a-\delta^H} \tag{1.105}$$

where $u^{0,PV}_{j,i}$ and $u^{0,PT}_{j,i}$ equal respectively to u^0_j and $u^0_j \left(\frac{\nu^{PV}_{j,i}}{\nu^{PT}_{j,i}}\right)^{\delta^a} \left(\frac{Y_j - P^{a,PV}_{j,i}}{Y_j - P^{a,PT}_{j,i}}\right)^{1-\delta^a}$ [see eq.(1.22) and eq.(1.23)].

In the case of workers having the choice between the two modes, the first principle of Wardrop (1952) is formalized from the equalization of the two transport modes specific utility functions, specified as in equations (1.104) and (1.105). This yields a transport-modes-indifference condition, according to which making the choice of one transport mode or the other is equivalent, as asserted below for a given place i of urban area j

$$u_{j,i}^{PV} = u_{j,i}^{PT} \Rightarrow u_j^0 \left(\upsilon_{j,i}^{PV}\right)^{\delta^a} \left(Y_j - P_{j,i}^{a,PV}\right)^{1-\delta^a} = u_{j,i}^{0,PT} \left(\upsilon_{j,i}^{PT}\right)^{\delta^a} \left(Y_j - P_{j,i}^{a,PT}\right)^{1-\delta^a} \tag{1.106}$$

which reduces to

$$\frac{\upsilon_{j,i}^{PV}}{\upsilon_{j,i}^{PT}} = \frac{\nu_{j,i}^{PV}}{\nu_{j,i}^{PT}} \tag{1.107}$$

1.C Website's variables descriptions

The large amount of data (used and) generated by the model are available online at https://gemse.alwaysdata.net for each country, for each scenario, for each variable, for each year, for each urban area as well as for each of its constitutive place. The goal is to provide readers with the possibility to apprehend and graphically check the coherence of the mechanisms that archetype the theoretical background of GEMSE. The following two tables map the names used on the website to the names used within this thesis.

Table 1.2: Global scale related variables

	webname	name	description	unit
1	CD p index	$I = \frac{\sum_r^{N+1} I_r L_r}{\sum_r^{N+1} L_r}$	Nation average cost of living index, also called perfect price index.	
2	CES elasticity	ε	Armington (1969)'s elasticity of substitution between heterogeneous goods.	
3	CO2eq price	$p^{CO_2^{eq}} = 10^3 \sum_{m\in\{PV,PT\}} s^{m,CO_2^{eq}} p_{\mathcal{D}_{\forall j}^m}^{m,CO}$	Nation transport-mode-weighted CO2eq social cost per ton having implicitly currency in the transport sector.	(\$|€)/ton
4	Hsqm tot	$H = \sum_j^N \sum_i^{N_j} H_{j,i}^*$	Nation total quantity of profit maximizing supplied square meters over urban areas.	m²
5	L	L	Nation number of workers.	
6	L HHI	HHI^L	Workers related Herfindahl-Hirschman Index.	
7	L growth	Δ^L	Nation annual growth of the number of workers.	
8	L pt	L^{PT}	Number of workers commuting in public transport.	
9	L pv	Λ	Number of workers commuting in private vehicle.	
10	T	T	Stable travel time budget of Zahavi and Talvities (1980) per business day.	min/bday

11	U	$u = \frac{\sum_r^{N+1} u_r L_r}{\sum_r^{N+1} L_r}$	Nation average workers utility.	
12	U0 pt	$u^{0,PT} = \frac{\sum_j^N u_j^{0,PT} L_j}{\sum_j^{N+1} L_j}$	Nation average urban public transport amenity.	
13	W	$W = \frac{\sum_r^{N+1} W_r \mathcal{P}_r}{\sum_r^{N+1} \mathcal{P}_r}$	Nation average labor forces welfare.	
14	act per pop ratio	$s^{\mathcal{P}}$	Employable share of the population.	
15	av bdays	b_d	Number of business days implied to get the annual number of browsed kilometers consistent with data.	
16	beta het	β	Share of δ^C attributed to consumption of heterogeneous goods.	
17	beta hom	$(1-\beta)$	Share of δ^C attributed to consumption of homogeneous goods.	
18	c fuel	c_a	Average consumption of liters of fuel per km.	liters/km
19	const surface	$S = \sum_j^N \sum_i^{N_j} S_{j,i}$	Nation total urban constructible surfaces.	km²
20	delta A	δ^a	Share of income attributed to mobility services.	
21	delta C	δ^C	Share of income attributed to consumption of goods.	
22	delta H	δ^H	Share of income attributed to housing services.	
23	delta HA	$(\delta^H + \delta^a)$	Share of income attributed to land/transport services.	
24	emp rate	$\widetilde{E}$	Nation employment rate.	
25	gdp	$\widetilde{\mathcal{Y}}$	Nation gross domestic product.	(\$|€)/year
26	gdp CO2 intensity	$\mathrm{d}q^{CO_2^{eq}}/\widetilde{\mathcal{Y}}$	Nation CO2 intensity of the gross domestic product.	kg/(\$|€)/year

27	gdp HHI	$\mathrm{HHI}^{\widetilde{\mathcal{Y}}}$	GDP related Herfindahl-Hirschman Index.	
28	gdp adj F	$1+\Delta\widetilde{\mathcal{Y}}$	Nation income average growth factor.	
29	gdp per L	$\widetilde{\mathcal{Y}}/L$	Nation gross domestic product per nation worker.	(\$|€)/worker/year
30	gup	$\mathcal{Y}_{\forall j}=\sum_j^N \mathcal{Y}_j$	Nation gross urban product.	(\$|€)/year
31	gup HHI	$\mathrm{HHI}^{\mathcal{Y}_{\forall j}}$	GUP related Herfindahl-Hirschman Index.	
32	h budget	$Y^H=\frac{\sum_j^N \sum_i^{N_j} L_{j,i}h_{j,i}R_{j,i}}{L}$	Nation average housing budget over urban areas.	(\$|€)/worker/year
33	implicit tr speed unit value	$\xi_{\mathrm{km/min}}$	Nation km/min value over urban areas.	(\$|€)/km/min
34	inst cap flow	$\mathrm{d}\chi^*=\sum_j^N \sum_i^{N_j} \mathrm{d}\chi_{j,i}^*$	Nation total developers installed flow of capital.	unit
35	inv	$K=\sum_r^{N+1} K_r$	Nation total capital invested in the Cobb-Douglas good sector.	(\$|€)
36	inv adj F	$1+\Delta^K$	Signal multiplying areas investment needed for in the Cobb-Douglas sector.	
37	knowledge spillover	η	Ostbye (2010)'s exogenous spillover reducing cost à la Brakman (1996).	
38	min wages	w^{min}	Nation minimum possible wages on average, i.e. nation wage curve intercept.	(\$|€)/worker/year
39	min wages adj F	Δ^w	Factor adjusting areas minimum wages, which redefines labor markets negociation terms.	
40	nb firms	n	Nation number of firms.	
41	nb firms HHI	HHI^n	Firms related Herfindahl-Hirschman Index.	
42	nb firms growth	Δ^K	Nation growth rate of firms.	

43	p HM	R_z	Price of transport/housing goods in rural area.	(\$|€)/unit
44	p fuel	p_a	Domestic fuel price per liter.	(\$|€)/liter
45	pop	$\mathcal{P}/s_{\mathcal{P}}$	Nation population.	
46	pop act	$\mathcal{P}$	Nation active population or labor force (unemployed+employed).	
47	pop act HHI	$\text{HHI}^{\mathcal{P}}$	Labor force related Herfindahl-Hirschman Index.	
48	pop growth	$\Delta^{\mathcal{P}}$	Nation annual population growth.	
49	production	$q = q_z + \sum_j^N n_j q_j$	Nation total produced quantity of heterogeneous and homogeneous goods.	unit
50	pt CO2eq flow	$\mathrm{d}q^{PT,CO_2^{eq}} = \sum_j^N \mathrm{d}q_j^{PT,CO_2^{eq}}$	Urban areas public transport CO2eq flows.	kg/year
51	pt CO2eq flow per Lkm	$\mathrm{d}q_{\mathcal{D}^{PT}}^{PT,CO_2^{eq}} = \mathrm{d}q^{PT,CO_2^{eq}}/\mathcal{D}^{PT}$	Average consumption of kilograms of CO2eq imputed per km browsed per public transport user.	kg/pass/year/km
52	pt CO2eq price per Lkm	$p_{\mathcal{D}_{\forall j}^{PT}}^{PT,CO_2^{eq}} = \frac{\sum_j^N p_{j,\mathcal{D}^{PT}}^{PT,CO_2^{eq}} \mathrm{d}q_j^{PT,CO_2^{eq}}}{\mathrm{d}q^{PT,CO_2^{eq}}}$	Nation average CO2eq social cost imputed per km browsed per public transport user.	(\$|€)/pass/km
53	pt CO2eq stock	$q^{PT,CO_2^{eq}} = \sum_j^N q_j^{PT,CO_2^{eq}}$	Urban areas public transport CO2eq stocks.	kg
54	pt Lkm	$\mathcal{D}^{PT} = b_d T \sum_j^N \sum_i^{N_j} v_{j,i}^{PT} L_{j,i}^{PT}$	Nation annual distance browsed in urban areas by workers in public transport.	km/year
55	pt budget	$Y^{a,PT} = \sum_j^N \frac{\sum_i^{N_j} L_{j,i}^{PT} P_{j,i}^{a,PT}}{L_j^{PT}}$	Nation average annual public transport budget.	(\$|€)/worker/year
56	pt budget growth	$\Delta^{P^{a,PT}}$	Nation annual public transport cost growth.	
57	pt round trips	$b_d a^{PT}$	Nation average number of back and forths in public transport.	1/worker/year

58	pt round trips per day	$a^{PT} = \sum_j^N \frac{\sum_i^{N_j} a_{j,i}^{PT} L_{j,i}}{L_j}$	Nation average number of back and forths per business day in public transport.	1/worker/bday
59	pt speed	$v^{PT} = \sum_j^N \frac{\sum_i^{N_j} v_{j,i}^{PT} L_{j,i}^{PT}}{L_j^{PT}}$	Nation average public transport speed over urban areas.	km/min
60	pv CO2eq flow	$\mathrm{d}q^{PV,CO_2^{eq}} = \sum_j^N \mathrm{d}q_j^{PV,CO_2^{eq}}$	Urban areas private vehicle CO2eq flows.	kg/year
61	pv CO2eq flow per Lkm	$\mathrm{d}q_{\mathcal{D}^{PV}}^{PV,CO_2^{eq}} = \mathrm{d}q^{PV,CO_2^{eq}}/\mathcal{D}^{PV}$	Average consumption of kilograms of CO2eq imputed per km browsed per private vehicle user.	kg/pass/year/km
62	pv CO2eq fshare	$s^{PV,CO_2^{eq}} = \mathrm{d}q^{PV,CO_2^{eq}}/\mathrm{d}q^{CO_2^{eq}}$	Nation share of CO2eq flows per user stemming from private vehicles.	
63	pv CO2eq price per Lkm	$p_{\mathcal{D}_{\forall j}^{PV}}^{PV,CO_2^{eq}} = \frac{\sum_j^N p_{j,\mathcal{D}^{PV}}^{PV,CO_2^{eq}} \mathrm{d}q_j^{PV,CO_2^{eq}}}{\mathrm{d}q^{PV,CO_2^{eq}}}$	Nation average CO2eq social cost imputed per km browsed per private vehicle user.	(\$|€)/pass/km
64	pv CO2eq rev	$Y^{CO_2^{eq}} = \sum_j^N Y_j^{CO_2^{eq}}$	Nation income generated by the kilometric CO2eq tax.	(\$|€)/year
65	pv CO2eq stock	$q^{PV,CO_2^{eq}} = \sum_j^N q_j^{PV,CO_2^{eq}}$	Urban areas private vehicle CO2eq stocks.	kg
66	pv Lkm	$\mathcal{D}^{PV} = b_d T \sum_j^N \sum_i^{N_j} v_{j,i}^{PV} \Lambda_{j,i}$	Nation annual distance browsed in urban areas by workers in private vehicle.	km/year
67	pv budget	$P^{a,PV}$	Nation average annual private vehicle budget.	(\$|€)/worker/year
68	pv free speed	$v^{0,PV} = \sum_j^N \frac{\sum_i^{N_j} v_{j,i}^{0,PV} \Lambda_{j,i}}{\Lambda_j}$	Nation average private vehicle free speed over urban areas.	km/min
69	pv km per L	$\mathcal{D}^{PV}/\Lambda$	Nation annual distance browsed in urban areas per worker in private vehicle.	km/pass/year
70	pv mshare	$\alpha = \Lambda/L$	Nation average cars modal share.	
71	pv round trips	$b_d a^{PV}$	Nation average accessibility in private vehicle.	1/worker/year

72	pv round trips per day	$a^{PV} = \sum_j^N \frac{\sum_i^{N_j} a_{j,i}^{PV} L_{j,i}}{L_j}$	Nation average accessibility in private vehicle.	1/worker/bday
73	pv speed	$v^{PV} = \sum_j^N \frac{\sum_i^{N_j} v_{j,i}^{PV} \Lambda_{j,i}}{\Lambda_j}$	Nation average private vehicle speed over urban areas.	km/min
74	real gdp	$\widetilde{\mathcal{Y}}/I$	Nation real gross domestic product.	($|€)/year
75	real gdp CO2 intensity	$\mathrm{d}q^{CO_2^{eq}} I/\widetilde{\mathcal{Y}}$	Nation CO2 intensity of the real gross domestic product.	kg/unit/year
76	real gdp per L	$\widetilde{\mathcal{Y}}/L/I$	Real nation gross domestic product per nation worker.	units/worker/year
77	real wages	w/I	Nation average real wages.	units/worker/year
78	roads capacity utilisation	$\sum_j^N \sum_i^{N_j} L_{j,i} \frac{(\mathbf{\Pi}_j \mathbf{\Lambda}_j)_{j,i}}{\mathcal{K}_{j,i}} / L_j$	Nation average roads capacity utilisation rate.	
79	roads infra value	NPV^a	Nation total value of roads considered in urban areas between places and their j-CBD.	($|€)
80	roads infra value per km	NPV^a/d	Roads infrastructures value per km over all urban areas.	($|€)/km
81	roads km	$d = \sum_j^N d_j$	Nation unique kilometers of roads considered between places and their respective j-CBD.	km
82	roads km per mn	$v = d/t$	Nation average travel speed on roads considered between places and their respective j-CBD.	km/min
83	roads mn	$t = \sum_j^N t_j$	Nation unique unique travel time on roads considered between places and their respective j-CBD.	min
84	tau elasticity	$\nabla^{\boldsymbol{\tau}}$	Oil price to shipment cost elasticity of tradable goods.	
85	tr CO2eq flow	$\mathrm{d}q^{CO_2^{eq}} = \sum_j^N \mathrm{d}q_j^{CO_2^{eq}}$	Urban areas CO2eq flows over modes.	kg/year

86	tr CO2eq flow HHI	$\mathrm{HHI}^{\mathrm{d}q^{CO_2^{eq}}}$	Transport CO2eq emissions related Herfindahl-Hirschman Index..	
87	tr CO2eq stock	$q^{CO_2^{eq}} = \sum_j^N q_j^{CO_2^{eq}}$	Urban areas CO2eq stocks over modes.	kg
88	tr CO2eq stock HHI	$\mathrm{HHI}^{q^{CO_2^{eq}}}$	Transport CO2eq emissions related Herfindahl-Hirschman Index..	
89	tr Lkm	$\mathcal{D} = \mathcal{D}^{PV} + \mathcal{D}^{PT}$	Nation (over modes) average annual browsed distance by workers.	km/year
90	tr budget	$Y^a = \sum_j^N Y_j^a L_j/L$	Nation average transport budget per worker over modes.	(\$|€)/worker/year
91	tr fuel exp	$Y^f = c_a p_a(\mathcal{D}^{PV} + \mathcal{D}^{PT}/o^{PT})$	Nation transport expenditures in liters of fuel.	(\$|€)/year
92	tr round trips	$a = \sum_j^N \frac{\sum_i^{N_j} L_{j,i}(\alpha_{j,i}a_{j,i}^{PV} + (1-\alpha_{j,i})a_{j,i}^{PT}}{L_j}$	Nation average number of back and forths over modes.	1/worker/year
93	tr round trips per day	a/b_d	Nation average number of back and forths per business day over modes.	1/worker/bday
94	uinv	$(k_z q_z + \sum_j^N \varkappa_j n_j)/q$	Nation total stock of invested capital per unit produced in the Cobb-Douglas good sector.	(\$|€)/unit
95	ulr	$l = \sum_r^{N+1} l_r L_r/L$	Nation average unitary labor requirement for production.	workers/unit
96	ulr adj F	Δ^l	Exogenous signal multiplying areas unitary labor requirement.	
97	urb L HHI	$\mathrm{HHI}^{L_{j=1,\ldots,N}}$	Urban workers related Herfindahl-Hirschman Index.	
98	urb ROE C	$r = \sum_j^N r_j n_j/n$	Nation average urban return on equity of the CES-sector.	

99	urb firms spov ucost	$\frac{\sum_{k=1}^{N} n_k^{\eta} L_k}{\sum_{k=1}^{N} L_k}$	Nation average knowledge spillover effect unitary-production-cost divider.	
100	urb firms ucost	c_j^L	Nation average urban unitary production cost.	(\$|€)/unit/year
101	urb het production	$\mathcal{Q}_{\forall j} = \sum_j^N n_j q_j$	Nation total produced quantity of heterogeneous goods.	unit
102	urb income	$\sum_{k=1}^{N} n_k p_k q_k$	Nation total produced value of heterogeneous goods.	(\$|€)/year
103	urb income HHI	$\mathrm{HHI}^{\sum_j^N n_j p_j q_j}$	Firms' income related Herfindahl-Hirschman Index.	
104	urb inv per firm	$\varkappa$	Non space-differentiated fixed input requirement of capital per firm.	(\$|€)/firm
105	urb pop act HHI	$\mathrm{HHI}^{\mathcal{P}_{\forall j}}$	Urban labor force related Herfindahl-Hirschman Index.	
106	urban cost	$Y^{HA} = \sum_j^N \sum_i^{N_j} L_{j,i}(Y_{j,i}^H + Y_{j,i}^a)/L_j$	Nation average urban cost over urban areas.	(\$|€)/worker/year
107	wages	$w = \frac{\sum_r^{N+1} w_r L_r}{L}$	Nation average wages.	(\$|€)/worker/year
108	wages elasticity	σ	Wages to unemployment elasticity.	

Table 1.3: Inter-areas scale related variables

	webname	name	description	unit
1	CD p index js	$I_r = \frac{Y_r}{u_r}$ for $r = 1, ..., N+1$	All areas cost of living index, also called perfect price index.	
2	CES p index js	$P_r = \left(\sum_{k=1}^{N} n_k p_{kr}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ for $r = 1, ..., N+1$	All areas index of the cost of consuming manufactured goods.	
3	CO2eq price js	$p_j^{CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas CO2eq price per ton.	(\$|€)/year
4	Hprices psqm js	R_j/r_H for $j = 1, ..., N$	All urban areas average housing prices per square meter.	(\$|€)/m²
5	Hrents psqm js	R_j for $j = 1, ..., N$	All urban areas average annual housing rents per square meter.	(\$|€)/year/m²

6	Hsqm js	$h_j = \sum_i^{N_j} L_{j,i} h_{j,i} / L_j$ for $j = 1, ..., N$	All urban areas average (over j-places) housing demand in j-agglomeration.	m^2/worker
7	Hsqm tot js	$H_j^* = \sum_i^{N_j} H_{j,i}^*$ for $j = 1, ..., N$	All urban areas total quantity of supplied square meters.	m^2
8	L growth js	Δ_j^L for $j = 1, ..., N$	All areas growth of the number of workers.	
9	L js	L_j for $j = 1, ..., N$	All areas number of workers.	
10	L pt js	L_j^{PT} for $j = 1, ..., N$	All urban areas numbers of workers commuting in public transport.	
11	L pv js	Λ_j for $j = 1, ..., N$	All urban areas numbers of workers commuting in private vehicle.	
12	L share js	$s_r^{L \forall r}$ for $r = 1, ..., N+1$	All areas share of the nation number of workers.	
13	ROE C js	r_r for $r = 1, ..., N+1$	All areas Cobb-Doulgas sector investment returns.	
14	ROE C rank js		All areas attractiveness rank for firms regarding local CES-sector returns on equity.	
15	U0 pt js	$u_j^{0,PT}$ for $j = 1, ..., N$	All urban areas average public transport amenity.	
16	U js	u_j for $j = 1, ..., N$	All areas workers utility.	
17	W growth js	Δ_j^W for $j = 1, ..., N$	All areas labor forces welfare growth.	
18	W js	W_j for $j = 1, ..., N$	All areas labor forces welfare.	
19	W rank js		All areas labor forces welfare attractiveness rank.	
20	Y js	Y_r for $r = 1, ..., N+1$	All areas income per worker.	(\$|€)/worker/year
21	const surface js	$S_j = \sum_i^{N_j} S_{j,i}$ for $j = 1, ..., N$	All urban areas constructible surfaces.	km^2
22	density js	$\sum_i^{N_j} \mathcal{P}_{j,i} / s_{\mathcal{P}} / \sum_i^{N_j} \mathrm{S}_{j,i}$ for $j = 1, ..., N$	All urban areas density.	pop/km^2
23	emp rate js	E_j for $j = 1, ..., N$	All areas employment rate.	

24	firms net migration js	$\mathrm{d}n_j$ for $j = 1, ..., N$	All areas' net migration of firms.	
25	gdp js	$\mathcal{Y}_r$ for $r = 1, ..., N+1$	All areas gdp.	(\$|€)/year
26	gdp share js	$s_r^{\widetilde{\mathcal{Y}}} = \mathcal{Y}_r / \sum_k^{N_k+1} \mathcal{Y}_k$ for $r = 1, ..., N+1$	All areas respective gdp share.	
27	gup CO2 intensity js	$\mathrm{d}q_j^{CO_2^{eq}} / \widetilde{\mathcal{Y}_j}$ for $j = 1, ..., N$	All urban areas gross urban product CO2 intensity.	kg/(\$|€)/year
28	gup js	$\mathcal{Y}_j$ for $j = 1, ..., N$	All urban areas gross urban product.	(\$|€)/year
29	gup share js	$s_j^{\mathcal{Y}_{\forall j}} = \mathcal{Y}_j / \sum_k^{N_k} \mathcal{Y}_k$ for $j = 1, ..., N$	All urban areas respective share of the nation's gross urban product.	
30	h budget js	$Y_j^H = \frac{\sum_i^{N_j} L_{j,i} h_{j,i} R_{j,i}}{L_j}$ for $j = 1, ..., N$	All urban areas average housing budget.	(\$|€)/worker/year
31	iceberg melting index js	$\left(\left(\sum_{k=1}^{N} n_k \tau_{kr}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)_r$ for $r = 1, ..., N+1$	All areas index of iceberg-melting.	
32	implicit tr speed unit value js	$\xi_{\mathrm{km/min},j}$ for $j = 1, ..., N$	All urban areas km/min value.	(\$|€)/km/min
33	inst cap flow js	$\mathrm{d}\chi_j^* = \sum_i^{N_j} \mathrm{d}\chi_{j,i}^*$ for $j = 1, ..., N$	All urban areas total developers installed flow of capital.	unit
34	intra connection js	$\sum_i^{N_j} (\mathbf{\Pi}_j \mathbf{1}_{N_j \times 1})_{j,i} / N_j$ for $j = 1, ..., N$	All urban areas intraconnection index.	
35	inv js	K_r for $r = 1, ..., N+1$	All areas total capital invested in the Cobb-Douglas good sector.	(\$|€)
36	min wages js	w_r^{min} for $r = 1, ..., N+1$	All areas minimum wages, i.e. all areas wage curve intercepts.	(\$|€)/worker/year
37	nb firms growth js	Δ_j^K for $j = 1, ..., N$	All urban areas growth rate of the number of firms.	
38	nb firms js	n_j for $j = 1, ..., N$	All urban areas number of firms.	
39	nb firms share js	$s_j^n = n_j / \sum_k^{N_k} n_k$ for $j = 1, ..., N$	All urban areas nation share of the number of firms.	
40	nb floors js	$f_j = H_j / S_j$ for $j = 1, ..., N$	All urban average number of residential floors.	
41	p js	p_r for $r = 1, ..., N+1$	All areas specific good price.	(\$|€)/unit

42	pass per pt js	$o_j^{PT} = o^{PT}$ for $j = 1, ..., N$	All urban areas number of workers per public transport.	pass/pt
43	pass per pv js	$o_j^{PV} = o^{PV}$ for $j = 1, ..., N$	All urban areas number of workers per private vehicle.	pass/pv
44	pop act js	$\mathcal{P}_r$ for $r = 1, ..., N+1$	All areas active population.	
45	pop act share js	$s_r^{\mathcal{P}} = \mathcal{P}_r / \sum_k^{N_k+1} \mathcal{P}_k$ for $r = 1, ..., N+1$	All areas share of the nation labor force.	
46	pop growth js	$\Delta_r^{\mathcal{P}}$ for $r = 1, ..., N+1$	All areas population growth.	
47	production js	for $r = 1, ..., N + 1, \begin{cases} n_r q_r & \text{if } r \neq z \\ q_r & \text{otherwise} \end{cases}$	All areas total produced quantity of heterogeneous and homogeneous goods.	unit
48	productivity js	q_r / L_r for $r = 1, ..., N+1$	All areas labor productivity.	units/worker
49	pt CO2eq flow js	$\mathrm{d}q_j^{PT,CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas public transport CO2eq flows.	kg/year
50	pt CO2eq price per Lkm js	$p_{j,\mathcal{D}^{PT}}^{PT,CO_2^{eq}} = p_j^{CO_2^{eq}} 10^{-3} \mathrm{d}q_{\mathcal{D}^{PT}}^{PT,CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas CO2eq social cost imputed per km browsed per public transport user.	(\$|€)/pass/km
51	pt CO2eq stock js	$q_j^{PT,CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas public transport CO2eq stocks.	kg
52	pt Lkm js	$\mathcal{D}_j^{PT} = b_d T \sum_i^{N_j} v_{j,i}^{PT} L_{j,i}^{PT}$ for $j = 1, ..., N$	All urban areas annual distance browsed by workers in public transport.	km/year
53	pt budget js	$Y_j^{a,PT} = \frac{\sum_i^{N_j} L_{j,i}^{PT} P_{j,i}^{a,PT}}{L_j^{PT}}$ for $j = 1, ..., N$	All urban areas average public transport budget.	(\$|€)/worker/year
54	pt round trips js	$b_d a_j^{PT}$ for $j = 1, ..., N$	All urban areas average number of back and forths in public transport.	1/worker/year
55	pt round trips per day js	$a_j^{PT} = \frac{\sum_i^{N_j} a_{j,i}^{PT} L_{j,i}}{L_j}$ for $j = 1, ..., N$	All urban areas average number of back and forths per business day in public transport.	1/worker/bday
56	pt speed js	$v_j^{PT} = \frac{\sum_i^{N_j} v_{j,i}^{PT} L_{j,i}^{PT}}{L_j^{PT}}$ for $j = 1, ..., N$	All urban areas average public transport speed.	km/min
57	pv CO2eq flow js	$\mathrm{d}q_j^{PV,CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas private vehicle CO2eq flows.	kg/year

58	pv CO2eq price per Lkm js	$p_{j,\mathcal{D}^{PV}}^{PV,CO_2^{eq}} = p_j^{CO_2^{eq}} 10^{-3} \mathrm{d}q_{\mathcal{D}^{PV}}^{PV,CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas CO2eq social cost imputed per km browsed per private vehicle user.	($|€)/pass/km
59	pv CO2eq rev js	$Y_j^{CO_2^{eq}} = \mathcal{D}_j^{PV} p_{j,\mathcal{D}^{PV}}^{PV,CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas income generated by the kilometric CO2eq tax.	($|€)/year
60	pv CO2eq stock js	$q_j^{PV,CO_2^{eq}}$ for $j = 1, ..., N$	All urban areas private vehicle CO2eq stocks.	kg
61	pv Lkm js	$\mathcal{D}_j^{PV} = b_d T \sum_i^{N_j} v_{j,i}^{PV} \Lambda_{j,i}$ for $j = 1, ..., N$	All urban areas annual distance browsed by workers in private vehicle.	km/year
62	pv budget js	$Y_j^{a,PV} = \frac{\sum_i^{N_j} \Lambda_{j,i} P_{j,i}^{a,PV}}{\Lambda_j}$ for $j = 1, ..., N$	All urban areas average private vehicle budget.	($|€)/worker/year
63	pv free speed js	$v_j^{0,PV} = \frac{\sum_i^{N_j} v_{j,i}^{0,PV} \Lambda_{j,i}}{\Lambda_j}$ for $j = 1, ..., N$	All urban areas average private vehicle free speed.	km/min
64	pv mshare js	$\alpha_j = \Lambda_j / L_j$ for $j = 1, ..., N$	All urban areas average modal share of private vehicle.	
65	pv round trips js	$b_d a_j^{PV}$ for $j = 1, ..., N$	All urban areas average number of back and forths in private vehicle.	1/worker/year
66	pv round trips per day js	$a_j^{PV} = \frac{\sum_i^{N_j} a_{j,i}^{PV} L_{j,i}}{L_j}$ for $j = 1, ..., N$	All urban areas average number of back and forths per business day in private vehicle.	1/worker/bday
67	pv speed js	$v_j^{PV} = \frac{\sum_i^{N_j} v_{j,i}^{PV} \Lambda_{j,i}}{\Lambda_j}$ for $j = 1, ..., N$	All urban areas average private vehicle speed.	km/min
68	real Y js	Y_r / I_r for $r = 1, ..., N+1$	All areas real income per worker.	units/worker/year
69	real gdp js	$\mathcal{Y}_r / I_r$ for $r = 1, ..., N+1$	All areas real gross urban product.	($|€)/year
70	real gup CO2 intensity js	$\mathrm{d}q_j^{CO_2^{eq}} I_j / \widetilde{\mathcal{Y}_j}$ for $j = 1, ..., N$	All urban areas real gross urban product CO2 intensity.	kg/unit/year
71	real gup js	$\mathcal{Y}_j / I_j$ for $j = 1, ..., N$	All urban areas real gross urban product.	($|€)/year
72	real wages js	w_r / I_r for $r = 1, ..., N+1$	All areas real wages.	units/worker/year
73	roads capacity utilisation js	$\sum_i^{N_j} L_{j,i} \frac{(\mathbf{\Pi}_j \mathbf{\Lambda}_j)_{j,i}}{\mathcal{K}_{j,i}} / L_j$ for $j = 1, ..., N$	All urban areas roads capacity utilisation rates.	
74	roads infra value js	NPV_j^a for $j = 1, ..., N$	All urban areas roads infrastructures value.	($|€)

75	roads infra value per km js	NPV^a_j/d_j for $j=1,...,N$	All urban areas roads infrastructures value per kilometer.	(\$|€)/km
76	roads km js	d_j for $j=1,...,N$	All urban areas unique kilometers of roads considered between places and their respective j-CBD.	km
77	roads km per mn js	$v_j=d_j/t_j$ for $j=1,...,N$	All urban areas average travel speed on roads considered between places and their respective j-CBD.	km/min
78	roads mn js	t_j for $j=1,...,N$	All urban areas unique travel time on roads considered between places and their respective j-CBD.	min
79	tr CO2eq flow js	$\mathrm{d}q_j^{CO_2^{eq}}$ for $j=1,...,N$	All urban areas CO2eq flows over modes.	kg/year
80	tr CO2eq flow share js	$\mathrm{d}q_j^{CO_2^{eq}}/\mathrm{d}q^{CO_2^{eq}}$ for $j=1,...,N$	All urban areas respective share of nation's transport related CO2eq emissions.	
81	tr CO2eq stock js	$q_j^{CO_2^{eq}}$ for $j=1,...,N$	All urban areas CO2eq stocks over modes.	kg
82	tr CO2eq stock share js	$q_j^{CO_2^{eq}}/q^{CO_2^{eq}}$ for $j=1,...,N$	All urban areas respective share of nation's transport related CO2eq emissions.	
83	tr Lkm js	$\mathcal{D}_j=\mathcal{D}_j^{PV}+\mathcal{D}_j^{PT}$ for $j=1,...,N$	All urban areas annual distance browsed by workers on average (over modes).	km/year
84	tr budget js	$Y_j^a=(1-\alpha_j)Y_j^{a,PT}+\alpha_jY_j^{a,PV}$ for $j=1,...,N$	All urban areas average transport budget per worker over modes.	(\$|€)/worker/year
85	tr budget tot js	$\widetilde{Y_j^a}$ for $j=1,...,N$	All urban areas total average transport budgets over modes.	(\$|€)/year
86	tr fuel exp js	$Y_j^f=c_ap_a(\mathcal{D}_j^{PV}+\mathcal{D}_j^{PT}/o^{PT})$ for $j=1,...,N$	All urban areas transport expenditures in liters of fuel.	(\$|€)/year

87	tr round trips js	$a_j = \frac{\sum_i^{N_j} L_{j,i}(\alpha_{j,i} a_{j,i}^{PV} + (1-\alpha_{j,i}) a_{j,i}^{PT})}{L_j}$ fo 1, ..., N	All urban areas average number of back and forths over modes.	1/worker/year
88	tr round trips per day js	a_j / b_d for $j = 1, ..., N$	All urban areas average number of back and forths per business day over modes.	1/worker/bday
89	uinv js	for $r = 1, ..., N +$ 1, $\begin{cases} k_r & \text{if } r \neq z \\ \varkappa_r n_r / q_r & \text{otherwise} \end{cases}$	All areas total capital invested per produced unit in the Cobb-Douglas good sector.	(\$|€)/unit
90	ulr js	l_r for $r = 1, ..., N + 1$	All areas unitary labor requirement for production.	workers/unit
91	urb L share js	$s_j^{L_{\forall j}}$ for $j = 1, ..., N$	All urban areas share of the nation urban number of workers.	
92	urb firms cost js	$c_j^L q_j$ for $j = 1, ..., N$	All urban areas total production costs per firm.	(\$|€)/firm/year
93	urb firms income js	$p_j q_j$	All urban areas income per firm from the heteorogeneous production.	(\$|€)/firm/year
94	urb firms profits js	$p_j q_j / \varepsilon$ for $j = 1, ..., N$	All urban areas profits per firm.	(\$|€)/firm/year
95	urb firms spov ucost js	n_j^{η} for $j = 1, ..., N$	All urban areas knowledge spillover effect unitary-production-cost divider.	
96	urb firms ucost js	c_j^L for $j = 1, ..., N$	All urban areas unitary production cost.	(\$|€)/unit/year
97	urb het firms production js	q_j	All urban areas produced quantity per firm.	unit
98	urb het production js	$n_j q_j$ for $j = 1, ..., N$	All urban areas produced quantity.	unit
99	urb het production share js	$s_j^{Q_{\forall j}} = n_j q_j / \sum_k^N n_k q_k$ for $j = 1, ..., N$	All urban areas shares of the nation heterogeneous produced quantity.	
100	urb het spendings js	$\beta \delta^C / (1 - \delta^a) \mathcal{S}_j$ for $j = 1, ..., N$	All urban areas total spendings in consumption of heterogenous goods.	(\$|€)/year
101	urb hom spendings js	$(1 - \beta) \delta^C / (1 - \delta^a) \mathcal{S}_j$ for $j = 1, ..., N$	All urban areas total spendings in consumption of homogeneous goods.	(\$|€)/year

102	urb income js	$n_j p_j q_j$ for $j = 1, ..., N$	All urban areas income from the heteorogeneous production.	(\$|€)/year
103	urb income share js	$n_j p_j q_j / \sum_k^N n_k p_k q_k$ for $j = 1, ..., N$	All urban areas share of the total produced value of heterogeneous goods.	
104	urb netTr Y js	$(Y_j - Y_j^a)$ for $j = 1, ..., N$	All urban areas income net of transport costs per worker.	(\$|€)/worker/year
105	urb pop act share js	$s_j^{\mathcal{P}_{\forall j}} = \mathcal{P}_j / \sum_k^{N_k} \mathcal{P}_k$ for $j = 1, ..., N$	All urban areas share of the nation urban labor force.	
106	urb spendings js	$\mathcal{S}_j = \sum_i^{N_j} (Y_j - P_{j,i}^a) L_{j,i}$ for $j = 1, ..., N$	All urban areas total spendings in consumption of goods.	(\$|€)/year
107	urban cost js	$Y_j^{HA} = \sum_i^{N_j} L_{j,i} (Y_{j,i}^H + Y_{j,i}^a) / L_j$ for $j = 1, ..., N$	All average urban costs.	(\$|€)/worker/year
108	wages js	w_r for $r = 1, ..., N + 1$	All areas wages.	(\$|€)/worker/year

References

Allio, C. (2016). “Interurban population distribution and commute modes”. *The Annals of Regional Science* 1991.

Alonso, W. (1964). *Location and land use: toward a general theory of land rent.* Harvard University Press, p. 204.

Behrens, K. and F. Robert-Nicoud (2011). “Tempora mutantur: In search of a new testament for NEG”. *Journal of Economic Geography* 11.2, pp. 215–230.

Blanchflower, D. G. and A. J. Oswald (1995). “The Wage Curve”. *Journal of Economic Literature* 33.2, pp. 785–799.

Bosi, S., E. Iliopulos and H. Jayet (2010). “Optimal Immigration Policy When the Public Good Is Rival”.

Brakman, S., H. Garretsen, R. Gigengack, C. van Marrewijk and R. Wagenvoort (1996). *Negative Feedbacks in the Economy and Industrial Location*.*

Dixit, A. K. and J. E. Stiglitz (1977). “Monopolistic Competition and Optimum Product Diversity”. *The American Economic Review* 67.3, pp. 297–308.

Fujita, M., P. R. Krugman and A. Venables (1999b). *The Spatial Economy: Cities, Regions, and International Trade.* MIT Press, p. 367.

Gaspar, M. J. L. (2017). “New Economic Geography: Perspectives, multiple regions and individual heterogeneity”. PhD thesis, p. 182.

Geroliminis, N. and C. F. Daganzo (2008). “Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings”. *Transportation Research Part B: Methodological* 42.9, pp. 759–770.

Hungerland, W.-F. (2017). “The Gains from Import Variety in Two Globalisations: Evidence from Germany”. *Institute of Economic History, School of Business and Economics, Humboldt-University of Berlin*, mimeo.

Krugman, P. (1991). “Increasing Returns and Economic Geography”. *Journal of Political Economy* 99.3, pp. 483–499.

Martin, P. and C. A. Rogers (1995). “Industrial location and public infrastructure”. *Journal of International Economics* 39.3–4, pp. 335–351.

McNally, M. G. (2008). “The Four-Step Model”. *Handbook of Transport Modelling.* Ed. by David A. Hensher and Kenneth J. Button, pp. 35–52.

Mills, E. S. (1967). “An Aggregative Model of Resource Allocation in a Metropolitan Area”. *The American Economic Review* 57.2, pp. 197–210.

Muth, R. F. (1961). “The spatial structure of the housing market”. *Papers of the Regional Science Association* 7.1, pp. 207–220.

Muth, R. F. (1969). *Cities and Housing: The Spatial Pattern of Urban Residential Land Use.* University of Chicago Press, p. 355.

Østbye, S. (2010). *Knowledge spillovers in spatial equilibrium.* Tech. rep. Mimeo, University of Tromsø.

Ottaviano, G. I. P. and P. Martin (2001). "Growth and Agglomeration". *International Economic Review* 42.4, pp. 947–968.

Samuelson, P. A. (1952). "The Transfer Problem and Transport Costs: The Terms of Trade When Impediments are Absent". *The Economic Journal* 62.246, pp. 278–304.

Wardrop, J. G. (1952). "Road Paper. Some theoretical aspects of road traffic research." en. *Proceedings of the Institution of Civil Engineers.*

Wilson, A. G. (1970). *Entropy in Urban and Regional Modelling.* London : Pion, p. 166.

Zahavi, Y. and A. Talvitie (1980). "Regularities in travel time and money expenditures". *59th Annual Meeting of the Transportation Research Board.* 750.

Chapter 2

Data and Calibration

Contents

2.1 Data **67**

2.1.1 USA's and France's data-related commonalities 67

2.1.2 American data 68

2.1.3 French data 71

2.2 Calibration methods **71**

2.2.1 Supra-areas scale 72

2.2.2 Inter-areas scale 72

2.2.3 Intra-areas scale 75

2.3 Econometric validation of the budget shares **78**

2.3.1 Theoretical background 79

2.3.2 A proposal of lag operator 81

2.3.3 Involving oknn in spatial ARIMA process 84

2.3.4 An application to the metropolitan area of Paris using the specification oknn 91

2.3.5 Main findings 104

Appendix 2.A Models selected for estimation **104**

Appendix 2.B Models development **111**

2.B.1 From the linear to the SEM 111

2.B.2 From the SEM to SARIMA 111

2.1 Data

All data used and generated by GEMSE for each country, each scenario, each variable, each year and each urban area are available online.[1] Note that all data are both presented at the regional and urban scales.

2.1.1 USA's and France's data-related commonalities

Step 1 boundary conditions of each nation urban system, defined by national macroeconomic projections such as GDP ($\widetilde{\mathcal{Y}}$), employment rate ($\widetilde{E}$) and oil price (p^a) are based empirically. The consumption of fuel liters per km (c^a) is considered identical for both the United States and France, and is taken from the *CARBECO* database.[2] Thus, the two nations follow the same annual variations in these above aggregates, and only differ from each other by their initial conditions that are empirically established. Other commonalities relate to:

(i) the *source* of transport data, such as private vehicle speeds with and without congestion, (respectively $\boldsymbol{v}_j^{PV}$ and $\boldsymbol{v}_j^{0,PV}$), public transport speeds ($\boldsymbol{v}_j^{PT}$), percentages matrices of road travel time in common between places to reach the CBD of their agglomeration ($\boldsymbol{\Pi}_j$) that have been bot-scrapped from *GoogleMaps*;[3]

(ii) the *source* of data concerning the average investment rates of return in the USA and France ($\bar{r}$) values that are respectively set to 30% and 15% as it reads in Askenazy and Timbeau (2003, p.169, Fig.1);

(iii) the *value* of the elasticity of wages with respect to the log-linearized rate of unemployment (σ), of -0.1, taken from the econometric study of Blanchflower and Oswald (1995);

(iv) the *value* of the elasticity of Armington (1969) (ε), of 4, derived from a review of econometric studies made by McDaniel and Balistreri (2003, p.3-6);[4]

(v) the *value* of the elasticity of labor costs with respect to the size of the manufactured sector (η), of 0.05, chosen at a level comparable to those of studies like Autant-Bernard and LeSage (2011, p.486, Table 1) or Kaiser (2002, p.26-29, Tables 3-6), *i.e.* often belonging to $[0, 0.1]$.[5] *(vi)* the *value* of the probability of domestic migration of population ($\nabla^{\mathcal{P}}$) of 2% stems from the *United States Census Bureau*'s 2010 *State to State migrations tables*[6] in the case of the USA and from Brigitte Baccaïni (2007, p.141, Table 1) for France about inter-regions

[1] Available at `https://gemse.alwaysdata.net/`.

[2] Developed and maintained by Franck Nadaud, economist at CIRED-CNRS. Email : nadaud@centre-cired.fr.

[3] Googlemaps website is available at `https://www.google.com/maps`.

[4] This high level of elasticity stands for the market depth of the entire non-homogeneous sector of the economy. See Rivera-Batiz (1988) for an intuition about the level of elasticity that might be chosen and its economic meaning.

[5] Other studies such as Dechezleprêtre et al. (2013, p.45-46, Tables 14-15) give interesting insights about the quantification of (sectors) knowledge spillovers, but use functional specifications not close enough to ours to be comparable.

[6] Available at `https://gemse.alwaysdata.net/static/gemse/USA/state_to_state_mig_table_2010.xls`.

migrations;

(vii) the *value* of the stable daily travel time budget of Zahavi and Talvitie (1980), set to 110 minutes to ensure that all modeled commuters are able to do at least one daily return trip;

(viii) the *value* of the annual distance browsed per *capita*, of 13000km – only used for calibration purpose –, derives from *Federal Highway Administration*'s tables in the case of the USA,[7] and from the *CARBECO* database in the case of France.

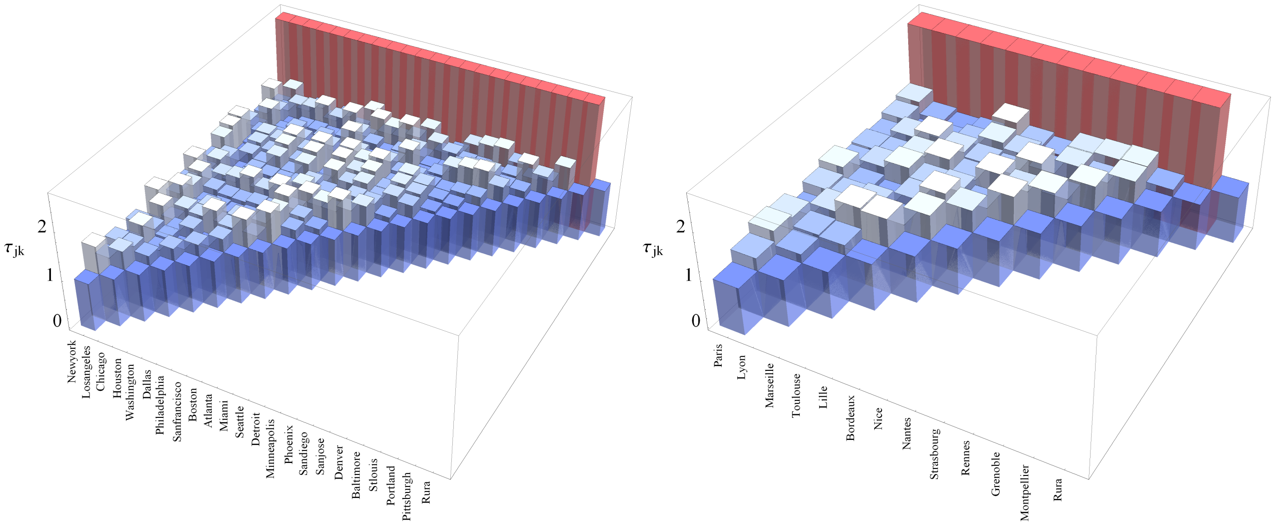


Figure 2.1: Iceberg melting cost symmetric matrix of the USA and France

(ix) the *approach followed to determine* the trade cost matrix ($\boldsymbol{\tau}$) (Samuelson, 1952), grounded on geographical distances between urban areas. The rural area is set subsequently to twice the maximum distance and $\boldsymbol{\tau}$ is finally obtained by normalizing and exponentiating these distances.[8] Figure 2.1 plots each nation's trade cost matrix.

(x) the *approach followed to determine* the stock of capital invested in the good-producing sector of each area $r \in [\![1; N]\!] \cup \{z\}$ (the K_rs), deduced according to its share of national GDP.

2.1.2 American data

In the USA, twenty-two urban areas are explicitly represented. These urban areas are responsible for about 55% of the national GDP ($\widetilde{\mathcal{Y}}$). Calibration (in 2010) is based on macroeconomic and demographic data taken from time series of the *United States Census Bureau*. The total number of firms is deduced using the total number of employees and their average number per firm, set to 16.1, as it reads in Choi and Spletzer (2012, p.1, Fig.1). The annual private vehicle budget ($\boldsymbol{P}_j^{a,PV}$) of \$8000 derives from the 2010 edition of the *AAA's 'Your driving cost'* report.[9] The annual public transport budget ($\boldsymbol{P}_j^{a,PT}$), based on average on a wide range of

[7] Available at `https://www.fhwa.dot.gov/ohim/onh00/bar8.htm`.

[8] The normalization consists of dividing all distances between urban areas by the distance that separates the rural area from the rest of the system of areas.

[9] Available at `https://gemse.alwaysdata.net/static/gemse/USA/AAA_Your_driving_costs_2010.pdf`.

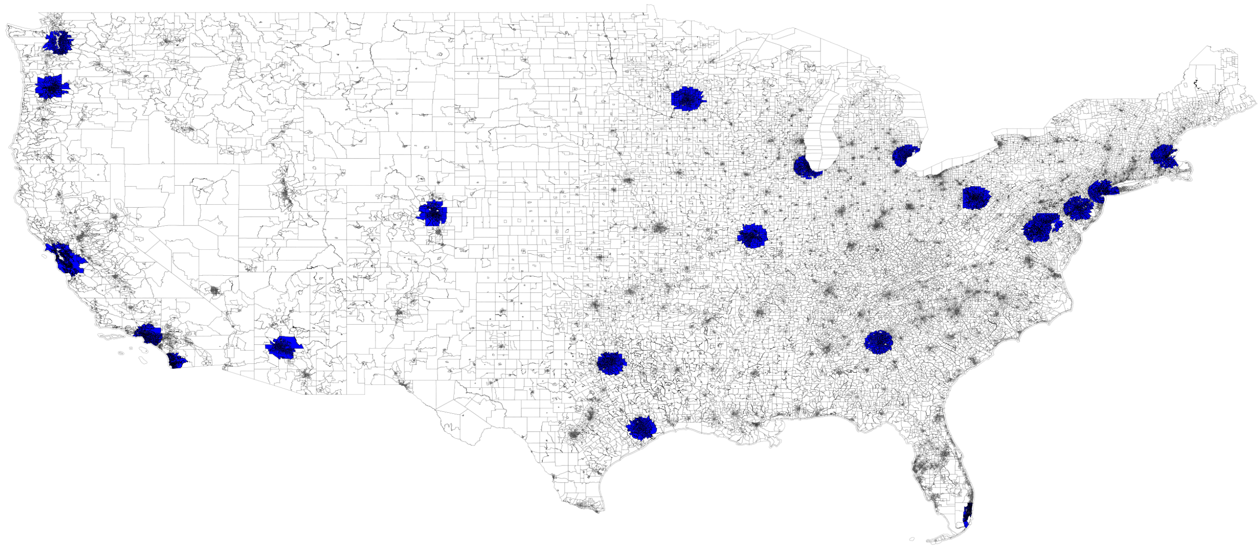


Figure 2.2: The 22 American agglomerations' 8785 places (merged census block groups) considered by GEMSE

differentiated services, is assumed to represent 5% of $\boldsymbol{P}_j^{a,PV}$.[10] The shares of income attributed to housing and transportation (δ^H and δ^a) are respectively set to 30% and 15%, as it reads, *e.g.* in U.S. Bureau of Labor Statistics (2013, p.3, Table B) or U.S. Bureau of Labor Statistics (2012, p.3, Table B).

In the first phase of information collection at the urban scale, each area comprised between 543 (Stlouis) and 4341 (Newyork) places, yielding to a total of 29752 places. These places actually are census block groups, whose definition come from shapefiles also distributed by the *United States Census Bureau*.[11] Each census block groups surface and population have been bot-scrapped from *USA.com*.[12] For computational and mathematical reasons,[13] these census block groups have then been merged to reduce the standard deviation of their surface, as illustrated in the case of Newyork in Figure 2.3. Of course, numerical variables have been merged considering their meaning (*i.e.* on average by default), *e.g.* populations and surfaces have been merged in an additive way. After merging, each agglomeration comprises between 322 and 569 places, yielding to a total of 8785 places, all mapped in Figure 2.2.

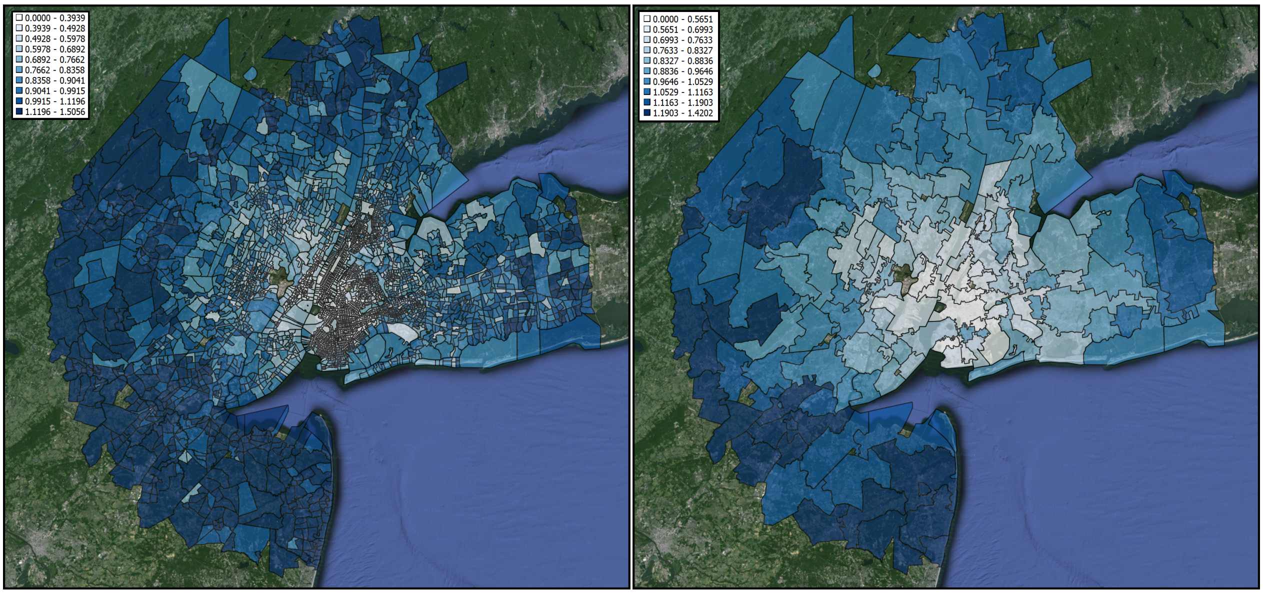


Figure 2.3: Illustration on New-York (private vehicle speed deciles km/min) of the census block groups merging process

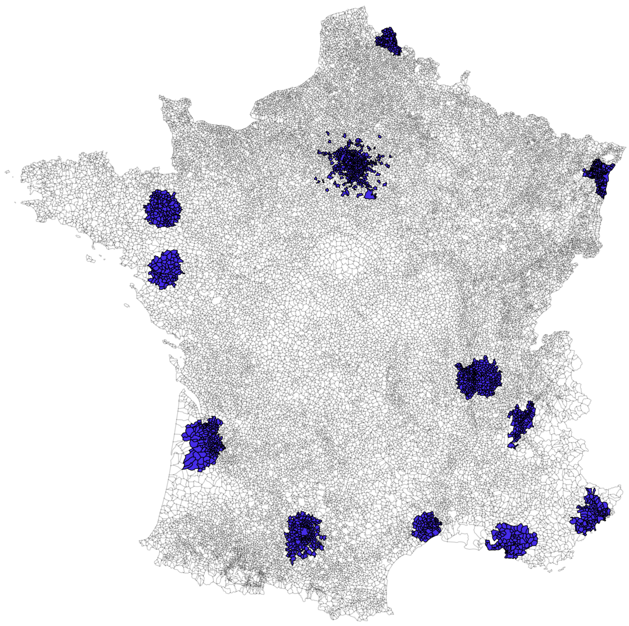


Figure 2.4: The 12 French agglomerations' 2253 places (communes) considered by GEMSE

2.1.3 French data

In France, twelve urban areas are explicitly represented. These urban areas are responsible for about 45% of the national GDP ($\widetilde{\mathcal{Y}}$). Calibration (in 2008) is based on macroeconomic and demographic data that derive from *Institut national de la statistique et des études économiques (INSEE)* time series. The total number of firms derives from Calatayud et al. (2011, p.8, Table 1): it consists of the sum over all sectors but in the construction one.[14] The annual private vehicle budget ($\boldsymbol{P}_j^{a,PV}$) of 6000€ derives from the *CARBECO* database. The shares of income attributed to housing and transportation (δ^H and δ^a) are respectively set to 25% and 10%, as it reads *e.g.* in Guidetti and INSEE (2012, p.3, Table 3) or Morer and INSEE (2015, p.3, Table 4). The latter settings undergo an econometric validation for Paris in Section 2.3.

At the scale of urban areas, the geographical data (each place surface and population) derive from *Institut national de l'information géographique et forestière (IGN)*'s shapefiles,[15] in which places superimpose to communes, *i.e.* cities. For each place, annual public transport budgets ($\boldsymbol{P}_j^{a,PT}$) are supplied by the *Commissariat général au Développement durable (CGDD)* and originate from l'*Enquête nationale transports et déplacements (ENTD) 2008*.[16]. Each urban area comprises between 94 (Nantes) and 428 (Paris) places, yielding to a total of 2253 places, all mapped in Figure 2.4.

2.2 Calibration methods

The phase of calibration that instantiates steps 1 [see Subsection 1.2.1] and 2 [see Subsection 1.2.2] of the GEMSE microcasting process, is divided into three parts: the first part relates to the national scale, the second relates to the inter-areas scale, *i.e.* the *NEG* scale, and the third one relates to the intra-areas scale, *i.e.* the *UE* scale. In the following subsections, the variables are presented according to a calibration-dependence logical order.

[10] Although in vector-like bold style and subscripted, the two annual budgets of public and private transport are actually the same across the country.

[11] Available at `https://www.census.gov/geo/reference/garm.html`.

[12] Available at `http://www.usa.com`.

[13] The computational reason refers to the time and the random access memory it takes to run a spatial equilibrium model that includes 29752 spatial units in network interaction. The mathematical reason refers to the singularity of some metropolitan areas congestion matrix, which contains census block groups both too close from each other and too organized in a Manhattan-style network structure so as not to induce strict colinearity when matrix-wise considered.

[14] Indeed, the housing sector is distinctly represented in GEMSE [see subsection 1.1.1.5].

[15] Available at `http://professionnels.ign.fr/geofla`.

[16] Available via `http://www.statistiques.developpement-durable.gouv.fr/sources-methodes/enquete-nomenclature/1543/139/enquete-nationale-transports-deplacements-entd-2008.html`.

2.2.1 Supra-areas scale

At this scale, all variables are freely set on the basis of empirical studies [see Section 2.1], except one variable, namely the share β of manufactured goods (M) in workers expenditures of "Cobb-Douglas goods".[17] Indeed, given GEMSE's shareholding structure, a too small (too high) β would result in manufacture (homogeneous) sector dividends being insufficient to justify the average income configured in the urban (rural) space, thus leading subsequent equilibria [see from eq.(1.41) to eq.(1.45)] to deal with negative wages (negative price of the homogeneous good). Hence the following calibration for β at $t = 0$:

$$\beta_0 = \frac{\sum_{j=1}^{N} \widetilde{\mathcal{Y}}_{j,0}^{u}}{\widetilde{\mathcal{Y}}_0} \tag{2.1}$$

where $\sum_{j=1}^{N} \widetilde{\mathcal{Y}}_{j,0}^{u} = \sum_{j=1}^{N} Y_{j,0} L_{j,0}$ is the gross urban product (GUP) and $\widetilde{\mathcal{Y}}_0$ is the top-down GDP that GEMSE microcasts at $t = 0$ [see eq.(1.63)].[18]

2.2.2 Inter-areas scale

The calibration of the drivers responsible for the uneven spatial distribution of economic activities are; *(i)* the variables characterizing the rural area such as population, employment rate, average income per worker, the rural unitary labor requirement and wages, the price of the homogeneous good; *(ii)* the variables characterizing urban areas such as wages and the price of each agglomeration-specific heterogeneous good; *(iii)* the migration dynamic drivers such as all areas welfare amenity and all urban areas investment returns in the manufactured sector.

Rural variables The calibration of rural variables such as population, employment rate and the average income per worker is straightforward, involving just to negate the analogous and data-sourced urban aggregates. The rural labor force $\mathcal{P}_{z,0}$ is deduced as follows:

$$\mathcal{P}_{z,0} = \mathcal{P}_0 - \sum_{j=1}^{N} \mathcal{P}_{j,0} \tag{2.2}$$

[17] Admittedly, qualifying a good of being "Cobb-douglas" is abusive strictly speaking. However, this linguistic liberty is relatively common, even more when the good is aggregated using a CES specification, see *e.g.* Bosi et al. (2010, p.13), Behrens and Robert-Nicoud (2011, p.218), Hungerland (2017, p.33), Ottaviano and Martin (2001, p.951) or Gaspar (2017, p.13).

[18] Presently, $Y_{j,0}$ is deduced from $\widetilde{\mathcal{Y}}_{j,0}^{u}$ *i.e.* the GUPs are inputed to GEMSE during the phase of income-calibration. Thus, Y_j is the average income per j-worker, L_j, and not the average income directly taken from the French or the American data sources, those ones being per *capita*. With the number of *capitas* and the average income per *capita* denoted respectively by $\mathcal{C}_j$ and I_j, note that the following relation holds: $\mathcal{C}_{j,t} I_{j,t} = Y_{j,t} L_{j,t} \quad \forall \{j,t\} \in [\![1; N]\!] \times [\![0; \infty[\![$.

the rural employment rate, $E_{z,0}$:

$$E_{z,0} = \frac{\widetilde{E}_0 \mathcal{P}_0 - \sum_{j=1}^{N} L_{j,0}}{\mathcal{P}_{z,0}} \tag{2.3}$$

and the rural average income per worker, $Y_{z,0}$:

$$Y_{z,0} = \frac{(1-\beta_0)\widetilde{\mathcal{Y}}_0}{E_{z,0}\mathcal{P}_{z,0}} \tag{2.4}$$

where $\widetilde{E}_0$ and $\mathcal{P}_0$ are respectively the top-down employment rate [see eq.(1.64)] and the national labor force [see eq.(1.79)] at $t = 0$.

The determination of the rural unitary labor requirement for production, $l_{z,0}$, first implies to express the total spending devoted to the consumption of goods (homogeneous or manufactured) through the country at $t = 0$, $\mathcal{S}_0$, which is:

$$\mathcal{S}_0 = \delta_0^c \left(L_{z,0} Y_{z,0} + \frac{1}{1-\delta_0^a} \sum_{j=1}^{N} \sum_{i=1}^{N_j} L_{j,i,0}(Y_{j,0} - P_{j,i,0}^a) \right) \tag{2.5}$$

recalling that δ_0^c is related to the share of Cobb-Douglas goods C in workers' expenditures, N_j is the number of places considered in agglomeration j and $P_{j,i,0}^a$ is the effective transport cost over the two modes that is endured by workers living in j-agglomeration place i at $t = 0$ [see eq.(1.26)]. Once $\mathcal{S}_0$ expressed, $l_{z,0}$ is straightforward to determine:

$$l_{z,0} = \frac{L_{z,0}}{q_{z,0}} = L_{z,0} \frac{p_{z,0}}{(1-\beta_0)\mathcal{S}_0} \tag{2.6}$$

where $(1-\beta_0)\mathcal{S}_0$ and $p_{z,0}$ respectively stand for the total spending devoted to the consumption of homogeneous goods through the country and the homogeneous good price at $t = 0$.

Rural wages are deduced from the income redistribution rule assumed in the economy for rural workers [see eq.(1.35)]. Subtracting capital income from the total one, leads to deduce rural wages,[19] $w_{z,0}$, in terms of the other data-sourced and calibrated variables, as follows:

$$w_{z,0} = \left(\sum_{j}^{N} L_{j,0} \right)^{-1} \left(\delta_0^C Y_{z,0} L_0 - \frac{\beta_0 \mathcal{S}_0}{\varepsilon} - (1-\beta_0)\mathcal{S}_0 \right) \tag{2.7}$$

where L_0 and ε respectively stand for the total nation number of workers and the Armington (1969)'s constant elasticity of substitution. Furthermore, still visible despite the rearranging, note that $\frac{\beta_0 \mathcal{S}_0}{\varepsilon}$ stands for the total profit stemming from the sector of manufactured goods.

The fact that rural firms production is done under constant returns to scale makes the

[19] The determination of the minimum wages simply stems from the inversion of the wages-curve function [see eq.(1.12)] and eq.(1.36)].

return on equity, r_z, be per unit of invested capital, k_z [see eq.(1.29)], imposing:

$$k_{z,t} = k_z = 1 \ \forall t \in [\![0; \mathcal{T}]\!] \tag{2.8}$$

The marginal cost pricing rule, implied in perfect competition, thus constitutes the basis for the calibration of the homogeneous good price at $t = 0$:

$$p_{z,0} = l_{z,0}w_{z,0} + 1r_{z,0} = r_{z,0}\left(1 - \frac{L_{z,0}w_{z,0}}{(1-\beta_0)\mathcal{S}_0}\right)^{-1} \tag{2.9}$$

Urban variables Wages in urban areas are deduced via the same method used for rural ones, *i.e.* by subtracting dividends from the total income and rearranging, which leads to express $w_{j,0}$ as follows:

$$w_{j,0} = \frac{\delta_0^C}{1-\delta_0^a}Y_{j,0} + \left(\frac{\delta_0^H}{1-\delta_0^a} - 1\right)\frac{\sum_{i=1}^{N_j} L_{j,i,0}P_{j,i,0}^a}{L_{j,0}} - \frac{\beta_0\mathcal{S}_0/\varepsilon - (1-\beta_0)\mathcal{S}_0 - L_{z,0}w_{z,0}}{L_0} \tag{2.10}$$

To be calibrated, the prices of the heterogeneous goods at $t = 0$ involve a highly non linear system whose solutions both *(i)* balance production capacities and effective demand [see eq.(1.41)]; and *(ii)* qualify the situation as profit-maximized [see eq.(1.42)]. The effective demand of manufactured goods that is asked to a firm settled in the agglomeration j at $t = 0$, $q_{j,0}$, is:

$$q_{0,j} = \delta_0^c\beta_0\left(\left(\frac{p_{jz,0}}{P_{z,0}}\right)^{1-\varepsilon}L_{z,0}Y_{z,0}\right) + \frac{1}{1-\delta_0^a}\sum_{k=1}^{N}\left(\left(\frac{p_{jk,0}}{P_{k,0}}\right)^{1-\varepsilon}\sum_{i=1}^{N_k}L_{k,i,0}(Y_{k,0} - P_{k,i,0}^a)\right) \tag{2.11}$$

where $p_{jk,0}$ ($p_{jz,0}$) stands for the manufactured good price p_j plus a transport cost (Samuelson, 1952) due to the shipment from agglomeration j to agglomeration k (to the rural area, z) [see eq.(1.39)]. Recalling that $P_{j,0}$ and $P_{z,0}$ are the regional price indexes of CES goods, respectively in a urban area j [see eq.(1.10)] and in rural area z [see eq.(1.34)], the system is solved for $p_{j,0}^* \ \forall j \in [\![1; N]\!]$ when:

$$\frac{\varepsilon}{\varepsilon - 1}w_{j,0}L_{j,0} = n_{0,j}q_{0,j}(p_{j,0}^*) \quad \forall j \in [\![1; N]\!] \tag{2.12}$$

The equation (2.12) is however still underdetermined, given that each solving argument $p_{j,0}$ is in a ratio-relationship with a functional of itself, *i.e.* *via* $P_{j,0}$ and $P_{z,0}$. Aesthetic[20] arbitrariness is therefore involved here to choose the wanted level of price, adding the following constraint to equation (2.12):

$$p_{z,0} = \frac{\sum_{j=1}^{N} n_{j,0}p_{j,0}}{\sum_{j=1}^{N} n_{j,0}} \tag{2.13}$$

[20]The aestheticism here consists of having prices of the homogeneous and manufacture goods with similar scale values.

which centers on average the prices of the heterogeneous goods on the homogeneous one.

Migration drivers What drives the migration of population in one area is its differential of welfare with respect to its deviation from the nation average [see eq.(1.82)]. Thus the amenity term of any area r, $\forall r \in [\![1; N+1]\!]$, that is u_r^0 (u_j^0 in urban areas [see eq.(1.6)] and u_z^0 in the rural area [see eq.(1.30)]), is calibrated so that its welfare differential leads to a growth of its population that fits the observed one, that is $\Delta\mathcal{P}_{r,0}$, involved between $t = -1$ and $t = 0$. Thus, the amenity terms of all areas r are deduced by solving the equation below for u_r^{0*} $\forall r \in [\![1; N+1]\!]$:

$$\Delta\mathcal{P}_{r,0} = -1 + \frac{\mathcal{P}_{r,0}}{\mathcal{P}_{r,-1}} = \nabla^{\mathcal{P}}\Delta W_{r,-1}(u_r^{0*}) \quad \forall r \in [\![1; N+1]\!] \tag{2.14}$$

Note that multiple solution vectors exist. Indeed, since only the relative differences between the utility levels are significant, the solving of equation (2.14) is done by choosing amenity-terms scaling factors that are comprised between 1 and 100.[21]

On the side of production, what drives the migration of firms in one urban area is the differential of return on equity that is stated therein [see eq.(1.81)]. The method used here consists of centering the average return of capital over urban areas on the nation one,[22] considered as also being the rural area's, $r_{z,0}$. To center all urban areas firms return on equity on $r_{z,0}$, one has to determine the corresponding fixed input requirement of capital per firm, $\varkappa$ [see eq.(1.2)], as follows:

$$\varkappa = \frac{\sum_{k=1}^{N} n_{k,0}p_{k,0}q_{k,0}}{\varepsilon r_{z,0}\sum_{k=1}^{N} n_{k,0}} = \frac{\sum_{k=1}^{N} n_{k,0}\pi_{k,0}}{r_{z,0}\sum_{k=1}^{N} n_{k,0}} = \frac{\overline{\pi}_0}{r_{z,0}} \tag{2.15}$$

where $\overline{\pi}_0$ stands for the average profit of firms settled in the urban space at $t = 0$.

2.2.3 Intra-areas scale

At the intra-agglomeration scale, the instantiation of the GEMSE microcasting process involves, in each j-agglomeration place i, the calibration of variables related to *(i)* transports, such as the modal shares of car and the capacities of the roads network; and to *(ii)* the housing market, such as the total stock of square meters, the rents per square meter and the production function specificities of housing developers, *i.e.* the decreasing return to scale of installed capital.

[21] This is ensured by using a log-it function like $a + (b-a)/(1+e^{-x})$, which thus constrains the image of the solving argument, x, to be between a and b.

[22] Rather than translating arbitrarily each urban area level of profits to induce the wanted flows of migrating firms, which is *a priori* impossible with identical firms.

Transport Albeit adapted in various ways, the method used to calibrate the modal share of car in each j-agglomeration place i, is done *à la* Joly et al. (2002),[23] which is itself theoretically grounded on Zahavi (1979)'s Unified Mechanism of Travel (UMOT) project.[24] Hence, in the case of an exclusive use of each of the two transport modes, the gain in utility level stemming from the log-linear speed differential in favor of public transports in any j-agglomeration place i, is formalized as follows:

$$\delta_0^a \ln\left(\frac{a_{j,i,0}^{PT}}{a_{j,i,0}^{PV}}\right) \quad \forall i \in [\![1; N_j]\!] \tag{2.16}$$

where $a_{j,i,0}^{PT}$ ($a_{j,i,0}^{PV}$) stands for the number of back and forth that a worker can perform in public transport (private vehicle) at $t = 0$ given her place i in j-agglomeration [see eq.(1.21)]. In the same manner, the gain in utility level stemming from the differential in the log-linear residual incomes in any j-agglomeration place i, is:

$$(1-\delta_0^a)^{-1} \ln\left(\frac{Y_{j,0} - P_{j,i,0}^{a,PT}}{Y_{j,0} - P_{j,i,0}^{a,PV}}\right) \quad \forall i \in [\![1; N_j]\!] \tag{2.17}$$

As stated in Joly et al. (2002), the public transport is beneficial if

$$(1-\delta_0^a)^{-1} \ln\left(\frac{Y_{j,0} - P_{j,i,0}^{a,PT}}{Y_{j,0} - P_{j,i,0}^{a,PV}}\right) > -\delta_0^a \ln\left(\frac{a_{j,i,0}^{PT}}{a_{j,i,0}^{PV}}\right) \quad \forall i \in [\![1; N_j]\!] \tag{2.18}$$

Rearranging the inequation (2.18) in terms of zero-centered relative difference, the choice of using the car is likely to occur when:

$$\Gamma_{j,i,0} = -1 + \frac{\delta_0^a \ln\left(\frac{a_{j,i,0}^{PV}}{a_{j,i,0}^{PT}}\right)}{(1-\delta_0^a)^{-1} \ln\left(\frac{Y_{j,0} - P_{j,i,0}^{a,PT}}{Y_{j,0} - P_{j,i,0}^{a,PV}}\right)} > 0 \quad \forall i \in [\![1; N_j]\!] \tag{2.19}$$

A logistic function is then used to calibrate the modal share of cars in any j-agglomeration place i at $t = 0$, $\alpha_{j,i,0}$, as follows:

$$\alpha_{j,i,0} = \frac{1}{1 + e^{-\Gamma_{j,i,0}}} \quad \forall i \in [\![1; N_j]\!] \tag{2.20}$$

It is now straightforward to calibrate the roads network capacity[25] of the j-agglomeration in any place i, $\mathcal{K}_{j,i,0}$, via the inversion of the fundamental diagram of traffic flow [see eq.(1.19)

[23]See section 5.Détermination du niveau d'équipement du ménage, mainly p.37

[24]As explained in Joly and Crozet (2006), UMOT is a model developed by Zahavi (1979) for predicting the mobility of people in urban areas around the two constraints faced by a mobile person: the budgetary and the time constraints.

[25]Involved to reach the j-CBD in a pendulum manner.

and eq.(1.20)], as follows:

$$\mathcal{K}_{j,i,0} = \left(-1 + \frac{v^{PV}_{0j,i,0}}{v^{PV}_{j,i,0}}\right)^{-0.25} \sum_{o=1}^{N_j} L_{j,o,0}\alpha_{j,o,0}\Pi_{j,io} \quad \forall i \in [\![1; N_j]\!] \tag{2.21}$$

Finally, reference speeds denoted by $\nu^{PV}_{j,i}$ and $\nu^{PT}_{j,i}$ [see eq.(1.23)], are respectively calibrated such that $\nu^{PV}_{j,i} = v^{PV}_{j,i,0}$ and $\nu^{PT}_{j,i} = v^{PT}_{j,i,0}$.

Housing The method used to calibrate the housing rents is, from A to Z, exactly the same as the one developed by Allio (2015, p.113-114), to whom I refer for more details. The idea is to compute utility levels and its inner housing/transport trade-off only in the center of the agglomeration, *i.e.* in places regarded as located in the j-CBD, wherein housing rents and transport speeds are both known. Then the assumption according to which the agglomeration is at its equilibrium state [see eq.(1.51)], allows for determining housing rents in the remaining agglomeration places. It is then straightforward to determine the total stock of square meters in each place i, $H_{j,i,0}$, since the total spending devoted to housing services and rent are known.

One element that differs is the calibration of the housing developers specificities,[26] whose decreasing return to scale of installed capital is space-differentiated [see eq.(1.27)]. This is done so to provide housing developers with the ability to maintain the stock of square meters at its equilibrium state, *i.e.* to replace the depreciated stock in a situation of stable expected housing rents. This consists of qualifying as profit-maximized, the replacement of the depreciated stock of capital, which is effective for $\gamma^*_{j,i}$ $\forall\{j,i\} \in [\![1; N]\!] \times [\![1; N_j]\!]$ when

$$S_{j,i}\left(\frac{c^H(1+r_H)}{\widetilde{R}_{j,i,0}\gamma^*_{j,i}(1-\delta_H)^{\gamma^*_{j,i}}}\right)^{\frac{1}{-1+\gamma^*_{j,i}}} = \left(\frac{H_{j,i,0}}{S^{1-\gamma^*_{j,i}}_{j,i}}\right)^{\frac{1}{\gamma^*_{j,i}}} \tag{2.22}$$

This concludes the calibration of the short-run model.

[26] Whose dynamic is explained from eq.(1.83) to eq.(1.87).

2.3 Econometric validation of the budget shares

A large amount of data has been gathered to calibrate GEMSE. Among those, note that housing rents are however not exogenously inputed at the calibration stage into GEMSE to initiate simulations, unlike, *e.g.* travel times. As explained in the housing related paragraph of subsection 2.2.3, housing rents are endogenerated by GEMSE, be it at the calibration or simulation stage. The assumption of calibration according to which the agglomeration is at its equilibrium state [see eq.(1.51)] allows for the deduction of housing rents in the entire urban area by only extrapolating those in the center.

In this section, observed (private vehicle) travel times in France in Paris are used in conjunction with observed housing rents to estimate the elasticities of utility with respect to accessibility, δ^a, and to housing services, δ^H. Put differently, we exploit the very nature of the transport/housing trade-off to estimate these elasticities. The aim is to assess whether the values of δ^H and δ^a that are chosen for France – respectively of 25% and 10% – are econometrically valid at the level of Paris. Note that this econometric validation could be declined into a spatial panel data applied to all urban areas of France and/or the USA. However, inclined not to resort to any kind of automatic selection techniques on the one hand and given that the approach that we follow requires careful handling on the other hand,[27] we decided to first concentrate on only one urban area, *i.e.* Paris.

Furthermore, this section takes part in the theoretical spatial-econometric (SE) debate on the right choice of the (so-denoted $\mathbf{W}$) spatial weight matrix (Anselin, 2007; Corrado and Fingleton, 2012; Elhorst et al., 2012; Gibbons and Overman, 2012) and makes a new proposal for it.

The proposal is implemented using a homemade Python library, named `PyOKNN`. The sourcecode and documentation are available online[a] and the package is easily installable via pip[b] opening a session in your OS shell prompt and typing `pip install pyoknn`. This library is though of as a potential extension of another library called `PySAL` (Anselin and J. Rey, 2007).[c] The lag operator is involved within the maximum likelihood estimator to conduct the estimation of the spatial ARIMA (SARIMA) model, which consists of the combination of the spatial error model (SEM), the spatial lag model (SAR) and the spatial moving average (SMA) model. It returns the same coefficients as those of `PySAL` when estimating monovariate SAR or SEM models, and allows for the computation of their multivariate version – where the terms *monovariate* and *multivariate*, refer to the number of parameters involved in spatial filters. Also, the confidence intervals that are computed in `PySAL` are analytically derived, while those of our library rely on numerical

[27] Indeed, as will be explained subsequently, this study deals with the contingent determination of the structure of autoregression of housing rents.

hessian approximation and/or spatial bootstrap sampling.

[a]Available at https://github.com/lfaucheux/PyOKNN
[b]It is a package management system used to install and manage software packages written in Python.
[c]See http://pysal.readthedocs.io/en/latest/.

Subsection 2.3.1 reviews the relevant literature, Subsection 2.3.2 offers the proposal and Subsection 2.3.4 applies this proposal to estimate δ^H and δ^a and assesses their representativity and coherence regarding the values that are chosen in Subsection 2.1.1.

2.3.1 Theoretical background

Fingleton (2009) and Corrado and Fingleton (2012) remind the analogies between temporal and spatial processes, at least when considering their lag operators. In the spatial econometric (SE) case, the lag operator is always explicitly involved *via* the use of a $n \times n$ matrix $\mathbf{W}$, where n is the number of interacting positions ***in the scope of the econometric study only***.[28] The chosen space can be geographic, economic, social or of any other type. In the temporal case, which is seen as a space like no other given its inescapable anisotropic nature, the lag operator is in practice never explicitly considered. Any variable to lag, say, a $n \times 1$ vector $\mathbf{y}$, is formed over components that are beforehand sorted according to their position on the timeline.[29] This allows the lag-procedure to simply consist of offsetting down these components by a lag-determined number of rows, say, one row. In matrix terms, this offsetting procedure would be entirely equivalent to pre-multiplying an unsorted version of $\mathbf{y}$ by a boolean $n \times n$ matrix $\mathbf{H}$ with 1s indicating the immediate and unilateral proximity between temporal positions.

The so-structured data generating process (DGP) thus involves $\mathbf{H}$ as primarily observed, *i.e.* with no restructuring hypothesis or transformation. For each lag, this provides the statistician with a straightforward parameter space definition, whose knowledge of the exact boundary is important, both for estimation and inference (Elhorst et al., 2012).

By opposition to the time series (TS) case, specifying $\mathbf{W}$ involves a lot of arbitrariness. Apart from $\mathbf{W}$'s non-nilpotency,[30] these hypotheses deal with $\mathbf{W}$'s isotropy (Cressie, 1993) and finding $\mathbf{W}$'s true entrywise specification through a very large number of competing ones, be it functional[31] or binary. Some famous entrywise specifications are the negative exponential function (Haggett, 1965), the inverse-distance function (Wilson, 1970), the combined distance-

[28]Indeed, let's recall that in the major part of this thesis, n_j and n respectively stand for the number of firms in agglomeration j and in the country of interest. The GEMSE-consistent manner to denote the number of interacting positions would have been N_j. This is reminded in subsection 2.3.4 when starting to deal with the application.

[29]The temporal lag operator illustrated by Corrado and Fingleton (2012) locates the most recent observation at the bottom of the to-be-lagged vector, *e.g.* at y_n in the case of a $n \times 1$ vector $\mathbf{y}$.

[30]Which means that there is no permutation of the observational units that would make $\mathbf{W}$ triangular, see Martellosio (2011).

[31]Whose parameters need to be estimated.

boundary function (Cliff and Ord, 1973) and the weighted logistic accessibility function (Bodson and Peeters, 1975).[32] Binary weights specifications are either based on the k^{th}-nearest neighbor ($k$nn), on the k^{th}-order of contiguity or on the radial distance. Then, to ensure the unique definition of any to-be-lagged variable in terms of the other variables of the model, $\mathbf{W}$ is scaled depending on the choice one makes among three competing normalization techniques. The first one makes $\mathbf{W}$ row-stochastic, but does not necessarily preserve its symmetry. The second one pre- and post-multiplies $\mathbf{W}$ by the negative square root of a diagonal matrix reporting its row-totals (Cliff and Ord, 1973). The last one scales $\mathbf{W}$ by its largest characteristic root (Elhorst, 2001).

But the choice of $\mathbf{W}$ and of its transformation is not innocuous. For a maximum likelihood (ML) estimation to be consistent, the estimated spatial model must involve the true $\mathbf{W}$ (Dogan, 2013; Lee, 2004). When dealing with autoregressive disturbances, both estimators ML and spatial generalized moments (GM) (Anselin, 2011; Arraiz et al., 2010; Drukker et al., 2013; Kelejian and Prucha, 2010)[33] theoretically base their knowledge of unobservable innovations upon the knowledge of $\mathbf{W}$. When facing endogeneity problems in non-autoregressive specifications and resorting to, *e.g.* Kelejian and Prucha (1999)'s generalized moments estimator (GM), the definition of the exogeneity constrains heavily relies on $\mathbf{W}$, which yields consistent and efficient estimations for sure, but potentially not with respect to the true DGP. If resorting to the instrumental variables (IV) method – in which space is conceived as providing ideal instruments (Das et al., 2003; Lee, 2003; Pinkse and Slade, 2010) –, the strength of instruments is far from being ensured with $\mathbf{W}$ in its most common specification, *i.e.* whose lag consists of neighbors-averaging. Moreover, as discussed by Gibbons and Overman (2012), the inclusion of the product of higher powers of the spatial lag operator in the set of instruments is very likely to lead to a problem of colinearity, which in turn leads to the weaknesses of both identification and instruments. Finally, when computing LeSage and Pace (2009)'s total direct and indirect effects, the correctness of the true derivative of the regressand with respect to any spatially filtered[34] variable is a direct result of the correctness of $\mathbf{W}$.

In the following subsection, we propose a specification method for the spatial lag operator whose properties are as close as possible to that of its time series (TS) counterpart, *i.e.* usable as primarily observed without modifications. Nonetheless we follow Pinkse and Slade (2010, p.105)'s recommendation of developing *tools that are not simply extensions of familiar TS techniques to multiple dimensions*. We do so by proposing a specification-method that is fully grounded on the observation of the empirical characteristics of space, while minimizing as much as possible the set of hypotheses that are required. As clarified previously, this is from

[32]Note that inverse-distance and negative exponential functions, as well as the continuum beyond and between the two, can be unified into the negative exponential of a modified Box-Cox transformation of any non-negative distance d. Formally, $e^{-\gamma(\min(\lambda,1)+d_\lambda)}$ with $d_\lambda = \frac{d^\lambda - 1}{\lambda}$ if $\lambda \neq 0$ and $d_\lambda = \ln d$ otherwise.

[33]Among who, Arraiz et al. (2010) formalize moment conditions that allow for spatial lags in the dependent variable, the exogenous variables, and disturbances, the latter being assumed to be based on unknown-form heteroskedastic innovations.

[34]The so-called spatial Cochrane-Orcutt style transformation.

the get-go in the way that $\mathbf{W}$ is observed, which is made under hypotheses.

2.3.2 A proposal of lag operator

As Anselin (2010) states, Paelinck and Klaassen (1979) is the first to elaborate five rules designed to guide the formulation of spatial econometric models. With no elusive omission, two of these rules are *(ii)* the asymmetry in spatial relations and *(v)* the explicit modelling of space (topology) in spatial models.

These rules are actually straightforwardly violated by some entrywise specifications of $\mathbf{W}$. The negative exponential function, the distances-inverse function or binary weights based on radial distance are prominent examples. Indeed, when formalizing the spatial relationship between two given spatial units i and j, these purely distance-based entrywise specifications fall short of formalizing asymmetric relations for the simple reason that $\mathrm{d}_{ij} = \mathrm{d}_{ji}$ is traditionally assumed. Incidentally, preserving this symmetry when making $\mathbf{W}$ row-stochastic is seen as desirable.[35]

These specifications furthermore constrain the explicit modelling of space upon an arbitrarily parameterized functional form. From the positions' standpoint, this inevitably instills non-neutral and rigid *kernels* about the masses of autoregressive effects that flow from them over their neighbors. Figure 2.5 illustrates this point upon Anselin (1988)'s Columbus, Ohio,[36] polygon 25,[37] respectively with the distance inverse function, $d_{i.}^{-\gamma}$, the exponential negative function, $e^{-\gamma d_{i.}}$, and the first-order contiguity, $c_{i.}^{1st}$. From the point of view of neighbors – over

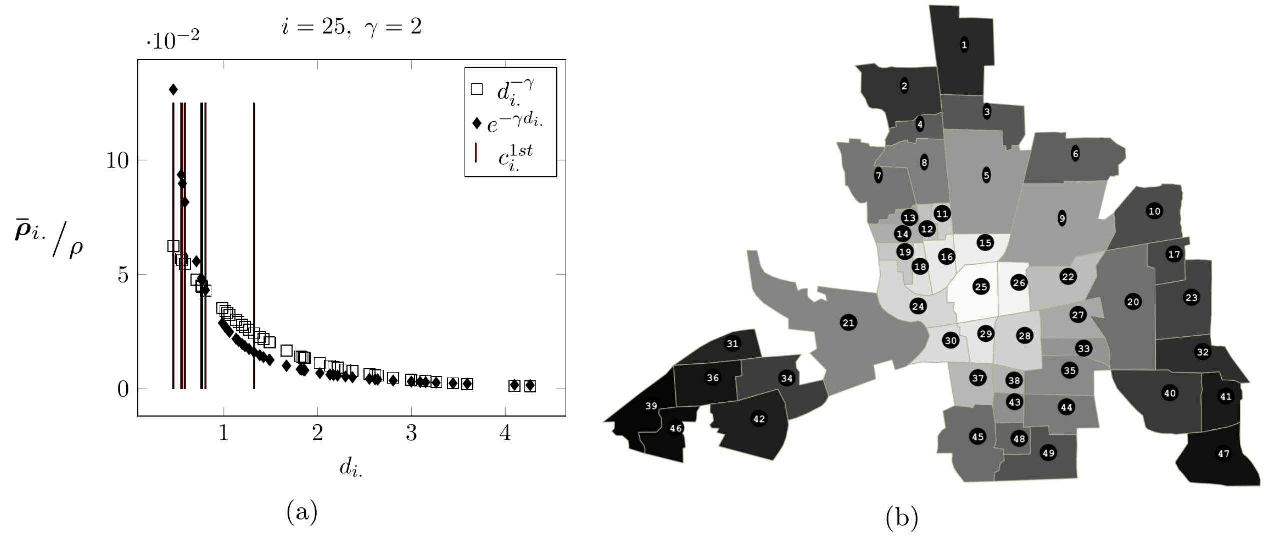


Figure 2.5: Columbus polygon 25's (a) normalized weights and (b) mapped distances from it

all positions – these kernels are very likely to be heterogeneous since for each order of proximity,

[35]This is seen as desirable since having $\mathbf{W}$ symmetric also means not having its eigenvalues in the complex domain, $\mathbb{C}$, which keeps the pro-stationarity parameter space definition entirely included in $\mathbb{R}$.

[36]This data set is used since it is one of the most famous demonstration data sets in Spatial Econometric.

[37]The polygon is selected for its central position in the lattice.

it is uncommon with $\{(i,j) : \mathrm{w}_{ij} > 0\}$ to get $\bar{\rho}_{ij} = \rho$ as in the TS case, since $\frac{\bar{\rho}_{i.}}{\rho} = \frac{\mathbf{w}_{i.}}{\sum_{j=1}^{n} \mathrm{w}_{ij}}$, where ρ is a scalar autoregressive parameter and $\mathbf{w}_{i.}$ is the i-th row of the weights matrix $\mathbf{W}$. Note that even flat/uniform kernels may instill non-neutral heterogeneity as long as the numbers of neighbors differ from one position to another. If the number of neighbors is identical across positions, still remains the impossibility to distinguish one order of proximity from the other. Finally, all entrywise specifications that imply non-binary weights and/or more than one nonzero weight per row, unlike *e.g.* a 1-nearest neighbor matrix,[38] are concerned by this problem.

It is worthwhile wondering why autoregressive effects should flow in that so-structured and smooth manner from polygon 25 over its neighborhood?[39] What about decomposing this outflow in an atomic fashion, *e.g.* by considering each order of proximity in separated correlation structures rather than in one piece? Avoiding this non-neutral kernel is the main *raison d'être* of the proposal that follows, baptized *k-nearest neighbor only* (*oknn*).

First, denote by $\mathbb{D}(n)$ the set of all $n \times n$ distance matrices and by $\mathbf{D} \in \mathbb{D}(n)$ a matrix whose typical element, $\mathbf{d}_{ij}$, is the distance from any spatial unit $i = 1, \ldots, n$ to any spatial unit j of the lattice. Second, for each order of autoregression, say, for each $k = 1, \ldots, n-1$, denote by $\mathbf{D}_k$ an $n \times n$ matrix whose typical element $\mathbf{d}_{ij,k}$ equates $\mathbf{d}_{ij}$ instead of zero if $\mathbf{d}_{ij}$ is the kth smallest distance from the spatial unit i, *i.e.* $\mathbf{d}_{ij,k} = \mathbf{d}_{ij}$ if $\mathrm{r}_i(\mathbf{d}_{ij}) = k$ and $\mathbf{d}_{ij,k} = 0$ otherwise, where r_i is a function that ranks distances from the spatial unit i to all units $j \neq i$. Furthermore, write $\mathbf{d}_{i.,k}$ for the ith row of $\mathbf{D}_k$, $\boldsymbol{\mathcal{I}}_{\mathbf{D}_k}$ for the set of all neighbors that are considered through $\mathbf{D}_k$ at rank k and $\boldsymbol{\mathcal{I}}_{i,\mathbf{D}_k}$ or $\boldsymbol{\mathcal{I}}_{\mathbf{d}_{i.,k}}$ for the set of neighbors of the spatial unit i that are considered through the ith line of $\mathbf{D}_k$ at rank k, where $\boldsymbol{\mathcal{I}}_{\mathbf{D}_k} = \bigcup_{i=1}^{n} \boldsymbol{\mathcal{I}}_{\mathbf{d}_{i.,k}}$. Note that any $n \times n$ distance matrix $\boldsymbol{\mathcal{D}} \in \mathbb{D}(n)$ can be seen as coordinates that characterize the corresponding lattice by locating it in $\mathbb{R}_{\geq 0}^{n(n-1)/2}$.[a]

Assumption 1. There exists a surjective triple $\left(\mathbb{R}_{\geq 0}^{n(n-1)/2}, \mathrm{D}, \mathbb{R}_{\geq 0}\right)$, *i.e.* a surjective function, D, that takes $\boldsymbol{\mathcal{D}} \in \mathbb{D}(n)$ as argument and increases indefinitely in the lattice irregularity.

Assumption 2. For $k = 1, \ldots, n-1$, any spatial unit $i = 1, \ldots, n$ always has at least 1 kth neighbor no matter how far they find themselves from each other, *i.e.* $\mathrm{card}(\boldsymbol{\mathcal{I}}_{i,\boldsymbol{\mathcal{D}}_k}) \geq 1 \; \forall \boldsymbol{\mathcal{D}} \in \mathbb{D}(n)$. This means that any spatial unit i has exactly one kth neighbor when the lattice is perfectly irregular, *i.e.* $\mathrm{card}(\boldsymbol{\mathcal{I}}_{i,\boldsymbol{\mathcal{D}}_k}) \to 1$ as $\mathrm{D}(\boldsymbol{\mathcal{D}}) \to \infty$.

Proposition 1. For $k = 1, \ldots, n-1$, $\mathrm{card}(\boldsymbol{\mathcal{I}}_{\boldsymbol{\mathcal{D}}_k}) = O(n)$ as $\mathrm{D}(\boldsymbol{\mathcal{D}}) \to \infty$.

[38] Another name of entrywise specification that represents the same object is the binary contiguity matrix of the first-order restricted to 1 neighbor.

[39] Tobler's first law of geography is a the very first answer to this question.

Proof of Proposition 1. For $k = 1, \ldots, n-1$, by definition $\boldsymbol{\mathcal{I}}_{\boldsymbol{\mathcal{D}}_k} = \bigcup_{i=1}^{n} \boldsymbol{\mathcal{I}}_{i,\boldsymbol{\mathcal{D}}_k}$ and with assumption 2, it follows that $\text{card}(\boldsymbol{\mathcal{I}}_{\boldsymbol{\mathcal{D}}_k}) = O(n)$ as $\text{D}(\boldsymbol{\mathcal{D}}) \to \infty$. $\square$

Assumption 3. $\mathbf{D}$ is such that $\text{card}(\boldsymbol{\mathcal{I}}_{\mathbf{D}_k}) = O(n)$ for $k = 1, \ldots, n-1$.

This assumption is mandatory to characterize our proposal of lag operator, whose rational is the irregularity of the lattice under consideration.

Proposition 2. For $k = 1, \ldots, n-1$, $\mathbf{D}_k$ contains exactly n non-zero elements.

Proof of Proposition 2. From assumptions 2 and 3, it follows that any position $i \in [\![1;n]\!]$ has exactly one kth neighbor for $k = 1, \ldots, n-1$. Put differently, it follows that $\forall i \in [\![1;n]\!]$, $\text{card}(\boldsymbol{\mathcal{I}}_{\mathbf{d}_{i.,k}}) = 1$ for $k \in [\![1;n[\![$. Thus $\sum_{i=1}^{n} \text{card}(\boldsymbol{\mathcal{I}}_{\mathbf{d}_{i.,k}}) = n$. $\square$

To ultimately define $\mathbf{W}_k$ from $\mathbf{D}_k$ for each $k = 1, \ldots, n-1$, simply replace each $\mathbf{D}_k$'s nonzero typical element, $\mathbf{d}_{ij,k}$, by 1. It follows that the assumption about the diagonal elements of the $\mathbf{W}_k$s is redundant since directly attributable to the distance-matrices intrinsic zero diagonal elements.

Proposition 3. For $k = 1, \ldots, n-1$, $\mathbf{W}_k$ is row-stochastic.

Proof of Proposition 3. From assumption 2, it follows that any position $i \in [\![1;n]\!]$ has exactly one kth neighbor for $k = 1, \ldots, n-1$, *i.e.* $\text{card}(\boldsymbol{\mathcal{I}}_{\mathbf{d}_{i.,k}}) = 1$. Thus for $k = 1, \ldots, n-1$, $\mathbf{W}_k$ has exactly one 1 per row, it is row-stochastic. $\square$

Proposition 4. For $k = 1, \ldots, n-1$, no $\mathbf{W}_k$ can be a linear combination of the others.

Proof of Proposition 4. From assumptions 2 and 3, it follows that any position $i \in [\![1;n]\!]$ has exactly $n-1$ different neighbors. Put differently, from these two assumptions it follows that $\text{card} \bigcup_{k=1}^{n-1} \boldsymbol{\mathcal{I}}_{\mathbf{w}_{i.,k}} = \text{card}(\{\mathbf{w}_{i.,k} : k \in [\![1;n[\![\}) = n-1$, *i.e.* a kth neighbor cannot also be a $(k+j)$th neighbor, where $j \in [\![-k+1; n-k[\![\backslash \{0\}$. Since the (row-) components of $\mathbf{W}_k$ cannot be a linear combination of themselves at different neighborhood orders, so is $\mathbf{W}_k$. $\square$

[a]Where $\frac{n(n-1)}{2}$ is the exact number of degrees of freedom over which a lattice can be uniquely defined, given that $\boldsymbol{\mathcal{D}}$, as a distance matrix, is intrinsically Hollow and symmetric.

As far as we know, whereas the oknn specification of $\mathbf{W}_k$ is the strict spatial counterpart of the k-order TS lag operator, $\mathbf{H}^k$, it had surprisingly never been proposed. The likely reason for this fact is the usual assumption of regular lattice, on which the autoregression structure superimposes.[40] Frequently seen as an issue, the irregularity of the lattice is the rational for this specification. Moreover, in realistic spatial configurations, the lattice regularity is the ex-

[40]In the regular lattice case, spatial lag operators differ from TS's by locating more than one neighbor for a given separating distance.

ception rather than the rule.

2.3.3 Involving oknn in spatial ARIMA process

The oknn specification for $\mathbf{W}_k$ possesses a high level of similarities with its TS counterpart, previously denoted as $\mathbf{H}^k$, *e.g.* they are non-normalized since originally row-stochastic, they are boolean permutation matrix and do not overlap with other lag orders. However, a major difference persists: the absence of the (linear) algebraic relationship that links lag orders between them, *e.g.* to the first one within an integer power transformation such that $\mathbf{H}^k = (\mathbf{H}_1)^k$ for $k = 1, ..., n-1$.[41] A direct consequence of this is the non-trivial choice of studying high-order polynomial in k spatial weights matrices $\mathbf{W}_k$ for $k = 1, ..., n-1$. To illustrate this point, consider a process with two orders of partial integration in $\mathbf{y}$. In the TS case, the second-order polynomial has the property that $\left(\mathbf{I} - \rho_1\mathbf{H}^1\right)\left(\mathbf{I} - \rho_2\mathbf{H}^2\right)\mathbf{y} = \left(\mathbf{I} - \rho_1\mathbf{H}^1 - \rho_2\mathbf{H}^2 + \rho_1\rho_2\mathbf{H}^3\right)\mathbf{y}$.[42] A contrario, in the SE case, it is (astronomically) unlikely that $\mathbf{W}_1\mathbf{W}_2 = \mathbf{W}_3$. As stated by Elhorst et al. (2012), "a logical implication of this view of modeling spatial dependence [...] implies that extending the first order model to include more that one spatial weights matrix requires that we consider the cross product term". However, in that case, what about considering the (explosive) combinatorial of the cross product with other matrices as well, *i.e.* the transposed cross product, the cross product of $\mathbf{W}_1\mathbf{W}_2$ with $\mathbf{W}_1$, $\mathbf{W}_2$, and so on ?[43] The questions that this poses are not dealt with in this study and remain a topic for future research.

One advantage of the oknn specification is that there is no need when involving *only one lag* to consider avoiding singularity or explosive processes by altering the parameter space definition. For example, if one considers a first order spatial autoregressive process (SAR), the pro-stationarity parameter space of the underlying autoregression coefficient –expressed in terms of the minimum and maximum eigenvalues of $\mathbf{W}_k$, respectively $\omega_{\min}$ and $\omega_{\max}$ – is $]\omega_{\min}^{-1}, \omega_{\max}[$. Note that for row-stochastic matrices one has $\omega_{\max}^{-1} = 1$, but no general result holds for $\omega_{\min}^{-1}$ (Anselin, 1982). However, when in addition to being row-stochastic, the lag operator is a permutation matrix as in the TS or oknn case, its eigenvalues necessarily lie on the unit circle,[44] which means that one actually has a general result in the oknn case, that is that $\omega_{\min}^{-1} \leq -1$. In the case of *multiple lags*, say, m, the definition of the admissible parameter space within $\mathbb{R}^m$ is less straightforward. As pointed out by Elhorst et al. (2012), the naive adoption of parameter space restricted such that $\sum_{i=1}^{m} |\rho_i| < 1$ "proves to be too restrictive".

[41] Which is the only reason why $\mathbf{W}_k$ is k-subscripted while $\mathbf{H}^k$ is k-superscripted since the beginning of this work.

[42] Putting aside the discussion about testing that $\rho_1\rho_2 + \rho_3 = 0$.

[43] Note that in the TS case, the number of possible unique combinations is finite, *i.e.* it is n, with $(\mathbf{H}_1)^n = \mathbf{0}_{n,n}$

[44] A permutation matrix can always be expressed as a product of independent rotation matrices with eigenvalues that lie on the unit circle, $e^{2ik\pi/n} \forall k \in \mathbb{Z}$, and thus whose non-imaginary part necessarily lies on the (rational) segment $[-1, 1]$. Note that we consider only the real part of eigenvalues since only those influence spatial filters' singularity (LeSage and Pace, 2009).

It is nevertheless far more desirable than the opposite situation in which this naive adoption would be not restrictive enough. Thus, this is both for the sake of caution and simplicity that we follow Lee and Liu (2010) and Badinger and Egger (2011) by making the assumption that the sum of the absolute values of the parameters (involved in the same spatial filter) should be less than one. Making this assumption also has the positive consequence of ensuring that each parameter, taken individually, complies with its own one-dimensional space definition.

With a little loss of generality since we neglect k-order polynomials, we put the focus on k-order spatial processes and we directly transpose in space the three-stage modeling approach of Box and Jenkins (1976) that consists of *(i)* identifying and selecting the model,[45] *(ii)* estimating the parameters and *(iii)* checking the model.

Know however that we do not use the usual TS notation with both nonseasonal and seasonal factors, *i.e.* ARIMA$(p,d,q) \times (P,Q,D)_s$,[46]. By analogy, we rather denote any model by using sets of lags, as ARIMA$(\{1,...,p,P\},\{1,...,d,D\},\{1,...,q,Q\})$. The reason for this notation choice is intended to be informative. As explained by Anselin (2002a, p.253-254), spatial pro-stationarity differences can only be performed partially compared to the TS case. Indeed, these differences must include autoregressive parameters – thus not implicitly set to 1 as in the TS case – so as to avoid singularities when handling models in their reduced form. Anselin mentions in addition that, unlike the TS case, any spatial filter estimation must be carried out jointly with that of the other model parameters. Put in technical terms, this means that during estimations, the two TS steps of stationarity differencing and model identification are merged together.

The development that follows is clarified in Section 2.B. We initiate the discussion with the traditional linear model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{v} \tag{2.23}$$

with $\mathbf{y}$ as a $n \times 1$ vector of observations on the endogenous variable, $\mathbf{X}$ as an $n \times k$ matrix of observations on exogenous variables, $\boldsymbol{\beta}$ as a $k \times 1$ vector of coefficients and $\mathbf{v}$ as a homoskedastic and non-autocorrelated $n \times 1$ vector of disturbances, such that $E[\mathbf{v}\mathbf{v}'] = \sigma^2_{\mathbf{v}}\mathbf{I}$.

If the disturbance terms of the model are more forcefully assumed i.i.d. normal with mean zero, *i.e.* $\mathbf{v}|\mathbf{X} \sim \mathcal{N}(0, \sigma^2_{\mathbf{v}}\mathbf{I})$, its multivariate density is

$$\mathcal{L}(\mathbf{v}) = (2\pi\sigma^2)^{-\frac{n}{2}} e^{-\frac{1}{2\sigma^2_{\mathbf{v}}}(\mathbf{v}'\mathbf{v})} \tag{2.24}$$

The likelihood for $\mathbf{y}$ conditional on $\mathbf{X}$ is then

$$\mathcal{L}(\mathbf{y}|\mathbf{X}) = \mathcal{L}(\mathbf{v}) \left|\frac{\partial \mathbf{v}}{\partial \mathbf{y}}\right| \tag{2.25}$$

[45] It is interesting to note that this phase may as well easily be thought of as the identification/selection procedure of the full autocorrelation structure of the variables involved in the data-generating process.

[46] The terms (p,d,q) and $(P,D,Q)_s$ respectively give the orders of the nonseasonal and seasonal parts, where s is the number of observations in a seasonal cycle, *e.g.* 12 for monthly series, 4 for quarterly series, 7 for daily series with day-of-week effects.

where $|(\partial\mathbf{v}/\partial\mathbf{y})|$ is the absolute value of the determinant formed from the $n \times n$ matrix of partial derivatives of the elements of $\mathbf{v}$ with respect to the elements of $\mathbf{y}$.

One may then think that eq.(2.25) should be augmented to entail different types of spatial effects. Considering these effects can be done through different spatial model specifications, the most notorious (and elementary) of which are the spatial autoregressive model (SAR), the spatial error model (SEM), the spatial moving average model (SMA) and *different combinations of those.*[47] However the way of combining them is not unique. This is why, as mentioned previously, the chosen combination and its analytic development is driven by the first phase of identification/selection. We recall that even if the step of pro-stationarity differencing presently initiates the selection phase – as classically performed in the TS case –, it will always be re-carried out jointly with the estimation of the other model parameters. This is clarified in the following respects.

Identifying the order and the structure of differencing

In the TS case, the first step is about making sure that the variables are stationary. In the SE context, this means assuming that the data-generating process (DGP) can be specified as a SEM, or less particularly, as a spatial Durbin model (SDM). Indeed, the SEM is a special case of SDM in which both $\mathbf{y}$ and $\mathbf{X}$ are assumed to be (partially) identically integrated, *i.e.* the difference structure is assumed to be the same on both the exogenous and endogenous variables (Anselin, 1980; Elhorst and Vega, 2013; Gibbons and Overman, 2012). However, the declination of the SEM into a SDM within the search of the order of differencing is not undertaken in this study.[48] Moreover, note that this subsection is – for the sake of simplicity – more about *differencing structure* than about differencing order. Actually, we chose to restrict the latter to the binary case of differencing or not differencing. Put differently, we chose to restrict the latter to contingently consider *how relatively high* or *how absolutely high* model's variables are over the urban space. Thus, still remains to determine the spatial structure of this first difference, if any. For the sake of clarity, let first denote

$$\begin{aligned}
\mathbf{G}(\boldsymbol{\gamma}) &= \sum_{i \,\in\, \mathcal{I}_{\gamma}} \gamma_i \mathbf{W}_i \\
\boldsymbol{\Gamma}(\boldsymbol{\gamma}) &= \mathbf{I} - \mathbf{G}(\boldsymbol{\gamma}) \\
\mathbf{y}_{\gamma} &= \boldsymbol{\Gamma}(\boldsymbol{\gamma})\mathbf{y} \\
\mathbf{X}_{\gamma} &= \boldsymbol{\Gamma}(\boldsymbol{\gamma})\mathbf{X} \\
\boldsymbol{\Omega}_{\gamma} &= \sigma^2_{\mathbf{u}} \left(\boldsymbol{\Gamma}'\boldsymbol{\Gamma}\right)^{-1}
\end{aligned}$$

[47] At the combo-top of which one has the general nesting spatial model (GNS). See Elhorst and Vega (2013, p.24, Fig.1) for a general-to-specific three-like comparison of different spatial econometric model specifications. Incidentally, the spatial moving average model is a notable absentee.

[48] The declination could be performed one step further by considering how a SDM boils down to a spatial lag (only) of $\mathbf{X}$ (SLX) model. Although this is not related to the notion of differencing.

where $\mathcal{I}_{\gamma}$ is the set of lag orders that are considered for estimation, $\gamma_i \ \forall i \in \mathcal{I}_{\gamma}$ is the i-th spatial autoregressive parameter involved in the auto-generation of $\mathbf{v}$, $\boldsymbol{\gamma}$ is the Card$(\mathcal{I}_{\gamma}) \times 1$ vector formed over these autoregressive parameters and $\mathbf{W}_i$ is the i-associated $n \times n$ spatial oknn-lag operator.

Henceforth it is assumed that $\mathbf{v} = \mathbf{G}(\boldsymbol{\gamma})\mathbf{v} + \mathbf{u}$ with $\mathbf{u} \sim \mathcal{N}(0, \sigma^2_{\mathbf{u}}\mathbf{I})$ and $\mathbf{v} \sim \mathcal{N}(0, \boldsymbol{\Omega}_{\boldsymbol{\gamma}})$ [see Subsection 2.B.1 for details]. This leads to rearrange eq.(2.23) as

$$\mathbf{y}_{\boldsymbol{\gamma}} = \mathbf{X}_{\boldsymbol{\gamma}}\boldsymbol{\beta} + \mathbf{u} \tag{2.26}$$

Substituting closed form solutions from the first order conditions for the parameters $\boldsymbol{\beta}$ and $\sigma^2_{\mathbf{u}}$ leads eq.(2.25) to be concentrated on $\boldsymbol{\gamma}$ as follows

$$\mathcal{L}(\boldsymbol{\gamma}) = \left(\frac{2\pi e}{n}\mathbf{u}'\mathbf{u}\right)^{-\frac{n}{2}} \left|\boldsymbol{\Gamma}(\boldsymbol{\gamma})\right| \tag{2.27}$$

Recalling that eq.(2.27) reduces to eq.(2.25) for $\mathcal{I}_{\gamma} = \varnothing$.

Identifying the numbers of AR or MA terms

One may then have reasons to think that, specified as in eq.(2.26) – or as in eq.(2.23) if $\mathcal{I}_{\gamma} = \varnothing$ –, one omits observable and/or non-observable characteristics at work in the DGP. On the one hand, one of those observable characteristics is very likely to be the long-distance auto-determining nature of the spatially-filtered regressand, $\mathbf{y}_{\boldsymbol{\gamma}}$. On the other hand, if the long-distance auto-regressive nature of the error vector is already considered by the SEM specification, a non-observable characteristic that is likely to remain is its short-distance autocorrelation. For the sake of clarity relatively to the SAR terms, let denote

$$\mathbf{Q}(\boldsymbol{\rho}) = \sum_{i \,\in\, \mathcal{I}_{\boldsymbol{\rho}}} \rho_i \mathbf{W}_i$$
$$\mathbf{P}(\boldsymbol{\rho}) = \mathbf{I} - \mathbf{Q}(\boldsymbol{\rho})$$
$$\mathbf{y}_{\boldsymbol{\gamma},\boldsymbol{\rho}} = \mathbf{P}(\boldsymbol{\rho})\mathbf{y}_{\boldsymbol{\gamma}}$$

where $\mathcal{I}_{\boldsymbol{\rho}}$ is the set of lag orders that are considered for estimation, $\rho_i \ \forall i \in \mathcal{I}_{\boldsymbol{\rho}}$ is the i-th spatial autoregressive parameter involved in the auto-generation of $\mathbf{y}_{\boldsymbol{\gamma}}$ and $\boldsymbol{\rho}$ is the Card$(\mathcal{I}_{\boldsymbol{\rho}}) \times 1$ vector formed over these autoregressive parameters.

Identically for the SMA terms, let denote

$$\begin{aligned}
\mathbf{L}(\boldsymbol{\lambda}) &= \sum_{i\,\in\,\mathcal{I}_{\boldsymbol{\lambda}}} \lambda_i \mathbf{W}_i \\
\boldsymbol{\Lambda}(\boldsymbol{\lambda}) &= \mathbf{I} + \mathbf{L}(\boldsymbol{\lambda}) \\
\mathbf{y}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}} &= \boldsymbol{\Lambda}(\boldsymbol{\lambda})^{-1}\mathbf{y}_{\gamma,\boldsymbol{\rho}} \\
\mathbf{X}_{\gamma,\boldsymbol{\rho}} &= \boldsymbol{\Lambda}(\boldsymbol{\lambda})^{-1}\mathbf{X}_{\gamma} \\
\boldsymbol{\Omega}_{\boldsymbol{\lambda}} &= \sigma^2_{\mathbf{r}}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^{'} \\
\boldsymbol{\Omega}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}} &= \mathbf{Q}\mathrm{cov}[\mathbf{y}_{\gamma}, \mathbf{y}_{\gamma}]\mathbf{Q}^{'} \\
&\quad + 2\mathbf{Q}\mathbb{E}[\mathbf{y}_{\gamma}\mathbf{r}^{'}]\boldsymbol{\Lambda}^{'} \\
&\quad + \boldsymbol{\Omega}_{\boldsymbol{\lambda}}
\end{aligned}$$

where $\mathcal{I}_{\boldsymbol{\lambda}}$ is the set of lag orders that are considered for estimation, λ_i $\forall i \in \mathcal{I}_{\boldsymbol{\lambda}}$ is the i-th spatial parameter involved in the local auto-regression of residuals and $\boldsymbol{\lambda}$ is the $\mathrm{Card}(\mathcal{I}_{\boldsymbol{\lambda}}) \times 1$ vector formed over these autoregressive parameters. Note that the MA filter, $\boldsymbol{\Lambda}(\boldsymbol{\lambda})$, is expressed following Anselin as a positive sum.[49]

Without loss of generalities, considering simultaneously the SAR and SMA specific terms while still opting for a SEM($\mathcal{I}_{\gamma}$) is equivalent to suspecting that $\mathbf{u}$ entails $\mathbf{Q}(\boldsymbol{\rho})\mathbf{y}_{\gamma} + \boldsymbol{\Lambda}(\boldsymbol{\lambda})\mathbf{r}$ with $\mathbf{r} \sim \mathcal{N}(0, \sigma^2_{\mathbf{r}}\mathbf{I})$, $\mathbf{u} \sim \mathcal{N}(0, \boldsymbol{\Omega}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}})$ and $\mathbf{v} \sim \mathcal{N}\left(0, \boldsymbol{\Gamma}^{-1}\boldsymbol{\Omega}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}}\boldsymbol{\Gamma}^{-1'}\right)$ [see Subsection 2.B.2 for details]. Note that the inclusion of SAR terms is also related to the identification of the structure of partial differencing, since it relates to only considering $\mathbf{y}_{\gamma}$ independently from the other variables of the model. As pointed out by Anselin (2002b, p.254), "this can be interpreted as a way to clean $\mathbf{y}_{\gamma}$ of the effects of spatial correlation, while maintaining the [...] estimates for $\boldsymbol{\beta}$". This leads to rearrange eq.(2.26) as

$$\mathbf{y}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}} = \mathbf{X}_{\gamma,\boldsymbol{\lambda}}\boldsymbol{\beta} + \mathbf{r} \tag{2.28}$$

and eq.(2.27) to be concentrated on $\boldsymbol{\gamma}$, $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ as follows

$$\mathcal{L}(\boldsymbol{\theta}) = \left(\frac{2\pi e}{n}\mathbf{r}^{'}\mathbf{r}\right)^{-\frac{n}{2}} \frac{\left|\boldsymbol{\Gamma}(\gamma)\right| \left|\mathbf{P}(\boldsymbol{\rho})\right|}{\left|\boldsymbol{\Lambda}(\boldsymbol{\lambda})\right|} \tag{2.29}$$

where $\boldsymbol{\theta}$ denotes the $(\mathrm{Card}(\mathcal{I}_{\gamma}) + \mathrm{Card}(\mathcal{I}_{\boldsymbol{\rho}}) + \mathrm{Card}(\mathcal{I}_{\boldsymbol{\lambda}})) \times 1$ hyperparameter that vertically stacks $\boldsymbol{\gamma}$, $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$, such that $\boldsymbol{\theta} = \left[\boldsymbol{\gamma}^{'}, \boldsymbol{\rho}^{'}, \boldsymbol{\lambda}^{'}\right]^{'}$.

[49] Indeed, probably following the convention introduced by Box and Jenkins, it is very common to read authors who express it exhibiting a minus sign. We prefer the form with a positive sum since it reminds a kind of truncated Leontief inverse, from which the interpretation in terms of auto-regression order is (subjectively) direct.

Dispersion for the parameters

In addition to the statistical inference required during the phase of selection/identification of the model, we recall the need to conduct inference on the budget shares, $\widehat{\delta^H}$ and $\widehat{\delta^a}$, that are set in GEMSE. The estimated values of these parameters derive from the estimated components of $\widehat{\boldsymbol{\beta}}$ – whose functional relations are explicited in subsection 2.3.4.

The models that are subject to selection in the present work are likely to involve a large number of parameters whose distributions, probably not symmetrical, are cumbersome to derive analytically. This is why in addition to the (normal-approximation-based) observed confidence intervals,[50] (non-adjusted and adjusted) bootstrap percentile intervals are provided. However, the existence of fixed spatial weight matrices prohibits the use of traditional bootstrapping methods.[51] So as to compute (normal approximation or percentile-based) confidence intervals for all the parameters, be them derived like the budget shares, we use a special case of bootstrap method, namely Lin et al. (2007)'s hybrid version of residual-based recursive wild bootstrap.[52] This method is particularly appropriate since it *(i)* "accounts for fixed spatial structure and heteroscedasticity of unknown form in the data" and *(ii)* "can be used for model identification (pre-test) and diagnostic checking (post-test) of a spatial econometric model". As mentioned above, non-adjusted percentile intervals as well as bias-corrected and accelerated (BCa) percentile intervals (Efron and Tibshirani, 1993) are provided. An issue-based summary of the calculation methods of all the intervals that are presented in this work follows.

Confidence intervals that are based on the observed Fisher information matrix rely on the symmetry of the distribution of the parameters under question. The (observed) standard errors and confidence bounds of these parameters, respectively denoted by $\mathbf{se}[\widehat{\boldsymbol{\Theta}}]^{\text{obs}}$ and $\mathbf{c}[\alpha]_{\pm}^{\text{obs},\boldsymbol{\Theta}}$,[a] are computed as

$$\mathbf{se}[\widehat{\boldsymbol{\Theta}}]^{\text{obs}} = \text{diag}\left[\left(-\frac{\partial \ln \mathcal{L}^{\text{full}}}{\partial^2 \boldsymbol{\Theta}}\bigg|_{\widehat{\boldsymbol{\Theta}}}\right)^{-1}\right]^{1/2} \quad ; \quad \mathbf{c}[\alpha]_{\pm}^{\text{obs},\boldsymbol{\Theta}} = \widehat{\boldsymbol{\Theta}} \pm \Psi_{n-k}^{-1}[\alpha/2]\mathbf{se}[\widehat{\boldsymbol{\Theta}}]^{\text{obs}}$$

where $\widehat{\boldsymbol{\Theta}} = \left[\widehat{\boldsymbol{\beta}}', \widehat{\boldsymbol{\theta}}', \widehat{\sigma^2_{\mathbf{r}}}\right]'$, α is the chosen probability of making type I error and Ψ_{n-k} is the student distribution function with $n-k$ degrees of freedom. To compute the observed standard errors of $\widehat{\delta^H}$ and $\widehat{\delta^a}$, one needs to redefine $\ln \mathcal{L}^{\text{full}}$ in terms of these two parameters

[50] The term "observed" stands for – as it reads in open-source codes –, the computation of the log-likelihood Fisher Information matrix evaluated/observed at its estimated maximum.

[51] Seminally without dependence (Bradley, 1979) or with (nilpotent anisotropic) temporal dependence by moving blocks (Kunsch, 1989). See Gonçalves and Politis (2011) for a short but very instructive review of bootstrap methods for TS.

[52] The way to mimick heteroskedasticity of unknown form is detailed and tested in Davidson and Flachaire (2008).

such that *(i)* $\boldsymbol{\delta} = f(\boldsymbol{\beta})$, where f is a vector isomorphic function, *(ii)* $\boldsymbol{\Xi} = \left[f^{-1}(\boldsymbol{\delta})', \boldsymbol{\theta}', \sigma^2_{\mathbf{r}}\right]'$, *(iii)* $\boldsymbol{\xi} = \left[\boldsymbol{\delta}', \boldsymbol{\theta}', \sigma^2_{\mathbf{r}}\right]'$ and *(iv)* $\widehat{\boldsymbol{\xi}} = \left[f(\widehat{\boldsymbol{\beta}})', \widehat{\boldsymbol{\theta}}', \widehat{\sigma^2_{\mathbf{r}}}\right]'$. These observed standard errors and confidence bounds, respectively denoted by $\mathbf{se}[\widehat{\boldsymbol{\Xi}}]^{\text{obs}}$ and $\mathbf{c}[\alpha]^{\text{obs},\boldsymbol{\Xi}}_{\pm}$, are

$$\mathbf{se}[\widehat{\boldsymbol{\Xi}}]^{\text{obs}} = \text{diag}\left[\left(-\frac{\partial \ln \mathcal{L}^{\text{full}}\left(\boldsymbol{\Xi}\right)}{\partial^2 \boldsymbol{\xi}}\bigg|_{\widehat{\boldsymbol{\xi}}}\right)^{-1}\right]^{1/2} \quad ; \quad \mathbf{c}[\alpha]^{\text{obs},\boldsymbol{\Xi}}_{\pm} = \widehat{\boldsymbol{\Xi}} \pm \Psi^{-1}_{n-k}[\alpha/2]\mathbf{se}[\widehat{\boldsymbol{\Xi}}]^{\text{obs}}$$

Unlike the above two approaches, mutually exclusive, confidence intervals that are based on bootstrap sampling distributions allow for the parallel estimation of the confidence bounds of $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$. Define $\widehat{\boldsymbol{\Theta}}^{\text{all}}$ as a vector that vertically stacks all the parameters, be them directly estimated like $\boldsymbol{\beta}$ or indirectly like $\boldsymbol{\delta}$, such that $\widehat{\boldsymbol{\Theta}}^{\text{all}} = \left[\widehat{\boldsymbol{\beta}}', \widehat{\boldsymbol{\delta}}', \widehat{\boldsymbol{\theta}}', \widehat{\sigma^2_{\mathbf{r}}}\right]'$. BCa percentile intervals are used to turn the bootstrap sampling distribution of $\widehat{\boldsymbol{\Theta}}^{\text{all}}$ into its analogous unbiased and constant variance version. The transformation implies two independent correction factors, commonly denoted by $\mathbf{z}$ and $\mathbf{a}$,[b] which consider respectively the bias and the skewness of the bootstrap sampling distribution. $\mathbf{z}$ and $\mathbf{a}$ are defined as

$$\mathbf{z} = \Phi^{-1}\left(\frac{1}{r}\overset{r}{\underset{b}{\#}}\left(\widehat{\boldsymbol{\Theta}}^{\text{all}}_b < \widehat{\boldsymbol{\Theta}}^{\text{all}}\right)\right) \quad ; \quad \mathbf{a} = \frac{1}{6}\frac{\sum_{i=1}^{n}\left(\widehat{\boldsymbol{\Theta}}^{\text{all}}_{-i} - \overline{\boldsymbol{\Theta}}^{\text{all}}\right)^3}{\left(\sum_{i=1}^{n}\left(\widehat{\boldsymbol{\Theta}}^{\text{all}}_{-i} - \overline{\boldsymbol{\Theta}}^{\text{all}}\right)^2\right)^{3/2}}$$

where, on the one hand, b, r and Φ respectively stand for the bootstrap replication index, the total number of replications and the cumulative standard normal distribution function and, on the other hand, $\widehat{\boldsymbol{\Theta}}^{\text{all}}_{-i} - \overline{\boldsymbol{\Theta}}^{\text{all}}$ is the mean deviation of the ith jackknife estimate of (the true) $\boldsymbol{\Theta}^{\text{all}}$. Note that, if any,[c] oknn matrices involved during the n jackknife estimation procedures are resampled as well by *(i)* removing their ith row and column and *(ii)* computing the corresponding oknn auto-correlation structure. Finally, define the correcting percentile (confidence interval) function, $\mathbf{c}[\alpha]^{\text{BCa}}_{\pm}$, as

$$\mathbf{c}[\alpha]^{\text{BCa}}_{\pm} = \mathcal{Q}\left(\Phi\left(\mathbf{z} + \frac{\mathbf{z} \pm \Phi^{-1}\left(\alpha/2\right)}{1 - \mathbf{a}\left(\mathbf{z} \pm \Phi^{-1}\left(\alpha/2\right)\right)}\right)\right)$$

where $\mathcal{Q}$ is the empirical quantile function that returns $\mathbf{c}[\alpha]_-$ or $\mathbf{c}[\alpha]_+$ such that, respectively, $\Pr(\boldsymbol{\Theta}^{\text{all}} < \mathbf{c}[\alpha]_-) = \alpha/2$ or $\Pr(\mathbf{c}[\alpha]_+ < \boldsymbol{\Theta}^{\text{all}}) = \alpha/2$, depending on whether or not $\mathbf{c}[\alpha]_{\pm}$ are BCa-based.

Thus, the confidence and percentile intervals that are computed in this work are four in number, above denoted as $\mathbf{c}[\alpha]^{\text{obs},\boldsymbol{\Theta}}_{\pm}$, $\mathbf{c}[\alpha]^{\text{obs},\boldsymbol{\Xi}}_{\pm}$, $\mathbf{c}[\alpha]_{\pm}$ and $\mathbf{c}[\alpha]^{\text{BCa}}_{\pm}$. Those will be presented for each coefficient in table format and referred respectively to as *D1*, *D2*, *D3* and *D4*, with D standing for (underlying) *Distribution*. Moreover, note that the definitions of the (BCa and non-BCa) percentile intervals are declined to be used for hypothesis testing. The aim is to consider the potential asymmetry of coefficients distributions. To do so, we first compute the one-sided p-value α^* that makes $\mathbf{c}[\alpha^*]_-$ equalize the value tested under the null, $\tilde{\theta}$. Formally,

$$\alpha^* = \underset{\alpha \in [0,1]}{\text{argeq}} \left(\mathrm{c}[\alpha^*]_- = \tilde{\theta} \right) = \Pr(\theta < \tilde{\theta})$$

We then compute $2\min(\alpha^*, 1-\alpha^*)$ to get the two-sided p-value associated to the bilateral version of the test.[d]

[a]Where **se** and **c** are denoted by bold lowercase symbols to signal that they have the same dimension as their (column) vector input, with which they have a direct component-to-component mapping.

[b]Where, as for **se** and **c**, **z** and **a** are denoted by bold lowercase symbols to signal that they have the same dimension as their (column) vector input, with which they have a direct component-to-component mapping.

[c]Indeed, if $\mathcal{I}_{\boldsymbol{\gamma}} \cup \mathcal{I}_{\boldsymbol{\rho}} \cup \mathcal{I}_{\boldsymbol{\lambda}} = \varnothing$, eq.(2.28) boils down to eq.(2.23).

[d]An interesting discussion about taking twice the minimum as it stands can be found at `https://stats.stackexchange.com/a/140517`. As it reads there, the main justification of this approach is about complying with the fact that *cumulative distribution functions are invariant to order-preserving transformations*.

2.3.4 An application to the metropolitan area of Paris using the specification oknn

First, recall that in the scope of the econometric study *only*, one denotes the number of place by n instead of N_j as in the rest of the thesis.[53] Then, recall the expression of the indirect utility function in its private-vehicle specified form, *i.e.* considering eq.(1.102) with $u^0_{j,i} = u^0_j$, that is

$$u_{j,i} = u_j^{0,2} R_{j,i}^{-\delta^H} \left(\frac{T}{d_{j,i}} v_{j,i} \right)^{\delta^a} \left(Y_j - P^a_{j,i}(a_{j,i}) \right)^{1-\delta^a}$$

where $u_j^{0,2} = u_j^0 u_j^2$. Assuming that the metropolitan area of Paris is at equilibrium, *i.e.* $u_{\text{Paris},i} = u_{\text{Paris}}\ \forall i \in [\![1; N_{\text{Paris}}]\!]$, and rearranging the expression to obtain housing rents per square meter on the LHS returns – one takes $j =$ Paris and omits the index in what follows for the sake of clarity,

$$\ln R_i = \beta_0 + \beta_1 \ln (Y - P^a_i) + \beta_2 \ln \left(\frac{T}{t_i} \right) + \varepsilon_i \tag{2.30}$$

with $\delta^H = 1/(\beta_1 + \beta_2)$ and $\delta^a = \beta_2/(\beta_1 + \beta_2)$.

Georeferenced Data

In Figure 2.6, we are shown that the number of places that are considered for the econometric study, 300, is not the one involved in deterministic simulations, 428. The reason of this

[53]In the major part of this thesis, n_j and n respectively stand for the number of firms in agglomeration j and in the country of interest.

difference is two-fold: first, it is related to the different position-filters that have been used. Second, it is due to computation time concerns.

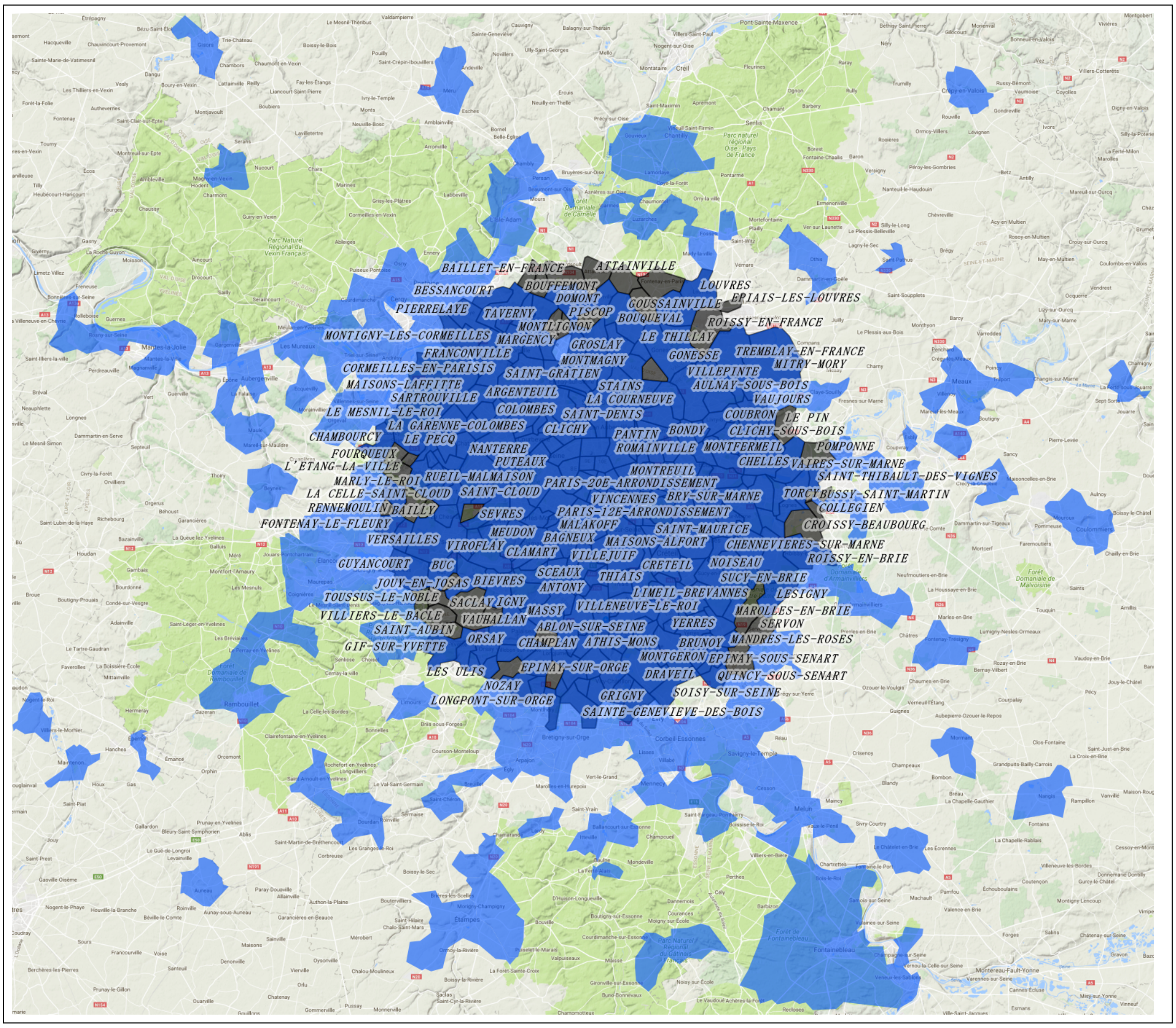


Figure 2.6: The 428 places considered in the simulation (blue) *versus* the 300 ones considered in the econometric study.

The 428 places that are considered during the simulation have been subject to the double filter of income availability on the one hand and of 95% population representativity on the other hand, which led to the emergence of geographic islands. On the contrary, the places that are considered in the econometric study are simply the 300 nearest ones to Place de l'Hôtel de ville.[54]

The bootstrap resampling process in itself takes time, which is even more true when implementing Lin et al. (2007)'s, whose "main disadvantage is the high computing cost of large matrix inversion". Regardless of the chosen number of resamplings, SARIMA models are intrinsically costly to compute. Putting aside the cases of models based on long-distance autoregression

[54] Wich may have caused the emergence of islands in other spaces, *e.g.* the plane formed by the two dimensions of populations and income.

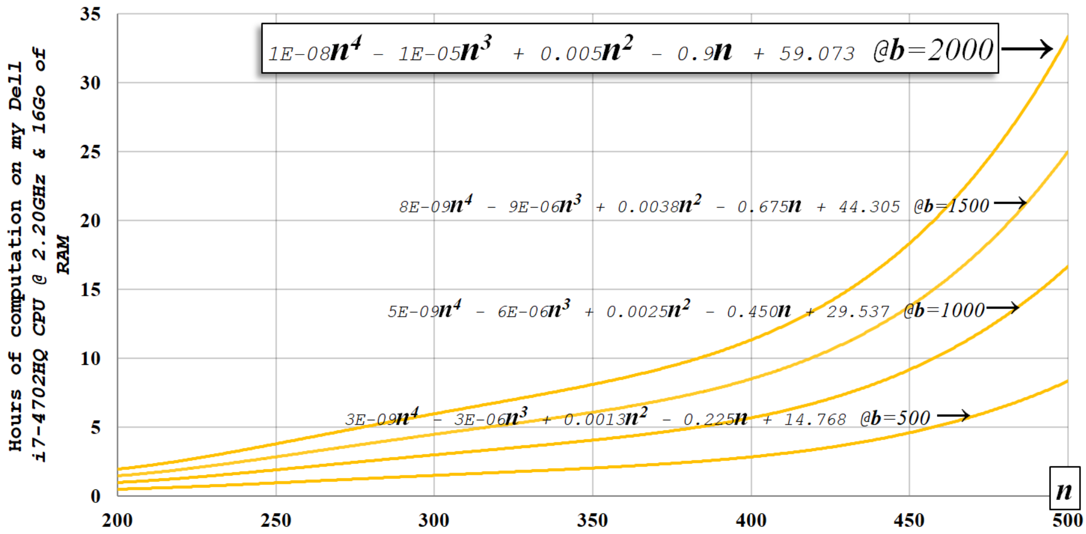


Figure 2.7: Polynomial Times of n

processes, *i.e.* SARIMA$(\mathcal{I}_\rho, \varnothing, \varnothing)$ and/or SARIMA$(\varnothing, \mathcal{I}_\gamma, \varnothing)$,[55] – that involves no $n \times n$ matrix inversions –, full SARIMA specifications analogous to that that are performed in TS are costly in terms of computation because of the MA terms. Indeed the consideration of the MA coefficients cannot be decoupled from the inversion of their framing $n \times n$ spatial filter, $\mathbf{\Lambda}(\boldsymbol{\lambda})$, that is performed as many times as needed by the maximization of the log-likelihood. In place of the simplex search algorithm (Nelder and Mead, 1965) chosen for the maximization, techniques such as LeSage (1999, p.59-60)'s lattice logarithmic search could be envisaged when dealing with little sets of parameters, but has the disadvantage of becoming difficult to handle as the sets of parameters increase in size.[56] To finally justify the choice of 300 for the sample size, Figure 2.7 illustrates how costly the estimation procedure of a SARIMA$(\{1\}, \{1\}, \{1\})$ is. We are shown that the time complexities of the input-size, n, and of the number of bootstrap samples, b, respectively are $O(n^4)$ and $O(b)$. b is set to 2000 in this work.

On these bases, we are shown in Figure 2.8 the two geographies of monthly housing rents and travel times, respectively bot-scrapped from *LaCoteImmo*[57] and *GoogleMaps*. As can be easily seen from these two maps, housing rents and travel times both possess an auto-correlation of the same nature, *i.e.* following a long-distance process with a positive sign. Considered together under the (credible) assumption of no spurious relation, the two scatter graphs of these variables clearly illustrate the housing/transport trade-off given the opposite sign of their (road-distance supported) trends. The geography of rents - monotonously decreasing from the

[55]Note that in the case in which the autoregression is in the endogenous variable, Anselin (1988) suggests some additional tricks to minimize at most the number of objects to compute when maximizing the log-likelihood.

[56]First, note that the denomination that LeSage (1999) uses for his grid search is "multiple pass grid search" and second, it is primarily designed to avoid the repeated Jacobian evaluation involved when brute force maximizing the log-likelihood, be it SAR- or SEM- related thus. By citing this, we simply assume that such a techniques could also have been used to minimize the number of $n \times n$ matrix inversions needed when maximizing the log-likelihood of SMA models.

[57]LaCoteImmo website is available at, *e.g.* `http://www.lacoteimmo.com/prix-de-l-immo/location/pays/france.htm`.

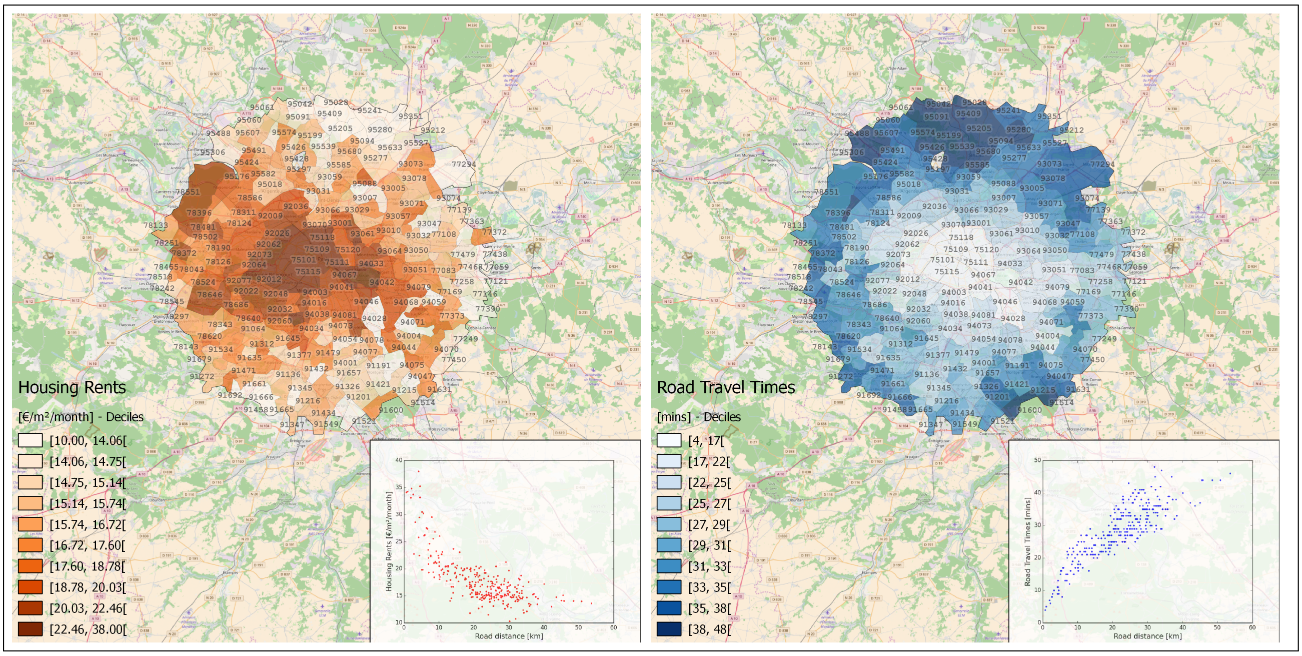


Figure 2.8: Housing rents, R_i, and Road travel times, t_i

center to the periphery – confirms that the choice of representing the metropolitan area of Paris with a monocentric LUTI-based spatial structure[58] is non-abusive, which suggests that the existence of a unique utility level of (urban) equilibrium covering the whole area is empirically valid.

In Figure 2.9, one is shown the two geographies of incomes net of transport cost and accessibility. Income, Y, derives on average[59] from income net of personal tax available at the level of municipalities in 2011, and is taken from *Direction Générale des Finances Publiques (DGFiP)*'s database.[60] In addition, note that Y is the result of a readjustment performed on incomes net of personal tax, originally calculated per fiscal household.[61] P_i^a, computed as shown in eq.(1.25), is composed of a distance-variable part, that is $c_a p_a 251 d_i T/t_i$ where d_i is the road distance between place i and Place de l'Hôtel de ville and 251 stands for the number of business days used (and assumed constant over the prospective horizon) in GEMSE. c_a and p_a are set to their 2011 value, respectively 0.0678liter/km and 1.518€/liter. Table 2.1 reports the summary statistics of the variables implied in the study, sometimes implicitly as the private vehicle speeds are. This concludes the presentation of data used in the econometric study.

[58] Recalling that the only numerical/mathematical objects that embody the spatial structure are the two geographies of rents and travel times, hence the expression "LUTI-based spatial structure".

[59] Indeed, the average is considered since GEMSE does not space-differentiate incomes within a given urban area, hence $Y_i = Y \ \forall i = 1, ..., N_{\text{Paris}}$.

[60] Available at https://gemse.alwaysdata.net/static/gemse/FRANCE/impot_rev_2011.xlsx.

[61] In INSEE's 2011-data, income net of personal tax is denoted by RNETFF11, while the number of households is denoted by NBFF11. This adjustement is also performed so as to comply with the nature of agents that are described at the urban level in GEMSE. Also recall that incomes that are involved in GEMSE's calibration are not directly taken from any database, but come from the spatial disaggregation of the GDP [see eq.(2.1) and eq.(2.4)].

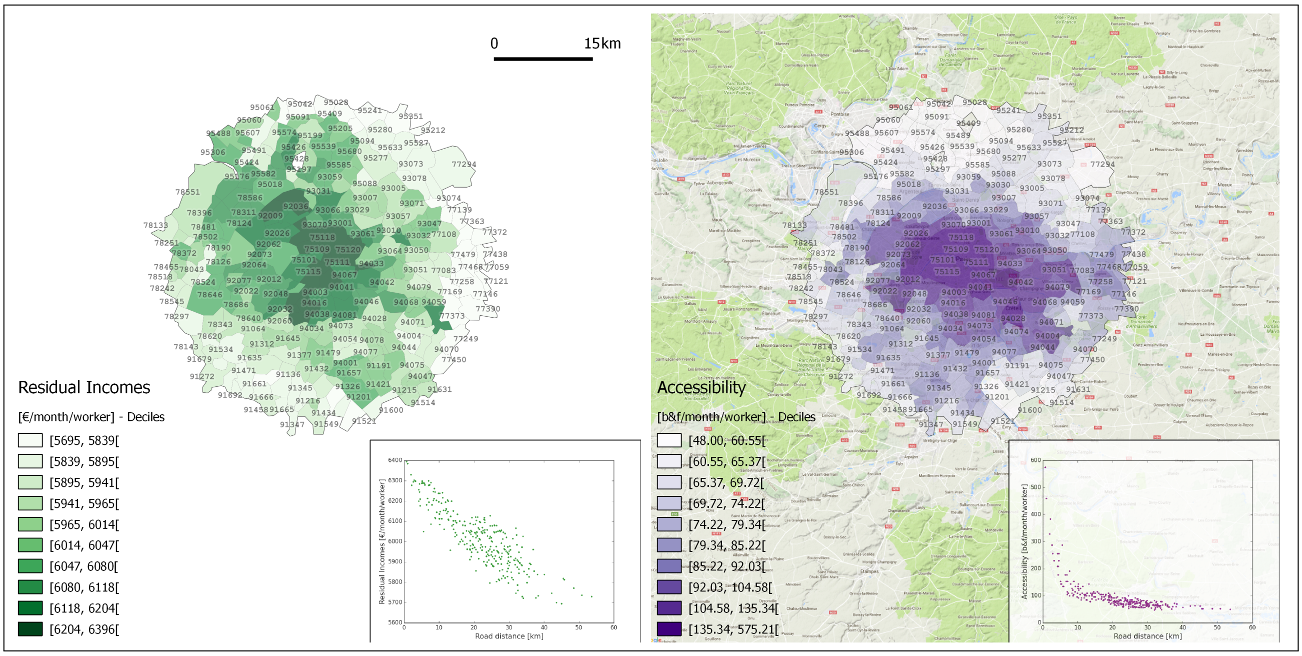


Figure 2.9: Residual incomes, $Y - P_i^a$, and Accessibility, T/t_i

Table 2.1: Summary statistics

$i = 1, ..., 300$	Mean	St. dev.	Min	Max
R_i [€/m²/year]	213.49	52.87	123.96	455.94
d_i [km]	22.10	9.72	0.70	53.60
t_i [mins]	28.51	8.34	4.00	48.00
v_i [km/h]	44.59	12.43	10.50	72.97
P_i^a [€/year]	6000.00	1673.26	1412.89	9819.28
$Y - P_i^a$ [€/year]	72160.44	1673.26	68341.16	76747.56
$251T/t_i$ [b&f/year]	1123.08	669.44	575.21	6902.50

SARIMA$(\mathcal{I}_{\boldsymbol{\rho}}, \mathcal{I}_{\boldsymbol{\gamma}}, \mathcal{I}_{\boldsymbol{\lambda}})$ model *(i)* Identification

We initiate the model identification with Table 2.2 that presents a first round estimation of eq.(2.30), with neither spatial components nor modeled residuals autocorrelation.[62] Let's figure out whether there is a need to explicit the structure of autoregression or/and autocorrelation.

In Figure 2.10, one is shown four maps of SARIMA$(\varnothing, \varnothing, \varnothing)$'s residuals for different definition of quantiles, that are, from the upper left to the lower right corner, biciles, terciles, quartiles and deciles. From this observation, one sees that the autocorrelation of residuals is very likely

[62] Qualified as unmodeled, or equivalently, as modeled with an $n \times n$ identity matrix in lieu of correlation structure. Recall how important the assumptions of homoscedastic and unautocorrelated residuals are, since coefficients standard errors (and significance tests) directly inherit their reliability from the non-rejection of those. Indeed, in the general case (and $\mathbf{X}$ considered fixed), recall that $\mathbf{se}[\widehat{\beta}]^2 = \text{diag}\left[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\widehat{\boldsymbol{\Omega}}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\right]$ where $\widehat{\boldsymbol{\Omega}}$ is the (presumably misspecified) square correlation structure of residuals.

Table 2.2: SARIMA $(\varnothing, \varnothing, \varnothing)$

	Coeff.	Dist.[a]	Std. Err.	P>|z|	[95% C.I.]	
β_0	-37.5566	D1	3.919830	0.000	-45.2708	-29.8425
		D2	.	.	.	.
		D3	3.952075	0.000	-45.5031	-29.6167
		D4	4.049645	0.000	-45.6052	-29.7309
β_1	3.6404	D1	0.357978	0.000	2.9359	4.3449
		D2	.	.	.	.
		D3	0.360783	0.000	2.9101	4.3689
		D4	0.368348	0.000	2.9339	4.3778
β_2	0.3133	D1	0.022422	0.000	0.2692	0.3574
		D2	.	.	.	.
		D3	0.022358	0.000	0.2692	0.3562
		D4	0.021910	0.000	0.2684	0.3542
δ^H	0.2529	D1	.	.	.	.
		D2	0.022155	0.000	0.2093	0.2965
		D3	0.022840	0.000	0.2148	0.3073
		D4	0.023482	0.000	0.2143	0.3064
δ^a	0.0792	D1	.	.	.	.
		D2	0.010986	0.000	0.0576	0.1009
		D3	0.011251	0.000	0.0602	0.1057
		D4	0.011360	0.000	0.0602	0.1047
$\sigma^2_{\mathbf{r}}$	0.0146	D1	0.001189	.	.	.
		D2	.	.	.	.
		D3	0.001342	.	0.0118	0.0171
		D4	0.001350	.	0.0120	0.0173

[a] C.I.-related objects derive from four types of distribution:
*D1 stands for normal-approximation-based distribution.
*D2 is as D1, but derives from an information matrix expressed in terms of δ^H and δ^a.
*D3 stands for bootstrap-based distribution. Associated std. are computed over bootstrap distributions.
*D4 stands for BCa bootstrap-based distribution. Associated std. are deduced by reversing the symmetry-based C.I. formula.

to be effective in the geographic space, and to follow a short distance process.

When observing biciles, which in this case is a manner to consider high orders of neighbors, it appears obvious that long-distance pairs of residuals do not follow a random process, but rather a negative autoregressive one. When considering terciles, which is a way of considering lower-than biciles related orders of neighbors, the spatial fragmentation occurs more among greater-than-0.002 (blue) biciles than among others (red), which suggests the existence of non-constant spatial variance that depends on the groups one forms. When considering quartiles,

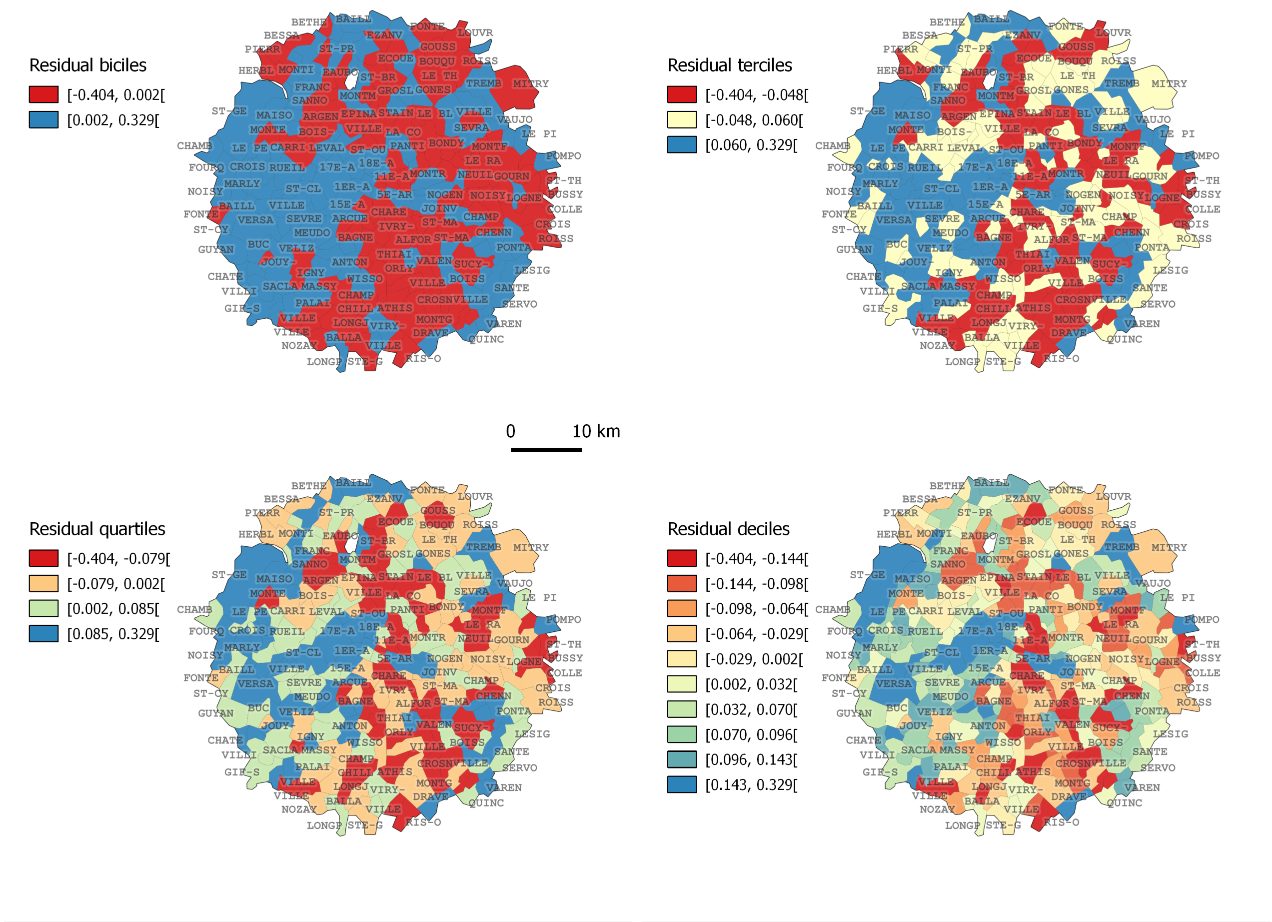


Figure 2.10: SARIMA($\varnothing, \varnothing, \varnothing$)-residual's mapped biciles, terciles, quartiles and deciles.

the most salient fact is that, while the blue space fragments, the red one does not change that much compared to the previous higher quantiles, *i.e.* rather than fragmenting, the red space, as a mountain range, undergoes a water-like surge that makes emerge islands. This suggests that this space concentrates rather outlying places. When observing deciles, one sees no long-distance gradient of colors, for example from the center to the periphery, which also enforces the assumption of the short-distance nature of the residual's autoregression. Note that this absence of long-distance trend is a notable difference with that of housing rents (see Map 2.8's left dial), residual incomes and accessibility (see Map 2.9's left and right dials) that do exhibit long-distance trends. This opposition strongly suggests that these three variables are cointegrated in space and that applying a SEM filter can improve the properties of the vector of residuals.

Undertaken within the stage of the identification of the differencing structure, Figure 2.11 shows SARIMA($\varnothing, \varnothing, \varnothing$)'s residuals plot and full autocorrelogram. Note the space-specific characteristics of the correlogram. It shows the so-called I of Moran (1950) and C of Geary (1954) that are associated to each neighbor-order, both transformed to be interterpretable as

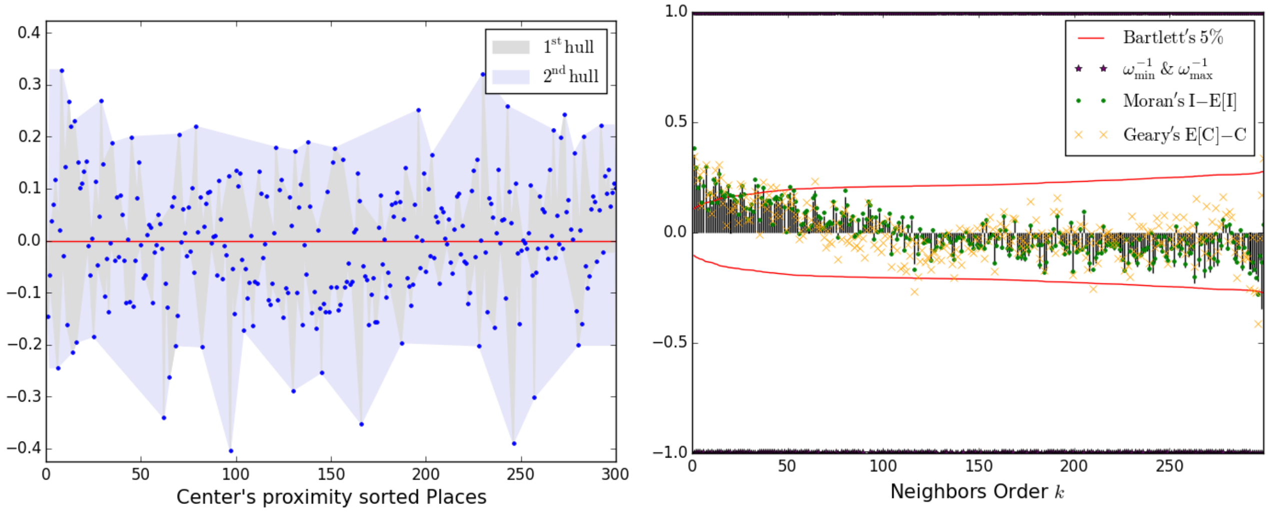


Figure 2.11: SARIMA($\varnothing, \varnothing, \varnothing$) residual's plot and (full) autocorrelogram

the regression coefficients are.[63]Looking closely at the definitions of I and C, one sees that the former – as a Pearson-like correlation coefficient – involves deviation from the sample mean while the latter involves deviations from neighbors values. Put differently, the former is more a global measure of autocorrelation than the latter.

The residuals plot confirms what was previously suggested regarding the (non-fragmenting) red space, *i.e.* this space contains places that instill heteroscedasticity. This is highlighted by the two hulls of the residuals,[64] especially the second one, in which one perceives the strong influence of some outlying negative deviations. The autocorrelogram in addition shows that this non-constant variance may also be accompanied by multiple local spatial trends, given the multiple slow linear decay patterns starting from, *e.g.* the 1st, 7th, 14th, 19th, 26th and 37th orders of neighbors.

Figures 2.12 and 2.13 zoom at the 50 first neighbors orders, and juxtapose two versions of correlogram versions whose difference lays in their abscissa. In these two figures, the (partial and full) Auto Correlation Function (ACF) plots on the left dial are – as in the TS case – based on neighbors orders while that on the right dial is based on average inter-distance neighbors.[65] This allows us to rephrase the interpretation of the ACF in terms of kilometers in the *one-dimensional space of inter-distances*, as follows: there exist multiple local spatial trends that start from, approximately, 1.8km, 3.8km, 5.5km, 6.5km, 7.7km and 9.3km. *What is*

[63]Moran's I usually ranges from -1 to 1 but is *a fortiori* not 0-centered, hence $\mathrm{I}-\mathbb{E}[\mathrm{I}]$, where $\mathbb{E}[\mathrm{I}] = -(n-1)^{-1}$ is the expected value of I under the null hypothesis of no spatial autocorrelation. Geary's C usually ranges from 0 to 2. Values that are below (resp. above) 1 indicate positive (resp. negative) spatial autocorrelation. Thus, $\mathbb{E}[\mathrm{C}] - \mathrm{C}$ ranges from -1 to 1 and is 0-centered, where $\mathbb{E}[\mathrm{C}] = 1$ is the expected value of C under the null hypothesis of no spatial autocorrelation.

[64]The first hull is formed over the local extrema, while the second hull is formed over the local extrema of the first-hull zero-centered extrema.

[65]Note that in the TS case, this nuance makes no sense since temporal positions are traditionally chosen to be equidistant.

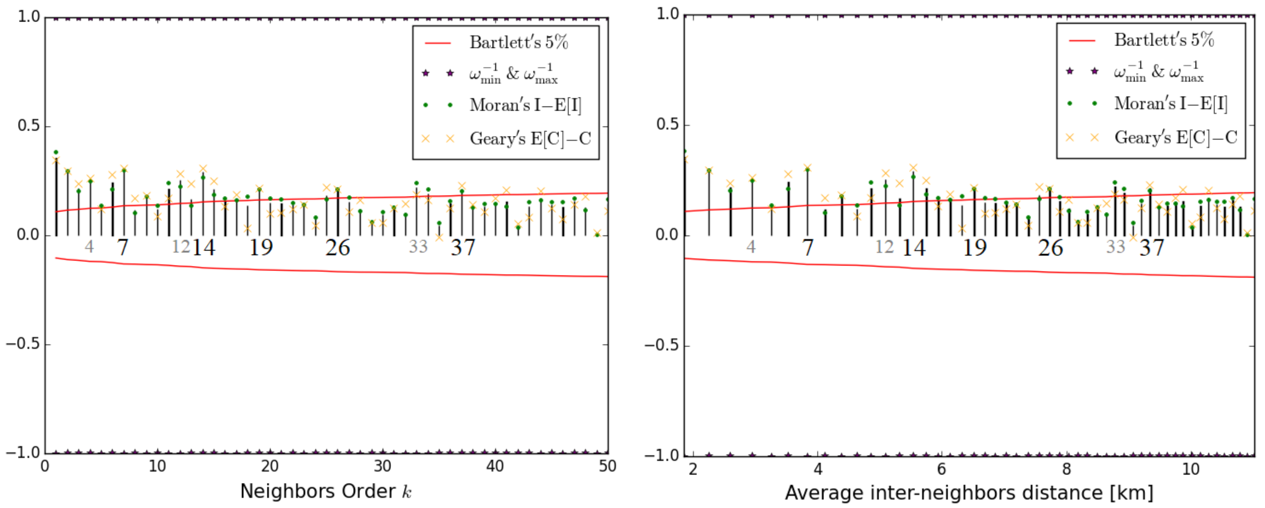


Figure 2.12: SARIMA(∅, ∅, ∅) residual's full autocorrelogram against orders (left) and kilometers (right)

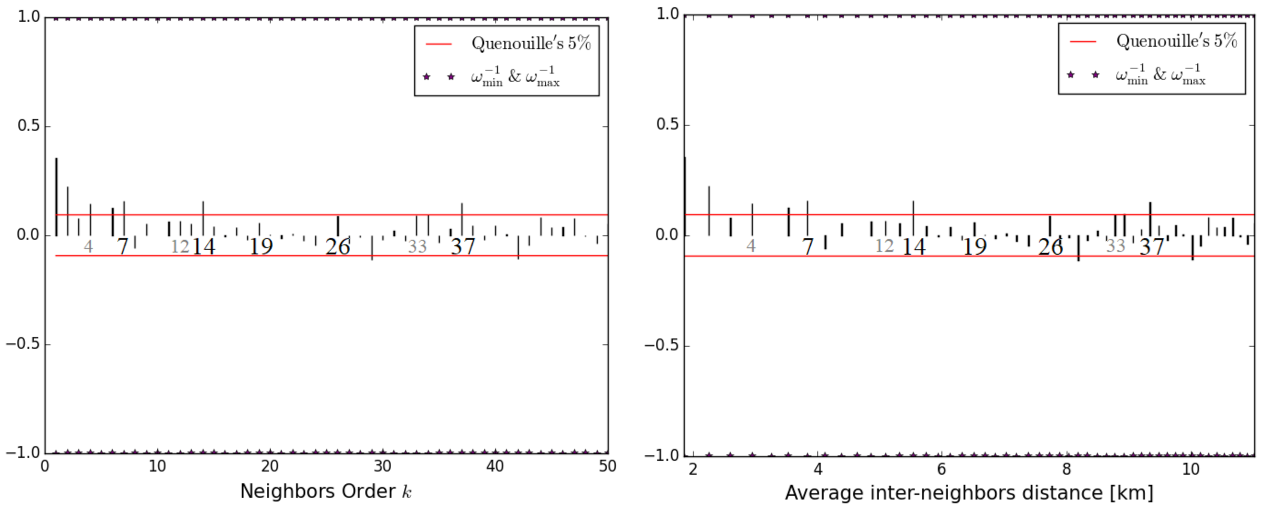


Figure 2.13: SARIMA(∅, ∅, ∅) residual's partial autocorrelogram against orders (left) and kilometers (right)

happening there? At order 1 or at km 1.8, wherever one is located in the agglomeration, one is positioned on the average centroid of the two closest neighbors. Interpreted in terms of travel times or housing rents, it appears reasonable to think that no clearly perceptible structural break exists (on average) at such a short distance. Thus, this first neighbor-order is likely to be representative of the reciprocal short distance effect between places. At order 7 or at km 3.8, if one takes the 1st arrondissement as a distance origin, one is just before the *north-south* frontier of intra-mural Paris. Still at order 7, but this time taking an example outside of intra-mural Paris, one is very likely to have moved to a new quartile (see Figure 2.10). This suggests that the 7th neighbor-order may coincide with a structural break both in housing rents and traffic conditions. The existence of such a structural break is also suggested when taking a look at the geographies of housing rents (see Map 2.8's left dial), residual incomes and accessibility (see Map 2.9). Indeed, by randomly drawing a place and considering a 3.8km-radius circle around it, one is likely to get a group of places belonging to the same decile. At order 14 – which in the TS case would seasonally superimposes to the just-explained 7th order – or at km 5.5, one is very likely to capture the same sort of information as the one that is captured at order 7 and whose value-added stems from the irregularity of the lattice. Note that by taking again the 1th arrondissement as a distance origin, the 14th order delimits the *north-south outer-side* frontier of intra-mural Paris. Averaged together, the 7th and the 14th neighbors orders are, remarkably, equivalent to the km 4.6, which is the real *north-south* radius of intra-mural Paris. At order 19 (km 6.5) or 26 (km 7.7), the information captured is not that clear and we thus skip these neighbors order. At order 37 or at km 9.3, one is actually looking at the last significant order stated by the ACF plot. Remarkably, once again with the 1st arrondissement as a distance origin, know that 9.3km is the real *east-west* radius of intra-mural Paris.

Particularly keen not to resort to any kind of automatic selection techniques, it is on these geographically established fundamentals, that we define the structure of the 1st difference over lags 7, 14 and 37, *i.e.* $\mathcal{I}_{\boldsymbol{\gamma}} = \{7, 14, 37\}$, all three representative of the north-south and east-west dimensions of intra-mural Paris. We justify our choice of defining $\mathcal{I}_{\boldsymbol{\gamma}}$ as it stands by the significance of these 3 lags both on the PACF and ACF sides. We then consider the 1st, 4th and 12th neighbor-orders as a starting point to structure the SMA part of the DGP, *i.e.* $\mathcal{I}_{\boldsymbol{\lambda}} = \{1, 4, 12\}$.

SARIMA($\varnothing, \{7, 14, 37\}, \{1, 4, 12\}$)-grounded models parameters *(ii)* Estimation and *(iii)* Checking

Figure 2.14 shows the ACF and PACF resulting from the inclusion of these lags in the model. Based on the interpretation of these two correlograms, multiple possibilities arise. Table 2.3 shows some selective properties of the so-suggested models. Multiple model-specifications may reasonably well explain the generative process of housing rents, explicited in eq.(2.30). It thus appears informative to compare their results.

We do not compute the bootstrap intervals of all the models that are enumerated in Table 2.3, only the first three ranked ones. The ranking process is five-dimensional and performed according to the so-defined lexical order: SH-pv> 5%, BP-pv> 5%, BIC, SH-pv, BP-pv. For example, the 5d rank of the 1st model is $True$, $True$, -443.18, 7.13%, 11.26%. That of the 14th is $False$, $True$, -444.91, 3.3%, 17.52%. Note the importance given to the non-rejection of prediction errors' normality and residuals homoscedasticity. For the former non-rejection, this is justified by the Gaussian nature of the maximized (log) likelihood, on which information criteria rely. The importance given to the latter non-rejection is justified by the simple fact that

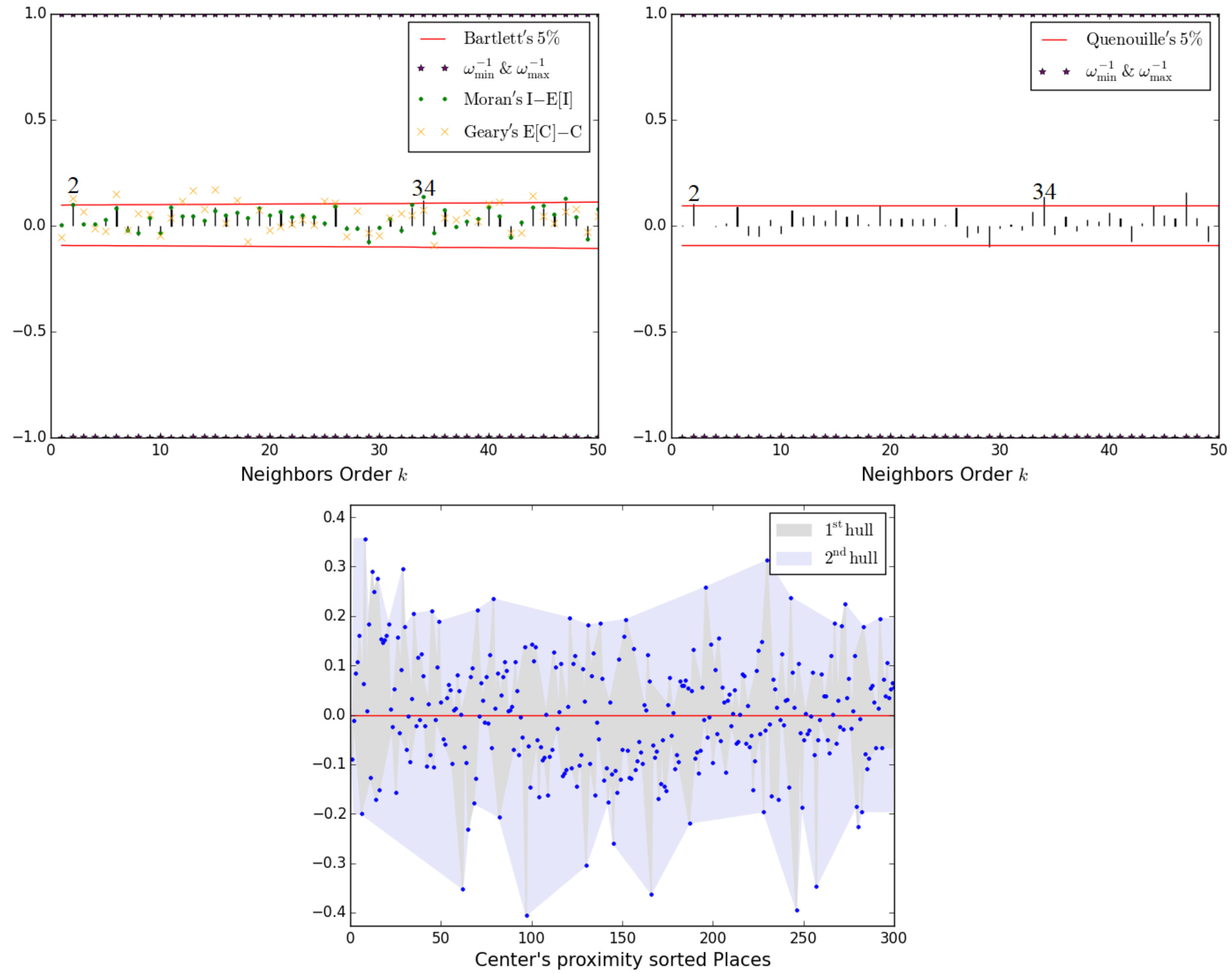


Figure 2.14: SARIMA($\varnothing$, $\{7, 14, 37\}$, $\{1, 4, 12\}$) residual's full (left), partial autocorrelograms and residual plot

the reliability of the non-percentile-based standards errors heavily depends on it.[66] In what follows, we first check the properties of these three models, and we then consider especially the estimated values for δ^H and δ^a, whose bootstrap-based density plots are shown in Figure 2.15.

Relatively to model checking, we first refer to the (vertically rotated) left parts of Tables 2.4, 2.5 and 2.6. In those, one can read that all the unit-root tests lead to strongly reject the null according to which spatial coefficients are equal either to the lower or the upper bound of their stationary space. But it is not sufficient and one must ensure that all spatial filters are non-singular. To do so, we then consider the sum of the absolute values of the estimated coefficients belonging to the same family, *i.e.* involved in the same spatial filter. It appears that none of these sums approach 1.

Relatively to housing services, the estimated values for δ^H – all greater than that of 25% set in GEMSE for France –, are in accordance with a study realized by *Institut national de la*

[66] As already mentioned in a previous footnote, in the general case (and $\mathbf{X}$ considered fixed), recall that $\mathbf{se}[\widehat{\boldsymbol{\beta}}]^2 = \operatorname{diag}\left[(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\widehat{\boldsymbol{\Omega}}\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\right]$ where $\widehat{\boldsymbol{\Omega}}$ is the (presumably misspecified) square correlation structure of residuals.

Table 2.3: SARIMA($\varnothing, \{7, 14, 37\}, \{1, 4, 12\}$)-grounded eligible models

SARIMA $(\mathcal{I}_\rho, \mathcal{I}_\gamma, \mathcal{I}_\lambda)$	(P)ACF score[a]	Rank	Selection criteria BIC	AIC	HQC[b]	$\ln \mathcal{L}$	Fit criteria MSE	R^2	Residuals properties BP-pv[c]	SH-pv[d]
$(\varnothing, \{2, 7, 14, 34, 37\}, \{1, 4, 12\})$	62.97	1	-443.18	-483.93	-467.62	252.96	0.0114	0.6800	0.1126	0.0713
$(\varnothing, \{7, 14, 37\}, \{1, 4, 12\})$	64.35	2	-442.61	-475.94	-462.60	246.97	0.0118	0.6826	0.0902	0.0995
$(\{2\}, \{7, 14, 37\}, \{1, 4, 12\})$	62.23	3	-442.40	-479.44	-464.62	249.72	0.0115	0.6886	0.0897	0.0734
$(\{2\}, \{7, 14, 34, 37\}, \{1, 4, 12\})$	62.29	4	-441.75	-482.49	-466.19	252.25	0.0113	0.6895	0.1119	0.0569
$(\varnothing, \{2, 7, 14, 37\}, \{1, 4, 12\})$	64.38	5	-441.33	-478.37	-463.55	249.19	0.0116	0.6815	0.0970	0.0831
$(\{2\}, \{7, 14, 37\}, \{1, 4, 12, 33\})$	63.70	6	-440.08	-480.82	-464.51	251.41	0.0114	0.6881	0.0990	0.0654
$(\{2\}, \{7, 14, 37\}, \{1, 4, 12, 34\})$	59.41	7	-439.29	-480.03	-463.73	251.02	0.0114	0.6896	0.0843	0.0546
$(\{33\}, \{7, 14, 37\}, \{1, 2, 4, 12\})$	64.38	8	-438.21	-478.95	-462.64	250.47	0.0117	0.6924	0.0653	0.0584
$(\{34\}, \{7, 14, 37\}, \{1, 2, 4, 12\})$	60.14	9	-434.48	-475.22	-458.91	248.61	0.0117	0.6847	0.1355	0.0732
$(\varnothing, \{7, 14, 37\}, \{1, 4, 12, 33\})$	62.97	10	-431.75	-468.78	-453.96	244.39	0.0121	0.6841	0.0922	0.1149
$(\{33\}, \{7, 14, 37\}, \{1, 4, 12\})$	58.69	11	-435.76	-472.79	-457.97	246.40	0.0119	0.6904	0.0420	0.0676
$(\varnothing, \{7, 14, 37\}, \{1, 2, 4, 12\})$	59.41	12	-434.94	-471.98	-457.16	245.99	0.0123	0.6838	0.0298	0.1041
$(\varnothing, \varnothing, \varnothing)$	48.83	13	-400.38	-411.49	-407.04	208.74	0.0147	0.6872	0.0367	0.1716
$(\{2, 34\}, \{7, 14, 37\}, \{1, 4, 12\})$	62.94	14	-444.91	-485.65	-469.35	253.83	0.0113	0.6861	0.1752	0.0330
$(\{2, 33\}, \{7, 14, 37\}, \{1, 4, 12\})$	62.97	15	-441.81	-482.55	-466.24	252.27	0.0113	0.6959	0.0799	0.0323

[a] (P)ACF score *norm*-considers the number of non-significant lags in the ACF and in the PACF, $\#_{\text{lag}_{\text{(P)ACF}}}$, via $(\#^2_{\text{lag}_{\text{ACF}}} + \#^2_{\text{lag}_{\text{PACF}}})^{.5}$.
[b] HQC stands for the Hannan-Quinn information criterion (Hannan and Quinn, 1979).
[c] BP-pv stands for the p-value associated to Breusch and Pagan (1979)'s test statistic whose null hypothesis is *model residuals*' homoskedasticity.
[d] SH-pv stands for the p-value associated to Shapiro and Wilk (1965)'s test statistic whose null hypothesis is *prediction errors*' normality.

statistique et des études économiques (INSEE) (Durand, 2012, p.9) with 2008-data. As it reads in there, "la part du loyer moyen dans le revenu des ménages interrogés s'élève à 34%". When referring to the (vertically rotated) right parts of Tables 2.4, 2.5 and 2.6, it appears that the possibility of having 25% as housing budget share is always rejected at a 5% significance level. However, it appears reasonable to assume that expanding the number of places from 300 to the one set in GEMSE, 428, would exert an upward pressure on these p-values, since the places that would be so-considered are very likely to exhibit economic characteristics that would in turn downward-pressure Paris's averages (of these economic characteristics) toward France's. Relatively to transport services, the level of 10% that is set in GEMSE is remarkably close to those yielded by the three specifications, all highly significant.

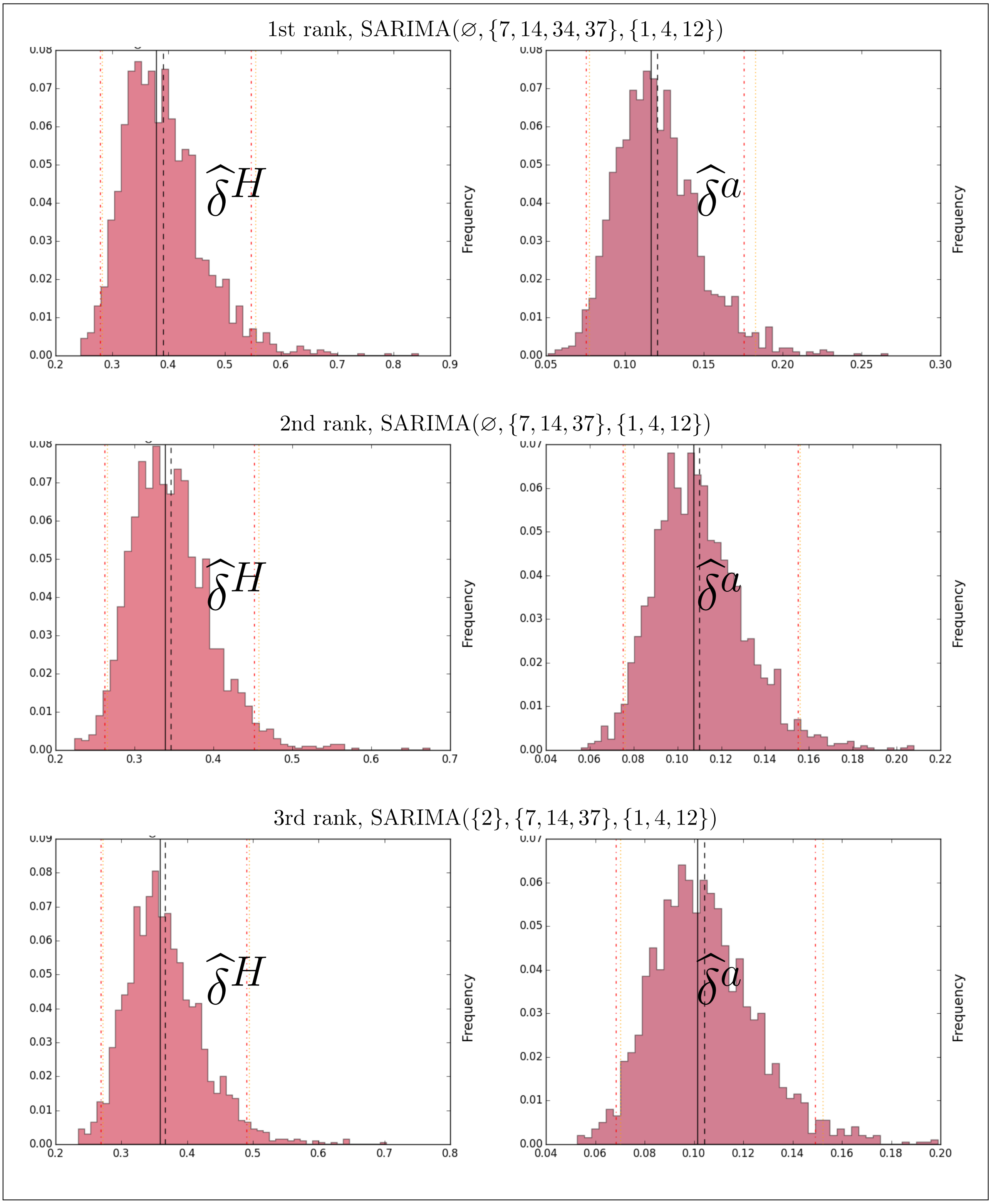


Figure 2.15: Bootstrapped budget shares' density plots

2.3.5 Main findings

Three mains findings are presented to conclude this section. The first is related to the simplicity of the structural equation that is estimated. The second relates to the estimated values themselves, similar to what one finds in the literature. The third finding relates to the proposal of lag operator that is implemented to conduct the validation of the budgets shares that are set in GEMSE.

The equation used in the study is really simple since only bivariate. It derives from the (indirect) utility function employed in GEMSE, rearranged to express housing rents [see eq.(2.30)]. This is a notable difference with the technique that is traditionally employed when dealing with housing rents or prices, such as hedonic regressions that rely on a (very) large set of characteristics, rarely publicly or easily available.

This structural equation remarkably yields estimated values for parameters that are similar to what one finds in the literature regarding France and Paris in particular. Indeed, the values found in the literature for France [see *e.g.* Guidetti and INSEE (2012, p.3, Table 3) or Morer and INSEE (2015, p.3, Table 4)] (and set in GEMSE) of 10% and 25% respectively for δ^a and δ^H are statistically supported in this study on the transportation side, and not so far from being so on the housing side. For Paris, which differs from the rest of the country on the housing side, our study remarkably finds (confidence and percentile) intervals that contain almost into their center the values found in the literature of 34% [see (Durand, 2012, p.9)].

On the theoretical plan, our econometric study generalizes in space an approach traditionally used in time series and presents the strict spatial counterpart of the well-known time-lag operator, baptized *k-nearest neighbor only* (*oknn*). Remarkably, it allows us to identify some key elements of the spatial structure of Paris, namely its North-South and East-West intramural dimensions. Given that the approach that we develop requires careful handling, it is only applied to this urban area. Its (automation if feasible and) declination into spatial panel is a promising area for future research.

2.A Models selected for estimation

The models that have been chosen for estimation on the basis of their characteristics [see Table 2.3] are presented sequentially in what follows.

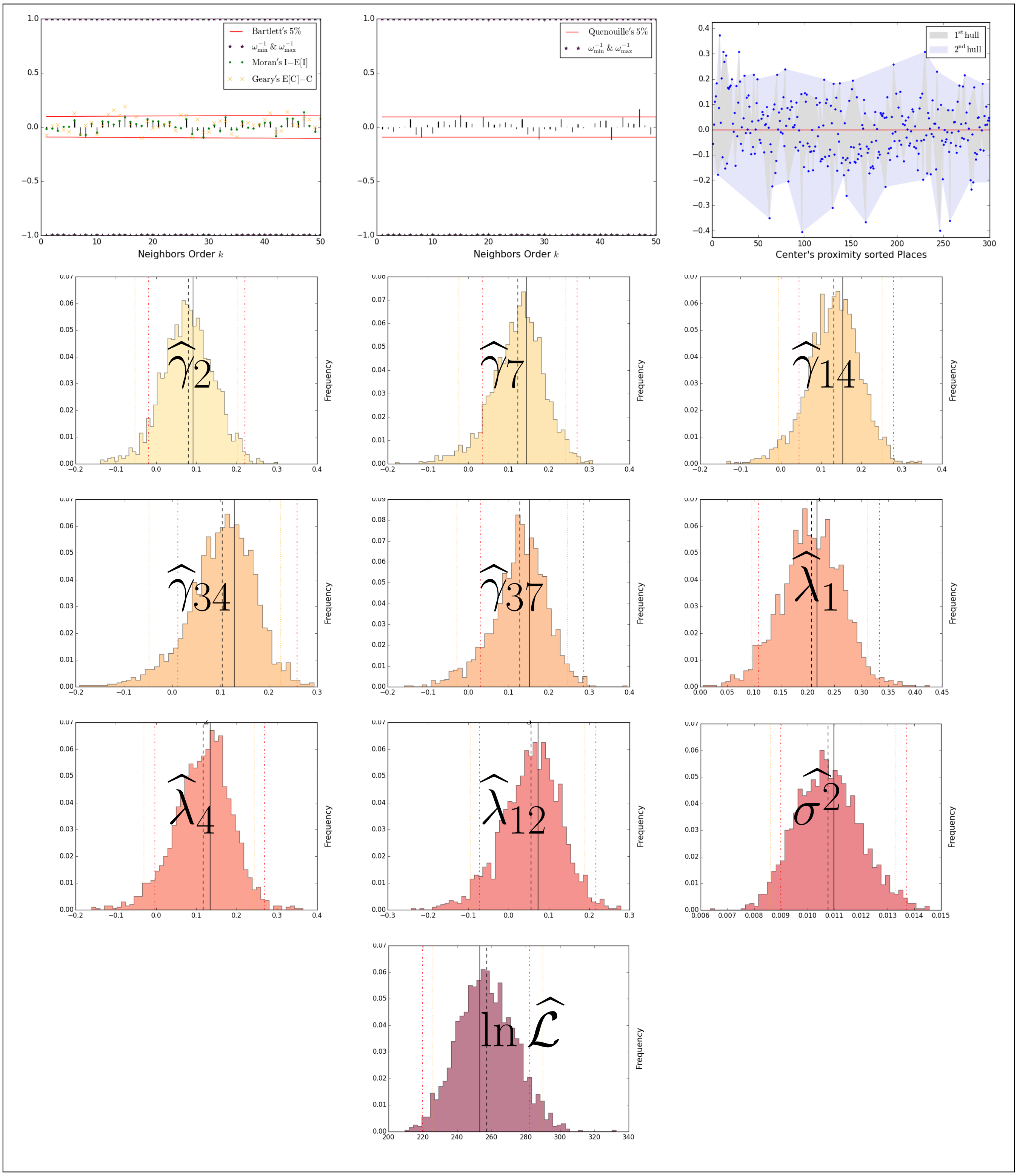


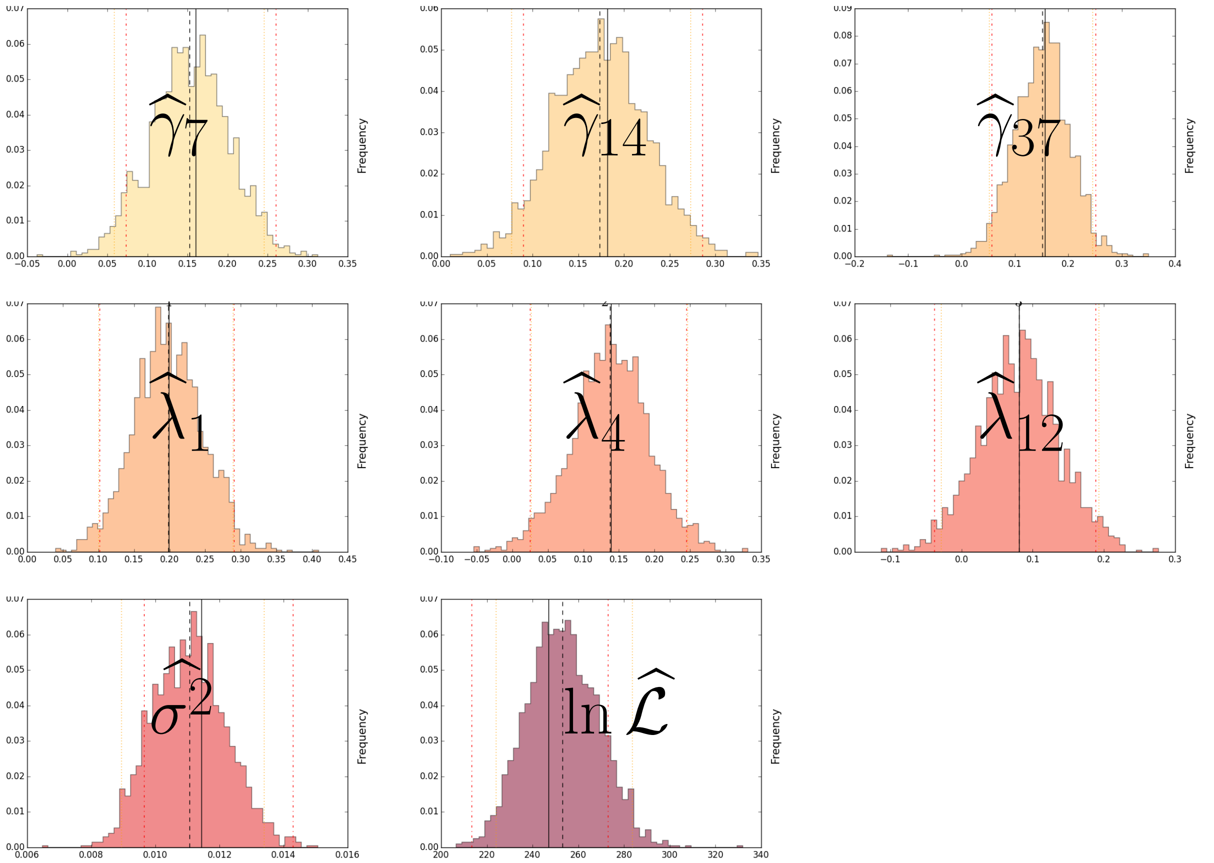
Figure 2.16: SARIMA($\varnothing, \{7, 14, 34, 37\}, \{1, 4, 12\}$)'s (P)ACFs, residual plot and bootstrap-based density plots of spatial parameters (1st rank)

Table 2.4: SARIMA $(\varnothing, \{2, 7, 14, 34, 37\}, \{1, 4, 12\})$

Coeff.		Dist.[a]	Std. Err.	P>|z|	[95% C.I.]	
β_0	-22.9356	D1	4.677358	0.000	-32.1416	-13.7295
		D2	.	.	.	.
		D3	4.984052	0.000	-32.9913	-13.4897
		D4	5.022955	0.000	-33.5484	-13.8588
β_1	2.3373	D1	0.420121	0.000	1.5104	3.1641
		D2	.	.	.	.
		D3	0.448576	0.000	1.4777	3.2423
		D4	0.452586	0.000	1.5387	3.3128
β_2	0.3085	D1	0.033398	0.000	0.2427	0.3742
		D2	.	.	.	.
		D3	0.034433	0.000	0.2401	0.3761
		D4	0.034243	0.000	0.2396	0.3738
γ_2	0.0914	D1	0.050415	0.070	-0.0078	0.1906
		D2	.	.	.	.
		D3	0.063333	0.185	-0.0533	0.2027
		D4	0.060998	0.088	-0.0195	0.2196
γ_7	0.1431	D1	0.049822	0.004	0.0451	0.2412
		D2	.	.	.	.
		D3	0.064895	0.081	-0.0238	0.2410
		D4	0.059817	0.019	0.0351	0.2696
γ_{14}	0.1531	D1	0.050293	0.002	0.0541	0.2521
		D2	.	.	.	.
		D3	0.064489	0.064	-0.0065	0.2510
		D4	0.059783	0.014	0.0446	0.2789
γ_{34}	0.1281	D1	0.046504	0.006	0.0366	0.2197
		D2	.	.	.	.
		D3	0.067534	0.145	-0.0488	0.2242
		D4	0.062970	0.039	0.0111	0.2579
γ_{37}	0.1513	D1	0.049874	0.002	0.0532	0.2495
		D2	.	.	.	.
		D3	0.067472	0.094	-0.0286	0.2453
		D4	0.065531	0.024	0.0291	0.2860
λ_1	0.2169	D1	0.048663	0.000	0.1211	0.3127
		D2	.	.	.	.
		D3	0.056515	0.000	0.0960	0.3110
		D4	0.057333	0.000	0.1083	0.3331
λ_4	0.1343	D1	0.052664	0.011	0.0306	0.2379
		D2	.	.	.	.
		D3	0.070150	0.114	-0.0311	0.2437
		D4	0.069426	0.254	-0.0037	0.2685
λ_{12}	0.0722	D1	0.052539	0.170	-0.0312	0.1756
		D2	.	.	.	.
		D3	0.073198	0.427	-0.0959	0.1883
		D4	0.073365	0.247	-0.0722	0.2154
δ^H	0.3780	D1	.	.	.	.
		D2	0.059715	0.000	0.2604	0.4955
		D3	0.070674	0.000	0.2819	0.5541
		D4	0.068306	0.000	0.2793	0.5470
δ^a	0.1166	D1	.	.	.	.
		D2	0.022648	0.000	0.0720	0.1612
		D3	0.026235	0.000	0.0776	0.1826
		D4	0.025519	0.000	0.0755	0.1756
$\sigma_{\mathbf{r}}^2$	0.0110	D1	0.000913	.	.	.
		D2	.	.	.	.
		D3	0.001191	.	0.0086	0.0133
		D4	0.001202	.	0.0090	0.0137

[a] C.I.-related objects derive from four types of distribution:
*D1 stands for normal-approximation-based distribution.
*D2 is as D1, but derives from an information matrix expressed in terms of δ^H and δ^a.

Coeff.	Dist.[a]	P>|z|						
		$\mathcal{H}_0 : \hat{}: = -1.00$	$\mathcal{H}_0 : \hat{}: = 0.05$	$\mathcal{H}_0 : \hat{}: = 0.12$	$\mathcal{H}_0 : \hat{}: = 0.15$	$\mathcal{H}_0 : \hat{}: = 0.25$	$\mathcal{H}_0 : \hat{}: = 0.35$	$\mathcal{H}_0 : \hat{}: = 1.00$
γ_2	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_7	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_{14}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_{34}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_{37}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_1	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_4	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_{12}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
δ^H	D3	.	.	.	0.0000	0.0041	0.6232	.
	D4	.	.	.	0.0000	0.0055	0.6724	.
δ^a	D3	.	0.0000	0.9234	.	.	.	.
	D4	.	0.0000	0.8481	.	.	.	.



See Figure 2.14 for the corresponding (P)ACFs and residual plot.

Figure 2.17: SARIMA($\varnothing, \{7, 14, 37\}, \{1, 4, 12\}$)'s (P)ACFs, residual plot and bootstrap-based density plots of spatial parameters (2nd rank)

Table 2.5: SARIMA $(\varnothing, \{7, 14, 37\}, \{1, 4, 12\})$

Coeff.		Dist.[a]	Std. Err.	P>|z|	[95% C.I.]	
β_0	-26.3143	D1	4.876331	0.000	−35.9117	−16.7170
		D2	.	.	.	.
		D3	4.627975	0.000	−35.4831	−17.7473
		D4	4.617045	0.000	−36.1738	−18.0754
β_1	2.6342	D1	0.439771	0.000	1.7686	3.4997
		D2	.	.	.	.
		D3	0.417639	0.000	1.8658	3.4698
		D4	0.412382	0.000	1.8936	3.5101
β_2	0.3166	D1	0.030931	0.000	0.2557	0.3775
		D2	.	.	.	.
		D3	0.030925	0.000	0.2584	0.3776
		D4	0.030529	0.000	0.2574	0.3771
γ_7	0.1601	D1	0.050056	0.001	0.0616	0.2586
		D2	.	.	.	.
		D3	0.048355	0.001	0.0586	0.2452
		D4	0.047785	0.000	0.0733	0.2606
γ_{14}	0.1817	D1	0.052825	0.001	0.0777	0.2856
		D2	.	.	.	.
		D3	0.050199	0.000	0.0767	0.2725
		D4	0.050028	0.000	0.0897	0.2858
γ_{37}	0.1561	D1	0.051381	0.002	0.0549	0.2572
		D2	.	.	.	.
		D3	0.051040	0.005	0.0519	0.2449
		D4	0.049602	0.005	0.0566	0.2511
λ_1	0.1987	D1	0.045590	0.000	0.1090	0.2885
		D2	.	.	.	.
		D3	0.048764	0.000	0.0999	0.2892
		D4	0.048213	0.000	0.1014	0.2904
λ_4	0.1385	D1	0.051791	0.007	0.0366	0.2405
		D2	.	.	.	.
		D3	0.054354	0.016	0.0263	0.2464
		D4	0.056146	0.017	0.0249	0.2449
λ_{12}	0.0811	D1	0.052018	0.119	−0.0213	0.1834
		D2	.	.	.	.
		D3	0.056203	0.150	−0.0290	0.1929
		D4	0.057675	0.170	−0.0380	0.1881
δ^H	0.3389	D1	.	.	.	.
		D2	0.049880	0.000	0.2407	0.4371
		D3	0.050861	0.000	0.2650	0.4574
		D4	0.048467	0.000	0.2618	0.4518
δ^a	0.1073	D1	.	.	.	.
		D2	0.020214	0.000	0.0675	0.1471
		D3	0.020579	0.000	0.0759	0.1558
		D4	0.020385	0.000	0.0750	0.1550
$\sigma^2_{\mathbf{r}}$	0.0114	D1	0.000944	.	.	.
		D2	.	.	.	.
		D3	0.001151	.	0.0089	0.0134
		D4	0.001188	.	0.0096	0.0143

		P>|z|						
Coeff.	Dist.[a]	$\mathcal{H}_0 : \hat{} = -1.00$	$\mathcal{H}_0 : \hat{} = 0.05$	$\mathcal{H}_0 : \hat{} = 0.12$	$\mathcal{H}_0 : \hat{} = 0.15$	$\mathcal{H}_0 : \hat{} = 0.25$	$\mathcal{H}_0 : \hat{} = 0.35$	$\mathcal{H}_0 : \hat{} = 1.00$
γ_7	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_{14}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_{37}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_1	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_4	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_{12}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
δ^H	D3	.	.	.	0.0000	0.0272	0.8696	.
	D4	.	.	.	0.0000	0.0214	0.7956	.
δ^a	D3	.	0.0000	0.5634	.	.	.	.
	D4	.	0.0000	0.5276	.	.	.	.

[a] C.I.-related objects derive from four types of distribution:
*D1 stands for normal-approximation-based distribution.
*D2 is as D1, but derives from an information matrix expressed in terms of δ^H and δ^a.
*D3 stands for bootstrap-based distribution. Associated std. are computed over bootstrap distributions.
*D4 stands for BCa bootstrap-based distribution. Associated std. are deduced by reversing the symmetry-based C.I. formula.

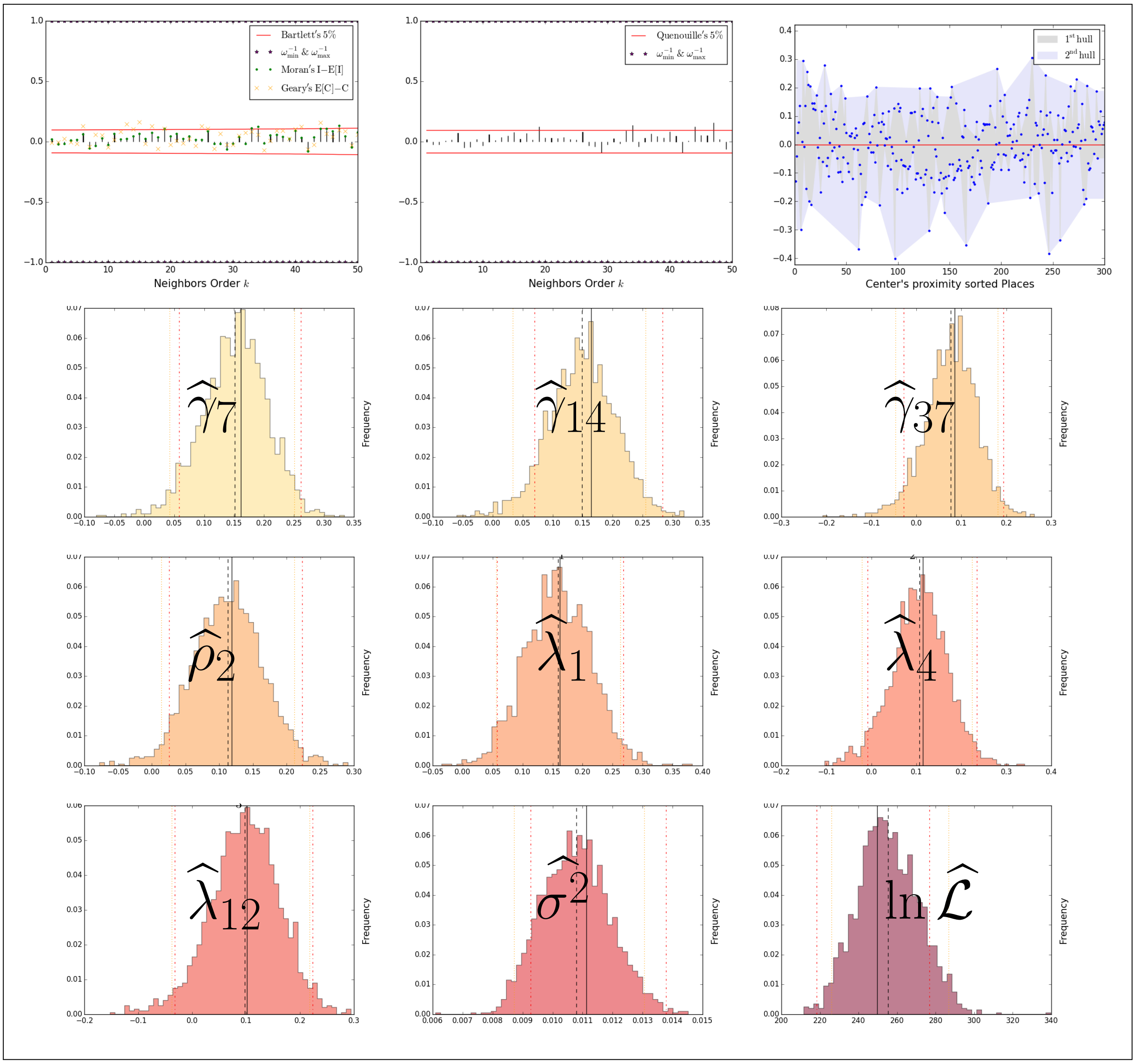


Figure 2.18: SARIMA($\{2\}, \{7, 14, 37\}, \{1, 4, 12\}$)'s (P)ACFs, residual plot and bootstrap-based density plots of spatial parameters (3rd rank)

Table 2.6: SARIMA $(\{2\}, \{7, 14, 37\}, \{1, 4, 12\})$

Coeff.		Dist.[a]	Std. Err.	P>|z|	[95% C.I.]	
β_0	-25.2986	D1	4.525859	0.000	−34.2063	−16.3909
		D2	.	.	.	.
		D3	4.651981	0.000	−35.3598	−16.7302
		D4	4.735276	0.000	−35.5907	−17.0287
β_1	2.5071	D1	0.411140	0.000	1.6979	3.3163
		D2	.	.	.	.
		D3	0.424019	0.000	1.7352	3.4201
		D4	0.431791	0.000	1.7654	3.4580
β_2	0.2830	D1	0.032517	0.000	0.2190	0.3470
		D2	.	.	.	.
		D3	0.032544	0.000	0.2226	0.3497
		D4	0.032271	0.000	0.2195	0.3460
γ_7	0.1612	D1	0.054314	0.003	0.0543	0.2681
		D2	.	.	.	.
		D3	0.053774	0.011	0.0424	0.2506
		D4	0.051831	0.006	0.0580	0.2612
γ_{14}	0.1638	D1	0.057506	0.004	0.0507	0.2770
		D2	.	.	.	.
		D3	0.055596	0.012	0.0339	0.2547
		D4	0.054434	0.003	0.0699	0.2833
γ_{37}	0.0855	D1	0.058350	0.143	−0.0294	0.2003
		D2	.	.	.	.
		D3	0.057416	0.183	−0.0462	0.1811
		D4	0.056600	0.120	−0.0284	0.1935
ρ_2	0.1189	D1	0.049549	0.016	0.0214	0.2164
		D2	.	.	.	.
		D3	0.051456	0.030	0.0142	0.2115
		D4	0.050322	0.016	0.0261	0.2234
λ_1	0.1621	D1	0.049255	0.001	0.0652	0.2591
		D2	.	.	.	.
		D3	0.054161	0.003	0.0558	0.2631
		D4	0.053734	0.003	0.0575	0.2682
λ_4	0.1149	D1	0.054839	0.036	0.0070	0.2229
		D2	.	.	.	.
		D3	0.062115	0.100	−0.0213	0.2236
		D4	0.061811	0.067	−0.0082	0.2341
λ_{12}	0.1015	D1	0.052822	0.055	−0.0024	0.2055
		D2	.	.	.	.
		D3	0.064349	0.130	−0.0379	0.2175
		D4	0.065256	0.113	−0.0321	0.2237
δ^H	0.3584	D1	.	.	.	.
		D2	.	.	.	.
		D3	0.057641	0.000	0.2717	0.4938
		D4	0.056590	0.000	0.2688	0.4907
δ^a	0.1014	D1	.	.	.	.
		D2	0.011030	0.000	0.0794	0.1228
		D3	0.020867	0.000	0.0701	0.1522
		D4	0.020605	0.000	0.0684	0.1492
$\sigma^2_{\mathbf{r}}$	0.0111	D1	0.000920	.	.	.
		D2	.	.	.	.
		D3	0.001136	.	0.0087	0.0131
		D4	0.001149	.	0.0093	0.0138

[a] C.I.-related objects derive from four types of distribution:
*D1 stands for normal-approximation-based distribution.
*D2 is as D1, but derives from an information matrix expressed in terms of δ^H and δ^a.
*D3 stands for bootstrap-based distribution. Associated std. are computed over bootstrap distributions.
*D4 stands for BCa bootstrap-based distribution. Associated std. are deduced by reversing the symmetry-based C.I. formula.

		P>|z|						
Coeff.	Dist.[a]	$\mathcal{H}_0 : \hat{\ } = -1.00$	$\mathcal{H}_0 : \hat{\ } = 0.05$	$\mathcal{H}_0 : \hat{\ } = 0.12$	$\mathcal{H}_0 : \hat{\ } = 0.15$	$\mathcal{H}_0 : \hat{\ } = 0.25$	$\mathcal{H}_0 : \hat{\ } = 0.35$	$\mathcal{H}_0 : \hat{\ } = 1.00$
γ_7	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_{14}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
γ_{37}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
ρ_2	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_1	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_4	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
λ_{12}	D3	0.0000	.	.	.	.	.	0.0000
	D4	0.0000	.	.	.	.	.	0.0000
δ^H	D3	.	.	.	0.0000	0.0104	0.8593	.
	D4	.	.	.	0.0000	0.0123	0.8930	.
δ^a	D3	.	0.0000	0.4078	.	.	.	.
	D4	.	0.0000	0.3622	.	.	.	.

2.B Models development

This section has a computational scope and shows successively how to compute all the objects that are involved during the declination of the model from its simplest form into that of a SARIMA process. Let's start with the traditional linear model, as presented in eq.(2.23):

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{v}$$

Under the assumption of unmodeled correlation structure for $\mathbf{v}$, *i.e* uncorrelated and homoscedastic, its covariance matrix is $\mathbb{E}[\mathbf{v}\mathbf{v}^{'}] = \sigma^2_{\mathbf{v}}\mathbf{I}$.

2.B.1 From the linear to the SEM

If one has good reasons to think that the above model should be turned into an autoregressive process in $\mathbf{v}$, *i.e.* that one should augment eq.(2.23) into eq.(2.26), one gets the following system to consider

$$\begin{cases} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \mathbf{v} \\ \mathbf{v} &= \mathbf{G}(\boldsymbol{\gamma})\mathbf{v} + \mathbf{u} \end{cases}$$

which leads to

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \mathbf{G}(\boldsymbol{\gamma})\mathbf{v} + \mathbf{u} \\ &= \mathbf{X}\boldsymbol{\beta} + \mathbf{G}(\boldsymbol{\gamma})\left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right) + \mathbf{u} \\ &\Leftrightarrow \mathbf{u} = \left(\mathbf{I} - \mathbf{G}(\boldsymbol{\gamma})\right)\left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right) \\ &= \boldsymbol{\Gamma}(\boldsymbol{\gamma})\left(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}\right) \\ &\Leftrightarrow \mathbf{y}_{\boldsymbol{\gamma}} = \mathbf{X}_{\boldsymbol{\gamma}}\boldsymbol{\beta} + \mathbf{u} \end{aligned}$$

Where the above line is uniquely defined only if $\boldsymbol{\Gamma}(\boldsymbol{\gamma})$ is invertible. If so, and assuming that $\mathbf{u}$ has no explicitly modelled covariance, it follows that that of $\mathbf{v}$ now becomes

$$\begin{aligned} \mathbb{E}[\mathbf{v}\mathbf{v}^{'}] &= \mathbb{E}[\boldsymbol{\Gamma}(\boldsymbol{\gamma})^{-1}\mathbf{u}\mathbf{u}^{'}\boldsymbol{\Gamma}(\boldsymbol{\gamma})^{-1^{'}}] \\ &= \sigma^2_{\mathbf{u}}\left(\boldsymbol{\Gamma}(\boldsymbol{\gamma})^{'}\boldsymbol{\Gamma}(\boldsymbol{\gamma})\right)^{-1} \\ &= \boldsymbol{\Omega}_{\boldsymbol{\gamma}} \end{aligned}$$

2.B.2 From the SEM to SARIMA

If one then has good reasons to think that the above model should in turn be turned into an autoregressive process in $\mathbf{y}_{\boldsymbol{\gamma}}$ with a moving average process in $\mathbf{u}$, *i.e.* that one should augment eq.(2.26) into eq.(2.28), the system to consider is

$$\begin{cases} \mathbf{y}_\gamma & = \mathbf{Q}(\boldsymbol{\rho})\mathbf{y}_\gamma + \mathbf{X}_\gamma\boldsymbol{\beta} + \mathbf{u} \\ \mathbf{u} & = \mathbf{L}(\boldsymbol{\lambda})\mathbf{r} + \mathbf{r} \end{cases}$$

which leads to

$$\begin{aligned} \mathbf{y}_\gamma = \mathbf{Q}(\boldsymbol{\rho})\mathbf{y}_\gamma + \mathbf{X}_\gamma\boldsymbol{\beta} + \mathbf{L}(\boldsymbol{\lambda})\mathbf{r} + \mathbf{r} \\ \Leftrightarrow \mathbf{y}_{\gamma,\boldsymbol{\rho}} = \mathbf{X}_\gamma\boldsymbol{\beta} + \boldsymbol{\Lambda}(\boldsymbol{\lambda})\mathbf{r} \end{aligned}$$

If the above equality is uniquely defined, it follows that

$$\begin{aligned} \mathbf{r} = \boldsymbol{\Lambda}(\boldsymbol{\lambda})^{-1}\left(\mathbf{y}_{\gamma,\boldsymbol{\rho}} - \mathbf{X}_\gamma\boldsymbol{\beta}\right) \\ \Leftrightarrow \mathbf{y}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}} = \mathbf{X}_{\gamma,\boldsymbol{\rho}}\boldsymbol{\beta} + \mathbf{r} \end{aligned}$$

If one assumes that $\mathbf{r}$ has been totally filtered, *i.e.* $\mathbb{E}[\mathbf{r}\mathbf{r}^{'}] = \sigma^2_{\mathbf{r}}\mathbf{I}$, it follows that the covariance matrices respectively of $\mathbf{u}$ and $\mathbf{v}$ are

$$\begin{aligned} \mathbb{E}[\mathbf{u}\mathbf{u}^{'}] &= \mathbb{E}[\left(\mathbf{Q}(\boldsymbol{\rho})\mathbf{y}_\gamma + \boldsymbol{\Lambda}(\boldsymbol{\lambda})\mathbf{r}\right)\left(\mathbf{Q}(\boldsymbol{\rho})\mathbf{y}_\gamma + \boldsymbol{\Lambda}(\boldsymbol{\lambda})\mathbf{r}\right)^{'}] \\ &= \mathbf{Q}\mathrm{cov}[\mathbf{y}_\gamma, \mathbf{y}_\gamma]\mathbf{Q}^{'} + 2\mathbf{Q}\mathbb{E}[\mathbf{y}_\gamma\mathbf{r}^{'}]\boldsymbol{\Lambda}^{'} + \sigma^2_{\mathbf{r}}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^{'} \\ &= \boldsymbol{\Omega}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}} \\ \mathbb{E}[\mathbf{v}\mathbf{v}^{'}] &= \boldsymbol{\Gamma}^{-1}\boldsymbol{\Omega}_{\gamma,\boldsymbol{\rho},\boldsymbol{\lambda}}\boldsymbol{\Gamma}^{-1'} \end{aligned}$$

References

Allio, C. (2015). "Local Policies, Urban Dynamics and Climate Change: Development of a Multiscale Modeling Framework". PhD thesis, AllioT2015.

Anselin, L (1982). "A Note on Small Sample Properties of Estimators in a First-Order Spatial Autoregressive Model". *Environment and Planning A* 14, pp. 1023–1030.

Anselin, L. (1988). *Spatial Econometrics: Methods and Models*, p. 289.

Anselin, L. (2002a). "Under the hood:Issues in the specification and interpretation of spatial regression models". *Agricultural Economics* 27, pp. 247–267.

Anselin, L. (2002b). "Under the hood:Issues in the specification and interpretation of spatial regression models". *Agricultural Economics* 27, pp. 247–267.

Anselin, L. (2007). "Spatial econometrics in RSUE: Retrospect and prospect". *Regional Science and Urban Economics* 37.4, pp. 450–456.

Anselin, L. (2010). "Thirty years of spatial econometrics". *Papers in Regional Science* 89.1, pp. 3–25.

Anselin, L. (2011). *GMM Estimation of Spatial Error Autocorrelation with and without Heteroskedasticity.* Tech. rep. GeoDa Center for Geospatial Analysis and Computation – Arizona State University, pp. 1–23.

Anselin, L. and S. J. Rey (2007). "PySAL: A Python Library of Spatial Analytical Methods". *The Review of Regional Studies* 37.1, pp. 5–27.

Anselin, L. and others (1980). "Estimation methods for spatial autoregressive structures." *Regional Science Dissertation & Monograph Series, Program in Urban and Regional Studies, Cornell University* 8, p. 273.

Armington, P. S. (1969). "A Theory of Demand for Products Distinguished by Place of Production". *Staff Papers (International Monetary Fund)* 16.1, pp. 159–178.

Arraiz, I., D. M. Drukker, H. H. Kelejian and I. R. Prucha (2010). "A spatial cliff-ord-type model with heteroskedastic innovations: Small and large sample results". *Journal of Regional Science* 50.2, pp. 592–614.

Askenazy, P. and X. Timbeau (2003). "Partage de la valeur ajoutée et rentabilité du capital en France et aux États-Unis : une réévaluation". *Economie et statistique* 363.1, pp. 167–189.

Autant-Bernard, C. and J. P. LeSage (2011). "Quantifying Knowledge Spillovers Using Spatial Econometric Models". *Journal of Regional Science* 51.3, pp. 471–496.

Badinger, H. and P. Egger (2011). "Estimation of higher-order spatial autoregressive cross-section models with heteroscedastic disturbances". *Papers in Regional Science.*

Behrens, K. and F. Robert-Nicoud (2011). "Tempora mutantur: In search of a new testament for NEG". *Journal of Economic Geography* 11.2, pp. 215–230.

Blanchflower, D. G. and A. J. Oswald (1995). "The Wage Curve". *Journal of Economic Literature* 33.2, pp. 785–799.

Bodson, P. and D. Peeters (1975). "Estimation of the Coefficients of a Linear Regression in the Presence of Spatial Autocorrelation. An Application to a Belgian Labour-Demand Function". *Environment and Planning A* 7.4, pp. 455–472.

Bosi, S., E. Iliopulos and H. Jayet (2010). "Optimal Immigration Policy When the Public Good Is Rival".

Box, G. E. P. and G. M. Jenkins (1976). *Time Series Analysis: Forecasting and Control.*

Bradley, E. (1979). "Bootstrap Methods: Another Look at the Jackknife". *The Annals of Statistics* 7.1, pp. 1–26.

Breusch, T. S. and A. R. Pagan (1979). "A Simple Test for Heteroscedasticity and Random Coefficient Variation". *Econometrica.*

Brigitte Baccaïni, C. D. (2007). "Inter-Regional Migration Flows in France over the Last Fifty Years". *Population (English Edition, 2002-)* 62.1, pp. 139–155.

Calatayud, P., J. Duvey-Pilate and D. Malody (2011). *Bilan économique et social de la région Centre*. Tech. rep. INSEE, p. 45.

Choi, E. J. and J. Spletzer (2012). "The declining average size of establishments: evidence and explanations". *Monthly Labor Review* March, pp. 50–65.

Cliff, A. D. and J. K. Ord (1973). *Spatial Autocorrelation.* Pion, London.

Corrado, L. and B. Fingleton (2012). "Where is the economics in spatial econometrics?" *Journal of Regional Science* 52.2, pp. 210–239.

Cressie, N. A. C. (1993). "Statistics for Spatial Data". *Computational Statistics & Data Analysis* 14.4, p. 547.

Das, D., H. H. Kelejian and I. R. Prucha (2003). "Finite sample properties of estimators of spatial autoregressive models with autoregressive disturbances". *Papers in Regional Science* 82.1, pp. 1–26.

Davidson, R. and E. Flachaire (2008). "The wild bootstrap, tamed at last". *Journal of Econometrics* 146.1, pp. 162–169.

Dechezleprêtre, A., R. Martin and M. Mohnen (2013). "Knowledge spillovers from clean and dirty technologies: A patent citation analysis". *Grantham Research Institute and the Environment Working Paper No 151* December, pp. 1–47.

Dogan, O. (2013). "Heteroskedasticity of Unknown Form in Spatial Autoregressive Models with Moving Average Disturbance Term". *Journal of Econometrics* December, pp. 101–127.

Drukker, D. M., P. Egger and I. R. Prucha (2013). "On Two-Step Estimation of a Spatial Autoregressive Model with Autoregressive Disturbances and Endogenous Regressors". *Econometric Reviews* 32, pp. 686–733.

Durand, D. (2012). *Le marché du logement à Paris 2001-2011: retour sur dix ans d'accession.* Tech. rep., p. 22.

Efron, B and R. J. Tibshirani (1993). "An introduction to the bootstrap". *Refrigeration And Air Conditioning.*

Elhorst, J. P., D. J. Lacombe and G. Piras (2012). “On model specification and parameter space definitions in higher order spatial econometric models”. *Regional Science and Urban Economics* 42.1-2, pp. 211–220.

Elhorst, J. (2001). “Dynamic models in space and time”. *Geographical Analysis* 33.2, pp. 119–140.

Elhorst, P and S. Vega (2013). “On spatial econometric models, spillover effects, and W”. *ERSA conference papers* February, pp. 1–28.

Fingleton, B. (2009). “Spatial autoregression”. *Geographical Analysis* 41.4, pp. 385–391.

Gaspar, M. J. L. (2017). “New Economic Geography: Perspectives, multiple regions and individual heterogeneity”. PhD thesis, p. 182.

Geary, R. C. (1954). “The Contiguity Ratio and Statistical Mapping”. *The Incorporated Statistician.*

Gibbons, S. and H. G. Overman (2012). “Mostly pointless spatial econometrics?” *Journal of Regional Science* 52.2, pp. 172–191.

Gonçalves, S. and D. Politis (2011). “Discussion: Bootstrap methods for dependent data: A review”. *Journal of the Korean Statistical Society* 40.4, pp. 383–386.

Guidetti, F. and INSEE (2012). *En 2011, la consommation des ménages marque le pas.* Tech. rep., pp. 1–4.

Haggett, P. (1965). *Locational analysis in human geography.* Edward Arnold, p. 339.

Hannan, E. J. and B. G. Quinn (1979). “The Determination of the Order of an Autoregression”. *Journal of the Royal Statistical Society.*

Hungerland, W.-F. (2017). “The Gains from Import Variety in Two Globalisations: Evidence from Germany”. *Institute of Economic History, School of Business and Economics, Humboldt-University of Berlin*, mimeo.

Joly, I. and Y. Crozet (2006). “Budgets temps de transport et vitesses : de nouveaux enjeux pour les politiques de mobilité urbaine”. *La ville aux limites de la mobilité Actes du colloque "La ville aux limites de la mobilité"; Paris; 2006.* Ed. by M. BONNET and P. AUBERTEL. Presses Universitaires de France, pp. 287–296.

Joly, I., Y. Crozet, P. Bonnel and C. Raux (2002). *La "Loi" de Zahavi, quelle pertinence pour comprendre la contraction ou la dilatation des espaces-temps de la ville ?* Tech. rep.

Kaiser, U. (2002). “Measuring knowledge spillovers in manufacturing and services: an empirical assessment of alternative approaches”. *Research Policy* 31.1, pp. 125–144.

Kelejian, H. H. and I. R. Prucha (1999). “A generalized moments estimator for the autoregressive parameter in a spatial model”. *International Economic Review.*

Kelejian, H. H. and I. R. Prucha (2010). “Specification and estimation of spatial autoregressive models with autoregressive and heteroskedastic disturbances”. *Journal of Econometrics* 157.1, pp. 53–67.

Kunsch, H. R. (1989). “The Jackknife and the Bootstrap for General Stationary Observations”. *The Annals of Statistics* 4.2, pp. 1217–1241.

Lee, L.-f. (2003). “Best Spatial Two Stage Least Squares Estimators for a Spatial Autoregressive Model with Autoregressive Disturbances”. *Econometric Reviews* 22.4, pp. 307–335.

Lee, L. F. (2004). “Asymptotic Distributions of Quasi-Maximum Likelihood Estimators for Spatial Econometric Models”. *Econometrica* 72, pp. 1899–1926.

Lee, L.-f. and X. Liu (2010). *Efficient Gmm Estimation of High Order Spatial Autoregressive Models With Autoregressive Disturbances*. Vol. 26. 01, pp. 187–230.

LeSage, J. and R. K. Pace (2009). *Introduction to Spatial Econometrics*, p. 374.

LeSage, J. P. (1999). “The Theory and Practice of Spatial Econometrics”. *International Journal of Forecasting* 2.2, pp. 245–246.

Lin, K.-p., Z.-h. Long and W. Mei (2007). “Bootstrap Test Statistics for Spatial Econometric Models”.

Martellosio, F. (2011). “Efficiency of the OLS estimator in the vicinity of a spatial unit root”. *Statistics and Probability Letters* 81.8, pp. 1285–1291.

McDaniel, C. a. and E. Balistreri (2003). “A review of Armington trade substitution elasticities”. *Economie internationale* 2.94/95, pp. 301–313.

Moran, P. A. P. (1950). “Notes on Continuous Stochastic Phenomena”. *Biometrika*.

Morer, N. and INSEE (2015). *La consommation des ménages est encore convalescente en 2014*. Tech. rep., pp. 1–4.

Nelder, J. and R. Mead (1965). “A simplex method for function minimization”. *The computer journal*.

Ottaviano, G. I. P. and P. Martin (2001). “Growth and Agglomeration”. *International Economic Review* 42.4, pp. 947–968.

Paelinck, J. and P. Klaassen (1979). *Spatial econometrics*. Saxon House, Farnborough.

Pinkse, J. and M. E. Slade (2010). “The future of spatial econometrics”. *Journal of Regional Science* 50.1, pp. 103–117.

Rivera-Batiz, F. (1988). “Increasing returns, monopolistic competition, and agglomeration economies in consumption and production”. *Regional Science and Urban Economics* 18.1, pp. 125–153.

Samuelson, P. A. (1952). “The Transfer Problem and Transport Costs: The Terms of Trade When Impediments are Absent”. *The Economic Journal* 62.246, pp. 278–304.

Shapiro, S. S. and M. B. Wilk (1965). “An Analysis of Variance Test for Normality (Complete Samples)”. *Biometrika* 52.3/4, pp. 591–611.

U.S. Bureau of Labor Statistics (2012). *Consumer Expenditure in 2010: Lingering Effects of the Great Recession*. Tech. rep. August. US Bureau of Labor Statistics.

U.S. Bureau of Labor Statistics (2013). *Consumer Expenditures in 2011*. Tech. rep. April, pp. 1–23.

Wilson, A. G. (1970). *Entropy in Urban and Regional Modelling*. London : Pion, p. 166.

Zahavi, Y. (1979). *The ‘UMOT’ Project*. Tech. rep. l’U.S. Department of Transportation and the Ministry of Transport of Federal Republic Of Germany.

Zahavi, Y. and A. Talvitie (1980). “Regularities in travel time and money expenditures”. *59th Annual Meeting of the Transportation Research Board*. 750.

Chapter 3

Simulations

Contents

3.1 Agglomerations and the economy: the baseline case **119**

3.1.1 Transport cost resilience to rising fuel price 129

3.1.2 Transport cost under migratory pressure 134

3.2 Agglomerations and the economy under alternative policy scenarios **137**

3.2.1 Urban transportation measures 138

3.3 Main findings **150**

Chapters 1 and 2 have presented a methodological proposal that, obviously, cannot take all economic phenomena into account. An important number of functional links are implicitly assumed at the initialization stage of the model and then projected constant over the prospective horizon, *e.g.* agents' perceptions and behaviors, CBDs' position, transport energy source – fossil fuels –, dynamics of investment such as those in public transit infrastructure and equipment. Thus, the methodological challenge is that the functional links retained by the model allow for a sufficiently effective representation of reality to support policy reasoning. What follows must therefore be regarded as a prototype example of the types of exercise that can be carried out when using GEMSE.

3.1 Agglomerations and the economy: the baseline case

In this section we describe the aggregate trends and urban trajectories that occur in the baseline scenario, defined as the continuation of the current economic status in absence of specific urban policy. These baseline trends then serve as a referential from which all deviations due to policy implementation are isolated by difference.[1] We identify the likely determinants of the differences exhibited by these deviations between the USA and France. Any temporal trajectory presented in this section is (dynamically) viewable online for the USA and France by selecting `baseline` in the field called `Scenarios`.[2] Moreover, note that in what follows, USA's (France's) trends and maps ***are always displayed on figures left (right) dials***.[3]

For each country, we are interested in how urban areas (whose specificities are captured in the model) react to an identical shock. We thus made the methodological choice of assuming identical exogenous macroeconomic evolutions: *(i)* the baseline trends feature a doubling of each nation's GDP over the prospective horizon, in parallel with a regular but moderate increase of both labor productivity and capital investment, which are sufficiently high to jointly ensure a steady increase of economic activity; *(ii)* each nation's labor market state unemployment rates that have been maintained constant since their level in 2015; *(iii)* the unitary fuel liter consumption per kilometer from vehicles has a consensual low negative slope over the period, from 7 liters per 100 kilometers in 2008 to 4.5 in 2050 and; *(iv)* following Waisman and Grazi (2013), the domestic price per liter of fuel in the two countries states a positive variation over the prospective horizon [Figure 3.1],[4] whose rather moderate increases are permitted by an average annual energy efficiency gain of 1.1%, over the first half of the century.

GEMSE provides urbanization trends that are consistent with the evolutions of the aggrega-

[1] This is equivalent to totally differentiate GEMSE with respect to urban policies.

[2] Respectively available at `http://gemse.alwaysdata.net/usa/` and France `http://gemse.alwaysdata.net/france/`.

[3] See Section 1.C's tables that make the concordance between the variables of the model and their online names.

[4] Admittedly, making the hypothesis of an identical evolution of the price of fuel for the two countries constitutes a lack of realism on the historical and predictive levels. *E.g.* USA's energy prices and growth have already been, and will continue to be, changed by the increasing exploitation of shale gas.

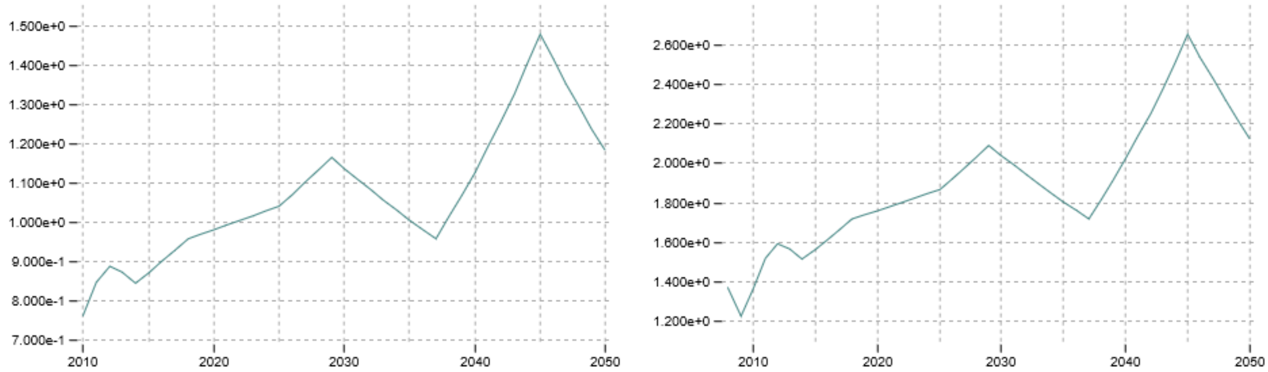
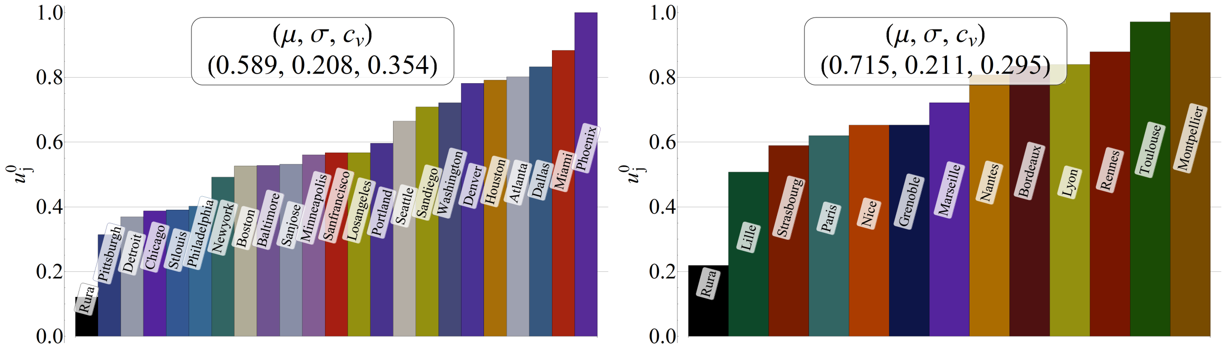


Figure 3.1: US ($) and French (€) domestic prices per liter of fuel

tes that are described above. The first determinants of the spatial distribution of these trends are the levels of amenity, described as the utility derived from simply living in agglomeration j, the u_j^0s. Recall that those are calibrated [see Subsection 2.2.2's eq.(2.14)] so as to yield welfare



μ stands for the sample average.
σ stands for the sample standard deviation.
c_v stands for the coefficient of variation. It is computed as σ/μ.

Figure 3.2: Levels of amenity in American and French areas

differentials that lead to growths of areas' population that fit that that are observed before the first year of the prospective horizon [see eq.2.14], *i.e.* each area's population growth between $t = -1$ and $t = 0$. The levels of amenity that are obtained are shown in Figure 3.2, respectively for the USA and France.

These levels of amenity are the first signals of attractiveness for population, which are then tempered by considerations related to urban cost, home market and employment.[5] The latter two considerations – home market and employment – are at the interface of migration decisions of populations and firms, each driven by *their location-specific attractiveness index* [see from eq.(1.80) to eq.(1.82)]. Figures 3.3 and 3.4 report the evolution of some of these indexes

[5]The *home market* is defined from local firms' standpoint. For firms, a market is qualified as the "home" one if most of its products are consumed therein, which consequently allow them to minimize transportation costs. Note that the so-called home market *effect* is a direct consequence of modeling firms that produce goods subject to shipment costs under increasing returns to scale (Krugman, 1980).

expressed in terms of ranks,[6] respectively for population and firms. These two parallel mecha-

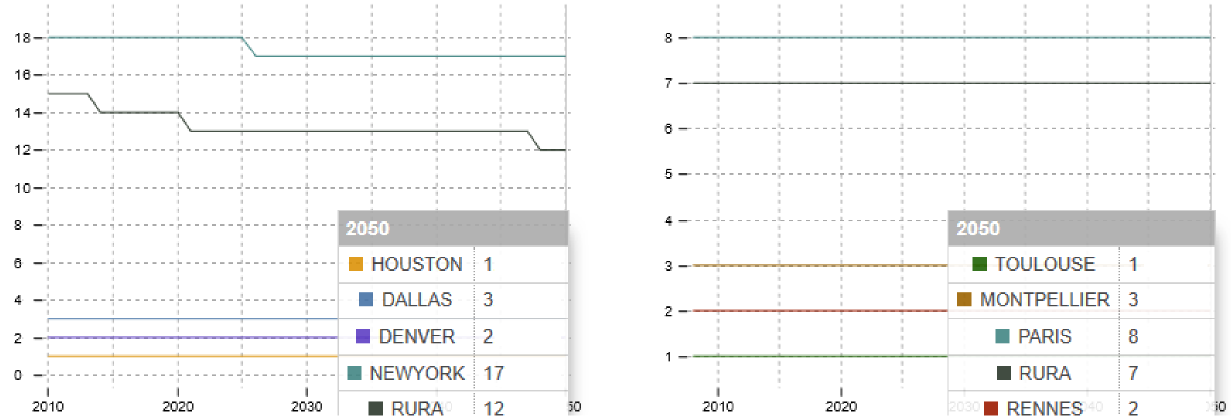


Figure 3.3: Population attractiveness ranks of each nation largest urban space, the top three and rural

nisms of migration jointly determine the spatial distribution of: *(i)* workers, firms and incomes across urban areas and; *(ii)* the dynamics of elements that are constitutive of urban costs – as captured by temporal changes on average agglomeration density, housing rents, mobility habits and transport costs.

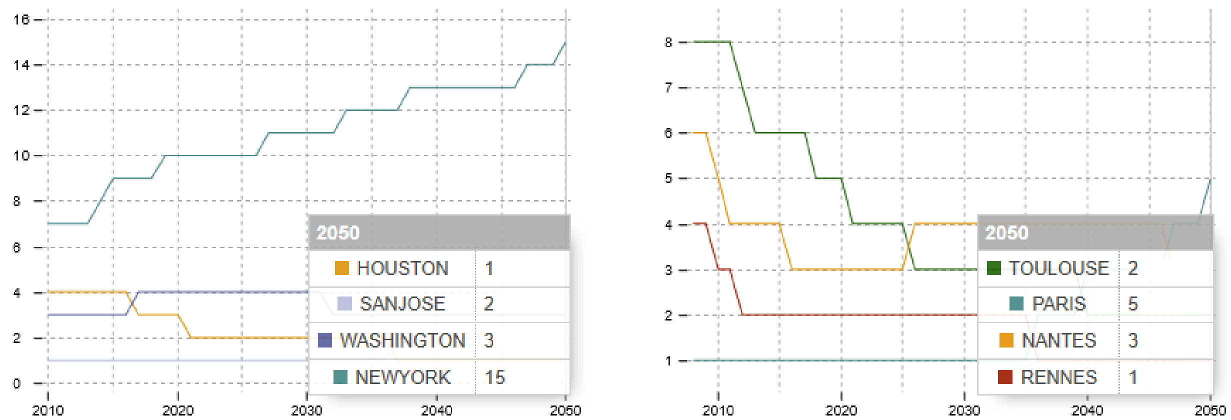


Figure 3.4: Firms attractiveness ranks of each nation largest urban space and the top three

Degree of centralization of economic activity

Let's first figure out how the concentration of economic activity behaves over the horizon in each country. A first approach to do so is by focusing on the trajectories of the urban shares of each nation's largest urban space that are related to some sensitive economic indicators.

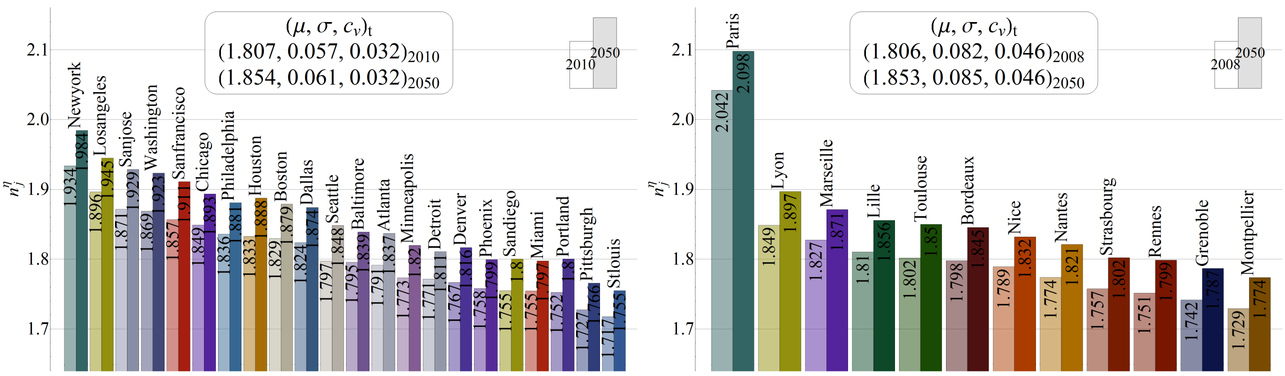
[6]Note that only some areas' evolution (of the variable under consideration) are reported here for the sake of clarity. Indeed, the illustration of the USA's 22 (France's 12) curves in a single chart would complicate legibility. Note that these charts are available online and that area's trends can be removed dynamically by clicking on their name.

NewYork's and Paris's urban shares of workers and of firms are shown in Table 3.1. It appears that for both population and firms, NewYork and Paris do not exhibit the same temporal profiles of attractiveness. Even if both possess capital and labor/consumer stocks permitting

Urban share of	NEW YORK					PARIS				
	2010	*2020*	*2030*	*2040*	*2050*	*2008*	*2020*	*2030*	*2040*	*2050*
Labor force	13.89	13.36	12.87	12.42	11.98	50.16	49.75	49.48	49.24	49.03
Workers	13.97	13.5	13.08	12.65	12.19	50.54	50.43	50.35	50.18	50.01
Active firms	14.6	14.62	14.6	14.54	14.45	55.58	55.87	56.09	56.28	56.45

Table 3.1: Shares in each nation largest urban space (%)

economies of scale, the strength of their respective attractiveness if far more inert in France than in the USA. This more inert geography of production in France than in the USA suggests a less severe competition among France's urban centers than among USA's. Recalling that (spillover-adjusted) unitary labor requirements (ULR), l_j/n_j^η [see eq.(1.1)], are *ceteris paribus* reduced in larger markets –,



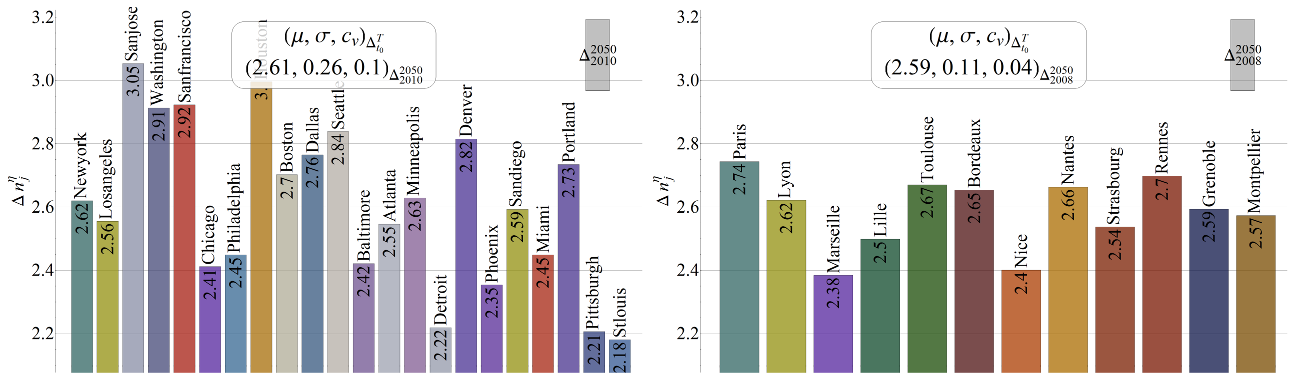
μ stands for the sample average.
σ stands for the sample standard deviation.
c_v stands for the coefficient of variation. It is computed as σ/μ.

Figure 3.5: ULR dividers in the USA and France

Figure 3.5 shows this competition intensity (in terms of knowledge spillover), via all areas effective ULR dividers, the n_j^ηs. As stated by each nation's related coefficient of variation, c_v, the n_j^ηs are clearly much closer to each other in the USA than in France, where Paris's is far ahead of that of other urban areas.[7]

Maintaining the same display order of ranking as that of Figure 3.5, Figure 3.6 shows the relative evolution of each (urban) area's ULR over the prospective horizon. It confirms the indisputable Paris-centralized geography of production in France on the one hand and the (NewYork-) decentralization process that takes place in the USA on the other hand. Indeed, in addition to benefiting from the highest external economies of scale over the entire prospective horizon, Paris also experiences the greatest growth in those, which enforces its indisputable

[7]We recall that in rural area, this divider is implicitly set to 1.



μ stands for the sample average.
σ stands for the sample standard deviation.
c_v stands for the coefficient of variation. It is computed as σ/μ.

Figure 3.6: Variation in ULR dividers in the USA and France (%)

central position within the French production system. In the background, the dynamism of urban areas such as Rennes, Nantes, Bordeaux and Toulouse influences the relocation of the gravity center of the production-geography toward the west-southwest of France at the expense of Lyon, Marseille and Lille. In the USA, NewYork's dynamism is only ranked 11th from this "variation" standpoint, notably surrendering to Sunbelt's urban areas such as SanFrancisco, SanJose, Houston and Dallas.

In our framework, comparing the ULR dividers actually boils down to studying the spatial concentration of firms. A common indicator to deal with this sort of consideration is the

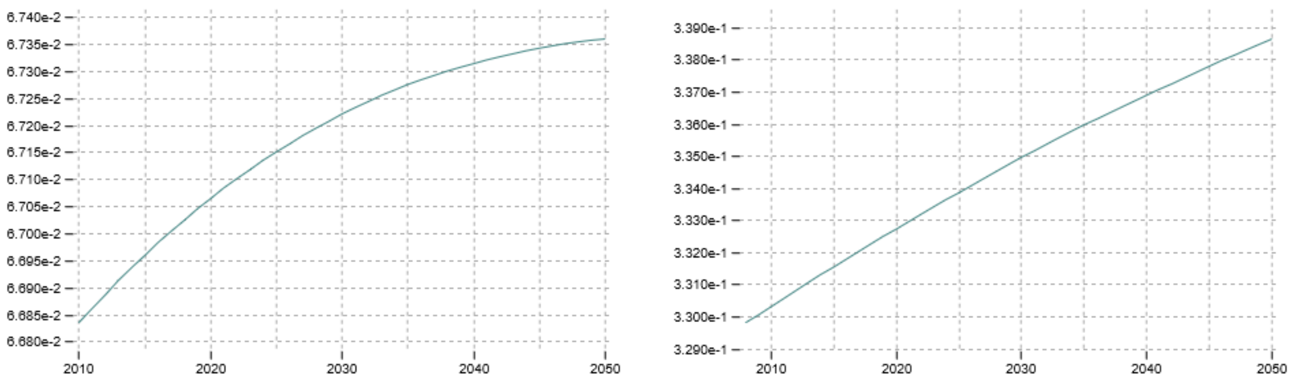


Figure 3.7: Firms-number related HHI of the USA and France

Herfindahl-Hirschmann index (HHI), which describes the degree of concentration (of anything) between $k > 1$ entities and lies in the interval $[1/k, 1]$.[8] Figure 3.7 shows this indicator with the spatial distribution of firms in the USA and France. In the case of USA – where $1/k = 1/22 \approx 0.045$ –, the indicator starts from a fairly dispersed geography of firms, barely increases

[8]In the case of $k > 1$ entities, an HHI equal to $1/k$ stands for the most even possible distribution between the k entities, while an HHI equal to 1 stands for a distribution in which one entity concentrates everything. Note that as put forward by Bosker et al. (2007), "using other more sophisticated measures [of concentration] does not change our results qualitatively".

and finally perceptibly operates an inflection before the end of the prospective horizon. In the case of France – where $1/k = 1/12 \approx 0.083$ –, the indicator starts from an already high level of concentration and follows a positive and quasi-constant slope until the end of the simulation.

We can conclude differently regarding USA's and France's spatial distribution of firms: the former is on the verge of (re-) spreading across the country while the latter is gaining momentum around Paris. This explains Table 3.1's urban shares in active firms of NewYork and Paris that respectively decrease and increase over the prospective horizon.

Degree of concentration of population

In Figure 3.8, we are however shown that the USA and France are at similar stages relatively to the dynamic of dispersion of their respective population, *i.e.* of their respective labor force *supply*. The upper dial of this figure deals with an HHI related to the concentration of their populations over all areas, $\text{HHI}^{\mathcal{P}}$, while the lower dial shows the same indicator computed over urban areas only, $\text{HHI}^{\mathcal{P}_{j=1,\ldots,N}}$. When interpreting $\text{HHI}^{\mathcal{P}}$ and $\text{HHI}^{\mathcal{P}_{j=1,\ldots,N}}$ together, it appears that each country's "rural" share of population restarts to grow around the period 2025-2030. Put differently, USA's and France's geography of people localizations is reconcentrating over the long term in small urban agglomerations after a long and historical phase of depopulation. We insist on the fact that this inversion over the long term needs not be strictly interpreted as a result of migration patterns from urban to "rural" areas, but rather from larger to smaller urban agglomerations, projected to become attractive.

This decrease in the attractiveness of some of the largest urban areas is one of the consequences of the secular general urban in-migration. To understand the logic behind people's dynamic of migration, let's first consider what drives them: the levels of attractiveness of areas [see eq.(1.82)]. Remember that this variable summarizes in real terms both the people's level of (indirect) utility and their job opportunities [see eq.(1.50)].[9] Figure 3.9 shows all areas' levels of welfare as well as the national attractiveness threshold – *i.e.* the population-weighted average over areas, $\overline{W}_t$ – above (or below) which an area sees its in-migration be positive (or negative). As it reads in Figure 3.9, USA's and France's largest urban area, *i.e.* NewYork and Paris, are differently positioned relatively to their national attractiveness-threshold. Below this attractiveness-threshold, both NewYork and Paris experience out-migration, which explains their negative trend reported in Table 3.1's urban shares of labor force. However, Paris's out-migration is moderate since the urban area remains at a very short distance from the threshold, which in turn does not help to reduce urban costs. On the contrary, NewYork is distant from the threshold, which exerts a downward pressure on urban costs.

Figure 3.10 shows the variations of these urban costs over the prospective horizon. Fostered

[9]In this micro-based context, the term *real* refers to using inverse levels of utility as deflation factors, *i.e.* as *exact price index*. For example of uses, see *e.g.* Eaton and Kortum (2002, p.1749), Cavailhès et al. (2007, p.388), Murata and Thisse (2005, p.141) or simply eq.(1.99) that would be the price index in an economy with only one (heterogeneous) good.

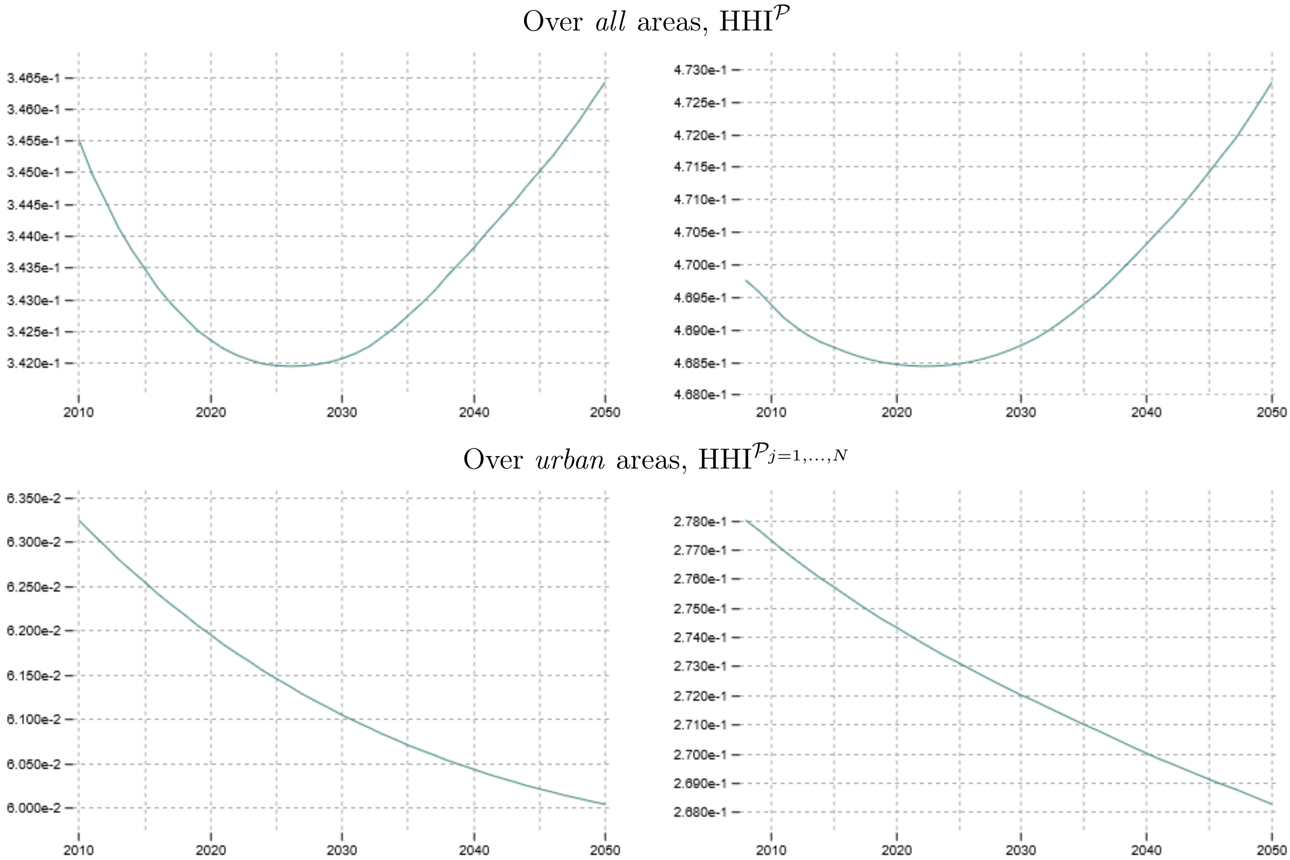


Figure 3.8: Population related HHI of the USA and France

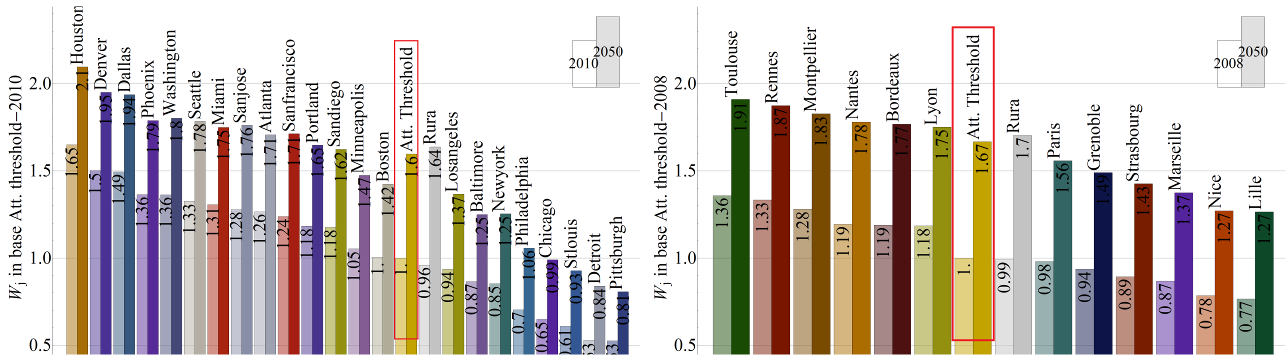
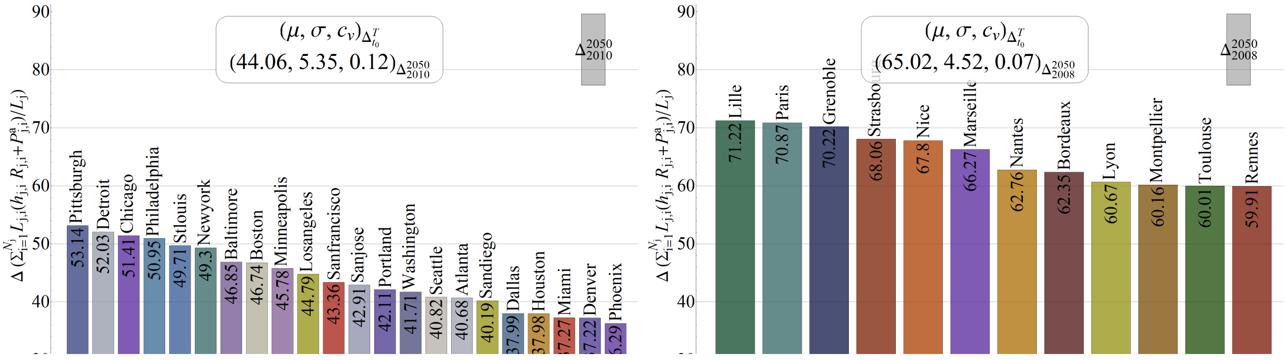


Figure 3.9: Welfares in the USA and France

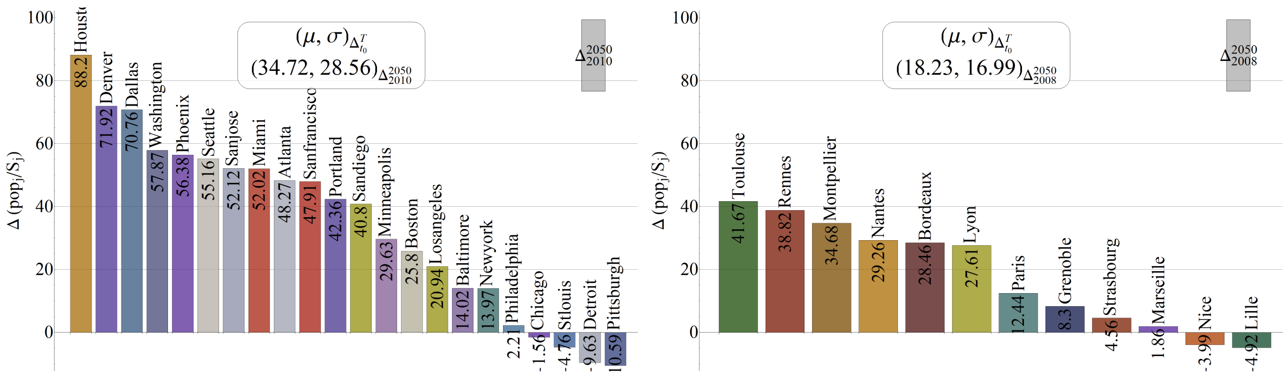
by the volatile yet strongly increasing price profile of fuel [see Figure 3.1], potential augmentations of densities that mechanically follow in-migrations are tempered by the preexisting urban costs as well as their sensibility. This is confirmed when considering Figures 3.10 and 3.11 together, in which the relation between urban costs and densities is more or less strictly proportional, depending on agglomerations' empirical characteristics, *i.e.* their transport network-structure and infrastructure capacity to dilute congestion (and housing pressure) over modes and space.

The consequences of these changes in the labor market conditions are not neutral for employ-



μ stands for the sample average.
σ stands for the sample standard deviation.
c_v stands for the coefficient of variation. It is computed as σ/μ.

Figure 3.10: Variation in average urban costs in the USA and France (%)



μ stands for the sample average.
σ stands for the sample standard deviation.

Figure 3.11: Variation in densities in the USA and France (%)

ment rates and wages.[10] As shown in Figure 3.12, those metropolitan areas share a downward

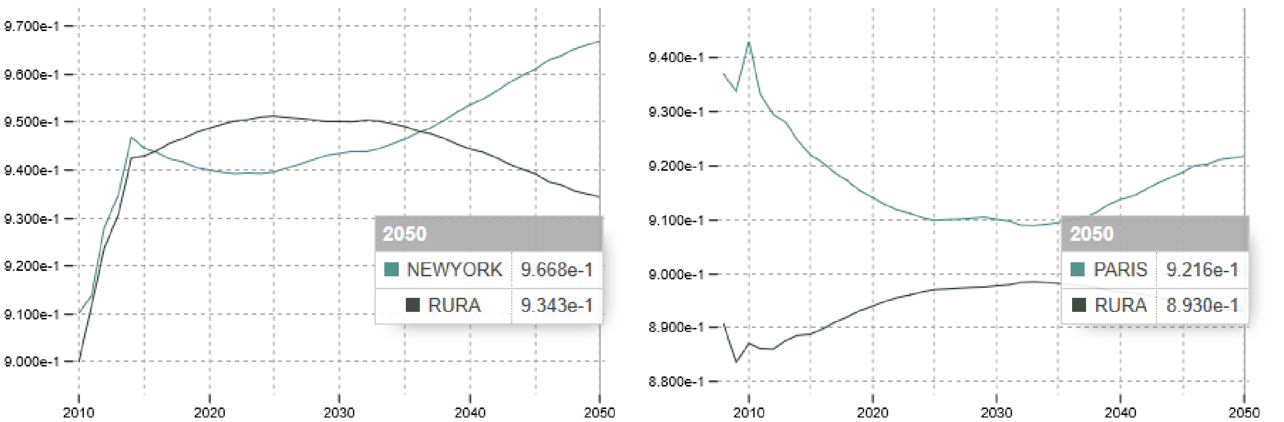


Figure 3.12: Employment rates of each nation largest urban space and rural

[10]We recall that each area's labor force supply and labor force demand [see eq.(1.48) and eq.(1.49)] jointly formalize a local labor market whose equilibrium is reached via wages [eq.(1.12) and eq.(1.36)].

trend of their employment rate over the short term, before a reversal occurs over the long term when inflows of population from small towns end, which inverts labor market frictions with net job creation. Thus, GEMSE predicts that big urban centers are likely to experience an increase of their employment rate around the year 2030 on the sole basis of a dynamic change in population migrations.

The necessary consideration of the urban scale

The interactions of the consequences of population and firms migrations progressively reveal new geographies of attractiveness [see Figure 3.3] and competitiveness [see Figure 3.4]. The magnitude of these consequences depends on the parallel adjustment of the utilization rate of transport infrastructures to changing local demographic conditions. Put differently, a *rising fuel cost* and *agglomerations in-migrations* do not necessarily lead to higher urban costs, depending on the capacity of urban areas' transport infrastructures to dilute congestion over modes and space.

If one compares the fuel price evolution [see Figure 3.1] over the prospective horizon with that of each country's average (over modes and workers) transport budget, shown in Figure 3.13, it is clear that spatial reorganization of economic activity can absorb nearly any fuel price shock. In the USA, where – as explained previously – the geographies (of population and firms) are far less locked than in France, it is even more acute. The answer to *why* there is such a great difference between the USA's and France's profile is tackled in subsection 3.1.2.

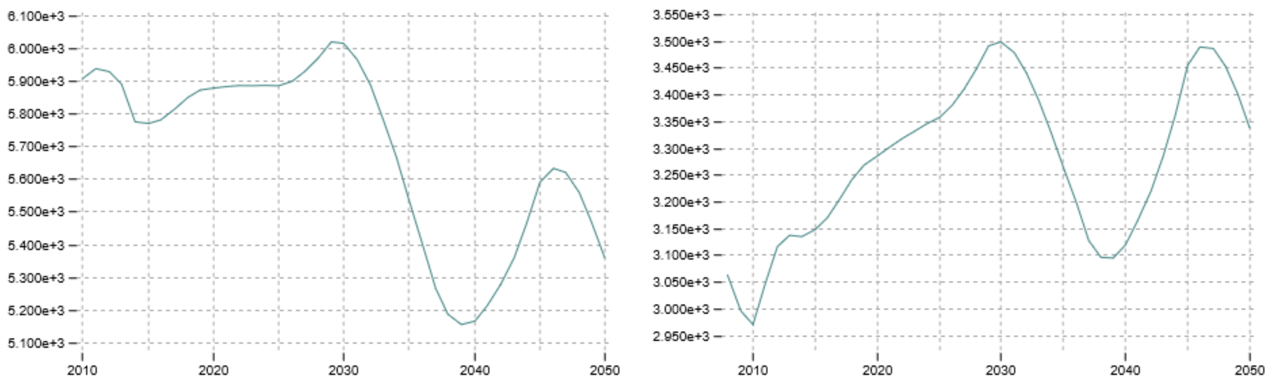


Figure 3.13: Average transport budget in the USA and France

The obtention of the profiles shown in Figure 3.13 necessarily relies on some urban areas being sufficiently equipped in terms of transport infrastructures. These well-equipped urban areas could become the corner stones of national strategies to absorb (on average) fuel price shocks. Examples of such urban areas are Houston and Toulouse, where the capacity and connectivity (to the CBD) of public transport in a given settlement pro-environmentally drives the modal share of cars [Figure 3.14], transport costs [Figure 3.15] and density [Figure 3.16]. It is worth mentioning that, relatively to other urban areas, these pro-environmental changes do not affect that much their internal dynamic since they monopolize the first attractiveness [Figure 3.3] and competitiveness [Figure 3.4] ranks.

In a policy-relevant perspective, it is interesting to quantify how likely a given urban area

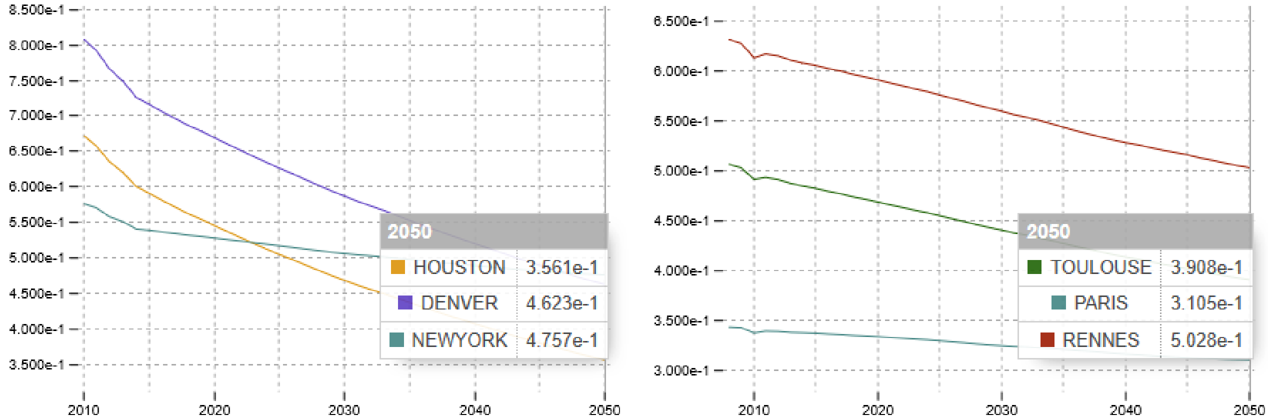


Figure 3.14: Modal share of car of each nation largest urban space and first ranked

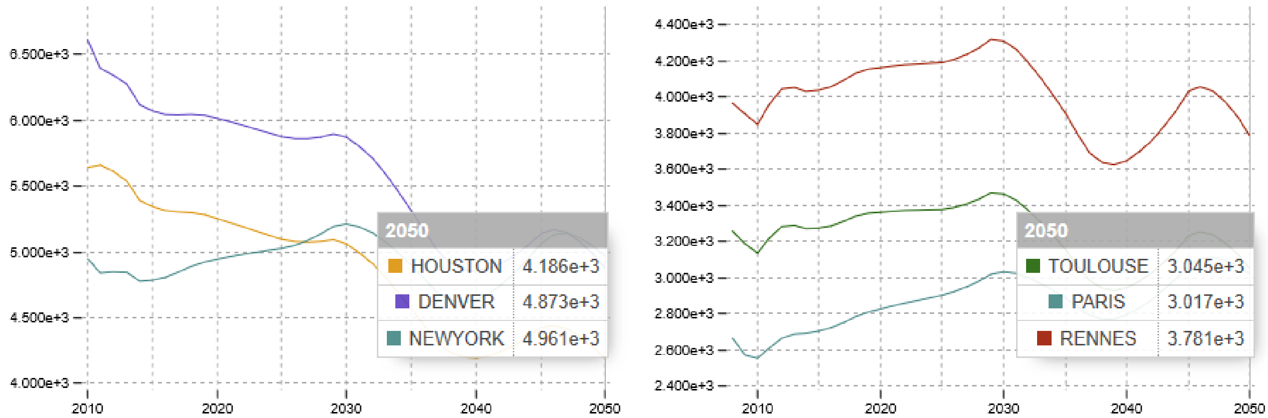


Figure 3.15: Transport budgets of each nation largest urban space and first ranked

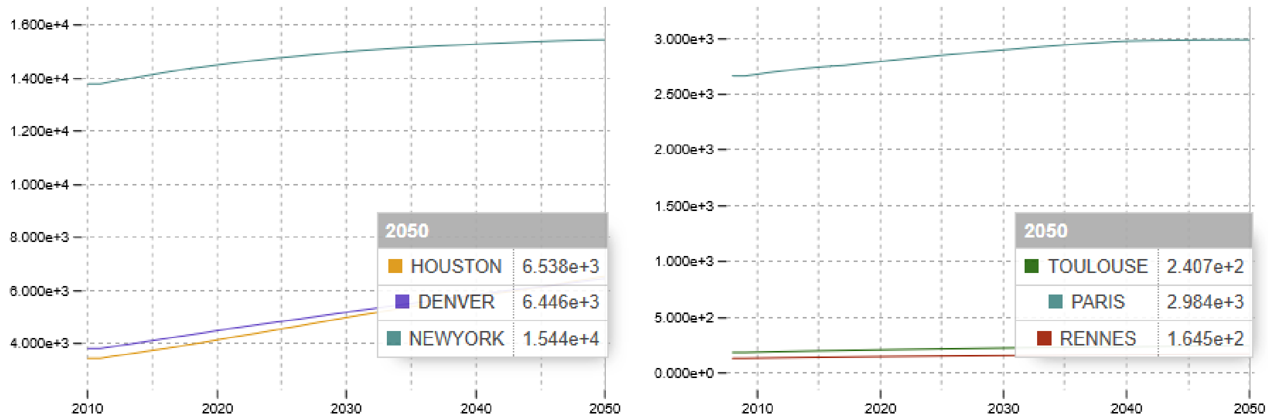


Figure 3.16: Density of each nation largest urban space and first ranked (hab/km²)

is to conduce to cost-minimizing and pro-environmental behaviors, given the specificities of its spatial structure and transport infrastructure.

3.1.1 Transport cost resilience to rising fuel price

In this subsection, we *probabilize* how transport-costs minimization behaviors occur in each urban area in presence of an increase in fuel price. For each urban area, the sample needed to conduct the probabilization is built from local places.

Remember the definition of the average transport cost of a representative worker living in place i of agglomeration j, $P^{a}_{j,i}$ [see eq.(1.26)] and of its determinants: *(i)* the equilibrium modal share of cars, $\alpha^{*}_{j,i}$ [see eq.(1.62)], *(ii)* the equilibrium number of workers who take public transport, $L^{PT*}_{j,i}$ [see eq.(1.61)]; *(iii)* the equilibrium utility level, u^{*}_{j} [see eq.(1.60)] and; *(iv)* the numbers of workers who take the car in the overall agglomeration j, $\mathbf{\Lambda}^{*}_{j}$ [see eq.(1.58)]. More formally, the question posed in this subsection is

$$\forall\{j,i\} \in [\![1;N]\!] \times [\![1;N_j]\!], \text{ how likely is } P^{a*}_{j,i}\Big|_{\substack{P^{a,PV}\\ \alpha^{*}_{j,i}}} = P^{a*}_{j,i}\Big|_{\substack{P^{a,PV}\left(1+\Delta^{P^{a,PV}}\right)\\ \alpha_{j,i}+\mathrm{d}\alpha^{*}_{j,i}}} ? \tag{3.1}$$

where $\Delta^{P^{a,PV}}$ stands for a (scenarized) relative variation in the cost of fuel. Note that $\Delta^{P^{a,PV}} \neq 0$ stands for the (relative) shock in fuel price. $\mathrm{d}\alpha^{*}_{j,i}$ is the differential in the modal share of cars and represents the resilience of the transport costs against the rise in fuel cost in place i of agglomeration j.

Note that addressing eq.(3.1) is not possible without simplifying some urban-scale elements of representation in GEMSE. Indeed, answering to the problem stated in eq.(3.1) has no interest if the $\mathrm{d}\alpha^{*}_{j,i}$s that are obtained are not those of a urban area in equilibrium, *i.e.* if the $\mathrm{d}\alpha^{*}_{j,i}$s that are obtained are transitory. But it is very unlikely that an urban equilibrium – among those in the set of all urban equilibria – is concomitant with a situation in which all places would have operated a modal shift that promotes the resilience of transport costs. However, this concomitance is possible if the numerical exercise outlined by eq.(3.1) is conducted for one place at a time. The necessary simplifications are presented in Box 3.

Box 3. When shocking only one place at a time, modal shifts that occur locally (are likely to) change the overall congestion state of the urban road network. This would necessarily lead other road-connected places to be moved from their urban equilibrium state, in turn making transitory the state of the shocked place. One must thus find the ante-$\mathrm{d}\alpha^{*}_{j,i}$ geography of congestion that, once $\mathrm{d}\alpha^{*}_{j,i}$ is effective, both neutralizes locally the rise in fuel cost and balances – via network effects [see eq.(1.52)] – the whole urban area.

The second element of representation that is simplified is related to the circular dependence that articulates the triptych {transport, housing, utility level of equalization} [see Subsection 1.1.4] and that ultimately leads to the equalization of all utility levels, characteristic of an urban area at equilibrium. To break this circularity, we chose to simplify the behavior of housing developers – open to discussion, *e.g.* the determinants of housing rents and the expectations of developers on future rents –, temporarily assumed to be different so as to facilitate the present numerical exercise.

Identification of the ante-d$\alpha^*_{j,i}$ geography of mobility

One wants place i's (transport-cost-immunizing) variation in the number of cars, $\Delta^{\Lambda *}_{j,i}$, to return accessibility levels in other places identical to what they are with no i-increase in fuel cost. This implies adjusting other places' number of cars so that $\Delta^{\Lambda *}_{j,i}$ puts the network in an identical congestion state between the shocked and the non-shocked situations, at least with regard to all places but i. The excess of cars on the road network – recalling that these excesses are negative –, $\mathrm{d}\mathbf{V}_j$, of which this adjustment is a function, is

$$\mathrm{d}\mathbf{V}_j = \mathbf{\Pi}_j(\mathbf{e}_{j,i}\mathrm{d}\Lambda^*_{j,i})$$

where $\mathbf{e}_{j,i}$ is a $N_j \times 1$ vector whose elements all equal 0 except the ith one that is equal to 1. $\mathrm{d}\Lambda^*_{j,i} = \Delta^{\Lambda *}_{j,i}\Lambda_{j,i}$ is the (scalar) differential in the number of cars in place i.
With $-i$ standing for the index of the component that is removed from the vector in question, define $\mathrm{d}\mathbf{\Lambda}_{j,-i}$, a $(N_j - 1) \times 1$ vector whose components compensate for the variation in the number of cars that occurs in place i in order to keep the same state of congestion between the shocked post-d$\alpha^*_{j,i}$ situation, and the non-shocked situation. $\mathrm{d}\mathbf{\Lambda}_{j,-i}$ is computed as follows

$$\mathrm{d}\mathbf{\Lambda}_{j,-i} = \left(\mathbf{\Pi}_{j,-i-i}\right)^{-1}\mathrm{d}\mathbf{V}_{j,-i}$$

where $\left(\mathbf{\Pi}_{j,-i-i}\right)^{-1}$ allocates/transforms over all places but i, the flow-differentials, $\mathrm{d}\mathbf{V}_{j,-i}$, into stock-differentials, $\mathrm{d}\mathbf{\Lambda}_{j,-i}$.
Finally, a geography of accessibility is *ceteris paribus* d$\alpha^*_{j,i}$-immunized if the number of cars in all places but i at equilibrium, $\hat{\mathbf{\Lambda}}_{j,-i}$, is such that

$$\hat{\mathbf{\Lambda}}_{j,-i} = \mathbf{\Lambda}_{j,-i} - \mathrm{d}\mathbf{\Lambda}_{j,-i}$$

The version of $\hat{\mathbf{\Lambda}}_{j,-i}$ over all places, say, $\mathbf{\Lambda}^*_j$, is simply formed over $\hat{\mathbf{\Lambda}}_{j,-i}$ with $\Lambda_{j,i} + \mathrm{d}\Lambda^*_{j,i}$

inserted as ith element. It follows that $\mathbf{\Lambda}_j^*$ has the following intrinsic property

$$\left\| \left(\mathbf{\Pi}_j \left(\mathbf{\Lambda}_j^* \Big|_{\mathrm{d}\Lambda_{j,i}^*=0} - \mathbf{\Lambda}_j^* \Big|_{\mathrm{d}\Lambda_{j,i}^* \in \mathbb{R}} \right) \right)_{-i} \right\| = 0$$

Put differently,

$$\mathbf{\Pi}_{j,\hat{\imath}.} \left(\mathbf{\Lambda}_j^* \Big|_{\mathrm{d}\Lambda_{j,i}^*=0} - \mathbf{\Lambda}_j^* \Big|_{\mathrm{d}\Lambda_{j,i}^* \in \mathbb{R}} \right) \begin{cases} = 0 & \text{if } \hat{\imath} \neq i \\ \neq 0 & \text{otherwise (if } \hat{\imath} = i) \end{cases}$$

where $\mathbf{\Pi}_{j,\hat{\imath}.}$ stands for the $\hat{\imath}$th row of matrix $\mathbf{\Pi}_j$. It follows that the vector of PV-speeds in all places but i, $\boldsymbol{v}_{j,-i}^{PV}$, is $\mathrm{d}\alpha_{j,i}^*$-immunized, and so is the utility that derives from it.

Simplification on the housing side

The flow of cars that were coming from place i have been taken off the road and then disseminated over the whole urban area to maintain the network in an identical state of congestion. This means that, admittedly, the urban equilibrium is henceforth undermined in all other (network-connected) places, but in an epsilonic range the higher the number of places. It is thus assumed – as outlined previously – that housing markets equilibria adapt to ensure the constancy of the utility level of equalization, u_j^* [eq.(1.60)], at its ante-shocked value. This both concludes the presentation of the analytic framework and initiates the presentation of the numerical evidences.

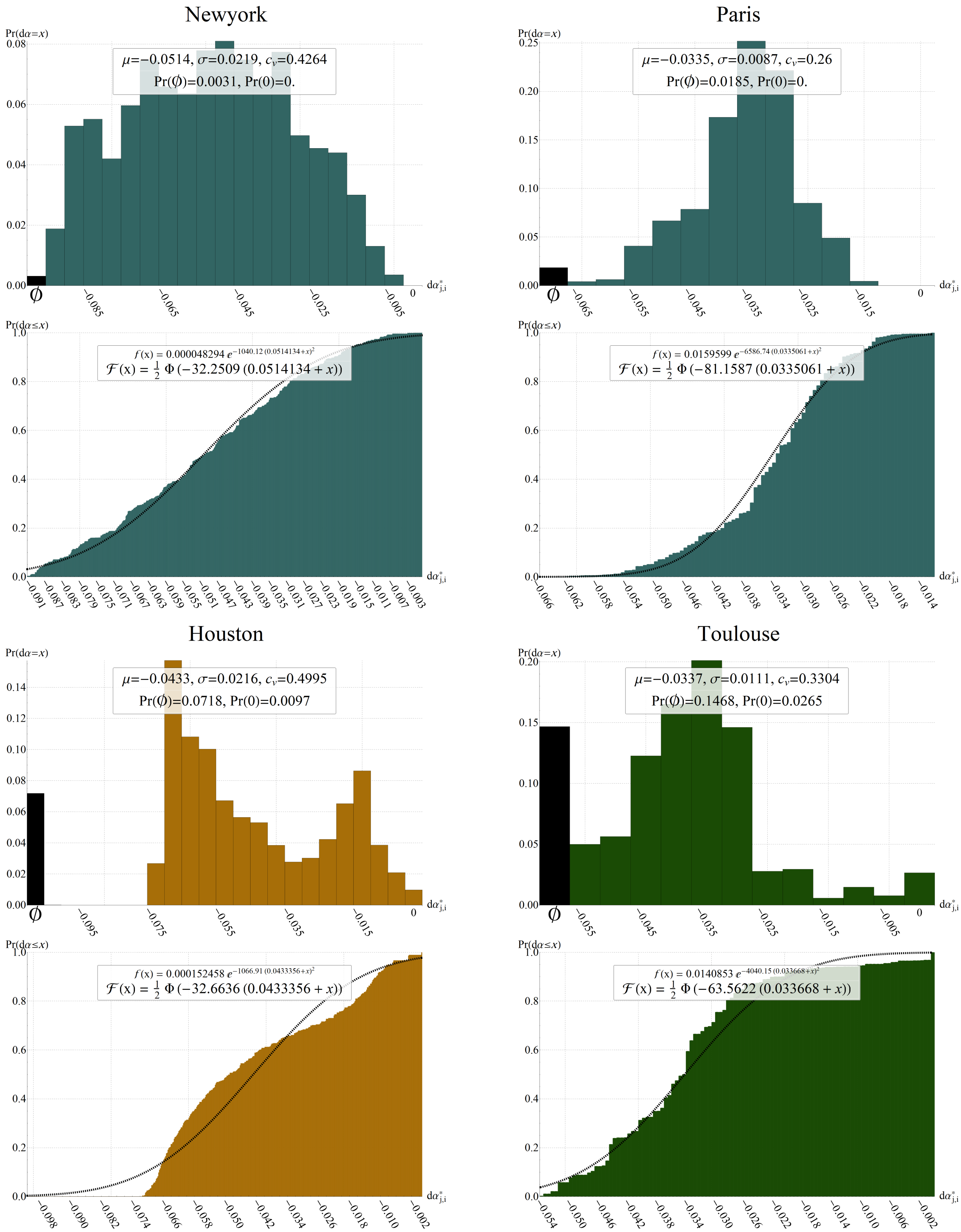
Probabilizing eq.(3.1)'s event for $\Delta^{P^{a,PV}} = 0.1$

In what follows, we present the results that are obtained in 2030 when studying eq.(3.1) for $\Delta^{P^{a,PV}} = 0.1$, *i.e.* for a 10% increase in fuel cost. The results consist of urban areas' distributions of the $\mathrm{d}\alpha_{j,i}^*$. Note that all urban areas' distributions were computed and are available online for the USA and France.[11] Normal[12] (probability and cumulative) density functions have been fitted to provide readers with the possibility to model these distributions in independent numerical exercises.

Figure 3.17 shows these distributions for NewYork, Houston, Paris and Toulouse. In the upper and lower dials of these figures, we show, respectively, the probability and cumulative

[11] The two files are in rar format, available at https://gemse.alwaysdata.net/static/gemse/USA/US_MShifts_dist.rar and https://gemse.alwaysdata.net/static/gemse/FRANCE/FR_MShifts_dist.rar.

[12] Which should not be the case in view of the (not reported) p-values of each fit to the test of Shapiro-Wilk – whose null hypothesis is the normality of the tested distribution –, never greater than $1e^{-10}$. A more appropriate approach would be to provide the best fits by testing a range of distributions, selecting the one associated to the highest p-value (to the most powerful test), and this over samples that may have been cut beforehand if they significantly have multiple modes.



[a] $\Pr(\emptyset)$ is the probability that a worker has no alternative but car.
[b] $\Pr(0)$ is the probability that a worker already takes public transport and is thus not subject to the 10% increase in fuel cost.
[c] $f(x)$ stands for the PDF fitted from the (population weighted) distribution of the $d\alpha^*_{j,i}$s.
[d] $\Phi(z(x))$ stands for the complementary error function integrated between its argument, z, and ∞.
[e] $\mathcal{F}(x)$ stands for the CDF fitted from the (population weighted) distribution of the $d\alpha^*_{j,i}$s. Note that when $z(x) < 0$, $\mathcal{F}(x)$ and $z(x)$ actually turn into $1 - \mathcal{F}(x)$ and $|z(x)|$ respectively.

Figure 3.17: Distribution of $\Delta^{P^{a,PV}}$-resilient modal shifts in 2030

histograms of $\mathrm{d}\alpha^*_{j,i} \forall i \in [\![1; N_j]\!]$.[13] Two indicators are of first interest in these charts: $\Pr(\emptyset)$ and $\Pr(0)$, respectively the probability that a worker has no alternative but car-driving and the probability that a worker is not subject to increases in fuel cost because already using public transports. In NewYork or in Paris, $\Pr(\emptyset)$ is respectively around 0.3% and 1.9%, which shows that in these two areas, almost everyone has the possibility to react over the very short term so as to compensate for an increase in fuel price. Put differently, in those two areas, few people are captive when faced with an increase in fuel price, which would lead them in the worst-case scenario to (first intra-area and then possibly inter-area) relocate. At the opposite end, as attested by the fact that $\Pr(0) = 0$ in these two areas, no one is living in places in which $\Delta^{P^{a,PV}} \neq 0$ is of no consequence *ceteris paribus*. This shows that even if weak, a change in fuel price actually does have an impact wherever people live within NewYork and Paris, respectively with a pro-resilient modal shift level – from cars to public transport – of around 5.1% and 3.6% on average. On the contrary, in Houston or Toulouse, $\Pr(\emptyset)$ is large, around 7% and 15% respectively, which suggests that these two areas are very likely to experience non-negligible changes of their spatial distribution of population when faced with changes in fuel price. That being explained, note that Houston and Toulouse already have places where $\Delta^{P^{a,PV}} \neq 0$ has no impact *ceteris paribus*, with respectively around 1% and 2.6% of people who are only using public transports.

Remark that in all of the above, means, standard deviations and coefficients of variation – that are displayed in Figure 3.17 – are all the more questionable as $\Pr(\emptyset)$ is great, since $\Pr(\emptyset)$ cannot be taken into account in the computation of these moment-statistics. One way of considering $\Pr(\emptyset)$ in relation to these moment-statistics, is by thinking of it as an indicator of their reliability, *i.e.* as $1 - \Pr(\emptyset)$.

Figure 3.18 boxplot-summarizes all the characteristics of these 2030-distributions for the USA and France. Via this figure, we see that urban areas have unequal ($\Delta^{P^{a,PV}}$-resilient) reactions to an increase in fuel price, with median levels of effort ranging from low to high by a factor of almost 1.5 in the USA and of almost 3 in France. We have also shown that America's most attractive urban area, Houston – whose rank is stable over the prospective horizon [see Figure 3.3] –, is also the most capable of costlessly absorbing a 10% positive shock in fuel price. Implicitly, we have also shown that the level of exposure of some urban areas to sudden increases in fuel price are likely to have costly consequences on their internal dynamic: a part of these $\Delta^{P^{a,PV}}$-resilient modal shifts are hardly achievable, which under non-simplifying assumptions [see Box 3] would lead to losses of real income, in turn translating into an economic decline of the home market.

[13]Note that probabilities have been modified so as to reflect places' population size. The modification is equivalent to repeating each occurrence a number of times equal to the local number of individuals.

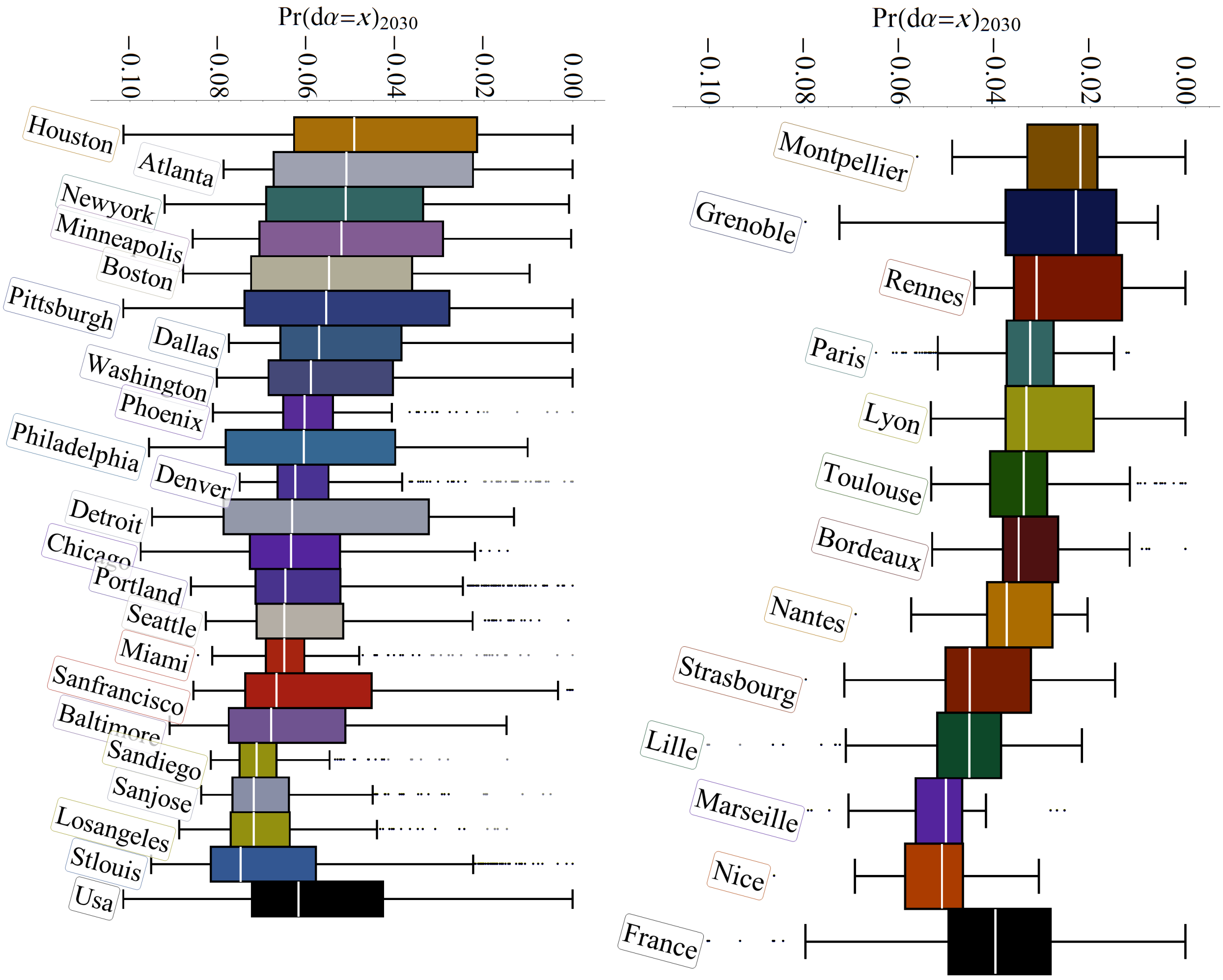


Figure 3.18: Boxplots of all urban areas distributions of $\Delta^{P^{a,PV}}$-resilient modal shifts in the USA and France in 2030

3.1.2 Transport cost under migratory pressure

Another determinant of unitary transport costs is the pressure of migration on mobility habits. In GEMSE, this pressure means road congestion, lower PV-speeds, a decrease in the real price associated to public mobility and by extension a decrease in marginal utility derived from private vehicle. From a policy-relevant perspective, it is interesting to quantify for a given urban area how migration of population exerts a downward pressure on unitary transport costs. The numerical framework developed to address this question is explained in Box 4.

Box 4. Recall the definitions of private vehicle speed, $v_{j,i}^{PV}$ [see eq.(1.20)] and of utility function of workers declined according to the mode of transport they use, $u_{j,i}^{PV}$ and $u_{j,i}^{PT}$ [see eq.(1.104) and eq.(1.105)]. Formalizing at urban equilibrium the first principle of

Wardrop (1952) in its discrete and relative form, *i.e.* $-1 + u^{PT}_{j,i}/u^{PV}_{j,i} = 0$, yields

$$-1 + \left(1 + \left(\frac{\sum_{o=1}^{N_j} \alpha_{j,o} L_{j,o} \Pi_{j,io}}{\mathcal{K}_{j,i}}\right)^4\right)^{\delta^a} \left(\frac{\upsilon^{PT}_{j,i}}{\upsilon^{0,PV}_{j,i}} \frac{\nu^{PV}_{j,i}}{\nu^{PT}_{j,i}}\right)^{\delta^a} = 0 \ \forall i \in [\![1; N_j]\!]$$

To study in a simple manner how this transport indifference condition behaves under migratory pressure for any place i of agglomeration j, one increases one place i's population at a time, $L_{j,i}$, within a $(1 + \Delta^L)$ factor for any value of modal share. This is done on average, studying the condition, say, $\mathcal{U}(\alpha_{j,i}, \Delta^L)$, at and around zero, $\forall i \in [\![1; N_j]\!]$.

$$\begin{aligned} \mathcal{U}(\alpha_{j,i}, \Delta^L) = & -1 + \left(\frac{\upsilon^{PT}_{j,i}}{\upsilon^{0,PV}_{j,i}} \frac{\nu^{PV}_{j,i}}{\nu^{PT}_{j,i}}\right)^{\delta^a} \\ & \frac{1}{N_j} \sum_{i=1}^{N_j} \left(1 + \left(\frac{\alpha_{j,i} L_{j,i}(1 + \Delta^L) + \sum_{o \in [\![1;N_j]\!] \setminus i} \alpha_{j,o} L_{j,o} \Pi_{j,io}}{\mathcal{K}_{j,i}}\right)^4\right)^{\delta^a} \end{aligned}$$

Migratory pressure is represented by a relative variation, Δ^L, itself entailed in $\mathcal{U}$ that is zero-centered relatively. This makes all urban areas indifference curves directly comparable thereby representable in a single chart [see Figure 3.19]: all urban areas curves may be future or past versions of each other, whose only difference is their position on the one-dimensional space of the population sizes, *i.e.* $L_{j,i} \forall \{j, i\} \in [\![1; N]\!] \times [\![1; N_j]\!]$ on $\mathbb{R}_{\geq 0}$.

Quantifying urban areas' transport mode indifference curves

Figure 3.19 shows for the USA and France,[14] all urban areas' sensitivity of the transport mode trade-off to migratory pressure. This is done *ceteris paribus* with 2017- and 2030-running values, numericizing $\mathcal{U}$ over $\{\alpha_{j,i}, \Delta^L\} \in [0; 1] \times [-.2; .2]$.

In 2017 and 2030, San Jose and Nice have the most sensitive transport mode trade-off to migration. Indeed in these two areas, the $(\Delta^L \neq 0)$-consecutive changes of $u^{PT}_{j,i} \forall i [\![1; N_j]\!]$ are the largest of their respective nation. It would thus be good from a pro-environmental standpoint that these areas are also those whose attractiveness is improved, so as to be pressured by migration and thus to foster the mutualization of transport GHGs emissions. *A contrario*, *e.g.* NewYork and Grenoble are the urban areas that offer the least interesting opportunities in terms of pro-environmental modal shifts since a population increase in these two areas generates the smallest increase in utility derived from the use of public transports. To see to what extent

[14]Two video files showing the annual evolution of these indifference curves for the USA and France over their prospective horizon are available respectively at `https://gemse.alwaysdata.net/static/gemse/USA/baseline_Proposition2.mp4` and `https://gemse.alwaysdata.net/static/gemse/FRANCE/baseline_Proposition2.mp4`.

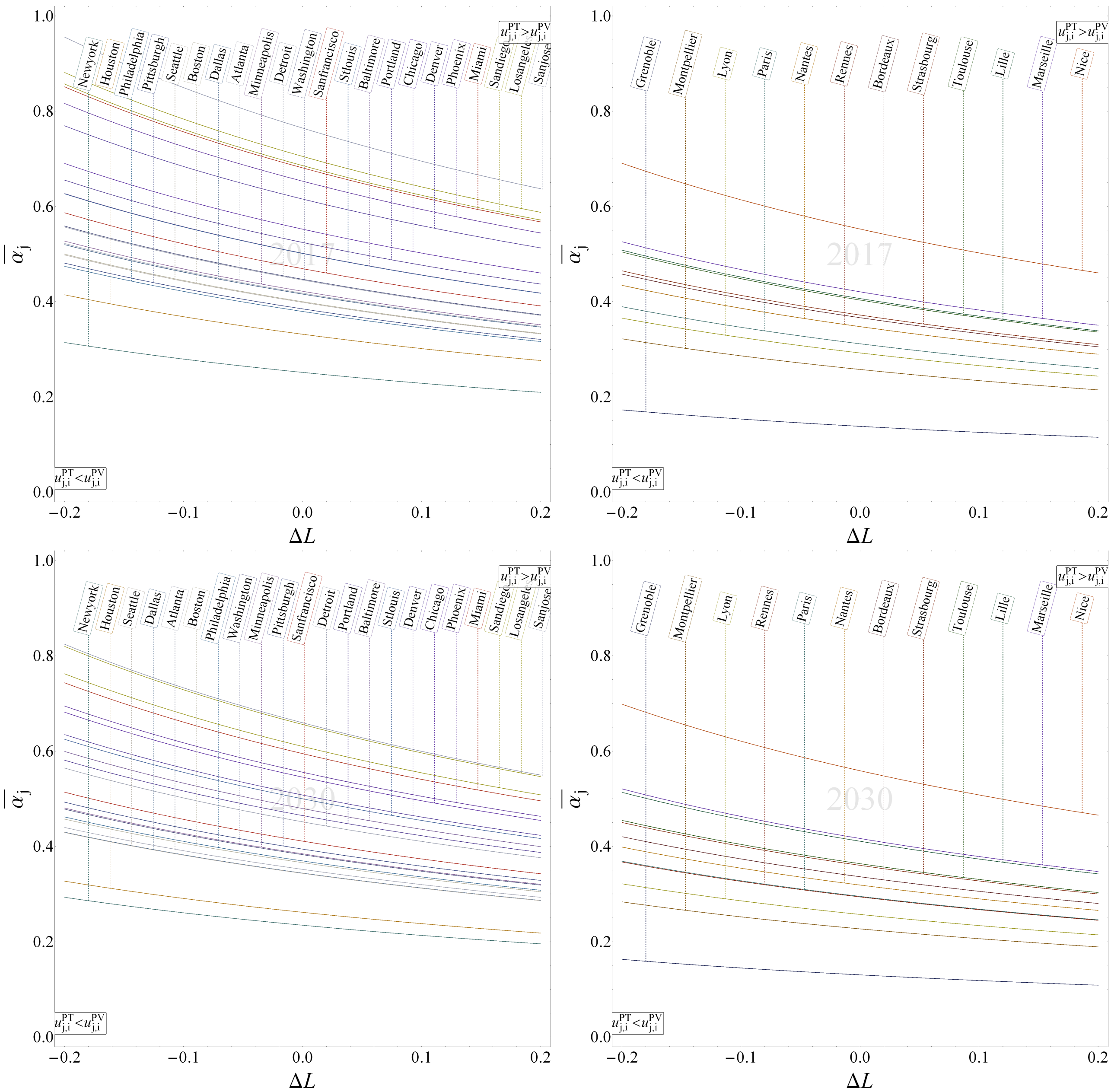
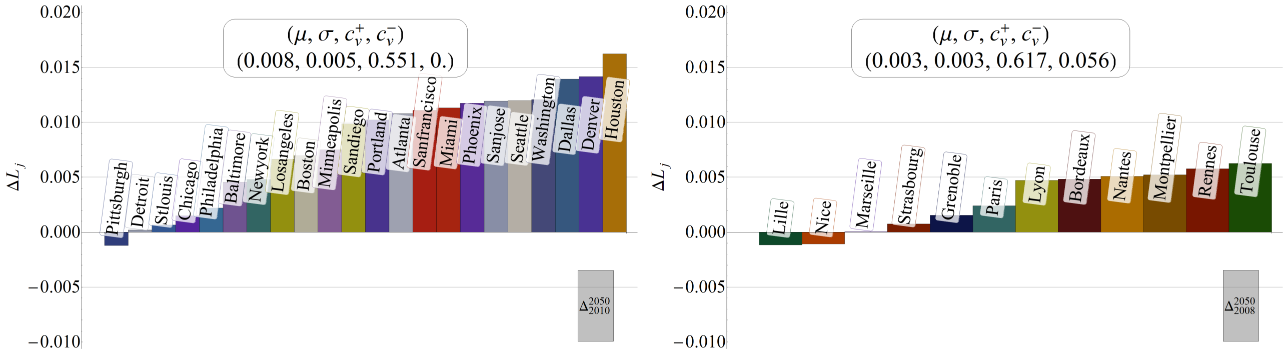


Figure 3.19: Weighted average (over places by population) transport modes indifference levels in the USA and France

these opportunities are *de facto* seized by migratory dynamics over the prospective horizon, Figure 3.20 shows the annualized growths of the number of workers of each nation's urban areas.

Considering jointly Figures 3.19 and 3.20, we are henceforth capable of explaining why the USA's and France's average (over modes and workers) transport budget exhibit so different (temporal) profiles [see Figure 3.13]. In the USA, population mainly migrate *to* urban areas where private vehicle's relative attractiveness is easy to deteriorate, which thus promotes the use of public transport. In France on the contrary, population migrate *from* this type of areas, *i.e.* Nice, Marseille and Lille.



μ stands for the sample average.
σ stands for the sample standard deviation.
c_v^- stands for the coefficient of variation computed over negative values. It is computed as σ^-/μ^-.
c_v^+ stands for the coefficient of variation computed over non-negative values. It is computed as σ^+/μ^+.

Figure 3.20: Annualized variations in workers in the USA and France

3.2 Agglomerations and the economy under alternative policy scenarios

In this section, we try and demonstrate how the modeling architecture constructed in this thesis can deliver relevant insights on the impacts of specific policy measures at the urban scale in view of achieving sustainability goals for the long-term development of urban and metropolitan areas. Urban activity occurring in the largest urban agglomerations in the USA and France is associated with environmental concerns, both at the local and global levels in terms of (air pollution and) GHG emissions. To shed light on what measures improve the sustainable nature of urban areas in the long-term, we need to assess the pro-environmental nature of specific transport policies while considering in parallel their consequences on the organization of economic activity. These consequences are captured by deviations with respect to baseline trends and thus consist of differentials and relative variations.

Given the large amount of data (used and) generated by the model, we only analyze the main findings in the following sections. The entire set of data is available online for each country, for each scenario, for each variable, for each year, for each urban area as well as for each of its constitutive place. The goal is to provide readers with the possibility to apprehend and graphically check the coherence of the mechanisms that archetype the theoretical background of GEMSE.

Box 5. On GEMSE's website, all policy-scenario's names are prefixed by *abs* or *rel*. These two prefixes stand for the type of deviation that is computed from baseline trajectories, respectively, in terms of absolute differences,

$$\text{d[value]} = \text{value}^{\text{policy}} - \text{value}^{\text{bau}}$$

or (discrete) relative

$$\Delta[\text{value}] = -1 + \frac{\text{value}^{\text{policy}}}{\text{value}^{\text{bau}}}$$

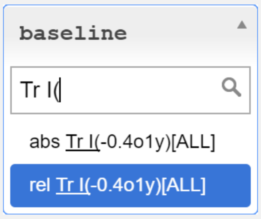
In what follows, d[value] is referred to as *difference* while Δ[value] is referred to as *variation.*[a]

[a]On the website, note that relative differences, Δ[value], may return (misleading) empty charts or zero-trajectories if the corresponding baseline values are 0.

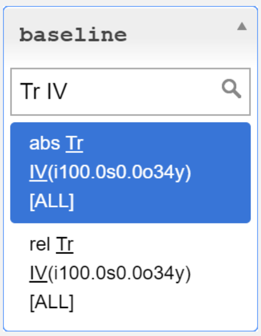
3.2.1 Urban transportation measures

In Box 6 we describe the standards (Std) and taxes (Tx) measures.[15]

Box 6. Tr^{Std} consists of a one-shot 40% decrease in private vehicle speed limitation ($v_{j,i}^{0,PV}$) implemented in 2017 in the center of all urban areas.[a] The 40% decrease approximately means, *e.g.*, at most -16km/h in NewYork and -12km/h in Paris. Tr^{Std}-related trajectories are viewable online for the USA and France via the field called `Scenarios` by selecting



Tr^{Tx} consists of a CO_2-price-based tax applied only to private vehicles in 2017 in the two nations.[b] The sums received are used to finance an increase in public transport speeds in places characterized by both the 95% lowest public transport speeds and the 95% highest population. Set to 100\$/ton in the USA and to 100€/ton in France, the price is turned into a distance-based tax by considering the CO_2eq content of one kilometer, subject to technical progress over the prospective horizon. Tr^{Tx}-related trajectories are viewable online for the USA and France via the field called `Scenarios` by selecting



[15]While the first measure is regulatory, the latter measure is market-based.

[a] Tr^{Std}'s name on the USA or French companion website is `Tr I(-0.4o1y)[ALL]`. It reads "−40% over 1 year in all urban areas".

[b] Tr^{Tx}'s name on the USA or French companion website is `Tr IV(i100.0s0.0o34y)[ALL]`. It reads "initiated at 100$ or € with a 0-rate of evolution – no slop – over 34 years in all urban areas".

Tr^{Std} Decreasing private transport speed

Tr^{Std}-like measures has fueled the public policy debate for some years now. The reduction of traffic speeds is known to have positive consequences at the *local* scale on traffic congestion, GHG emissions, health, road safety and noise. However, the *global* scale impact of such a measure on the geographical distribution of economic activity is rarely considered. GEMSE is thus employed here to fill this knowledge gap thanks to its capacity to describe interacting urban areas and to produce global scale results that are based on spatial dynamics captured at a high level of granularity.

The reduction of speeds in some urban areas may influence migration flows in such a way that workers[16] would actually "bring their congestion charge with them" when migrating. In this case, the reduction of speeds may yield a zero-sum gain at the national scale in terms of congestions and/or of GHG abatements (in the transport sector) since in a decrease in these terms somewhere can be counterbalanced by an increase of the same magnitude elsewhere.

In economic terms, the consequences on growth and employment are likely to be different from an urban area to another according to whether people's and firms' relocation contradict each other, e.g. firms leave a area while people enter that area. This would locally change labor market conditions, wages, income formation and ultimately the home market. Thus, let's see what GEMSE returns as results when decreasing private speeds limits in all the CBDs in the USA and France.

Figure 3.21 illustrates for NewYork and Paris the places that are concerned by the reductions of speed limitations. When this policy is implemented, it directly triggers congestion in CBDs with the consequence of inducing changes in mobility habits towards public transports. Aggregated at the national level, this yields a difference in the modal share of cars of around -1% for the USA and -5% for France, immediately after the implementation of Tr^{Std} and until the end of the prospective horizon.

For the remaining private vehicle users, Tr^{Std} has a positive impact on private speeds in the whole agglomeration by making traffic flows smoother on their daily trips. However, everyone's accessibility is necessarily undermined on average since people are forced in the broad sense to use slower transport modes. It follows that this undermined accessibility modifies local

[16] Remember that "workers" – labor demand – and population or people – labor supply – are used interchangeably. This is because population means workers within an employment rate factor. In the same manner, recall that the utility is defined at the level of workers and that of people is equal this utility level within an employment rate factor [see eq.(1.50)].

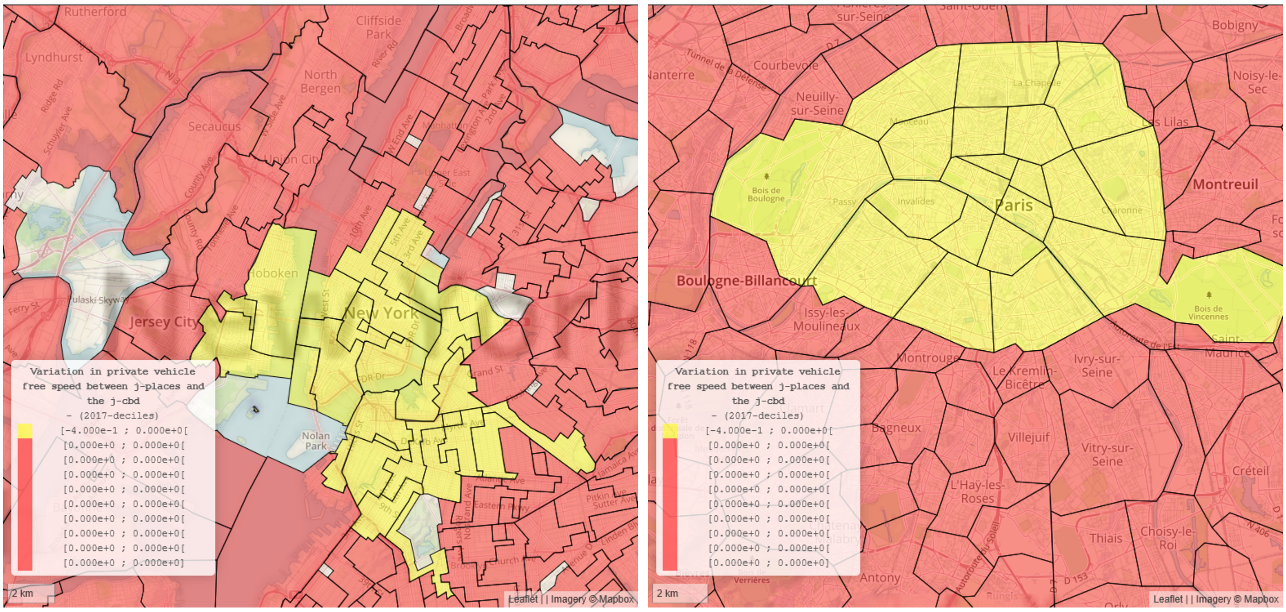


Figure 3.21: Places concerned (yellow) in 2017 by the Tr^{Std} 40% decrease in free private vehicle speeds limitation in NewYork and Paris

transport/housing trade-offs and intra-area location decisions in a way that strongly depends on housing prices.[17] If housing prices are low enough, local trade-offs are stabilized and no migration occurs. Otherwise, the resilience of the urban system if put to the test: people migrate to peripheral places, which respectively pulls private speeds and housing prices upward and downward in the places they left. This occurs until both remaining and newly-relocated people's housing/transport trade-offs underlie an identical utility-level everywhere in the urban area, *i.e.* until the urban area is in equilibrium. Thus, if this process of intra-area relocation can be *perfectly* performed in such a *homogeneous manner* across the urban area, no change in housing price is perceptible at an aggregated level. If this can only be performed *imperfectly*, the resilience of the urban system is exceeded and the equilibrium level of utility decreases until it is achievable by all places.

A decrease in the local level of utility of workers does not necessarily translate into a loss of attractiveness (of the urban area) since the former is linked to the latter within an employment rate. For example, an area with a low workers' utility-level associated to a high employment rate can underlie an area's attractiveness equal to that of another area with a high worker's utility level associated to a low employment rate. This is ultimately depending on the specificities of the labor market under consideration that people migrate subsequently to having (employment-)probabilized the qualities of life in other areas.

As shown in Figure 3.22, Baltimore and Grenoble are two urban areas that homogeneously provide their people with the possibility to intra-area relocate, which allows the housing sector

[17] Recalling that prices and rents-like flows are equal within a discount factor. Thus, dealing with housing prices or rents is actually strictly equivalent.

to recover a quantity-level of equilibrium comparable to what it was before the implementation of Tr^{Std}, partially in the case of Grenoble.

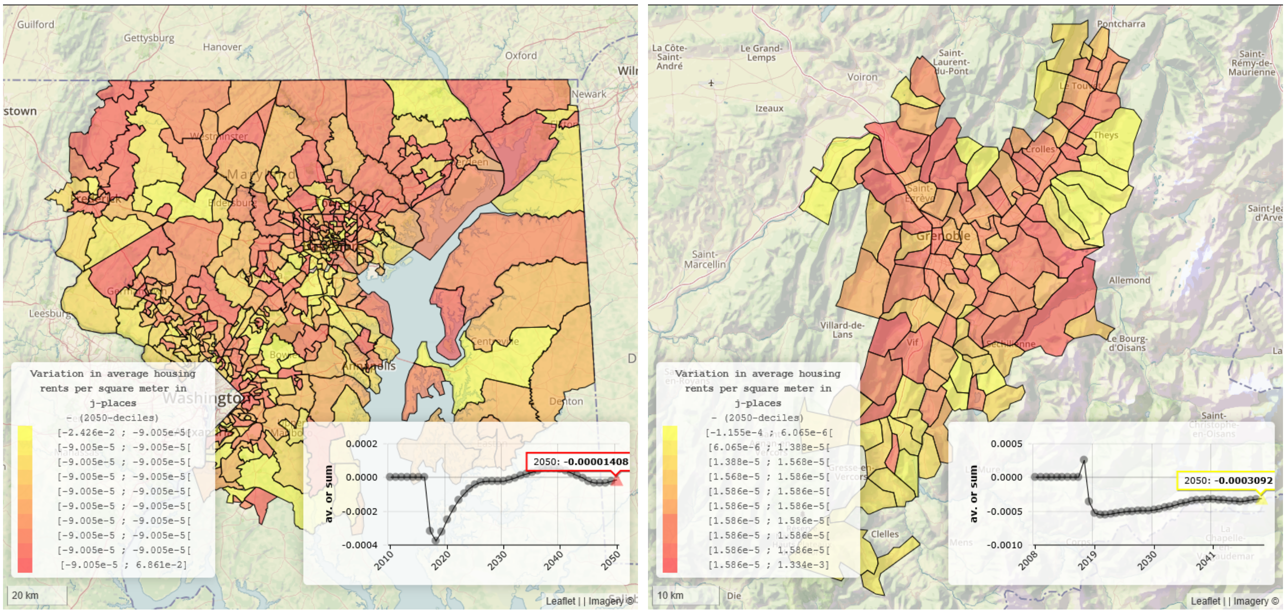


Figure 3.22: Consequences of Tr^{Std} on housing rents in terms of variations in Baltimore and Grenoble

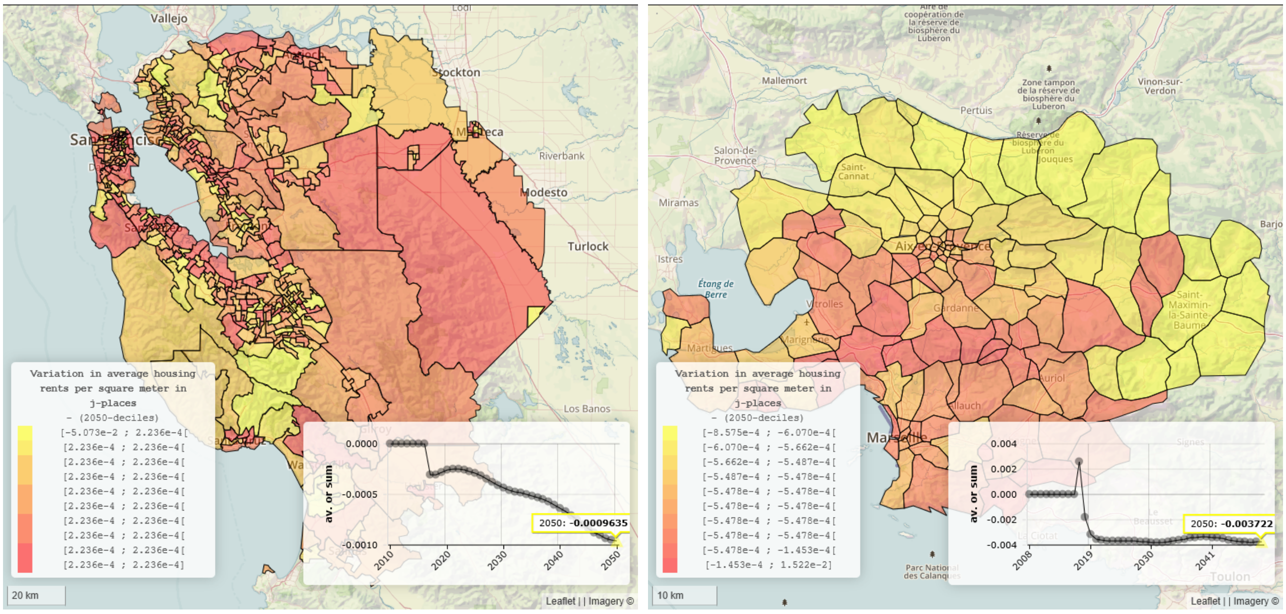


Figure 3.23: Consequences of Tr^{Std} on housing rents in terms of variations in SanJosé and Marseille

Not all urban areas are endowed with such a capacity to homogeneously dilute shocks in transport/housing trade-offs through space. To be capable of doing so, the agglomeration must possess a public transport infrastructure that allows people to decouple their intra-area choice of relocation from their change in mobility habits. Put differently, if people have to relocate (from places with no public transport connections to places that are well connected) to be able to modal-shift and that public transport services are confined to a small number of places, this results into a too slow (short-term) adaptation of the housing market. This is notably what happens in Marseille as shown in Figure 3.23's rigth dial, whose variations in housing rents are not even starting to recover their ante-Tr^{Std} levels before the end of the prospective exercise: some people have left Marseille, which has subsequently downward-adjusted the housing-market quantity level of equilibrium. In the USA, the situation just described does not exist since *all* the urban areas of the country actually possess a geography of public-transport-availability that coincides to that of the transport/housing trade-offs that are highly-shocked. In Figure 3.23's left dial, we have shown SanJosé, which as Marseille exhibits a temporal profile of variations in housing rents (with respect to the corresponding baseline trend) that does not reabsorb, but for a different reason from that invoked for Marseille. In SanJosé (and Washington) housing rents are the highest in the country and, while people do not particularly leave the area, they tend to relocate less close to the center where rents are lower. These relocations reweight SanJosé's average housing rent downward. Different for the USA and France, the dynamics of inter-areas people's relocation that are shown in Figure 3.24 over Baltimore, SanJosé, Marseille and Grenoble, are thus explained.

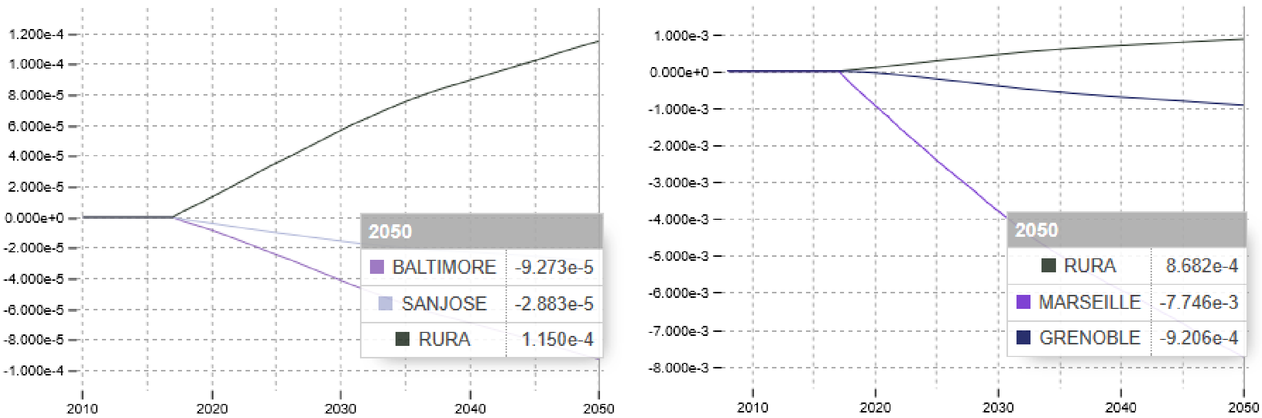


Figure 3.24: Consequences of Tr^{Std} on population shares in terms of variations in the USA and France

When people – the labor supply – leave a given region, this increases the local employment rate, increases wages and decreases the profitability of local economic activity. By the same token, the reduction in local returns on equity (ROE) pushes some firms – the labor demand – to inter-areas relocate,[18] which in turn decreases wages and makes it possible for the ROEs of the remaining firms to recover a level that cancels the opportunity cost of not relocating. Put differently, immediately after Tr^{Std} is implemented, the long term equilibrium of the nation's

[18]We recall that firms cannot relocate within a urban area [see subsection 1.1.1]. Only people can both relocate between areas and within a given area.

economy is moved away from its baseline position. The dynamic as well as the spatial distribution of economic activity, previously at a cruising speed, must reorient themselves, which takes time. Tr^{Std} is thus a *mini-crisis* that, however, offers a higher potential in terms of future reachable level of GDP. The major dynamic at play here is that changes in mobility habits of people, releases income, leads people to relocate, changes the geography of production and improves external economies of scale. This is what we are shown in Figure 3.25, in which the USA's and France's GDP recover their pre-Tr^{Std} level respectively after 9 and 7 years. Note that it takes more time in the USA than in France because of the degree of competition superior in the USA than what it is in France, where Paris's little "black hole" increases the average speed-resilience of France's urban system [see Figure 3.5]. Figure 3.26 reports each country's dynamics that are related to GHG emissions and GDP's CO_2eq intensity over the prospective horizon under Tr^{Std}. It shows that Tr^{Std} stimulates economic activity in a pro-environmental fashion by first, admittedly, contracting the American and French GDP, but by then allowing them to reach higher levels, which translates into a non-zero sum game, and this even before 2050.

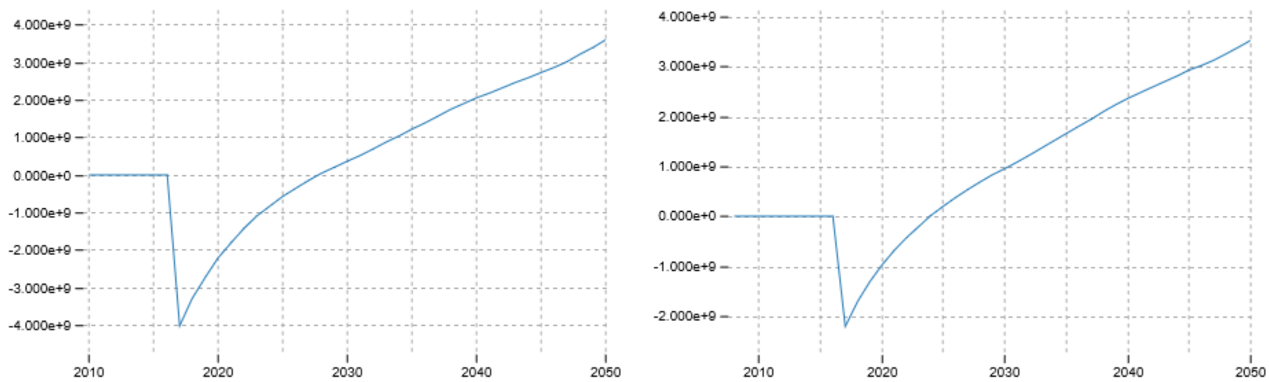


Figure 3.25: Consequences of Tr^{Std} on GDP in terms of differences in the USA ($) and France (€)

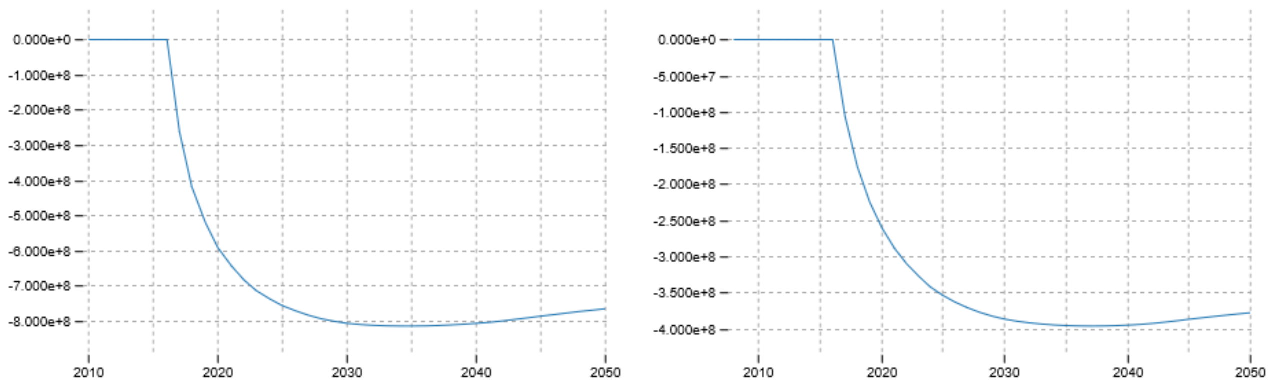


Figure 3.26: Consequences of Tr^{Std} on GHG emissions in terms of differences in the USA and France (kg/year)

Tr^{Tx} Financing public speed upgrades by taxing private vehicles' CO_2eq emissions

Tr^{Tx} is first designed to internalize at national level a share of the total costs of CO_2eq emissions by tying them to their private-vehicle-related sources. Second, like Tr^{Std}-like measures, it aims to shock the modal trade-offs by degrading the advantages provided by cars in terms of flexibility and speed while trying in parallel to capture car drivers by upgrading public speeds. This policy also has the indirect objective that dovetails with subsection 3.1.1, namely that of reducing the exposition of economic activity to unpredictable shocks in fuel price. This way, the potential future (imported) inflation in fuel price is turned into a monitored and domestic one that can be invested in public infrastructures. In other words, Tr^{Tx} hedges the American and French economy against the uncertainty of future energy prices.

The implementation of this policy implies the modeling of a national price of the speed-unit. Indeed, this policy aims to improve the quality of public infrastructures. Numerically, this means adding a certain amount of speed units (km/min) here and there by spending the (CO_2eq tax based) budget envelope, legitimately allocated in space. To put a price on speed units, we use a well known pricing method in finance, the dividend discount model (DDM). The approach adopted in this pricing technique amounts to computing net-present-value (NPV) of speed-units by imputing them the total incomes generated in the transport sector. The description of the pricing approach is given in Box 7.

Box 7. The idea that is described below is about *(i)* making the inventory of transport infrastructure in terms of kilometers, travel times and speeds and *(ii)* imputing all incomes that are generated in the transport sector to this infrastructure. Note that when public transport data such as distances and travel times have been collected [see subsection 2.1.1], they were no distinctions between *e.g.* buses, subways, tramways, trains, all sequentially mixed together depending on the (shortest) route proposed by *GoogleMaps*. This means that any urban area's public transport infrastructure only partially superimposes to its road network. However, for the sake of implicity, we use road networks as proxy to do the inventories of areas' public transport infrastructure.

Recall the way that road-network structures are modeled in each urban area $j = 1, ..., N$, denoted by $\mathbf{\Pi}_j$ [see Section 1.1.1.4's Figure 1.5]. Define $\mathbf{d}_j$, a $N_j \times 1$ vector of distances between each place i and its j-CBD. The total distance on the road-network, d_j, is

$$d_j = \|(\mathbf{\Pi}_j^{-1})' \mathbf{d}_j\|_1$$

where $\|.\|_1$ is the l_1-norm operator. Identically, define $\mathbf{t}_j$, a $N_j \times 1$ vector of average (over modes) travel times between each place i and its j-CBD. The total time it takes to browse the entire road-network, t_j, is

$$t_j = \|(\mathbf{\Pi}_j^{-1})' \mathbf{t}_j\|_1$$

On this basis, it is direct to compute the average transport speed one faces on the national road-network over all urban areas, such that

$$v = \frac{\sum_{j=1}^{N} d_j}{\sum_{j=1}^{N} t_j} = \frac{d}{t}$$

Then we assume that all income generated by this infrastructure is equal to the total sum of transport budget in agglomeration j, as follows

$$\widetilde{Y^a} = \sum_{j=1}^{N} \widetilde{Y^a_j}$$

where $\widetilde{Y^a_j}$ is the total transport cost of workers living in urban area j [see eq.(1.17)]. For the sake of simplicity, we consider this income flow as being representative of all future income flows generated by the transport sector, *i.e.* we consider it as a perpetuity. Resorting to the DDM, it follows that the nation's NPV of the transport infrastructure, NPV^a, is

$$\text{NPV}^a = \widetilde{Y^a} \sum_{t=1}^{\infty} \left(\frac{y_a}{1 + r_a} \right)^t = \widetilde{Y^a} \frac{y_a}{1 + r_a - y_a}$$

where r_a is the discount rate of public investment projects set to 4%,[a] and y_a is the net return on equity in the transport sector set to that of the heterogeneous sector. It is now straightforward to price speed supplied by each country's transport infrastructure, $\xi_{\text{km/min}}$, as

$$\xi_{\text{km/min}} = \frac{\text{NPV}^a}{v}$$

Since figuring out how such $\xi_{\text{km/min}}$ makes sense is anything but obvious, we also carry out the account assignment of NPV^a over nation's kilometers of infrastructure, as follows

$$\xi_{\text{km}} = \frac{\text{NPV}^a}{d}$$

which is far more easy than $\xi_{\text{km/min}}$ to compare given the numerous references that exist on this matter.[b]

[a] As it reads regarding the USA and France in Guesnerie et al. (2017, p.7).

[b] As it reads in *e.g.* Cazala et al. (2006, p.6) or Global BRT Data (2017), the kilometer of transport infrastructure is of the order of a million euro or US dollar.

Figure 3.27 shows what the pricing technique (presented above) returns regarding the NPVs of one kilometer of road in the USA and France, denoted by ξ_{km}. Figure 3.28 does as well regarding the NPVs associated to one unit of speed (in km/min) in the USA and France, denoted by $\xi_{\text{km/min}}$. While the values that are shown for ξ_{km} in the USA and France rather make sense – around a million euros –, this is at first less likely regarding each country's $\xi_{\text{km/min}}$.

Nevertheless, even though it might seem unlikely, it is actually quite telling that increasing the speed of more than half of the population of each country represents such high values. The number of workers of a given urban area is actually considered by proportioning $\xi_{\text{km/min}}$ such that the j-unitary-cost of speed becomes $\frac{L_j}{\sum_{k=1}^{N} L_k}\xi_{\text{km/min}}$.

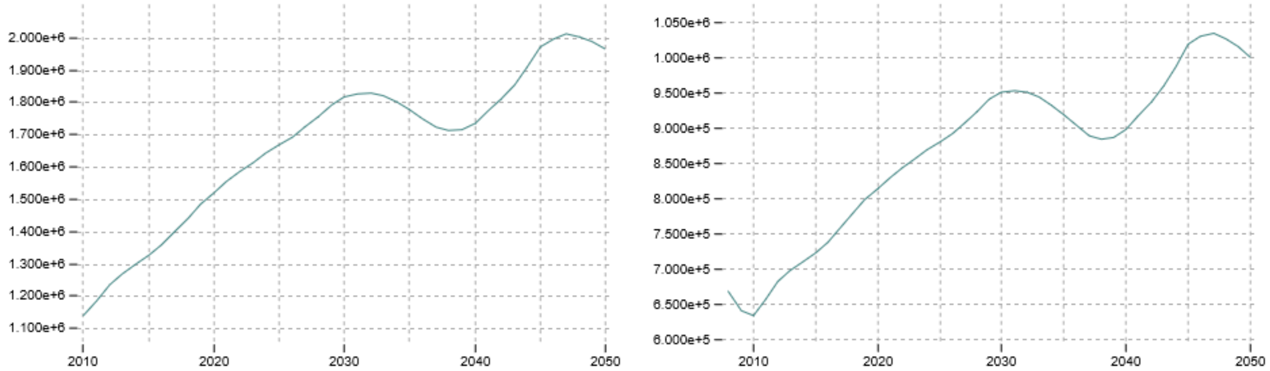


Figure 3.27: NPV of one kilometer of road, ξ_{km}, in the USA and France

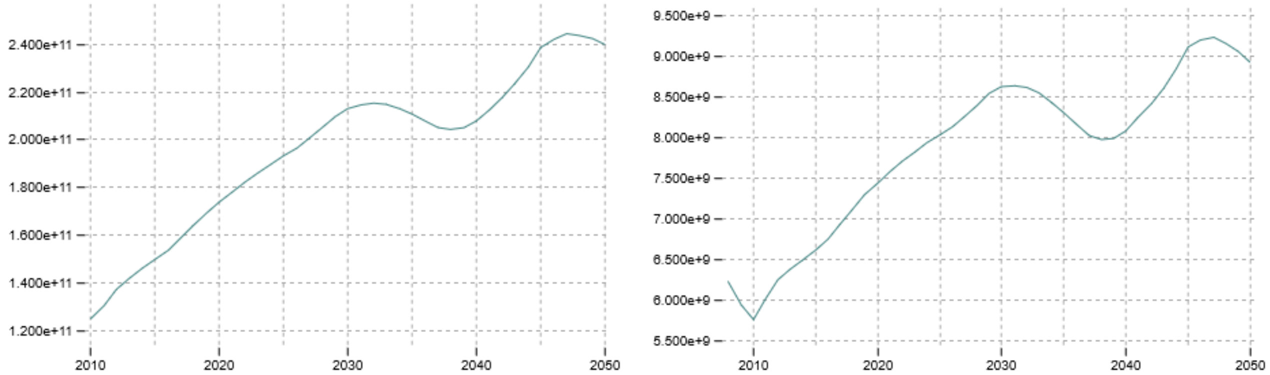


Figure 3.28: NPV of one kilometer per minute, $\xi_{\text{km/min}}$, in the USA and France

Implemented in 2017, the tax-revenues (generated by the CO_2-price-based tax applied to private vehicles) are shown in Figure 3.29 for the USA and France, whose yearly total amount respectively to around 19 billions dollars and 950 millions euros. However, it is hard to tell in what ways the tax-related component of Tr^{Tx} supports the main goal of achieving a change in mobility habits without getting an idea of the disincentive it represents per user per kilometer. Figure 3.30 shows that setting a price of 100 per ton of CO_2eq, be it in dollars or in euros, actually represents almost nothing once traduced per commuter-kilometer, not even 2 cents or centimes. It can thus hardly be argued that Tr^{Tx}'s tax deter from the use of cars.[19]

Be that as it may, the investment of these tax-revenues are illustrated in Figure 3.31 over SanDiego and Marseille – selected for their high response to Tr^{Tx} –, in which we see the public speed upgrades that are financed by the revenues of the tax. Given that the eligible amount of

[19]The two charts are identical since the price that is set in dollars or in euros in the same.

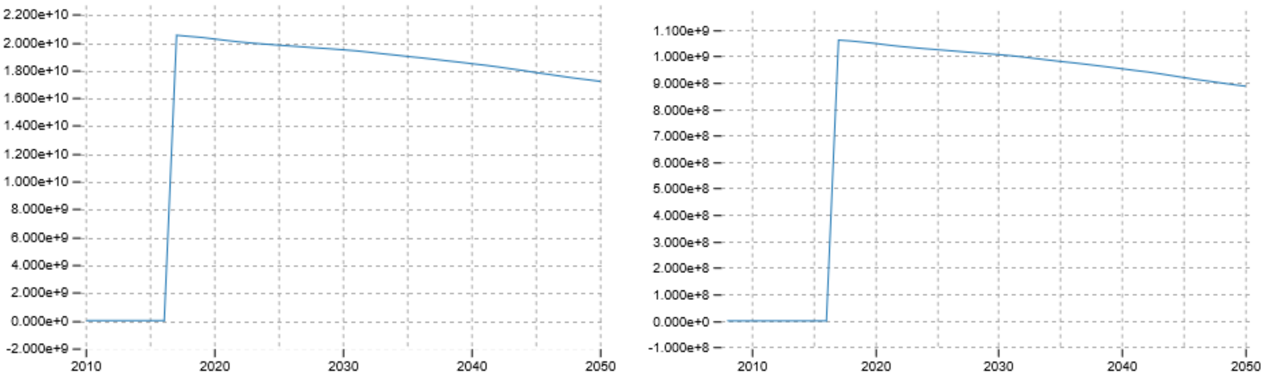


Figure 3.29: Revenues of the tax in the USA and France

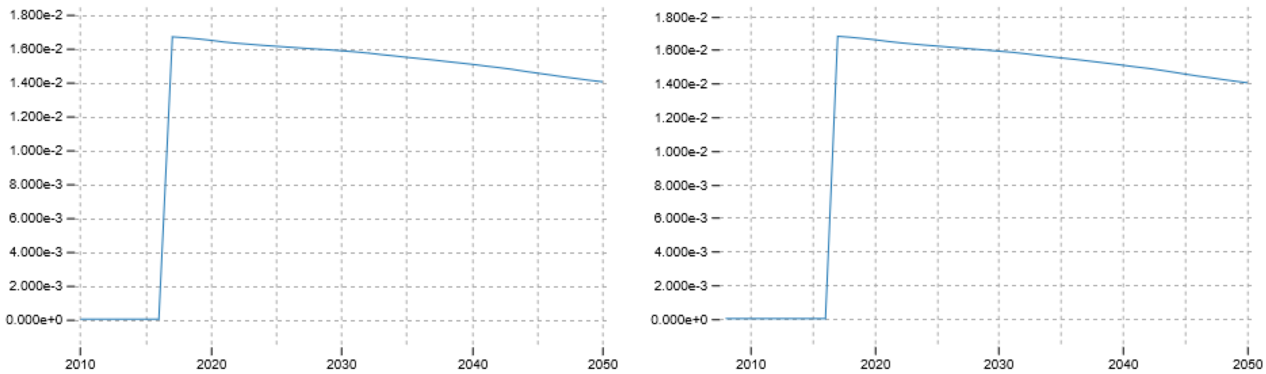


Figure 3.30: Translation of 100 \$ or € per ton of CO_2eq into its car-kilometer-based counterpart in the USA (\$) and France (€)

speed upgrades to which a place is entitled depends on its (public speed) inferiority-distance from the 95th percentile of the considered urban area, the maps that are shown in Figure 3.31 are indicative of the heterogeneity of public speeds. Regarding Marseille, the lack of transport services in the northern and eastern places is striking and Tr^{Tx} is likely to change the deal there. In SanDiego, the situation is less clear-cut than in Marseille and (as it reads from deciles) the dispersion of public speeds is weak admittedly, but is the highest compared to other urban areas of the country.

One direct consequence of Tr^{Tx} is to induce changes in mobility habits towards the use of public transports. This occurs in an order of magnitude that matches what is shown in Figure 3.19, whose curves are moved leftward by Tr^{Tx}. In urban areas such as Marseille and SanDiego, shown in Figure 3.32, this means respectively a reduction of the average modal share of cars of about 20% and 3% at the end of the prospective horizon.

This modal shift is exacerbated in Marseille (and Nice) but actually occurs in every urban area, be it in France or in the USA. Tr^{Tx} thus releases an important mass of income at the national scale, which consequently stimulates demand for other types of goods and services. In the housing sector, this reorientation of income increases housing rents in a differentiated manner, which in turn also increases urban costs in a differentiated manner, ultimately changing

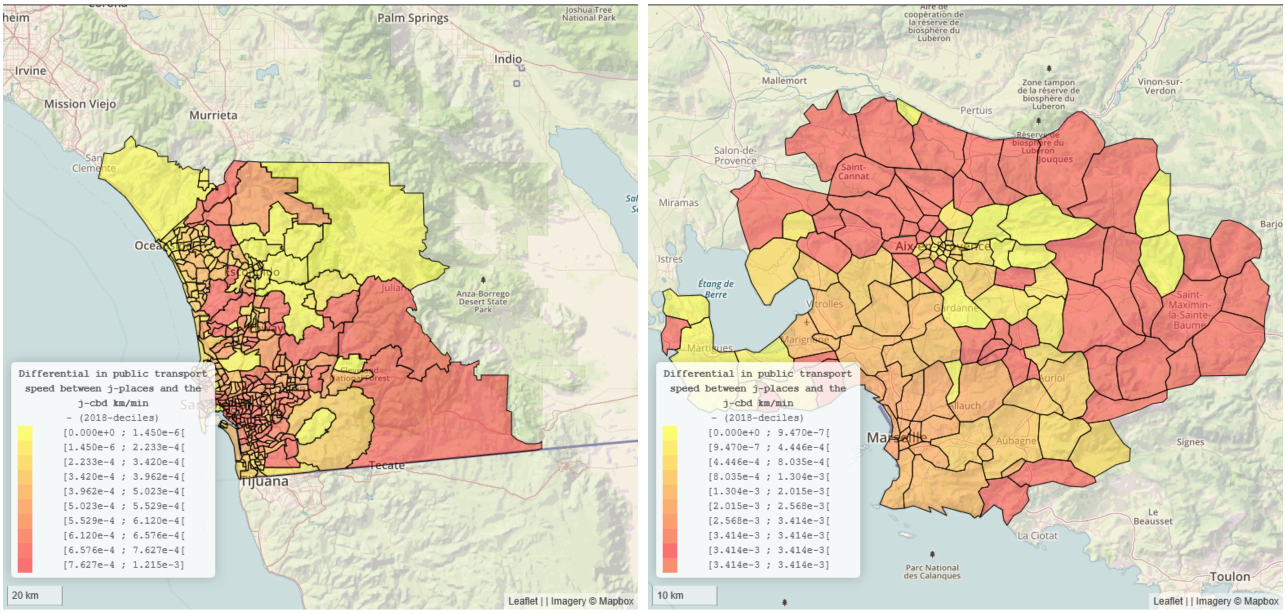


Figure 3.31: Tr^{Tx}-financed speed upgrades in SanDiego and Marseille in 2018

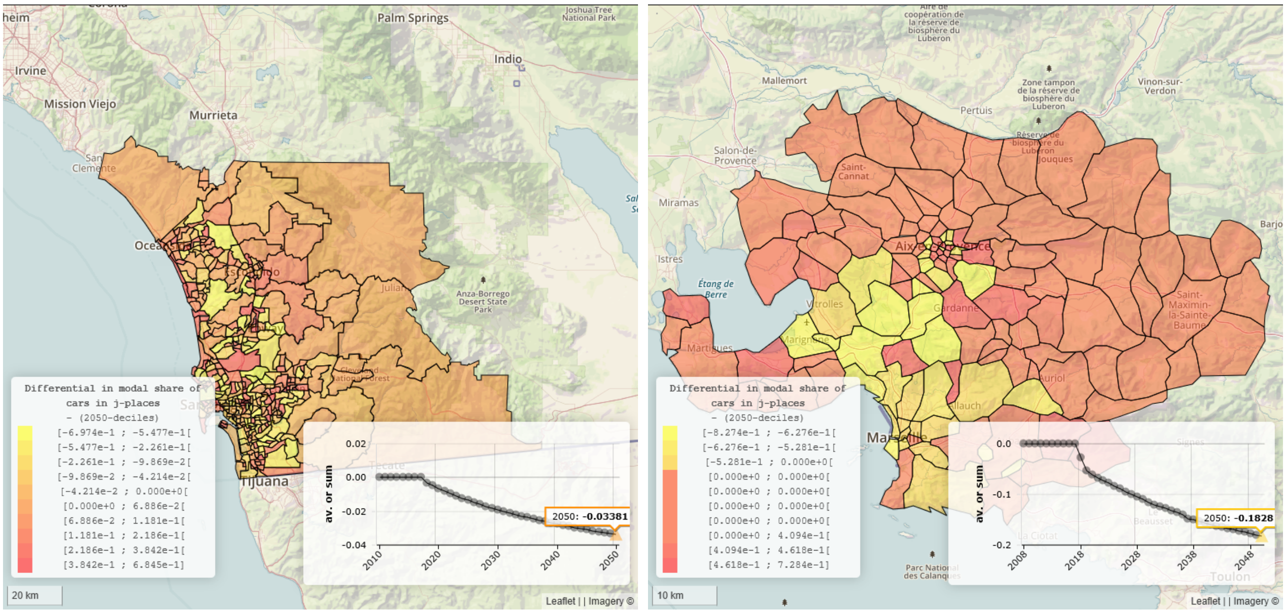


Figure 3.32: Consequences of Tr^{Tx} on cars modal shares in terms of differences in SanDiego and Marseille

the geography of attractiveness. As in the case of Tr^{Std}, this have direct consequences on migrations flows, areas' labor market, ROEs, spillover effects and at the end, the national geography of production.

Figure 3.33 shows the consequence on the attractiveness of some urban areas. In the USA, the urban area that benefits the most from this spatial reorganization is NewYork that, both thanks to its capital and labor/consumer stocks (permitting economies of scale) and to the ante-Tr^{Tx} mobility habits of its inhabitant is not that impacted by the implementation of

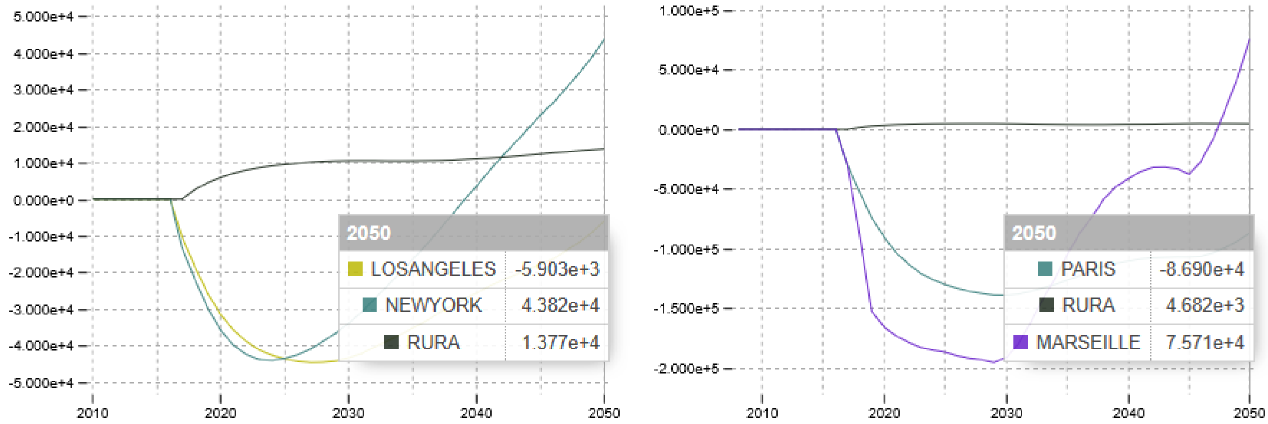


Figure 3.33: Consequences of Tr^{Tx} in terms of differences on population welfare in the USA and France

the tax, while seeing its home market benefiting in the same time of the massive increases in residual – of transport costs – income that take place across the country. In France, the urban area that benefits the most from Tr^{Tx} is Marseille, for which the policy has a liberating effect by decreasing more transport costs than increasing housing rents, stimulating accessibility, wages and employment rates.

As shown in Figures 3.34 and 3.35, by pro-environmentally changing mobility habits and reducing urban costs on average, Tr^{Tx} results in lower emissions and higher GDP in a non negligible order of magnitude.

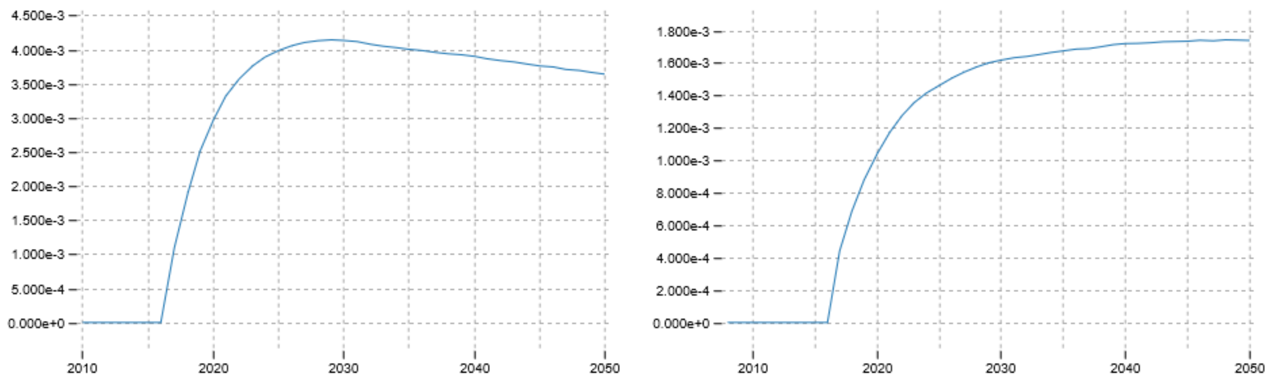


Figure 3.34: Consequences of Tr^{Tx} on GDP in terms of variations in the USA and France

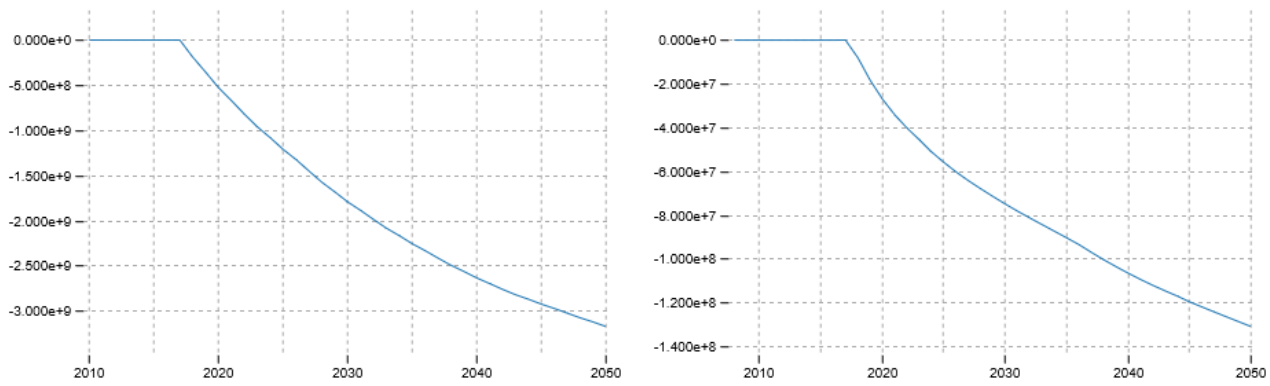


Figure 3.35: Consequences of Tr^{Tx} on GHG emissions in terms of differences in the USA and France (kg/year)

3.3 Main findings

Multiple policy-relevant indicators are presented, such as *(i)* degrees of competition intensity between urban areas (expressed in terms of knowledge spillover and) that quantify the magnitude of the centralization of economic activity in the USA and France, *(ii)* future geographies of people localizations and their concentration between small and large urban areas, *(iii)* fitted distributions of the levels of effort required in each urban area to hedge its home market against external shocks in fuel price and *(iv)* capacities of urban areas to favor public transports under migratory pressure. Such indicators show that GEMSE can be used to help decision-makers to anticipate the consequences of shocks in fuel price on urban areas' resilience in view of protecting economic activity that may be strategic at the national scale. These indicators also show that the model can help to anticipate the consequences of migrations of people on urban areas' transport sector emissions so as to take measures to incentivize individuals to settle in areas capable of promoting sustainable mobility habits.

Under alternative policy scenarios, the model was applied to study the consequences of two transport measures that feed the public policy debate for some years now. The first measure relates to the decrease in private vehicle speed limitation. We conclude that such a policy stimulates economic activity in a pro-environmental fashion by first contracting GDP but then allowing it to reach higher levels resulting in a positive sum game. The second measure relates to the implementation of a CO_2-price-based tax applied to private vehicle whose received sums are used to finance an increase in public transport speeds in some places. We argue that setting a price of 100 per tonne of CO_2eq, be it in dollars or in euros, represents virtually nothing once converted per commuter-kilometer. Thus, this policy does not deter from the use of cars. However, the investment of tax revenues in public transport infrastructure does release an important mass of income at the national scale, ultimately changing the geography of attractiveness and production. Thus, it is not through the tax itself but through the recycling of the tax that consumers change their behavior. Globally, this policy is both pro-environmental and pro-growth by resulting in lower emissions and higher GDP.

Be it regarding the baseline or policy scenarios, the urban-scale-related results can hardly be generalized and can only be considered on a case-by-case basis, say, in the light of a specific question asked by a decision-maker. Indeed, urban areas are strongly identified entities in numerical terms. They are geographically and historically characterized and possess their own set of inertia that, yet within a same baseline world, lead them to evolve on very differentiated paths. This is how generalizing, *e.g.*, the consequences on people's transport budgets of fuel price increases can easily be shown to be inadequate, not to say *utterly wrong*, if doing so over differently public-transport-covered areas, say, Houston *versus* Pittsburgh in the USA or Toulouse *versus* Nice in France.[20] Moreover, these paths of evolution are dependent since co-endogenous. This makes urban areas be highly unlikely to converge toward an equiweighted situation (in terms of whatever one may consider) even though influenced by a same baseline-set of strengths.[21] It naturally follows that this non-generalizability is transmitted to the global scale. This is why, *e.g.* the evolution of centralization of economic activity, yet under the influence of the same baseline-strengths, cannot be neither generalized nor even schematized in a similar fashion, without, once again, leading to *utterly wrong* conclusions.[22] This is also why the consequences of a same measure on different urban areas can often be found contradictory, yielding a whole range of effects (from strongly positive to strongly negative), be that at the urban or the global scale.[23]

[20]To figure this out you can select the `baseline` scenario and the variable `tr budget js` in the field called `NEGs`.

[21]Of course if their evolutions are initiated with empirical conditions and that they are unbalanced at each recursion.

[22]To figure this out you can select the `baseline` scenario and, in the field called `MACROs`, the variable `gup HHI` (which stands for the HHI related to the gross urban product) or, to echo the previous example, the variable `tr budget`.

[23]To figure this out at the global scale you can select the `abs Tr I(-0.4o1y)[ALL]` scenario and the variable `gup HHI` in the field called `MACROs`. To do so at the local scale you can select the `abs Tr IV(i100.0s0.0o34y)[ALL]` scenario and the variable `tr budget js` in the field called `NEGs`.

References

Bosker, M, S Brakman, H Garretsen and M Schramm (2007). "Adding geography to the new economic geography". *Cesifo Working Paper* 2038.

Cavailhès, J., C. Gaigné, T. Tabuchi and J. F. Thisse (2007). "Trade and the structure of cities". *Journal of Urban Economics* 62.3, pp. 383–404.

Cazala, A., G. Crespy, J. Deterne, P. Garnier and P. Rimattei (2006). *Comparaison au niveau européen des coûts de construction, d'entretien et d'exploitation des routes*. Tech. rep. Ministère des transports, de l'équipement, du tourisme et de la mer 1., p. 48.

Eaton, J. and S. Kortum (2002). "Technology, Geography, and Trade". *Econometrica* 70.5, pp. 1741–1779.

Global BRT Data (2017). *Infrastructure cost per kilometer.*

Guesnerie, R., C. Abraham, O. Al Tarabichi, J.-P. Arduin, D. Auverlot and M. Auzanneau (2017). *The discount rate in the evaluation of public investment project*. Tech. rep. Paris: France Stratégie, p. 70.

Krugman, P. (1980). "Scale Economies, Product Differentiation, And The Pattern Of Trade". *The American Economic Review* 70.5, pp. 950–959.

Murata, Y. and J.-F. Thisse (2005). "A simple model of economic geography à la Helpman–Tabuchi". *Journal of Urban Economics* 58.1, pp. 137–155.

Waisman, H. and F. Grazi (2013). "Cities in International Climate Policy : A Dynamic CGE Approach Cities in International Climate Policy : A Dynamic CGE Approach".

Wardrop, J. G. (1952). "Road Paper. Some theoretical aspects of road traffic research." en. *Proceedings of the Institution of Civil Engineers.*

Conclusions

Contents

Methodological contributions **154**
Substantive contributions **155**
Limits and future research **157**
An attempt to overcome the 'blackbox' syndrome **159**

This thesis was motivated by a diagnosis about the drawbacks – for climate policies deliberations – of the non-explicit consideration of the spatial dimension in Integrated Assessment Models (IAMs).[24] It relies on two intuitions. The first is that the core theoretical elements of the New Economic Geography (NEG) and Urban Economy (UE) can be embedded within a single modeling tool thanks to the progress in computational capabilities. The second is that the data-intensive nature of the modeling structures suited to operate this integration poses by itself a problem of readability, transparency and interpretability of the results.

Most prospective models utilized in the field of energy, environment and economy incorporate an implicit consideration of space. But, they rely on the use of inter-scale correlations to describe the influences of the urbanization dynamics on macro trends and/or vice-versa. Even though these approaches provide quantitative insights about the responsibility of urban areas in climate change, they do not picture the existence of effects at the local scale that potentially turn out to generate unexpected interactions with significant consequences at the aggregated level. Given this diagnosis, we aimed to consider space in an explicit and empirical manner through the representation of the internal structures and shapes of multiple and interacting urban areas, all constitutive of a national system. This is why we have created a modeling tool capable of explicitly considering the geo-economical characteristics of national, regional and local economies in a unified multi-scale architecture (Allio, 2015; Scott and Storper, 2015; Thisse, 2010).

Our contribution, although primarily methodological in nature, delivers also some substantive insights on the mechanisms at play that are of interest for public deliberations.

Methodological contributions

We first propose a solution to overcome a difficulty pointed out by Simmonds et al. (2013) about the causal order in which submodels are chained. While this causal order is very likely to have non trivial effects on modeling results and can lead to biased information for policy making processes, most models do not explicit these implications.[25] For this reason, we created a solving process that avoids any kind of "chicken-or-the-egg" issue by running submodels in parallel until their convergence on the indicators that are computed in each submodel. This has the obvious drawback of requiring higher amount of computational resources (or time)[26] but has the advantage of calculating the transitory equilibrium at each point in time without pre-judging the sequence of the causal chain behind this equilibrium. Like in comparative

[24] IAMs only have an implicit consideration of space *via* the geographical divisions that come from the borders of countries.

[25] Examples of models that imply composite-modeling techniques and/or do not discuss the implications of the module-processing order are TIGRIS XL (Zondag and Jong, 2011, p.57, Fig.1) or MOLE (Tikoudis and Oueslati, 2017, p.9, Fig.1).

[26] Indeed, under constant computational resource, the required time to perform a complete run of the model in the case of serially processed submodels is roughly ten times greater for France and fifty times greater for the USA.

static analysis, this keeps the door opened for in depth discussions, outside the model, about the more realistic sequence.

Second, the calibration of the model led us to take part in the theoretical debate in spatial econometric about the right choice of the weight matrix, whose elements numerically represent the strength of interaction at work between pairs of positions, *e.g.* between pairs of municipalities. We consequently made proposal through the development of a numerical tool independent from GEMSE, namely, `PyOKNN`. The rationale behind this proposal lies in the fact that by opposition to the time-series case, specifying the lag operator (in space) involves a lot of arbitrariness that our proposal minimizes at most since it calls for a binary specification exclusively implied as *primarily observed*, *i.e. with no modifications nor transformations*.

A good signal of the reliability of the so-developed numerical tool lies in the fact that it significantly identified some interpretable key elements of Paris's spatial autocorrelation structure. These interpretable key elements are then revealed as major structural drivers of the spatial-dependences of which the agglomeration is the arena. Another good signal of the reliability of the implemented approach lies in the fact that it yielded parameters' (bias-corrected and accelerated bootstrap) intervals that broadly contain point-estimates found in the literature.

The third methodological contribution lies obviously in the architecture of GEMSE itself. The model, "transplantable" as possible to any preexisting modeling architecture,[27] provides the research community with an operational tool that embodies Thisse (2010)'s recommendation of unifying New Economic Geography (NEG) (Krugman, 1991) and Urban Economics (UE) (Alonso, 1964; Mills, 1967; Muth, 1969).

Substantive contributions

We tried and demonstrated the applicability of the model to contemporaneous political reflections about the sustainable planning of urban space. The objective was to provide decision-makers with indicators that incorporate indigenous specificities of space at multiple scales.

Given the multiple scales at which economic variables were considered when performing simulations, we have shown that GEMSE can be used to provide decision-makers with spatio-temporal baseline trajectories at various levels of governance. Even though debatable, those at least have the merit of submitting quantified objects that can be criticized through the consideration of their fully-indigenous empirical underliers and the introduction of alternative conjectures about their evolution.

Moreover most of the objects that are modeled at the local scale can easily be questioned in light of everyone's everyday life experience,[28] which highlights one of the characteristics of

[27] Modeling architecture whose zero-level version is a set of arbitrarily forecasted trends to which the model would be plugged.

[28] Indeed, through the website, readers can see how GEMSE models multiple comprehensible everyday life information relative to their home city, *e.g.* public transport and private speeds, housing rents , density, etc...

GEMSE, *i.e.* modeling *tangible spaces*. We then have shown that grounding simulations on such a granular description of local and interacting economies allows us to make non-trivial conclusions that are not readily adducible, not to say not addressable, when inferring from the top with macroeconomic aggregates.

The results obtained though the simulation exercises illustrate these latter points. In the baseline scenario, we have characterized the USA and France as systems of interacting areas. Regarding the future degrees of competition intensity between urban areas,[29] we have exhibited different behavioral patterns for the two countries: in the USA, this competition intensity tends to reduce the weight of NewYork, while in France it gains momentum around Paris. Despite this difference, the USA and France show a similar dynamic of the distribution of their respective rural and urban population, which portrays a future in which the secular migratory tendency in favor of large urban areas fade by 2030 for the benefit of smaller agglomerations. Typically, this finding is not a forecast. It is a prediction given the set of parameters embarked by the model and, obviously counter-tendencies might emerge to falsify it. But this reversal of secular trends appears so systematically in our results that we venture to say that this is a substantive conclusion that deserves further in depth discussion.

Passing down to the local scale, we have probabilized *all* urban areas' capacity to absorb a fuel price shock of 10% only via modal shift providing readers with fitted probability distributions that are usable in external exercises.[30] We have then quantified the sensitivities of the attractivenesses of urban areas' public transport to migratory pressures. This quantification reveals interestingly two contrasted behavioral patterns in the USA and France: USA's (average) transport budget declines over the long-run while France's increases. This surprising nationwide divergence obviously result from empirically calibrated complex interactions within each country's set of areas, with a net immigration in some agglomerations and net emigrations in other. But the main reason of this aggregate difference lies in the relation between migrations, the impact of migrations on mobility choices, the cost of transport that is associated to a specific mobility choice and, finally, the changes in the weights associated to each type of mobility when computing transport cost on average (both over modes and over urban areas when reaching the national scale). Having that in mind, the net migration pattern in France is such that people mainly migrate *to* urban areas where their mobility habits have almost no consequences on the mobility habits of the locals, not incentivized to modal-shift from cars to public transports, hence to reduction of their transport bill. *A contrario*, in the USA, the net migration pattern is such that people mainly migrate *from* this type of urban area or, equivalently, migrate to areas in which their arrival create an incentive for the locals to modal-shift in favor of public transports, which ultimately decreases transport costs on average over modes.

The results that have been obtained by studying the consequences of (two) transport meas-

[29]Where the term "competition intensity" refers to the degree of dissimilarity of external economies of scale enjoyed by firms in each urban area.

[30]For each urban area, the sample needed to conduct the probabilization is built from its places.

ures provide also policy relevant information. The first measure – the decrease in private vehicle speed limitation – stimulates economic activity in a pro-environmental fashion by contracting GDP in a first phase but then allowing it to reach higher levels and resulting in a positive sum game. The major dynamic at play is that changes in mobility habits increase the income disposable for buying other goods and services, leads people to relocate, changes the geography of production and finally improves external economies of scale. This suggest a mechanism through which regulatory instruments are capable of moving upward nations' production-possibility frontiers by eliminating costly organizational frictions if they release income that becomes available for other socially more profitable uses. The second measure is the implementation of a CO_2 tax to private vehicle whose collected revenues are used to finance an increase in public transport speeds in some places. The main policy insight is that setting a price of 100 per tonne of CO_2eq – be it in Dollars or Euros – represents virtually nothing once converted per commuter-kilometer and deters only marginally the use of cars, whereas the recycling of the tax in public transport infrastructure induces a low-carbon growth. Indeed, the implementation of a duly recycled carbon tax releases an important mass of income at the national scale that can be used to change the geography of attractiveness and stimulate production. In this case, a carbon tax is less interesting as a "signal" than as a component of complex policy designs where recycled revenues help to support policies aiming at redirecting behavioral evolutions.

This calls for more complex policy designs that internalize all distortive effects, *e.g.* infrastructure policy and changes in mobility habits, stimulations and reorientations of demands, unbalances in labor markets via people's relocations and firms' improvements in terms of economies of scale. It also calls for the full consideration of local specificities since, both in the baseline or policy scenarios, results can hardly be generalized. One key question here is obviously related to the geographical distribution of policy impacts with effects ranging from strongly positive to strongly negative depending on the (set of) analysed urban areas.

Limits and future research

The first and major limit of the current version of GEMSE is a too simple shareholding structure. It makes impossible to study measures related to, say, fiscal switch-overs, e.g. from labor taxation to green taxation including land-taxation, and to include capital flows. For instance, in its current form, the only labor market that is explicit is that involved in the so-called "Cobb-Douglas" sector. Given that this sector is equitably owned by everyone, any reduction of the burdens on (local) employment turns into nationwide impacts cascading through the dividend-channel, which is actually anything but realistic. This means that the next version of the model must actually work through a representation of the income-formation that is based on empirical and regionalized social-accounting matrices (SAM) rather than on putative national income circuits. The direct corollary of this complexification is the explicit consideration of social classes, dissimilar in respects other than geographical, as it is in current GEMSE's form and of their sources of income (wages, rents, dividends, social security).

Forcefully in coherence with the just-described SAM complexification, a future model development is the explicit representation of a capital market. This would allow for making these markets not only outlined through the attractiveness criterion.[31] This would also allow for studying the consequences of measures on existing (or emerging) industry clusters (Porter, 1990).

A second limit of GEMSE's current version, is that transport of goods and passengers *between areas* is only implicitly represented. Indeed, putting aside the realism of the shape of each nation's urban-areas system, moves between areas are represented with no consideration for the heterogeneity of transport mode in terms of quality and availability, as well as regarding their GHGs emissions. Yet, this may have contradictory results from the economical and environmental standpoint, either directly via the GHG emissions due to inter-region transportation or indirectly via the changes in GHG emissions due to the impacts on the geography of economic activity.

At *the urban scale*, the most salient improvement to be made is probably to introduce polycentric urban areas. Such an improvement is needed for multiple reasons. A first one is the possibility of having a more realistic representation of housing markets, which are empirically far from always exhibiting prices that monotonously decrease from center to periphery. A second reason is that this would allow to endogenize the emergence of industry clusters within a single urban area. A last reason is that modeling polycentric urban areas would pave the way for going beyond *unifying* – by progressive numerical convergence – the scales of analysis specific to NEG- and UE- models as operationalized in GEMSE's current version. It would be possible to *merge* these two scales in a single framework where an entire region, say, a country, could be represented as a giant conurbation whose tissue-discontinuities would only be explained with mono-scale determinants. Indeed, in such a formalism, all centers of the giant conurbation would be treated as belonging to the same space as any other center.

Finally, one notable GEMSE's limit is its lack of consideration of changes in the type of motorizations. The consequences of modeling new types of vehicles may be of first importance for studying the transformation of urban shapes or the dynamic of urban sprawl. In the case of electric vehicles, the cost of browsing one kilometer may be substantially weaker than that of its fuel counterpart. Areas' periphery would densify, simultaneously influencing housing markets via changes in transport/housing tradeoffs. From the environmental standpoint however, such a consideration is critical to correctly picture the link between mobility and GHGs emissions in function of nations' energy mixes.

[31] Reminding that the attractiveness criterion is defined to endogenize the migrations of firms that move towards the agglomeration markets that offer the most attractive rate of return.

An attempt to overcome the 'blackbox' syndrome

However, all these attempts to overcome GEMSE's limits might themselves confront the limit emerging from the transparency problem posed by such a data-intensive model. Indeed, the often-mentioned drawback of the research based on such complex models is that it is not possible to share or browse the results in a user-friendly fashion, say, for interrogative readers who would like to apprehend the coherence of the mechanisms put at stake. In reaction to this critic and to increase transparency, we have developed a geowebsite – companion to this thesis document –, which makes available all data used and generated by GEMSE for each country considered, each scenario, each variable, each year, each urban area as well as for each of its constitutive place.[32] The website can also be HATEOAS- (Hypermedia As Engine of Application State) driven,[33] where the term HATEOAS simply involves rendering website-like content (in JavaScript Object Notation) with no superfluous interface since it is addressed to remote programs.

Moreover, this model has been designed to be adopted by the open-source community. It has been developed in Python in an object-oriented programming fashion by following the state-of-the-art practices in terms of maintainability and transmission. This is an attempt to concentrate the modeling efforts related to space and economics within a unique tool, just as what the C40 initiative on the political side is. In accordance with this, a revised version of GEMSE will be available at https://github.com/lfaucheux.

The tool developed to implement the new proposal of spatial lag operator and to conduct the econometric study presented in Section 2.3, PyOKNN, is already available online. The sourcecode and documentation are publicly accessible.[34] The package can easily be installed via the notorious system used to manage software packages written in Python, pip, by opening a session in your OS shell prompt and typing `pip install pyoknn`.

Identically, PyGWP, the tool developed to consider the global warming potential (GWP) of the GHGs that are considered by GEMSE – CO_2, CH_4 and N_2O – and that implements Levasseur et al. (2010)'s dynamic GWPs is available online as well. The sourcecode and example of usage are publicly accessible.[35] As for PyOKNN, the installation of PyGWP simply requires to open a session in your OS shell prompt and to type `pip install pygwp`.

We are conscious that these attempts will not suffice in removing doubts about the control of complex integrated assessment models like those evoked by Pindyck (2013). However, they hopefully demonstrate why and how part of the progresses in computational power could be

[32] USA's urban areas are Atlanta, Baltimore, Boston, Chicago, Dallas, Denver, Detroit, Houston, Losangeles, Miami, Minneapolis, Newyork, Philadelphia, Phoenix, Pittsburgh, Portland, Sandiego, Sanfrancisco, Sanjose, Seattle, Stlouis and Washington and France's are Bordeaux, Grenoble, Lille, Lyon, Marseille, Montpellier, Nantes, Nice, Paris, Rennes, Strasbourg and Toulouse. Note that urban areas' maps are available online according to the syntax https://gemse.alwaysdata.net/<nation-name>/<area-name>, *e.g.* https://gemse.alwaysdata.net/france/paris.

[33] See *e.g.* https://gemse.alwaysdata.net/france/?wSce=baseline.

[34] Available at https://github.com/lfaucheux/PyOKNN.

[35] Available at https://github.com/lfaucheux/PyGWP.

devoted to improve the transparency of results, and their control by both modeling experts and by non specialists, so that they can really be of use in policy debates.

References

Allio, C. (2015). "Local Policies, Urban Dynamics and Climate Change: Development of a Multiscale Modeling Framework". PhD thesis, AllioT2015.

Alonso, W. (1964). *Location and land use: toward a general theory of land rent.* Harvard University Press, p. 204.

Bosker, M, S Brakman, H Garretsen and M Schramm (2007). "Adding geography to the new economic geography". *Cesifo Working Paper* 2038.

Cavailhès, J., C. Gaigné, T. Tabuchi and J. F. Thisse (2007). "Trade and the structure of cities". *Journal of Urban Economics* 62.3, pp. 383–404.

Cazala, A., G. Crespy, J. Deterne, P. Garnier and P. Rimattei (2006). *Comparaison au niveau européen des coûts de construction, d'entretien et d'exploitation des routes.* Tech. rep. Ministère des transports, de l'équipement, du tourisme et de la mer 1., p. 48.

Eaton, J. and S. Kortum (2002). "Technology, Geography, and Trade". *Econometrica* 70.5, pp. 1741–1779.

Global BRT Data (2017). *Infrastructure cost per kilometer.*

Guesnerie, R., C. Abraham, O. Al Tarabichi, J.-P. Arduin, D. Auverlot and M. Auzanneau (2017). *The discount rate in the evaluation of public investment project.* Tech. rep. Paris: France Stratégie, p. 70.

Krugman, P. (1980). "Scale Economies, Product Differentiation, And The Pattern Of Trade". *The American Economic Review* 70.5, pp. 950–959.

Krugman, P. (1991). "Increasing Returns and Economic Geography". *Journal of Political Economy* 99.3, pp. 483–499.

Levasseur, A., M. Margni and L. Desche (2010). "Considering Time in LCA : Dynamic LCA and Its Application to Global Warming Impact Assessments". 44.8, pp. 3169–3174.

Mills, E. S. (1967). "An Aggregative Model of Resource Allocation in a Metropolitan Area". *The American Economic Review* 57.2, pp. 197–210.

Murata, Y. and J.-F. Thisse (2005). "A simple model of economic geography à la Helpman–Tabuchi". *Journal of Urban Economics* 58.1, pp. 137–155.

Muth, R. F. (1969). *Cities and Housing: The Spatial Pattern of Urban Residential Land Use.* University of Chicago Press, p. 355.

Pindyck, R. S. (2013). "Climate Change Policy: What Do the Models Tell Us?" *Journal of Economic Literature.*

Porter, M. E. (1990). "Chapter 12 Government Policy". *The Competitive Advantage of Nations.*

Scott, A. J. and M. Storper (2015). "The nature of cities: The scope and limits of urban theory". *International Journal of Urban and Regional Research* 39.1, pp. 1–15.

Simmonds, D., P. Waddell and M. Wegener (2013). "Equilibrium versus Dynamics in Urban Modelling". *Environment and Planning B: Planning and Design* 40.6, pp. 1051–1070.

Thisse, J.-F. (2010). “Toward a Unified Theory of Economic Geography and Urban Economics”. *Journal of Regional Science* 50.1, pp. 281–296.

Tikoudis, I. and W. Oueslati (2017). “Multi-objective local environmental simulator”. 122.

Waisman, H. and F. Grazi (2013). “Cities in International Climate Policy : A Dynamic CGE Approach Cities in International Climate Policy : A Dynamic CGE Approach”.

Wardrop, J. G. (1952). “Road Paper. Some theoretical aspects of road traffic research.” en. *Proceedings of the Institution of Civil Engineers*.

Zondag, B. and G. de Jong (2011). “The development of the TIGRIS XL model: A bottom-up approach to transport, land-use and the economy”. *Research in Transportation Economics* 31.1, pp. 55–62.